

ANALYSES FOR A BALANCED DESIGN IN A MARKETING EXPERIMENT*

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The analyses given below are for an experiment conducted in four different grocery stores on five products (facial tissues, towels, kotex, bathroom tissue and table napkins) for one company as well as the effect on competing products of different companies. Eight treatments involving display, advertising, and pricing in a 2^3 factorial were tested in the experiment. The four treatments for one price level were arranged in a four x four latin square design in a four week period and for four stores. These same treatments were arranged in another four x four latin square for each product. Thus, a four x four x four latin cube design of first order was used for the four treatments for one price level. The same sort of design was used for the four treatments of the second price level. It should be noted that two products, table napkins and bathroom tissue, appeared together. The experimental design and the treatment design are given in table 1.

The data involved are retail sales of a product; all product sales were converted to equivalent units. Prices on all brands were controlled, as was advertising and display. Precautions were taken to be certain that sufficient supplies of all brands of all products were available at all times to customers. Since the variance of the number of units sold might be related to the mean number of units sold, it is possible that a square root or logarithmic transformation should be applied to the data prior to analysis of variance calculations. Also, in week 10 and store 69, a foul-up on the advertising with the newspaper resulted in a missing plot value. This treatment was re-run in the 14th week and for the analyses over all 53 weeks, the results for the 14th, 15th, 16th and 17th weeks should be substituted for the 10th, 11th, 12th, and 13th weeks, respectively for the product towels on all brands. The results could be left as they are, but in order to simplify the analysis this procedure will be followed.

Some of the simplest analyses that could be run would be 4×4 latin square analyses. For example, consider product 1, facials, in weeks 4, 17, 30 and 43. conducted in the four stores. The analysis of variance partitioning of degrees

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Table 1: Experimental Design and Treatment Design

Test of the Effects of Special Price, Special Displays, Newspaper Advertising and Combinations thereof on Kimberly-Clark Products (Nov. 10, 1959 - Nov. 14, 1960)

Test week	Test product	Store			
		8	22	24	69
4	Kleenex Facial Tissue 400's	000	001	011	010
5	Kleenex Twin-pack Towels	001	010	000	011
6	Kotex Reg. and Super 12's	010	011	001	000
7	Delsey 4-roll bathroom tissue	011	000	010	001
7	Kleenex Table Napkins	011	000	010	001
9	Kleenex Facial Tissue 400's	100	101	111	110
10	Kleenex Twin-pack Towels	111	100	110	101*
11	Kotex Reg. and Super 12's	110	111	101	100
12	Delsey 4-roll bathroom tissue	101	110	100	111
12	Kleenex Table Napkins	101	110	100	111
17	Kleenex Facial Tissue 400's	001	010	000	011
18	Kleenex Twin-pack Towels	010	011	001	000
19	Kotex Reg. and Super 12's	011	000	010	001
20	Delsey 4-roll bathroom tissue	000	001	011	010
20	Kleenex Table Napkins	000	001	011	010
22	Kleenex Facial Tissue 400's	111	100	110	101
23	Kleenex Twin-pack Towels	110	111	101	100
24	Kotex Reg. and Super 12's	101	110	100	111
25	Delsey 4-roll bathroom tissue	100	101	111	110
25	Kleenex Table Napkins	100	101	111	110
30	Kleenex Facial Tissue 400's	011	000	010	001
31	Kleenex Twin-pack Towels	000	001	011	010
32	Kotex Reg. and Super 12's	001	010	000	011
33	Delsey 4-roll bathroom tissue	010	011	001	000
33	Kleenex Table Napkins	010	011	001	000
35	Kleenex Facial Tissue 400's	101	110	100	111
36	Kleenex Twin-pack Towels	100	101	111	110
37	Kotex Reg. and Super 12's	111	100	110	101
38	Delsey 4-roll bathroom tissue	110	111	101	100
38	Kleenex Table Napkins	110	111	101	100
43	Kleenex Facial Tissue 400's	010	011	001	000
44	Kleenex Twin-pack Towels	011	000	010	001
45	Kotex Reg. and Super 12's	000	001	011	010
46	Delsey 4-roll bathroom tissue	001	010	000	011
46	Kleenex Table Napkins	001	010	000	011
48	Kleenex Facial Tissue 400's	110	111	101	100
49	Kleenex Twin-pack Towels	101	110	100	111
50	Kotex Reg. and Super 12's	100	101	111	110
51	Delsey 4-roll bathroom tissue	111	100	110	101
51	Kleenex Table Napkins	111	100	110	101

*Re-run week 14.

