



Power, Patents and Peer-Review: Essays in Applied Microeconomic Theory

by Vidya Atal

This thesis/dissertation document has been electronically approved by the following individuals:

Basu, Kaushik (Chairperson)

Bar, Talia (Minor Member)

Mitra, Tapan (Minor Member)

POWER, PATENTS AND PEER-REVIEW:
ESSAYS IN APPLIED MICROECONOMIC THEORY

A Dissertation

Presented to the Faculty of the Graduate School
of Cornell University

in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy

by

Vidya Atal

August 2010

© 2010 Vidya Atal
ALL RIGHTS RESERVED

POWER, PATENTS AND PEER-REVIEW:
ESSAYS IN APPLIED MICROECONOMIC THEORY

Vidya Atal, Ph.D.

Cornell University 2010

My dissertation is a collection of four essays in applied microeconomic theory.

The first chapter develops a theory of female labor supply in a general equilibrium framework where decisions are taken by the households and the power distribution among the members is determined endogenously. It is shown that female labor supply can take different shapes due to the structural differences between economies, and multiple equilibria might occur in the labor market. Policy implications of tax-benefits, subsidies for labor-saving household durables, and reservation for women at employment are worked out. The results found here resonate well with previous empirical findings and suggest additional testable implications.

In the next chapter, Talia Bar and I study and criticize the current patent system in the United States, particularly, prior art search and its disclosure policy. To determine patentability, inventions are evaluated in light of existing prior art. Innovators have a duty to disclose any prior art that they are aware of, but have no obligation to search. We identify conditions in which innovators have no incentive to search. Search intensity increases with R&D cost, the examination intensity, and patenting fees. In the later half of the chapter, we study determinants of patent quality and the volume of patent applications. We model a policy reform proposal to establish a two-tiered patent system. Introducing a second patent-tier can reduce patent applications and the incidence of bad

patents. We claim that innovators with high ex-ante probability of validity will more likely apply to the more valuable patent-tier, but sorting in the dimension of economic significance is not obvious.

The last chapter provides a theoretical model for analyzing the behavior of peer-reviewed journals. It finds that, apart from natural human errors, inefficiencies arise purely for reasons of inter-journal strategic behavior. Specifically, as a result of competition, journals tend to set their quality cut-off points excessively low.

BIOGRAPHICAL SKETCH

Vidya Atal was born and brought up in a small town in West Bengal, India. After finishing high school, she did her Bachelor's and Master's degree with Economics major from two renowned institutions in Calcutta — the St. Xavier's College and the Indian Statistical Institute, respectively. At the beginning, she was not as passionate about economics as some of her other friends were. But the more she learnt, the more she enjoyed it.

She joined the Ph.D. program in the Department of Economics at Cornell University in 2005. There she started working on various topics of industrial organization and development economics using the game theoretic approach. She has been working on theoretically analyzing different issues such as female labor supply, the strategic behavior of peer-review journals, and issues related to the United States patent system. Her current and future research agenda includes studying various policies in developing economies such as women empowerment, education and employment rights. In addition, she is interested in studying the effects of changes in H1-B visa policies on the service sector in India.

In memory of my father, Late Jagadish Prasad Atal

ACKNOWLEDGEMENTS

I am grateful to my Ph.D. advising committee, Kaushik Basu (chair), Talia Bar and Tapan Mitra, for helping me and encouraging me in every possible way to work on my dissertation. I would like to thank Haim Bar, Levon Barseghyan, Paul Carpenter, Phoebe Chan, Madhumita Datta, Norm Gilman from Gilman Research Services LLC, Shawn Xianzheng Kong, Corinne Langinier, Aija Leiponen, Oskar Liivak, Yann Mnire, Alan Paau, Joshua Teitelbaum and the participants of the Advanced Graduate Workshop (2009) conducted by Joseph Stiglitz at Manchester University, the participants of the seminars at Cornell University, and the participants in the International Industrial Organization Conference (2008 and 2009) for numerous extremely helpful comments. I also thank an anonymous patent researcher for insightful conversations and three anonymous referees for comments. My special thanks to Dipankar Dasgupta, Sunil Datta and Mukul Majumdar. Finally, I would like to convey my gratitude to my mother Nirmala Devi Atal, brother Vivek Atal and my husband Souptik Datta for providing me the limitless support without which I could not have reached here.

TABLE OF CONTENTS

Biographical Sketch	iii
Dedication	iv
Acknowledgements	v
Table of Contents	vi
List of Figures	viii
1 Introduction	1
1.1 Chapter Two: A Theory of Female Labor Supply	1
1.2 Chapter Three: Current Patent Policies And The Reform Proposals	2
1.2.1 Prior Art – To Search or Not to Search	3
1.2.2 Patent Quality and a Two-Tiered Patent System	3
1.3 Chapter Four: Do Journals Accept Too Many Papers?	4
2 A Theory of Female Labor Supply	6
2.1 Introduction	6
2.2 The Model	10
2.2.1 Steeper Indifference Curves of Women than Men’s	15
2.2.2 Flatter Indifference Curves of Women than Men’s	18
2.3 Children and Female Labor Force Participation	23
2.4 Technological Improvement and Household-work	25
2.5 Effects of Tax Benefit on Female Labor Supply	28
2.6 Dependence between Labor Markets of Men and Women: The General Equilibrium Framework	30
2.7 Reservation for Women at Work	35
2.8 Conclusion	38
Bibliography	40
3 Current Patent Policies And The Reform Proposals	43
3.1 Introduction	43
3.2 Prior Art: To Search or Not to Search	43
3.2.1 The Model	49
3.2.2 Innovator’s Optimal Search	53
3.2.3 Think Like an Examiner	58
3.2.4 Search Intensity and its Timing	61
3.2.5 Policy Implications	66
3.2.6 Prior Art Disclosure	72
3.2.7 Remarks	79
3.3 Patent Quality and a Two-Tiered Patent System	81
3.3.1 The Model	85
3.3.2 The Single-Tiered Patent System	87
3.3.3 Two-Tiered Patent System	97
3.3.4 Remarks	112

Bibliography	114
4 Do Journals Accept Too Many Papers?	117
4.1 Introduction	117
4.2 The Model	119
4.2.1 First-time Submissions	121
4.2.2 Second-time Submissions	122
4.2.3 Cut-off Quality in Nash Equilibrium	123
4.3 Cut-off Quality while under Same “Management”	126
4.4 Conclusion	129
Bibliography	131
A Proofs of the Lemmas and Propositions in Chapter 2	132
A.1 Lemma 1	132
A.2 Sufficient Condition for Investment	135
A.3 Proposition 1	136
A.4 Proposition 2	137
A.5 Proposition 3	138
A.6 Proposition 4	139
A.7 Proposition 5 and 6	140
A.8 Lemma 5 and its Proof	141
A.9 Proposition 7	142
A.10 Proposition 8	144
A.11 Proposition 9	145
A.12 Proposition 10	145
A.13 Proposition 11	146
A.14 Proposition 12	148
A.15 Proposition 13	150
A.16 Proposition 14	151
A.17 Proposition 15	154
A.18 Proposition 16	155
A.19 Proposition 17	158
A.20 Proposition 18	164

LIST OF FIGURES

2.1	Unique household equilibrium in Case I	15
2.2	Changes in household equilibrium with increase in w	16
2.3	Multiple equilibria in female-labor market	17
2.4	Multiple household equilibria in Case II	19
2.5	Changes in household equilibria with increase in w and female labor supply correspondence per household	20
2.6	Tax-benefit to the employers for employing women at work may give a huge boost to female labor force participation	21
2.7	Continuum of equilibria in female-labor market	22
2.8	Facilitating availability (or affordability) of household-labor saving goods may give a boost to female labor force participation	27
2.9	Introduction of tax-benefit might have different effects at different equilibria	29
2.10	The general equilibrium	34
2.11	Effects of a quota for women at work-place on the general equilibrium	37
2.12	Quota may reduce equilibrium female labor force participation	37
3.1	Equilibrium in a single-tiered system	90
3.2	Effect of an increase in patenting fee	92
3.3	Effect of an increase in examination intensity	92
3.4	Economically significant patents	110

CHAPTER 1

INTRODUCTION

An economic agent, be it an individual in a household or a firm in a market, is usually interested in enhancing its power and domain of control in order to use these to increase its pay-offs. The nexus between power and utility is different for different kinds of agents and can take complex forms at times. For example, in a household, a woman can acquire more power by bringing in to the household a larger share of the household income; and this, in turn, can enable her to buy the kind of goods she considers valuable for the household. In particular, the amount of labor a woman supplies depends on the intra-household bargaining power of the woman, which, in turn, is influenced by the amount of labor she supplies. On the other hand, a firm in a monopolistic market can have more market-power and control by having more valuable patents and this can influence its decision to invest in R&D and file for patents. In my dissertation, I analyze these and other kinds of interaction between power and control, on the one hand, and market outcomes and earnings, on the other, in three chapters.

1.1 Chapter Two: A Theory of Female Labor Supply

This chapter of my dissertation studies the nature of female labor supply in an economy and how it is affected by policy reforms and technological advances. It also studies the nature of female power within the household. Both these topics interact with each other and the aim of the chapter is to analyze this interaction in a setting more general than has been attempted thus far. In this chapter, I construct a comprehensive model of the status of women and the behavior of female labor in the market place. The aim is to understand the na-

ture of female labor supply in a set-up where the decision to work is not that of the woman herself but of the household to which she belongs. The set-up used is a very general one, where market wages of both men and women are fully flexible and influence each other. Interesting implications emerge from the recognition that the more a woman contributes to the family income, the more power she gains; and, as the power of the woman increases in the household, she has more freedom to do what she prefers – house-work or outside job. It is shown that female labor supply can take different forms due to structural differences between economies, and the occurrence of multiple equilibria. The latter implies that, two economies, similar in every fundamental aspect, might end up at two different equilibria and thus they may look very different in behavioral terms. Effects of children and technological improvement on female labor supply are also studied. Policy implications of tax-benefit programs, subsidies for labor-saving household durables (like cooking oven), facilitating availability of drinking water and reservation for women at work-place are worked out as well. The results found here resonate well with the range of empirical findings reported in the literature.

1.2 Chapter Three: Current Patent Policies And The Reform Proposals

The next chapter of my dissertation is based on papers that I have written with Talia Bar. It has two parts. The first part studies and criticizes one particular feature – the prior art search and its disclosure – in the current patent system in the United States. The second part analyzes a proposal of the patent system reform

– a two-tiered patent system, in existence of a negative externality imposed by bad patents on the value an innovator gets for holding a good patent.

1.2.1 Prior Art – To Search or Not to Search

To determine patentability, inventions are evaluated in light of existing prior art. Innovators have a duty to disclose any prior art that they are aware of, but have no obligation to search. This section studies innovators' incentives to search for prior art, and their search intensities and timing. We distinguish between early state of the art search—conducted before R&D investment, and novelty search—conducted right before applying for a patent. We identify conditions in which innovators have no incentive to search for prior art. Hence the entire responsibility of searching for any invalidating prior art passes to the patent examiner and, given the backlog of patent applications, examiners do not spend enough time on each application. Thus the economy may end up having a lot of bad patents. We find that the search intensity of applicants increases with the R&D cost, the examiners' expected search effort, and with patenting fees. We also find that innovators prefer to correlate their search technology with that of the patent office. In light of our model, we discuss the implications of some proposed policy reforms.

1.2.2 Patent Quality and a Two-Tiered Patent System

This part of the chapter provides an equilibrium model of the patent system. We study determinants of patent quality and the volume of patent applications. The

value of a patent for innovators may depend on the overall quality of granted patents, which in turn depends on the volume of applications, as well as on the examination process. It is found that, in equilibrium, an increase in patenting fee reduces the number of patent applications and increases the quality of patents. Interestingly, a more stringent examination process can have ambiguous effects on the volume of patent applications. We study the effects of patent system reforms on the equilibrium volume of applications and on patent quality. We model and evaluate a reform proposal that has recently captured the attention of policy makers to establish a two-tiered patent system in which applicants can choose a more costly and more stringent examination process – “gold-plate patents”. It is developed on the founding idea of a two-tiered patent system as described by Lemley, Lichtman and Sampat (2005). Introducing a second patent-tier can reduce patent applications, and the incidence of bad patents. We claim that innovators with high ex-ante probability of validity will more likely apply for the gold-plate patents, but sorting in the dimension of economic significance is not obvious.

1.3 Chapter Four: Do Journals Accept Too Many Papers?

The final chapter of my dissertation provides a theoretical model for analyzing the behavior of peer-reviewed journals. There has been rising concern about the flaws in the peer review system. Due to errors in the refereeing process, many papers of good quality get rejections from journals. The reverse mistake of accepting papers that ought not to be set in print also no doubt occurs. It is demonstrated in this chapter that, apart from natural human errors, inefficiencies can arise purely for reasons of inter-journal strategic behavior. Specifi-

cally, it is shown that, as a result of competition to improve their ranks, journals tend to set their quality cut-off points excessively low. Apart from these specific findings, this chapter also tries to contribute to providing a general theoretical structure for analyzing journal behavior – a subject that has not received as much attention as it should.

CHAPTER 2
A THEORY OF FEMALE LABOR SUPPLY

"Men are from Mars, Women are from Venus." – John Gray (1992)

2.1 Introduction

There exist differences between the preferences of men and women. These lead them to take different decisions in similar situations. Many empirical studies find that giving household subsidies to a woman rather than a man leads to different outcomes in the household expenditures, notably, child nutrition and schooling (see Senauer, Garcia & Jacinto, 1988; Hopkins, Levin and Haddad, 1994; Hoddinott and Haddad, 1995; Handa, 1999; Duflo, 2003; Gitter and Barham, 2008). Recently, there have been empirical studies suggesting differences in the household-decisions that can be attributed to differences in the power distribution between husbands and wives within households (Felkey, 2005; Lancaster, Maitra and Ray, 2006; Gitter and Barham, 2008). It is, therefore, necessary to study the economic issues driven by decisions of women separately from those influenced by men; labor supply is one such decision.

Labor supply plays a very important role in an economy's development. A robust and ample labor force promotes development, and development, in turn, feeds back on labor market conditions. Studying the behavior of labor market can give rise to important policy implications. There have been many studies focusing on labor supply and in recent times, there has also been a fair amount of research on female labor supply in particular. Most of these studies are, however, empirical (Blundell, Ham and Meghir, 1987; Arellano and Meghir,

1992; Nakamura and Nakamura, 1994; Eissa and Liebman, 1996; Greenwood, Seshadri and Yorukoglu, 2005). The theoretical foundations of this topic has not been well explored in the existing literature and this chapter attempts to make amends for that.

The goal of this chapter is to develop a general theory of female labor supply—a theory that shows how the nature of female labor supply can take different forms and shapes due to cultural or structural differences between economies. Hence similar policies might have different economic implications. Therefore, before considering any proposal for policy reforms for raising female labor force participation, it is necessary to understand its behavior in that particular economy.

Usually a woman's labor supply decision is not taken by herself alone. All adult members of the household, and especially the adult males would typically participate in this decision. To study the behavior of female labor supply, it is thus important to understand the household's decision making process. On one hand, a working woman's income adds to the household's total income which increases the collective utility; on the other hand, working outside leaves a woman with less time to spend on household-work which in turn decreases household-utility. Therefore a woman's labor supply decision depends on the collective utility of the household, the power distribution between the members of the household and, of course, the market wages. The power of a woman may be determined endogenously. The more a woman contributes to the family income compared to the other members of the family, the more power she gains; again, as the power of the woman increases in the household, she has more freedom to do what she prefers—household work or outside job.

This chapter works with a general equilibrium model in which consumption and female labor supply decisions are made by households and power is determined endogenously. The producers employ both men and women to produce the consumption good; and households own equal shares of profits earned by the firms. Using this model, it is shown that female labor supply can be increasing, or decreasing, or backward-bending, with respect to a rise in the market wage rate. Under some circumstances, multiple equilibria might occur in the female labor market so that two economies with exactly same fundamental characteristics might end up at two very different equilibria: one with a high female labor force participation and the other with a low participation. Sometimes multiple equilibria might occur within households which give rise to the female labor supply taking the form of a correspondence. In such a situation, a slight rise in female labor demand may cause a huge increase in female-labor employment.

The chapter also derives some important comparative statics results and policy implications. It is found, somewhat unsurprisingly, that women reduce their labor force participation when they have child-birth, or if they have children with disabilities. I also study the effects of technological innovations of consumer durable goods, which help in reducing household work, on the female labor supply and women's time spent on house-work. I find that, as a result, women tend to work more outside home since household work becomes less labor-intensive. The model also follows us to study the policy implications of tax-benefit programs and reservation for women. These are found to have ambiguous effects. It can be shown, using the model in this chapter, that the effects of these policies on female labor force participation are not necessarily positive, contrary to what we would be led to believe if we rely solely on intuition.

Even in economies with similar fundamental characteristics, the equilibrium female labor force participation may rise in one and fall in the other as a result of tax-benefits given to women or work-place reservation policy for them. This occurs because of the multiplicity of equilibria. Although only one policy implication—reservation for women—is actually worked out in a general equilibrium framework where there exist some substitutability between female labor and male labor, it is argued that all other results hold in the general set-up.

There is a growing literature on collective models of household behavior (Bourguignon and Chiappori, 1992, 1994; Vermeulen, 2002; Lundberg, 2005). However, very few papers relate female labor to the structure of household decision making. Francois (1998), Basu (2006) and, hopefully, mine are contributions to this. Francois' (1998) paper was focused on gender discrimination. He showed that even in the absence of any gender-specific inefficiencies, gender discrimination in the labor market may arise just "from the interaction between women and men within the household."

The model developed in the present chapter is more closely related to the one in Basu (2006). Using a collective utility model, he showed how a household might end up with multiple equilibria while choosing the effort-level of the woman for working outside home. However, he assumed that wages are fixed which can be justified as long as we are considering one household at a time. One household (consisting of one woman) cannot have any significant impact on the wages. But when we aggregate all the household decisions to get the total female labor supply, we cannot take female and, for that matter, male wages to be fixed because market wages are determined endogenously. They depend on the labor demand and the total labor supply. Allowing wages to

vary, it can be shown that the multiplicity of household equilibria described in Basu's model might vanish. And, more interestingly, we might have multiple equilibria in the female-labor market although there is a unique equilibrium for each of the identical households.

2.2 The Model

There are N identical households in a society. Each household consists of two adults: a male (m) and a female (f). They have different utility functions. However, they take the household-decisions collectively. Their objective is to maximize a weighted average of the utility each of them gets from their collective decisions. The weights depend on the power distribution in the household. Let $\theta \in [0, 1]$ denote the power of the woman in the household. Hence $(1 - \theta)$ is the power of the man. Following the arguments of Agarwal (1997) and Basu (2006), it will be assumed that this index of power is endogenous to the household, that is, while θ influences household decisions, the decisions in turn influence θ . The woman may gain more power by earning money from an outside job and thus increasing the total household income; on the other hand, she can choose to do more what she likes—outside job or household work—if she has more power. This endogeneity of power is not at odds with empirical findings; see Bittman, England, Sayer, Folbre and Matheson (2003). Let $e \in [0, 1]$ denote the woman's effort put to work outside home and $h \in [0, 1]$ be her effort on household work, $(e + h) \in [0, 1]$. Let α denote the woman's pain or disutility from outside job in terms of household work, i.e., the pain from working for one hour outside is equivalent to the pain from working α hours in the household, $\alpha > 0$. Basically, working at home or outside are perfectly substitutable choices for the woman

and α works as a preference parameter here. Hence working one hour outside is equivalent to working α hours at home.

Let w be the market wage rate for female labor and fix, for the time-being, the wages for men at \bar{w} . Unlike Basu (2006), wages in this chapter are not fixed. This allows us to study the labor markets, especially the female labor supply and to do comparative statics in a general equilibrium framework. I initially assume that the labor markets for women and men are two completely separate markets independent of each other—changes in the wages and employment in one has no influence on the other. This of course is not true in reality and later on I relax this assumption to allow for dependence between them. To focus on the analysis of female labor supply, assume that the man always puts effort 1 for outside work.

Let x be the consumption good and normalize its price at 1. For technical ease, assume that there is only one consumption good and both agents gain some utility from it. Let $v_i(\cdot)$ denote the utility of a person of gender $i \in \{m, f\}$ from the household work done by the woman and assume $v'_i(\cdot) > 0, v''_i(\cdot) \leq 0$. Let us denote the pain caused by i 's effort on outside work by $c_i(\cdot)$, where $c'_i(\cdot) > 0, c''_i(\cdot) \geq 0$, i.e., the disutility increases at an increasing rate. Now we can write down the utility functions for the female and the male in a household in the following form:

$$\bar{u}_f(x, e, h) = x + v_f(h) - c_f(h + \alpha e),$$

$$u_m(x, h) = x + v_m(h) - c_m(1).$$

Assume that $v'_f(1) > c'_f(1), i \in \{m, f\}$, i.e., for all h , the woman's marginal utility from her work at home is more than her marginal disutility from that. This guarantees that the optimum choice of e and h by the household are such that $h > 0$

and $(e + h) = 1$, i.e., the woman puts her entire effort 1 on work—household and outside. Let

$$u_f(x, h) = \bar{u}_f(x, 1 - h, h).$$

Hence the household's objective is to choose (x, h) or, equivalently, (x, e) such that the weighted average of the utilities of the man and the woman

$$\begin{aligned} U(x, h) &= \theta u_f(x, h) + (1 - \theta) u_m(x, h) \\ &= x + \theta [v_f(h) - c_f(\alpha + (1 - \alpha)h)] + (1 - \theta) [v_m(h) - c_m(1)] \end{aligned} \quad (2.1)$$

is maximized subject to the household's budget constraint

$$x \leq \bar{w} + (1 - h)w.$$

Since the household's collective utility is strictly increasing in x , the budget constraint will hold with equality. Substituting for x from the budget constraint, let

$$\begin{aligned} \tilde{u}_i(h) &= u_i(\bar{w} + (1 - h)w, h), i \in \{m, f\}, \\ \tilde{U}(h) &= \theta \tilde{u}_f(h) + (1 - \theta) \tilde{u}_m(h). \end{aligned}$$

Hence, $\tilde{U}(h)$ is maximized w.r.t. $h \in [0, 1]$. Therefore, when the woman's power is θ and the market wage rate for her labor is w , the collective utility maximizing effort (e) by the woman for her outside job is given by the solution of the first order condition:¹

$$\begin{aligned} w &= \theta [v'_f(h) + (\alpha - 1)c'_f(\alpha + (1 - \alpha)h)] + (1 - \theta)v'_m(h) \\ \text{or, } w &= \theta [v'_f(1 - e) + (\alpha - 1)c'_f(1 + (\alpha - 1)e)] + (1 - \theta)v'_m(1 - e). \end{aligned} \quad (2.2)$$

The equation above gives us the household-utility maximizing effort supplied by the woman for outside job, e , as a function of θ for a given wage w :

$$e = e(\theta, w).$$

¹If $w < \theta [v'_f(1) + (\alpha - 1)c'_f(1)] + (1 - \theta)v'_m(1)$, then $e = 0$.

Implicitly differentiating the first order condition (2.2), find that

$$\frac{\partial h}{\partial \theta} > 0 \left(\text{or, } \frac{\partial e}{\partial \theta} < 0 \right)$$

if and only if

$$\frac{\partial u_f(x, h)}{\partial h} > \frac{\partial u_m(x, h)}{\partial h}$$

or, $MRS_f^{h,x} > MRS_m^{h,x}$,

where $MRS_i^{h,x}$ is person i 's marginal rate of substitution between the woman's household work and the consumption good. The above statement simply means that if the woman's marginal rate of substitution between her household work and the consumption good is more than that of the man's, i.e., if the woman prefers working at home more than the man likes her household work, then the more power she gains, the more she can choose to work at home.

The woman can acquire more power by earning more. Suppose the power of a woman (θ) in the household depends not only on the relative wages she earns compared to the man ($\frac{e w}{w}$), but also on the prevailing relative market wage for female labor ($\frac{w}{w}$). If $\frac{w}{w}$ is very high, then even a woman who does not actually go outside for a job (i.e., $e = 0$), can enjoy a pretty high power by the mere availability of a very good outside option. On the other hand, if $\frac{w}{w} = 0$ (or a very low value), then the woman cannot gain a lot of power by working outside even for full-time. Therefore, since \bar{w} is fixed, we can write the power of a woman (θ) as a function of (e, w) so that θ is increasing in e and as w increases, θ shifts up.

$$\theta = \theta(e, w).$$

Definition 1 *A household equilibrium in this model, for a given market wage rate for female labor w , is described by $(e^*(w), \theta^*(w))$ where*

$$e^*(w) = e(\theta^*(w), w),$$

$$\theta^*(w) = \theta(e^*(w), w).$$

Definition 2 *An equilibrium in the female labor market, or simply market equilibrium, occurs when total female labor supply equals the demand for female labor.*

Let $L_f^D(w)$ be the female labor demand when market wage rate for female labor is w and let $L_f^S(w)$ be the female labor supply at that wage rate. Hence, given that there are N identical households (and thus N women) in the economy, from the household equilibrium described above, we can say that the total female labor supply in the economy is

$$L_f^S(w) = N \cdot e^*(w).$$

Therefore, the market equilibrium is given by the equilibrium wage rate for female labor w^* such that

$$L_f^D(w^*) = N \cdot e^*(w^*).$$

Next, I shall assume the single crossing property between the man's and the woman's indifference curves. By this, I mean to assume either of the following two cases. In case I, $MRS_f^{h,x} > MRS_m^{h,x}$ for all h so that the indifference curves of the woman for her household work and consumption good are always steeper than those of the man. As a result, the household's collective utility maximizing effort supplied by the woman for outside job (e) is decreasing with her power: $\frac{\partial e}{\partial \theta} < 0$. In case II, I assume exactly the opposite, i.e., $MRS_f^{h,x} < MRS_m^{h,x}$ for all h which means steeper indifference curves for the man compared to those for the woman. Hence, in case II, the more power she gains, the more she chooses to work outside: $\frac{\partial e}{\partial \theta} > 0$. Let us analyze these two cases in the following two sections and describe the household equilibrium and female labor market equilibrium in each case.

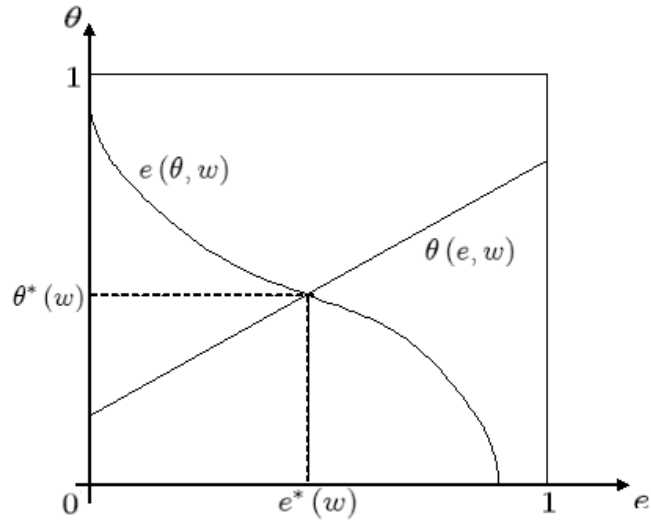


Figure 2.1: Unique household equilibrium in Case I

2.2.1 Steeper Indifference Curves of Women than Men's

In this case, the woman's marginal rate of substitution between her household work and consumption good is more than that of the man. Since the woman likes working at home more than the man likes her household work, the household's collective utility maximizing effort supplied by the woman for outside job decreases as her power increases: $\frac{\partial e}{\partial \theta} < 0$. However, the woman can acquire more power by working more outside (thus earning more): $\frac{\partial \theta}{\partial e} > 0$. Thus, plotting $e(\theta, w)$ and $\theta(e, w)$ in the $e - \theta$ space, it is easy to see that there exists a unique household equilibrium in this case, as shown in Figure 2.1.

To find the market equilibrium, we first need to construct the female labor supply from the household equilibria at different market wages for female labor. From the first order condition of the household's utility maximization problem

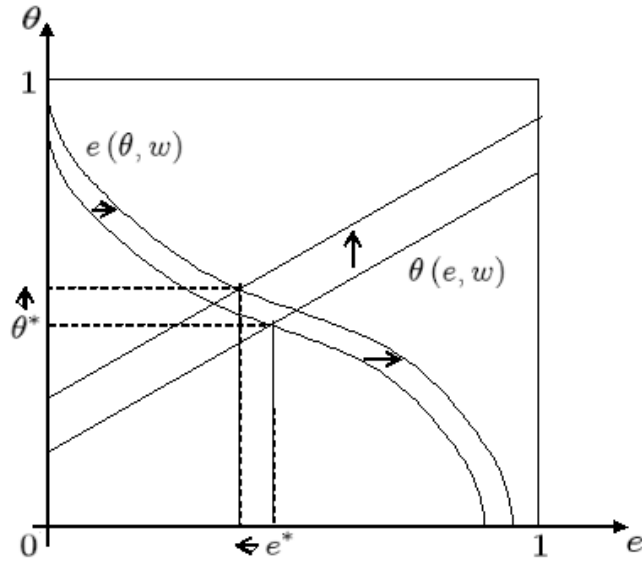


Figure 2.2: Changes in household equilibrium with increase in w

given by Equation (2.2), it is easy to check that

$$\frac{\partial e}{\partial w} > 0.$$

We can think of it as a substitution effect of a price-rise. The market wage-rate w is nothing but the price of working one hour at home for the woman. Hence, as a result of a rise in wages, she will want to work less at home and work more outside. There is a “power-gain effect” as well. As we have argued earlier, the more the market wage is, the more power the woman earns:

$$\frac{\partial \theta}{\partial w} > 0,$$

and the more power she earns, the less she wants to work outside. Therefore, the total effect of the increased wages on her outside work choice is ambiguous. As the wage-rate for female labor rises, both $e(\theta, w)$ and $\theta(e, w)$ shift up in the $e - \theta$ space. This may cause e^* to either increase or decrease or remain unchanged. If $e^*(w)$ increases as w increases, then the female labor supply curve is increasing

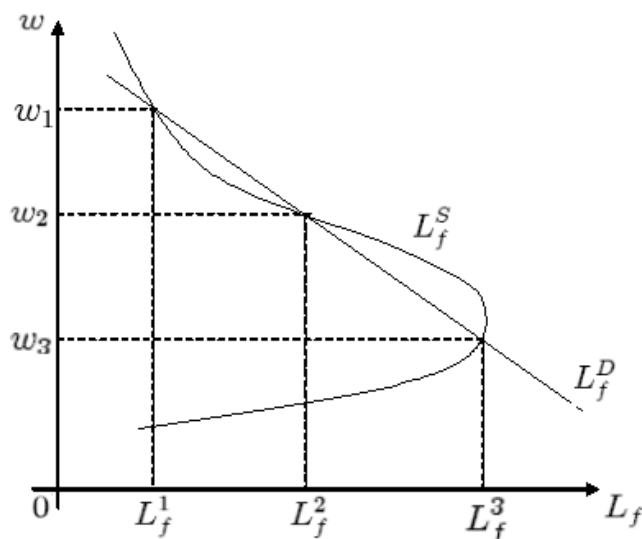


Figure 2.3: Multiple equilibria in female-labor market

as usual. But if $e^*(w)$ decreases as w increases, then interesting outcomes may occur since the female labor supply curve is now decreasing. In Figure 2.2 we can see a situation where the effort-level in the household-equilibrium falls as female-wages increase.

This may give rise to a downward sloping or backward bending supply curve for female labor. Assuming downward sloping demand curve for female labor, we might have multiple equilibria in some situations in the female labor market although there exists a unique household equilibrium.² One such situation is shown in Figure 2.3.

Therefore, two economies, similar in every fundamental aspect, might end up at two different equilibria and thus they look very different from outside in terms of the outcome. One of them might have a very high female labor force

²I have elsewhere, along with co-authors, established a different setting where, again with feasible wages, one gets multiple equilibria though through a very different mechanism (Atal, Basu, Gray and Lee, 2010).

participation in equilibrium and low market wage rate. And in the other one, women may spend more time at household work in equilibrium although the market wage rate is very high.

2.2.2 Flatter Indifference Curves of Women than Men's

Recall that, in this case, when the marginal utility from the outside job for the woman is more than the marginal utility of the man for her effort put outside, then as the power of the woman increases, her effort supply for outside job increases: $\frac{\partial e}{\partial \theta} > 0$. Since both the "power-earning curve" $\theta(e, w)$ and "effort supply curve" $e(\theta, w)$ are increasing in this case, we might have multiple equilibria in a household as shown in Basu (2006). Plotting $e(\theta, w)$ and $\theta(e, w)$ in the $e - \theta$ space as we did earlier, we can get the following figure which shows one such instance where for a given wage w for female-labor, there exist three household equilibria: $E_1 = (e_1, \theta_1)$, $E_2 = (e_2, \theta_2)$ and $E_3 = (e_3, \theta_3)$.

As we did in the previous case, let us now find the market equilibrium in this case. For that, we need to construct the female labor supply first, that is, allow the wages to change and check what happens to the household equilibrium.

If we incorporate changes in wages for female-labor, the multiplicity of household equilibria might not exist even in the example shown in Figure 2.4. To see this, first note that as wage-rate w increases, the power-earning curve $\theta(e, w)$ shifts up and the effort-supply curve $e(\theta, w)$ moves to the right (or down) (see Figure 2.5) since for $w'' > w'$, we have

$$\theta(e, w'') > \theta(e, w') \text{ for all } e \text{ and}$$

$$e(\theta, w'') > e(\theta, w') \text{ for all } \theta.$$

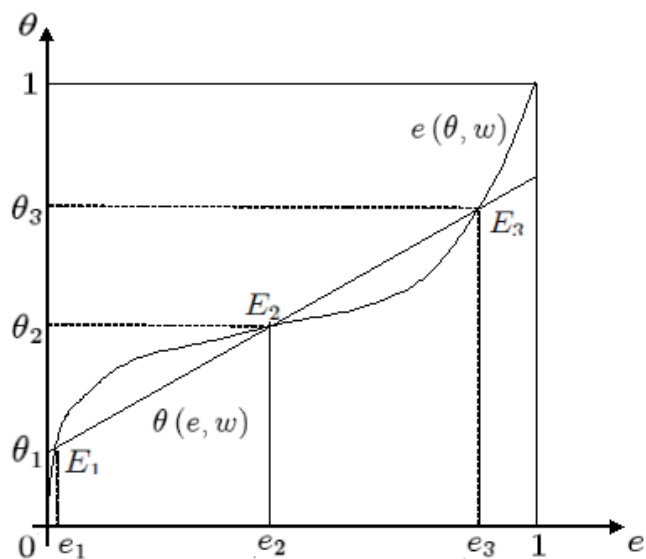


Figure 2.4: Multiple household equilibria in Case II

Hence the equilibrium effort-levels e_1 and e_2 come closer and e_2 and e_3 move farther apart. After a sufficient increase in w , two of the three equilibria E_1 and E_2 vanish and the household ends up at the unique equilibrium with a very high effort-level e . Similarly, for sufficiently low wages for female-labor, the household may have a unique equilibrium with very low effort-level. Since for wages in some particular range we might have multiple equilibria for each household, the female labor supply for each household in such a situation is given by a correspondence as shown in Figure 2.5.

