UNRAVELING COMMODITY MARKET DYNAMICS: INSIGHTS FROM CONVENIENCE YIELDS AND IMPLIED STORAGE COSTS

A Thesis

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Master of Science

by

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ABSTRACT

This thesis investigates convenience yields and storage costs across five major commodities—wheat, soybean, corn, crude oil, and natural gas—over the period from 2018 to 2023. Using an option-based approach and extensive data analysis, I examined the dynamics of commodity markets, identifying key factors influencing convenience yields such as supply and demand dynamics, geopolitical events, and market sentiment. Through regression analyses and examination of implied storage costs and the Working Curve, I studied relationships between convenience yields and variables including spot prices, futures prices, and interest rates. Findings also include insights into the relationship between storage supplied and the cost of storage. This research contributes to our understanding of commodity market efficiency, risk management strategies, and offers practical implications for investors navigating volatile commodity markets. Further research could explore drivers behind observed deviations and refine existing models for improved predictive accuracy.

BIOGRAPHICAL SKETCH

Yining Zhu is a dedicated researcher with a passion for understanding the intricacies of financial markets and economic phenomena. Born and raised in Beijing, China, Yining developed an early interest in economics, driven by a curiosity to unravel the complexities of global markets and their impact on society.

After completing a Bachelor of Arts in Economics from New York University, Yining pursued further academic endeavors, presently pursuing a Master of Science in Applied Economics and Management at Cornell University's Charles H. Dyson School of Applied Economics and Management, with graduation expected in May 2024.Throughout his academic journey, he demonstrated a keen aptitude for quantitative analysis and a meticulous approach to research methodologies.

Yining Zhu’s research interests center around the intersection of finance and economics, with a particular focus on commodity markets.

ACKNOWLEDGMENTS

I extend my deepest gratitude to my advisors, Professor Calum Turvey and Professor David Just, for their invaluable guidance, support, and mentorship throughout the course of this research. Their expertise and insights have been pivotal in shaping both the direction and the substance of this work.

Professor Turvey's profound knowledge in the field of agricultural finance and risk management has greatly enriched my understanding and appreciation of the complexities within commodity markets. His thoughtful advice and rigorous academic standards have been a constant source of inspiration and motivation. In addition, Professor Just's expertise in applied economics has provided a broader perspective that has deeply informed my approach to this research. His encouragement to pursue innovative solutions and rigorous economic analysis has been instrumental in the successful completion of this thesis.

I am truly thankful for the opportunity to work under the guidance of such distinguished scholars, and I am appreciative of the patience, wisdom, and academic rigor they have shared with me.

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INTRODUCTION

The study of convenience yields in commodity markets is crucial for understanding the complex interplay between supply, demand, and storage dynamics. In this analysis, I delve into the estimation and implications of convenience yields for five key commodities: wheat, soybeans, corn, crude oil, and natural gas. Through a combination of theoretical frameworks, empirical analysis, and regression modeling, I aim to uncover the underlying trends, anomalies, and relationships shaping convenience yields across different time horizons and market conditions.

The methodology employed in this research draws on the option-based approach detailed in seminal works such as Heaney (2002), providing a robust framework for estimating convenience yields. By analyzing data spanning from 2018 to 2023, I aim to study the nuanced interactions within commodity markets and provide insights into the drivers of convenience yields.

The analysis begins with an examination of convenience yield trends over time, highlighting the distinct patterns observed for each commodity. From the stability observed for soybeans to the volatility in natural gas observations, I explore how convenience yields respond to changes in market fundamentals, including price volatilities, demand-supply dynamics, and shifts between contango and backwardation.

Descriptive statistics provide further insights into the magnitude and variability of convenience yields across different maturities and commodities. Notably, natural gas emerges as the most volatile commodity, with convenience yield estimates exhibiting significant variability and skewness. Correlation analysis reveals the degree of co-movement between convenience yield estimates for different maturities, shedding light on the interconnectedness of short- and long-term storage dynamics for each commodity.

Moving forward, the focus shifts to investigating the implied storage costs derived from convenience yield estimates. My goal is to ascertain whether these costs align with the theoretical framework proposed by the Working Curve. Through an examination of the relationship between storage costs and inventory levels, I aim to uncover distinct patterns for each commodity. I found that some commodities exhibit behavior akin to the Working Curve, as observed in soybeans, others may deviate, such as the "V" shape for wheat.

Finally, regression analysis offers further insights into the determinants of convenience yield estimates, including interest rates, futures prices, spot prices, and storage costs. While the results generally align with theoretical expectations, anomalies highlight the complexity of real-world market dynamics and the limitations of theoretical models.

Overall, this analysis provides a comprehensive understanding of convenience yields in commodity markets. Through empirical analysis and theoretical insights, I aim to contribute to the ongoing discourse surrounding commodity market dynamics and the role of convenience yields in shaping market behavior.

LITERATURE REVIEW

Convenience yield, a fundamental concept in commodity markets, has garnered considerable attention and scrutiny from scholars over the years, owing to its profound implications for market dynamics, risk management, and pricing mechanisms. Its historical roots can be traced back to seminal works such as that of Kaldor (1939), whose pioneering exploration introduced the notion of convenience yield within the context of agricultural stocks. Kaldor noted that holding inventory provides a benefit to the producer by allowing immediate access to goods, thus eliminating the need for frequent orders or delays in receiving supplies. Building upon this foundation, Blau (1944) provided further explanation, defining convenience yield as the aggregate of additional benefits accruing to manufacturers from maintaining stocks on hand, as opposed to holding cash and procuring stocks at a later time. This additional utility, Blau argued, arises from the intrinsic liquidity advantage of physical commodities over cash equivalents.

The importance of convenience yield in comprehending commodity market dynamics is underscored by Working (1948), whose seminal study identified it as a pivotal determinant of backwardation (inverse carrying charge) in futures markets. Working (1948) discussed four possible explanations for backwardation in his article *Theory of Inverse Carrying Charge in Futures Markets*. He discredited three of the four explanations and points out that an inverse carrying charge is a true negative price of storage, arising from the fact that stocks may have a high marginal convenience yield. This perspective garnered further validation in the empirical research conducted by Fama and French (1987), who studied two leading views of commodity futures prices. The first is the theory of storage proposed by Kaldor (1939) and Working (1948). The alternative views futures price as a composition of expected risk premium and forecast of future spot price. The authors used data from 21 commodities and found that the reaction of futures prices to storage-cost variables is easier to detect than evidence of futures prices that contain risk premiums or future forecasts in more powerful statistical tests. Their findings corroborated the theory of storage, positing a negative relationship between convenience yields and inventories.

Researchers have also delved into the modeling of convenience yield. The classic cost of carry model implies that:

$$F = S * e^{(r+s-c)t} \tag{1}$$

where F is the futures price, S is the spot price, r is the risk-free interest rate, s is storage cost, c is the convenience yield, and t is the time to maturity of the futures contract. However, storage costs, a critical component of this model, are often unobservable, presenting a hurdle in accurately estimating convenience yields. In response to this limitation, scholars have increasingly turned to option-based models as an alternative approach. These models offer a more flexible framework for estimating convenience yields by leveraging derivative pricing techniques. By treating convenience yield as an option-like feature embedded in commodity prices, researchers aim to capture the additional benefits of holding physical commodities.

Milonas and Thomadakis (1998) were the first to model convenience yields as options. Examining behavior and determinants of convenience yields for four commodities, they proposed treating convenience yields as call options and found strong evidence for it. They also showed that convenience yields exhibit a term structure that resembles seasonality within a crop cycle. It increases as a new cycle approaches. Then, Heaney (2002) built on a model proposed by Longstaff (1995) and suggested an approximation of convenience yield. The model assumes that a profit-maximizing trader has perfect foresight and essentially treats convenience yield as the difference between two lookback options[1]. The author calculated convenience yield for copper, lead, and zinc and compared observed future price with theoretical future prices adjusted for convenience yield. The results are consistent with the convenience yield effect found by others.

Further study in the estimation and modeling of convenience yield were championed by Hochradl and Rammerstorfer (2011). They approximated convenience yields using three methods. The first method is the standard cost-of-carry model. Since storage cost is not observable, the authors estimated the convenience yield net of storage costs, referred to as the net convenience yield. For the second method, the authors used the approach outlined in Heaney (2002) and derived convenience yield as the difference between two lookback put options. In addition, they argued that convenience yield can be derived as the difference between two average strike Asian put options[2]. The authors then compared results of the three approaches using data from three major natural gas markets in Europe. They found similarities between the two option-based approaches, whereas the traditional approach led to different results. The correlation between the two option-based approximations is positive and higher than the correlation between either of the two option-based approaches and the traditional approach.

Last but not least, Omura and West (2014) compared the traditional cost-of-carry approach with an option-based approach for modeling convenience yield. They studied whether the relationship between convenience yield and inventories can be better quantified using the option-based approach relative to the traditional approach by examining price behavior and

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[1] A lookback option allows the holder to "look back" over the life of the option to determine the optimal payoff based on the maximum or minimum asset price reached during the period

[2] An Asian option's payoff depends on the average price of the underlying asset over a specified period, rather than the price at maturity.

convenience yield for six metals. For the option-based approach, they followed the method developed in West (2012) to model the convenience yield using a contingent-claim approach. They conducted two separate regression analyses to evaluate the two models. The dependent variable in each regression is the monthly convenience yield return, while the independent variable is the monthly return on inventory levels. Their results demonstrate that the option-based model offers a more robust and consistent model for convenience yields relative to inventory levels than the cost-of-carry approach.

METHODOLOGY

In this thesis, I employ the option-based approach detailed in Heaney (2002) to estimate the convenience yield for five commodities, namely corn, soybeans, wheat, crude oil, and natural gas. This estimation is conducted using data from 2018 to 2023, and details regarding the data are described in the next section. Subsequently, I leverage the estimated convenience yield to calculate implied storage costs, applying the cost-of-carry model. By plotting storage cost against inventory volume data, I analyze whether these results provide support for the Working Curve. Additionally, I use regression analyses to examine how convenience yield correlates with factors such as spot prices, futures prices, storage costs and interest rates.

