THE ROLE OF KEY FINANCIAL INSTRUMENTS AND NEW FINANCIAL TECHNOLOGIES IN INVESTMENT AND POLICY DECISIONS

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This dissertation is composed of three essays that study the role of key financial instruments and new financial technologies in investment and policy decisions.

In the first essay, I investigate convenience yields, which are non-pecuniary benefits in the form of liquidity, of portfolio debt assets or safe assets for emerging market economies (EMEs). Along with the United States, the Euro area, the United Kingdom, and Japan act as the major safe asset providers to an EME. Empirically, I establish that convenience yield of various safe assets differs for an EME investor, convenience yield of a particular safe asset varies during different time periods, convenience yield plays a crucial role in explaining covered interest parity deviations, and forecasts of the convenience yield for various safe asset providers are dissimilar. I develop a simple safe assets portfolio model for an EME investor that helps to understand their investment behavior. The framework predicts a positive relationship between the convenience yield and the share of the safe assets, and it is validated empirically for the United States, the Euro area, and Japan safe assets in an EME investor portfolio. I demonstrated through structural estimation that convenience yield is the second-most significant channel in explaining the interest rate differentials between an EME and a safe asset provider.

The second essay is co-authored with Eswar Prasad. We explore the implications of adoption of the Central Bank Digital Currency (CBDC) in an economy.

CBDC is electronic, universally accepted, and central bank issued money. We present a general equilibrium model that highlights the economic trade-offs between cash and CBDC. The key differences between cash and CBDC include transaction efficiency, possibilities for tax evasion, and, potentially, nominal rates of return. We examine the differential sensitivity of the CBDC share to changes in different parameters. Our model suggests that the highest degree of sensitivity of the CBDC share is due to the nominal rate of return on CBDC. We also evaluate the welfare implications of different settings of the policy parameters. Increases in the transaction efficiency and rate of return on CBDC generate welfare gains. We establish conditions under which cash and CBDC can co-exist and show how government policies can influence relative holdings of cash, CBDC, and other assets. We also illustrate how a CBDC can facilitate negative nominal interest rates and helicopter drops, and how a CBDC can be structured to prevent capital flight from other assets.

In the third essay, I document a direct benefit of circulation of CBDC. A quantitative model is developed to understand the effects of the introduction of CBDC on tax evasion and the relative sizes of formal and informal economies. This paper proposes the conditions under which CBDC helps in reducing the amount of tax evasion in an economy. Monitoring and detecting the process of tax evasion in an economy is a costly business. Introduction of CBDC leads to reduction in monitoring cost which enhances the probability of detection of tax evaders. Eventually, this mechanism results in lower level of tax evasion in the economy. The agent faces a penalty cost when detected for tax evasion. CBDC increases transparency and induces higher penalty cost. This theory suggests an agent is motivated to move towards high penalty cost CBDC regime due to low tax rate.

BIOGRAPHICAL SKETCH

Bineet Mishra is a PhD candidate in the Department of Economics at Cornell University, working under the supervision of Eswar Prasad, Kristoffer Nimark, and Mathieu Taschereau-Dumouchel. His research lies at the intersection of macroeconomics and finance. In particular, he studies how safe assets affects the investment decisions of an EME investor and how central bank digital currencies bring positive outcomes in an economy. He holds a Master of Arts degree in Economics from Duke University, a Post Graduate Diploma in Management from Xavier Institute of Management Bhubaneswar, India, and a Bachelor of Technology degree in Electronics and Communication Engineering from National Institute of Rourkela, India.

To my family who are my lifelines.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

DECOMPOSING COVERED INTEREST RATE PARITY DEVIATIONS FOR AN EMERGING MARKET ECONOMY

1.1 Introduction

Safe assets are financial instruments that have low risk and high liquidity (Eichengreen, 2017). Safe assets are held by private entities such as banks and investors as well as by public institutions such as the central bank of an economy. Safe assets provide a convenience yield in addition to the assured financial rate of return. Convenience yields are the non-pecuniary benefits due to the liquidity properties of the safe assets. This paper examines the role of the convenience yield in the safe assets demand for private investors of emerging market economies (EMEs). This study also aims to understand the significance of the convenience yield in the dynamics of the covered interest parity condition of EMEs.

The government bonds of an economy are considered as safe assets (Brunnermeier and Huang, 2018). The private international safe assets demand across EMEs has been increasing for the last fifty years. EMEs' private demand for safe assets is valued at approximately 7.6 trillion U.S. Dollars in 2020, which is almost 4 times higher than in 2000. The average share of safe assets demand by EME investors is about 20 percent of the total private safe assets demand from 2000 to 2020. What constitutes the major safe asset providers for an EME? There are only a few advanced economies (AEs), namely the United States of America (U.S.), Euro area (EU), United Kingdom (U.K.), and Japan (JA), that act as the major safe asset providers to the EMEs. The composition of these four safe assets across the EMEs ranges between 50 percent and 70 percent over the last two decades.

The financial openness of an economy leads investors to invest in home and international safe assets. The nominal interest rate differentials, measured as the difference between the nominal interest rate of short term government bonds of an EME and that of a major safe asset provider, which are positive over the last two decades. A higher return on the government bonds of an EME compared to that of safe assets would seem to make safe assets not as lucrative an investment opportunity for an EME investor. Thus, the increasing demand of international safe assets in EMEs cannot be fully justified by the positive interest rate differentials. This paper explores and identifies the channels, other than the interest rate differential channel, that explain the demand for safe assets by an EME investor. Specifically, this work shows the importance of the convenience yields in the demand structure for a portfolio of safe assets in an EME. [1]

The positive interest rate differentials between EME and safe asset providers measure the exchange rate risk, default risk, liquidity risk, and risk premia of the EME government bonds. The covered interest parity condition states that the interest rate for an EME government bond is equal to the interest rate on an EME currency converted AE government bond. The covered interest parity condition is derived from an assumption of no arbitrage opportunity between EME and AE interest rates using forward contracts. Empirically, covered interest parity deviations have been observed between an EME and AEs. The presence of covered interest parity deviations suggests that the EME government bonds are subject to default and liquidity risks. The liquidity benefit of

[1] In this paper, the short term government bonds of the U.S., Euro area, U.K., and Japan are considered as safe assets for simplicity. The long term government bonds add a layer of complexity of measuring recurring convenience yield over multiple years.

safe assets over the government bonds of EME is captured after factoring in the default risk of the EME. The empirical international finance literature identifies this liquidity benefit, which is a non-pecuniary benefit of the safe assets, as the convenience yield. Thus, empirically the convenience yield is defined as the interest rate difference between a default risk adjusted EME government bond and an EME currency converted AE government bond of same duration.[2] This research not only shows the role of convenience yield in explaining the covered interest parity condition but also quantifies the significance of the convenience yield channel for different EME portfolio of safe assets.

This paper analyzes of effects of the convenience yield for EMEs through empirical investigation, a simple analytical model, and quantitative analysis. First, the empirical investigation focuses on understanding the reasons behind the positive interest rate differentials, identifying the presence of covered interest parity deviations for an EME, and measuring systematically the convenience yield of various safe asset providers. While the international finance literature has primarily focused on the U.S. convenience premium for advanced economies, this study documents the convenience yields of four major safe assets providers for fifteen EMEs. Interestingly, the United Kingdom provides the highest convenience yield, on an average of 30 basis points, among the four safe asset providers to an EME during the last two decades. The convenience yield of the United States for an EME is 25 basis points. However, the U.S. convenience yield shot up to 66 basis points during the Global Financial Crisis (GFC). The time evolution of the convenience yields of the four safe asset providers

[2]The empirical literature has followed a nomenclature of stating the convenience yield of an AE government bond. But it actually discusses the relative convenience yield which is the difference between the convenience yield of a safe asset provider's government bonds and that of an EME's government bonds. There is a subtle difference between absolute and relative convenience yields. In this paper, convenience yield is referred to as the relative convenience yield to avoid confusion.

varies for different EMEs suggesting the heterogeneity properties of the safe assets. The convenience yield of the safe asset providers for an EME is not always positive. The time series future forecast of the convenience yield of different safe asset providers for an average EME doesn't follow the same trend.

Second, a simple analytical framework is presented to tease out the channels of the safe assets demand for EME. In this paper there are two major departures from the traditional international capital flow models. The first departure is that the model features heterogeneity across the access of safe assets that enriches the dynamics of the safe assets flows in an EME. A small open economy (SOE) is considered where investors of the SOE can hold home risky asset, home government bonds, and multiple international safe assets.[3] The international safe assets provide a liquidity premium over home assets which is prominent during a financial crisis.[4] The second departure relates to the income process shocks and exchange rate shocks faced by the SOE investor. The appreciation or depreciation of the exchange rate is well studied in the literature for the demand of international safe assets. There are two new mechanisms explored here. The first mechanism corresponds to the role of risk in the exchange rate and the second mechanism corresponds to the role of the covariance between the risk of income and exchange rate processes. Both mechanisms influencing the demand for the international safe assets are studied.[5] Overall, a safe asset provider which faces appreciation against the EME, has low risk in the exchange rate process,

[3]Investors harbor their investment in international safe assets since the home economy faces financial crisis or turmoil.

[4]The safe assets provide convenience yields which are a combination of non-pecuniary liquidity and safe benefits. While short term government bonds exhibit the liquidity premium, long term government bonds feature the safety premium (Gorton, 2017). Henceforth, the liquidity premium is equivalent to convenience yield.

[5]Appreciation or depreciation is the percentage drift of the exchange rate. Exchange rate risk is the percentage volatility of the exchange rate. Mathematically, risk is the volatility in the geometric brownian process defining the exchange rate.

and offers high liquidity premium is an ideal candidate in the portfolio of the investors.

Third, the quantitative implications impart the magnitude of the convenience yield of the safe assets and tie the relationship of the convenience yield to investment decision. Is there a relationship between the shares and convenience yields of the safe assets? The model predicts that an increase in the convenience yield of the safe asset leads to an increase in the share of that safe asset. This is verified through a panel regression between the shares of the safe asset and convenience yields of that safe asset for different EMEs. The regression shows a positive association between the two variables.

The model helps to estimate the values of the liquidity premium for the international safe assets after factoring in the exchange rate risk, default risk, and risk premia of the home country. The liquidity premium can be used to understand the dynamics of the interest rate differentials between an EME and major safe asset providers. The liquidity premium of U.S. safe assets contributes 26 basis points towards a 100 basis points interest rate differential between Brazil and the U.S. Also, the model demonstrates the importance of different channels though variance decomposition of the safe assets flows. Liquidity premiums of the U.S., Euro area, U.K., and Japan explain about 8, 10, 11, and 7 percent variations of the Brazil interest rate differentials against the corresponding safe asset providers. This shows the liquidity benefits are another reason for holding the safe assets even in the presence of positive interest rate differentials between the EMEs and the safe asset providers.

The model reinforces the understanding of the behavior of the EME investor in rebalancing their portfolio during the time of financial crisis. EMEs are characterized by underdevelopment of the financial market and face financial turmoil. Such crises often feature a sudden collapse in the home risky asset increasing its volatility and dropping its rate of return as well as decline in the rate of return of home government bonds. Through the lens of the model, such a shock leads the investor to increase the demand for the international safe assets. An investor rebalances their portfolio to increase the share of the U.S. safe assets when their home economy faces a crisis. This investment strategy depicts the special position of the U.S. government bonds in the international monetary system.

This work is related to two strands of literature. First, the international finance literature has extensively studied the convenience yield of the U.S. for advanced economies. The Euro area, U.K., and Japan, along with the U.S., act as safe asset providers to EMEs. In this paper the convenience yield of various safe asset providers for different EMEs is characterized. Second, the macro finance literature analyzes a two country framework to understand the investment decisions. In this paper an N-countries safe assets model, which includes exchange rate and income risks, is developed to understand the dynamics of the portfolio choice of safe assets for an EME investor. The model helps to structurally determine the convenience yield of the safe asset providers after factoring in pertinent risks. This framework also shows the relationship between the demand for the safe asset and the convenience yield of that safe asset.

Valchev (2020) studies the U.S. bonds convenience yield and builds a tractable model to determine the dynamics of uncovered interest rate parity deviations. Du, Im, and Schreger (2018) study the U.S. Treasuries premium for advanced countries. They documented that the U.S. convenience yield to other advanced economies is about 10-26 basis points. Du, Tepper, and Verdelhan (2018) document the covered interest parity deviation for advanced countries and investigate reasons for the deviations. Engel and Wu (2022) establish evidence that liquidity yield is significant in explaining exchange rate changes for all of the G10 countries. In this paper, a similar empirical strategy of using interest parity deviation is followed to determine the convenience yield, not only of the U.S., but also of the Euro area, U.K., and Japan for fifteen EMEs.

Farhi and Maggiori (2018) provide a theoretical framework to understand the supply and demand of reserve assets in a two period and two country framework. The model developed in this paper differs from Farhi and Maggiori (2018) by incorporating not only the level of income process but also the volatility of the income process and of exchange rate process as a determinant of the safe asset demand. Also, this paper builds the model from the classic portfolio papers of Merton (1969), Merton (1971), and Solnik (1974) with additional features of imperfect international capital markets having liquidity costs and an additional risk of the income process. Brunnermeier and Sannikov (2019) focus on the currency acting as a safe asset and show the portfolio weight of the international currency depends on the risk of investment in physical capital. On the contrary, this paper shows that the portfolio weight of international government bonds, which are considered as safe assets, depends on the exchange rate and income risks.

The model encompasses the idea of need for international safe assets in an EME due to liquidity reasons which is in line with Caballero and Krishnamurthy (2006). Additionally, this paper borrows the view from He, Krishnamurthy, and Milbradt (2019) and Hassan (2013) of relative advantage of one country having strong fundamentals, which is reflected by higher liquidity in its government bonds, compared to other countries make that particular country's government bonds as the dominant safe asset. Jiang, Krishnamurthy, and Lustig (2021) provide that the formulation of the spot exchange rate of a safe asset currency is reflected by the cumulative value of all future convenience yields earned by foreign investors. Their model assume an investor's Euler equation which explains the failure of Uncovered Interest rate Parity (UIP). Similarly, this paper documents a key result of UIP failure from the optimality conditions. Davis and van Wincoop (2021) develop a multi-country setup with heterogeneous agents to explain the nature of gross and net capital flows across economies during the global financial crisis. In similar fashion, this paper attempts to understand only the gross portfolio debt flows for an EME.

The remainder of the paper is structured as follows. Section 2 shows the empirical characteristics of the safe assets in the EMEs and the convenience yield of the safe assets for the EMEs. Section 3 demonstrates a safe assets portfolio model that determines the safe assets demand for the investors in an EME. Section 4 presents the theoretical results that provide structure to the channels of the safe assets demand and shows failure of uncovered interest rate parity condition. Section 5 explores a quantitative estimation of the model through the lens of the EMEs' international safe assets holdings. Section 6 concludes. All proofs of the propositions and corollaries as well as additional empirical observations for the EMEs are detailed in the appendix.

1.2 Empirically quantifying convenience yields

The data used for identifying the empirical characteristics of the safe assets in EMEs and for computing the convenience yield are described. Certain features of safe assets such as distribution of safe assets for AEs and EMEs, gross safe assets demand for the EMEs, and contribution by the investors of the EMEs in the safe assets demand are presented. The interest rate differentials between the safe asset providers and an EME are reported. The key safe asset provider countries are shown. The convenience yield of the safe asset providers to the EMEs are documented.

Following empirical strategy as in Du et al. (2018), the convenience yield of the four major safe asset providers to the fifteen different EMEs are computed. It helps to understand how the convenience yield of various safe asset providers to EMEs differs.

1.2.1 Data Description

The Coordinated Portfolio Investment Survey (CPIS) provides an economy's portfolio investment securities data, which are categorized by the instrument type and issuer. Spot and forward exchange rates of an EME against the safe asset provider countries, interest rate on short term government bonds for different economies, and credit default swap (CDS) rates of the EMEs are obtained from Datastream. Gross domestic product of various countries are retrieved from the IMF.[6]

[6] Data for fifteen EMEs namely Brazil, Colombia, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Peru, Poland, Russia, South Africa, South Korea, Thailand, and Turkey are collected. This sample closely follows Du et al. (2018)

1.2.2 Safe assets characteristics in EMEs

The total demand for safe assets in EMEs is approximately 7.6 trillion U.S. dollars in 2020, which is about 4 times that in 2000. The average demand for safe assets as a percentage of EME GDP is 24 percent over the last two decades.[7] The private safe assets are held by individual investors and banks. The average share of safe assets held by EME investors is about 20 percent from 2000 to 2020. The detailed evolution of safe assets properties in EMEs are outlined in section A.1.1.

1.2.3 Nominal interest rate differentials between safe asset providers and EMEs

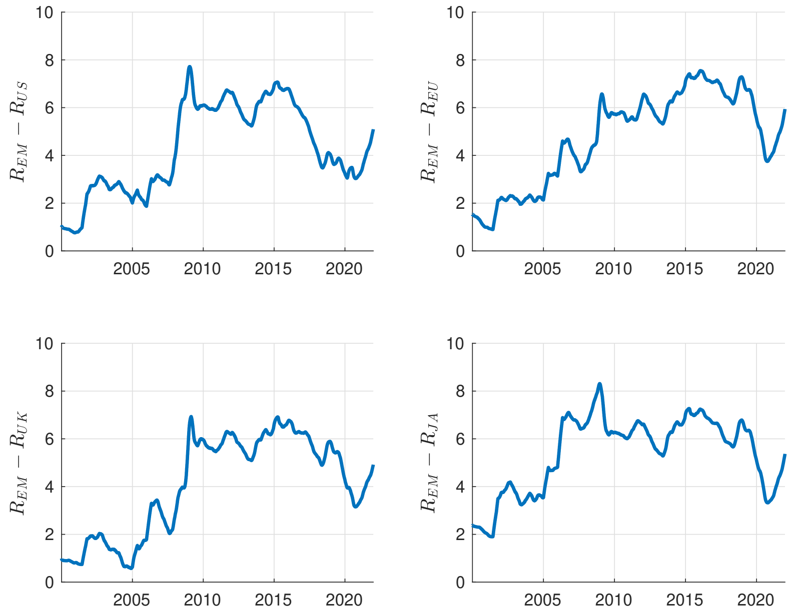
The nominal interest rate on short term government bonds of EMEs and safe asset providers are obtained from Datastream. A representative EME denotes a synthetically constructed EME as the weighted average of the fifteen EMEs. The weights are calculated from the gross domestic product of the EME. Figure 1.1 shows the yearly nominal interest differentials between a representative EME and the major safe asset providers.

Empirical Observation 1: The nominal interest rate differentials between a representative EME and the safe asset providers are positive for the last two decades.

The interest rate differential is one of the major drivers for the financial capital movement across economies. The positive interest rate differentials imply that the nominal interest rate on the government bonds of an EME is higher

[7] Also, the cross-sectional median demand for the safe assets in EMEs is about 1.5 times higher than that in AEs.

Figure 1.1: Nominal interest rate differentials



Notes: EM denotes a representative EME. US, EU, UK, and JA denote the United States, Euro area, United Kingdom, and Japan respectively. The nominal interest rate differentials between short term government bonds of a representative EME and the United States, Euro area, United Kingdom, Japan are shown from 2000 to 2021. The y-axis is in percentage scale.

than that of safe asset providers.

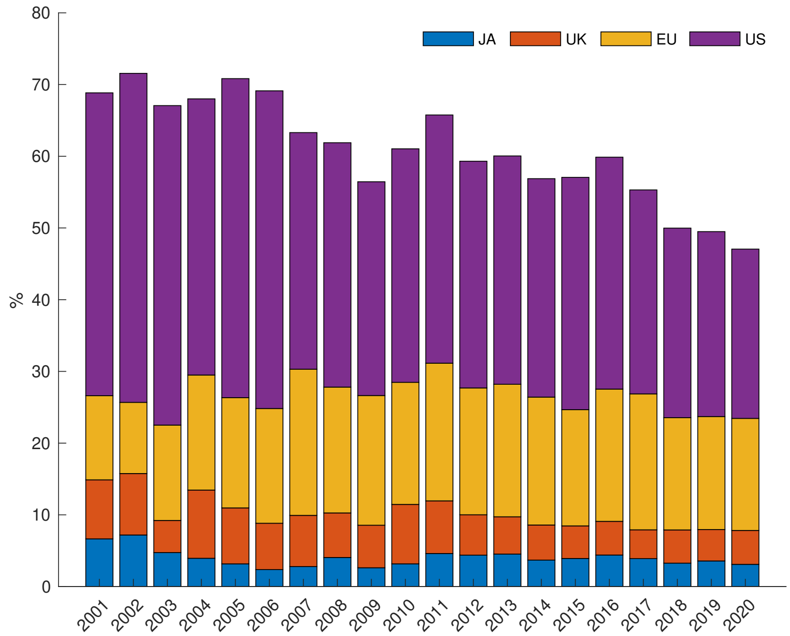
1.2.4 Safe asset provider countries

Data on the safe assets holdings by various EMEs are obtained from the CPIS. The shares of the safe assets held by the EME investors are evaluated.[8]

Empirical Observation 2: The major safe asset provider countries are the United States, Euro area, United Kingdom, and Japan.

[8] Denote the share of an AE safe assets to the EME investors' safe assets portfolio, the holding of AE safe assets by the investors of EME, and the total holding of safe assets by the EME investors as $ISAS_t^{AE}$, ISA_t^{AE}, and ISA_t^{E} respectively. Thus, $ISAS_t^{AE} = \frac{ISA_t^{AE}}{ISA_t^{E}} \times 100$.

Figure 1.2: Safe asset providers



Notes: The evolution of the shares of the United States, Euro area, United Kingdom, and Japan safe assets held in a representative EME.

Figure 1.2 shows that the composition of the safe assets for a representative EME for the last two decades. The investors of a representative EME hold the U.S. safe assets in majority, thus corroborating the special status of the U.S. in the international finance system. A representative EME investor holds about 50% -70% of their international bonds portfolio in the four major safe assets. The rest is held in the government bonds of other advanced economies and emerging market economies.

Figure 1.1 shows that the positive interest rate differentials makes the safe asset not as lucrative investment opportunity for an representative EME. However, Figure 1.2 and characteristics of safe assets as highlighted in the section 1.2.4 depict a substantial demand for safe assets in the EMEs. Therefore, the positive interest rate differentials confound the safe asset demand for the EMEs.

1.2.5 Convenience yield of various safe asset providers to different EMEs

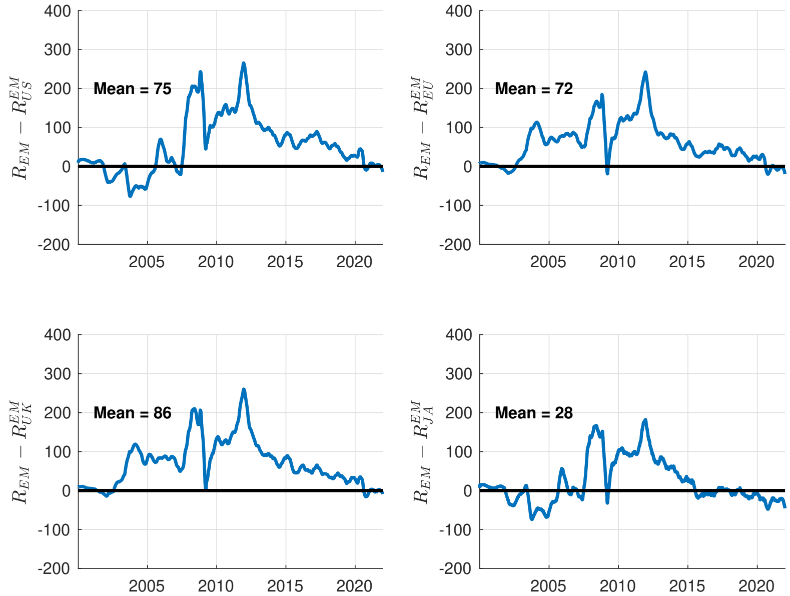
The positive interest rate differentials between a representative EME and the safe asset providers capture the exchange rate, default, and liquidity risks of EME government bonds. Covered interest parity (CIP) states that an investment in EME government bonds is equivalent to investment in EME currency converted safe assets. CIP condition captures the exchange rate fluctuations of the EME. There must be no CIP deviations (CIPD) when CIP condition holds. Let R_i and R_h denote nominal rate of return on a safe asset and an EME government bond respectively. Let e_{hit} and $\mathbb{E}(e_{hit+1})$ denote the log of spot exchange rate of an EME against the safe asset provider and the expectation of the log of the future exchange rate of an EME against the safe asset provider respectively. CIPD is calculated as the interest rate differential between an EME government bonds and an EME currency converted safe assets. R^h_{it} represents the interest rate of an EME currency converted safe asset.

$$CIPD = R_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R_{it}) = R_{ht} - R^h_{it}$$

Empirical Observation 3a: The CIP for an EME fails.

The forward rates of an EME against the safe asset provider act as proxy for $\mathbb{E}(e_{hit+1})$. Data on the forward rates and spot exchange rates are retrieved from Datastream. Figure 1.3 shows the CIP deviations of a representative EME against the major safe asset providers. Section A.2.1 tabulates the CIP deviations of fifteen EMEs against the U.S., Euro area, U.K., and Japan safe assets. Presence of CIP deviations for an EME signify that the exchange rate risks of the EME cannot fully explain the interest rate differentials between an EME and a safe

Figure 1.3: CIP deviations in basis points



Notes: EM denotes a representative EME. US, EU, UK, and JA denote the United States, Euro area, United Kingdom, and Japan. CIP deviations for a representative EME are plotted from 2000 to 2021. The mean is the time average CIP deviations for the whole sample.

asset provider.

CIP deviations indicate the presence of default and liquidity risks of the EME government bonds. The default probability of an EME is computed from the credit default swaps rates. The interest rate on the EME government bond is adjusted for its implied default. Let R_h^d and R_i^d denote the default risk adjusted nominal rate of return on an EME government bond and that on an safe asset respectively. Let p_h^d and p_i^d denote the probability of default on an EME government bond and that on an safe asset respectively. Thus, the relationship between the default risk adjusted nominal rate and the nominal rate of return on the EME government bonds is as follows.

$$(1 + R^d_{ht}) = p_{ht}0 + (1 - p_{ht})(1 + R_{ht}) = (1 - p_{ht})(1 + R_{ht})R^d_{ht} = R_{ht} - p_{ht}(1 + R_{ht}) \approx R_{ht} - p_{ht}$$

Similar relationship also holds between the default risk adjusted nominal rate and the nominal rate of return on the safe asset. The liquidity benefit (LB) of the safe asset is defined as the interest rate differential between an default risk adjusted EME government bonds and an EME currency converted default risk adjusted safe assets. This liquidity benefit is referred as the convenience yield of the safe asset. Safe assets provide convenience yield, a non-pecuniary benefit due to the liquidity benefit, in addition to the financial return. Section A.2.1 shows the derivation of the relationship. ϕ_i and ϕ_h represent convenience yield of a safe asset and an EME government bond respectively. R^{dh}_{it} represent the interest rate of an EME currency converted default risk adjusted safe asset. There is a tight relationship between the liquidity benefit and the relative convenience yield between a safe asset and an EME government bond which is captured below.[9]

$$LB = CY^h_i = \phi_{it} - \phi_{ht} = R^d_{ht} - R^{dh}_{it}$$

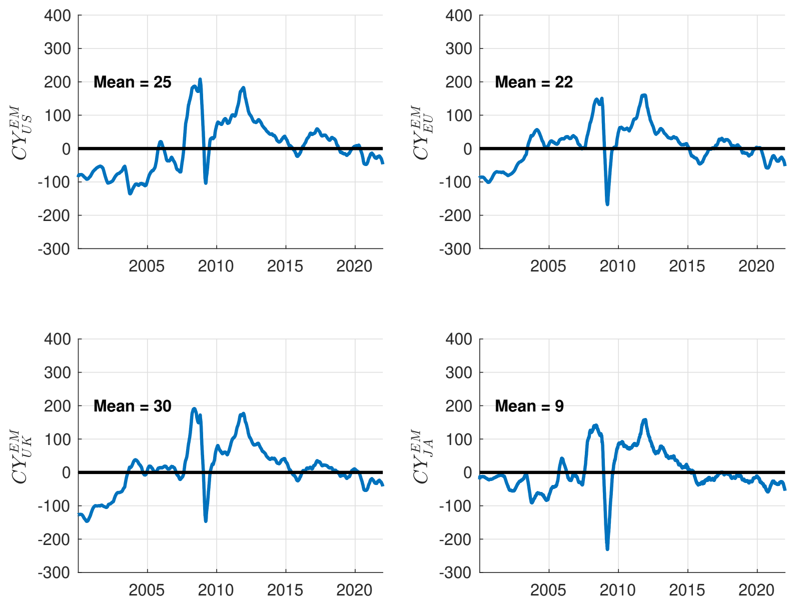
Using the data on spot and forward exchange rates of an EME against the safe asset provider countries, interest rate on short term government bonds for different economies, and credit default swap rates, the convenience yield of the four major safe assets for different EMEs is measured.

Empirical Observation 3b: The convenience yield of various safe asset providers differ to a representative EME.

Empirical Observation 3c: The safe assets provide different convenience yield to the EME during different time-periods.

[9] $R^{dh}_{it} = \mathbb{E}(e_{hit+1}) - e_{hit} + R_{it} - p_{it}(1 + R_{it}) = R^h_{it} - p_{it}(1 + R_{it}) \approx R^h_{it} - p_{it}$

Figure 1.4: Convenience yields in basis points



Notes: EM denotes a representative EME. US, EU, UK, and JA denote the United States, Euro area, United Kingdom, and Japan. Convenience yields for a representative EME are plotted from 2000 to 2021. The mean is the time average convenience yield for the whole sample.

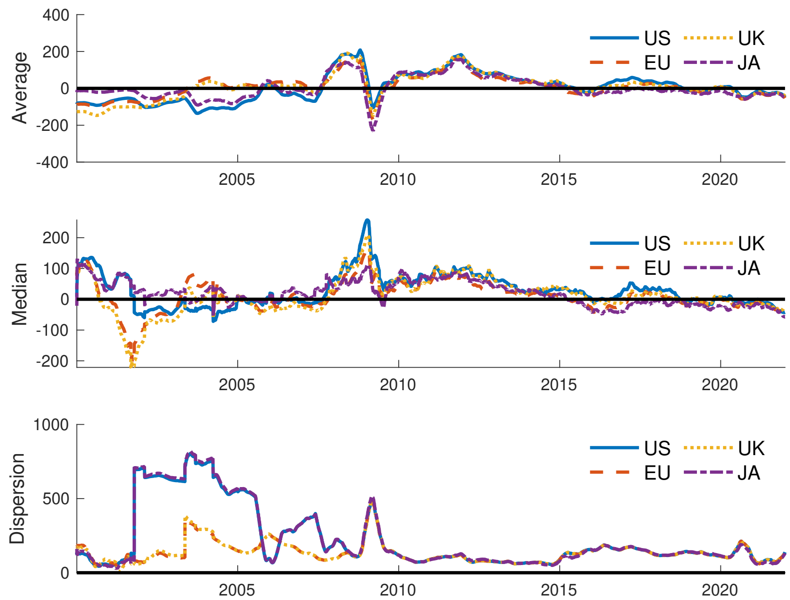
Table 1.1: Convenience yield in basis points

	2000-2021				**Pre GFC**				**GFC**				**Post GFC**			
Statistics	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA
Mean	25	22	30	9	49	54	85	3	66	19	41	23	32	15	27	16
25th	-23	-16	-9	-59	-45	-64	-74	-35	-46	-96	-66	-141	-30	-47	-35	-39
50th	16	7	14	6	3	-14	-23	19	132	58	81	48	11	-4	8	2
75th	52	35	42	54	133	66	103	95	193	175	189	165	51	35	47	35

Notes: US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. Relative convenience yield weighted mean, 25th percentile, 50th percentile, and 75th percentile are shown in basis points for sample periods from 2000 to 2021, Pre Global Financial Crisis, Global Financial Crisis (GFC), and Post Global Financial Crisis periods.

Empirical Observation 3d: There exists heterogeneity in the convenience yield of the safe assets to the EMEs.

Figure 1.5: Convenience yields distribution in basis points



Notes: US, EU, UK, and JA denote the United States, Euro area, United Kingdom, and Japan. Relative convenience yield for weighted average (representative) EME, median EME, and dispersion among EMEs are plotted from 2000 to 2021.

The relative convenience yield of the four safe asset providers for fifteen EMEs are measured. Section A.2.2 tabulates the convenience yields of the U.S., Euro area, U.K., and Japan safe assets for fifteen EMEs. The mean row of Table 1.1 shows the convenience yields of the U.S., Euro area, U.K., and Japan safe assets from 2000 to 2021 and during different sub-samples to a representative EME. Japan's safe asset provides the least convenience yield to the EMEs for the last two decades. In general, the convenience yield of the safe asset providers increased during the global financial crisis period. Figure 1.5 shows there is heterogeneity in the patterns of the convenience yields. The top and middle panels show the convenience yield of various safe assets to a representative and median EME respectively. The bottom panel shows the dispersion of the convenience yield of various safe assets among the EMEs. There is heterogeneity in the relative convenience yield provided by the U.S., Euro area, U.K., and Japan. The convenience yield is a potential channel for the sharp increase in the safe assets demand in the EMEs.

A safe asset exhibits safety and liquidity properties. The CIP deviations can be decomposed into liquidity and safety benefits. Section A.2.2 shows the decomposition of the CIP deviations into the safety and liquidity benefits of the safe asset. The relationship is summarized below.

$$CIPD = \underbrace{R^{d}_{ht} - R^{dh}_{it}}_{Liquidity} + \underbrace{p_{ht}(1 + R_{ht}) - p_{it}(1 + R_{it})}_{Safety}$$

Empirical Observation 3e: The safety and liquidity components of the safe asset varies.

Table 1.2: Liquidity and Safety Components

	2000-2021				**Pre GFC**				**GFC**				**Post GFC**			
Variable	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA
Liquidity	25	22	30	9	49	54	85	3	66	19	41	23	32	15	27	16
Safety	50	50	56	19	40	40	58	12	76	76	76	101	42	42	42	12
CIP deviation	75	72	86	28	89	94	143	15	142	95	117	124	74	57	69	28
Liquidity (% of CIPD)	33	31	35	32	55	57	59	20	46	20	35	19	43	26	39	57
Safety (% of CIPD)	67	69	65	68	45	43	41	80	54	80	65	81	57	74	61	43

Notes: US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. Safety benefit, liquidity benefit, and CIP deviation are shown in basis points for sample periods from 2000 to 2021, Pre Global Financial Crisis, Global Financial Crisis, and Post Global Financial Crisis periods for a representative EME.

The safety component of the CIP deviations captures the relative default risks of the EME government bonds and safe asset. Table 1.2 shows the safety component dominates over the liquidity component when measuring the covered interest rate parity deviations.