If all the households choose exactly the same equilibrium at a given wage, then the total female labor supply looks exactly like the effort supply correspondence for each household (as in Figure 2.5). The total labor supply is N times the effort exerted by the woman from each household where N is the total number of households in the society. In this case, a slight rise in female labor demand might give a huge boost to the female labor force participation (in hours). In

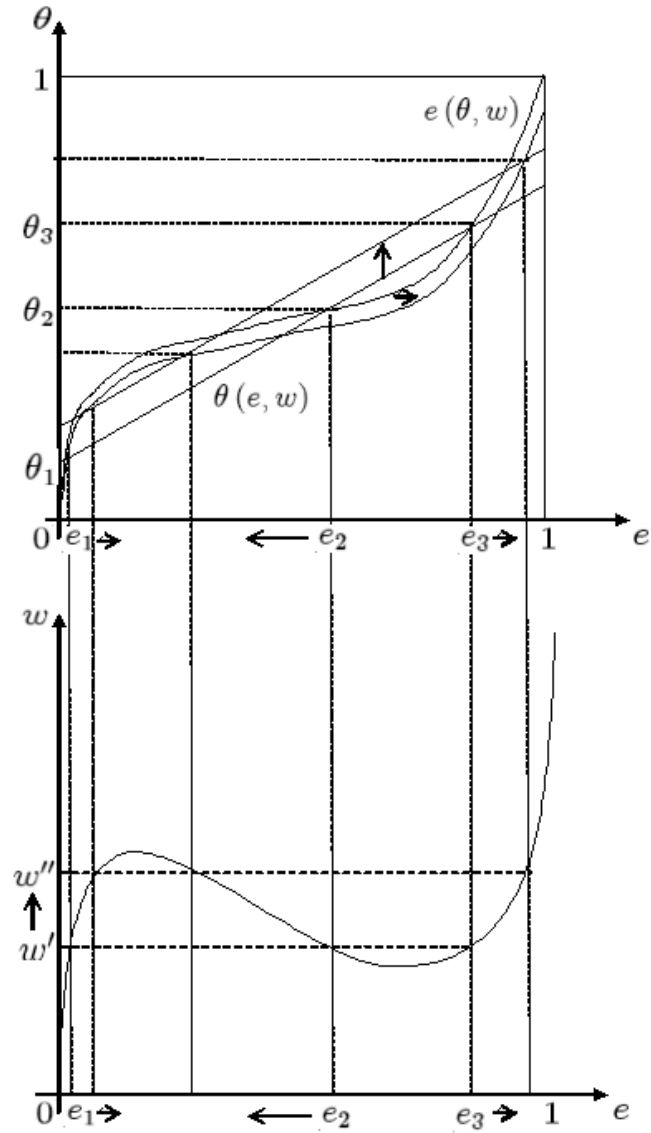


Figure 2.5: Changes in household equilibria with increase in w and female labor supply correspondence per household

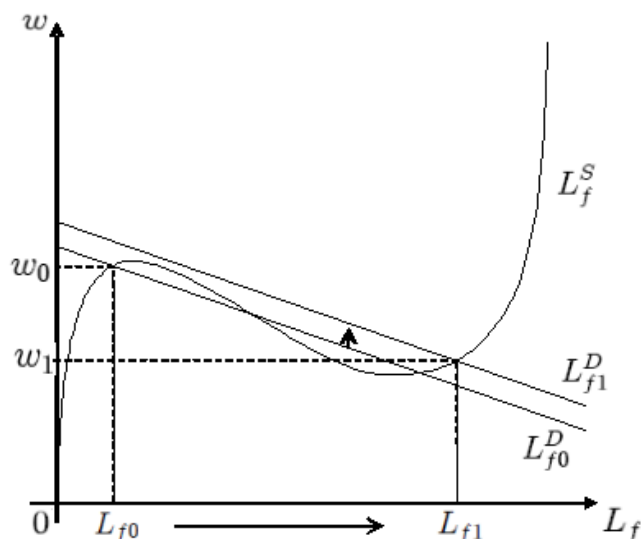


Figure 2.6: Tax-benefit to the employers for employing women at work may give a huge boost to female labor force participation

Figure 2.6, consider a situation where an economy starts with a labor demand L_{f0}^D and it is at a low participation equilibrium L_{f0} . Then, due to a tax-benefit to the employers for employing women at work, suppose the demand for female labor shifts up to the one given by L_{f1}^D in Figure 2.6. As a result, the economy reaches at a new equilibrium L_{f1} where both the supply and demand for female labor are much more compared to the initial equilibrium causing a major increase in female labor force participation (in hours). In fact, in this case, if the government decides to rescind on the policy slowly, i.e., by gradually reducing the tax-benefits so that the demand for female labor moves back to the initial one, the economy may end up being at the high participation equilibrium instead of the low one where it originally started.

Instead of all households behaving the same way, if households are assigned one of the multiple equilibria in appropriate proportions, then the total female

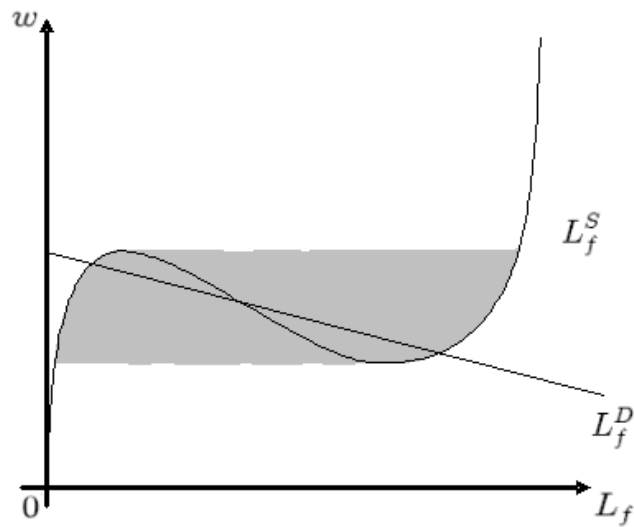


Figure 2.7: Continuum of equilibria in female-labor market

labor supply correspondence (denoted by L_f^S) looks like the one in Figure 2.7. For each w for which there exist multiple equilibria in the household, the total female-labor supply can be any rational number between $Ne_{\min}(w)$ and $Ne_{\max}(w)$ where $e_{\min}(w)$ is the minimum effort-level supplied by a woman at wages w according to the multiple household-equilibria and $e_{\max}(w)$ gives the maximum effort-level by a woman in the household-equilibrium for the same wages. If N is very large, then since rational numbers are dense on real line, the total female labor supply correspondence L_f^S has a thick area as shown in Figure 2.7. Let L_f^D be the demand curve for female-labor which is down-ward sloping. Then, from Figure 2.7, it is evident that a society might have a continuum of equilibria in the female labor market.

In the entire analysis above we have seen some situations where multiple equilibria may occur in the female labor market and in some other instances we may have a continuum of equilibria. However, all these analyses have been

done keeping the labor market for men out of the picture. We assumed that movements in the labor market for women does not have any impact on the market wage for men. I shall relax this assumption later in Section 2.6 and analyze in a general equilibrium framework.

2.3 Children and Female Labor Force Participation

While studying women and their labor force participation (as hours of work outside home), one usual question arises always: how does it change when women have kids? Many people have studied empirically the effect of children on the work choices of the mothers (Nakamura and Nakamura, 1994; Porterfield, 2002; Boushey, 2008). All of them, unsurprisingly, found a negative impact, specially when a child is young or with disabilities. Young children or children with disabilities usually require more attention of their mothers compared to the older healthier children. Hence a woman with a young child³ (along with the entire household) face more difficulties if the woman spends more time for the outside job. The marginal utility of the household from the woman's household-work is now larger than before. Hence the household ends up choosing more time at house-work (and less time outside) for the woman.

To see this outcome from the model in this chapter, suppose a new baby is born. Although it is unrealistic to assume that newborn babies will consume the same good as adults, for algebraic simplicity, let us assume that the baby also gets some utility from the household consumption good x . Let $v_b(h)$ be the utility the child gets from the mother's house-work, $v'_b(\cdot) > 0$ and $v''_b(\cdot) \leq 0$.

³Note that fertility is exogenous in this model.

Hence the total utility of the child from the consumption good and the mother's house-work is given by

$$u_b(x, h) = x + v_b(h).$$

Let b be the importance the baby's preferences get in the household-utility, and thus $(1 - b)\theta$ and $(1 - b)(1 - \theta)$ are the weights attached with the woman's and the man's utilities, respectively, in the household. Therefore, the collective utility of the household becomes

$$\begin{aligned} \widehat{U}(x, h) \\ = bu_b(x, h) + (1 - b) \left[\theta u_f(x, h) + (1 - \theta) u_m(x, h) \right]. \end{aligned}$$

Since the income of the family does not change, the budget constraint remains unchanged. Hence, as we have done earlier, the first order condition of the household's collective utility maximization problem is

$$\begin{aligned} w &= bv'_b(h) + (1 - b) \left[\theta \{v'_f(h) + (\alpha - 1) c'_f(\alpha + (1 - \alpha)h)\} + (1 - \theta) v'_m(h) \right] \\ &= b \frac{\partial u_b(x, h)}{\partial h} + (1 - b) \left[\theta \frac{\partial u_f(x, h)}{\partial h} + (1 - \theta) \frac{\partial u_m(x, h)}{\partial h} \right]. \end{aligned}$$

Therefore, if $b > 0$ and if the marginal utility of the baby is more than the marginal utility of the rest of the household from the woman's house-work, i.e., if

$$\frac{\partial u_b(x, h)}{\partial h} > \left[\theta \frac{\partial u_f(x, h)}{\partial h} + (1 - \theta) \frac{\partial u_m(x, h)}{\partial h} \right],$$

which is usually expected in case of young children or children with disabilities, then the household's utility maximizing effort supply of the woman falls or $e(\theta, w)$ shifts left. We are not done yet. To find out the effect on the female labor supply, we have to find out what happens to the household equilibria. From Figure 2.1 in Section 2.2.1 and Figure 2.4 in Section 2.2.2, it is easy to check that given w , equilibrium effort supply (and thus total female labor supply) falls

as $e(\theta, w)$ shifts left.⁴ Hence, the model in this chapter is able to establish, theoretically, the same result as expected by all intuitively and shown empirically, that existence of children, who need more attention of their mothers, reduces the female labor supply.

2.4 Technological Improvement and Household-work

In the 20th century and late 19th century, following the industrial revolution, people observed a huge technological revolution in their homes. The introduction of household consumer durables like washing machine, vacuum cleaner, refrigerator, etc. made the household work a lot easier. Hence, many people argued, women could spend less time at household and work more outside. In 1912, Thomas Edison said in an interview that “(t)he housewife of the future will be neither a slave to servants nor herself a drudge. She will give less attention to the home, because the home will need less; she will be rather a domestic engineer than a domestic labourer, with the greatest of all handmaidens, electricity, at her service. This and other mechanical forces will so revolutionize the woman’s world that a large portion of the aggregate of woman’s energy will be conserved for use in broader, more constructive fields.”

However, there has been a debate among social scientists over this issue. Some find that, as a result of the technological revolution, women indeed spent less time in the household and labor force participation had increased (Gershuny and Robinson, 1988; Greenwood, Seshadri and Yorukoglu, 2005). Others, on the other hand, show evidence that this was not the case. Vanek (1974) ar-

⁴In Figure 2.4, e_2 (the equilibrium effort supply in the middle one of the three equilibria, E_2) goes up, but we can ignore it since it is an unstable equilibrium.

gued that women's time spent on household work has increased marginally since the revolution. Bittman, Rice and Wajcman (2004) showed that "domestic technology rarely reduces women's unpaid working time and even, paradoxically, produces some increases in domestic labour." It would be interesting what theory would have to say on this matter and that is what I proceed to investigate now.

First of all, recall the first order condition from the household's collective utility maximization problem after substituting for x from the budget constraint:

$$\theta \bar{u}'_f(h) + (1 - \theta) \bar{u}'_m(h) = 0,$$

where

$$\bar{u}_i(h) = u_i(\bar{w} + (1 - h)w, h), i \in \{m, f\}.$$

Suppose η is a parameter in the model. Then, for finding the comparative statics results of η on h , we have to differentiate the first order condition given above⁵ with respect to η .

$$\frac{\partial}{\partial \eta} [\theta \bar{u}'_f + (1 - \theta) \bar{u}'_m] + [\theta \bar{u}''_f + (1 - \theta) \bar{u}''_m] \frac{dh}{d\eta} = 0$$

or,

$$[\theta \bar{u}''_f + (1 - \theta) \bar{u}''_m] \frac{dh}{d\eta} = -\frac{\partial}{\partial \eta} [\theta \bar{u}'_f + (1 - \theta) \bar{u}'_m].$$

This implies, since $\bar{u}''_i < 0$ for $i \in \{m, f\}$,

$$\begin{aligned} \text{sign}\left(\frac{dh}{d\eta}\right) &= \text{sign}\left[\frac{\partial}{\partial \eta} (\theta \bar{u}'_f + (1 - \theta) \bar{u}'_m)\right] \\ &= \text{sign}\left[\frac{\partial}{\partial \eta} \left[\theta \{v'_f(h) + (\alpha - 1) c'_f(\alpha + (1 - \alpha)h)\} + (1 - \theta) v'_m(h) - w\right]\right]. \end{aligned}$$

Technological improvement in household's consumer durables helps increase the efficiency at household work. Therefore, the output from the same

⁵Dropping the variables in the parentheses for ease of writing.

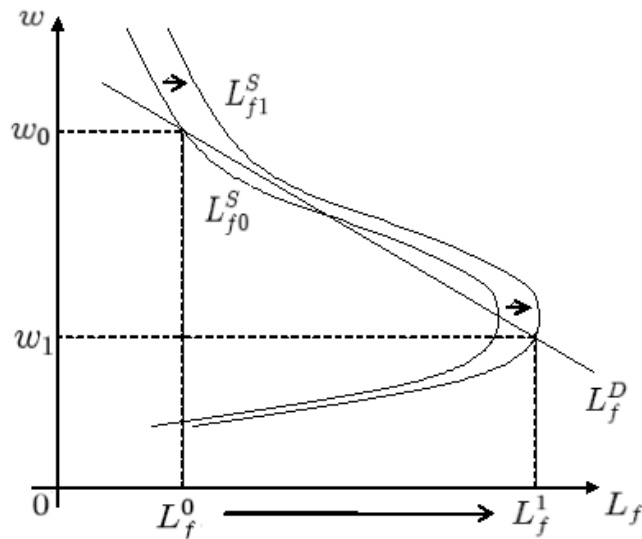


Figure 2.8: Facilitating availability (or affordability) of household-labor saving goods may give a boost to female labor force participation

time spent on the house-work is expected to be much more than before. Hence, the marginal utility from the same time spent at house-work is now less than before. This means, in terms of our model, a lower $v'_i(\cdot)$ for $i \in \{m, f\}$. Since we know that $(e + h) = 1$ always, it is easy to check that when η represents fall in $v'_i(\cdot)$, then the collective utility maximizing effort (or time) spent at house-work h falls. Therefore, according to our model, technological improvement in household's consumer durable goods causes an increase in female labor force participation, as found in Gershuny and Robinson (1988), Greenwood, Seshadri and Yorukoglu (2005).

This may give rise to important policy implications. Subsidizing consumer durable goods that increase efficiency at household-work, for example washing machines, or facilitating the availability of drinking water, or any such improvement that is labor-saving in household-work, may raise the female labor

supply and this might lead to a huge—in fact, disproportionate—increase in the female labor force participation (in hours) in equilibrium. To see this, suppose the economy has a backward bending female labor supply curve L_{f0}^S as in Figure 2.8 and initially the economy is situated at a low participation equilibrium (L_f^0, w_0) . Then suppose the government starts distributing washing machines or cooking ovens at a subsidized rate. This leads to a rise in female labor supply and it shifts right to L_{f1}^S . As a result, as it can be seen in Figure 2.8, the economy reaches at a new equilibrium with a massive increase in female labor force participation from L_f^0 to L_f^1 .

2.5 Effects of Tax Benefit on Female Labor Supply

In many countries, there are policy programs specially aimed at increasing the participation of women in the labor force. One of these policies include giving tax-benefits to women on their incomes. To see the implications of this policy in our model, we can do similar exercise as we have done in the previous section.

In this case, η represents w because w is basically the net wage rate (gross wage rate minus taxes) and a tax-benefit simply means a rise in net wages. As a result, from Equation (2.2), it is easy to find that given θ , the household's utility maximizing effort supply of the woman rises or $e(\theta, w)$ shifts right. Again, since her net wage rate is now higher, she can gain more power from the same amount of effort put on outside job, i.e., $\theta(e, w)$ shifts up. This exercise has been worked out in Figure 2.2 in Section 2.2.1 and in Figure 2.5 in Section 2.2.2. From these figures, it is evident that the equilibrium effort supply of the woman (and thus total female labor supply) might go up or go down as a result of the tax-

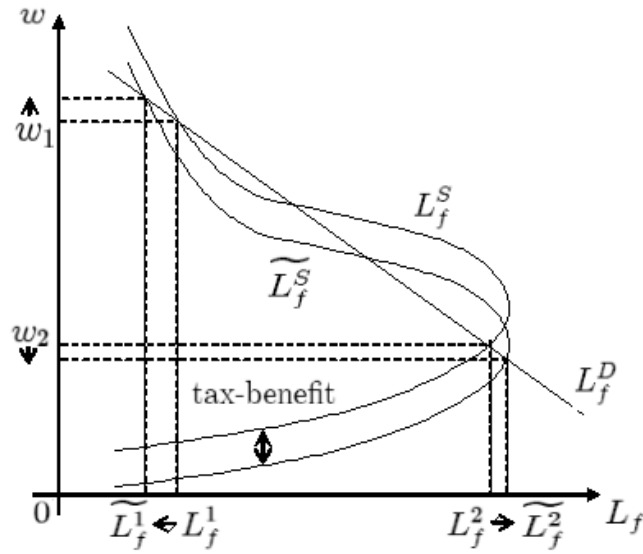


Figure 2.9: Introduction of tax-benefit might have different effects at different equilibria

benefit to women. Note that since a tax-benefit of value τ on net wage-rate w is equivalent to a rise in the net wage rate by the same amount, the entire female labor supply curve (or correspondence) shifts down by that amount:

$$L_f^S(w + \tau) = \widetilde{L}_f^S(w),$$

where L_f^S and \widetilde{L}_f^S denote the female labor supply before and after the introduction tax-benefit, respectively. Hence, in an economy with a backward-bending female labor supply curve as shown in Figure 2.3 and multiple equilibria in the female labor market, introduction of a tax-benefit program for women might have different outcomes depending on which market equilibrium the economy is at. In Figure 2.9, we can see that a tax-benefit causes an increase in female labor force participation (in terms of hours) in one equilibrium (from L_f^2 to \widetilde{L}_f^2), whereas, in the other equilibrium, it falls, $\widetilde{L}_f^1 < L_f^1$. There have been many empirical works for measuring the effectiveness of some tax-benefit programs (Eissa

and Liebman, 1996; Blundell, Duncan and Meghir, 1998; Grogger, 2003). Most of them find a positive impact on women's labor force participation. Eissa and Liebman (1996) found that for one group of women, the effect is positive and for another group of women, it is zero. But, to the best of my knowledge, the negative impact of tax-benefits on the equilibrium female labor force participation has not been observed by any empirical work yet.

2.6 Dependence between Labor Markets of Men and Women:

The General Equilibrium Framework

In the previous sections, the female and male labor markets were treated as two completely separate markets, changes in one having no influence on the other. In this section, I allow for some substitutability between male and female labor. Let us bring in the production side as well to get the demand for labor. In the partial equilibrium framework one done in Basu (2006), we don't consider all the economic agents in the economy and thus we might miss the feedback effects from them. Hence we might over-estimate or under-estimate some policy implications which is misleading for the government while considering a policy. Therefore, it is necessary to study the policy issues in the general equilibrium framework.

Suppose there are N identical households (each consists of a man and a woman) who own M identical firms producing the consumption good x with equal share of profits. As before, let x be the numeraire good in this model, i.e., price of the consumption good is 1. Let w_f be the wages for female labor and w_m be the wages for male labor. Assume that the man always works full-time

outside, however, the woman's effort for outside job (e) is chosen in the household equilibrium by maximizing household utility. The utility functions take the same form as before (given by equation (2.1)), but the budget constraint is now different. Each of the N household's objective function is

$$\max_{x \geq 0, e \in [0,1]} U(x, e) = \theta u_f(x, e, 1 - e) + (1 - \theta) u_m(x, e)$$

subject to the budget constraint

$$x \leq (w_m + ew_f) + \frac{M}{N} \pi(w_f, w_m)$$

where π is the profit of each firm. Typically, because of the dependence of the budget constraint on the non-wage income π , we should have a family of labor supply curves (each curve indexed by the π level). However, in this case, the household-utility maximizing labor supply will be independent of the level of non-wage income. This arises because we have a utility function which is quasi-linear in the consumption good, and we are assuming an interior solution. That is, any increase in non-wage income (wage rates being given) would be fully reflected in a corresponding increase in the consumption good, x , to restore budget equality.

As found in Section 2.2, the first order condition of the household's collective utility maximization problem is given by

$$w_f = \theta \left[v'_f(1 - e) + (\alpha - 1) c'_f(1 + (\alpha - 1)e) \right] + (1 - \theta) v'_m(1 - e)$$

which in turn gives us the effort supplied by the woman for outside job, e , as a function of θ for a given female-wage w_f :

$$e = e(\theta, w_f)$$

where

$$\frac{\partial e}{\partial \theta} \leq 0$$

accordingly as

$$\frac{\partial u_f(x, e, 1 - e)}{\partial e} \geq \frac{\partial u_m(x, 1 - e)}{\partial e} \text{ for all } e.$$

Suppose the power of a woman (θ) in the household depends on her total earnings relative to the man ($\frac{ew_f}{w_m}$) and also on the prevailing relative market wage ($\frac{w_f}{w_m}$). At very high relative wages, the woman can exercise a lot of power in the household even though she does not actually go outside for a job (i.e., $e = 0$) just because she has a very good outside option available. On the other hand, if relative wages are very low, then the woman cannot gain a lot of power by working outside even for full-time. Therefore,

$$\theta = \theta\left(e, \frac{w_f}{w_m}\right).$$

A household equilibrium in this model, for given wages (w_f, w_m) , is described by $(e^*(w_f, w_m), \theta^*(w_f, w_m))$ where

$$\begin{aligned} e^*(w_f, w_m) &= e\left(\theta^*(w_f, w_m), w_f\right) \text{ and} \\ \theta^*(w_f, w_m) &= \theta\left(e^*(w_f, w_m), \frac{w_f}{w_m}\right). \end{aligned}$$

Hence, since the man always goes out to work for the entire 1 unit of time, the total supply of female and male labor in this economy are

$$\begin{aligned} L_f^S(w_f, w_m) &= N \cdot e^*(w_f, w_m), \\ L_m^S(w_f, w_m) &= N. \end{aligned}$$

Now let us look at the producers' side. Suppose that there exists some substitutability between male labor and female labor. Each one of the M identical producers choose the amount of inputs (or the two kinds of labor) to maximize profit:

$$\pi = F(L_f, L_m) - w_m L_m - w_f L_f,$$

where $F(L_f, L_m)$ is the production function with two inputs—female labor and male labor, with positive marginal products.⁶ Assuming decreasing returns to scale in both the inputs or a strictly concave production function, i.e.,

$$\begin{aligned} \frac{\partial^2 F}{\partial L_f^2} < 0, \frac{\partial^2 F}{\partial L_m^2} < 0 \text{ and} \\ \frac{\partial^2 F}{\partial L_f^2} \frac{\partial^2 F}{\partial L_m^2} > \left(\frac{\partial^2 F}{\partial L_f \partial L_m} \right)^2. \end{aligned}$$

Therefore, solving the two first order conditions of the profit maximization problem, we get the demand for both kinds of labor by each firm:

$$\begin{aligned} \frac{\partial F(L_f, L_m)}{\partial L_f} &= w_f \text{ and } \frac{\partial F(L_f, L_m)}{\partial L_m} = w_m \\ \Rightarrow L_f &= L_f(w_f, w_m) \text{ and } L_m = L_m(w_f, w_m). \end{aligned}$$

Concavity of the production function guarantees downward sloping labor demand curves with an upward shift caused by the increase in wages for the other kind of labor since male labor and female labor are substitutes to some extent. Hence, the total demand for both kinds of labor is given by

$$\begin{aligned} L_f^D(w_f, w_m) &= M \cdot L_f(w_f, w_m), \\ L_m^D(w_f, w_m) &= M \cdot L_m(w_f, w_m). \end{aligned}$$

Since the price of the consumption good is assumed to be 1, the equilibrium of this model is described by the equilibrium wage rates for female and male labor (w_f^*, w_m^*) where the demand for labor equals its supply, i.e.,

$$\begin{aligned} L_f^D(w_f^*, w_m^*) &= L_f^S(w_f^*, w_m^*), \\ L_m^D(w_f^*, w_m^*) &= L_m^S(w_f^*, w_m^*). \end{aligned}$$

With this general equilibrium set-up in hand, note that, while studying the

⁶To avoid any kind of complementarity between the inputs, assume that the elasticity of substitution is at least as much as 1, i.e., $\frac{\frac{\partial F}{\partial L_f} / \frac{\partial F}{\partial L_m}}{(L_f/L_m)} \geq -\frac{d\left(\frac{\partial F}{\partial L_f} / \frac{\partial F}{\partial L_m}\right)}{d(L_f/L_m)}$.

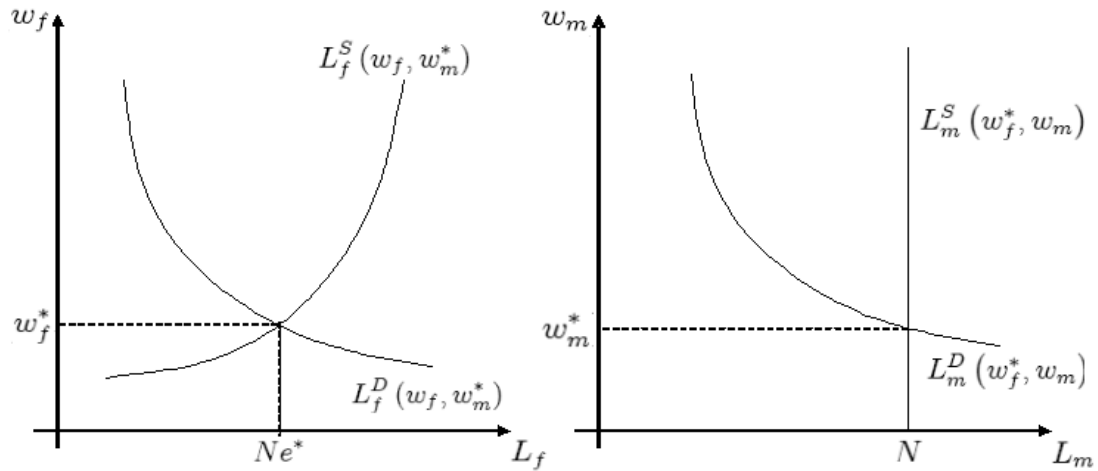


Figure 2.10: The general equilibrium

impacts of various changes like child-birth, technological improvement, or tax-benefits on female labor supply, as done in Sections 2.3, 2.4 and 2.5 respectively, it has been argued that only female labor supply gets affected and that leads to a change in the equilibrium female labor force participation and female wage-rate. Since the demand-side for both female and male labors remain unchanged, the analysis leads to similar results in the current general equilibrium framework as well. ⁷ If, however, labor-demand changes, the outcome may be different. For example, in case of technological improvement, if the production technology also changes and becomes less female-labor intensive, then the demand for female-labor decreases along with an increase in the supply (due to the availability of labor-saving household durables), and hence the overall effect on the equilibrium participation rate becomes ambiguous.

⁷The direction of change will be the same, but the exact quantitative measurement of the policy implications will be different between the partial and the general equilibrium ones.

2.7 Reservation for Women at Work

Although almost half of the population in the world is female, they occupy a much smaller proportion of population in terms of employment. In 2000, women held only 30% share of the total employed positions. The average hourly wage-rate of women is just three-quarters of that of men. Aiming at reducing the gender-inequality, various countries have been considering different policies to increase female labor force participation and reduce the wage-gap. They have been trying to do so by providing micro-credit facilities targeted at women, facilitating vocational training programs, raising general awareness of the society, and so on. In India, reservation of political posts for women is one such policy. According to this, at least one-third of all the villages should have a woman as their council-leaders. This has led to a significant increase in women's involvement in as well as impact on policy-making (Chattopadhyay and Duflo, 2004). This gives us a hint that a similar quota for women at work-places may encourage female labor force participation and reduce the wage-gap between men and women. Let us check the implication of this kind of a policy by using the model in this chapter.

Suppose, initially, the economy was at a general equilibrium as described in the previous section. Then the government makes a law by which the ratio between the number of female employees to the number of male employees, at each of the M firms producing the consumption good, has to be at least as large as the fraction $r \in (0, 1)$, i.e.,

$$\frac{L_f}{L_m} \geq r.$$

As a result, the producer cannot always choose the profit-maximizing levels of both kinds of labor. When female wage-rate is high enough compared to the

male wage-rate, then although profit-maximization requires the ratio between female labor and male labor to be strictly less than r , the producer cannot do that due to the quota and in that situation, he simply maintains the ratio exactly. For example, let us take a specific CES production function with decreasing returns to scale:

$$F(L_f, L_m) = \left[\frac{1}{3}L_f^{\frac{1}{2}} + \frac{2}{3}L_m^{\frac{1}{2}} \right]^{\frac{2}{3}}.$$

Let the quota for female employees be $r = 0.36$. Hence the ratio between the demands for the two kinds of labor will be

$$\frac{L_f^D(w_f, w_m)}{L_m^D(w_f, w_m)} = \begin{cases} \left(\frac{w_m}{2w_f} \right)^2, & \text{if } \frac{w_m}{w_f} \geq \frac{6}{5} \\ 0.36 & \text{otherwise.} \end{cases}$$

Therefore, a portion of the female labor demand curve shifts right when female wages are high compared to existing male wages. And, for the similar reason, a portion of the demand curve for male labor shifts down when male wages are low. As a result, as evident from Figure 2.11, female labor force participation in equilibrium may rise and the wage-gap between male labor and female labor reduces.

However, we have to be careful with this policy. A quota for women at work-place does not necessarily raise female labor force participation. For example, in case of a backward-bending female labor supply with an initial equilibrium at the downward sloping part of the curve, see Figure 2.12, an upward shift in the demand for female labor reduces the equilibrium participation rate. Also, the wage-gap may not necessarily go down as a result of a quota for women at work-places. If the productivity of women is sufficiently low compared to that of men, then it might be more profitable for a producer to cut the production down rather than hiring more women.

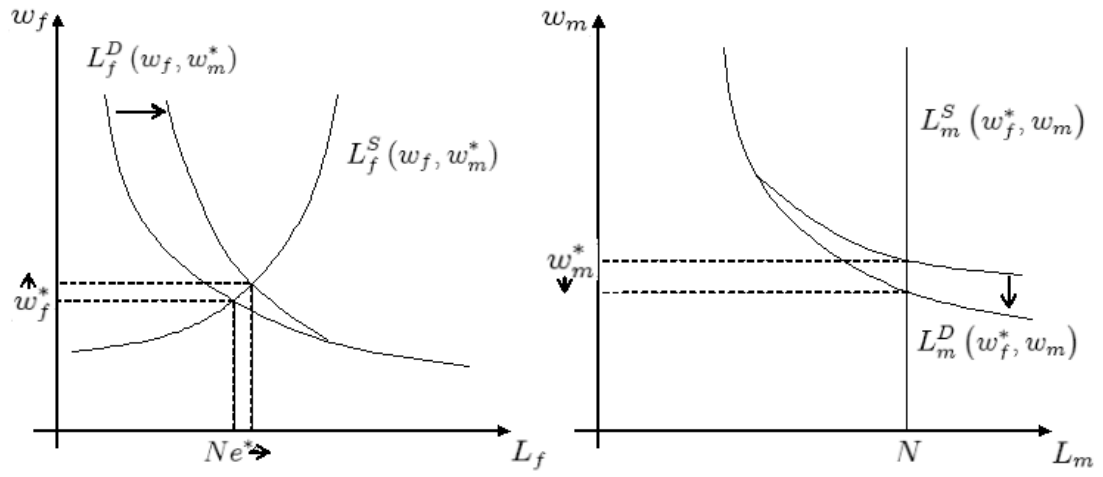


Figure 2.11: Effects of a quota for women at work-place on the general equilibrium

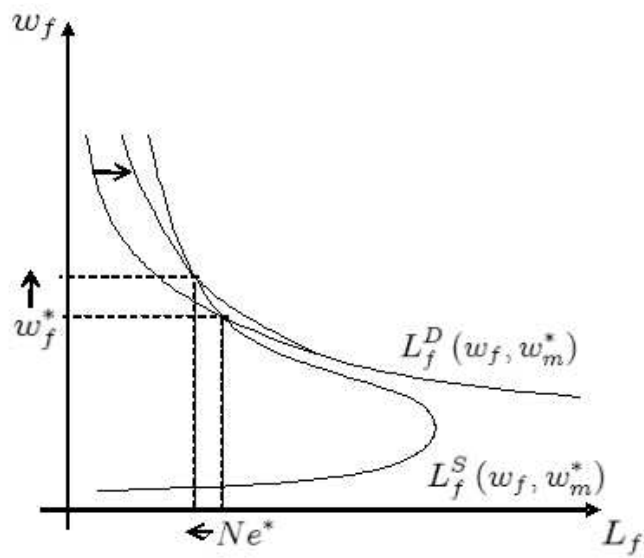


Figure 2.12: Quota may reduce equilibrium female labor force participation

Hence, if women's empowerment is the objective of the policy-makers, then reservation for women at work-place may be a policy they might consider. However, if the goal is to increase the female labor force participation or reduce the wage-gap, then they have to be careful.

2.8 Conclusion

In this chapter, I have tried to develop a theoretical model for studying the nature of female labor supply in an economy. Since the labor supply decision of a woman is taken by the entire household instead of just the individual herself, I have considered a collective utility model to explain the behavior of female labor supply. The power of the woman, and thus the power distribution between all members of the household, has been taken to be endogenous here. Under this setting, it has been shown that female labor supply can take various shapes as the market wage rate changes. Sometimes multiple equilibria might occur in the female labor market. Hence we can have different policy implications for different economies depending on the behavior (or shapes) of their female labor supply (and also their demand for female labor). Not only that, policy implications might differ for the same economy at different time-points depending on the initial equilibrium before the policy-imposition.

In the entire analysis above, we have assumed similar characteristics for all the households in an economy. We assumed that women have same efficiency-level across all the households which is far from reality. Further research on the theory of female labor supply can be done where women have heterogeneous abilities. We can think of the scope of education as well in this context. Educa-

tion can help an individual in acquiring more skill and thus gain more power to bargain for higher wages from the employer. However, getting some education is costly. Even if basic primary education may be freely available in many countries, acquiring education may involve an opportunity cost because of the time spent for it. This might give rise to interesting outcomes in women's participation decisions in skilled or unskilled labor force and the literacy rate among them in an economy.

BIBLIOGRAPHY

- [1] Agarwal, B., ““Bargaining” and Gender Relations: Within and beyond the Household.” *Feminist Economics*, 3(1): 1–51, 1997.
- [2] Arellano, M. and C. Meghir, “Female Labour Supply and On-the-Job Search: An Empirical Model Estimated Using Complementary Data Sets.” *Review of Economic Studies*, 59: 537-557, 1992.
- [3] Atal, V., K. Basu, J. Gray and T. Lee, “Literacy Traps: Society-wide Education and Individual Skill Premia.” Forthcoming, *International Journal of Economic Theory*, 2010.
- [4] Basu, K., “Gender and Say: A Model of Household Behavior with Endogenously-Determined Balance of Power,” *The Economic Journal*, 116(4): 558-580, 2006.
- [5] Bittman, M., J. M. Rice and J. Wajcman, “Appliances and Their Impact: The Ownership of Domestic Technology and Time Spent on Household Work.” *The British Journal of Sociology*, 55(3): 401-423, 2004.
- [6] Bittman, M., P. England, L. Sayer, N. Folbre and G. Matheson, “When Does Gender Trump Money? Bargaining and Time in Household Work,” *American Journal of Sociology*, 109(1): 186-214, 2003.
- [7] Blundell, R., A. Duncan and C. Meghir, “Estimating Labor Supply Responses Using Tax Reforms.” *Econometrica*, 66(4): 827-861, 1998.
- [8] Blundell, R., J. Ham and C. Meghir, “Unemployment and Female Labour Supply.” *The Economic Journal*, 97: 44-64, 1987.
- [9] Bourguignon, F. and P. Chiappori, “Collective Models of Household Behavior: An Introduction.” *European Economic Review*, 36: 355-364, 1992.
- [10] Bourguignon, F. and P. Chiappori, “The Collective Approach to Household Behavior.” *The Measurement of Household Welfare* (Blundell, R., I. Preston and I. Walker eds.), Cambridge University Press, 1994.
- [11] Boushey, H., ““Opting out?” The effect of children on women’s employment in the United States.” *Feminist Economics*, 14(1): 1-36, 2008.

- [12] Chattopadhyay, R. and E. Duflo, "Women as Policy Makers: Evidence from a Randomized Policy Experiment in India." *Econometrica*, 72(5): 1409-1443, 2004.
- [13] Duflo, E., "Grandmothers and Granddaughters: Old-Age Pensions and Intra-household Allocations in South Africa." *The World Bank Economic Review*, 17(1): 1-25, 2003.
- [14] Eissa, N. and J. B. Liebman, "Labor Supply Response to the Earned Income Tax Credit" *The Quarterly Journal of Economics*, 111(2): 605-637, 1996.
- [15] Felkey, A. J., "Husbands, Wives and the Peculiar Economics of Household Public Goods." <http://campus.lakeforest.edu/felkey/marketpaper.pdf>, 2005.
- [16] Francois, P., "Gender Discrimination without Gender Difference: Theory and Policy Responses." *Journal of Public Economics*, 68: 1-32, 1998.
- [17] Gershuny, J. and J. P. Robinson, "Historical Changes in the Household Division of Labor." *Demography*, 25(4): 537-52, 1988.
- [18] Gitter, S. R. and B. L. Barham, "Women's Power, Conditional Cash Transfers, and Schooling in Nicaragua." *The World Bank Economic Review*, 22(2): 271-290, 2008.
- [19] Greenwood, J., A. Seshadri and M. Yorukoglu, "Engines of Liberation." *The Review of Economic Studies*, 72(1): 109-133, 2005.
- [20] Grogger, J., "The Effects of Time Limits, the EITC, and Other Policy Changes on Welfare Use, Work, and Income among Female-Headed Families." *The Review of Economics and Statistics*, 85(2): 394-408, 2003.
- [21] Handa, S., "Maternal Education and Child Height." *Economic Development and Cultural Change*, 47: 421-439, 1999.
- [22] Hoddinott, J. and L. Haddad, "Does Female Income Share Influence Household Expenditures? Evidence from Cote D'Ivoire." *Oxford Bulletin of Economics and Statistics*, 51: 77-96, 1995.
- [23] Hopkins, J., C. Levin and L. Haddad, "Women's Income and Household Expenditure Patterns: Gender or Flow? Evidence from Niger." *American Journal of Agricultural Economics*, 76: 1219-25, 1994.

