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Specification Bias in Translog Cost Systems

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ABSTRACT

A Monte Carlo experiment comparing translog cost and production systems indicates that the cost system's biased estimates of returns to scale and input substitution elasticities result from an errors in variables problem and a disturbance with non-zero mean. Guidelines for when the cost or production system is more appropriate are suggested.

A Monte Carlo Experiment Demonstrating Specification Bias in Translog Cost Systems

With the help of the translog and other flexible forms, economists have increasingly applied dual cost and profit functions in studies of production and factor demand analysis. The attractiveness of the dual approach stems from its methodological simplicity; it avoids the need to invert the first-order conditions from the production problem in calculating substitution and factor demand elasticities (Silberberg). Berndt and Christensen, Lau and Binswanger have argued that the dual avoids simultaneity problems, facilitates data collection and relies on a data matrix which is not ill-conditioned.

These arguments are based on *a priori* statistical considerations and convenience, but the relative performance of the dual system compared with a production approach is not well understood. Burgess, in modeling the U.S. economy, finds that the translog production and cost systems yield widely divergent factor demand elasticities. Humphrey and Moroney and Lessner report similar experiences with industry and firm-level data. These two translog models are not self-dual but if treated as approximations to the true functions, one would hope the implications from both are similar (Theil). Failing this, one must understand the circumstances under which each is more suitable. This paper reports on a Monte Carlo experiment comparing the performance of translog cost and production systems for synthetic technologies with known characteristics and draws implications for empirical analysis.

The Monte Carlo Design

The Monte Carlo experiments are similar to Guilkey et al.'s study of several flexible cost functions. To begin, 50 observations of three inputs each are chosen from a log normal distribution,

$$\text{(1)}\quad \ln \underline{X} \sim N \begin{bmatrix} 0.582 \\ 0.239 \\ 0.391 \end{bmatrix}, \begin{bmatrix} 0.066 & & \\ 0.039 & 0.026 & \\ 0.063 & 0.041 & 0.065 \end{bmatrix}$$

, representative of manufacturing and agricultural time series data (e.g. Berndt and Wood; Brown). They are used to calculate output from a generalized CES function

(2) $Y_t\, e^{\theta Y_t} = (\sum \delta_i X_{it}^{-\rho_i})^{-\gamma/\rho}\, e^{\epsilon_{yt}}$, where X_{it} are inputs, Y_t is output and ϵ_{yt} is a stochastic term and is distributed N ~ (0,.0025). Output varies across experiments as the parameters of (2) are changed to exhibit properties ranging from linear homogeneous and separable to non-homothetic and inseparable and different returns to scale.

Next, a set of prices (w) is derived from first-order conditions:

$$\text{(3)}\quad w_{it} = \frac{\gamma}{\rho}\,[e^{\theta y_t}(1+\theta y_t)](\sum_j \delta_j\, X_{jt}^{-\rho_j})^{-\gamma/\rho-1}\,(\rho_i \delta_i X_{it}^{-\rho_i-1})e^{\epsilon_{it}} \qquad i=1,\ldots,3;\ t=1,\ldots,t,$$

where y_t, the expected value of output, excludes the disturbance ϵ_{yt}. A stochastic term (ϵ_{it}) reflects errors in optimizing decisions and differences between observed and expected prices (Zellner et al.).

For each experiment, the values of four production characteristics (marginal products, factor shares, returns to scale and substitution elasticities) are determined at each of the fifty sample points (Driscoll) and stored for comparison with the corresponding estimates derived from parameters estimated for translog cost and production systems:

$$\text{(4)}\quad \ln Y = \alpha_0 + \sum \alpha_i \ln X_i + 1/2\ [\sum_i \sum_j \beta_{ij} \ln X_i \ln X_j] + \epsilon;$$

$$\text{(5)}\quad M_i = \alpha_i + \sum_j \beta_{ij} \ln X_i + \epsilon_i \qquad \text{for all } i = 1,\ldots,n; \text{ and}$$

$$\text{(6)}\quad \ln C = \delta_o + \delta_y \ln Y + \delta_{yy}(\ln Y)^2 + 1/2\ [\sum_i \sum_j \gamma_{ii} \ln w_i \ln w_j] + \sum_i \gamma_{iy} \ln w_i \ln Y + \epsilon_o;$$