In Heaney (2002), the author came up with a method of modeling convenience yield as the difference of the value of two lookback options. They assumed that the trader opting to store the commodity possesses the ability to predict when the commodity price will peak between the current time *t* and the end of the period *T*. The model assumes the trader has perfect timing and price prediction abilities, so it should be considered as presenting an upper limit for the trading strategy. Consider a commodity buyer at a time t. This buyer has the option to sell the commodity before the maturity of the futures contract. Therefore, if a sale of the commodity takes place, it occurs at the highest observed price S_τ at the time τ ($t < \tau < T$). The income generated from selling the commodity at time τ is subsequently invested at the risk-free rate until time T. At T, the commodity is repurchased at the market price. The highest observed price during the period from t to T, compounded to the maturity of the futures contract, is defined as

$$M_{iT} = \max_{0 \le \tau \le T} \{ e^{[r(T-\tau)] S_{i\tau}} \} \quad (2)$$

The value for a trader who purchases the commodity and holds it is approximated by the value of the option to sell it, should the price increase significantly enough to yield a profit when repurchased at time T. It can be defined as

$$TS(S_{it}, T) = e^{[-\mathrm{r}(\mathrm{T}-\mathrm{t})]E(M_{iT})} - e^{[-\mathrm{r}(\mathrm{T}-\mathrm{t})]E(S_{iT})} \quad (3)$$

Longstaff (1995) solved this problem:

$$TS(S_{it}, T) = S_{it}\left\{\left[2 + \frac{\sigma^2(T-t)}{2}\right] N\left[\frac{\sqrt{\sigma^2(T-t)}}{2}\right] + \frac{\sqrt{\sigma^2(T-t)}}{2\pi} e^{[-\frac{\sigma^2(T-t)}{8}]} - 1\right\} \quad (4)$$

Dividing by S_{it}, adding one to both sides, and taking natural log led to

$$ts_{tT}(S_{it}, T) = \ln\left[1 + \frac{TS(S_{it},T)}{S_{it}}\right] = \ln\left\{\left[2 + \frac{\sigma^2(T-t)}{2}\right] N\left[\frac{\sqrt{\sigma^2(T-t)}}{2}\right] + \sqrt{\frac{\sigma^2(T-t)}{2\pi}} e^{[-\frac{\sigma^2(T-t)}{8}]}\right\} \quad (5)$$

where σ is the volatility of price returns and N() is the cumulative normal distribution. We understand $ts_{tT}(S_{it}, T)$ as the value of holding the physical asset. In addition to commodity buyers, this opportunity to trade is equally accessible to the futures contracts buyers. For them, a similar line of reasoning leads to $ts_{tT}(F_{it}, T)$. The advantages of possessing the physical commodity, which are not achievable through the purchase of a futures contract, namely, the convenience yield, can be estimated as

$$C_{tT} = ts_{tT}(S_{it}, T) - ts_{tT}(F_{it}, T) \quad (6)$$

In addition, by taking logs and rearranging equation (1), we get

$$ln\ (F_{tT}/S_t) - r_t t = (s_t - c_t)t \equiv iab_t \quad (7)$$

The interest-adjusted basis (iab_t) is a function of storage cost and the convenience yield estimate. It is then regressed on the convenience yield estimate

$$iab_t = \beta_0 + \beta_1 c_t + \varepsilon_t \tag{8}$$

Therefore, we have the fitted value $\widehat{iab}_t = \widehat{\beta_0} + \widehat{\beta_1} c_t$, and as a result we can calculate an approximation of F_t with cost of carry adjusted for convenience yield

$$\widehat{F_t} = S_t * e^{[(r_t + s_t - c_t)*t]} = S_t * e^{\,(r_t + \widehat{ia\ \ t})} \tag{9}$$

By comparing $\widehat{F_t}$ with F_t, I can evaluate whether $\widehat{F_t}$ is an accurate approximation of F_t and thus whether the convenience yield estimate is accurate.

Also, with the estimated convenience yields, I calculate implied storage costs using the cost of carry model from equation (1) and graph the storage costs against inventory levels to study whether the result conforms with the Working Curve. Since Working only provided the conceptual idea of this relationship between storage cost and storage supplied. A mathematical relationship proposed by Zhang and Turvey (2024) is used to analyze the results.

Furthermore, to study the relationship between risk-free interest rate, spot price, futures price, storage costs, and convenience yield, I run regressions of the convenience yield estimates on these variables and analyze the results. To address endogeneity issues, a two-stage least square regression is carried out.

DATA

Data used for this study spans five commodities: wheat, soybean, corn, crude oil, and natural gas. Corn USD/bu physical futures price, IL north central No. 2 yellow corn spot price, soybean USD/bu physical future price, No.1 yellow soybeans spot price, SRW Wheat futures price, No.2 SRW wheat Toledo Ohio spot price, Crude Brent futures price, Crude Dated Brent Spot price, US Natural gas Henry hub futures price, and Henry Hub Natural gas trade date spot price over the period from 2018 to 2023 are obtained from Bloomberg. Daily futures prices for each contract during the period along with the corresponding spot prices from four months prior to contract maturity are observed.

For crude oil and natural gas, there are contracts maturing every month in a year. As a result, over the five-year period, price data from 60 contracts are retrieved. For wheat and corn, contracts mature in March, May, July, September, and December, resulting in five contracts per year and twenty-five contracts over the five-year period. For soybeans, there are contracts for January, March, May, July, August, September, and November, totaling seven contracts per year and thirty-five contracts in the five-year period. To estimate convenience yield, the dates for 1-month, 2-month, and 3-month to maturity of each contract are recorded, and spot and futures return volatilities of the 20-trading-day period prior to these dates are calculated and then annualized.

Additionally, quarterly storage data for wheat, corn, and soybean over the period are obtained from the U.S Department of Agriculture. They are then interpolated to a weekly basis for ease of further analysis, assuming linear changes through the period. Weekly storage data for natural gas and crude oil are available and obtained from the U.S. Energy Information

Administration. For the risk-free interest rate, daily 10-year treasury yields are retrieved from Federal Reserve Economic Data. The treasury yields range from 4.98% to 0.52% over the period, with a mean of 2.39%, a median of 2.38%, and a standard deviation of 1.04%.

Table I: Inventory Level - Descriptive Statistics

	Mean	Median	Max	Min	Standard Deviation
Wheat (Million Bushels)	203.83	196.38	356.00	95.76	67.52
Soybean (Thousand Bushels)	1718856.98	1630061.05	3598670.50	362730.73	838756.54
Corn (Million Bushels)	6593.53	6399.04	12036.17	1828.96	2735.56
Crude Oil (Thousand Barrels)	1034196.59	1080012.50	1188940.00	765343.00	114391.81
Natural Gas (Bcf)	2631.82	2664.50	3955.00	1107.00	729.67

Table I presents the descriptive statistics of inventory data for the five commodities. For all of them, the data indicates a diverse range of variability and storage behaviors. Soybeans exhibit the greatest fluctuation. In contrast, wheat shows more stability, suggesting a steadier market.

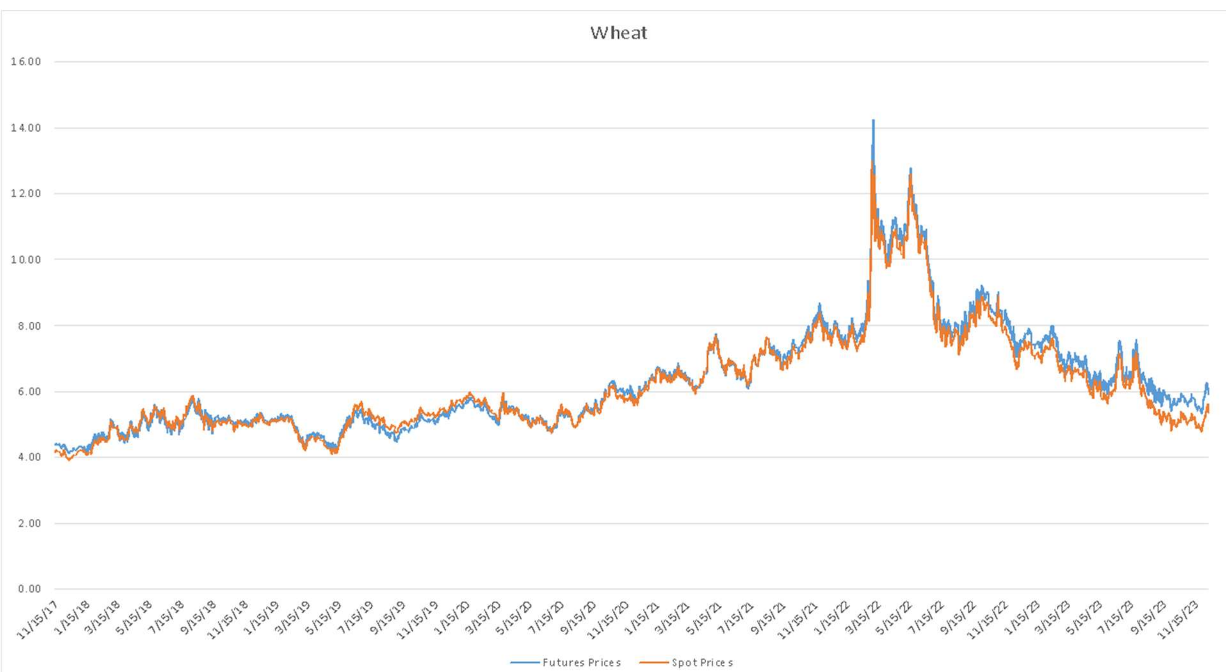

Figure 1: Wheat Price

Figure 1 displays the futures and spot prices of wheat. Over the period of 2018 to 2023, it appears that periods of backwardation (it occurs when spot price is higher than the futures price for contracts maturing in the coming months) and contango (it occurs when spot price is lower than the futures price for contracts maturing in the coming months) alternates, but overall, the difference between them is small. From early 2018 to early 2022, spot and futures prices gradually increased from around $4/bu to $8/bu. Since the commencement of the Russian invasion of Ukraine in February 2022, Ukrainian grain exports have faced significant disruptions. For a period spanning over four months, Ukrainian ports along the Black Sea were blockaded by Russian military vessels. However, from July 2022 to July 2023, an agreement was brokered between the United Nations, Turkey, and Russia, known as the Black Sea Grain Initiative. This initiative allowed for the resumption of grain and foodstuff exports from Ukraine via three Ukrainian ports, which led to the stabilization of prices. However, on July 17, 2023, Russia announced its decision to terminate the Black Sea Grain Initiative and the prices rose again. [3]

[3] How the Russian invasion of Ukraine has further aggravated the global food crisis (Council of the EU, 2024).

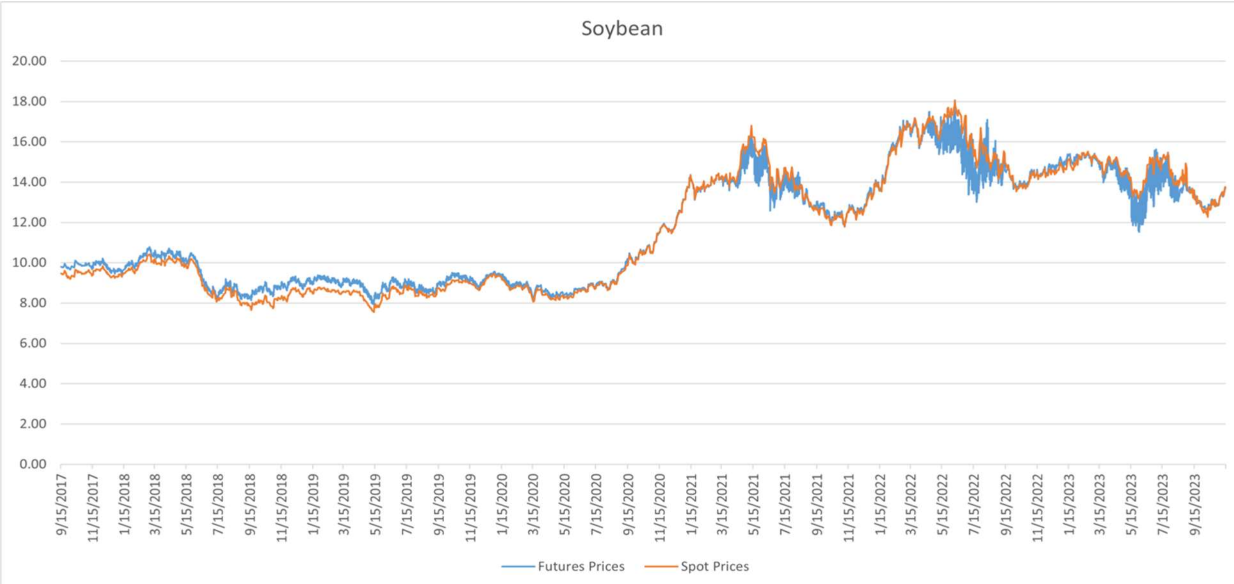

Figure 2: Soybean Price

For soybeans, we can see from figure 2, prices were mostly stable prior to September 2020 and were in contango. Since September 2020, prices began to increase while backwardation starts to occur in the market, indicating a higher demand for soybeans in the spot market. Both spot and futures prices remain high till the end of 2023.

