Factor shares, the price markup, and the elasticity of substitution between capital and labor *

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Abstract

The labor income share is constant under the assumptions of a Cobb-Douglas production function and perfect competition. This paper relaxes these assumptions and investigates to what extent the actual non-constant behavior of this factor share is explained by (i) a non-unitary elasticity of substitution between capital and labor and (ii) non-perfect competition in the product market. We focus on Spain and the U.S. and estimate a constant elasticity of substitution production function under imperfect competition in the product market. The degree of imperfect competition is measured through a time series computation of the price markup following the dual approach. We show that the elasticity of substitution is above one in Spain and below one in the US. We also show that the price markup drives the elasticity of substitution away from one, upwards in Spain, downwards in the U.S. These results are used to explain the declining path of the labor income share, common to both economies, and their contrasted patterns in terms of capital deepening.

JEL Classification: E22, E24, E25.

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1 Introduction

The labor income share (LIS) is constant under the assumptions of a Cobb-Douglas (CD) aggregate production function and perfect competition. Although these are widely used assumptions, the prediction of a constant LIS is at odds with empirical evidence showing that the LIS is time-varying in most countries at least in the medium run (see, among many others, Bentolila and Saint-Paul, 2003; Ríos-Rull and Santaeulàlia-Llopis, 2007; and Choi and Ríos-Rull, 2008). Figure 1 presents the Spanish and U.S. series and makes clear that they have been far from constant in last decades.¹

![Figure 1. Labor income shares in Spain and the U.S.](image)

This wide evidence suggests that at least one of the assumptions has to be removed. The immediate question, ‘Which one?’, is implicitly and ex-ante answered in most papers when one (or both) of these standard assumptions is just dropped. However, surrounding this question there a number of crucial issues which should deserve most attention from the profession. For example, when diverging from the CD framework, what are the different implications for the LIS of having an elasticity of substitution above or below one? What are consequences of diverging from a situation of perfect competition in the product market? When none of the assumptions hold, to what extent does a non-unitary substitution in technology explain the time-varying path of the LIS? To what extent does it the degree of imperfect competition? What are the implications of the value of the elasticity of substitution² and the degree of imperfect competition for capital accumulation?

¹To check whether the labor shares in Spain and the U.S. can be treated as stationary or not we use the Augmented Dickey-Fuller (ADF) and the Kwiatkowski - Phillips - Schmidt - Shin (KPSS) tests. We obtain consistent results (which available upon request): according to the ADF test the null of a unit root cannot rejected; according to the KPSS the null of stationarity is rejected. We conclude that during our sample period these series have not evolved around a constant value.
The aim of this paper is to provide answers to these questions by examining two very different economies like Spain and the U.S. In particular, we analyze how much of the LIS variation in these countries is explained by a non-unit elasticity of substitution and how much by non-perfect competition. In doing so, we contribute to the literature in several respects such as the computation of an aggregate time-series measure of the degree of imperfect competition in the product market, the estimation of the elasticity of substitution under such imperfect competition, and the assessment of how the outcome of these empirical exercises enlightens the characterization of the Spanish and U.S. labor markets and of their aggregate technologies.

The non-constant behavior of the LIS has already been studied, often by considering a single-departure from the two standard assumptions. For example, departures from the CD production function are examined in Driver and Muñoz-Bugarín (2009), and Arpaia, Pérez and Pichelmann (2009), who show the implied dynamics of the LIS when technology is characterized by a constant elasticity of substitution (CES) production function. In turn, departures from perfect competition in the product market are explored, among others, in Bentolila and Saint-Paul (2003) and Estrada (2005), who also appraise the dynamics of LIS when the price markup is non-constant. The work by Choi and Rios-Rull (2008) considers departures from the CD production function and perfect competition in the labor market by introducing wage setting and frictions. They investigate whether the dynamics of the LIS are better explained by non-competitive factor prices or by a non-unit elasticity of substitution, and find the latter to be more important. This finding is obtained through a stochastic dynamic general equilibrium model where the value of the elasticity of substitution is assumed and the path of the price markup is calibrated accordingly.

Here we depart from the CD production function and perfect competition in the product market and provide quantitative estimates of the LIS determinants. In this way, our paper offers two main contributions to this literature. First, a time-series calculation of the aggregate price markup which we take as the aggregate proxy of the degree of imperfect competition in the product market. Second, the estimation of the elasticity of substitution under such product market imperfections.

The calculation of the price markup has attracted attention since the seminal contribution by Rotemberg and Woodford (1999). However, their methodology requires information on the shape of the production function, in particular on the value of the elasticity of substitution which is obviously unknown and must be estimated ex-ante. To bypass these information requirements we track Roeger (1995), who obtains the price markup from the difference between the primal and dual measures of the total factor

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2 See, among many others, Banerjee and Rusell (2004); Rawn, Schmitt-Grohe, and Uribe (2004); Altug and Filiztekin (2002); and Jamovich and Floetotto (2007).
productivity (TFP). We follow a similar method and obtain the time path of the price markup from a dual approach taking advantage of available information on factor prices.

This approach requires the obtainment of the rental price of capital and implies dealing with data on capital stock, interest rates, and depreciation rates. For this, we rely mainly on the OECD database to ensure comparable data, and thus comparable results. Once the price markups are computed, we check their cyclical properties and we find them to be countercyclical in both countries, with an average value around 31% in Spain and 35% in the U.S. According to the literature these are standard properties of the price markup (see Rotemberg and Woodford, 1999; and Estrada, 2005).

Regarding the estimation of the elasticity of substitution between capital and labor, we assume a CES production function and face two methodological possibilities. On the one hand, this elasticity can be directly obtained by applying non-linear methods, as done by Duffy and Papageorgious (2000), and Klump, McAdam, and Willman (2007). On the other hand, it can be obtained from the input demands, as in Antràs (2004), by applying linear methods to the log-linearization of the CES input demands. We follow this second approach.

We extend Antràs’ work by considering imperfect competition and augment the estimation of the input demands by considering our computation of the price markup as a proxy of the time-varying aggregate degree of product market imperfection. This econometric exercise yields two main findings.

First, the elasticity of substitution is larger than one in Spain and smaller than one in the U.S. To explain this finding, we show that the elasticity of substitution measures the effect of capital accumulation on the LIS. In Spain the LIS has decreased while the ratio of capital to GDP has increased. These two facts imply an elasticity of substitution larger than one. In contrast, both the ratio of capital to GDP and the LIS have decreased in the U.S., which implies an elasticity of substitution lower than one.

Our second main finding is that consideration of the price markup drives the value of the elasticity of substitution away from one and, therefore, provides a further cause of rejection of the CD specification (Antràs, 2004). We show that if the price markup is not considered the estimates of the elasticity of substitution are biased because of a misspecification of the output elasticity of labor. This main result holds both for Spain and the U.S. but goes in opposite direction: it yields an upward bias in Spain and a downward bias in the U.S.

To check the robustness of our findings we perform an extra exercise. In the absence of data measurement errors, a perfect estimation of the elasticity of substitution would yield the same result no matter the price markup had been computed using the primal or the dual approach. To have a sense of how far we are from this ideal situation, we derive and regress a simple equation providing estimates of the values of the elasticity of substitution
that minimize the difference between the primal- and dual-approach calculations of the price markups. We find these new estimates to be broadly consistent with the ones obtained by the estimation of the production function, especially in the U.S. where a robust estimate is obtained.

Finally, we make use of the computed price markups and estimated values of the elasticity of substitution to perform two simulations. In the first one we also take data on GDP per worker and simulate the LIS in four different scenarios that are used to decompose the LIS and examine to what extent the price markup and the elasticity of substitution account for significant portions of its actual trajectory. Because underlying this simulation the TFP grows at a constant rate, we look at the permanent components of the series and abstain from business cycle considerations. We find that the price markup accounts for 63% of the LIS variation in Spain and 57% in the U.S., whereas the elasticity of substitution explains, respectively, 27% and 39% of its variation. This implies that the elasticity of substitution has less than half the explanatory power of the price markup in Spain, and about two thirds of it in the U.S.