- [24] Lancaster, G., P. Maitra and R. Ray, "Endogenous Intra-household Balance of Power and its Impact on Expenditure Patterns: Evidence from India." *Economica*, 73: 435–460, 2006.
- [25] Lundberg, S., "Gender and Household Decision-making." *Frontiers in Gender Economics* (Bettio, F. and A. Verashchagina eds.), Routledge, 2005.
- [26] Nakamura, A. and M. Nakamura, "Predicting Female Labor Supply: Effects of Children and Recent Work Experience." *The Journal of Human Resources*, 29(2): 304-327, 1994.
- [27] Porterfield, S., "Work Choices of Mothers in Families with Children with Disabilities." *Journal of Marriage and Family*, 64(4): 972-981, 2002.
- [28] Senauer, B., G. Garcia and E. Jacinto, "Determinants of the Intrahousehold Allocation of Food in the Rural Philippines." *American Journal of Agricultural Economics*, 70: 170-180, 1988.
- [29] Vanek, J., "Time Spent in Housework." *Scientific American*, 231(5): 116–120, 1974.
- [30] Vermeulen, F., "Collective household models: principles and main results." *Journal of Economic Surveys*, 16: 533-564, 2002.

CHAPTER 3

CURRENT PATENT POLICIES AND THE REFORM PROPOSALS

3.1 Introduction

This chapter is based on two papers I have co-authored with Talia Bar. The first part of this chapter studies and criticizes the prior art search and disclosure policies in the current patent system in the United States. The second part studies the effect of bad patents in the economy, especially the negative externality imposed by them on the value of a good patent holder. We also model and analyze a recent patent system reform proposal for setting up a two-tiered system.

3.2 Prior Art: To Search or Not to Search

The patent system was designed to provide incentives to innovate and to disclose research findings. Two central conditions for patentability of an invention—novelty and non-obviousness—are evaluated in light of the existing prior art. Broadly speaking, prior art could refer to any prior knowledge. However, for the purpose of determining patentability, prior art is defined in the U.S. Patent Act by stating that an invention is *not* patentable if “the invention was known or used by others in this country, or patented or described in a printed publication in this or a foreign country, before the invention thereof by the applicant for patent,” and if such knowledge existed more than one year before the filing of the patent (35 U.S.C. §102). According to Rule 56 of the Rules of Practice in Patent Cases (37 CFR §1.56), “each individual associated with the filing

and prosecution of a patent application has a duty of candor and good faith in dealing with the Office, which includes a duty to disclose to the Office all information known to that individual to be material to patentability..."¹ Thus, Rule 56 requires a patent applicant and his representatives not to intentionally omit any information they have that appears to be "by itself or in combination with other information" relevant for determining patentability. Violation of Rule 56 is considered "inequitable conduct" in court. However, there is no duty to search for prior art, only to disclose what is known. According to Cotropia (2007), "(t)he immediate results from a finding of inequitable conduct create a tremendous deterrent against nondisclosure," and there is a "perverse incentive for the relevant parties to remain ignorant about relevant information since the more the party knows, the greater is their exposure under the doctrine."

Patent examination is imperfect. Patents on "innovations" that are either not novel or obvious are often granted. Had the examiner been sufficiently informed, such patent would not have been granted. These "bad patents"—for which invalidating prior arts exist but are not found—might curtail future innovation, unnecessarily limit market activities and unduly create welfare reducing market power. Bad patents are also likely to result in waste due to litigation costs and disadvantage those who cannot afford it. Amid concerns over the patent office granting a growing number of bad patents, many have called for reform of the patent system and proposed remedies, such as a patent opposition system (Merges (1999)), patent bounties (Thomas (2001)), "gold-plate" patents (Lemley, Lichtman and Sampat (2005)), and community patent review (Noveck (2006)).

In August 2007, the United States Patent and Trademark Office (USPTO)

¹See Manual of Patent Examining Procedure <http://www.uspto.gov/web/offices/pac/mpep/>

published a set of new rules that included a requirement to submit, with any application that has more than five independent claims or twenty-five total claims, an examination support document (ESD) that contains a detailed prior art search statement by the innovator. On October 31, 2007, just before the new rules were set to become effective, the United States District Court for the Eastern District of Virginia issued a decision temporarily enjoining the USPTO from implementing the new rules.² On April 1, 2008, the court handed down a decision that permanently blocks implementation of the USPTO's proposed new rules.³ These proposed rules could be seen as an attempt to shift the duty of prior art search from examiners to innovators (at least in some instances).

According to Alcacer and Gittelman (2006), more than 500,000 utility patents were issued by the USPTO between 2001 and 2003, of which, around forty percent had all the prior art references inserted by the examiners. Additionally, two-thirds of all the citations on an average utility patent are contributed by the examiners. The goal of this section is to better understand innovators' incentives to search for (and thus reveal) prior art and the policy levers that affect these incentives. We study the benefits, intensity and the timing of prior art search and the potential implications of related proposed policy changes.

Our analysis distinguishes between *ex ante* search (conducted before R&D investment), which we refer to as "*early state of the art search*" and *ex post* or "*novelty search*" (conducted after successful R&D but before filing for the patent). Early state of the art search might help avoid duplication when it is not profitable to duplicate (saving investment cost) and it could shape innovation

²Tafas v. Dudas, No. 1:07 Civ 846 (E.D. va. Oct. 31, 2007), see memorandum opinion 10-31-2007, available at <http://www.jsslaw.com/publications.aspx>

³Tafas v. Dudas, No. 1:07 Civ 846 (E.D. va. Apr.1, 2008), available at <http://www.patentlyo.com/patent/2008/04/tafas-v-dudas-p.html>

by guiding the researcher to a path that is more likely to be novel, whereas novelty search can save on patenting costs. Since search lowers the probability of being granted a patent, and even bad patents may be profitable to the awardee, an innovator might prefer to avoid or limit prior art search. We derive payoff maximizing search intensities and compare them to the socially optimal ones.

We study prior art search strategies in a sequential decision process. In the model, an innovator chooses her early state of the art search intensity before investing in R&D. She learns from search results and updates her belief on patentability. As more search effort produces no invalidating prior art, she becomes increasingly optimistic. After this initial search, she decides whether to invest in risky R&D. If R&D is successful, the innovator chooses the intensity of novelty-search and files for a patent if no invalidating prior art was found. At the patent office, an examiner follows a pre-determined search routine and grants the patent if no invalidating prior art was found.

We determine the innovators' optimal prior art search strategies under different policy rules and patent examination regimes. We find that the innovator's effort level is weakly increasing with the examiner's expected search effort. Innovators search more when R&D investment and patenting costs are higher. We identify conditions under which an innovator would prefer not to search at all. If the cost of patenting is sufficiently low compared to the gain from a bad patent, then the innovators under-invest in search compared to the social optimum. There are conditions under which a suitable patent fee can give innovators incentives for optimal search.

Patent policy has long been a subject of interest and debate in the economic literature. Such work examined various aspects of patent policy, for example,

optimal patent length and breadth (Klemperer, 1990; Gilbert and Shapiro, 1990), the novelty or patentability requirement (O'Donoghue, 1998; Scotchmer and Green, 1990), infringement and litigation (Chang, 1995; Crampes and Langinier, 2002). Prior art search and disclosure incentives have been discussed by many legal scholars. Yet, these issues have received relatively little formal consideration in the economic literature. To the best of our knowledge, the first model of prior art search and disclosure is due to Langinier and Marcoul (2003). Their paper examines "the strategic non-revelation of information by innovators when applying for patents." They recommended that a patent examiner should undertake identical scrutiny effort on all patent applications irrespective of the number of citations by the applicant. In our analysis, we assume this is the case. Lampe (2008) also considers innovators' incentives not to disclose prior art. He predicts that innovators would conceal information about prior arts which are most "closely related" to their invention and thus, the most important pieces of prior art are not cited by the patent applicants.

In contrast to these contributions, our main focus is on the incentives to search for prior art, its timing and intensity. In most of our analysis, innovators comply with the duty to disclose, but they may choose not to search. This premise is in line with the writing of legal scholars such as Thomas (2001): "(a)lthough Rule 56 mandates that the applicants disclose known prior art, it does not require them to search in the first place. Coupled with the draconian consequences of a holding of inequitable conduct, many applicants are discouraged from conducting prior art searches in the first place." Our private communications with innovators, IP attorneys and search experts also suggested that more often search is strategically avoided rather than its results illegally not disclosed. We argue that in fact, even if the consequences of inequitable conduct

are not severe, as long as prior art search requires effort, it is in the researcher's best interest to remain ignorant rather than search and conceal. Given no legal obligation to search, a researcher would not have an incentive to invest in prior art search in the first place unless, in the event prior art is found, she would change her actions—either not investing in this particular innovation, or not filing for the patent.

Caillaud and Duchêne (2007) examine the impact of the patent office on firms' incentives to innovate and to apply for patent protection, and the overload problem patent examiners face. They show that given imperfections in the examination process, some granting of bad patents are inevitable. In their model, innovators know the quality of their patents before deciding whether or not to apply. In contrast, since we focus on incentives to search for prior art, in our model innovators can learn about their innovations' quality by investing in prior art search. Caillaud and Duchêne (2007) also consider the role of patent fees as a policy instrument. They consider the effects of patenting fees on R&D investment and on incentives to apply for patents. This chapter, on the other hand, shows that patenting fees can also provide incentives to search for prior art.

Finally, we mention that there is a relatively recent body of empirical research on prior art search. From 2001, the USPTO began indicating which prior art references were inserted by the examiner. This newly available data on prior art enabled empirical analysis of prior art (see, for example, the contributions in Sampat (2004), Alcacer and Gittelman (2006), Lampe (2008), Alcacer and Gittelman (2006) and Alcacer, Gittelman and Sampat (2009)). There exist some works as well trying to improve our understanding about the operating procedure of

the USPTO, for example, Cockburn, Kortum and Stern (2002) and Langinier and Lluís (2009).

3.2.1 The Model

In our model, there is an innovator or a researcher (R) and an examiner (E). The researcher has an innovation idea which she at first believes to be patentable with probability $(1 - \alpha) \in (0, 1)$. With probability α , there exists invalidating prior art. There is a fixed cost for R&D denoted by I . R&D is risky, success occurs with a probability $\theta > 0$. The innovator can apply for a patent on her invention. A patent application costs P including patenting fees and legal costs. We account for the cost of prior art search separately.

Patent applications are examined in the patent office. We assume that the patent office commits to prior art search intensities and examiners pursue this search.⁴ We model prior art search technology with a function $F(X)$. If there was no search by the innovator, then conditional on the existence of invalidating prior art, examiner's search effort $X_E \in [0, \infty)$ reveals it with a probability $F(X_E) \in [0, 1]$. This probability increases with search effort, $F'(X) = f(X) > 0$, at a decreasing rate, $F''(X) = f'(X) < 0$. We denote by $\lambda(X) = \frac{f(X)}{1-F(X)}$, the hazard rate of the distribution F . We assume that the hazard rate (which represents the probability of finding invalidating prior art with effort X given that it is not found with a lower search effort) is non-increasing. Search technology F likely varies by field. In matured technological areas, where a lot of the prior art is patented, search is likely to be more efficient than in areas where most of the prior arts are not patented.

⁴We further discuss this assumption in Section 3.2.5.

The researcher can also search for prior art. The researcher’s search technology could be correlated with that of the examiner. For example, both the innovator and the examiner might start with examining the USPTO database and use similar keywords in their search. If the innovator’s search does not reveal prior art and if the examiner follows roughly the same search path as the innovator, then the examiner is not likely to find any invalidating prior art either. However, having been exposed to different research related experiences (interactions with colleagues, prior research or examination experience etc.), the researcher and the examiner could be using different data sources, different search engines, different search keywords and so on. Hence, our model accounts for the possibility that search technologies of the innovator and the examiner are somewhat but perhaps not perfectly correlated. To model varying levels of correlation, we assume that with probability ρ , the innovator has the same search path as the examiner, that is, examiner’s and researcher’s searches are perfectly correlated. But, with probability $(1 - \rho)$, the innovator has a different search path which is independent from that of the examiner. Search technologies are chosen by “nature” (i.e. determined by events not in the researcher’s or examiner’s control), but we discuss in Section 3.2.4 why researchers, if they can, might want to influence the degree of correlation between their search and that of the examiner.

Search efficiency of the examiner and the innovator could also differ. For simplicity, we take the same functional form for their search technologies $F(\cdot)$, but differences in search efficiency can be captured by differences in search costs. For the innovator, we assume that a search effort X_R costs $C_R(X_R) = X_R$.⁵ The examiner’s search cost is an increasing function $C_E(X_E)$ for search effort X_E .

⁵We assume here that search cost is incurred for a single innovation. It is possible however that innovators experience returns-to-scale when they engage in multiple innovation projects. The amount invested to search for prior art in one project can be used for another project as well. This is beyond the scope of this chapter.

When the examiner is less efficient than the innovator, this cost can be higher than X_E . For a given amount of examination time allocated to each application, examiner's "effective" units of search effort X_E would be lower in fields where his search technologies are less efficient, for example in emerging fields, where much of the prior art is not patented and examiners are less experienced.

Accounting for innovator's search and the correlation in search technologies, we find that if the researcher's search effort was X_R and the examiner's search effort is X_E , then the probability that the examiner finds invalidating prior art (IPA) conditional on invalidating prior art existing but it was not found by the innovator, is given by

$$p(X_R, X_E) = pr(E \text{ finds IPA} | \exists \text{ IPA and } R \text{ did not find it}), \quad (3.1)$$

or,

$$p(X_R, X_E) = \begin{cases} (1 - \rho) F(X_E) & \text{if } X_R \geq X_E \\ \frac{\rho(F(X_E) - F(X_R)) + (1 - \rho)(1 - F(X_R))F(X_E)}{1 - F(X_R)} & \text{if } X_R < X_E \end{cases}. \quad (3.2)$$

When $\rho = 0$, the search technologies are independent and the probability that the examiner finds prior art conditional on its existence is

$$p(X_R, X_E) = F(X_E) \quad (3.3)$$

which only depends on the examiner's effort. When $\rho = 1$, the search technologies are perfectly correlated and

$$p(X_R, X_E) = \begin{cases} 0 & \text{if } X_R \geq X_E \\ \frac{F(X_E) - F(X_R)}{1 - F(X_R)} & \text{if } X_R < X_E \end{cases}. \quad (3.4)$$

In the perfectly correlated case, if the examiner's search does not exceed that of the innovator, then if the innovator does not find any prior art, the examiner does not either.

We consider two stages of search. Early state of the art search, conducted before R&D investment, and novelty search, conducted after success in innovation but before filing for a patent. Before investment in R&D, the researcher chooses her early state of the art search intensity x_1 . Observing the results of this initial search, she decides whether to invest in R&D. If any invalidating prior art is found, she does not engage in research.⁶ When no invalidating prior art is found, the researcher updates her belief that her innovation is patentable. If innovation succeeds, the researcher chooses novelty search intensity level x_2 . Search at this stage accumulates with the early state of the art search, that is, conditional on the existence of invalidating prior art, if the innovator exerted early search effort x_1 and novelty search effort x_2 , then invalidating prior art is not found with probability $[1 - F(x_1 + x_2)]$.⁷ After conducting novelty search, the researcher further updates her belief on the patentability of her innovation and chooses whether to file for a patent.

After the examination process, the patent examiner decides whether to grant the patent. Since the examination process is not perfect, it is possible that bad patents would be granted. A bad patent refers to a patent granted when invalidating prior art exists but the examiner was not aware of it. The researcher enjoys a benefit G if she is granted a patent which is truly novel and a benefit $g < G$ if she is granted a bad patent. An awardee may benefit from bad patents because of the reputation value of having a patent. Larger patent portfolios can be useful in cross-licensing agreements with other firms or as signals

⁶We start by assuming that the innovator complies with the duty to disclose. Therefore, she does not invest if she finds invalidating prior art. We argue in Section 3.2.5 that we do not need this assumption.

⁷We implicitly assumed here, for simplicity, that the innovator's available search technology is the same before and after innovation. It is possible, however, that after successful innovation the innovator knows more and is better able to search. We generalize the model to allow for different search technologies ex-ante and ex-post under the assumption that $\rho = 0$ in Proposition 7 in Section 3.2.4 as well as in Section 3.2.6.

to investors. Patents, even bad ones, may also deter competitors from use of the innovation in fear of infringement suits, especially if the competitor is also unaware of the existing invalidating prior art or is unable to cover large litigation costs. But, it is reasonable to assume that the value of a bad patent is lower than that of a good patent since invalidating prior art can be exposed after its issuance. In particular, if a patent-holder plans to enforce it, the alleged infringer would likely make an effort to prove it invalid.

3.2.2 Innovator's Optimal Search

An innovator faces the following decisions: a choice of her early state of the art search effort x_1 , investment decision, novelty search effort after innovation x_2 and patent filing. We derive the innovator's optimal search effort for prior art using backward induction.

Consider first a successful researcher who did not find any invalidating prior art and is facing the decision whether to file for a patent or not. If invalidating prior art does not exist, then the innovator's benefit from the patent is G . If invalidating prior art exists (but the researcher's search effort did not reveal it), then the innovator's expected benefit from the patent application is $[1 - p(X_R, X_E)]g$, since with probability $[1 - p(X_R, X_E)]$ the patent examiner does not find invalidating prior art either. Having invested search efforts x_1 and x_2 and not found invalidating prior art (IPA), the innovator's belief that such prior art exists can be derived using Bayes' rule:

$$q(x_1 + x_2) = pr(\text{IPA exists} | \text{IPA not found}) = \frac{\alpha [1 - F(x_1 + x_2)]}{1 - \alpha F(x_1 + x_2)}. \quad (3.5)$$

Hence, the expected payoff from filing for a patent on an innovation for which

invalidating prior art was not found with search efforts (x_1, x_2) is

$$q(x_1 + x_2) [1 - p(X_R, X_E)] g + [1 - q(x_1 + x_2)] G - P - I - (x_1 + x_2). \quad (3.6)$$

The first two terms capture the expected benefits from a bad or a good patent application using updated belief, then we subtracted patenting costs, R&D costs and search costs.⁸ Given that the cost of investment and search are already sunk at this time, the innovator files for a patent only if

$$q(x_1 + x_2) [1 - p(X_R, X_E)] g + [1 - q(x_1 + x_2)] G \geq P. \quad (3.7)$$

We now consider the choice of effort for validity prior art search, x_2 . The innovator who has exerted effort x_1 and yet did not find any invalidating prior art has the belief that such prior art exists with probability

$$q(x_1) = \frac{\alpha [1 - F(x_1)]}{1 - \alpha F(x_1)}. \quad (3.8)$$

This probability equals α if no search effort was exerted, it declines to zero as $x_1 \rightarrow \infty$. That is, the innovator is increasingly optimistic that her innovation is good the more search effort she exerted without finding invalidating prior art. Let the net expected gain from a bad patent application be

$$B(X_R, X_E) = (1 - p(X_R, X_E)) g - P. \quad (3.9)$$

Using our definition of $p(X_R, X_E)$ from (3.2), we obtain

$$B(X_R, X_E) = \begin{cases} B + \rho g & \text{if } X_R \geq X_E \\ B + \left(\frac{1 - F(X_E)}{1 - F(X_R)} \right) \rho g & \text{if } X_R < X_E \end{cases}, \quad (3.10)$$

⁸Our analysis abstracts from the possibility that financial constraints limit innovator's ability to search for prior art or alter the amount invested in the R&D project. While it is possible that such financial constraints are sometimes in effect, we believe it is a reasonable simplification because in many cases R&D investments are on a much larger scale than the costs of prior art search. Basic prior art searches with search professionals cost \$1000 on average. Such cost is not likely to explain the large share of applicants who insert no prior art citations. Moreover, note that, large firms, who are less likely to be budget constrained, are more likely not to include prior art references (see Alcacer et al., 2009).

where

$$B = (1 - \rho)(1 - F(X_E))g - P. \quad (3.11)$$

The innovator will choose her novelty search effort x_2 to maximize her expected payoff:

$$\begin{aligned} \pi(x_1, x_2) = & q(x_1) \frac{[1 - F(x_1 + x_2)]}{[1 - F(x_1)]} B(X_R, X_E) \\ & + (1 - q(x_1))(G - P) - I - (x_1 + x_2). \end{aligned} \quad (3.12)$$

This payoff function is continuous in x_2 , it is everywhere differentiable except at the kink $x_2 = X_E - x_1$. For any given state of the art search x_1 , we can derive the optimal level of novelty search $x_2^*(x_1)$. In Lemma 1, we identify a condition under which the innovator would not engage in novelty search before patenting. The proof of Lemma 1, and all other proofs, are provided in the Appendix.

Lemma 1 *For any x_1 , there is a unique level of novelty search $x_2^*(x_1)$ that maximizes (3.12). When the net benefit from a bad patent is large enough ($B \geq 0$), the innovator does not invest in novelty search, $x_2^* = 0$.*

We now consider the decision to invest in R&D. Having invested x_1 in early state of the art search and not found invalidating prior art, the researcher invests in R&D if

$$\theta \left[q(x_1) \frac{[1 - F(x_1 + x_2^*)]}{[1 - F(x_1)]} B(X_R, X_E) + (1 - q(x_1))(G - P) - x_2^* \right] \geq I. \quad (3.13)$$

Let us assume that the expected benefit from the innovation is high enough so that the innovator invests in R&D if she found no prior art in her early search.

A sufficient condition (see Lemma 4 in the Appendix) for this to hold is:

$$\theta[\alpha B + (1 - \alpha)(G - P)] \geq I. \quad (3.14)$$

This condition states that the expected benefit from R&D investment, if the innovator does not search at all, exceeds its cost.

Consider now the choice of effort for initial prior art search, x_1 . Before conducting any search, the researcher has a prior belief that with probability α there exists prior art that can invalidate her innovation. Thus, her expected payoff from the initial search is

$$\begin{aligned} \Pi(x_1, x_2^*(x_1)) = & (1 - \alpha)[\theta(G - P - x_2^*) - I] + \\ & \alpha[1 - F(x_1)] \left[\theta \left(\frac{[1 - F(x_1 + x_2^*)]}{[1 - F(x_1)]} B(X_R, X_E) - x_2^* \right) - I \right] - x_1. \end{aligned} \quad (3.15)$$

Maximizing (3.15) with respect to early state of the art search intensity x_1 , taking into account its effect on x_2^* as derived in Lemma 1, yields the optimal search intensities.⁹

We now identify some properties of optimal search efforts. Clearly the intensity of search would depend on parameter values. In Proposition 1, we find that when innovator's and examiner's search technologies are not independent ($\rho > 0$), then there is a non-negligible range of parameter values for which the innovator's total search exactly matches that of the examiner. This result holds because when ($\rho > 0$), equation (3.15) has a kink at $X_R = X_E$. For an intermediate range of B (net value of a bad patent) and I (R&D cost), innovator's payoff is maximized at this kink. If B is very low and I is large, innovator's search effort could exceed examiner's effort while if B is high enough and I is low, innovator's search effort would be lower than the examiner's.

⁹This profit function is continuous. Search effort would never exceed the highest benefit $G - P$, hence x_1 is bounded in $[0, G - P]$. Therefore, a maximum is achieved.

Proposition 1 *When $\rho > 0$, there is a range of parameter values for which the researcher matches the examiner's search effort: $(x_1^* + x_2^*) = X_R = X_E$.*

In Proposition 2, we find conditions under which innovators have no incentive to search for prior art.

Proposition 2 *If the expected benefit from a bad patent is large enough so that $B \geq \max \left\{ -\frac{1}{\alpha\lambda(0)}, \frac{\alpha\lambda(0)I-1}{\alpha\theta\lambda(0)} \right\}$, then the innovator would not exert any effort searching for prior art, $x_1^* = x_2^* = 0$.*

The innovator is more likely not to search for prior art at all when patenting fee P is low and the examiner's search effort is low, when the cost of investment is small and the probability that invalidating prior art exists is small. There are also ranges of the parameter values for which the innovator might search either only before innovation ($x_1^* > 0$ and $x_2^* = 0$), or only prior to patenting ($x_1^* = 0$ and $x_2^* > 0$). Intuitively, early state of the art search is more important for innovations that require large R&D investment. If investment cost is large, the innovator would never engage only in novelty search. Thus, if she has an incentive to engage in novelty search, she must also have searched ex ante. On the other hand, when investment cost is low, if the innovator has no incentive to search ex post, then she has no incentive to search ex ante either.

Proposition 3 (i) *When investment cost is high enough $(I \geq \frac{(1-\theta)}{\alpha\lambda(0)})$, then an innovator who has no incentive for an early search, has no incentive for a novelty search either: $x_1^* = 0$ implies $x_2^* = 0$.*

(ii) *When investment cost is low enough $(I < \frac{(1-\theta)}{\alpha\lambda(0)})$, then an innovator who has no*

incentive for a novelty search, has no incentive for early search either: $x_2^ = 0$ implies $x_1^* = 0$.*

3.2.3 Think Like an Examiner

In this section, we take the possibility of correlated search technologies into account. Empirical evidence by Alcacer and Gittelman (2006) point to a striking similarity between the distributions of examiner and inventor citations, suggesting a “tracking scenario.” Their paper suggests that “(a)ttorneys anticipate citations most likely to be added by examiners, so that examiner and inventor citations may come to resemble each other closely.”

A prior art search professional who took pride in his company’s ability to “think like an examiner” motivated us to consider the possibility that correlation in prior art search can arise strategically when innovators seek to correlate their search effort with that of the examiner. If examiners follow a somewhat predictable search technology, then the researcher has an incentive to choose a search technology that is correlated with that of the examiner. In the industry, this is also sometimes referred to as “being in alignment with” the examiner.

We measured the degree of correlation of innovator’s search with examiner’s search with the parameter ρ . The higher is the ρ , the more correlated search technologies are. In equation (3.2), we derived the probability that the examiner finds invalidating prior art when it exists but was not found by the researcher $p(X_R, X_E)$. For fixed search efforts, this probability decreases with the degree of correlation ρ . This implies that if $g \geq 0$, then the net expected benefit from a bad patent $B(X_R, X_E)$ increases with ρ , which in turn implies that for fixed levels of

search, the researcher's payoff increases with correlation.

Proposition 4 *If the gain from a bad patent is positive, $g > 0$, and the researcher invests in search $X_R > 0$, then her payoff is higher the more correlated her search is with the examiner's search (i.e. the higher ρ).*

“Thinking like an examiner” increases the expected value of patent application when invalidating prior art exists and thus increases the researcher's payoff. If the researcher could choose the level of correlation between search technologies, then when $g > 0$, among equally efficient search technologies, one that is perfectly correlated with that of the examiner would maximize the researcher's payoff.

Varying levels of correlation in search technologies can affect the innovator's choice of search intensities. Let us consider, for simplicity, the levels of early state of the art search when $B > 0$, in this case $x_2^* = 0$ (see Lemma 1). The effect of correlation on search depends on whether the optimal level of search exceeds that of the examiner or not. For a researcher who (perhaps due to high investment costs) invests in early state of the art search more than the examiner, a more correlated search technology would reduce search. However, for a researcher who exerts less than examiner's effort, correlation increases search efforts. Search is more beneficial to the innovator in this situation because with higher correlation, the examiner is less likely to find invalidating prior art conditional on the innovator not having found any. We summarize this discussion in the following proposition.

Proposition 5 *When $B > 0$, if innovator's optimal early state of the art search exceeds examiner's search ($x_1^* > X_E$), then it is (locally) decreasing with ρ (the measure of corre-*

lation between innovator's and examiner's search technologies); while if $x_1^* < X_E$, then early state of the art search increases with correlation ρ .

From a policy perspective, it might be possible for the patent office to have some control over level of correlation between search technologies. If it were desirable by the patent office to decrease correlation between search technologies, then this might be possible by making examination less predictable (for example guiding examiners to search more for non-patented prior art and use less conventional search technologies), reducing transparency about the examination process and perhaps signing contractual agreements with examiners that limit their ability to work as prior art searchers in the private sector when they leave the patent office.¹⁰

For a given effort by the examiner X_E , under the conditions of Proposition 5, we show (see Lemma 5 in the Appendix) that

$$\frac{dp(X_R(\rho), X_E, \rho)}{d\rho} < 0. \quad (3.16)$$

This implies that less correlated search technologies (or lower ρ) result in a higher conditional probability of rejecting a bad patent. When the social value of a bad patent is negative, an increase in the probability of rejecting a bad patent is socially desirable. Note, however, that in the range where search efforts increase with ρ , innovator's own search can lead to less bad applications. Hence, we cannot unambiguously determine the effect of reduced correlation on welfare.

¹⁰This idea would be similar to "non-compete clauses" or "covenant not to compete" which in contract law refer to a contract by which an employee agrees not to pursue a similar profession which competes with the employer.

3.2.4 Search Intensity and its Timing

In this section, we examine determinants of the timing and intensity of prior art search. We first examine factors that affect the level of early state of the art search (x_1) when $B > 0$ (which implies $x_2^* = 0$).

Proposition 6 *When $B > 0$, early state of the art search (weakly) increases with investment cost I , the probability of a bad patent α , patenting fee P and examiner's search intensity X_E . Search (weakly) decreases with the value of a bad patent g . The value of a good patent G does not affect prior art search effort.*

Intuitively, early state of the art search helps the innovator to avoid investment in an innovation that is bad. Hence, the innovator has more to benefit from search the higher is the investment cost and the higher is the probability that her innovation is bad. The innovator is less likely to search the more she benefits from a bad patent. The net benefit from a bad patent decreases with patenting fee and examination effort and it increases with g . Higher benefits from a good patent G make the innovator more likely to invest. However, as long as this benefit is large enough so that the investment condition holds, its value will not affect search because conditional on the innovation being good, invalidating prior art will not be recovered regardless of the level of search.

To further investigate the intensity of search and its timing, we specialize the model to assume an exponential search technology function: $F(X) = 1 - e^{-\lambda X}$, where $\lambda > 0$ is a constant hazard rate. The parameter λ measures the ease of locating invalidating prior art when it exists. For a given search effort x , the higher λ is, the more likely it is to find an invalidating prior art when it exists. A high λ might, for example, prevail in fields where patenting is heavily relied on,

more prior art is patented and is thus easier to find. λ may also be high for innovators who have multiple projects in the same technological area. Emerging fields are expected to have search technologies with a low λ . We also focus on the case of independent search technologies. This offers tractability as well as a benchmark (and the limit as $\rho \rightarrow 0$). These simplifications allow us to derive optimal search efforts and conduct a more comprehensive analysis of comparative statics. In this setting, we can also more easily account for differences in ex ante and ex post search technologies. We assume novelty search technology is at least as efficient as early state of the art search technology. Hence, the hazard rate for novelty search is at least as large, $\lambda_n \geq \lambda_s$, where λ_n and λ_s are the hazard rates for the novelty search technology and the early state of the art search technology, respectively. We begin with a discussion of our findings and summarize them in Proposition 7 in the end of the section.

The intensity of search depends on the gain from a bad patent. In situations when the applicant is expecting a large gain from a bad patent (g), she is less inclined to conduct both early state of the art search and novelty search, that is, x_1^* and x_2^* decrease if g increases. In a survey of R&D labs in the U.S. manufacturing sector, Cohen et al. (2000) found that in complex industries firms are “much more likely to use patents to force rivals into negotiations.” In such industries, the size of the patent portfolio matters and firms are less likely than in discrete technologies (e.g. drugs) to use a single patent to block a rival or to generate licensing fees. In terms of our model, this seems to suggest a higher value of a bad patent g in complex industries (as any single patent is not likely to be involved in litigation). Hence our model predicts relatively less prior art search in complex industries. To the extent that patent citations not inserted by the examiner proxy the inventor’s prior art search, this prediction is supported by

the empirical findings of Alcacer et al. (2009) who found that in complex technologies (such as computers and electronics), patents have a higher share of examiner citation (which may indicate less search). Lanjouw and Schankerman (2004) found that “litigation risk is much higher for patents owned by individuals and firms with small patent portfolios.” This suggests that small firms likely have lower values of bad patent and therefore, our model predicts that all else equal, small firms would search more for prior art. Indeed, the empirical work of Alcacer et al. (2009) found that small firms and those who are less experienced (measured by their number of patents) had a significantly lower share of examiner inserted citations.

For innovations that require high investments, the benefit of early state of the art search is higher. Early search can help save large R&D spending on duplication. As the cost of investment (I) rises, the intensity of early state of the art search x_1^* rises; this substitutes in part for the later novelty search, thus x_2^* drops. Overall, there is more search (larger $x_1^* + x_2^*$) for innovations that require large investment. This suggests, for example, that for patents of pharmaceutical drugs that are known to require large R&D investments, we should expect significant search effort, particularly early state of the art search. Indeed, Alcacer et al. (2009) found that, compared to other fields, the share of examiner inserted citations was significantly lower in the drug, medical and chemical fields. Sampat (2005) also found that “the share of applicant inserted citations to U.S. patents is significantly higher for chemical and biomedical patents than for patents in other technological fields. This is an intriguing result, especially in light of empirical research suggesting that patents are more important as mechanisms for appropriating returns to R&D in chemicals and pharmaceuticals than in other fields.” Note that these technology areas are likely to be ones in which inno-

vation requires large investments. Additionally, for a patent on an innovation that is likely to be commercialized, we can expect a small value to a bad patent due to the risk of infringement suits and the inability to enforce it. These two forces (high I and low g) work in the same direction suggesting firms in the drug industry would have more incentive to search for prior art.

In the current model, as long as the expected benefit from innovation is high enough so that the investment condition holds, the gross benefit of a valid innovation, G , has no effect on search intensity.¹¹ Intuitively, in our setting, search affects payoff through its effect on the expected benefit in the event that invalidating prior art exists. Thus the optimal search effort depends on parameter values that play a role determining payoff in that event. This feature of the model is not, however, in odds with empirical findings that more important patents include more prior art citations (suggesting perhaps more prior art search), see Sampat (2005) and Lampe (2008). Two of the parameters of the model, investment cost I and the value of a bad patent g , are likely to be related to the benefit from a valid innovation G . First, for the investment condition to hold, G needs to be high enough compared to the investment cost. Hence, high-investment patents likely also have a high value of a good patent. Second, the value of a bad patent might also be related to the value of a good patent, but it is not clear in what direction this relation goes. On one hand, it seems that owning intellectual property rights on a more important innovation could be more valuable and hence g is large for large G . On the other hand, if an innovation is important, it is also more important for others who would then have stronger incentives to challenge the patent. Thus, there is likely to be more risk of litigation and exposure of invalidating prior art after the granting of the patent and hence g can be

¹¹In Section 3.2.6, we offer a generalization of the model in which search increases with G .

small for large G .

All else equal, innovators tend to search more for prior art, both before investment and after investment but before patenting, when there is a higher probability that invalidating prior art exists (α). In some fields, like software patenting, there may be a high probability that invalidating prior art exists, but still low search efforts since at the same time investment cost is low and the examiner's probability of finding invalidating prior art is also low.

We summarize the results of this section in the following proposition. In the proof, we solve for the optimal search efforts and then derive comparative statics with respect to various parameters of the model.

Proposition 7 *Assume search technology is given by $F_s(X) = 1 - e^{-\lambda_s X}$ for early state of the art search and $F_n(X) = 1 - e^{-\lambda_n X}$ for novelty search and assume $\rho = 0$ (i.e., search technologies of the examiner and innovator are independent), then all else equal, optimal search efforts weakly¹² satisfy the following:*

- (i) *As investment (I) rises, x_1^* rises, x_2^* falls, but $(x_1^* + x_2^*)$ rises.*
- (ii) *As the value of a bad patent (g) rises, both x_1^* and x_2^* fall. Search is not directly affected by the value of a good patent (G) if the investment condition holds.*
- (iii) *As the patenting fees (P) rise or the examiner's search effort (X_E) increases, both x_1^* and x_2^* rise.*
- (iv) *As the probability of invalidating prior art (α) rises, x_1^* and x_2^* rise.*

¹²By "weakly" we mean that when we say search effort "rises," search effort could either increase or remain unchanged. The qualification accounts for ranges of parameters with corner solution.