The surge in soybean prices during 2020 can be attributed to several factors that collectively drove up demand and tightened global supplies. One significant factor was China's unexpectedly rapid recovery from African Swine Fever. This recovery led to a surge in Chinese demand for soybeans, as soybean is a key component of feed for swine. China's increased soybean imports, particularly from Brazil, strained global soybean supplies and contributed to price hikes.

Furthermore, despite reduced purchases from China during the ASF outbreak and trade tensions resulted in an accumulation of surplus soybeans in 2018/19, the subsequent resurgence in Chinese demand and delays in the South American harvest, combined with modest production levels in 2019/20, diminished U.S. soybean stocks.

Additionally, the competition for acreage between soybeans and corn in the United States played a role in boosting soybean prices. Spikes in corn prices incentivized farmers to allocate more acreage to corn, thereby limiting soybean supply and pushing up prices.[4]

We also see prices fall temporarily from May 2021 to Jan 2022, but prices increased again as Russia invaded Ukraine in Feb 2022.

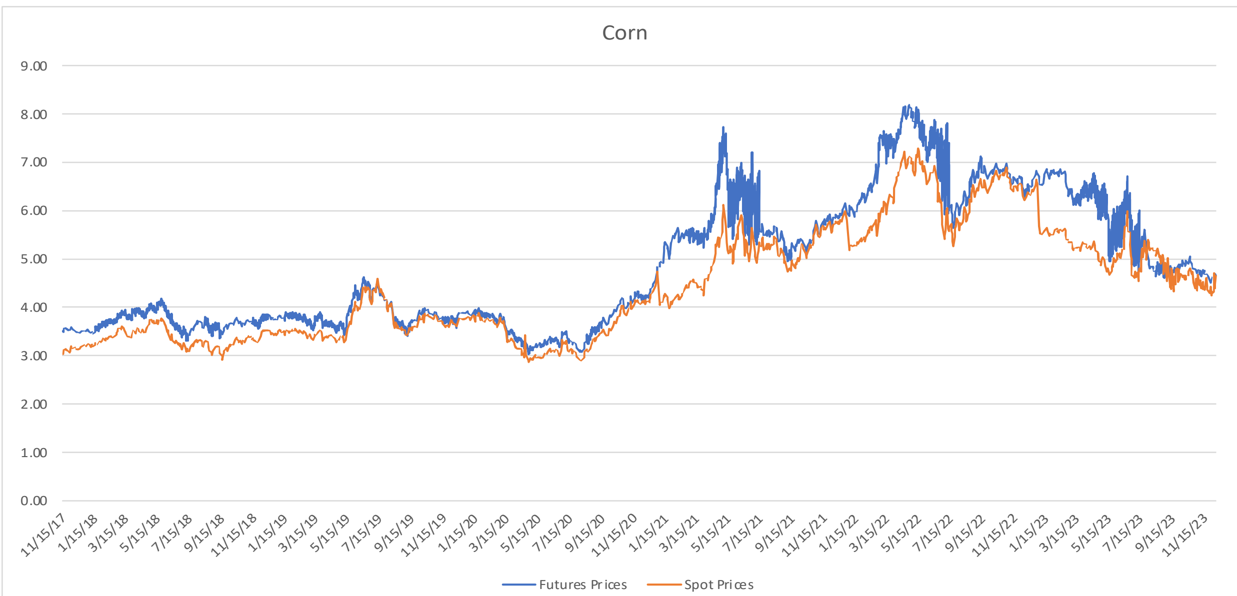

Figure 3: Corn Price

For corn, futures prices exceed spot prices for the most part, indicating that the market is in contango mostly. Corn prices follow a similar trend to soybeans, with prices increasing substantially from September 2020 to May 2021 due to high demand from China and a drought

[4] 2021 U.S. Soybean Outlook Remains Strong After Record First Quarter Export Volume (USDA, 2021).

in Brazil[5], decreasing briefly from May 2021 to Feb 2022, and increasing again from Feb 2022 due to the war in Ukraine.

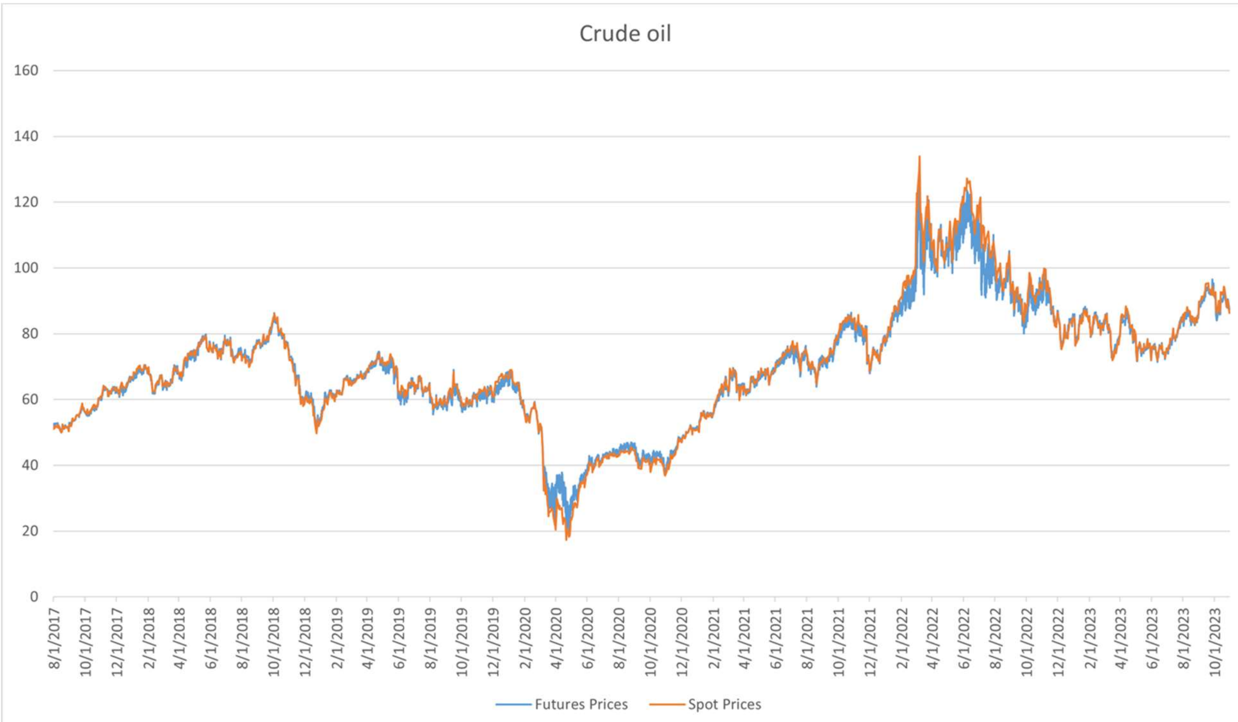

Figure 4: Crude Oil Price

Crude oil futures and spot prices are displayed in figure 4. In general, futures and spot prices are close and periods of contango and backwardation alternate. Notably, from December 2019 to May 2020, price decreased steeply, reaching the price of about $20 per barrel at its lowest.

The decline in crude oil prices during the first half of 2020 was primarily driven by the significant drop in global petroleum demand resulting from the COVID-19 pandemic. As lockdown measures and travel restrictions were implemented worldwide to curb the spread of the virus, demand for transportation fuels plummeted. This sharp reduction in demand led to an

[5] Corn 2021 review, highlights and what to expect for 2022 (Eduardo Tinti, 2022).

oversupply of crude oil in the market, as refineries cut back on operations and crude oil inventories surged.

In particular, the United States experienced a notable decrease in petroleum product demand starting in mid-March, prompting refiners to scale back their operations. This reduction in refinery activity led to a buildup of crude oil inventories. The imbalance between supply and demand was exacerbated by the slow response of U.S. crude oil producers to the sudden decline in demand, further contributing to the accumulation of crude oil stocks.[6]

In 2021, crude oil prices saw an uptick due to the acceleration of COVID-19 vaccination rates, easing of pandemic restrictions, and an expanding economy, which collectively led to a surge in global petroleum demand outpacing petroleum supply.

In 2022, crude oil prices initially surged due to geopolitical tensions surrounding Russia's invasion of Ukraine and low global inventories. However, prices later declined amid concerns of a possible economic downturn and reduced demand, exacerbated by high petroleum prices and COVID-19 containment measures in China. [7]

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[6] Crude oil prices briefly traded below $0 in spring 2020 but have since been mostly flat (Matt French, 2021)
[7] Crude oil prices increased in first-half 2022 and declined in second-half 2022 (Jimmy Troderman, 2023)

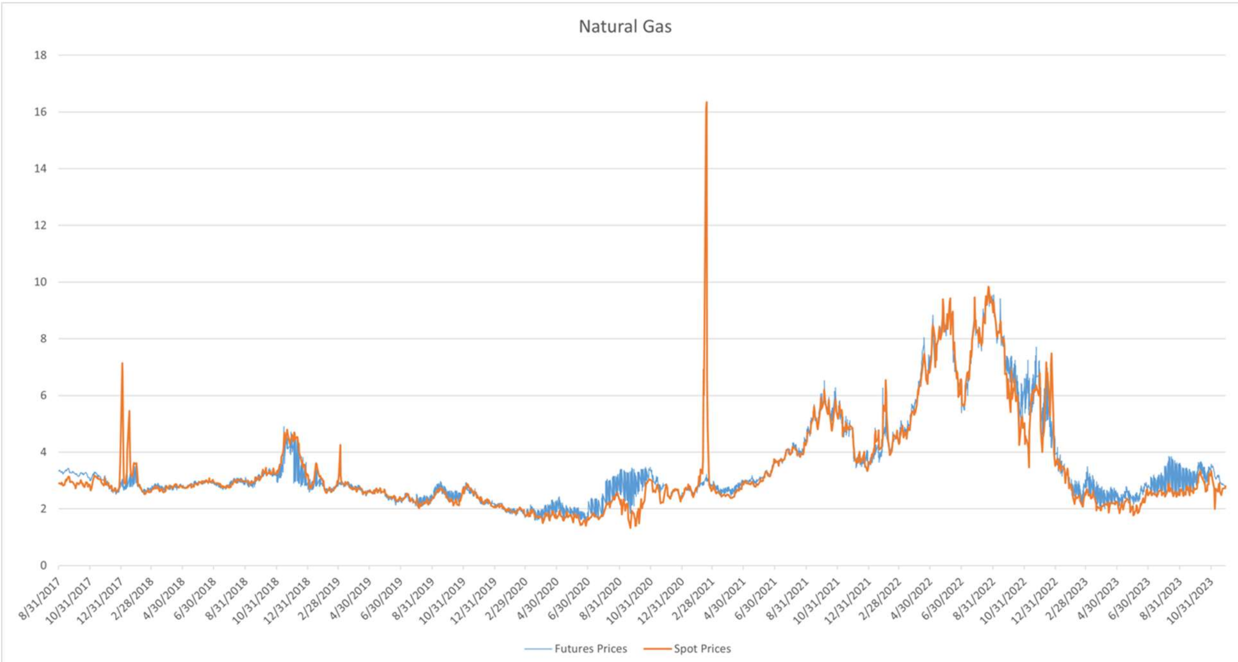

Figure 5: Natural Gas Price

For natural gas, prices generally increased each year during winter months, due to higher demand for heating purposes, limited storage capacity leading to supply constraints, and potential disruptions to production and transportation caused by winter weather conditions. Notably, in February 2021, severe winter weather in Texas and Oklahoma caused spikes in natural gas prices and depleted inventories. In 2022, natural gas prices soared due to concerns over supply shortages following Russia's invasion of Ukraine. However, in 2023, prices plummeted as record-high production levels and warmer winter temperatures led to a surplus in domestic storage. Prices dropped to an average of approximately $2.82/MMBtu, marking a significant decrease from the previous year.