Treatment Code:

First digit 0 - Regular price	1 - Special price	A - 001	E - 101
Second digit 0 - Shelf display	1 - Special display	B - 000	F - 100
Third digit 0 - No advertising	1 - Newspaper advertising	C - 011	G - 111
		D - 010	H - 110

of freedom is:

<u>Source of variation</u>	<u>df</u>
Total (uncorrected)	16
Correction for mean	1
Columns = stores	3
Weeks (4,17,30,43) = rows	3
Treatments	3
Display vs no display	1
Newspaper advertising vs no advertising	1
Interaction	1
Residual	6

The above analysis is the standard one for a 4x4 latin square where the treatments are arranged in a 2x2 factorial.

There are 9 other analyses with the above form; these are:

2. Facials in weeks 9,22,35 and 48 with a special price.
3. Towels in weeks 5,18,31 and 44 with a regular price.
4. Towels in weeks 10,23,36 and 49 with a special price.
5. Kotex in weeks 6,19,32 and 45 with a regular price.
6. Kotex in weeks 11,24,37 and 50 with a special price.
7. Bathroom tissue in weeks 7,20,33 and 46 with a regular price.
8. Bathroom tissue in weeks 12,25,38 and 51 with a special price.
9. Table napkins in weeks 7,20,33 and 46 with a regular price.
10. Table napkins in weeks 12,25,38 and 51 with a special price.

The stores, treatments for one price level, the products, and the time periods of four consecutive weeks for one price level were arranged in a latin cube design. There are several ways to analyze the data from such a design, but it is thought that one of the following two analyses will suffice for these data:

<u>Source of variation</u>	<u>df</u>	
Stores	3	
Weeks (e.g., 4-7, 17-20, 30-33, 43-46)	15	
Products (e.g., facials, towels, kotex, and bathroom tissue) = P	3	F tests
Time periods (4-7=1, 17-20=2, etc.) = T	3	
PxT	9	
Weeks x Stores	45	
Stores x P	9	F tests
Stores x T	9	
Stores x P x T	27	
Treatments (display and advertising)	3	F tests
Remainder	24	

An alternative breakdown for the above is given below:

<u>Source of variation</u>	<u>df</u>
Stores = S	3
Products = P	3
Time periods = T	3
Treatments = V	3
Display = D	1
Advertising = A	1
AD = interaction	1
VxP (= part of TxS)	9
VxS (= part of PxT)	9
Residual	33
Correction for mean	1
Total (uncorrected)	64

To determine the type of analyses that can be run, tables 2 and 3 were prepared. From table 3 we note that the time period x product interaction has 5 of the 9 degrees of freedom confounded with the treatment x store interaction, the time period x treatment has 5 of the 9 degrees of freedom confounded with the product x store interaction, and the time period x store interaction has 5

Table 2. Designation of time periods (digits 1 and 2), products (digits 3 and 4), treatments (display = digit 5 and advertising = digit 6), and stores (digits 7 and 8) as combinations of a 2^8 factorial (these 64 combinations result in a 1/4 replicate of the 2^8 factorial) for regular pricing.

Weeks 4-7	Weeks 17-20	Weeks 30-33	Weeks 43-46
0000000	01000100	10001100	11001000
00000101	01001001	10000001	11001101
00001110	01000010	10001010	11000110
00001011	01001111	10000111	11000011
00010100	01011000	10010000	11011100
00011001	01011101	10010101	11010001
00010010	01010110	10011110	11011010
00011111	01010011	10011011	11010111
00101000	01101100	10100100	11100000
00101101	01100001	10101001	11100101
00100110	01101010	10100010	11101110
00100011	01100111	10101111	11101011
00111100	01110000	10111000	11110100
00110001	01110101	10111101	11111001
00111010	01111110	10110110	11110010
00110111	01111011	10110011	11111111

Table 3. Full and one-half aliases for the one-fourth replicate of a 2^8 factorial in table 2.