(7) $M_i = \alpha_i + \sum_j \gamma_{ij} \ln w_j + \gamma_{iy} \ln Y + e_i$ for $i = 1,\ldots,n-1$, where Y, X_i, and w_i are as above; M_i = ith factor share of output (cost); and C = cost. Symmetry of the coefficients was imposed in both models, while linear homogeneity was also imposed on the cost system. Variables are scaled by dividing by the geometric means prior to taking logarithms; an iterative Zellner (IZEF) procedure is used to estimate both systems.

The experiment covers 28 technologies. Each of 50 replications for every technology differs because of the stochastic disturbances on output and prices. Estimates of the production characteristics are averaged over the 50 observations for 50 replications. Following Guilkey et al., the absolute deviation of the characteristic's estimate from the true characteristic value is noted at each observation and the mean absolute deviation (MAD) is determined as well. In evaluating performance, one is ideally looking for average estimates close to the average of actual values, with small MAD statistics. When the average of the estimates is not close to the average of actual values and MAD is close to this difference, there is a consistent bias.

Empirical Results

This discussion focuses on eight of the technologies involved in the simulation experiment. They represent a wide range in technological conditions and are representative of the results across all simulations. In table 1, the first 3 technologies exhibit constant returns to scale and have constant elasticities of substitution among inputs. In 4, 5 and 6, returns to scale average less than unity over the observations. In 4 and 5, the underlying cost functions are separable in two prices, while in technologies 7 and 8, inputs 2 and 3 are complementary.

Table 1. Monte Carlo Estimates of Scale and Allen Partial Substitution Elasticities for Selected Technologies