Table II: Price and Returns - Descriptive Statistics

		S	*F(1m)*	*F(2m)*	*F(3m)*
Panel I: Wheat					
Prices					
	Mean	6.223	6.355	6.322	6.326
	Median	5.660	5.828	5.883	5.718
	Std. dev.	1.607	1.687	1.779	1.692
Returns					
	Mean	0.001	-0.002	0.007	-0.002
	Median	0.002	-0.001	0.006	-0.002
	Std. dev.	0.017	0.015	0.017	0.016
Penal II: Corn					
Prices					
	Mean	4.470	4.955	4.893	4.896
	Median	4.160	4.529	4.738	4.356
	Std. dev.	1.171	1.440	1.350	1.375
Returns					
	Mean	0.000	-0.003	0.004	-0.001
	Median	0.000	-0.001	0.006	0.001
	Std. dev.	0.019	0.014	0.019	0.015
Penal III: Soybean					
Prices					
	Mean	11.674	11.709	11.687	11.600
	Median	10.434	11.119	10.901	10.611
	Std. dev.	3.043	2.802	2.824	2.643
Returns					
	Mean	0.000	-0.002	0.000	0.001
	Median	0.000	-0.001	-0.002	0.000
	Std. dev.	0.015	0.015	0.012	0.014
Penal IV: Natural Gas					
Prices					
	Mean	3.383	3.471	3.523	3.555
	Median	2.820	2.845	2.896	2.904
	Std. dev.	1.643	1.694	1.703	1.664
Returns					
	Mean	-0.001	-0.005	-0.002	-0.001
	Median	-0.002	0.000	-0.002	0.000
	Std. dev.	0.062	0.035	0.035	0.028

Penal V: Crude Oil

Prices					
	Mean	70.094	70.592	69.616	68.716
	Median	69.025	69.330	68.865	68.300
	Std. dev.	19.419	18.617	17.748	17.086
Returns					
	Mean	-0.002	-0.002	-0.003	-0.003
	Median	-0.002	-0.002	-0.002	-0.003
	Std. dev.	0.026	0.024	0.022	0.022

Table II reports the descriptive statistics of the prices and returns. In general, spot prices are less than futures prices, except for crude oil and soybean. Mean crude oil spot price is higher than mean 2-month futures price and 3-month futures price, indicating the market is on average in backwardation over the period. Similarly, the mean soybean spot price is higher than the mean 3-month futures contract price. In addition, standard deviations of returns are all higher than the means, a signal that these markets are quite volatile.

ANALYSIS

Figures 6 to 10 display estimated convenience yields of contracts for the five commodities with 1-month, 2-month, and 3-month maturities. The x-axis represents the dates when contracts mature, with the data points indicating the convenience yields of carrying the assets one month to three months until the maturity date shown on the x-axis. This explains why, for commodities with monthly contracts such as crude oil and natural gas, their convenience yield graphs showcase very similar trends across different time-to-maturities, but the spikes do not overlap; instead, there is a one-month lag.

Figure 6 displays estimated convenience yields of wheat futures. It appears that they generally follow similar trends, with 3-month futures contracts having relatively higher convenience yields than contracts with shorter time-to-maturities, except for a spike in the case of the May 2022 contract with two months until maturity. This spike is caused by an increase in spot price volatility from February to March 2022, which resulted from the Russian-Ukrainian war. The spike is not observed for contracts with one month or three months until maturity due to the lack of April and June contracts for wheat. Spikes in convenience yield are also observed for May and July 2020 contracts, as well as contracts in late 2018 and early 2019. These are periods when spot and future prices were close, and the spikes in convenience yield result from higher spot price volatility compared to futures prices.

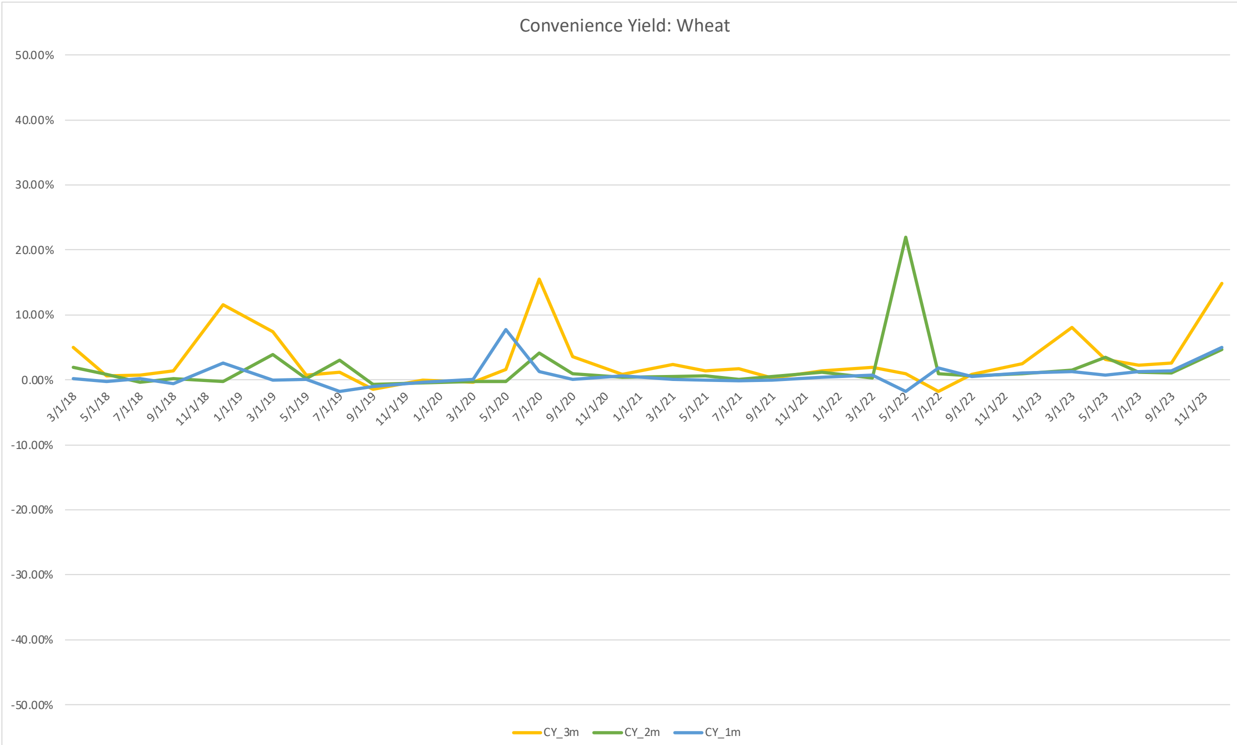

Figure 6: Wheat Convenience Yield

As for soybeans, its convenience yields were mostly stable, revolving around zero prior to September 2021. Since then, periods of more substantial convenience yields have been observed. These periods of high convenience yields align with times when the market was in backwardation. This is consistent with the theory of storage, which suggests that backwardation should be associated with high convenience yield because asset holders are rewarded for holding the assets on hand when there is a high demand for spot assets.

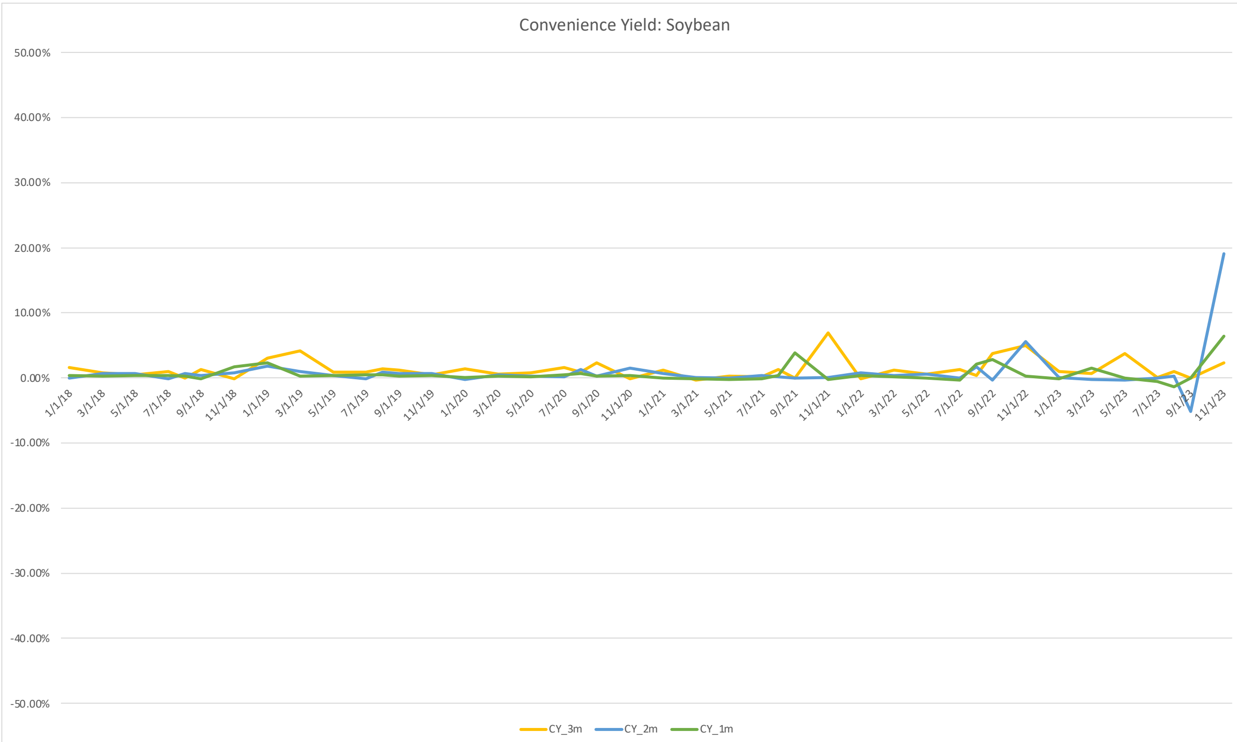

Figure 7: Soybean Convenience Yield

Turning our attention to corn, as depicted in Figure 3, corn prices remained in contango for most of the period from 2018 to 2023. According to the theory of storage, in contango markets, where futures prices exceed spot prices, convenience yield should be minimal, as individuals holding stock on hand should not receive additional rewards. Figure 8 illustrates the estimated convenience yield of corn. Indeed, for the majority of the time leading up to September 2023, convenience yield estimates hovered around zero, with few exceptions. However, as convenience yields began to increase in mid-2023, the market shifted from contango to weak-contango and eventually to backwardation.