	Effect	Full alias	Fractional Aliases							
			-1/2	1/2	1/2	-1/2	1/2	1/2	1/2	1/2
Time periods 3df	I	ABDFGH	ACDEG	ACEFG	BCDEH	BCEFH	BCEG	ACDEFH	BCDEFG	ACEH
	A	BDFGH	CDEG	CEFG	ABCDEH	ABCFEH	ABCEG	CDEFH	ABCDEFG	CEH
	B	ADFGH	ABCDEG	ABCEFG	CDEH	CEFH	CEG	ABCDEFH	CDEFG	ABCEH
	AB	DFGH	BCDEG	BCEFG	ACDEH	ACEFH	ACEG	BCDEFH	ACDEFG	BCEH
Products 3df	C	ABCDFGH	ADEG	AIEFG	BDEH	BEFH	BEG	ADEFH	BDEFG	AEH
	D	ABFGH	ACEG	ACDEFG	BCEH	BCDEFH	BCDEG	ACEFH	BCEFG	ACDEH
	CD	ABCFGH	AEG	AIEFG	BEH	BDEFH	BDEG	AIEFH	BEFG	AIEH
Treatments 3df	E	ABDEFGH	ACDG	ACFG	BCDH	BCFH	BCG	ACDFH	BCDFG	ACH
	F	ABDGH	ACDEFG	ACEG	BCDEFH	BCEH	BCEFG	ACDEH	BCDEG	ACEFH
	EF	ABDEGH	ACDFG	ACG	BCDFH	BCH	BCFG	ACDH	BCDG	ACFH
Stores 3df	G	ABDFH	ACDE	ACEF	BCDEGH	BCEFGH	BCE	ACDEFGH	BCDEF	ACEGH
	H	ABDFG	ACDEGH	ACEFGH	BCDE	BCEF	BCEGH	ACDEF	BCDEFGH	ACE
	GH	ABDF	ACDEH	ACEFH	BCDEG	BCEFG	BCEH	ACDEFG	BCDEFH	ACEG
Time periods x Products 9df	AC	BCDFGH	DEG	EFG	ABDEH	ABEFH	ABEG	DEFH	ABDEFG	EH
	AD	BFGH	CEG	CDEFG	ABCEH	ABCDEFH	ABCDEG	CEFH	ABCEFG	CDEH
	ACD	BCFGH	EG	DEFG	ABEH	ABDEFH	ABDEG	EFH	ABIEFG	DEH
	BC	ACDFGH	ABDEG	ABIEFG	DEH	EFH	EG	ABDEFH	DEFG	ABEH
	BD	AFGH	ABCEG	ABCIEFG	CEH	CDEFH	CDEG	ABCFEH	CEFG	ABCIEH
	BCD	ACFGH	ABEG	ABIEFG	EH	DEFH	DEG	ABIEFH	EFG	ABIEH
	ABC	CDFGH	BDEG	BEFG	AIEH	AIEFH	AIEG	BIEFH	AIEFG	BEH
	ABD	FGH	BCEG	BCIEFG	ACEH	ACIEFH	ACIEG	BIEFH	ACEFG	BCIEH
ABCD	CFGH	BEG	BIEFG	AIEH	AIEFH	AIEG	BEFH	AIEFG	BIEH	
Time periods x treatments 9df	AE	BIEFGH	CDG	CFG	ABCDH	ABCFH	ABCG	CDFH	ABCDFG	CH
	AF	BDGH	CIEFG	CEG	ABCIEFH	ABCEH	ABCEFG	CIEH	ABCIEG	CFH
	AIEF	BIEGH	CDFG	CG	ABCDFH	ABCH	ABCFG	CDH	ABCIEG	CFH
	BE	AIEFGH	ABCIEG	ABCIEFG	CDH	CFH	CG	ABCDFH	CDFG	ABCH
	BF	ADGH	ABCIEFG	ABCEG	CIEFH	CEH	CEFG	ABCIEH	CIEG	ABCFEH
	BEF	AIEGH	ABCDFG	ABCG	CDFH	CH	CFG	ABCDH	CDG	ABCFH
	ABE	DEFGH	BCDG	BCFG	ACDH	ACFH	ACG	BCDFH	ACDFG	BCH
	ABF	DGH	BCIEFG	BCEG	ACIEFH	ACEH	ACEFG	BCIEH	ACIEG	BIEFH
ABEF	DEGH	BCDFG	BCG	ACDFH	ACH	ACFG	BCDH	ACIEG	BCFH	

Table 3. (Continued)

