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A COMPONENT OF VARIANCE DUE TO COMPETITION[1]

by

W. T. Federer and O. O. Ladipo

Abstract

The existence of a component of variance for competition among sampling units or among individuals in a group was discussed by Yates and Zacopany in 1935. No procedure was suggested for estimating this component of variance. It is the purpose of this paper to give a procedure for estimating the component of variance due to competition and to apply the procedure to a

set of data on weaning weights of pigs with 116 litters of various sizes and for Yorkshire, Chester-White, and Berkshire breeds. The first problem was to define litter size. Within this definition then, litters sizes of 3 to 14 pigs per litter were obtained. The variation among pigs within a litter of size h was considered to have an expected value equal to $V_s + V_{ch}$ where V_s is the sampling variance component and V_{ch} is the competition variance component for a litter of size h. In order to obtain an estimate of V_{ch}, a polynomial relation between h and V_{ch} was postulated. In particular, it was postulated that

$$E(V_{ch}) = \beta_1(h-1) + \beta_2(h-1)^2 + \beta_3(h-1)^3 + \cdots$$

or

$$E(V_{ch}) = \beta_1 h + \beta_2 h^2 + \beta_3 h^3 + \cdots$$

where E( ) denotes expected value. The first form states that V_{ch} goes to zero for one pig per litter while the second form puts V_{ch} equal to zero for h = 0.

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[1] Paper No. BU-154 of the Biometrics Unit and No. 548 of the Department of Plant Breeding and Biometry, Cornell University.

The first form may be more appropriate as long as small litter sizes (say 1 and 2 at least) are omitted from the analysis as was done in the present instance. Likewise, one could postulate a polynomial relation between the sampling variance component V_s and litter size. This polynomial would have an intercept α and would need to have different powers in the polynomial in order to estimate the two variance components; e.g. one could postulate that $E(V_{sh}) = \alpha + \Pi \sqrt{h}$.

Using the assumption that V_s was unaffected by litter size and that $E(V_{ch}) = \beta_1(h-1) + \beta_2(h-1)^2$, it was found that $\hat{V}_{ch}$ reached a maximum for a litter size of 10 for the odd litter sizes and 6 for the even litter sizes after the weaning weights had been adjusted by covariance for birth weights. It appeared that $V_{ch} + V_s$ = within litter mean square followed a different pattern for odd sizes than for even sizes of litter. The biological reason for this is unknown.

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W. T. Federer and O. O. Ladipo[2]

Introduction

Competition exists among the individuals of a litter, of a family, a class of students, animals in a fixed size feed-lot, a team, or any group of individuals competing for food, space, attention, etc. In some instances the competition stimulates the individual toward a higher yield which may be weight, scholastic achievement, or performance on a test. If the competition is limited as in small groups or is too severe as in large groups the individual may not perform well. If this situation holds then there should be a group size that leads to maximum achievement. One obvious way to determine optimum group size is to plot mean achievement per individual against group size and to estimate the optimum group size from a polynomial regression fitted to the data. Although this method holds for the above, it will not suffice for all purposes, as it may be desirable to estimate a component of variance due to competition separately from the sampling and/or genetic components of variance associated with individuals within a group. Such estimates yield information on group size leading to a maximum component of variance due to competition. Also, investigations on the relation of size of this component of variance to performance can then be made.

Yates and Zacopanay [1935] described a component of variance due to competition but did not present a method for its estimation. The present paper describes a method of estimating this variance component and its application to a

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set of data consisting of weaning (56 day) weights of swine for 116 litters with litter sizes of h=3,4,5,···,14. Possible applications to other experimental situations are described.

The data used in this study were obtained from W. G. Pond, Animal Science Department, Cornell University, and were collected from 1946 through 1958. Birth weights and 56 day weights were recorded for each piglet in the litter. Litter size is not a simple count of animals in a litter but had to be defined; it was defined to be the number of piglets alive in a litter from seven days after birth to weaning time at 56 days of age. The death of a piglet within the first week after birth was assumed to not materially affect competition among the remaining piglets in the litter. If a piglet died at any time between 7 and 56 days that litter was not utilized since this would change the level of competition within a litter. The data are from three breeds, viz. Berkshire, Yorkshire, and Chester-White resulting in data for 116 litters. The cross-breeds were not utilized in the present study; a study of data on the Yorkshire breed only was made by Ladipo [1965].

2. A Method of Estimating a Component of Variance Due to Competition

Yates and Zacopanay [1935] made the assumption that the sum of the competition effects within a group would be zero. This means that what one individual gains from the other individuals, is compensated for by a corresponding loss in other individuals. Jensen and Federer [1964] have found that this model does not hold for wheat planted in adjacent rows. The loss in yield in certain varieties of wheat was less than the gain in others. Thus, there was a bonus in yield from intervarietal competition over varieties planted separately. However, for the data at hand we shall use the model of Yates and Zacopanay [1935] and assume the linear yield equation model is

$$Y_{ij} = p_i + s_{ij} + c_{ij} \quad ,$$

where the p_i are random variates associated with litters and are identically and independently distributed with mean μ and variance V_p, the s_{ij} are random variates relating to sampling variation among individuals within a litter and are independently and identically distributed with mean zero and variance V_s, the c_{ij} are random variates associated with competition effects between individuals of a litter and are identically distributed with mean $\bar{c}_i$ and variance V_c, $i=1,2,\cdots,$ ℓ_h = number of litters of size h, $j=1,2,\cdots,h$, and the effects are independent of each other.