The convenience yield of various safe asset providers shows a downward trend for last decade. A forecast of the convenience yield helps to understand its possible dynamics. The forecasting of the convenience yield of the safe asset providers for a representative EME is conducted by the following time series model.

$$\Upsilon(L)(D(CY_t)) = \Omega(L)\xi_t$$

$$\Upsilon(L) = 1 - \mathit{\Upsilon}_1 L - ... - \mathit{\Upsilon}_p L^p$$

$$\Omega(L) = 1 - \mathit{\Omega}_1 L - ... - \mathit{\Omega}_p L^q$$

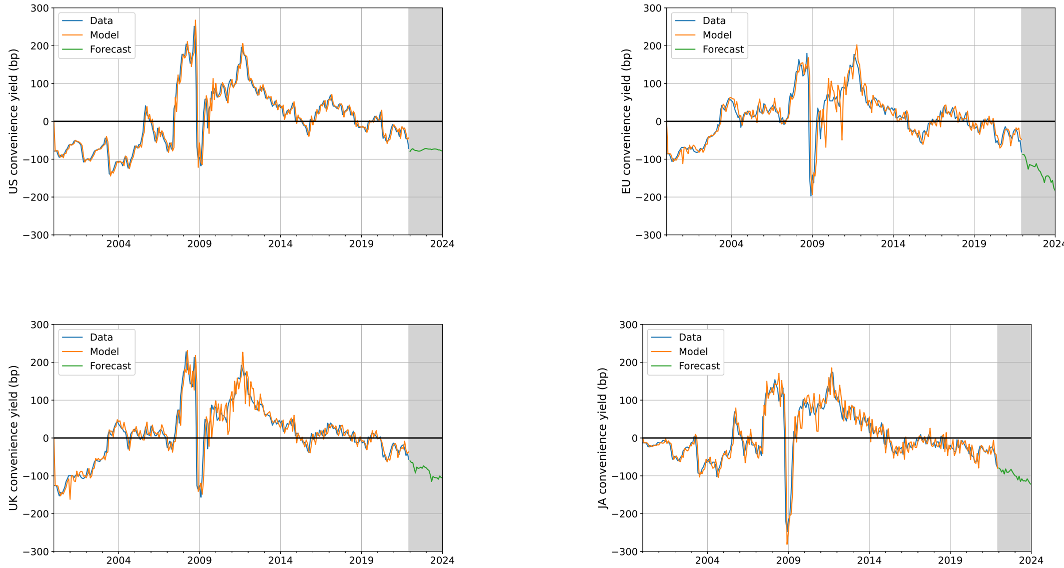
$$D(CY_t) = CY_t - CY_{t-d}$$

The hyper-parameters p, d, and q represent the auto regressive order, differencing, and moving average order respectively. The hyper-parameters to estimate the ARIMA model for forecasting each convenience yield time series are different. Section A.2.3 shows the algorithm for forecasting the time series of the convenience yield of the safe asset providers.

Empirical Observation 3f: The forecasting of the convenience yield of the safe asset providers for next few years is dissimilar as compared to the last decade trend.

Figure 1.6 shows the forecasting of the convenience yield of the safe asset providers for a representative EME. While the Euro area, U.K., and Japan safe assets show a sharp downward trend, the U.S. safe assets have no trend in their forecasts of the convenience yield.

Figure 1.6: Forecast of the convenience yield



Notes: US, EU, UK, and JA are the four international safe asset providers and stands for the United States, Euro area, United Kingdom, and Japan. Forecast is shown in shaded grey region.

1.3 Safe assets portfolio model

A simple analytical model is presented to characterize the portfolio of safe assets allocation for an investor in an EME. This is a partial equilibrium framework where the price process of the safe assets are known. This framework determines the optimal demand of the safe assets. Also, the model is used to estimate the liquidity benefit of the international safe asset after factoring in different types of risks.

1.3.1 Environment

Time is continuous. The economic system consists of a home country (EME) and N-1 international countries (AE). A country is represented by j. A government issued short term bond of a country, B_j, is considered as a safe asset. An investor in a home country holds both home and international safe assets as well as a home risky asset. The investor objective is to maximize their present discounted utility of consumption subject to budget and inequality constraints. The choice variables for the investor are the consumption and share of wealth invested in the home and international safe assets as well as a home risky asset.

1.3.2 Country characteristics

The heterogeneity among the countries is due to different rates of return on the short term government bonds, exchange rate of the home country against the international country, and liquidity costs incurred on the safe assets. Denote the nominal rate of return on a country j safe asset as R_j . The evolution of the price of safe assets in a country j follows:

$$\frac{dB_{jt}}{B_{jt}} = R_{jt}dt$$

The price of home risky asset, A_h, follows a geometric brownian motion with constant drift α_h and volatility of β_h. Z^A_{ht} is a Wiener process governing the price of home risky asset.

$$\frac{dA_{ht}}{A_{ht}} = \alpha_h dt + \beta_h dZ^A_{ht}$$

The exchange rate, E_{hj}, of home country h with respect to international country j also follows a geometric brownian motion with constant drift μ_{hj} and volatility of σ_{hj}. Z_{hjt} is a Wiener process governing the exchange rate dynamics.

$$\frac{dE_{hjt}}{E_{hjt}} = \mu_{hj}dt + \sigma_{hj}dZ_{hjt}$$

For example, $E_{IU} = \frac{INR}{Dollar}$ represents the exchange rate between Indian Rupee and U.S. Dollar. Thus, a positive (negative) drift implies depreciation (appreciation) of the Indian Rupee against the U.S. dollar. Thus, an investor in their home country h investing in country j safe asset will obtain the following return process from the investment.[10] This formulation makes all the investment in the safe assets implicitly converted into the home currency.

$$\frac{dB^h_{jt}}{B^h_{jt}} = \frac{dB_{jt}E_{hjt}}{B_{jt}E_{hjt}} = (R_{jt} + \mu_{hj})dt + \sigma_{hj}dZ_{hjt}$$

1.3.3 Representative investor in the home country

Let ρ denote the time preference rate of the investor. A representative investor of the home country h maximizes their present discounted value of all future utilities. The investor derives utility from the consumption goods, C^h_t. The following assumptions hold for the utility function, $U(C^h_t)$. The utility function is increasing in consumption goods and has diminishing marginal returns from the consumption good and exhibits Inada conditions for C_t.[11] CRRA utility function will be assumed to carry out the analysis and γ represents the constant risk aversion coefficient of an investor.[12]

Let Y^h_t represents the idiosyncratic income process of the investor and fol-

[10]This formulation handles the home safe asset investment where the $\mu_{hh} = 0$ and $\sigma_{hh} = 0$.
[11]$U_C(C_t) > 0$,$U_{CC}(C_t) < 0$, $\lim_{C_t \to 0} U_C(C_t) = \infty$, and $\lim_{C \to \infty} U_C(C_t) = 0$.
[12]CRRA utility function: $\frac{C^{1-\gamma}}{1-\gamma}$.

lows an Ornstein-Uhlenbeck process with long term mean μ_y, volatility σ_y, and reversion speed to mean χ. Z_{ht} is a Wiener process governing the income process dynamics.

$$dY_t^h = \mu(Y_t^h)dt + \sigma(Y_t^h)dZ_{ht} = \chi(\mu_y - Y_t^h)dt + \sigma_y dZ_{ht}$$

The wealth of the investor is represented by W_t^h. The investor can only save in the safe assets of any country or in a risky home asset. They hold N_{Bjt} number of safe assets of country j and N_{At} number of home risky assets. The share of wealth invested in the safe asset of country j is θ_{jt}^h and in the home risky asset is $\tilde{\theta}_{ht}^h$. An investment in a safe asset depends on its liquidity. Liquidity cost formuation is adopted in this set up rather than bond in utility function formulation following Valchev (2020) and Feenstra (1986). The investor pays a cost, due to the liquidity reasons, for their investment in the safe assets. The liquidity cost is assumed as a convex function of the total amount of the investment in the safe assets. L_t^h denotes the liquidity costs.[13]. An illiquid safe asset entails a higher proportional liquidity cost. Let η_s represents the financial state of the home country. The return and liquidity costs on the home safe asset as well as return and volatility of the home risky asset are a function of the state of the economy.[14]

So, the investor maximizes the present discounted value of all future utility flows by choosing the consumption as well as share of wealth invested in different assets and is subjected to budget and inequality constraints.

$$\max_{C_t^h, \tilde{\theta}_{ht}^h, \{\theta_{jt}^h\}_{j=1,\ldots,N}} \int_0^\infty e^{-\rho t} U(C_t^h)$$

[13]$L_t^h = L(\tilde{\phi}_{ht}^l N_{At} A_{ht} + \sum_{j=1}^N \phi_{jt}^l N_{Bjt} B_{jt}^h)$

[14]The parameters of different processes are function of η_s. To be precise, the rate of return on home safe asset is $R_h(\eta_s)$, the drift of the home risky asset is $\alpha_h(\eta_s)$, the volatility of the home risky asset is $\beta_h(\eta_s)$, and the liquidity costs on the home safe asset and home risky asset are $\phi_h^l(\eta_s)$ and $\tilde{\phi}_h^l(\eta_s)$ respectively.

subject to

$$dW_t^h = (Y_t^h - C_t^h - L_t^h)dt + N_{At}dA_{ht}^h + \sum_{j=1}^{N} N_{Bjt}dB_{jt}^h$$

$$W_t^h \geq 0$$

$$\tilde{\theta}_{ht}^h = \frac{N_{At}A_t^h}{W_t^h}, \theta_{jt}^h = \frac{N_{Bjt}B_{jt}^h}{W_t^h}$$

$$0 \leq \tilde{\theta}_{jt}^h \leq 1, 0 \leq \theta_{jt}^h \leq 1 \forall j$$

$$\tilde{\theta}_{ht}^h + \sum_{j=1}^{N} \theta_{jt}^h = 1$$

It is assumed that the exchange rates of home country against various international country are correlated. The correlation between the exchange rate of country j and exchange rate of country k is r_{jk}. Let Ψ^h denotes the covariance matrix of exchange rates for the home country. Ψ^h is a (N-1,N-1) matrix where its element $\psi_{jk}^h = cov(E_{hj}, E_{hk}) = r_{jk}\sigma_{hj}\sigma_{hk}$. Suppose the correlation between the exchange rate of country j and income process is given as r_{jY} and the correlation between the home risky asset and income process is given as r_{hY}.

The share vector, Θ^h is a N-1 column vector whose element gives the share of wealth invested in the safe asset of an international country j. The return vector, R^h is a N-1 column vector whose element gives return of the safe asset of an international country j. The drift vector, μ^h is a N-1 column vector whose element gives appreciation or depreciation of exchange rate of home country h with respect to an international country j. The liquidity cost vector, Φ^{lh} is a N-1 column vector whose element gives the marginal liquidity cost of an international safe asset. The income exchange rate correlation vector, r_Y^h is a N-1 column vector whose element gives the correlation between the income process and the exchange rate of an international country j.[15]

[15] $\Theta^h = [\theta_1^h, ..., \theta_{h-1}^h, \theta_{h+1}^h, ..., \theta_N^h]'$. Θ^h doesn't include the share of wealth invested in

Let λ represents the transition rate between the financial crisis and no financial crisis states. V represents the value function which depends on the wealth and income of the representative investor as well as the financial state of the economy. Section A.3.1 details the setting up of Hamilton-Jacobi-Bellman (HJB) equation and finding the optimal conditions. The HJB equation in a particular financial state of the home country is as follows.

$$\begin{aligned}\rho V(W^h, Y^h; \eta_s) = & U(C^h) \\ & + V_W(-C^h + (R_h(\eta_s) + (\alpha_h(\eta_s) - R_h(\eta_s))\tilde{\theta}_h^h \\ & + \sum_{j \neq h}(R_j - R_h(\eta_s) + \mu_{hj})\theta_j^h)W^h) \\ & - L((\phi_h^l(\eta_s) + \tilde{\phi}_h^l(\eta_s)\tilde{\theta}_h^h + \sum_{i \neq h}(\phi_j^l - \phi_h^l(\eta_s))\theta_j^h)W^h) \\ & + \frac{1}{2}V_{WW}(\beta_h(\eta_s))^2(\tilde{\theta}_h^h W^h)^2 + \frac{1}{2}V_{WW}\sum_j\sum_k \psi_{jk}^h\theta_j^h\theta_k^h(W^h)^2 \\ & + V_Y\mu(Y^h) + \frac{1}{2}V_{YY}\sigma^2(Y^h) \\ & + V_{WY}r_{hY}\beta_h\sigma(Y^h)\tilde{\theta}_h^h W^h + V_{WY}\sum_{j \neq h} r_{jY}\sigma_{jh}\sigma(Y^h)\theta_j^h W^h \\ & + \lambda(V(W^h, Y^h; \eta_{s'}) - V(W^h, Y^h; \eta_s))\end{aligned}$$

Denote V_W and V_{WW} as the partial first-derivative and second-derivate of the value function with respect to wealth respectively. Similarly, denote V_Y and V_{YY} as the partial first-derivative and second-derivate of the value function with respect to income respectively. Finally, denote V_{WY} as the cross derivative of the value function with respect to wealth and income. The optimal consumption and share of wealth invested in the safe asset of international countries are given as follows.

$$U_C(C^h) = V_W$$

its home country safe asset, θ_h^h. One can calculate $\theta_h^h = 1 - \tilde{\theta}_h^h - \sum_{i \neq h}\theta_i^h$. $R^h = [R_1, ..., R_{h-1}, R_{h+1}, ..., R_N]'$. R^h doesn't include the rate of return of the home country safe asset, R_h. $\mu^h = [\mu_{h1}, ..., \mu_{hh-1}, \mu_{hh+1}, ..., \mu_{hN}]'$. $\Phi^{lh} = [(\phi_1^l), ..., (\phi_{h-1}^l), (\phi_{h+1}^l), ..., (\phi_N^l)]'$. $r_Y^h = [r_{1Y}, ..., r_{h-1Y}, r_{h+1Y}, ..., r_{NY}]'$.

$$
\begin{aligned}
\tilde{\theta}_h^h =& \frac{-V_W}{V_{WW}W^h}\frac{(\alpha_h(\eta_s) - R_h(\eta_s))}{(\beta_h(\eta_s))^2} \\
&+ \frac{-V_W}{V_{WW}W^h}\frac{L'(\tilde{\phi}_h^l(\eta_s) - \phi_h^l(\eta_s))}{(\beta_h(\eta_s))^2} \\
&+ \frac{-V_{WY}}{V_{WW}W^h}\frac{r_{hY}\sigma(Y^h)}{\beta_h(\eta_s)} \\
\Theta^h =& \frac{-V_W}{V_{WW}W^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h) \\
&+ \frac{-V_W}{V_{WW}W^h}(\Psi^h)^{-1}L'(.)(\Phi^{lh} - \phi_h^l(\eta_s)) \\
&+ \frac{-V_{WY}}{V_{WW}W^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h))^{\frac{1}{2}}\sigma(Y^h)
\end{aligned}
$$

Augmented share vector, $\hat{\Theta}^h$ is a N column vector which includes the share of wealth invested in the safe asset of home country and international countries. Thus, the demand vector (D^h) for the safe assets by the representative investor is

$$D^h = W^h\hat{\Theta}^h$$

The wealth and income process of the representative investor changes due to the investment decision and the exogenous process. Let $g^h(W^h, Y^h)$ denote the joint distribution of the wealth and income for the representative investor. This distribution will change based on the financial state of the economy. Note $\int_0^\infty \int_0^\infty g^h(W^h, Y^h)dW^h dY^h = 1$. Thus, the following relation characterize the expected demand ($\bar{D^h}$) for the safe assets in home country h.

$$\bar{D^h} = \int_0^\infty \int_0^\infty g^h(W^h, Y^h)D^h dW^h dY^h$$

1.4 Results from the model

This section shows two main results implied by the framework. The liquidity cost is assumed to be a linear function for closed form solutions. The first result establishes the channels that drive demand for safe assets in a home country. The second result shows violation of the uncovered interest parity and provides the reasoning behind it.

1.4.1 Optimal demand of international safe assets

Let ϵ_{WC} and ϵ_{YC} represent the wealth elasticity of consumption and the income elasticity of consumption respectively.$\epsilon_{WC}(\epsilon_{YC})$ is the percentage change in the consumption of the investor from a one-percent increase in wealth (income).[16] The optimal demand of international safe assets is given as the sum of the speculative, the exchange rate driven, the liquidity driven, and the hedging demands. Section A.4.1 shows the closed form solution for the optimal demand of safe assets.

$$\Theta^h = \underbrace{\frac{1}{\gamma\epsilon^h_{WC}}(\Psi^h)^{-1}(R^h - R_h)}_{Speculative} + \underbrace{\frac{1}{\gamma\epsilon^h_{WC}}(\Psi^h)^{-1}(\mu^h)}_{Exchange-rate} + \underbrace{\frac{1}{\gamma\epsilon^h_{WC}}(\Psi^h)^{-1}(-(\Phi^{lh} - \phi^l_h))}_{Liquidity} + \underbrace{\frac{-\epsilon^{ha}_{YC}}{\epsilon^{ha}_{WC}Y^h}(\Psi^h)^{-1}r^h_Y(diag(\Psi^h))^{\frac{1}{2}}\sigma(Y^h)}_{Hedging}$$

Investor chooses an optimal demand vector of international safe assets (Θ^h). The right hand side of the above equation shows the reasons behind the demand decision. The first three terms are in the form of sharpe ratios. The first, second, and third terms encapsulate the role of the interest rate differential vector ($R^h - R_h$), the exchange rate fluctuation vector (μ^h) , and the liquidity premium vector ($\phi^l_h - \Phi^{lh}$) respectively. These three terms are controlled by the exchange rate volatility, risk-aversion, and wealth elasticity of consumption. The last term

[16] $\epsilon_{WC} = \frac{\frac{dC}{C}}{\frac{dW}{W}}$ and $\epsilon_{YC} = \frac{\frac{dC}{C}}{\frac{dY}{Y}}$. In general, the wealth and income elasticities of the investor are positive.

dictates the demand due to the variability of the income process.[17]

Proposition 1 : The optimal demand of an international safe asset by an investor in a home country increases with the

- *i) increase in the interest rate differential between the international and home safe asset,*
- *ii) depreciation of home country exchange rate, and*
- *iii) decrease in the liquidity costs differential between the international and home safe asset.*

and is ambiguous with the

- *i) increase in the risk aversion of the investor,*
- *ii) increase in the volatility of exchange rate between the home and international country, and*
- *iii) increase in the volatility of the income.*

Section A.4.2 shows the proof. The demand for the international safe assets increases when the financial return in the form of interest rate of the international government bond increases as compared to the home government bond. The depreciation of the home country exchange rate increases the valuation of the synthetic home currency international safe assets that cause an enhanced demand of the international safe assets. The increase in the liquidity of the

[17] $R_h(\eta_s)$ and $\phi^l_h(\eta_s)$ are function of financial state of η_s. R_h and ϕ^l_h represent the expected value of the return and the liquidity cost on the home safe asset. Therefore, R_h in this equation is very closely related to the default risk adjusted nominal interest rate of EME government bonds.

international safe asset decreases their relative liquidity cost, equivalent to increase in the liquidity premium, which leads to an increase in the demand for the international safe asset. Risk aversion and volatility of exchange rate affect the interest rate differential, exchange rate fluctuation, and liquidity premium in the same manner. Hence, the impact of an increase in risk aversion and the volatility of the exchange rate would depend on the cumulative effect of interest rate differential, exchange rate fluctuation, and liquidity premium. Finally, an increase in the income volatility would decrease (increase) the demand of a safe asset when the income process and exchange rate of that country co-move in the same (opposite) direction.

1.4.2 Uncovered interest rate parity condition

The interest rate differential between the home and international safe asset can be obtained by rearranging the optimal demand for safe assets formulation.

$$R_h - R^h = \underbrace{\mu^h}_{Exchangerate} + \underbrace{(-(\Phi^{lh} - \phi_h^l))}_{Liquidity} + \underbrace{-\Theta^h \gamma \epsilon_{WC}^h \Psi^h}_{Wealth} + \underbrace{\frac{-\epsilon_{YC}^h}{Y^h} \gamma r_Y^h (diag(\Psi^h))^{\frac{1}{2}} \sigma(Y^h)}_{Income}$$

The above equation is in vector formulation. The left hand side of the equation is the interest rate differential vector where the interest rate on home safe asset is R_h and interest rate vector of the international safe assets is R^h. The right hand side of the equation shows different types of premium that justify the interest rate differentials between the home and the international safe asset. While the vector of exchange rate (μ^h) and liquidity ($\Phi^{lh} - \phi_h^l$) premiums are solely influenced by macroeconomic factors, the vector of wealth and income premiums are due to the investor characteristics.

Corollary 1. The interest rate differential between the home and international safe asset is the sum of the

i) exchange rate premium,

ii) liquidity premium from the international safe asset,

iii) wealth premium from the international safe asset, and

iv) income premium from the international safe asset.

The proof is detailed in Section A.4.3. The interest rate differential between the government bond of a home and international country can be explained by conventional channel of depreciation or appreciation of the exchange rate and key channels of liquidity premium, wealth premium, and income premium. Uncovered interest parity condition is satisfied only when the last three terms are absent or cumulative sum of the last three terms is zero. An EME facing depreciation against an international safe asset provider will contribute positively towards the interest rate differentials. The liquidity cost of an international safe asset are lower than that of the home safe asset. Hence, the liquidity premium from the international safe assets relative to the home safe assets will play positively towards the interest rate differentials. A positive share of the international safe assets in the portfolio of an investor provides a wealth effect that impacts negatively the interest rate differentials. The income hedge premium shows up when the correlation between the income process and the exchange rate is negative and it adds to the interest rate differentials.

1.5 Quantitative inferences from testable predictions

The model generates the following testable predictions. First, proposition 1.4.1 shows the relationship between the shares of the safe asset and convenience yields of the safe asset in an EME. Second, corollary 1.4.2 suggests the channels for the interest rate differentials. Third, the rebalancing effect on different asset classes due to an impact of the financial crisis in an EME is analyzed.

1.5.1 Empirical justification of safe assets demand and convenience yields

Proposition 1.4.1 states an increase in the liquidity premium results in an increase in the share of the international safe asset. This is tested and verified in the data available.

Relationship between safe asset providers share and convenience yield

Panel regression is performed to understand the relationship between share and convenience yield of the safe assets. Data on these variables for fifteen EMEs and four major safe asset providers are obtained as outlined in 1.2.2 and 1.2.5. The following regression is run to examine the relationship between the share of the safe asset and the convenience yield of that safe asset.

$$S^h_{it} = \beta_i LB^h_{it} + \alpha_h + \delta_t + \Gamma_i c_{ht} + \epsilon_{ht}$$

S^h_{it} is the share of the i safe asset for the h EME. LB^h_{it} is the liquidity benefit of the i safe asset for the h EME.α_h and δ_t capture the EME and time fixed effect respectively. c_{ht} are the control variables which include the interest rate differentials of the EME and the safe asset provider and appreciation and depreciation of the exchange rate which are closely related to the proposition 1.4.1. β_i is the coefficient of interest that shows the relationship between the share and liquidity benefit of the safe asset.

Table 1.3: Relationship between the share and convenience yield of the safe asset

	US	EU	UK	JA
US CY	22.54* (12.78)			
EU CY		19.19* (10.61)		
UK CY			-4.69 (8.49)	
JA CY				1.50** (0.76)
Observation	212	212	212	212
R-squared	0.68	0.78	0.56	0.35
Controls	✓	✓	✓	✓
Country FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓

Notes: US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. Interest rate differentials and exchange rate fluctuations are the control variables. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

1.5.2 Explanation of the interest rate differentials

Brazil, Colombia, Hungary, and India are chosen as the EMEs for the quantitative exercise. The safe asset provider countries are the United States, Euro area, United Kingdom, and Japan.

The drift and volatility of the exchange rate of EMEs against the safe assets provider countries, the parameters of the price process of the EMEs risky assets, the return on the safe assets, and the parameters on the EMEs income process are estimated by the time series method. The model is solved using the finite difference value function iteration method. The policy functions of consumption and portfolio holdings of the home risky, the home safe, and the international safe assets along with the invariant distribution of investor's wealth and income are computed.

The liquidity cost parameters (ϕ_j^l) are estimated by the simulated method of moments (SMM) using EME's observations of the portfolio debt assets investment from the United States, Euro area, United Kingdom, and Japan for the period from 2000 to 2020. The moments used to estimate the liquidity cost parameters are the auto covariances, variances, and covariances of the portfolio investment from the major safe asset providers.

Parameters

Table 1.4 shows the parameter space of the model. Except the risk aversion coefficient, most of the parameters are estimated from the data available. The values for the parameters change for different EMEs. The range column of the table 1.4 provides the region of the parameter values for different EMEs.

Liquidity parameters

Table 1.5 provides the estimated average values of liquidity benefit parameters for various safe asset providers. These values show that liquidity benefit of a safe asset providers for different EMEs varies. The US short term government

Table 1.4: Parameters

Symbol	Name	Range	Source
γ	Risk aversion coefficient	2	Literature
ρ	Discount factor	5.58 - 10.45	Internal estimation
R_h	Return on home safe asset	5.38 - 10.25	Internal estimation
R^h	Vector of return on international safe assets	0.09 - 2.04	Internal estimation
μ^h	Vector of drift of exchange rate	2.31 - 9.31	Internal estimation
Ψ^hV	Covariance matrix of exchange rate	0.36 -1.52	Internal estimation
α	Return on home risky asset	7.38 - 12.25	Internal estimation
β	Risk on home risky asset	0.65 - 1.01	Internal estimation
$\sigma(Y^h)$	Standard deviation of the income process	0.03 - 0.28	Internal estimation
r_Y^h	Correlation of income and exchange rate process	-0.22 - 0.82	Internal estimation

Notes: Discount factor, return in home safe asset, vector of return on international safe assets, vector of drift of exchange rate, covariance matrix of exchange rate, return on home risky asset, and risk on home risky asset are in percentage.

bond provides liquidity benefits ranging from 48 to 212 basis points. These values also depict that an EME investor faces different liquidity benefits from different safe asset providers. A Brazilian investor obtains 212, 192, 213, and 170 basis points in the form of liquidity benefits from investing in the U.S., Euro area, U.K., and Japan safe assets respectively.

Table 1.5: Liquidity premium in basis points

	US	EU	UK	JA
BR	212	192	213	170
CO	48	26	37	7
HU	105	80	97	70
IN	110	103	97	87

Notes: BR, CO, HU, and IN denote Brazil, Colombia, Hungary, and India respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively.

Validation of the model calibration

Table 1.6 depicts how the model compares the average wealth held in different assets against data statistics. Qualitatively, the model does a decent job in matching the wealth shares in different asset classes. The model generally overestimates the wealth share in the home safe assets and underestimates the wealth share in the home risky assets. Section A.5.1 shows the time path comparison of different assets between actual data source and model generated data for Brazil, Colombia, Hungary, and India.

Table 1.6: Other moments

EME	Source	HRAW	HSAW	ISAW
BR	Model	7	65	28
	Data	6	56	38
CO	Model	4	79	16
	Data	20	46	35
HU	Model	3	79	17
	Data	17	57	26
IN	Model	15	48	37
	Data	35	46	19

Notes: BR, CO, HU, and IN denote Brazil, Colombia, Hungary, and India respectively. HRAW, HSAW, and ISAW stand for the share of wealth in the home risky asset, home safe asset, international safe assets respectively. All numbers are in percentage.

The role of liquidity premium of a safe asset in understanding the interest rate differentials between an EME and that safe asset is studied. A factor decomposition that shows the percentage contribution of each channels towards the interest rate differentials and variance decomposition that explains the variation of the interest rate differentials by various channels are analyzed.

Factor decomposition for the interest rate differentials

Result in the corollary 1.4.2 shows that the interest rate differentials is the sum of the exchange rate, liquidity, wealth, and income premiums. Table 1.7 shows the decomposition of 100 basis points in the interest rate differential into exchange rate premium, liquidity premium, wealth premium, and income premium channels. As an example, exchange rate premium contributes 86 basis points, liquidity premium contributes 24 basis points, income premium contributes -0.13 basis points, and income premium contributes -10 basis points towards the 100 basis points interest rate differential between Brazil and the United States.

Table 1.7: Basis Decomposition

EME	AE	IRD	EP	LP	WP	IP
BR	US	100	86.12	24.45	-0.13	-10.44
	EU	100	91.90	21.21	-0.12	-12.99
	UK	100	89.41	25.95	-0.13	-15.23
	JA	100	91.60	16.72	-0.11	-8.21
CO	US	100	82.42	10.94	-0.25	6.89
	EU	100	93.64	5.33	-0.21	1.24
	UK	100	88.82	9.48	-0.24	1.94
	JA	100	92.74	1.16	-0.17	6.27
HU	US	100	71.14	27.64	-0.26	1.48
	EU	100	78.77	19.15	-0.22	2.31
	UK	100	69.07	29.05	-0.25	2.13
	JA	100	86.87	13.22	-0.15	0.06
IN	US	100	78.56	21.74	-0.06	-0.24
	EU	100	83.92	18.99	-0.05	-2.86
	UK	100	77.80	21.15	-0.07	1.12
	JA	100	84.97	13.34	-0.05	1.74

Notes: BR, CO, HU, and IN denote Brazil, Colombia, Hungary, and India respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. IRD, EP, LP, WP, IP denote interest rate differential, exchange rate premium, liquidity premium, wealth premium, and income premium respectively.

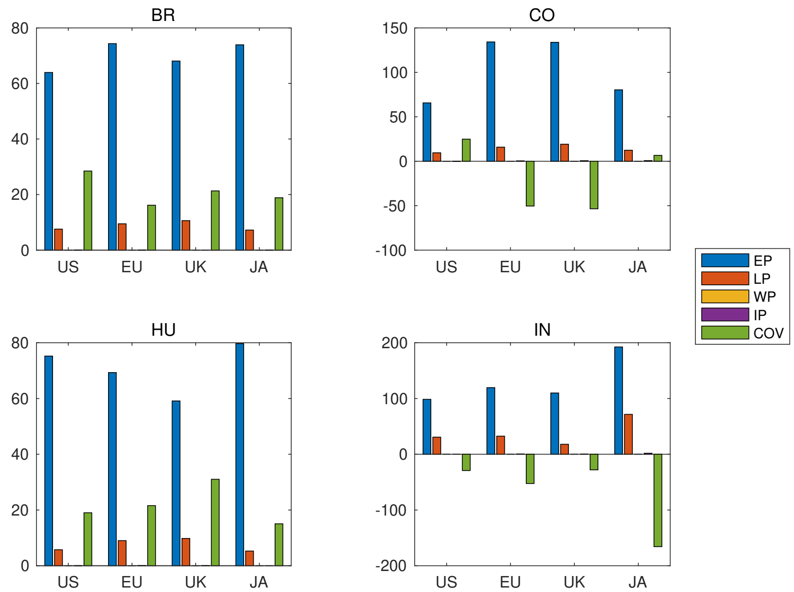
The fluctuations in the exchange rate majorly factor into the interest rate differentials. However, there is not a complete pass through of exchange rate fluctuations into the interest rate differentials. This reaffirms the CIP deviations as highlighted in the empirical observation 1.2.5.

Variance decomposition for the interest rate differentials

The variation in the interest rate differentials can be explained by the movements and co-movements of the different channels. Figure 1.7 shows the variance covariance contribution of the exchange rate premium, liquidity premium, wealth premium, and income premium channels towards the variance of interest rate differentials. For example, variance of exchange rate premium and liquidity premium contribute 64 and 7 percentage points respectively towards the variance of interest rate differentials between Brazil and the United States.

The combination of liquidity, wealth, and income premiums are measured as convenience yield in the data. Thus, empirical observation may be overestimating or underestimating the convenience yield. The estimation from the model not only factor in exchange rate and default risk as in empirical calculations but also factor in the risk premia, as measured by wealth premium and income premium, due to the investment decisions. The liquidity premium is thus an important factor for the covered interest parity condition which is shown by the above decompositions.

Figure 1.7: Variance Decomposition



Notes: BR, CO, HU, and IN denote Brazil, Colombia, Hungary, and India respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. EP, LP, WP, IP denote interest rate differential, exchange rate premium, liquidity premium, wealth premium, and income premium respectively. COV stands for the sum of the covariance terms between the exchange rate and the liquidity premiums, the exchange rate and the wealth premiums, the exchange rate and the income premiums, the liquidity and the wealth premiums, the liquidity and the income premiums, the wealth and the income premiums.

1.6 Behavior of an Indian investor during financial crisis

Estimating the model for India helps to understand the patterns of the assets holdings for an Indian investor during different financial conditions. Table 1.8 shows the financial wealth of the aggregate economy and its decomposition under different financial conditions of the economy for India. The financial wealth is decomposed into the home risky assets, the home safe assets, and the international safe assets. The international safe assets are further segregated into the four safe assets provider countries.[18] First, the percentage of home-based assets (risky and safe assets) is greater than 50% for the aggregate economy, irrespective of the financial conditions, confirming home bias hypothesis. Second, the holding of international safe assets increases as the financial crisis hits the home country. Third, the U.S. and the Euro area are the dominant safe asset providers for India. Fourth, there is heterogeneity in holdings across the international safe assets for different states of the economy. The reallocation of the wealth in different international safe assets varies with the financial condition of the economy.

Table 1.8: Financial Wealth

	Aggregate	State of the economy	
		No Financial Crisis	Financial Crisis
Home Risky Assets	15	18	9
Home Safe Assets	48	50	45
International Safe Assets	37	32	46
US	44	42	47
EU	54	57	49
UK	0	0	0
JA	2	1	4

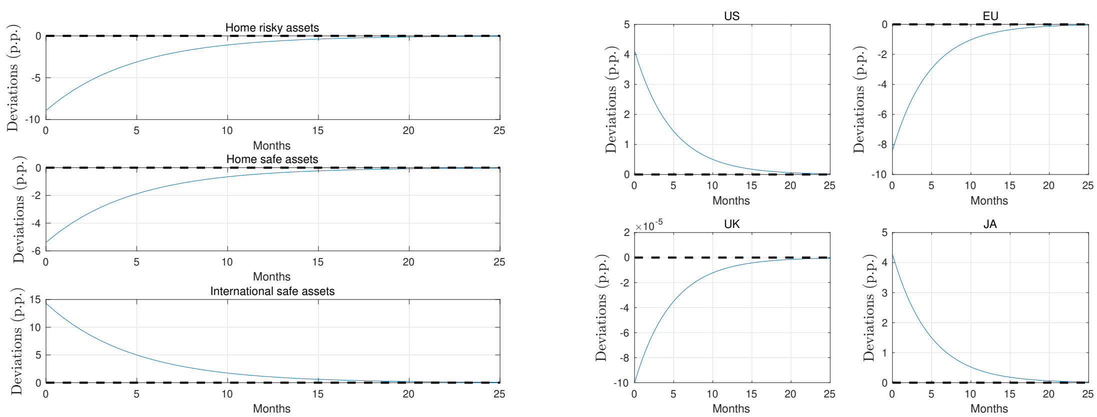
Notes: All the numbers are in percentages. US, EU, UK, and JA are the four international safe assets provider countries and stands for the United States, Euro area, the United Kingdom, and Japan.

