A NOTE ON A METHOD OF CONSTRUCTION OF DESIGNS FOR TWO-WAY ELIMINATION OF HETEROGENEITY

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ABSTRACT

In this note we present a mathematical proof for a method of constructing a balanced generalized two-way elimination of heterogeneity design. The method was given in a paper by H. Agrawal in 1966.

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INTRODUCTION

Of the methods of construction of two-way elimination of heterogeneity designs, Agrawal (1966) had no mathematical proof for the following construction procedure:

Method. Let D be a symmetrical balanced incomplete block (BIB) design (cf. Raghavarao [1971], pp. 63) with parameters $v = b$, $r = k$, λ. Let D_1 be the derived design (cf. Raghavarao [1971], pp. 65) of D with parameters

$$v_1 = v-k,\ b_1 = v-1,\ r_1 = k,\ k_1 = k-\lambda,\ \lambda_1 = \lambda$$

and let D_2 be the residual design (cf. Raghavarao [1971], pp. 65) of D with incidence matrix N_2 having the parameters

$$v_2 = k,\ b_2 = v-1,\ r_2 = k-1,\ k_2 = \lambda,\ \lambda_2 = \lambda-1.$$

In the incidence matrix of the complementary design (cf. Raghavarao [1971], pp. 65) of D_2, by replacing 1's in the i^{th} column by the symbols of the i^{th} set of D_1 and rearranging the symbols in such a way that each of the $v-k$ symbols occurs exactly once in each row, a balanced generalized two-way elimination of heterogeneity design will be obtained.

The proof that the symbols could be arranged such that every row is a complete replication of $v-k$ symbols will be supplied by us in this communication with the help of systems of distinct representatives (SDR). For the definition and results of SDRs, we refer to chapter 6 of Raghavarao (1971).

PROOF OF THE METHOD

A moment's consideration reveals that we will be through with the proof if we show that SDRs exist for the sets of D_2 in which each of the symbols of the first set of D does not occur in D_1. Let $\theta_1, \theta_2, \ldots, \theta_k$ be the k symbols in the first set of D and let $\phi_1, \phi_2, \ldots, \phi_{v-k}$ be the other v-k symbols. Let $B_1^{(1)}$, $B_2^{(1)}, \ldots, B_{v-k}^{(1)}$ be the sets of D_2 not containing θ_1 in the corresponding sets of D_1. Since (θ_1, ϕ_j) pair occurs exactly λ times in D, each ϕ_j can occur at most $k-\lambda$ times in $B_1^{(1)}, B_2^{(1)}, \ldots, B_{v-k}^{(1)}$ for $j = 1, 2, \ldots, v-k$. One can easily show that the necessary and sufficient condition for the existence of SDR will be satisfied by sets $B_1^{(1)}, B_2^{(1)}, \ldots, B_{v-k}^{(1)}$ and this SDR will be a permutation of $\phi_1, \phi_2, \ldots, \phi_{v-k}$. Bringing these symbols in the first column and leaving blanks in the first column of other sets, we omit the first column and analogously show that for the deleted sets $B_1^{(2)}, B_2^{(2)}, \ldots, B_{v-k}^{(2)}$, where θ_2 does not occur in the corresponding sets of D_1, an SDR exists. This could then be brought to the second position leaving blanks in the other sets. Following this method we can show that the rearrangement as given by the method could be achieved.

It may be noted that this series of designs could be constructed by omitting a column and the symbols in it from the corresponding Youden Square designs. However, in this method also we use SDRs to show the existence of Youden Squares from symmetrical BIB designs.

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