Suppose that there are h individuals, piglets, in each group, litter, that there are ℓ_h randomly selected litters for each size of litter h, and that k individuals, piglets, are randomly selected within each group, litter. Then, the expected mean squares in the analysis of variance are of the following form for a litter of size h:

Source of variation	Degrees of freedom	Mean square	Expected value of mean square
Among groups (litters)	ℓ_h-1	A_h	$V_{sh} + \frac{h-k}{h} V_{ch} + kV_{ph}$
Within groups (litters)	$\ell_h(k-1)$	W_h	$V_{sh} + V_{ch}$

In a single analysis of variance we can only estimate $V'_{sh} = V_{sh} + V_{ch}$ and

$V'_{ph} = V_{ph} - V_{ch}/h$. If k = h then we can only estimate $V_{sh} + V_{ch}$ and $V_{sh} + hV_{ph}$. Unless we have an estimate of V_{sh} we cannot estimate V_{ph}.

Now suppose we have ℓ_h litters available for p litter sizes, say h = g+1, g+2,$\cdots$,g+p, for litter or group sizes above a certain size, say g. Further suppose that there is a b-polynomial relationship between group size and V_{ch} of the following nature:

$$V_{ch} = \sum_{u=1}^{b} \beta_u h^u \tag{1}$$

or

$$V_{ch} = \sum_{u=1}^{b} \beta_u (h-1)^u \tag{2}$$

Equation (1) indicates that V_{ch} is zero for h = 0 whereas equation (2) indicates that V_{ch} is zero for h = 1 individual per group. The latter equation may be more appropriate; also, it is possible that competition within small (say h=1,2,3) litters is quite different from that within larger litters. If g is fairly large, say greater than 10 to 15, either equation (1) or (2) may be utilized as both would give approximately the same results.

Furthermore, it may be possible that V_{sh} is not a constant for all h but is related to h by a polynomial of the following form:

$$V_{sh} = \alpha + \sum_{u=1}^{c} \delta_u h^{u-\frac{1}{2}} \quad . \tag{3}$$

The forms of the polynomial in equations (1) (or (2)) and (3) need not be as above, but it is necessary that the powers of h in (1) (or (2)) be different from those in (3) in order to estimate the β_u, the δ_u, and α. Also, it is necessary to have α in only (1) or in (3) in order that it can be estimated.

Likewise, a polynomial relationship of V_{ph} with h could be utilized, but here again the polynomial coefficients must be different than for equations (1) and (3) if both A_h and W_h observations are utilized. In many situations it would appear that V_{ph} should equal V_p = a constant and be independent of h.

Let $\underline{B}' = (\alpha\ \beta_1\ \beta_2 \cdots \beta_b\ \delta_1\ \delta_2 \cdots \delta_c)$, let $\underline{Y}' = (W_{g+1}\ W_{g+2} \cdots W_{g+p})$ for $b + c < p$, and let X be a $p \times (1+b+c)$ matrix with ones in the first column, $h = g+1, g+2, \cdots, g+p$ (or h-1) in the second column, values of h^2 (or $(h-1)^2$) in the third column, etc. up to the b+2nd column which has values of $h^{\frac{1}{2}}$ (or some power not unity), values of $h^{3/2}$ in the b+3rd column, etc. The form of the average value of the observation equation is then $E(\underline{Y}) = X\underline{B}$. The least squares estimate of $\underline{B}$ from the normal equations is then $\hat{\underline{B}} = (X'X)^{-1}X'\underline{Y}$. If the various mean squares W_h are weighted by degrees of freedom then the weighted regression solution is $\hat{\underline{B}}_w = (X'FX)^{-1}X'F\underline{Y}$ where F is a $p \times p$ diagonal matrix with diagonals equal to degrees of freedom associated with the mean squares in Y.

Weaning weights should be adjusted for birth weights as weight increases geometrically in this period. Hence, the following linear yield equation is used:

$$Y_{ij} = p_i + s_{ij} + c_{ij} + \beta(Z_{ij} - \bar{Z}) \quad ,$$

where Z_{ij} is the birth-weight corresponding to the weaning-weight Y_{ij} and β is the linear regression coefficient of Y_{ij} on Z_{ij}. To estimate the parameters use is made of the mean squares adjusted for the covariate and the associated degrees of freedom in F are reduced by one. Therefore, let $\underline{Y}_a$ equal the column vector of the p adjusted mean squares and then the solutions for the parameters in $\underline{B}$ are $\hat{\underline{B}}_{aw}=(X'(F-I)X)^{-1} = X'(F-I)\underline{Y}_a$ where I is the p x p identity matrix, X and F are as defined previously, $\underline{Y}_a$ contains the p adjusted mean squares, and the 1+b+c elements of $\hat{\underline{B}}_{aw}$ are the least squares estimates of the parameters using weighted regression and the adjusted mean squares as the observations.