A financial crisis causes a decrease in the return on the home safe asset and home risky asset as well as an increase in the volatility of the home risky asset. The financial crisis hits the Indian economy at period 0 and then gradually return back to its original state. This shock results in the portfolio rebalancing behavior of the Indian investor. Figure 1.8 shows the reallocation of wealth share between home assets and international safe assets as well as among the various

[18]The sum of the first, second, and third rows add up to one hundred as the total financial wealth is decomposed into home risky assets, home safe assets, and international safe assets. The sum of the last four rows add up to one hundred as there are four safe assets provider countries.

safe assets when the EME is hit by a financial crisis shock. The Indian investor not only assigns their wealth to international safe assets but also apportions their wealth diversely among various safe assets with the onset of financial crisis. The positive change in the wealth allocated to the U.S. safe assets supports the special status of the U.S. in the international financial system.

Figure 1.8: Impulse Response Function



Notes: All the numbers are in percentage points. US, EU, UK, and JA are the four international safe assets provider countries and stands for the United States, Euro area, United Kingdom, and Japan.

1.7 Conclusion

Investors of emerging market economies hold various safe assets in their portfolio. This research provides the characteristics of the convenience yield of the various safe asset providers for different EMEs. The convenience yield for an investor in an EME varies across the safe asset providers. Additionally, the convenience yields of the safe asset providers amplify during the time of financial crisis.

This paper builds a partial equilibrium open economy model with multiple countries to understand the demand for safe assets in an EME. The interest rate differential between the government bonds of an EME and an advanced economy is explained by conventional channel of exchange rate depreciation or appreciation and by key channels of liquidity premium, wealth premium, and income premium. Empirically, the convenience yields from different safe assets providers, such as the United States, Euro area, United Kingdom, and Japan, vary across the EMEs. Through the lens of the model, convenience yield is the second most important factor explaining the interest rate differential between an EME and an advanced economy and understanding the covered interest parity deviations for an EME.

The United States has a special status of dominant safe asset provider in the international monetary system. The dominance of the U.S. safe assets in the portfolio of EME investor is validated by the model, especially during a financial crisis. Ex-ante an EME investor has a menu of safe assets to choose when an EME is hit by financial crisis. Interestingly, the investor reallocates their wealth towards the U.S. safe assets.

This theory is crucial to understanding the share of different safe assets for an economy when the exchange rate, income, and price of various assets processes are known. It shows the macroeconomic and investment channels that are crucial to understand the dynamics of the interest parity mechanism. This is a starting point for promising future research work on an endogenous open economy framework to study the share of safe asset, interest rate, exchange rate, and convenience yield dynamics.

CHAPTER 2

A SIMPLE MODEL OF A CENTRAL BANK DIGITAL CURRENCY

2.1 Introduction

Many central banks around the world are considering or already experimenting with the issuance of retail central bank digital currencies (CBDCs), digital versions of their fiat currencies. The Bahamas, the Eastern Caribbean Currency Union, and Nigeria have already introduced CBDCs. CBDC experiments and pilots are underway in many other economies including China, India, Japan, and Sweden.[1] This concept has gained traction as the use of physical cash (currency banknotes and coins) is declining around the world with digital payment technologies gaining prevalence. There are a few major economies such as Japan, Switzerland, and the United States, where cash remains an important (although increasingly less important) payment mechanism for low-value retail transactions. But in most other advanced and emerging market economies, the proliferation of innovative low-cost retail payment systems is rapidly displacing cash.

A retail CBDC would in principle be available to all agents in an economy, as distinct from the electronic balances (reserves) that financial institutions have access to at central banks. The principal motivations for central banks to consider issuing CBDC appear to be two-fold. In some developing economies, a key

[1]For updates on the status of CBDC trials and implementation around the world, please see the following websites: https://www.atlanticcouncil.org/cbdctracker/ and https://kiffmeister.com/category/special/. For evidence on the declining use of cash (and its continued prevalence in some major economies, see Prasad (2021). For a discussion of design issues concerning validation, finality of settlement, anonymity, and security of CBDC transactions, see Allen et al. (2020).

objective is to increase financial inclusion by giving households easy and low-cost access to an electronic form of payment. In other countries, the objective is to create a central bank-managed backstop to a payment system and infrastructure managed entirely by the private sector, which could be vulnerable to confidence shocks.

Our focus in this paper is on account-based (also referred to as register-based) retail CBDC, which could be backed up by a payment technology that provides instantaneous and low-cost settlement. CBDC that can be held in digital wallets that, in effect, serve as CBDC accounts are becoming the norm in CBDC experiments and proofs-of-concept that are being undertaken in different countries. Cash and CBDC share some similar features–they would both remain liabilities of the central bank but could be distributed and circulated by commercial banks and other payment providers. Rogoff (2017) highlights the differences between cash and CBDC. He argues that a CBDC makes it easier to limit the use of central bank money for illicit activities and to relax the zero lower bound constraint on nominal interest rates. The emergence of CBDC could also have implications for financial inclusion, financial stability, and monetary policy implementation and transmission.

We develop a framework that highlights the economic trade-offs between cash and CBDC. In our model, agents have access to a broad set of assets that they can use for intertemporal consumption smoothing: cash, CBDC, government bonds, and physical capital. The first two are necessary for consumption transactions, i.e., they serve as mediums of exchange. Our model attempts to capture the key attributes that differentiate the two forms of retail central bank money. We model CBDC as providing greater transaction efficiency (lower transaction costs) relative to cash. However, using cash makes it possible to avoid taxes; by contrast, CBDC holdings used for purchases of consumption goods are subject to a tax. Cash provides a zero nominal rate of return while CBDC can have a variable nominal interest rate that can be either positive or negative.

Our model establishes conditions under which the two types of currency issued by a central bank can co-exist and indicates how government policies can affect their relative attractiveness. We show that the share of CBDC in total assets is positively related to the rate of return on CBDC, the relative transaction efficiency of CBDC, degree of monitoring of transactions, and penalties for tax evasion. We also examine the differential sensitivity of the CBDC share to changes in various policy parameters. Our model suggests that the highest degree of sensitivity of the CBDC share is to the nominal rate of return on CBDC. Additionally, we evaluate the welfare implications of different settings of the policy parameters. Increases in the transaction efficiency and rate of return on CBDC generate significant welfare gains.

One of the key attractions of a CBDC is that it allows the central bank, when facing a situation of economic or financial crisis, to impose a negative nominal interest rate on outside money, rather than being constrained by the zero bound on nominal interest rates. In our model, CBDC demand does not collapse to zero even with a negative nominal interest rate because CBDC serves as a more efficient medium of exchange than cash. We also find that negative and positive rates of return on CBDC result in asymmetric effects on allocations across different assets. We show how the probability of helicopter drops of money (as a counter-cyclical policy tool in the face of a deep recession), which in our model amount to direct transfers into CBDC accounts rather than money-financed fiscal stimulus, can influence the holdings of CBDC relative to other assets. In another extension, we show how the government can encourage CBDC holdings by imposing a tax on cash holdings above a predetermined threshold, a proxy for various policies adopted by countries such as Greece and India to limit the transaction use of cash for tax evasion.

A key concern about a retail CBDC is that it could disintermediate the banking system through capital flight away from bank deposits into CBDC, thereby precipitating financial instability. Our model does not incorporate bank deposits but we show how the rate of return on CBDC can be structured ex-ante to avoid flight from other assets such as bonds, essentially by setting a fee on CBDC holdings above a threshold level.

One question raised by our analysis is why, if a CBDC has lower transaction costs and lower levels of tax evasion than cash, the government would not simply eliminate cash. Even proponents of CBDC note that there are privacy concerns and other political considerations that make it infeasible or, at a minimum, unwise for a government to do away with cash altogether. For instance, in a recent report the BIS lays out a “foundational principle" that a CBDC would need to co-exist with and complement existing forms of money (BIS, 2020). There are also concerns that elimination of cash could disadvantage the poor, who are more prone to exclusion from the formal financial system and have lower access to electronic means of payment. Our analysis could be viewed as showing how the government could, through its policies, endogenously reduce consumers' preference for cash rather than doing this through a directive. The analysis also provides a quantitative assessment of how different factors could affect the evolution of the relative shares of cash and CBDC in an economy.

Our paper abstracts from issues of competition between fiat currencies and private decentralized cryptocurrencies such as Bitcoin. However, by shedding light on the relative importance of a currency's attributes in multiple dimensions, the framework that we develop in the paper has the potential to be extended to study the co-existence of multiple forms of physical and electronic currencies, both official and private.[2]

2.1.1 Related literature

Digital central bank money has already existed for a long time. Electronic balances held by commercial banks (and certain other financial institutions) at central banks, referred to as reserves, facilitate payments and settlement through interbank payment systems managed by the central bank. Kumhof and Noone (2021) distinguish CBDC from reserves and cash by defining it as electronic central bank money that (i) can be accessed more broadly than reserves, (ii) has functionality for retail transactions, (iii) can be interest bearing, and (iv) has a separate operational structure relative to other forms of central bank money. Yao (2018), former head of the Institute of Digital Money at the People's Bank of China, refers to a CBDC as "a credit-based currency in terms of value, a cryptocurrency from a technical perspective, an algorithm-based currency in terms of implementation, and a smart currency in application scenarios." Auer, Cornelli, and Frost (2020) and Lovejoy et al. (2022) assess the drivers, technical designs, and architecture of different variants of CBDC.[3]

[2]Auer and Claessens (2018, 2021) discuss cryptocurrency regulation.

[3]For further discussion, see BIS (2018), Brunnermier, James, and Landau (2019), and Allen et al. (2020). Gnan and Masciandaro (2018) and Keister and Sanches (2023) discuss the rationale

A number of papers have modeled specific differences between cash and CBDC. Agur, Ari, and Dell'Ariccia (2022) model the difference between cash and CBDC as hinging on two features–anonymity and security. Garratt and Lee (2021), Garratt and van Oordt (2021) and Ahnert, Hoffman, and Monnet (2022) discuss the implications of digital payments for privacy. Our contribution to this literature is to incorporate other features that distinguish CBDC from cash, and also to illustrate some design features that are under discussion in ongoing CBDC experiments.

Another topic that has received attention is how a CBDC can be positioned within an array of other payment options and the factors that determine the relative demand for each option. A useful framework for this exercise is provided by Kahn and Roberds (2009), who highlight the essential function of payments and provide a taxonomy of alternative forms of payments, as well as their financial and macroeconomic implications. Bijlsma et al. (2021) and Li (2022) argue that the adoption rate of CBDC increases with the interest rate it bears, the extent of public knowledge about it, and the degree of trust in the banking system.

An important concern is whether a CBDC would disintermediate the banking system. In early contributions, Bjerg (2017) and Bordo and Levin (2017) discuss alternative designs for a CBDC in terms of whether or not it would be interest-bearing and be complementary to or directly compete with bank deposits. Andolfatto (2021) studies the implications of CBDC in an overlapping generations model with a monopolistic banking sector. In this model, the introduction of interest-bearing CBDC increases the market deposit rate, leads to an expansion of the deposit base, and reduces bank profits. However, the CBDC

for a CBDC.

has no effect in terms of bank lending activity and lending rates. Chiu et al. (2022) reach a similar conclusion.

By contrast, Whited, Wu, and Xiao (2022), using a model with frictions that bind deposits and lending, argue that introduction of a CBDC could modestly reduce bank lending. The effect is mitigated by the ability of banks to replace deposits with wholesale funding. In a different setting, Nyffenegger (2022) also finds a modest degree of bank disintermediation when a CBDC is introduced. Fernandez-Villaverde et al. (2020) show how, in an economy with CBDC, depositors can internalize the relative stability of the central bank relative to commercial banks, leading to the central bank becoming a deposit monopolist even in normal times. Williamson (2022) argues that a CBDC can increase welfare by shifting safe assets from the private banking sector to what is effectively a narrow banking facility, which allows for more efficient use of the aggregate stock of safe collateral.

Recent papers have begun to grapple with the implications of CBDC for financial markets and monetary policy.[4] Some authors argue that a CBDC will not materially affect the implementation of monetary policy, although there could be other macroeconomic effects. The conclusions depend on the model structure and the manner in which the CBDC is introduced into the economy. Barrdear and Kumhof (2022) develop a DSGE model with multiple sectors and several nominal and real rigidities. In a model calibrated to U.S. data, they find that infusing CBDC on the order of 30 percent of GDP through central bank purchases of government bonds could result in substantial steady state output gains of nearly 3 percent due to reductions in real interest rates, distortionary taxes, and

[4]Cong and Mayer (2022) and Ferrari Minesso, Mehl, and Stracca (2022) discuss the cross-border implications of retail CBDC introduction.

monetary transaction costs. Bordo and Levin (2017) conclude that a CBDC could bolster the effectiveness of monetary policy and enhance the stability of the financial system. Davoodalhosseini (2022) finds that an interest-bearing CBDC environment can help the central bank attain a more efficient allocation than with cash. Our contribution to this literature is to explicitly model how the government and central bank can affect the choice of cash versus CBDC and what the welfare implications are.

The literature on CBDC of course draws on an extensive literature about models of money. Early approaches include money in the utility function (Sidrauski, 1967), cash-in advance models (Svensson, 1985), shopping-time models (Brock, 1990), and the turnpike model of spatially separated agents (Townsend, 1980). Search-theoretic models of money pioneered by Kiyotaki and Wright (1993) represent a major step forward in this literature. Kocherlakota (1998) highlights the role played by money in environments with incomplete information and limited commitment.

2.2 Model

In this section, we sketch the main features of our model. We focus the exposition on the main differences between cash and CBDC, and show how their relative desirability is affected by key features of the environment, such as relative transaction costs, and also government policy variables such as tax rates, the probability of detecting tax evasion, and the penalty for being detected evading taxes.

2.2.1 Environment

For ease of exposition, we show the representative agent's maximization problem entirely in real terms. The representative agent derives utility from private consumption goods (c_t) and public consumption goods (c_t^g). The utility functions for the private and public consumption goods are assumed to be additive separable; both functions are assumed to be strictly increasing, strictly concave, and satisfy the Inada conditions. CRRA utility function satisfies the following properties and will be used in our analysis. β is the subjective discount rate ($0 < \beta < 1$). The representative agent maximizes the present discounted value of utility by choosing private consumption goods (c_t), public consumption goods (c_t^g), physical capital (k_t), and holdings of bonds (b_t), cash (ca_t), and CBDC (dc_t).

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)] \tag{2.1}$$

subject to

$$\begin{aligned} c_t + c_t^g + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{2.2}$$

$$k_t - (1-\delta)k_{t-1} \leq i_t \tag{2.3}$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{2.4}$$

In the budget constraint (equation (2.2)), the LHS denotes the uses of funds while the RHS shows the sources of funds.[5] The available funds are used to purchase private and public consumption goods, undertake investment, purchase bonds, bear transaction costs, pay a penalty for tax evasion, hold cash

[5]Appendix B.1 shows the derivation of the real budget constraint.

and central bank digital currency, and pay taxes. The terms τ and m denote the constant tax rate and probability of monitoring due to tax evasion, respectively. π_t denotes the inflation rate.[6] It is assumed that the relative prices of public consumption goods to private consumption goods and of investment to private consumption goods are both one. Note that the subscript *t-1* denotes beginning of period real stocks of assets and the subscript t denotes end of period stocks.

Transaction costs are a function of consumption goods, cash, and CBDC. The properties of the transaction cost function are as follows:[7]

(i) $\phi \geq 0$, $\phi(0, ca, dc) = 0$

(ii) $\phi_c \geq 0$, $\phi_{ca} \leq 0$, $\phi_{dc} \leq 0$

(iii) $\phi_{cc} \geq 0$, $\phi_{caca} \geq 0$, $\phi_{dcdc} \geq 0$, $\phi_{cac} \leq 0$, $\phi_{dcc} \leq 0$

Cash has a higher transaction cost than CBDC, a zero nominal rate of return, and allows for tax evasion.[8] CBDC has a lower transaction cost than cash, could pay interest (which can be positive or negative), and transactions using it are subject to taxation. The agent pays a penalty on tax evasion with a constant probability of being monitored. Thus, the penalty function depends on real cash balances. The penalty function, denoted by $\psi(\frac{ca_{t-1}}{1+\pi_t})$, is assumed to be strictly increasing.[9]

[6] $(1 + \pi_t) = \frac{P_t}{P_{t-1}}$, where P_t is the price level in period t.

[7] Feenstra (1986) has a similar set of assumptions on the transaction cost function. Fried and Howitt (1983) propose an early model in which both government bonds and cash provide liquidity functions. Our model can be thought of as introducing an additional asset that has both these features while bonds cannot be directly used for transactions.

[8] Alternatively, we could model cash as providing the benefit of privacy rather than facilitating tax evasion, although this would require us to include the utility benefits from privacy in the model.

[9] We maintain the generality of the model at this stage and describe the equilibrium and steady state conditions before specifying particular functional forms for the utility, production, transaction cost, and tax evasion penalty functions in Section 2.2.4.

The source of funds in a given period is given by the sum of real income from output, interest payments accrued from bond holdings, interest payments (if any) from CBDC, and cash holdings from the previous period. Output in the economy equals $f(k_{t-1})$. Physical capital is the only factor of production; the production function $f(k_{t-1})$ is strictly increasing, strictly concave, and satisfies the Inada conditions. I_t^b and I_t^d denote the gross nominal interest rates on bond holdings and on CBDC, respectively.[10]

Equation (2.3) represents the evolution of the stock of physical capital, with δ being the depreciation rate. Equation (2.4) denotes a liquidity constraint–the purchase of private consumption goods cannot exceed the amount of cash holdings and CBDC holdings at the beginning of the period.[11] The state variables are physical capital (k_{t-1}), bond holdings (b_{t-1}), real cash balances (ca_{t-1}), and real CBDC (dc_{t-1}).[12]

The government consumes g_t units of output each period and produces η units of public goods per unit consumed.[13] Thus, equation (2.5) represents the government provision of public consumption goods.

$$c_t^g = \eta g_t \tag{2.5}$$

[10] $I^b = R^b\Pi$, $I^d = R^d\Pi$, and $\Pi = (1 + \pi)$. R^b, R^d represent the gross real interest rates on bonds and CBDC. $I^b = 1 + i^b$, $I^d = 1 + i^d$, $R^b = 1 + r^b$, and $R^d = 1 + r^d$, where i^b, i^d, r^b, and r^d represent net nominal interest rate on bond holdings, net nominal interest rate on CBDC, net real interest rate on bond holdings, and net real interest rate on CBDC, respectively.

[11] Appendix B.1 shows the derivation of the real liquidity constraint. The liquidity constraint does not bind in equilibrium, implying that agents always have sufficient real balances in the form of cash and CBDC to obtain consumption goods. The liquidity constraint and the transaction function together rule out the possibility of bonds serving as a medium of exchange.

[12] An endowment economy with no physical capital would be a special case of our model (see section 2.4.7). We incorporate physical capital to have a more generalized model that includes a relatively safe asset (bonds) and a riskier one (physical capital), although we do not explicitly model the risky return from physical capital in the present paper.

[13] Setting the parameter $\eta = 1$ implies that the government does not waste any resources. Thus, $g_t - c_t^g$ represents the administrative costs of running the government that do not benefit the representative agent.

The consolidated government budget constraint is given by equation (2.6). The LHS and RHS represent the government's expenditure and income, respectively. The government spends on own expenditure net of provision of public goods and pays returns on bond and CBDC holdings. The government obtains revenue from issuing bonds, seigniorage from issuing CBDC and cash (both of which have a zero cost of issuance), taxation on consumption transactions using CBDC, and penalties collected from tax evaders.

$$\begin{aligned} &g_t - c_t^g + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \\ &= b_t + dc_t + ca_t + \frac{\tau}{1+\pi_t} dc_{t-1} + m\psi\left(\frac{ca_{t-1}}{1+\pi_t}\right) \end{aligned} \tag{2.6}$$

The monetary authority controls the circulation of money supply (sum of cash and CBDC). We assume that the nominal stock of cash and CBDC grows at an exogenous and constant rate μ. We use upper case variables CA_t, DC_t, and P_t to denote nominal cash, nominal CBDC, and the price level, respectively.

$$CA_t + DC_t = (1+\mu)(CA_{t-1} + DC_{t-1}) \tag{2.7}$$

The real stock of money supply grows at a rate given by μ minus the rate of inflation.

$$ca_t + dc_t = \frac{1+\mu}{1+\pi_t}(ca_{t-1} + dc_{t-1}) \tag{2.8}$$

The change in real money supply, given by equation (2.9), depends on the inflation rate.

$$\begin{aligned} \Delta(ca_t + dc_t) = \Delta\left(\frac{CA_t + DC_t}{P_t}\right) &= \frac{CA_t + DC_t}{P_t} - \frac{CA_{t-1} + DC_{t-1}}{P_{t-1}} \\ &= \frac{\mu - \pi_t}{1+\pi_t}(ca_{t-1} + dc_{t-1}) \end{aligned} \tag{2.9}$$

2.2.2 Equilibrium

Solving the optimization problem by using the Bellman equation yields the following inter-temporal conditions. The detailed solution using the dynamic optimization method is shown in Appendix B.2.

$$\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = v_{c^g}(c_t^g) \tag{2.10}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}(f_k(k_t)+1-\delta) \tag{2.11}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} = \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}\frac{I_t^b}{1+\pi_{t+1}} \tag{2.12}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \\ & (\frac{1}{1+\pi_{t+1}} - \frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} - \frac{\phi_{ca}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{2.13}$$

$$\begin{aligned}\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \\ & (\frac{I_t^d}{1+\pi_{t+1}} - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}})\end{aligned} \tag{2.14}$$

The optimal paths of c_t, c_t^g, b_t,k_t, ca_t, and dc_t are characterized by equations (2.10)-(2.14), the budget constraint of the representative agent by equations (2.2)-(2.4), the provision of public consumption goods by equation (2.5), the government budget constraint by equation (2.6), and the change in real cash balances by equation (2.9).[14] Equation (2.10) represents the intra-temporal relationship between private and public consumption goods. Equations (2.11)-(2.14) represent the Euler equations for capital, bonds, cash, and CBDC, respectively.

[14]The liquidity constraint (equation(2.4)) doesn't bind in equilibrium. Thus, the Lagrange multiplier associated with this inequality is zero according to the complementary slackness condition.

2.2.3 Steady state

The key equations that characterize the steady state of the economy are as follows:

$$\frac{u_c(\bar{c})}{1+\phi_c(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})} = v_{c^g}(\bar{c^g}) \tag{2.15}$$

$$1 = \beta(f_k(\bar{k}) + 1 - \delta) \tag{2.16}$$

$$1 = \beta\frac{\bar{I}^b}{1+\bar{\pi}} \tag{2.17}$$

$$1 = \beta(\frac{1}{1+\bar{\pi}} - \frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}} - \frac{\phi_{ca}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \tag{2.18}$$

$$1 = \beta(\frac{\bar{I}^d}{1+\bar{\pi}} - \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \tag{2.19}$$

$$\begin{aligned}\bar{c} + \bar{c^g} + \delta\bar{k} + \bar{b} + \phi(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}}) + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) + \bar{ca} + \bar{dc} + \frac{\tau}{1+\bar{\pi}}\bar{dc} \\ = f(\bar{k}) + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}}\end{aligned} \tag{2.20}$$

$$(\bar{g} - \bar{c^g}) + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} = \bar{b} + \bar{dc} + \bar{ca} + \frac{\tau}{1+\bar{\pi}}\bar{dc} + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) \tag{2.21}$$

$$\bar{c^g} = \eta\bar{g} \tag{2.22}$$

$$0 = \Delta(\bar{ca} + \bar{dc}) = \frac{\mu - \bar{\pi}}{1+\bar{\pi}}(\bar{ca} + \bar{dc}) \tag{2.23}$$

$\bar{k}$, $\bar{c}$, $\bar{g}$, $\bar{c^g}$, $\bar{ca}$, $\bar{dc}$, $\bar{b}$, $\bar{I}^b$, and $\bar{\pi}$ are determined from the above steady-state conditions.

2.2.4 Analysis

We assume the following functional forms for the utility function, the production function, the transaction cost function, and the tax evasion penalty function.

$$u(c) = \frac{c^{1-\epsilon_c}}{1-\epsilon_c} \tag{2.24}$$

$$v(c^g) = \frac{(c^g)^{1-\epsilon_g}}{1-\epsilon_g} \tag{2.25}$$

$$f(k) = Ak^{\alpha} \tag{2.26}$$

where $0 < \alpha < 1$ and A is productivity.

$$\phi(c, ca, dc) = \theta_1 \frac{c^a}{ca^{\gamma} dc^{\rho}} \tag{2.27}$$

where $a > 1$, $\gamma > 0$, $\rho > 0$, and $\gamma < \rho$

$$\psi(ca) = \theta_2 ca^{\nu} \tag{2.28}$$

where $\theta_2 > 1$ and $\nu > 1$

We have kept the structure of these functions simple to the extent possible and such that they meet the general conditions on each of the functions mentioned in Section 2.2.1. The transaction cost function allows for an interior solution with cash and CBDC co-existing as mediums of exchange.[15] The structure of this function allows us to easily evaluate the effects of changes in the relative transaction efficiency of cash and CBDC. We also incorporate a non-negativity

[15]This is in the spirit of what central banks are considering, i.e., the co-existence of cash and CBDC, at least for the foreseeable future.

constraint on bond holdings; this constraint is not binding for most of our parameter settings (we discuss some exceptions to this later in the paper).

The detailed solution for the steady state is provided in Appendix B.3. Solving for the above functional form yields the following for $\bar{k}$, $\bar{c}$, $\bar{g}$, $\bar{c^g}$, $\bar{ca}$, $\bar{dc}$, $\bar{b}$, $\bar{I^b}$,and $\bar{\pi}$. The steady state value of physical capital is given by equation (2.29).

$$\bar{k} = (\frac{A\alpha\beta}{1-\beta(1-\delta)})^{\frac{1}{1-\alpha}} = \Theta_k(A,\alpha,\beta,\delta) \tag{2.29}$$

The inflation rate in steady state is derived from the condition of no change in real money supply in the economy (equation (2.9)). Hence, the inflation rate is equal to the growth rate of cash and CBDC.

$$\bar{\pi} = \mu = \Theta_\pi(\mu) \tag{2.30}$$

The steady state value of the gross nominal interest rate of bonds is given by equation (2.31).

$$\bar{I^b} = \frac{1+\bar{\pi}}{\beta} = \frac{1+\mu}{\beta} = \Theta_{I^b}(\beta,\mu) \tag{2.31}$$

The steady state levels of private and public consumption goods and private consumption goods are obtained by substituting equation (2.21) into equation (2.20).

$$\bar{c} + \bar{g} + \phi(\bar{c},\bar{ca},\bar{dc}) = f(\bar{k}) - \delta\bar{k} = \bar{k}^\alpha - \delta\bar{k} = \Theta_3(A,\alpha,\beta,\delta) \tag{2.32}$$

The government sets the nominal interest rate of CBDC in steady state. Using the intra-temporal relationship between private and government public consumption goods, the inter-temporal equation for cash holdings, and the inter-temporal equation for CBDC holdings, one can determine the levels of cash, CBDC, private consumption goods, public consumption goods, and government expenditure as functions of the model primitives. The level of bond holdings in the steady state is determined from the government's consolidated budget constraint.

2.3 Results

We now conduct five types of experiments using the model. First, we evaluate the impacts of changes in specific policy parameters. Second, we present alternative baseline scenarios as a robustness test for our main conclusions. Third, we analyze the effects of varying a particular parameter over a range of possible values. Fourth, we provide a welfare comparison of alternative policy choices. Fifth, we explore the effects of imposing a negative nominal interest rate on CBDC. These are all meant to be illustrative exercises and, since there are no major economies operating with retail CBDC, at this stage we do not attempt to match any specific data moments.

The results of the quantitative exercises we undertake are described in terms of the composition of asset holdings of the representative agent. These assets are composed of physical capital and holdings of bonds, cash, and CBDC. The value of total assets in steady state ($\bar{f}a$) can be defined as the sum of the steady state values of these four assets.

$$\bar{f}a = \bar{k} + \bar{b} + \bar{c}a + \bar{d}c$$

We examine the share of each asset in total assets in the steady states implied by different parameter settings. For instance, the share of CBDC in total assets is given by

$$\bar{dc}^{share} = \frac{\bar{dc}}{\bar{fa}}$$

The parameters of the model influence the steady state values of physical capital, private consumption goods, public consumption goods, cash holdings, CBDC holdings, bond holdings, the rate of return on bonds, and the inflation rate. As our main objective is to understand the trade-offs between holdings of cash and CBDC, we restrict our analysis to parameters that influence these holdings. These parameters are γ, ρ, m, θ_2, τ, and $\bar{I}^d$. The influence of these parameters on the choice variables is determined by the inter-temporal equations for cash holdings and CBDC holdings, the consolidated budget constraint, and the intra-temporal equation for private and public consumption goods. Note that these parameters have no effect on the steady state values of capital, the nominal rate of return on bonds, and the inflation rate.

The parameter settings for the baseline model (BM) for determining the steady state are shown in Table 2.1. We adopt conventional values for standard parameters and, where feasible, rely on empirical observations for others. For instance, the tax rate parameter is set at 0.08 based on the average state sales tax in the U.S., which is roughly 8 percent. As noted above, our objective is to conduct some illustrative exercises rather than match any specific data moments. Hence, for the remaining parameters for which there are no clear analogues in the data (as no economy has a full-fledged CBDC as yet), we pick arbitrary values and then attempt to provide some economic intuition about how varying those values affects the dynamics of the model.[16] The time period in our

[16]Some papers contain indirect estimates of the relative efficiency of digital payments and the demand for CBDC but these are difficult to map into our model. Biljsma et al. (2021), Li (2022), and Whited, Wu, and Xiao (2022) contain estimates of the digital premium and the demand for CBDC. These are not easy to compare even relative to each other and are sensitive to the

model is equivalent to one year and we pick parameters corresponding to this frequency.

Table 2.1: Parameters for the baseline model

Parameter	Description	
β	Discount factor	0.95
η	Proportion of government spending on public good	0.80
ϵ_c	Inverse of IES for private consumption goods	2.00
ϵ_g	Inverse of IES for public consumption goods	1.50
m	Probability of monitoring	0.15
τ	Tax rate	0.08
$\bar{I}_d$	Steady state nominal rate of return on CBDC	1.05
μ	Growth rate of money supply	0.01
α	Capital share	0.33
A	Productivity	1.00
δ	Depreciation rate	0.10
θ_1	Level parameter of transaction function	2.00
a	Transaction costs for consumption goods	2.00
γ	Transaction efficiency for cash	1.05
ρ	Transaction efficiency for CBDC	1.75
θ_2	Level parameter of penalty function	2.00
ν	Sensitivity to cash in penalty function	3.00

Notes: Conventional values for β, ϵ_c, α, and δ. τ set at 8% as the average U.S. state sales tax is around 8%. ϵ_g is set lower than ϵ_c. μ is set at 1% to have an inflation rate at 1%. γ is set at 0.6 times ρ. Other parameters are set arbitrarily.

We consider six alternative parameter settings, in each of which we vary one parameter relative to the BM. Each of these changes can be viewed as a government policy intervention to increase the representative agent's relative holdings of CBDC. Alternative model-I (AM-I), with a lower γ parameter, corresponds to a decrease in the transaction efficiency of cash. Alternative model-II (AM-II) corresponds to an increase in the transaction efficiency for CBDC (higher value for ρ); alternative model-III (AM-III) corresponds to an increase in the rate of monitoring, i.e., a higher probability of detection of tax evasion (higher m); alternative model-IV (AM-IV) corresponds to an increase in the penalty for tax

model specification. Our sensitivity experiments cover a range of possibilities for the relative transaction efficiency of CBDC compared to cash.

evasion (higher θ_2); alternative model-V (AM-V) corresponds to a decrease in the tax rate (lower τ); and alternative model-VI (AM-VI) corresponds to an increase in the rate of return on CBDC (higher $\bar{I}^d$).

2.3.1 Comparisons across policies

To standardize the changes in the policy parameters, each alternative model involves a 1 percent change in the value of the relevant parameter relative to its baseline value shown in Table 2.1. Table 2.2 shows the shares of the different assets in steady state for each parameter setting.

Table 2.2: Steady state shares (in percent) for baseline and alternative models

Description	BM	AM-I	AM-II	AM-III	AM-IV	AM-V	AM-VI
Physical capital	37.44	37.46	37.28	37.41	37.41	37.53	38.70
Bond holdings	16.80	16.73	16.88	16.84	16.84	16.38	10.67
Cash holdings	6.48	6.46	6.45	6.46	6.46	6.49	6.60
CBDC holdings	39.28	39.35	39.39	39.29	39.29	39.61	44.03
CBDC (% change)	-	0.17	0.27	0.03	0.03	0.83	12.08

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

There are two important points to note in the results in Table 2.2. First, both cash and CBDC co-exist in the economy under both baseline and alternative parameter settings. Second, the representative agent holds a low share of cash in all of these models, consistent with the shift toward a near cashless economy that many developed and developing economies seem to be experiencing. The elasticity of the CBDC share with respect to the parameters in the alternative model varies widely. For example, the elasticity of the CBDC share with respect to the rate of return on CBDC is 12.08. This implies that a one percent increase in the rate of return on CBDC results in a 12.08 percent increase in the CBDC share. The elasticity of the CBDC share with respect to changes in the tax rate is lower. A one percent decrease in the tax rate results in an increase of 0.83 percent in the CBDC share.

The share of CBDC in total financial assets increases significantly for four policies and barely changes for the other two policies. The share of CBDC in total assets can change through direct channels that affect CBDC holdings or through indirect channels that affect cash holdings. The direct channels include increases in the transaction efficiency of CBDC, which could be affected by policy measures, as well as more explicit policy interventions such as a decrease in the tax rate or increase in the nominal rate of return on CBDC. Agents increase their share of CBDC in total assets when it becomes a more lucrative store of value or a more efficient medium of exchange.

The indirect channels include a decrease in the transaction efficiency of cash and policy interventions such as an increase in the probability of monitoring and an increase in the penalty for being caught evading taxes. Interestingly, not all of these indirect channels result in the expected change in the CBDC share of total assets. When cash becomes a less efficient medium of exchange, the CBDC share rises. However, the other two indirect channels related to tax evasion have virtually no effect on the CBDC share. This is perhaps because, while these channels directly impact the tax evasion role of cash, they have no effect on the medium of exchange or store of value aspects of CBDC.