3.2.5 Policy Implications

Simple Interventions

Using our solution for the optimal search efforts as derived in the proof of Proposition 7 (with exponential search technologies and independent search efforts), we found that a decrease in the net expected benefit from a bad patent B would result in an increase in both the early state of the art search and novelty search efforts, x_1^* and x_2^* . The net expected value of a bad patent is given by $B = [(1 - F(X_E))g - P]$ which depends negatively on the patenting fee P and on the examiner's search intensity when reviewing applications X_E . B also increases with the value of a bad patent g . Hence, all else equal, in our simple model, an increase in examiner effort or in the patenting fee would result in higher search for prior art by the applicant. The three parameters that determine the net value of a bad patent X_E, P and g can serve as policy levers to influence search.

One element is common to several proposals to reform the patent system and reduce the number of bad patents granted, namely, the patent office should gather prior art information from third parties. We do not describe any of the suggested reforms in detail, but we briefly discuss how these can be thought of in terms of our model. Noveck (2006) advocates a "Community Patent Review" system. In this proposal, for each patent application, there would be a window of time during which patent examination is open to the public. Facilitating the addition of prior art by the public pre-granting of the patent can be seen as an increase in the probability that invalidating prior art would be detected when it exists, that is, an increase in X_E . Thomas' (2001) proposal combines a pre-

examination period in which informants might submit pertinent prior art, with a bounty to any party who succeeds in providing invalidating prior art. The bounty would be financed by charging a fine to the applicant, thus, in addition to an increase in the probability of finding invalidating prior art X_E , the expected value of a bad patent B further decreases due to the possibility of being fined in the event such prior art is found. Merges (1999) considers the possibility of establishing a patent opposition system. This would increase the probability that invalidating prior art be revealed after the granting of a patent. Hence, it can be seen as a decrease in the value of the patent conditional on it being a bad patent, g .

All these proposals suggest a decline in the net expected benefit of a bad patent B , which would result in an increase in both early prior art search and novelty search. Note, however, that we have taken the gross value of a bad patent g to be fixed. Improvements in the examination process as described in the policy reforms mentioned could result in an increase in value of any granted patent, including the value of a bad patent g , which in turn has a positive effect on B . Hence, these policies would result in an increase in search efforts as long as this latter effect is small enough not to offset the decline in B .

Lemley, Lichtman and Sampat's (2005) "gold-plate" patent-reform policy proposes that applicants should have an option to certify their patent with a "gold plate" by opting for an examination procedure that would have more careful examination (higher X_E) for a higher patenting fee P . "Gold plated" patents are likely to have significantly higher value (first, since the gold plate would signal a more carefully examined patent; second, since the authors expect selection of higher value innovations for this option). Hence, an increase in

g is expected for gold plated patents. The effect on applicant's prior art search is therefore ambiguous, but we expect it to be positive with a sufficient increase in patenting fee.

Finally, note that a policy intervention that weakens the presumption of validity would likely lower the value of a bad patent g and increase the incentive to search.

Social Planner's Problem

The question we address here is as follows: if a hypothetical social planner could mandate certain search intensities as well as disclosure of all relevant prior art, then what would the social planner's choice of search efforts be? We then compare innovators' search efforts to these "first best" levels of search and discuss policy levers that could motivate optimal search.

Innovators might not have socially optimal incentives to search since the private values of innovations are different than their social values. First, an innovator is unable to appropriate the full surplus generated by a novel invention. Hence, the social value of a true innovation is larger than its private value, $\hat{G} > G$. Second, while we argued that there are private benefits to be made from a bad patent, from a social point of view these benefits are likely offset by losses to others. Hence, we assume that the social value of a bad patent is lower than its private value, $\hat{g} < g$, and possibly, $\hat{g} < 0$. Finally, an innovator's patenting fees P might be different than the social cost of patenting \hat{P} . Assume a common probability α that there exists invalidating prior art, and a common probability of success in R&D, θ .

Note that if the planner could dictate search intensities and disclosure, then, as long as the examiner's search technology is not more efficient than the innovator's, the planner would put the burden of search entirely on the innovator rather than on the examiner. Innovator's search can help save investment costs by avoiding duplication as well as patenting cost. Thus, it is better to find invalidating prior art before patent application rather than after. The social planner's choice now amounts to applying our earlier findings on the optimal search efforts only using the social parameter values \hat{g} , \hat{P} and $X_E = 0$. We can then compare the socially optimal search effort to that chosen by the payoff maximizing innovator. In Proposition 8, we show that when the value of a bad patent for the innovator net of patenting fee ($g - P$) is lower than the social net value of a bad patent, innovators have too little incentive to search.

Proposition 8 *If the cost of patenting is sufficiently low compared to the gain from a bad patent such that $(\hat{g} - \hat{P}) < (g - P)$, then the researcher always under-invests in search compared to the socially optimal search level.*

If the benefits of innovation are high enough to ensure that the investment condition (3.14) holds, patent policy could induce efficient search with a high enough patenting fee $P = \hat{P} + (g - \hat{g})$. This, however, is not likely to be a practical policy to implement. First, because such patenting fees can be very high (when the researcher's private benefit from a bad patent is significantly larger compared to the social value of a bad patent) which might lead to under-investment in R&D. Second, because the right choice of patenting fees requires information on the value of a bad patent as well as an ability to charge differentiated patenting fees. It is impossible to do this for every single innovation. Patent policy typically sets rules that apply to the universe of patent applications, or to large

subsets of patent applications (e.g. a uniform patent length on most patents and a uniform patent fee, with a lower fee for small innovators). Finally, as we will see in Section 3.2.6, in reality, there may be situations where the innovator has an incentive not to disclose prior art. Nevertheless, even if the first best is not feasible, patenting fees that depend on the technological field could help induce more search in fields where we suspect search is inefficiently low and where an increase in fee would not significantly lower the incentive to innovate.

Commitment to Examination Procedure

Our premise in this chapter is that the examination process is not influenced by search and disclosure of prior art. This requires that the patent office would be able to commit to an examination process. The following questions thus arise. Can the patent office commit? Should the patent office commit to an examination process that is independent of prior art disclosure? And if examiners respond to applicants' prior art, how would this affect the incentives to search and disclose prior art?

We argue that it is reasonable to assume that the patent office can commit to a search process. The patent office is a government agency and it interacts with innovators repeatedly. Thus it is likely to be able to create a reputation on examination procedures. The budget of the patent office, the number of its employees and the time allocated to patent examination (at least on average) can be made public. According to Cockburn et al., "examiners are allocated fixed amounts of time for completing the initial examination of the application, and for disposal of the application." Examiners can however average these times over their case-loads. While individual examiners are heterogenous and may

use different examination technologies, a patent examiner is assigned to each application not chosen by the innovator. Cockburn et al. also document that “USPTO operates various internal systems to ensure “quality control” through auditing, reviewing and checking examiner’s work.” Additionally, for the first several years of their career, examiners are routinely reviewed by a more senior primary examiner. It seems reasonable that by and large the patent office can make sure its employees follow the guidance provided to them for examination procedure and intensity.

Should the patent office commit to an examination process that is independent of prior art disclosure? Note that the innovator’s search effort cannot directly be observed by the examiner. Hence, the examiner could make search contingent on the volume of prior art disclosure, but not on actual search effort. Innovators are likely to strategically choose the amount of prior art they disclose if this could affect the intensity of examination to their benefit. Langinier and Marcoul (2008) focus on innovators’ strategic non-disclosure of prior art. In their model, prior art disclosure by the innovator lowers the examiner’s search cost and the examiner exerts more search effort the more prior art the innovator discloses. Under this complementarity assumption on innovators’ and examiners’ search efforts, they find that “an examiner should not have different scrutiny levels but rather, should commit to an equal screening intensity across all applications. This simple rule has two advantages: first, it requires a limited commitment and, second, it induces truthful information transmission from applicants.”

If, instead, prior art disclosure by the innovator would induce less search by the examiner, then the innovator might have an incentive to increase the vol-

ume of prior art disclosed. More citations do not necessarily imply more search, for a given search level, innovators could be more permissive in their decision what to include as relevant citations. Concerns over excess disclosure of prior art (although for a different reason) were raised in a symposium on the Federal Circuit in March 2009. Senator Orrin Hatch (speaking on the issue of inequitable conduct) said that “(e)xaminers are buried in references by patent applicants for fear that they will be found to have withheld something. If the applicant does anything to try to focus the examiner on the closest prior art, this is also considered fodder for inequitable conduct claims.” Thus some innovators may disclose excessive volumes of prior art, not all of it highly relevant. Assessing the quality and relevance of prior art citations also requires examiner effort. The volume of disclosure does not necessarily indicate higher search intensity. If examination procedure were to be tied to the level of disclosure, then, depending on how examiners respond, this may create incentive to manipulate the level of disclosure and the informativeness of the number of applicant added prior art citations would be reduced.

3.2.6 Prior Art Disclosure

Existing literature has emphasized on the innovator’s strategic choice not to *disclose* prior art. In Langinier and Marcoul’s (2003) work, the main driver of this incentive is their assumption that higher information transmission increases examiner’s search intensity. Lampe (2008) assumes that disclosure of prior art information increases the probability that the applicant will be found to have willfully infringed upon an existing patent. In the model we analyzed thus far, innovators do not have an incentive not to disclose prior art, rather they might

choose not to *search* for it in the first place. We first explain why this is true here and then suggest circumstances when strategic non-disclosure of prior art may arise. We then pursue an extension of our model in which R&D process is influenced by early state of the art search. In this case, strategic non-disclosure of information may arise.

Ignorance is Bliss

Consider novelty search. Suppose a successful innovator is deciding how much to invest in novelty search before the filing of a patent application. Suppose that the innovator could choose not to disclose prior art. The innovator would engage in novelty search if this could save the cost of patenting in the event she finds invalidating prior art. Such search is worthwhile only if she would refrain from patenting in the event she finds invalidating prior art. If she is better off patenting even when invalidating prior art is found (only not disclosed), then she is better off not searching for it in the first place. Similarly, the innovator only engages in early state of the art search if she intends to save on R&D investment in case invalidating prior art is found. She would not invest in search only to ignore her findings.

The argument above relies on the assumption that finding prior art requires a conscious effort. If, however, in some circumstances, innovators could stumble on prior art without searching for it, an incentive not to disclose might arise. If R&D investment is costly enough, still it is likely that if invalidating prior art is found before investment then the innovator would not invest. But, if the innovator unintentionally comes across invalidating prior art for innovations that require only small R&D investment or after R&D investment is sunk, and

if the expected value of a bad patent is positive, $B \geq 0$, then an incentive not to disclose prior art might arise. Recall, however, as we discussed in the introduction, that knowingly concealing prior art is considered inequitable conduct and would be very risky practice on part of the innovators. Thus, in fact, innovators could even have an incentive to make conscious efforts not to accidentally find prior art after innovation and prior to filing for a patent.¹³

It is hard to tell empirically whether innovators strategically concealed prior art or whether they did not search for it. The overwhelming proportion of patents that have only examiner inserted citations (40% according to Alcacer and Gittelman (2006)) seems to us as strong evidence of a weak incentive to search for prior art. Sampat (2005) as well as Alcacer and Gittelman (2006) provide evidence on examiners' and assignees' propensity to add assignee-assigned self citations. According to Sampat, "(t)he fact that examiners insert a significant share of self-citations provides prima facie evidence that a significant share of applicants do not search for, or fail to disclose, material prior art." While such cases may seem more likely consistent with non-disclosure (as one expects an assignee to be aware of her own patents), other explanations are also plausible. Some assignees (for example, big software companies) have a lot of patents and they may not be fully aware of their own portfolios. Moreover, given that the assignee is not likely to fear litigating herself, she might be less careful searching her own patents. It is also possible that there is not always full agreement on the relevance of previous patents. An assignee who is familiar with the details of her own innovation may consider it sufficiently distant from the new invention not to be material to patentability.

¹³As an anecdotal example, an individual in a high technological industry told us that some companies in his industry block their employees' access to the patent office database to avoid finding prior art and risk inequitable conduct allegations.

When Search Shapes Innovation

In the model we analyzed in the previous sections, researchers never had an incentive not to disclose prior art. This was partly because we abstracted from some of the potential benefits from prior art search, particularly in the early stages of research. Prior art searches might help the innovator decide in what direction research will go. Finding that one path of research is not novel can lead the researcher to invest in another related direction. An early state of the art search may help shape the innovation, not just decide whether or not to invest. Hence, search can interact with the innovation process. With such additional potential benefits, an incentive not to disclose prior art may arise.

We illustrate this idea with a modified version of our model. Suppose the innovator has two research paths to choose from. As before, the cost of innovation in either path is I and the probability of success is θ . The prior probability that invalidating prior art exists for the innovation pursued in path i is α_i , $i \in \{1, 2\}$, with path 1 being the more promising choice, $\alpha_1 \leq \alpha_2$. We assume that early search is not yet focused, early state of the art search effort x_1 reveals prior art relevant to either path with a probability $F_s(x_1)$ which satisfies the earlier assumptions we made. If the search reveals no invalidating prior art, then the researcher would invest in research path 1—the more promising direction. If search reveals invalidating prior art on one path, then pursuing that path—imitating it—costs less, $I_m < I$, and uncertainty about the probability of success is reduced, we assume the success probability becomes 1. If search reveals invalidating prior art for one path but not the other, the researcher faces a choice between investing in the path for which no prior art was found, or investing in the bad path (which is now less costly and more certain) with the intention

not to disclose the invalidating reference. If search reveals invalidating prior art on both paths, the researcher could abandon the project, or invest with the intention not to disclose.

The researcher decides whether or not to invest and which path to pursue. We assume that the researcher can only pursue one path of innovation. If she does not succeed with the path she chose or if she finds invalidating prior art during the ex post novelty search, she abandons the project.

After successful innovation, the researcher chooses how much to invest in novelty search before filing for a patent. Novelty search technology can be more focused than the early state of the art search as the researcher is more informed at this point. Nevertheless, the earlier search effort still contributes to novelty prior art search. We denote the novelty search technology by $F_n(X)$, with $F_n(X) > F_s(X)$ for any $X > 0$. To account for the contribution of the early state of the art search to the novelty search stage, we express early search effort x_1 in terms of equivalent novelty search effort units as follows: an investment of x_1 in search before R&D is equivalent to an effort \tilde{x}_1 which satisfies $F_s(x_1) = F_n(\tilde{x}_1)$, that is $\tilde{x}_1 = F_n^{-1}(F_s(x_1))$. Hence, an innovator needing to decide how much to invest in novelty search faces the same decision as if early search had the same technology as novelty search F_n and she had exerted effort \tilde{x}_1 .

We first consider the choice of novelty search. If the researcher chooses path i , then her expected pay-off from x_2 is

$$(1 - q_i(x_1))(G - P) + q_i(x_1) \frac{[1 - F_n(\tilde{x}_1 + x_2)]}{[1 - F_n(\tilde{x}_1)]} B - I - (x_1 + x_2) \quad (3.17)$$

where

$$q_i(x_1) = \frac{\alpha_i [1 - F_s(x_1)]}{[1 - \alpha_i F_s(x_1)]}. \quad (3.18)$$

Maximizing researcher's payoff results in novelty search effort given by:

$$x_{2i}^*(\tilde{x}_1) = \begin{cases} f_n^{-1}\left(\frac{[1-\alpha_i F_n(\tilde{x}_1)]}{-\alpha_i B}\right) - \tilde{x}_1, & \text{if } B < \frac{[1-\alpha_i F_n(\tilde{x}_1)]}{-\alpha_i f_n(\tilde{x}_1)}, \\ 0, & \text{if } B \geq \frac{[1-\alpha_i F_n(\tilde{x}_1)]}{-\alpha_i f_n(\tilde{x}_1)}. \end{cases} \quad (3.19)$$

Novelty search effort is the same function of early search effort as we derived in the proof of Proposition 7.

Again, we assume a sufficient condition for the researcher to invest in R&D:

$$\theta[\alpha_2 B + (1 - \alpha_2)(G - P)] \geq I. \quad (3.20)$$

Consider now the situation in which an early state of the art search has revealed prior art to invalidate both research paths. In this case, the researcher either abandons her innovation idea, or pursues it with the intension of not disclosing the invalidating prior art. Abstracting from the risks associated with inequitable conduct, the researcher would invest with the intention not to disclose if the net expected value of a bad patent exceeds the cost of imitation, $B > I_m$.

Proposition 9 (i) If $B < I_m$, then the researcher never has an incentive not to disclose prior art. (ii) If $B > I_m$, an incentive not to disclose invalidating prior art that was revealed in early state of the art search may arise; in this situation, the innovator does not invest in novelty search.

When the net value of a bad patent is low compared to the cost of imitation, early state of the art search can help the innovator avoid "stepping on" existing innovations and either choose a path that is more likely to be novel, or avoid R&D spending altogether when both paths are not novel. In this situation,

strategic non-disclosure does not arise, invalidating prior art alters the innovator's choice of path of investment and she avoids investing in a non-novel path. However, when the net value of a bad patent is high compared to the cost of imitation, early search can result in imitation and non-disclosure of prior art.

Considering the optimal choice of ex ante search, we find, as in the earlier version of our model, that there are parameter values for which the innovator has no incentive to search for prior art: $x_1^* = x_2^* = 0$. Focusing on the range of parameters for which the innovator does not imitate and only has an incentive for early search, we derive comparative statics results that help us understand the determinants of early search in the two paths model. We describe these results in the following proposition.

Proposition 10 *Suppose $0 < B < I_m$ (implying no imitation and no ex post search). In an interior solution ($x_1^* > 0$), early state of the art search increases with investment cost (I), examination effort (X_E), the probability that path 1 is bad (α_1), patenting fee (P) and the value of a good patent (G). Early state of the art search decreases with the value of a bad patent (g). The increase in the probability that path 2 is bad (α_2) has an ambiguous effect on x_1^* .*

These results are the similar to what we found in Proposition 7 (when we had a single path) except that in the two paths model, early state of the art search increases with the value of a good patent, whereas in the single path model G had no effect on search. Search in this version of the model helps shape the path of innovation making it more likely to pursue a good path. This benefit is more significant when the value of a good innovation is larger and which explains why there is more incentive to search when the value of a good patent is larger.

3.2.7 Remarks

In this section, we strive to better understand what drives prior art search by innovators. We focus on two motivations for search: innovators might engage in early state of the art search to avoid spending on costly R&D, and/or conduct novelty search to save on patenting costs. While earlier work focused on incentives not to disclose, we show that when revealing invalidating prior art requires search effort, innovators may refrain from searching rather than avoid disclosure. In the current patent system, where innovator's net private benefit from a bad patent is likely to be higher than its social value, innovators have too little incentive to search. Policy interventions that lower the net expected benefit of a bad patent would induce more search and may increase social welfare. An increase in patenting fee, for example, would serve this purpose (as long as it does not discourage innovation). Several recently proposed policy interventions such as a patent-opposition system, community patent review or patent bounties are likely to decrease the net value of bad patents. Thus, such interventions not only make bad patents less likely to be granted, but also create incentives for prior art search by innovators before filing for a patent, which would reduce the number of bad patent applications and increase the quality of patents. Our analysis also found that innovators are better off if they can correlate their search technology with that of patent examiners. Higher correlation between innovators' and examiners' search technologies results in a lower conditional probability of rejecting a bad patent application.

We also consider an extension of our model in which early state of the art search can influence the choice of research path. Early search can help the innovator avoid research paths that are not novel. When cost of imitation is low and

the value of a bad patent is high, innovators might pursue non-novel research paths with the intention of applying for the patent without disclosing invalidating prior art references. Hence, when early state of the art search shapes innovation, incentives not to disclose prior art may arise.

Our analysis simplifies on several dimensions that could be interesting for future research. We assumed a simple state space—an invalidating prior art reference either exists or it does not exist. In reality, however, there could exist prior art references that invalidate some but not all claims of a patent, or that invalidate the patent in combination with other references but not alone. We have also assumed a simple binary investment decision. However, finding related prior art before innovation can have an effect on the process and cost of innovation. We provided one simple extension of the model in which search affects innovation, but did not fully account for the possibility that innovators can learn from others' experiences and build on existing knowledge to lower costs of innovation, even when this knowledge does not invalidate their own innovation. Knowledge of patented prior art could also guide the innovator how to innovate around or tailor the patent application so as not to infringe on existing patents. Such additional benefits from search might provide additional incentives for ex ante search, but as the two-paths version of our model suggests, possibly also additional incentives not to disclose prior art. Finally, we mention that we have assumed that the patent office commits to a uniform examination process. A more careful look at the inside operation on the patent office and its relation to prior art search is another important direction for future work.

3.3 Patent Quality and a Two-Tiered Patent System

The quality of patents has been a subject of growing concerns. Bad patents—patents that would have failed the novelty or non-obviousness patentability requirements had their examiners been more informed—likely have adverse effects on our society. Patents, good or bad, have social costs. Patent holders can exclude others from use of inventions which might hinder future innovations and commercialization of products, or induce unnecessary costs of duplication and inventions around the patent. In the case of good patents, such social costs may be offset by the benefits of increased incentives to innovate and to disclose new information. But bearing the social cost of a patent can hardly be justified for bad patents. Moreover, bad patents are presumably more likely to be associated with litigation costs (see Merges, 1999 for a more detailed discussion on the social costs of bad patents).

This chapter highlights an additional, largely overlooked, cost of bad patents – the negative externality imposed by bad patents on other patent holders. We argue that bad patents undermine the goals of the patent system because they reduce the value of holding patents. Third parties (competitors, investors, etc.) are likely less informed than the innovator about the probability of validity of a specific issued patent, but they have an overall perception of patent quality. How third parties perceive the quality of patents may affect a patentee’s ability to deter entry, negotiate licensing fees, bargain, or secure venture capital funding. For example, if patent quality is perceived to be low, a potential entrant might be less worried about infringement and an investor might be less impressed by the fact that a start-up company holds a patent. The quality of patents can be thought of as the probability that an issued patent is good (or

valid). If a patent system allows many bad patents, the perceived quality of patents declines lowering the value of holding patents and thus limiting the ability of the patent system to reward the true innovators.

This chapter studies the determinants of patent quality. Patent quality is endogenously determined in equilibrium together with innovators' decisions whether to apply for a patent. In our model, every innovator has private information on the ex-ante (before examination) probability of validity of his own invention. A patent gives its owner a larger benefit if others believe that patents are of high quality. Quality depends on all innovators' decisions to apply for patents as well as on the examination process. For any given perceived patent quality, we find that innovators will apply for patents if the probability of validity of their patent applications exceed a certain threshold. An increase in patent quality lowers the threshold. However, a lower threshold implies lower quality because the pool of applicants becomes inferior. Hence, the quality of patents and the decision to apply for patents are jointly determined in equilibrium.

We study the effects of patent system reforms on the volume of patent applications and on the quality of patents. Particularly, we examine how changes in patenting fees and in the intensity of examination affect equilibrium outcomes. We find that an increase in patenting fee reduces the number of patent applications that have a low ex-ante probability of validity. It also increases the quality of patents (that is, the probability that granted patents are good) and thus, their value. Interestingly, making the examination process more stringent could have ambiguous effects on the volume of patent applications. On one hand, stringent examination reduces the probability of any applicant to receive a patent, which deters low probability of validity applicants. On the other hand, the increase in

expected patent quality makes holding a patent more valuable and thus more attractive.

Concerns about the abundance of bad patents prompted several proposals for patent system reforms such as establishing a patent opposition system (Merges (1999)), patent bounties (Thomas (2001)), “gold-plate” patents (Lemley, Lichtman and Sampat (2005); Lemley and Lichtman (2007)), and community patent review (Noveck (2006)). This section provides insights on the potential effects of such policies. Particularly, we focus on a formal analysis of the Lemley et. al. (2005) proposal to establish a two-tiered patent system which, according to the authors, “would dramatically improve the quality of economically significant patents.” In such system, applicants would be allowed to “gold-plate” their patents by paying a higher fee and being subject to a more thorough review process. This proposal has also attracted the attention of the new administration: “[w]ith better informational resources, the Patent and Trademark Office could offer patent applicants, who know they have significant inventions, the option of a rigorous and public peer-review that would produce a “gold-plate” patent much less vulnerable to court challenge. Where dubious patents are being asserted, the PTO could conduct low-cost, timely administrative proceedings to determine patent validity.”¹⁴

This section formally models a two-tiered patent system and examines its outcomes compared to the standard (single-tiered) system. We show that introducing a two-tiered system, in which the lower tier is the same as the original patent system but a second more stringently examined patent tier is also offered, results in a decline in the volume of patent applications, with low quality

¹⁴See Barack Obama on Technology and Innovation at http://www.barackobama.com/pdf/issues/technology/Fact_Sheet_Innovation_and_Technology.pdf

applicants less likely to apply. The volume of bad patents issued would decline for two reasons, the more stringent examination in the second tier, and the reduced number of low ex-ante probability of validity applicants. Sorting of applicants between regular and gold-plate patents depends on the innovator's ex-ante probability of validity. Innovators of higher ex-ante probability of validity are more likely to gold-plate their patents. The sorting of innovators between the two tiers could additionally depend on the economic value of the invention. Lemley et. al. (2005) hypothesized that economically significant patents will sort into the gold-plated tier. We examine this in the context of our model to find conditions under which this relationship holds. Finally, we study the effect of changes in patent policy on the overall volume of patent applications and on gold-plate patents.

This chapter relates to a large body of literature on innovation and patent policy. An excellent review of many of these contributions can be found in Scotchmer (2006). More specifically, this chapter contributes to a fledgling body of literature which recognizes imperfections in the functioning of the patent examination process. Among such theoretical contributions is Caillaud and Duchêne (2007). They examine the impact of the patent office on firms' incentives to innovate and to apply for patent protection, and the overload problem patent examiners face. They show that given imperfections in the examination process, some granting of bad patents are inevitable. They also consider the role of patent fees as a policy instrument and examine their effects on R&D investment and on incentives to apply for patents. Langinier and Marcoul (2008) concentrate on incentives to search and disclose prior art given an imperfect examination process. Atal and Bar (2010) focus on innovators' timing and intensity of prior art search.

3.3.1 The Model

There exists a large heterogeneous population of innovators. Each innovator has one invention which is characterized by a parameter $\theta \in [0, 1]$ that represents the ex-ante (before patent application) probability that the invention is good. That is, θ is the probability that there does not exist prior art that invalidates this invention.¹⁵ Innovators' types θ are independently drawn from a distribution described by the cumulative distribution function $F(\theta)$ on $[0, 1]$, and positive density $f(\theta) > 0$.

An innovator can choose whether or not to apply for a patent. Patent application fee is given by P . Any application is examined in the patent office to determine if it is patentable. For simplicity, we assume patentability only depends on whether or not invalidating prior art is found in the examination process. The examiner searches for prior art exerting search effort such that, if there exists invalidating prior art, it would be found with a probability p . If no invalidating prior art is found, a patent is granted. Hence, a good innovation is always granted a patent and a bad invention is granted a patent with probability $(1 - p)$.

By making the assumption that invalidating prior art is found with a probability p , we implicitly assume that the patent office is committed to (in expectation) a uniform examination intensity. The patent office is a government agency and it interacts with innovators repeatedly. Thus it is likely to be able to create a reputation on examination procedures. The budget of the patent office, the number of its employees and the time allocated to patent examination (at least on average) can be made public. According to Cockburn et al., "examiners are

¹⁵Patent applications often include several claims. Validity is determined claim by claim. We simplify here by assuming the patent is either valid—all its claims are valid—or it is not.

allocated fixed amounts of time for completing the initial examination of the application, and for disposal of the application.” Examiners can however average these times over their case-loads. While individual examiners are heterogenous and may use different examination technologies, a patent examiner is assigned to each application not chosen by the innovator. Cockburn et al. also document that “USPTO operates various internal systems to ensure “quality control” through auditing, reviewing and checking examiner’s work.” Additionally, for the first several years of their career, examiners are routinely reviewed by a more senior primary examiner. It seems reasonable that by and large the patent office can make sure its employees follow the guidance provided to them for examination procedure and intensity.

Patents are often used in negotiations with other firms, as signals to potential investors, or to exclude others from use of the patented technology. We assume that the value of a patent to an innovator depends on the quality it is perceived to have by less informed third parties. Our model captures such dependence in a simple stylized way. Let q be the probability that an innovation is good (i.e., there exists no invalidating prior art) conditional on it having been granted a patent:

$$q = \Pr[\text{good patent} \mid \text{patent was granted}]. \quad (3.21)$$

We will refer to this probability as the *perceived patent quality*, or simply as *patent quality*. An innovator does not know if he will be granted a patent and if the patent is good, but he forms an expectation taking into account the ex-ante probability of validity θ and the examination process. Benefits from good or bad patents depend on perceived patent quality q . An innovator’s expected payoff from a patent application is given by

$$V(\theta, q) = \theta G(q) + (1 - \theta)(1 - p)B(q) - P \quad (3.22)$$

where $G(q)$ and $B(q)$ denote the benefit from a patent as a function of perceived quality q conditional on the innovation being good or bad respectively. We assume $G(q) \geq B(q) \geq 0$ for all q . The assumption that the benefits from patents are lower if the patent is bad could capture a probability that the patent would be contested and fail to be defended. We also assume that the functions B and G are differentiable and that the value of a bad patent is more sensitive to the perceived quality of patents, $B'(q) > 0, G'(q) \geq 0$. If perceived quality is low, third parties may be more likely to contest the patent which would be more detrimental to the holder of a bad patent. For a patent quality $q = 1$, we assume $B(1) = G(1)$, that is, if perceived patent quality is perfect, the values of a good and a bad patent are the same. Finally, for simplicity, the value to the innovator from an invention which is not protected by a patent is normalized to be zero.

3.3.2 The Single-Tiered Patent System

In this section, we examine the single-tiered patent system – the standard patent system in which innovators face the decision to apply for a patent protection or not to apply, but have no choice regarding the intensity of the examination process.

Equilibrium

The decision to apply for a patent depends on the innovator's expected benefit. The innovator applies for a patent if this expected benefit exceeds that of not applying, $V(\theta, q) \geq 0$. Using (3.22), we find that for any quality q , there exists a

cut-off probability $\theta_1(q)$ defined by

$$\theta_1(q) = \begin{cases} 1 & \text{if } P > G(q) \\ \frac{P - (1-p)B(q)}{G(q) - (1-p)B(q)} & \text{if } G(q) \geq P \geq (1-p)B(q) \\ 0 & \text{if } (1-p)B(q) > P \end{cases} \quad (3.23)$$

so that innovators with ex-ante probability of validity $\theta \geq \theta_1$ apply for the patent and those with $\theta < \theta_1$ do not apply. If $P > G(q)$, then patents are too costly and no one applies for a patent. If $P < (1-p)B(q)$, then patenting fee is low enough and the expected benefit of holding even a bad patent is high enough so that everyone applies for a patent. In the interior range, the threshold $\theta_1(q)$ decreases with q , that is, the higher the perceived patent quality is, the more innovators apply for patents. Particularly, more low ex-ante probability of validity applicants would find it worthwhile to apply.

Taking into account a threshold ex-ante probability of validity θ above which innovators apply for a patent, the probability that a granted patent is good is given by

$$q_1(\theta) = \Pr(\text{good} \mid \text{granted}) = \frac{\Pr(\text{good and granted})}{\Pr(\text{granted})} = \frac{\int_{\theta}^1 \bar{\theta} f(\bar{\theta}) d\bar{\theta}}{\int_{\theta}^1 [1 - p(1 - \bar{\theta})] f(\bar{\theta}) d\bar{\theta}}. \quad (3.24)$$

This is the belief that an uninformed person has on the probability that a patent is good. The possible values of q_1 are in the range $[E(\theta), 1]$. The lowest value $q_1 = E(\theta)$ is obtained if all innovators apply for a patent and examiners never find invalidating prior art ($p = 0$), and the highest value $q_1 = 1$ is obtained if the examination is perfect ($p = 1$). Note here that when $p < 1$, the perceived quality of patents increases with the threshold θ . A higher threshold implies an overall better pool of applicants and thus a higher probability that a granted patent is good.

We now define an equilibrium in our model.

Definition 3 (*Patenting Equilibrium*) *A patenting equilibrium is characterized by a pair $\{\theta_1^*, q_1^*\}$, such that*

$$\theta_1^* = \theta_1(q_1^*) \text{ and } q_1^* = q_1(\theta_1^*). \quad (3.25)$$

Innovators of type $\theta > \theta_1^$ apply for a patent and innovators of type $\theta < \theta_1^*$ do not apply for a patent. The equilibrium is interior when $\theta_1^* \in (0, 1)$.*

Our first result establishes existence of an equilibrium and derives conditions for an interior equilibrium where some but no all innovators apply for patents. We also show that given our assumptions, when an interior equilibrium exists, it is unique.

Proposition 11 *If $P > G(1)$, then in equilibrium no one applies for a patent ($\theta_1^* = 1$). If $P < (1 - p)B(q_1(0))$, then in equilibrium all innovators apply for a patent ($\theta_1^* = 0$). For intermediate levels of the patenting fee, there is a unique interior equilibrium $\{\theta_1^*, q_1^*\}$.*

Detailed proofs are provided in the appendix. To prove this proposition, we show that the equilibrium is defined as an intersection between two functions $\theta_1(q)$ which is decreasing in q (because with a higher perceived quality more innovators apply) and $q_1(\theta)$ which is increasing in θ (because quality increases when fewer low probability of validity applicants apply.) When these two functions intersect, there is a unique interior equilibrium. Figure 3.1 illustrates the equilibrium, it depicts the downward sloping $\theta_1(q)$, the upward sloping $q_1(\theta)$ and their intersection which defines the equilibrium.

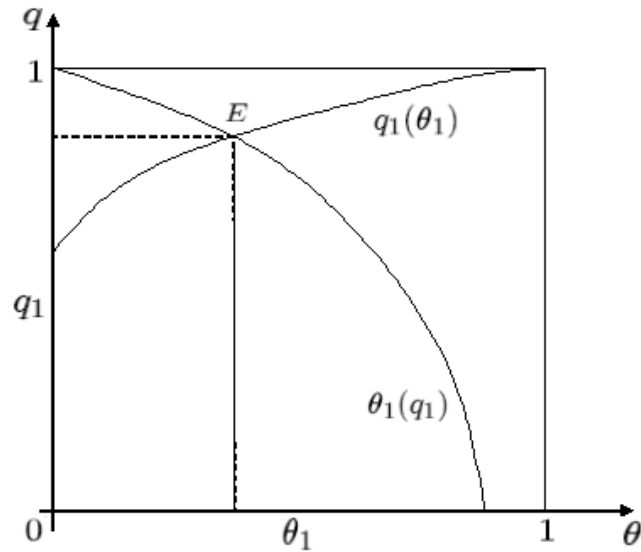


Figure 3.1: Equilibrium in a single-tiered system

Policy Implications

The equilibrium pair $\{\theta_1^*, q_1^*\}$ depends on the patent policy parameters p (examination intensity) and P (patenting fee). Equilibrium also depends on the benefit functions $G(\cdot)$ and $B(\cdot)$. These functions reflect the economic value of the innovation, and can be affected by patent policies that strengthen patent protection (such as increased patent breadth and longer patent term).¹⁶ In Proposition 12, we examine the relationship between policy levers and equilibrium outcomes. We examine how policy changes affect θ_1^* which determines the volume of patent applications $\left(\int_{\theta_1^*}^1 f(\theta) d\theta\right)$ and the quality of patents q_1^* .

Proposition 12 *In the range of an interior equilibrium, (i) the equilibrium volume of patent applications decreases with patenting fee (P) and the quality of patents increases*

¹⁶Patent breadth and patent length are extensively discussed in the literature on patents; see, for example, Gilbert and Shapiro (1990), Klemperer (1990), O'Donoghue (1998), O'Donoghue, Scotchmer and Thisse(1998).

with patenting fee; (ii) patent quality increases with the examination intensity (p) but the effect of examination intensity on the equilibrium volume of patent applications is ambiguous; (iii) a policy that strengthens patent protection to increase $G(\cdot)$ and $B(\cdot)$ would result in lower patent quality and more patent applications.

The effect of an increase in patenting fee is intuitive. A higher patenting fee makes patent applications less attractive as fewer low ex-ante probability of validity innovators apply for the patent and thus the quality of patents increases. The increase in patent quality makes applying for a patent more attractive, but not enough to reverse the decline in applications due to the increase in fee. Figure 3.2 illustrates two equilibrium points $E_1(p, P_1)$ and $E_2(p, P_2)$ which correspond to two systems with the same examination intensity p but different levels of patent fee $P_1 < P_2$. A higher patent fee results in a shift of the curve $\theta_1(q_1)$ to the right, but no change in the curve $q_1(\theta_1)$. The intersection E_2 has less applications (higher θ_1) and higher patent quality q_1 .