For the particular set of data involving 116 litters of swine we shall use equations (1) and (2) using only β_1 = linear regression coefficient and β_2 = quadratic regression coefficient, i.e. $V_{ch} = \beta_1(h-1) + \beta_2(h-1)^2$ or $V_{ch} = \beta_1 h + \beta_2 h^2$ depending upon which form is considered appropriate. Likewise, in equation (3) we shall assume that all δ_u are zero and that $V_{sh} = V_s = \alpha$ is independent of litter size. This formulation states that V_{ch} is a quadratic regression function of litter size and V_s is independent of litter size. V_s is positive, $\hat{\beta}_1$ should be positive, $\hat{\beta}_2$ should be negative, and the expected value of the within litters mean square is $V_s + \beta_1(h-1) + \beta_2(h-1)^2$.

3. The Experimental Data

The experimental data for the 116 litters utilized in this study are presented in Table 1. The average birth weights (times 10) and the average weaning (56 day) weight from Table 1 are plotted against the number of pigs per litter in Figure 1. Both plots follow the same pattern with the lower weights being associated with the larger litter sizes. The mean per pig 56 day weights for litter sizes of 13 and 14 appear to deviate from the pattern set by other litter sizes. This is further borne out in Figure 2, where the total weight per litter is plotted against litter size; there appears to be an almost constant total weight for litters of size 10, 11, and 12, i.e. the extra pigs did not increase the total weight of the litter. Thus, maximum pig size and total weight of a litter appear to be about 10 pigs per litter up to litter sizes of 12. This pattern was not exemplified by litter sizes of 13 and 14. This could be sampling variation as there were only three litters in these litter sizes, or it could be that the pigs in litter sizes of 13 and 14 became independent of food from the dam sooner than pigs from smaller litter sizes. Forty-day weights, e.g., may all have the same pattern for all litter sizes but data were not available to check this conjecture.

The within litter regression coefficients of 56 day weights on birth weights were computed for each litter size (Table 1). A plot of the within regression coefficients on litter size is given in Figure 3 indicating an increase in the regression coefficient with litter size. The litter size of 5 appears to be different from the pattern exhibited by other litter sizes. Except for litter size 5, this relationship indicates that birth weight has an effect on 56 day weights which is proportional to litter size.

The within litters mean squares in Table 1, column 4, form the $p = 12$ elements of the vector $\underline{Y}$ and those in column 6 form the elements of the vector $\underline{Y}_a$. The values of h in column 1 of Table 1 are used to set up the elements in the X matrix. Since the within litter mean squares for even sized litters appear to follow a different pattern from the odd sized litters the data are treated separately (see Figure 4). Using equation (2) for the odd sized litters

$$\underline{X\bar{B}}_a = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \\ 1 & 8 & 64 \\ 1 & 10 & 100 \\ 1 & 12 & 144 \end{bmatrix} \cdot \begin{bmatrix} \bar{V}_s \\ \bar{\beta}_1 \\ \bar{\beta}_2 \end{bmatrix} = \begin{bmatrix} 14.5 \\ 31.2 \\ 31.6 \\ 37.7 \\ 36.0 \\ 33.3 \end{bmatrix} = \underline{Y}_a$$

and

$$(F-I) = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 & 0 \\ 0 & 0 & 120 & 0 & 0 & 0 \\ 0 & 0 & 0 & 128 & 0 & 0 \\ 0 & 0 & 0 & 0 & 120 & 0 \\ 0 & 0 & 0 & 0 & 0 & 24 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 19 & 0 & 0 & 0 & 0 \\ 0 & 0 & 119 & 0 & 0 & 0 \\ 0 & 0 & 0 & 127 & 0 & 0 \\ 0 & 0 & 0 & 0 & 119 & 0 \\ 0 & 0 & 0 & 0 & 0 & 23 \end{bmatrix}$$

The solutions for $\underline{\bar{B}}'_a = (\bar{V}_s\ \bar{\beta}_1\ \bar{\beta}_2)$ and for the corresponding $\underline{\bar{B}}_{aw}$ are given in the top part of Table 2 under equations (vii) and (viii), respectively. Least squares estimates for the parameters V_s, β_1, and β_2 are obtained from the above equations for V_s equal a constant independent of litter size and for equations (1) and (2) as $\underline{\hat{B}}$ and $\underline{\bar{B}}$, respectively, when the competition variance is a quadratic function of litter size (see Table 2). As stated before V_s and β_1 should be positive and β_2 should be negative. The only estimates satisfying these conditions are for equations (vii) and (viii) in Table 2. Here the competition variance is assumed to be zero for one piglet per litter and the adjusted within litter mean squares are considered to be the appropriate observations. These assumptions are considered to be the most appropriate from a biological point of view and since the observations are subject to different variances weighting by degrees of freedom and using a weighted regression approach is statistically desirable. Hence, the results in the last column of Table 2 are deemed the appropriate estimates of the parameters.