Among these policy changes, an increase in the nominal rate of return on CBDC has the largest impact on CBDC holdings. Table 2.2 yields the following ordering in terms of the effects of different policy changes on CBDC shares: increase in rate of return on CBDC > decrease in tax rate > increase in transaction efficiency of CBDC > decrease in transaction efficiency of cash > increase in the probability of monitoring ~ increase in the penalty function.

2.3.2 Alternative baselines

A significant challenge we face is the lack of empirical data even from CBDC experiments and pilots to discipline the key policy parameters in our model. Hence, we now turn to an alternative approach by evaluating the robustness of our results to different combinations of the main policy parameters. Table 2.3 shows five of these combinations and the associated steady state shares of various assets. This table confirms that the relative shares of different assets, and the effects of changes in policy parameters on those shares, are not sensitive to the specific baseline that we assume. This is further confirmed by an extensive battery of experiments we conducted with sensitivity tests that involve changing specific parameters around alternative baselines (these results are shown in Appendix B.5). For instance, reducing CBDC transaction costs and increasing the tax evasion penalty increases the share of CBDC across alternative settings of most other parameters, while reducing the rate of return on CBDC has the opposite effect. This table does illustrate interesting interactions among various parameter settings. For example, the share of CBDC holdings rises when the transaction efficiency of CBDC is higher even if the rate of return on CBDC is reduced (comparing column BM with columns BM-I, BM-II, BM-IV, and BM-V). These alternative calibrations might prove useful when data from CBDC trials become available and make it possible to pin down at least some of the policy parameters in our model.

Table 2.3: Parameters and steady state shares (in percent) for baseline models

Description	BM	BM-I	BM-II	BM-III	BM-IV	BM-V
γ	**1.05**	1.50	1.75	2.00	1.00	0.95
ρ	**1.75**	2.00	2.00	2.50	1.50	1.20
m	**0.15**	0.10	0.20	0.30	0.25	0.10
θ_2	**2.00**	3.00	2.50	1.50	2.50	3.00
τ	**0.80**	0.50	0.30	0.40	0.50	0.30
$\bar{I}_d$	**1.05**	1.04	1.03	1.03	1.04	1.02
Physical capital	**37.44**	37.10	36.73	32.22	40.58	46.61
Bond holdings	**16.80**	7.99	2.84	13.21	4.96	0.41
Cash holdings	**6.48**	6.90	6.13	5.73	5.59	7.93
CBDC holdings	**39.28**	48.01	54.30	48.84	48.87	45.05

Notes: BM refers to the baseline model as stated in Table 2. BM-I to BM-V corresponds to five different combinations of policy parameters.

2.3.3 Asymmetric responses to changes in policy parameters

Next, we examine possible asymmetries and nonlinearities in the shares of CBDC in total assets ($\bar{dc}^{share}$) across different settings of each policy parameter. We show what happens when we vary the value of each relevant parameter over a range from minus to plus 1 percent of its baseline value.

Table 2.4 shows the change (in percentage points) in the steady state share of CBDC in total assets for the two parameter settings (plus/minus 1 percent) in each alternative model relative to the baseline model. Most of the changes in CBDC shares are relatively modest and symmetric. The highest degree of sensitivity of this share is to the rate of return on CBDC. We explore the effects of this parameter in Figure 2.1, which varies it across a much broader range. It appears that the relationship between $\bar{dc}^{share}$ and the nominal rate of return on CBDC (I_{ss}^d) is captured well by a cubic function. The asymmetry arises from the fact that, even when its rate of return falls, the CBDC remains useful as a medium of exchange. When the interest rate rises, its store of value function and transactional efficiency reinforce each other and lead to more substitution away from other assets.

Figure 2.1 shows that the share of CBDC in total assets is 26.03% when the gross nominal return on CBDC is one. This implies that the net nominal rate of return on CBDC is zero, the same as cash, but CBDC accounts for a larger share of assets than cash (6.06% of total assets) as it is a more efficient medium of exchange.

Table 2.4: Steady state shares (in percent) for positive and negative changes to policy parameters

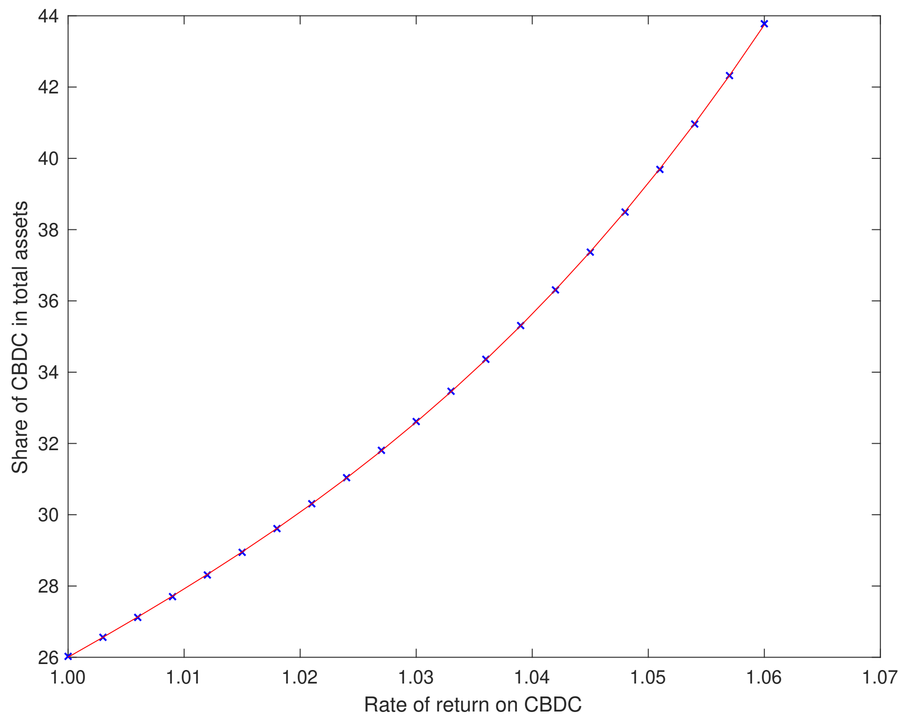
	AM-I (γ)		**AM-II (ρ)**		**AM-III (m)**		**AM-IV (θ_2)**		**AM-V (τ)**		**AM-VI(I^d)**	
Variable	**-1%***	**1%**	**-1%**	**1%***	**-1%**	**1%***	**-1%**	**1%***	**-1%***	**1%**	**-1%**	**1%***
CBDC	39.35	39.21	39.18	39.39	39.27	39.29	39.27	39.29	39.60	38.96	35.47	44.03
Deviation	0.07	-0.07	-0.10	0.11	-0.01	0.01	-0.01	0.01	0.32	-0.32	-3.81	4.75

Notes: The results reported in this table are the shares of CBDC in total assets based on positive and negative deviations of specific parameters from their baseline values. The symbol * indicates the results of the experiments reported in Table 2.2. The share of CBDC in the baseline model is 39.28.

2.3.4 Welfare comparison

One normative question raised by our analysis is whether a benevolent government should attempt to raise the relative share of CBDC through its policy choices. To answer this question requires an evaluation of welfare outcomes under different policy settings. While recognizing that our model abstracts from many advantages of cash, especially in economies in which the population has limited access to the formal financial system, we now turn to such an evaluation using our model.

Figure 2.1: Relationship between the steady state share of CBDC and nominal rate of return of CBDC



Note: This figure shows how the steady state share of CBDC is influenced by the gross nominal rate of return on CBDC, keeping other parameters at their baseline levels. The relationship is well approximated by the following cubic function: $\bar{dc}^{share} = -21887.75 + 65898.21\bar{I}^d - 66239.27\bar{I}^{d2} + 22254.81\bar{I}^{d3}$

We define welfare in the baseline model as the discounted sum of the present and future utility streams of the representative agent:

$$V_0^b = \sum_{t=0}^{\infty} \beta^t (u(c_t^b) + v(c_t^{gb}))$$

Let ω^a represent the conditional welfare gain from adopting an alternative model. That is, ω^a denotes the fraction of private consumption goods that have to be added under a particular alternative model to achieve the same welfare as in the baseline model (Schmitt-Grohé and Uribe, 2007). As before, each alternative model that we analyze involves changing one parameter and keeping all other parameters the same as in the baseline model. Welfare in the alternative model is given by:

$$V_0^a = \sum_{t=0}^{\infty} \beta^t (u(c_t^a) + v(c_t^{ga})) = \sum_{t=0}^{\infty} \beta^t (u((1+\omega^a)c_t^b) + v(c_t^{gb}))$$

Denote $G_0^b = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}$. The conditional welfare gain (ω^a) is then given by equation (2.33)

$$\omega^a = (\frac{V_0^a - G_0^b}{V_0^b - G_0^b})^{\frac{1}{1-\epsilon_c}} - 1 \tag{2.33}$$

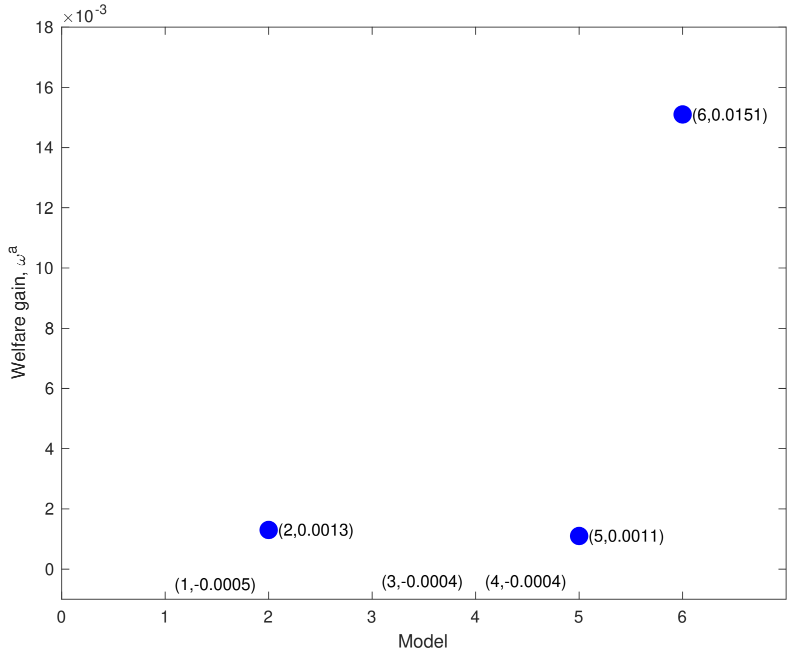
Note that, for ease of exposition, we redefine the welfare gain as a positive number if an alternative policy yields a higher level of welfare than the baseline policy. The derivation of the measure of the conditional welfare gain is shown in Appendix B.4.

Figure 2.2 shows the welfare gain under different models. There are two main points to be taken from the results. First, the following changes in policy parameters increase welfare: an increase in the transaction efficiency of CBDC, a decrease in the tax rate, and an increase in the nominal rate of return on CBDC. Second, it is possible to rank these three changes as follows based on the relative welfare gains from implementing them: increase in the nominal rate of return on CBDC > increase in the transaction efficiency of CBDC > decrease in the tax rate. The following changes in policy parameters marginally reduce welfare: a decline in the transaction efficiency of cash, an increase in the probability of detection of tax evasion, and an increase in the penalty for evading taxes.

2.3.5 Negative nominal rate of return on CBDC

One of the key attractions of a CBDC is that it allows the central bank to impose a negative nominal interest rate even on outside money, which is not possible

Figure 2.2: Welfare gain in alternative models relative to baseline model



Notes: This figure shows that there are welfare gains in alternative model 2 (increase in the transaction efficiency of CBDC), alternative model 5 (reduction in tax rate applied to transactions using CBDC), and alternative model 6 (higher rate of return on CBDC) relative to the baseline model.

with cash. This would allow a central bank facing deflationary risks to circumvent the constraints on conventional monetary policy implied by the zero lower bound. In our model, such a policy would not necessarily cause CBDC demand to collapse to zero because CBDC still has a number of advantages that allow it to retain value as a medium of exchange. That is, even when the CBDC's nominal rate of return is dominated by those of other assets available to the representative agent, the model still yields steady states with positive CBDC holdings (our preliminary analysis shows that there are no steady states when the return on CBDC becomes excessively negative; in future work, we intend to more carefully pursue a characterization of this threshold).

A change in the rate of return on CBDC, which is set by the government/central bank, will of course affect the entire structure of rates of return on the available menu of assets as well as their shares of total assets. Table 2.5 lists the different definitions and representations of rates of return for various assets.

Table 2.5: Formulas for rates of return (in percent)

Description	Shorthand	Capital	Bond	CBDC	Cash
Gross nominal rate of return	GNRR	$f_k(\bar{k})(1+\bar{\pi})$	$\bar{I}^b$	$\bar{I}^d$	$\bar{I}^c$
Gross real rate of return	GRRR	$f_k(\bar{k})$	$\frac{\bar{I}^b}{1+\bar{\pi}}$	$\frac{\bar{I}^d}{1+\bar{\pi}}$	$\frac{\bar{I}^c}{1+\bar{\pi}}$

Notes: This table shows the definitions of different rates of return for various assets.

As an illustrative exercise, we analyze the steady state values of the model when the central bank sets the gross nominal rate of return on CBDC at 0.95 percent (this corresponds to a 10 percent decrease in the gross nominal rate of return on CBDC relative to the baseline model). The results in the block AM in Table 2.6 show that the share of CBDC falls by about 20 percentage points (from 39.28 to 19.02). The shares of cash and capital decrease while the share of bonds increases. This can be seen more clearly in the last block of the table, which shows the percentage changes in asset shares relative to the baseline model. When the gross nominal rate of return of CBDC (0.95%) is lower than that of bonds (1.05%), bonds behave as a better storage technology. Hence, agents shift from CBDC towards bond holdings. Cash holdings increase when the nominal rate of return on CBDC is negative. However, since bond holdings increase substantially as they become the dominant store of value, the share of cash holdings falls.[17]

One question is whether negative and positive changes to the CBDC interest

[17] Additionally, private consumption falls by 9 percent. This is because of the decline in CBDC holdings, which is the efficient medium of exchange, and the only modest increase in cash holdings. Agents switch to saving through bonds and reduce their consumption.

Table 2.6: Steady state implications of negative nominal interest rate on CBDC

Model	BM				AM				Δ%			
Variable	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**
GNRR	0.15	1.06	1.00	1.05	0.15	1.06	1.00	0.95				
GRRR	0.15	1.05	0.99	1.04	0.15	1.05	0.99	0.94				
Position	3.23	1.45	0.56	3.39	3.23	4.74	0.61	2.01				
Share	37.44	16.80	6.48	39.28	30.48	44.73	5.77	19.02	-6.96	37.93	-0.71	-20.26

Notes: BM refers to the baseline model in which the gross nominal rate of return on CBDC is 1.05 percent. AM is the alternative model with a gross nominal rate of return of 0.95 percent on CBDC. The first two rows show various rates of return, the third row shows asset positions, and the last row shows share of each of the assets in total assets. The Δ% column block shows the percentage change in the shares of various assets between baseline and alternative models.

rate have symmetric effects on asset shares. In Table 2.7, we examine the effects of a 1 percent increase in the gross rate of return on CBDC. This increase in the CBDC interest rates results in an increase in the share of CBDC that is much greater (in proportional terms) than the fall in its share in response to a cut in the CBDC interest rate. Holdings of bonds decline significantly when the CBDC interest rate rises. That is, the asset held primarily as a store of value, rather than as a factor of production or a medium of exchange, is most affected by changes in the CBDC interest rate.

Table 2.7: Steady state implications of positive nominal interest rate on CBDC

Model	BM				AM				Δ%			
Variable	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**	**Capital**	**Bond**	**Cash**	**CBDC**
GNRR	0.15	1.06	1.00	1.05	0.15	1.06	1.00	1.06				
GRRR	0.15	1.05	0.99	1.04	0.15	1.05	0.99	1.05				
Position	3.23	1.45	0.56	3.39	3.23	0.89	0.55	3.67				
Share	37.44	16.80	6.48	39.28	38.70	10.67	6.60	44.03	1.26	-6.13	0.12	4.75

Notes: BM refers to the baseline model. AM is the alternative model with a gross nominal rate of return of 1.06 percent on CBDC. Rate of return, asset position, and asset shares when the nominal rate of return on CBDC is positive. The Δ% column block of last row represents the percentage change between baseline and alternative models.

2.4 Extensions

In this section, we consider extensions to the basic model. First, we examine the effects of setting the CBDC interest rate at zero, the same rate of return as cash. Second, we study the transition dynamics of private consumption in response to productivity shocks. Third, we analyze how the government can influence holdings of CBDC relative to other assets via direct helicopter drops that increment the stock of CBDC held by agents. Fourth, we show how to structure the rate of return on CBDC ex-ante to avoid capital flight from bonds into CBDC. Fifth, we investigate the effects of introducing a tax on earnings from bond holdings. Sixth, we show how the government can encourage CBDC holdings by imposing a tax on cash holdings above a predetermined threshold. Finally, we consider some special cases of our more general model.

2.4.1 Zero net interest rate on CBDC

In our baseline analysis, we have assumed a positive net interest rate on CBDC. Central banks considering the issuance of CBDC seem to view it, at least in the initial stages, as being a cash-like instrument, which would imply a zero interest rate (see BIS, 2020).

Table 2.8 shows the counterparts of our main results when the net interest rate on CBDC is set to zero. A few points are worth noting. First, agents still hold CBDC for transaction purposes but the steady state shares are lower than in the baseline results (Table 2.2). Second, cash and CBDC co-exist in this equilibrium but, since neither of them serves as a store of value, the shares of bond

Table 2.8: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	33.27	33.29	33.13	33.25	33.25	33.32	33.93
Bond holdings	34.64	34.59	34.75	34.68	34.68	34.44	32.03
Cash holdings	6.07	6.05	6.03	6.04	6.04	6.07	6.13
CBDC holdings	26.03	26.07	26.09	26.03	26.03	26.17	27.90
CBDC (% change)	-	0.17	0.26	0.03	0.03	0.54	7.21

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1 percentage point increase in $\bar{I}_d$, i.e. $\bar{I}_d^{BM} = 1.00$ and $\bar{I}_d^{AM-VI} = 1.01$ (rate of return on CBDC).

holdings are in general higher as well. Thus, all of our key results are preserved and do not hinge on the assumption of a positive interest rate on CBDC.[18]

2.4.2 Transition dynamics in response to productivity shocks

Our analysis thus far provides comparisons across steady states. In our model, capital and, hence, output remain the same across different steady states. In order to better understand how economic activity evolves between two steady states due to various shocks, we now conduct a simple exercise to determine the impulse response function for private consumption in response to productivity shocks. This allows us to characterize, in an admittedly crude manner, the transition path between two steady states in response to a particular shock. Let productivity, denoted by A, follow an AR(1) process

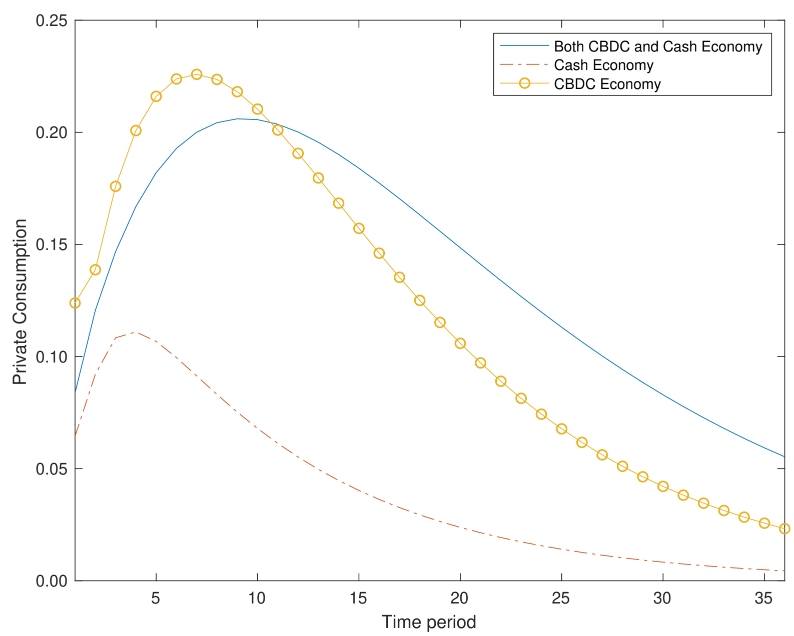
$$log(A_t/\bar{A}) = \rho_a log(A_{t-1}/\bar{A}) + e_a$$

where $0 < \rho_a < 1$ and $e_a \sim N(0, \sigma_a^2)$.

[18]The experiments conducted later in this section are also robust to eliminating the positive interest rate on CBDC.

Figure 2.3 shows the impulse response functions of private consumption in response to a (transitory) one standard deviation positive shock to productivity. The blue solid line depicts the consumption profile for an economy with both cash and CBDC, the red dashed line shows the consumption profile for an economy with cash only, and the yellow bubble line displays the consumption profile for an economy with CBDC only. The positive impact on consumption is largest in the case of an economy with both cash and CBDC. More formal welfare calculations suggest that, relative to an economy with both cash and CBDC, the welfare gain is about 18 percent lower in the case of an economy with only CBDC and 73 percent lower in an economy with only cash. The smaller welfare gain in the case of a cash economy is probably on account of the CBDC being usable as both a medium of exchange and store of value while cash is less efficient in both those dimensions.

Figure 2.3: Consumption profiles in different cash-CBDC regimes



Notes: This figure shows the profiles of private consumption under different regimes in response to a one standard deviation shock to productivity.

The response to a negative productivity shock would be symmetric. This sets the stage, in the next sub-section, for illustrating the use of helicopter drops of money as a counter-cyclical policy tool in an environment with CBDC.

2.4.3 Helicopter drops

In our model, helicopter drops of money (as a counter-cyclical policy tool) can be implemented directly by affecting the holdings of CBDC rather than through the conventional view of helicopter drops as fiscal policy measures financed by monetary expansion. Similarly, a negative interest rate on CBDC can take the form of a haircut, in which agents face an effective nominal negative rate of return on CBDC as the balances on their accounts at the central bank shrink. The model in this paper does not incorporate endogenous output responses but we can still use it to show how such policies affect the entire portfolio of assets held by agents and their inter-temporal allocation decisions.

Define h as the probability of a recession that triggers a policy response in the form of a helicopter drop of CBDC and s as the size of the helicopter drop. More precisely, h is the probability that an agent gains an s percent increment to her/his CBDC holdings. The central bank implements a helicopter drop of CBDC or sets a negative nominal rate of return on CBDC in bad states of the economy. The parameter h is set based on the frequency of recessions in the past 100 years in the United States, i.e., $h = \frac{\#Recession}{100}$. The central bank sets s and the nominal rate of return on CBDC. Thus, the helicopter drops in this model are ex-ante probabilistic increments to CBDC holdings.

Modification of baseline model

As before, the representative agent maximizes the present discounted value of utility by choosing private consumption goods, public consumption goods, physical capital, bonds, cash, and CBDC holdings. However, the nominal rate of return on CBDC in the RHS of the budget constraint differs from that in the baseline model. The revised budget constraint is as follows:

$$
\begin{aligned}
c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\
\leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}(h(1+s)dc_{t-1} + (1-h)dc_{t-1}) + \frac{ca_{t-1}}{1+\pi_t}
\end{aligned}
$$

All the FOCs remain the same, except with respect to the choice variable dc. The FOC for dc yields

$$
\begin{aligned}
\frac{u_c(c_t)}{1+\phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \Big(\frac{I_t^d}{1+\pi_{t+1}}(h(1+s)+(1-h)) \\
& - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} \Big)
\end{aligned}
$$

In the steady state,

$$
1 = \beta\Big(\frac{\bar{I}^d}{1+\bar{\pi}}(h(1+s)+(1-h)) - \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}\Big)
$$

Comparison of steady state relative to the baseline model

The parameter values in this exercise are the same as in the baseline model, except for the new parameters h and s, which are set as follows:

Table 2.9: Additional parameters for helicopter drop model

Parameter	Description	Baseline Model	Helicopter drop Model
h	Probability of helicopter drop	-	0.01
s	Percentage of helicopter drop	-	0.15

Notes: These are the new parameters in addition to those listed in Table 2.1.

The steady state shares of different assets are shown in Table 2.10. We observe that the steady state share of CBDC increases in comparison with that in the baseline model. The representative agent has an incentive to increase the share of CBDC holdings due to the probability of a helicopter drop increment.

Table 2.10: Shares in steady state for helicopter drop model

Variable	Description	Baseline	Helicopter drop
$\bar{k}^{share}$	Physical capital	37.44	38.14
$\bar{b}^{share}$	Bond holdings	16.80	15.68
$\bar{ca}^{share}$	Cash holdings	6.48	6.62
$\bar{dc}^{share}$	CBDC holdings	39.28	39.56

Notes: Comparison of asset shares between baseline and helicopter drop models.

Implementing helicopter drops

The probability of a helicopter drop, which depends on the state of the economy, is taken as an exogenous parameter by the central bank. This parameter h, is assumed to take any one of the following values: $h \in \{0.01, 0.05, 0.10, 0.15, 0.20\}$. The central bank then has two instruments under its discretion in implementing helicopter drops. First, it can set different sizes of helicopter drops (increments to the beginning of period CBDC stocks) for a given nominal rate of return. Second, it can set different nominal rates of return on CBDC for a given helicopter drop percentage. We determine the relationship between the helicopter drop percentage (s) and share of CBDC ($\bar{dc}^{share}$) under different states of the economy (captured by h) for a fixed gross nominal rate of return ($\bar{I}^d$). The gross nominal rate of return on CBDC is chosen from among the following values: $\bar{I}_d \in \{1.04, 1.05, 1.06\}$. The steady state CBDC shares are determined for each of those values.

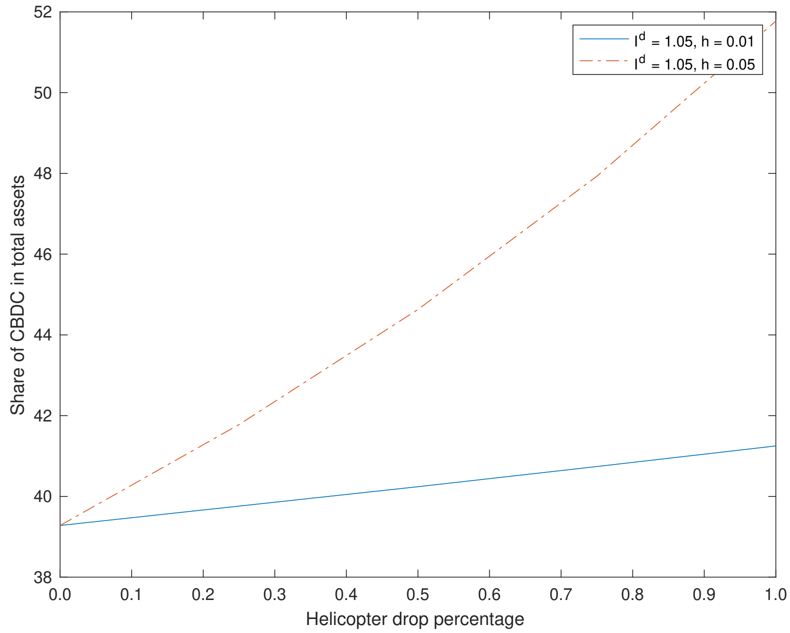
Table 2.11 and Figure 2.4 capture the relationship between s and $\bar{dc}^{share}$. While the first column block of Table 2.11 shows the effect of s on $\bar{dc}^{share}$ at $\bar{I}^d = 1.04$ and $h = 0.01$, the second column block shows the effect of s on $\bar{dc}^{share}$ at $\bar{I}^d = 1.04$ and $h = 0.05$. Similarly, the third and fourth column blocks represent the effects of s on $\bar{dc}^{share}$ for different states of the economy when $\bar{I}^d = 1.05$. For a fixed value of $\bar{I}_d$ and h, the CBDC share increases with the size of the helicopter drop. The rise in the CBDC share is much steeper as the probability of a bad state of the economy rises. Also, for a fixed nominal rate of return on CBDC (I^d) and a given value of the helicopter drop percentage (s), the CBDC share rises with increases in the probability of a helicopter drop (h).

Table 2.11: Effects of probability and size of helicopter drop on CBDC shares in total asset holdings

$\bar{I}^d = 1.04$ & $h = 0.01$		$\bar{I}^d = 1.04$ & $h = 0.05$		$\bar{I}^d = 1.05$ & $h = 0.01$		$\bar{I}^d = 1.05$ & $h = 0.05$	
s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$	s	$\bar{dc}^{share}$
0.00	35.64	0.00	35.64	0.00	39.28	0.00	39.28
0.25	36.02	0.25	37.67	0.25	39.76	0.25	41.78
0.50	36.42	0.50	39.96	0.50	40.24	0.50	44.63
0.75	36.83	0.75	42.57	0.75	40.74	0.75	47.93
1.00	37.24	1.00	45.55	1.00	41.25	1.00	51.77

Notes: CBDC shares for different values of the size of the helicopter drop (increment, in percent, to the beginning of period CBDC stocks). $\bar{I}^d$ denotes the gross nominal rate of return on CBDC, h denotes probability of helicopter drop, s denotes size of helicopter drop, and $\bar{dc}^{share}$ denotes the CBDC share.

Figure 2.4: Relationship between size of helicopter drop and CBDC share



Notes: This figure shows how the share of CBDC in total assets varies with size of the helicopter drop, with the gross nominal rate of return on CBDC set at 1.05% and the probability of the helicopter drop set at 0.01 (solid blue line) and 0.05 (dotted red line), respectively.

2.4.4 Limiting flight into CBDC

We now show how the rate of return on CBDC can be designed ex-ante to avoid capital flight into CBDC, which could be triggered by flight to safety during a financial panic or a severe recession.[19] The model in this paper does not feature bank deposits, so the flight would be out of bonds (and, potentially, other assets) but the mechanism we discuss below would be equally relevant for limiting flight from bank deposits into CBDC. Of course, any flight into CBDC for reasons of safety would hinge on the credibility of the central bank issuing the

[19] An alternative approach, adopted by the Central Bank of the Bahamas in the design of its CBDC called the “sand dollar," is to impose a quantity restriction on the size of retail CBDC accounts. See https://www.centralbankbahamas.com/news/press-releases/project-sand-dollar-the-central-bank-identifies-preferred-technology-solutions-provider-for-bahamas-digital-currency.

CBDC and also on the fiscal soundness of the country's government, both of which might be tested in periods of financial crisis.

Design

The monetary authority randomly chooses a cut-off CBDC value, dc^c, that pins down the value of CBDC above which agents are required to pay a fee to hold CBDC. Let agents' CBDC holdings at a given point of time be denoted by dc_t. CBDC holdings provide a constant rate of return, as in the baseline model, when dc_t is less than or equal to dc^c. However, agents pay an increasing fee on CBDC holdings when dc_t is greater than dc^c. This fee translates into a decreasing return when dc_t is greater than dc^c. The rate of return on CBDC is designed as follows:

$$I_t^d(dc_t, dc^c) = \begin{cases} \bar{I}^d, & \text{if } dc_t \leq dc^c \\ \bar{I}^d - I^f(dc_t - dc^c), & \text{if } dc_t > dc^c \end{cases}$$

where $I^f(dc_t - dc^c)$ is an increasing function of $dc_t - dc^c$.

Denote $\bar{dc}$ as the steady state value of CBDC holdings. The steady state rate of the return on CBDC is then determined as

$$I_t^d(\bar{dc}, dc^c) = \begin{cases} \bar{I}^d, & \text{if } \bar{dc} \leq dc^c \\ \bar{I}^d - I^f(\bar{dc} - dc^c), & \text{if } \bar{dc} > dc^c \end{cases}$$

Functional form for the penalty function

Assume $I^f(dc_t - dc^c) = \omega_f(dc_t - dc^c)$. That is, the fee rises linearly with the excess of CBDC holdings above the cut-off level.

$$I_t^d(dc_t, dc^c) = \begin{cases} \bar{I}^d, & \text{if } dc_t \leq dc^c \\ \bar{I}^d - \omega_f(dc_t - dc^c), & \text{if } dc_t > dc^c \end{cases}$$

Modification of the basic model

The only modification relative to the baseline model is the revised budget constraint:

$$\begin{aligned} c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \\ \leq f(k_{t-1}) + \eta g_t - g_t + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \mathbb{1}_{dc_t \leq dc^c}\frac{I_{t-1}^{dl}}{1+\pi_t}dc_{t-1} + \mathbb{1}_{dc_t > dc^c}\frac{I_{t-1}^{du}}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned}$$

where $\mathbb{1}$ is the indicator function, $I_{t-1}^{dl} = \bar{I}^d$, and $I_{t-1}^{du} = \bar{I}^d - \omega_f(dc_{t-1} - dc^c)$. All the FOCs remain the same, except with respect to the choice variable dc. The FOC with respect to CBDC holdings under the transformed budget constraint takes the following form:

$$\begin{aligned} \frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})} =& \beta \frac{u_c(c_{t+1})}{1 + \phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})} \\ & (\mathbb{1}_{dc_t \leq dc^c}(\frac{I_t^{dl}}{1+\pi_{t+1}} + \frac{dc_{t+1}}{1+\pi_t}\frac{\partial I_t^{dl}}{\partial dc_t}) \\ & + \mathbb{1}_{dc_t > dc^c}(\frac{I_t^{du}}{1+\pi_{t+1}} + \frac{dc_t}{1+\pi_{t+1}}\frac{\partial I_t^{du}}{\partial dc_t}) \\ & - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}) \end{aligned}$$

In the steady state,

$$1 = \beta(\mathbb{1}_{\bar{d}c \leq dc^c}(\frac{\bar{I}^{dl}}{1+\bar{\pi}} + \frac{\bar{d}c}{1+\bar{\pi}}\frac{\partial \bar{I}^{dl}}{\partial \bar{d}c}) + \mathbb{1}_{\bar{d}c > dc^c}(\frac{\bar{I}^{du}}{1+\bar{\pi}} + \frac{\bar{d}c}{1+\bar{\pi}}\frac{\partial \bar{I}^{du}}{\partial \bar{d}c})$$
$$- \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \frac{\bar{c}a}{1+\bar{\pi}}, \frac{\bar{d}c}{1+\bar{\pi}})}{1+\bar{\pi}})$$

where $\frac{\partial \bar{I}^{dl}}{\partial \bar{d}c} = 0$ and $\frac{\partial \bar{I}^{du}}{\partial \bar{d}c} = -\omega_f$.