The effect of an increase in examination intensity is more complex. Figure 3.3 illustrates equilibria for two different examination intensities $E_1(p_1, P)$ and $E_2(p_2, P)$ with $p_1 < p_2$. A higher examination intensity results in a shift of the curve $\theta_1(q_1)$ rightward, but also a shift up of the curve $q_1(\theta_1)$ to the right. The equilibrium $E_2(p_2, P)$ has a higher patent quality q_1 , but the effect on patent applications is ambiguous. On one hand, tougher examination reduces the probability of any applicant to secure a patent, making a patent application less attractive, particularly to low probability of validity innovators; on the other hand, one can expect an increase in the perceived quality of patents due to the tougher examination process making patent applications more attractive (when innovators care about the perceived quality of a patent). The overall effect on

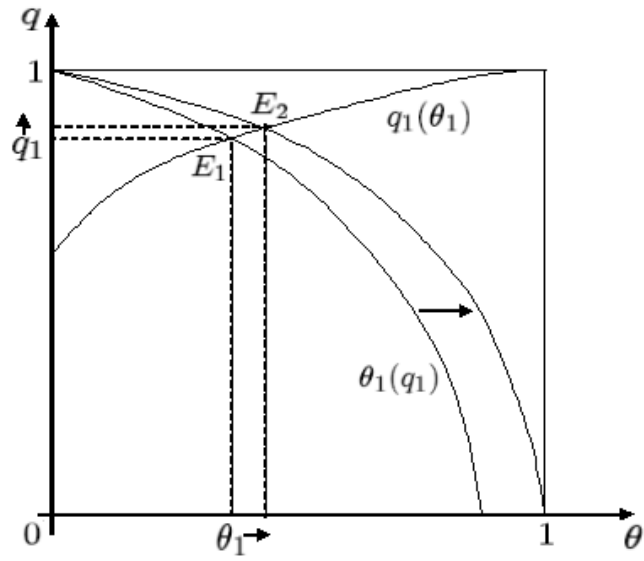


Figure 3.2: Effect of an increase in patenting fee

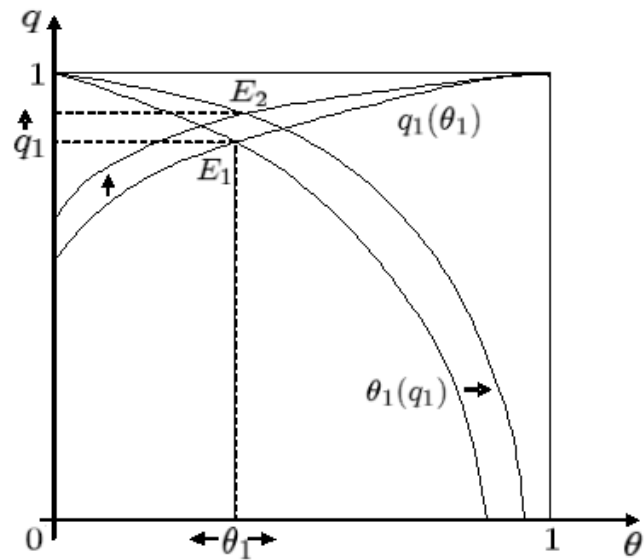


Figure 3.3: Effect of an increase in examination intensity

the volume of patent applications is ambiguous. If the benefits from a patent were not sensitive to perceived patent quality, then the increase in examination intensity would only reduce the probability of securing a patent and hence the volume of applications would decline. However, when the benefit from a patent is sensitive enough to perceived patent quality, then the volume of applications might increase with examination intensity. The perceived quality of patents must increase, whether or not the volume of application decreases. If perceived quality were to decrease, there would necessarily be a decline in patent applications which in turn would imply higher quality.

Changes in the benefit functions $G(q)$ and $B(q)$ change the function $\theta_1(q)$ but do not have an effect on $q_1(\theta)$. A strengthening of patent protection, for example by increasing patent length or breadth, increases the conditional benefit functions $G(q)$ and $B(q)$. As a result, more innovators apply and the quality of patents declines. In contrast, a post grant opposition system as proposed by Merges (1999), or a weakening of the presumption of validity are likely to lower $B(q)$.¹⁷ This will shift $\theta_1(q)$ up resulting in an equilibrium with less patent applications and a higher patent quality. However, accounting for more invalidation of bad patents post patent granting suggests an additional effect which is similar to that of increased examination intensity. In this case, the combined effect would be an increase in patent quality, but an ambiguous effect on the volume of applications.

Noveck (2006) advocates a “Community Patent Review” system. In this proposal, for each patent application, there would be a window of time during which patent examination is open to the public. Facilitating the addition of

¹⁷In the USA, patents are presumed valid. This implies that the burden of establishing invalidity of a patent rests on the party asserting invalidity. We discuss this issue further in Section 3.3.3.

prior art by the public pre-granting of the patent can be seen as an increase in the probability that invalidating prior art would be detected when it exists, that is, an increase in p . Hence, this reform is expected to result in an increase in patent quality, but an ambiguous effect on the volume of patent applications.¹⁸

Caillaud and Duchêne (2007) propose a policy in which the patent office would penalize rejected applicants to induce more investment in R&D which is assumed to increase the probability that the innovation quality is high. Thomas' (2001) proposal combines a pre-examination period in which informants might submit pertinent prior art, with a bounty to any party who succeeds in providing invalidating prior art. The bounty would be financed by charging a fine to the applicant, thus, in Thomas' proposal, a penalty for a bad patent is combined with an increase in the probability of finding invalidating prior art p . In the context of our model, a penalty for a bad patent P_B would appear in the applicant's payoff function as

$$V(\theta, q) = \theta G(q) + (1 - \theta)(1 - p)B(q) - (1 - \theta)pP_B - P. \quad (3.26)$$

The penalty for a bad patent results in a similar effect to that of an increase in patenting fee P . Increasing the penalty will result in less patent applications and higher patent quality. However, a one dollar increase in patenting fee would result in a larger reduction in patent applications and in the volume of bad patents than the same increase in penalty. The reason is that the patent applicants in our model do not know for sure if they have a good or a bad application. The ex-ante probability of validity is θ , and so, for any applicant, an increase of one dollar in patenting fee results in an increase of one dollar in the cost of patent-

¹⁸Allowing public review also requires that the patent application be made public. In the current US system, applications are typically published 18 month after the effective filing date. If in implementing this reform this period is shortened, there would be an additional ambiguous effect on the value of patent applications which we have not accounted for.

ing, while an increase of one dollar in the penalty for a rejected application only results in an expected cost increase of $(1 - \theta)p$. We note however that our model does not account for the effect fees might have on the incentive for R&D and hence it does not fully capture the potential benefits from penalties.

In Section 3.3.3, we will more closely examine one more policy reform, the Lemley, Lichtman and Sampat's (2005) proposal of a two-tiered patent system. First, we turn to an examination of welfare and the optimal examination intensity and patenting fee.

Welfare

Suppose that in terms of social welfare, the value of a good patent is at least as large as its private value, $\tilde{G}(q) \geq G(q)$, and the social value of a bad patent is lower than its private value, $\tilde{B}(q) \leq B(q)$. This depicts the idea that private innovators cannot capture the full benefits of their innovation, nor do they take into account the social costs of bad patents. Social welfare in the single-tiered system is

$$W = \int_{\theta_1}^1 \left[\theta \tilde{G}(q_1) + (1 - \theta)(1 - p)\tilde{B}(q_1) - c(p) \right] f(\theta) d\theta \quad (3.27)$$

where $c(p)$ is the social cost of patent examination such that the probability of finding an invalidating prior art is p if there exists any. Assume, for any θ_1 , welfare increases with patent quality.¹⁹ In an optimal policy with $P > 0$ and $1 > p > 0$ that maximize the social welfare, the following conditions hold:

$$\frac{dW}{dp} = 0 \text{ and } \frac{dW}{dP} = 0. \quad (3.28)$$

¹⁹This holds true when the social benefit functions increase with quality $\tilde{G}'(\cdot) > 0, \tilde{B}'(\cdot) > 0$, or when the weaker sufficient condition $\int_0^1 \left[\theta \tilde{G}'(q_1) + (1 - \theta)(1 - p)\tilde{B}'(q_1) \right] f(\theta) d\theta > 0$ holds.

Rearranging these first order conditions, we obtain

$$\frac{\partial W}{\partial q_1} \frac{dq_1}{dp} + c(p)f(\theta_1) \frac{d\theta_1}{dp} = \int_{\theta_1}^1 c'(p)f(\theta)d\theta + [\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1)] f(\theta_1) \frac{d\theta_1}{dp}. \quad (3.29)$$

and

$$\frac{\partial W}{\partial q_1} \frac{dq_1}{dP} + c(p)f(\theta_1) \frac{d\theta_1}{dP} = [\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1)] f(\theta_1) \frac{d\theta_1}{dP}. \quad (3.30)$$

These condition for an optimal policy reflects the balance between marginal costs and marginal benefits of changes in policy parameters. We focus on the first order condition with respect to patenting fee (3.30). The marginal benefits of an increase in patent fee include the increase in social benefits due to the increase in patent quality, as well as the saved examination costs on marginal applicants. The marginal cost of the increase in patent fee is the lost surplus from marginal applicants.

We make two observations based on the condition (3.30) for optimal patenting fee.

Proposition 13 *Given an examination intensity p , if patenting fee is chosen optimally, then*

(i) *the net social surplus from the marginal applicant θ_1 is positive:*

$$[\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1) - c(p)] > 0; \quad (3.31)$$

(ii) *if the private expected benefit of the marginal applicant exceeds the social benefit of his application, i.e., if*

$$\theta_1 G(q_1) + (1 - \theta_1)(1 - p)B(q_1) \geq \theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1), \quad (3.32)$$

then the optimal patenting fee is larger than the cost of examination, $P > c(p)$.

The marginal applicant is the lowest probability of validity applicant θ_1 , who, in equilibrium, is indifferent between applying for a patent or not applying for a patent. The first claim in the proposition follows immediately from (3.30). The condition in the second claim (3.32) is more likely to hold when bad patents are costly for the society and the volume of patent applications is high (low θ_1). The condition holds, for example, if private and social benefits of a good project are the same, $\tilde{G}(q) = G(q)$, but bad projects have a lower social value, $\tilde{B}(q) \leq B(q)$.

While examining the first order conditions, as we did above, help understand some of the forces involved, as a practical matter, patent policy is not likely to be set to its optimal levels. Lack of information and resource constraints may be among the reasons why an optimal policy is not feasible, and in practice, it is often more uniform than it should be from a social perspective.

3.3.3 Two-Tiered Patent System

In an article titled “What to Do about Bad Patents?” Lemley et. al. (2005) propose establishing a two-tiered patent system. The proposal was further discussed in Lemley and Lichtman (2007). Their rationale is that the two-tiered system will allow the patent office to “focus its examination resources on important patents and pay little attention to the rest.” The two-tiered patent system would give applicants a choice between a low cost patent application which is not examined thoroughly, and a high cost patent application which would be subject to a thorough examination and thus earn more presumption of validity by getting a “gold-plate” patent. They suggested that innovators would

likely pay for serious review of their most economically important patents. This self selection mechanism would allow the patent office to focus resources on most important patents (those whose innovators choose to gold-plate). In this section, we extend our basic model to analyze a two-tiered patent system. We examine innovators' equilibrium selection of a patent tier and how it is affected by patent policies. We establish conditions under which the conjectured positive correlation between economic value of an innovation and gold-plate patent applications holds.

Extending the Model

We maintain most of our assumptions from the previous section, but now we introduce the two-tiered patent system. Innovators can choose to apply for a "regular" patent, or for a "gold-plate" patent. A gold-plate patent is associated with a higher fee $P_{gp} > P_r$ and a more thorough examining procedure $p_{gp} > p_r$. We assume that p_{gp} is sufficiently high (or B only moderately steeper than G) so that

$$G'(q) - (1 - p_{gp})B'(q) > 0. \quad (3.33)$$

This condition will help us establish the existence of equilibrium in the two-tiered system. The condition clearly holds when $p_{gp} = 1$. Which patent was granted is public information. The value of a patent to an innovator depends on both its ex-ante probability of validity θ and on the perceived quality of patents of its tier – q_r for a regular patent or q_{gp} for a gold-plate patent. This dependence arises for two reasons: the different examination intensities and the different endogenously determined selection of patent applicants.

Given patent policy parameters and choices by all other innovators, an inno-

vator whose ex-ante probability of validity is θ and who applies for a patent-tier $i \in \{r, gp\}$, obtains a value:

$$V_i(\theta, q_i) = \theta G(q_i) + (1 - \theta)(1 - p_i)B(q_i) - P_i. \quad (3.34)$$

An innovator would file for a regular patent if the value of a regular patent is greater than zero (value of not patenting), $V_r(\theta, q_r) \geq 0$ and also greater than the value of a gold-plate patent, $V_r(\theta, q_r) \geq V_{gp}(\theta, q_{gp})$.

Let us denote the set of innovators who apply for a regular patent by Θ_r and the set of innovators who file for a gold-plate patent by Θ_{gp} . The probability that a patent of type $i \in \{r, gp\}$ is good is then given by

$$q_i = \Pr(\text{good} \mid \text{patent-tier } i \text{ granted}) = \frac{\int_{\Theta_i} \theta f(\theta) d\theta}{\int_{\Theta_i} [1 - p_i(1 - \theta)] f(\theta) d\theta}. \quad (3.35)$$

Under the assumption of rational expectations, the probability q_i is the belief others hold about the quality of a patent-tier i . As before, we refer to q_i as perceived patent quality, or simply patent quality. We now define an equilibrium in our model. For simplicity (and without loss of generality), we assume that when indifferent between patenting or not patenting, innovators choose to patent and when indifferent between a regular patent and a gold-plate patent, innovators apply for the regular patent.

Definition 4 *A two-tiered patent system equilibrium is given by two disjoint sets of innovators Θ_r and Θ_{gp} in $[0,1]$ and patent qualities q_r and q_{gp} such that:*

1. for all $\theta \in \Theta_r$, $V_r(\theta, q_r) \geq 0$ and $V_r(\theta, q_r) \geq V_{gp}(\theta, q_{gp})$;
2. for all $\theta \in \Theta_{gp}$, $V_{gp}(\theta, q_{gp}) \geq 0$ and $V_{gp}(\theta, q_{gp}) > V_r(\theta, q_r)$;

3. q_r and q_{gp} satisfy equation (3.35).

The first two conditions imply that innovators choose optimally between applying for a regular patent, a gold-plate patent or no patent; the third condition states that expectations about patent quality are rational given all innovators' choices and the existing patent policy.

The equilibria we identify are such that the sets Θ_r and Θ_{gp} can be defined using thresholds so that high types apply for gold-plate patents, and intermediate types apply for a regular patent.

Definition 5 (i) A “thresholds equilibrium” is a two-tiered patent system equilibrium such that there exist $0 \leq \theta_* \leq \theta^* \leq 1$ so that in equilibrium $(\theta^*, 1) \subset \Theta_{gp}$, $(\theta_*, \theta^*) \subset \Theta_r$, $(0, \theta_*)$ and innovators with types $(0, \theta_*)$ do not apply for a patent.

(ii) A thresholds equilibrium is interior if $0 < \theta_* < \theta^* < 1$.

In an interior threshold equilibrium, at least some innovators apply for each patent tier, and some do not apply for a patent. In the next proposition, we show that an equilibrium exists and that every interior equilibrium is a threshold equilibrium.

Proposition 14 An equilibrium for the two-tiered system exists. Any interior equilibrium is a threshold equilibrium. Moreover, in the interior equilibrium, the probability that a patent is good is higher for gold-plate patents, $q_{gp} > q_r$.

In the interior equilibrium, the innovator with ex-ante probability of validity θ_* is indifferent between applying for a regular patent and not applying for the

patent at all. Hence, if q_r is the equilibrium perceived quality of regular patents, then $V_r(\theta_*, q_r) = 0$. An innovator with ex-ante probability of validity θ^* is indifferent between applying for a regular patent and applying for a gold-plate patent. Hence, $V_{gp}(\theta^*, q_{gp}) = V_r(\theta^*, q_r)$. Payoffs $V_r(\cdot)$ and $V_{gp}(\cdot)$ are defined in (3.34). By the definition of interior equilibrium and by Proposition 14, to find an equilibrium, we need to find θ_* , θ^* , q_r and q_{gp} such that the following system is satisfied:

$$\begin{aligned}
\theta_* &= \frac{P_r - (1 - p_r)B(q_r)}{G(q_r) - (1 - p_r)B(q_r)}, \\
\theta^* &= \frac{(P_{gp} - P_r) - [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)]}{[G(q_{gp}) - G(q_r)] - [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)]}, \\
q_r &= \frac{\int_{\theta_*}^{\theta^*} \theta f(\theta) d\theta}{\int_{\theta_*}^{\theta^*} [1 - p_r(1 - \theta)] f(\theta) d\theta}, \\
q_{gp} &= \frac{\int_{\theta^*}^1 \theta f(\theta) d\theta}{\int_{\theta^*}^1 [1 - p_{gp}(1 - \theta)] f(\theta) d\theta}.
\end{aligned} \tag{3.36}$$

The first equation states the indifference of an applicant, with an ex-ante probability of validity θ_* , between applying for a regular patent or not applying; the second states the indifference of an applicant, with an ex-ante probability of validity θ^* , between applying for a regular patent or for a gold-plate one; the last two equations define perceived patent quality for regular and gold-plate patents based on the equilibrium choices of innovators.

Consequences of Gold-plating Patents

In this section, we consider the effect of introducing gold-plate patents to a standard single-tiered patent system. As a benchmark for comparison of the two-tiered patent system with the single-tiered patent system, we will assume that

examination effort and patenting fee for the regular patent in the two-tiered system remain the same as those in the single-tiered patent system. But in the two-tiered system, the innovators have an additional choice: they can gold-plate their patents which provides a more thorough examination process at a higher fee.

Suppose that in the single-tiered patent system, in equilibrium, innovators with ex-ante probability of validity $\theta \geq \theta_1$ apply for the patent, and average patent quality is given by q_1 . Suppose now that the patent office adopts the new two-tiered system. Let us consider the effect of the availability of the second patent-tier on the volume of patent applications and on the quality of patents. We assume (the more interesting case) that the new system would result in an interior equilibrium, where both patent-tiers are applied for. As described earlier, θ_* and θ^* denote the cut-off ex-ante probabilities of validity for regular patent application and for gold-plate patents, respectively (see (3.36)). In the next proposition, we show that the addition of the gold-plate patent option results in a decline in the overall volume of patent applications (with less low quality patents being applied for). While regular patents would be of lower quality than a patent in the single-tiered system, gold-plate patents are of higher quality and the overall quality of patents would increase as a result of this policy change. The latter effect results from the decline in low quality patent applications as well as the more thorough examination of gold-plate patents.

Proposition 15 *Consider a single-tiered system with examination intensity p and a patent fee P and consider a two-tiered system where regular patents have the same fee and examination intensity as the single-tiered system ($P_r = P$ and $p_r = p$). Assume each system has an interior equilibrium. Then, (i) in the two-tiered system, the vol-*

ume of patent applications is lower than in the single-tiered system ($\theta_* > \theta_1$); (ii) the perceived quality of a regular patent is lower than that of a patent in the single-tiered system; (iii) the perceived quality of a gold-plate patent is higher; (iv) overall, there are less bad patents in the two-tiered system.

A two-tiered system involves more intense examination. This potentially increases the patent office's expenditure. If the gold-plate patent fee is set high enough to cover the higher examination costs, the expenditure of the patent office would not increase. For a very stringent gold-plate patent examination process, this cost may be prohibitively high so that no interior equilibrium exists. However, under reasonable technical conditions stated in the next proposition, if the single-tiered system has an interior equilibrium and imperfect examination ($p < 1$), then there is a two-tiered patent system with an interior equilibrium in which the expenditure of the patent office does not exceed that of the single-tiered system.

Proposition 16 *Consider a single-tiered system with examination intensity p and a patent fee P that has an interior equilibrium $\{\theta_1, q_1\}$. Assume the Jacobian matrix of the system of equations (3.36) has a positive determinant²⁰ at the point $(\theta_* = \theta_1, \theta^* = 1, q_r = q_1, q_{gp} = 1)$. Then there exist $p_{gp} > p$ and $P_{gp} > P$ such that the two-tiered system, where $P_r = P$ and $p_r = p$, has an interior equilibrium in which the expenditure of the patent office does not exceed that of the single-tiered system.*

To assure that the expenditure of the patent office does not exceed that in the single-tiered system, we could set gold-plate patent fee to cover the additional

²⁰The Jacobian matrix is the matrix of first derivatives of the system of equilibrium equations. A positive Jacobian guarantees that the solution to the equilibrium equations system exists and is locally unique.

cost of examination: $P_{gp} \geq P_r + [c(p_{gp}) - c(p_1)]$. A sufficiently small increase in examination intensity p_{gp} in the second tier would allow a small enough gold-plate patent fee so that there is an interior equilibrium in the two-tiered system.

Patent Policy and Its Effect on the Two-Tiered System

In this section, we consider the effect of changes in patent policy on the two-tiered patent system, particularly, the effect of changes in patenting fees and the intensity of examination on the volume of patent applications, on the choice between regular and gold-plate patents and on patent quality. Gold-plate patents are intended to be ones for which the patent office thoroughly examines the applications. To simplify the analysis while capturing this idea, we will consider the extreme case in which examination of gold-plate patents is perfect, $p_{gp} = 1$. With such stringent examination, the perceived quality of gold-plate patents is maximized, $q_{gp} = 1$, since any bad application for a gold-plate patent is rejected. The system of equilibrium equations (3.36) is then reduced to

$$\begin{aligned}\theta_* &= \frac{P_r - (1 - p_r)B(q_r)}{G(q_r) - (1 - p_r)B(q_r)}, \\ \theta^* &= \frac{(P_{gp} - P_r) + (1 - p_r)B(q_r)}{[G(1) - G(q_r)] + (1 - p_r)B(q_r)}, \\ q_r &= \frac{\int_{\theta_*}^{\theta^*} \theta f(\theta) d\theta}{\int_{\theta_*}^{\theta^*} [1 - p_r(1 - \theta)] f(\theta) d\theta}.\end{aligned}$$

There exists an interior equilibrium when there is a solution to the system of equilibrium equalities with $(\theta_*, \theta^*, q_r) \in (0, 1)^3$. Interior equilibria exist for some functional forms (G , B and F), and parameter values (P_{gp}, P_r, p_r) , but not always. From the second equilibrium condition, it is easy to see that a necessary

condition for the existence of an interior equilibrium is that

$$G(1) - G(0) > (P_{gp} - P_r). \quad (3.37)$$

If this condition fails, then no innovator applies for a gold-plate patent. If innovators' payoffs are independent of perceived quality, then $G(1) = G(0)$ and the two-tiered system fails.

We now investigate how patenting fees P_{gp} and P_r and the examination intensity for regular patents p_r affect the (interior) equilibrium outcomes in the two-tiered patent system. To simplify derivations, from now on, consider a linear model which is defined as follows.

Definition 6 (*Linear model*). *In the linear model, innovator types are uniformly distributed: $F(\theta) = \theta$, benefit-functions are linear: $G(q) = (G + G'q)$ and $B(q) = (B + B'q)$, with parameters that satisfy $(G + G') = (B + B')$ and $2G' \geq B' > G' > 0$. (Hence, the benefit from a good patent is at least as high as that from a bad patent, these benefits are equal at $q = 1$, and $B(q)$ is moderately steeper.)*

Consider first the effect of an increase in the fee for a gold-plate patent. With a rise in cost of applying for a gold-plate patent, we can expect some applicants to apply for a regular patent instead of a gold-plate patent. This would result in a decline of gold-plate patent applications (θ^* increases). Applicants who switch from gold-plate patents to regular patents are of higher ex-ante probability of validity than those who applied for regular patents in the first place. Therefore the perceived quality of regular patents (q_r) increases. If benefits from patent applications did not depend on the perceived quality, there would be no reason for the overall volume of patent applications to change. However, since benefits increase with perceived quality, with higher gold-plate patent fee there is an

increase in perceived quality of regular patents which results in an increase in regular patent applications also from marginally low quality holders (a decline in θ_*). Hence, overall there is an increase in the number of patent applications. Because there is a decline in (stringently examined) gold-plate patents and rise in low-quality applications, the number of bad patents granted increases. However, the quality of regular patents increases despite the possible increase in number of applications.

An increase in the fee for a regular patent would make regular patents less attractive for some high ex-ante probability of validity innovators who would now prefer a gold-plate patent instead, as well as from some low ex-ante probability of validity innovators who would now prefer not to apply for a patent at all. Therefore, the prevalence of bad patents would decline both due to the decrease in applications by some low ex-ante probability of validity innovators and because of the increase in applications for the more stringently examined gold-plate patents. The perceived quality of a regular patent increases.

The effect of an increase in patent examination intensity is more complex. On one hand, the increase in examination effort reduces the probability that a patent is granted which lowers the value of applying for regular patents. On the other hand, because patentees benefit from a higher perceived value, if the quality of patents increases with examination intensity, it would have a positive effect on the value of applying for regular patents. In our analysis, the first effect (which makes regular patents less attractive) dominates for low ex-ante probability of validity innovators, thus there will be a decline in overall applications for patents. As a result of the increase in examination intensity, there is an ambiguous effect on the volume of gold-plate patent applications. The effect on the

prevalence of bad patents with respect to examination intensity is negative and on the perceived quality of a regular patent is positive.

We summarize these findings in the following proposition.

Proposition 17 *In the linear model, if regular patent examination intensity is not too high ($p_r \leq \frac{1}{2}$), and gold-plate patents are perfectly examined ($p_{gp} = 1$), then in an interior equilibrium,*

(i) the overall volume of patent applications increases (θ_ declines) with gold-plate patenting fees (P_{gp}), it decreases with regular patents' fees (P_r) and with the intensity of the examination process (p_r);*

(ii) the volume of gold-plate patent applications increases (θ^ declines) with the fees of regular patents; it decreases with gold-plate patenting fees. The effect of the intensity of examination of regular patents is ambiguous;*

(iii) the quality of regular patents (q_r) increases with gold-plate patenting fees, with regular patents' fees and with its examination intensity;

(iv) the prevalence of bad patents increases with gold-plate patenting fees and decreases with regular patents' fees and with their examination intensity.

Presumption of Validity

The patent system is a part of the executive branch of the government. Its decisions are subject to review by courts. Patent law states however that "a patent shall be presumed valid" and that the "burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity" (35

USC 282). Lemley and Lichtman (2007), in their examination of the two-tiered patent system proposal discuss at length the issue of presumption of validity. They suggest that in the two-tiered system, gold-plate patent holders should enjoy a presumption of validity as their patents were thoroughly examined, but patents in the other tier should not be presumed valid. They explain further that “[w]e know far less than we should about how presumptions affect litigation decisions. ... it is far from a simple matter to predict how changes in a legal presumption would change actual case outcomes.” Our model of the two-tiered patent system therefore focused on the main characteristics of the two-tiered system – more thorough examination and higher fees for gold-plate patents, but did not explicitly address the issue of presumption of validity.

Presumption of validity may, to some extent, have been captured in our model by the fact that in our model we take into account perceived patent quality and its positive effect on the value of patents. Benefit functions were assumed to have the same functional forms $G(q)$ and $B(q)$, but gold-plate patents have a higher perceived quality ($q_{gp} > q_r$) which we can think of as also capturing a “presumption of validity”. Another way to think about modeling more explicitly a legal change in presumption of validity is to assume different benefit functions in the two tiers $G_i(q)$ and $B_i(q)$ for $i \in \{r, gp\}$, with higher benefits in the case of gold-plate patents because of the stronger presumption of validity. In our system of equilibrium inequalities, the change would be reflected in the equation defining θ^* —the threshold between types who apply for the regular and the gold-plate patents. For simplicity of the notation and analysis, we chose not to incorporate this in the model. We conjecture that reducing presumption of validity for regular patents and increasing it for gold-plate patents, to the extent that this is described by tier-specific values for holding a patent,

is expected to make gold-plate patents at least marginally more attractive and regular patents less attractive. As long as the difference in benefits is not large, we would not expect selection patterns into the two tiers to change.

Economic Importance and Gold-Plate Patents

Lemley et. al. (2005) stated that “most likely applicants would pay for serious review with respect to their most important patents but conserve resources on their most speculative entries.” To examine this assertion in light of our model, we first need to ask what characterizes the economically “most important” patents? If one thinks of these as being innovations that are ex-ante most likely to be valid, then our previous analysis establishes the suggested relation. This holds because we found that only innovators with a high enough ex-ante probability of validity apply for a gold-plate patent.

However, economic importance is probably better interpreted in terms of the economic value of the innovation rather than in terms of the probability of it being good. In our model, the values of a patent conditional on it being good or bad are given by the functions $G(q)$ and $B(q)$. We have assumed that these two functions are increasing in the perceived quality of patents q , that the value of a bad patent is steeper but that at $q = 1$, $G(1) = B(1)$. What would distinguish the values associated with an economically important patent from a less important one? It seems reasonable to assume that the value of the patent conditional on it being good would be higher for an economically more significant patent. Denoting two innovations 1 and 2, the first being more significant, we expect $G_1(q) > G_2(q)$. It also seems reasonable that for economically significant innovations, the value of at least a bad patent is more sensitive to perceived

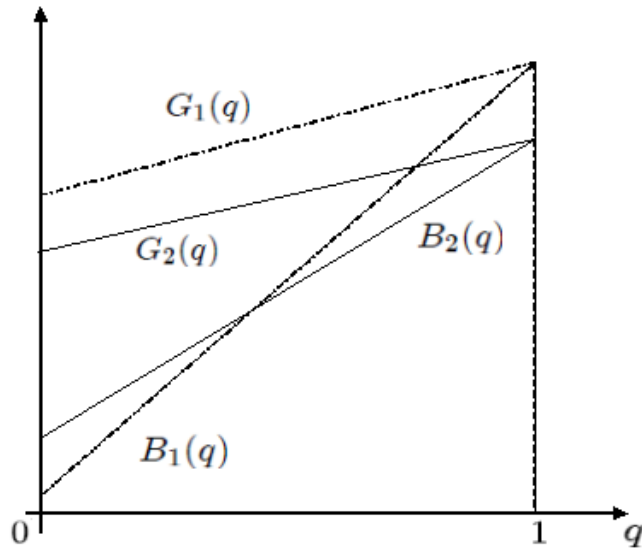


Figure 3.4: Economically significant patents

quality. This is because if the patent is economically significant, a competitor would have more to lose from being excluded and perhaps more to gain from trying to invalidate it. How likely the competitor is to challenge the patent can depend on the perceived value of patents. Hence, our second assumption is that for economically significant patents, $B(q)$ is steeper ($B'_1(q) \geq B'_2(q)$), and possibly also $G(q)$ (or, $G'_1(q) \geq G'_2(q)$). For low perceived patent quality, the value of a bad patent may be lower. But for high perceived patent quality, the value is higher: $B_1(1) > B_2(1)$. Figure 3.4 illustrates this comparison between the values of an economically significant patent and an economically insignificant one.

Proposition 18 *If "economic significance" is characterized by higher benefit functions and steeper value of bad patents, the effect of economic significance on the volume of gold-plate patent applications is ambiguous.*

We establish this ambiguity in the appendix using two numerical examples.

In both examples, we assume a linear model. We vary the degrees of increase in the benefit and the change in slope of the function $B(q)$. The examples establish the ambiguity of the result on economically significant patents – applicants with economically more significant patents are not necessarily more likely to apply for gold-plate patents than the ones with economically less significant patents.

We computed an equilibrium for innovations that are heterogenous in their ex-ante probability of validity, but otherwise homogeneous. This best describes a situation in which innovators and their competitors can observe innovations' economic significance, even if the patent office does not. Allowing heterogenous benefit functions would clearly make the analysis significantly more complex. However, to get a sense of what the potential effect of economic significance might be when applicants have heterogenous benefit functions, we consider two innovators' choices given the equilibrium levels of perceived patent qualities $q_{gp}^* > q_r^*$. The difference between each of the innovator's benefit in the two tiers is

$$\Delta V(\theta, q_{gp}, q_r) = \theta [G(q_{gp}) - G(q_r)] + (1-\theta) [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)] - [P_{gp} - P_r]. \quad (3.38)$$

An innovator applies for a gold-plate patent when $\Delta V(\theta, q_{gp}, q_r) > 0$. Suppose that innovator 1 has an economically more significant patent which is captured as a parallel shift up of the benefit functions compared to those of innovator 2: $G_1(q) = G_2(q) + \delta$ and $B_1(q) = B_2(q) + \delta$. The effect of δ on $\Delta V(\theta, q_{gp}, q_r)$ is

$$\frac{\partial \Delta V(\theta, q_{gp}, q_r)}{\partial \delta} = -(1 - \theta)(p_{gp} - p_r). \quad (3.39)$$

This implies that for fixed equilibrium levels of patent qualities, the innovator with an economically more significant patent might be less likely to apply for

a gold-plate patent. As we discussed above, economic significance might have a more complex effect than the one we described here, but this provides an intuition for the ambiguity of the result even if we were to allow heterogeneous patent values in the model.

3.3.4 Remarks

Patent policy reform has been a subject of intense policy debate in recent years. The quality of patents is a central issue in this debate. This chapter sheds lights on the determinants of equilibrium patent quality. In our model, the value of a patent to an innovator depends on what a third party would perceive its quality to be. This quality depends on overall patent applications and the examination procedure. Hence, patent quality and innovators' decisions to patent are determined together in equilibrium. Bad patents impose an externality on good patent holders as they decrease the perceived patent quality and hence the value of patents of all innovators.

We examined the effects of policy changes on patent quality and the volume of patent applications. Intuitively, increasing patenting fees would lower the volume of applications and increase patent quality. Interestingly, we find that making the examination procedure more stringent might result in an increase in the volume of applications.

We formally examine a policy reform proposal of establishing a two-tiered patent system. We show that such policy could result in a decrease in the volume of applications and in the probability that bad patents be issued. It is not obvious however that the two-tiered system would help sort economically im-

portant patents and focus examination efforts on these innovations. An important dimension which determines selection between regular patents and gold-plate patents is the ex-ante quality of the innovation. Innovators who believe that invalidating prior art is less likely to exist are more likely to sort into the gold-plate patent tier. This would imply that more intense examination effort would be aimed precisely at those applications that are least likely to be invalid. Additionally, if cost of examination is convex in the probability of finding invalidating prior art conditional on its existence, the marginal benefit of an additional dollar spent on examination effort might be higher if it is spent on the less thoroughly examined applications. This raises concerns over the efficiency of the two-tiered system, and may raise some doubt over how credible can the patent office be in maintaining a commitment for high examination standards in the second tier.

In our analysis, we have not accounted for a possible liquidity constraint that would prevent some innovators from patenting if patenting fee is too high. Binding liquidity constraints on the part of small innovators may prevent them from gold-plating patents even if their ex-ante probability of validity is high. In this case, the two-tiered system might disadvantage small and financially constrained innovators who would be pooled with lower quality applicants in the first tier. A fee schedule that allows discounts for small entities, as exists in the current system, can alleviate this concern.

BIBLIOGRAPHY

- [1] Alcacer, J. and M. Gittelman, "Patent Citations as a Measure of Knowledge Flows: The Influence of Examiner Citations." *Review of Economics and Statistics*, 88(4): 774 - 779, 2006.
- [2] Alcacer, J., M. Gittelman and B. Sampat, "Applicant and Examiner Citations in U.S. Patents: An Overview and Analysis." *Research Policy*, 38(2): 415-427, 2009.
- [3] Atal, V. and T. Bar, "Prior Art : To Search or Not to Search." *International Journal of Industrial Organization*, 2010.
- [4] Caillaud, B. and A. Duchêne, "Patent Office and Innovation Policy." Working paper, 2007.
- [5] Chang, H., "Patent Scope, Antitrust Policy and Cumulative Innovation." *RAND Journal of Economics*, 26: 34–57, 1995.
- [6] Cockburn, I., S. Kortum and S. Stern, "Are All Patent Examiners Equal? The Impact of Examiner Characteristics." NBER Working Paper No. W8980. Available at: <http://www.nber.org/papers/w8980.pdf>, 2002.
- [7] Cohen, W., R. Nelson and J. Walsh, "Protecting Their Intellectual Assets: Appropriability Conditions and Why U.S. Manufacturing Firms Patent (or Not)," NBER Working Paper No. W7552, 2000. Available at: <http://www.nber.org/papers/w7552.pdf>
- [8] Cotropia, C., "Recent Developments in the Inequitable Conduct Doctrine and Their Impact on Patent Quality." Available at: <http://www.ipo.org/AM/Template.cfm?Section=Calendar&Template=/CM/ContentDisplay.cfm&ContentID=15882>, 2007.
- [9] Crampes, C. and C. Langinier, "Litigation and Settlement in Patent Infringement Cases." *RAND Journal of Economics*, 33: 228–274, 2002.
- [10] Gilbert, R. and C. Shapiro, "Optimal patent length and breadth." *RAND Journal of Economics*, 21(1): 106-112, 1990.
- [11] Klemperer, P., "How broad should the scope of patent protection be." *RAND Journal of Economics*, 21(1): 113-130, 1990.