In a second simulation we analyze whether the time paths of GDP, capital accumulation, employment growth and the LIS implied by the estimated values of the elasticity of substitution and the computed price markup are consistent with the time series of these variables (the value of the elasticity of substitution determines the relationship between GDP in efficiency units and capital accumulation, while the value of the price markup relates these two variables with the LIS and employment growth). In this exercise, we extend the Solow model by considering (i) a CES production function; (ii) product market imperfections, which are summarized by the price markup; and (iii) labor market imperfections, which are introduced through a simple wage equation arising from a standard efficiency wage model. Accordingly, the labor market does not clear because wages are set above their competitive value. Moreover, to obtain efficiency units of labor we compute the Solow residual from an accounting exercise that takes into account that the Solow residual is affected by the price markup (Hall, 1988). Given that the TFP is not restricted to grow at constant rate, in this simulation we work with the whole series and do not abstain from their business cycle component.

This version of the Solow model is solved numerically in a base run scenario of non-perfect competition and non-unit elasticities of substitution (i.e., in the presence of price markups) and a scenario of perfect competition and elasticities of substitution close to one (i.e., in the absence of price markups). The comparison of how the two scenarios predict

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3The macroeconomic implications of the elasticity of substitution between capital and labor have been stressed by several authors. For example, Klump and Preisler (2000), Duffy and Papageorgious (2000), and Acemoglu (2002), examine its implications for capital accumulation and long-run growth. In turn, Rowthorn (1999) shows that capital accumulation affects the long-run unemployment rate when the elasticity of substitution differs from one.
the actual trajectories of the main macroeconomic variables reveals how important is to take into account the degree of imperfect competition and its influence on technology. When this is overlooked, the LIS displays a constant trajectory and the macroeconomic predictions are seriously flawed. We conclude that economists should carefully design their modeling assumptions before embarking on sophisticated analyses built up on excessively unrealistic premises. This is specially important if the resulting outcomes are used for policy advice.

The plan of the paper is as follows. Section 2 shows the determinants of the LIS. Section 3 computes the time path of the price markup which is used, in Section 4, for the estimation of the production function. Section 5 studies the determinants of the LIS. Section 6 augments the Solow model to examine the consequences of a non-unit elasticity of substitution and non-perfect competition on capital accumulation and labor market performance. Section 7 concludes.

2 The labor income share

In this section we derive the equation of the LIS when there is non-perfect competition and the aggregate technology is characterized by the following CES production function:

$$Y_t = F(K_t, L_t) = \left[ \alpha (K_t)^{\frac{\sigma}{\sigma - 1}} + (1 - \alpha) (A_t L_t)^{\frac{\sigma}{\sigma - 1}} \right]^{\frac{1}{\sigma - 1}},$$  \hspace{1cm} (1)

where $Y_t$ is GDP, $K_t$ is the aggregate stock of capital, $L_t$ is employment, $A_t$ is technological augmenting labor, and $\sigma > 0$ is the elasticity of substitution between capital and efficiency units of labor.

Under imperfect competition in the product market, profit maximization implies

$$m_t w_t = F_L(K_t, L_t) = (1 - \alpha) \left[ \alpha (K_t)^{\frac{\sigma}{\sigma - 1}} + (1 - \alpha) (A_t L_t)^{\frac{\sigma}{\sigma - 1}} \right]^{\frac{\sigma}{\sigma - 1}} (A_t L_t)^{-\frac{1}{\sigma - 1}} A_t,$$  \hspace{1cm} (2)

where $m_t$ measures the price markup, $w_t$ is the wage per unit of labor and $F_L$ is the marginal product of labor. This equation is presented in Galí (1996) and corresponds to the first order condition for a symmetric equilibrium. It is derived assuming monopolistic competition with $n$ symmetric sectors with $m = \frac{\xi}{\xi - 1}$, where $\xi$ is the elasticity of substitution of consumers’ and firms’ demand, that we assume to be equal.

Combining equations (1) and (2), we obtain the following expression for the LIS:

$$\psi_t = \frac{w_t L_t}{Y_t} = \left( \frac{1 - \alpha}{m_t} \right) \left( \frac{Y_t}{A_t L_t} \right)^{\frac{1}{\sigma - 1}}.$$  \hspace{1cm} (3)

This equation shows that the LIS depends: (i) on the time path of the price markup
(always); and (ii) on the average labor productivity in efficiency units whenever $\sigma \neq 1$. Using (1), the average productivity can be rewritten as

$$\left( \frac{Y_t}{A_t^\ell L_t} \right)^{\frac{1-\sigma}{\sigma}} = \frac{1}{\alpha \left( \frac{K_t}{A_t^\ell L_t} \right)^{\frac{\sigma+1}{\sigma}} + (1 - \alpha)}.$$ 

so that

$$\psi_t = \left( \frac{1}{m_t} \right) \left( \frac{1 - \alpha}{\alpha \left( \frac{K_t}{A_t^\ell L_t} \right)^{\frac{\sigma+1}{\sigma}} + (1 - \alpha)} \right). \tag{4}$$

Note that average productivity depends on the capital labor ratio in efficiency units (i.e., the ratio between capital and efficiency units of labor) and, thus, it is related to capital deepening. Note also that equation (4) implies a relationship between the LIS and capital deepening that depends on the value of $\sigma$. If the elasticity of substitution is larger than one, capital deepening reduces the LIS. If it is smaller than one, capital deepening increases the LIS.

From this perspective it is now easy to see the strong assumptions introduced in related literature when explaining the dynamics of the LIS. On the one hand, Bils (1987) and Galí (1995) assume $\sigma = 1$ (i.e., a Cobb-Douglas production function) so that these dynamics can only arise from the evolution of the price markup. On the other hand, under the assumption that $m = 1$ (i.e., perfect competitive markets), the price markup effect vanishes and the dynamics of the LIS are explained just by capital deepening. The latter is the route followed by Antràs (2004), who assumes perfect competitive markets and uses data on the LIS to estimate the U.S. production function. In our analysis, which is free from these restrictions ($\sigma \neq 1$, $m > 1$), we use the dynamics of the LIS to estimate the elasticity of substitution when the price markup is time-varying (Section 4). The main difficulty at this point lies in the calculation of the time-varying price markup. Section 3 deals with this issue.

## 3 The price markup

Most of the related literature follows Rotemberg and Woodford (1999) and obtains the price markup using the Solow residual, which is

$$\frac{\Delta A_t}{A_t} = \frac{1}{\alpha_L} \frac{\Delta Y_t}{Y_t} - \left( \frac{1 - \alpha_L}{\alpha L} \right) \frac{\Delta K_t}{K_t} - \frac{\Delta L_t}{L_t}, \tag{5}$$
where $\alpha_L$ is the output elasticity of labor. Given that $\alpha_L = m_t \psi_t$ (from the first order conditions of the firms’ problem) and using equation (3) we obtain

$$\frac{\Delta m_t}{m_t} + \frac{\Delta w_t}{w_t} = \frac{1}{\sigma} \frac{\Delta Y_t}{Y_t} + \left(1 - \frac{1}{\sigma}\right) \left(\frac{\Delta A_t}{A_t} + \frac{\Delta L_t}{L_t}\right) - \frac{\Delta L_t}{L_t}.$$  

When the latter is combined with equation (5) we have

$$\frac{\Delta m_t}{m_t} = \left(\frac{m_t \psi_t + \sigma - 1}{\sigma m_t \psi_t}\right) \frac{\Delta Y_t}{Y_t} - \left(\frac{\sigma - 1}{\sigma m_t \psi_t}\right) \frac{\Delta K_t}{K_t} - \frac{\Delta L_t}{L_t} - \frac{\Delta w_t}{w_t}. \quad (6)$$

Although his expression is commonly used in the literature to compute the growth rate of the price markup, it is not useful for us. The problem lies in the unobservable nature of the values of the aggregate price markup and the elasticity of substitution. This problem may be solved by introducing assumptions on these values. However, these unknowns are precisely the two variables we seek to quantitatively approach in this paper.

In view of these problems, we follow a dual approach and compute the price markup directly from its definition

$$m_t = \frac{Y_t}{w_t L_t + R_t K_t}, \quad (7)$$

where $R_t$ is the rental price of capital. From equation (7), we obtain

$$\frac{\Delta m_t}{m_t} = \frac{\Delta Y_t}{Y_t} - m_t \frac{L_t w_t}{Y_t} \left(\frac{\Delta w_t}{w_t} + \frac{\Delta L_t}{L_t}\right) - m_t \frac{R_t K_t}{Y_t} \left(\frac{\Delta R_t}{R_t} + \frac{\Delta K_t}{K_t}\right),$$

so that

$$\frac{\Delta m_t}{m_t} = \frac{\Delta Y_t}{Y_t} - m_t \psi_t \left(\frac{\Delta w_t}{w_t} + \frac{\Delta L_t}{L_t}\right) - (1 - m_t \psi_t) \left(\frac{\Delta R_t}{R_t} + \frac{\Delta K_t}{K_t}\right). \quad (8)$$

Equations (7) and (8) characterize the dual approach to compute the price markup. Note that the path of the markup can be characterized without any assumption on the aggregate production function. The only requirement is data availability on GDP, capital stock, employment, wages and the rental price of capital. The first three variables, the quantities, are directly available through the OECD database. The latter two, the prices, require some extra work.