The estimate of V_s from even sized litters is approximately five times that from odd sized litters. There appears to be no biological explanation for this result although the results are much more discrepant than would ordinarily be expected from random sampling. If β_i (i=1,2) from even sized litters should be the same as from odd sized litters it would appear that the results should be pooled. However, the results are quite discrepant statistically and it may be that there is a higher V_s and that the β_i are different in the even sized litters from those in odd sized litters. Also, V_s may be the same for odd and even sized litters but the linear and quadratic coefficients may be different. In this event, α would be estimated from both sets of data, and $\underline{X}\underline{B} = \underline{Y}_a$ for h=2,3,4,5,6,7,8 would take the form

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 4 \\ 1 & 3 & 9 & 0 & 0 \\ 1 & 0 & 0 & 4 & 16 \\ 1 & 5 & 25 & 0 & 0 \\ 1 & 0 & 0 & 6 & 36 \\ 1 & 7 & 49 & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta_{1e} \\ \beta_{2e} \\ \beta_{1o} \\ \beta_{2o} \end{pmatrix} = \begin{pmatrix} W_2 \\ W_3 \\ W_4 \\ W_5 \\ W_6 \\ W_7 \\ W_8 \end{pmatrix}$$

where W_h = the within mean squares adjusted for the covariate birth weight for litters of size h, β_{1e} and β_{2e} are linear and quadratic coefficients for even sized litters, and β_{1o} and β_{2o} are linear and quadratic coefficients for odd sized litters. Then, $\hat{\underline{B}}_{aw} = (X'(F-I)X)^{-1}X'(F-I)\underline{Y}_a$ in the same manner as described above. These computations were not made for these data, but this may be a more realistic approach if there is no biological explanation that V_s should be different for odd and even sized litters. Alternatively, V_s could be estimated from column 5 of Table 1 if the expected value of each of the mean squares is $V_s + hV_p$; there appears to be a linear relation when the values of column 5 in Table 1 are plotted against litter size. Here again one could obtain V_s from all of the data in columns 5 and 6 in the same manner as described above for odd and even sized litters.

With an estimate of V_s as $\hat{V}_s$ and of V_{ch} as $\hat{V}_{ch} = \hat{\beta}_1(h-1) + \hat{\beta}_2(h-1)^2$ an estimate of V_p, the variance due to litter means, is available. Likewise, values of $\hat{V}_{ch}$ may be computed as $\hat{V}_{ch} = 7.28(h-1) - 0.40(h-1)^2$ for odd sized litters and $\hat{V}_{ch} = 1.54(h-1) - 0.15(h-1)^2$ for even sized litters. The maximum $\hat{V}_{ch}$ was attained for h=6 for even sized litters and h=10 for odd sized litters. Plots of $\hat{V}_{ch}$ values against mean weight per pig for litters of size h or against total weight of a litter of size h, did not reveal any simple relationships except as noted above. Some of the plots gave rather curious configurations, e.g. the plot of average weight of a litter against V_{ch} values computed from an average β_1 and β_2 resembled an inverted question mark, for which no ready explanation was available.

4. Discussion

Using a weighted regression approach with V_{ch} as a quadratic function of h and hence with the expected value of the within litter mean squares equated to $V_s + V_{ch} = V_s + \beta_1(h-1) + \beta_2(h-1)^2$, estimates of the parameters were obtained which were considered to be of the correct nature, i.e. $\hat{V}_s$ and $\hat{\beta}_1$ positive and $\hat{\beta}_2$ negative. The data (weaning weight within litter mean squares adjusted for birth weights) appeared to fit this model quite well except that even sized and odd sized litters followed different patterns and have different within litter sampling variances.

A considerable number of situations involve competition among individuals in a group, e.g. students in a classroom, members of a military air, ground or marine patrol, members of a farm settlement in an emerging nation, animals in an area, etc. The problem is to determine the optimum group size to make maximum achievement per individual or per group of individuals in order to utilize resources in an efficient manner. In a classroom one goal could be to maximize achievement per student per dollar spent or independent of money. Students, animals, etc. may perform better in an atmosphere of competition than in one freed of competition. In an atmosphere of competition then, it would be desirable to determine the group size as well as the size of the variance component due to competition which yields maximum gain or achievement. If the variance among individuals within a group can be formulated as a specified function of the group size, then estimates of the various variance components are available as described above.

5. Summary

The existence of a component of variance for competition among sampling units or among individuals in a group was discussed by Yates and Zacopany in 1935. No procedure was suggested for estimating this component of variance. It is the purpose of this paper to give a procedure for estimating the component of variance due to competition and to apply the procedure to an extensive set of data on weaning weights of pigs with 116 litters of various sizes and for Yorkshire, Chester-White, and Berkshire breeds. The first problem was to define litter size. Within this definition then, litters sizes of 3 to 14 pigs per litter were obtained. The variation among pigs within a litter of size h was considered to have an expected value equal to $V_s + V_{ch}$ where V_s is the sampling variance component and V_{ch} is the competition variance component for a litter of size h. In order to obtain an estimate of V_{ch}, a polynomial relation between h and V_{ch} was postulated. In particular, it was postulated that

$$E(\hat{V}_{ch}) = \beta_1(h-1) + \beta_2(h-1)^2 + \beta_3(h-1)^3 + \cdots$$

or

$$E(\hat{V}_{ch}) = \beta_1 h + \beta_2 h^2 + \beta_3 h^3 + \cdots$$

where E( ) denotes expected value. The first form states that V_{ch} goes to zero for one pig per litter while the second form puts V_{ch} equal to zero for h = 0. The first form may be more appropriate as long as small litter sizes (say 1 and 2 at least) are omitted from the analysis as was done in the present instance. Likewise, one could postulate a polynomial relation between the sampling variance component V_s and litter size. This polynomial would have an intercept α and

would need to have different powers in the polynomial in order to estimate the two variance components; e.g. one could postulate that $E(V_{sh}) = \alpha + \Pi \sqrt{h}$.