Generalized CES Parameters $\theta = 0,\ \sigma = 1$		Returns to Scale	σ_{11}	σ_{22}	σ_{33}	σ_{12}	σ_{13}	σ_{23}
	Technology 1							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	1.000	-6.052	-8.103	-4.680	3.030	3.030	3.030
$\delta_3=.4$	PROD SYS	1.001	-6.071	-8.269	-4.711	3.067	3.023	3.097
$\rho_i=-.67,\ \rho=-.67$	MAD STAT	0.005	0.242	0.798	0.304	0.280	0.216	0.476
	COST SYS	1.050	-5.778	-6.160	-3.931	2.706	3.025	1.960
	MAD STAT	0.066	0.284	1.943	0.750	0.335	0.118	1.070
	Technology 2							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	1.000	-2.332	-2.331	-1.498	0.999	0.999	0.999
$\delta_3=.4$	PROD SYS	1.001	-2.343	-2.390	-1.531	1.006	1.003	1.037
$\rho_i=.001,\ \rho=.001$	MAD STAT	0.006	0.065	0.169	0.095	0.067	0.043	0.125
	COST SYS	1.051	-2.223	-1.805	-1.256	0.898	0.994	0.679
	MAD STAT	0.068	0.111	0.526	0.244	0.105	0.039	0.320
	Technology 3							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	1.000	-1.247	-0.349	-0.388	0.250	0.250	0.250
$\delta_3=.4$	PROD SYS	1.002	-1.180	-0.351	-0.390	0.244	0.236	0.255
$\rho_i=3.0,\ \rho=3.0$	MAD STAT	0.008	0.131	0.026	0.028	0.033	0.045	0.026
	COST SYS	1.065	-0.846	-0.255	-0.289	0.197	0.173	0.178
	MAD STAT	0.091	0.627	0.096	0.112	0.112	0.190	0.081
	Technology 4							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	0.967	-2.622	-3.913	-27.429	3.108	1.080	1.080
$\delta_3=.4$	PROD SYS	0.969	-2.727	-4.097	-28.701	3.243	0.851	1.473
$\rho_1=-.67,\ \rho_2=-.67$	MAD STAT	0.006	0.285	0.494	5.678	0.357	0.774	1.274
$\rho_3=-.05,\ \rho=-.47$	COST SYS	1.029	-1.752	-2.309	-14.930	1.964	2.038	-1.198
	MAD STAT	0.086	0.870	1.604	12.858	1.144	1.000	2.277
	Technology 5							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	0.949	-5.685	-5.130	-0.198	1.538	0.500	0.500
$\delta_3=.4$	PROD SYS	0.950	-5.355	-6.636	-0.107	1.755	0.287	0.541
$\rho_1=.3,\ \rho_2=.3$	MAD STAT	0.049	1.202	2.462	0.152	1.214	0.368	0.381
$\rho_3=3.0,\ \rho=1.0$	COST SYS	1.162	126.557	100.053	-0.074	-4.200	0.073	-0.211
	MAD STAT	0.384	132.256	105.187	0.126	10.450	0.600	0.875
	Technology 6							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	0.997	-9.438	-14.930	-8.361	7.742	5.161	2.581
$\delta_3=.4$	PROD SYS	0.997	-9.882	-16.068	-8.664	8.128	5.274	2.978
$\rho_1=-.9,\ \rho_2=-.8$	MAD STAT	0.006	1.394	2.768	0.908	1.225	1.190	1.674
$\rho_3=-.7,\ \rho=-.8$	COST SYS	1.047	-6.716	-9.033	-5.180	5.382	3.794	0.698
	MAD STAT	0.060	2.723	6.028	3.181	2.396	1.367	1.943
	Technology 7							
$\delta_1=.3,\ \delta_2=.3$	ACTUAL	1.100	-0.382	-19.204	-18.524	1.984	1.736	-1.240
$\delta_3=.4$	PROD SYS	1.105	-0.389	-21.722	-20.308	2.032	1.717	0.873
$\rho_1=-1.5,\ \rho_2=-.3$	MAD STAT	0.010	0.041	3.991	3.607	0.190	0.285	3.215
$\rho_3=-.2,\ \rho=-.75$	COST SYS	1.148	-0.095	-2.853	-4.683	0.387	0.509	-0.756
	MAD STAT	1.131	0.286	16.352	13.841	1.597	1.227	2.011
	Technology 8							
$\delta_1=.1,\ \delta_2=.45$	ACTUAL	1.007	-1.825	-8.463	-4.711	2.157	2.427	-0.270
$\delta_3=.45$	PROD SYS	1.011	-1.927	-8.679	-4.852	2.255	2.543	-0.266
$\rho_1=-1.1,\ \rho_2=-.1$	MAD STAT	0.011	2.628	1.993	2.390	2.110	2.445	2.104
$\rho_3=-.2,\ \rho=-.3$	COST SYS	1.054	-0.468	-4.919	-1.904	0.555	0.646	1.218
	MAD STAT	0.101	1.357	3.544	2.807	1.602	1.780	1.487

As reported by Driscoll, both translog systems produced good factor share estimates. For seven technologies, the MAD errors for the cost and production systems' estimates of factor shares were less than 3% of the true values. For technology 5, the MAD errors average about 15% and 7% for the cost and production systems, respectively.

For marginal products, MAD errors of the production system estimates never exceed 7% and usually average between 1 and 2% of the actual values. The MAD error in the marginal products from one cost system is in excess of 50%; an upward bias of from 10 to 20% exists for most of the 28 technologies. This is not unexpected. In the production system, marginal products are simple functions of production elasticities (left-hand sides of share equations) which are estimated accurately, while in the cost system, marginal products are more complicated expressions in output, cost and the term for returns to scale (Driscoll, p. 136).