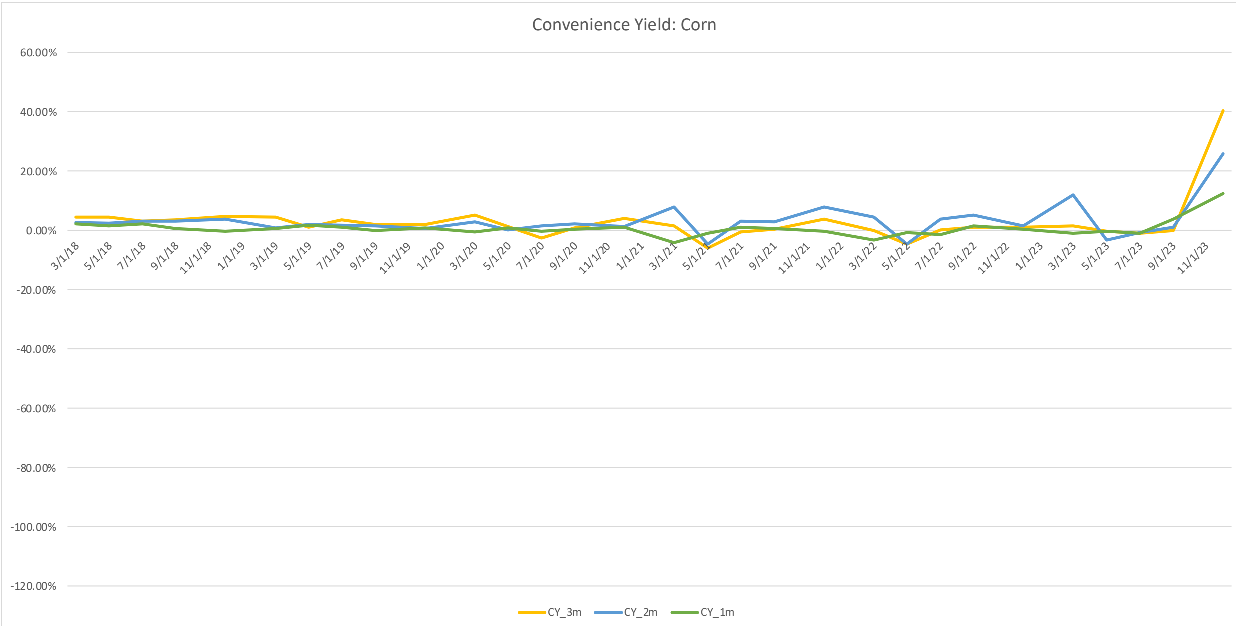

Figure 8: Corn Convenience Yield

For crude oil, the most noticeable element in the graph of its convenience estimates is the spike in April 2020. As discussed earlier, a sharp reduction in demand led to an oversupply of crude oil in the market at the time, resulting in a rapid decrease in price. From Figure 4, it is evident that the market was in hyper-contango[8] at that moment, indicating ultra-low demand for spot assets. In fact, at its lowest point on April 20th, crude oil was trading at negative prices due to a lack of storage supply at its lowest point. Under the traditional theory of storage, one would expect convenience yields to be very low during such a supply surplus, since holding spot assets would ostensibly offer little benefit. However, contrary to this expectation, the observed convenience yield estimates were anomalously high. This could be interpreted as a contradiction of the theory, but a deeper analysis suggests that during the hyper-contango observed, much of

[8] Hyper-contango is when the price of futures contracts for delivery further in the future is significantly higher than for nearer deliveries, indicating an extreme surplus of a commodity and limited storage options. This typically leads to high storage costs being factored into futures prices.

the market activity was concentrated in the cash and carry market rather than in the futures market itself.

This context is crucial because hyper-contango often intensifies as contracts approach expiration and storage capacity becomes constrained. Therefore, the high convenience yields observed may not solely reflect the typical market dynamics but rather capture extraordinary market conditions where the pricing dynamics are influenced more by logistical constraints and speculative actions than by inherent asset values. In this scenario, the theory of storage still holds valid; it's the exceptional circumstances that are being captured in the convenience yield estimates, reflecting the extreme pressures on storage rather than the usual market dynamics. On the other hand, as the market normalized and went into backwardation in early 2022, convenience yield estimates increased noticeably, conforming to the theory in this case.

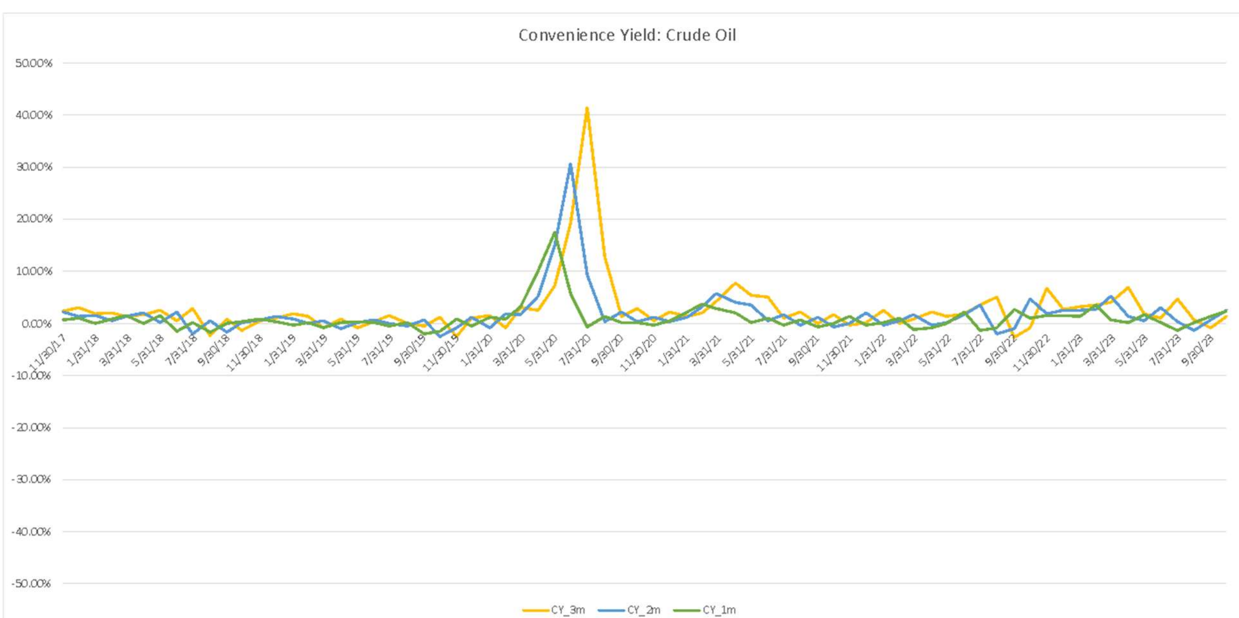

Figure 9: Crude Oil Convenience Yield

As for natural gas, it is a highly volatile commodity in terms of price movements. Especially during winter months, local demand and supply situations can lead to drastic changes in spot prices in a short period of time. Consequently, its convenience yield estimates are also quite volatile. Figure 10 displays multiple peaks over the five-year period. In general, the convenience yield estimates align with expectations based on the theory of storage. Spikes in spot prices lead to backwardations in the market, and they correspond to the spikes observed in the graph for convenience yields.

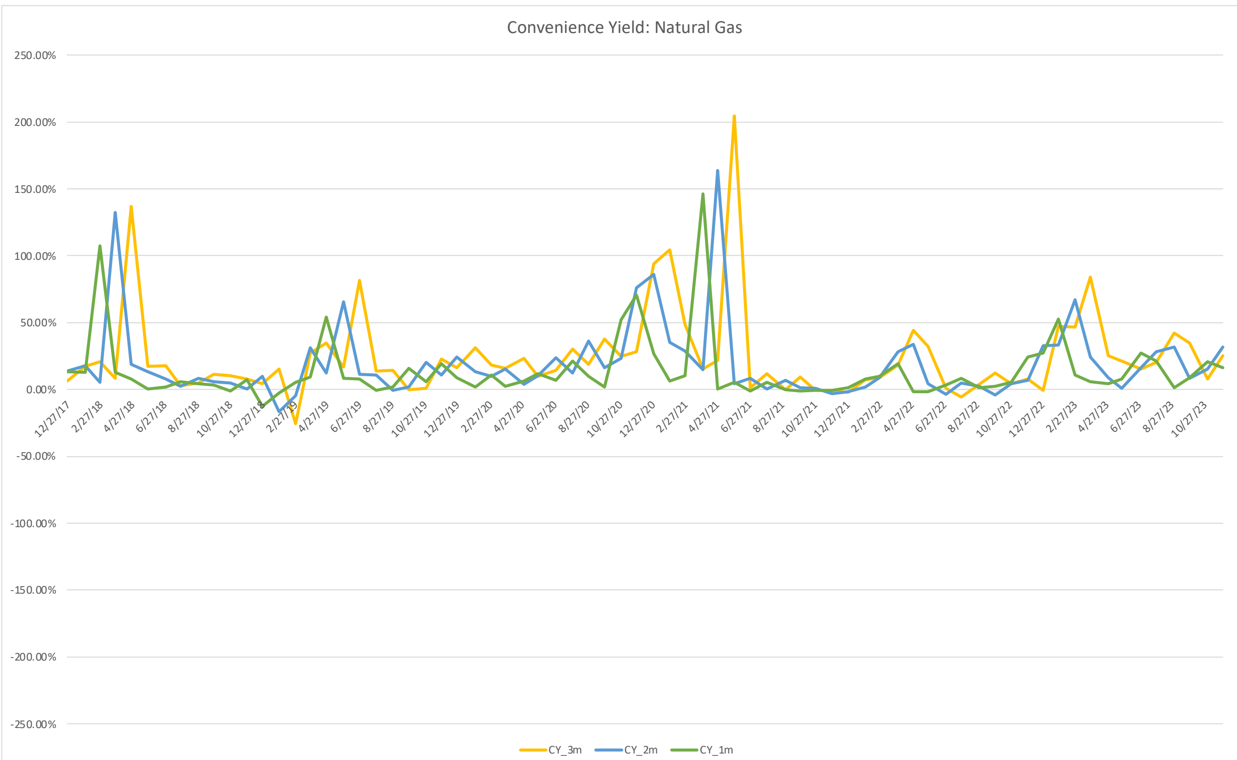

Figure 10: Natural Gas Convenience Yield

In Table III, I present the descriptive statistics of the convenience yield estimates. Among the five commodities, natural gas exhibits the most volatile and significant convenience yield, with much higher mean, median, and standard deviation compared to others. The estimates for all five commodities are positively skewed. Additionally, standard deviations exceed means for all contracts, indicating the high volatility of the convenience yields. In terms of the term structure, as the time to maturity increases, the mean, median, and standard deviation also increase, except for the mean and median of corn, and the standard deviation of soybean. Furthermore, for all commodities, I recorded negative convenience yields for minimum values. To understand the phenomenon, it is crucial to consider market conditions such as contango, where future prices are expected to be higher than current prices. In such markets, the costs and incentives to store commodities increase. This is particularly relevant in strong contango markets where there is a pronounced incentive to store, potentially driving up storage costs substantially. Negative convenience yield occurs in situations where the costs of holding the commodity exceed the benefits of physical possession, and it is more costly to carry the physical commodity over time.