Using the assumption that V_s was unaffected by litter size and that $E(V_{ch}) = \beta_1(h-1) + \beta_2(h-1)^2$, it was found that $\hat{V}_{ch}$ reached a maximum for a litter size of 10 for the odd litter sizes and 6 for the even litter sizes after the weaning weights had been adjusted by covariance for birth weights. It appeared that $V_{ch} + V_s$ = within litter mean square followed a different pattern for odd sizes than for even sizes of litter. The biological reason for this is unknown.

6. Literature Cited

Jensen, N. F. and Federer, W. T. [1964]. Adjacent row competition in wheat. Crop Science 4:641-645.

Ladipo, O. O. [1965]. A component of variance due to competition. B.S. Thesis, Biometrics Unit, Cornell University, Ithaca, N.Y.

Yates, F. and Zacopanay, I. [1935]. The estimation of the efficiency of sampling for yield in cereal experiments. Journal of Agricultural Science 25: 545-577.

Table 1. Number of litters, within and among mean squares on weaning weights both adjusted and unadjusted for birth weights, weaning weight and birth weight means, and within litter regressions for 116 litters of pigs for litters sizes 3 to 14.

Litter size	No. of litters	Weaning weight mean squares unadjusted		Mean squares adjusted for the covariate birth weight		Weaning weight mean (lbs.)	Birth weight mean (lbs.)	Within litter regression	Total weight of a litter
		Among litters	Within litters	Among litters	Within litters				
3	3	302.3	14.3	36.7	14.5	-	3.3	3.4	-
4	4	362.3	33.1	36.6	20.0	43.2	3.4	6.3	172.8
5	5	86.2	38.6	42.1	31.2	36.8	2.9	10.1	184.0
6	13	150.2	29.5	141.8	26.5	35.0	3.1	4.2	210.0
7	20	186.4	38.0	190.6	31.6	37.3	3.0	7.5	261.1
8	20	150.3	32.5	116.7	26.1	34.0	2.9	5.8	272.0
9	16	339.5	37.7	203.7	37.7	34.9	3.0	4.1	314.1
10	13	249.1	26.8	184.2	21.1	32.9	2.7	9.2	329.0
11	12	280.6	55.6	256.1	36.0	30.4	2.9	8.7	334.4
12	7	705.4	31.4	363.5	21.2	28.4	2.2	10.9	340.8
13	2	167.5	51.2	246.9	33.3	34.4	2.4	11.6	447.2
14	1	-	25.8	-	16.6	32.6	2.6	8.7	456.4

Table 2. Estimates of parameters under different models.

Parameter	Equation (1)				Equation (2)			
	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
Odd sizes								
V_s	-3.88	21.47	-6.99	-3.80*	4.13	24.60+	1.70	4.02
β_1	8.32	1.32	9.16	8.12*	7.70	1.12+	8.22	7.28
β_2	-0.31	0.12	-0.47	-0.40*	-0.31	0.15+	-0.47	-0.40
Even sizes								
V_s	33.43	34.06+	10.23	19.57	33.18	33.60+	13.75	21.74
β_1	-0.23	-0.54+	3.76	1.95	-0.26	-0.54+	3.28	1.54
β_2	-0.02	0.01+	-0.24	-0.16	-0.02	0.01+	-0.24	-0.15

(i)	$\hat{\underline{B}} = (X_1'X_1)^{-1}X_1'\underline{Y}$	$V_{ch} = \beta_1 h + \beta_2 h^2$	unadjusted mean squares
(ii)	$\hat{\underline{B}}_w = (X_1'FX_1)^{-1}X_1'F\underline{Y}$	$V_{ch} = \beta_1 h + \beta_2 h^2$	unadjusted mean squares
(iii)	$\hat{\underline{B}}_a = (X_1'X_1)^{-1}X_1'\underline{Y}_a$	$V_{ch} = \beta_1 h + \beta_2 h^2$	adjusted mean squares
(iv)	$\hat{\underline{B}}_{aw} = (X_1'(F-I)X_1)^{-1}X_1'(F-I)\underline{Y}_a$	$V_{ch} = \beta_1 h + \beta_2 h^2$	adjusted mean squares
(v)	$\bar{\underline{B}} = (X_2'X_2)^{-1}X_2'\underline{Y}$	$V_{ch} = \beta_1(h-1) + \beta_2(h-1)^2$	unadjusted mean squares
(vi)	$\bar{\underline{B}}_w = (X_2'FX_2)^{-1}X_2'F\underline{Y}$	$V_{ch} = \beta_1(h-1) + \beta_2(h-1)^2$	unadjusted mean squares
(vii)	$\bar{\underline{B}}_a = (X_2'X_2)^{-1}X_2'\underline{Y}_a$	$V_{ch} = \beta_1(h-1) + \beta_2(h-1)^2$	adjusted mean squares
(viii)	$\bar{\underline{B}}_{aw} = (X_2'(F-I)X_2)^{-1}X_2'(F-I)\underline{Y}_a$	$V_{ch} = \beta_1(h-1) + \beta_2(h-1)^2$	adjusted mean squares

The values of h in X_1 are replaced by h-1 to obtain X_2.

