ESSAYS ON FIRM DYNAMICS AND RESOURCE ALLOCATION

A Dissertation
Presented to the Faculty of the Graduate School
of Cornell University
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy

by
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May 2022

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Cornell University 2022

This dissertation consists of three essays in the areas of Macroeconomics and Firm Dynamics, examining the role of resource (mis)allocation on business dynamism and market outcomes.

The first essay examines the role of input specificity in affecting firm-to-firm trade dynamics and how business dynamism and a sticky input market interact. I construct a novel panel data of supply chain relationships with an additional layer of input specificity measures on each linkage across US firms and point out the foreclosure effect of input specification contracts. I then propose a Schumpeterian model where each firm endogenously chooses whether to produce a specific input for an exclusive customer firm at the expense of giving up larger customer base. Bringing the micro-evidence to the equilibrium model, I show that the input specificity channel is quantitatively important and can exaggerate the downturn in business dynamism. I show that shutting down the foreclosure effect can lead to a 10% and 25% increase in growth and entry rates.

In the second essay, I study the role of local vintage capital market in driving co-locations of entrants and capitalists and shaping the spatial disparities in business dynamism and productivity. I combine the US-based venture capital (VC) investment records with local business vintage capital supply across metropolitan statistical areas (MSAs). I find that the VC investment elasticity with respect to local vintage capital is about 9% - 14%. Based on the motivating empirical evidence, I integrate vintage capital induced co-location of entrants

and VC investment into a theoretical framework. I highlight a straightforward mechanism where a vintage capital market with abundant supply can lower the capital cost and thus increase firms' profits, attracting VC investment, encouraging entrepreneurship and further leading to a selection-induced agglomeration effect. I show that a larger city intensifies such allocative forces and thus amplifies the agglomeration effect, making the city further attractive to VC investment and entrants relative to others.

The third essay, which is joint work with Qinshu Xue, examines how firm-to-firm partnership shapes a firm's innovation strategy and its implications for business dynamism. My coauthor and I combine the firm-to-firm partnership information with the patent data filed by the US publicly listed firms from 2003 to 2013. We find that the degree of firm-to-firm partnership engagement along the supply chain is positively associated with stronger incentives in conducting exploitative innovations, leading to less knowledge spillover than exploratory innovations do. To mitigate the potential selection bias, we further adopt a double debiased machine learning technique to allow for a more flexible relationship between partnership variables and control variables and confirm the positive correlation. We then propose a quality-ladder framework where firms optimally choose to either continue exploitative innovations by leveraging possible collaborations with other firms or stop them and turn to exploratory innovations for its product. Our model predicts that partnership engagement can induce more technologies that are too sophisticated to generate spillover and thus lead to a lower entry rate and precludes growth.

BIOGRAPHICAL SKETCH

Bin Zhao grew up in Shanghai, China. He earned a Bachelor of Science in Economics from the Texas A&M University, College Station in 2016. He joined the Ph.D. program in economics at Cornell University in the fall of 2016, and aspires to earn his Ph.D. in May 2022. His primary research interests are macroeconomics, firm dynamics and trade.

To my grandparents and my parents.

ACKNOWLEDGEMENTS

First and foremost, I am extremely grateful to my thesis advisors, Nancy Chau, Justin Johnson, and Ravi Kanbur for their invaluable advice, tremendous inspiration, continuous encouragement, and patience during my PhD study. I owe my development throughout graduate school, both professionally and personally, to them.

I would like to thank Julieta Caunedo, Kristoffer Nimark, Mathieu Taschereau-Dumouchel for their valuable comments and suggestions that inspire and improve my work at different stages. I wish to extend thanks to my peers at Cornell Zhou Fan, Yizhou Kuang, Raul Morales, Kevin Ng, Yu She, Qinshu Xue and Pengfei Zhang for their incredibly valuable supports and helpful comments. I also acknowledge the Department of Economics, the Graduate School for financial support.

Finally, I would like to express my gratitude to my parents and my girlfriend Qinshu Xue. None of these would have been possible without their tremendous encouragement and understanding in the past few years.

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CHAPTER 1

INPUT SPECIFICITY, FIRM-TO-FIRM TRADE, AND FIRM GROWTH

1.1 Introduction

...[We] use some custom components that are not commonly used by our competitors. When a component or product uses new technologies, initial capacity constraints may exist...The continued availability of these components at acceptable prices, or at all, may be affected if suppliers decide to concentrate on the production of common components instead of components customized to meet the Company's requirements...The Company has entered into agreements for the supply of many components...

Annual Report (2020) from Apple.Inc.[1]

Input specification is commonly believed to be crucial in modern manufacturing practices, particularly when firms attempt to conduct specific tasks or designs to adopt new technologies or launch new features. Along with the increasingly sophisticated contract enforcement, downstream firms are being granted more access to implement input specifications without being held up. Economists traditionally view this positively: partnering with suppliers to leverage input specificity can realize synergies, effectively enable specialization, and subsequently achieve growth and increase productivity (Nunn, 2007; Boehm and Oberfield, 2020).

This paper argues that input specification may, in turn, harm growth and productivity — for other firms and hence discourage firm creation, intensify market concentration and slow productivity growth, setting the macroeconomic

[1]Details can be found at https://investor.apple.com/sec-filings/sec-filings-details/default.aspx?FilingId=14468826

stage for reinforcing dynamic interaction between higher firm-to-firm matching frictions and lower aggregate-level industrial dynamism. The hypothesis is established based on two empirical observations. First, a vertical contract for input specification between a downstream firm and a supplier exploits the supplier's capacity, resulting in foreclosure against other downstream customers of the supplier. Second, firms actively engaging in input specification have enjoyed a longer life span and taken the dominant market share across finely defined sectors.

To assess the important macroeconomic implication of vertical contracts for input specification, this paper proposes a Schumpeterian firm dynamics model where the vertical contracting for input specificity takes center stage. Incumbent firms search for suppliers to source components that take some measure of tasks. A highly specific component involves more tasks and prevents suppliers from supplying others due to capacity constraints, which ultimately induces Bertrand competition among firms for supplier capacities. Given heterogeneous productivity across firms, scarce capacities resources are allocated to the more productive firms that demand a high degree of input specificity. This raises the cost of entry as new firms, which either seek common or specific components for production, may have to bid up surplus promised to the suppliers to obtain the input components. Consequently, a lower entry rate leads to slower growth as the propagation of productivity distribution is powered by technology spillover unfavorable toward entrants.

The theory, in addition, highlights a dynamic general equilibrium through the production network where input specificity-induced matching frictions interact with lower entry and growth rates. A lower growth corresponds to incumbent firms' slower technological obsolescence rate in the spirit of the creative-destruction effect of Aghion and Howitt (1992). Lower obsolescence, in turn, affords relatively more productive incumbent firms more time to find better-matched suppliers to implement contracts for input specification, which further amounts to higher market concentration, tighter component sourcing market and a decline in supply chain dynamism, referring to a dynamic crowding-out effect. A tighter component market further pushes up the sourcing cost and thus depresses the entry value. Since entrants are both the customers of some firms and the suppliers of others in the production network, lower entry further exacerbates the supply shortage and slows the growth through the network effect, leading to a vicious loop.

To quantify the importance of this mechanism, I firstly calibrate the model to production network data in the United States. Specifically, I combine firm-level financial data and production network data with the ability to observe connections among firms about their downstream customers, upstream suppliers and additional research collaborating partners on the top of vertical relationships indicating the extent of component specificity. Along the process of economic growth, more productive firms expand faster and generate more significant surplus through forming vertical relations for input specification yet foreclosing other customer firms. I therefore ensure that the model replicates the moments of contracting and separation rate of vertical relation for specifically designed components together with those of common vertical relationship exposed to foreclosure risk. Economic growth also requires a Schumpeterian-style of replacing less productive incumbent firms with relatively more productive entrants due to technological spillover. Hence the model parameters are calibrated to generate aggregate moments, including growth and entry rate.

I use the calibrated model to evaluate a counterfactual impact of increasing difficulty in entrants' technology adoption on the set of moments capturing the industry dynamism. First, economic growth slows down because the main driving forces by entrants diminish, given the lower value of the entry. Second, the linkage creation rate for both specifically designed and common components declines. Less new entrants imply fewer new capacities for the component provision and hence lower the linkage creation rate due to matching frictions. Furthermore, a slower-moving economy gives relatively more productive incumbent more time to find a better-matched upstream supplier, which is more likely to result in foreclosure. Ultimately, the upstream capacities become more concentrated in relatively more productive firms for component specification.

Lastly, I further evaluate the roles of the foreclosure effect induced by the vertical relationship for component specification. By relaxing the capacity constraints, the firm-to-firm trade regains the dynamics, further encouraging entry by more than 25% and contributing to higher growth by more than 10%. This concludes the importance of scrutinizing the vertical contracting associated with a high degree of specificity, which has been such a prevalent commercial practice with few regulations.

1.2 Related Literature

This paper is related to three strands of literature and incorporates the workhorse canonical models and quantitative approaches into a unified framework. Firstly, it connects to and extends beyond the burgeoning works on knowledge spillover and growth in several aspects, including Klette and Kortum (2004), Luttmer (2007), and Perla and Tonetti (2014). I allow firm-to-firm matching and input specification-induced matching frictions, enabling the rich interactions between differential vertical linkages churnings and productivity growth. Furthermore, I impose exclusivity through the input specification channel, providing new micro-foundations of resource concentration toward relatively more productive firms. This aligns with the spirit of recent works on the decline in the knowledge diffusion between the frontier and the laggard firms including Akcigit and Ates (2021) and Andrews et al. (2016). Lastly, the model provides closed-form solutions along the balanced growth equilibrium allowing for analytical characterization of firms and linkages dynamism and confronting rich dynamic framework to firm-level production network panel data.

Second, I relate to literature on firm-to-firm trade in an environment with matching frictions. Recent papers, including Bernard et al. (2019), Demir et al. (2021), have emphasized that firms' performance benefits from more accessibility to larger suppliers pool. I reach a similar conclusion yet highlight an endogenous foreclosure effect through vertically specific contracts in the spirit of Allain et al. (2015). This paper also connects to the production network literature in which many works examine the propagation of shocks along the supply chains, including Barrot and Sauvagnat (2016) and Carvalho et al. (2020). My framework focuses on shocks originating from inter-firm contracting for input specification. As relatively more productive firms have more substantial incentives to conduct input specifications. Therefore, the input specification shocks are more likely to affect smaller firms, proposing new channels and reasoning of heterogeneous risk exposure planted in the production network.

Third, this paper contributes a novel mechanism to the vast and growing literature on documenting and understanding the declining trends in US business dynamism. In terms of empirical evidence, Akcigit and Ates (2021) and Davis and Haltiwanger (2014) provides an overview of the recent declines based on census data on labor flow and establishments' employment growth. Inferring from firm-level employment data, Decker et al. (2020) suggest a close empirical relationship between the weak employment response of firms to shocks and economy-wise dynamism decline. This paper departs from inferring from employment data by looking at rich information embodied in the firm-to-firm supply chain and providing micro-evidence on how an exclusive production network forged by vertical contract for input specification facilitates a weak linkage dynamism viewed as micro-foundations behind the weak employment response.

Inspired by the stellar and puzzling empirical patterns of decline in business dynamism, many papers attempt to explore various factors behind it more structurally. Led by Aghion et al. (2019), De Ridder (2019), many works emphasize the increasing importance of intangible assets and information and communications technology (ICT) as a major explanation as these technological factors exhibit a strong increasing return to scale. Other works explore non-technological culprits, including aging demographic changes stressed by Peters and Walsh (2019), Karahan et al. (2019), and Engbom (2019). My paper deviates from examining the very root of the recent decline and instead emphasizes an essential mechanism through vertically specific contracts that can intensify a decline, which can be quantitatively sizable. Furthermore, although those papers evaluate heterogeneous firm-level decisions on the aggregate industry-level performance through a general equilibrium channel, inter-firm interactions are largely absent due to a lack of tractable framework. This paper attempts to fill the gap by introducing a heterogeneous firm dynamics model that admits firms' competition for suppliers' capacities for specific components.

To summarize, this paper's main contribution is to first develop a theory of firm-to-firm linkages and life-cycle dynamics that incorporate entry, the exit of firms, separation and creation of vertical relations, endogenous growth and shapes of the productivity distribution. Second, I apply the theory to quantitatively assess the roles of vertical contracts for input specification over the decades in the US on business dynamism, market concentration and economic growth.

The following section summarizes three sets of facts that motivate my study and provides empirical evidence on the roles of vertical relationships for input specification in shaping production networks. Section 1.3 introduced a stylized two-period model to demonstrate the critical implication of vertical contracts in affecting firm dynamics. Section 1.4 develops a formal and general theory of firms and firm-to-firm linkages dynamics along an infinite horizon, and Section 1.5 layouts the equilibrium of the model along a balanced growth path and brings the model to the data to replicate firms' and their linkages life-cycles. Section 1.6 uses the quantitatively calibrated result to conduct a counterfactual analysis and evaluate the impact of foreclosure effect of contracts for input specification. Finally, Section 1.7 concludes.

1.3 Data and Motivating Facts

I combine three data sets at the firm level from the US publicly listed companies: (1) domestic firm-to-firm supply chain relationships, (2) data on an additional layer on customer-supplier relationship indicating a high degree of input specificity, and (3) data on firm's financial data. These data are constructed by merging two separate datasets: S&P Compustat and FactSet Revere, using common identifiers of CUSIP and ISIN codes.

The supply chain data is the FactSet Revere Supply Chain Relationships Data, which records and reports detailed annual data from 2004 to 2018. Analysts collect these data from various sources, including 10-Q, 10-K filings, annual reports, investor presentations and earnings calls. Since the data company gradually collects the data for international linkages, our primary empirical analysis thus focuses on domestic linkages for consistency.[2]

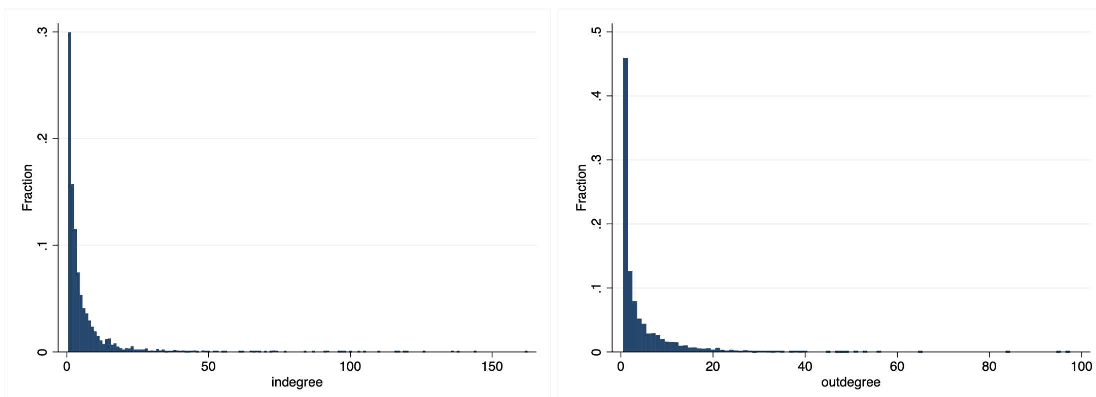

Figure 1.1: Sample Distribution of Indegree and Outdegree in Year 2012

Figure 1.1 shows the scale of the production network of US firms in 2012, as described by the FactSet data. Both the number of suppliers (in-degree) and the number of customers (out-degree) exhibit long tails, with some single firms having more than 150 suppliers or having more than 100 customers.

To construct a measure on whether a vertical relationship bears input specification, I leverage the additional layer of inter-firm relationship provided by

[2]The sample data includes almost 4,000 non-financial firms and more than 14,000 distinct vertical relationships in an average year.

FactSet Revere as it documents research collaboration partnerships between firms. That is, I term a vertical contract engages in input specification as long as the two parties also commit to investing in research and development in production. In an average year, the fraction of firms engaging in input specification with their suppliers reaches almost 50%. [3]

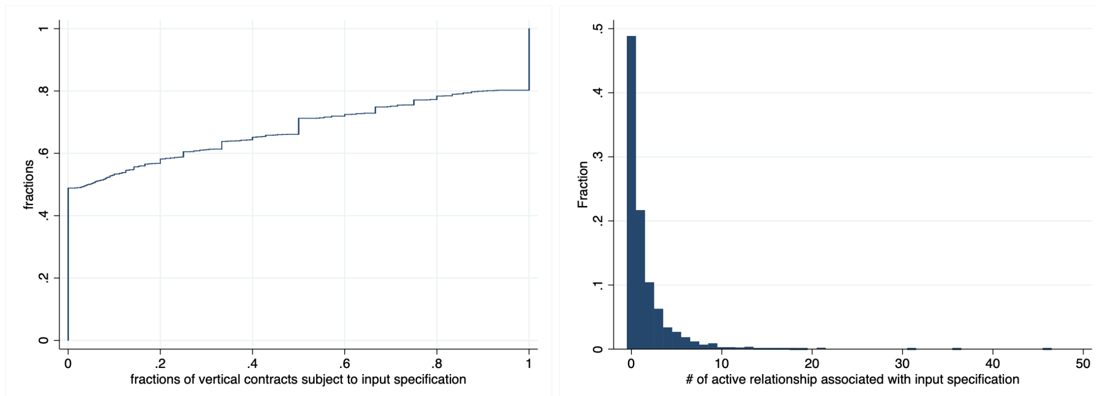

Figure 1.2: Input Specification Intensity in Year 2012

Figure 1.2 demonstrates both the intensive and extensive prevalence of vertical contracts associated with input specification. Conditional on having at least one contract associated with input specification, almost half of the firms implement input specification with all their suppliers. The distribution of the number of active vertical contracts subject to input specification also exhibits long-tail, demonstrating uneven demand or capability in sustaining such relationships across firms.

Given the observed vertical relationship landscape across US firms, two natural questions arise: (1) which are the firms investing in such vertical relationships associated with input specification; (2) what is the role of dynamics of vertical contracts for input specification in shaping overall in-degree and out-

[3]Conditional on engaging in input specification, less than 50% of suppliers in the data have multiple specification contracts.

degree dynamics across firms? To those ends, I first investigate the correlation between a firm's market share concerning its primary industry defined by the 4-digit SIC code and the relative engagement of vertical contracts associated with input specification to its industry level. As illustrated in Figure 1.3, there is a strong positive relationship between a firm's market share and its engagement in input specification, coinciding with the argument by Akcigit and Ates (2021) on close links between market concentration and resources concentration.

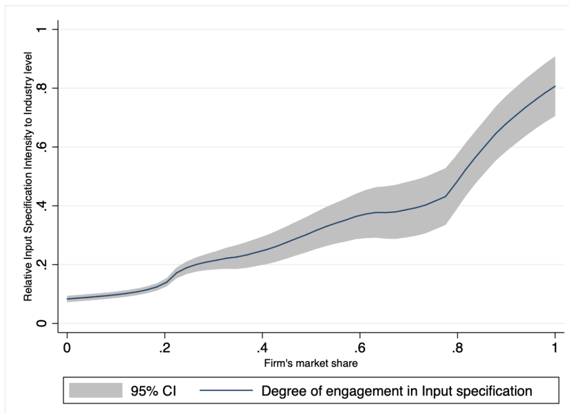

Figure 1.3: Correlation between Market Share and Input Specification Engagement

Second, to further stress how input specification can be a key factor causing the observed resource concentration, I investigate how the establishment of a vertical contract associated with input specification affects the other vertical linkage with the same supplier. Put it differently, I study whether vertical relationships are more likely to break after the supplier enters a vertical contract with some other customer for input specification. To this end, I take an event study approach and estimate the probit regression on the set of tuples consisting of upstream suppliers u $\times$ year t given that the year in which upstream u announces customized input contract with some downstream (not d) is denoted by T_u:

$$Pr(\text{LinkBreaks})_{ut} = \sum_{h=1}^{4} \beta_h 1\{t = T_u - h\} + \sum_{k=0}^{4} \beta_k 1\{t = T_u + k\} + \alpha_{ut} + \alpha_{i(u)} + \alpha_t + \epsilon_{ut} \tag{1.1}$$

where $Pr(\text{LinkBreaks})_{ut}$ is a probit function that takes one if and only if the vertical contracts between upstream u and its downstream customers are active at year t, but at least one of the contract is no longer active in year $t+1$. On the right-hand side of the equation, the first set of coefficients $\{\beta_h\}_{h=1}^{4}$ indicates the effect of suppliers entering a contract associated with input specification at year T_u on linkage status h year ahead of given year t. Similarly, $\{\beta_k\}_{k=0}^{k=4}$ captures the foreclosure effect from the year of the event to 4 years after the event. I include (i) fixed effects for the industry, $\alpha_{i(u)}$, which absorbs the industry-pair specific shocks that amount to the linkage separation, (ii) year fixed effect α_t taking out the impact from some year specific shocks that may affect linkage hazard rate, and (iii) time-varying upstream fixed effect.

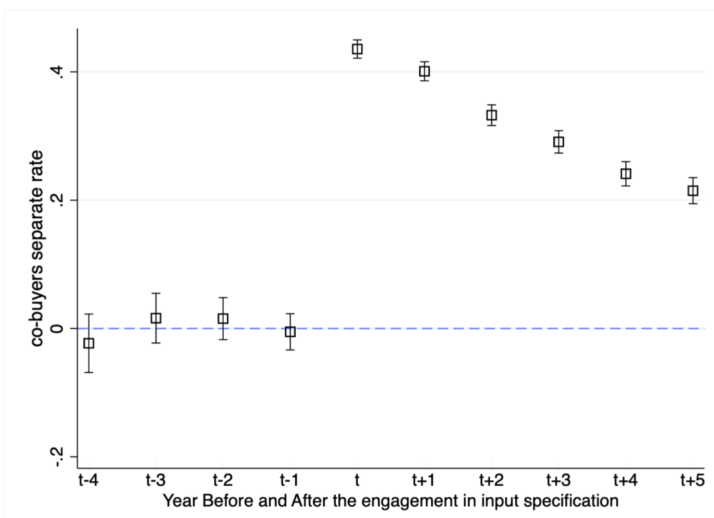

Figure 1.4: Foreclosure Effect of Input Specification

Figure 1.4 visualizes the estimated marginal effect of input specification engagement on linkage hazard rate for other downstream firms sharing the same supplier.[4] The probability that a supplier separates from its other customers when it enters a vertical contract associated with input specification increases by more than 40%. Beyond the immediate impact, the foreclosure effect lasts through the subsequent years after the event, and there is no pre-trending.

One possible argument against the negative externality of input specification engagement is that upstream may benefit from the new technology and equipment invested by the downstream partner and thus attract new customers to compensate for the loss of previous customers. I implement the event study approach on the new customer acquisition probability with a similar specification.

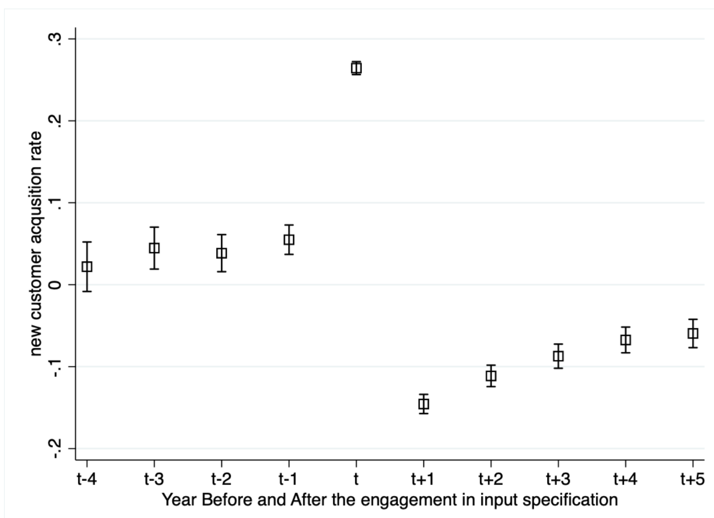

Figure 1.5: Foreclosure Effect of Input Specification Against New Customer Acquisition

Figure 1.5 clearly illustrates the decline in new customer acquisition after implementing input specification for some downstream customers at time t, consistent with the capacity constraint-induced resource concentration hypothesis.

[4]The separation rate is demeaned by taking out the fixed effect.

1.4 A Stylized Two-Period Model

The empirical evidence motivates us to study a theoretical framework that incorporates the foreclosure effect due to vertical contracts associated with input specification. This section introduces a simple two-period partial equilibrium model that demonstrates a key implication on firm and linkage dynamics after combining the specificity-induced competition for capacity with a frictional firm-to-firm match market.

For the simple exposition, I abstract from the firm's life-cycle and productivity growth and assume that the economy begins with one unit fixed mass of firms indexed by j. As in the production network literature, each firm is matched with some mass of suppliers and customers. [5]. Alternatively speaking, a firm can be viewed as an entity that consists of an upstream sector and a downstream sector. For the upstream sector of a firm, it can provide either common input components for up to K_c customer firms or specifically designed components for one particular customer firm. On the other hand, the downstream sector of a firm makes efforts to search for other firms to source components for production. I assume each firm can take a fixed mass of search efforts $\nu_c > K_c$ for common input components at each period. [6] Matching technology governing the meeting between buyers and sellers for common components follows the Leontief function:

$$\mathcal{M}_{c,t} = \max\{\mathcal{V}_{c,t}, \quad \mathcal{K}^*_{c,t}\}, \quad t \in \{1,2\}$$

where $\mathcal{M}_{c,t}$ is the mass of meetings between buyers and sellers for common

[5]A large body of literature presumes that each firm is serving both upstream and downstream simultaneously in a production network setting. See Acemoglu et al. (2018) and Demir et al. (2021).

[6]One can view such a search effort as either costless or sunk. Endogenizing search efforts in a symmetric environment does not alter the implications.

components in period t and $\mathcal{K}^*_{c,t}$ is the total mass of available capacities posted by firms in the economy in period t while $\mathcal{V}_{c,t}$ is the total mass of searching efforts taken by firms in the economy for common components in period t. Denote by q_c the rate at which each unit of searching effort yields a contact with an open capacity of common capacity,

$$q_{c,t} = \min\{\frac{\mathcal{K}^*_{c,t}}{\mathcal{V}_{c,t}}, 1\}$$

The mass of common component capacities that a firm contact after taking ν_c mass of search efforts in period t is hence $\nu_c q_{c,t}$ and common for all firms by construction. I assume each successfully signed contract for common components yields exogenous joint surplus at π, which implies sourcing common components demonstrates a constant return to scale in the mass of such contracts.

Each firm enters with an idea with low quality and high quality at the rate of $(1 - h)$ and h for their downstream product, respectively, and the quality of its idea is then fixed. Regardless of idea quality, each firm can source common components by making efforts to search for sellers as described above. However, only those firms with high-quality ideas are eligible to make additional efforts to search for and source from upstream for customized components. The marginal joint surplus of a contract for a customized component is $z \cdot \pi$, which is assumed to be greater than $K_c \cdot \pi$ to avoid trivial cases. Moreover, I assume the matching technology for the customized components market is also Leontief for simplicity.

Now I layout the timing of the economy. The economy in period 1 consists of three stages. Recall that, in the first period, there are no entry decisions to be made as the economy is initially endowed with one unit mass of firms.

[Period 1] Stage 1: Each firm exerts ν_c search efforts (costlessly) to find sellers for common components while each firm also posts K_c mass of capacities to attract buyers searching for common components.

[Period 1] Stage 2: Firms with high-quality idea further initiate searches for sellers and then propose a take-it-or-leave-it offer for customized components conditional on a successful meeting. The offer is high enough to induce foreclosure against the customer firms of the seller for common components contacted in the first stage.

[Period 1] Stage 3: Each firm signs the contracts which maximize their surplus and starts to produce and splits the surplus accordingly: (1) the upstream sector of a firm gets $\beta\pi$ of surplus for each common component contract; (2) upstream gets $\beta K_c \pi$ from implementing input specification.[7]

In the first period, all capacities on the supply side are initially unemployed, which implies $\mathcal{K}^*_{c,t} = K_c$, and the total mass of searching efforts in the economy follows $\mathcal{V}_{c,t} = \nu_c$ given the unit mass of firms in the first period is fixed at unity. This implies the mass of capacities a firm contact in the first stage is

$$q_{c,1} \cdot \nu_c = \min\{\frac{K_c}{\nu_c}, 1\} \cdot \nu_c = \frac{K_c}{\nu_c} \cdot = K_c$$

The probability at which a firm with a high-quality idea meets an upstream for specifically designed components is

$$q_{s,1} = \min\{\frac{1}{h}, 1\} = 1$$

This immediately follows that h fraction of upstream sectors of firms choose to implement input specification for firms with high-quality ideas and foreclosure

[7] I abstract from bargaining with incumbent common component customer firms. Therefore, a firm with a high-quality idea proposes exactly $\beta K_c \pi$ to match the outside option of the upstream.

of those buyers for common components contacted in the first stage. This implies that the ex-post mass of contracts for common components attained by each firm after the foreclosure in the first period is

$$q_{c,1} \cdot \nu_c \cdot (1-h) = (1-h)K_c$$

Consequently, the payoff of a firm with a low-quality idea in the first period is

$$\pi_1^l = \underbrace{(1-\beta)(1-h)K_c \cdot \pi}_{\text{surplus gain from adopting common components}} + \underbrace{\beta K_c \pi}_{\text{surplus gain from upstream sector}},$$

where the first term captures the surplus gains from sourcing common components after taking the foreclosure event into account, while the second term reflects the return to the upstream sector of a firm. [8]

$$\pi_1^h = \pi_1^l + \underbrace{(z - \beta K_c) \cdot \pi}_{\text{surplus gains from adopting specific components}}$$

The payoff of a firm with a high-quality idea earns additional return from adopting component specifications after compensating the seller for giving up the common component contracts.

The first period of economy sets up a stage for understanding how the landscape of firm-to-firm contracts for common and specific components can affect subsequent payoffs of firms and entry decisions. I layout the timing in the second period and assume there is no discounting factor.

[Period 2] Stage 1: Potential entrants determine whether to enter the economy by paying an entry cost r^e. Let the mass of entrants denote by $E \in (0,1)$.

[Period 2] Stage 2-4: Identical to Stage 1-3 from Period 1

[8] Given the absence of a bargaining game between incumbent buyers for common components contacted in the first period and a high-quality customer firm looking for component specification, a seller always receives $\beta K_c \pi$.

Although the actions taken by firms in the economy are almost identical to the period 1 except for the entry decision, the availability of upstream capacity has changed to $\mathcal{K}_{c,2}^{*} = E \cdot K_c$ as the capacities of incumbents are already filled up. Moreover, besides the entrants, incumbents are also competing for the capacity, which makes the competition for capacity more arduous. The rate at which a firm meets an open slot for common component provision is given by

$$q_{c,2} = \frac{K_c \cdot E}{(1+E) \cdot \nu_c} < q_{c,1}$$

On the other hand, the event of a successful meeting with an upstream for specific components for a buyer with a high-quality idea is guaranteed.

$$q_{s,2} = \min\{\frac{(1+E)}{h(1+E)}, 1\} = 1$$

Nevertheless, the ex-post contracting probability for specific components is lower as $\frac{h}{1+E}$ fractions of upstream in the economy at this stage are under component specification contracts already. When meeting an upstream subject to specification contract, there are no additional surplus gains for a potential buyer even if bidding/bargaining is allowed. Therefore, the ex-post contracting probability of a buyer with a high-quality idea for specific components is

$$\tilde{q}_{s,2} = \frac{1+E-h}{1+E} < 1$$

The foreclosure events following the acceptance of proposals for specific components in the second period results in a lower the ex-post mass of component contracts for each firm in the second period than that in the first period:

$$(1-h) \cdot \frac{K_c E}{1+E}$$

The increased contracting difficulties for both common and specific components therefore suggest that competition for capacity induced by input specification extends its impact on the subsequent sourcing environment, indicating a dynamic crowding-out effect.

Furthermore, another indirect channel of input specification on the sourcing cost is through affecting entry rate as the ex-post foreclosure effect endogenously links to the value of entry. To see this, firstly, note that the value of entry conditional on drawing a low-quality and high-quality idea in the second period are

$$V_e^l = \underbrace{(1-h) \cdot \frac{K_c E}{1+E} \cdot (1-\beta)\pi}_{\text{surplus gains from adopting common components}} + \underbrace{\beta K_c \pi}_{\text{surplus gains from upstream sector}}$$

$$V_e^h = V_e^l + \underbrace{\frac{1+E-h}{1+E}(z-\beta K_c)}_{\text{surplus gains from adopting specific components}}$$

Regardless of an entry's idea quality, its value increases in the entry rate as there is a positive externality due to the network effect: new entrants are not only customers but also component suppliers. More inflow of entrants brings more capacities to the economy, which ultimately can ease the supply shortage for both common and customized components. Nevertheless, the network effect is not internalized by entrants, and thus the incentives of entry can be adversely affected by a high intensity of conduct of input specification.

Proposition 1: The value of entry is strictly increasing and concave in entry rate. Furthermore, if $h > \frac{1+E}{2}$, then the value of entry is strictly decreasing in h. That is, more frequent/prevalent conducts of input specification discourage entry.
Proof: See Appendix 1.A.2.