Table 1 reports average values for returns to scale and Allen partial elasticities of substitution (σ_{ij}) for the eight technologies with their corresponding cost and production system estimates. MAD errors appear below the estimates. Direct and cross Allen partial elasticity terms are of interest in that they could be derived for both the production and cost systems and they are related to factor demand elasticities (Ferguson, p. 152). (Direct comparison of factor demand elasticities was not possible. Closed form solutions to factor demands were unobtainable for some technologies.) It is evident that the cost system's estimates of scale and Allen partial elasticities are relatively more biased and exhibit larger MAD errors than the corresponding production system's estimates. Returns to scale from the cost system are biased upward by at least 5% for every technology. Cost side estimates of the Allen own partials are biased upward; since

(8) $\sum_j M_j \sigma_{ij} = 0$ for all i (Allen, p. 504),

at least one and usually both cross partials are biased downward.

The sources of the consistent biases are examined by comparing the coefficient estimates of the cost function under different disturbance assumptions (table 2). First, the models were estimated by setting all price and output disturbances to zero, leaving only the error involving truncation of higher-order terms in the translog approximations. With no disturbances, the cost system estimates of _all_ production characteristics are unbiased with much smaller MAD error as long as the technologies are not too complex (Driscoll, tables C.1-C.28). These same translog coefficient estimates, which may be interpreted as first and second-order log derivatives, were found to approximate the mean of the corresponding log derivatives (evaluated at all sample points) of the underlying production function (across all 28 technologies) and the cost function (for 7 CES technologies where a dual form exists). For complex technologies, large variances in the log derivatives across sample points account for relatively poorer quality estimates of both systems (Driscoll, tables 5.3-5.7). Thus, for all technologies, except possibly those involving complementarities, these coefficient estimates based on the undisturbed data are a good base for comparison.

Focusing first on the returns to scale

(9) $r = 1/(\delta_y + \delta_{yy} \ln Y + \sum_i \gamma_{iy} \ln w_i)$,

one can begin to identify the sources of bias. The data are scaled around their geometric means; at the means ln Y and ln w_i are zero at this point.

Table 2. Coefficient Estimates of the Translog Cost System Under Different Disturbance Regimes

Disturbances	δ_0	δ_y	δ_{yy}	γ_{1y}	γ_{2y}	δ_1	δ_2	γ_{11}	γ_{12}	γ_{22}
Technology 1										
N	.0004	1.00	.005	.00001	-.0001	.334	.272	-.449	.184	-.401
w,y	-.004	.931	.190	.004	-.016	.334	.272	-.401	.122	-.252
w	.0000	1.00	.211	-.006	-.007	.337	.268	-.388	.121	-.323
Technology 2										
N	.000	1.00	-.002	.000	.000	.300	.300	.000	.000	.000
w,y	-.004	.931	.189	.004	-.016	.300	.299	.016	-.02	.050
Technology 3										
N	-.020	1.00	-.206	-.002	.001	.173	.427	.108	-.055	.181
w,y	-.027	.930	-.04	-.001	.020	.174	.423	.113	-.058	.193
Technology 4										
N	.005	1.03	.183	.048	-.032	.529	.432	-.420	.392	-.359
w,y	-.006	.952	.488	.053	-.044	.530	.431	-.206	.192	-.168
Technology 5										
N	-.078	1.06	.716	.256	-.201	.129	.147	.055	-.041	.179
w,y	-.066	1.04	1.44	-.079	.132	.132	.144	.080	-.007	.115
Technology 6										
N	.000	1.00	-.058	.221	-.061	.402	.267	-.127	.713	-.847
w,y	.000	.932	.000	.162	-.069	.402	.267	-.82	.399	-.400
Technology 7										
N	.007	.917	-.566	.158	-.087	.837	.086	.001	.006	-.019
w,y	-.012	.854	.118	.118	-.066	.837	.085	.035	-.021	.032
Technology 8										
N	.005	1.00	-.78	.353	-.104	.574	.138	-.128	.018	-.013
w,y	-.005	.928	.052	.161	-.060	.574	.138	.073	-.031	.024

Note: N = no disturbances; w,y = prices and output disburbed; w = prices disturbed. Because of the adding up conditions to impose linear homogeneity, the remaining parameters γ_{3y}, γ_{13}, γ_{23} and γ_{33} are calculated as residuals.