* F was used instead of (F-I).

\+ (F-I) was used instead of F.

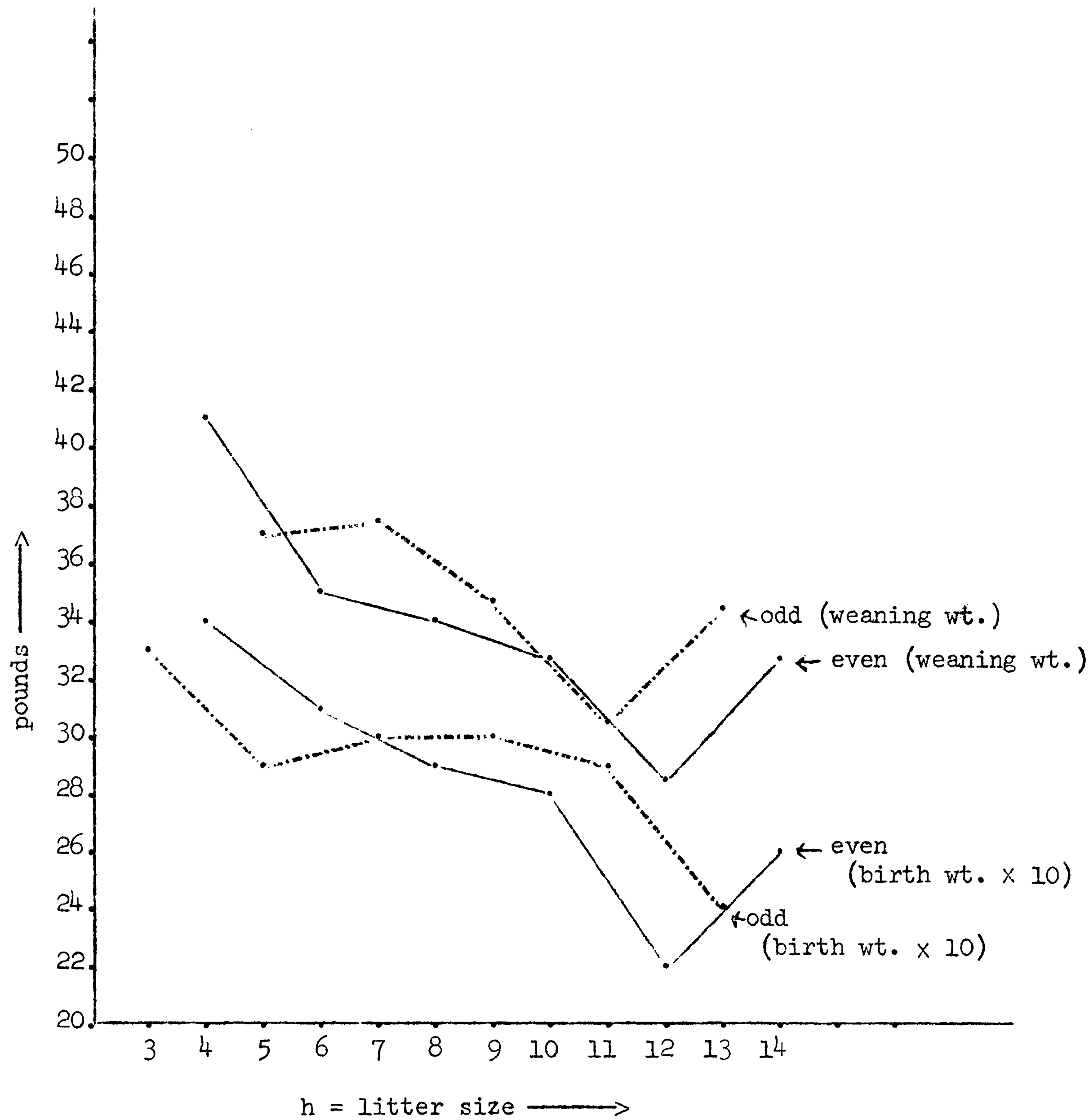

Figure 1. Average birth and weaning weights per pig for each litter size.