An interesting departure from the earlier setting is that the rate of return on CBDC is now determined endogenously:

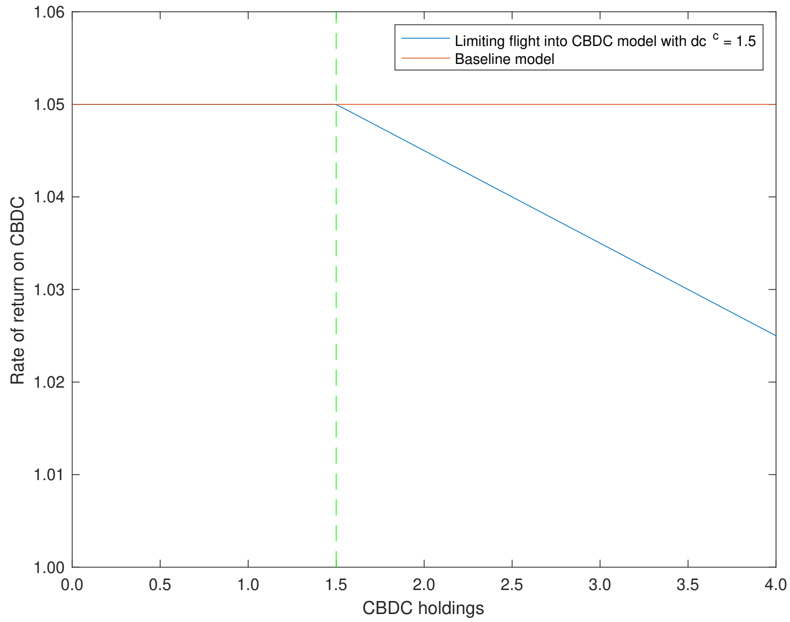
$$\bar{I}^d = \mathbb{1}_{\bar{d}c \leq dc^c}(\bar{I}^d) + \mathbb{1}_{\bar{d}c > dc^c}(\bar{I}^d - \omega_f(\bar{d}c - dc^c))$$

Results

We set the penalty fee parameter $\omega_f = 0.01$. That is, the CBDC interest rate decreases by 1 percentage point for a unit increase in CBDC holdings above the cut-off CBDC threshold. Figure 2.5 shows the relationship between the nominal rate of return of CBDC and CBDC holdings for the baseline model and for the extended model with the cut-off CBDC value set at 1.50. This shows how the monetary authority can design the nominal rate of return on CBDC ex-ante to disincentivize CBDC holdings above the cut-off value.

The steady state shares of different assets are reported below in Table 2.12. The steady state share of CBDC in the model with the fee on excess CBDC holdings (Column A) is lower than that in baseline model. In a period of financial distress, agents might wish to move completely to CBDC. This phenomenon is similar to having the CBDC steady state value above the cut-off CBDC. However, the increasing fee (or decreasing return on CBDC) regime kicks in that

Figure 2.5: Relationship between CBDC holdings and rate of return of CBDC



Notes: This figure shows how the rate of return is designed by the central bank ex-ante to limit flight into CBDC. The central bank imposes a fee on CBDC holdings above a given threshold. This fee is linear in the amount by which CBDC holdings exceed the threshold.

results in the gross nominal return on CBDC being lower than that of bonds. This prevents bond holdings from collapsing to zero.

2.4.5 Tax on bond holdings

In our baseline model, the only tax is on CBDC holdings, which captures taxes on consumption transactions using CBDC. In this extension, we modify our baseline model by imposing a 5 percent tax on agents' interest earnings from bond holdings. The non-negativity constraint on bond holdings now becomes pertinent since in this environment a CBDC might, under some settings, be a better store of value than bonds, in addition to being an efficient medium of

Table 2.12: Steady state shares (in percent) for model with fee on CBDC holdings above threshold

Description	**BM**	**A**
Share		
Physical capital	37.44	40.72
Bond holdings	16.80	18.37
Cash holdings	6.48	8.19
CBDC holdings	39.28	32.72
Gross Nominal Rate of Return		
CBDC	1.05	1.04
Bond	1.06	1.06
Positions		
CBDC holdings	3.39	2.59
CBDC cut-off	-	1.50

Notes: BM refers to the baseline model. Column A refers to the alternative model with a fee on CBDC holdings above the threshold, which is set at 1.5. The table shows the steady state shares of different assets in total assets. The last two blocks of the table show the gross nominal rate of return on CBDC, which is endogenous in the alternative model, and the absolute level of CBDC holdings.

exchange.

Modification of baseline model

The representative agent's maximization problem now incorporates the tax on bond returns in the RHS of the budget constraint and (as before) a separate non-negativity constraint on bond holdings.

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)]$$

subject to

$$c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t$$
$$+\frac{\tau}{1+\pi_t}dc_{t-1} \le f(k_{t-1}) + (1-\tau_b)\frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$
$$b_t \ge 0$$
$$c_t \le \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t}$$

Table 2.13 shows the steady state shares of different assets under various taxation regimes. Setting a positive tax rate on bond yields has no impact on the shares of bonds in total assets (Column A). Rather there is an increase in the gross nominal rate of return on bonds, which offsets the positive tax rate on bond yields.[20] When the nominal rate of interest on CBDC is increased such that, after accounting for the tax on bond returns, the rate of return on CBDC approaches (but is still below) that of bonds, bond holdings hit the boundary condition (Column B). Thus, the increase in the CBDC interest rate and the tax on bond holdings reinforce each other in making CBDC more attractive both as a store of value and medium of exchange.

2.4.6 Influencing CBDC holdings by taxing cash

In principle, the government can levy a tax on cash holdings to encourage a greater use of CBDC in consumption and other transactions. For example, in its 2019 budget session, the Indian government considered the introduction of a tax of 2% on cash withdrawals of more than 10 million Indian rupees (INR; roughly

[20] The after-tax gross nominal rate of return on bonds for column A is $(1-\tau_b)I^b = (1-0.05)1.12 = 1.06$. Thus, the after-tax gross nominal rate of return on bonds in column A is equal to the gross nominal rate of return on bonds in baseline model (which has no taxation on interest income from bond holdings).

Table 2.13: Steady state shares (in percent) for model with tax on bond yields and non-negative bond holdings

Description	**BM**	**A**	**B**
Share			
Physical capital	37.44	37.44	12.36
Bond holdings	16.80	16.80	0.00
Cash holdings	6.48	6.48	2.78
CBDC holdings	39.28	39.28	84.86
Gross Nominal Rate of Return			
CBDC	1.05	1.05	1.09
Bond	1.06	1.12	-
Tax rate			
CBDC	0.08	0.08	0.08
Bond	0.00	0.05	0.05

Notes: BM refers to the baseline model. The next two columns show shares of various assets when a tax is levied on interest earnings from bonds (Column A) and, in addition, a non negativity restriction is imposed on bond holdings in conjunction with an increase in the CBDC interest rate (Column B).

$135,000 at the November 2020 exchange rate) in a year. In 2005, the government had introduced a Banking Cash Transaction Tax (BCTT) that imposed a tax of 0.1% on cash withdrawals exceeding 25 thousand INR in a single day.[21] In 2019, the Greek government mandated that 30 percent of an individual's income be allocated to digital spending, with a tax imposed on any shortfall from the target. This is an alternative mechanism for penalizing the use of cash in the purchase of goods and services.[22]

We now examine how a tax on cash holdings, if that were feasible to imple-

[21]https://www.financialexpress.com/budget/budget-2019-govt-looks-to-drive-digital-payments-with-2-tax-on-cash-withdrawals-above-rs-1-crore/1633224/;https://economictimes.indiatimes.com/wealth/personal-finance-news/budget-2019-will-the-banking-cash-transaction-tax-make-a-comeback/articleshow/69929250.cms?

[22]https://www.dontmesswithtaxes.com/2019/12/greece-to-force-more-digital-transactions-new-tax-penalty-to-thwart-tax-cheats.html

ment, could be used to affect CBDC holdings. Let h_c be the probability that an agent has cash holdings that surpass a threshold value, ca^c. Let τ_c be the tax that is imposed when an agent's cash holdings exceed this threshold. The probability of the imposition of such a tax on cash holdings and the structure of the tax are designed as follows:

$$Probability = \begin{cases} h_c, & \text{if } ca_t > ca^c \\ 1 - h_c, & \text{if } ca_t \leq ca^c \end{cases}$$

$$\tau_c(ca_t) = \begin{cases} \tau_c, & \text{w.p. } h_c \\ 0, & \text{w.p. } (1 - h_c) \end{cases}$$

This results in the following revised budget constraint:

$$c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t$$
$$+ h_c\tau_c ca_t + (1-h_c)0ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$

All the FOCs remain the same, except with respect to the choice variable ca. The FOC for ca yields

$$\frac{u_c(c_t)}{1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})}(1 + h_c\tau_c) = \beta\frac{u_c(c_{t+1})}{1 + \phi_c(c_{t+1}, \frac{ca_t}{1+\pi_{t+1}}, \frac{dc_t}{1+\pi_{t+1}})}$$
$$(\frac{1}{1+\pi_{t+1}} - \frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} - \frac{\phi_{ca}(c_{t+1}, ca_t, dc_t)}{1+\pi_{t+1}})$$

In the steady state,

$$1 + h_c\tau_c = \beta(\frac{1}{1+\bar{\pi}} - \frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}} - \frac{\phi_{ca}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}})$$

Comparison of steady state relative to the baseline model

The parameter values are the same as in the baseline model, except for the new parameters h_c and τ_c, which are chosen as follows:

Table 2.14: Additional parameters for model with tax on cash holdings

Parameter	Description	BM	Tax on Cash Model
h_c	Probability of cash holding above threshold	-	0.05
τ_c	Tax-rate on cash holding above threshold	-	0.08

Notes: BM stands for baseline model.These are the new parameters in addition to those listed in Table 2.1.

The steady state values of different assets are reported in Table 2.15. The steady state value of CBDC is higher than in the baseline model. Taxing cash holdings causes agents to use relatively more CBDC than cash to purchase private consumption goods, which results in an increase in the CBDC position. The tax on cash (along with the existing tax on CBDC) also leads to a shift away from consumption toward higher savings in the form of bonds.

Table 2.15: Positions of various assets in model with tax on cash

Variable	Description	Baseline	Tax on Cash	Percent change
$\bar{k}$	Physical capital	3.2274	3.2274	0.00
$\bar{b}$	Bond holdings	1.4483	1.4822	2.34
$\bar{ca}$	Cash holdings	0.5589	0.5565	-0.42
$\bar{dc}$	CBDC holdings	3.3866	3.3918	0.15

Notes: Comparison of asset positions in the model with a tax on cash holdings relative to the baseline model.

Effects of probability of cash holdings above threshold level

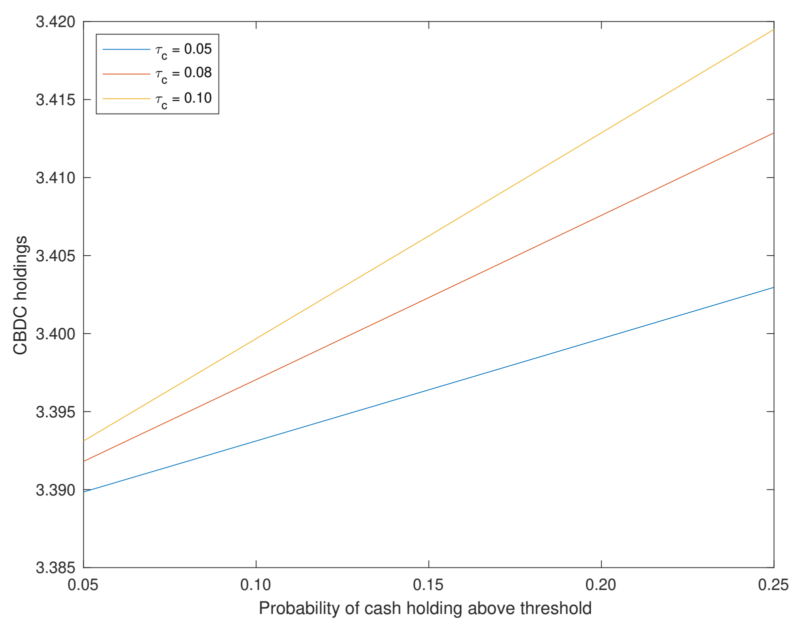
For this experiment, we keep the tax rate on cash holdings constant at one of the following values: $\tau_c \in \{0.05, 0.08, 0.10\}$. For a given value of τ_c, CBDC holdings in the steady state are determined for different values of the probability of cash holdings being above the threshold value. Table 2.16 and Figure 2.6 capture the relationship between τ_c and $\bar{d}c$. For a given value of τ_c, CBDC holdings rise with the increase in the probability of cash holdings above the threshold value. Also, for a given value of h_c, CBDC holdings rise when there is an increase in the tax rate on cash holdings. This shows that a higher level of cash holdings and/or a higher tax rate on cash holdings incentivizes agents to increase holdings of CBDC for transaction purposes.

Table 2.16: Effects of cash holdings above threshold level on CBDC holdings

Variable			**τ_c=0.05**					**τ_c=0.08**					**τ_c=0.10**		
h_c	0.05	0.10	0.15	0.20	0.25	0.05	0.10	0.15	0.20	0.25	0.05	0.10	0.15	0.20	0.25
$\bar{d}c$	3.390	3.393	3.396	3.400	3.403	3.392	3.397	3.402	3.407	3.413	3.393	3.400	3.406	3.413	3.420

Notes: Steady state values of CBDC for different values of the probability of cash holdings being above threshold at which tax on cash kicks in.

Figure 2.6: Relationship between probability of cash holding above threshold and CBDC holdings



Notes: This figure shows how CBDC holdings vary with the probability of cash holdings being above the threshold level, with the tax rates on cash holdings above the threshold set at 5% (blue line), 8% (red line), and 10% (yellow line), respectively.

2.4.7 Model variations

In this sub-section, we present results from one additional sensitivity test and two variants of the model that could be considered special cases of the more general baseline model presented in Section 2.

Level parameter of the transaction function

The transaction function includes a level parameter, θ_1, which captures the standard of the payment infrastructure for purchasing goods and services and could be interpreted as a crude measure of the degree of households' access to the payment system. Currently, the value of θ_1 is set to 2 in all the analysis. We now change this parameter in a systematic manner over the range shown in Table 2.17, holding all other parameters (including policy parameters constant.

Table 2.17: Level parameter of the transaction function

Parameter	**AM-I**	**AM-II**	**AM-III**	**BM**	**AM-IV**	**AM-V**	**AM-VI**
θ_1	0.5	1	1.5	2	2.5	3	3.5
%change	-75%	-50%	-25%	0%	25%	50%	75%

Notes: θ_1 varies from -75% to 75% of the original value used in the baseline model.

Table 2.18 shows the steady state shares of different assets for various settings of θ_1. The share of CBDC increases and that of cash decreases when θ_1 is higher. When the costs of transactions using both cash and CBDC rise (higher value of θ_1), bonds become more preferred assets while the share of physical capital declines. Thus, the overall efficiency of the retail payment infrastructure (which could be taken as a crude proxy for financial inclusion) affects holdings of all types of financial and physical assets.

Table 2.18: Steady state shares (in percent)

Asset	$\theta_1 = 0.5$	$\theta_1 = 1$	$\theta_1 = 1.5$	$\theta_1 = 2$	$\theta_1 = 2.5$	$\theta_1 = 3$	$\theta_1 = 3.5$
Physical capital	55.27	45.45	40.56	**37.44**	35.20	33.49	32.12
Bond holdings	0.51	9.43	13.92	**16.80**	18.87	20.47	21.75
Cash holdings	7.97	7.21	6.78	**6.48**	6.26	6.09	5.94
CBDC holdings	36.25	37.91	38.75	**39.28**	39.66	39.96	40.19

Notes: Steady state share of various assets are compared across different values of θ_1, the level parameter in the transaction cost function, holding all other parameters (including policy parameters) at their respective baseline levels. The column in bold corresponds to the baseline model in Table 2.2.

Model variant excluding public goods and government expenditure

We now turn to a special case of our general model by excluding public goods (c_g) and government expenditure (g) to examine if that influences the results. The utility function and the budget constraints of the representative agent and the government are then modified as follows:

Representative Agent

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1}$$
$$\leq f(k_{t-1}) + \frac{I^b_{t-1}}{1+\pi_t} b_{t-1} + \frac{I^d_{t-1}}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$

$$k_t - (1-\delta)k_{t-1} \leq i_t$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t}$$

Government

$$\frac{I^b_{t-1}}{1+\pi_t} b_{t-1} + \frac{I^d_{t-1}}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$
$$= b_t + dc_t + ca_t + \frac{\tau}{1+\pi_t} dc_{t-1} + m\psi(\frac{ca_{t-1}}{1+\pi_t})$$

The steady state shares in this more restrictive model show one important difference relative to the baseline model, which is that the share of bond holdings is comparable to that of CBDC holdings (Table2.19) This is because, without any expenditure on public goods, the government pays a higher interest rate on bonds in the steady state. The effects of changes in policy parameters around the new baseline are, however, quite similar to those presented in Table 2.2.

Table 2.19: Steady state shares (in percent) for baseline and alternative models

Description	BM	AM-I	AM-II	AM-III	AM-IV	AM-V	AM-VI
Physical capital	19.21	19.22	19.13	19.20	19.20	19.25	19.79
Bond holdings	39.78	39.72	39.77	39.79	39.79	39.43	34.79
Cash holdings	4.16	4.15	4.14	4.15	4.15	4.17	4.23
CBDC holdings	36.85	36.91	36.95	36.86	36.86	37.15	41.19
CBDC (% change)	-	0.17	0.29	0.03	0.03	0.81	11.78

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Endowment economy with no physical capital

In this variant of our baseline model, we drop physical capital and investment, switching instead to an endowment economy. The representative agent is bestowed with an exogenous, deterministic stream of income (y_t) every period. The maximization problem then simplifies to:

$$\max \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(c_t^g)]$$

subject to

$$c_t + c_t^g + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1}$$
$$\leq y_t + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t}$$

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t}$$

The steady state value of income ($\bar{y}$) is set at 1.15. To enable comparisons with the baseline (more general) model, this value is set such that $f(\bar{k}) - \delta\bar{k} = \bar{y}$, where $\bar{k}$ is the steady state value of capital obtained from the baseline model with capital.

The results, as shown in Table 2.20, indicate that the ordering of the relative shares of bonds, cash, and CBDC remains the same as in the baseline model with capital. Moreover, the effects of changing various policy parameters are qualitatively similar to those in Table 2.2.

Table 2.20: Steady state shares (in percent) for baseline and alternative models

Description	**BM**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Bond holdings	26.89	26.79	26.96	26.94	26.94	26.26	17.46
Cash holdings	10.37	10.35	10.29	10.33	10.33	10.40	10.78
CBDC holdings	62.73	62.86	62.74	62.73	62.73	63.34	71.76
CBDC (% change)	-	0.13	0.01	0.00	0.00	0.61	9.03

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

2.5 Conclusion

In this paper, we have developed a simple general equilibrium model to capture the trade-offs between cash and CBDC. The key differences between these two forms of central bank-issued (outside) money include transaction costs (lower for CBDC), possibilities for tax evasion (higher for cash, but with a positive probability of being caught and penalized), and nominal rates of return (zero for cash; potentially positive or negative for CBDC). We showed how different combinations of government policies—such as the level of taxes, the penalty for being caught undertaking tax evasion, and the rate of return on CBDC—can influence the relative holdings of cash and CBDC. We also examined other policies—such as helicopter drops and taxes imposed on cash holdings beyond a predetermined threshold—that can influence the shares of CBDC in total assets (which include CBDC, cash, government bonds, and physical capital). The model provides a framework that can be expanded to incorporate further attributes of various types of official and nonofficial currencies.

To highlight the trade-offs between cash and CBDC under different policy environments, we have abstracted from a number of other considerations. One important aspect that is worth exploring further is the potential role for a CBDC in broadening financial inclusion, although this would require a heterogeneous agent model in order to capture differential levels of access to digital payments. We do not explicitly model one important advantage of cash, which is that it provides anonymity in financial transactions unlike most digital forms of payment. Incorporating this explicitly (rather than viewing our general formulation as capturing various benefits of using cash) would require assigning a utility value for privacy, which could be an interesting exercise in itself.

In addition, we have posited an exogenous growth rate of central bank money (cash plus CBDC) in order to pin down the inflation rate. Making monetary policy endogenous and explicitly modeling the transmission of monetary policy to economic activity would be useful extensions. In the present formulation of the model, activity is determined by supply-side fundamentals since our main interest is in characterizing policies that affect the distribution of asset holdings. Another exercise would be to examine if the share of CBDC in total assets is affected by the uncertainty of returns on CBDC (in contrast to cash, which has a fixed zero nominal rate of return). We leave these and other extensions for future work.

CHAPTER 3

INTRODUCTION OF CENTRAL BANK DIGITAL CURRENCY: EFFECT ON TAX EVASION IN THE ECONOMY

3.1 Introduction

Cryptocurrency is a buzzword in the world of finance and economics in present time. The deregulated market of cryptocurrency is captured by different players such as bitcoin, ethereum, ripple, and so on. Under such situation central bank of different countries are accessing the role of circulating its own digital currency in the economy. The Central Bank Digital Currency (CBDC) will play a pivotal role for changes in monetary policies and agent behavior as most countries are moving towards cashless economy. Recently, there has been different discussions and some frameworks to understand the implications of introduction of central bank digital currency in the economy.

Following the definition by Bjerg (2017), Central Bank Digital Currency (CBDC) is electronic, universally accepted, and central bank issued money. Bjerg discussed the implementation of CBDC under three scenario: namely, the money user, the money manager, and the money maker.[1] First design as *the money user* reveals CBDC serves as electronic cash in complement to cash and bank deposits. Thus, CBDC would fulfill the role of medium of exchange. The central bank would simultaneously fulfill the objective of free convertibility and maintaining parity among CBDC, cash, and bank deposit. However, the central bank would lose its monetary sovereignty of designing any interest based mon-

[1]The money user, the money manager, and the money maker scenario correspond to the commodity theory, the credit theory, and the state theory of money respectively.

etary policy. Second design as *the money manager* reveals CBDC serves as universal reserve and fulfill the role of store of value. In this scenario, CBDC can be positioned as a replacement for cash. The central bank would position itself to satisfy the objective of establishing monetary sovereignty and maintaining parity between CBDC and bank deposit, but unable to maintain free convertibility between CBDC and bank deposit. Finally, third design as *the money maker* reveals CBDC serves as sovereign account money and perform the function as the unit of account. In this scenario, CBDC can be positioned as a replacement for bank deposit and the central bank takes the sole responsibility of creating and issuing money in the economy. The central bank would act to satisfy the objective of establishing monetary sovereignty and maintaining free convertibility between CBDC and bank deposit. So, in this scenario the central bank could effectively use monetary policy to create or destroy liquidity in the system based on the state of economy.

Bordo and Levin (2017) explained the design features of CBDC in order to satisfy the function of an currency: namely medium of exchange, store of value, and unit of account. The authors present two design mechanism for *medium of exchange*. According to first design, the central bank circulates CBDC tokens into economy supported with costly distributed ledger technology for ownership verification and payment transactions. Alternatively in second design, the central bank maintains CBDC accounts that facilitate electronic holding of funds for individuals and follow a simple debiting and crediting transaction protocol that is instantaneous and costless. The authors explored three alternatives for secure *store of value*. First, similar to paper currency, the central bank would issue CBDC with constant nominal value and earning zero interest. On the flip side, the central bank would be tightly constrained on negative nominal interest rate regime as individuals would substitute towards CBDC. Second, central bank would retain stable real value of CBDC through price level indexation of CBDC. However, such indexation puts central bank heavily constrained due to zero lower bound on real interest rates during recession. Third, the central bank would provide interest-bearing CBDC where the interest rate would be positive in a growing and stable price economy. The authors considered CBDC would serve stable *unit of account* with the help of flexible price-level targeting monetary policy. They mentioned a policy strategy focus towards the adjustment of CBDC interest rate for stabilizing the price level through Taylor Rule.

Barrdear and Kumhof (2016) developed a DSGE model with multiple sectors and several nominal and real rigidities to understand the effect of introduction of CBDC in the economy. The authors found an economy infused with CBDC equal to 30% of GDP result in substantial steady state output gains. Additionally, this effect persist if central bank issues large amount of CBDC against government bonds.

Traditionally, paper currency serves as the form of money. Central bank digital currency will serve as second form of money with its introduction into economy. The paper currency and CBDC fundamentally differ in the following properties. Though both physical cash and CBDC will be used in transaction process, physical cash has higher transaction cost than CBDC. While CBDC holding yields interest, there is no interest earning from cash holding. Cash-transaction provides a benefit of tax evasion, whereas CBDC-transaction entails tax payment.

Bordo and Levin (2017) mentioned that CBDC discourage tax evasion, corruption, money laundering, and other illegal activities in an economy. Developing countries faces rampant problems due to bribery, money laundering, low percentage of tax payer population, and so on. Hence, it is pertinent to ask: Can introduction of CBDC change the game plan for the economy? In other words, does introduction of CBDC generates positive behavior change in people and thus economy shifts towards more law-binding outcomes?

The present paper focus on the difference in tax evasion property between cash and CBDC. This paper examines the reduction in amount of tax evasion with the introduction of CBDC in the economy. A general equilibrium model with an representative agent, representative firm, and government is developed. The results of model is analyzed to examine the difference in outcomes on probability of detection of tax evasion and amount of tax evasion under cash regime and CBDC regime. The important implications of the model are investigated through monitoring and penalty cost function. Monitoring an agent for tax evasion is an costly business for the government. CBDC helps in reducing the variable cost of monitoring process that improves the probability of detection and diminishes the amount of tax evasion in the economy. The CBDC based transactions are more transparent and more easily tracked down. Though an agent faces a higher penalty cost in CBDC regime, (s)he enjoys a lower tax rate in CBDC regime. A higher penalty cost and lower tax rate imposed on agent under CBDC regime help in increasing the probability of detection and decreasing the amount of tax evasion in the economy.

The paper is organized as follows. Section 2 provides the model to study the relationship among the consumption by agent, probability of detection of tax evasion, and amount of tax evasion. Section 3 presents the analysis of the model by assuming functional forms and parameters values for cash regime and CBDC regime. The computation algorithm is described and key summary statistics in cash regime and CBDC regime is presented. Section 4 concludes.

3.2 Model

In this section, a dynamic general equilibrium model of tax evasion feature is developed. The model studies the probability of detecting tax evasion, the penalty for getting detected while evading taxes, monitoring cost for the government, and the amount of tax evasion in the economy.

3.2.1 Environment

The economy consists of a representative agent, representative firm, and government. This is a continuous time model and thus each variable is a function of time.[2] Each player solves her/his optimization problem. The role of each player and the assumptions for characterization of each player is described as follows.

The representative agent (RA) maximizes her/his present discounted value of all future utilities. RA derives utility from consumption goods (C). The following assumptions hold for the utility function ($U(C)$). The utility function is increasing in consumption goods and has diminishing marginal returns from the consumption good.[3] The utility function exhibits Inada conditions for C.[4]

[2]The functional notation for time,for example C(t), is dropped for brevity whenever necessary.

[3]$U_C(C) \geq 0$ and $U_{CC}(C) \leq 0$

[4]$\lim_{C\to 0} U_C(C) = \infty$; $\lim_{C\to\infty} U_C(C) = 0$

The representative agent has an asset position, A. Here C is assumed to be numeraire and the price of C is set to 1. Let r represents the real return in asset. RA decides to evade a proportion of her/his asset to reduce tax-burden. Let E represents the amount of tax evasion by the RA. The agent faces a penalty function, P which is an increasing convex function of E.

The representative firm (RF) maximizes the profit every period. RF produces output using only capital (K) as the input.The production function is assumed to be linearly homogeneous. The above assumption is satisfied by taking the functional form of production function as $F(K) = \Theta K$. Also, it is assumed that RA owns RF.

The government levy a flat rate tax τ on capital income. Government maximizes the amount of tax collected in the economy. Tax revenues collected by government are returned to RA as lump-sum transfer G. The government determines the probability of detection of tax evasion for RA as π. The monitoring process to detect the tax evasion is a costly business for the government. The government incurs a cost M which is an increasing convex function of π.

3.2.2 Dynamics of probability of detection and amount of tax evasion

Representative agent

The representative agent derives utility from consumption good. ρ is the subjective discounting factor of the representative agent. So, the agent maximizes the present discounted value of all future utility flows given by equation (3.1).

$$U_0 = \int_0^{\infty} e^{-\rho t} U(C(t)) \tag{3.1}$$

The agent faces the following budget constraint, represented by equation (3.2).

$$\dot{A}(t) = (1-\tau)r(t)(A(t) - E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t) \tag{3.2}$$

The budget constraint is interpreted as following. The LHS represents the change in the asset position.[5] The RHS denotes net asset position of RA. While the first three terms on RHS show the source of income for the RA, the last two terms on RHS denote the expenditure for the RA. The first term on RHS represents the net capital gain after tax for RA. The second and third term on RHS represent the government transfer and amount of tax evasion respectively. The fourth term indicates the amount of penalty paid for tax evasion. Finally, the last term denotes expenditure on consumption goods.

Let $r^{\tau}(t) = (1-\tau)r(t)$ for short-hand representation. Assume $\lambda(t)$ denote the shadow value of asset. The present value Hamiltonian is given by equation (3.3). The detailed process of solving this optimization problem using the dynamic optimization method is shown in Appendix C.1

$$\begin{aligned} PVH = \; & e^{-\rho t}(U(C(t))) \\ & + \lambda(t)((r^{\tau}(t)(A(t) - E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t)) \end{aligned} \tag{3.3}$$

The optimal path of consumption and amount of tax evasion is characterized as follows.[6]

$$C(t) = C(0)e^{(\frac{r^{\tau}(t)-\rho}{\sigma})t} \tag{3.4}$$

[5] $\dot{A}(t) = \frac{dA(t)}{dt}$

[6] The probability of detection is taken as given by RA. The optimal value of probability of detection is determined by the optimization problem of the government.

$$1 = \pi(t)P_E(E(t)) + r^{\tau}(t) \tag{3.5}$$

$$C(0) = \mu(0)(PV^G(0) + PV^E(0) + A(0)) \tag{3.6}$$

Equation (3.4) shows that the consumption profile of the RA grows at a rate $\frac{r^{\tau}(t)-\rho}{\sigma}$. Hence, the growth rate of consumption is a function of subjective discounting rate, risk aversion(σ), rate of return on asset, and tax-rate. Equation (3.5) represents the intra-temporal relationship for tax evasion. While LHS of equation (3.5) denotes the marginal benefit for tax evasion, RHS of same equation indicates marginal cost of the tax evasion as the sum of additional penalty payment and rate of return on assets. Equation (3.6) shows the initial consumption of RA. $\mu(0)$, $PV^G(0)$, $PV^E(0)$, and $A(0)$ is marginal propensity to consume, present discounted value of future earning from government transfers, present discounted value of future earning from expected benefits of tax evasion, and asset position at time, t=0 respectively.[7]

Representative Firm

The representative firm maximizes profit in each period. Let $\Pi(t)$ denotes the profit for RF.

$$\Pi(t) = \Theta K(t) - r(t)K(t) \tag{3.7}$$

RF chooses the optimal capital to maximizes the profit. Taking first order condition of profit with respect to capital yields

$$r(t) = \Theta \tag{3.8}$$

The RF rents capital until the marginal product of capital is equal to rental rate of

[7]The initial value of C(0) can also be obtained by using transversality condition. Refer appendix C.5 to see the derivation of C(0) from transversality condition.

capital. So, rental rate of capital is equal to a constant,Θ,based on the assumption of the model.

Relationship between tax evasion amount and probability of detection

Lemma: The amount of tax evasion is a decreasing function of probability of detection.

Proof: In equilibrium, the rate of return of asset is equal to the rental rate of capital and the asset holding of RA is equal to capital employed by RF. Thus, the relationship between the probability of detection and amount of tax evasion is obtained by differentiating the log of equation (3.5). The detailed process of determining the tax evasion amount as a function of probability of detection is carried in Appendix C.2.

$$E_{\pi}(\pi(t)) = -\frac{1}{\pi(t)}\frac{P_E(E(t))}{P_{EE}(E(t))} \tag{3.9}$$

Thus, the amount of tax evasion for the agent is a decreasing function of probability of detection since the penalty function is assumed to be increasing and convex.

Government

The government maximizes tax revenue of the economy. Government imposes a penalty on tax evader, which is an increasing function of amount of tax evasion. Also, the government wants to increase the probability of detection of tax evasion in the economy. However, monitoring requires strict institutional framework which is a costly business. The associated cost function is increasing and convex of probability of detection. The government has two sources of funds. First, it receives tax from the asset holding of the agents. Second, it captures the agent who evade tax and that agent pays the penalty. However, the government has two ways to use the funds. First, it provides back a lump-sum transfer to the household. Second, it incurs a cost for the monitoring process.

The government maximization problem is given by (3.10) subject to (3.11),(3.12), and (3.13)

$$\tau r(t)A(t) \tag{3.10}$$

s.t.

$$G(t) + M(\pi(t)) = \pi(t)P(E(t)) + \tau r(t)A(t) \tag{3.11}$$

$$r(t) = \Theta \tag{3.12}$$

$$E_{\pi}(\pi(t)) = -\frac{1}{\pi(t)}\frac{P_E(E(t))}{P_{EE}(E(t))} \tag{3.13}$$

The optimization problem is solved in Appendix C.3. The optimal value of probability of detection is determined by the following equation(3.14)

$$M_{\pi}(\pi(t)) = P(E(\pi(t))) - \frac{P_E(E(\pi(t)))^2}{P_{EE}(E(\pi(t)))} \tag{3.14}$$

3.2.3 Equilibrium

An equilibrium is defined as a path of consumption, $C(t)$, capital stock for the economy, $K(t)$, amount of tax evasion, $E(t)$, probability of detection of tax evasion, $\pi(t)$, return on assets, $r(t)$, tax, τ, penalty function, $P(E(t))$, monitoring cost function, $M(\pi(t))$ such that:

(a) Given $r(t), \tau$, $P(E(t))$, and $M(\pi(t))$; $C(t)$ and $E(t)$ maximizes the RA utility maximization problem.

(b) Given $r(t)$; $K(t)$ maximizes the RF profit maximization problem.

(c) Given $r(t), \tau$, $P(E(t))$, and $M(\pi(t))$; $\pi(t)$ maximizes the government optimization problem.