Therefore, a low entry rate discouraged by a high intensity of input specification further depresses the value of entry through the network effect, lowering the entry rate further. To close the simple economy with a stable solution, I parameterize the entry cost r^e as a function of E. In particular, I assume $r^e = r \cdot E$, and there exists a set of parameters admitting a stable equilibrium where $h > \frac{1+E^*}{2}$.

Corollary 1: If $r^e = r \cdot E$, there exists a $r > 0$ such that there exists unique solution $E^* \in (0, 1)$ with $h > \frac{1+E^*}{2}$. Furthermore, in this case, the entry rate is strictly decreasing in h.
Proof: See Appendix 1.A.2.

Figure 1.6 summarizes the role of input specification in the two-period model. A higher fraction of firms with high-quality ideas shifts down the curve of entry value, capturing a dynamic crowding-out effect. The upward-sloping value of entry due to a network effect further leads to a lower equilibrium entry.

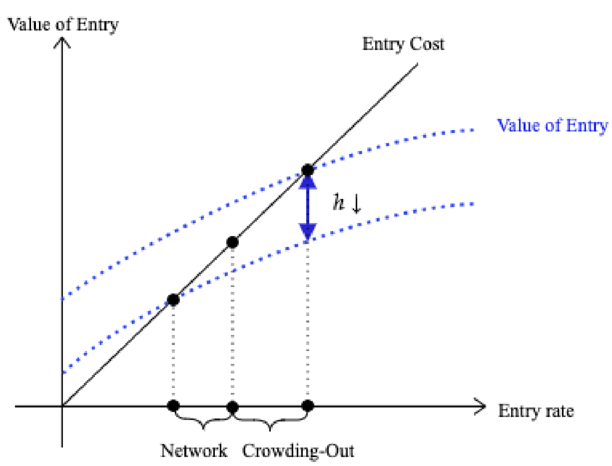

Figure 1.6: Entry and Input Specification

Discussion: The primary assumption $h > \frac{1+E}{2}$ can be supported empirically as the average entry rate is around 8% while almost half of the innovative publicly listed firms are engaging in input specification. Another assumption on the absence of bargaining ensures that the expected return to the upstream sector of a firm is independent of the idea draw h. In the Appendix 1.A.1, I characterize an extension of the two-period framework by allowing bargaining between buyers with high-quality ideas and buyers of common components and argue that the main implications of the theory can be supported by imposing additional assumptions on the premium of conducting specification. Regardless of whether parameter values are such that $h > \frac{1+E}{2}$ holds, the stylized example sheds enough light on the implication of input specification in affecting firm dynamics. Notice that at the end of period 2, the fraction of upstream capacities exploited by input specification increases to $h + \frac{h(1-h)}{1+E}$ which is decreasing in entry rate. Holding other constant, a lower entry rate in period 2 further aggravates the dynamic crowding-out effect as more fractions of incumbent capacities are already in input specification contracts, which dampens the expected return from exercising input specification and thus decreases the expected value of entry in the next period. [9] The dynamic crowding-out effect further invokes a network effect that refers to a deterred inflow of capacities, leading to an even lower entry value. Carrying such insights, I further embed them into a more complete and general framework that allows for a richer life-cycle of firms and endogenous growth along an infinite horizon.

1.5 The General Framework

Now I move to construct a more general dynamic framework by incorporating the novel features of vertical contracts associated with input specification and study its aggregate implication along the infinite horizon. Specifically, this sec-

[9] This implies that the value of entry is shifted down by a lower entry rate of the previous period, demonstrating the same implication as shown in Figure 1.6

tion outlines a rich equilibrium model of the life-cycle of firms with three main features. The first is a firm dynamics model of technology diffusion in the spirit of Perla and Tonetti (2014), admitting endogenous growth together with entry dynamism. The second is a firm-to-firm matching embedding heterogeneous premium of input specification close to Demir et al. (2021). Lastly, the model adopts the classic IO theory on foreclosure, borrowing from Bolton and Whinston (1993).

1.5.1 Time and Demand

Time is continuous, and its horizon is infinite. The demand side of the economy is summarized by a simple utility function of a representative consumer (household):

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \ln C(\tau) d\tau.$$

The utility function is the present discounted value of the instantaneous utility of consuming the final good. The discount rate is $\rho > 0$, and the instantaneous utility takes the logarithmic form. A competitive final goods producer transforms the final consumption good from an aggregate bundle of manufacturing varieties called manufacturing composite.

Consumers supply labor to manufacturing firms for the production of manufacturing varieties. Labor is supplied inelastically, and the total mass of labor is fixed at $\overline{\mathcal{L}}$. Consumers own a diversified portfolio of all operating firms and lands. Therefore the total consumer income consists of net surplus from firms, rents of land, and total wages:

$$P_{final}(t)C(t) \leq W(t)\bar{L} + R(t)\overline{K} + P_{final}(t)\bar{\Pi}(t)$$

where $P_{final}(t)$ is the price of homogeneous final good. $\bar{\Pi}(t)$ summarizes aggregate firms' static surplus that nets the rent of land, entry cost, and fixed cost associated with vertical relationships along the supply chain in the unit of final good. Lastly, $R(t)\overline{K}$ is the total revenue from leasing the total mass of land (normalized to unity), $\overline{K} = 1$, for business usage in the economy in the unit of the final good.

1.5.2 Final Good Producer

Supply Side. The supply side comprises two sectors: the service/retail sector and the manufacturing sector. The service sector is perfectly competitive and serves to transform manufacturing composite into homogeneous final goods. In order to underscore the implication of rich interactions and dynamic linkages among manufacturing firms, I simply treat the sector as a representative competitive final good producer with simple linear technology without involving labor:

$$\mathcal{Y}_{final}(t) = \mathcal{Q}^{M}_{final}(t)$$

where $\mathcal{Q}^{M}_{final}$ is the composite bundle that aggregates manufacturing varieties produced by heterogeneous manufacturing firms used for final good production. Furthermore, given the simple linear technology and perfectly competitive final good, the zero-profit condition implies that the price of final good is equal to its cost:

$$P_{final}(t) = P^{M}(t)$$

1.5.3 Manufacturing Firms

The manufacturing sector is populated by some mass of heterogeneous firms involved in the production of manufacturing varieties. Each manufacturing firm participates in the production of its own manufacturing variety and the production of other firms' varieties through two distinct channels. First, each manufacturing firm uses some manufacturing composite that aggregates manufacturing varieties produced by an entire set of operating firms per instant. Second, each manufacturing firm further demands productivity-enhancing technological components supplied by other manufacturing firms. The first channel captures those less directed sourcing relationships, e.g., procurement from wholesalers and ready-to-assemble manufacturing inputs. The latter reflects a relatively more pair-specific relation, which plays a specific role in enhancing productivity or quality in variety production. The production function of a manufacturing variety, therefore, takes a nested Cobb-Douglas form to reflect such inter-connectedness across firms which is jointly shaped by a firm's demand for manufacturing composites and technology-embodied components:

Production Function of a given manufacturing variety:

$$\boldsymbol{y}_j\big(Z_j(t),\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi,\boldsymbol{\mathcal{A}}_j(t))\big) = Z_j(t)\bar{\epsilon}\left[\bar{\alpha}\,\mathcal{Q}_j^M(t)^{\alpha}L_j^M(t)^{1-\alpha}\right]^{\epsilon}\cdot\left[\boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)})\cdot\int_0^{\boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)})}\boldsymbol{x}_{j\omega(j')}(t)d\omega(j')\right]^{1-\epsilon},$$

$$\textit{with}\quad \boldsymbol{\mathcal{H}}(\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}) \equiv \boldsymbol{\mathcal{H}}\big(\textstyle\sum_{j'\in\mathcal{J}_j(t)} n_{j'}\big(\varphi, A_{jj'}(t)\big)\big),\quad \bar{\alpha} = [\alpha^{\alpha}(1-\alpha)^{1-\alpha}]^{-1},\quad \bar{\epsilon} = [\epsilon^{\epsilon}(1-\epsilon)^{1-\epsilon}]^{-1}$$

where, formally speaking, the production of a variety j at a given time t bears a time-evolving Hicks-neutral baseline technology $Z(t)$ adopted upon the firm's entry and takes three factors: manufacturing composite $\mathcal{Q}_j^M(t)$, labor demand $L_j^M(t)$ and a set of technological components $\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}$ which is further structured as a set of vertical technological tasks indexed by $\{j\omega(j')\}$ with its production level $\{x_{j\omega(j')}(t)\}_{\omega(j')\in\{\mathcal{N}_{\mathcal{J}_j}(t)\}}$ conducted by some distinct firms $\{j'\}_{\{j'\in\mathcal{J}_j(t)\}}$. [10]

For instance, the firm j has established vertical contracts with a set of firms (suppliers) $\mathcal{J}_j(t)$ and asks each $j' \in \mathcal{J}_j(t)$ to perform a corresponding set of technological tasks $\{j\omega(j')\}$. The entire mass of tasks given $\mathcal{J}_j(t)$ depends on the complexity level of each technological component, $\{n_{j'}(\varphi, A_{jj'}(t))\}_{j'\in\mathcal{J}_j(t)}$ with $\{A_{jj'}\}_{j'\in\mathcal{J}_|}(t) \equiv \boldsymbol{\mathcal{A}}_j(t)$. Given a stock of vertical contracts, a firm organizes the complexity of each partner and design tasks to assign to them in the form of $\{j\omega(j')\}$ via a task organizing function. Specifically, I assume a common task organizing technology $\boldsymbol{\mathcal{H}}(\cdot)$ that maps the aggregate complexity across relational vertical linkages to effective unit mass of tasks.[11] The lower bound of complexity is normalized to be 1. It follows $\boldsymbol{\mathcal{H}}(0) = 1$, which corresponds to the case where a manufacturing firm has not yet established any relation for technological components.

Production function of a given task $j\omega(j')$: the production level of a given task indexed by $j\omega(j')$ is [12]

$$\boldsymbol{x}_{j\omega(j')}(t) = \bar{\alpha}\,\mathcal{Q}^M_{x_{j\omega(j')}}(t)^{\alpha} L_{x_{j\omega(j')}}(t)^{1-\alpha}.$$

Production takes manufacturing composite and labor as input factors without heterogeneity as pair-wise states are absorbed by the complexity degree. This follows that the unit production cost of any task by any $j' \in \mathcal{J}_j(t)$ at any t is symmetric, while different vertical contracts owned by firm j may receive different unit mass of tasks.

[10] The task-based setting is built on canonical work of Autor et al. (2003), Acemoglu and Zilibotti (2001), Acemoglu et al. (2007), Bernard et al. (2019) among others.

[11] I introduce $\mathcal{H}(\cdot)$ in front of the integral to ensure the output is linear in $\mathcal{H}(\cdot)$ following Benassy (1998) and Acemoglu et al. (2007).

[12] Notice that the production of technology components is abstract from the firm's technological class Z for simple exposition. The analytical result remains the same when allowing technological class-dependent production function: $x(t) = Z\alpha\mathcal{Q}^M(t)^{\alpha}L(t)^{1-\alpha}$

Determination of complexity and its mass of tasks: the complexity of a technological component produced by firm j' for firm j, $\{n_{j'}(\cdot)\}_{j' \in \mathcal{J}_j(t)}$, is characterized by two states. First, it depends on a pair-wise state of the relational contract is subject to $A_{j\omega(j')}(t) = A_{jj'}(t)$, which summarizes the degree of specificity intensity over any given technological task ω that is chosen by the pair. A higher specificity level means greater complexity and implies not only a higher surplus but also greater pressure on the supplier's capacity. The set of specificity degrees denotes by $\mathbf{A}$ takes finite discrete values for simplicity. Second, how complex a technological component is produced depends on downstream firm's design/organization ability, which is captured by φ, a firm-specific but time-invariant draw from an exogenous distribution $F(\varphi)$ with support $[\underline{\varphi}, \overline{\varphi}]$. It captures the capability of a given firm in implementing the technological task, which plays a key role in shaping the ex-post heterogeneity and dynamics of firm sizes and vertical relationships. Throughout the paper, I interpret φ as a measure of an entrepreneur's ability that allows a firm to better organize resources within the firm and across the supply chain so that it can conduct more complex specific tasks and achieve a greater return. Alternative interpretations in related literature include efficiency in deploying and pooling intangibles along the vertical relationship and advantage in relational linkages creations due to greater commitment power, lower financial default risk, or business culture of collaboration. [13]

Contracts for Technological Components: The task-based sourcing for technological components involves contracts that settle the transaction terms as the tasks need to be specified at the pair level per technological component. For the simple exposition, I assume away distortion from hold-up and that firm j obtain

[13]See De Ridder (2019) and Gibbons and Henderson (2011).

$(1-\beta)$ share of ex-ante **net surplus**.[14] This implies the two parties jointly agree with choosing $x_{j\omega(j')}$ at its marginal cost:

$$P^x(t) = (P^M(t))^\alpha W(t)^{1-\alpha} \tag{1.2}$$

Ultimately, the manufacturing firm takes the composite cost P^M, wage W and unit cost of task P^x as given, and thus yields the marginal cost of production of a variety j as:

$$MC_j\big(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))\big) = \Big[Z_j(t) \cdot \big[\boldsymbol{\mathcal{H}}\big(\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big)\big]^{\frac{1}{1-\epsilon}}\Big]^{-1} C$$

where $C \equiv W^{(1-\alpha)}(P^M)^\alpha$ summarizes the unit cost of input factors in variety production and $Z \cdot \big[\boldsymbol{\mathcal{H}}\big(\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big)\big]^{\frac{1}{1-\epsilon}}$ captures the ex-post productivity of firm j in variety production. It naturally demonstrates how the dynamics of stock of vertical contracts for technological components generate firm's expansion and contraction over time. In order to obtain tractable derivation for analytical insights, I assume the mapping from effective unit of distinct technology components (aggregate complexity given the stock of vertical contracts) to the mass of tasks follow specific power rule:

$$\boldsymbol{\mathcal{H}}\big(\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big) = \Big[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\Big]^{\frac{(1-\epsilon)}{(\sigma-1)}} \tag{1.3}$$

The marginal cost, therefore, amounts to

$$MC_j\big(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))\big) = \Big[Z_j(t) \cdot \big[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]^{\sigma-1}\Big]^{-1} C \tag{1.4}$$

[14] Recall that hold-up problem can emerge when either party can re-negotiate surplus split ex-post, which distorts ex-ante input demand. Nevertheless, such distortions do not alter our implications.

Solve Firm's Static Problem

Manufacturing composite is a standard CES bundle of manufacturing varieties:

$$\mathcal{Q}^M(t) = (\int_{\mathcal{J}_t^M} \boldsymbol{y}_j(t)^{\frac{\sigma-1}{\sigma}} dj)^{\frac{\sigma}{\sigma-1}} \tag{1.5}$$

The parameter $\sigma > 1$ governs the elasticity of substitution across input varieties. $\mathcal{J}_t^M$ is the set of manufacturing varieties at time t. The real revenue of a firm from supplying varieties with technology Z is

$$[\frac{p_j(t)}{P^M(t)}] \cdot \left[(\frac{p_j(t)}{P^M(t)})^{-\sigma}\overline{\boldsymbol{\mathcal{D}}}(t)\right] \tag{1.6}$$

where $\overline{\boldsymbol{\mathcal{D}}}$ is the total demand for manufacturing composites, which is further composed of demand from manufacturing firms, final good producers and total capital cost Ξ including entry cost, rent for lands and vertical relation associated fixed-cost:

$$\overline{\boldsymbol{\mathcal{D}}}(t) = \boldsymbol{\mathcal{D}}_M(t) + \boldsymbol{\mathcal{D}}_{final}(t) + \Xi(t) \tag{1.7}$$

Given the constant markup emerged from the CES bundling, the price of each manufacturing variety j is

$$p_j(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))) = \overline{\sigma} \cdot MC_j\big(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))\big) \tag{1.8}$$

where $\overline{\sigma} = \frac{\sigma}{\sigma-1}$. Together with equation (1.6), the ex-post output $\boldsymbol{y}_j^*$, the revenue $\mathcal{R}_j$ and the static surplus of manufacturing variety j given $Z_j(t)$ and technology component profile $\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))$ are:

$$\boldsymbol{y}_j^*(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t)))) = \overline{\sigma}^{-\sigma}\left[Z_j(t)^{\sigma-1}\big[1 + \textstyle\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]\right](\frac{P^M(t)}{W(t)})^{\sigma(1-\alpha)}\overline{\boldsymbol{\mathcal{D}}}(t) \tag{1.9}$$

$$\mathcal{R}_j(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t)))) = \overline{\sigma}^{1-\sigma}\left[Z_j(t)^{\sigma-1}\big[1 + \textstyle\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]\right](\frac{P^M(t)}{W(t)})^{(\sigma-1)(1-\alpha)}\overline{\boldsymbol{\mathcal{D}}}(t) \tag{1.10}$$

$$\Pi_j(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t)))) = \frac{1}{\sigma} \cdot R_j(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t)))) \tag{1.11}$$

Given the specific imposition on the functional form of $\mathcal{H}(\cdot)$, firm's output level, revenue, static surplus from variety production is linear in $\big[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]$. In other words, the complexity degree $n'_j(\varphi, \boldsymbol{\mathcal{A}}(t))$ of a technology component served by given firm j' demonstrates constant return to scale:

$$\partial_{n_{j'}} \Pi(Z(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t)))) = \tfrac{1}{\sigma}\overline{\sigma}^{1-\sigma} Z_j(t)^{\sigma-1}(\tfrac{P^M(t)}{W(t)})^{(\sigma-1)(1-\alpha)}\overline{\boldsymbol{\mathcal{D}}}(t) \equiv \Pi(Z(t)) \tag{1.12}$$

Taking $\overline{\boldsymbol{\mathcal{D}}}(t)$ as given, firm j's total demand for manufacturing composite at time t is given by

$$\begin{aligned}\overline{\mathcal{Q}}_j^M(Z_j, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))) = & \underbrace{\mathcal{Q}_j^M(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\cdot))}_{\text{composite demand for variety production}} + \underbrace{\int_0^{\boldsymbol{\mathcal{H}}\left(\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, A_{jj'}(t))\right)} \mathcal{Q}^M_{\boldsymbol{x}_{j\omega(j')}}(t) d\omega}_{\text{composite demand for tasks}} \\ &= \alpha\overline{\sigma}^{-\sigma}\Big[Z_j(t)^{\sigma-1}\big[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]\Big](\frac{P^M(t)}{W(t)})^{(\sigma-1)(1-\alpha)}\overline{\boldsymbol{\mathcal{D}}}(t).\end{aligned} \tag{1.13}$$

The total demand for manufacturing composite from manufacturing firms therefore amounts to:

$$\boldsymbol{\mathcal{D}}_M(t) = \int_{\mathcal{J}_t^M} \overline{\mathcal{Q}}_j^M\big(Z_j, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t), t)\big) dj \tag{1.14}$$

Similarly, firm j's total demand for labor at given time t is:

$$\begin{aligned}\overline{L}_j(Z_j, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))) = & \underbrace{L_j^M(Z_j(t), \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\cdot))}_{\text{labor demand for variety production}} + \underbrace{\int_0^{\boldsymbol{\mathcal{H}}\left(\sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, A_{jj'}(t))\right)} L^M_{\boldsymbol{x}_{j\omega(j')}}(t) d\omega}_{\text{labor demand for tasks}} \\ &= (1-\alpha)\overline{\sigma}^{-\sigma}\Big[Z_j(t)^{\sigma-1}\big[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t))\big]\Big](\frac{P^M(t)}{W(t)})^{(\sigma-1)(1-\alpha)}\overline{\boldsymbol{\mathcal{D}}}(t).\end{aligned} \tag{1.15}$$

Capital/Fixed Costs: Each manufacturing firm must incur a fixed-cost in the unit of final good at quantity R^e upon entry. Conditional on the entry cost, each firm is further subject to rent per instant for a piece of land to operate. The supply of land is restricted and normalized to be unity. The land market is assumed to be competitive, implying that the landlords (owned by households) set the rent of land at the value of entry. Let the rent at given time t denote by $R(t)$. Since the total supply of land is fixed at unity, the aggregate cost of land rent is

$$\int_{\mathcal{J}_t^M} R(t) = R(t). \tag{1.16}$$

In addition to payments to landlords for operation, each firm pays the fixed cost in the unit of final good to establish and then maintain the linkages with other firms for technological components. The fixed cost of a given vertical contract jj' depends on the mass of tasks assigned at the pair level which therefore is denoted by $R^M_{jj'}(n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t)))$. Thus the aggregate fixed cost for technology components is given by

$$\int_{\mathcal{J}_t^M} \sum_{j' \in \mathcal{J}_|(t)} R^M_{jj'}(n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t)))dj \tag{1.17}$$

Static Aggregation and Market Clearing: Given equation (1.5) and (1.6), one can derive the aggregate price index of manufacturing composite:

$$P^M = \left(\int_{\mathcal{J}_t^M} p_j^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \tag{1.18}$$

Manufacturing composite market clearing condition implies aggregate demand must be identical with aggregate supply:

$$\overline{\boldsymbol{\mathcal{D}}}(t) = \int_{\mathcal{J}_t^M} \boldsymbol{y}_j^* dj \tag{1.19}$$

Using equation (1.7), (1.14) and representative household's budget constraint, this implies the final good market clearing condition must satisfy :

$$C(t) = \boldsymbol{\mathcal{D}}_{final}(t) + R(t)\overline{K} = \int_{\mathcal{J}_t^M} \boldsymbol{y}_j^* dj - \int_{\mathcal{J}_t^M} \overline{\mathcal{Q}}_j^M (Z_j, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t), t))dj - \Xi(t) + R(t)\overline{K} \tag{1.20}$$

where the cost of land is equal to the lease revenue of households, and the aggregate fixed cost incurred in the economy at time t is

$$\Xi(t) = R(t)\overline{K} + \int_{\mathcal{J}_t^M} \sum_{j' \in \mathcal{J}_|(t)} R_{jj'}^M(n_{j'}(\varphi, \boldsymbol{\mathcal{A}}(t)))dj + E(t) \cdot R^e(t) \tag{1.21}$$

Lastly, the labor market clears using equation (1.19)

$$\int_{\mathcal{J}_t^M} \overline{L}_j dj = \overline{\mathcal{L}} \tag{1.22}$$

Firm Dynamics

Given the static surplus function, the set of fixed costs associated with linkage management and rents and a perceived law of motion for the operating technology distribution and linkages creation and churning, each firm face two folds of dynamic problem: (1) match and search for new suppliers for technological components and (2) timing to exit the market. Since the frictions in the technological component market affect the present discounted expected value of an operating firm, it is a factor in a firm's exiting strategy. On the other hand, the exit and entry dynamics generate fluctuations which in turn affect the extent of matching frictions. Overall, the solutions to the two problems are endogenously determined. To better layout the mechanism, I begin by separately introducing the two essential elements: the search and match environment and entry/exit dynamics, as follows.

Technological components linkages dynamics: As briefly introduced in the two-period model, firms expand through accumulating technological components by searching for others to install technology components. Recall that a given set of technological components that firm j adopts at time t, $\boldsymbol{\mathcal{N}}_{\mathcal{J}_j(t)}(\varphi, \boldsymbol{\mathcal{A}}(t))$, amounts to $\left[1 + \sum_{j' \in \mathcal{J}_j(t)} n_{j'}\left(\varphi, \boldsymbol{\mathcal{A}}(t)\right)\right]$ mass of tasks. Thanks to the simple linear form, the firm's heterogeneity can be easily attributed to technology Z and the portfolio of productivity-enhancing technological components. The heterogeneity that emerged from a stock of vertical linkages for technological components can be further grouped into an extensive and intensive margin. The heterogeneous number of linkages corresponds to the extensive margin, while the degree of specificity at the linkage level reflects the intensive margin.

Search and match (Extensive Margin): The number of linkages to source technological components that a manufacturing firm manages is evolving subject to search and match frictions intertwined with capacity competitions and firms' life cycles. Following the search and match literature, I assume that each manufacturing firm takes search efforts to purchase feasible technological components. In particular, I assume a given manufacturing firm with technology $Z(t)$ can only acquire technological components from other manufacturing firms operating with the same technology class $Z(t)$ at any given time t. Furthermore, conditional on technology $Z(t)$, each firm searches for upstream suppliers in two sub-markets: (1) market of vertically specific components and (2) market of common components. Let the firm-level mass of search efforts exerted in specific and common component markets be exogenous and denoted by ν_s and ν_c, respectively. The aggregate amounts of ads posted in both markets are given by

$$\mathcal{V}_s(Z,t) = \int_{j\in\mathcal{J}(Z,t)} \nu_s(Z,t)dj, \quad \mathcal{V}_c(Z,t) = \int_{j\in\mathcal{J}(Z,t)} \nu_c(Z,t)dj$$

where $\mathcal{J}(Z,t)$ is the set of operating manufacturing firms with technology $Z(t)$. One the other hand, let the mass of the available capacity associated with specific and common component provisions by firms with technology (Z,t) denote by $\mathcal{K}_s(Z,t) = \int_{j\in\mathcal{J}^s(Z,t)} K_s(Z,t)dj$ and $\mathcal{K}_c(Z,t) = \int_{j\in\mathcal{J}^c(Z,t)} K_c(Z,t)dj$, where $\mathcal{J}^s(Z,t)$ and $\mathcal{J}^c(Z,t)$ are the set of firms with available capacity for input specification and common components with technology (Z, t) respectively. I define the market tightness for specific and common technological components at technology $Z(t)$ is defined by the average search efforts exerted by the customer firm per capacity: $\theta_s(Z, t) = \frac{\mathcal{V}_s(Z,t)}{\mathcal{K}_s(Z,t)}$ and $\theta_c(Z, t) = \frac{\mathcal{V}_c(Z,t)}{\mathcal{K}_c(Z,t)}$.

The market for specificity is subject to standard search frictions with a Cobb-Douglas matching function $\mathcal{M}^s(\mathcal{K}_s, \mathcal{V}_s) = \chi \mathcal{K}_s^{\xi} \mathcal{V}_s^{1-\xi}$. The equilibrium rate at which a manufacturing firm operating at technological class $Z(t)$ meets a potential customer firm with the same $Z(t)$ for specific technological components, $\lambda_s(Z, t)$, as well as the equilibrium rate at which a searching customer firm with search effort ν_s for component specificity meets a manufacturing upstream at same $Z(t)$, $\tilde{q}_s(Z, t)$, are given by

$$\lambda_s(Z, t) = \chi \theta_s^{1-\xi}(Z, t), \quad \tilde{q}_s(Z, t) = \nu_s \cdot \chi \theta_s^{-\xi}(Z, t) \equiv \nu_s q_s(Z, t) \tag{1.23}$$

In addition, upon the meeting, a match-specific shock is realized and informs both parties whether the match is good or not. The conditional probability of having a good match is fixed at a rate of Δ_g^s. If the match is good, two parties sign the contract and incur a fixed cost per instant to maintain the relationship at $R^M(\varphi, A_{jj'}(t))$ in the unit of the final good.

The market for common components is distinct from its counterpart as the matching function is assumed to be Leontief, capturing the differential matching frictions between the two markets. An upstream supplier can always find a match immediately once posting an available capacity for the common component market. On the other hand, a manufacturing firm with search effort ν_c

finds a vacant common component capacity at a rate of $q^c(Z,t)$:[15]

$$\tilde{q}_c(Z,t) = \nu_c \frac{\mathcal{K}_c(Z,t)}{\mathcal{V}_c(Z,t)} \equiv \nu_c q_c(Z,t) \tag{1.24}$$

Degree of Specificity (Intensive Margin): To recall, the extent to which a given technological component produced by manufacturing firm j' complements customer firm j is summarized by pair-wise specificity intensity $A_{jj'}$ and customer firm's ability φ_j. The two-party joint agrees on a choice of specificity intensity which can take three possible values: common, medium, and high intensity $A_{jj'} \in \{c, s_{med}, s_{high}\}$. The return of sourcing technological components is increasing in specificity intensity, and the complementarity between the return and specificity intensity is increasing in the customer firm's ability.

$$n_{j'}(\varphi, c) = 1; \quad n_{j'}(\varphi, s_{med}) = \varphi^{\eta_m}; \quad n_{j'}(\varphi, s_{high}) = \varphi$$

The specificity choice is closely related to the capacity and demand for technological components. On the supply side, each manufacturing firm is capacity-constrained and confronted with options between either focusing on serving common technology components or vertically specific technology components, which is the cause of capacity bidding among manufacturing firms. To be in line with the firms' life-cycle and empirical evidence, each manufacturing firm face learning shock and can become "mature" and recover from capacity shortage *partially* at Poisson rate of κ ever since entering a relational contract for a specific component. A mature upstream can choose to supply common components at the possible expense of discounting returns from serving specific components simultaneously. The extent of discounting is governed by η_m. If $\eta_m = 0$, then

[15] I assume that the customer firm always seeks for vacant capacity. This is equivalent to the scenario where I allow for meeting with occupied capacity. The reason is that the return from sourcing common components is homogeneous across firms which means Bertrand competition eliminates surplus for customer firms.

it implies there is no room for a firm to expand capacity for common components while producing specific components simultaneously. If $\eta_m = 1$, once a manufacturing firm becomes seasoned in serving specific components, it can serve customer firms for specific and common technological components without trade-offs. Therefore, the discount can also be interpreted as the strength of capacity exploitation of input specificity, which in turn affects the persistence of the foreclosure effect.

Linkage churnings: Each linkage associated with technological components can break either endogenously or exogenously. Serving specific components exploits the firm's capacity, which forecloses the other incumbent contracts associated with less surplus, such as common technology components. Without losing generality, I assume each manufacturing firm is at most able to serve one customer firm for specific components. That is, I set $K_s = 1$ for all j. If a firm under contract to provide either common or specific technological components meets a new customer firm, the current and new customer firm engages in Bertrand competition for the capacity for the technological component.[16]. This competition is won by the customer(s) with a higher joint surplus of the match, and the second-highest value becomes the firm outside option in a new alternating offers game with the winning customer. I follow Postel-Vinay and Robin (2002) in assuming there is no further renegotiation when the customer firm fully compensates the firm with its outside option. Owing to the limited capacity, firms always opt for the better match and separate with the relatively worse match.

Besides endogenous linkage separations due to capacity competition, each

[16]I allow upstream to switch between customer firms to implement specifications for tractability of the framework. The equilibrium rate of such a switch is controlled by Δ_g^s. An alternative model allowing for different match shock conditional on whether the upstream is subject to specification already or not generates little change in implications.

firm confronts with an exogenous Poisson churning shock of capacity retirement shock at δ and has not been able to provide technological components since then.[17]

Entry and Exit Dynamics: The model features dynamic entry and exit following the Schumpeterian paradigm. Each incumbent manufacturing firm is facing an optimal stopping problem as the re-incurring rent of land is increasing at faster pace along the balanced growth path in the model. Specifically, each incumbent firm faces exogenously evolving technology progress following simple geometric motion:

$$d \ln Z(t) = \mu dt, \quad \mu \geq 0$$

On the entry side, a new entrant, conditional on incurring the entry cost, draws $\varphi \sim F(\varphi)$ and adopts incumbent's operating technology $Z(t)$ from $\Phi(Z, t)$. Intuitively speaking, the entire technology distribution is moving forward at a faster pace as the pushing-forward force is not only backed by the natural progression rate μ but also fueled with the "leap" from the entrants. The faster-moving aggregate economy generates selection through the rent of land in the spirit of Melitz (2003) but in a dynamic fashion. To simplify the economy, I restrict the mass of firms to 1 by fixing the land supply to be unity. Therefore, a fixed land supply implies that the entry rate must coincide with the exit rate. I assume the land market is competitive, so each landlord chooses the rent as the value of entry given the free entry condition. Consequently, there exists $\underline{Z}(\varphi, t)$ such that a manufacturing firm with φ chooses to exit whenever its technology hits the threshold.

[17] Alternative types of exogenous churning shocks, such as those causing firms to directly exit, generates the same implications but with more complicated algebra.

Firm's Dynamic Problem: recursive formulation along balanced growth path

I now resume the characterization of the economy along the infinite horizon. I focus the exposition on a *balanced growth path* (BGP), in which relevant variables and all quantiles of the technology distribution $\Phi(Z,t)$ grow at the same endogenously determined rate g. That is, the distributions evolve as "traveling waves," shifting out over time while preserving their shape. Given the property of BGP, it is convenient to study a transformed, stationary version of the model. I normalize z to capture a firm's technology Z relative to the exit threshold. On a BGP, the exit threshold must grow at the rate of the economy, $d\ln z = (\mu - g)dt \equiv -\tilde{g}$, where μ is the exogenous progression (drift) of technology of incumbent firms and $\tilde{g}$ hence captures the endogenous rate of obsolescence.