Table III: Convenience Yield - Descriptive Statistics

	Mean	*Median*	*Max.*	*Min.*	*Std. dev.*
Panel I: Wheat					
1m	0.70%	0.20%	7.70%	-1.77%	1.83%
2m	1.76%	0.72%	22.00%	-0.66%	4.08%
3m	3.03%	1.51%	15.45%	-1.74%	4.30%
Penal II: Corn					
1m	0.62%	0.45%	12.32%	-4.18%	2.70%
2m	3.08%	2.37%	25.83%	-4.58%	5.44%
3m	2.66%	1.38%	40.37%	-5.93%	7.59%
Penal III: Soybean					
1m	0.59%	0.28%	6.39%	-1.41%	1.30%
2m	0.84%	0.36%	19.07%	-5.16%	3.16%
3m	1.31%	0.94%	6.93%	-0.37%	1.52%
Penal IV: Natural Gas					
1m	13.53%	7.07%	146.28%	-12.58%	24.01%
2m	19.18%	10.86%	163.51%	-16.78%	28.63%
3m	24.29%	15.98%	204.60%	-25.45%	33.79%
Penal V: Crude Oil					
1m	0.93%	0.33%	17.40%	-2.09%	2.65%
2m	1.77%	0.94%	30.62%	-2.48%	4.28%

3m	2.67%	1.59%	41.50%	-2.77%	5.66%

Table IV reports the correlations between the convenience yield estimates with different maturities for the five commodities. In general, correlations between these contracts are positive, except for the correlation between convenience yields of 1-month and 2-month wheat contracts. For crude oil and corn, correlations are strong, while for natural gas, correlations are much weaker.

Table IV: Convenience Yield - Correlations

Panel I: Wheat				***Panel II: Soybean***			
	3m	*2m*	*1m*		*3m*	*2m*	*1m*
3m	1.000			*3m*	1.000		
2m	0.143	1.000		*2m*	0.213	1.000	
1m	0.400	-0.201	1.000	*1m*	0.079	0.675	1.000
Panel III: Oil				***Panel IV: Natural Gas***			
	3m	*2m*	*1m*		*3m*	*2m*	*1m*
3m	1.000			*3m*	1.000		
2m	0.545	1.000		*2m*	0.167	1.000	
1m	0.149	0.554	1.000	*1m*	0.025	0.110	1.000
Panel V: Corn							
	3m	*2m*	*1m*				
3m	1.000						
2m	0.396	1.000					
1m	0.819	0.108	1.000				

Next, with the estimated convenience yields, I calculate the implied storage costs using equation (1).

Table V: Implied Storage Costs - Descriptive Statistics

	Mean	**Median**	**Max**	**Min**	**Standard Deviation**
Wheat	2.50%	2.58%	15.71%	-5.89%	4.48%
Soybean	1.56%	1.82%	10.07%	-16.38%	3.70%
Corn	10.60%	9.74%	30.31%	-5.08%	6.76%
Crude Oil	1.56%	0.29%	64.17%	-14.74%	8.47%
Natural Gas	4.00%	1.94%	69.36%	-21.80%	10.96%

Descriptive statistics of the implied storage costs are reported in Table V. Across the five commodities, corn shows higher average costs than the others. Crude Oil and Natural Gas display considerable volatility in storage costs, with both reaching substantial highs. The negative minimums across the commodities, suggest there are significant benefits to storage at times, likely due to factors such as market shortages.

Working (1948) suggested the "Working Curve" to study the relationship between storage supplied and the cost of storage. Working stated that the relationship should resemble a supply curve, illustrating minimal levels of storage service provided when the storage price is low, with the quantity of storage service increasing as the price of storage rises. He believed that this relationship remains intact even if the price goes into negative values.

One goal of this paper is to study whether the implied storage cost calculated from the cost of carry model with option-based convenience yield estimate features the relationship with storage level that Working outlined. However, Working only provided the conceptual idea of this relationship. There lacks a mathematical model that represents this relationship. Zhang and Turvey (2024) proposed that this relationship between storage cost and storage supplied can be represented by

$$K(S_t) = k - a(h - S_t)^3$$

where *k* is the fixed storage cost, a is a mean-reversion adjustment multipliers, h is the optimal storage capacity when the market is in equilibrium, and $S_t$is the level of storage supplied in the market at time *t*.

Table VI reports the result of regressing the implied storage costs $K(S_t)$ on $(h - S_t)^3$ for all commodities. The constant provides an estimate of *k* and the coefficient provides an estimate of *a* for each commodity. All constants are statistically significant, and apart from the coefficient for corn, the results are all statistically significant. In addition, the coefficients all take on extremely small values, which is as expected due to the setup of the model.

Table VI: Regression of Implied Storage Cost on the Mean Reversion Term

	Wheat	**Soybean**	**Corn**	**Crude Oil**	**Natural Gas**
Constant	0.023***	0.014***	0.106***	0.019***	0.041***
	(0.005)	(0.003)	(0.007)	(0.006)	(0.007)
Mean Reversion	3.04E-08***	1.02E-20***	1.54E-13	2.15E-18*	2.76E-11***
	(5.45E-09)	(2.36E-21)	(1.64E-13)	(1.27E-18)	(8.32E-12)
R-Squared	0.2609	0.1471	0.0099	0.0133	0.0488

With the estimated *k* and *a* values, I fit them to the model and obtain the fitted storage costs. Additionally, using storage data from the five commodities, I am also able to calculate the estimated storage costs based on the model. In this case, I used the average implied storage cost for *k* and an ad-hoc *a* value that fits the data the best. Then, the estimated storage costs, along with the implied storage costs, are graphed against the inventory data. The results are shown in Figures 11 to 15. Blue dots represent implied storage costs calculated from the cost of carry model, orange dots represent estimated storage costs based on Zhang and Turvey's model, and gray dots represent the fitted storage costs using regression results.

The graph for wheat is displayed in Figure 11. There are indeed observable negative storage costs, but instead of resembling a supply curve as Working suggested, it features a "V" shape. At lower inventory levels, the implied storage costs were mostly positive, and then started to decrease as wheat stock increased. Negative storage costs became more common, and then as wheat stock further increased, storage costs started to pick up again. Comparing with the estimated and fitted storage costs under Zhang and Turvey's model, the difference is obvious. For estimated and fitted storage costs, when the storage level is low, the estimated storage cost is low. They increase initially at a decreasing rate as the storage level increases and then at an increasing rate as the storage approaches its capacity.

Notably, this "V" shape in implied storage costs is observed solely for wheat among the five commodities. It raises intriguing questions about the unique characteristics of the wheat market and potential factors driving this phenomenon.

One potential explanation could be the specific supply-demand dynamics and storage infrastructure peculiar to the wheat market. For instance, it's possible that wheat storage facilities exhibit different capacity constraints or operational inefficiencies compared to those of other commodities, leading to distinct patterns in implied storage costs.

Additionally, the seasonal nature of wheat production and consumption may play a significant role. Wheat is subject to distinct planting and harvesting seasons (i.e. winter wheat and spring wheat), which can lead to pronounced fluctuations in inventory levels throughout the year. These seasonal variations in supply and demand may influence market participants' storage decisions and speculative behavior, contributing to the observed "V" shape in implied storage costs.

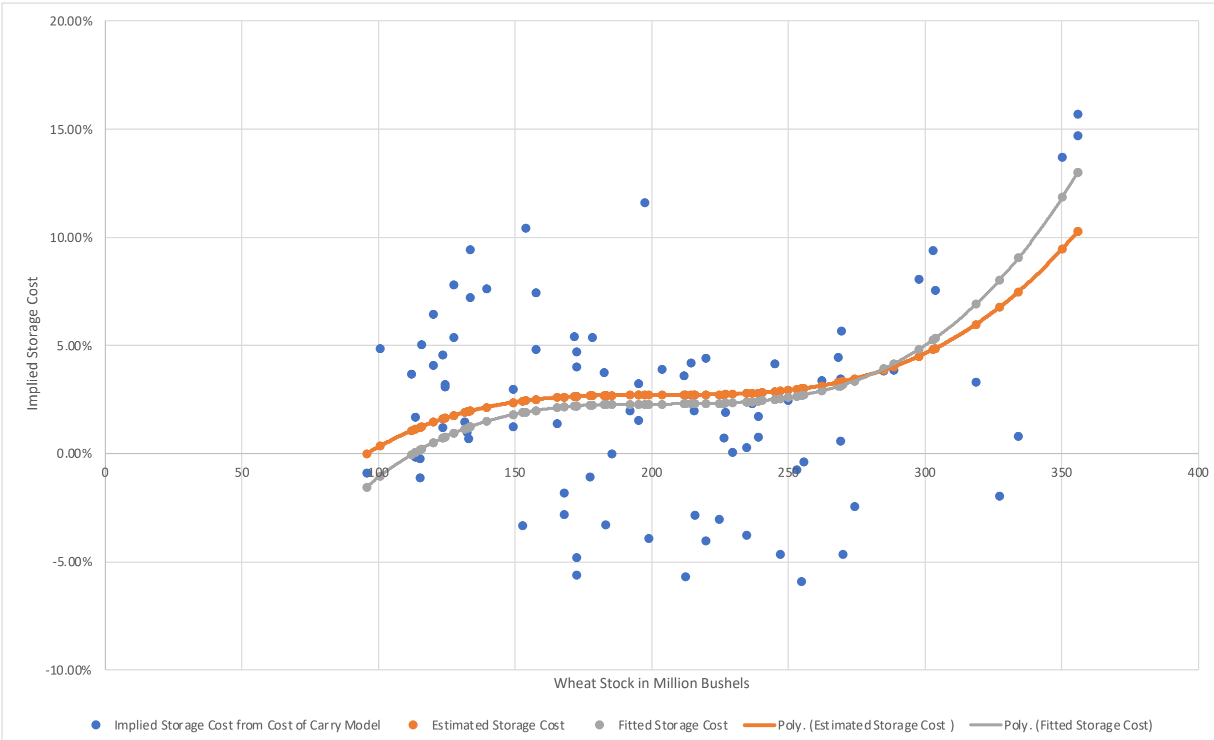

Figure 11: Wheat Storage Cost

In the case of soybeans, the result is a much closer representation of the Working Curve. At low inventory levels, negative storage costs are common and significant. As inventory levels grow, storage costs turn positive and showcase a slight positive slope. The relationship closely matches the trend observed in the estimated and fitted storage costs using Zhang and Turvey's model.