- [12] Lampe, R., "Strategic Citation." Working paper, 2008. Available at SSRN: <http://ssrn.com/abstract=984123>
- [13] Langinier, C. and S. Lluís, "Mobility and Career Concerns of USPTO examiners." Working paper, 2009.
- [14] Langinier, C. and P. Marcoul, "Patents, Search of Prior Art and Revelation of Information." Working paper, 2008. Available at <http://www.ualberta.ca/langinie/papers.html>
- [15] Lanjouw, J. O. and M. Schankerman, "Protecting Intellectual Property Rights: Are Small Firms Handicapped?" *The Journal of Law and Economics*, 47(1): 45-74, 2004.
- [16] Lemley, M., "Rational Ignorance at the Patent Office." *Northwestern University Law Review*, 95: 1-34, 2001.
- [17] Lemley, M., D. Lichtman and B. Sampat, "What to do About Bad Patents." *Regulation*, 28(4): 10-13, 2005.
- [18] Lichtman, D. and M. Lemley, "Rethinking Patent Law's Presumption of Validity." *Stanford Law Review*, 60(1): 45-72, 2007.
- [19] Manual of Patent Examining Procedure. 2008. <http://www.uspto.gov/web/offices/pac/mpep/>
- [20] Merges, R. P., "As Many as Six Impossible Patents before Breakfast: Property Rights for Business Concepts and Patent System Reform." *Berkeley Technology Law Journal*, 14(2), 1999.
- [21] Noveck, B., "Peer to Patent: Collective Intelligence, Open Review, and Patent Reform." *Harvard Journal of Law and Technology*, 20(1): 123-162, 2006.
- [22] O'Donoghue, T., "A patentability requirement for sequential innovation." *RAND Journal of Economics*, 29: 654-679, 1998.
- [23] O'Donoghue, T., S. Scotchmer and J. Thisse, "Patent Breadth, Patent Life, and the Pace of Technological Progress," *Journal of Economics and Management Strategy*, 7(1): 1-32, 1998.

- [24] Sampat, B., "Examining Patent Examination: An Analysis of Examiner and Applicant Generated Prior Art." Working paper, 2005.
- [25] Scotchmer, S. and J. Green, "Novelty and Disclosure in Patent Law." *RAND Journal of Economics*, 21(1): 131-146, 1990.
- [26] Scotchmer S., *Innovation and Incentives*, MIT Press Cambridge, MA, 2006.
- [27] Thomas, J. R., "Collusion and Collective Action in the Patent System: A Proposal for Patent Bounties." *University of Illinois Law Review*, 2001(1): 305-353, 2001.

CHAPTER 4

DO JOURNALS ACCEPT TOO MANY PAPERS?

4.1 Introduction

Everyone makes mistakes. Journal referees are no exceptions. Refereeing a paper for a journal to judge whether it is of high enough quality and whether it satisfies the goals of the journal is difficult enough in the best of times and it is sometimes even tougher because of the innovative and unfamiliar methods used in papers trying to scale scientific heights. Hence many papers of really good quality get rejections from journals. For example, George Akerlof's Nobel Prize winning paper, "The Market for 'Lemons': Quality, Uncertainty and the Market Mechanism," was rejected by three different journals before it was finally accepted by the Quarterly Journal of Economics (Gans and Shepherd, 1994; Shepherd, 1994). Paul Samuelson admits that some of his 'classic' papers were initially rejected by some journals. Paul Krugman said, "I would estimate that 60% of my papers sent to refereed journals have been rejected on the first try." (Gans and Shepherd, 1994; Shepherd, 1994). The reverse mistake of accepting papers that ought not to be set in print also no doubt occurs. Error in the refereeing process is not confined to the field of economics only, it happens in every area. In 1977, Jerzy Kosinski, the National Book Award winner for one of his novels in 1969, allowed a free-lancer to resubmit the manuscript of the same novel with a different title and author's name. Surprisingly, all the publishers refused to publish it and more astonishingly, the original publisher of the novel did not notice the disguise and also rejected it (Peters and Ceci, 1980).

The larger problems of peer-reviewed journals will need a lot of research and

effort to address. However, discrepancies between ideal acceptance rate and the actual acceptance rate can also occur for reasons of pure 'market structure,' that is, stemming from strategic matters of inter-journal competition. The present chapter is a contribution to this latter source of inefficiency.

What the chapter does is to take this tiny slice of the large problem of efficient peer review and analyze if there are inefficiencies that stem from this. Indeed, I find that there are such inefficiencies that can be corrected. In particular, I find that in the presence of errors in estimating the true quality of a paper, journals lower their quality cutoffs while competing against each other. Apart from these specific findings, the chapter also tries to contribute to providing a theoretical structure for analyzing journal behavior—a subject that has not received as much attention as it should.

There has been a rising concern about the flaws in the peer-review system (Blank, 1991; Hamermesh, 1994; Ellison, 2002). Ellison (2002) found that there has been an increasing time-gap between the submission and acceptance of a paper. However, he found that, this increasing review time is not a result of more thorough review or more complex papers, rather because the journals require more extensive revisions. In addition, Ellison (2002) concludes that "top general-interest journals have raised their quality threshold (relative to the other journals), which suggests that there is now more competition for their space." Note that this finding is not in contrast with the results of my model. In this chapter, there is a constant pool of paper submissions. However, in case of top journals, the competition is much severe among the authors compared to the competition among journals. And my chapter does confirm that as severity of competition between journals decreases, they raise their quality cut-offs.

4.2 The Model

Consider two journals - A and B , similar in every aspect to start with, i.e., they have the same rank initially in a particular field of publication, same cost of publishing a given number of papers, they make similar mistakes in reviewing an article and so on. Suppose that each journal receives a continuum of papers, each of quality $q \in [0, 1]$, for review and the journal publishes it if it is of a sufficiently high quality. Assume that, quality-wise, the papers are uniformly distributed over $[0, 1]$. After review, journal k , $k \in \{A, B\}$, observes quality q_i^k for paper i ; q_i^k is uniformly distributed over $[q_i - \varepsilon, q_i + \varepsilon]$ where q_i is the true quality of the paper and ε is the error in the review process while assessing the true quality. Assume that the errors are the same for both the journals. Each journal k sets a cut-off q^k on the observed quality of a paper for publishing it. Thus, paper i is accepted by journal k if and only if $q_i^k \geq q^k$, otherwise rejected. If a journal rejects a “first-time submission,” then the paper is submitted to the other journal. I shall refer this as “second-time submission” in this chapter. If a paper is rejected by both journals, it is abandoned.

For each journal, cost of publishing n papers is

$$C(n) = cn. \tag{4.1}$$

Linear cost is just for the technical simplification of the model. However, all the main results of this chapter hold (qualitatively) with a strictly convex cost function as well.

It is reasonable to assume that a journal’s objective is to improve its rank.¹ Different analysts suggest different ways to rank a journal. Some rank journals

¹Note that this is just a horizontal competition between journals.

according to the number of citations others make to the articles in the particular journal (see, for example, Liebowitz and Palmer, 1984), whereas some others rank journals according to the ranks of the economic departments affiliating the corresponding authors of the articles published in that particular journal (see Moore, 1972). I take the first approach. Hence, in this chapter, the objective of a journal is to maximize the total quality of the papers published each period² minus the cost of publishing.

For journal k , let Q^k denote the expected total quality of accepted papers and n^k denote the expected volume of papers accepted. By “volume of papers,” I mean the total number of papers accepted in the journal, not the size of each paper. Therefore, n^k can be written in two parts:

$$n^k = f_1(q^k) + f_2(q^k, q^l), l \in \{A, B\}, l \neq k, \quad (4.2)$$

where $f_1(q^k)$ denotes the expected volume of papers accepted by journal k from the pool of first-time submissions and $f_2(q^k, q^l)$ denotes the expected volume of papers accepted by journal k from the pool of second-time submissions. Similarly, we can break Q^k into two parts:

$$Q^k = \theta_1(q^k) + \theta_2(q^k, q^l) \quad (4.3)$$

where $\theta_1(q^k)$ denotes the expected total quality of papers accepted by journal k from the pool of first-time submissions and $\theta_2(q^k, q^l)$ denotes the expected total quality of papers accepted by journal k from the pool of second-time submissions.

Let π^k be journal k 's payoff:

$$\pi^k = Q^k - C(n^k). \quad (4.4)$$

²For simplicity, there is only one period.

Therefore, each journal's objective is to set the cut-off on observed quality of a paper q^k , $k \in \{A, B\}$, such that its pay-off is maximized. Assume that $q^k \in (2\varepsilon, 1 - 2\varepsilon)$, that is, journals set the cut-off neither very low so that they end up accepting most of the papers submitted to it, nor very high to avoid rejection of too large a number of submissions.

Assume that the error in the review process is small enough:

$$0 \leq \varepsilon \leq \frac{1}{5} \quad (4.5)$$

and the cost of publishing is neither very high so that journals don't want to publish at all, nor very low so that the cost does not affect the pay-off much:

$$2\varepsilon \leq c \leq (1 - 3\varepsilon). \quad (4.6)$$

4.2.1 First-time Submissions

Since, for each journal, the observed quality of a paper of true quality q_i is uniformly distributed over $[q_i - \varepsilon, q_i + \varepsilon]$, the probability of acceptance of this paper by journal $k \in \{A, B\}$, is given by

$$\Pr(q_i^k \geq q^k \mid q_i) = \begin{cases} 1, & \text{if } q_i > q^k + \varepsilon \\ 0, & \text{if } q_i < q^k - \varepsilon \\ \frac{1}{2\varepsilon}(q_i + \varepsilon - q^k), & \text{if } q_i \in [q^k - \varepsilon, q^k + \varepsilon] \end{cases}$$

Let us now find the expected volume of papers to be accepted by journal k from the pool of first-time submissions.

$$f_1(q^k)$$

$$\begin{aligned}
&= \int_0^1 \Pr(q_i^k \geq q^k \mid q_i) dq_i \\
&= (1 - q^k).
\end{aligned} \tag{4.7}$$

Similarly, the expected total quality (for this pool) of accepted papers is given by

$$\begin{aligned}
&\theta_1(q^k) \\
&= \int_0^1 q_i \Pr(q_i^k \geq q^k \mid q_i) dq_i \\
&= \frac{1}{2} \left[1 - q^{k2} - \frac{\varepsilon^2}{3} \right].
\end{aligned} \tag{4.8}$$

4.2.2 Second-time Submissions

As seen in the previous section, the probability of rejection of a paper of true quality q_i by journal l , $l \in \{A, B\}$, $l \neq k$, is given by

$$\begin{aligned}
&\Pr(q_i^l < q^l \mid q_i) \\
&= \begin{cases} 0, & \text{if } q_i > q^l + \varepsilon \\ 1, & \text{if } q_i < q^l - \varepsilon \\ \frac{1}{2\varepsilon}(q^l + \varepsilon - q_i), & \text{if } q_i \in [q^l - \varepsilon, q^l + \varepsilon] \end{cases}
\end{aligned}$$

Hence, the probability of acceptance of the paper of true quality q_i by journal k , which has been rejected by journal l , is given by

$$\begin{aligned}
&\Pr(q_i^k \geq q^k \mid q_i \text{ and } q_i^l < q^l) \\
&= \begin{cases} 0, & \text{if } q_i > q^l + \varepsilon \text{ or } q_i < q^k - \varepsilon \\ 1, & \text{if } q^k + \varepsilon < q_i < q^l + \varepsilon \\ \frac{1}{2\varepsilon}(q_i + \varepsilon - q^k), & \text{if } q_i \in [q^k - \varepsilon, \min\{q^k + \varepsilon, q^l + \varepsilon\}] \end{cases}
\end{aligned}$$

Therefore, the expected volume of papers accepted by journal k from the pool of second-time submissions is

$$\begin{aligned}
& f_2(q^k, q^l) \\
&= \int_0^{q^l + \varepsilon} \Pr(q_i^k \geq q^k \mid q_i \text{ and } q_i^l < q^l) \Pr(q_i^l < q^l \mid q_i) dq_i \\
&= \begin{cases} 0, & \text{if } q^l + 2\varepsilon \leq q^k \\ \frac{1}{24\varepsilon^2} (q^l - q^k + 2\varepsilon)^3, & \text{if } q^l \leq q^k \leq q^l + 2\varepsilon \\ (q^l - q^k) + \frac{1}{24\varepsilon^2} (q^k - q^l + 2\varepsilon)^3, & \text{if } q^l - 2\varepsilon \leq q^k \leq q^l \\ (q^l - q^k), & \text{if } q^k \leq q^l - 2\varepsilon \end{cases} \quad (4.9)
\end{aligned}$$

Similarly, expected total quality of accepted papers in journal k for this pool of papers is

$$\begin{aligned}
& \theta_2(q^k, q^l) \\
&= \int_0^{q^l + \varepsilon} q_i \Pr(q_i^k \geq q^k \mid q_i \text{ and } q_i^l < q^l) \Pr(q_i^l < q^l \mid q_i) dq_i \\
&= \begin{cases} 0, & \text{if } q^l + 2\varepsilon \leq q^k \\ \frac{1}{24\varepsilon^2} (q^l - q^k + 2\varepsilon)^3 \frac{(q^l + q^k)}{2}, & \text{if } q^l \leq q^k \leq q^l + 2\varepsilon \\ \frac{1}{2} (q^{l2} - q^{k2}) + \frac{1}{24\varepsilon^2} (q^k - q^l + 2\varepsilon)^3 \frac{(q^l + q^k)}{2}, & \text{if } q^l - 2\varepsilon \leq q^k \leq q^l \\ \frac{1}{2} (q^{l2} - q^{k2}), & \text{if } q^k \leq q^l - 2\varepsilon \end{cases} \quad (4.10)
\end{aligned}$$

4.2.3 Cut-off Quality in Nash Equilibrium

We have now all the information needed to find out the Nash equilibrium of this model. Recall that the pay-off of journal k , $k \in \{A, B\}$ is given by:

$$\begin{aligned}
& \pi(q^k, q^l) \\
&= \theta_1(q^k) + \theta_2(q^k, q^l) - c [f_1(q^k) + f_2(q^k, q^l)]
\end{aligned}$$

where $l \in \{A, B\}$, $l \neq k$. Each journal k maximizes its pay-off by choosing an appropriate cut-off for the observed quality of a paper, $q^k \in (2\varepsilon, 1 - 2\varepsilon)$, taking

the cut-off quality of the other journal as given.

A Nash equilibrium of this model is given by a pair of cut-off qualities $(q^A, q^B) \in (2\varepsilon, 1 - 2\varepsilon)^2$ such that each journal maximizes its pay-off given the other journal's cut-off and no one wants to deviate unilaterally to choose some other cut-off level. Hence, (q^A, q^B) are such that the following conditions³ hold:

$$\frac{\partial \pi(q, q^B)}{\partial q} \Big|_{q=q^A} = 0 \text{ and } \frac{\partial \pi(q, q^A)}{\partial q} \Big|_{q=q^B} = 0. \quad (4.11)$$

From equations (4.7), (4.8), (4.9) and (4.10), we get the first order condition as follows:

$$\frac{\partial \pi(q^k, q^l)}{\partial q^k} = \begin{cases} -(q^k - c), & \text{if } q^l + 2\varepsilon \leq q^k \\ -(q^k - c) - \frac{1}{24\varepsilon^2} (q^l - q^k + 2\varepsilon)^2 (2q^k + q^l - 3c - \varepsilon), & \text{if } q^l \leq q^k \leq q^l + 2\varepsilon \\ -2(q^k - c) + \frac{1}{24\varepsilon^2} (q^k - q^l + 2\varepsilon)^2 (2q^k + q^l - 3c + \varepsilon), & \text{if } q^l - 2\varepsilon \leq q^k \leq q^l \\ -2(q^k - c), & \text{if } q^k \leq q^l - 2\varepsilon \end{cases} \quad (4.12)$$

In the following lemma, it is shown that the cut-off on the observed quality of a paper for each journal has to be same in a Nash equilibrium.

Lemma 2 *There does not exist any Nash equilibrium with $|q^A - q^B| > 0$.*

Proof. This can be proved in two steps.

First, suppose that there is a Nash equilibrium with $|q^A - q^B| \geq 2\varepsilon$. Without loss of generality, let $q^A \geq (q^B + 2\varepsilon)$. Then from the first order condition (4.12) and (4.11), we have

$$q^A = c \quad (4.13)$$

³These conditions are necessary and sufficient since the pay-off functions are strictly concave.

and

$$q^B = c = q^A, \quad (4.14)$$

a contradiction.

Next, suppose that there is a Nash equilibrium with $2\varepsilon > |q^A - q^B| > 0$. Without loss of generality, let $q^A > q^B > (q^A - 2\varepsilon)$. Then, from (4.12) and (4.11), the two first order conditions are given by

$$-q^A - \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^A + q^B - \varepsilon) + c \left[1 + \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] = 0 \quad (4.15)$$

and

$$-2q^B + \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^B + q^A + \varepsilon) + c \left[2 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] = 0. \quad (4.16)$$

Since $q^A > q^B > (q^A - 2\varepsilon)$, we have

$$\begin{aligned} & -2q^B + \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^B + q^A + \varepsilon) + c \left[2 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] \\ &= \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (q^A - q^B + \varepsilon) + \left[2 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] (-q^B + c) \\ &> \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (q^A - q^B + \varepsilon) + \left[1 + \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] (-q^A + c) \\ &= -q^A - \frac{1}{24\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^A + q^B - \varepsilon) + c \left[1 + \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right]. \end{aligned}$$

Therefore, both first order conditions cannot hold together when $q^A > q^B > q^A - 2\varepsilon$ and hence there is no Nash equilibrium with $2\varepsilon > |q^A - q^B| > 0$. ■

The intuition behind the lemma is that if a journal sets its cut-off quality higher than that of the other journal, then it can strictly increase its profit by reducing its cut-off marginally. This happens because although the fall in cut-off will cause the average quality of the papers to be published in this journal to fall slightly, however, the total quality of accepted papers increase sharply since

the number of papers accepted increases. Not only that, the rise in total quality more than offsets the extra cost of publication.

With this lemma in hand, we can now proceed to describe the Nash equilibrium in this model.

Proposition 19 *There exists a unique Nash equilibrium, $(q^A, q^B) = (q^*, q^*)$ where $q^* = c + \frac{\varepsilon}{9}$. The equilibrium cut-off quality increases with the error in the review process ε and the cost parameter c .*

Proof. From Lemma 2, we know that in a Nash equilibrium, $q^A = q^B = q^*$. Therefore, from the first order conditions given by (4.12) and (4.11), find that

$$-\frac{3}{2}q^* + \frac{\varepsilon}{6} + \frac{3}{2}c = 0 \tag{4.17}$$

and there is a unique solution to the equation above:

$$q^* = c + \frac{\varepsilon}{9}. \tag{4.18}$$

■

4.3 Cut-off Quality while under Same “Management”

This section tries to work out something akin to the social planner’s problem. However, since I have totally ignored the benefits the authors get from publishing a paper and the larger public’s need for “knowledge,” I cannot get a complete social welfare function. Instead of maximizing the social welfare, let the social planner choose the quality cutoffs for each journal to maximize the sum of their payoffs. Alternatively, this can be thought of the pay-off maximization

exercise of the management had both the journals been under the same management. One should be a little careful in noting that this is not monopoly or collusion or merger. They are still two separate journals.⁴

If a journal rejects a paper, then the author submits the paper to the other journal. Depending on the quality of the paper revealed after review and the cut-off quality, the second journal accepts or rejects the paper. If it accepts, then the pay-off of the journal changes—increases or decreases depending on the cost of publishing the paper and its quality. Hence, rejection to a paper by a journal has an externality on the pay-off of the other journal. While choosing their pay-off maximizing cut-off level on the observed quality of a paper, journals do not consider this externality on the other journal, however, the management does. The objective of the management is to maximize the total pay-off for both the journals keeping this externality in mind. Therefore, the objective function of the “management” can be written as follows:

$$\begin{aligned} & \max_{(q^A, q^B) \in (2\varepsilon, 1-2\varepsilon)^2} \Pi(q^A, q^B) \\ = & \max_{(q^A, q^B) \in (2\varepsilon, 1-2\varepsilon)^2} [\pi(q^A, q^B) + \pi(q^B, q^A)] \end{aligned}$$

The first order conditions for this maximization problem are given by

$$\frac{\partial \Pi(q^A, q^B)}{\partial q^A} = 0 \text{ and } \frac{\partial \Pi(q^A, q^B)}{\partial q^B} = 0. \tag{4.19}$$

From equations (4.7), (4.8), (4.9) and (4.10), we get that

$$\frac{\partial \Pi(q^k, q^l)}{\partial q^k}$$

⁴However, due to linear cost function, the management’s optimum choice of quality cut-offs match exactly with those chosen when the two journals collude but do not know if a paper has been rejected by the other journal or not.

⁵Second order conditions hold since the pay-off function is strictly concave.

$$= \begin{cases} 0, & \text{if } q^l + 2\varepsilon \leq q^k \\ -\frac{1}{12\varepsilon^2} (q^l - q^k + 2\varepsilon)^2 (2q^k + q^l - 3c - \varepsilon), & \text{if } q^l \leq q^k \leq q^l + 2\varepsilon \\ -2(q^k - c) + \frac{1}{12\varepsilon^2} (q^k - q^l + 2\varepsilon)^2 (2q^k + q^l - 3c + \varepsilon), & \text{if } q^l - 2\varepsilon \leq q^k \leq q^l \\ -2(q^k - c), & \text{if } q^k \leq q^l - 2\varepsilon \end{cases} \quad (4.20)$$

Analogous to Lemma 2 in the previous section, we have the following lemma for the management's optimization problem.

Lemma 3 *The "optimum" choice of cutoffs on the observed qualities for both the journals have to be same, $q^A = q^B$.*

Proof. It is very east to check that optimum (q^A, q^B) cannot be such that $|q^A - q^B| \geq 2\varepsilon$, because if so happens, then the total pay-off can be increased by reducing the cut-off of the journal with higher one.

To see that $|q^A - q^B| \not\geq 2\varepsilon$, let us suppose that there exists (q^A, q^B) that maximizes the total pay-off and $2\varepsilon > |q^A - q^B| > 0$. Without loss of generality, let $q^A > q^B > (q^A - 2\varepsilon)$. Then, from (4.19) and (4.20), the two first order conditions are given by

$$-\frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^A + q^B - \varepsilon) + 2c \left[\frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] = 0 \quad (4.21)$$

and

$$-2q^B + \frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^B + q^A + \varepsilon) + 2c \left[1 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] = 0 \quad (4.22)$$

Since $q^A > q^B > (q^A - 2\varepsilon)$, we have

$$\begin{aligned} & -2q^B + \frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^B + q^A + \varepsilon) + 2c \left[1 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] \\ &= \frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (q^A - q^B + \varepsilon) + 2(c - q^B) \left[1 - \frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] \end{aligned}$$

$$\begin{aligned}
&> \frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (q^A - q^B + \varepsilon) + 2(c - q^A) \left[\frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right] \\
&= -\frac{1}{12\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 (2q^A + q^B - \varepsilon) + 2c \left[\frac{1}{8\varepsilon^2} (q^B - q^A + 2\varepsilon)^2 \right].
\end{aligned}$$

Therefore, when $q^A > q^B > (q^A - 2\varepsilon)$, $\Pi(q^A, q^B)$ can be increased either by reducing q^A slightly or by raising q^B a little. Hence, at the optimum, $q^A = q^B$. ■

Proposition 20 *There exists a unique cut-off level for both the journals that maximizes the total pay-off and it is higher than the Nash Equilibrium cut-off level on the observed qualities. The gap between them decreases as the error in the review process decreases.*

Proof. By Lemma 3, we know that $q^A = q^B = \widehat{q}$, at optimum. Therefore,

$$-\widehat{q} + \frac{\varepsilon}{3} + c = 0 \tag{4.23}$$

and there is a unique solution to the equation above:

$$\widehat{q} = c + \frac{\varepsilon}{3}. \tag{4.24}$$

From equation (4.18), comparing \widehat{q} and q^* , find that

$$\widehat{q} - q^* = \frac{2\varepsilon}{9} > 0 \tag{4.25}$$

and as $\varepsilon \rightarrow 0, \widehat{q} \rightarrow q^*$. ■

So journals, left to themselves, accept too many papers.

4.4 Conclusion

Using a simple duopoly model, I have shown that how the error in review process leads journals to choose a lower cut-off on observed quality than the one optimal for them collectively.

Admittedly, the analysis in this chapter is restrictive. I assumed that the error term ε is same for both the journals across all individual papers. However, first of all, different journals might have different errors because their referee pools are different. Secondly, errors might vary author-wise. It has been found that there is a significant difference between the referee reports when the author's identity is unknown and when the author is well renowned (see Blank, 1991; Hamermesh, 1994; Peters and Ceci, 1980). This suggests different ε for each individual paper. My analysis above does not address this issue. There are some more serious assumptions. I considered only two journals and that too of same rank which is far from reality. I did not consider the benefits that an author gets from publishing a paper. The benefits of the larger public from the knowledge diffusion by publishing a paper also do not enter into the set-up of the model in this chapter.

The peer review system has its problems, but it keeps a lot of "bad"-quality papers out. The journal review process has a large impact on the growth of the economics literature and it has a very significant impact on the growth of knowledge, the design of policy and the careers of the researchers. Hence, it is very important to raise awareness of journal editors and referees. The error in the system might make them compromise on the quality threshold which in turn may increase the number of low quality paper submissions raising the pressure on the referees and thus increasing the error even more. This calls for a reform of the peer review system. This chapter provides no blueprint for reform but a basic analytical structure that can be a first step towards such a blueprint.

BIBLIOGRAPHY

- [1] Atal, V., "Do Journals Accept Too Many Papers? " *Economics Letters*, 107(2): 229-232, 2010.
- [2] Blank, R. M., "The Effects of Double-Blind versus Single-Blind Reviewing: Experimental Evidence from The American Economic Review." *The American Economic Review*, 81(5): 1041-1067, 1991.
- [3] Ellison, G., "The Slowdown of the Economics Publishing Process." *Journal of Political Economy*, 110(5): 947-993, 2002.
- [4] Gans, J. S. and Shepherd, G. B., "How Are the Mighty Fallen: Rejected Classic Articles by Leading Economists." *The Journal of Economic Perspectives*, 8(1): 165-179, 1994.
- [5] Hamermesh, D. S., "Facts and Myths about Refereeing." *The Journal of Economic Perspectives*, 8(1): 153-163, 1994.
- [6] Liebowitz, S. J. and Palmer, J. P., "Assessing the Relative Impacts of Economics Journals." *Journal of Economic Literature*, 22(1): 77-88, 1984.
- [7] Moore, W. J., "The Relative Quality of Economics Journals: A Suggested Rating System." *Economic Inquiry*, 10(2): 156-169, 1972.
- [8] Peters, D. P. and Ceci, S. J., "A Manuscript Masquerade: How well does the review process work?" *Sciences*, 20(7): 16-19, 1980.
- [9] Shepherd, G. B., "Rejected: Leading Economists Ponder the Publication Process." Thomas Horton and Daughters, 1994.

APPENDIX A

PROOFS OF THE LEMMAS AND PROPOSITIONS IN CHAPTER 2

A.1 Lemma 1

Proof. The innovator's payoff when she faces the choice of novelty search effort is given in (3.12). Using the definition of $B(X_R, X_E)$ in (3.10) we write the payoff in two ranges of search efforts. In the range $x_1 + x_2 = X_R \geq X_E$, the profit of the researcher is given by

$$q(x_1) \frac{[1 - F(x_1 + x_2)]}{[1 - F(x_1)]} (B + \rho g) + (1 - q(x_1))(G - P) - I - (x_1 + x_2) \quad (\text{A.1})$$

and in the range $x_1 + x_2 = X_R < X_E$, the profit of the researcher is given by

$$q(x_1) \frac{[1 - F(x_1 + x_2)]}{[1 - F(x_1)]} \left[B + \frac{(1 - F(X_E))}{1 - F(x_1 + x_2)} \rho g \right] + (1 - q(x_1))(G - P) - I - (x_1 + x_2). \quad (\text{A.2})$$

Differentiating with respect to x_2 in each range, we find that

$$\frac{\partial \pi(x_1, x_2)}{\partial x_2} = \begin{cases} q(x_1) \frac{[-f(x_1 + x_2)]}{[1 - F(x_1)]} (B + \rho g) - 1 & \text{if } x_2 > X_E - x_1 \\ \text{undefined} & \text{if } x_2 = X_E - x_1 \\ q(x_1) \frac{[-f(x_1 + x_2)]}{[1 - F(x_1)]} B - 1 & \text{if } x_2 < X_E - x_1 \end{cases} \quad (\text{A.3})$$

and

$$\frac{\partial^2 \pi(x_1, x_2)}{\partial x_2^2} = \begin{cases} q(x_1) \frac{[-f'(x_1 + x_2)]}{[1 - F(x_1)]} (B + \rho g) & \text{if } x_2 > X_E - x_1 \\ \text{undefined} & \text{if } x_2 = X_E - x_1 \\ q(x_1) \frac{[-f'(x_1 + x_2)]}{[1 - F(x_1)]} B & \text{if } x_2 < X_E - x_1 \end{cases} \quad (\text{A.4})$$

We need to find the optimal novelty search effort x_2 given the early state of the art search level x_1 . We consider several cases.

Case 1: $0 \geq X_E - x_1$

We are necessarily in the range $x_2 \geq X_E - x_1$. In this range,

$$\frac{\partial \pi(x_1, x_2)}{\partial x_2} = q(x_1) \frac{[-f(x_1 + x_2)]}{[1 - F(x_1)]} (B + \rho g) - 1. \quad (\text{A.5})$$

If $(B + \rho g) \geq \frac{[1 - F(x_1)]}{-q(x_1)f(x_1)}$, then $\pi(x_1, x_2)$ decreases everywhere and there is a corner solution. Otherwise, $(B + \rho g) < 0$ which implies that the payoff function is concave and there is a unique solution that solves the first order condition.

$$x_2^*(x_1) = \begin{cases} f^{-1}\left(\frac{[1 - F(x_1)]}{-(B + \rho g)q(x_1)}\right) - x_1, & \text{if } B < \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} - \rho g, \\ 0, & \text{if } B \geq \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} - \rho g. \end{cases} \quad (\text{A.6})$$

Case 2: $X_E - x_1 > 0$

Case 2.1: Solution in the range $x_2 > X_E - x_1$.

If there is a solution in the range $x_2 > X_E - x_1$, then $x_2 = f^{-1}\left(\frac{[1 - F(x_1)]}{-(B + \rho g)q(x_1)}\right) - x_1$ and $B < \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} - \rho g$.

Because $(B + \rho g) < 0$ and $X_E - x_1 > 0$, payoff function is concave on each range $x_2 < X_E - x_1$ or $x_2 > X_E - x_1$ separately. In this case,

$$\frac{\partial \pi(x_1, x_2)}{\partial x_2} \Big|_{X_E - x_1} = \frac{-[q(x_1) f(X_E)]}{[1 - F(x_1)]} B - 1 > 0 \quad (\text{A.7})$$

so the proposed x_2 is a global Max.

Case 2.2: Solution with $x_2 = X_E - x_1$

For this to be a solution, we need

$$\begin{aligned} \text{from below} & : \frac{\partial \pi(x_1, x_2)}{\partial x_2} \Big|_{X_E - x_1} = \frac{-[q(x_1) f(X_E)]}{[1 - F(x_1)]} B - 1 \geq 0 \text{ and} \\ \text{from above:} & \frac{\partial \pi(x_1, x_2)}{\partial x_2} \Big|_{X_E - x_1} = \frac{-[q(x_1) f(X_E)]}{[1 - F(x_1)]} (B + \rho g) - 1 \leq 0 \end{aligned}$$

or,

$$\frac{[1 - \alpha F(x_1)]}{-\alpha f(X_E)} \geq B \geq \frac{[1 - \alpha F(x_1)]}{-\alpha f(X_E)} - \rho g. \quad (\text{A.8})$$

If the above condition holds, then there is no solution in the range $x_2 > X_E - x_1$ (from case 2.1). Additionally, in this case, $B < 0$. So $\pi(x_1, x_2)$ is concave and with a positive derivative from below, thus we know there is also no solution with $x_2 < X_E - x_1$ either.

Case 2.3: Solution in the range $0 < x_2 < X_E - x_1$

If there is such a solution, then we have

$$x_2 = f^{-1}\left(\frac{[1 - F(x_1)]}{-Bq(x_1)}\right) - x_1. \quad (\text{A.9})$$

For this to exist, $B < 0$ and thus $\pi(x_1, x_2)$ in this range is concave. Also, to be in the range, we need

$$\begin{aligned} \frac{\partial \pi(x_1, x_2)}{\partial x_2} \Big|_{X_E - x_1} &= \frac{-[q(x_1) f(X_E)]}{[1 - F(x_1)]} B - 1 < 0 \\ \text{and } \frac{\partial \pi(x_1, x_2)}{\partial x_2} \Big|_0 &= \frac{-[q(x_1) f(x_1)]}{[1 - F(x_1)]} B - 1 > 0 \end{aligned}$$

or,

$$\frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} > B > \frac{[1 - \alpha F(x_1)]}{-\alpha f(X_E)}. \quad (\text{A.10})$$

Case 2.4: Solution with $x_2 = 0$

We have a solution with $x_2 = 0$ if

$$B \geq \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)}. \quad (\text{A.11})$$

Summarizing the results, for any level of early state of the art search effort $x_1 \geq X_E$, the payoff maximizing novelty search is given by:

$$x_2^*(x_1) = \begin{cases} 0, & \text{if } B \geq \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} - \rho g, \\ f^{-1}\left(\frac{[1 - \alpha F(x_1)]}{-\alpha(B + \rho g)}\right) - x_1, & \text{if } B < \frac{[1 - \alpha F(x_1)]}{-\alpha f(x_1)} - \rho g. \end{cases} \quad (\text{A.12})$$

For any level of early state of the art search effort $x_1 < X_E$, the payoff maximizing novelty search is given by:

$$x_2^*(x_1) = \begin{cases} 0, & \text{if } B \geq \frac{[1-\alpha F(x_1)]}{-\alpha f(x_1)}, \\ f^{-1}\left(\frac{[1-\alpha F(x_1)]}{-\alpha B}\right) - x_1, & \text{if } \frac{[1-\alpha F(x_1)]}{-\alpha f(x_1)} > B > \frac{[1-\alpha F(x_1)]}{-\alpha f(X_E)}, \\ X_E - x_1, & \text{if } \frac{[1-\alpha F(x_1)]}{-\alpha f(X_E)} \geq B \geq \frac{[1-\alpha F(x_1)]}{-\alpha f(X_E)} - \rho g, \\ f^{-1}\left(\frac{[1-\alpha F(x_1)]}{-\alpha(B+\rho g)}\right) - x_1, & \text{if } B < \frac{[1-\alpha F(x_1)]}{-\alpha f(X_E)} - \rho g. \end{cases} \quad (\text{A.13})$$

Therefore, when $B \geq 0$, then $x_2^* = 0$. ■

A.2 Sufficient Condition for Investment

Lemma 4 *A sufficient condition for the innovator to choose to invest (that is for (3.13) to hold) is $\theta[\alpha B + (1 - \alpha)(G - P)] \geq I$.*

Proof. From (3.13), we find that after putting some effort on the early state of the art search x_1 , the researcher invests in the R&D project if the following condition holds:

$$\theta \left[q(x_1) \frac{[1 - F(x_1 + x_2^*)]}{[1 - F(x_1)]} B(X_R, X_E) + (1 - q(x_1))(G - P) - x_2^* \right] \geq I. \quad (\text{A.14})$$

Now, consider the left hand side of the above condition:

$$\begin{aligned} & \theta \left[q(x_1) \frac{[1 - F(x_1 + x_2^*)]}{[1 - F(x_1)]} B(X_R, X_E) + (1 - q(x_1))(G - P) - x_2^* \right] \\ & \geq \theta \left[q(x_1) \frac{[1 - F(x_1 + x_2^*)]}{[1 - F(x_1)]} B(X_R, X_E) + (1 - q(x_1))(G - P) - x_2^* \right]_{x_2=0} \\ & = \theta [q(x_1) B(X_R, X_E) + (1 - q(x_1))(G - P)] \\ & \geq \theta [\alpha B + (1 - \alpha)(G - P)]. \end{aligned}$$

The first inequality comes from the fact that x_2^* is the optimum ex post search effort that maximizes the total expected pay-off and the second inequality holds

because $\alpha \geq q(x_1) \forall x_1$ and $(G - P) > (g - P) \geq B(X_R, X_E) \geq B$. Thus we get the sufficient condition for investment by the researcher as

$$\theta[\alpha B + (1 - \alpha)(G - P)] \geq I. \quad (\text{A.15})$$

■

A.3 Proposition 1

Proof. We show that there are parameter values for which $X_R = X_E$. Recall that by Lemma 1, $x_2 = X_E - x_1$ if $X_E > x_1$ and

$$\frac{[1 - \alpha F(x_1)]}{-\alpha f(X_E)} \geq B \geq \frac{[1 - \alpha F(x_1)]}{-\alpha f(X_E)} - \rho g \quad (\text{A.16})$$

which is a non empty range for all $\rho > 0$.