Normalizing the Economy: Formally, I normalize the economy to be stationary by defining the following normalized, real, per-capita values (drop t):

$$z \equiv \frac{Z}{\underline{Z}}, \quad w \equiv \frac{W}{P^M \underline{Z}^{\frac{1}{1-\alpha}}}, \quad y_j \equiv \frac{\mathcal{Y}_j}{\overline{L}\underline{Z}^{\frac{1}{1-\alpha}}}, \quad \overline{D} \equiv \frac{\overline{\mathcal{D}}}{\overline{L}\underline{Z}^{\frac{1}{1-\alpha}}}, \quad \pi_j(Z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}}))) \equiv \frac{\Pi_j(Z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})))}{\overline{L}\underline{Z}^{\frac{1}{1-\alpha}} w}$$

Given the normalization, the explicit expressions of prices, labor demand, output of variety j are given by:

$$\frac{p_j(Z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})))}{P^M} = \overline{\sigma} \underbrace{\left[\left(1 + \sum_{j' \in \mathcal{J}_j} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}) \right)^{\frac{1}{\sigma-1}} \cdot z \right]^{-1}}_{\textit{ex-post normalized productivity}} w^{1-\alpha}$$

$$\frac{L_j(Z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})))}{\overline{L}} = (1-\alpha)\overline{\sigma}^{-\sigma} \left[\left(1 + \sum_{j' \in \mathcal{J}_j} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}) \right) \cdot z^{\sigma-1} \right] w^{-[\alpha + \sigma(1-\alpha)]} \overline{D}$$

$$y_j(z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})) = \overline{\sigma}^{-\sigma} \left[\left(1 + \sum_{j' \in \mathcal{J}_j} n_{j'}(\varphi, \boldsymbol{\mathcal{A}}) \right) \cdot z^{\sigma-1} \right] w^{-\sigma(1-\alpha)} \overline{D}$$

Define the normalized ex-post productivity as $\mathcal{Z}_j(z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})) = \Big[1 + \sum_{j' \in \mathcal{J}_j} n_{j'}(\varphi, A_{jj'})\Big]^{\frac{1}{\sigma-1}} \cdot z$, and let the distribution of normalized ex-post productivity denote by $\mathcal{F}(\mathcal{Z}, \boldsymbol{\mathcal{N}}(\varphi, \boldsymbol{\mathcal{A}}))$. It follows the normalized real wage satisfies:

$$w^{1-\alpha} = \frac{1}{\bar{\sigma}} \mathbb{E}_{\mathcal{F}}(\mathcal{Z})$$

The real normalized static surplus therefore can be expressed as a function of relative ex-post productivity:

$$\pi_j(z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})) = \pi(\mathcal{Z}) = \frac{1}{\sigma} \Big[\frac{\mathcal{Z}}{\mathbb{E}_{\mathcal{F}}(\mathcal{Z})}\Big]^{\sigma-1} \frac{\overline{D}}{w}$$

Using labor market clearing condition:

$$\int_j \overline{L}_j dj = (1-\alpha)(\overline{\sigma})^{-\sigma} [\mathbb{E}_{\mathcal{F}}(\mathcal{Z})]^{\sigma-1} w^{-[\alpha+\sigma(1-\alpha)]} \overline{D} \cdot \overline{L} = \overline{L}$$

This implies the normalized total demand for manufacturing composites is growing at the same speed as the normalized wage:

$$\frac{\overline{D}}{w} = \frac{\overline{\sigma}}{1-\alpha}$$

The normalized static surplus from producing variety of firm j can be further simplified to

$$\pi_j(z, \boldsymbol{\mathcal{N}}_{\mathcal{J}_j}(\varphi, \boldsymbol{\mathcal{A}})) = \frac{1}{(\sigma-1)(1-\alpha)} \Big[\frac{\mathcal{Z}}{\mathbb{E}_{\mathcal{F}}(\mathcal{Z})}\Big]^{\sigma-1} \equiv \pi_{min} \cdot [\mathcal{Z}]^{\sigma-1} = \pi_{min} \Big[1 + \sum_{j' \in \mathcal{J}_j} n_{j'}(\varphi, A_{jj'})\Big] z^{\sigma-1}$$

where $\pi_{min} \equiv \frac{1}{(\sigma-1)(1-\alpha)} \Big[\frac{1}{\mathbb{E}_{\mathcal{F}}(\mathcal{Z})}\Big]^{\sigma-1}$ represents the static surplus from variety production of the *marginal firm*. Given our normalization and entry/exit dynamics, a manufacturing firm with $z = 1$ and $\boldsymbol{\mathcal{H}}(0) = 1$ is the firm that is on the margin between keep operating or exiting.

Value Functions: I now formulate a set of key Bellman equations that characterize manufacturing firms' behaviors. I split the problem of manufacturing firm into that of variety production and technological component supply. Firstly, I characterize the value of match along the firm's life cycle as an upstream supplier for technological components.

Joint value of a vertical contract: When a firm is fresh/immature in the technological component provision, it initially can only supply common component at exogenously given capacity K_c as a default state, and the value of a match is given by

$$(\rho+\delta)J^0(z) = [\pi(z) - r^M(\varphi, c)] + (\mu - g)zJ^0_z(z) \tag{1.25}$$

$$\text{s.t.} \quad J^0(\underline{z}^c) = J^0_z(\underline{z}^c) = 0$$

where $r^M(\varphi,\cdot) \equiv \frac{R^M(\varphi,\cdot)}{\bar{L}\underline{Z}^{\frac{1}{1-\alpha}}w}$ is normalized fixed cost/perishable investment to incur necessary for the linkage maintenance. $\underline{z}^c$ is the productivity threshold at which the two parties agree to dissolve the common component contract. Recall that with some probability λ^s, a manufacturing firm will meet a customer firm seeking technology components. Following Postel-Vinay and Robin (2002), the customer firm bid against the incumbent firm, which is under common component sourcing contract. Consequently, the "poacher" offers $J^0(z)$ precisely, and the specification contract is signed, leaving the value of match unchanged.

When a firm is immature in technological component provision yet is under a specification contract at (z, φ), the net joint value is given by

$$(\rho+\delta)J^i(z,\varphi) = [\varphi\pi(z) - r^M(\varphi, s_{high})] + (\mu - g)zJ^i(z,\varphi) + \kappa[J^m(z,\varphi,s_{high}) - J^i(z,\varphi)] \tag{1.26}$$

$$\text{s.t.} \quad J^i(\underline{z}^i,\cdot) = J^i_z(\underline{z}^i,\cdot) = 0, \quad \forall\varphi$$

where $r^M(\varphi, s_{high})$ is the normalized unit cost for vertically specific contracting per instant. The last term of the bellman equation captures the value flow from maturity shock at rate κ.

Given the notion of maturity and the feature of common component market, a customer manufacturing firm seeking for common component has to compensate the incumbent value by $J^m(z,\varphi,s_{high}) - J^m(z,\varphi,s_{med})$, again leaving the value of match invariant:

$$(\rho+\delta)J^m(z,\varphi,s_{high}) = [\varphi\pi(z) - R^M(\varphi,,s_{high})] + (\mu-g)zJ^m(z,\varphi,s_{high}) \quad (1.27)$$

$$\text{s.t.} \quad J^m(\underline{z}^m,\cdot) = J^m_z(\underline{z}^m,\cdot) = 0, \quad \forall\varphi$$

Overall, given the bargaining protocol, the value of match does not change against maturity shock as

Lemma 1: $J^m(z,\varphi,s_{high}) = J^i(z,\varphi) \equiv J(z,\varphi)$, for all (z,φ).

Proof: See Appendix 1.A.2.

Joint value of a manufacturing firm as customer: To layout the dynamic problem of variety production, I now characterize the joint value of a manufacturing firm together with its portfolio of vertical contracts. Recall that the state space is summarized by the technology z, ability φ, and the technological component contract vector represented in the portfolio of complexity degree embodied $\boldsymbol{\mathcal{N}}_{\mathcal{J}_j}$. I do not pursue discrete choices between searching for the specific and common component at a given time. Each manufacturing firm can search for specific and common components simultaneously. Since the complexity vector demonstrates additive-linear return, it would be convenient to transform the vector as a three-point: $\{n^s(i), n^s(m), n^c\}$. Specifically, $n^s(i), n^s(m)$ represent the number of linkages maintained by φ-firm for sourcing specific components when the upstream partners are immature and mature, respectively. The number of linkages for common components held by such a firm is counted as n^c. Let the joint value of a downstream denote by $V(z,\varphi,n^s(i),n^s(m),n^c) \equiv V(z,\varphi,\vec{n})$. The marginal value of vertical contract for specific component with state of upstream at $a \in \{i, m\}$ is given by $V_{n^s(a)}(\cdot)$. Similarly, the marginal value of a common technology component contract is $V_{n^c}(\cdot)$.

The value inflow can be broadly categorized into three channels. First, a manufacturing firm obtains the baseline static surplus and capital gain from operating. Second, it includes the collection of the joint values of the current vertical contract portfolio. Lastly, as a customer firm, the third part is captured by the flow value from expected future contracting for both specific and common components, given the competition and matching friction.

$$
\begin{aligned}
\rho V(z,\varphi,\vec{n}) = {} & \pi(z) - r + (\mu - g) z V_z(z,\varphi,\vec{n}) \\
& + q_s \nu_s \Delta_g^s \Big\{ (1-\mathcal{S}) \underbrace{\int \left[V_{n^s(i)}(z,\varphi,\vec{n}) - K_c \cdot V_{n^c}(z,\varphi',\vec{n}) \right]^+ d\hat{\mathcal{G}}^c(z,\varphi',\vec{n})}_{\text{expected return with foreclosing common component provision}} \\
& + \underbrace{\mathcal{S} \sum_{a\in\{i,m\}} \zeta(a) \int \left[V_{n^s(a)}(z,\varphi,\vec{n}) - V_{n^s(a)}(z,\varphi',\vec{n}') \right]^+ d\hat{\mathcal{G}}^s(z,\varphi',\vec{n}',a)}_{\text{expected return from winning specification contracts against incumbent}} \Big\} \\
& + q_c \nu_c \Big\{ \underbrace{\iota \int \left[V_{n^c}(z,\varphi,\vec{n}) + \frac{1}{K_c} [J(z,\varphi^{\eta_m}) - V_{n^s(m)}(z,\varphi,\vec{n})] \right]^+ d\hat{\mathcal{G}}^i(z,\varphi',\vec{n},i)}_{\text{expected return from sourcing common component with compensation}} \\
& + \underbrace{(1-\iota)(1-\beta) V_{n^c}(z,\varphi,\vec{n})}_{\text{expected return from sourcing common component without compensation}} \Big\} \\
& + n^s(i) \cdot \underbrace{\kappa \left[V_{n^s(m)}(z,\varphi,\vec{n}) - V_{n^s(i)}(z,\varphi,\vec{n}) \right]}_{\text{maturity shocks arrive}} \\
& + n^s(i) \cdot \underbrace{\left[\pi(z,\varphi) - \varphi r^M(\varphi) - \delta V_{n^s(i)}\big(z,\varphi,\vec{n}\big) \right]}_{\text{expected value flow from sourcing one vertically specific component with immature partner}} \\
& + n^s(m) \cdot \underbrace{\left[\pi(z,\varphi) - r^M(\varphi) - \delta V_{n^s(m)}\big(z,\varphi,\vec{n}\big) \right]}_{\text{expected value flow from sourcing one vertically specific component with mature partner}} \\
& + n^c \cdot \underbrace{\left[\pi(z) - r^M - \delta V_{n^c}\big(z,\varphi,\vec{n}\big) \right]}_{\text{expected value flow from one common component relation}}
\end{aligned}
\tag{1.28}
$$

$$
\text{s.t.} \quad V(1,\cdot) = V_z(1,\cdot) = 0
$$

where r is the normalized rent of land that a firm is paying for the operation. $\mathcal{G}^s(\varphi, z, a)$ is the distribution of vertical contract for specific components across manufacturing customer firms with different φ conditional on the upstream firms' maturity states are a along the stationary equilibrium. Let the fraction of manufacturing upstream capacity that is already subject to input specification denote by $\mathcal{S}$. Each manufacturing customer firm either competes with another for component specificity or foreclosure K_c number of common component customers by compensating $K_c V_{n^c}(z, \cdot)$ with probability $q_s \nu_s \Delta_g^s (1 - \mathcal{S})$. Let the fraction of common capacity availability that is not subject to an existing vertical contract for specification denote by $(1 - \iota)$. This plays a role in determining the degree of persistent effect of foreclosure.

Thanks to the linearity in φ and the independence across technological component contracts, the marginal contribution of a linkage for a technological component net the corresponding fixed cost is the net joint value of such match

$$V_{n^s(i)}(z, \varphi, \vec{n}(\varphi, a)) - r^M(z, \varphi, s_{high}) = J^i(z, \varphi) \tag{1.29}$$

$$V_{n^s(m)}(z, \varphi, \vec{n}(\varphi, a)) - r^M(z, \varphi, s_{high}) = J^m(z, \varphi), \tag{1.30}$$

$$V_{n^c}(z, \varphi, \vec{n}(\varphi, a)) - r^M(z, \varphi, c) = J^0(z) \tag{1.31}$$

$$V(z, \varphi, \vec{n}) = O(z, \varphi) + \left[\Sigma_{a \in \{i,m\}} n^s(a) J^a(z, \varphi) + n^c J^0(z) \right] \tag{1.32}$$

The linearity offers a clear additive form that distinct customer firm baseline inflow value and discounted future value flow from searching for technological components from the match value of all existing technological component linkages. This immediately follows the value of entry net the value of technological component provision as a firm has no immediate linkages for technological component upon entry:

$$\rho O(z,\varphi) = \pi(z) - r + (\mu - g) z O_z(z,\varphi)$$
$$+ q^s \nu^s \Delta_g^s \Big\{ (1-\mathcal{S})\big(J^i(z,\varphi) - K_c \cdot J^0\big) + \mathcal{S}\Sigma_{a\in\{i,m\}}\zeta(a) \int [J^a(z,\varphi) - J^a(z,\varphi')]^+ d\tilde{\mathcal{G}}^s(\varphi', z, a) \Big\}$$
$$+ q^c \nu^c \{(1-\iota)(1-\beta) J^0(z) + \iota \int \big[J^0(z) + \frac{1}{K_c}[J^m(z,(\varphi^{\eta_l}) - J^m(z,\varphi)]\big]^+ d\tilde{\mathcal{G}}^s(\varphi, z, i)\} \tag{1.33}$$

$$\text{s.t.} \quad O(1,\cdot) = O_z(1,\cdot) = 0$$

In order to reach analytical solution to the systems of value functions, we impose parameter restriction on the fixed cost:

Assumption 1: $r^M(\varphi, c) = r, \quad r^M(\varphi, s) = \varphi r$

That is, the baseline fixed cost of a vertical linkage is set to be equal to the rent of land. This assumption allows the perfect scaling and easy characterization of a set of optimal stopping problem.

Lemma 2: If Assumption 1 holds, then we have $\underline{z}^c = \underline{z}^m = \underline{z}^i = \underline{z} = 1$

When the fixed cost exhibit the perfect scaling, each match breaks up voluntarily at common threshold regardless of mass of tasks involved in the given match.

Value of Entry: Each firm, upon entry, has no immediate technology component firm linkages. Thus the value of entry taking the rent as given is the sum of the expected return from producing a variety without established linkages and the immediate surplus received from supplying common technological components netting the fixed entry cost:

$$V^e = \int\int \big[O(z,\varphi) + \beta K_c J^0(z)\big] dF(\varphi) d\Gamma(z) - r^e \tag{1.34}$$

At the equilibrium, the competitive land market ensures that

$$r = \pi_{min} \text{ and } \quad V^e = 0 \tag{1.35}$$

Aggregation, Law of Motion, Entry and Growth

The aggregate entry rate is the same as the aggregate exit rate, given the limited supply of lands. Let the normalized cumulative distribution function of the operating technological class denote by $\Gamma(z,t)$. The dynamics of entry and exit decisions of firms shape the distribution endogenously:

$$\partial_t \Gamma(z,t) = E(t)\Gamma(z,t) + (g-\mu)z\Gamma_z(z,t) - Exit(t) \tag{1.36}$$

Along the BGP together with $E = Exit$, the solution to the above differential equation illustrates the relationship between growth and firms' entry and exit dynamics:

$$E = \gamma(g-\mu), \quad \Gamma(z) = 1 - z^{-\gamma} \tag{1.37}$$

The stationary distribution of operating technology is Pareto with pre-determined tail parameter γ which also can be interpreted as the difficulty of adopting technology as higher γ implies a thinner tail.

The distribution of firms with operating technological component department over ability is composed of unconditional distribution of firms implementing specification with some customer firms, $\mathcal{G}^s(z,\varphi)$ and that of firms are not subject to specification contracts $\mathcal{G}^0(z)$:

$$\mathcal{G}(z,\varphi) = \mathcal{G}^s(z,\varphi) + \mathcal{G}^0(z) \tag{1.38}$$

The Kolmogorov Forward Equations (KFEs) of $\mathcal{G}^S(z,\varphi), \mathcal{G}^0(z)$ are given by:

$$(g-\mu)z\mathcal{G}^S(z,\varphi) + \Delta_g^s\lambda^s\mathcal{G}^0(z)F(\varphi) - (\delta + \Delta_g^s\lambda^s[1-F(\varphi)])\mathcal{G}^s(z,\varphi) - \int_0^{\varphi} Exit^s(\varphi')d\mathcal{G}^s(\underline{z},\varphi') = 0 \tag{1.39}$$

The first term is due to the de-trended drift productivity as obsolescence rate; the second term is due to the inflow from successful matching for specification; the third term is due to capacity retirement shock to the firms' technological component provision and the outflow to matching with other firms; the last term captures optimal exiting.

$$(g-\mu)z\mathcal{G}^0(z) - \delta\mathcal{G}^0(z) + E\Gamma(z) - Exit^0 - \Delta_g^s\lambda^s\mathcal{G}^0(z) = 0 \tag{1.40}$$

Similarly, the first term is due to the drift; the second term is due to the capacity retirement; the third term reflects the business's quits; the last term represents the outflow of entering specification contracts with some firms.

Recall that the search efforts for technological components exerted by firms are exogenously fixed at $\{\nu^s, \nu^c\}$ which is a sufficient assumption that ensures that the above KFEs can be solved analytically and that the joint distribution $\mathcal{G}^s(z,\varphi)$ is separable.

Proposition 1: $\mathcal{G}^S(z,\varphi) = \psi\mathcal{S}G^s(\varphi)\Gamma(z)$ with $\mathcal{S} = \frac{\Delta_g^s\lambda^s}{(E+\delta)+\Delta_g^s\lambda^s}$, $\psi = \frac{E}{E+\delta}$ and $\mathcal{G}^0(z) = \psi(1-\mathcal{S})\Gamma(z)$, $\lambda^s = \chi\big[\frac{[1-\tilde{F}(K_c)]\nu_s}{\psi}\big]^{1-\xi}$, where

$$G^s(\varphi) = \frac{(\delta+E)\tilde{F}(\varphi)}{(\delta+E)+\Delta_g^s\lambda^s[1-\tilde{F}(\varphi)]} \tag{1.41}$$

$$\tilde{F}(\varphi) = \frac{F(\varphi)-F(K_c)}{1-F(K_c)}$$

where ψ is the mass of firms with operating technological component capacity given the presence of capacity retirement shock. Note that the finding rate at which a manufacturing firm meets a customer firm λ^s is decreasing in entry rate as the lower entry induces a relatively tighter specific technological component market. Furthermore, the fraction of firms serving specific components decrease in entry rate because a higher entry rate corresponds to a relatively more significant inflow of new entrants. The entry rate thus drives the overall right-skewness of the distribution through the two channels: (1) lower entry induces a higher share of firms engaging in providing specification $\mathcal{S}$ (a dynamic crowding-out effect/intensive margin); (2) lower entry induces a lower share of firms with available technological component capacities (a network effect/extensive margin). The lower entry sheers the distribution to the right tail as the firms are more likely to hunt for specification through the crowding-out margin. The network effect margin intensifies the competition for specification capacities as a lower entry rate makes the likelihood of re-allocating pairs from common component transactions toward specification greater.

Further decomposing into the conditional distribution of mature and immature firms over vertical-specific contract distribution gives a closer look at how the aggregate variables affect sourcing cost by affecting the shapes of these distributions.

Proposition 2: $\tilde{\mathcal{G}}^s(\varphi, z, a) = \Gamma(z)\tilde{G}^s(\varphi, a), \quad a \in \{i, m\}$, with

$$\tilde{G}^s(\varphi, i) = \frac{(\delta + E + \kappa)\tilde{F}(\varphi)}{\Delta_g^s \lambda^s [1 - \tilde{F}(\varphi)] + \delta + E + \kappa}, \tag{1.42}$$

$$\tilde{G}^s(\varphi, m) = \frac{\delta + E}{\Delta_g^s \lambda^s [1 - \tilde{F}(\varphi)] + \delta + E}\tilde{G}^s(\varphi, i) \tag{1.43}$$

and the fractions of firms as component suppliers that are subject to specification contracts are immature and mature are given by $\zeta(i) = \frac{\delta+E}{\delta+E+\kappa}, \quad \zeta(m) = \frac{\kappa}{\delta+E+\kappa}$ respectively.

Corollary 1: For both $\tilde{G}^i$ and $\tilde{G}^m$, the lower the entry rate, the fatter the tail.

Entrants bring new capacities, which ultimately increase both the common and specific technological components, just as we have shown in the two-period model. The new inflow of technological component capacities is a force that allocates resources relatively more evenly given the random meeting technology and flat marginal return. Therefore, the greater the provision, the lower the matching quality, yet the cheaper average sourcing costs because the distribution of specification contracts' matches are less right-skewed.

Given the Leontief matching technology for the common component market, the meeting probability at which customer firm meets an open capacity is

$$q^c = \frac{\psi \mathcal{S} \zeta(i) \kappa + E}{v^c}; \tag{1.44}$$

This implies that the equilibrium rate at which a manufacturing customer firm finds a common component capacity is increasing in entry rate. Furthermore, the conditional probability at which a customer firm meets a common component capacity posted by a mature firm is

$$\iota = \frac{\psi \mathcal{S} \zeta(i) \kappa}{\psi \mathcal{S} \zeta(i) \kappa + E} \tag{1.45}$$

1.5.4 Equilibrium

Definition 1 (Stationary equilibrium). *A balanced growth path (BGP) equilibrium consists of value functions $\{V, J^0, J^i, J^m, O\}$; decisions rule of firm's capacity allocation $\{\underline{z}^0, \underline{z}^i, \underline{z}^m, \hat{\varphi}\}$; meeting rates $\{\lambda^s, \lambda^c\}$, $\{q^s, q^c\}$ and firms (customer) searching distribution $\mathcal{F}(\varphi, z)$, aggregate mass of searching efforts $\{v^s, v^c\}$, a rental rate of land r, an aggregate entry rate E and rate of net obsolescence $-\tilde{g} = \mu - g$; a distribution of firms as component suppliers over specification ladder $\{\tilde{\mathcal{G}}^s(\varphi, z, a), \psi, \mathcal{S}\}_{a \in \{i,m\}}$; the distribution of operating technology $\Gamma(z)$ and labor and input composite demand by firms $\{L(z, \tilde{\boldsymbol{n}}(\varphi)), \mathcal{Y}^I(z, \tilde{\boldsymbol{n}}(\varphi)\}$ with equilibrium wage and aggregate price W, P^I, P^{final} such that*

1. *The value function J^0 and reservation $\underline{z}^0$ solve the optimal stopping problem (1.25); he value function $\{J^a\}_{a \in \{i,m\}}$ and reservation $\underline{z}^0 a$ solve the optimal stop-*

ping problem (1.26)-(1.27), and value function O and the exit threshold $\underline{z}$ solve the stopping problem (1.33) with $\hat{\varphi}$;

2. *The meeting rate $\{\lambda^s, \lambda^c\}, \{q^s, q^c\}$ are given by (1.23)-(1.24) and the searching distribution $\mathcal{F} = F(\varphi)\Gamma(z)$ and aggregate number of searching efforts ν^s and price of land by (1.35) and entry rate by (1.37);*

3. *The distribution of upstream capacity allocation $\{\mathcal{G}^0(z), \mathcal{G}^s(\varphi, z, a), \psi, \mathcal{S}\}_{a\in\{i,m\}}\}$ by (1.40)-(1.41) and firm's technological distribution Γ and rate of obsolescence $\mu - g$ solved by (1.34)-(1.36) jointly;*

4. *the labor and input composite demand with $P^I, P^{\{final\}}$ derived from the static surplus maximization problem by (1.13)-(1.15) and W solves the labor market clearing condition.*

1.6 Quantitative Exploration

This section extends the theoretical analysis in a quantitative direction. Beginning with a calibration of the model, I demonstrate salient features which reflect the correlation between firm dynamics and vertical linkages dynamism. Then I perform a counter-factual quantitative exercise with the calibrated model by focusing on how the declining economy driven by the increasing difficulty in ideas/technologies acquisitions can be further dampened by the exclusivity of vertical relationship for input specifications.

1.6.1 Calibration

I calibrate the model to match key moments in 2005 to 2016. I choose this time period because it is the span for which I have access to the firm-to-firm linkages dynamics data combining FactSet Revere and Compustat.

The model is parametrized at an annual frequency. I first determine a set of standard parameters based on a mix of commonly adopted values in the literature and value directly estimated from data, summarized in Table 1. The discount rate is set to satisfy the average annual real interest rate over the 2005-2016 time period through the consumers' Euler equation, $\rho + g = \textit{real interest rate}$, along with the average annual growth rate of economy at 2%. I borrow the elasticity of substitution parameters estimated by Broda and Weinstein (2006) for U.S. tradable products at 0.5. Since the elasticity of matching function is not identifiable as the searching efforts by the downstream are unobservable. Hence, I set a commonly estimated value when one allows for search on the contract in the match and search literature led by Petrongolo and Pissarides (2001). Finally I proxy the technology Pareto shape parameter by directly computing the mean of total factor productivity (TFP) of firms $\frac{\gamma}{\gamma-1}$ in the sample. I

Table 1.1: Externally Calibrated Parameters

	Description	Target	Value
ρ	Discount rate	Annual interest rate of 3%	0.01
σ	Elasticity of substitution parameter	Broda and Weinstein (2006)	5
ξ	Elasticity of matching function	Petrongolo and Pissarides (2001)	0.5
β	Bargaining power	Miyauchi (2018)	0.5
γ	Technological Pareto shape parameter	Average TFP of firms	6.99

further choose the remaining parameters by using a mix of normalization and simulated method of moments. Firstly, I normalize the discounts in specificity degree η_l when choosing to reallocate internal resources for the provision of common technological components at 0.5 as it is not separately identified from the recovered capacities K^c. For the same reason, I set matching efficiency χ for the input specification market at 0.3 following Engbom (2019) and choose the searching effort in common technological component markets ν_c to be 1. Furthermore, since the value functions are all linear in $\pi_{min} = r$ at equilibrium, I normalize $r = 1$ and interpret the entry cost r^e as a relative cost to the minimum profit that an incumbent can make in the economy. Lastly, I fix the rate of capacities recovery κ after engaging in input specification at 0.5. For the remaining 7 parameters $\mathbf{P} = \{\delta, f, \mu, r^e, \nu_s, \Delta_g^s, K^c\}$, I internally choose them by minimizing the sum of squares percentage deviations between 8 moments in the data and the counterparts constructed from the model following the simulated method of moments (SMM),

$$\mathbf{P} = argmin \sum_{M=1}^{8} \left[\frac{\text{data}_M - \text{model}_M(\mathbf{P})}{\text{data}_M} \right]^2$$

The estimated results fit the data sufficiently well by simply assigning the same weights to each moment, as illustrated in the panel below. Although the estimation is joint, it still leaves some room to discuss what moments particularly inform what parameter heuristically. The idiosyncratic rate of having a good match for input specification Δ_g^s governs the average contracting rate of vertical linkage with an upstream associated with input specification. The searching efforts ν_s for vertical relationships for high input specificity not only affect the contracting probability but also affect the separation rate as firms are poaching the capacity. The capacity for common component provision K^c is informed by the contracting rate of common vertical relation. The capacity retirement shock δ together with the creation rate of exclusive vertical relation for input specification (foreclosure effect) jointly determine the separation rate of common ver-

Table 1.2: Internally Calibrated Parameters

	Parameter	Value	Moment	Data	Model
Δ_g^s	good match rate	0.306	Spec linkage creation rate	0.393	0.409
ν_s	searching efforts for spec	1.959	Spec linkage separation rate	0.155	0.170
K^c	capacity for common components	15	Common linkage creation rate	1.406	1.400
δ	capacity retirement rate	0.153	Common linkage separation rate	0.195	0.187
f	Firm's ability Pareto shape	1.204	Shares of upstream s.t. spec	0.150	0.150
η_l	Discounts in specificity degree	0.5	Shares of upstream w/ exclusive spec	0.42	0.45
r^e	relative entry cost	54.225	Growth rate	0.02	0.02
μ	average incumbent productivity drift rate	0.008	Entry rate	0.084	0.084
χ	Matching efficiency (input specification)	0.3	Normalization		
r	Rent and linkage-level unit fixed cost	1	Normalization		
κ	Capacity recovery rate	0.5	Normalization		

tical contract. The thickness of the tail of entrepreneurs' ability draws f plays a crucial role in driving the linkage dynamism and is informed by the shares of upstream engaging in contracts for input specification. The discount in the degree of specificity η_l is closely related to the ex-post recovery rate from capacity constraint induced by the high degree of input specificity and is informed by the fractions of upstream engaging in input specification that have no other customer firms.

The key underlying parameters for internal calibration in the endogenous growth part of the model are $\{r^e, \mu\}$. The relative entry cost r^e affects the value of entry at equilibrium along the (de-trended) balanced growth path. In other words, the cost of entry is informed by the growth rate induced by the dynamic entry/exit through the mechanism of technological spillover in the framework. Lastly, the productivity drift μ is informed by the obsolescence/exit rate, which is given by $g - \mu$ as the exit rate in the model is always equal to the entry rate along the equilibrium path.

In terms of fit, the model-implied moments with calibrated parameters match the empirical counterpart pretty well. Equipped with these estimates, I further investigate how a counterfactual increasing difficulty in acquiring technology Z affects the aggregate industry dynamism with firms' entry and exit decisions. Such a shock has a sizeable indirect effect through the extensive linkages' responses as the slower-moving economy allows firms with greater advantage in leveraging specifications to afford more time to find better-match upstream to implement input specifications.

1.6.2 Counterfactual Analysis

The key counterfactual practice is a comparative static change in the thickness of the tail parameter of technological draw γ, which governs how easy to acquire an operating technology. Figure 1.7 plots the rate of growth rate when increasing γ from the previously calibrated value. A higher γ induces a lower growth rate without a surprise. In the rest of the section, I examine the impact of shocks on the other relevant aggregate moments, including business dynamism and technological/input linkage dynamism. Lastly, I evaluate the economic responses to the same shock when the foreclosure effect of input specification is shut down.

Business dynamism and Input market dynamism

Fixing the calibrated parameters of the economy, Figure 1.8 shows a stellar decline in firms' entry rates. It is noticeable that the entry rate drops faster than the growth rate in response to the shocks. Generally speaking, as implied by

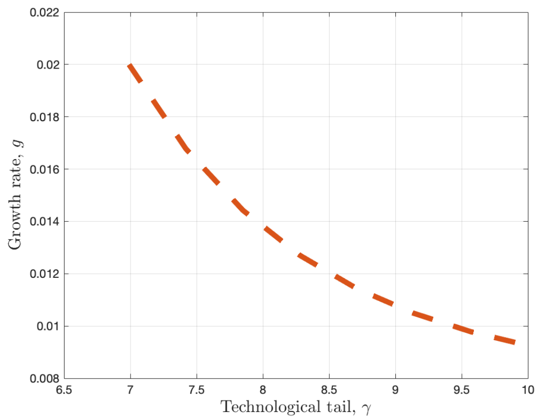

Figure 1.7: Growth Rate When Varying γ

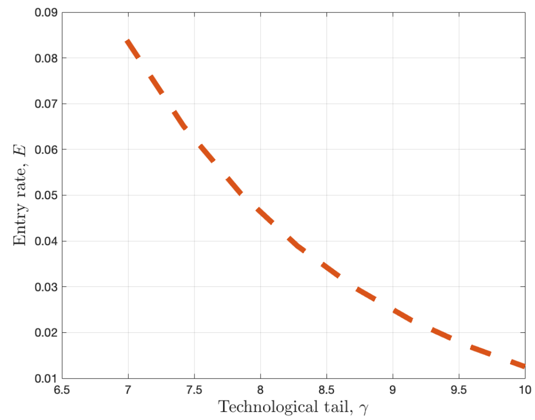

Figure 1.8: Entry Rate When Varying γ

the model, the value of entry decreases when the adoption of technology becomes harder. This is immediately followed by a lower entry rate. Consistent with the argument on aging U.S. economy, the lower entry rate implies higher proportion of 'mature' incumbent upstream as $\frac{\partial \zeta(m)}{\partial E} < 0$.

Beyond the slowdown of business dynamism, higher γ also intensifies the concentration of the resources reflected by declining extensive linkage dynamics for technological components. Figures 1.9 and 1.10 plot the average downstream firm's contracting and separation rate for input specification as a function of γ, respectively. As the technologies become harder to adopt, there are fewer linkages created, including those for input specification. On the other hand, the separation rate remains almost the same for two reasons. First, the slower-moving economy affords incumbent firms more time to match and end up with higher match quality, which in turn implies a robust linkage relationship. Second, the slower economy results in less amount of upstream with capacity for input specification that has not yet retired. This makes intensifies the downstream competition for capacity for input specification, which forms a pushing force behind the separation rate. Overall, the dynamics of linkages for specific components are in decline.