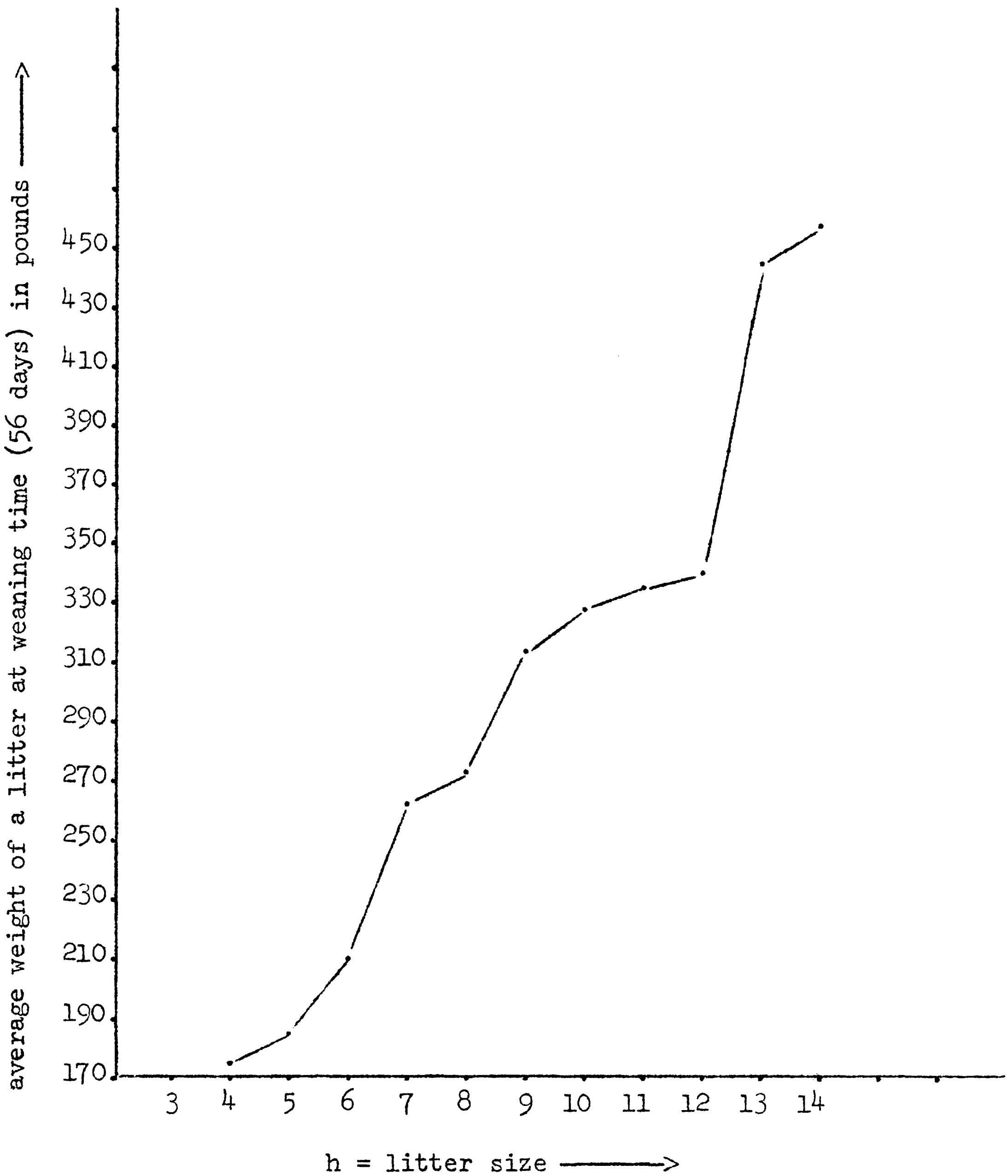

Figure 2. Total litter weaning weight (56 days) for each litter size.