Time periods x stores 9df	AG	BDFH	CDE	CEF	ABCDEGH	ABCEFGH	ABCE	CDEFGH	ABCDEF	CEGH
	AH	BDFG	CDEGH	CEFGH	ABCDE	ABCEF	ABCEGH	CDEF	ABCDEFHG	CE
	AGH	BDF	CDEH	CEFH	ABCDEG	ABCEFG	ABCEH	CDEFG	ABCDEFH	CEG
	BG	ADFH	ABCDE	ABCEF	CDEGH	CEFGH	CE	ABCDEFHG	CDEF	ABCEGH
	BH	ADFG	ABCDEGH	ABCEFGH	CDE	CEF	CEGH	ABCDEF	CDEFGH	ABCE
	BGH	ADF	ABCDEH	ABCEFH	CDEG	CEFG	CEH	ABCDEFG	CDEFH	ABCEG
	ABG	DFH	BCDE	BCEF	ACDEGH	ACEFGH	ACE	BCDEFGH	ACDEF	BCEGH
	ABH	DFG	BCDEGH	BCEFGH	ACDE	ACEF	ACEGH	BCDEF	ACDEFHG	BCE
	ABGH	DF	BCDEH	BCEF	ACDEG	ACEFG	ACEH	BCDEFG	ACDEFH	BCEG
Products x treatments 9df	CE	ABCDEFGH	ADG	AFG	BDH	BFH	BG	ADFH	BDFG	AH ¹
	CF	ABCDGH	ADEFG	AEG	BDEFH	BEH	BEFG	ADEH	BDEG	AEFH
	CEF	ABCDEGH	ADFG	AG	BDFH	BH	BFG	ADH	BDG	AFH ²
	DE	ABEFGH	ACG	ACDFG	BCH	BCDFH	BCDG	ACFH	BCFG	ACDH
	DF	ABGH	ACEFG	ACDEG	BCEF	BCDEH	BCDEFG	ACEH	BCEG	ACDEFH ³
	DEF	ABEGH	ACFG	ACDG	BCFH	BCDH	BCDFG	ACH	BCG	ACDFH
	CDE	ABCEFGH	AG	ADFG	BH	BDFH	BDG	AFH	BFG	ADH ⁴
	CDF	ABCGH	AIEFG	AIEG	BEFH	BDEH	BIEFG	AIEH	BEG	AIEFH
	CDEF	ABCEGH	AFG	ADG	BFH	BDH	BDFG	AH	BG	ADFH ⁵
Products x stores 9df	CG	ABCDFH	ADE	AIEF	BDEGH	BEFGH	BE	AIEFGH	BIEF	AIEGH ⁶
	CH	ABCDFG	ADEGH	AIEFGH	BDE	BEF	BEGH	AIEF	BIEFGH	AIE ⁷
	CGH	ABCDF	AIEH	AIEFH	BIEG	BEFG	BEH	AIEFG	BIEFH	AIEG
	DG	ABFH	ACE	ACIEF	BIEGH	BCIEFGH	BCIE	ACEFGH	BIEF	ACIEGH
	DH	ABFG	ACEGH	ACIEFGH	BIE	BCIEF	BCIEGH	ACEF	BIEFGH	ACIE
	DGH	ABF	ACEH	ACIEFH	BIEG	BCIEFG	BCIEH	ACEFG	BIEFH	ACIEG ⁸
	CDG	ABCFH	AIE	AIEF	BEGH	BIEFGH	BIE	AIEFGH	BIEF	AIEGH ⁹
	CDH	ABCFG	AIEGH	AIEFGH	BE	BIEF	BIEGH	AIEF	BIEFGH	AIE ¹⁰
	CDGH	ABCF	AIEH	AIEFH	BEG	BIEFG	BIEH	AIEFG	BIEFH	AIEG ¹¹
Treatments x stores 9df	EG	ABCIEFH	ACD	ACF	BCDGH	BCFGH	BC	ACIEFGH	BCDF	ACGH ¹²
	EH	ABCIEFG	ACDGH	ACFGH	BCD	BCF	BCGH	ACIEF	BCIEFGH	AC ¹³
	EGH	ABCIEF	ACDH	ACFH	BCDG	BCFG	BCH	ACIEFG	BCIEFH	ACG
	FG	ABDH	ACIEF	ACE	BCIEFGH	BIEGH	BIEF	ACIEGH	BCIE	ACEFGH
	FH	ABDG	ACIEFGH	ACEGH	BCIEF	BIE	BIEFGH	ACIE	BCIEGH	ACEF
	FGH	ABD	ACIEFH	ACEH	BCIEFG	BIEG	BIEFH	ACIEG	BCIEH	ACEFG ¹⁴
	EFG	ABIEH	ACIEF	AC	BCIEFGH	BCGH	BCF	ACDGH	BCD	ACFGH ¹⁵
	EFH	ABIEG	ACIEFGH	ACGH	BCIEF	BC	BCFGH	ACD	BCDGH	ACF ¹⁶
	EFGH	ABIE	ACIEFH	ACH	BCIEFG	BCG	BIEFH	ACDG	BCDH	ACFG