Figures 1.11 and 1.12 show the change in creation and separation rate for common component linkages in response to varying γ. Highlighted by the mechanism, the strengthening foreclosure effect drives the decreasing creation rate and increasing separation rate for common vertical linkages since resources (capacity) for technological components are concentrated toward more capable firms for specification.

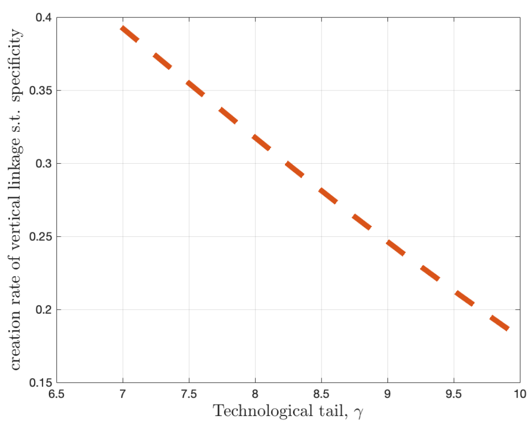

Figure 1.9: Spec-link Creat.Rate and γ

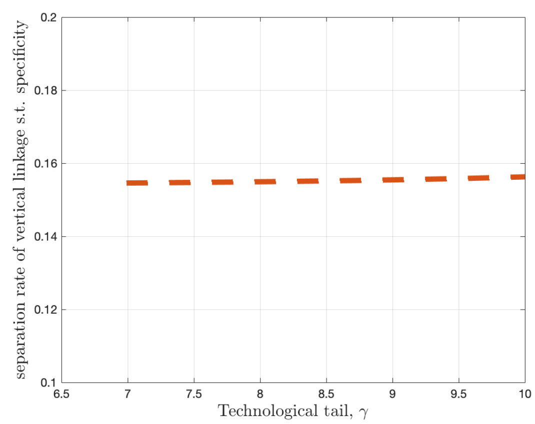

Figure 1.10: Spec-link Sep.Rate and γ

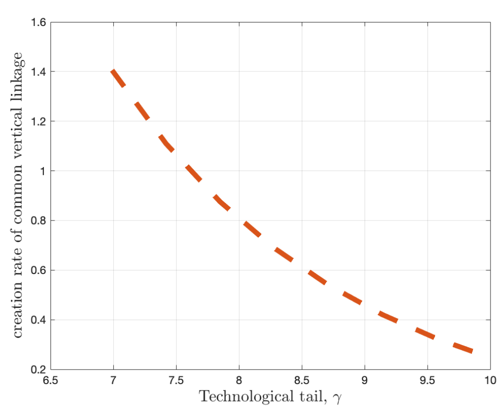

Figure 1.11: Common-link Creat.Rate and γ

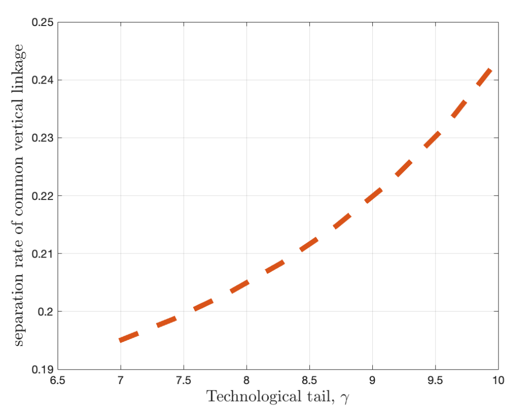

Figure 1.12: Common-link Sep.Rate and γ

Figures 1.13 and 1.14 confirm the trend in resources concentration endogenously related to the slower growth and lower entry rate. The increasing fractions of upstream engaging in input specification leads to an “older" economy, with more upstream suppliers being able to meet well-matched customer firms to implement the specification. The shares of upstream choose to entirely focus on serving the relational customer firms for input specification climbs up as the technological components are allocated toward more capable firms, which results in a lower incentive for upstream to allocate capacity to supply common components even when capacity expansion is available.

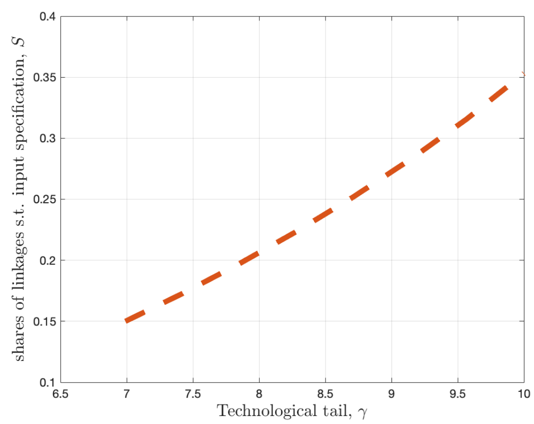

Figure 1.13: Share of Upstream Engaging in Specification and γ

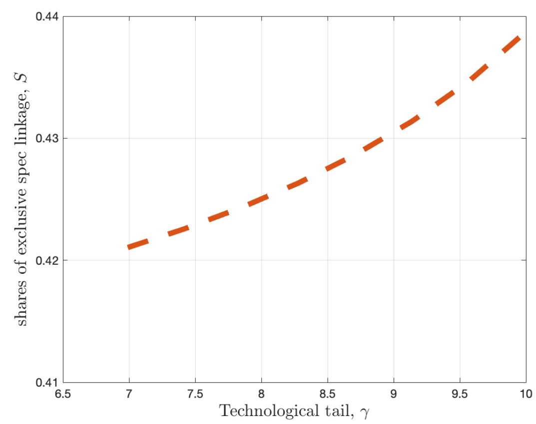

Figure 1.14: Share of Upstream Engaging in Exclusive Specification γ

Role of foreclosure effect

A natural question arises: what is the role of the foreclosure effect induced by the input specification contracts. To evaluate its impact, I construct the additional counterfactual changes on the previous exercise by setting $\eta_l = \kappa = 1$. This means there is no trade-off when firms face no capacity constraint after engaging in input specification. Figure 1.15 and 1.16 shows how the shutdown of foreclosure effect affects firms’ common linkage creation and separation rate. Specifically, the contracting rate of common vertical relationships is much higher than the case subject to foreclosure shock. The separation rate of

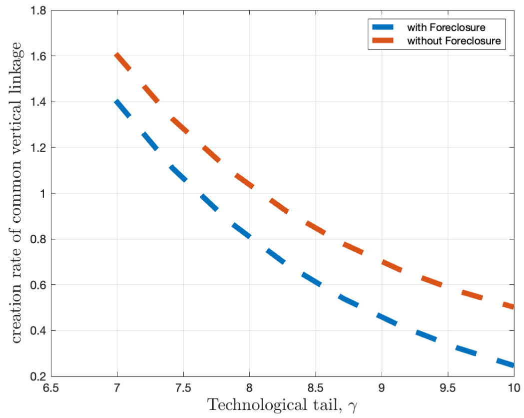

Figure 1.15: Common-link Creat.Rate and γ

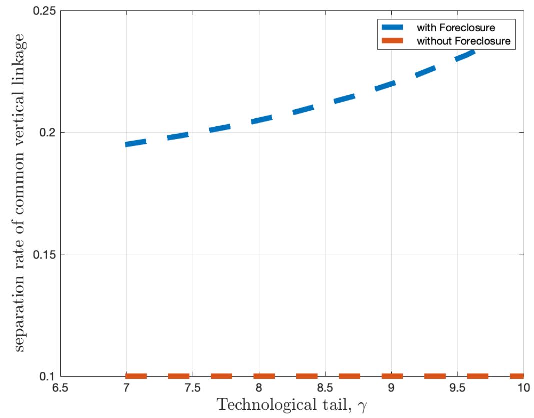

Figure 1.16: Common-link Sep.Rate and γ

common relationship is solely driven by exogenous separation shock, which is half of that when there is a foreclosure effect.

Figures 1.17 and 1.18 demonstrate the aggregate implication of the foreclosure effect. The growth rate is 10% greater than the benchmark case, suggesting that the capacity constraint induced by input specification is quantitatively important in exaggerating the decline in business dynamism. Similarly, the entry rate is more than 25% higher, implying the foreclosure channel plays a significant role in deterring entry. Taking stock, the input specification induced capac-

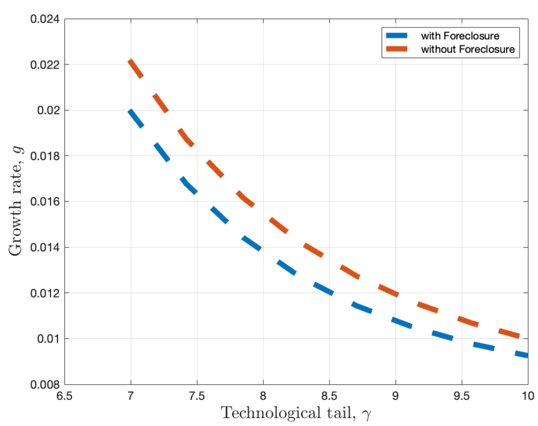

Figure 1.17: Growth Rate and γ

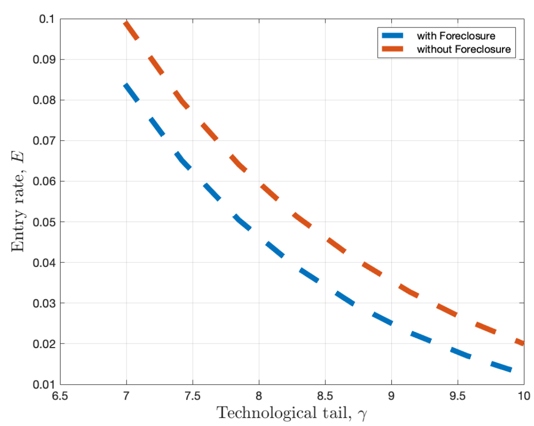

Figure 1.18: Entry Rate and γ

ity shocks to the suppliers has significant aggregate implications in determining the firm-to-firm trade dynamism and entry/exit dynamics, which ultimately connect to the long-run growth. Policies targeting relaxing capacity constraints of pairs contracting for specific technological components for production can encourage entry and energize the industry dynamism.

1.7 Conclusion

This paper studies the aggregate implication of vertical contracts for input specification by embedding the foreclosure effect through capacity exploitation due to specificity in a Schumpeterian growth model. The foreclosure effect induces tougher competition for upstream capacity, which leads to higher sourcing costs, thus discouraging entry. A lower entry rate, in turn, generates less capacity inflow which makes the competition tougher through the network effect. Furthermore, a slower economy due to lower entry gives the incumbent a longer lifespan, intensifying the competition for upstream capacity, which amounts to market concentration and deterring entry further. Leveraging US production network data, I provide empirical support for the foreclosure effect. Bringing the data into the model, I show that the input specificity channel is quantitatively important and that removing the foreclosure effect can contribute to a 10% more increase in growth rate.

Many questions related to the interactions across firms along the supply chain and the aggregate economic dynamism remain unanswered. For instance, it would be interesting to understand the implication of input specification in shaping production networks in an open economy and thus trade flow. Furthermore, it is important to exploit policy implications and discuss the optimal policy to mitigate friction and encourage entry.

1.A Appendix—Chapter 1

1.A.1 A Simple Two-Period Model with Bargaining

This section considers a simple model where bargaining between incumbent customers of common components and a buyer with high-quality idea looking for specific components. It is sufficient to focus on the entry's problem in the second period as the bargaining protocol does not alter the matching and contracting outcome but change the ex-post payoffs. Specifically, the payoff received by an entrant with low-quality idea is

$$V_e^l = (1-h)(1-\beta)\frac{E}{1+E}K_c\pi + (1-h)\beta K_c\pi + \underbrace{hK_c\pi}_{\text{gains from bidding}}$$

Conditional on contacting with a buyer demanding customized components, the offer to an upstream sector is bidded up to the full surplus from serving common components, which is captured by the additional third term. The corresponding payoff received by an entrant with high-quality idea is

$$V_e^h = V_e^l + \frac{1+E-h}{1+E}(z-_c)\pi$$

where the second term is lower than the benchmark case as the buyer has to compensate more to the seller for the provision of specific components. The expected value of entry thus is

$$V_e = (1-h)V_e^l + hV_e^h,$$

and it is easy to verify that $\frac{\partial V_e}{\partial h} < 0$ if $h > \frac{1+E}{2}$ and $z > (1 + \frac{1-\beta}{2h-(1+E)})K_c$.

1.A.2 Proofs

Proof of Proposition 1: Note that the expected value of entry reads:

$$\begin{aligned}
V_e &= (1-h)V_e^l + hV_e^h \\
&= (1-h)\left[(1-h)\cdot\frac{K_cE}{1+E}\cdot(1-\beta)\pi + \beta K_c\pi\right] + h\left[V_e^l + \frac{1+E-h}{1+E}(z-\beta K_c)\pi\right] \\
&= (1-h)\cdot\frac{K_cE}{1+E}\cdot(1-\beta)\pi + \beta K_c\pi + h\frac{1+E-h}{1+E}(z-\beta K_c)\pi
\end{aligned}$$

It is easy to verify $\frac{\partial V_e}{\partial E} > 0$ and $\frac{\partial^2 V_e}{\partial E^2} < 0$. For the relationship between value of entry and frequency of input specification conducts h, taking partial derivative with respect to h, one can obtain

$$\frac{\partial V_e}{\partial h} = -(1-\beta)\frac{K_cE}{1+E}\pi + (\beta K_c - z)\pi\cdot\frac{2h-(1+E)}{(1+E)^2}$$

Since $z > K_c$ together with $h > \frac{1+E}{2}$, we have $\frac{\partial V_e}{\partial h} < 0$. □

Proof of Corollary 1: Note that V_e is increasing and strictly concave in entry rate, and $V_e(E=0) > 0, \quad V_e(E=\infty) = \text{constant} > 0$, the equilibrium is unique and stable when entry cost is linear in entry rate. Equating the value of entry and entry cost and rearrange the equation, one can obtain:

$$(1-h)\frac{\beta+E}{E(1+E)}K_c\pi + h\frac{1+E-h}{E(1+E)}(z-\beta K_c)\pi = r$$

Since the left hand side strictly decreases in entry rate and converges to zero as the entry rate goes to infinity, there exists $r(E^*)$ such that the solution $E^* \in (0,1)$. Furthermore, evaluating the left-hand side with $h = \frac{1+E}{2}$, the left hand side of equation is still strictly decreasing in E. Denote by $\hat{E}$ the solution to the equation. There, again, exists $r(\hat{E})$ such that $\hat{E} \in (0,1)$. In this case, for any $h > \frac{1+\hat{E}}{2}$ will generate lower equilibrium entry rate $E^* < \hat{E}$ by proposition 1 and thus $h > \frac{1+E}{2}$ still holds. □

Proof of Lemma 2: It is sufficient to show that the value functions are proportional to φ and their optimal stopping problem therefore are identical.

$$(\rho+\delta)J^i(z,\varphi) = [\pi(z,\varphi) - \varphi r] + (\mu - g)zJ^i_z(z,\varphi)$$

subject to optimal stopping condition and smooth-pasting condition. Guess $J^i(z,\varphi) = \varphi J(z)$. It is easy to verify that the above problem is identical to

$$(\rho+\delta)J(z) = \pi(z) - r + (\mu - g)J(z)$$

Both $J^m andJ^c$ follows the same proof, which leads to the desired result. □

Proof of Proposition 2: I firstly characterize the steady state of mass of firms with operating upstream sector ψ and the fraction of firms who upstream sector is engaging in providing specific component.

The inflow of upstream is simply E while the outflow of operating upstream is given by $\delta\psi + Ex^{up}$ where Ex^{up} is the mass of upstream exits due to firm's optimal stopping choice. This follows that, along the balanced flow, $\psi = \frac{E - Ex^{up}}{\delta}$. Using the fact that $E^{up}_x = \psi E_x$ along the steady state, one then can obtain $\psi = \frac{E}{E+\delta}$.

To compute $\mathcal{S}$, firstly note that along the steady state, the fraction of upstream serving no specification must be stable. Specifically, its inflow is E by construction, and its outflow is $(\delta + \Delta^s_g\lambda^s)(1-\mathcal{S})\psi + (1-\mathcal{S})Ex^{up}$. At balanced flow, we obtain $E = \frac{1-\mathcal{S}}{\mathcal{S}}\Delta^s_g\lambda^s\psi$. Given the result obtained for ψ previously, we have

$$\mathcal{S} = \frac{\Delta^s_g\lambda^s}{(E+\delta) + \Delta^s_g\lambda^s}.$$

Then we move on to characterize $\mathcal{G}^0(z)$. Recall that $\mathcal{G}^0(z)$ solves KFE

$$(g-\mu)z\mathcal{G}^0_z(z) - \Delta^s_g\lambda^s + \delta\mathcal{G}^0(z) + E\Gamma(z) - E^0_x = 0$$

Guess $\mathcal{G}^0(z) = \psi(1-\mathcal{S})\Gamma(z)$ and $E_x^0 = \psi(1-\mathcal{S})E$. Note that $\psi = \frac{E}{E+\delta}$, and the KFE of productivity propagation is

$$(g-\mu)z\Gamma_z(z) + E\Gamma(z) - E = 0$$

It is easy to verify the guess is the solution.

Lastly, we guess $\mathcal{G}^s(z,\varphi) = \psi\mathcal{S}G^s(\varphi)\Gamma(z)$. Note that $\mathcal{G}^s(z,\varphi)$ solves the following KFE

$$(g-\mu)z\mathcal{G}^s_z(z,\varphi) + \Delta^s_g\lambda^s\mathcal{G}^0(z)\tilde{F}(\varphi) - [\delta + [1-\tilde{F}(\varphi)]\Delta^s_g\lambda^s]\mathcal{G}^s(z,\varphi) - \int_{\underline{\varphi}}^{\varphi} Ex(\varphi')d\tilde{F}(\varphi') = 0$$

Given that $\int_{\underline{\varphi}}^{\varphi} Ex(\varphi')d\tilde{F}(\varphi') = G^s(\varphi)\mathcal{S}\psi E$, substitute the terms with the guess and one can obtain the desired result. □

Proof of Proposition 3: I firstly characterize fraction of upstream under specification contracts that are mature, $\zeta(m)$. The inflow is $\kappa(1-\zeta(m))$ while the outflow is $(\delta+E)\zeta(m)$. This amounts to $\zeta(m) = \frac{\kappa}{\kappa+\delta+E}$ and $\zeta(i) = \frac{\delta+E}{\kappa+\delta+E}$. Now we move on to characterize $\tilde{\mathcal{G}}(z,\varphi,i)$. Guess $\tilde{\mathcal{G}}(z,\varphi,i) = \Gamma(z)\tilde{G}(\varphi,i)$. Note that the inflow in absolute mass is given by $\lambda^s(1-\mathcal{S})\psi\tilde{F}(\varphi)$ while its outflow is

$$(\Delta^s_g\lambda^s[1-\tilde{F}(\varphi)+\delta)\mathcal{S}\psi\zeta(i)\tilde{G}(\varphi,i) + \mathcal{S}\psi\zeta(i)E\tilde{G}(\varphi,i) + \kappa\mathcal{S}\psi\zeta(i)\tilde{G}(\varphi,i)$$

Rearrange the term and equate the inflow and outflow, we have

$$\tilde{G}(\varphi,i) = \frac{(E+\delta+\kappa)\tilde{F}(\varphi)}{(E+\delta+\kappa) + \Delta^s_g\lambda^s[1-\tilde{F}(\varphi)]}$$

It is easy to verify the guess is true then.

Similarly, it is sufficient to compute $\tilde{G}(\varphi,m)$. Its inflow is $\kappa\mathcal{S}\psi\zeta(i)\tilde{G}(\varphi,i)$ while its outflow is

$$\left(\Delta^s_g\lambda^s[1-\tilde{F}(\varphi)] + \mathcal{S}\psi\delta\right)\zeta(m)\tilde{G}(\varphi,m) + \mathcal{S}\psi\zeta(m)E\tilde{G}(\varphi,m)$$

Rearrange the equation, and one can obtain the desired result. □

1.A.3 Steps to Solve the Equilibrium

1. Closed-form solution to the joint net value of a match (25)-(27):

$$J(z) = \frac{\pi_{min}}{(\rho+\delta)+(\sigma-1)(g-\mu)}\left[z^{\sigma-1} + \frac{(\sigma-1)(g-\mu)}{\rho+\delta} z^{-\frac{\rho+\delta}{g-\mu}}\right] - \frac{\pi_{min}}{g-\mu}$$

Match associated with specificity with (z, φ) is given by

$$J(z, \varphi) = \varphi J(z)$$

2. Value of variety production at (z, φ) (33):

$$O(z, \varphi) = A^O\left[z^{\sigma-1} - 1\right] + B^O\left[z^{-\frac{\rho+\delta}{g-\mu}} - 1\right] + D^O\left[z^{-\frac{\rho}{g-\mu}} - 1\right]$$

where

$$A^O = \frac{\pi_{min} + \Psi(\varphi) \cdot \frac{\pi_{min}}{(\rho+\delta)+(\sigma-1)(g-\mu)}}{\rho + (g-\mu)(\sigma-1)},$$

$$B^O = -\left[\Psi(\varphi) \cdot \frac{B^J}{\delta}\right],$$

$$B^J = \frac{\pi_{min}}{(\rho+\delta)+(\sigma-1)(g-\mu)} \frac{(\sigma-1)(g-\mu)}{\rho+\delta},$$

$$D^O = \frac{g-\mu}{\rho}\left[A^O(\sigma-1) + \Psi(\varphi) \cdot \frac{(\rho+\delta)}{\delta(g-\mu)} B^J\right]$$

$\Psi(\varphi, \cdot)$ is a function that summarizes the the expected contracting cost of linkages creations given :

$$\Psi(\varphi, \cdot) = q_s \nu_s \Delta_g^s \left\{(1-\mathcal{S})(\varphi-1) + \mathcal{S} \sum_{a \in \{i,m\}} \zeta(a) \int_1^{\varphi} \left[\varphi - \varphi'\right] d\tilde{G}^s(\varphi', a)\right\}$$
$$+ q_c \nu_c \left\{\iota \int_1^{\hat{\varphi}} (1 + \varphi^{\eta_l} - \varphi) d\tilde{G}^s(\varphi, i) + (1-\iota)(1-\beta)\right\}$$

where

$$\hat{\varphi}^{\eta_l} + 1 - \hat{\varphi} = 0$$

3. Key equations to solve for growth rate g and entry rate E

$$E = \gamma(g-\mu),$$

$$V^e = \int\int \left[O(z, \varphi) + \beta J(z)\right] dF(\varphi) d\Gamma(z) - r^e = 0, \quad \Gamma(z) = 1 - z^{-\gamma}$$

CHAPTER 2

VINTAGE CAPITAL AND VENTURE CAPITAL INVESTMENT CONCENTRATION

2.1 Introduction

Recent works on business dynamism and productivity have highlighted the roles of entrants and financial intermediaries separately. Entrants play a significant role in job creation in the US economy, as documented in Haltiwanger et al. (2013), and are the main body of the most high-growth and innovative firms (Lerner and Nanda, 2020). Meanwhile, financial intermediaries, especially venture capital, provide essential financing support to mitigate finance frictions for young firms, encourage firm creation, and reduce resource misallocation. Nevertheless, these two important drivers of firm dynamics and productivity growth are highly concentrated in specific areas (Bay-Boston-NY), leaving the economic activity across the space strikingly uneven (Gaubert, 2018) and also leaving an open question for place-based policies to attract new entrepreneurship and capital to counter-balance the spatial inequality.

So, why are the geographic choice of new entry and inflow of capitalists highly clustered? In this paper, to understand the tremendous spatial disparities, I link the motives of co-locating by entrants and capitalists via a core feature of vintage capital reallocation toward young firms mediated by venture capital. The hypothesis is motivated by two empirical patterns. First, venture capital investment is increasing in the availability of local vintage supply, even when the overall asset specificity or asset immobility demanded by the VC-backed firms is high. This supports the localness of vintage capital transactions, which can drive the spatial concentration. Second, venture capital investment in an early stage of firms illustrates a higher positive response to local vintage capital supply than that in a later stage. This coincides with the well-documented empirical pattern that young firms are more financially constrained and dependent on used capital seasoned by those older, established firms from the same region (Ma, Murfin and Pratt 2021). Therefore, the local vintage capital supply attracts entrants, further attracting VC investment because of more deal-flows and lower financing costs (more profits).

To formalize the mechanism, this paper then integrates vintage capital induced co-location of entrants and VC investment into a theoretical framework where entrepreneurship and VC capital flow are endogenously determined. Beginning with a simple static partial equilibrium model with exogenous city size and availability of vintage capital that features used capital reallocation and firm-to-VC matching, I highlight a straightforward mechanism where the supply of local vintage capital leads to more entrepreneur-VC matches, which thus encourages ex-post entry. Following this, more business opportunity powered by greater financial accessibility, in turn, yields tougher selection and thus higher productivity, leveling up the surplus of the operating business. This, in turn, invites more VC investment, amounting to a virtuous circle.

Guided by the baseline mechanism with exogenous vintage capital supply, I assess the important spatial implications of local vintage capital supply by extending to a simple infinite-horizon steady-state equilibrium model with endogenous cities and vintage capital supply. Beyond the insight derived from the partial equilibrium approach, a simple model with endogenous vintage capital supply sheds additional light on understanding how the firm-VC matching, selection, agglomeration, and sorting interact and emerge at equilibrium: More talented individuals sorting into larger cities invites more VC investment to incubate profitable business. Increasing commercial opportunities induces tough competition, thus leading to greater selection and lower misallocation. Less misallocation implies higher capital demanded due to increased output, which yields larger stock of vintage capital along the horizon. As a result, lower capital cost further pushes up the profitability and wages, thus attracting more talents and thus VC investment.

Finally, to close the theory, I examine the endogenous location choices made by individuals with heterogeneous talents. More talented individuals sort into larger cities which invite more VC investment due to more promising return of investment in those more productive start-ups. More financial accessibility amplifies the local economic performance through a selection-induced agglomeration channel, ultimately leaving the most talented individual and mass of VC investment concentrated in largest cities, such as Bay-Boston-NY areas.

2.2 Literature Review

This paper connects and contributes to two threads of literature. First, I expand an additional dimension on the literature on agglomeration. This body of works explores the role of resource allocation in contributing to agglomeration through the labor market. Behrens et al. (2014) and Gaubert (2018) study the agglomeration effect when internalizing the endogenously spatial sorting of entrepreneurs and workers. Moretti (2021) provides empirical support on a large scale of agglomeration as a source of knowledge spillover by which the talents are further attracted. Bilal (2021) micro-found the agglomeration in alternative respect by introducing the heterogeneous job separation rate across cities, which affects entrepreneurs' location choices and the local ex-post agglomeration effect exhibited in job-finding rate, implying more efficient labor allocation in a larger city.

A key implication of this literature is that the populated city is more productive as the individual allocation is more efficient when a city is large. This paper extends the resource allocation channel by introducing a capital market that highlights interactions between the productivity of a city, entrepreneurship, and capital market efficiencies powered through venture capitalist engagement. A more efficient allocation of used capital market mediated by venture capitalists reduces labor misallocation through two margins: (1) it reduces misallocation by offering individuals more financial accessibility, which amounts to more entrepreneurship opportunities. (2) More entrepreneurship opportunities amount to the more arduous selection, and more productive firms emerge. A larger city intensifies the allocative forces, making the city larger relative to others.

Second, this paper connects to the literature on the role of financial constraints in firm dynamics. This paper leverages the seminal contribution by Midrigan and Xu (2014), who argue that the aggregate impact of finance frictions primarily affects the economy through distorting the firms' entry decisions, and further elaborates on the competition effect and agglomeration effect induced by the distorted entry decisions due to VC-firm matching frictions. Another closed related body of literature studies the implication of used capital reallocation. Eisfeldt and Rampini (2006) study how the aggregate capital reallocation is intertwined with the business cycle. Lanteri (2018) studies how costly capital reallocation affects efficiency. Ma et al. (2021) provide empirical evidence to argue that used capital reallocation is a vital capital acquisition channel for young firms. The major take-away of this literature focuses on the critical roles of (used/vintage) capital reallocation in supporting entrants. I adopt the idea and further examine the linkage from local capital reallocation efficiency to VC-firm matching, which further generates stark spatial disparities in young firms' activities and industry dynamism.

The rest of the paper is organized as follows: Section 2.3 presents empirical evidence on VC-entrants' co-location and the relationship between VC investment and local vintage capital supply; Section 2.4 describes a partial equilibrium model that demonstrates how the local vintage capital market attracts VC investment which amounts to local agglomeration effect; Section 2.5 generalizes the framework by extending it into a steady-state general equilibrium model along infinite time horizon with endogenous vintage capital supply and location choices by individuals; Section 2.6 concludes.

2.3 Empirical Patterns

Data Sources: I source the data from VentureXpert database provided by Thomson Financial. It contains detailed information about the dates of venture financing rounds, the investors and portfolio companies involved, the estimated amounts invested by each party, and the ultimate portfolio outcomes. The primary sample includes all VC investments made between 1980 and 2016 and focuses on the venture stage (seed, early, expansion, or later stage). I focus on investments made by US-based VC in private companies headquartered in the

US and exclude those by angels and buyout funds.

The key empirical pattern derived from the data is the concentration of VC investment. I examine two different measures of concentration: (1) measured by the number of VCs' headquarters by regions; (2) measured by the number of start-ups invested by VCs by regions.

From figure 2.1, it is easy to see that US venture capital is heavily clustered in four MSA: San Jose, San Francisco, Boston, and NY. (We later refer these four cities the venture capital centers.) More than half of all venture capital offices in the US are located in those metropolitan areas.

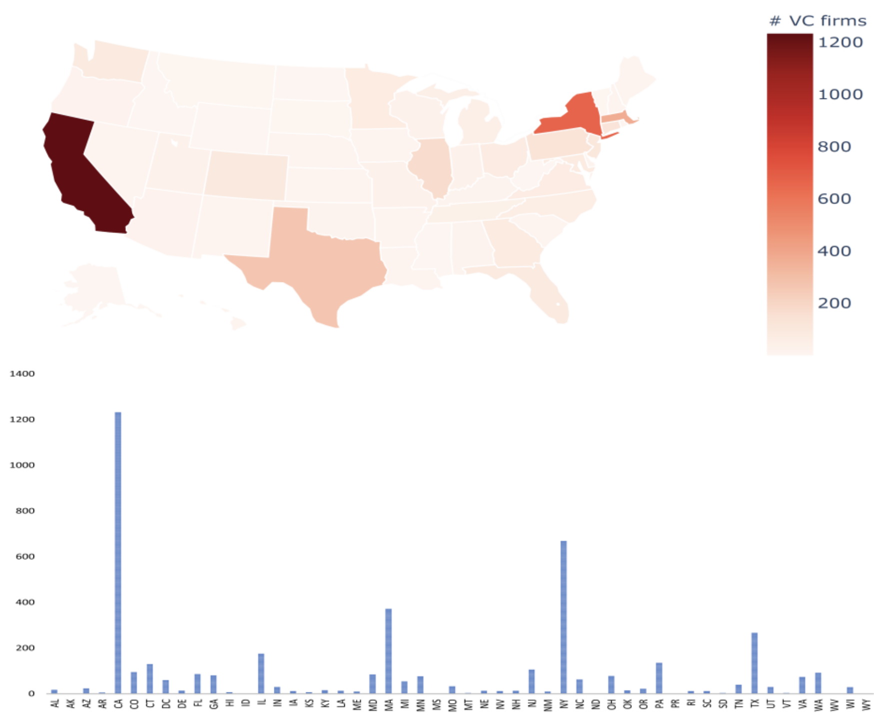

Figure 2.1: VC Concentration

Figure 2.2 illustrates similar implication of VC investment concentration: more than half of all companies financed by venture capital are located in those venture capital centers areas. The distribution of VC-backed companies are slightly more concentrated than VC firms.