When $x_2^*(x_1) = X_E - x_1$, innovator's payoff is given by

$$\begin{aligned} \Pi(x_1, x_2^*(x_1)) &= (1 - \alpha)\theta(G - P) + \alpha\theta[1 - F(X_E)](B + \rho g) \\ &\quad - [1 - \alpha F(x_1)](I + X_E - x_1) - x_1 \end{aligned}$$

and

$$\begin{aligned} \Pi'(x_1, x_2^*(x_1)) &= \alpha f(x_1)(I + \theta X_E - \theta x_1) - 1 + \theta[1 - \alpha F(x_1)], \\ \Pi''(x_1, x_2^*(x_1)) &= \alpha[(I + \theta X_E - \theta x_1)f'(x_1) - 2\theta f(x_1)] < 0. \end{aligned}$$

Hence, the payoff is a concave function of x_1 when $x_2^*(x_1) = X_E - x_1$ and has a solution $x_1 \in [0, X_E]$ whenever $\Pi'(0, X_E) > 0$ and $\Pi'(X_E, 0) < 0$ which holds true if I is such that $\frac{1-\theta}{\alpha f(0)} - \theta X_E < I < \frac{1-\theta}{\alpha f(X_E)} + \frac{\theta F(X_E)}{f(X_E)}$. This is a non-empty range. The solution x_1^* does not depend on P . Hence we can always find P so that (A.16) holds for this solution. ■

A.4 Proposition 2

Proof. By Lemma 1, we know that when $B \geq \frac{[1-\alpha F(x_1)]}{-\alpha f(x_1)} = \frac{-1}{q(x_1)\lambda(x_1)}$, then $x_2^* = 0$. This holds true for $x_1 = 0$ when

$$B \geq \frac{-1}{\alpha\lambda(0)}. \quad (\text{A.17})$$

Since $q(x_1)$ and $\lambda(x_1)$ decline with x_1 when the condition holds at $x_1 = 0$, it holds for $x_1 > 0$ too. Hence, $x_2^*(x_1) = 0$ and the researcher's payoff in the range $x_1 < X_E$ becomes

$$\begin{aligned} \Pi(x_1, 0) &= \alpha\theta[(1 - F(x_1))B + (1 - F(X_E))\rho g] \\ &\quad + (1 - \alpha)\theta(G - P) - [1 - \alpha F(x_1)]I - x_1. \end{aligned}$$

Differentiating, we get

$$\Pi'(x_1, 0) = \alpha(I - \theta B)f(x_1) - 1. \quad (\text{A.18})$$

If $B \geq \frac{I}{\theta}$, then the profit is decreasing in x_1 in the range $x_1 < X_E$ as well as in $x_1 \geq X_E$. Therefore profit is maximized at $x_1 = 0$. If $B < \frac{I}{\theta}$, then the profit is concave in the range $x_1 \leq X_E$. Therefore, its maximum in the range $x_1 \leq X_E$ is at $x_1 = 0$ if and only if

$$\Pi'(0, 0) = \alpha(I - \theta B)f(x_1) - 1 \leq 0 \quad (\text{A.19})$$

or,

$$B \geq \frac{\alpha\lambda(0)I - 1}{\alpha\theta\lambda(0)}. \quad (\text{A.20})$$

Under these conditions, profit also decreases in the range $x_1 > X_E$ since the derivative close to X_E is negative from the left and it is even lower from the right.

To sum up, if

$$B \geq \max \left\{ \frac{-1}{\alpha\lambda(0)}, \frac{\alpha\lambda(0)I - 1}{\alpha\theta\lambda(0)} \right\}, \quad (\text{A.21})$$

then $x_1^* = 0$ and $x_2^* = 0$. ■

A.5 Proposition 3

Proof. Consider the profit function given in (3.15). Differentiating this function in each of its regions, we obtain

$$\frac{\partial \Pi(x_1, x_2^*(x_1))}{\partial x_1} = \begin{cases} \begin{bmatrix} -\alpha\theta(B + \rho g) f(x_1 + x_2^*) \left(1 + \frac{\partial x_2^*}{\partial x_1}\right) \\ +\alpha f(x_1) (I + \theta x_2^*) - (1 - \alpha F(x_1)) \theta \frac{\partial x_2^*}{\partial x_1} - 1 \end{bmatrix} & \text{if } X_R > X_E \\ \begin{bmatrix} -\alpha\theta B f(x_1 + x_2^*) \left(1 + \frac{\partial x_2^*}{\partial x_1}\right) \\ +\alpha f(x_1) (I + \theta x_2^*) - (1 - \alpha F(x_1)) \theta \frac{\partial x_2^*}{\partial x_1} - 1 \end{bmatrix} & \text{if } X_R < X_E \end{cases} \quad (\text{A.22})$$

(i) Suppose $x_1^* = 0$ and $x_2^* > 0$. This implies that $x_1^* < X_E$ and $X_R = x_2^*$. Using (A.13) in Lemma 1 for the range $x_2^* > 0$, we have

$$x_2^*(0) = \begin{cases} f^{-1}\left(\frac{1}{-\alpha B}\right) < X_E, & \text{if } \frac{1}{-\alpha f(0)} > B > \frac{1}{-\alpha f(X_E)}, \\ X_E, & \text{if } \frac{1}{-\alpha f(X_E)} \geq B \geq \frac{1}{-\alpha f(X_E)} - \rho g, \\ f^{-1}\left(\frac{1}{-\alpha(B + \rho g)}\right) > X_E, & \text{if } B < \frac{1}{-\alpha f(X_E)} - \rho g. \end{cases} \quad (\text{A.23})$$

Substituting into (A.22), we get

$$\frac{\partial \Pi(0, x_2^*(0))}{\partial x_1} = \alpha f(0) (I + \theta x_2^*(0)) - (1 - \theta). \quad (\text{A.24})$$

Now, $x_1^* = 0$ implies that

$$\frac{\partial \Pi(0, x_2^*(0))}{\partial x_1} = \alpha f(0) (I + \theta x_2^*(0)) - (1 - \theta) \leq 0 \quad (\text{A.25})$$

which can hold true only if

$$I \leq \frac{(1 - \theta)}{\alpha f(0)} - \theta x_2^*(0), \quad (\text{A.26})$$

where $x_2^*(0)$ is given in (A.23). Hence, if I is large enough ($I \geq \frac{(1-\theta)}{\alpha\lambda(0)}$), then $x_1^* = 0$ implies $x_2^* = 0$.

(ii) Suppose $x_1^* > 0$ and $x_2^* = 0$. This implies that $X_R = x_1^*$. Substitution into the first order condition and setting $\frac{\partial \Pi(x_1, x_2^*(x_1))}{\partial x_1} = 0$, we obtain

$$\begin{cases} -\alpha\theta(B + \rho g)f(x_1^*) + \alpha f(x_1^*)I - 1 = 0 & \text{if } x_1^* \geq X_E \\ -\alpha\theta Bf(x_1^*) + \alpha f(x_1^*)I - 1 = 0 & \text{if } x_1^* < X_E \end{cases} \quad (\text{A.27})$$

which implies

$$I = \begin{cases} \frac{1 + \alpha\theta(B + \rho g)f(x_1^*)}{\alpha f(x_1^*)} & \text{if } x_1^* \geq X_E \\ \frac{1 + \alpha\theta Bf(x_1^*)}{\alpha f(x_1^*)} & \text{if } x_1^* < X_E \end{cases} \quad (\text{A.28})$$

By Lemma 1 and the fact that $x_2^* = 0$, we have

$$\begin{cases} (B + \rho g) \geq \frac{[1 - \alpha F(x_1^*)]}{-\alpha f(x_1^*)} & \text{if } x_1^* \geq X_E \\ B \geq \frac{[1 - \alpha F(x_1^*)]}{-\alpha f(x_1^*)} & \text{if } x_1^* < X_E \end{cases} \quad (\text{A.29})$$

and therefore it must be that

$$I \geq \frac{1 - \theta [1 - \alpha F(x_1^*)]}{\alpha f(x_1^*)}, \quad (\text{A.30})$$

where x_1^* is derived from (A.27). Hence, if I is low enough ($I < \frac{(1-\theta)}{\alpha\lambda(0)}$), then $x_2^* = 0$ implies $x_1^* = 0$. ■

A.6 Proposition 4

Proof. Given fixed search efforts (x_1^*, x_2^*) , payoff is higher the higher is the ρ :

$$\begin{aligned} & \frac{\partial \Pi(x_1^*, x_2^*, \rho)}{\partial \rho} \\ = & \alpha(1 - F(x_1^*))\theta \left(\frac{1 - F(X_R)}{1 - F(x_1^*)} \frac{\partial B(X_R, X_E, \rho)}{\partial \rho} \right) > 0 \end{aligned}$$

because

$$\frac{\partial B(X_R, X_E, \rho)}{\partial \rho} = \begin{cases} F(X_E)g & \text{if } X_R \geq X_E \\ (1 - F(X_E)) \frac{F(X_R)}{1 - F(X_R)}g & \text{if } X_R < X_E \end{cases} \quad (\text{A.31})$$

and hence

$$\frac{\partial B(X_R, X_E, \rho)}{\partial \rho} \geq 0 \text{ if } g \geq 0. \quad (\text{A.32})$$

Let $x_i^*(\rho)$ denote the optimal search efforts given ρ . Then for two correlation parameters $\rho_h > \rho_l$, we have

$$\Pi(x_1^*(\rho_h), x_2^*(\rho_h), \rho_h) \geq \Pi(x_1^*(\rho_l), x_2^*(\rho_l), \rho_h) \geq \Pi(x_1^*(\rho_l), x_2^*(\rho_l), \rho_l). \quad (\text{A.33})$$

■

A.7 Proposition 5 and 6

Proof. In the assumed range of parameters, $x_2^*(x_1) = 0$ and thus

$$\Pi(x_1, 0) = (1 - \alpha) [\theta(G - P) - I] + \alpha [1 - F(x_1)] [\theta B(x_1, X_E) - I] - x_1, \quad (\text{A.34})$$

where,

$$B(x_1, X_E) = \begin{cases} B + \rho g & \text{if } x_1 \geq X_E \\ B + \left(\frac{1 - F(X_E)}{1 - F(x_1)} \right) \rho g & \text{if } x_1 < X_E \end{cases}. \quad (\text{A.35})$$

In an interior solution with $0 < x_1 < X_E$, the following first order condition must hold:

$$\Pi'(x_1, 0) = -\alpha f(x_1) (\theta B - I) - 1. \quad (\text{A.36})$$

In an interior solution with $x_1 > X_E$, the following first order condition must hold:

$$\Pi'(x_1, 0) = -\alpha f(x_1) [\theta(B + \rho g) - I] - 1. \quad (\text{A.37})$$

Implicitly differentiating $\Pi'(x_1^*, 0)$ with respect to any parameter η and using the second order condition, we find that

$$\text{sign}\left(\frac{dx_1^*}{d\eta}\right) = \text{sign}\left(\frac{\partial\Pi'(x_1^*, 0)}{\partial\eta}\right). \quad (\text{A.38})$$

We now differentiate with respect to each of the parameters in the range $0 < x_1^* < X_E$:

$$\begin{aligned} \frac{\partial\Pi'(x_1^*, 0)}{\partial I} &= \alpha f(x_1^*) > 0. \\ \frac{\partial\Pi'(x_1^*, 0)}{\partial\alpha} &= f(x_1^*)(I - \theta B) = \frac{1}{\alpha} > 0. \\ \frac{\partial\Pi'(x_1^*, 0)}{\partial P} &= \alpha\theta f(x_1^*) > 0. \\ \frac{\partial\Pi'(x_1^*, 0)}{\partial X_E} &= \alpha f(x_1^*)\theta(1 - \rho)f(X_E)g > 0. \\ \frac{\partial\Pi'(x_1^*, 0)}{\partial g} &= -\alpha\theta f(x_1^*)(1 - \rho)(1 - F(X_E)) < 0. \end{aligned}$$

Similar derivatives confirm these results when $x_1^* > X_E$.

The effect of ρ depends on the optimal level of search:

$$\frac{\partial\Pi'(x_1^*, 0)}{\partial\rho} = \begin{cases} -\alpha\theta f(x_1^*)F(X_E)g < 0 & \text{if } x_1^* > X_E \\ \alpha\theta f(x_1^*)(1 - F(X_E))g > 0 & \text{if } x_1^* < X_E \end{cases}. \quad (\text{A.39})$$

■

A.8 Lemma 5 and its Proof

Lemma 5 *When $B > 0$, then*

$$\frac{dp(X_R(\rho), X_E, \rho)}{d\rho} < 0. \quad (\text{A.40})$$

Proof. We have shown that

$$p(X_R, X_E) = \begin{cases} (1 - \rho) F(X_E) & \text{if } X_R \geq X_E \\ \frac{\rho(F(X_E) - F(X_R)) + (1 - \rho)(1 - F(X_R))F(X_E)}{(1 - F(X_R))} & \text{if } X_R < X_E \end{cases}. \quad (\text{A.41})$$

Differentiating in each range, we find that

$$\frac{dp(X_R(\rho), X_E, \rho)}{d\rho} = \begin{cases} -F(X_E) & \text{if } X_R \geq X_E \\ -\frac{(1 - F(X_E))}{(1 - F(X_R))} \left[F(X_R) + \rho \frac{f(X_R)}{(1 - F(X_R))} \frac{dX_R}{d\rho} \right] & \text{if } X_R < X_E \end{cases}. \quad (\text{A.42})$$

When $B > 0$, then $x_2^* = 0$. In Proposition 5, we established that in this case, when $x_1^* < X_E$, search increases with correlation, $\frac{dx_1}{d\rho} > 0$. Under the conditions of this lemma, $X_R = x_1^*$, hence $\frac{dX_R}{d\rho} > 0$ and therefore $\frac{dp(X_R(\rho), X_E, \rho)}{d\rho} < 0$. ■

A.9 Proposition 7

Proof. Assume that $\rho = 0$, i.e., innovator's search process is independent of examiner's search process. Assume an exponential search technology which is more efficient after the innovation, i.e., the search technology for ex ante search is given by $F_s(x_1) = 1 - e^{-\lambda_s x_1}$ and for novelty search is given by $F_n(\tilde{x}_1 + x_2) = 1 - e^{-\lambda_n(\tilde{x}_1 + x_2)}$, where $\lambda_s < \lambda_n$ and $\tilde{x}_1 = F_n^{-1}[F_s(x_1)] = \frac{\lambda_s}{\lambda_n} x_1$.

We first derive the optimal search efforts. We show that, in this set-up, for parameter values satisfying the investment condition (3.14), the payoff maximizing search intensities by the researcher are given by:

1. if $I > \left(\frac{\lambda_n - \lambda_s \theta}{\alpha \lambda_s \lambda_n} \right)$, then

$$\begin{cases} x_1^* = \frac{1}{\lambda_s} \ln \left[\frac{\alpha \lambda_s (\theta + \lambda_n I + \lambda_n \theta \tilde{x}_2)}{\lambda_n - \theta (1 - \alpha) \lambda_s} \right], x_2^* = \tilde{x}_2, & \text{if } -\frac{1 + (1 - \alpha) \lambda_s I}{\lambda_n - \theta (1 - \alpha) \lambda_s} > B, \\ x_1^* = \frac{1}{\lambda_s} \ln [\alpha \lambda_s (I - \theta B)], x_2^* = 0, & \text{if } \frac{\alpha \lambda_s I - 1}{\alpha \lambda_s \theta} > B \geq -\frac{1 + (1 - \alpha) \lambda_s I}{\lambda_n - \theta (1 - \alpha) \lambda_s}, \\ x_1^* = 0, x_2^* = 0, & \text{if } B \geq \frac{\alpha \lambda_s I - 1}{\alpha \lambda_s \theta}; \end{cases} \quad (\text{A.43})$$

2. if $I \leq \left(\frac{\lambda_n - \lambda_s \theta}{\alpha \lambda_s \lambda_n}\right)$, then

$$\left\{ \begin{array}{ll} x_1^* = \frac{1}{\lambda_s} \ln \left[\frac{\alpha \lambda_s (\theta + \lambda_n I + \lambda_n \theta \bar{x}_2)}{\lambda_n - \theta(1-\alpha)\lambda_s} \right], x_2^* = \bar{x}_2, & \text{if } -\frac{1}{\alpha \lambda_n} e^{\frac{(\lambda_n - \lambda_s \theta) - \alpha \lambda_s \lambda_n I}{\alpha \lambda_s \theta}} > B, \\ x_1^* = 0, x_2^* = \frac{1}{\lambda_n} \ln [-\alpha \lambda_n B], & \text{if } -\frac{1}{\alpha \lambda_n} > B \geq -\frac{1}{\alpha \lambda_n} e^{\frac{(\lambda_n - \lambda_s \theta) - \alpha \lambda_s \lambda_n I}{\alpha \lambda_s \theta}}, \\ x_1^* = 0, x_2^* = 0, & \text{if } B \geq -\frac{1}{\alpha \lambda_n}, \end{array} \right. \quad (\text{A.44})$$

where $B = [(1 - F_E(X_E))g - P]$ and \bar{x}_2 is the unique solution to

$$x_2 = \frac{1}{\lambda_n} \ln \left[\frac{-B \{\lambda_n - \theta(1-\alpha)\lambda_s\}}{1 + (1-\alpha)\lambda_s(I + \theta x_2)} \right]. \quad (\text{A.45})$$

To derive these search efforts, we use the optimal novelty search x_2 as we derived in Lemma 1, and consider the optimal choice of x_1 given $x_2^*(x_1)$ that maximize the researcher's payoff as given in (3.15). From Lemma 1, we know that

$$\begin{aligned} x_2^*(x_1) &= \begin{cases} f_n^{-1} \left(\frac{[1 - \alpha F_n(\bar{x}_1)]}{-\alpha B} \right) - \bar{x}_1, & \text{if } B < \frac{[1 - \alpha F_n(\bar{x}_1)]}{-\alpha f_n(\bar{x}_1)}, \\ 0, & \text{if } B \geq \frac{[1 - \alpha F_n(\bar{x}_1)]}{-\alpha f_n(\bar{x}_1)}. \end{cases} \\ &= \begin{cases} \frac{1}{\lambda_n} \ln \left[\frac{-\alpha \lambda_n B}{[(1-\alpha)e^{\lambda_s x_1} + \alpha]} \right], & \text{if } B < \frac{(1-\alpha)e^{\lambda_s x_1} + \alpha}{-\alpha \lambda_n}, \\ 0, & \text{if } B \geq \frac{(1-\alpha)e^{\lambda_s x_1} + \alpha}{-\alpha \lambda_n}. \end{cases} \end{aligned} \quad (\text{A.46})$$

Maximizing the expected payoff from early state of the art search given by equation (3.15), we get the first order condition as

$$-\alpha \theta B f_n(\bar{x}_1 + x_2^*) \left(\frac{\lambda_s}{\lambda_n} + \frac{dx_2^*}{dx_1} \right) - \theta [1 - \alpha F_s(x_1)] \frac{dx_2^*}{dx_1} + \alpha f_s(x_1) (I + \theta x_2^*) - 1 = 0. \quad (\text{A.47})$$

Therefore, when x_2^* is interior, then substituting (A.46) into (A.47), we get that

$$\begin{aligned} -\theta [1 - \alpha F_s(x_1)] \frac{dx_2^*}{dx_1} + \theta [1 - \alpha F_s(x_1)] \left(\frac{\lambda_s}{\lambda_n} + \frac{dx_2^*}{dx_1} \right) + \alpha f_s(x_1) (I + \theta x_2^*) &= 1 \\ \text{or, } \theta \frac{\lambda_s}{\lambda_n} [1 - \alpha F_s(x_1)] + \alpha f_s(x_1) (I + \theta x_2^*) &= 1 \\ \text{or, } \theta (1 - \alpha) \frac{\lambda_s}{\lambda_n} + \alpha \lambda_s e^{-\lambda_s x_1} \left(I + \theta x_2^* + \frac{\theta}{\lambda_n} \right) &= 1 \\ \text{or, } \frac{\alpha \lambda_s \left(I + \theta \bar{x}_2 + \frac{\theta}{\lambda_n} \right)}{1 - \theta (1 - \alpha) \frac{\lambda_s}{\lambda_n}} &= e^{\lambda_s x_1} \end{aligned}$$

where (using (A.46)) \bar{x}_2 is the unique solution to

$$x_2 = \frac{1}{\lambda_n} \ln \left[\frac{-\lambda_n B \left\{ 1 - \theta (1 - \alpha) \frac{\lambda_s}{\lambda_n} \right\}}{1 + (1 - \alpha) \lambda_s (I + \theta x_2)} \right]. \quad (\text{A.48})$$

Similarly, when $x_2^* = 0$, then substituting (A.46) into (A.47), we get the first order condition as

$$\begin{aligned} -\alpha \theta B f_n(\bar{x}_1) \frac{\lambda_s}{\lambda_n} + \alpha f_s(x_1) I - 1 &= 0 \\ \text{or, } \alpha \lambda_s e^{-\lambda_s x_1} (I - \theta B) &= 1. \end{aligned}$$

Therefore,

$$x_1^* = \frac{1}{\lambda_s} \ln [\alpha \lambda_s (I - \theta B)] > 0 \text{ and } x_2^* = 0 \text{ if } \alpha \lambda_s (I - \theta B) > 1. \quad (\text{A.49})$$

Combining all the above, we obtain the optimal search efforts as stated earlier.

Now, from the optimal solutions listed above (or the first order conditions), it is easy to find the comparative static results (all results are weak, e.g. rises could mean rise or remain unchanged):

1. as I increases, x_1^* rises, x_2^* falls, but $(x_1^* + x_2^*)$ rises;
2. as B increases, i.e., as g rises or P falls or X_E falls, we have lower x_1^* and x_2^* ;
3. as α increases, both x_1^* and x_2^* rise. ■

A.10 Proposition 8

Proof. Using the social parameter values \hat{g}, \hat{P} and $X_E = 0$, we see that $B = (g - P) > (\hat{g} - \hat{P}) = \hat{B}$. In Proposition 7, we have seen that as B increases, the

search efforts by the researcher, both before investment and after investment but before filing for a patent, decrease. Since $\hat{B} < B$, we can conclude that the researcher under-invests in prior art search than the socially optimal level. ■

A.11 Proposition 9

Proof. The innovator never has an incentive not to disclose results of novelty search, or else she would have been better off not to have searched. Suppose the innovator has searched for prior art before innovation and revealed invalidating prior art. Pursuing a bad path and applying for a patent (not disclosing the invalidating prior art references) yields payoff $(B - I_m)$. If $B > I_m$, and if when both paths were found to be bad, pursuing a bad path and not disclosing is better than not pursuing any path. If $B < I_m$, pursuing a bad path is inferior to not investing, hence if invalidating prior art is revealed, the innovator does not pursue that path. Therefore, no non-disclosure issue arises. ■

A.12 Proposition 10

Proof. Suppose $0 < B < I_m$, then $x_{2i}^* = 0$ for $i \in \{1, 2\}$. Therefore the pay-off from the ex ante search $\Pi(x_1, x_{21}, x_{22})$ is given by

$$\begin{aligned} \Pi(x_1, 0, 0) &= (1 - \alpha_1) [\theta(G - P) - I] + \alpha_1 [1 - F_s(x_1)] (\theta B - I) \\ &\quad + \alpha_1 F_s(x_1) [(1 - \alpha_2) [\theta(G - P) - I] + \alpha_2 [1 - F_s(x_1)] (\theta B - I)] - x_1 \end{aligned}$$

Differentiating the payoff w.r.t. x_1 , we get

$$\Pi'(x_1, 0, 0) = \alpha_1 (1 - \alpha_2) \theta f_s(x_1) [G - (1 - F_E(X_E)) g]$$

$$+2\alpha_1\alpha_2f_s(x_1)F_s(x_1)(I-\theta B)-1.$$

Assume that we are in a range with interior solution $x_1^* > 0$. Then we have

$$\text{sign}\left(\frac{dx_1}{d\eta}\right) = \text{sign}\left(\frac{\partial\Pi'(x_1, 0, 0)}{\partial\eta}\right) \quad (\text{A.50})$$

Differentiating w.r.t. each of the parameters, we get the following:

$$\frac{\partial\Pi'(x_1^*, 0, 0)}{\partial I} = 2\alpha_1\alpha_2f_s(x_1^*)F_s(x_1^*) > 0.$$

$$\begin{aligned} \frac{\partial\Pi'(x_1^*, 0, 0)}{\partial\alpha_1} &= (1-\alpha_2)\theta f_s(x_1^*)[G-(1-F_E(X_E))g] + 2\alpha_2f_s(x_1^*)F_s(x_1^*)(I-\theta B) \\ &= \frac{1}{\alpha_1} > 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial\Pi'(x_1^*, 0, 0)}{\partial\alpha_2} &= -\alpha_1\theta f_s(x_1^*)[G-(1-F_E(X_E))g] + 2\alpha_1f_s(x_1^*)F_s(x_1^*)(I-\theta B) \\ &= \frac{1}{\alpha_2}[1-\alpha_1\theta f_s(x_1^*)[G-(1-F_E(X_E))g]] \geq 0. \end{aligned}$$

$$\frac{\partial\Pi'(x_1^*, 0, 0)}{\partial G} = \alpha_1(1-\alpha_2)\theta f_s(x_1^*) > 0.$$

$$\begin{aligned} \frac{\partial\Pi'(x_1^*, 0, 0)}{\partial g} &= -\alpha_1(1-\alpha_2)\theta f_s(x_1^*)(1-F_E(X_E)) - 2\alpha_1\alpha_2\theta f_s(x_1^*)F_s(x_1^*)(1-F_E(X_E)) \\ &< 0. \end{aligned}$$

$$\frac{\partial\Pi'(x_1^*, 0, 0)}{\partial P} = 2\alpha_1\alpha_2\theta f_s(x_1^*)F_s(x_1^*) > 0.$$

$$\frac{\partial\Pi'(x_1^*, 0, 0)}{\partial X_E} = \alpha_1(1-\alpha_2)\theta f_s(x_1^*)f_E(X_E)g + 2\alpha_1\alpha_2\theta f_s(x_1^*)F_s(x_1^*)f_E(X_E)g > 0.$$

■

A.13 Proposition 11

Proof. If $P > G(1)$, then the innovators' payoffs from applying for a patent, as given by (3.22), is always negative, hence innovators do not apply for a patent.

If, on the other hand, $P < (1 - p)B(q_1(0))$, then any innovator's payoff from patent application is positive and hence everyone applies.

Consider the range of interior equilibria $(1 - p)B(q_1(0)) \leq P \leq G(1)$. Let us define

$$\begin{aligned} h_1(p, P, q_1) &= \frac{P - (1 - p)B(q_1)}{G(q_1) - (1 - p)B(q_1)}, \\ h_2(p, P, \theta_1) &= \frac{\int_{\theta_1}^1 \theta f(\theta) d\theta}{\int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta}. \end{aligned} \quad (\text{A.51})$$

We denote $\theta_1(p, P)$ and $q_1(p, P)$ the equilibrium outcomes that solve

$$\begin{aligned} \theta_1 &= h_1(p, P, q_1), \\ q_1 &= h_2(p, P, \theta_1). \end{aligned} \quad (\text{A.52})$$

Consider now the function $h_1(p, P, q_1)$.

$$\begin{aligned} \frac{\partial h_1}{\partial q_1} &= -\frac{(1 - p)B'(q_1) + \theta_1 [G'(q_1) - (1 - p)B'(q_1)]}{[G(q_1) - (1 - p)B(q_1)]} \\ &= -\frac{(1 - \theta_1)(1 - p)B'(q_1) + \theta_1 G'(q_1)}{[G(q_1) - (1 - p)B(q_1)]} \leq 0. \end{aligned}$$

Therefore, $h_1(p, P, q_1)$ changes continuously (weakly decreases) from $h_1(p, P, q_1(0)) > 0$ to $h_1(p, P, 1) < 1$ (these inequalities follow from the assumption that $(1 - p)B(q_1(0)) \leq P \leq G(1)$).

Taking the derivative of h_2 with respect to θ_1 , we get

$$\begin{aligned} \frac{\partial h_2}{\partial \theta_1} &= \frac{-\theta_1 f(\theta_1) \int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta + [1 - p(1 - \theta_1)] f(\theta_1) \int_{\theta_1}^1 \theta f(\theta) d\theta}{\left(\int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta \right)^2} \\ &= f(\theta_1) \frac{\int_{\theta_1}^1 [\theta(1 - p(1 - \theta_1)) - \theta_1(1 - p(1 - \theta))] f(\theta) d\theta}{\left(\int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta \right)^2} \\ &= (1 - p) f(\theta_1) \frac{\int_{\theta_1}^1 (\theta - \theta_1) f(\theta) d\theta}{\left(\int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta \right)^2} > 0. \end{aligned}$$

Hence, the function h_2 is strictly increasing in θ_1 . Moreover, $h_2(p, P, 0) = q_1(0) \in (0, 1)$ and $h_2(p, P, 1) \rightarrow 1$.

An equilibrium is an intersection between the increasing function $h_2(p, P, \theta_1)$ and the weakly decreasing function $h_1(p, P, q_1)$ in the (θ_1, q_1) -space. Thus the graph of $h_1(p, P, q_1)$ must intersect with that of $h_2(p, P, \theta_1)$ and only once. Hence we can conclude that there always exists an interior equilibrium for this range of parameter values and the equilibrium is unique. ■

A.14 Proposition 12

Proof. We derive comparative statics around an interior equilibrium. In such equilibrium, $G(q_1) > P$. We denote by $\theta_1(p, P)$ and $q_1(p, P)$ the equilibrium outcomes that solve (A.52).

For any parameter $\eta \in \{p, P\}$,

$$\begin{aligned} \frac{d\theta_1}{d\eta} &= \frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_1} \frac{dq_1}{d\eta} \\ &= \frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_1} \left(\frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial \theta_1} \frac{d\theta_1}{d\eta} \right), \end{aligned}$$

where h_i are defined in (A.51). Rearranging, we find

$$\frac{d\theta_1}{d\eta} = \frac{\frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial \eta}}{1 - \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial \theta_1}}. \quad (\text{A.53})$$

Similarly,

$$\frac{dq_1}{d\eta} = \frac{\frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial \theta_1} \frac{\partial h_1}{\partial \eta}}{1 - \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial \theta_1}}. \quad (\text{A.54})$$

We first determine the sign of the denominator. We have shown in the proof

of Proposition 11 that in an interior equilibrium,

$$\frac{\partial h_2}{\partial \theta_1} > 0 \text{ and } \frac{\partial h_1}{\partial q_1} \leq 0. \quad (\text{A.55})$$

These results imply that

$$1 - \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial \theta_1} > 0. \quad (\text{A.56})$$

Hence,

$$\begin{aligned} \text{sign}\left(\frac{d\theta_1}{d\eta}\right) &= \text{sign}\left(\frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial \eta}\right), \\ \text{sign}\left(\frac{dq_1}{d\eta}\right) &= \text{sign}\left(\frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial \theta_1} \frac{\partial h_1}{\partial \eta}\right). \end{aligned}$$

(i) Consider the effect of patenting fee P . Differentiating h_1 and h_2 with respect to P , we find that

$$\begin{aligned} \frac{\partial h_1}{\partial P} &= \frac{1}{[G(q_1) - (1-p)B(q_1)]} > 0, \\ \frac{\partial h_2}{\partial P} &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} \text{sign}\left(\frac{d\theta_1}{dP}\right) &= \text{sign}\left(\frac{\partial h_1}{\partial P}\right) > 0, \\ \text{sign}\left(\frac{dq_1}{dP}\right) &= \text{sign}\left(\frac{\partial h_2}{\partial \theta_1} \frac{\partial h_1}{\partial P}\right) > 0. \end{aligned}$$

(ii) Next we consider the effect of examination intensity p .

$$\begin{aligned} \frac{\partial h_1}{\partial p} &= \frac{B(q_1) - \theta B(q_1)}{[G(q_1) - (1-p)B(q_1)]} \\ &= \frac{(1-\theta)B(q_1)}{[G(q_1) - (1-p)B(q_1)]} > 0. \end{aligned}$$

$$\begin{aligned} \frac{\partial h_2}{\partial p} &= \frac{-\int_{\theta_1}^1 \theta f(\theta) d\theta}{\left(\int_{\theta_1}^1 [1-p(1-\theta)] f(\theta) d\theta\right)^2} \left(-\int_{\theta_1}^1 (1-\theta) f(\theta) d\theta\right) \\ &= \frac{q_1 \int_{\theta_1}^1 (1-\theta) f(\theta) d\theta}{\int_{\theta_1}^1 [1-p(1-\theta)] f(\theta) d\theta} > 0. \end{aligned}$$

Therefore,

$$\text{sign}\left(\frac{d\theta_1}{dp}\right) = \text{sign}\left(\frac{\partial h_1}{\partial p_+} + \frac{\partial h_1}{\partial q_1} \frac{\partial h_2}{\partial p_+}\right) \quad (\text{A.57})$$

This effect is ambiguous. If benefit functions were such that $G = 1$ and $B = q$, then the increase in p would result in an increase in θ . However, if $G = 1$ and $B = q^n$, then for a large enough n we obtain a decline in θ .

The effect of increased examination intensity on patent quality, however, must be positive.

$$\text{sign}\left(\frac{dq_1}{dp}\right) = \text{sign}\left(\frac{\partial h_2}{\partial p} + \frac{\partial h_2}{\partial \theta_1} \frac{\partial h_1}{\partial p}\right) > 0. \quad (\text{A.58})$$

■

A.15 Proposition 13

Proof. (i) Since the exam intensity is chosen optimally, for the given p , we have

$$\frac{dW}{dP} = 0 \quad (\text{A.59})$$

or,

$$\left[\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1) - c(p)\right] f(\theta_1) \frac{d\theta_1}{dP} = \frac{\partial W}{\partial q_1} \frac{dq_1}{dP}, \quad (\text{A.60})$$

hence, by assumption that $\frac{\partial W}{\partial q_1} > 0$, we have

$$\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1) - c(p) > 0. \quad (\text{A.61})$$

(ii) We know that

$$\begin{aligned} \frac{dW}{dP} &= -\left[\theta_1 \tilde{G}(q_1) + (1 - \theta_1)(1 - p)\tilde{B}(q_1) - c(p)\right] f(\theta_1) \frac{d\theta_1}{dP} \\ &\quad + \frac{dq_1}{dP} \int_{\theta_1}^1 \left[\theta \tilde{G}'(q_1) + (1 - \theta)(1 - p)\tilde{B}'(q_1)\right] f(\theta) d\theta \end{aligned}$$

$$\begin{aligned}
&> -[\theta_1 G(q_1) + (1 - \theta_1)(1 - p)B(q_1) - c(p)] f(\theta_1) \frac{d\theta_1}{dP} \\
&= [c(p) - P] f(\theta_1) \frac{d\theta_1}{dP}.
\end{aligned}$$

The inequality follows from our assumption in the proposition and the equality sign follows from the definition of θ_1 where we know that $\theta_1 G(q_1) + (1 - \theta_1)(1 - p)B(q_1) = P$. Therefore, if $c(p) \geq P$, then $\frac{dW}{dP} > 0$, that is, welfare increases with P . Hence, the optimal policy must be such that $P > c(p)$. ■

A.16 Proposition 14

Proof. We first show that an equilibrium exists. Our proof of existence will rely on a fixed point argument. An interior threshold equilibrium would be characterized by the cut-off θ_* below which no patent is applied for, θ^* above which gold-plate patents are applied for and patent qualities in each patent tier: q_r, q_{gp} . We will define a function $H : [0, 1]^4 \rightarrow [0, 1]^4$ so that its fixed point $(\theta_*, \theta^*, q_r, q_{gp})$ is a threshold equilibrium. If θ_* were interior, then it would be derived from $V_r(\theta_*, \theta^*, q_r, q_{gp}) = 0$, however we cannot guarantee an interior solution and hence the cut-off below which no applicant applies for a patent may be 0 or 1. The following definitions will help define the function H taking into account possible corner solutions.