Wages need to be computed because the total compensation of dependent employees must be adjusted by the share of the pie corresponding to self-employment (Gollin, 2002). For this, we use the GDP at factor costs and compute self-employed income as effectively labor income. On this basis, $w_t$ is defined as $\psi_t Y_t$, where $\psi_t$ is the (adjusted) LIS.

For the rental price of capital, $R_t$, we have two possibilities. The first one is based on the National Accounts so that $R_t$ is computed as the share of payments to capital in total income divided by the capital-output ratio. This measure is directly at hand since
we have data on $Y_t$ and $K_t$ (in real terms), and we have just computed the (adjusted) LIS, $\psi_t$. Thus, the solid line in Figures 2a (for Spain) and 3a (for the U.S.) is just the rental price of capital computed as $\frac{(1-\psi_t)Y_t}{K_t}$. Obviously, this measure implies a situation of perfect competition and no price mark-up.

The second possibility is to compute a price-based measure of $R_t$ following Hsieh (1999). In front of the problems of bad national statistics, Hsieh argues that price-based estimates have the advantage of being “based on market prices (namely, wages and interest rates) paid by agents who have every incentive to get the prices right” [p. 134]. Using this approach, the rental price of capital is obtained from the following non-arbitrage condition:

$$R_t = \frac{r^i_t}{p_t} = \frac{p^k_t}{p_t} \left( i_t - \frac{\Delta p^k_t}{p^k_t} + \delta^j \right),$$

where $r^i_t$ is the nominal rental price of capital, $p^k_t$ is the nominal price of one unit of capital, $\delta^j$ is the depreciation rate of capital, $i_t$ is the nominal interest rate, and $p_t$ is a price index. For capital stock, the OECD database only supplies the aggregate series. Thus, to compute $p^k_t$ and $\delta^j$ we use the FBBVA-IVIE database for Spain and, the NIPA (National Income and Product Accounts) for the U.S.4 As a measure of the annual interest rate, for Spain we use the nominal long-term interest rate on government bonds, while for the U.S. we use the Federal Funds rate.5 Finally, for $p_t$ we use the GDP deflator.

Relying on this data, we obtain a homogeneous and, for our purposes, sufficiently long time series which we plot as a dotted line in Figures 2a and 3a. Based on this measure of $R_t$, Figures 2b and 3b show the implied price markup for Spain and the U.S.

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4These databases provide the following depreciation rates. For Spain (average for 1964-2007): (i) Residential buildings: 1.10%; (ii) Other construction: 1.53%; (iii) Transport equipment: 14.43%; (iv) Machinery equipment: 10.97%. Overall, the depreciation rate is 2.49%. For the U.S. (average for 1960-2007), these rates are: (i) Residential buildings: 1.47%; (ii) Nonresidential buildings: 4.68%; (iii) Structures: 2.32%; Equipment and Software: 12.17%. The overall depreciation rate is 3.34%. Note that they are consistent with Hsieh’s (1998) reported ones: (i) Residential buildings: 1.3%; (ii) Non-residential buildings: 2.9%; (iii) Other construction: 2.1%; (iv) Transportation equipment: 18.2%; (v) Machinery equipment: 13.8%.

5For Spain the long-term government bond yield corresponds to the weighted average yields of bonds with maturities of more than two years, weighting the yield of each operation by the negotiated amount. This series is originally supplied by the Bank of Spain. We also experimented with various other series. For Spain, we used a short-run interest rates consisting on the 3-month interbank loans computed as the average of the 3-month interbank rate weighted by the value of credit granted, which is also originally supplied by the Bank of Spain. For the U.S., the rental price of capital was also computed for a short and a long-run interest rate. For the first one, we used the 3-month LIBOR (London Interbank Offered Rate), which is the rate of interest at which banks offer to lend money to one another in the wholesale money markets in London, and is a standard financial index used in U.S. capital markets. For the second one, we used the rate on the 10-year government bonds. It is important to note that all these series yield a similar picture both in terms of $R_t$ and in terms, consequently, of the price markup. We opted for the long-run series given that the series on the short-term interest rates for Spain only starts at the end of the 70s, and we thus loose too many degrees of freedom.
This price markup has four noteworthy characteristics. First, it follows a sort of U shaped trajectory, with a downward path in the aftermath of the oil price crisis, which subsequently turns into a rise. Second, this path is overall stationary since, according to the KPSS test, the null of stationarity cannot be rejected at a 5% critical value. This null, in contrast, is rejected in the case of the labour share $\psi_t$. This implies, for example, that a mark-up measure for Spain based on the inverse of the LIS would probably mix information on the price markup and the downward-pushing capital deepening. The third characteristic is that this price-based markup evolves well above 1. This indicates the existence of imperfect competition and significant price markups which are, on average, 31% in Spain and 35% in the U.S. It seems a sensible measure given the 38% value at which Ravn, Schmitt-Grohe and Uribe (2004), based on GMM estimates, place the steady state price mark-up in the U.S. in 1967-2003. Note, also, that $m = 1.35$ implies that around a quarter of the national income (GDP) is generated through market power and thus corresponds to monopolistic rents $\left(\frac{Y-(wL+rK)}{1} = \frac{m-1}{m} = 0.26\%\right)$. The fourth characteristic is a
countercyclical behavior. As stressed by Rotemberg and Woodford (1999), this countercyclical behavior reconciles theory and empirical evidence on the procyclical behavior of wages. We follow these authors and compute the correlation of our cyclical indicator of the price markup—the growth rate of the price markup—with the HP filtered GDP, the linearly detrended hours, and the HP filtered hours (HP stands for Hodrick-Prescott). For Spain we find, respectively, the following correlation coefficients: -0.28, -0.20, and -0.23. For the U.S., in turn, we find -0.25, -0.08, and -0.04. In view of these results, we conclude that our price markup time series is countercyclical.

4 The production function

In this section we estimate the production function and obtain the elasticity of substitution between capital and labor. We follow Antràs’ (2004) methodology, but diverge in one important respect. Rather than assuming perfect competition, we consider the price markup as a relevant determinant of the relationships at work. Thus, our contribution lies in the obtainment of new estimates of the elasticity of substitution under imperfect competition.

Following Antràs, we assume a functional form of the technological parameters so that labor efficiency increases at a constant growth rate, i.e. \( A_t = A_0 e^{\lambda t} \), where \( \lambda \) is the growth rate of technological change. From the first order conditions of the firms’ maximization problem, we obtain the labor demand

\[
m_t w_t = (1 - \alpha) \left( \frac{Y_t}{L_t} \right)^{\frac{1}{\sigma}} (A_t)^{\frac{\sigma - 1}{\sigma}}, \tag{9}
\]

which can be rewritten as

\[
\ln \left( \frac{Y_t}{L_t} \right) = \beta_1 + \sigma \ln m_t w_t + (1 - \sigma) \lambda t, \tag{10}
\]

or

\[
\ln (m_t w_t) = \beta_2 + \frac{1}{\sigma} \ln \left( \frac{Y_t}{L_t} \right) - \left( \frac{1 - \sigma}{\sigma} \right) \lambda t, \tag{11}
\]

where \( \beta_1 \) and \( \beta_2 \) are constants and (10) and (11) are, respectively, the inverse and direct labor demands.

4.1 The elasticity of substitution between capital and labor

We next estimate the labor demand equation both under perfect competition, as Antràs (2004), and under imperfect competition by considering the price markup time series
computed in the previous section.\textsuperscript{6} To conduct the estimation, we follow the cointegration-autoregressive distributed lag (ARDL) bounds testing procedure developed in Pesaran and Shin (1999) and Pesaran \textit{et al.} (2001). Its main advantage is that it can be used irrespective of whether the underlying regressors are integrated of order one, zero, or are fractionally integrated. In this way we avoid the pretesting problem implicit in the standard cointegration techniques, that is, in the Johansen maximum likelihood, and the Phillips-Hansen semi-parametric fully-modified OLS procedures. Nevertheless, we will also check that our ARDL estimates are consistent with the ones that would be obtained using Johansen’s procedure (see Tables 2 and 4, below). As shown in Harris and Sollis (2003), other advantages of the ARDL method are that it yields consistent long-run estimates of the equation parameters even for small size samples and under potential endogeneity of some of the regressors.