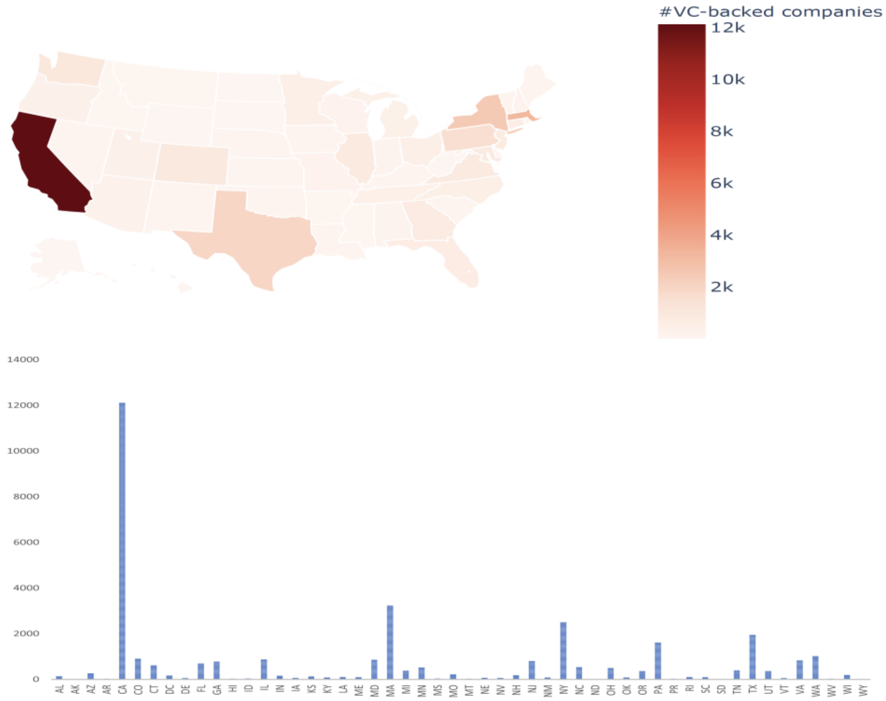

Figure 2.2: VC-backed Companies Concentration

2.3.1 VC investment and Vintage Capital Reallocation

To understand to what extent the local vintage capital market plays a role in driving the secular pattern, I begin with documenting the evidence that reveals the relationship between aggregate VC investment in a given area at a given year and the local measure of aggregate used capital reallocation. [1] I firstly follow Eisfeldt and Rampini (2006) by adopting sales of property, plant, and

[1]Since we are not able to observe the inter-companies capital transaction records mediated by venture capitalists nor the detailed vintage of capital reallocated, I, therefore, focus on the response of aggregate VC investment by region to the local vintage capital supply.

equipment from Compustat as a measure of the amount of capital reallocation. Specifically, I run the following linear regression:

$$\log(\mathrm{I}^{VC} + 1)_{m,s,t} = \beta \cdot \log(K^{\text{Resale}} + 1)_{m,s,t} + \delta_{FE} + \epsilon_{m,s,t}$$

I aggregate VC firm × VC-backed company × round investment flow up to yearly Metropolitan Statistical Area-Industry (SIC) level denoted by $\mathrm{I}^{VC}_{m,s,t}$. I sum up the sales of property, plant, and equipment reported by companies whose headquarter is located in the given region by MSAs across years to measure the amount of used capital reallocation at a given industry level. The simple regression includes fixed effect for year, and industry fixed effect.

Table 2.1: VC & Capital Resale

$\log(K^{\text{Resale}})_{m,s,t}$	0.106***
	(0.009)
Fixed Effect	Yes
N	59015
adj. R^2	0.011

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

The result illustrates a positive correlation between the investment of venture capital and the local capital reallocation activity. Specifically, a 10 percent increase in the value of the capital sale is associated with a one percent increase in VC investment inflow into the area. However, the observed capital reallocation is an equilibrium outcome, and it thus is subject to endogeneity problems. For instance, an increase in VC investment may increase the value of old capital held by the incumbent companies in the area and result in more sales of assets, amounting to reverse causality. Therefore, to study the factors that drive the VC investment, instead of using the capital sales data, I examine the relationship between VC investment and local old capital supply in the spirit of Ma et al.

(2021). I proxy the availability of old local capital by aggregating incumbent companies' local total capital value in the previous year.

Formally, I hypothesize that the local old capital supply will attract VC investments (concentration of VC investment) and thus benefit new start-ups (concentration of VC-backed companies). I construct the local old capital availability based on *type* by leveraging the BEA inter-sector input-output table following Kermani and Ma (2020).

$$K^{e}_{\varphi,m,t} = \sum_{s} \omega_{s\varphi} \sum_{j \in \mathcal{J}_{m,s,t}} K^{e}_{j,t-1}$$

For any given company in a given industry s, it requires capital goods with different *types*. The BEA input-output table provides information on how many fractions of capital goods employed by companies in a given sector s are type φ denoted by $\omega_{s\varphi}$. Notice that we use the capital held by the company in $t-1$ as a proxy for the old capital available at time t.

In the same manner, I construct the VC investment for capital goods type φ:

$$I^{VC}_{\varphi,m,t} = \sum_{s} \omega_{s\varphi} \sum_{i \in \mathcal{I}_{m,s,t}} I^{VC}_{m,s,t}$$

Armed with the constructed measures, I run the analog linear regression as before.

$$\log(\mathrm{I}^{VC} + 1)_{\varphi,m,t} = \beta \cdot \log(K^{e} + 1)_{\varphi,m,t} + \delta_{FE} + \epsilon_{\varphi,m,t}$$

The fixed effect absorbs the year-specific and capital type-specific effects. The estimated result yields a positive correlation between the old capital availability. However, it is still insufficient to conclude causality as potential confounding factors exist. For example, VC investments may be driven by the technology spillover while the companies working with frontier technologies are capital-intensive. Furthermore, in general, large firms have more capital and more talents. More talents tend to generate more spin-offs and thus run more start-ups, which can attract VC investments.

Table 2.2: VC & Capital Supply

$\log(K^e + 1)_{\varphi,s,t}$	0.096***
	(0.004)
Fixed Effect	Yes
N	59142
adj. R^2	0.019

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

2.3.2 Heterogeneous Demand of VC investment for Vintage Capital

To alleviate the endogeneity concern, I further examine the VC-firm match level data by exploiting the potential factors that amount to heterogeneous VC investment demand for local vintage capital. In particular, I leverage the corporate finance literature on the differential demand for vintage capital across firms' ages and the specificity degree of asset demanded.

The firm's financing stages along its life cycle have important implications on its demand for used capital. Young firms are riskier and financially constrained and thus have a stronger preference over vintage capital (Ma et al., 2021). This motivates to exploit the variation in the VC investment across seniority of firms since VC investment in immature firms is very likely to prefer vintage capital as the early round of capital inflow is very likely insufficient. If the intangible spillover from capital-intensive firms solely drives the VC investment, we should see the negligible effect of local vintage capital supply in VC investment across different stages of VC-backed firms in the same industry.

With this identification strategy in mind, I show that a more abundant supply of vintage capital influences VC investment differently across different stages of VC-backed firms' financing/life cycle. Specifically, I estimate a VC investment decision model for firms in the early and later stages based on the latent supply of local capital across sectors. As an alternative indicator of the financing stage of start-ups, I also adopt the age of VC-backed firms since its registration to supplement the estimated results.

$$\log(I^{VC}_{i,a,s,m,t}) = \beta_1 \cdot \log(K^e_{\varphi(s),m,t} + 1) \times \textit{Financing Stage}_{s,m,t} + \beta_2 \cdot \log(K^e_{\varphi(s),m,t} + 1) + \beta_3 \cdot \textit{Financing Stage}_{s,m,t} + \delta_{FE} + \epsilon_{\varphi,m,t}$$

where i indexes individual venture capitalists and a indexes the staging in which a given VC engages. The unit of observation is a potential VC investment by VC i in location m for sector s at staging a in year t. Since the financing stage indicator cannot be reconstructed to capital type level, I assign input weight to the latent capital supply faced by a firm in a given sector s before aggregating:

$$K^e_{\varphi(s),m,t} = \sum_{\varphi} \omega^D_{\varphi,s} K^e_{\varphi,m,t}$$

where $\omega^D_{\varphi,s}$ is the fraction of type φ capital used by sector$s$ on average given by the BEA input-output table. The layer of heterogeneous demand for a specific type of capital across sectors captures additional variations in the latent supply of vintage capital.

The differential effects of latent vintage capital supply across financing staging are summarized by the coefficient β_1. The second coefficient β_2 demonstrates, again, how local supply shapes venture capital investment choice. The third term controls for the staging fixed effect, which is supposed to be increasing in the staging as higher staging implies expansion of business. Regressions include fixed effects that control for location (MSA level) $\times$ year and sector (2-digit SIC level), netting out the industrial differentials or local economic trends correlated with both supply and new entrants.

Table 2.3 presents the results. In column (1), I report the baseline sensitivity of all VC investments to the local vintage capital measure. The estimates yield a similar result to our previous specification. In column (2), I use the staging information reported by the VentureXpert database to measure the financing stage of VC-backed firms. This confirms the stylized fact about the firm's life cycle that it enters rapid expansion during later financing stages. The interaction with the financing stage is significant and negative. As start-ups are mature and less financially constrained, the VC investment in such firms becomes less sensitive to local vintage capital supply. In column (3), I adopt the firm's age as an alternative proxy of the financing stage and yield very similar estimates.

Table 2.3: VC Investment Response to Vintage Capital Supply

	(1)	(2) *(staging)*	(3) *(age)*
$\log(K^e_{\varphi(s),m,t}+1)$	0.082***	0.142***	0.102***
	(0.004)	(0.005)	(0.003)
Financing Stage		1.978***	0.825***
		(0.019)	(0.013)
$\log(K^e_{\varphi(s),m,t}+1) \times \mathit{Financing\ Stage}_{s,m,t}$		-0.064***	-0.052***
		(0.003)	(0.002)
Fixed Effect	Yes	Yes	Yes
N	81973	81973	81973
adj. R^2	0.683	0.801	0.753

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

To supplement the argument for the causal effects of vintage capital supply on VC investment choice, I then exploit the demand variation due to asset specificity. As argued by Kermani and Ma (2020) and Lanteri (2018), asset specificity is an important component in determining the demand for such assets. Investment in sectors that demand more specific assets should prefer less local vintage capital as the seasoned capital has been at least partially specifically designed or manufactured for the original owner.

To this end, I construct two measures of asset specificity at the capital type level. First, I follow Rauch (1999) and classify the asset type based on whether they are traded on organized exchanges, which has been used as a proxy for specificity (Nunn, 2007; Barrot and Sauvagnat, 2016). For robustness, I adopt an alternative proxy for specificity using the mobility index of assets proposed by Kermani and Ma (2020). A highly specific piece of asset is more likely to involve expensive re-installation costs and transportation costs. I extract the transportation cost share from the BEA input-output table at the commodity level.

With these additional variations in demand for local vintage capital, I run the analog investment choice model estimation:

$$\log(I^{VC}_{i,s,m,t}) = \beta_1 \cdot \sum_{\varphi \in \Psi(s)} \log(K^e_{\varphi,m,t} + 1) \times \textit{Asset Specificity}_{\varphi,m,t} + \beta_2 \cdot \log(K^e_{\varphi(s),m,t} + 1)$$
$$+ \beta_3 \cdot \sum_{\varphi \in \Psi(s)} \textit{Asset Specificity}_{\varphi,m,t} + \delta_{FE} + \epsilon_{\varphi,m,t}$$

where $\Psi(s)$ is the set of assets that are demanded in sector s. The regression includes the direct effect of asset specificity on VC investment. It is not surprising to see a positive correlation between asset specificity and VC investment as such capital can still be expensive conditional on the firm-asset match is good. The interaction term captures the differential effects of local capital supply across sectors that are dependent on asset specificity differently.

Table 2.4 presents similar results even though the sensitivity of local capital is diluted after controlling for asset specificity. The estimates of interacted terms are negative and significant, reflecting lower sensitivity to local capital supply when the asset specificity demanded is high.

Table 2.4: VC Investment Response to Asset Immobility and Specificity

	(1)	(2)
$\log(K^e_{\varphi(s),m,t}+1)$	0.022***	0.021***
	(0.005)	(0.004)
Transportation cost share	1.043***	
	(0.256)	
$\sum_{\varphi\in\Psi(s)}\log(K^e_{\varphi,m,t}+1)\times \textit{Transportation cost share}_{\varphi,m,t}$	-0.082**	
	(0.036)	
Asset Specificity		0.570***
		(0.163)
$\sum_{\varphi\in\Psi(s)}\log(K^e_{\varphi,m,t}+1)\times \textit{Asset Specificity}_{\varphi,m,t}$		-0.034***
		(0.010)
Fixed Effect	Yes	Yes
N	30275	30275
adj. R^2	0.241	0.155

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

2.4 A Model of Vintage Capital induced VC investment

Building on the empirical evidence presented above, I build a model that connects the empirical finding on VC investment choice as a function of local vintage capital supply to entrepreneurship and resources allocation. In this section, I firstly begin with a simple static partial equilibrium model that features used capital reallocation and firm-to-VC matching. I impose market clearing in the market for used capital and derive analytical results on the equilibrium VC investment and entrepreneurship (entry). To better highlight the role of local vintage capital in interacting with firms and venture capitalists, I abstract from endogenous location choices by both individuals and venture capitalists, and I further take the local vintage capital as exogenously given at the beginning of the economy.

2.4.1 Environment

I start by focusing on a single location with an exogenous mass of individuals residing in the region denoted by N. Conditional on location, each individual is ex-ante homogeneous in talents z at birth. Once the economy begins to operate, each individual subsequently draws type $x \sim G(\cdot)$. This heterogeneity can be interpreted as serendipity shock or a pairwise location-individual level shock that subsumes many uncertain local interactions and affect productivity, such as being acquainted with the right people at the right time. The ex-post productivity of an individual is captured by a simple multiplicity of her talent and serendipity: $\mathcal{Z} \equiv z \cdot x$. For simplicity, let the ex-post productivity distribution denote by $F(\cdot)$ with a lower bound at 0. Upon the realization of ex-post shock, an individual may face an occupational choice between becoming an entrepreneur by converting the productivity to managerial ability in a one-to-one fashion and working as local labor at $\mathcal{Z}^{\gamma}$ effective working hours. The occupational choice shock depends on whether the individual successfully obtains financing from a venture capitalist, which is extensively discussed later. The power parameter γ captures the elasticity of effective labor supply with respect to underlying individual ex-post productivity.

Preference: For the simple exposition, I assume the preference takes simple logarithm form on final good consumption without leisure utility:

$$U = \log(C) \tag{2.1}$$

This implies that each individual simply maximizes their net surplus/income and supplies their labor inelastically whenever possible. Furthermore, there is no additional residing cost in the simple single-location model for individuals.

Technological Assumptions and Production: A competitive final good manufacturer is assumed to produce a final good bundle using locally produced differentiated intermediate inputs via a standard constant elasticity of substitution technology with parameter $\sigma > \gamma + 1$.

$$Y = \left[\int_{\mathcal{J}} y(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \tag{2.2}$$

where $y(j)$ is the quantity of input j demanded for the final good bundle, and $\mathcal{J}$ is the endogenously determined set of intermediate inputs produced by the entrepreneurs in the resided location.

Production - Intermediate input: Recall that entrepreneurs bear ex-post heterogeneous productivity $\mathcal{Z}$ which amounts to hicks-neutral managerial ability in a standard Cobb-Douglas production function:

$$y(j) = \overline{\alpha}\mathcal{Z}(j) \cdot k(j)^{1-\alpha} l(j)^{\alpha} \tag{2.3}$$

where $l(j)$ is labor demand in efficiency units for the production of variety j and $k(j)$ is capital demand for the production of variety j and $\overline{\alpha} \equiv \alpha^{-\alpha}(1-\alpha)^{\alpha-1}$ is a demand shifter for normalization.

Financial Frictions for Capital Acquisition: To operate, an entrepreneur needs to acquire capital to materialize the production. Yet each entrepreneur is assumed to possess no wealth or credibility to obtain finance in the credit market before operation.[2] Thus, each potential entrepreneur must first search for a venture capitalist to receive financing ability. I further assume there is no extra cost in searching for venture capitalists, which immediately follows that all individuals first participate in the venture capital market for a match before making an occupational choice. Once a match is successful, the two parties split the surplus through simple Nash bargaining with bargaining power β assigned to the entrepreneur given the nature of equity holding relationship between venture capitalist and entrepreneur.

Matching with Venture Capitalist: Let the endogenously determined mass of VCs chosen to invest in the location denoted by V. The market tightness for venture capital investment is given by the ratio of the mass of venture capitalists over the mass of individuals in the location:

$$\theta = \frac{V}{N} \tag{2.4}$$

The matching function is given by $\mathcal{M}(\theta)$ which follows the probability at which an individual successfully match with a venture capitalist is $\lambda(\theta) = \frac{\mathcal{M}(\theta)}{N}$ and the probability at which a venture capitalist meet with an individual is $q(\theta) = \frac{\mathcal{M}(\theta)}{V} = \frac{\lambda(\theta)}{\theta}$. I impose concavity on the meeting technology such that $\lambda(\theta) \to 1$ and $\lambda'(\theta) \to 0_+$ when $\theta \to \infty$ and $\lambda(\theta) \to 0$ and $\lambda'(\theta) \to \infty$ when $\theta \to 0$.

Conditional on a match, an individual decides whether to start a business or not. If deciding to become an entrepreneur, such an individual is then granted the financing capability for capital acquisition by her paired VC. In particular, each entrepreneur paired with VC can then source capital through two channels in the static framework. First, a firm can invest newly produced capital, supplied perfectly elastically from some capital goods producer outside of this

[2]Each entrepreneur can pay wage after realizing revenue, which is a common timing assumption that fits empirical facts.

economy or self-produced using simple linear technology at one unit of the final good bundle. Alternatively, a firm can invest in used capital available in the local region. Furthermore, following Lanteri (2018), the substitutability between new and vintage capital is imperfect, which means a firm needs to bundle the vintage capital with some newly produced capital to make the capital workable.

Formally, the investment technology takes the sum of the capital amount sourced from two capital sourcing channels:

$$k(j) = \tilde{I}_n(j) + \Delta(I_n(j), I_v(j)) \tag{2.5}$$

where $\tilde{I}_j^n$ is the investment in newly-produced capital and $\Delta(\cdot)$ is the investment in capital bundle which takes both new capital $I_n(j)$ and vintage capital $I_v(j)$. I take a special case of capital bundle function for a simple exposition of the mechanism:

$$\Delta(I_n(j), I_v(j)) = \overline{\eta} I_n^{\eta}(j) I_v^{1-\eta}(j), \quad \overline{\eta} = \eta^{-\eta}(1-\eta)^{\eta-1} \tag{2.6}$$

The cost of a unit of capital bundle in the unit of final good in the location is

$$r_\Delta = \frac{1}{\chi} r_v^{1-\eta} \tag{2.7}$$

where r_v is the unit cost of vintage capital. This immediately follows that the capital rent at equilibrium must take the lower cost of capital choice given the investment technology:

$$r = \min\{r_\Delta, 1\} \tag{2.8}$$

Throughout this paper, I focus on the scenario where there is no pure new investment channel.[3]

[3]Or alternative assumptions on parameters is required to ensure $r_\Delta < 1$, e.g. sufficiently large χ.

2.4.2 Selection and Matching

Solving the cost minimization problem of the final good bundle by taking the set of entrepreneurs as given yields the demand for intermediate input:

$$y(j) = (\frac{P}{p(j)})^{\sigma} Y \tag{2.9}$$

where P summarizes the final good price index $P = \left(\int_{\mathcal{J}} p(j)^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}$ and Y captures the economic size of the location in the unit of final good bundle.

Similarly, solving the cost minimization problem of entrepreneur obtains the marginal cost of input:

$$mc(\mathcal{Z}) = \frac{1}{\mathcal{Z}} r^{\alpha} w^{1-\alpha} \equiv \frac{c}{\mathcal{Z}} \tag{2.10}$$

where both cost of capital r and wage rate w are in the unit of final good. Given the demand function derived as in (9) and monopolistic competition structure, the profit-maximizing price for each intermediate input displays a constant markup over marginal cost:

$$p(\mathcal{Z}) = \frac{\sigma}{\sigma - 1} mc(\mathcal{Z}) \equiv \overline{\sigma} \cdot mc(\mathcal{Z}) \tag{2.11}$$

Substituting out $p(j)$ in the price index with (11) allows us to express the aggregate productivity of the economy as a function of wage and capital cost:

$$\mathbb{Z} \equiv \left[\lambda(\theta) N\right]^{\frac{1}{\sigma-1}} \left(\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})\right)^{\frac{1}{\sigma-1}} = \overline{\sigma} c \tag{2.12}$$

where $c = r^{\alpha} w^{1-\alpha}$ and $\underline{\mathcal{Z}}$ is the cutoff above which the individual paired with a VC chooses to become an entrepreneur. The aggregate productivity of the economy depends on the population size of the location N and the probability at which an individual match with a venture capitalist. The multiplicity of the two gives the mass of individuals who meet a venture capitalist while $\lambda(\theta)N(1 -$

$F(\underline{\mathcal{Z}}))$ captures the mass of entrepreneurs. The expression in (2.12) also suggests that both real wage and real capital rent are increasing in aggregate productivity, indicating the selection forces stemming from stronger labor demand from more productive entrepreneurs/firms.

Leveraging the connection between marginal cost and aggregate productivity allows us to further rewrite the demand function:

$$y(\mathcal{Z}) = (\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma} Y \tag{2.13}$$

the relative productivity $\frac{\mathcal{Z}}{\mathbb{Z}}$ plays a key role in determining its revenue share. Combine with equations (2.10) and (2.11), the operating profit becomes:

$$\pi(\mathcal{Z}) = \frac{1}{\sigma}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1} Y \tag{2.14}$$

Occupational choice and selection: Individuals choose their occupation by comparing their prospective entrepreneurial profit given by (2.14) with their labor income $w \cdot \mathcal{Z}^{\gamma}$ conditional on meeting with a venture capitalist successfully in the first place. The indifference condition characterizes the selection cutoff $\underline{\mathcal{Z}}$:

$$\beta\pi(\underline{\mathcal{Z}}) = w \cdot \underline{\mathcal{Z}}^{\gamma} \quad \Rightarrow \quad \underline{\mathcal{Z}}^{\sigma-1-\gamma} = \frac{\sigma}{\beta}\mathbb{Z}^{\sigma-1}\frac{w}{Y} \tag{2.15}$$

Given that $\sigma > \gamma + 1$, the selection is tougher when the average productivity is higher since it is more difficult to compete against more productive and numerous firms for labor if holding wage and total income constant. Finally, holding other constant, higher wage also levels up the selection cutoff because higher wage means a more favorable outside option. Nevertheless, note that the equilibrium effect induced by a higher selection cutoff is endogenously average productivity and real wage, which means a better understanding of the relationship across selection cutoff, average aggregate productivity, and real wage requires solving the equilibrium via labor market cleaning.

The selection cutoff not only demonstrates individual occupational choice when taking wage, demand and aggregate productivity as given but also links to the effective labor demand and supply. Firms with ex-post productivity above $\underline{\mathcal{Z}}$ demand labor at

$$l(\mathcal{Z}) = \frac{1-\alpha}{\bar{\sigma}w}\left(\frac{\mathcal{Z}}{\mathbb{Z}}\right)^{\sigma-1} Y$$

Thus the aggregate labor demand given selection cutoff $\underline{\mathcal{Z}}$ is

$$L^D = \lambda(\theta) N \int_{\underline{\mathcal{Z}}} l(\mathcal{Z}) dF(\mathcal{Z}) = \frac{1-\alpha}{\bar{\sigma}w} Y \tag{2.16}$$

It is noticeable that the aggregate labor demand is simply derived from the fact that the labor income share in the local economy is $\frac{1-\alpha}{\bar{\sigma}}$ of the aggregate demand/income.

On the other hand, the aggregate labor supply given the selection cutoff is given by

$$L^S = \underbrace{(1-\lambda(\theta)) \cdot N \int \mathcal{Z}^\gamma dF(\mathcal{Z})}_{\text{individuals fail to match with VC}} + \underbrace{\lambda(\theta) \cdot N \int_0^{\underline{\mathcal{Z}}} \mathcal{Z}^\gamma dF(\mathcal{Z})}_{\text{matched but below selection cutoff}} \tag{2.17}$$

Imposing the labor market clearing condition with $L^S = L^D$ together with the indifference condition (2.15) characterize the selection cutoff equilibrium as a function of VC finance market tightness θ.

Proposition 1 (Selection and Matching): Given the population size of the location, N, the productivity distribution $F(\cdot)$ and VC-to-population ratio θ, the selection cutoff exists and is unique. Furthermore, the selection is tougher when more VCs decide to invest in the location.

Proof: Using equation (2.15),(2.16),(2.17) to eliminate $w, Y, \mathbb{Z}$ yileds an implicit solution for $\underline{\mathcal{Z}}$:

$$\beta\underline{\mathcal{Z}}^{\sigma-\gamma-1}\left[\int \mathcal{Z}^{\gamma}dF(\mathcal{Z})-\lambda(\theta)\int_{\underline{\mathcal{Z}}}\mathcal{Z}^{\gamma}dF(\mathcal{Z})\right]=\lambda(\theta)(1-\alpha)(\sigma-1)\int_{\underline{\mathcal{Z}}}\mathcal{Z}^{\sigma-1}dF(\mathcal{Z}) \tag{2.18}$$

Since $\sigma > \gamma + 1$, this implies the left-hand side of the equation is strictly increasing in $\underline{\mathcal{Z}}$ from 0 to infinity while the right-hand side is strictly decreasing in $\underline{\mathcal{Z}}$ and goes to zero when taking the selection cutoff to infinity. Furthermore, both sides are differentiable (thus continuous) in $\underline{\mathcal{Z}}$, ensuring the unique solution of $\underline{\mathcal{Z}}$. Moreover, note that the population size is taken exogenously given. It is easy to show that the left-hand side of an equation (2.18) is strictly decreasing in θ while the right-hand side is strictly increasing in θ. Therefore, a higher market tightness due to a greater mass of VC investing in the location leads to a higher selection cutoff. □

The basic intuition is that greater finance accessibility means more entrants, leading to tougher competition and thereby lifting the selection cutoff. The following Corollary highlights the implications of more VC investment.

Corollary 1: Given population size N, the aggregate productivity is strictly increasing in VC investment.

Proof: See Appendix 2.A.

The greater finance accessibility contributes to higher aggregate productivity in two channels: (1) it increases entrepreneurship at an extensive margin; (2) more entrepreneurship opportunities induce tougher selections, increasing the productivity at the intensive margin. Nevertheless, the higher selection cutoff can offset excessively offset the entrepreneurship from the extensive margin as the VC choose over relatively more productive start-ups trading off a large number of less profitable business opportunities when the distribution of ex-post productivity is a heavy tail, e.g., Pareto distribution.

The above two results summarize how the VC-firm matching market affects the selection cutoff and aggregate productivity. I move forward to examine the role of vintage capital supply in shaping the interplay across VC investment decisions, VC-firm matching induced selection, and the local agglomeration effect.

2.4.3 Vintage Capital Supply and Agglomeration

In the static framework, I fix the mass of vintage capital at K_v^S, which is not in the usage of any firms in the economy but owned by a latent local government. The capital demand for a firm with $\mathcal{Z}$ is given by:

$$k(\mathcal{Z}) = \frac{\alpha}{\overline{\sigma} r}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y \tag{2.19}$$

Given the investment technology, this implies that the demand of vintage capital at firm level is characterized as

$$I_v(\mathcal{Z}) = (1-\eta)\frac{r}{r_v}k(\mathcal{Z}) = (1-\eta)\frac{\alpha}{\overline{\sigma} r_v}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y$$

Integrating over the mass of entrepreneurs, the aggregate demand of vintage capital is

$$K_v^D = \lambda(\theta)N\int_{\underline{\mathcal{Z}}} I_v(\mathcal{Z})dF(\mathcal{Z}) = (1-\eta)\frac{\alpha}{\overline{\sigma} r_v}Y \tag{2.20}$$

I do not focus on the unemployed state of capital due to the match frictions in the vintage capital market and assume there is an equilibrium price r_v that clears the market. This amounts to the capital market clearing condition:

$$K_v^D = K_v^S$$

Given the fixed supply in the static framework, one can express the aggregate equilibrium demand/income as a function of K_v^S and its associated cost:

$$Y = \frac{\overline{\sigma} r_v}{(1-\eta)\alpha} K_v^S \tag{2.21}$$

Not surprisingly, holding other constant, a more abundant supply of vintage capital implies higher aggregate income for the local economy. This expression, together with the labor market clearing condition, allows us to connect the expenditure share of local vintage capital to the that of labor at equilibrium:

$$\alpha w L^S = \frac{(1-\alpha)}{(1-\eta)} r_v K_v^S \tag{2.22}$$

Since that equilibrium labor supply is a function of selection cutoff, this implies the relative cost of vintage capital in the unit of labor is a function of selection cutoff. A higher selection cutoff means tougher competition and higher aggregate productivity, which means the competition for vintage capital is also tougher, thus pushing up the relative cost of vintage capital in the unit of labor cost. On the other hand, recall that unit cost of input is a function of selection cutoff: $\overline{\sigma} r^{\alpha} w^{1-\alpha} = \mathbb{Z}$. Higher aggregate productivity pushes up the cost of labor and capital simultaneously. Therefore, the two conditions amount to the identification of real wage and vintage capital cost.

Lemma 1: Let $H(\underline{\mathcal{Z}};\theta) \equiv (1-\lambda(\theta)) \cdot \int \mathcal{Z}^{\gamma} dF(\mathcal{Z}) + \lambda(\theta) \cdot \int_0^{\underline{\mathcal{Z}}} \mathcal{Z}^{\gamma} dF(\mathcal{Z})$. Given the productivity distribution $F(\cdot)$, VC finance market tightness θ, the real wage is increasing in vintage capital stock and real rent of vintage capital is decreasing in the stock. Formally,

$$w = \overline{\chi}_w \cdot (K_v^S)^{\frac{\alpha(1-\eta)}{1-\alpha\eta}} H(\underline{\mathcal{Z}};\theta)^{-\frac{\alpha(1-\eta)}{1-\alpha\eta}} N^{\frac{1}{1-\alpha\eta}\left[\frac{1}{\sigma-1}-\alpha(1-\eta)\right]} \left[\lambda(\theta) \int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})\right]^{\frac{1}{(1-\alpha\eta)(\sigma-1)}} \tag{2.23}$$

$$r_v = \frac{\alpha(1-\eta)}{1-\alpha} \overline{\chi}_w \cdot (K_v^S)^{-\frac{1-\alpha}{1-\alpha\eta}} H(\underline{\mathcal{Z}};\theta)^{\frac{1-\alpha}{1-\alpha\eta}} N^{\frac{1}{1-\alpha\eta}\left[\frac{1}{\sigma-1}+1-\alpha\right]} \left[\lambda(\theta) \int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})\right]^{\frac{1}{(1-\alpha\eta)(\sigma-1)}} \tag{2.24}$$

Notice that the real wage is only strictly increasing in the population size N when the input market competition is not that tough (low σ), or labor share is low (high α). Since the population elasticity of aggregate productivity is $\frac{1}{\sigma-1}$, this implies the population elasticity of wage has to be adjusted to ensure the population elasticity of marginal cost is also $\frac{1}{\sigma-1}$ given equation (2.12). A high capital share implies a weak rent response to population size, leaving lower downward pressures on wages. Furthermore, a high capital share means a lower labor share, indicating an already high sensitivity of wages to the population. In other words, a higher capital share makes capital rent increase in population size with a 'milder' sense, which does not depress the wages when population-scale up. On the other hand, high elasticity of substitution ensures higher profits for firms which prevents making the wages decrease in population due to higher labor supply.

Throughout the paper, I assume that $(1-\alpha)(\sigma-1) \leq 1$ corresponds to the set of well-documented empirical facts on lower labor share and higher markup (less competition) in the US industry.

Proposition 2 (Agglomeration): (1) When both the population size and vintage capital stock are exogenously given, the economy demonstrates the agglomeration effect with respect to population size if $\frac{1}{\sigma-1} > (1-\alpha)$; (2) the agglomeration effect is complemented by vintage capital stock holding θ constant.

$$\frac{\partial w}{\partial N} > 0 \quad and \quad \frac{\partial \frac{Y}{N}}{\partial N} > 0$$
$$\frac{\partial^2 w}{\partial N \partial K_v^S} > 0 \quad and \quad \frac{\partial^2 \frac{Y}{N}}{\partial N \partial K_v^S} > 0$$

where

$$Y = \frac{\overline{\sigma}}{1-\alpha}\overline{\chi}_w \cdot (K_v^S)^{\frac{\alpha(1-\eta)}{1-\alpha\eta}} H(\underline{\mathcal{Z}};\theta)^{\frac{1-\alpha}{1-\alpha\eta}} N^{\frac{1}{1-\alpha\eta}\left[\frac{1}{\sigma-1}+1-\alpha\right]} \left[\lambda(\theta) \int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})\right]^{\frac{1}{(1-\alpha\eta)(\sigma-1)}} \tag{2.25}$$

By fixing the market tightness θ, the above result demonstrates the direct channel of availability of vintage capital goods in increasing per-capita income and real wage rate through cost reduction on the capital bundle.

Moreover, on the top of this conventional channel, this result features the causal relationship between vintage capital stock and venture capital participation/investment. Treating θ as a parameter, a higher matching rate with venture capitalists immediately implies a higher wage rate due to tougher competition and greater aggregate productivity. The higher aggregate productivity further pushes up the per-capita income.

Corollary 2: If $\frac{1}{\sigma-1} \geq (1-\alpha)$, wage rate and income per capita are strictly increasing in the VC investments given location population N:

$$\frac{\partial w}{\partial \theta} > 0, \quad \frac{\partial \frac{Y}{N}}{\partial \theta} > 0$$

Proof: See Appendix 2.A.