An upward bias in r requires an underestimate of δ_y. This parameter is important at other points because ln Y and ln w_i are positive and negative when above and below the mean. Thus, one would not expect any bias in δ_{yy} or γ_{iy} to affect r in the same direction at each point.

To understand why the coefficient of ln Y in the translog cost function is underestimated, recall that minimum cost prices from (3) are based on expected output (y), a reasonable assumption for many biological production processes (Zellner et al.). However, in general, only actual output data (ye^{ϵ_y}, not y) are available in empirical work. By specifying the cost equation in terms of expected output, equation (6) can be written (for θ = 0 from (2))

$$(10)\quad \ln C = \delta_o + \delta_y \ln(Ye^{-\epsilon_y}) + \delta_{yy}(\ln(Ye^{-\epsilon_y}))^2 + \sum_i \delta_i \ln w_i + \sum_i \sum_j \delta_{ij} \ln w_i \ln w_j$$

$$+ \sum_i \gamma_{iy} \ln(Ye^{-\epsilon_y}) \ln w_i + \epsilon_o = \delta_o + \delta_y \ln Y + \delta_{yy} (\ln Y)^2 + \sum_i \delta_i \ln w_i$$

$$+ \sum_i \sum_j \delta_{ij} \ln w_i \ln w_j + \sum_i \gamma_{iy} \ln y \ln w_i$$

$$+ [\epsilon_o - 2\delta_{yy} \ln Y \epsilon_y + \delta_{yy}(\epsilon_y)^2 - \delta_y \epsilon_y - \sum_i \gamma_{iy} \ln w_i \epsilon_y]$$

where y is the unobserved expected output

$$(11)\quad Y = ye^{\epsilon_y} \text{ or } y = Ye^{-\epsilon_y}.$$

Following the literature on errors in variables, this results in a cost equation whose disturbance (in brackets) is a function of observed output (ln Y) and ϵ_y, and bias results. Although estimates of all parameters in (10) will be inconsistent, δ_y will be particularly so, given the second component of the error. If the squared logarithm of output is removed from the cost equation, bias to δ_y will be reduced but not eliminated. If ϵ_y = 0, the errors in variable bias disappears. This latter proposition was tested by removing the output and price disturbances from technology 1.

When both disturbances were eliminated, the cost system produced nearly perfect estimates of returns to scale. This was also the case when the output disturbance was removed, in spite of the remaining price disturbances (Driscoll). One may compare the coefficient estimates for technology 1 under three different disturbance regimes to see which coefficients are affected by the output disturbance (table 2). The results indicate that only output disturbances affect the estimate of δ_y, that price disturbances affect δ_{yy}, γ_{iy} and γ_{ij} coefficients, and that output disturbances can further distort estimates of the γ_{iy} and γ_{ij} coefficients. Similar conclusions are evident from the results of the two disturbance regimes reported for the remaining seven technologies.

The effects of these distortions are transmitted to the Allen partial estimates as well. Notice that production system estimates are relatively unbiased and generally exhibit less than half the MAD error of their cost system counterparts (table 1). The cost system overestimates all Allen own partials across all eight technologies. Because of the adding up restrictions given in (8) the Allen cross partials are usually underestimated (σ_{13} in technology 4 and σ_{23} in technologies 7 and 8 are exceptions). The source of the cost system bias may be traced by examining expressions for the Allen partials from the cost system

$$(12) \quad \sigma_{ii} = \frac{\gamma_{ii} + M_i^2 - M_i}{M_i^2}, \quad \sigma_{ij} = \frac{\gamma_{ij} + M_i M_j}{M_i M_j} \quad .$$

These elasticities are affected by errors or biases in estimated factor shares (M_i) and γ's. They are particularly sensitive to small shares. Generally M_i's are estimated quite accurately; bias of the magnitude seen in σ_{ii} and σ_{ij} must be in large measure due to bias in γ_{ii} and γ_{ij}.