Table 1 presents labor demand estimates for Spain distinguishing two versions. The first one, named \(E1\), is equation (11) and has \(wm\) as dependent variable (the variables are always defined in logs). The second one, named \(E2\), is equation (10) having \(Y/L\) as dependent variable. The first block, in the left-hand side of the table, shows the results when the price markup is taken into account. The second block, in the right-hand side, shows the same estimates without considering the price markup. These extra equations are named \(E1'\) and \(E2'\).

The estimates of \(E1\) and \(E1'\) reveal similar levels of inertia, with persistence coefficients around 0.80 and alike short-run elasticities of the dependent variable with respect to \(Y/L\) (given the similar values of the persistent coefficients, the long-run ones are also alike). \(Y/L\) is significant at a 10\% critical value in \(E1\) but it is not significant in \(E1'\). The trend displays opposite signs and is not significant at conventional critical values. Both equations pass the standard misspecification and structural stability tests. Regarding \(E2\), the persistence coefficient is extremely large and close to a unit root (0.97), the coefficient on \(wm\) is not significant, and the residuals are homoscedastic. Thus, the preferred estimated version of the labor demand is \(E1\), which is the one we use to derive the estimates of \(\sigma\).

According to equation \(E1\), the elasticity of substitution in Spain is 1.58 (= 1/0.63, where 0.63 is the long-run elasticity of \(wm\) with respect to \(Y/L\)).\textsuperscript{7} This is in fact the value we credit, since the results from equation \(E2\) are highly inconclusive due to the two above mentioned problems (residuals not normal and lack of significance of the \(wm\) \textsuperscript{6}Following Antràs (2004), we have also estimated the demand of capital, both for Spain and the U.S. However, since the corresponding results are very poor and no significant conclusions can be obtained, we have chosen no to present them, but make them available upon request.

\textsuperscript{7}This long-run elasticity is obtained as the long-run coefficient of \(Y/L\), that is 0.13/(1-0.79). It is important to note that we always work with the exact values of the coefficients. Therefore, even though the ratio above yields 0.62, the true long-run elasticity is 0.63. And the accurate estimate of \(\sigma\) is 1.58, not the 1.59 resulting from 1/0.63.
coefficient). Its counterpart with no price markup, $E1'$, yields $\sigma = 1.21 = 1/0.83$, where 0.83 is the long-run elasticity of $w$ with respect to $Y/L$.\(^8\) We conclude that (i) Spain has an elasticity of substitution above 1, which we place at 1.58; and, (ii) failure to consider the price markup generates a downward bias in the estimation of $\sigma$.

\section*{Table 1. Spanish labor demand. 1967-2007.}

<table>
<thead>
<tr>
<th></th>
<th>Price markup considered</th>
<th>Price markup not considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>0.73</td>
<td>0.14</td>
</tr>
<tr>
<td>$\ln (w_{t-1}m_{t-1})$</td>
<td>0.79</td>
<td>0.97</td>
</tr>
<tr>
<td>$\ln (\frac{Y_t}{L_t})$</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>$\Delta \ln (\frac{Y_t}{L_t})$</td>
<td>0.91</td>
<td>-0.13</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.988</td>
<td>0.998</td>
</tr>
<tr>
<td>$s.e.$</td>
<td>0.027</td>
<td>0.013</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>$c$</td>
<td>0.24</td>
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<tr>
<td>$\ln (w_{t-1})$</td>
<td>0.84</td>
<td>0.85</td>
</tr>
<tr>
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<td>0.15</td>
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<tr>
<td>$\Delta \ln (\frac{Y_t}{L_t})$</td>
<td>0.51</td>
<td>-0.07</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.997</td>
<td>0.998</td>
</tr>
<tr>
<td>$s.e.$</td>
<td>0.014</td>
<td>0.011</td>
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</table>

<table>
<thead>
<tr>
<th></th>
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<tr>
<td>$SC$</td>
<td>1.11</td>
<td>1.87</td>
</tr>
<tr>
<td>$LIN$</td>
<td>0.64</td>
<td>0.175</td>
</tr>
<tr>
<td>$NOR$</td>
<td>1.00</td>
<td>1.93</td>
</tr>
<tr>
<td>$HET$</td>
<td>1.58</td>
<td>5.87</td>
</tr>
</tbody>
</table>

\section*{Notes:} Dependent variables: $\ln (wm_t)$ in $E1$, $\ln (w_t)$ in $E1'$, and $\ln (Y_t/L_t)$ in $E2$ and $E2'$; $\Delta$ is the difference operator; $s.e.$ the standard error; p-values in square brackets.

Misspecification tests: Serial correlation $SC$, Linearity $LIN$, Normality $NOR$, and Heteroscedasticity $HET$.

Because our interest lies in the implied long-run relationships between $wm$ and $Y/L$, in Table 2 we present the coefficients of the error correction model ($ecm$) and the long-run relationships (or cointegrating vectors) underlying the estimated equations (they are obtained from the reparameterizing equations $E1$ and $E2$ in error correction form). Next to these results, obtained via the ARDL method, we also present the cointegrating vector resulting from conducting the same analysis using the Johansen procedure.\(^9\) Finally, we

\(^8\)The estimate of the elasticity of substitution could be wrongly placed around 1 in case of ignoring the price markup. In particular, following equation $E2'$, $\sigma$ would be estimated at 0.98, which is the long-run elasticity of $Y/L$ with respect to $w$.

\(^9\)Underlying this exercise is (i) the performance of unit root tests; (ii) the estimation of VAR models having the same lag structure and containing the same variables than the structural equations; and (iii)
show the results of a likelihood ratio (LR) test following a $\chi^2$ distribution that restricts the Johansen values to take the ARDL values. Non rejection of the LR test provides evidence on the results’ consistence across econometric methodologies.

In Spain this consistency cannot be rejected for any of the two versions of the labor demand. However, note that for $E2$ the second term of the ARDL cointegrating vector (i.e., the long-run elasticity of elasticity $Y/N$ with respect to $wm$) is not significant. These results reinforce our choice of 1.58 as our best estimate of $\sigma$.

Table 2. Long-run relationships in Spain.

<table>
<thead>
<tr>
<th></th>
<th>ARDL</th>
<th>Johansen</th>
<th>LR Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ecm_{t-1}$</td>
<td>$\ln(wm)$</td>
<td>$\ln(Y_t)$</td>
<td>$\chi^2(1)=0.35[0.553]$</td>
</tr>
<tr>
<td>$[E1]$</td>
<td>$-0.21$</td>
<td>$0.63[0.004]$</td>
<td></td>
</tr>
<tr>
<td>$ecm_{t-1}$</td>
<td>$Y_t$</td>
<td>$wm$</td>
<td>$\chi^2(1)=5.34[0.021]$</td>
</tr>
<tr>
<td>$[E2]$</td>
<td>$-0.03$</td>
<td>$0.78[0.434]$</td>
<td></td>
</tr>
</tbody>
</table>

Notes: p-values in square brackets; 5% critical value: $\chi^2(1)=3.84$.

Tables 3 and 4 are, respectively, the US counterparts of Tables 1 and 2 for Spain. Regarding the estimation of equation (11), the version with the price markup ($E3$) results in a much lower persistence coefficient, 0.50, than the version with perfect competition ($E3'$), where it attains 0.73. This would result on different long-run elasticities of $wm_t$ and $w_t$ with respect to $Y_t/L_t$ for equal short-run coefficients on $Y_t/L_t$. However, even the short-run coefficients, 0.79 and 0.27, differ substantially. As in Spain, the trend (not significant at conventional critical values) displays opposite signs and both equations pass the standard misspecification and structural stability tests. The estimation of equation (10), named $E4$, yields a very similar picture than its counterpart $E3$, with a similar persistence coefficient around 0.50 and a short-run coefficient of $wm_t$ with respect to $Y_t/L_t$ of 0.21. This results in relatively close estimates of $\sigma$. However, given that the residuals in equation $E2$ are not normal, we credit the elasticity of substitution obtained through $E3$, which will be compared to the one obtained through $E3'$. Note, also, that the selected estimates of $\sigma$ for Spain and the U.S. are taken from the same specification. This ensures consistent results across countries and warrants comparability.