Residual = three and higher factor interactions = 12 df

Table 3. (Continued) Footnotes

1. See BG and AH above (1/2 aliases)
2. See BH and AG above (1/2 aliases)
3. See ABGH above (full alias)
4. See BH and AG above (1/2 aliases)
5. See AH and BG above (1/2 aliases)
6. See AEF and BE above (1/2 aliases)
7. See AE and BEF above (1/2 aliases)
8. See ABF above (full alias)
9. See BEF and AE above (1/2 aliases)
10. See AEF and BE above (1/2 aliases)
11. See BE and AEF above (1/2 aliases)
12. See ACD and BC above (1/2 aliases)
13. See AC and BCD above (1/2 aliases)
14. See ABD above (full alias)
15. See AC and BCD above (1/2 aliases)
16. See BC and ACD above (1/2 aliases)

of the 9 degrees of freedom confounded with the product x treatment interaction.

A full analysis of variance for main effects and two-factor interactions would be:

<u>Source of variation</u>	<u>df</u>
Stores = S	3
Time periods = T	3
Products = P	3
Treatments = V	3
Display = D	1
Advertising = A	1
AD	1
SxT	4
PxV	4
SxT and PxV (completely confounded)	5
PxT	4
SxV	4
PxT and SxV (completely confounded)	5
TxV	4
PxS	4
TxV and PxS (completely confounded)	5
Three-, four- and higher factor interactions	12
Correction for mean	1
Total (uncorrected)	64

If some three-factor interactions are desired these could be partitioned out to determine the aliases (full and fractional) for each degree of freedom of the interaction.

The above analysis or analyses will be repeated for the four products on facials, towels, kotex, and table napkins. Then, two similar additional analyses are to be run on the 16 weeks involving products with the special pricing treatment; however, we need to check confounding here as for regular pricing. It should be noted that these four analyses could be repeated on each brand of a product and for the sum of all brands.

Another type of analysis that could be run on each brand of each product is the following:

<u>Source of variation</u>		<u>df</u>
Weeks	52	
Treatment-product periods = Q		8
Pretreatment vs post-treatment = A		1
Nos. 2 to 5 vs 6 to 9 = B		1
Among 2 to 5 = C		3
Among 6 to 9 = D		3
Remainder		44
Stores	3	
Stores x Weeks	156	
Advertising and Display treatments = T		3
TxQ		24
TxA		3
TxB		3
TxC		9
TxD		9
T x Stores		9
Q x Stores		24
T x Q x Stores		72
Remainder		24

where the treatment-periods = Q are as follows:

- 1 = week sales before the treatment (00,01,10 or 11) was applied to a given product
- 2 = week sales during the week the treatment was applied for regular pricing
- 3 = week sales during the week after the treatment was applied for regular pricing
- 4 = week sales during the second week after the treatment was applied for regular pricing
- 5 = week sales during the third week after the treatment was applied for regular pricing
- 6 = treatment-period 2 above but with the special price
- 7 = treatment-period 3 above but with the special price
- 8 = treatment-period 4 above but with the special price
- 9 = treatment-period 5 above but with the special price

In order to obtain simpler analyses than the above, consider the set-up in table 4 which is for treatment-periods 1 to 5 and treatments 00, 01, 10, and 11, in the 4 stores. The analysis of variance for the set-up in table 4 is:

Table 4
Facials - Regular Price

Pre- and Post-treatment periods		Store			
		18 (week no.)	29 (week no.)	16 (week no.)	69 (week no.)
1 week prior to treatment = 1	Treat. 00	3	29	16	42
	01	16	3	42	29
	10	42	16	29	3
	11	29	42	3	16
week of treat- ment = 2	Treat. 00	4	30	17	43
	01	17	4	43	30
	10	43	17	30	4
	11	30	43	4	17
week after treatment = 3	Treat. 00	5	31	18	44
	01	18	5	44	31
	10	44	18	31	5
	11	31	44	5	18
2 weeks after treatment = 4	Treat. 00	6	32	19	45
	01	19	6	45	32
	10	45	19	32	6
	11	32	45	6	19
3 weeks after treatment = 5	Treat. 00	7	33	20	46
	01	20	7	46	33
	10	46	20	33	7
	11	33	46	7	20