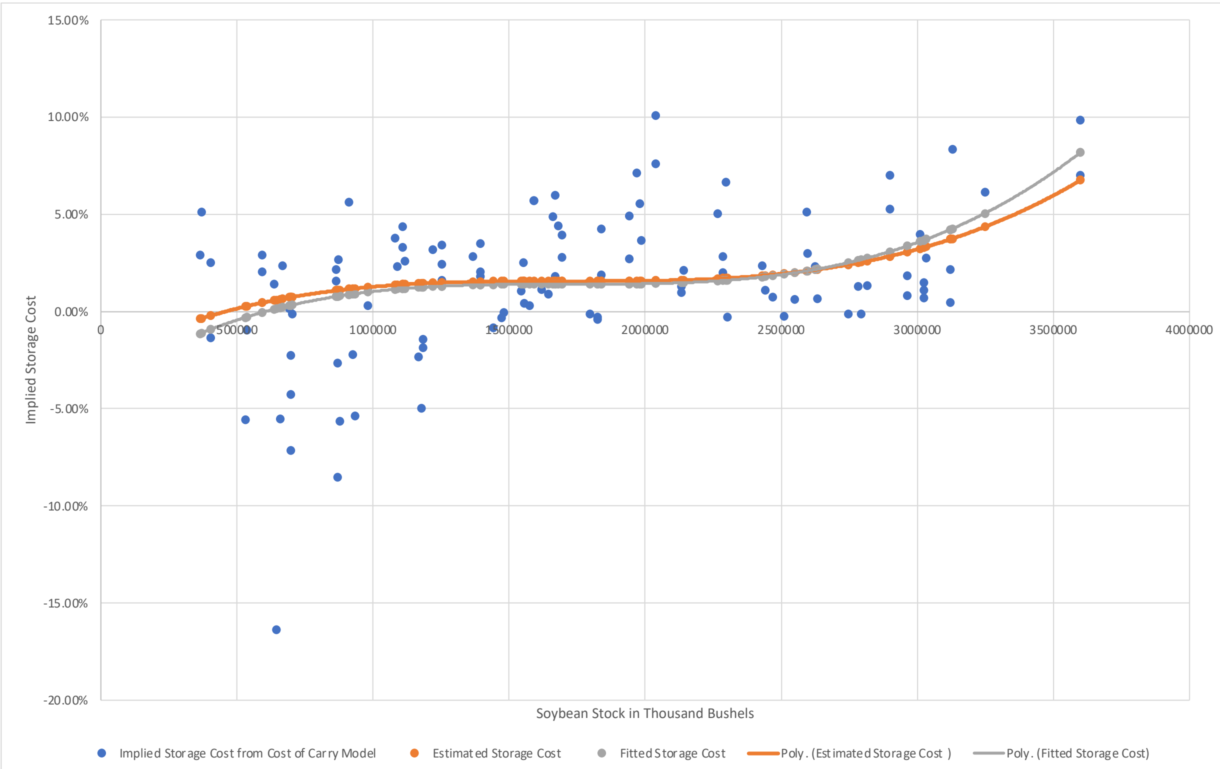

Figure 12: Soybean Storage Cost

In the case of corn, negative storage costs were recorded only once, notably when the inventory level of corn reached its lowest point. Beyond this isolated occurrence, the costs remained relatively stable and exhibited minimal fluctuation as the stock level of corn increased during the period. It is possible that during the period, corn storage did not hit capacity, and thus no high storage cost was observed. It also did not reach low enough levels to induce negative storage costs. Indeed, negative values are also not found in the estimated and fitted storage costs under Zhang and Turvey's model and the fitted storage costs also showcased a flat shape.

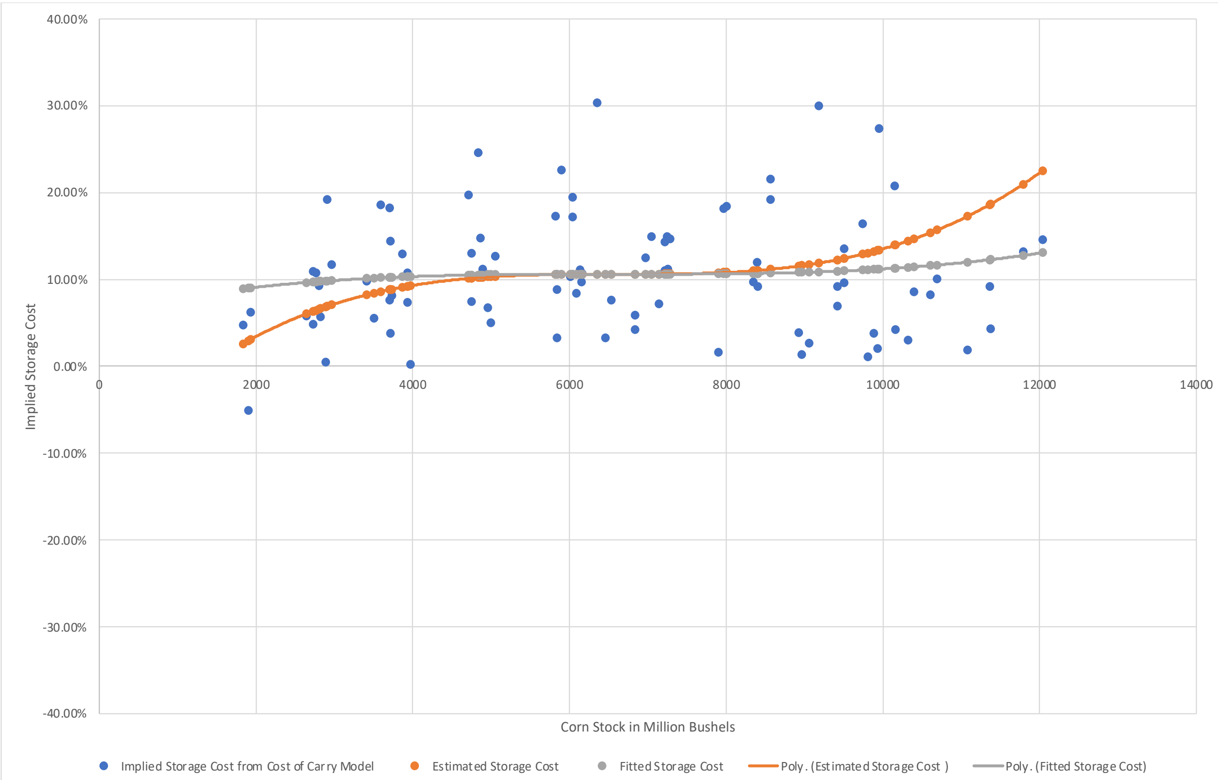

Figure 13: Corn Storage Cost

The graph for crude oil, however, exhibits an inverted "L" shape that has not been seen with agricultural commodities. At low inventory levels, negative storage costs also exist. They gradually increase as the inventory level rises. However, they eventually sharply rise to over 60% as the inventory reaches a high enough level. One explanation for this could be that crude oil is not disposable, so asset holders would have to pay the high storage cost as facilities reach their limit, while agricultural commodities like soybean are easily disposable, thus avoiding ultra-high storage costs. However, both the estimated and fitted storage costs derived from Zhang and Turvey's model do not adequately capture this steep increase in implied storage costs for crude oil as it reaches high storage levels. These conditions were so exceptional that they stretched beyond the predictive capabilities of the model. The fitted storage cost in particular exhibits a very flat shape.

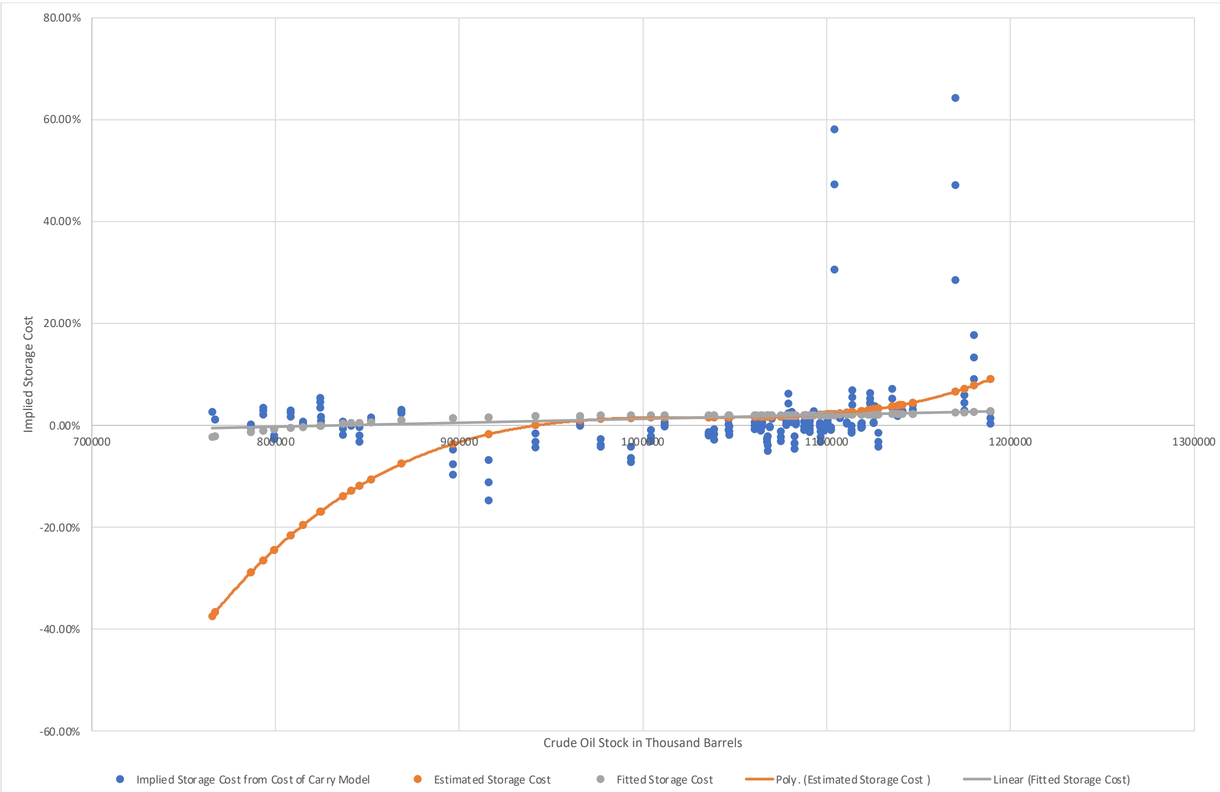

Figure 14: Crude Oil Storage Cost

For natural gas, a similar trend can be identified. At low storage levels, storage costs are near zero. They increase as the inventory level grows and eventually spike when capacity is reached. This is as expected, as natural gas is also non-disposable, leading to exceptionally high storage costs when facilities reach capacity. The results largely matched the relationship exhibited in the estimated storage cost and fitted costs under Zhang and Turvey's representation of the Working curve.