(d) Market clears during all time

(i) (Goods Market): C(t)=F(K(t))

(ii) (Asset Market): A(t)=K(t)

3.3 Analysis

Equation(3.5), (3.8), and (3.14) are key equations that determine the probability of detection and tax evasion amount in the economy. We need to assume certain functional form of the penalty and monitoring cost functions to understand the relationship. Suppose the penalty and monitoring cost function consist of fixed part and variable part. Hence, the functional specifications take the following form.

$$P(E) = \begin{cases} \alpha_1 + \alpha_2 E^\gamma, & \text{if } E > 0. \\ 0, & \text{if } E = 0. \end{cases} \tag{3.15}$$

$$M(\pi) = \begin{cases} \beta_1 + \beta_2 \pi^\eta, & \text{if } 0 < \pi \leq 1. \\ 0, & \text{if } \pi = 0. \end{cases} \tag{3.16}$$

Assume $\gamma > 1$ and $\eta > 1$. $\alpha_1, \alpha_2, \beta_1$, and β_2 are all assumed to be greater than 0. Hence, the increasing and convex property is satisfied for both penalty and monitoring cost function. Given the functional specification, the optimal value of probability of detection is determined using equation(3.14).[8]

$$M_\pi(\pi) = P(E(\pi)) - \frac{P_E(E(\pi))^2}{P_{EE}(E(\pi))}$$

[8]For simplicity, time in brackets is dropped.

$$
\begin{aligned}
\eta\beta_2\pi^{\eta-1} &= \alpha_1 + \alpha_2 E^{\gamma} - \frac{(\gamma\alpha_2 E^{\gamma-1})^2}{\gamma(\gamma-1)\alpha_2 E^{\gamma-2}} \\
&= \alpha_1 - \frac{\alpha_2 E^{\gamma}}{\gamma - 1}
\end{aligned}
\tag{3.17}
$$

Using equation(3.5) and (3.8), we obtain the relationship between amount of tax evasion and probability of detection.

$$
1 = \pi P_E(E) + (1-\tau)\Theta
$$

$$
\begin{aligned}
1 &= \pi\gamma\alpha_2 E^{\gamma-1} + (1-\tau)\Theta \\
E &= [\frac{1-(1-\tau)\Theta}{\pi\gamma\alpha_2}]^{\frac{1}{\gamma-1}}
\end{aligned}
\tag{3.18}
$$

Substituting the above result in equation(3.17), the optimal probability of detection is determined as a function of the model primitives.

$$
\eta\beta_2\pi^{\eta-1} = \alpha_1 - \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\pi\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}} \tag{3.19}
$$

Proposition 1: Given the functional form of penalty and monitoring cost and under certain parameter restriction, the optimal level of probability of detection increases if (a) the variable monitoring cost decreases, or (b) the fixed penalty cost increases, or (c) the variable penalty cost increases.[9]

Proof: Proof of proposition is shown in Appendix C.4. This proposition states that optimal level of probability of detection is a decreasing function of the variable monitoring cost (β_2), an increasing function of the fixed penalty cost (α_1), and an increasing function of the variable penalty cost (α_2).

[9]Functional form is given in equations(3.15)-(3.16) and parameter restriction is $\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}$.

Corollary 1: Given the functional form of penalty and monitoring cost and under certain parameter restriction, the optimal level of amount of tax evasion decreases if (a) the variable monitoring cost decreases, or (b) the fixed penalty cost increases, or (c) the variable penalty cost increases.

Proof: Lemma states that the amount of tax evasion is decreasing function of probability of detection. Combining the results of proposition 1 and lemma provide the proof of corollary. This corollary states that amount of tax evasion is an increasing function of the variable monitoring cost (β_2), a decreasing function of the fixed penalty cost (α_1), and a decreasing function of the variable penalty cost (α_2).

The asset position held by the representative agent is valued in physical cash. Suppose the government decides to pass a law that makes CBDC as the unit of account and the asset position will be valued in CBDC. The government would like to study the effect of passing the law,which implements CBDC as unit of account, on the tax evasion behavior of the agent. Thus, the economy can be categorized into two regimes: namely cash regime and CBDC regime. While only physical cash is circulated in the economy in cash regime, only CBDC is circulated in the economy in CBDC regime.[10] The properties of penalty and monitoring cost function differ for cash regime and CBDC regime. Hence, the implications of the model will vary by putting more structures on penalty and monitoring function for both regimes. Let $i = ca$ or dc stands for cash regime or CBDC regime respectively. The functional specifications (equations (15) and

[10]There exist another regime: named mixed regime in which both cash and CBDC are circulated in the economy. The mixed regime is not considered in this paper.

(16)) take the following form.

$$P(E) = \begin{cases} \alpha_1^i + \alpha_2^i E^{\gamma}, & \text{if } E > 0. \\ 0, & \text{if } E = 0. \end{cases} \tag{3.20}$$

$$M(\pi) = \begin{cases} \beta_1^i + \beta_2^i \pi^{\eta}, & \text{if } 0 < \pi \leq 1. \\ 0, & \text{if } \pi = 0. \end{cases} \tag{3.21}$$

Thus, the optimal value of probability of detection and amount of tax evasion in two regimes is given as following.

$$\eta \beta_2^i \pi^{\eta - 1} = \alpha_1^i - \frac{\alpha_2^i}{\gamma - 1}[\frac{1 - (1 - \tau)\Theta}{\pi \gamma \alpha_2^i}]^{\frac{\gamma}{\gamma - 1}} \tag{3.22}$$

$$E = [\frac{1 - (1 - \tau)\Theta}{\pi \gamma \alpha_2^i}]^{\frac{1}{\gamma - 1}} \tag{3.23}$$

Equation(3.22) represents a non-linear equation for probability of detection. It is solved using computer programming language. Assume the fixed and variable cost of penalty and monitoring function is constant over time. Equation(3.22) implies that the probability of detection and hence the amount of tax evasion in the economy is constant over time. The monitoring cost and penalty cost also becomes constant. Plugging those values back into the budget constraint of the RA, the equilibrium path of capital and consumption are determined. The equilibrium path of capital and consumption are given below for a given initial value of capital, $K(0)$. Appendix C.5 details the equilibrium path of capital.

$$K(t) = \vartheta_1 + \vartheta_3 e^{(\frac{(1-\tau)\Theta - \rho}{\sigma})t} \tag{3.24}$$

$$C(t) = C(0) e^{(\frac{(1-\tau)\Theta - \rho}{\sigma})t} \tag{3.25}$$

where $\vartheta_1 = -\frac{B}{\Theta}$, $\vartheta_3 = K(0) + \frac{B}{\Theta}$, $B = -M(\pi) + E[1 - (1 - \tau)\Theta]$, and $C(0) = -\frac{\sigma\Theta+\rho-(1-\tau)\Theta}{\sigma}(K(0) + \frac{B}{\Theta})$. The equilibrium path of consumption is different in both regimes.

The result of the model depends on the parameters of monitoring and penalty cost function. So, the government can engineer different outcomes for the economy based upon the choice of the parameters value under cash and CBDC regime. Government can implement six policy interventions to analyze the implications of the model. The first case involves the variation in variable cost of monitoring process for cash and CBDC regime. The second case involves the difference in fixed cost of monitoring process for cash and CBDC regime. The third case involves the difference in fixed cost and variable cost of monitoring process for cash and CBDC regime. The fourth case involves the difference in variable cost of penalty process for cash and CBDC regime. The fifth case involves the difference in fixed cost of penalty process for cash and CBDC regime. The final case involves the variation in fixed cost and variable cost of penalty function for cash and CBDC regime.

3.3.1 Policy

Difference in variable cost of monitoring process

The tax record documented in a centralized digital database improves the supervision process. The ease of monitoring is higher in CBDC regime than that in cash regime. So the variable cost, captured by β_2^i, is lower in CBDC regime than that in cash regime. In short, $\beta_2^{dc} < \beta_2^{ca}$. While the variable cost of monitoring process for cash regime varies between 6 and 8, the variable cost of monitoring process for CBDC regime varies between 4 and 6.

Difference in fixed cost of monitoring process

The CBDC regime requires a sophisticated infrastructure,which includes IT framework, Fintech, and wireless network facilities, for monitoring. Thus, the fixed cost for CBDC regime is assumed to be greater than that for cash regime. So, $\beta_1^{dc} > \beta_1^{ca}$. While the fixed cost of monitoring process for cash regime varies between 1 and 3, the fixed cost of monitoring process for CBDC regime varies between 3 and 5.

Difference in fixed and variable cost of monitoring process

The fixed and variable cost of monitoring process is varied for both regime. On the one hand, the fixed cost of monitoring process for cash regime varies between 1 and 3 and the variable cost of monitoring process for cash regime varies between 6 and 8. On the other hand, the fixed cost of monitoring process for CBDC regime varies between 3 and 5 the variable cost of monitoring process for CBDC regime varies between 4 and 6.

Difference in variable cost of penalty function

CBDC would lead to high transparent transaction and low unfair practice among agents. The government can impose a higher penalty function in CBDC regime than that in cash regime. The variable cost, captured by α_2^i, is higher in CBDC regime than that in cash regime. In short, $\alpha_2^{dc} > \alpha_2^{ca}$. While the variable cost of penalty function for cash regime varies between 2 and 4, the variable cost of penalty function for CBDC regime varies between 4 and 6.

Difference in fixed cost of penalty function

A better tractability under CBDC regime provides benefit in terms of higher penalty cost on agents. The fixed cost, captured by α_1^i, is higher in CBDC regime than that in cash regime. In short, $\alpha_1^{dc} > \alpha_1^{ca}$. While the fixed cost of penalty function for cash regime varies between 4 and 6, the fixed cost of penalty function for CBDC regime varies between 6 and 8.

Difference in fixed and variable cost of penalty process

Both fixed cost and variable cost of the penalty function differs in cash and CBDC regime under sixth policy intervention. The CBDC regime entails a higher penalty cost than cash regime. On the one hand, the fixed cost of penalty function for cash regime varies between 4 and 6 and the variable cost of penalty process for cash regime varies between 2 and 4. On the other hand, the fixed cost of penalty process for CBDC regime varies between 6 and 8 and the variable cost of penalty process for CBDC regime varies between 4 and 6.

3.3.2 Computational Algorithm

Each policy implication is studied by comparing the probability of detection and amount of tax evasion under cash and CBDC regime. The model is solved for each policy intervention using computer programming language.[11] A simple computational algorithm is developed to compute the variables under both regimes. The following steps are followed for each policy implementation.

(i) The number of simulation is set to 100.

(ii) The total number of time horizon of analysis is set to 5 years.

(iii) The model parameters are assumed for cash and CBDC regime. The parameters of penalty and monitoring function differs for cash and CBDC regime. Additionally, the parameters of penalty and monitoring function is assumed to follow an uniform distribution.

(iv) Non-linear equation(3.22) is solved using numerical analysis to determine the optimal probability of detection under both regimes at each simulation round.

(v) Plugging the optimal value of probability of detection in equation(3.23), the amount of tax evasion is determined under both regimes at each simulation round.

(vi) Using the optimal value of probability of detection and amount of tax evasion, the equilibrium consumption and capital path of RA are calculated for 5 years under both regimes at each simulation round.

(vii) The present discounted value of utility at time,t=0 is determined for both regimes at each simulation round.

(viii) The summary statistics of probability of detection, amount of tax evasion, and present discounted value of utility under cash and CBDC regime is reported.

(ix) The average consumption profile under cash and CBDC regime is shown.

[11]MATLAB is used to solve the model.

3.3.3 Parameters

Following parameters are used in different policy implementation.[12]

Table 3.1: Parameters

	Policy-I		Policy-II		Policy-III		Policy-IV		Policy-V		Policy-VI	
Parameter	**Cash**	**CBDC**	**Cash**	**CBDC**	**Cash**	**CBDC**	**Cash**	**CBDC**	**Cash**	**CBDC**	**Cash**	**CBDC**
ρ	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
σ	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Θ	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05	1.05
τ	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
γ	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
η	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
K_0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
α_1	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	[4,6]	[6,8]	[4,6]	[6,8]
α_2	4.00	4.00	4.00	4.00	4.00	4.00	[2,4]	[4,6]	4.00	4.00	[2,4]	[4,6]
β_1	3.00	3.00	[1,3]	[3,5]	[1,3]	[3,5]	3.00	3.00	3.00	3.00	3.00	3.00
β_2	[6,8]	[4,6]	6.00	6.00	[6,8]	[4,6]	6.00	6.00	6.00	6.00	6.00	6.00

Notes: Policy-I corresponds to difference in variable cost of monitoring function. Policy-II corresponds to difference in fixed cost of monitoring function. Policy-III corresponds to difference in fixed and variable cost of monitoring function. Policy-IV corresponds to difference in variable cost of penalty function. Policy-V corresponds to difference in fixed cost of penalty function. Policy-VI corresponds to difference in fixed and variable cost of penalty function.

3.3.4 Result

Using the above parameter values, the probability of detection and amount of tax evasion is determined using numerical approximation. Using those values, the consumption and capital profiles are computed. The present discounted value of utility is determined from consumption profile.Table 3.2 and 3.3 document the summary statistics of probability of detection, amount of tax evasion, and present discounted value of utility. Thus, on an average, the probability of

[12]The parameter value are chosen to satisfy the assumptions made for the model.

detection of tax evasion is higher in CBDC regime than that in cash regime and amount of tax evasion in CBDC regime is lower than that in cash regime.

Table 3.2: Summary Statistics for variation in monitoring cost

		Policy-I(VMC)			**Policy-II(FMC)**			**Policy-III(TMC)**		
Var	**Stat**	**Cash**	**CBDC**	**Change**	**Cash**	**CBDC**	**Change**	**Cash**	**CBDC**	**Change**
π	Mean	0.5344	0.6338	18.59%	0.5766	0.5766	0.00%	0.5370	0.6328	17.83%
	SD	0.0231	0.0395	0.0164	0.0000	0.0000	0.0000	0.0213	0.0330	0.0117
	Min	0.4995	0.5786	15.83%	0.5766	0.5766	0.00%	0.5006	0.5777	15.41%
	Max	0.5732	0.7055	23.08%	0.5766	0.5766	0.00%	0.5754	0.7019	21.98%
E	Mean	0.2034	0.1869	-8.10%	0.1957	0.1957	0.00%	0.2029	0.1870	-7.84%
	SD	0.0044	0.0058	0.0014	0.0000	0.0000	0.0000	0.0040	0.0049	0.0008
	Min	0.1963	0.1769	-9.86%	0.1957	0.1957	0.00%	0.1959	0.1774	-9.46%
	Max	0.2103	0.1954	-7.08%	0.1957	0.1957	0.00%	0.2100	0.1955	-6.91%
U_0	Mean	-0.8446	-0.8014	5.12%	-1.1895	-0.6664	43.98%	-1.2651	-0.6523	48.44%
	SD	0.0107	0.0162	0.0055	0.2615	0.0837	-0.1778	0.2815	0.0773	-0.2041
	Min	-0.8609	-0.8246	4.23%	-1.7003	-0.8206	51.74%	-1.8569	-0.8128	56.23%
	Max	-0.8269	-0.7724	6.59%	-0.8259	-0.5433	34.21%	-0.8284	-0.5385	35.00%

Notes: π, E, and U_0 refer to probability of detection, the amount of tax evasion, and welfare (computed as the present discounted value of consumption) respectively. VMC, FMC, TMC refer to the difference in variable monitoring cost, in fixed monitoring cost, and in fixed and variable monitoring cost process respectively.

The consumption profile over 5 periods is calculated. Figures 3.1 and 3.2 present the consumption profile for RA under cash and CBDC regime in different policy implementation. Graphically, it is observed that RA derives higher consumption levels in CBDC regime than that in cash regime. The present discounted value of utility for the RA at time, t=0, is calculated for both regimes in different policy implementation. While the present discounted value of utility for the RA at time, t=0, in CBDC regime is -0.8041 utils, the present discounted value of utility for the RA at time, t=0, in cash regime is -0.8446 utils for policy-I. Consider the present discounted value of utility for the RA at time, t=0, as a measure of welfare. Comparatively, RA has a higher welfare in CBDC regime.[13]

Numerically, it can be inferred from the six policy cases that CBDC circula-

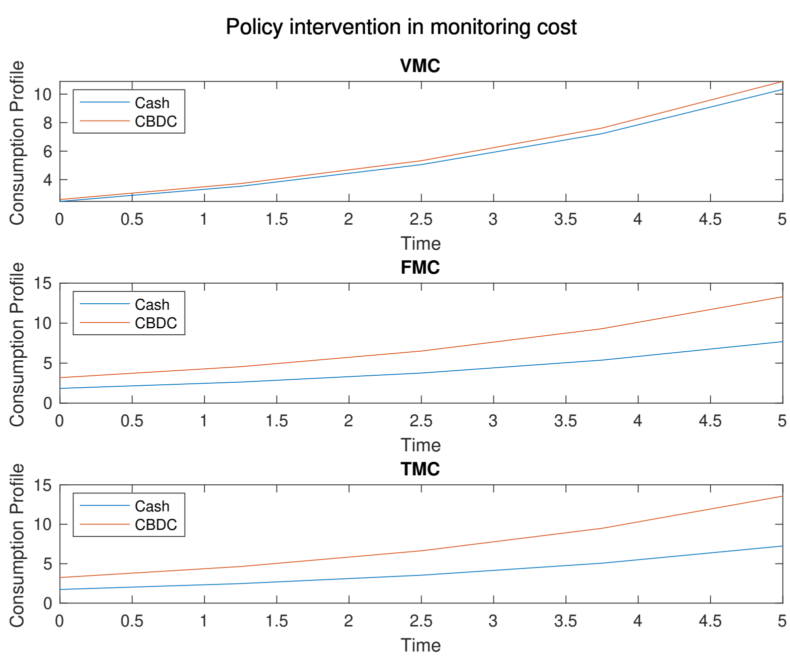
[13]Similar results are observed in other policies.

Table 3.3: Summary Statistics for variation in penalty cost

Var	Stat	Policy-IV(VPC)			Policy-V(FPC)			Policy-VI(TPC)		
		Cash	CBDC	Change	Cash	CBDC	Change	Cash	CBDC	Change
π	Mean	0.5765	0.5767	0.04%	0.5283	0.6230	17.93%	0.5257	0.7027	33.67%
	SD	0.0001	0.0000	-0.0000	0.0308	0.0262	-0.0046	0.0314	0.0221	-0.0093
	Min	0.5763	0.5766	0.05%	0.4709	0.5776	22.66%	0.4701	0.6665	41.77 %
	Max	0.5766	0.5768	0.02%	0.5762	0.6660	15.60%	0.5759	0.7448	29.34%
E	Mean	0.2302	0.1751	-23.93%	0.2047	0.1884	-7.97%	0.2440	0.1592	-34.76%
	SD	0.0216	0.0101	-0.0115	0.0060	0.0040	-0.0021	0.0243	0.0097	-0.0146
	Min	0.1959	0.1598	-18.40%	0.1958	0.1821	-6.99%	0.1990	0.1432	-28.06%
	Max	0.2742	0.1955	-28.69%	0.2166	0.1955	-9.71%	0.2955	0.1794	-39.30%
U_0	Mean	-0.8276	-0.8241	0.41%	-0.8869	-0.7652	13.72%	-0.8928	-0.6631	25.72%
	SD	0.0013	0.0006	-0.0007	0.0387	0.0339	-0.0047	0.0396	0.0272	-0.0123
	Min	-0.8303	-0.8254	0.59%	-0.9576	-0.8242	13.93%	-0.9625	-0.7088	26.36%
	Max	-0.8254	-0.8232	0.27%	-0.8260	-0.7096	14.09%	-0.8295	-0.6114	26.29%

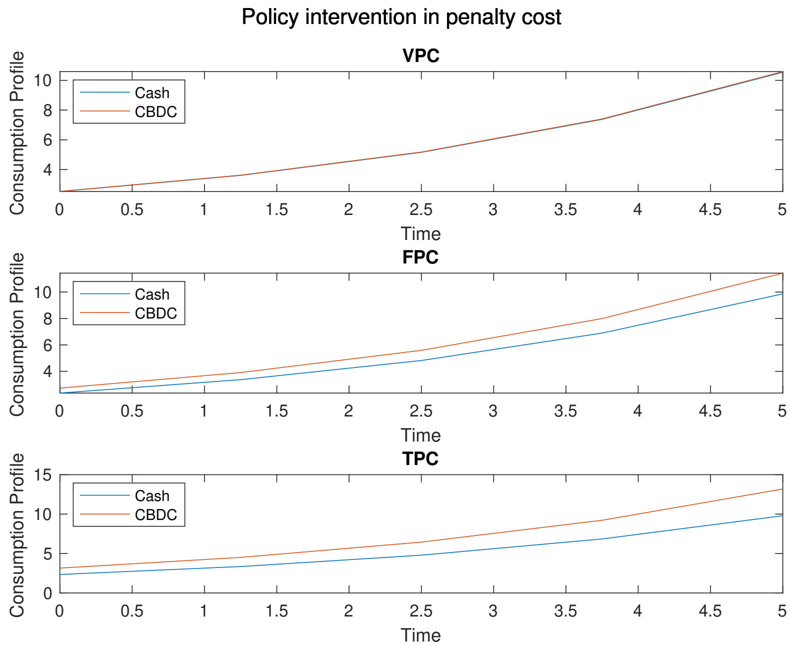
Notes: π, E, and U_0 refer to probability of detection, the amount of tax evasion, and welfare (computed as the present discounted value of consumption) respectively. VPC, FPC, TPC refer to the difference in variable penalty cost, in fixed penalty cost, and in fixed and variable penalty cost process respectively.

Figure 3.1: Consumption path in Cash and CBDC Regime for variation in monitoring cost



Notes: This figure shows that consumption profile of the representative agent for difference in variable monitoring cost, in fixed monitoring cost, and in fixed and variable monitoring cost process.

Figure 3.2: Consumption path in Cash and CBDC Regime for variation in penalty cost



Notes: This figure shows that consumption profile of the representative agent for difference in variable penalty cost, in fixed penalty cost, and in fixed and variable penalty cost process.

tion lowers the amount of tax evasion and has potential benefits.

3.3.5 Extension

Equation 3.19 provides the relationship between probability of detection and tax rate. The effect of tax rate on the probability of detection can be designed to generate favorable outcomes in CBDC regime.

Proposition 2: Given the functional form of penalty and monitoring cost and under certain parameter restriction, the optimal level of probability of detection increases if the tax rate decreases.[14]

[14] Functional form is given in equations(3.15)-(3.16) and parameter restriction is $\eta(\eta-1)\beta_2\pi^{\eta-2}-$

Proof: Proof of proposition is shown in Appendix C.6. This proposition states that optimal level of probability of detection is a decreasing function of the tax rate.

Corollary 2: Given the functional form of penalty and monitoring cost and under certain parameter restriction, the optimal level of amount of tax evasion decreases if the tax rate decreases.

Proof: Lemma states that the amount of tax evasion is decreasing function of probability of detection. Combining the results of proposition 2 and lemma provide the proof of corollary. This corollary states that amount of tax evasion is an increasing function of the tax rate.

The representative agent faces higher penalty cost in CBDC regime than that in cash regime. The government can motivate the agent to move towards CBDC regime by lowering the tax rate in CBDC regime. The CBDC economy will exhibit a higher probability of detection, lower amount of tax evasion, and higher present discounted value of utility than cash economy. These effects are outcomes of the combined results of proposition 1 and 2.

A CBDC regime with higher penalty cost and lower tax rate will induce agents move from cash regime to CBDC regime. The penalty cost in CBDC regime is chosen higher than that in cash regime. While the tax rate is fixed at 0.30 in cash regime, the tax rate is chosen between 0.10 and 0.30 in CBDC regime. All other parameters are kept the same as in previous policy implementation. Following parameters are used for tax policy implementation.

The computational method as described in 3.3.2 is followed with few modi-

---

$\frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}$.

Table 3.4: Parameters

Parameter	Cash	Policy-VII CBDC
ρ	0.02	0.02
σ	2.00	2.00
Θ	1.05	1.05
τ	0.30	[0.10,0.15,0.20,0.25,0.30]
γ	3.00	3.00
η	3.00	3.00
K_0	0.25	0.25
α_1	5.00	7.00
α_2	3.00	5.00
β_1	1.00	1.00
β_2	8.00	8.00

Notes: Policy-VII corresponds to the variations in the tax rates.

fications. In this exercise, no simulation is considered. The non-linear equation (3.22) is solved for tax rate = 0.30 in cash regime to pin down the probability of detection in cash regime. Similarly, the non-linear equation (3.22) is solved for different values of tax rate in CBDC regime to pin down the probability of detection in CBDC regime. Thereafter, the amount of tax evasion, present discounted value of utility, and present discounted value of government transfer are determined.[15] Table 3.5 records those variable values in cash and CBDC regime.

While the probability of detection, present discounted value of utility, and present discounted value of government transfer for different values of tax rates in CBDC regime are higher than those in cash regime, the amount of tax evasion in CBDC is lower than that in cash regime. When the tax rate is reduced, the tax evasion is reduced. Government collects higher tax revenue and transfers greater amounts to the agent. The agent are motivated by lower tax rate and

[15] $G_0 = e^{-\rho t}(\pi(t)P(E(t)) + \tau r(t)A(t) - M(\pi(t))) = e^{-\rho t}(\pi P(E) + \tau\Theta K(t) - M(\pi))$

Table 3.5: Result

	Cash	CBDC				
Variable	$\tau = 0.30$	$\tau = 0.10$	$\tau = 0.15$	$\tau = 0.20$	$\tau = 0.25$	$\tau = 0.30$
π	0.4553	0.5400	0.5399	0.5398	0.5397	0.5395
E	0.2543	0.0824	0.1152	0.1406	0.1620	0.1810
U_0	-1.7078	-1.6197	-1.6125	-1.6117	-1.6168	-1.6274
G_0	0.3831	4.4481	4.0474	3.7405	3.5510	3.4638

Notes: π, E, U_0, and G_0 refer to probability of detection, the amount of tax evasion, welfare (computed as the present discounted value of consumption), and government surplus.

higher government transfer to move towards CBDC regime.

3.3.6 Discussion

The first, second, and third policy interventions can be broadly categorized into monitoring process changes. The fourth, fifth, and sixth policy interventions can be broadly categorized into penalty imposed changes. The seventh policy intervention includes a higher penalty function but a lower tax rate for the agent in CBDC regime. In general, every policy intervention leads to higher probability of detection of tax evasion, lower amount of tax evasion, and higher welfare implications for economy in CBDC regime than those in cash regime.

It is important to understand that the CBDC regime results are better than cash-regime results only under appropriate choice of parameters for cash regime and CBDC regime.[16] In economic sense, the government need to make necessary changes in monitoring process and penalty system along with circulation of CBDC in the system.

[16]Parameter restriction in propositions refer to the parameter selection which leads to favorable outcomes for CBDC regime. It can be shown under certain choice of parameter values, cash regime results are more favorable than CBDC regime results

The focal point of the system can be attributed by the optimal value of probability of detection of tax evasion and amount of tax evasion in the economy. Introduction of CBDC leads to change in the structure of economy. Specifically, CBDC would improve the monitoring process and penalty imposed with the suitable choices of parameters. These changes result in moving the salience point of the economy from a lower probability of detection of tax evasion and higher amount of tax evasion in cash regime to a higher probability of detection of tax evasion and lower amount of tax evasion in CBDC regime. Thus, CBDC brings positive behavior changes in people of economy.

3.4 Conclusion

This paper documents lower tax evasion amount with the introduction of CBDC in the economy. This paper identifies the difference in characteristics of penalty and monitoring cost function for cash regime and CBDC regime. On the one hand, the model predicts the probability of detection of tax evasion is higher in CBDC regime than that in cash regime. On the other hand, the model predicts the amount of tax evasion is lower in CBDC regime than that in cash regime.

The paper also indicates that the welfare of consumer as measured in terms of present discount value of utility is higher in CBDC regime than that in cash regime. The implementation of monitoring process to capture tax evaders under CBDC regime requires high fixed cost. However, the benefit is realized with a higher consumption profile for the consumer in CBDC regime. This result infers that the well-being of consumer based on her consumption profile is enhanced with the circulation of CBDC in the economy.

The theory nicely extends to the implication where the agent in incentivized to move towards the high penalty function and low tax rate CBDC regime. The government can lower the tax rate and transfer higher funds to the agents in CBDC regime than those in cash regime. This implementation suggests the benefits in terms of better consumption profile and higher government transfer for consumers.

This model examines only the tax evasion property difference between cash and CBDC. Hence, the model predicts clearly that the results in CBDC regime is better than those in cash regime. However, the model did not capture the transaction process and interest-bearing property of CBDC and cash. The present model paves the path to a enriched model that captures the trade-offs among the transaction process, interest-bearing, and tax evasion properties of CBDC and cash. The enriched model would also provide the conditions for presence of cash-only, CBDC-only, both cash and CBDC in the economy.

APPENDIX A

APPENDIX OF CHAPTER 1

A.1 A Empirical observation appendix

A.1.1 Safe assets evolution

Safe assets are defined, following Davis and van Wincoop (2021), as the sum of portfolio debt assets and other debt assets. This definition caters to private safe assets. The economies are categorized into advanced economies, emerging market economies, and low income developing countries following the IMF country classification.[1] The External Wealth of Nations database is the primary source for presenting the safe assets evolution. This database collates the external assets and liabilities as well as gross domestic product panel data for two hundred fifteen economies from 1970 to 2020. The time series mean, adjusted for the standard deviation, of safe assets demand in AEs and EMEs are determined.[2] Figure A.1 shows the safe assets distribution for the advanced economies and emerging market economies.

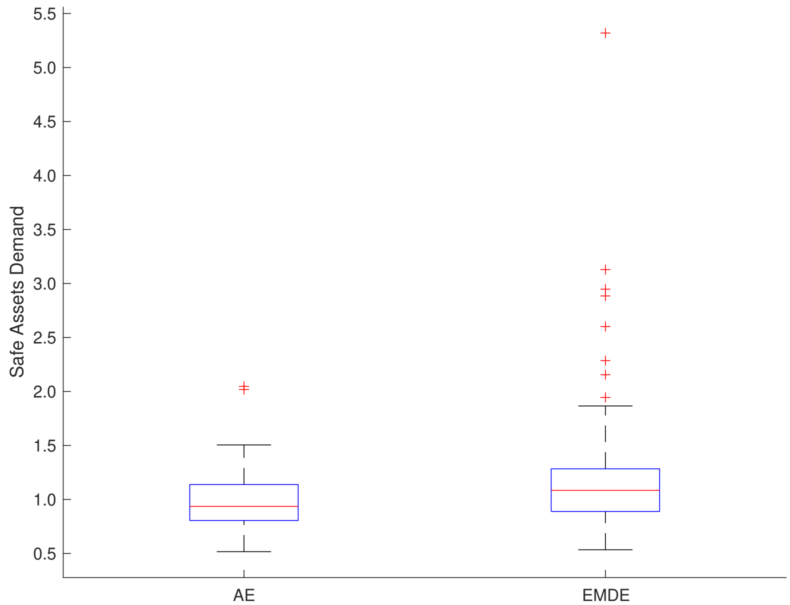
The cross-sectional coefficient of variation for the EMEs are calculated.[3] Figure A.2 depicts the dispersion of safe assets demand for the EMEs.

[1] IMF identifies 39 countries as advanced economies, 96 countries as emerging market economies, and 59 countries as low income developing countries.

[2] $\mu_A = \frac{\sum_{t=1}^{T} SA_t^A}{T}, \mu_E = \frac{\sum_{t=1}^{T} SA_t^E}{T}, \sigma_A = \frac{\sum_{t=1}^{T}(SA_t^A - \mu_A)}{T-1}, \sigma_E = \frac{\sum_{t=1}^{T}(SA_t^E - \mu_E)}{T-1}, SA_A = \frac{\mu_A}{\sigma_A}, and SA_E = \frac{\mu_E}{\sigma_E}$ represent the time series average of safe assets for an AE, average of safe assets for an EME, standard deviation of safe assets for an AE, standard deviation of safe assets for an EME, mean adjusted for standard deviation for an AE, and mean adjusted for standard deviation for an EME respectively.

[3] $\mu_t^E = \frac{\sum_{i=1}^{N} SA_t^i}{N}, \sigma_t^E = \frac{\sum_{i=1}^{N}(SA_t^i - \mu_t^e)}{N-1}, and CEV_t^E = \frac{\mu_t^E}{\sigma_t^E}$ represent the cross-sectional mean of safe assets, standard deviation of safe assets, and coefficient of variation for an EME respectively.

Figure A.1: AE and EME safe assets demand distribution



Notes: AE represents advanced economies and EME represents emerging market economies. The central mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The whiskers extend to the most extreme data points not considered outliers, and the outliers are plotted individually using the '+' marker symbol.

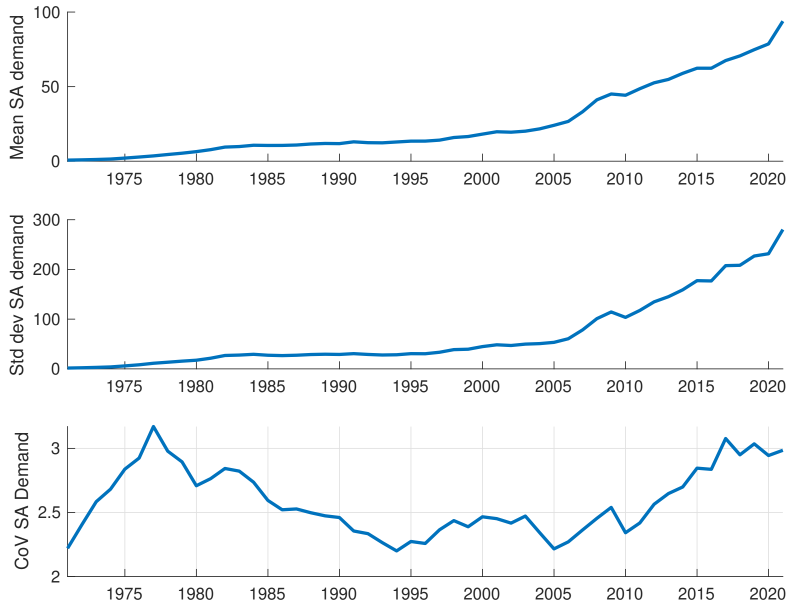
The time series of the demand for safe assets across all EMEs and safe assets normalized by the gross domestic product for EMEs are calculated.[4] Figure A.3 shows the evolution of safe assets in EMEs.

The stake of EME investors in the total safe assets demand is quantified.[5] Figure A.4 shows that safe assets held by investors of EMEs is rising and is about one-fifth of the total private safe assets holdings of the EMEs.

[4]The evolution of the safe assets for EMEs is calculated as follows. Let N be the number of emerging market and developing economies. $SA_t^E = \sum_{i=1}^{N} SA_t and SAGDP_t^E = \frac{\sum_{i=1}^{N} SA_t}{\sum_{i=1}^{N} GDP_t} \times 100$

[5]The investors' safe assets and share of investor safe assets in an EME is given by $ISA_t^E = \sum_{i=1}^{N} ISA_t and ISAS_t^E = \frac{ISA_t^E}{SA_t^E} \times 100$ respectively.