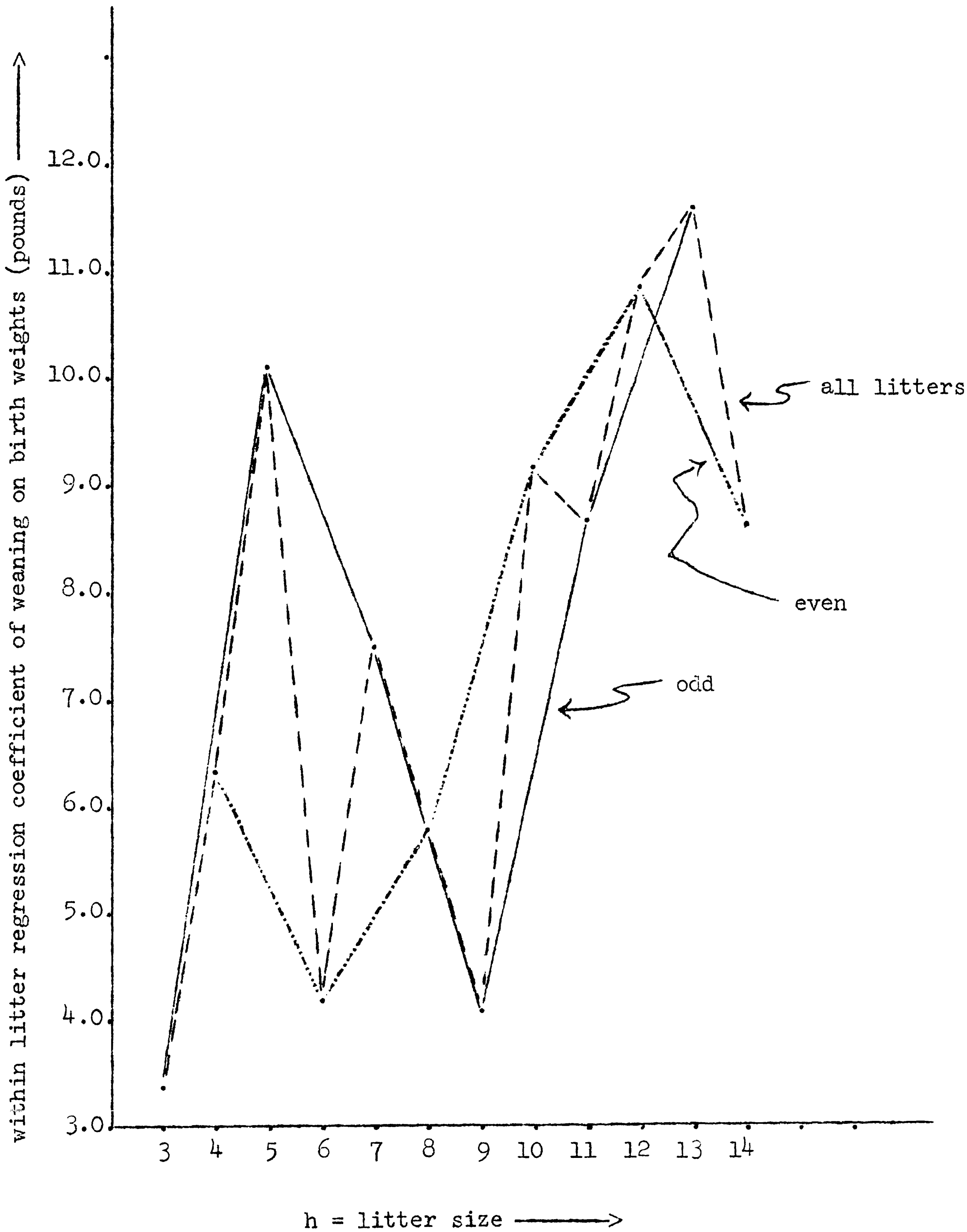

Figure 3. Within litter regression coefficient for each litter size.

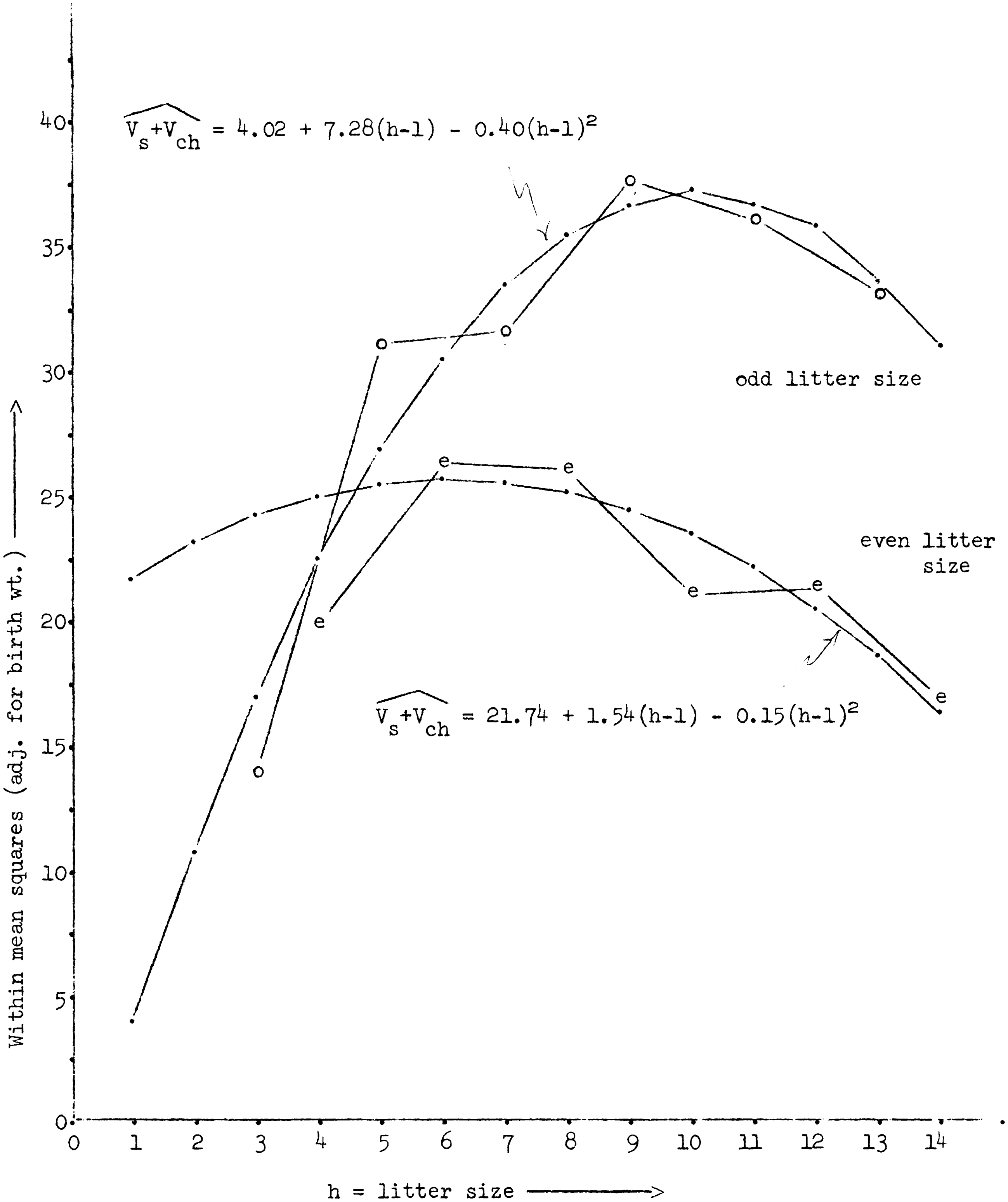

Figure 4. Within litter mean squares (adjusted for birth weight) for various litter sizes.