Notice that the γ's exhibit bias under the price, output disturbance regime and that this bias does not disappear if the output disturbance is removed (see technology 1, table 2). Remaining price disturbances here mimic imperfect price foresight or ignorance of the true production technology, but in either case, the resulting price-input combinations are seldom minimum cost combinations of producing the specified output (Varian, p. 39). However, it is only this minimum cost surface that is dual to the production surface. Given this disturbance regime, a cost surface is generated which lies <u>on or above</u> the minimum cost surface and the error structure (ϵ_o) associated with the translog cost function (10) has a positive rather than a zero expected value. For $E[\epsilon_o] > 0$ the bias in the vector of parameters of the cost function (β) is given by

$$(13)\quad E[\hat{\beta}] - \beta = (x'(\hat{\Sigma}^{-1} \otimes I)x)^{-1}\, x'(\hat{\Sigma}^{-1} \otimes I)\, E[\epsilon_o].$$

Although it is not evident from this expression which parameters reflect bias, the empirical results indicate a persistent upward bias to the δ_{yy} and γ_{ii} coefficients (compare the estimates of the undisturbed and disturbed scenarios in table 2).

Concentrating on the translog cost equation, this appears reasonable. To raise cost estimates up to the cost surface generated under producer error, some coefficients in equation (6) must adjust. The intercept is a candidate, but at the mean of the data, δ_o should be close to zero since the dependent and all independent variables are zero at that point. The coefficient δ_o does remain close to zero. With the exception of technologies 5 and 8, the γ_{iy} coefficients exhibit no appreciable bias and essentially, no bias accrues to parameters δ_y, and δ_i as a result of the erroneous assumption on cost error structure.

The remaining candidates are the coefficients δ_{yy} and γ_{ii}. Because the data are scaled by the geometric mean, the squared terms associated with these coefficients are the only ones that are uniquely signed at all observations. An upward bias to these coefficients is the only way to raise cost estimates consistently. It appears that only very high positive (negative) correlations among output and prices will upset this generalization, in that under these conditions, the two logarithmic variables of the cross product terms would have the same (opposite) sign and their product would be positive (negative). In such cases, it is difficult to say how the δ_{yy}, γ_{ii} and γ_{ij} coefficients are affected.

Because linear homogeneity in prices is imposed on the cost system, the expected positive bias in the γ_{ii} terms leads to a compensating bias in γ_{ij} in the opposite direction. In a three-input technology, if a disproportionate amount of the bias falls on any pair of γ_{ii} coefficients, say γ_{11} and γ_{22}, adding up restrictions are most easily satisfied through negative bias to γ_{12}. This phenomenon occasionally produces a real outlier in the estimate of the Allen cross partial; sometimes, as in technologies 4 and 5, a complementary input pair is identified when none exists.

Implications for Empirical Research

Through a careful examination of the results from a Monte Carlo experiment across 28 technologies (eight of which are reported here), differences in the performance of the translog production and cost systems are shown to be attributable to two important specification errors in the cost system, one associated with an errors in variable problem and the other with a positive expectation on the cost function's error term. The direction of the bias is predictable, suggesting that estimates of returns to scale and direct partial elasticities of substitution estimated from cost functions should be interpreted as upper bounds, whereas estimates of cross partial elasticities are probably lower bounds. Thus, in empirical situations where output disturbances and producer decision errors are expected to be large, a translog production system may well outperform the cost system in estimating important characteristics of production technology and factor demands. Where these errors are not expected to be large, the choice of system may well depend on the availability of and the relative dispersion in input and input price data (Driscoll).

In drawing these conclusions, it must be kept in mind that they are based on averages across 50 replications for each separate synthetic technology. One can legitimately ask whether these results will be sustained for any given sample of actual empirical data. Too few comparisons of these two approaches to measuring input substitution exist to provide a definitive answer to this question and more work is needed. However, two of the important empirical comparisons to date (e.g. Burgess and Humphrey and Moroney) offer substantial support for these propositions. In Burgess, for example, the estimates of the direct Allen partials from the cost system were consistently higher than for the production system for all three inputs; the cross partials were lower between capital and labor and capital and imports and higher between labor and imports. The production system indicates easy substitution between capital and imports, while the cost system identifies a weak complementarity.

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