Figure A.2: EME safe assets demand



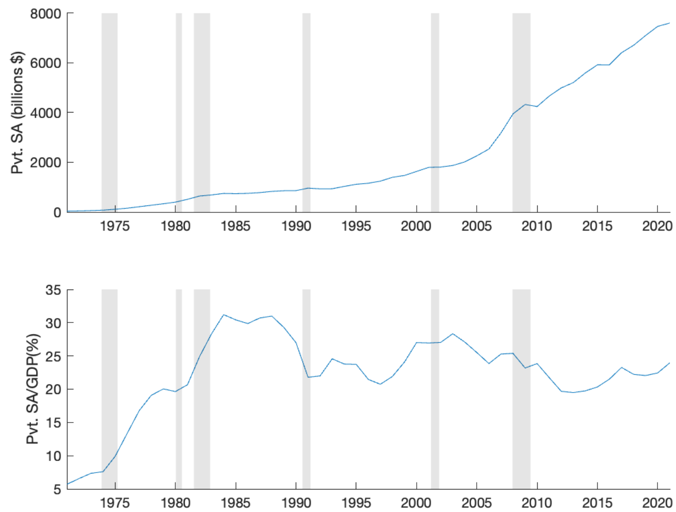
Notes: The top panel shows the time series of mean safe assets demand of EME, middle panel shows the time series of standard deviation safe assets demand among EMEs, and bottom panel shows the time series of coefficient of variation of EMEs. The units for top and middle panels are billions of U.S. dollars.

A.2 Convenience yield

A.2.1 Derivation of convenience yield

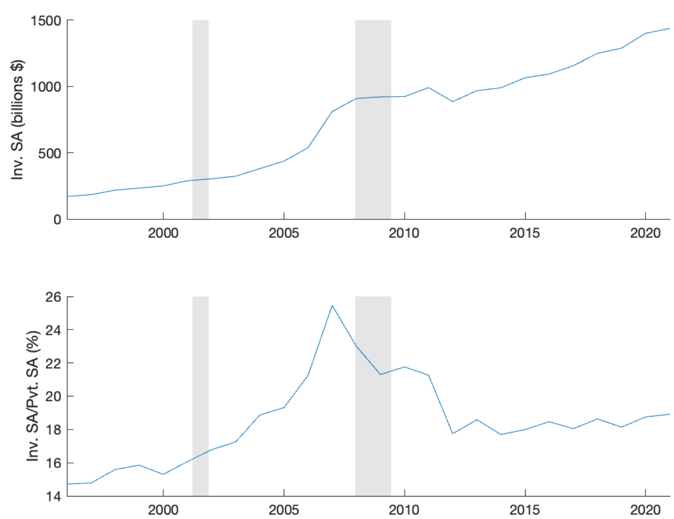
Covered interest parity (CIP) deviations between the emerging market economy and safe assets providers can be used to derive the implied relative convenience yield. An investor can invest in home government bonds. Alternatively, the investor can convert a unit home currency at the given exchange rate for international currency, invest in international government bonds, and reap the returns back into the home currency at the future exchange rate. This strategy is defined as synthetic home currency converted international government bonds. CIP states that a unit home currency invested in home government bonds is

Figure A.3: Safe assets evolution in EMEs



Notes: The top panel shows the total private safe assets position across all EMEs. The bottom panel shows the total private safe assets position normalized by GDP across all EMEs. The grey shaded bars denote the NBER-dated recession.

Figure A.4: Investors Safe assets evolution in EMEs



Notes: The top panel shows the total investor-held safe assets position across all EMEs. The bottom panel shows the share of investors' safe assets across all EMEs. The grey shaded bars denote the NBER-dated recession.

equivalent to the synthetic home currency converted international government bonds.

Let R_{ht} and R_{it} denote the nominal interest rate on the government bonds of home country and the nominal interest rate on the government bonds of home international country respectively. Let exchange rate be E_{hit}. $\mathbb{E}(E_{hit+1})$ is the expected value of future exchange rate. Mathematically, CIP condition is as given by following equation.

$$1 + R_{ht} = \frac{\mathbb{E}(E_{hit+1})}{E_{hit}}(1 + R_{it})$$

Taking logs on both sides and using first order approximation, we get

$$R_{ht} = (\mathbb{E}(e_{hit+1}) - e_{hit}) + R_{it}$$

Let CIP deviations be defined as the difference between the yields on two investments. Mathematically,

$$CIPD = R_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R_{it})$$

Table A.1 shows the mean of CIP deviations of fifteen EMEs against the U.S, Euro, U.K, and Japan safe assets from 2000 to 2021 and during different subsamples.

CIP deviations capture default and liquidity risks of the home government bonds. In the context of this paper, the international government bond refers to the safe assets and the home government bond refers to an EME government bond. Let the probability of default of an EME government bond be p_{ht} and that of a safe asset be p_{it}. The default risk adjusted rate of return on an EME government bond is $(1 + R^d_{ht}) = p_{ht}0 + (1 - p_{ht})(1 + R_{ht})$. Similarly, the default risk adjusted rate of return on the safe asset is $(1 + R^d_{it}) = p_{it}0 + (1 - p_{it})(1 + R_{it})$. Thus, the liquidity benefit (LB) of the safe asset is measured after factoring the default

Table A.1: Covered interest rate deviations in basis points

	2000-2021				Pre GFC				GFC				Post GFC			
	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA
BR	215	194	207	168	170	148	158	140	325	275	301	263	206	189	201	161
CO	45	23	37	0	47	26	40	18	118	67	95	49	34	17	29	-12
HU	102	80	94	63	72	50	64	51	291	236	261	203	77	63	74	53
IN	109	90	104	96	127	104	130	133	281	244	260	329	84	69	81	46
ID	-154	93	107	-204	-620	469	493	-678	55	2	29	-17	-7	-25	-13	-53
IS	65	44	58	21	13	-8	5	-18	58	3	33	-13	81	64	76	37
MA	46	50	52	-209	13	31	3	-782	151	103	126	86	70	54	66	25
ME	43	25	37	-19					-39	-74	-58	-96	44	26	39	-18
PE	-269	-284	-271	-305									-269	-284	-271	-305
PO	65	36	49	27	54	16	30	31	226	210	224	154	52	25	37	8
RU	16	-6	8	-35	70	50	63	44	-261	-314	-289	-93	33	15	27	-22
SA	21	-1	13	-23	4	-20	-6	-23	13	-37	-7	-62	27	10	22	-19
SK	124	105	118	81	327	95	375	60	366	314	340	280	103	87	99	66
TH	123	76	90	82	191	87	103	165	201	101	123	127	68	63	75	22
TU	88	66	79	32	219	196	205	159	341	288	316	244	28	11	23	-24
AV	75	72	86	28	89	94	143	15	142	95	117	124	74	57	69	28

Notes: BR, CO, HU, IN, ID, IS, MA, ME, PE, PO, RU, SA, SK, TH, TU, and AV denote Brazil, Colombia, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Peru, Poland, Russia, South Africa, South Korea, Thailand, Turkey, weighted cross sectional average respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. CIP deviation mean are shown in basis points for sample periods from 2000 to 2021, Pre Global Financial Crisis, Global Financial Crisis, and Post Global Financial Crisis periods. The first observation for Mexico is 26-May-2009 and Peru is 20-Jan-2010 respectively. Thus, there are gaps in the Pre GFC period for Mexico and in the Pre GFC and GFC periods for Peru.

risk into the CIP deviations.

$$LB = R^d_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R^d_{it}) = (R_{ht} - p_{ht}(1+R_{ht})) - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + (R_{it} - p_{it}(1+R_{it})))$$

Convenience yield is the non-pecuniary benefit of the government bonds. Let ϕ_i and ϕ_h represent convenience yield of a safe asset and an EME government bond.

$$1 + R^d_{ht} + \phi_{ht} = \frac{\mathbb{E}(E_{hit+1})}{E_{hit}}(1 + R^d_{it} + \phi_{it})$$

The relative convenience yield is defined as the difference between the convenience yield of international government bonds and that of home government bonds. The relative convenience yield is measured by the liquidity benefit.

$$\phi_{it} - \phi_{ht} = LB$$

A.2.2 Safety and liquidity components

CIP deviations represent the default and liquidity risks of the EME. CIP deviation is decomposed into the safety and liquidity benefits of the safe asset.

$$\begin{aligned}
CIPD =& R_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R_{it}) \\
=& R^{d}_{ht} + p_{ht}(1 + R_{ht}) - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R^{d}_{it} + p_{it}(1 + R_{it})) \\
=& R^{d}_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R^{d}_{it}) + p_{ht}(1 + R_{ht}) - p_{it}(1 + R_{it}) \\
=& \underbrace{R^{d}_{ht} - ((\mathbb{E}(e_{hit+1}) - e_{hit}) + R^{d}_{it})}_{Liquidity} + \underbrace{p_{ht}(1 + R_{ht}) - p_{it}(1 + R_{it})}_{Safety}
\end{aligned}$$

Table A.2 and A.3 show the mean of safety and liquidity benefits of the U.S., Euro area, U.K., and Japan safe assets from 2000 to 2021 and during different sub-samples for fifteen EMEs.

A.2.3 Forecasting the convenience yield

The algorithm used for the forecasting is as follows.

1. There are four safe asset providers, namely the U.S., Euro area, U.K., and Japan.
2. The time series of the convenience yield of a safe asset provider for an average EME, CY^{sa}_{t}, is chosen.
3. The stationarity condition of CY^{sa}_{t} is checked.

Table A.2: Safety benefits in basis points

	2000-2021				Pre GFC				GFC				Post GFC			
	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA
BR	54	54	56	20	45	45	64	13	43	43	43	57	56	56	56	16
CO	41	41	46	16	44	44	63	13	53	53	53	70	37	37	37	11
HU	86	86	102	30	129	129	184	37	103	103	103	138	49	49	49	14
IN	52	52	56	21	171	171	244	49	88	88	88	118	35	35	35	10
ID	47	47	52	23	41	41	58	12	127	127	127	169	32	32	32	9
IS	29	29	33	10	55	55	78	16	35	35	35	46	22	22	22	6
MA	24	24	27	9	16	16	23	5	34	34	34	45	19	19	19	5
ME	34	34	34	9					47	47	47	63	33	33	33	9
PE	26	26	26	7									26	26	26	7
PO	29	29	35	11	41	41	58	12	35	35	35	47	22	22	22	6
RU	70	70	78	29	64	64	91	18	142	142	142	189	64	64	64	18
SA	53	53	57	20	50	50	71	14	59	59	59	79	52	52	52	15
SK	30	30	36	13	65	65	93	19	59	59	59	78	16	16	16	5
TH	62	62	70	21	33	33	47	9	54	54	54	72	67	67	67	19
TU	76	76	79	26	52	52	75	15	59	59	59	79	83	83	83	24
AV	50	50	56	19	40	40	58	12	76	76	76	101	42	42	42	12

Notes: BR, CO, HU, IN, ID, IS, MA, ME, PE, PO, RU, SA, SK, TH, TU, and AV denote Brazil, Colombia, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Peru, Poland, Russia, South Africa, South Korea, Thailand, Turkey, weighted cross sectional average respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. Safety benefit mean are shown in basis points for sample periods from 2000 to 2021, Pre Global Financial Crisis, Global Financial Crisis, and Post Global Financial Crisis periods. The first observation for Mexico is 26-May-2009 and Peru is 20-Jan-2010 respectively. Thus, there are gaps in the Pre GFC period for Mexico and in the Pre GFC and GFC periods for Peru.

4. The CY_t^{sa} is split into train and test samples.

5. A seasonal ARIMA (SARIMA) model is examined to CY_t^{sa}

   (a) Create a cartesian space for the combination hyper-parameters.

   (b) Fit SARIMA model on the train sample based on a particular hyper-parameter combination.

   (c) Predict the series for the same sample period as that of test sample.

   (d) Compute the root mean square error (RMSE) between the predicted and test samples.

Table A.3: Liquidity benefits in basis points

	2000-2021				Pre GFC				GFC				Post GFC			
	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA	US	EU	UK	JA
BR	161	140	151	148	125	103	94	127	282	232	258	206	150	133	145	145
CO	4	-18	-9	-16	3	-18	-23	5	65	14	42	-21	-3	-20	-8	-23
HU	16	-6	-8	33	-57	-79	-120	14	188	133	158	65	28	14	25	39
IN	57	38	48	75	-44	-67	-114	84	193	156	172	211	49	34	46	36
ID	-201	46	55	-227	-661	428	435	-690	-72	-125	-98	-186	-39	-57	-45	-62
IS	36	15	25	11	-42	-63	-73	-34	23	-32	-2	-59	59	42	54	31
MA	22	26	25	-218	-3	15	-20	-787	117	69	92	41	51	35	47	20
ME	9	-9	3	-28					-86	-121	-105	-159	11	-7	6	-27
PE	-295	-310	-297	-312									-295	-310	-297	-312
PO	36	7	14	16	13	-25	-28	19	191	175	189	107	30	3	15	2
RU	-54	-76	-70	-64	6	-14	-28	26	-403	-456	-431	-282	-31	-49	-37	-40
SA	-32	-54	-44	-43	-46	-70	-77	-37	-46	-96	-66	-141	-25	-42	-30	-34
SK	94	75	82	68	262	30	282	41	307	255	281	202	87	71	83	61
TH	61	14	20	61	158	54	56	156	147	47	69	55	1	-4	8	3
TU	12	-10	0	6	167	144	130	144	282	229	257	165	-55	-72	-60	-48
AV	25	22	30	9	49	54	85	3	66	19	41	23	32	15	27	16

Notes: BR, CO, HU, IN, ID, IS, MA, ME, PE, PO, RU, SA, SK, TH, TU, and AV denote Brazil, Colombia, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Peru, Poland, Russia, South Africa, South Korea, Thailand, Turkey, weighted cross sectional average respectively. US, EU, UK, and JA denote United States, Euro area, United Kingdom, and Japan respectively. Liquidity benefit mean are shown in basis points for sample periods from 2000 to 2021, Pre Global Financial Crisis, Global Financial Crisis, and Post Global Financial Crisis periods. The first observation for Mexico is 26-May-2009 and Peru is 20-Jan-2010 respectively. Thus, there are gaps in the Pre GFC period for Mexico and in the Pre GFC and GFC periods for Peru.

(e) Repeat step (b) - (d) for all possible combination of hyper-parameters.

(f) Choose the optimal hyper-parameters that minimizes the RMSE.

6. The best SARIMA model, which has the optimal hyper-parameters, is used to forecast the CY_t^{sa}.

7. Steps (2) - (6) are performed for $sa \vee$ the U.S., Euro area, U.K., and Japan.

A.3 Model appendix

A.3.1 Representative investor

Investor's evolution of wealth is given by

$$\begin{aligned} dW_t^h =&(Y_t^h - C_t^h - L_t^h)dt + W_t^h \tilde{\theta}_h^h \frac{dA_{ht}}{A_{ht}} + W_t^h \sum_{i=1}^{N} \theta_{it}^h \frac{dB_{it}^h}{B_{it}^h} \\ =&(Y_t^h - C_t^h - L_t^h)dt + (R_{ht} + (\alpha_h - R_{ht})\tilde{\theta}_{ht}^h + \sum_{i \neq h}(R_{it} - R_{ht} + \mu_{hi})\theta_{it}^h)W_t^h dt \\ &+ \beta_h \tilde{\theta}_{ht}^h W_t^h dZ_{ht}^A + \sum_{i \neq h} \sigma_{ih}\theta_{it}^h W_t^h dZ_{hit} \end{aligned}$$

Let η_s represent the financial state of the home economy and it follows a two-state poisson process $\{\eta_{fc}, \eta_{nfc}\}$ with transition rate between the states equal to λ.[6] The rate of return on the home safe asset, the rate of return and volatility on the home index fund, and the liquidity cost of the home assets depend on η. The Hamilton-Jacobi-Bellman equation of the investor in a home country h is

$$\begin{aligned} \rho V(W^h, Y^h; \eta_s) =&U(C^h) \\ &+ V_W(-C^h + (R_h(\eta_s) + (\alpha_h(\eta_s) - R_h(\eta_s))\tilde{\theta}_h^h \\ &+ \sum_{i \neq h}(R_i - R_h(\eta_s) + \mu_{hi})\theta_i^h)W^h) \\ &- L((\phi_h^l(\eta_s) + \tilde{\phi}_h^l(\eta_s)\tilde{\theta}_h^h + \sum_{i \neq h}(\phi_i^l - \phi_h^l(\eta_s))\theta_i^h)W^h) \\ &+ \frac{1}{2}V_{WW}(\beta_h(\eta_s))^2(\tilde{\theta}_h^h W^h)^2 + \frac{1}{2}V_{WW}\sum_i \sum_k \psi_{ik}^h \theta_i^h \theta_k^h (W^h)^2 \\ &+ V_Y \mu(Y^h) + \frac{1}{2}V_{YY}\sigma^2(Y^h) \\ &+ V_{WY} r_{hY}\beta_h \sigma(Y^h)\tilde{\theta}_h^h W^h + V_{WY}\sum_{i \neq h} r_{iY}\sigma_{ih}\sigma(Y^h)\theta_i^h W^h \\ &+ \lambda(V(W^h, Y^h; \eta_{s'}) - V(W^h, Y^h; \eta_s)) \end{aligned}$$

[6] States of the economy, denoted by s, can be in financial crisis (fc) or in no financial crisis (nfc).

The first order conditions with respect to consumption and share of wealth invested in the home risky asset and safe asset of international country i are

$$U_C(C^h) = V_W(W^h, Y^h; \eta_s)$$

$$V_W(\alpha_h(\eta_s) - R_h(\eta_s))W^h - V_W L'(.)(\phi_h^l(\eta_s) - \tilde{\phi}_h^l(\eta_s))W^h$$
$$+V_{WW}(\beta_h(\eta_s))^2(W^h)^2\tilde{\theta}_h^h + V_{WY} r_{hy}(\beta_h(\eta_s))\sigma(Y^{ha}; \eta_s)W^h = 0$$

$$V_W(R_i - R_h(\eta_s) + \mu_{hi})W^h - V_W L'(.)(\phi_h^l(\eta_s) - \phi_i^l)W^h$$
$$+V_{WW}(W^h)^2 \sum_{k \neq h} \psi_{ik}^h \theta_k^h + V_{WY} r_{iY} \sigma_{ih} \sigma(Y^h) W^h = 0$$

A.4 Result appendix

A.4.1 Optimal demand of assets in the portfolio of the investor

Let the marginal increase in consumption due to wealth (income) be $C_w = \frac{\partial C}{\partial W}(C_y = \frac{\partial C}{\partial Y})$. Let ϵ_{WC} and ϵ_{YC} represent the wealth elasticity of consumption and the income elasticity of consumption respectively. The optimal demand of the home risky index asset is given as

$$\begin{aligned}\tilde{\theta}_h^h =& \frac{-V_W}{V_{WW}W^h}\frac{\alpha_h(\eta_s) - R_h(\eta_s)}{(\beta_h(\eta_s))^2} + \frac{-V_{WY}}{V_{WW}W^h}\frac{r_{hY}\sigma(Y^h)}{\beta_h(\eta_s)} \\ =& \frac{1}{\gamma\epsilon_{WC}^h}\frac{\alpha_h(\eta_s) - R_h(\eta_s)}{(\beta_h(\eta_s))^2} + \frac{-\epsilon_{YC}^h}{\epsilon_{WC}^{ha}Y^h}\frac{r_{hY}\sigma(Y^h)}{\beta_h(\eta_s)}\end{aligned}$$

The optimal demand of the safe assets is given as

$$
\begin{aligned}
\Theta^h =& \frac{-V_W}{V_{WW}W^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-V_{WY}}{V_{WW}W^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h) \\
=& \frac{-U_C}{U_{CC}C_W^h W^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-U_{CC}C_Y^h}{U_{CC}C_W^h W^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h) \\
=& \frac{-C^{h^{-\gamma}}}{-\gamma C^{h^{-\gamma-1}}C_W^h W^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-\gamma C^{h^{-\gamma-1}}C_Y^h}{\gamma C^{h^{-\gamma-1}}C_W^h W^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h) \\
=& \frac{1C^h}{\gamma C_W^h W^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-C_Y^h}{C_W^h W^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h) \\
=& \frac{1}{\gamma \epsilon_{WC}^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-C_Y^h \frac{Y^h}{C^h}\frac{C^h}{Y^h}}{C_W^h \frac{W^{ha}}{C^{ha}}C^{ha}}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h) \\
=& \frac{1}{\gamma \epsilon_{WC}^h}(\Psi^h)^{-1}(R^h - R_h(\eta_s) + \mu^h + \phi_h^l(\eta) - \Phi^{lh}) \\
&+ \frac{-\epsilon_{YC}^h}{\epsilon_{WC}^h Y^h}(\Psi^h)^{-1}r_Y^h(diag(\Psi^h)^{\frac{1}{2}}\sigma(Y^h)
\end{aligned}
$$

A.4.2 Proof of Proposition 1

Let us assume there is one international safe asset. The optimal demand for that international safe asset is given by

$$
\begin{aligned}
\theta_i^h =& \frac{1}{\gamma \epsilon_{WC}^h}(\sigma_{hi}^2)^{-1}(R_i - R_h) + \frac{1}{\gamma \epsilon_{WC}^h}(\sigma_{hi}^2)^{-1}\mu_{hi} \\
&+ \frac{1}{\gamma \epsilon_{WC}^h}(\sigma_{hi}^2)^{-1}(-(\phi_i^l - \phi_h^l)) + \frac{-\epsilon_{YC}^h}{\epsilon_{WC}^h Y^h}(\sigma_{hi}^2)^{-1}r_{iY}\sigma_{hi}\sigma_Y
\end{aligned}
\tag{A.4.2}
$$

Taking the derivative of A.4.2 with respect to interest rate differential yields

$$\frac{\partial\theta_i^h}{\partial(R_i - R_h)} = \frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-1} > 0$$

Taking the derivative of A.4.2 with respect to depreciation of home country exchange rate yields

$$\frac{\partial\theta_i^h}{\partial\mu_{hi}} = \frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-1} > 0$$

Taking the derivative of A.4.2 with respect to liquidity cost differential yields

$$\frac{\partial\theta_i^h}{\partial(\phi_i^l - \phi_h^l)} = -\frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-1} < 0$$

Taking the derivative of A.4.2 with respect to risk aversion of the investor yields

$$\frac{\partial\theta_i^h}{\partial\gamma} = -[\frac{1}{\gamma^2\epsilon_{WC}^h}(\sigma_{hi}^2)^{-1}(R_i - R_h + \mu_{hi} + \phi_h^l - \phi_i^h)] \gtrless 0$$

Taking the derivative of A.4.2 with respect to volatility of home country exchange rate yields

$$\begin{aligned}\frac{\partial\theta_i^h}{\partial\sigma_{hi}^2} = -&[\frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-2}(R_i - R_h) + \frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-2}(\mu_{hi}) \\ &+ \frac{1}{\gamma\epsilon_{WC}^h}(\sigma_{hi}^2)^{-2}(-(\phi_i - \phi_h))] - \frac{1}{2}\frac{-\epsilon_{YC}^h}{\epsilon_{WC}^h Y^h} r_{iY}(\sigma_{hi}^2)^{-\frac{3}{2}}\sigma(Y^h) \gtrless 0\end{aligned}$$

Taking the derivative of A.4.2 with respect to volatility of income yields

$$\frac{\partial\theta_i^h}{\partial\sigma_Y^2} = \frac{1}{2}\frac{-\epsilon_{YC}^h}{\epsilon_{WC}^h Y^h}(\sigma_{hi}^2)^{-1} r_{iY}(\sigma_{hi})^{-1}(\sigma_Y^2)^{-\frac{1}{2}} \gtrless 0$$

A.4.3 Proof of Corollary 1

Uncovered interest parity states that the interest rate differential between an international government bond and home government bond must be equal to the appreciation or depreciation of the exchange rate. There is clear violation of uncovered interest parity which is seen by rearranging the optimal demand for the safe assets. So, the interest rate differential is the sum of the exchange rate premium, liquidity premium, wealth premium, and income premium channels.

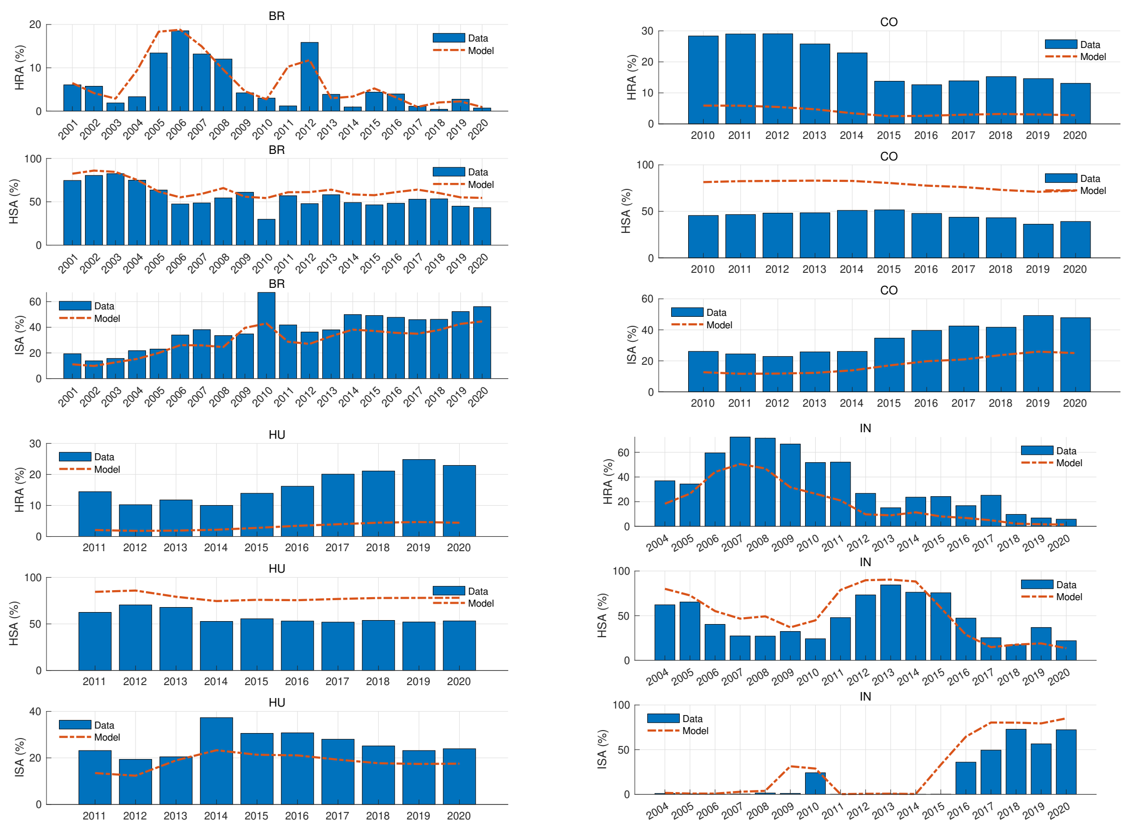
$$R_h - R^h = \underbrace{\mu^h}_{Exchangerate} + \underbrace{(-(\Phi^{lh} - \phi_h^l))}_{Liquidity} + \underbrace{-\Theta^{ha}\gamma\epsilon_{WC}^{ha}\Psi^h}_{Wealth} + \underbrace{\frac{-\epsilon_{YC}^{ha}}{Y^{ha}}\gamma r_Y^h(diag(\Psi^h))^{\frac{1}{2}}\sigma(Y^{ha})}_{Income}$$

A.5 Quantitative Inferences

A.5.1 Model and Data Variables

The index fund of the EME, the one year government bond of the EME, and the total international portfolio of debt assets for the EME represent the home risky, the home safe, and the international safe assets respectively in the model. The index fund and government bond holdings are obtained from the CEIC and the international portfolio of debt assets are obtained from the CPIS. Figure A.5 shows the comparison of the share of the home risky asset, the home safe asset , and the international safe asset for Brazil, Colombia, Hungary, and India from the actual data and model generated counterpart.

Figure A.5: Shares of different assets in EMEs



Notes: BR, CO, HU, and IN represent Brazil, Colombia, Hungary, and India respectively.

APPENDIX B

APPENDIX OF CHAPTER 2

B.1 Real budget constraint and real liquidity constraint

The nominal budget constraint is as follows:

$$\begin{aligned} P_t c_t + P_t c_t^g + P_t i_t + B_t + P_t \phi(c_t, \frac{CA_{t-1}}{P_t}, \frac{DC_{t-1}}{P_t}) + P_t m\psi(\frac{CA_{t-1}}{P_t}) + CA_t \\ + DC_t + \tau DC_{t-1} \leq P_t f(k_{t-1}) + I_{t-1}^b B_{t-1} + I_{t-1}^d DC_{t-1} + CA_{t-1} \end{aligned} \tag{B.1.1}$$

Divide P_t throughout equation B.1.1

$$\begin{aligned} c_t + c_t^g + i_t + \frac{B_t}{P_t} + \phi(c_t, \frac{CA_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}, \frac{DC_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}) + m\psi(\frac{CA_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}) + \frac{CA_t}{P_t} \\ + \frac{DC_t}{P_t} + \tau\frac{DC_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} \leq f(k_{t-1}) + I_{t-1}^b \frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + I_{t-1}^d \frac{DC_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{CA_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} \end{aligned}$$

Thus, the real budget constraint is as follows:

$$\begin{aligned} c_t + c_t^g + i_t + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t} dc_{t-1} \\ \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t} b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t} dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \end{aligned} \tag{B.1.2}$$

The nominal liquidity constraint is as follows:

$$P_t c_t \leq CA_{t-1} + DC_{t-1} \tag{B.1.3}$$

Divide P_t throughout equation B.1.3

$$P_t c_t \leq \frac{CA_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} + \frac{DC_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}$$

Thus, the real liquidity constraint is as follows:

$$c_t \leq \frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} \tag{B.1.4}$$

B.2 Dynamic optimization of representative agent problem

The Bellman equation for the optimization problem is as follows:

$$V(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \max\{u(c_t) + v(c_t^g) + \beta V(k_t, b_t, ca_t, dc_t) + \lambda_t(\frac{ca_{t-1}}{1+\pi_t} + \frac{dc_{t-1}}{1+\pi_t} - c_t)\} \tag{B.2.1}$$

s.t.

$$c_t + c_t^g + k_t - (1-\delta)k_{t-1} + b_t + \phi(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t}) + m\psi(\frac{ca_{t-1}}{1+\pi_t}) + ca_t + dc_t + \frac{\tau}{1+\pi_t}dc_{t-1} \leq f(k_{t-1}) + \frac{I_{t-1}^b}{1+\pi_t}b_{t-1} + \frac{I_{t-1}^d}{1+\pi_t}dc_{t-1} + \frac{ca_{t-1}}{1+\pi_t} \tag{B.2.2}$$

The first order conditions with respect to the choice variables yields

$$u_c(c_t) = \beta V_k(k_t, b_t, ca_t, dc_t)(1 + \phi_c(c_t, \frac{ca_{t-1}}{1+\pi_t}, \frac{dc_{t-1}}{1+\pi_t})) - \lambda_t \tag{B.2.3}$$

$$v_{c^g}(c_t^g) = \beta V_k(k_t, b_t, ca_t, dc_t) \tag{B.2.4}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_b(k_t, b_t, ca_t, dc_t) \tag{B.2.5}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_{ca}(k_t, b_t, ca_t, dc_t) \tag{B.2.6}$$

$$\beta V_k(k_t, b_t, ca_t, dc_t) = \beta V_{dc}(k_t, b_t, ca_t, dc_t) \tag{B.2.7}$$

The envelope conditions with respect to the state variables yields

$$V_k(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)(f_k(k_{t-1}) + 1 - \delta) \tag{B.2.8}$$

$$V_b(k_{t-1}, b_{t-1}, ca_{t-1}, dc_{t-1}) = \beta V_k(k_t, b_t, ca_t, dc_t)\frac{I_{t-1}^b}{1+\pi_t} \tag{B.2.9}$$

$$V_{ca}(k_{t-1},b_{t-1},ca_{t-1},dc_{t-1}) = \beta V_k(k_t,b_t,ca_t,dc_t)(\frac{1}{1+\pi_t} - \frac{m\psi_{ca}(\frac{ca_{t-1}}{1+\pi_t})}{1+\pi_t} - \frac{\phi_{ca}(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})}{1+\pi_t}) + \frac{\lambda_t}{1+\pi_t} \quad \text{(B.2.10)}$$

$$V_{dc}(k_{t-1},b_{t-1},ca_{t-1},dc_{t-1}) = \beta V_k(k_t,b_t,ca_t,dc_t)(\frac{I^d_{t-1}}{1+\pi_t} - \frac{\tau}{1+\pi_t} - \frac{\phi_{dc}(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})}{1+\pi_t}) + \frac{\lambda_t}{1+\pi_t} \quad \text{(B.2.11)}$$

Updating the envelope conditions by one time period, multiplying by β, and using the first order conditions yields the inter-temporal conditions.[1]

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = v_{c^g}(c^g_t) \quad \text{(B.2.12)}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(f_k(k_t)+1-\delta) \quad \text{(B.2.13)}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}\frac{I^b_t}{1+\pi_{t+1}} \quad \text{(B.2.14)}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{1+\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(\frac{1}{1+\pi_{t+1}} - \frac{m\psi_{ca}(\frac{ca_t}{1+\pi_{t+1}})}{1+\pi_{t+1}} - \frac{\phi_{ca}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}) \quad \text{(B.2.15)}$$

$$\frac{u_c(c_t)}{1+\phi_c(c_t,\frac{ca_{t-1}}{1+\pi_t},\frac{dc_{t-1}}{1+\pi_t})} = \beta\frac{u_c(c_{t+1})}{\phi_c(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}(\frac{I^d_t}{1+\pi_{t+1}} - \frac{\tau}{1+\pi_{t+1}} - \frac{\phi_{dc}(c_{t+1},\frac{ca_t}{1+\pi_{t+1}},\frac{dc_t}{1+\pi_{t+1}})}{1+\pi_{t+1}}) \quad \text{(B.2.16)}$$

[1]Assuming that the liquidity constraint does not hold in the equilibrium, the Lagrange multiplier associated with that constraint (λ_t) is zero.

B.3 Steady state analysis

The utility function is assumed to be

$$u(c) = \frac{c^{1-\epsilon_c}}{1-\epsilon_c} \tag{B.3.1}$$

$$u_c(c) = c^{-\epsilon_c} > 0 \tag{B.3.2}$$

$$u_{cc}(c) = -\epsilon_c c^{-\epsilon_c - 1} < 0 \tag{B.3.3}$$

$$\lim_{c\to 0} u_c(c) = \lim_{c\to 0} c^{-\epsilon_c} = \infty; \lim_{c\to\infty} u_c(c) = \lim_{c\to\infty} c^{-\epsilon_c} = 0 \tag{B.3.4}$$

$$v(c^g) = \frac{(c^g)^{1-\epsilon_g}}{1-\epsilon_g} \tag{B.3.5}$$

$$v_{c^g}(c^g) = (c^g)^{-\epsilon_g} > 0 \tag{B.3.6}$$

$$v_{c^g c^g}(c^g) = -\epsilon_g (c^g)^{-\epsilon_g - 1} < 0 \tag{B.3.7}$$

$$\lim_{c^g\to 0} v_{c^g}(c^g) = \lim_{c^g\to 0} (c^g)^{-\epsilon_g} = \infty; \lim_{c^g\to\infty} v_{c^g}(c^g) = \lim_{c^g\to\infty} (c^g)^{-\epsilon_g} = 0 \tag{B.3.8}$$

Utility functions are strictly increasing, strictly concave, and satisfy the Inada conditions, which is confirmed by equations (B.3.2)-(B.3.4) and (B.3.6)-(B.3.8).