According to equations $E3$ and $E3'$, the elasticity of substitution between capital and labor in the U.S. is below 1. When the price markup is considered, we find it to be the use of LR tests based on the maximal eigenvalue and the trace of the stochastic matrix to determine the existence of cointegrating vectors (and its number and values). Given that we estimate VAR models with unrestricted constants and trends and two I(1) variables, we need to test one restriction. All these underlying results are available upon request.
0.63 (= 1/1.58, where 1.58 is the long-run elasticity of \( w_m \) with respect to \( Y/L \)). In the absence of the price markup, we find it to be 0.96 (= 1/1.014, where 1.014 is the long-run elasticity of \( w \) with respect to \( Y/L \)). Antràs (2004) shows that the estimation of the equivalent equation in the U.S. yields \( \sigma = 0.89 \) when the preferred estimation method - Saikkonen’s one- is used. Therefore, our results are consistent with those in Antràs (2004) and, interestingly enough, uncover a new relationship: consideration of the price markup reduces the elasticity of substitution in the U.S.\(^{10}\)


<table>
<thead>
<tr>
<th></th>
<th>Price markup considered</th>
<th></th>
<th>Price markup not considered</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>( [E3] ) [0.164] c</td>
<td>( [E4] ) 2.46 [0.000]</td>
<td>( [E3'] ) 0.12 [0.814]</td>
<td>( [E4'] ) 1.32 [0.026]</td>
</tr>
<tr>
<td>( \ln(w_{t-1}m_{t-1}) )</td>
<td>0.50 [0.004]</td>
<td>0.56 [0.000]</td>
<td>0.73 [0.000]</td>
<td>0.71 [0.026]</td>
</tr>
<tr>
<td>( \Delta \ln(wm_t) )</td>
<td>0.27 [0.078]</td>
<td>0.21 [0.000]</td>
<td>0.19 [0.103]</td>
<td>0.17 [0.143]</td>
</tr>
<tr>
<td>( \ln\left(\frac{Y_t}{L_t}\right) )</td>
<td>0.79 [0.027]</td>
<td>( \Delta \ln\left(\frac{Y_t}{L_t}\right) ) 0.27 [0.004]</td>
<td>( r_t ) 0.57 [0.000]</td>
<td></td>
</tr>
<tr>
<td>( \Delta \ln\left(\frac{Y_t}{L_t}\right) )</td>
<td>0.70 [0.061]</td>
<td>( \Delta \ln\left(\frac{Y_t}{L_t}\right) ) 0.21 [0.107]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t \times 100 )</td>
<td>-0.45 [0.131]</td>
<td>( t \times 100 ) 0.33 [0.000]</td>
<td>( t \times 100 ) 0.08 [0.230]</td>
<td>( t \times 100 ) 0.20 [0.008]</td>
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<table>
<thead>
<tr>
<th></th>
<th>( R^2 ) 0.981 s.e. 0.025</th>
<th></th>
<th>( R^2 ) 0.997 s.e. 0.008</th>
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**Misspecification tests:**

<table>
<thead>
<tr>
<th></th>
<th>( SC ) 0.14 [0.712]</th>
<th>( SC ) 0.12 [0.734]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( LIN ) 0.001 [0.890]</td>
<td>( LIN ) 1.88 [0.352]</td>
</tr>
<tr>
<td></td>
<td>( NOR ) 0.31 [0.473]</td>
<td>( NOR ) 0.51 [0.384]</td>
</tr>
<tr>
<td></td>
<td>( HET ) 0.10 [0.474]</td>
<td>( HET ) 1.76 [0.488]</td>
</tr>
</tbody>
</table>

**Notes:** Dependent variables: \( \ln(wm_t) \) in \( E3 \), \( \ln(w_t) \) in \( E3' \), and \( \ln(Y_t/L_t) \) in \( E4 \) and \( E4' \); \( \Delta \) is the difference operator; s.e. the standard error; p-values in square brackets. Misspecification tests: Serial correlation \( SC \), Linearity \( LIN \), Normality \( NOR \), and Heteroscedasticity \( HET \).

As in the Spanish case, when checking for consistency between our ARDL estimates and the ones that would be obtained from the Johansen procedure, we cannot reject the LR test.

---

\(^{10}\)This conclusion also holds when looking at the \( \sigma \) estimates derived from estimation of equation \( E4 (0.47) \) and \( E4' (0.59) \), which also fit within the range of estimates provided by Antràs (2004).
This empirical analysis yields two important conclusions. First, the elasticity of substitution between capital and labor is larger than 1 in Spain and smaller than 1 in the U.S. Second, consideration of the price markup causes the estimates of this elasticity to be drawn apart from 1. In other words, the assumption of perfect competition introduces a bias on its estimate which differs depending on the estimated value. It is a downward bias in Spain and an upward bias in the U.S.

In what follows we rationalize these two conclusions by using an accounting exercise based on equation (6) rewritten as

$$\frac{\Delta \psi_t}{\psi_t} = h \left( \frac{1 - \sigma}{\sigma} \right) - \frac{\Delta m_t}{m_t}.$$ 

Note that the growth rate of the LIS depends on capital deepening (measured by the difference between the growth rates of capital and GDP)$^{11}$ and on the growth rate of the price markup. As mentioned, the effect of capital deepening on the LIS depends on the elasticity of substitution. Capital deepening increases the LIS when the elasticity is smaller than one and decreases when it is larger than one.

The latter expression allows us to rationalize the two main conclusions obtained from the econometric analysis. To this end, we can rewrite it as follows:

$$\frac{\Delta \psi_t}{\psi_t} + \frac{\Delta m_t}{m_t} = h \left( \frac{1 - \sigma}{\sigma} \right).$$ 

$^{11}$The CES production function implies a direct relationship between the ratio of capital to GDP and the ratio of capital to labor. Thus, capital deepening implies an increase in the ratio of capital to GDP.
The left hand side of this equation is on average negative both in Spain and the U.S., with sample period means at -0.15% and -0.29%, respectively, that are clearly dominated by the LIS growth rates (the growth rates of the price markup are much smaller and play a minor role). By implication, the right hand side must also take negative values. This situation is pictured in Figure 4, where the continuous line displays the right hand side of equation (12) as a function of $\sigma$ taking into account that $h$ takes a negative value in the U.S. and a positive one in Spain. As it is clear from this Figure, when the LIS shows a downward path and there is capital deepening, the elasticity of substitution must be larger than one. In turn, when both the LIS and the ratio GDP/capital falls, the elasticity of substitution must be smaller than one. This explains the first conclusion above regarding the elasticity of substitution between capital and labor in Spain and the U.S.

Figure 4. Equation (12).

Note: Continuous lines denote the function $(1-\sigma)h$ when the price markup is considered; dashed lines show the same function when the price markup is not considered.

Regarding the effect of the price markup on the estimated $\sigma$, note that omission of the markup underestimates the value of the labor-output elasticity which, in turn, generates an overestimation of the absolute value of $h$. The dashed lines in Figure 4 display the effect of the price markup on the estimated $\sigma$. Related with these figures, it is worth recalling that in both cases the LIS behaves as non-stationary in contrast with the stationary path of the price markup.

Note that $h$ is the product of two terms. The first one is positive, whereas the second one is positive when there is capital deepening and negative otherwise. Data from the OECD shows that during the period there is capital deepening in the Spanish economy, implying a positive value of $h$, and there is a reduction in the ratio capital to GDP in the US economy, implying a negative value of $h$. 

---

12 Related with these figures, it is worth recalling that in both cases the LIS behaves as non-stationary in contrast with the stationary path of the price markup.

13 Note that $h$ is the product of two terms. The first one is positive, whereas the second one is positive when there is capital deepening and negative otherwise. Data from the OECD shows that during the period there is capital deepening in the Spanish economy, implying a positive value of $h$, and there is a reduction in the ratio capital to GDP in the US economy, implying a negative value of $h$. 

---

17
right hand side of equation (12) as a function of $\sigma$ when $h$ is overestimated due to the absence of the price markup. As shown in Figure 4, this implies that, in the absence of the price markup, the estimated value of $\sigma$ is biased towards one. This explains the second conclusion.