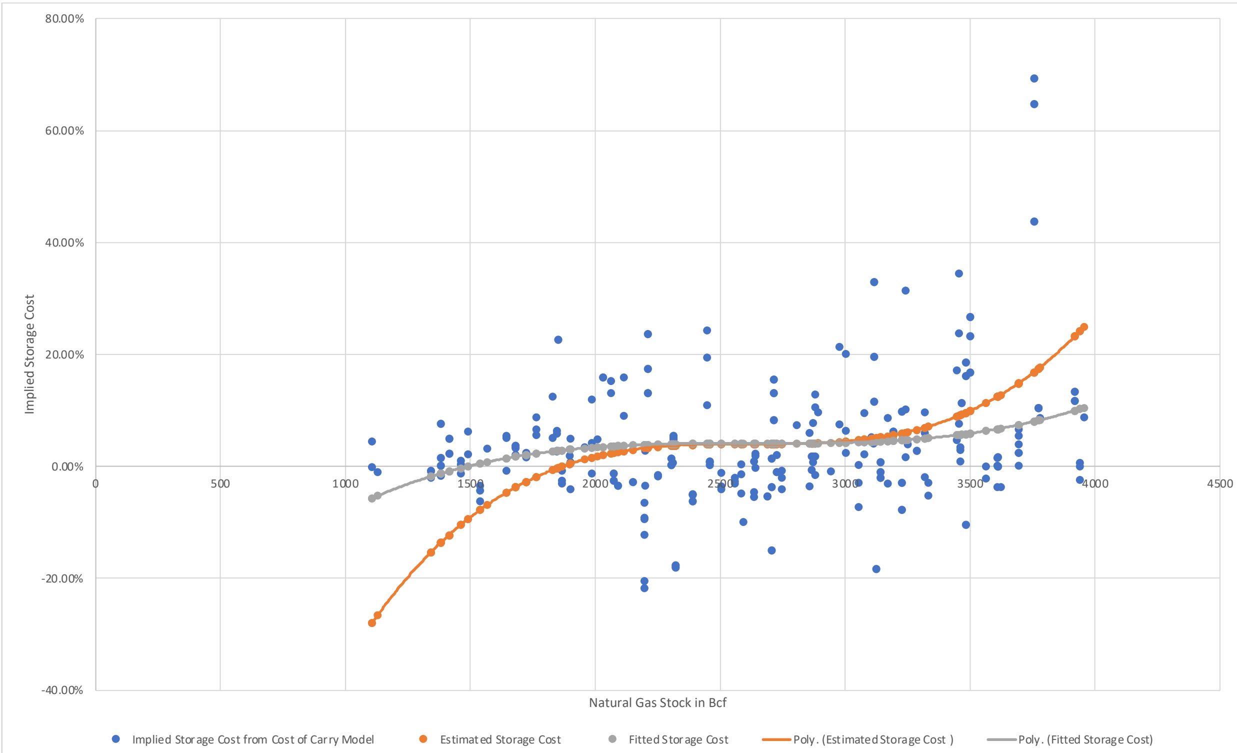

Figure 15: Natural Gas Storage Cost

In the next part of the analysis, the interested adjusted basis is regressed on the convenience yield estimate. According to equation (7), there is a negative relationship between the two variables. At the outset, I conducted the regression analysis using ordinary least squares. However, the Breusch–Godfrey LM test for autocorrelation indicates that the regression residuals exhibited serial correlation up to 10 lags at 5% level of significance. As a result, an AR(1) model is used for the analysis with the results reported in Table V.

Table VII: Regression of Interest-Adjusted Basis on Estimated Convenience Yield

Panel I: Wheat				***Panel II: Soybean***			
	1m	2m	3m		1m	2m	3m
Constant	0.030	0.016	0.049	Constant	0.010	-0.001	0.008
	(0.045)	(0.020)	(0.056)		(0.012)	(0.016)	(0.018)
C	-0.523***	-0.112	-0.323***	C	-0.181*	-0.841*	-0.112*
	(0.158)	(0.102)	(0.080)		(0.088)	(0.471)	(0.054)
R Square	0.334	0.052	0.423	R Square	0.138	0.102	0.163
Durbin–Watson statistic	1.303	1.568	1.897	Durbin–Watson statistic	2.012	1.886	1.790
Panel III: Crude Oil				***Panel IV: Natural Gas***			
	1m	2m	3m		1m	2m	3m
Constant	-0.004	-0.016***	-0.026***	Constant	0.019	0.042*	0.057*
	(0.003)	(0.005)	(0.008)		(0.015)	(0.023)	(0.032)
C	0.280**	0.357*	0.276*	C	-0.326***	-0.009	-0.422***
	(0.106)	(0.167)	(0.130)		(0.037)	(0.040)	(0.076)
R Square	0.441	0.419	0.354	R Square	0.377	0.124	0.452
Durbin–Watson statistic	2.017	2.033	2.075	Durbin–Watson statistic	1.761	1.952	1.991
Panel V: Corn							
	1m	2m	3m				
Constant	0.051*	0.075***	0.103***				
	(0.026)	(0.022)	(0.013)				
C	-0.208**	-0.307*	-0.342**				
	(0.086)	(0.137)	(0.133)				
R Square	0.171	0.266	0.231				
Durbin–Watson statistic	2.024	2.190	2.083				

The Durbin-Watson statistic reported in Table VII is a measure used to detect autocorrelation in the residuals of a regression model. A value of around 2 suggests no autocorrelation, while values deviating from 2 indicate positive or negative autocorrelation. In this case, the Durbin-Watson statistics provide little evidence of the presence of residual serial correlation within the AR(1) model. For wheat, soybean, corn and natural gas, the estimated coefficients for convenience yield are negative, ranging from -0.841 to -0.009. Based on the definition of interest-adjusted basis in equation (7), it increases as the market goes into contango and decreases as the market goes into backwardation. Hence, the negative relationship between the interest-adjusted basis and convenience yield estimate is consistent with the theory of storage since according to the theory, convenience yields decrease in contango markets and increase in backwardations. However, the coefficient for crude oil is positive, deviating from results of other commodities. A positive coefficient suggests that as the market demand for spot asset decreases (contango), convenience yield for holding the spot asset would increase, contradicting the theory. This deviation from the theory was also observed in figure (9) discussed earlier. In April 2020, there was a severe over supply of crude oil in the market and the market was in hyper-contango. Convenience yield should be low, as spot asset holders should not be rewarded for carrying spot assets. However, due to the higher spot price return volatility relative to future price return volatility, the convenience yield estimate was high. Note that this spike in convenience yield was also the most significant one during the five-year period, with maximum convenience yield reaching 40.1% while the median is 1.59%. Thus, as a result, it is likely that this exceptional hyper-contango market condition led to the skewed regression coefficient for crude oil.

Additionally, with the results from the regression, I calculated the fitted values of the interest-adjusted basis $\widehat{\iota ab}_t = \widehat{\beta_0} + \widehat{\beta_1} c_t$, and as a result I am able to get an approximation of F_t with cost of carry adjusted for convenience yield

$$\widehat{F_t} = S_t * \exp[(r_t + s_t - c_t) * t] = S_t * \exp\left(r_t + \widehat{\iota ab}_t\right) \tag{9}$$

Table VIII: Cost of Carry Adjusted by the Convenience Yield - Percentage of Price Difference

	3-month maturity			2-month maturity			1-month maturit y		
Commodity	**Mean**	**t-test mean = 0**	**P-value**	**Mean**	**t-test mean = 0**	**P-value**	**Mean**	**t-test mean = 0**	**P-value**
Wheat	-0.053%	-0.199	0.843	-0.047%	-0.359	0.722	-0.048%	-0.376	0.709
Soybean	-0.118%	1.059	0.296	-0.087%	1.033	0.308	-0.030%	1.023	0.312
Corn	-0.160%	-0.040	0.968	-0.198%	-0.426	0.673	-0.263%	-0.409	0.686
Crude Oil	-0.107%	1.469	0.146	-0.063%	1.434	0.156	-0.024%	1.348	0.182
Natural Gas	-0.784%	0.722	0.473	-0.563%	0.237	0.814	-0.309%	0.024	0.981

Note: the percentage difference is computed as $1-\frac{\widehat{F_{t,T}}}{F_{t,T}}$, where $\widehat{F_{t,T}}$ is calculated using equation (3) and $F_{t,T}$ is the actual futures price.

Table VIII reports the mean percentage difference between the estimated futures price and the actual futures price, as well as test results on whether the mean difference is zero. From the results, it is obvious that the percentage differences are very small, and in general, the difference gets smaller as time-to-maturity of the contracts decreases. In all cases, we fail to reject the null hypothesis that the mean difference is zero. It appears that $\widehat{F_t}$ is an accurate approximation of F_t and thus the convenience yield estimate is generally accurate.

Table IX: Two Stage Least Square Regression on 3-month Estimated Convenience Yield

	Wheat	**Soybean**	**Corn**	**Crude Oil**	**Natural Gas**
Constant	-0.031*	-0.022***	-0.072	0.020	0.326
	(0.017)	(0.006)	(0.093)	(0.013)	(0.360)
Interest Rate	0.394***	0.201*	3.992***	0.511	-2.382
	(0.140)	(0.116)	(1.288)	(0.449)	(3.960)
Futures Price	-0.171***	-0.057***	-0.116***	-0.009***	-0.020
	(0.038)	(0.004)	(0.040)	(0.004)	(0.169)

Spot Price	0.176***	0.059***	0.119**	0.009***	-0.003
	(0.040)	(0.004)	(0.049)	(0.003)	(0.204)
Storage Cost	-0.967***	-0.845***	-0.282	-0.630***	-0.201
	(0.117)	(0.053)	(0.243)	(0.107)	(1.051)
R-Squared	0.995	0.920	0.249	0.916	0.219

Furthermore, convenience yield estimates for 3-month contracts were regressed on interest rates, futures prices, spot prices, and storage costs. To address the plausible positive correlation between storage costs and inventory levels, a two-stage least squares regression was employed. The first stage involved regressing storage cost on inventory levels and using the fitted values in the subsequent analyses.

However, this approach might not fully escape concerns about tautology. This is because the storage costs were derived from an identity that already incorporates interest rates and prices, and these were then re-used in a model to predict convenience yields. Hence, the regression could be seen more as confirming the internal consistency of the model's components rather than explaining new relationships. That is also the reason why there are high R-squared values recorded.

Results reported in Table IX showed that interest rates have a positive and significant effect on convenience yield estimates for wheat, soybean, and corn. This might suggest that as the cost of holding inventory increases with rising interest rates, the market compensates with higher convenience yields. Conversely, the negative coefficients associated with futures prices across most commodities (except natural gas) align with the expectation of a market shifting towards contango, where higher future prices lead to lower convenience yields. For spot prices, the positive and significant coefficients (except in natural gas) support the theory of storage, indicating that higher spot prices, reflecting higher immediate demand, correspond to higher convenience yields. As for storage costs, the coefficients are negative and significant for most commodities except for corn and natural gas. This is the expected result as lower storage cost indicates lower inventory level. Lower inventory suggests it will be more costly for people to obtain spot assets and those who have the assets on hand will enjoy a higher convenience yield.

CONCLUSION

In conclusion, this study provides a comprehensive analysis of convenience yields, storage costs, and their determinants across five major commodities. By employing an option-based approach and leveraging extensive data spanning from 2018 to 2023, I have gained valuable insights into the dynamics of commodity markets and the factors influencing convenience yields.

The analysis reveals notable trends and patterns within each market, highlighting the impact of supply and demand dynamics, geopolitical events, and market sentiment on convenience yields. Through regression analyses, I have confirmed relationships between convenience yields and key variables, underscoring the importance of factors such as spot prices, futures prices, and interest rates in shaping market dynamics.

Furthermore, the examination of implied storage costs and the Working Curve offers insights into the relationship between storage supplied and the cost of storage, providing valuable implications for market participants and policymakers. The findings of this study not only contribute to our understanding of commodity market efficiency and risk management strategies but also offer practical implications for investors seeking to navigate volatile commodity markets.

Moving forward, further research could delve deeper into understanding the drivers behind these deviations and their implications for market participants. Additionally, exploring alternative frameworks or refining existing models may enhance our ability to predict and interpret commodity market behavior more accurately.

Overall, this study contributes to the ongoing discourse surrounding commodity pricing, with potential implications for risk management, investment strategies, and policy formulation.

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