The production function is given by

$$f(k) = Ak^{\alpha} \tag{B.3.9}$$

where $0 < \alpha < 1$

$$f_k(k) = A\alpha k^{\alpha-1} > 0 \tag{B.3.10}$$

$$f_{kk}(k) = A\alpha(\alpha-1)k^{\alpha-2} < 0 \tag{B.3.11}$$

$$\lim_{k\to 0} f_k(k) = \lim_{k\to 0} \alpha k^{\alpha-1} = \infty; \lim_{k\to\infty} f_k(k) = \lim_{k\to\infty} \alpha k^{\alpha-1} = 0 \tag{B.3.12}$$

The production function is strictly increasing, strictly concave, and satisfies the Inada conditions, which is confirmed by equations (B.3.10), (B.3.11), and (B.3.12).

The penalty function takes the form

$$\psi(ca) = \theta_2 ca^{\nu} \tag{B.3.13}$$

where $\theta_2 > 1$ and $\nu > 1$

$$\psi_{ca}(ca) = \nu\theta_2 ca^{\nu-1} > 0 \tag{B.3.14}$$

The penalty function is strictly increasing, which is guaranteed by equation (B.3.14).

The transaction function is assumed to be

$$\phi(c, ca, dc) = \theta_1 \frac{c^a}{(ca)^{\gamma}(dc)^{\rho}} \tag{B.3.15}$$

where $a > 1$ and $\gamma < \rho$

$$\phi(0, ca, dc) = \theta_1 \frac{0^a}{(ca)^{\gamma}(dc)^{\rho}} = 0 \tag{B.3.16}$$

$$\phi_c(c, ca, dc) = a\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{c} > 0 \tag{B.3.17}$$

$$\phi_{ca}(c, ca, dc) = -\gamma\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(ca)} < 0 \tag{B.3.18}$$

$$\phi_{dc}(c, ca, dc) = -\rho\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)} < 0 \tag{B.3.19}$$

$$\phi_{cc}(c, ca, dc) = a(a-1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{c^2} > 0 \tag{B.3.20}$$

$$\phi_{caca}(c, ca, dc) = \gamma(\gamma+1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(ca)^2} > 0 \tag{B.3.21}$$

$$\phi_{dcdc}(c, ca, dc) = \rho(\rho+1)\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)^2} > 0 \tag{B.3.22}$$

$$\phi_{cac}(c, ca, dc) = -a\gamma\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(ca)c} < 0 \tag{B.3.23}$$

$$\phi_{dcc}(c, ca, dc) = -a\rho\theta_1 \frac{c^a}{(ca)^\gamma (dc)^\rho} \frac{1}{(dc)c} < 0 \tag{B.3.24}$$

The transaction function satisfies all the conditions mentioned in section 2.1.

Solving for equilibrium in steady state yields the following.

$$\begin{aligned} 1 &= \beta(f_k(\bar{k}) + 1 - \delta) = \beta(A\alpha\bar{k}^{\alpha-1} + (1-\delta)) \\ \Rightarrow \bar{k} &= \left(\frac{A\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} = \Theta_k(A, \alpha, \beta, \delta) \end{aligned} \tag{B.3.25}$$

$$\frac{\bar{I}^b}{1+\bar{\pi}} = \frac{1}{\beta} \Rightarrow \bar{I}^b = \Theta_{I^b}(\beta, \bar{\pi}) \tag{B.3.26}$$

$$\bar{c} + \bar{c}^g + \delta\bar{k} + \bar{b} + \phi(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}}) + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) + \bar{ca} + \bar{dc} + \frac{\tau}{1+\bar{\pi}}\bar{dc} = f(\bar{k}) + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} \tag{B.3.27}$$

$$\bar{c}^g = \eta\bar{g} \tag{B.3.28}$$

$$\bar{g} - \eta\bar{g} + \frac{\bar{I}^b}{1+\bar{\pi}}\bar{b} + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} = \bar{b} + \bar{dc} + \bar{ca} + \frac{\tau}{1+\pi_t}\bar{dc} + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) \tag{B.3.29}$$

Substituting equations (B.3.28) and (B.3.29) in equation (B.3.27)

$$\bar{c} + \bar{g} + \phi(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}}) = f(\bar{k}) - \delta\bar{k} = A\bar{k}^{\alpha} - \delta\bar{k} = \theta_3(A, \alpha, \beta, \delta) \tag{B.3.30}$$

$$\frac{(\bar{c})^{-\epsilon_c}}{1 + \phi_c(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})} = (\eta\bar{g})^{-\epsilon_g} \tag{B.3.31}$$

$$\begin{aligned} 0 = \Delta(\bar{ca} + \bar{dc}) &= \frac{\mu - \bar{\pi}}{1+\bar{\pi}}(\bar{ca} + \bar{dc}) \\ \Rightarrow \bar{\pi} &= \mu = \Theta_{\pi}(\mu) \end{aligned} \tag{B.3.32}$$

$$\begin{aligned} 1 &= \beta(\frac{1}{1+\bar{\pi}} - \frac{m\psi_{ca}(\frac{\bar{ca}}{1+\bar{\pi}})}{1+\bar{\pi}} - \frac{\phi_{ca}(\bar{c}, \frac{\bar{ca}}{1+\bar{\pi}}, \frac{\bar{dc}}{1+\bar{\pi}})}{1+\bar{\pi}}) \\ \Rightarrow 1 &= \beta(\frac{1}{1+\mu} - \frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu} + \gamma\theta_1\frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}}\frac{1}{(\bar{ca})}) \\ \Rightarrow 1 - \beta\frac{1}{1+\mu} + \beta\frac{m\nu\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu} &= \beta\gamma\theta_1\frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^{\gamma}(\frac{\bar{dc}}{1+\mu})^{\rho}}\frac{1}{(\bar{ca})} \end{aligned} \tag{B.3.33}$$

The government sets the nominal interest rate of CBDC in steady state.

$$1 = \beta(\frac{\bar{I^d}}{1+\bar{\pi}} - \frac{\tau}{1+\bar{\pi}} - \frac{\phi_{dc}(\bar{c}, \bar{ca}, \bar{dc})}{1+\bar{\pi}})$$
$$\Rightarrow 1 = \beta(\frac{\bar{I^d}}{1+\mu} - \frac{\tau}{1+\mu} + \rho\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^\gamma (\frac{\bar{dc}}{1+\mu})^\rho} \frac{1}{(\bar{dc})}) \tag{B.3.34}$$
$$\Rightarrow 1 - \beta\frac{\bar{I^d}}{1+\mu} + \beta\frac{\tau}{1+\mu} = \beta\rho\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^\gamma (\frac{\bar{dc}}{1+\mu})^\rho} \frac{1}{(\bar{dc})}$$

Dividing equation (B.3.33) by equation (B.3.34)

$$\frac{1 - \beta\frac{1}{1+\mu} + \beta\frac{mv\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu}}{1 - \beta\frac{\bar{I^d}}{1+\mu} + \beta\frac{\tau}{1+\mu}} = \frac{\gamma}{\rho}\frac{(\bar{dc})}{(\bar{ca})} \tag{B.3.35}$$
$$\Rightarrow \bar{dc} = \theta_4(\bar{ca}, \alpha, \beta, \delta, \gamma, \rho, \tau, m, \bar{I^d}, \theta_2, \mu, \nu)$$

Substituting equation(B.3.35) in equation(B.3.33)

$$1 - \beta(\frac{1}{1+\mu} - \frac{mv\theta_2(\frac{\bar{ca}}{1+\mu})^{\nu-1}}{1+\mu}) = \beta\gamma\theta_1 \frac{\bar{c}^a}{(\frac{\bar{ca}}{1+\mu})^\gamma (\frac{\theta_4}{1+\mu})^\rho} \frac{1}{(\bar{ca})} \tag{B.3.36}$$
$$\Rightarrow \bar{ca} = \theta_5(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I^d}, \theta_1, \theta_2, \mu, \nu)$$

So,

$$\Rightarrow \bar{dc} = \theta_6(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I^d}, \theta_1, \theta_2, \mu, \nu) \tag{B.3.37}$$

Substituting the value of $\bar{ca}$ and $\bar{dc}$ in equation (B.3.31)

$$\frac{(\bar{c})^{-\epsilon_c}}{1 + \phi_c(\bar{c}, \frac{\theta_5}{1+\mu}, \frac{\theta_6}{1+\mu})} = (\eta\bar{g})^{-\epsilon_g} \tag{B.3.38}$$
$$\Rightarrow \bar{g} = \theta_7(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I^d}, \theta_1, \theta_2, \mu, \nu, \eta, \epsilon_c, \epsilon_g)$$

Finally, the steady state value of private consumption is obtained from equation (B.3.30)

$$\bar{c} + \theta_7(\bar{c}, a, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I^d}, \theta_1, \theta_2, \mu, \nu, \eta, \epsilon_c, \epsilon_g) + \phi(\bar{c}, \frac{\theta_5}{1+\mu}, \frac{\theta_6}{1+\mu}) = \theta_3(A, \alpha, \beta, \delta)$$

$$\bar{c} = \Theta_c(a, A, \alpha, \beta, \delta, a, \gamma, \rho, \tau, m, \bar{I^d}, \theta_1, \theta_2, \mu, \nu, \eta, \epsilon_c, \epsilon_g) \tag{B.3.39}$$

Thus, the steady state values of cash, CBDC, and government expenditure are obtained from equations (B.3.36), (B.3.37), and (B.3.38) and are functions of the model parameters.

The steady state level of bond holdings is obtained from the consolidated government budget constraint.

$$
\begin{aligned}
\bar{g} - \eta\bar{g} + \frac{\bar{I}^b}{1+\bar{\pi}}b + \frac{\bar{I}^d}{1+\bar{\pi}}\bar{dc} + \frac{\bar{ca}}{1+\bar{\pi}} &= \tau\bar{dc} + m\psi(\frac{\bar{ca}}{1+\bar{\pi}}) + \bar{b} + \bar{ca} + \bar{dc} \\
\Rightarrow \Theta_g - \eta\Theta_g + \frac{\Theta_{I^b}}{1+\mu}\bar{b} + \frac{\bar{I}^d}{1+\mu}\Theta_{dc} + \frac{\Theta_{ca}}{1+\mu} &= \tau\Theta_{dc} + m\psi(\frac{\Theta_{ca}}{1+\mu}) + \bar{b} + \Theta_{ca} + \Theta_{dc}
\end{aligned}
\tag{B.3.40}
$$

B.4 Welfare gain calculation

$$
\begin{aligned}
V_0^a =& \sum_{t=0}^{\infty}\beta^t(u((1+\omega^a)c_t^b) + v(c_t^{gb})) \\
=& \sum_{t=0}^{\infty}\beta^t(\frac{((1+\omega^a)c_t^b)^{1-\epsilon_c}}{1-\epsilon_c} + \frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}) \\
=&(1+\omega^a)^{1-\epsilon_c}\sum_{t=0}^{\infty}\beta^t\frac{(c_t^b)^{1-\epsilon_c}}{1-\epsilon_c} + \sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g} \\
=&(1+\omega^a)^{1-\epsilon_c}(\sum_{t=0}^{\infty}\beta^t(\frac{(c_t^b)^{1-\epsilon_c}}{1-\epsilon_c} + \frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g}) - (1+\omega^a)^{1-\epsilon_c}\sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g} + \sum_{t=0}^{\infty}\beta^t\frac{(c_t^{gb})^{1-\epsilon_g}}{1-\epsilon_g} \\
=&(1+\omega^a)^{1-\epsilon_c}V_0^b + (1-(1+\omega^a)^{1-\epsilon_c})G_0^b \\
=&(1+\omega^a)^{1-\epsilon_c}(V_0^b - G_0^b) + G_0^b \\
\implies \omega^a =&(\frac{V_0^a - G_0^b}{V_0^b - G_0^b})^{\frac{1}{1-\epsilon_c}} - 1
\end{aligned}
$$

B.5 Sensitivity tests for policy parameter choices around alternative baselines

In the next set of tables, we present sensitivity tests for the main policy parameters but around the alternative baseline scenarios shown in Table 2.3. The main results are all preserved. For instance, cash and CBDC co-exist under all of the parameter combinations. Similarly, the share of CBDC holdings rises and that of alternative assets falls when the rate of return on CBDC rises.

Table B.1: Steady state shares (in percent) for baseline and alternative models

Description	**BM-I**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	37.10	37.13	36.96	37.07	37.07	37.17	38.55
Bond holdings	7.99	7.88	8.02	8.02	8.02	7.59	0.00
Cash holdings	6.90	6.89	6.87	6.88	6.88	6.91	7.05
CBDC holdings	48.01	48.10	48.14	48.02	48.02	48.32	54.40
CBDC (% change)	-	0.09	0.13	0.01	0.01	0.31	6.39

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table B.2: Steady state shares (in percent) for baseline and alternative models

Description	**BM-II**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	36.73	36.77	36.60	36.70	36.70	36.78	37.21
Bond holdings	2.84	2.70	2.84	2.88	2.88	2.54	0.00
Cash holdings	6.13	6.12	6.10	6.11	6.11	6.13	6.17
CBDC holdings	54.30	54.42	54.47	54.32	54.32	54.55	56.62
CBDC (% change)	-	0.12	0.15	0.02	0.02	0.25	2.32

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table B.3: Steady state shares (in percent) for baseline and alternative models

Description	**BM-III**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	32.22	32.25	32.10	32.20	32.20	32.28	33.73
Bond holdings	13.21	13.09	13.22	13.24	13.24	12.89	3.92
Cash holdings	5.73	5.72	5.70	5.71	5.71	5.73	5.88
CBDC holdings	48.84	48.94	48.98	48.85	48.85	49.10	56.47
CBDC (% change)	-	0.10	0.14	0.01	0.01	0.26	7.60

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table B.4: Steady state shares (in percent) for baseline and alternative models

Description	**BM-IV**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	40.58	40.60	40.41	40.55	40.55	40.65	41.49
Bond holdings	4.96	4.86	5.02	5.00	5.00	4.56	0.00
Cash holdings	5.59	5.57	5.56	5.57	5.57	5.60	5.66
CBDC holdings	48.87	48.96	49.01	48.89	48.89	49.19	52.85
CBDC (% change)	-	0.09	0.13	0.02	0.02	0.32	3.98

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

Table B.5: Steady state shares (in percent) for baseline and alternative models

Description	**BM-V**	**AM-I**	**AM-II**	**AM-III**	**AM-IV**	**AM-V**	**AM-VI**
Physical capital	46.61	46.65	46.44	46.58	46.58	46.66	46.69
Bond holdings	0.41	0.30	0.47	0.45	0.45	0.18	0.00
Cash holdings	7.93	7.91	7.89	7.90	7.90	7.93	7.94
CBDC holdings	45.05	45.15	45.20	45.07	45.07	45.23	45.37
CBDC (% change)	-	0.10	0.15	0.02	0.02	0.18	0.32

Notes: BM refers to the baseline model. AM-I corresponds to 1% decrease in γ (transaction efficiency of cash); AM-II corresponds to 1% increase in ρ (transaction efficiency of CBDC); AM-III corresponds to 1% increase in m (probability of detection of tax evasion); AM-IV corresponds to 1% increase in θ_2 (tax evasion penalty); AM-V corresponds to 1% decrease in τ (tax rate on transactions using CBDC); and AM-VI corresponds to 1% increase in $\bar{I}_d$ (rate of return on CBDC).

APPENDIX C

APPENDIX OF CHAPTER 3

C.1 Dynamic optimization of representative agent problem

The representative agent maximizes the present discounted value of flow utility which is a function of consumption goods. The agent also takes a decision on amount of tax evasion during any period of time. So, the control variables are consumption goods(C) and tax evasion(E). The state variable is asset (A) of the representative agent. Hence, the maximization problem of represented by equation (C.1.1) subject to budget constraint given by equation (C.1.2).

$$U_0 = \int_0^\infty e^{-\rho t}(U(C(t))) \tag{C.1.1}$$

s.t.

$$\dot{A}(t) = (1-\tau)r(t)(A(t)-E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t) \tag{C.1.2}$$

Let $r^\tau(t) = (1-\tau)r(t)$ for short-hand representation. The present value Hamiltonian (PVH) is given by equation (C.1.3) where $\lambda(t)$ is the shadow price of asset.

$$\begin{aligned} PVH = \ & e^{-\rho t}(U(C(t))) \\ & + \lambda(t)((r^\tau(t)(A(t)-E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t)) \end{aligned} \tag{C.1.3}$$

Taking the derivative of PVH with respect to control variables and setting it to 0 yields the following:

$$[C(t)] : e^{-\rho t}U_C(C(t)) = \lambda(t) \tag{C.1.4}$$

$$[E(t)] : 1 = \pi(t)P_E(E(t)) + r^\tau(t) \tag{C.1.5}$$

Taking the derivative of PVH with respect to state variable and setting it to negative rate of change in shadow value of asset yields the following:

$$[A(t)] : \lambda(t)r^{\tau}(t) = -\dot{\lambda}(t) \tag{C.1.6}$$

The derivative of PVH with respect to shadow value of asset and set it to rate of change in asset position yields the following:

$$[\lambda(t)] : (1-\tau)r(t)(A(t)-E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t) = \dot{A}(t) \tag{C.1.7}$$

Transversality condition (TVC) states the value of asset i.e. the asset multiplied by its shadow value at the end of planning horizon is equal to 0.

$$\lim_{t\to\infty} \lambda(t)A(t) = 0 \tag{C.1.8}$$

Taking log of equation (C.1.4) yields

$$-\rho t - ln(U_C(C(t))) = ln\lambda(t)$$

Taking time derivative yields

$$-\rho - \frac{U_{CC}(C(t))}{U_C(C(t))}\dot{C}(t) = \frac{\dot{\lambda}(t)}{\lambda(t)}$$

Define the risk-aversion σ as following[1]

$$\sigma = -\frac{U_{CC}(C(t))}{U_C(C(t))}C(t)$$

Using equation (C.1.6) and definition of risk-aversion in above result, we obtain

$$\frac{\dot{C}(t)}{C(t)} = \frac{r^{\tau}(t)-\rho}{\sigma} \tag{C.1.9}$$

Thus, the path of optimal consumption path is given as following

$$C(t) = C(0)e^{(\frac{r^{\tau}(t)-\rho}{\sigma})t} \tag{C.1.10}$$

[1]The risk-aversion is assumed to be constant for RA over time.

where C(0) is the consumption good at time, t=0. Rearranging the budget constraint of the representative agent yields

$$\dot{A}(t) - r^{\tau}(t)A(t) = G(t) + E(t) - r^{\tau}(t)E(t) - \pi(t)P(E(t)) - C(t)$$

Multiplying by the integrating factor $e^{-r^{\tau}(t)t}$ and integrating from 0 to ∞ yields

$$\begin{aligned}\int_0^{\infty} (\dot{A}(t) - r^{\tau}(t)A(t))e^{-r^{\tau}(t)t}dt = & \int_0^{\infty} (G(t)e^{-r^{\tau}(t)t}dt \\ & + \int_0^{\infty} ((E(t) - r^{\tau}(t)E(t) - \pi(t)P(E(t)))e^{-r^{\tau}(t)t}dt \\ & - \int_0^{\infty} C(t)e^{-r^{\tau}(t)t}dt\end{aligned}$$

$$\begin{aligned}A(t)e^{-r^{\tau}(t)t}|_0^{\infty} = & \int_0^{\infty} (G(t)e^{-r^{\tau}(t)t}dt \\ & + \int_0^{\infty} ((E(t) - r^{\tau}(t)E(t) - \pi(t)P(E(t)))e^{-r^{\tau}(t)t}dt \\ & - \int_0^{\infty} C(t)e^{-r^{\tau}(t)t}dt \\ -A(0) = & PV^G(0) + PV^E(0) - \int_0^{\infty} C(t)e^{-r^{\tau}(t)t}dt\end{aligned}$$

where $PV^G(0) = \int_0^{\infty}(G(t)e^{-r^{\tau}(t)t}dt$ and $PV^E(0) = \int_0^{\infty}((E(t) - r^{\tau}(t)E(t) - \pi(t)P(E(t)))e^{-r^{\tau}(t)t}dt$ represent the present discounted value of future earning from government transfers and from expected benefits of tax evasion respectively. Using equation (C.1.10) in above result gives

$$\int_0^{\infty} C(0)e^{(\frac{r^{\tau}(t)-\rho}{\sigma})t}e^{-r^{\tau}(t)t}dt = PV^G(0) + PV^E(0) + A(0)$$

Define marginal propensity to consume at time, t=0 as $\mu(0) = \frac{1}{\int_0^{\infty} e^{(\frac{r^{\tau}(t)-\rho}{\sigma})t}e^{-r^{\tau}(t)t}dt}$. So, C(0) the consumption good at time, t=0 is given by following equation (C.1.11)

$$C(0) = \mu(0)(PV^G(0) + PV^E(0) + A(0)) \tag{C.1.11}$$

Thus, equation (C.1.2), (C.1.5), (C.1.10), and (C.1.11) pin down the optimal path of consumption and tax evasion.

C.2 Relationship between tax evasion amount and probability of detection

The following relationship is obtained from the optimization problem of RA and RF.

$$1 = \pi(t)P_E(E(t)) + r^{\tau}(t) = \pi(t)P_E(E(t)) + (1-\tau)r(t) \tag{C.2.1}$$

$$r(t) = \Theta \tag{C.2.2}$$

Substitute $r(t)$ from equation (C.2.2) in equation (C.2.1) and take log of resulting equation.

$$log(1-(1-\tau)\Theta) = log\pi(t) + logP_E(E(t))$$

Differentiating with respect to $\pi(t)$ yields

$$0 = \frac{1}{\pi(t)} + \frac{P_{EE}(E(t))}{P_E(E(t))}E_{\pi}(\pi(t))$$

$$E_{\pi}(\pi(t)) = -\frac{1}{\pi(t)}\frac{P_E(E(t))}{P_{EE}(E(t))} \tag{C.2.3}$$

Thus, the tax evasion is a decreasing function of probability of detection since the penalty function is assumed to be increasing and convex.

C.3 Static optimization of government problem

The government maximizes the tax revenue. On the one hand, the government has to pay lump-sum transfer to the households and incur a cost given by $M(\pi(t))$. On the other hand, the government receives tax from the asset holding of the households and captures the tax evasion amount. So, the optimization problem for government is given by (C.3.1) subject to (C.3.2),(C.3.3), and (C.3.4)

$$\tau r(t)A(t) \tag{C.3.1}$$

s.t.

$$G(t) + M(\pi(t)) = \pi(t)P(E(t)) + \tau r(t)A(t) \tag{C.3.2}$$

$$r(t) = \Theta \tag{C.3.3}$$

$$E_\pi(\pi(t)) = -\frac{1}{\pi(t)}\frac{P_E(E(t))}{P_{EE}(E(t))} \tag{C.3.4}$$

So, the optimization problem of government is reduced to the following.

$$G(t) + M(\pi(t)) - \pi(t)P(E(\pi(t))) \tag{C.3.5}$$

The probability of detection is the control variable for the government. Taking FOC with respect to $\pi(t)$ yields

$$M_\pi(\pi(t))) = P(E(\pi(t))) + \pi(t)P_E(E(\pi(t)))E_\pi(\pi(t)) \tag{C.3.6}$$

The above equation is simplified using (C.3.4) as

$$M_\pi(\pi(t))) = P(E(\pi(t))) - \frac{P_E(E(\pi(t)))^2}{P_{EE}(E(\pi(t)))} \tag{C.3.7}$$

C.4 Proof of Proposition 1

Equation (C.4.1) provides the relationship between the probability of detection and parameters of the economy.

$$\eta\beta_2\pi^{\eta-1} = \alpha_1 - \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\pi\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}} \tag{C.4.1}$$

Suppose $\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}} > 0$. Label this inequality as condition (A), which corresponds to the parameter restriction in the proposition. Loosely, this condition relates the convexity of monitoring cost to be greater than that of penalty cost. Differentiating the equation (C.4.1) with respect to variable monitoring cost, fixed penalty cost, and variable penalty cost helps to understand the effect of change of those variables to probability of detection.

(a) Effect of variable monitoring cost β_2: Differentiate equation (C.4.1) with respect to β_2.

$$\eta\pi^{\eta-1} + \eta(\eta-1)\beta_2\pi^{\eta-2}\frac{\partial\pi}{\partial\beta_2} = \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\frac{\gamma}{\gamma-1}\pi^{\frac{1-2\gamma}{\gamma-1}}\frac{\partial\pi}{\partial\beta_2}$$

$$\frac{\partial\pi}{\partial\beta_2} = \frac{-\eta\pi^{\eta-1}}{\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}} \tag{C.4.2}$$

With condition (A) assumed, equation (C.4.2) proves that the probability of detection decreases with the increase in variable monitoring cost.

(b) Effect of fixed penalty cost α_1: Differentiate equation (C.4.1) with respect to α_1.

$$\eta(\eta-1)\beta_2\pi^{\eta-2}\frac{\partial\pi}{\partial\alpha_1} = 1 + \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\frac{\gamma}{\gamma-1}\pi^{\frac{1-2\gamma}{\gamma-1}}\frac{\partial\pi}{\partial\alpha_1}$$

$$\frac{\partial\pi}{\partial\alpha_1} = \frac{1}{\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}} \tag{C.4.3}$$

With condition (A) assumed, equation (C.4.3) proves that the probability of detection increases with the increase in fixed penalty cost.

(c) Effect of variable penalty cost α_2: Differentiate equation (C.4.1) with respect to α_2.

$$\eta(\eta-1)\beta_2\pi^{\eta-2}\frac{\partial\pi}{\partial\alpha_2} = \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\frac{\gamma}{\gamma-1}\pi^{\frac{1-2\gamma}{\gamma-1}}\frac{\partial\pi}{\partial\alpha_2} + \frac{1}{\gamma-1}\frac{\alpha_2^{\frac{-\gamma}{\gamma-1}}}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\gamma\pi}]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\partial \pi}{\partial \alpha_2} = \frac{\frac{\alpha_2^{\frac{-\gamma}{\gamma-1}}}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\pi}]^{\frac{\gamma}{\gamma-1}}}{\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}} \tag{C.4.4}$$

With condition (A) assumed, equation (C.4.4) proves that the probability of detection increases with the increase in variable penalty cost.

C.5 Equilibrium path of capital

The budget constraint of consumer is

$$\dot{A}(t) = (1-\tau)r(t)(A(t) - E(t)) + G(t) + E(t) - \pi(t)P(E(t)) - C(t)$$

Government faces the following budget constraint.

$$G(t) + M(\pi(t)) = \pi(t)P(E(t)) + \tau r(t)A(t)$$

Substituting the government budget constraint into the consumer budget constraint yields

$$\dot{A}(t) - r(t)A(t) = -C(t) - M(\pi(t)) + E(t)[1 - (1-\tau)r(t)]$$

In equilibrium and with functional specification of penalty and monitoring process, the following conditions are satisfied :

(i) A(t) = K(t)

(ii) r(t) = Θ

(iii) $C(t) = C(0)e^{(\frac{r^{\tau}(t)-\rho}{\sigma})t}$

(iv) $\eta\beta_2^i\pi(t)^{\eta-1} = \alpha_1^i - \frac{\alpha_2^i}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\pi(t)\gamma\alpha_2^i}]^{\frac{\gamma}{\gamma-1}}$

(v) $E(t) = [\frac{1-(1-\tau)\Theta}{\pi(t)\gamma\alpha_2^i}]^{\frac{1}{\gamma-1}}$

The present discounted value of utility in equilibrium is

$$\begin{aligned} U_0 &= \int_0^\infty e^{-\rho t}\frac{C(t)^{1-\sigma}}{1-\sigma} \\ &= \int_0^\infty \frac{C(0)^{1-\sigma}}{1-\sigma} e^{(-\rho+(1-\sigma)(\frac{r^T(t)-\rho}{\sigma}))t} \\ &= \int_0^\infty \frac{C(0)^{1-\sigma}}{1-\sigma} e^{\frac{(1-\tau)\Theta-\rho-\sigma(1-\tau)\Theta}{\sigma}t} \end{aligned}$$

The present discounted value of utility has to be bounded. This entails the following parameter restriction, which is termed as bounded utility condition.

$$(1-\tau)\Theta - \rho - \sigma(1-\tau)\Theta < 0.$$

Assuming the parameters of penalty and monitoring function as constant over time, conditions iv. and v. implies that the probability of detection and the amount of tax evasion is constant over time. Thus,

$$\dot{K}(t) - \Theta K(t) = -C(t) - M(\pi) + E[1-(1-\tau)\Theta]$$

Denote $-M(\pi) + E[1-(1-\tau)\Theta]$ as some constant B. So,

$$\dot{K}(t) - \Theta K(t) = -C(t) + B$$

Multiply the integrating factor $e^{-\Theta t}$ and integrate from 0 to T.

$$\int_0^T (\dot{K}(t) - \Theta K(t))e^{-\Theta t}dt = -\int_0^T C(0)e^{(\frac{(1-\tau)\Theta-\rho-\sigma\Theta}{\sigma})t}dt + \int_0^T Be^{-\Theta t}dt$$

$$\Rightarrow K(t)e^{-\Theta t}|_0^T = -\frac{C(0)\sigma}{(1-\tau)\Theta-\rho-\sigma\Theta}e^{(\frac{(1-\tau)\Theta-\rho-\sigma\Theta}{\sigma})t}|_0^T - \frac{B}{\Theta}e^{-\Theta t}|_0^T$$

$$\Rightarrow K(T)e^{-\Theta T} - K(0) = -\frac{C(0)\sigma}{(1-\tau)\Theta-\rho-\sigma\Theta}e^{(\frac{(1-\tau)\Theta-\rho-\sigma\Theta}{\sigma})T} + \frac{C(0)\sigma}{(1-\tau)\Theta-\rho-\sigma\Theta} - \frac{B}{\Theta}e^{-\Theta T} + \frac{B}{\Theta}$$

$$\Rightarrow K(T)e^{-\Theta T} = (K(0) + \frac{C(0)\sigma}{\sigma\Theta+\rho-(1-\tau)\Theta} + \frac{B}{\Theta}) - \frac{C(0)\sigma}{\sigma\Theta+\rho-(1-\tau)\Theta}e^{(\frac{(1-\tau)\Theta-\rho-\sigma\Theta}{\sigma})T} - \frac{B}{\Theta}e^{-\Theta T}$$

$$\Rightarrow K(T) = (K(0) + \frac{C(0)\sigma}{\sigma\Theta + \rho - (1-\tau)\Theta} + \frac{B}{\Theta})e^{\Theta T} - \frac{C(0)\sigma}{\sigma\Theta + \rho - (1-\tau)\Theta}e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})T} - \frac{B}{\Theta}$$

Denote $-\frac{B}{\Theta}$ as ϑ_1, $(K(0) + \frac{C(0)\sigma}{\sigma\Theta+\rho-(1-\tau)\Theta} + \frac{B}{\Theta})$ as ϑ_2, and $-\frac{C(0)\sigma}{\sigma\Theta+\rho-(1-\tau)\Theta}$ as ϑ_3.

$$\Rightarrow K(T) = \vartheta_1 + \vartheta_2 e^{\Theta T} + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})T}$$

Using the transversality condition,

$$K(T)u'(C(T))e^{-\rho T} = (\vartheta_1 + \vartheta_2 e^{\Theta T} + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})T})C(T)^{-\sigma}e^{-\rho T}$$

$$\Rightarrow K(T)u'(C(T))e^{-\rho T} = (\vartheta_1 + \vartheta_2 e^{\Theta T} + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})T})C(0)^{-\sigma}e^{-\sigma(\frac{(1-\tau)\Theta-\rho}{\sigma})T}e^{-\rho T}$$

$$\Rightarrow K(T)u'(C(T))e^{-\rho T} = (\vartheta_1 + \vartheta_2 e^{\Theta T} + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})T})C(0)^{-\sigma}e^{-(1-\tau)\Theta T}$$

$$\Rightarrow K(T)u'(C(T))e^{-\rho T} = (\vartheta_1 e^{-(1-\tau)\Theta T} + \vartheta_2 e^{\tau\Theta T} + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho-\sigma(1-\tau)\Theta}{\sigma})T})C(0)^{-\sigma}$$

As time T tends to ∞, the RHS of above equation tends to 0. This implies

(i) $\vartheta_2 = 0$ which pins down C(0) for a given initial value of capital, K(0). So, $C(0) = -\frac{\sigma\Theta+\rho-(1-\tau)\Theta}{\sigma}(K(0) + \frac{B}{\Theta})$.

(ii) $(1-\tau)\Theta - \rho - \sigma(1-\tau)\Theta < 0$ which is true under bounded utility condition.

Thus, the equilibrium time of capital is given by equation (C.5.1)

$$\Rightarrow K(t) = \vartheta_1 + \vartheta_3 e^{(\frac{(1-\tau)\Theta-\rho}{\sigma})t} \tag{C.5.1}$$

where $\vartheta_1 = -\frac{B}{\Theta}$ and $\vartheta_3 = -\frac{C(0)\sigma}{\sigma\Theta+\rho-(1-\tau)\Theta} = K(0) + \frac{B}{\Theta}$

C.6 Proof of Proposition 2

Equation (C.4.1) provides the relationship between the probability of detection and tax rate. Suppose the condition (A), as mentioned in A4, holds. Differentiating equation (C.4.1) with respect to tax rate helps to understand the effect of change of tax rate to probability of detection.

$$\eta(\eta{-}1)\beta_2\pi^{\eta-2}\frac{\partial\pi}{\partial\tau} = \frac{\alpha_2}{\gamma-1}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\frac{\gamma}{\gamma-1}\pi^{\frac{1-2\gamma}{\gamma-1}}\frac{\partial\pi}{\partial\tau} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\pi\gamma\alpha_2}]^{\frac{1}{\gamma-1}}\frac{\Theta}{\pi\gamma\alpha_2}$$

$$\frac{\partial\pi}{\partial\tau} = \frac{-\frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\pi\gamma\alpha_2}]^{\frac{1}{\gamma-1}}\frac{\Theta}{\pi\gamma\alpha_2}}{\eta(\eta-1)\beta_2\pi^{\eta-2} - \frac{\alpha_2\gamma}{(\gamma-1)^2}[\frac{1-(1-\tau)\Theta}{\gamma\alpha_2}]^{\frac{\gamma}{\gamma-1}}\pi^{\frac{1-2\gamma}{\gamma-1}}} \tag{C.6.1}$$

With condition (A) assumed, equation (C.6.1) proves that the probability of detection decreases with the increase in tax rate.

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