**Remark 1** An interesting implication of our findings is that the high elasticity of substitution in the Spanish economy implies that higher wages (in efficiency units) will reduce the labor income share as firms respond to this rise by substituting labor for capital more than proportionally with respect to the wage rise. On the contrary, the low value of the elasticity of substitution in the U.S. implies that higher wages (in efficiency units) generate a less than proportional response by firms, which therefore allows a larger LIS. This remark follows from rewriting equation (4) as:

$$\psi_t = \left( \frac{1 - \alpha}{m_t^\sigma} \right) \left( \frac{w_t}{A_t} \right)^{1 - \sigma}.$$  \hfill (13)

### 4.2 The primal and dual paths of the markup

The price markup was computed in Section 3 following the dual approach. If data were perfect and the elasticity of substitution could be perfectly estimated, the dual approach-based markup would coincide with the primal approach-based markup obtained from equation (6). In this section we evaluate the extent to which these two measures differ. To this end, we rewrite equation (6) as equation (12) to obtain a relationship between the LIS and the growth of the price markup. We use this relationship to regress the following equation:

$$\frac{\Delta \psi_t}{\psi_t} = \alpha h_t - \beta \frac{\Delta m_t}{m_t} + \varepsilon_t,$$  \hfill (14)

where $\alpha = \frac{1 - \sigma}{\sigma}$ and $\beta$ are the parameters to be estimated, and $\varepsilon$ is the residual of the equation. The growth rate of the price markup in this expression is the one obtained from the dual approach. Therefore, from this regression we obtain the values of the elasticity of substitution that minimize the difference (or error) between the primal and dual measures of the price markup. The results of this estimation are presented in Table 5.

Our estimation is based on equation (14), rather than on equation (12), because the estimates of $\beta$ are significantly different from -1. When we restrict $\beta = -1$, which would be the implicit value of $\beta$ in equation (12), we find non-sensible results. However, when this parameter is left free (as in Table 5), it is interesting to observe that the estimated values of $\alpha$ for Spain and the U.S. are broadly consistent with the estimates of $\sigma$ obtained via the estimation of the production function. Of course, the drawback of this exercise is the poor performance of the Spanish and U.S. econometric versions of equation (14), with poor explanatory power and coefficients that in some cases are not significant. However,
we find remarkable that our finding of $\alpha = -0.327$ for Spain implies $\sigma = 1.49$, whereas our estimate of $\alpha = 0.967$ for the U.S. implies $\sigma = 0.51$. Recall that if data were perfect these values of $\sigma$ would coincide, respectively, with 1.58 and 0.63.

| Table 5. Primal approach-based estimates of $\sigma$. |
|---------------------------------|---------------------------------|----------------|----------------|-------------------------------|
| Spain                          | $\Delta \psi_t / \psi_t$       | $\alpha$      | $\beta$       | $R^2$ = 0.19, $DW = 1.95$    |
| $\Delta \psi_t / \psi_t$       | $-0.327$                       | -0.237        |                |                               |
|                                 | [0.492]                        | [0.002]        |                |                               |
| U.S.                           | $\Delta \psi_t / \psi_t$       | $0.967$       | -0.076         | $R^2 = 0.08$, $DW = 1.80$    |
|                                 | [0.102]                        | [0.242]        |                |                               |

Note: Probabilities in brackets; $DW$=Durbin-Watson statistic.

5 Simulated labor income shares

In this section, we answer some of the questions we were asking at the beginning of the paper. In particular, we examine how much of the variation of the LIS is explained by the value of $\sigma$ (capital deepening) and how much by the trajectory of the price markup. To address this question, we use equation (3) to simulate the path of the LIS. As inputs of the simulation we need the path of GDP per efficiency unit of labor, the value of parameter $\alpha$, our estimates of $\beta$, and our dual measure of the price markup. GDP per efficiency unit of labor is obtained from the ratio between GDP per worker and technology. To obtain the technological path we assume, as we did in the previous section, that it grows at a constant rate. This rate is set at the sample period average growth rate of per worker GDP, which is equal to 2.14% in Spain and 1.55% in the U.S. In turn, the value of $\alpha$ is set so that the simulated LIS coincides with actual LIS in the initial period. Since the growth rate of the TFP is constant, we abstain from business cycle considerations and conduct our simulation on the trend component of the actual LIS, which is obtained through the HP filter.

We distinguish four different scenarios that combine (i) the presence and absence of the price markup in the simulation; and (ii) the $\sigma$ estimates obtained when the price markup is, and is not, included in the regression. In this way we can infer to what extent the elasticity of substitution or the price markup play dominant roles in explaining the actual trajectories of the labor share in Spain and the U.S. The resulting simulations are plotted in Figure 5 and their fit is evaluated in Table 6.

In Scenarios I and III we consider the estimated $\sigma$ obtained in the presence of the price markup, 1.58 in Spain, and 0.63 in the US. Scenario I, where the simulation is conducted
in the presence of the markup (in contrast to Scenario III), is our base run model. The resulting time series provide the closest approximation to the actual labor share trajectories in both economies with residual sum of squares (RSS) and \( R^2 \) of, respectively, 0.017 and 0.65 in Spain, and 0.007 and 0.93 in the U.S.

The second scenario is one of perfect competition (the price markup is not considered neither in the estimated regression nor in the simulated time path of the labor share) and provides the closest situation to the Cobb-Douglas case. It is thus natural to obtain a relatively constant labor share in both countries with, nevertheless, significant differences: the simulated path in Spain evolves slightly downwards initially, when the actual labour share is rising, and upwards subsequently, when the labour share trends downwards. As a consequence, the correlation coefficient and explanatory power of the simulated series are virtually null. In contrast, the simulated series in the U.S. behaves more in accordance with the initially downward and finally constant actual path, thereby resulting in a much accurate fit than in Spain (see Table 6).

**Figure 5. Simulated labor shares.**

![Graph of simulated labor shares for Spain and the U.S.](image)

Note: Simulations I to IV correspond, respectively, to Scenarios I to IV in Table 6.

Simulations in Scenario III use the estimated sigma in the presence of the price markup, but the price markup series is not used when computing the simulated path. Therefore, the difference between the simulated series under Scenarios I and III accounts for the contribution of the price markup to the LIS trajectory. In turn, in Scenario IV we use the estimated sigma with no markup, but the price markup is considered in the simulation. Hence, the difference between the simulated series under Scenarios I and IV accounts for the contribution of capital deepening to the LIS trajectory. To approximate these two contributions we follow Karanassou *et al.* (2003, p. 261) and regress the contribution of the markup on a constant and the contribution of capital deepening. We save the residuals and regress the actual LIS over a constant and the saved residuals. The \( R^2 \)
of this regression gives us the portion of the LIS variation explained by the part of the markup contribution that is uncorrelated with the contribution of capital deepening. We find it to be 63% in Spain and 57% in the U.S. Similarly, when we regress the actual LIS on the residuals of a regression of the capital deepening contribution on a constant and the contribution of the markup, we find the $R^2$ to be 0.27 in Spain and 0.39 in the U.S. This result indicates that capital deepening, which reacts to $\sigma$, is also a driving force of the labor share trajectory as it explains 27% and 39% of the LIS variation.

Table 6. Simulated labor shares’ fit.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\sigma$</th>
<th>$m$</th>
<th>RSS</th>
<th>$R^2$</th>
<th>$\sigma$</th>
<th>$m$</th>
<th>RSS</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.58</td>
<td>✓</td>
<td>0.017</td>
<td>0.65</td>
<td>0.63</td>
<td>✓</td>
<td>0.007</td>
<td>0.93</td>
</tr>
<tr>
<td>II</td>
<td>1.21</td>
<td>X</td>
<td>0.047</td>
<td>0.03</td>
<td>0.90</td>
<td>X</td>
<td>0.013</td>
<td>0.47</td>
</tr>
<tr>
<td>III</td>
<td>1.58</td>
<td>X</td>
<td>0.048</td>
<td>0.03</td>
<td>0.63</td>
<td>X</td>
<td>0.013</td>
<td>0.47</td>
</tr>
<tr>
<td>IV</td>
<td>1.21</td>
<td>✓</td>
<td>0.023</td>
<td>0.53</td>
<td>0.90</td>
<td>✓</td>
<td>0.010</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Notes: RSS = Residual sum of squares; the $R^2$ and the RSS are obtained from regressing the actual trend-component of the LIS on a constant and the simulated LIS in each scenario.