<u>Source of variation</u>	<u>df</u>
Treatment - period = Q	4
Stores = S	3
Treatments = T	3
QxS	12
QxT	12
TxS	9
QxTxS (error)	36
Correction for mean	1
Total (uncorrected)	80

The above analysis could be run for each brand or for sum of all brands for all products (except bathroom tissues and table napkins with special pricing, where it would have been necessary that week 5⁴ be obtained or that we assume that treatment-period 9 has no meaning for these products) which would result in 5+3=8 analyses plus 2 additional analyses on bathroom tissue and table napkins either with treatment 9 omitted or with a non-orthogonal analysis for a three-way classification. Likewise, one could set up similar analysis for other brands if there were any promotional treatments on these brands and products.

In order to combine the results on all brands for each product, the following analysis of variance on data like that in table 4 is:

<u>Source of variation</u>	<u>df</u>
Treatment-periods = Q	4
Stores = S	3
Treatments = T	3
QxS	12
QxT	12
SxT	9
QxSxT (error a)	36
Brands (either 4 or 5 depending upon the product) = B	3 or 4
BxQ	12 or 16
BxS	9 or 12
BxT	9 or 12
BxQxS	36 or 48
BxQxT	36 or 48
BxSxT	27 or 36
BxQxSxT (error b)	108 or 144
Correction for mean	1
Total (uncorrected)	320 or 400

Unless results for week 5⁴ are available, there will be only 4 treatment-periods on bathroom tissue and table napkins. Although we could combine the entire set of data, it is doubtful if much additional information will be obtained. However, for completeness, the breakdown of the degrees of freedom for all the data of a given product for all brands with both special and regular pricing over the 53 week period is:

<u>Source of variation</u>	<u>df</u>
Treatment-periods (8 or 9 depending upon product and pricing, but 9 used here) = Q	8
Stores = S	3
Treatments = T	3
QxS	24
QxT	24
SxT	9
QxSxT	72
Remainder	68
Brands (4 or 5, but 5 used here)	4
BxQ	32
BxS	12
BxT	12
BxQxS	96
BxQxT	96
BxTxS	36
BxTxSxQ	288
B x remainder	272
Correction for mean	1
Total (uncorrected)	1060

It is possible to set up a combined analysis for 4 products with 5 brands each for one pricing level. Suppose we select regular pricing and the 4 products with 5 brands each; the analysis of variance is:

<u>Source of variation</u>	<u>df</u>
Products = P	3
Treatment-period = T	4
Stores = S	3
Treatments (display and advertising) = A	3
PxT	9
PxS	12
PxA	9
TxS	9
TxA	12
SxA	12
PxTxS	27
PxTxA	36
PxSxA	27
TxSxA	36
PxTxSxA (error a)	108
Brands = B	4
BxP	12
BxT	16
BxS	12
BxA	12
BxPxT	36
BxPxS	48
BxPxA	36
BxTxS	36
BxTxA	48
BxSxA	48
BxPxTxS	108
BxPxTxA	144
BxPxSxA	108
BxTxSxA	144
BxPxTxSxA (error b)	432
Correction for the mean	1
Total (uncorrected)	1600

Of the total number of observations ($4 \text{ stores} \times 53 \text{ weeks} \times 4 \text{ products} \times 5 \text{ brands} = 4240$) only 1600 are used for the above analysis; data from different weeks are also involved. The 1600 observations for all brands of each product will come from tables similar to table 4. Also, a second analysis with the regular pricing could be run on the four products, facial tissue, towels, kotex, and table napkins with 4 brands. Two similar analyses could be run for the special pricing, but here it would be necessary to use 4 treatment periods in order to use all products and to retain orthogonality in the analyses of variance.

The preceding sets of analyses could be combined with pricing as a 6th variable. Also, it should be noted that table napkins and bathroom tissue always appear together in the same weeks. These two products could be combined for several of the preceding analyses if desired. For example, consider the analysis for regular pricing for treatment periods, treatments, and stores which would be:

<u>Source of variation</u>	<u>df</u>
Treatment period = Q	4
Stores = S	3
Treatments = T	3
QxS	12
QxT	12
SxT	9
QxSxT (error (a))	36
Products (table napkins and bathroom tissues) = P	1
PxQ	4
PxS	3
PxT	3
PxQxS	12
PxQxT	12
PxSxT	9
PxQxSxT (error (b))	36
Correction for mean	1
Total (uncorrected)	160