2.4.4 Entry of Venture Capital and Equilibrium

Each venture capital is assumed to be ex-ante homogeneous. Upon residing in a location, each requires to pay a fixed cost in the unit of the local final good bundle at f_{VC} as a prerequisite for start-up searching. The expected value of the entry is then given by

$$J_{VC} = (1-\beta)q(\theta)\int_{\underline{\mathcal{Z}}} \pi(\mathcal{Z})dF(\mathcal{Z}) \tag{2.26}$$

The free entry condition implies

$$J_{VC} = f_{VC}$$

Recall that the selection cutoff is increasing in θ from equation (2.18). This implies a higher θ can reduce the value of venture capital in two channels: (1) lower probability of meeting a potential entrepreneur and (2) conditional on a meeting, the ex-post productivity of the young entrepreneur is more likely to be not competent enough due to higher selection cutoff. This amounts to a relationship between θ and per-capita income.

$$J_{VC} = \frac{1-\beta}{\sigma} q(\theta) \frac{1}{\lambda(\theta) N} Y = \frac{1-\beta}{\sigma\theta} \frac{Y}{N} \tag{2.27}$$

From this point, holding θ as a constant, together with the result derived in Proposition 2, the agglomeration effect not only leads to greater per-capita welfare but also the value of venture capital. This implies the negative impact of higher θ on VC's value is partially offset by the general equilibrium effect given that $\frac{\partial \frac{Y}{N}}{\partial \theta} > 0$ shown in Corollary 2.

Furthermore, recall that $\theta = \frac{V}{N}$, this leads to an explicit expression that links the participation of venture capitalists and residing/searching cost:

$$\frac{Y}{N} = \frac{\sigma f_{VC}}{1-\beta} \cdot \theta \quad \Rightarrow \quad V = \frac{1-\beta}{\sigma} \frac{Y}{f_{VC}} \tag{2.28}$$

Note that the demand curve for VC investment in equation (2.25) is non-decreasing in θ, and the supply curve captured by (2.28) is strictly increasing in θ. This follows that, for some f_{VC}, there exists a unique solution θ^* solving the equilibrium. Furthermore, since both demand and supply are (weakly) upward sloping, it immediately indicates an amplification channel facilitated by venture capitalists when the vintage capital stock increases.

Proposition 3 (Amplification): Higher vintage capital stock induces greater aggregate per-capita income, attracting more venture capitalists. More venture capital participation reduces misallocation and induces tougher competition and greater aggregate productivity, further pushing up per-capita income.

2.5 A Model of Endogenous Vintage Capital Stock and Location Choice

This section focuses on a simple infinite horizon environment along a steady-state, inheriting the critical components from the static framework. On top of a dynamic setting, I further endogenize the vintage capital stock by linking the capital stock previously owned by the exiting incumbents to the locally available vintage capital supply.[4] Finally, this framework features a location choice decision by both venture capitalists and individuals.

Preference: Time is discrete, and its horizon is infinite. The discount factor is ρ. Each individual maximizes their income per period and is hand-to-mouth.

A Simple Dynasty: Newbie and Retirement: Each individual faces a retirement shock arriving at Poisson rate δ regardless of her occupation. Upon anyone's retirement, a new individual is born in the economy and draws her talent z from a distribution $\Phi(\cdot)$ and sorts into locations indexed by $o \in \mathcal{O}$. Upon residing, each individual draws location-specific shock x from an independent distribution $G(\cdot)$ which amounts to ex-post productivity $\mathcal{Z} = z \cdot x$. After realizing their ex-post productivity, each newbie of a given location has a chance to be paired with a venture capitalist, which amounts to a commercial opportunity as an entrepreneur of a start-up. I assume each individual can only enter the VC market once for the simple exposition. That is, if an individual fails to match with a VC, she has to be a worker since then.[5] Furthermore, relocation or

[4] I abstract from the case where the incumbent firms optimally sell the old capital to invest in new equipment or relax the financial distress due to idiosyncratic shocks and leave them for future study.

[5] Allowing individuals always to possess an opportunity to start a business generates no conflicting implications of our simple model.

immigrants are not allowed.

Location and Residing cost: There is a set of ex-ante homogeneous locations indexed $\mathcal{O}$. I assume no latent characteristics of the location, and the sorting and ex-post heterogeneity stems from assignment problems from the individual's talent z to the location's population $N(z)$ in the spirit of Behrens et al. (2014). That is, each city is assigned a specific talent at equilibrium. There is an upfront fixed cost for residing in a location o with population N at ψN^{ζ}.[6] Intuitive interpretation for the increasing residing cost in population size includes higher commuting costs and housing prices in population-dense areas.

2.5.1 Production and Capital Stocking

The production function and investment technology are summarized by the same settings from equations (2.2) - (2.8) in the static framework. For an entrepreneur with ex-post productivity $\mathcal{Z}$, she employs a measure of capital k_{-1} during the previous period and starts the current period with $(1-d)k_{-1}$ due to depreciation. She then chooses the amount of capital bundle $\Delta(I_n, I_v)$ to maximize its expected discounted profits/surplus taking the wages and capital cost as given. As in the static model, there is no additional fraction in sourcing for vintage capital.[7] After the investment in the current period, she starts to produce with capital stock k.

The period profit/surplus is therefore characterized by $\pi(\mathcal{Z}, k) =$

[6] An implicit assumption is that the economy admits a bank system can only provide housing loans to individuals but not to business.

[7] For example, allocation of vintage capital can be subject to search/match frictions. See Ottonello (2017), Ramey and Shapiro (2001), Gavazza (2016).

$p(\mathcal{Z})y(\mathcal{Z},k) - wl(\mathcal{Z}) - r\Delta(I_n, I_v)$, and an entrepreneur's problem (before surplus split with VC) conditional on $(\mathcal{Z}, k_{-1})$ and location o follows:

$$J_o^E(\mathcal{Z}, k_{-1}) = \max_{\{p,y,l,\Delta(\cdot)\}} p \cdot y(Z,k) - wl - r\Delta + \rho\big[\delta R_v(\mathcal{Z},k) + (1-\delta)J_o^E(\mathcal{Z},k)\big] \tag{2.29}$$

where δ is the rate of retirement, $R_v(\mathcal{Z},k)$ is the resale revenue of vintage capital conditional on receiving the retirement shock

$$R_v(\mathcal{Z},k) = r_v \cdot (1-d)k$$

and Δ is the investment in capital bundle subject to law of motion of capital stock:

$$k = (1-d)k_{-1} + \Delta$$

I am focusing on the steady-state equilibrium where each firm is continually operating at optimal capital stock as it can always purchase the capital bundle and recover to the optimal level immediately. It turns out that the recursive problem can be simplified into a simple static problem where each firm chooses the optimal capital level at a discounted presented cost of capital by internalizing the longevity of assets and the value of resale.

Lemma 2: In a steady-state Equilibrium, the dynamic problem faced by a firm with ex-post productivity $\mathcal{Z}$ is equivalent to

$$\max_{p,y,l,k} p \cdot y - wl - \big[1 - \rho(1-d)[\delta\frac{r_v}{r} + (1-\delta)]\big]rk \tag{2.30}$$

Proof: Rewrite the recursive problem in equation (2.29) by substituting Δ out:

$$\begin{aligned} J_o^E(\mathcal{Z},k_{-1}) &= \max_{\{p,y,l,\Delta(\cdot)\}} p\cdot y(Z,k) - wl - r[k-(1-d)k_{-1}] + \rho\big[\delta R_v(\mathcal{Z},k) + (1-\delta)J_o^E(\mathcal{Z},k)\big] \\ &= r(1-d)k_{-1} + \underbrace{\max_{\{p,y,l,\Delta(\cdot)\}}\{p\cdot y(Z,k) - wl - rk + \rho\big[\delta R_v(\mathcal{Z},k) + (1-\delta)J_o^E(\mathcal{Z},k)\big]\}}_{\text{constant denoted by } B} \end{aligned}$$

Given the steady state, the maximized sub-problem is a constant. It follows $J_o^E(\mathcal{Z},k) = r(1-d)k + B$ and

$$J_o^E(\mathcal{Z},k_{-1}) = r(1-d)k_{-1} + \max_{\{p,y,l,\Delta(\cdot)\}} \{p \cdot y(Z,k) - wl - rk + \rho\big[\delta R_v(\mathcal{Z},k) + (1-\delta)[r(1-d)k + B]\big]\}$$

Substituting out $R_v(\mathcal{Z},k)$ we have reduced firm's problem to

$$\max_{\{p,y,l,\Delta(\cdot)\}} p \cdot y(Z,k) - wl - rk + \rho\big[\delta(1-d)\frac{r_v}{r} + (1-\delta)(1-d)\big]rk$$

□

Firm recognizes the longevity of assets reflected by $(1-d)(1-\delta)rk$ as the capital purchase today also saves the investment next period. Furthermore, the resale value of asset as vintage capital summarized by $\delta(1-d)r_v k$. Define $\tilde{r}(r) \equiv \big[1 - \rho(1-d)[\delta\frac{r_v}{r} + (1-\delta)]\big] \cdot r$ as the adjusted cost of capital. The firm's marginal cost, revenue, labor and capital demand are

$$mc(\mathcal{Z}) = \frac{1}{\mathcal{Z}}\tilde{r}^{\alpha}w^{1-\alpha}$$

$$p \cdot y(\mathcal{Z}) = (\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma}Y$$

$$l(\mathcal{Z}) = \frac{1-\alpha}{\overline{\sigma}w}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y$$

$$k(\mathcal{Z}) = \frac{\alpha}{\overline{\sigma}\tilde{r}}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y$$

where $\mathbb{Z} = [\lambda(\theta)N]^{\frac{1}{\sigma-1}}(\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z}))^{\frac{1}{\sigma-1}}$ summarizing the aggregate productivity. [8]

[8] To see why the mass of firms in the local economy is $\lambda(\theta)N[1-F(\mathcal{Z})]$ along the path steady-state equilibrium, let the mass of firms denote by h and consider the outflow and inflow of firms:

$$\underbrace{\delta \cdot h}_{\text{outflow}} = \underbrace{\delta N\lambda[1-F(\mathcal{Z})]}_{\text{inflow}} \quad \Rightarrow \quad h = \lambda N[1-F(\mathcal{Z})]$$

By solving the privately optimal allocations, one can obtain the explicit expression of the value of a firm at state $(\mathcal{Z}, N)$ at a steady-state:

$$J_o^E(\mathcal{Z},k) = \frac{1}{\overline{\sigma}[1-\rho(1-\delta)]}\left[\overline{\sigma} - (1-\alpha) - \alpha\frac{r}{\tilde{r}}[d-(1-d)\rho\delta\chi r_v^{\eta}]\right](\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y$$

Notice that when the firm has optimal capital stock in the previous period, it invests dk^* amount of capital bundle to hire back to the optimal level after the depreciation for the next period. On the other hand, the new entrant with zero capital stock immediately stocks up to the optimal capital level. The recursive problem of a new entrant is given by

$$J_o^E(\mathcal{Z},0) = p^*y^*(\mathcal{Z}) - wl^* - rk^* + \rho[(1-\delta)J_o^E(\mathcal{Z},k) + \delta R_v(\mathcal{Z},k^*)] \tag{2.31}$$

where the revenue, labor demand and capital demand are the solutions to equation (2.30). Using the solution to $J_o^E(\mathcal{Z},k)$, the solution to the entrant problem ends up with a straightforward form:

$$J_o^E(\mathcal{Z},0) = \frac{1}{1-\rho(1-\delta)}\frac{1}{\sigma}(\frac{\mathcal{Z}}{\mathbb{Z}})^{\sigma-1}Y \tag{2.32}$$

The solution is intuitive since the value of an entrant is simply earning discounted present profit stream with the adjusted capital cost upon its birth.

2.5.2 Worker's Problem

If an individual with ex-post productivity $\mathcal{Z}$ chooses to work as labor, it provides $\mathcal{Z}^{\gamma}$ efficiency unit of labor at equilibrium local wage w. There is no unemployment state for a worker, but each individual is subject to retirement shock regardless of her occupation. Formally, the recursive problem of a worker is

$$J^W(\mathcal{Z}) = \mathcal{Z}^r \cdot w + \rho(1-\delta)J^W(\mathcal{Z})$$

Along the steady-state equilibrium, the solution is

$$J^W(\mathcal{Z}) = \frac{1}{1-\rho(1-\delta)} \mathcal{Z}^r \cdot w \tag{2.33}$$

2.5.3 Occupational Choice and Selection

The characterization of steady-state recursive problems yields precisely the same solution we derived in the state model.

$$\max\{\beta J^E(\mathcal{Z},0), J^W(\mathcal{Z})\} \Rightarrow \underline{\mathcal{Z}}^{\sigma-1-\gamma} = \frac{\sigma}{\beta} \mathbb{Z}^{\sigma-1} \frac{w}{Y}$$

Not surprisingly, together with the labor market clearing at each period, the steady-state selection cutoff in the dynamic setting confronts the same condition:

$$\beta \underline{\mathcal{Z}}^{\sigma-\gamma-1} \left[\int \mathcal{Z}^\gamma dF(\mathcal{Z}) - \lambda(\theta) \int_{\underline{\mathcal{Z}}} \mathcal{Z}^\gamma dF(\mathcal{Z}) \right] = \lambda(\theta)(1-\alpha)(\sigma-1) \int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})$$

Recall that the ex-post productivity is a multiplicity of an individual's talent z and location-specific matching shock $x \sim G(\cdot)$, and the model features a sorting equilibrium at which individuals reside a given location is talent-homogeneous. The following property ensures common selection cutoff conditional on financing probability $\lambda(\theta)$.

***Lemma* 3**: Let $F(\mathcal{Z}) = zG(x)$, then $\underline{\mathcal{Z}} = z \cdot \underline{x}$, where $\underline{x}$ is implicit solution to

$$\beta \underline{x}^{\sigma-\gamma-1} \left[\int x^\gamma dF(\mathcal{Z}) - \lambda(\theta) \int_{\underline{x}} x^\gamma dG(x) \right] = \lambda(\theta)(1-\alpha)(\sigma-1) \int_{\underline{x}} x^{\sigma-1} dG(x) \tag{2.34}$$

In other words, the selection cutoff is identified by location-specific shock conditional on talent z residing in the location. Note that the selection effect within a location is controlled by financing accessibility θ, which is endogenously connected to the efficiency of vintage capital reallocation.

2.5.4 Vintage Capital Market and Cost of Capital

Departure from the static model, the vintage capital supply at each period depends on the capital stock of incumbents which experience the retirement shock:

$$K_v^S = \delta\lambda(\theta)N\int_{\underline{\mathcal{Z}}} k_{-1}(\mathcal{Z})dF(\mathcal{Z}) = (1-d)\delta\lambda(\theta)N\int_{\underline{\mathcal{Z}}} k(\mathcal{Z})dF(\mathcal{Z}) \tag{2.35}$$

Before spelling the demand for vintage capital, it is convenient to start by examining the demand for the capital bundle at any given period conditional on location with population N:

$$K^D = (1-\delta)\lambda(\theta)N\int_{\underline{\mathcal{Z}}}\Delta(\mathcal{Z})dF(\mathcal{Z}) + \delta\lambda(\theta)N\int_{\underline{\mathcal{Z}}} k(\mathcal{Z})dF(\mathcal{Z}) = [(1-\delta)d+\delta]\frac{\alpha Y}{\overline{\sigma}\tilde{r}}$$

The capital demand is composed of two channels: the capital bundle demanded by the incumbent firms that invest $\Delta(\cdot)$ to maintain its optimal capital level and the entrants that purchase capital from 0 stock. Given the aggregate demand function for the capital bundle, the corresponding demand function for vintage capital is

$$k_v^D = (1-\eta)\frac{r}{r_v}K^D = (1-\eta)\frac{r}{r_v}[(1-\delta)d+\delta]\frac{\alpha Y}{\overline{\sigma}\tilde{r}} \tag{2.36}$$

The Cobb-Douglas form in investment technology results in the demand for vintage capital that is proportional to that for the capital bundle, which further pins down the cost of vintage capital immediately:

$$r_v = [\frac{(1-\eta)[(1-\delta)d+\delta]}{\delta(1-d)\chi}]^{\frac{1}{\eta}} \tag{2.37}$$

The cost of vintage capital decreases in the inverse of specificity friction χ and longevity of vintage asset $(1-d)$. The adjusted capital cost is then a constant as well, which allows us to examine the real wage as a function of aggregate productivity directly:

$$w = (\frac{1}{\overline{\sigma}})^{-\frac{1}{1-\alpha}}\chi^{\frac{\alpha}{(1-\alpha)\eta}}\left[(1-\eta)[\frac{(1-\delta)d+\delta}{\delta(1-d)}]\right]^{-\frac{\alpha}{1-\alpha}\frac{1-\eta}{\eta}}\cdot\mathbb{Z}^{\frac{1}{1-\alpha}} \tag{2.38}$$

This implies the wage rate is increasing in the location's population. Furthermore, the agglomeration effect for the local wage is decreasing in specificity frictions if holding financial accessibility θ constant:

$$\frac{\partial^2 w(\theta)}{\partial N \partial \chi} > 0, \quad \frac{\partial^2 w(\theta)}{\partial N \partial d} < 0$$

Using the real wage, indifference condition for occupational choice, and labor market clearing condition, one can back out the per-capita income by taking population N and talent in the location z as given:

$$\frac{Y}{N} = \tilde{\chi}_{\frac{Y}{N}} \cdot H(\underline{x}, \theta) \cdot \left[\lambda(\theta) N \int_{\underline{x}} x^{\sigma-1} dG(x)\right]^{\frac{1}{\sigma-1}\frac{1}{1-\alpha}} \cdot z^{r+\frac{1}{1-\alpha}} \tag{2.39}$$

with

$$\tilde{\chi}_{\frac{Y}{N}}(\delta, d) \frac{\overline{\sigma}^{\frac{\alpha}{1-\alpha}}}{1-\alpha} \chi^{\frac{\alpha}{(1-\alpha)\eta}} \left[(1-\eta)\left[\frac{(1-\delta)d+\delta}{\delta(1-d)}\right]\right]^{-\frac{\alpha}{1-\alpha}\frac{1-\eta}{\eta}}$$

where the selection cutoff $\underline{x}$ solves equation (2.34). The per capita income is proportional to real wage and thus exhibits an agglomeration effect when holding θ fixed. I close the local equilibrium by obtaining θ by solving the venture capitalist's free entry problem.

2.5.5 Venture Capitalist's Problem

The venture capital in the model provides necessary financing services to the business by enabling them to access finance for operation. I am abstract from the scenario where a venture capital simultaneously manages multiple deal flows. I follow the setting in the static model where the matches between VC and potential entrepreneur is governed by a random match technology $\mathcal{M}(\theta)$. The financing accessibility is summarized by the ratio of VC searching efforts over the mass of potential entrepreneurs (newbies residing in the location) $\theta = \frac{V}{\delta \cdot N}$.

A venture capitalist posts an ad and incurs a location-dependent search cost $f_{VC}(\cdot)$. Explicitly, the real cost of participating in searching for a start-up in a location with talent z is

$$f_{VC}(z) = \tilde{f} \cdot z^{\gamma} w \tag{2.40}$$

That is, the search cost is proportional to the (lowest) wage earned by the local individual. It follows that the expected value of VC investment is

$$J^{VC}(z) = (1-\beta) q(\theta) \int_{\mathcal{Z}} J^{E}(\mathcal{Z}, 0) dF(\mathcal{Z}) = \frac{1}{1-\rho(1-\delta)} \frac{1-\beta}{\sigma\theta} \frac{Y}{N} \tag{2.41}$$

where $\underline{x}$ solves (34). The free entry condition implies

$$J^{VC}(z) = f_{VC}(z) \tag{2.42}$$

This amounts to the supply curve of VC investment with respect to income per capita:

$$\frac{Y}{N} = \frac{\sigma[1-\rho(1-\delta)]}{1-\beta} \cdot f_{VC}(z) \cdot \theta \tag{2.43}$$

The higher per-capita income summarizes the expected value of VC investment, which therefore attracts more VC investment. The demand curve of VC investment is summarized by equation (2.39) together with the constant wage expenditure share of total income:

$$\frac{Y}{N} = \frac{\overline{\sigma}}{1-\alpha} \cdot H(\underline{x}, \theta) \cdot z^{r} \cdot w$$

Higher per-capita income is increasing in the aggregate productivity and therefore demands greater VC investment to induce tougher selection. To derive a sharp solution, I impose $G(\cdot)$ following Pareto distribution with tail parameter μ with lower support at 1. This allows us to characterize the VC market tightness in explicit form.

Lemma 4: If $G(x) = 1 - x^{-\mu}$ with $\mu > \sigma - 1$, for sufficient low β, there exists unique solution to the local economy equilibrium given the location population

N with the talent normalized selection cutoff satisfying

$$\underline{x}^{\mu-\gamma} = \lambda(\theta)\left[1 + \frac{(1-\alpha)(\sigma-1)(\mu-\gamma)}{\beta(\mu-\sigma+1)}\right]$$

and mass of venture capital is linear in location's population:

$$V = \frac{1}{1-\rho(1-\delta)} \frac{1-\beta}{(1-\alpha)(\sigma-1)} \cdot H^* \cdot N$$

where $H^* = \frac{\mu(1-\alpha)(\sigma-1)}{\beta(\mu-\sigma+1)+(1-\alpha)(\sigma-1)(\mu-\gamma)}$.

Proof: One can obtain $\underline{x}$ by solving equation (2.34) with imposing the Pareto distribution form of $G(\cdot)$. Then solve the demand and supply curve in equation (2.39) and (2.43) for VC investment V. A low β ensures the selecting cutoff to be greater than 1. □

The implication of the closed-form solution to selection cutoff is straightforward: greater financial accessibility generates tougher competition and selection. With the Pareto distribution assumption, the total labor supply at equilibrium is constant because the increase in the labor supply due to tougher selection cutoff is exactly offset by the decrease in the labor supply due to more financial accessibility. This implies the per-capita income is affected by financial accessibility only through the aggregate productivity channel. Since the fixed cost of VC entry is proportional to local wage, it means the cost is also proportional to the local aggregate productivity. This ensures that the overall VC investment in the location is linear in the population size as the demand and supply elasticity with respect to aggregate productivity is the same.

The above section has solved the local equilibrium conditional on the population N. I proceed to solve the endogenous size of location as a function of talent and evaluate how parameters that affect local vintage supply shape the location size and the sorting outcome.

2.5.6 Individual's Location Choice and Vintage Capital Market

Each individual draws talent z upon birth and then chooses the location with population N to reside. Before characterizing the location choice problem faced by each individual, it is helpful first to examine the expected value of an individual with talent z residing in a talent-homogeneous location with population N:

$$\mathbb{E}[J^0(z)] = \int \left[\lambda(\theta) \cdot \max\{J^E(z \cdot x, 0), J^W(z \cdot x)\} + [(1-\lambda(\theta)]J^W(z \cdot x)\right] dG(x) - \psi N^\zeta \tag{2.44}$$

From previously established results, both values of being an entrepreneur and being a worker exhibit an increasing return to population size. More population means more business, which implies greater average productivity and thus a higher real wage. On the other hand, more population is associated with more business, generating more vintage capital and thus more profits for firms. The following result further demonstrates how talent complements such local agglomeration channel, setting the stage for the positive assortative matching between talent and size of a location.

Lemma 5: Individuals with greater talents benefit more from residing in a location with a more significant population.

$$\frac{\partial^2 \mathbb{E}[J^0(z)]}{\partial z \partial N} > 0$$

Proof: Recall that $J^E(zx, 0) = \frac{\beta}{1-\rho(1-\delta)} \frac{1}{\sigma} (\frac{zx}{\bar{\sigma} c})^{\sigma-1} Y = \frac{1}{1-\rho(1-\delta)} z^\gamma (\frac{x}{\underline{x}})^{\sigma-1} \cdot w$. Rewrite the conditional expected value:

$$\mathbb{E}[J^0(z)] = \frac{1}{1-\rho(1-\delta)} z^\gamma \cdot w \cdot \left[\lambda(\theta)[\underline{x}^\gamma \int_{\underline{x}} (\frac{x}{\underline{x}})^{\sigma-1} dG(x) + \int_0^{\underline{x}} x^\gamma dG(x)] \right.$$
$$\left. + (1-\lambda(\theta)) \int x^\gamma dG(x) \right] - \psi N^\zeta$$

Note that $\underline{x}$ is a constant, and $\frac{\partial^2 w}{\partial z \partial N} > 0$ from equation (38). The result then immediately follows. □

The complementarity between an individual's talent and location size is fully summarized by the minimum local revenue of an individual at $z^{\gamma} w$. A natural question arises as to how an individual with some talent z' values a location populated by individuals with talent z. Formally, a location choice problem for an individual with talent z' is

$$\begin{aligned}\mathbb{E}[J^0(z', z)] = \max_z \frac{1}{1-\rho(1-\delta)} z'^{\gamma} w \Big[& \lambda(\theta)[(\frac{z'}{z \cdot \underline{x}})^{\sigma-1-\gamma} \int_{\frac{z \cdot \underline{x}}{z'}} x^{\sigma-1} dG(x) \\ & + \int_0^{\frac{z \cdot \underline{x}}{z'}} x^{\gamma} dG(x)] + (1-\lambda(\theta)) \int x^{\gamma} dG(x) \Big] \\ & - \psi N^{\zeta}(z)\end{aligned} \tag{2.45}$$

For an individual with talent z' resided in a location with talent z, her occupational choice is then disciplined by $\frac{z \cdot \underline{x}}{z'}$. Consider a case where $z' > z$, then an individual with z' can benefit from being more likely to become an entrepreneur, yet she is paid at a lower equilibrium wage given that others' talent at z and thus faces lower conditional expected value of both being worker and entrepreneur. The following result shows that the value of an individual is maximized when she chooses to reside in a location with others sharing a common talent.

Proposition 4 (Sorting and Agglomeration): For sufficiently large ζ, The endogenous optimal location population is increasing in talent z. Specifically,

$$N(z) = (A_N + \frac{A_z(\zeta - \tilde{\sigma})}{\tilde{\gamma}})^{\frac{1}{\zeta - \tilde{\sigma}}} \cdot z^{\frac{\tilde{\gamma}}{\zeta - \tilde{\sigma}}} \tag{2.46}$$

where $\tilde{\sigma} \equiv \frac{1}{(\sigma-1)(1-\alpha)}$ and $\tilde{\gamma} \equiv \gamma + \frac{1}{1-\alpha}$.
Proof: See Appendix 2.A.

The proposition, together with *Lemma 4*, identifies the equilibrium VC investment as a function of talent in the location. Moreover, the VC investment concentration in high-talent locations is intensified when vintage capital is more durable or bears lower specificity.

Corollary 3: The VC investment is strictly increasing in the talent residing in the location.

$$V(z) = \frac{1}{1-\rho(1-\delta)} \frac{1-\beta}{(1-\alpha)(\sigma-1)} \cdot H^* \cdot (A_N + \frac{A_z(\zeta-\tilde{\sigma})}{\tilde{\gamma}})^{\frac{1}{\zeta-\tilde{\sigma}}} \cdot z^{\frac{\gamma+\frac{1}{1-\alpha}}{\zeta-\tilde{\sigma}}} \tag{2.47}$$

Moreover, the complementarity between VC investment and talent is further increasing in vintage capital durability and decreasing in specificity:

$$\frac{\partial^2 V(z)}{\partial z \partial \chi} > 0, \quad \frac{\partial^2 V(z)}{\partial z \partial d} < 0$$

It is noticeable that the VC investment elasticity of talent depends on the capital share of the production function. In particular, a higher capital share implies greater dispersion in VC investment and population size across locations. Furthermore, since we do not allow for additional synergy between VC investors and start-ups, VC investment elasticity is independent of VC market tightness though the VC investment level will increase if the VC-entrepreneur matching friction is mitigated. Future work on incorporating a richer notion of VC-Entrepreneurship pairing with direct productivity enhancement is an essential next step.

2.6 Conclusion

The geographic concentration of new entry and inflow of capitalists is not just a curiosity—it has important implications for urban design and infrastructure investment policy. A dynamic and energetic business environment welcomes young firms and capital, in turn, plants seeds for long-term local economic prosperity. This paper attempts to join the motives of co-location of capitalists and entrants through a vintage capital market and stresses that the efficiency of local vintage capital reallocation plays an essential role in explaining the empirically observed spatial disparities in terms of economic activities. Firstly, I empirically document the concentration of VC investment in the US and then explore the positive response of VC investment to the local vintage capital supply. Given the empirical support, I then build a partial equilibrium model with exogenous local vintage supply to evaluate its roles in attracting VC investment which further generates a selection-induced agglomeration effect through financing more productive entrepreneurs. Finally, I extend the theoretical framework to allow endogenous local vintage capital supply and co-location choices by potential entrepreneurs and VC investment. The sorting mechanism generates striking even spatial heterogeneity in terms of capital investment, entrants, and population coupled with the heterogeneous local agglomeration effects. Such spatial inequality intensifies when the local vintage capital market is more efficient.

There are a number of interesting extensions and related topics to be explored in future work. Firstly, in this model, I do not allow for other financial institutions, such as banks function as alternative financing service providers. It would be an important question to ask about the roles of local vintage capital in distinguishing the two financing services. Secondly, the model does not allow for either idiosyncratic or aggregate shocks to firms that can generate more capital reallocation patterns between operating firms along the business cycle. A third potentially crucial point to be examined is how the exiting strategies of VC are affected by the vintage capital market, which will shape additional interactions between dynamism in the VC market and firms' dynamism. Lastly, constructing a more quantitatively tractable framework based on this theory can help derive crucial counterfactual policy implications given the detailed data. I hope that this work can shed light on those directions.