The conclusion we draw from this exercise is threefold. First, our base run case is able to proxy the path followed by the LIS in last decades in the two economies considered. The corresponding $R^2$s are 0.65 in Spain and 0.93 in the U.S. Second, both the price markup and capital deepening contribute to explain this path, although the price markup is clearly more determinant in Spain. Third, the explanatory power of $\sigma$, and thus of capital deepening, is less than half the explanatory power of the price markup in Spain, but two thirds of it in the U.S. This is consistent with the larger ratio of capital stock per employee in the U.S.\footnote{The process of capital deepening has been larger in Spain than in the US. However, since the capital/output is higher in the US throughout the whole sample period, smaller variations in capital deepening turn out to be more influential than the larger variations of the smaller Spanish ratio.}

6 Capital accumulation and labor income share

The elasticity of substitution determines the relationship between GDP (in efficiency units and capital accumulation), while the price markup relates these two variables with the LIS and employment growth. This leads us to bring the analysis into a broader macroeconomic context, which we do by augmenting the Solow model. We then simulate it and compare the resulting predictions with the actual time-series of GDP, capital accumulation, employment growth, and the LIS. We consider a base run scenario in the
presence of our price markups (so that there is imperfect competition and the elasticities of substitution differ from one) and a scenario in their absence (so that there is perfect competition and the elasticities of substitution are close to one). Comparison of how the two scenarios predict the actual trajectories of the main macroeconomic variables informs on the extent to which the degree of imperfect competition and its influence on technology are important ingredients of the model.

6.1 The model

We extend the Solow model by including (i) a CES production function; (ii) product market imperfections, which are summarized by the price markup; and (iii) labor market imperfections, which are introduced through a simple wage equation arising from a standard efficiency wage model and prevent the labor market to clear.

In particular, we assume that the wage is a constant markup over a reference wage \( w_t^R \), so that \( w_t = m_w w_t^R \).\(^{15}\) For the sake of simplicity, we assume that the reference wage depends positively on per capita GDP, \( \frac{Y_t}{N_t} \), and the employment rate, \( \frac{L_t}{N_t} \).\(^{16}\)

\[
w_t^R = \left( \frac{Y_t}{N_t} \right) \left( \frac{L_t}{N_t} \right),
\]

It is easy to see that the wage equation simplifies to

\[
w_t = m_w \left( \frac{Y_t}{L_t} \right) \left( \frac{L_t}{N_t} \right)^2,
\]

which can be rewritten in efficiency units as

\[
\tilde{w}_t = m_w \tilde{y}_t \left(1 - u_t\right)^2,
\]

where \( u_t \) is the unemployment rate, \( \tilde{y}_t = \frac{Y_t}{A_t L_t} \) and \( \tilde{w}_t = \frac{w_t}{A_t} \). To obtain the equilibrium unemployment, we use the labor demand equation (2) which can be rewritten in terms of the capital labor-ratio in efficiency units as follows:

\[
\tilde{w}_t = \left( \frac{1 - \alpha}{m_t} \right) \left[ \alpha \left( \frac{\tilde{c}_t}{\tilde{k}_t} \right)^{\frac{\alpha - 1}{\sigma}} + (1 - \alpha) \right]^{\frac{1}{\sigma - 1}},
\]

\(^{15}\)This constant mark up can be obtained in an efficiency wage model with effort function \( e = \left[ \frac{w_t - w_t^R}{w_t} \right]^\beta \)

if \( w_t > w_t^R \) and \( 0 < \beta < 1 \), which yields \( m_w = \frac{1}{1 - \beta} \).

\(^{16}\)When wages are set at the firm or sector level, the equilibrium wage depends on unemployment benefits and the unemployment rate (see, for example, Layard, Nickell and Jackman, 1991, pp.105-106). If we further relate unemployment benefits with income per capita as, for example, in Daveri and Tabellini (2000), we obtain a wage equation that depends positively on the employment rate and income per capita as in our postulated wage equation.

22
where $\tilde{k}_t = \frac{k_t}{A_t L_t}$. Using the wage equation (15) and the labor demand equation (16), we obtain the unemployment rate in equilibrium:

$$1 - u_t = \sqrt{\left(\frac{1 - \alpha}{m_w m_t}\right) \tilde{y}_t \left(\frac{1}{1 - \sigma} - 1\right)},$$  \hspace{1cm} (17)

where $\tilde{y}_t$ arises from rewriting the production function (1) in efficiency units of labor as

$$\tilde{y}_t = f(\tilde{k}_t) = \left[\alpha \left(\frac{\tilde{k}_t}{A_t}\right)^{\frac{1}{\sigma}} + (1 - \alpha)\right]^{\frac{\sigma}{\sigma - 1}}.$$  \hspace{1cm} (18)

Assuming an inelastic labor supply, $N_t$, that grows at the constant rate, $n$, equation (17) can be rewritten in terms of the growth rate of employment as

$$\frac{L_{t+1}}{L_t} = (1 + n) \sqrt{\left(\frac{m_t}{m_{t+1}}\right) \left(\frac{\tilde{y}_{t+1}}{\tilde{y}_t}\right)^{\left(\frac{1}{\sigma} - 1\right)}}.$$  \hspace{1cm} (19)

To close the model we characterize capital accumulation. For the sake of simplicity, we assume a constant savings rate so that capital evolves according to the following equation

$$K_{t+1} = sY_t + (1 - \delta) K_t,$$

where $s \in (0, 1)$ is the constant savings rate and $\delta \in (0, 1)$ is the constant depreciation rate. We rewrite this equation in labor efficiency units as

$$\tilde{k}_{t+1} = \left(\frac{L_t}{L_{t+1}}\right) \left(\frac{A_t}{A_{t+1}}\right) \left[s\tilde{y}_t + (1 - \delta) \tilde{k}_t\right].$$  \hspace{1cm} (20)

Then, we use equations (18) and (19) to obtain the following difference equation

$$\tilde{k}_{t+1} = (1 + n)^{-1} \sqrt{\left(\frac{m_{t+1}}{m_t}\right) \left(\frac{f(\tilde{k}_{t+1})}{f(\tilde{k}_t)}\right)^{\left(\frac{1}{1 - \sigma}\right)}} \left(\frac{A_t}{A_{t+1}}\right) \left[sf(\tilde{k}_t) + (1 - \delta) \tilde{k}_t\right].$$  \hspace{1cm} (21)

Equation (21) drives the accumulation of capital in this economy.

### 6.2 Numerical simulation

We first obtain the path of capital accumulation by solving numerically equation (21), and we then use equations (3), (18), and (19) to simulate the paths of the LIS, the ratio of capital to GDP, per worker GDP, and the growth rate of employment.

We calibrate the parameters as follows. First of all, we use the estimated values of $\sigma$, the average values of $\delta$ and $n$, and the computed values of the price markup for Spain.
and the U.S. To obtain efficiency units of labor we compute the Solow residual from an accounting exercise based on equation (5). We take into account that the Solow residual is affected by the price markup (Hall, 1988). We ensure that the simulated values depart from the actual values of the variables by setting the value of \( \alpha \) accordingly, and by fixing the initial amounts of capital stock and technology (in efficiency units) to match, respectively, the ratio of capital to GDP and per worker GDP. The values of the savings rate are set to calibrate the ratio of capital to GDP. To obtain a close simulation we have had to split the sample period into two. In this way, we are able to deal with the exceptionally high saving rates of Spain during the first 6 years of the sample, and the low U.S. rates of the first 10 years. This information is summarized in Table 7.

Table 7. Parameter values.