2.A Appendix—Chapter 2

Proof of Corollary 1: It is equivalent to show that the value of both two hand sides are increasing in θ at solution.

$$\beta\underline{\mathcal{Z}}^{\sigma-\gamma-1}\left[\int \mathcal{Z}^{\gamma}dF(\mathcal{Z}) - \lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\gamma}dF(\mathcal{Z})\right] = \lambda(\theta)(1-\alpha)(\sigma-1)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z})$$

Given that $\sigma - 1 > \gamma$, it is sufficient to show that the value of two hand sides are increasing in θ at solution in the following equation:

$$\beta\underline{\mathcal{Z}}^{\sigma-\gamma-1}\left[\int \mathcal{Z}^{\gamma}dF(\mathcal{Z}) - \lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z})\right] = \lambda(\theta)(1-\alpha)(\sigma-1)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z})$$

Rearrange the above solution, one can obtain:

$$\lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z}) = \frac{\beta\underline{\mathcal{Z}}^{\sigma-\gamma-1}\cdot\int \mathcal{Z}^{\gamma}dF(\mathcal{Z})}{(1-\alpha)(\sigma-1)+\beta\underline{\mathcal{Z}}^{\sigma-\gamma-1}}$$

After taking derivative w.r.t θ on two sides, it is easy to see the right hand side is strictly increasing in θ since proposition 1 has established that $\mathcal{Z}'(\theta) > 0$. Thus at solution the value of right-hand side increases at equilibrium. Therefore, for the original equation,

$$\frac{\partial[\lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z})]}{\partial\theta} > 0 \quad \Rightarrow \frac{\partial\mathbb{Z}}{\partial\theta} > 0$$

Now consider the case where $F(\mathcal{Z}) = 1 - \mathcal{Z}^{-\mu}$ with $\mu > \sigma - 1$ and lower support at 1. Using equation (18), one can obtain

$$\underline{\mathcal{Z}}^{\mu-\gamma} = \max\frac{(1-\alpha)(\sigma-1)(\mu-\gamma)}{\beta(\mu-\sigma+1)}\lambda(\theta), 1 \equiv \max B\cdot\lambda(\theta), 1$$

The mass of operating firms is then $(B\cdot\underline{\mathcal{Z}}^{\gamma})^{-1}$ if $\underline{\mathcal{Z}} > 1$ otherwise B^{-1}. □

Proof of Corollary 2: To save notations, let $B \equiv \int z^{\gamma}dF(z)$, $Q_{\gamma}(\underline{\mathcal{Z}},\theta) \equiv \lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\gamma}dF(\mathcal{Z})$ and $Q_{\sigma-1}(\underline{\mathcal{Z}},\theta) \equiv \lambda(\theta)\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1}dF(\mathcal{Z})$. Note that $Q_{\sigma}(\underline{\mathcal{Z}},\theta) >$

$Q_\gamma(\underline{\mathcal{Z}},\theta)$ for all $(\underline{\mathcal{Z}},\theta)$ and $\frac{Q'_\gamma(\theta)}{Q'_\sigma(\theta)} < \frac{\int_{\underline{\mathcal{Z}}} \mathcal{Z}^\gamma dF(\mathcal{Z})}{\int_{\underline{\mathcal{Z}}} \mathcal{Z}^{\sigma-1} dF(\mathcal{Z})} < \underline{\mathcal{Z}}^{\gamma-\sigma+1}$. Since $\frac{Y}{N} \propto w \propto H(\underline{\mathcal{Z}},\theta) \cdot Q_\sigma^{\frac{1}{(1-\alpha)(\sigma-1)}}$, it is sufficient to show that

$$\frac{\partial\left[H(\underline{\mathcal{Z}},\theta) \cdot Q_\sigma^{\frac{1}{(1-\alpha)(\sigma-1)}}\right]}{\partial\theta} = \frac{\partial\left[(R - Q_\gamma(\theta)) \cdot Q_\sigma^{\frac{1}{(1-\alpha)(\sigma-1)}}(\theta)\right]}{\partial\theta} > 0$$

Expand the expression, one can have

$$\begin{aligned}\frac{\partial\left[(R - Q_\gamma(\theta)) \cdot Q_\sigma^{\frac{1}{(1-\alpha)(\sigma-1)}}(\theta)\right]}{\partial\theta} &\propto R - \left[Q_\gamma + (\sigma-1)(1-\alpha)Q_\sigma \frac{Q'_\gamma(\theta)}{Q'_\sigma(\theta)}\right] \\ &> R - [Q_\gamma + (\sigma-1)(1-\alpha)\underline{\mathcal{Z}}^{\gamma-\sigma+1}Q_\sigma] \\ &> 0\end{aligned}$$

where the last inequality comes from $\beta \in (0,1)$. and equation (18):

$$R - Q_\gamma = \frac{(1-\alpha)(\sigma-1)}{\beta}\underline{\mathcal{Z}}^{\gamma-\sigma+1}Q_\sigma > (1-\alpha)(\sigma-1)\underline{\mathcal{Z}}^{\gamma-\sigma+1}Q_\sigma$$

□

Proof of Proposition 4: Firstly, note that real wage is a function of talent z:

$$w = (\overline{\sigma}r^\alpha)^{-\frac{1}{1-\alpha}} \cdot z^{\frac{1}{1-\alpha}} \cdot [\lambda(\theta)N(z)]^{\frac{1}{(\sigma-1)(1-\alpha)}}[\int_{\underline{x}} x^{\sigma-1}dG(x)]^{\frac{1}{(\sigma-1)(1-\alpha)}}$$

This implies $\frac{\partial w}{\partial z} = \frac{1}{1-\alpha}\frac{w}{z} + \frac{1}{(\sigma-1)(1-\alpha)}\frac{w}{N(z)}N'(z)$. A equilibrium at which all individuals resided in a given location share common talent must satisfy the first-order condition with respect to z equal to zero when evaluating at $z' = z$:

$$\frac{\partial \mathbb{E}[J^0(z',z)]}{\partial z}|_{z'=z} + \frac{\partial \mathbb{E}[J^0(z',z)]}{\partial N}N'(z)|_{z'=z} = 0$$

Expand two terms, we obtain:

$$\frac{\partial \mathbb{E}[J^0(z',z)]}{\partial z}|_{z'=z} = \tilde{A}_z z^{\gamma+\frac{\alpha}{1-\alpha}} N^{\frac{1}{(\sigma-1)(1-\alpha)}}$$

where

$$\tilde{A}_z = \frac{\lambda(\theta)^{\frac{1}{(\sigma-1)(1-\alpha)}+1}}{1-\rho(1-\delta)}[(\frac{1}{1-\alpha}+\gamma)+(\sigma-1)\frac{1-\beta}{\beta}]$$
$$\times(\frac{1}{\bar{\bar{\sigma}}}r^{\alpha})^{\frac{1}{1-\alpha}}\underline{x}^{\frac{1}{1-\alpha}+\gamma}[\int_{\underline{x}}(\frac{x}{\underline{x}})^{\sigma-1}dG(x)]^{1+\frac{1}{(\sigma-1)(1-\alpha)}}$$

and

$$\frac{\partial\mathbb{E}[J^0(z',z)]}{\partial N}|_{z'=z} = \tilde{A}_N z^{\gamma+\frac{1}{1-\alpha}} N^{\frac{1}{(\sigma-1)(1-\alpha)}-1} - \psi\zeta N^{\zeta-1}$$

where

$$\tilde{A}_N = \frac{\beta+(\sigma-1)(1-\alpha)}{(\sigma-1)\beta+\gamma(\sigma-1)(1-\alpha)\beta+(\sigma-1)^2(1-\alpha)(1-\beta)}\tilde{A}_z$$

Notice that both $\tilde{A}_N$ and $\tilde{A}_z$ are constant as θ and $\underline{x}$ are solved at local economy in the previous sections. Let $A_i \equiv \frac{\tilde{A}_i}{\psi\zeta}, \quad i \in \{z, N\}$, and let $\tilde{\sigma} \equiv \frac{1}{(\sigma-1)(1-\alpha)}$ and $\tilde{\gamma} \equiv \gamma + \frac{1}{1-\alpha}$. We can express the FOC in the form of ordinary differential equation:

$$[A_n z^{\tilde{\gamma}} - N(z)^{\zeta-\tilde{\sigma}}]\frac{N'(z)}{N(z)} + A_z z^{\tilde{\gamma}-1} = 0$$

Guess $N(z) = Az^{\nu}$, one can obtain the desired solution. □

CHAPTER 3

PARTNERSHIP, INNOVATION STRATEGY, AND ITS AGGREGATE IMPLICATIONS

3.1 Introduction

Knowledge spillover is a critical factor behind aggregate growth and firm dynamics. An increase in productivity from innovation can spur productivity gains for others. Yet, when a technology has been highly specific or developed, the effect of spillover diminishes as its extent of sophistication limits the insights that others can perceive. Along with the decline in productivity spillover in the US industry over the past decades (Akcigit and Ates, 2021), many innovative firms opt to form partnerships along their supply chains to expand their innovative capacity.

This new trend in organization structure has sparked many ongoing policy debates centering around whether the economy should relax more constraints on those integrating activities so that those incorporated firms can overcome the difficulties during their innovating activities and thus further recover the force of spillover in aggregate.

This paper argues that partnership activity may achieve the first goal, but it does not contribute to more spillover as a firm's engagement in a partnership may further encourage it to develop the product to be more specific and sophisticated. To support the argument, we use data from Factset Revere, Computat, and US Patent and Trademark Office to compile a micro-level dataset on firm-level financial information, partnership and supplier information, and patent filing for public firms in the US between 2003 and 2016. We further classify the innovation direction by a pattern's citations. A patent is classified as an exploratory (radical) innovation if at least 60% of its citations are based on new knowledge, while a patent is classified as an exploitative (incremental) innovation if at least 60% of its citations are based on old knowledge. Partnerships are associated with more incremental innovations, after controlling for a set of fixed effects and firm-level time-variant characteristics. On the other hand, it is accompanied by a slowing down innovation progress upon exploratory *R&D* which is commonly believed to generate more spillover and insights.

To understand the aggregate implication of partnership activity, we develop an endogenous growth model where the economy is populated by innovative firms which optimally choose innovation schedules over their life cycle and less-innovative firms which rely on insights derived from the technologies operated by the innovative firms. Specifically, each innovative firm optimally chooses between exploitative (incremental) and exploratory (radical) innovations at each instant. When conducting exploitative innovations, an innovative firm can either conduct in-house incremental innovation or leverage the know-how from others by forming a firm-to-firm partnership, both of which result in an upgrade in the technology and thus improve the product quality. Nevertheless, despite multiple channels an innovative firm can leverage, attempts in incremental innovation strategies suffer from decreasing return to scale That is, the more exploitative improvement made on a given product, the less improvement it can attain in the next round on average, capturing the natural life-cycle of a technology. Furthermore, as the technologies become more specific and sophisticated, the insights perceivable by the less-innovative firms are diminishing.

On the other hand, innovative firms can terminate their pursuit of exploitative innovations and re-direct their innovation strategy to be more radical, which may successfully replace their product lines with a brand new technology, restoring the technological potential and spillover power. The accessibility to firm-to-firm partnership may empower the incentives to conduct exploitative innovation and leads to a delay in implementing exploratory innovation, which, in aggregate, contribute to a more sophisticated technology landscape in the economy and thus discourage entry dynamism due to a weaker spillover.

Overall, our model remains tractable and admits rich implications after incorporating several features. First, we admit heterogeneous innovation types by extending Klette and Kortum (2004) framework. Furthermore, given the notions of the two types of innovations, it naturally motivates us to treat the innovation schedule as an optimal stopping problem which asks for the privately optimal switching time from exploitative innovations to exploratory innovations to maximize an innovative firm's value flow. Third, we nest a firm-to-firm partner market, of which the accessibility profoundly affects innovative firms' innovation strategies. On the one hand, accessibility to firm-to-firm partnerships provides incentives for innovative firms to conduct exploitative innovations as they can benefit from pooling knowledge to upgrade existing technology. On the other hand, pursuing more exploitative innovations deteriorates the return from collaboration, making exploratory innovation a more attractive option. Lastly, the tractability of the Schumpeterian framework accommodates the macroeconomic stage for interactions between the individually optimal innovation strategies and its resulted technology spillover toward less-innovative firms together with entry dynamism, highlighting a novel channel in explaining the recent decline in US business dynamism mechanically.

3.2 Literature Review

This paper connects three strands of topics. First, it links to the heterogeneity in innovations. The innovation literature is initially more concentrated on radical innovation solely. Initiated by Romer (1990), Aghion and Howitt (1992), the endogenous growth model was built on a radical technological change in the aggregate representative agent framework. It then has been extended to a firm-level framework as elaborated in Klette and Kortum (2004), Lentz and Mortensen (2008), and recently in Acemoglu et al. (2018). Until recently, researchers have started exploring incremental innovations' roles. Acemoglu and Cao (2015) extend the Schumpeterian endogenous growth model by allowing incumbents to undertake innovations to improve their products while entrants engage in more "radical" innovations to replace incumbents. Our model does not emphasize radical entry by itself but focuses on the value of radical innovation to others, given the spillover channel. Empirically, Garcia-Macia et al. (2019) present shreds of evidence on the differential contributions by incremental and exploratory innovations. Although they argue that incremental innovations drive most growth, our theory suggests that the value of radical innovation is not manifested immediately but reveals its potential through subsequent developments and its spillover impact, which is consistent with Brynjolfsson et al. (2021).

Second, our work is also related to the literature on the implications of resource misallocation. Hsieh and Klenow (2009), Restuccia and Rogerson (2008) are the pioneering efforts that empirically document the significant productivity losses due to resource misallocation. Extending their works, several strands of papers attempt to micro-found the sources of misallocations in various facets.

Misallocations due to the financial fraction are developed in Buera et al. (2011) and Midrigan and Xu (2014). Our model emphasizes a misallocation channel of coordination failure as firms fail to internalize the aggregate influences of jointly implementing incremental innovations on the spillover force. The coordination failure may be exacerbated when accessibility to firm-to-firm partnerships increases.

Lastly, this paper contributes a novel mechanism to the vast and growing literature on documenting and understanding the declining trends in US business dynamism and innovation activities. Bloom et al. (2020) argue that research efforts are rising substantially while research productivity is declining sharply across various industries, products, and firms. This paper provides additional support: when it becomes harder to conduct radical innovations, all firms implement more incremental innovations, and leads to strong negative externality, amounting to further lower growth and weaker knowledge spillover.

The rest of the paper is organized as follows: we present motivating empirical facts in Section 3.3 with data description and empirical strategy; we present the model in Section 3.4, starting with introducing heterogeneous firm types and solving the static profit maximization problem faced by a firm in Section 3.4.1 - 3.4.2. From Section 3.4.3 - 3.4.4, we outline the dynamic part of firms' problems and characterize the innovation direction decision-making. Section 3.4.5 - 3.4.7 closes the economy and characterizes the stationary economy with implications of firm-to-firm partnership in affecting innovations and the aggregate economy. We conclude in Section **??**. All related proof can be found in the 3.A.

3.3 Motivating Empirical Facts

We begin by examining the empirical implication of how firm-to-firm partnership along the supply chain correlates with a firm's innovation strategy.[1] Specifically, we investigate whether forming a partnership is associated with more exploitative innovations subsequently for a firm. We compile three datasets to implement our empirical strategy.

Factset Revere: The first source is Factset Revere Supply Chain Relationships Data, covering 2003 to 2016 and 13,000 US private and public firms. For each documented firm, we would observe the firm's network structure with specific relationships, including buyer-seller relationships, equity-holding relationships, and other strategic relationships, including research collaboration and marketing. We are particularly interested in firm-to-firm partnerships along the supply chain in the form of equity-holding and research collaboration. As argued in Atalay et al. (2014), integration tends to facilitate know-how and intangible transfer across integrated parties.[2] With similar empirical implications, Holmes et al. (2015) suggest that technology transfer can be settled through the device of a joint venture. Hence, such knowledge transfer or sharing makes this particular form of partnership more likely to expand a firm's innovation capacity, thus affecting its subsequent innovation activities and strategies.

Compustat North American Fundamentals: Another source is firm-level financial data, Compustat, which covers public firms in the United States since 1976.

[1]The data on the firm-to-firm partnerships that are independent of the production network is incomplete. For this reason, we thus focus on the firm-to-firm partnership along the vertical supply chains. Due to the closeness to the production process, such partnerships have more impact on firms' innovation policy. See Survey by Lafontaine and Slade (2007) for more comprehensive discussions.

[2]Specifically, Atalay et al. (2014) argue that know-how acquisition or transfer is very likely to occur in the case of vertical integration.

It reports a broad set of firm-level panel data such as employment, and firms' ages, which are used as our set of control variables.

United States Patent and Trademark Office: The last source is the USPTO dataset which reports detailed information on utility patents granted from 2003 to 2013. In particular, it includes the citations information of a given patent, based on which we construct two measures of innovation types:

- a patent is classified as an exploratory innovation if at least 60% of its citations are based on new knowledge; [3]
- a patent is classified as an exploitative innovation if at least 60% of its citations are based on existing knowledge. [4]

We match the patent ownership with firm information in the production network and financial data, which allows us to investigate the correlation between a firm's partnership exposure and innovation strategy. Specifically, we start by constructing the dependent variable. To capture the innovation direction/strategy, we compute the fractions of the exploitative and exploratory innovations by a given firm for each year whenever available.[5]

Secondly, we construct a measure to capture a firm's partnership exposure. One single partnership linkage maintained by a large firm may have little impact on the overall innovation strategy. To overcome the challenge, we explicitly control for the number of suppliers of a firm and the number of patents created

[3] New knowledge is defined as the citations are not recorded in the firm's other patents nor the associated citations.

[4] Existing knowledge is defined as the citations are recorded in the firm's other patents or the associated citations.

[5] By the definition of exploitative and exploratory innovations, one is not the conjugate of the other despite the fact that the two types are disjoint.

in the previous years as a proxy for the firm's technology clusters/portfolios. Specifically, we run the following baseline OLS regression:

$$y_{j,t+2} = \beta_0 + \beta_1 \log(\#partners_{j,t}) + B * X_{j,t} + \epsilon_{j,t}$$

where $\{y_{j,t+1}\}$ are the two fractions of innovations of firm j in two years ahead of year t: that are exploitative and exploratory, and $X_{j,t}$ are the controls including the log of the number of suppliers of a given firm j at year t, age of firms, log of employment size (size of firms), industry fixed effect at SIC2 level, year fixed effect and # citations received at t , # citations per patent received at t.[6]

A partner of a firm is defined as

$$S_{partner}(j) = S_{shareholder}(j) \cup S_{shareholding}(j) \cup S_{JV_{partner}}(j) \\ \cup S_{researchcollaboration}(j) \cup S_{poolinglicence_{partner}}(j)$$

which includes both equity-based partnership and non-equity partnership reported by a firm's upstream. We document the number of a given firm's partners reported by its suppliers maintained in a given year as our measure on a firm's partnership exposure.

Table 3.1 reports baseline OLS result suggests that the partnership exposure along a firm's supply chain is positively associated with relatively more exploitative innovations. Specifically, doubling the number of partners along a firm's supply chain corresponds to 38% more fractions of innovations to be more exploitative. On the other hand, it is accompanied by a slowing down innovation progress upon exploratory *R&D* despite its statistical insignificance.

Nevertheless, the baseline regression is subject to selection bias which is a crucial concern when evaluating the impact of the partnership on innovation di-

[6] Due to the lagged filing process of patents, we investigate the impact of partnership engagement in the patent filing in two years.

Table 3.1: OLS - Innovation Direction and Partnership Exposure

	$Exploratory_{j,t+2}$	$Exploitative_{j,t+2}$
$\log(\#partners_{j,t})$	-0.003	0.004*
	(0.004)	(0.002)
$\log(suppliers_{j,t})$	-0.034***	-0.007 **
	(0.004)	(0.002)
$\log(citation_{j,t})$	0.082***	0.024***
	(0.001)	(0.001)
$\log(citation\ per\ patent_{j,t})$	-0.001***	-0.0003***
	(0.000)	(0.00)
Controls	Yes	Yes
Fixed Effect	Yes	Yes
N	6,809	6,809

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

rections. Some unobserved variables would be driving both the partnership and innovation direction decisions. Even though we control for the firm's characteristics, innovation capability, and efficiency in our OLS regression, there might be other potential variables we fail to consider. We thus apply the machine learning technique to mitigate the selection problem. It will allow us to incorporate as many firm characteristics as possible and a more flexible relationship between the partnership variable and the controls characterized by function g_0. [7]

$$y_{j,t+2} = \beta_1 \log(\#partners_{j,t}) + g_0(X'_{j,t}) + \epsilon_{j,t}$$

On top of this, we also follow Chernozhukov et al. (2018) to adopt the double debiased machine learning method to uncover the structural parameter β_1. Instead of naively estimating g_o, we construct the orthogonalized regression $V = \log(\#partners_{j,t}) - \hat{m}_0(X'_{j,t})$ to partial out the effect of $X'_{j,t}$ on the partnership variable. It overcomes the regularization biases common in non-parametric

[7] $X'_{j,t}$ not only includes log of the number of suppliers of firm j at year t, age of firms, log of employment size (size of firms), industry fixed effect at SIC2 level, year fixed effect and # citations received at t , # citations per patent received at t, but also firm-level asset structure and profitability variables available in Compustat.

estimation by using orthogonalization. In addition, estimating the g_0 and m_0 functions separately also allows us to use the sample splitting method to remove the bias induced by over-fitting.

$$\log(\#partners_{j,t}) = m_0(X'_{j,t}) + \nu_{j,t}$$

Table 3.2 reports the results based on our double-biased specification, confirming the stronger incentives to conduct exploitative innovations when a firm's partnership exposure is higher.

Table 3.2: Double Debiased ML - Innovation direction and Partnership Exposure

	$Exploratory_{j,t+2}$	$Exploitative_{j,t+2}$
$\log(\#partners_{j,t})$	0.0039	0.0048**
	(0.003)	(0.002)

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

3.4 A Growth Model of Innovation Strategy with Partnership

Motivated by the empirical pattern, in this section, we provide a simple growth model of innovation strategy under partnership which shapes the spillover effect and thus the aggregate firm dynamics. The model is built upon the quality ladder framework by Klette and Kortum (2004) with three features. The first is a Schumpeterian model with heterogeneous innovations in the spirit of Akcigit and Kerr (2018), the second is a firm-to-firm partnership through a matching process following Akcigit et al. (2016), and the last is an optimal stopping problem which depicts firms' innovation strategies along their life cycle.

3.4.1 Environment

Time is continuous. A representative household consume final good and take simple preference form:

$$\int_0^\infty e^{-\rho t}\frac{C(t)^{1-\gamma}-1}{1-\gamma}$$

where γ captures the inverse of inter-temporal elasticity of substitution. The final good is produced by a competitive final good producer using labor and a unit mass of intermediate goods y_j:

$$Y(t)=\frac{1}{1-\alpha}\left[\int_0^1\sum_{f\in S_j(t)}z_{fj}(t)^{\alpha}y_{fj}(t)^{1-\alpha}dj\right]l^{\alpha} \tag{3.1}$$

where α captures the labor share and $z_{fj}(t)$ captures the quality of intermediate good j produced by firm f at time t, and $S_j(t)$ denotes the set of firms competing in variety j's market at time t. For the simple exposition, we assume the labor supply for final goods production is perfectly inelastic and fixed at an exogenous level l. Furthermore, final goods are either consumed by households or used as input for the production of intermediate goods. This amounts to the final good market-clearing condition: $C(t)=Y(t)-K(t)$ where $K(t)$ is the total amount of final good used in intermediate goods production. The price of a normal good is normalized to 1.

3.4.2 Static Profit Maximization and Competition

Each firm is a single product producer, and the economy is thus populated by one unit mass of heterogeneous firms. [8] There are two sources of heterogeneity

[8] We discuss the extension of the model into multi-product firms setting in the later section. Admitting multi-product firms does not change the major implications given the linearity of Klette and Kortum (2004) framwork.

across firms: (1) product-specific quality z_{fj}; and (2) firm-specific and the time-invariant marginal cost of production mc_f. In particular, a firm f producing product j with marginal cost mc_f produces output according to

$$y_{fj} = \frac{k}{mc_{fj}} \tag{3.2}$$

where k is the amount of final good used in the production of a variety j by firm f. Another interpretation of the production function is that each firm source varieties from all the other firms in the economy and bundle them with the technology described by (3.1).[9] As the problem is static, we drop the time subscript whenever possible. Each firm takes the marginal cost as given but competes in Bertrand fashion with entrants offering the same product given the single-product structure. Specifically, the inverse demand function of a given variety derived from final goods bundling equation (3.1) suggests:

$$p_j = (z_j l)^{\alpha} y_j^{-\alpha} \quad \Rightarrow \quad \frac{p_j}{z_j^{\alpha}} = (\frac{l}{y_j})^{\alpha} \tag{3.3}$$

This implies that for two firms competing in a given variety j market, the winner must be the one who is able to offer the lowest quality-adjusted price at any given quantity demanded at y_j

$$\min_{p_{fj}} \frac{p_{fj}}{z_{fj}^{\alpha}} = \frac{mc_f}{z_{fj}^{\alpha}} \tag{3.4}$$

In other words, possessing a leading-edge quality of a given variety/product, j does not guarantee to be the winner if such entrant faces higher marginal cost. To maintain the tractability of the theory, following Acemoglu et al. (2012), we further assume that there is a small fixed cost at $\epsilon > 0$ before the production of variety for both incumbent and entrant. This follows that, by backward induction, a firm chooses to exit if it finds it impossible to outbid its competitor

[9] This is a simplified version of firm-to-firm trade. See more general settings in Demir et al. (2021) and Lim (2017)

with a lower quality-adjusted price, which ensures that only one firm is producing any given variety with monopoly profits at equilibrium. Therefore, we can drop the firm index subscript, and the static profit earned by a winning firm with (z_j, mc_j) reads:

$$\Pi(z_j, mc_j) = \max_{y_j}(p_j - mc_j)y_j = (1-\alpha)^{\frac{1}{\alpha}} \frac{\alpha}{1-\alpha} L \cdot (\frac{1}{mc_j})^{\frac{1-\alpha}{\alpha}} z_j \equiv \pi \cdot s_j z_j \quad (3.5)$$

where we define $s_j \equiv (\frac{1}{mc_j})^{\frac{1-\alpha}{\alpha}}$ as the production efficiency of the firm which is producing variety j. Throughout the paper, we restrict to the cases where each firm is either low-type (L) or high-type (H). In terms of the heterogeneity in marginal cost, a low-type firm faces a production efficiency normalized to one while a high-type firm faces a weakly higher production efficiency at level $s \geq 1$.

3.4.3 Dynamics: Innovation and Spillover

A firm's heterogeneity is not only reflected by the different marginal costs of production across types but also by its innovation capacity. Specifically, a high type firm is innovative and can conduct R&D activity to improve the quality of its product lines, while a low type firm is not equipped with the ability to innovate original ideas and thus has to rely on the insights from a high-type firm's product lines to improve their own product lines. The following section elaborates on the firm's life cycles and innovation schedules across types.

Innovative (High-type) Firm's Innovation Strategy

Given the static profit structure in equation (3.5), each intermediate goods producer has profit incentives to improve the idea quality of their existing products. In the spirit of Akcigit and Kerr (2018), the innovations are heterogeneous in nature. Firms undertake exploratory innovations to replace the existing technology with a fundamental breakthrough, while exploitative innovations improve the quality based on the current technology. Hence a firm's innovation schedule is characterized by an array of discrete choices between climbing along the quality ladder through implementing exploitative (incremental) innovations and replacing the current technology embodied in the product with a novel idea via an exploratory innovation along the time horizon. Let $\theta_{jt} \in \Theta \equiv \{\theta_{jt,D}, \theta_{jt,E}\}$ denote by the exploitative (incremental) innovation ($\theta_{jt,D}$) or exploratory (radical) innovation ($\theta_{jt,E}$) directed in variety j at time t. Each firm thus evaluates the discounted present value flow from conducting $\theta_{jt,D}$ and that from implementing $\theta_{jt,E}$ at each instant t.

For tractability, we abstract from endogenizing *R&D* intensity (intensive margin), and focus on firms' decisions on the timing of switching innovation directions. That is, we are shaping the dynamic innovation decision as an optimal stopping problem. The delay in directing exploratory innovation on a given product means more incremental innovations to be taken.

Assumption 1: Innovation arrival rates of both types are constant across firms and time, and there is no explicit *R&D* cost for all firms. [10]

[10] As discussed in much relevant literature, constant *R&D* efforts per product line are sufficient to generate either constant or decreasing *R&D* intensity in firm sizes. See Klette and Kortum (2004), Acemoglu et al. (2018),Akcigit and Kerr (2018). In terms of the cost of R&D, one can think of the constant term *pi* in our profit function as it has already incorporated the

θ_D Exploitative (Incremental) Innovation

Conditional on directing the innovation strategy toward exploitative innovation on a given product j, an innovative (high-type) firm is able to access two channels to improve their product quality: (1) an improvement of current quality at exogenous Poisson rate i as a result of R&D activity within the firm itself, (2) a technological upgrade by partnering with other innovative firms via a firm-to-firm matching mechanism.

For the first channel, the step size of an improvement over the current quality level z_j exhibits a decreasing return to scale. In other words, the degree of improvement is diminishing with the number of in-house exploitative innovations which has been implemented so far. Specifically, the law of motion of variety j's quality is given by

$$z_j' = z_j + \Delta a^{n_j} \bar{z}$$

where Δ is a parameter that governs the upper bound of step size per improvement, $a \in (0, 1)$ summarizes the how fast the return deteriorates per more improvement through θ_D innovation, and n_j is the number of in-house exploitative innovations that have been successfully implemented so far. An alternative interpretation of n_j is the degree of sophistication of current technology for variety j. For shorthand, I thus term n_j as a sophistication index of a given variety j. Moreover, the step size is linear in $\bar{z}$, capturing the average/aggregate quality of all incumbent products: $\bar{z} = \int_0^1 z_j dj$.[11] Upon a successful in-house exploitative innovation, the sophistication index accumulates:

$$n_j' = n_j + 1 \tag{3.6}$$

associated R&D costs. We leave endogenous R&D efforts in the future works.

[11] This assumption ensures the tractability of the theory and allows us to derive a closed-form solution of the firm's problem.

For the second channel, an innovative firm is able to collaborate with other innovative firms to realize an upgrade based on the current technology. In particular, conditional on a successful match with a partner, the quality of the product line evolves as

$$z_j' = z_j + \eta a^{n_j} \bar{z}$$

where $\eta \geq \Delta$ indicates a synergy effect originated from the collaboration. Note that the likely increase in quality via a partnership is still affected by the sophistication index. Nevertheless, such quality gains from pooling the know-how of two parties do not accumulate the index, which is viewed as a way to slow down the deteriorating internal innovation efficiency.[12]

Firm-to-Firm Partnering

Each innovative firm is able to form a partnership with other innovative firms along its life cycle. Specifically, each innovative firm faces the Poisson flow rate $x \geq 0$ with which it perceives a collaboration idea and thus initiates a search for a potential partner to implement the collaboration. A Cobb-Douglas matching function governs the firm-to-firm partnering process following the Diamond-Mortensen-Pissarides (DMP) framework. Therefore, the mass of meetings for partnership at time t is given by

$$\mathcal{M}_t = \chi \cdot S_t^{\gamma} V_t^{1-\gamma}$$

where S_t is the mass of searchers for partnership at time t, V_t is the mass of innovative firms that can provide collaboration, χ is a parameter that captures the

[12]One can interpret it as another consequence of the synergy effect. The corporate finance and innovation literature has been arguing that such know-how acquisition behavior can "fix the weaknesses" and ""build on strength". (Nelson 1982; Telser 1982; Jovanovic and Rob 1989; and Ma 2019)

matching efficiency, and $\gamma \in (0,1)$ governs elasticity of matches.[13] Recall that the naive production network structure implies that a partner is also a supplier of each innovative firm. We thus interchangeably refer to a partner-seeking firm and a firm to be matched for partnership as a buyer firm and a seller firm, respectively. Furthermore, due to the continuous time setting, an innovative firm is either searching for a partner or waiting for being matched at each given instant t. Since x fraction of innovative firms receive the collaboration idea, this immediately follows that $V_t = \frac{1-x}{x} S_t$. Therefore, the rate at which a partner-seeking firm meets a partner, λ, as well as the rate at which an innovative firm meets a partner-seeking firm, q are constant and homogeneous for all innovative firms:

$$\lambda = \chi(\frac{1-x}{x})^{\gamma}, \quad q = \chi(\frac{1-x}{x})^{\gamma-1}. \tag{3.7}$$

Conditional on a successful meeting, the two parties bargain over the value of surplus of quality upgrade from collaboration following standard Nash bargaining protocol with buyer firm's bargaining power at parameter β. Specifically, let the value paid to a seller firm denote by $T(s,z,n)$ when it collaborates with a buyer firm whose production efficiency is s and technology quality is z with sophistication index n. The bargaining game is characterized by

$$\begin{aligned} T(s,z,n) &= \text{argmax}\big[V^H(s,z+\eta a^n \bar{z},n) - V^H(s,z,n) - T\big]^{\beta} T^{1-\beta} \\ &= (1-\beta)\big[V^H(s,z+\eta a^n \bar{z},n) - V^H(s,z,n)\big] \end{aligned} \tag{3.8}$$

where $\big[V^H(s,z+\eta a^n \bar{z},n) - V^H(s,z,n)\big]$ is the premium of value attained from the collaboration taking into its impact on buyer firm's future value flow. We formally elaborate on the value flow of an innovative (high-type) firm given the innovation strategy and its relevant payoff in the later section.

[13] The matching efficiency parameter χ can also be interpreted as a match-specific shock between the two firms, capturing the additional layer of uncertainty in forming a partnership.

θ_E Exploratory Innovation

When a firm has successfully implemented an exploratory innovation, its current variety j will be replaced with a new product with a new strand of technology that possess greater potential such that its sophistication index resets to 0:

$$n'_j = 0, \quad \text{after } \theta_E \text{ innovation}$$

To maintain tractability, I assume no more evolution in terms of technology quality. That is,

$$z'_j = z_j, \quad \text{after } \theta_E \text{ innovation}$$

This notion is aligned with the current discussion by Brynjolfsson et al. (2021), where they find an exploratory development on technology tends not to manifest its productivity advent until it becomes mature. The arrival rate of successful exploratory innovation is exogenous set at ν conditional on the event at which a firm directs innovation strategy to θ_E.

Value of an Innovative Firm

We formally elaborate the value flow of an innovative (high-type) firm given the innovation strategy and its relevant payoff. An innovative firm producing variety j at a given time t is summarized by its production efficiency s_j, the quality of product/technology z_{jt} and its associated sophistication index n_{jt} taking the aggregate quality level $\bar{z}_t$ as given. We, again, drop variety index j as (s, z, n) are sufficient to identify a firm's state. The value flow of an innovative firm is thus given by

$$
\begin{aligned}
r_t V_t^H(s,z,n) = sz\pi + \dot{V}_t^H(s,z,n) + \max_{(\theta_D,\theta_E)} \Big\{ & i\big[V_t^H(s,z+\Delta a^n \bar{z}, n+1) - V_t^H(s,z,n)\big] \\
& + x\lambda \cdot \big[V_t^H(s,z+\eta a^n \bar{z}, n) - V_t^H(s,z,n) - T_t(s,z,n))\big] \\
& + (1-x)q \cdot \mathbb{E}\big[T_t(s,\tilde{z},\tilde{n})\big]; \quad \nu \cdot \big[V_t^H(s,z,0) - V_t^H(s,z,n)\big] \Big\} \\
& - \tau_t^H V_t^H(s,z,n)
\end{aligned} \tag{3.9}
$$

The value of firm consists of four parts. First, three is the instantaneous payoff, which is just the static profit collected given firm's production efficiency and product quality. Second, firm benefits from economic growth through $\dot{V}_t^H(s,z,n)$ as the improvement of quality is a function of evolving $\bar{z}$, which can also be interpreted as a form of capital gains. The third part captures the future value from optimally choosing innovation strategy over the current product between θ_D and θ_E. Specifically, if an innovative firm optimally choose to implement exploitative innovation given current product state (s,z,n), then her return on the choice includes: (1) in-house quality improvement with arrival rate i captured by $i\big[V_t^H(s,z+\Delta a^n \bar{z}, n+1) - V^H(z,n)\big]$; (2) gains from collaboration with another innovative (seller) firm in the economy at effective rate of $x\lambda$, summarized by $x\lambda \cdot \big[V_t^H(z+\eta a^n \bar{z}, n) - V_t^H(s,z,n) - T_t(s,Z,n))\big]$; (3) gains from providing a collaboration service to improve other innovation (buyer) firms in the economy, $(1-x)q \cdot \mathbb{E}\big[T_t(s,\tilde{z},\tilde{n})\big]$, taking expectation over others' quality $\tilde{z}$ and the associated sophistication index $\tilde{n}$. On the other hand, if an innovative firm optimally choose θ_E innovation, its technology will be replaced with exploratory innovation with reset sophistication index, $\nu \cdot \big[V_t^H(s,z,0) - V_t^H(s,z,n)\big]$, implying regains of technological potential. The last part is a risk of being competed away by an entrant, $-\tau_t^H V_t^H(s,z,n)$, where the rate of losing a competition is taken as given by each firm and potentially depends on firm's type H, which is discussed in the later section on entrant's problem.