<table>
<thead>
<tr>
<th></th>
<th>( \sigma )</th>
<th>( m )</th>
<th>( \delta )</th>
<th>( n )</th>
<th>( \alpha )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Spain</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario I</td>
<td>1.58</td>
<td>√</td>
<td>0.043</td>
<td>0.0104</td>
<td>0.12</td>
<td>0.270</td>
<td>0.135</td>
</tr>
<tr>
<td>Scenario II</td>
<td>1.21</td>
<td>√</td>
<td>0.043</td>
<td>0.0104</td>
<td>0.27</td>
<td>0.240</td>
<td>0.160</td>
</tr>
<tr>
<td><strong>U.S.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario I</td>
<td>0.63</td>
<td>X</td>
<td>0.043</td>
<td>0.0144</td>
<td>0.22</td>
<td>0.155</td>
<td>0.185</td>
</tr>
<tr>
<td>Scenario II</td>
<td>0.90</td>
<td>X</td>
<td>0.043</td>
<td>0.0144</td>
<td>0.32</td>
<td>0.145</td>
<td>0.150</td>
</tr>
</tbody>
</table>

It is important to emphasize the different nature of the exercise undertaken in this section relative to the simulation performed in Section 5. Rather than checking the relative incidence of the price markup and capital deepening on the LIS trajectory, we have now developed a model in which capital accumulation is explained. By equation (20), this requires the use of Solow’s residual, as computed by equation (5), which is time-varying and entails the need to pay attention to both the trend and cyclical components of the series. This is the reason why we do not filter the series under scrutiny. Moreover, there is an important remark related to the restricted period for which the simulation exercise is conducted, from 1979 to 2007. When the whole sample period is considered, the model fails to produce a good fit in the 1960s and 1970s. We thus acknowledge that some relevant determinants of the macroeconomic scene in the first part of the sample are not well captured by our stylized analysis. In contrast, the model performs reasonably well

---

17 Rotemberg and Woodford (1999), and previously Hall (1988 and 1990), argue that the Solow residual is biased if the price markup is not considered. Its obtention, therefore, is also important in growth accounting, specifically when computing the Solow residual.

18 The values of the saving rates are fixed in each scenario to obtain the best possible fit of the capital to GDP ratio. Recall that this is the first step of the exercise. The predicted values of this ratio are then used to simulate the other variables.

19 Among other elements, we probably lack (i) the relevant influence of the inflationary oil price shocks and the subsequent deflationary interest rate shocks; (ii) a more realistic specification of the wage setting process; and (iii) an explicit specification of labor supply decisions which, in those years, were affected by demographic changes such as the baby-boom and the baby-bust. Although all of them are clearly
when explaining the evolution of the labor share, employment, GDP per worker, and the ratio of capital stock to GDP. This is shown in Figures 6, for Spain, and 7, for the U.S., while Table 8 evaluates these simulations through the RSS (to check their global fit) and the ratio between the actual and simulated standard deviations (to check their fit in terms of volatility).

Figure 6. Simulated selected variables in Spain.

Note: Simulations I and II correspond, respectively, to Scenarios I and II in Table 7.

We find scenario I to track reasonably well the evolution of the labor share, employment growth, and the ratio of capital stock to GDP in both Spain and the US. On the contrary, Scenario II fails to capture the downward trend in the labor share during these years, and yields unrealistic flat trajectories of the LIS and the growth rate of employment. In terms of GDP per worker, Scenario I allows a close replication of the actual upward trajectory in clear contrast with the flawed predictions of Scenario II. In the case of Spain it also important, consideration of these factors lies beyond the scope of the specific exercise we conduct in this Section.
captures the inflection point experienced in the second half of the 1990s, when the wild-ride years started and produced the unprecedented employment boost in response to the rapid economic growth that became the trademark of this economy.

Figure 7. Simulated selected variables in the U.S.

(a) Labor share

(b) Employment growth

(c) GDP per worker

(d) Capital stock / GDP

Note: Simulations I and II correspond, respectively, to Scenarios I and II in Table 7.

Table 8 confirms that Scenario I provides a better fit than Scenario II. Note that whenever the RSS is not conclusive (i.e. for the labor share and the ratio between capital stock and GDP in the U.S., and for employment growth in both economies), their relative volatilities are closer to 1 due to the low volatility characterizing Scenario II. The close fit delivered by Scenario II in terms of the volatilities of GDP per worker and the ratio of capital stock to GDP is remarkable for both economies. This implies that our simulations under Scenario I, that is when the markup is considered, are specially suitable to account for the facts when these facts involve a time-varying pattern of the variables. Given that this is generally the case, the role played by the price markup should not be disregarded.
Table 8. Simulated variables’ fit.

<table>
<thead>
<tr>
<th></th>
<th>$\psi$</th>
<th>$\Delta N$</th>
<th>$Y \cdot \Delta N$</th>
<th>$K/Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RSS</td>
<td>RV</td>
<td>RSS</td>
<td>RV</td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario I</td>
<td>0.007</td>
<td>1.89</td>
<td>0.023</td>
<td>0.55</td>
</tr>
<tr>
<td>Scenario II</td>
<td>0.016</td>
<td>0.09</td>
<td>0.020</td>
<td>0.01</td>
</tr>
<tr>
<td>U.S.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scenario I</td>
<td>0.003</td>
<td>1.92</td>
<td>0.003</td>
<td>1.43</td>
</tr>
<tr>
<td>Scenario II</td>
<td>0.003</td>
<td>0.02</td>
<td>0.003</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: RSS=Residual sum of squares; RV=Relative volatility ($\frac{\text{simulated std. dev.}}{\text{actual std. dev.}}$).
Scenario I: $\hat{\sigma} = 1.58$ in Spain; $\hat{\sigma} = 0.63$ in the U.S.; markup considered;
Scenario II: $\hat{\sigma} = 1.21$ in Spain; $\hat{\sigma} = 0.90$ in the U.S.; markup not considered.

Overall, it seems safe to claim that consideration of the price markup in the analysis provides a relevant insight when attempting to explain the evolution of the key macroeconomic variables in last decades.

7 Conclusions

We provide estimates of the elasticity of substitution between capital and labor under imperfect competition in the product market. This elasticity is larger than one in Spain and lower than one in the U.S.

An important contribution of the paper is the rationale behind these different values. Both economies have experienced a declining path of the LIS and, at the same time, differ in the evolution of capital deepening. The paper reconciles these facts by unveiling the connection between the elasticity of substitution, capital deepening, and the trajectory of the time-varying LIS.

One important aspect of this connection is that the elasticity of substitution determines the effect of capital deepening on the LIS which, in turn, depends on the time-varying price markup. In showing this, the paper uncovers the bias that the assumption of perfect competition introduces in the estimates of the elasticity of substitution. This bias drives the estimated elasticity of substitution towards one irrespective of whether this estimate is placed above or below one. We believe this is an important result of the paper.

The differences in the elasticity of substitution have interesting implications for the relationship between other aggregate variables. For example, higher wages imply a reduction of the LIS in Spain, as a result of the large substitution between capital and labor, while they drive it upwards in the U.S., on account of the small substitutability between capital and labor.

In the final part of the paper we use the estimated elasticities and the price markups
in Spain and the U.S. to conduct two simulation exercises. In the first one, we examine to what extent the trajectory of the LIS is explained by capital deepening and the price markup. For Spain we find that the former accounts for 63% of the changes in the LIS, while the price markup accounts for 27% of them. For the U.S. these values are, respectively, 57% and 39%.

In the second simulation, we extend the Solow model by considering a CES production function, and imperfect competition in the labor and product markets. We solve this model numerically in two relevant scenarios, and we conclude that neglecting the degree of imperfect competition—in our case measured by the time-series aggregate price markup—makes the model invalid for predicting the relationship between crucial macroeconomic variables such as capital, GDP, and the LIS. On the contrary, when the price markup is taken into account, the model yields predictions that are broadly consistent with the data.

The empirical findings in this paper open two relevant issues. The first one relates to the causes behind the different elasticity of substitution in Spain and the U.S. Among others, some potential candidates are differences in the sectoral composition of GDP, in the composition of the labor force (skilled/unskilled), and in the institutional environment.20 The second issue relates to the high correlation between capital deepening and the price markup. The explanation of this finding would obviously require to consider models with endogenous markups. These new and interesting research avenues will be the aim of future research.

References


20 We believe that a potentially crucial explanation could be the institutional environment in which firms have traditionally operated in these two countries. Spain has evolved from a highly rigid and regulated economy to a liberalized situation with a salient characteristic: its segmented labor market. Indeed, for the last 25 years, a third of dependent employment has been holding a temporary contract, while the other two thirds belonged to a highly protected permanent segment (see Dolado, García-Serrano, and Jimeno, 2002). Together with traditional difficulties in funding access (due to the small firms’ average size and, until recently, financial markets underdevelopment and scarce competition in the banking system), Spanish firms have tended to live in a situation of expensive capital and progressively cheap labor. This, we believe, may have led them to a relatively high sensitivity in factor substitution. On the contrary, the US economy is the paradigm of a deregulated environment in which firms are less constrained by regulations and, thus, less sensitive to changes in factor prices: prices in the U.S. have probably been driven by the market and not that much, as in Spain, by deregulation processes of different intensities in the product, labor, and financial markets. This provides less incentives to be opportunistic in search of the best factor combination resulting from changing regulations and institutions.