A Low-type Firm's Dynamic Problem

A low-type firm does not have the capability of implementing independent in-house innovation, which means it does not face the problem of choosing an innovation strategy. Nevertheless, a low-type firm can draw insights/ideas randomly from an innovative firm's technology and improve its product with such imperfect adoption. Alternatively speaking, a low-type firm can benefit from technology spillover from innovative firms. [14] We further assume no license payment toward innovative firms for adopting insights to simplify the framework.[15] The core mechanism sits on the insight-generating process, which depends on the distribution of sophistication degree of products produced by innovative firms in the economy. Specifically, let the insight arrival rate denote by ζ, exogenously given. Conditional on obtaining an insight, the improvement can be made upon adoption of insight from a product with quality $\tilde{z}$ and sophistication $\tilde{n}$ for a low-type firm with product quality z is captured by the following law of motion

$$z' = z + \Delta a^{\tilde{n}} \bar{z} \tag{3.10}$$

Note that the improvement is dependent on the sophistication of other's technology $\tilde{n}$. Since insights draw is random, greater domination of sophisticated technologies in the economy induces less degree of insights that a lowe-type firm can leverage on average. We thus summarize the HJB equation that char-

[14]Another interpretation can be the adoption of expiring patents or unpatented know-how from other innovative firms.

[15]Allowing a licensing market does not alter the main implications of the current simplified model.

acterizes a low-type firm's value function conditional on its quality z at time t:

$$r_t V_t^l(z) = z\pi + \dot{V}_t^L(z) + \zeta \left[\mathbb{E}_{\tilde{n}} \left[V_t^L(z + \Delta a^{\tilde{n}} \bar{z}) \right] - V_t^L(z) \right] - \tau_t^L V_t^L(z) \tag{3.11}$$

Similar to the value function of an innovative firm, the value of a low-type firm consists of four parts. The first is the firm's static profit. The next part is the capital gains from the evolution of the aggregate quality of the economy. Thirdly, a low-type firm can benefit from adopting insights generated from others' technologies possessed by innovation firms in the economy, taking the expectation over the sophistication index of other innovation firms' technologies. The fourth part is the discounts due to the risk of being competed away by entrants.

3.4.4 Entry and Exit

Similar to Klette and Kortum (2004) and Akcigit and Kerr (2018), a mass of entrants can enter the market by successfully improving the incumbent's product-adjusted quality. Each potential entrant has to choose effort κ^e with cost $f_e(\kappa^e)$ in the unit of final good before realizing its type and technology improvement. After incurring the entry cost, each entrant observes its type and then implements either in-house innovation or adoption of others' ideas given the type. Specifically, the probability that an entrant draws to be high-type is fixed at h.

Conditional on being a high-type entrant, such a firm can successfully conduct in-house exploitative innovation over an incumbent's technology randomly at a rate of κ^e chosen by itself. Note that since exploratory innovation in our set-up does not immediately improve the quality of a product, there are no incentives to conduct an exploratory innovation as it cannot compete against an incumbent.

$$z_j^e = z_j + \Delta a^n \bar{z}, \quad \text{for high-type entrant} \tag{3.12}$$

where n is the sophistication index of the incumbent firm being challenged by the high-type entrant when the incumbent firm is innovative, while n is assumed to be a random draw from other innovative firms in the economy. [16]

Hence, conditional on κ^e, the value function of a high-type entrant is determined according to to[17]:

$$V_t^{H,e} = \mathbb{E}\left[V_t^H(s, z + \Delta a^n \bar{z}, n)\right] \tag{3.13}$$

On the other hand, for a low-type entrant, the arrival rate of insight is κ as well. The quality of variety obtained by such low-type entrant conditional on perceiving an insight from a technology with sophistication $\tilde{n}$ is given by

$$z_j^e = z_j + \Delta a^{\tilde{n}} z_j, \quad \text{for low-type entrant} \tag{3.14}$$

where we impose the step-size as a function of incumbent technology z_j instead of $\bar{z}$ in order to ensure a closed-form solution for the general case. Since the innovation is random, this implies the ex-ante improvement made by low-type entrant is $\Delta a^{\tilde{n}} \bar{z}$.

Apart from being an innovative entrant, it does not always guarantee that a low-type firm can win the competition against the incumbent upon successful adoption of the perceived insight as the ex-post adjusted quality can be lower than that of the incumbent due to low production efficiency and imperfect adoption of idea due to the technological sophistication. To see this, consider the

[16]For simplicity, we do not let n be based on the insight drawn by low-type firm. We leave the case for future extension.

[17]In order to avoid excessive complication without novel implication in computing the distribution of sophistication index of technologies in the economy at the stationary equilibrium, we assume the entrant's exploitative innovation does not accumulate sophistication index.

lowest quality-adjusted price that a low-type entrant can post and that the corresponding high-type incumbent to be challenged can post are:

$$p^{L,e}_{min}(z,\tilde{n}) = \frac{1}{(1+\Delta a^{\tilde{n}})^{\alpha} z^{\alpha}} \quad , p^{H}_{min}(s,z) = \frac{1}{s z^{\alpha}} \tag{3.15}$$

This immediately follows that $p^{L,e}_{min}(z,\tilde{n}) < p^{H}_{min}(s,z)$ if and only if $a^{\tilde{n}} < \frac{s-1}{\Delta}$. There then exists n^L such that $p^{L,e}_{min}(z,n) < p^{H}_{min}(s,z)$ for all $n > n^L$. Note that the special case is $n^L \to \infty$ when $s = 1$.

Let the fraction of incumbent firms that are low-type at time t and high-type with sophistication n denote by μ^L_t and $\mu^H_t(n)$, respectively. Since the insights are only drawn from innovative firms without crowding-out, the sophistication degree of an insight perceived by low-type firms is the conditional distribution of sophistication index $\{\hat{\mu}^H_t(n) \equiv \mu^H_t(n)/(1-\mu^L_t)\}^{n^H}_{n=0}$. Given the competition structure, the value function of a low-type entrant conditional on receiving an insight is

$$V^{L,e}_t = \left[\mu^L_t \mathbb{E}\left[V^L_t(z+\Delta a^{\tilde{n}} z)\right] + (1-\mu^L_t) \sum_{\tilde{n}=0}^{n^L} \hat{\mu}^H_t(\tilde{n}) \mathbb{E}_z\left[V^L_t(z+\Delta a^{\tilde{n}} z)\right] \right] \tag{3.16}$$

where the first term in the bracket captures the event that a low-type firm draws an insight randomly and competes with another low-type firm, the second term summarizes the cases where a low-type firm draws insight from other's technology with a sophistication degree no greater than n^L and then compete with some random high-type firm. [18].

Thus ex-ante value of entry must satisfy:

$$rV^e_t - \dot{V}^e_t = \max_{\kappa^e} \quad \kappa^e \left[hV^{H,e}_t + (1-h)V^{L,e}_t \right] - f^e(\kappa^e) \tag{3.17}$$

[18] Alternative timing for a low-type entrant: (1) it firstly draws and adopts an insight from incumbent firm subject to sophistication $\tilde{n}$; (2) and then applies the adoption on another product/variety j with current quality z.

Furthermore, given entry efforts made at κ_t^e, the high-type and low-type firm's churning rate are given by:

$$\tau_t^L = \kappa_t^e, \quad \tau_t^H = \left[(1-h)\sum_{\tilde{n}=0}^{n^L} \hat{\mu}_t^H(\tilde{n}) + h\right]\kappa_t^e \tag{3.18}$$

3.4.5 Solving the Economy

Having spelled out all agents' behaviors in the economy, we now focus on solving the economy for a stationary equilibrium in this section. Thanks to the nice theoretical property of Klette and Kortum (2004) framework and the linearity of the Nash Bargaining protocol, we firstly show that the solution to the Hamilton–Jacobi–Bellman (HJB) equation (3.7) of an innovative firm has an explicit (partially) expression along a stationary equilibrium path.

Proposition 1: Along a stationary equilibrium path, the solution to HJB equation (7) is given by

$$V^H(s,z,n) = A^H(s)\cdot z + B^H(n)\cdot \bar{z}, \tag{3.19}$$

where

$$A^H(s) = \frac{s\pi}{r+\tau^H},$$

$$(r-g+\tau^H)B^H(n) = \begin{cases} a^n C^H + i[B^H(n+1) - B^H(n)] + D^H(\mathbb{E}[a^{\tilde{n}}]), & \textit{if } n \le n^H \\ \nu\Big[B^H(0) - B^H(n)\Big], & \textit{if } n > n^H \end{cases}$$

with

$$C^H = A^H(i\Delta + x\lambda\beta\eta),$$

$$D^H = A^H(1-x)q(1-\beta)\eta\mathbb{E}[a^{\tilde{n}}],$$

$$n^H = \min\left\{n \in \mathbb{Z}\cup\{0\} : a^n C^H + i[B^H(n+1) - B^H(n)] + D^H(\mathbb{E}[a^{\tilde{n}}]) \le \nu\Big[B^H(0) - B^H(n)\Big]\right\}.$$

Proof: See Appendix 3.A.

The solution is consistent with intuition. There exists a maximal degree of exploitative effort per technology. Upon reaching such a threshold, an innovative firm directs toward exploratory innovation to replace the technology that has little potential to improve. The benefit from implementing θ_D development is decreasing in the so-far number of successful exercises of θ_D innovations, which means the future value to be collected also diminishes per successful exercise of θ_D innovations. Moreover, note that $D^H(\mathbb{E}[a^{\tilde{n}}])$ diminishes to 0 if n^H goes to infinity because all firms keep building on existing technologies without resetting their sophistication index in this case. Ultimately, this implies the value flow of conducting θ_D innovation converges to null if all innovative firms never conduct exploratory innovation, which further indicates the exploratory innovation will be favored eventually.

It is noticeable that the expected return to partnership forming affects the term C^H (a quality improvement from the partnership) and D^H (providing collaboration service to other firms) when conducting exploitative innovation. Specifically, a more accessible firm-to-form partnering market (higher matching efficiency χ) and a greater step size parameter η can lead to a higher return to the partnership. An application to the monotonicity of contraction mapping theorem arrives at an intuitive result:

Corollary 1: A higher return to partnership leads to a delay in directing toward exploratory innovation for firms. Specifically, the flipping point n^H to exploratory innovation increases in accessibility to partnership χ and step size parameter η.

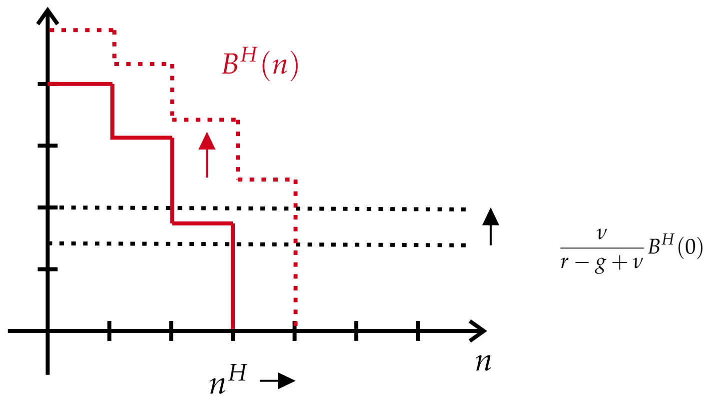

Figure 3.1: Role of Firm-to-Firm Partnership in Innovation Strategy

Proof: See Appendix 3.A.

The trade-off for this comparative statics on the optimal stopping problem is illustrated in Figure 3.1. On the one hand, an increase in return from partnership increases the value of conducting exploitative innovation as its deterioration is mitigated. On the other hand, it also increases the value of conducting exploratory innovation as a successful resetting sophistication index also benefits from a higher partnership return, making the option more attractive. Our result argues that incentives to conduct exploitative innovations are stronger than their counterpart when forming a partnership is more privately beneficial.

The value function of a low-type firm takes similar but simpler form after solving HJB equation (3.9) at stationary equilibrium.

$$V^L(z) = A^L \cdot z + B^L \cdot \bar{z}, \tag{3.20}$$

where

$$A^L = \frac{\pi}{r + \tau^L},$$

$$(r - g + \tau^H)B^H(n) = a^n C^L,$$

with

$$C^L = A^H \zeta \Delta \sum_{\tilde{n}=0}^{\infty} \hat{\mu}^H(n),$$

As expected, the value of being a low-type firm depends on the degree of spillover captured by the distribution of the sophistication index of innovation firms' technologies. When the economy is populated by more highly developed sophisticated technologies, the insights generated from them are limited, which depress the quality improvement for the low-type firms.[19]

Invariant Sophistication Index Distribution

We next compute the invariant sophistication index distribution $\{\mu^H(n)\}$ at the stationary equilibrium. In particular, we characterize the stationary mass of firms across sophistication index n. The first thing to notice is that the maximal sophistication index in the economy is $n^H + 1$. In order to ensure the existence of equilibrium, we allow ψ fractions of firms with sophistication index n^H choose to direct their innovation toward θ_E such that the optimal stopping solution satisfies complementary slackness condition at n^H:

$$\left[\nu\left[B^H(0) - B^H(n^H)\right] - a^n C^H - i[B^H(n^H+1) - B^H(n^H)] - D^H\right]\psi = 0$$

with $\psi > 0$ when $\nu\left[B^H(0) - B^H(n^H)\right] - \left[a^n C^H + i[B^H(n^H+1) - B^H(n^H)] + D^H\right] = 0$ and $\psi = 0$ when $\nu\left[B^H(0) - B^H(n^H)\right] - \left[a^n C^H + i[B^H(n^H+1) - B^H(n^H)] + D^H\right] > 0$.

Therefore, for the stationary mass of firms with the brand new technology $\mu^H(0)$, its inflow is contributed by the flow of successful θ_E innovations and

[19] We abstract from allowing high-type firms to enjoy the spillover effect for simplicity. Let high-type firm be able to learn insights from others means an increase in D^H in equation (3.16) and leads to a delay of innovation redirection to θ_E innovation with Corollary 1.

high-type entrants, while its outflow is due to exploitative innovation and successful entrants:[20]

$$\underbrace{\nu\left[\mu^H(n^H+1)+\mu^H(n^H)\psi\right]+\tau_H^L\mu^H(0)}_{\text{inflow to }\mu^H(0)}=\underbrace{(i+\tau_L^H)\mu^H(0)}_{\text{outflow from }\mu^H(0)}$$

where $\tau_L^H = h\kappa^e\mu^L$ is the fractions of low-type incumbents replaced by the high-type entrants while $\tau_H^L = (1-h)\kappa^e\sum_{n=0}^{n^L}\hat{\mu}^H(n)$ captures the probability that a high-type incumbent is replaced by a low-type entrant.

For those mass of firms with technology sophistication index $n \in (0, n^H)$, the law of motion at the stationary equilibrium is given by

$$\underbrace{i\mu^H(n-1)+\tau_H^L\mu^H(n)}_{\text{inflow from } n-1 \text{ through } \theta_D \text{ innovations and high-type entrants}} = \underbrace{(\tau_L^H+i)\mu^H(n)}_{\text{outflow due to exit and } \theta_D \text{ innovations}}$$

For completeness, we characterize the mass flow for $\mu^H(n^H)$ and $\mu^H(n^H+1)$ separately given ψ characterized by the complementary slackness condition.

$$\underbrace{i\mu^H(n^H-1)+\tau_H^L\mu^H(n^H)}_{\text{inflow}}=\underbrace{\tau_L^H\mu^H(n^H)+i(1-\psi)\mu^H(n^H)+\nu\psi\mu^H(n^H)}_{\text{outflow}}$$

The inflow to $\mu^H(n^H-1)$ comes from θ_D innovation and high-type entrants while the outflow consists of three parts: (1) competition lost to low-type entrants; (2) θ_D innovations by $(1-\psi)$ fractions of firms with n^H; (3) θ_E innovations by ψ fractions of firms with n^H.

$$\underbrace{i(1-\psi)\mu^H(n^H)+\tau_H^L\mu^H(n^H+1)}_{\text{inflow from } \theta_D \text{ innovations and high-type entrants}} = \underbrace{(\tau_L^H+\nu)\mu^H(n^H+1)}_{\text{outflow due to competition and } \theta_E \text{ innovations}}$$

The equilibrium mass of low-type firms in the economy is determined through the following inflow-outflow balancing equation:

$$(1-\mu^L)(1-h)\kappa^e\sum_{n=0}^{n^L}\hat{\mu}^H(n)=\mu^L h\kappa^e \quad\Rightarrow\quad \mu^L=\frac{(1-h)\sum_{n=0}^{n^L}\hat{\mu}^H(n)}{(1-h)\sum_{n=0}^{n^L}\hat{\mu}^H(n)+h} \tag{3.21}$$

[20] Recall that successful entrant as high-type does not accumulate the incumbent sophistication index upon entry by construction.

The mass of low-type firms depends on minimum necessary quality improvement n^L to overcome the lack of production efficiency. In particular, higher n^L implies greater μ^L holding other constant. The following lemma summarizes the stationary mass of innovative firms across the sophistication index .

Lemma 1: The mass of innovative firms across the sophistication index is summarized by the following equations:

$$\mu^H(0) = \frac{\nu\left[\mu^H(n^H+1) + \mu^H(n^H)\psi\right]}{i + \tau_L^H \mu^L} \tag{3.22}$$

$$\mu^H(n) = \frac{i}{i + \tau_L^H \mu^L}\mu^H(n-1), \quad \text{for } n \in (0, n^H) \tag{3.23}$$

$$\mu^H(n^H) = \frac{i}{i(1-\psi+\psi\nu) + \tau_L^H \mu^L}\mu^H(n^H-1) \tag{3.24}$$

$$\mu^H(n^H+1) = \frac{i(1-\psi)}{\nu + \tau_L^H \mu^L}\mu^H(n^H) \tag{3.25}$$

$$\sum_{n=0}^{(n^H+1)} \mu^H(n) + \mu^L = 1 \tag{3.26}$$

The entry and exit dynamics not only shape the landscape of technologies composition of the economy but also has implication on the growth rate. Given the distribution characterized in Lemma 1, the aggregate growth rate is determined by the frequency of innovations and the successful entry rate, summarized in the following Lemma.

Lemma 2: Given the entry efforts κ^e and the technology distribution $\{\mu^L, \{\mu^H(n)\}_{n=1}^{n^H+1}\}$, the steady state growth rate of the economy is

$$g = (1-h)\kappa^e\left[(1-\mu^L)\sum_{n=0}^{n^L}\hat{\mu}^H(n)\frac{\Delta}{s}a^n + \mu^L\sum_{n=0}^{\infty}\hat{\mu}^H(n)\frac{\Delta}{s}a^n\right] + h\kappa^e\sum_{n=0}^{\infty}\hat{\mu}^H(n)\Delta a^n$$
$$+ i\sum_{n=0}^{\infty}\hat{\mu}^H(n)\Delta a^n + x\lambda\sum_{n=0}^{\infty}\hat{\mu}^H(n)\eta a^n + \mu^L\zeta\sum_{n=0}^{\infty}\hat{\mu}^H(n) \tag{3.27}$$

The aggregate growth rate is composed of four sources: The first source is the growth contributed by low-type entrants captured by the first term, $(1-h)\kappa^e\left[(1-\mu^L)\sum_{n=0}^{n^L}\hat{\mu}^H(n)\frac{\Delta}{s}a^n+\mu^L\sum_{n=0}^{\infty}\hat{\mu}^H(n)\frac{\Delta}{s}a^n\right]$. The second channel is the improvement by the high-type entrant. The incumbent innovative firms grows with incremental innovation and collaboration while the last part is the growth contributed by spillover toward incumbent low-type firms.

Lastly, we close this section by summarizing the equilibrium.

Definition 1 (Stationary Equilibrium): *A stationary equilibrium of this economy is a tuple*

$$\left\{(n^H,n^L),\lambda,q,(\tau^H,\tau^L),\kappa^e,\{\mu^L,(\{\mu^H(n)\}_{n=0}^{n^H+1}\psi)\},\{V^H(s,z,n),V^L(z),T(n),V^e\},g,r\right\}$$

such that:

(i) n^H solves the optimal innovation policies solved in Proposition 1, and n^L is minimum improvement step size to win an innovative incumbent, which is derived from equation (3.15); (ii) (λ,q) summarizes the partnership meeting probability as in equation (3.7); (iii) (τ^H,τ^L) are the exit rate of high-type and low-type incumbents by equation (3.18); (iv) κ^e is the entry rate which solves equation (3.17); (v) $\{\mu^L,(\{\mu^H(n)\}_{n=0}^{n^H+1}\psi)\}$ summarizes the stationary technology distribution of the economy characterized by equations (3.21)-(3.26); (vi) a system of value functions $\{V^H(s,z,n),V^L(z),T(n),V^e\}$ are solved by equations (3.8),(3.9),(3.11),(3.17); (vii) growth rate specified in equation (**??**) and the real interest rate derived by Euler equation $r=\rho+g$.

3.4.6 Role of Partnership-Induced Exploitative Innovations

We next discuss the implication of forming a partnership in the economy. Specifically, we first establish an argument on the partnership's role in determining the entry rate.

Proposition 2: There exists $\hat{h}$ such that for all $h < \hat{h}$, we have $\frac{\partial \kappa^e}{\partial \chi} < 0$.

Proof: Given Corollary 1, a higher accessibility to partnership leads to a higher n^H holding other constant. Now think of the case where h is close to zero. This immediately follows that a higher χ lower V^e. Since the value of entry V^e is continuous in h, the result immediately follows. □

Higher accessibility of partnership leads to a more sophisticated technology landscape in the economy, which restricts the spillover and follow-up improvements by entrants, thus constituting a downward pressure on the entry incentives. On the other hand, such accessibility may benefit the entrants if a realization of its type is high. It follows that when the expected return on partnership is low due to a lower probability of drawing a high-type, the downward pressure dominates, leading to a lower equilibrium entry rate.

Now we evaluate how the accessibility to partnership further interacts with an economic downturn. Specifically, we consider how the entry rate is affected when the efficiency in conducting exploratory innovations decreases captured by a lower ν. Immediately, a decrease in ν lowers the value of being a high-type entrant. The following result shows that the accessibility to partnership exacerbates the weakening of entry incentives.

Proposition 3: A lower arrival rate of exploratory innovation leads to a delay in directing toward exploratory innovation, which further lower the entry rates.

Proof: See Appendix 3.A.

A lower incentive to conduct exploratory innovations induces more sophisticated technologies, which, again, generates lower spillover. Together with the availability of firm-to-firm partnering, this further pushes up the incentives to conduct exploitative innovations. This eventually corresponds to a scenario where the entry rate declines and the economy slows down.

3.4.7 Discussions

Each firm is assumed to be a single-product producer throughout this paper for the simple exposition. Leveraging the linearity property of Klette and Kortum (2004), this framework can be tractably extended to a model admitting multi-product producers by assuming at least some fractions of successful exploratory innovations can expand into other varieties using novel technologies. Similarly, the framework can accommodate creative destruction by new entrants. Furthermore, this paper assumes the heterogeneity in firms type is time-invariant, which can be extended to allow for a transition matrix for firm types to better capture each firm's life cycle. Therefore tractability of the framework can further allow for counter-factual policy implications when bringing more detailed data on ideas/patents transactions into a more general and quantitative-oriented model.

Furthermore, apart from recent literature on adopting optimal stopping problems in a dynamic setting, such as Benhabib et al. (2021) and Bilal (2021), our model allows for heterogeneous stopping points by heterogeneous agents without losing the accommodation for aggregation and characterization of growth along a stationary equilibrium path. This allows analysis on understanding firms' innovation strategies across borders and the aggregate impact in an open economy setting when internalizing heterogeneity of technology spillover across the globe.

3.5 Conclusion

This paper studies the role of firm-to-firm partnership in influencing an individual firm's innovation strategy and ultimately affecting the aggregate economy. Leveraging the data on publicly listed firms' patents data and partnership information revealed in 10-K, we argue that forming a partnership does lead to more innovations, yet most of which are exploitative and thus generate less insights for others. Then we propose a tractable Schumpeterian framework that allows firm-to-firm partnership nested in an optimal stopping problem, demonstrating how a delay in re-directing to exploratory innovations adversely affects the spillover and thus discourages the entry. It would also be interesting to bring more detailed and rich data into this framework to conduct quantitative exercises to derive more aggregate implications on interactions across entry and growth when implementing policies restricting or relaxing firm-to-firm partnerships.

3.A Appendix—Chapter 3

Proof of Proposition 1: To solve the value function, we conjecture $V^H(s,z,n) = A^H z + B^H(n)\bar{z}$ and verify the terms. We then show that there exists flipping point n^H such that

$$(r-g+\tau^H)B^H(n) = \begin{cases} a^n C^H + i[B^H(n+1) - B^H(n)] + D^H(\mathbb{E}[a^{\tilde{n}}]), & \textit{if } n \leq n^H \\ \nu\Big[B^H(0) - B^H(n)\Big], & \textit{if } n > n^H \end{cases}$$

We prove the existence by contradiction. Suppose it is optimal to never conduct θ_E innovation. This implies the problem can be reduced to

$$(r-g+\tau^H)\hat{B}^H(n) = a^n C^H + i[\hat{B}^H(n+1) - \hat{B}^H(n)] + D^H(\mathbb{E}[a^{\tilde{n}}]),$$

Rearrange the equation, we define a contraction mapping $\mathbb{T}$ such that

$$\mathbb{T}[\hat{B}^H(n)] = \frac{1}{\rho+\tau^H+i}a^n C^H + \frac{i}{\rho+\tau^H+i}\hat{B}^H(n+1) + \frac{i}{\rho+\tau^H+i}D^H$$

It is easy to verify that the contracting mapping satisfies Blackwell's conditions. Specifically, to verify the monotonicity condition, consider some $G(n) > \hat{B}(n)$ for all n, then it is straightforward to see $\mathbb{T}(G(n)) > \mathbb{T}(\hat{B}(n))$. Secondly, to check discounting condition, observe that:

$$\begin{aligned}\mathbb{T}[\hat{B}^H(n)+c] &= \frac{i}{\rho+i+\tau^H}[a^n C^H + \hat{B}^H(n+1)] + \frac{i}{\rho+i+\tau^H}c + \frac{i}{\rho+i+\tau^H}D^H \\ &< \mathbb{T}[\hat{B}^H(n)] + c, \textit{ for all } c > 0.\end{aligned}$$

Hence $\mathbb{T}$ is a well-defined contraction mapping, which implies $[B^H(n) - B^H(n+1)]$ diminishes as $n \to \infty$. In particular,

$$\hat{B}^H(\infty) = \frac{i}{\rho+i+\tau^H}\hat{B}^H(\infty) + \frac{1}{\rho+i+\tau^H}D^H(\infty) \Rightarrow \hat{B}^H(\infty) = \frac{D^H(\infty)}{\rho} = 0$$

where the last equation comes from the fact that when sophistication index of all available technologies go to infinity, the return from providing collaboration to such technology also diminishes to zero. While on the other hand, the redirecting to θ_E innovation implies

$$(\rho + \tau^H)\tilde{B}^H(n) = \nu[\tilde{B}^H(0) - \tilde{B}^H(n)]$$

of which $\tilde{B}^H(n)$ is strictly positive thus the right-hand side is positive. This implies there exists a cutoff n^H such that $a^n C^H + i[B^H(n^H+1) - B^H(n^H)] + D^H \leq \nu\Big[B^H(0) - B^H(n^H)\Big]$. To complete the proof, we rearrange the above inequality:

$$a^n C^H + i[B^H(n^H+1) - \frac{i-\nu}{i}B^H(n^H)] + D^H \leq \nu B^H(0)$$

and shows that left hand side constitute a contraction mapping. To do this, let $b(n; D^H) \equiv \frac{a^n C^H + D^H}{i} + B^H(n+1) - \frac{i-\nu}{i}B^H(n)$. This follows that

$$(\rho + \nu + \tau^H)B^H(n+1) = max\{ib(n+1); \nu\tilde{B}^H(0)\}$$

from which we can further construct the following equation

$$\begin{aligned}(\rho + \nu + \tau^H)\Big[B^H(n+1) - \frac{i-\nu}{i}B^H(n)\Big] &= \max\{ib(n+1); \nu\tilde{B}^H(0)\} \\ &\quad - \frac{i-\nu}{i}\max\{ib(n); \nu\tilde{B}^H(0)\}\end{aligned}$$

which further amounts to:

$$\begin{aligned}(\rho + \tau^H + \nu)b(n) + \frac{i-\nu}{i}\max\{ib(n); \nu\tilde{B}^H(0)\} &= (\rho + \nu + \tau^H)(\frac{a^n C^H + D^H}{i}) \\ &\quad + \max\{ib(n+1); \nu\tilde{B}^H(0)\}\end{aligned}$$

Let $F(b(n)) = LHS$ and therefore we have

$$\mathbb{T}(b(n)) = F^{-1}(RHS)$$

which clearly satisfies monotonicity condition as right-hand side is increasing in $b(n+1)$. We claim the discounting condition holds as well. Suppose that

$i(b(n+1)+c) \leq \nu\tilde{B}^H(0)$, it immediately follows:

$$T[b(n)+c] = F^{-1}[\frac{i}{i-\nu}\Big[(\rho+\nu+\tau^H)a^n C^H + \nu\tilde{B}^H(0)\Big]]$$
$$= T[b(n)]$$

Otherwise, we have:

$$T[b(n)+c] = F^{-1}[\frac{i}{i-\nu}\Big[(\rho+\nu+\tau^H)a^n C_L + ib(n+1) + D^H + ic\Big]$$
$$< T[b(n)] + \frac{\frac{i}{i-\nu}ic}{\frac{i}{i-\nu}(\rho+\nu+\tau^H)+i} = T[b(n)] + \frac{i}{\rho+i+\tau^H}c$$
$$< T[b(n)] + c$$

Therefore, T is a contraction mapping so that $b(n)$ is strictly decreasing in n. Therefore, for all $n > n^H$, we have $a^n C^H + i[B^H(n^H+1) - B^H(n^H)] + D^H < \nu\Big[B^H(0) - B^H(n^H)\Big]$. □

Proof of Corollary 1: Recall from the proof in Proposition 1 that $b(n; D^H) \equiv \frac{a^n C^H + D^H}{i} + B^H(n+1) - \frac{i-\nu}{i}B^H(n)$ which is strictly increasing in D^H. Consider $D^H(x)$ and $D^H(y)$ with $x > y$. By monotoncity condition of contraction mapping, we must have $T(b(n; D^H(x)) \geq T(b(n; D^H(y))$ for all n. The desired result immediately follows. The placeholder x can be either χ and η. □

Proof of Proposition 3: Given the solutions derived from Proposition 1, we can rewrite $B^H(0)$ by forward iterating:

$$B^H(0) = \frac{C^H}{ai}\sum_{k=1}^{n^H}(\frac{a}{\rho+i+\tau^H})^k + (\frac{i}{\rho+i+\tau^H})^{n^*_H}B_H(n^*_H) + \frac{D^H}{i}\sum_{k=1}^{n^*_H}(\frac{i}{\rho+i+\tau^H})^k$$
$$\equiv H_C + H_D + (\frac{i}{\rho+i})^{n^*_H}B_H(n^*_H) \equiv \tilde{H} + (\frac{i}{\rho+i})^{n^*_H}B_H(n^*_H)$$

Recall that $(\rho+\tau^H)B^H(n^H) = \nu[B^H(0) - B^H(n^H)]$, this implies $B^H(n^H) = \frac{\nu}{\rho+\tau^H+\nu}B^H(0)$. One can express $B^H(0)$ explicitly:

$$B^H(0) = \frac{\tilde{H}}{1 - (\frac{i}{\rho+\tau^H+i})^{n^H}\frac{\nu}{\rho+\tau^H+\nu}}$$

This implies $B^H(0)$ is increasing in ν. Then notice that for $n^H - 1$, we must have

$$a^{n^H}c^H + i[B^H(n^H) - B^H(n^H-1)] + D^H > \nu[B^H(0) - B^(n^H-1)]$$

Define $F \equiv a^{n^H}c^H + i[B^H(n^H) - B^H(n^H-1)] + D^H - \nu[B^H(0) - B^(n^H-1)]$ capturing the gap between the premium from θ_D innovation and that of θ_E innovation. Recall that

$$\begin{aligned}(\rho+\tau^H+i)B^H(n^H-1) &= a^{n^H-1}C^H + iB^H(n^H) + D^H \\ &= a^{n^H-1}C^H + D^H + \frac{i\nu}{\rho+\tau^H+\nu}B^H(0)\end{aligned}$$

Given this, $F = (\rho+\tau^H+\nu)B^H(n^H-1) - \nu B^H(0)$, and

$$\frac{\partial F}{\partial \nu} \propto a^{n^H-1}C^H + D^H - (\rho+\tau^H)B^H(0) - \frac{\rho+\tau^H}{\rho+\tau^H+i}\frac{\partial B^H(0)}{\partial \nu}$$

Note that $a^{n^H-1}C^H + D^H = (\rho+\tau^H+i)B^H(n^H-1) - iB^H(n^H) < (\rho+\tau^H)B^H(n^H-1) < (\rho+\tau^H)B^H(0)$. This follows $\frac{\partial F}{\partial \nu} < 0$, implying n^H increases when ν decreases. Furthermore, $\frac{\partial^2 F}{\partial \nu \partial D^H} > 0$. This implies the expected return to firm-to-firm partnership further pushes up the incentives to conduct θ_D innovation in the event of decline in ν. □

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