Let

$$\begin{aligned}
T_r(\theta_*, \theta^*, q_r, q_{gp}) &= \min \left\{ \max \left\{ 0, \frac{P_r - (1 - p_r)B(q_r)}{G(q_r) - (1 - p_r)B(q_r)} \right\}, 1 \right\}, \\
T_{gp}(\theta_*, \theta^*, q_r, q_{gp}) &= \min \left\{ \max \left\{ 0, \frac{P_{gp} - (1 - p_{gp})B(q_{gp})}{G(q_{gp}) - (1 - p_{gp})B(q_{gp})} \right\}, 1 \right\}, \\
T_{r,gp}(\theta_*, \theta^*, q_r, q_{gp}) &
\end{aligned}$$

$$= \min \left\{ \max \left\{ 0, \frac{(P_{gp} - P_r) - [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)]}{[G(q_{gp}) - G(q_r)] - [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)]} \right\}, 1 \right\}.$$

The intuition behind these definitions is as follows: when $T_r \in [0, 1]$ is interior, it represents the type who is indifferent between applying for a regular patent and not applying for any patent; when $T_{gp} \in [0, 1]$ is interior, it represents the type who is indifferent between applying for a gold-plate patent and not applying for any patent; when $T_{r,gp} \in [0, 1]$ is interior, it represents the type who is indifferent between applying for a gold-plate patent and applying for a regular patent. By our assumptions and the fact that the min and the max of two continuous functions are continuous functions, we know that T_r, T_{gp} and $T_{r,gp}$ define three continuous functions.

Let us now define a function $H : [0, 1]^4 \rightarrow [0, 1]^4$ as follows:

$$\begin{aligned} h_1(\theta_*, \theta^*, q_r, q_{gp}) &= \min \{T_r(\theta_*, \theta^*, q_r, q_{gp}), T_{gp}(\theta_*, \theta^*, q_r, q_{gp})\} \\ h_2(\theta_*, \theta^*, q_r, q_{gp}) &= \max \{T_{gp}(\theta_*, \theta^*, q_r, q_{gp}), T_{r,gp}(\theta_*, \theta^*, q_r, q_{gp})\} \\ h_3(\theta_*, \theta^*, q_r, q_{gp}) &= \frac{\int_{\theta_*}^{\theta^*} \theta f(\theta) d\theta}{\int_{\theta_*}^{\theta^*} [1 - p_r(1 - \theta)] f(\theta) d\theta}, \\ h_4(\theta_*, \theta^*, q_r, q_{gp}) &= \frac{\int_{\theta^*}^1 \theta f(\theta) d\theta}{\int_{\theta^*}^1 [1 - p_{gp}(1 - \theta)] f(\theta) d\theta}. \end{aligned} \tag{A.62}$$

The function $H(\theta_*, \theta^*, q_r, q_{gp}) = (h_1, h_2, h_3, h_4)$ is continuous on $[0, 1]^4$ which is a compact subset of the Euclidean space R^4 . Therefore it has a fixed point $(\theta_*, \theta^*, q_r, q_{gp})$. Now define $\Theta_r = [\theta_*, \theta^*]$ as the set of innovators that apply for a regular patent, and the set $\Theta_{gp} = (\theta^*, 1]$ as the set of innovators that apply for a gold-plate patent, and if $\theta_* > 0$, then the set of innovators $[0, \theta_*)$ do not apply for any patent. Because $(\theta_*, \theta^*, q_r, q_{gp})$ solves the system (A.62), it is easy to verify that the equilibrium conditions are met.

We now show that any interior equilibrium is a thresholds equilibrium. Consider an interior equilibrium so that some innovators apply for each type of patents and some do not apply for any patent. If it were the case that, in equilibrium, $q_{gp} \leq q_r$, then for all θ , the regular patent would be preferred and no one would apply for a gold-plate patent. Hence, in the interior equilibrium, $q_{gp} > q_r$.

Let the difference between the value of a gold-plate patent and a regular patent be

$$\Delta V(\theta) = V_{gp}(\theta, q_{gp}) - V_r(\theta, q_r), \quad (\text{A.63})$$

or, after substituting these values,

$$\Delta V(\theta) = \theta [G(q_{gp}) - G(q_r)] + (1 - \theta) [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)] - (P_{gp} - P_r). \quad (\text{A.64})$$

Given patent qualities q_r and q_{gp} , ΔV is linear in θ , and we argue that its slope must be positive. The slope is given by

$$[G(q_{gp}) - G(q_r)] - [(1 - p_{gp})B(q_{gp}) - (1 - p_r)B(q_r)]. \quad (\text{A.65})$$

Evaluating this at $q_{gp} = q_r$, we have $(p_{gp} - p_r)B(q_r) > 0$. Moreover, for a given q_r , this slope increases with q_{gp} because of our assumption (3.33). Therefore, it is positive for all $q_{gp} \geq q_r$.

We now argue that every interior equilibrium has the form of a threshold equilibrium. Because the equilibrium is interior, there is some value θ' for which a gold-plate patent is applied for. Then, because $\Delta V(\theta)$ and $V_{gp}(\theta)$ are increasing, a gold-plate patent is applied for all $\theta > \theta'$. Let θ^* denote the infimum of all θ such that gold-plate patent is preferred. We know $\theta^* > 0$ or else no regular patents are applied for. Because in an interior equilibrium not all innovators apply for a patent, there is a type θ'' that does not apply. Because $V_r(\theta)$ and

$V_{gp}(\theta)$ increase in θ , any lower type does not apply for a patent. Let θ_* denote the smallest θ for which $V_r(\theta_*, q_r) = 0$. This is a threshold level for applying for a regular patent, that is, $V_r(\theta, q) \geq 0$ for $\theta \geq \theta_*$ and $V_r(\theta, q) < 0$ for $\theta < \theta_*$. Finally, because at least some regular patents are applied for, $\theta_* < \theta^*$, otherwise, gold-plate patents are preferred wherever regular patents yield positive payoffs, and no regular patents would be applied for. ■

A.17 Proposition 15

Proof. (i) Suppose, by contradiction, that $\theta_* \leq \theta_1$. Then, in the two-tiered system, the quality of regular patents q_r would be lower than the quality of a patent in the single-tiered system for two reasons: (a) more low ex-ante probability of validity innovators apply ($\theta_* \leq \theta_1$ and $\frac{dq_r}{d\theta_*} > 0$) and (b) some high ex-ante probability of validity innovators apply for a gold-plate patent instead ($\theta^* < 1$ and $\frac{dq_r}{d\theta^*} > 0$). To establish these effects formally, we show that

$$\frac{dq_r}{d\theta_*} = (1-p) f(\theta_*) \frac{\int_{\theta_*}^{\theta^*} (\theta - \theta_*) f(\theta) d\theta}{\left(\int_{\theta_*}^{\theta^*} [1 - p(1-\theta)] f(\theta) d\theta \right)^2} > 0,$$

$$\frac{dq_r}{d\theta^*} = (1-p) f(\theta^*) \frac{\int_{\theta_*}^{\theta^*} (\theta^* - \theta) f(\theta) d\theta}{\left(\int_{\theta_*}^{\theta^*} (1 - p(1-\theta)) f(\theta) d\theta \right)^2} > 0.$$

This would imply that $q_r < q_1$. But then, applying for a regular patent becomes less appealing than applying for a patent in the single-tiered system which would imply $\theta_* > \theta_1$, a contradiction. Hence, in the new system, there must be less applications: $\theta_* > \theta_1$.

(ii) If $q_r > q_1$, then $\theta_* \leq \theta_1$ which, as we just saw, yields a contradiction. Thus, $q_r \leq q_1$.

(iii) As in the proof of Proposition 11, we can write that

$$q_1 = h_2(p, \theta_1) = \frac{\int_{\theta_1}^1 \theta f(\theta) d\theta}{\int_{\theta_1}^1 [1 - p(1 - \theta)] f(\theta) d\theta},$$

$$q_{gp} = h_2(p_{gp}, \theta^*).$$

We have already shown before (while proving Propositions 11 and 12) that $\frac{\partial h_2}{\partial \theta} > 0$ and $\frac{\partial h_2}{\partial p} > 0$. By assumption, $p_{gp} > p_r = p$. Again, by Proposition ?? and part (i) of this proof, $\theta^* > \theta_* > \theta_1$. Therefore, $q_{gp} > q_1$.

(iv) Expected number of bad patents in a single-tiered patent system is

$$N_1 = (1 - p) \int_{\theta_1}^1 (1 - \theta) f(\theta) d\theta \quad (\text{A.66})$$

and expected number of bad patents in a two-tiered patent system is

$$N_2 = (1 - p_r) \int_{\theta_*}^{\theta^*} (1 - \theta) f(\theta) d\theta + (1 - p_{gp}) \int_{\theta^*}^1 (1 - \theta) f(\theta) d\theta. \quad (\text{A.67})$$

Since $\theta^* > \theta_* > \theta_1$ and $p_{gp} > p_r = p$, we have

$$\begin{aligned} & N_1 - N_2 \\ &= (1 - p) \int_{\theta_1}^1 (1 - \theta) f(\theta) d\theta - (1 - p) \int_{\theta_*}^{\theta^*} (1 - \theta) f(\theta) d\theta - (1 - p_{gp}) \int_{\theta^*}^1 (1 - \theta) f(\theta) d\theta \\ &= (1 - p) \int_{\theta^*}^1 (1 - \theta) f(\theta) d\theta + (1 - p) \int_{\theta_1}^{\theta_*} (1 - \theta) f(\theta) d\theta - (1 - p_{gp}) \int_{\theta^*}^1 (1 - \theta) f(\theta) d\theta \\ &= (p_{gp} - p) \int_{\theta^*}^1 (1 - \theta) f(\theta) d\theta + (1 - p) \int_{\theta_1}^{\theta_*} (1 - \theta) f(\theta) d\theta \\ &> 0. \end{aligned}$$

■

A.18 Proposition 16

Proof. Consider a single-tiered system with examination intensity p and a patent fee P that has an interior equilibrium $\{\theta_1, q_1\}$.

The system of equations (3.36) can be written as

$$\begin{aligned}
\theta_* - h_1(q_r; P_r, p_r) &= 0, \\
\theta^* - h_2(q_r, q_{gp}; P_{gp}, p_{gp}, P_r, p_r) &= 0, \\
q_r - h_3(\theta_*, \theta^*; p_r) &= 0, \\
q_{gp} - h_4(\theta^*; p_{gp}) &= 0,
\end{aligned}$$

where the functions h_i represent the right hand side of equation i in the system.

The Jacobian matrix of the system is given by:

$$J = \begin{pmatrix} 1 & 0 & -\frac{\partial h_1}{\partial q_r} & 0 \\ 0 & 1 & -\frac{\partial h_2}{\partial q_r} & -\frac{\partial h_2}{\partial q_{gp}} \\ -\frac{\partial h_3}{\partial \theta_*} & -\frac{\partial h_3}{\partial \theta^*} & 1 & 0 \\ 0 & -\frac{\partial h_4}{\partial \theta^*} & 0 & 1 \end{pmatrix} \quad (\text{A.68})$$

Given our assumptions on the model, it is easy to show that

$$\frac{\partial h_1}{\partial q_r} < 0, \frac{\partial h_2}{\partial q_r} > 0, \frac{\partial h_2}{\partial q_{gp}} < 0, \frac{\partial h_3}{\partial \theta_*} > 0, \frac{\partial h_3}{\partial \theta^*} > 0 \text{ and } \frac{\partial h_4}{\partial \theta^*} \geq 0. \quad (\text{A.69})$$

The determinant of the matrix J is given by

$$\det(J) = 1 + \left(-\frac{\partial h_2}{\partial q_{gp}}\right) \frac{\partial h_4}{\partial \theta^*} + \left(-\frac{\partial h_1}{\partial q_r}\right) \left(-\frac{\partial h_2}{\partial q_{gp}}\right) \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_4}{\partial \theta^*} + \left(-\frac{\partial h_1}{\partial q_r}\right) \frac{\partial h_3}{\partial \theta_*} - \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta^*}. \quad (\text{A.70})$$

The first four terms in this sum are all positive. But the last term is $-\frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta^*} < 0$.

If gold-plate patenting fee is $P_{gp} = P_r + [G(1) - G(q_1)]$, then $(\theta_*, \theta^*, q_r, q_{gp}) = (\theta_1, 1, q_1, 1)$ is the (unique) solution to the system of equilibrium inequalities. In this special case, in equilibrium, the set of innovators that apply for a regular patent is the same as that in the single-tiered system. We assumed that for $(\theta_*, \theta^*, q_r, q_{gp}) = (\theta_1, 1, q_1, 1)$, $\det(J) > 0$. This can be shown to hold when the function G is not too steep at q_1 (i.e. $G'(q_1)$ is sufficiently small). We will now apply

the implicit function theorem to obtain a solution to the system of equilibrium equations in the neighborhood of $(\theta_1, 1, q_1, 1)$.

To apply the implicit function theorem, we need the system of inequalities to be continuously differentiable in a neighborhood of point $(\theta_1, 1, q_1, 1)$. In our application, we defined the functional forms in the range $q_r \leq 1$ and $\theta^* \leq 1$. We can augment the range of the system by defining the functions $B(q)$ and $G(q)$ for $q > 1$ as $B(q) = B'(1)q + [B(1) - B'(1)]$ and $G(q) = G'(1)q + [G(1) - G'(1)]$ and also defining $f(\theta) = f(1)$ for $\theta > 1$. Now the system is continuously differentiable in a neighborhood of $(\theta_1, 1, q_1, 1)$. Since $\det(J) > 0$, from the implicit function theorem, we can represent the solution of the system in the neighborhood of $(\theta_1, 1, q_1, 1)$ as functions of the parameters $(P_{gp}, p_{gp}, P_r, p_r)$. Moreover,

$$\frac{d\theta^*}{dP_{gp}} = \det(K) / \det(J) \quad (\text{A.71})$$

where K is the same as the matrix J only its second column is replaced with the vector of partial derivatives with respect to P_{gp} , $(0, \frac{\partial h_2}{\partial P_{gp}}, 0, 0)^T$. Under our assumption that $\det(J) > 0$, we have $\frac{d\theta^*}{dP_{gp}} > 0$. Hence, if P_{gp} is set close to but below $P_{gp} = P_r + [G(1) - G(q_1)]$, then there is an interior solution to the system of equilibrium equalities ($\theta^* < 1$ which also implies $q_{gp} < 1$).

For expenditures of the patent system not to increase with the introduction of the new policy, it is sufficient that $P_{gp} \geq P_r + [c(p_{gp}) - c(p_r)] = P_r + [c(p_{gp}) - c(p_r)]$. When p_{gp} is sufficiently close to p_r , then $P_r + [G(1) - G(q_1)] > P_r + [c(p_{gp}) - c(p_r)]$ so that there exists P_{gp} for which there is an interior equilibrium in the two-tiered system and the expenditure of the patent office declines. ■

A.19 Proposition 17

Proof. We derive comparative statics around an interior equilibrium. Given assumptions of the linear model and $p_{gp} = 1$, in an interior equilibrium, the system of equations become

$$\begin{aligned}\theta_* &= h_1(q_r, p_r, P_r) = \frac{P_r - (1 - p_r)(B + B'q_r)}{[G - (1 - p_r)B] + q_r[G' - (1 - p_r)B']}, \\ \theta^* &= h_2(q_r, p_r, P_r, P_{gp}) = \frac{(P_{gp} - P_r) + (1 - p_r)(B + B'q_r)}{(1 - q_r)G' + (1 - p_r)(B + B'q_r)}, \\ q_r &= h_3(\theta_*, \theta^*, p_r) = \frac{(\theta^* + \theta_*)}{2(1 - p_r) + p_r(\theta^* + \theta_*)}.\end{aligned}$$

From the second equation, $G' > (P_{gp} - P_r)$. We denote by $\theta_*(P_{gp}, P_r, p_r)$, $\theta^*(P_{gp}, P_r, p_r)$ and $q_r(P_{gp}, P_r, p_r)$ the equilibrium outcomes that solve

$$\begin{aligned}\theta_* &= h_1(q_r, p_r, P_r), \\ \theta^* &= h_2(q_r, p_r, P_r, P_{gp}), \\ q_r &= h_3(\theta_*, \theta^*, p_r).\end{aligned}$$

For any parameter $\eta \in \{P_{gp}, P_r, p_r\}$,

$$\begin{aligned}\frac{d\theta_*}{d\eta} &= \frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_r} \frac{dq_r}{d\eta} \\ &= \frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_r} \left(\frac{\partial h_3}{\partial \eta} + \frac{\partial h_3}{\partial \theta_*} \frac{d\theta_*}{d\eta} + \frac{\partial h_3}{\partial \theta^*} \frac{d\theta^*}{d\eta} \right)\end{aligned}$$

and

$$\begin{aligned}\frac{d\theta^*}{d\eta} &= \frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial q_r} \frac{dq_r}{d\eta} \\ &= \frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial q_r} \left(\frac{\partial h_3}{\partial \eta} + \frac{\partial h_3}{\partial \theta_*} \frac{d\theta_*}{d\eta} + \frac{\partial h_3}{\partial \theta^*} \frac{d\theta^*}{d\eta} \right).\end{aligned}$$

Rearranging, we find

$$\frac{d\theta_*}{d\eta} = \frac{\left(\frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \eta} \right) - \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial \eta} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial \eta} \right)}{\left(1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta_*} - \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta^*} \right)},$$

$$\frac{d\theta^*}{d\eta} = \frac{\left(\frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \eta}\right) + \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial \eta} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial \eta}\right)}{\left(1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta^*} - \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta^*}\right)}.$$

It is easy to find that

$$\begin{aligned}\frac{\partial h_1}{\partial q_r} &= -\frac{(1 - \theta_*)(1 - p_r)B' + \theta_*G'}{(G + q_rG') - (1 - p_r)(B + B'q_r)} < 0, \\ \frac{\partial h_2}{\partial q_r} &= \frac{(1 - \theta^*)(1 - p_r)B' + \theta^*G'}{(1 - q_r)G' + (1 - p_r)(B + B'q_r)} > 0, \\ \frac{\partial h_3}{\partial \theta^*} &= \frac{2(1 - p_r)}{[2(1 - p_r) + p_r(\theta^* + \theta_*)]^2} > 0, \\ \frac{\partial h_3}{\partial \theta^*} &= \frac{2(1 - p_r)}{[2(1 - p_r) + p_r(\theta^* + \theta_*)]^2} = \frac{\partial h_3}{\partial \theta^*} > 0.\end{aligned}$$

Now we can determine the sign of the denominator.

$$\begin{aligned}& 1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta^*} - \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta^*} \\ &= \frac{\partial h_3}{\partial \theta^*} \left(\frac{1}{\frac{\partial h_3}{\partial \theta^*}} - \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \right) \\ &= \frac{\partial h_3}{\partial \theta^*} \left[\frac{[2(1 - p_r) + p_r(\theta^* + \theta_*)]^2}{2(1 - p_r)} - \frac{(1 - \theta^*)(1 - p_r)B' + \theta^*G'}{(1 - q_r)G' + (1 - p_r)(B + B'q_r)} \right] \\ &\quad + \frac{\partial h_3}{\partial \theta^*} \frac{(1 - \theta_*)(1 - p_r)B' + \theta_*G'}{(G + q_rG') - (1 - p_r)(B + B'q_r)} \\ &= \frac{\partial h_3}{\partial \theta^*} (1 - \theta^*)(1 - p_r) \left(2 - \frac{B'}{(1 - q_r)G' + (1 - p_r)(B + B'q_r)} \right) \\ &\quad + \frac{\partial h_3}{\partial \theta^*} \theta^* \left(2 - \frac{G'}{(1 - q_r)G' + (1 - p_r)(B + B'q_r)} \right) \\ &\quad + \frac{\partial h_3}{\partial \theta^*} \left[2p_r\theta_* + \frac{p_r^2(\theta^* + \theta_*)^2}{2(1 - p_r)} + \frac{(1 - \theta_*)(1 - p_r)B' + \theta_*G'}{(G + q_rG') - (1 - p_r)(B + B'q_r)} \right] \\ &> 0 \text{ since } B' > G' \geq \frac{1}{2}B' \text{ and } p_r \leq \frac{1}{2}.\end{aligned}$$

Hence,

$$\begin{aligned}\text{sign}\left(\frac{d\theta^*}{d\eta}\right) &= \text{sign}\left[\left(\frac{\partial h_1}{\partial \eta} + \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \eta}\right) - \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial \eta} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial \eta}\right)\right], \\ \text{sign}\left(\frac{d\theta^*}{d\eta}\right) &= \text{sign}\left[\left(\frac{\partial h_2}{\partial \eta} + \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \eta}\right) + \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial \eta} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial \eta}\right)\right], \\ \text{sign}\left(\frac{dq_r}{d\eta}\right) &= \text{sign}\left(\frac{\partial h_3}{\partial \eta} + \frac{\partial h_3}{\partial \theta^*} \frac{d\theta^*}{d\eta} + \frac{\partial h_3}{\partial \theta^*} \frac{d\theta^*}{d\eta}\right).\end{aligned}$$

From the proof of Proposition 15, the number of bad patents in this two-tiered system is

$$\begin{aligned} N_2 &= (1 - p_r) \int_{\theta_*}^{\theta^*} (1 - \theta) f(\theta) d\theta \\ &= (1 - p_r) \left[\left(\theta^* - \frac{\theta^{*2}}{2} \right) - \left(\theta_* - \frac{\theta_*^2}{2} \right) \right]. \end{aligned}$$

Using these findings, we now derive all the results in Proposition 17. For convenience of proof, we follow a different order of derivations than the statement in the proposition, (implicitly) taking derivatives with respect to each parameter at a time.

(a) Everything else being fixed, differentiating h_1, h_2 and h_3 with respect to P_{gp} , we get

$$\begin{aligned} \frac{\partial h_1}{\partial P_{gp}} &= 0, \\ \frac{\partial h_2}{\partial P_{gp}} &= \frac{1}{(1 - q_r) G' + (1 - p_r) (B + B' q_r)} > 0, \\ \frac{\partial h_3}{\partial P_{gp}} &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} \text{sign} \left(\frac{d\theta_*}{dP_{gp}} \right) &= \text{sign} \left[\frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_2}{\partial P_{gp+}} \right] \\ &< 0, \\ \text{sign} \left(\frac{d\theta^*}{dP_{gp}} \right) &= \text{sign} \left[\frac{\partial h_2}{\partial P_{gp+}} \left(1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta_*} \right)_+ \right] \\ &> 0, \\ \text{sign} \left(\frac{dN_2}{dP_{gp}} \right) &= \text{sign} \left[(1 - \theta^*) \frac{d\theta^*}{dP_{gp+}} - (1 - \theta_*) \frac{d\theta_*}{dP_{gp-}} \right] \\ &> 0, \end{aligned}$$

and

$$\begin{aligned}
\text{sign}\left(\frac{dq_r}{dP_{gp}}\right) &= \text{sign}\left(\frac{d\theta_*}{dP_{gp}} + \frac{d\theta^*}{dP_{gp}}\right) \\
&= \text{sign}\left(\frac{\partial h_2}{\partial P_{gp+}}\right) \\
&> 0.
\end{aligned}$$

(b) As we have done above, everything else being fixed, differentiating h_1, h_2 and h_3 with respect to P_r , we get

$$\begin{aligned}
\frac{\partial h_1}{\partial P_r} &= \frac{1}{(G + q_r G') - (1 - p_r)(B + B' q_r)} > 0, \\
\frac{\partial h_2}{\partial P_r} &= -\frac{1}{(1 - q_r) G' + (1 - p_r)(B + B' q_r)} < 0, \\
\frac{\partial h_3}{\partial P_r} &= 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{sign}\left(\frac{d\theta_*}{dP_r}\right) &= \text{sign}\left[\frac{\partial h_1}{\partial P_{r+}}\left(1 - \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta_*}\right)_+ + \frac{\partial h_1}{\partial q_{r-}} \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_2}{\partial P_{r-}}\right] \\
&> 0,
\end{aligned}$$

$$\begin{aligned}
\text{sign}\left(\frac{d\theta^*}{dP_r}\right) &= \text{sign}\left[\frac{\partial h_2}{\partial P_{r-}}\left(1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta_*}\right)_+ + \frac{\partial h_2}{\partial q_{r+}} \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_1}{\partial P_{r+}}\right] \\
&< 0,
\end{aligned}$$

because

$$\begin{aligned}
&\frac{\partial h_2}{\partial P_r} \left(1 - \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial \theta_*}\right) + \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_1}{\partial P_r} \\
&= \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_2}{\partial P_r} \left[\frac{1}{\frac{\partial h_3}{\partial \theta_*}} - \frac{\partial h_1}{\partial q_r} + \frac{\frac{\partial h_2}{\partial q_r} \partial h_1}{\frac{\partial h_2}{\partial P_r}}\right] \\
&= \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_2}{\partial P_r} \left(\frac{[2(1 - p_r) + p_r(\theta^* + \theta_*)]^2}{2(1 - p_r)} + \frac{(1 - \theta_*)(1 - p_r)B' + \theta_* G'}{(G + q_r G') - (1 - p_r)(B + B' q_r)}\right) \\
&\quad - \frac{\partial h_3}{\partial \theta_*} \frac{\partial h_2}{\partial P_r} \frac{(1 - \theta^*)(1 - p_r)B' + \theta^* G'}{(G + q_r G') - (1 - p_r)(B + B' q_r)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\partial h_3}{\partial \theta_{*+}} \frac{\partial h_2}{\partial P_{r-}} \left(\frac{(\theta^* + \theta_*)}{q_r} - \frac{(\theta^* - \theta_*) [G' - (1 - p_r) B']}{(G + q_r G') - (1 - p_r) (B + B' q_r)} \right) \\
&\quad + \frac{\partial h_3}{\partial \theta_{*+}} \frac{\partial h_2}{\partial P_{r-}} \left(p_r (\theta^* + \theta_*) + \frac{p_r^2 (\theta^* + \theta_*)^2}{2(1 - p_r)} \right) \\
&< 0 \text{ since } \frac{1}{q_r} > \frac{[G' - (1 - p_r) B']}{[G - (1 - p_r) B] + q_r [G' - (1 - p_r) B']} \text{ and } (\theta^* + \theta_*) > (\theta^* - \theta_*).
\end{aligned}$$

Hence,

$$\begin{aligned}
\text{sign} \left(\frac{dN_2}{dP_r} \right) &= \text{sign} \left[(1 - \theta^*) \frac{d\theta^*}{dP_{r-}} - (1 - \theta_*) \frac{d\theta_*}{dP_{r+}} \right] \\
&< 0
\end{aligned}$$

and

$$\begin{aligned}
\text{sign} \left(\frac{dq_r}{dP_r} \right) &= \text{sign} \left(\frac{d\theta_*}{dP_r} + \frac{d\theta^*}{dP_r} \right) \\
&= \text{sign} \left(\frac{\partial h_1}{\partial P_{r+}} + \frac{\partial h_2}{\partial P_{r-}} \right) \\
&> 0 \text{ since } (G + G') = (B + B') \text{ and } 2G' \geq B'.
\end{aligned}$$

(c) Finally, keeping everything else fixed, differentiating h_1, h_2 and h_3 with respect to p_r , we get

$$\begin{aligned}
\frac{\partial h_1}{\partial p_r} &= \frac{(1 - \theta_*) (B + B' q_r)}{(G + q_r G') - (1 - p_r) (B + B' q_r)} > 0, \\
\frac{\partial h_2}{\partial p_r} &= -\frac{(1 - \theta^*) (B + B' q_r)}{(1 - q_r) G' + (1 - p_r) (B + B' q_r)} < 0, \\
\frac{\partial h_3}{\partial p_r} &= \frac{q_r [2 - (\theta^* + \theta_*)]}{2(1 - p_r) + p_r (\theta^* + \theta_*)} > 0.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\text{sign} \left(\frac{d\theta_*}{dp_r} \right) &= \text{sign} \left[\left(\frac{\partial h_1}{\partial p_r} + \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial p_r} \right) - \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial p_r} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial p_r} \right) \right] \\
&> 0
\end{aligned}$$

since

$$\left(\frac{\partial h_1}{\partial p_r} + \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial p_r} \right) - \frac{\partial h_3}{\partial \theta^*} \left(\frac{\partial h_1}{\partial p_r} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial p_r} \right)$$

$$\begin{aligned}
&= \frac{1}{q_r} \frac{\partial h_3}{\partial p_r} \frac{\left((1-p_r)(1-\theta_*)B + \frac{(\theta^*+\theta_*)}{[2-(\theta^*+\theta_*)]} (1-\theta_*)(B+B'q_r) - \theta_*G'q_r \right)}{\left((G+q_rG') - (1-p_r)(B+B'q_r) \right)} \\
&\quad - \frac{\partial h_3}{\partial \theta^*} \left(\frac{(B+B'q_r)}{\left((G+q_rG') - (1-p_r)(B+B'q_r) \right)} \frac{(\theta^*-\theta_*)G'}{\left((1-q_r)G' + (1-p_r)(B+B'q_r) \right)} \right) \\
&= \frac{(1-q_r) \left[\theta_*G'(1-q_r) + \frac{(\theta^*+\theta_*)}{[2-(\theta^*+\theta_*)]} (1-\theta_*)(1-p_r)(B+B'q_r) \right] [B+q_r(B'-G')]}{\left((1-p_r) \left[(G+q_rG') - (1-p_r)(B+B'q_r) \right] \left[(1-q_r)G' + (1-p_r)(B+B'q_r) \right] \right)} \\
&\quad + \frac{(1-\theta_*)(1-q_r)B}{\left[(G+q_rG') - (1-p_r)(B+B'q_r) \right]} \\
&> 0.
\end{aligned}$$

Similarly,

$$\begin{aligned}
\text{sign} \left(\frac{d\theta^*}{dp_r} \right) &= \text{sign} \left[\left(\frac{\partial h_2}{\partial p_{r-}} + \frac{\partial h_2}{\partial q_{r+}} \frac{\partial h_3}{\partial p_{r+}} \right) + \frac{\partial h_3}{\partial \theta_{*+}} \left(\frac{\partial h_1}{\partial p_{r+}} \frac{\partial h_2}{\partial q_{r+}} - \frac{\partial h_1}{\partial q_{r-}} \frac{\partial h_2}{\partial p_{r-}} \right) \right] \\
&= \text{ambiguous}
\end{aligned}$$

since

$$\begin{aligned}
&\left(\frac{\partial h_2}{\partial p_r} + \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial p_r} \right) + \frac{\partial h_3}{\partial \theta_*} \left(\frac{\partial h_1}{\partial p_r} \frac{\partial h_2}{\partial q_r} - \frac{\partial h_1}{\partial q_r} \frac{\partial h_2}{\partial p_r} \right) \\
&= \frac{1}{q_r} \frac{\partial h_3}{\partial p_r} \frac{\left(\theta_*G'q_r - \frac{(\theta^*+\theta_*)}{[2-(\theta^*+\theta_*)]} (1-\theta^*)(B+B'q_r) - (1-p_r)(1-\theta^*)B \right)}{\left((1-q_r)G' + (1-p_r)(B+B'q_r) \right)} \\
&\quad + \frac{\partial h_3}{\partial \theta^*} \left(\frac{(B+B'q_r)}{\left((G+q_rG') - (1-p_r)(B+B'q_r) \right)} \frac{(\theta^*-\theta_*)G'}{\left((1-q_r)G' + (1-p_r)(B+B'q_r) \right)} \right) \\
&= \frac{(1-q_r) \left[\theta_*G'q_r - \frac{(\theta^*+\theta_*)}{[2-(\theta^*+\theta_*)]} (1-\theta^*)(B+B'q_r) \right] (G+G')}{\left((1-p_r) \left[(1-q_r)G' + (1-p_r)(B+B'q_r) \right] \left[(G+q_rG') - (1-p_r)(B+B'q_r) \right] \right)} \\
&\quad + \frac{(1-q_r) \left[\theta^*(1-q_r)G' + \frac{(\theta^*+\theta_*)}{[2-(\theta^*+\theta_*)]} (1-\theta^*)(1-p_r)(B+B'q_r) \right] (B+B'q_r - G'q_r)}{\left((1-p_r) \left[(1-q_r)G' + (1-p_r)(B+B'q_r) \right] \left[(G+q_rG') - (1-p_r)(B+B'q_r) \right] \right)} \\
&\quad - \frac{(1-\theta^*)(1-q_r)B}{\left((1-q_r)G' + (1-p_r)(B+B'q_r) \right)} \\
&= \begin{cases} < 0 & \text{if } (\theta^* - \theta_*) \rightarrow 0 \\ > 0 & \text{if } (\theta^* - \theta_*) \rightarrow 1. \end{cases}
\end{aligned}$$

However,

$$\begin{aligned}
\text{sign} \left(\frac{dq_r}{dp_r} \right) &= \text{sign} \left(\frac{\partial h_3}{\partial p_{r+}} + \frac{\partial h_3}{\partial \theta_{*+}} \frac{d\theta_*}{dp_{r+}} + \frac{\partial h_3}{\partial \theta_{*+}} \frac{d\theta^*}{dp_{r\pm}} \right) \\
&> 0
\end{aligned}$$

Although the last term can be negative, the following derivation establishes that the sum is positive.

$$\begin{aligned}
& \frac{\partial h_3}{\partial p_r} + \frac{\partial h_3}{\partial \theta_*} \frac{d\theta_*}{dp_r} + \frac{\partial h_3}{\partial \theta^*} \frac{d\theta^*}{dp_r} \\
= & \frac{\partial h_3}{\partial p_r} + \frac{\partial h_3}{\partial \theta_*} \left[\left(\frac{\partial h_1}{\partial p_r} + \frac{\partial h_1}{\partial q_r} \frac{\partial h_3}{\partial p_r} \right) + \left(\frac{\partial h_2}{\partial p_r} + \frac{\partial h_2}{\partial q_r} \frac{\partial h_3}{\partial p_r} \right) \right] \\
= & \frac{\partial h_3}{\partial p_r} \frac{\partial h_3}{\partial \theta_*} \left[\frac{[2(1-p_r) + p_r(\theta^* + \theta_*)]}{(1-p_r q_r)} + \frac{(\theta^* G' - \frac{(1-p_r)}{(1-q_r)}(1-\theta^*)B'q_r - \frac{(1-p_r)}{(1-q_r)}(1-\theta^*)\frac{B}{q_r})}{(1-q_r)G' + (1-p_r)(B+B'q_r)} \right] \\
& + \frac{\partial h_3}{\partial p_r} \frac{\partial h_3}{\partial \theta_*} \left[\frac{(\frac{(1-p_r)}{(1-q_r)}(1-\theta_*)\frac{B}{q_r} + \frac{(1-p_r)}{(1-q_r)}(1-\theta_*)B'q_r - \theta_* G')}{(G+q_r G') - (1-p_r)(B+B'q_r)} \right] \\
= & \frac{\partial h_3}{\partial p_r} \frac{\partial h_3}{\partial \theta_*} \left[\left(\frac{2(1-p_r)}{(1-p_r q_r)} - \frac{\frac{(1-p_r)}{(1-q_r)}(1-\theta^*)B'q_r}{(1-q_r)G' + (1-p_r)(B+B'q_r)} \right) + \frac{p_r(\theta^* + \theta_*)}{(1-p_r q_r)} \right] \\
& + \frac{\partial h_3}{\partial p_r} \frac{\partial h_3}{\partial \theta_*} \frac{(1-p_r)B}{q_r(1-q_r)} \left[\frac{(1-\theta_*)}{(G+q_r G') - (1-p_r)(B+B'q_r)} - \frac{(1-\theta^*)}{(1-q_r)G' + (1-p_r)(B+B'q_r)} \right] \\
& + \frac{\partial h_3}{\partial p_r} \frac{\partial h_3}{\partial \theta_*} \left[\frac{\theta^* G'}{(1-q_r)G' + (1-p_r)(B+B'q_r)} + \frac{\frac{(\theta^* + \theta_*)}{[2-(\theta^* + \theta_*)]}(1-\theta_*)B' - \theta_* G'}{(G+q_r G') - (1-p_r)(B+B'q_r)} \right] \\
> & 0 \text{ since } (G+G') = (B+B') \text{ and } 2G' \geq B' > G'.
\end{aligned}$$

And, for similar reasons, we have

$$\begin{aligned}
\text{sign} \left(\frac{dN_2}{dp_r} \right) &= \text{sign} \left[(1-p_r) \left((1-\theta^*) \frac{d\theta^*}{dp_{r\pm}} - (1-\theta_*) \frac{d\theta_*}{dp_{r+}} \right) - (\theta^* - \theta_*) \left(1 - \frac{\theta^* + \theta_*}{2} \right) \right] \\
&< 0.
\end{aligned}$$

■

A.20 Proposition 18

Proof. To establish the ambiguity of the effect of economic significance on the volume of gold-plate patent applications, we provide two numerical examples with effects going in opposite directions.

Let the patent policy be such that the examination intensity $p_r = 0.25$ for regular patents and $p_{gp} = 1$ for gold-plate patents; application fees $P_r = 2.3$ for regular patents and $P_{gp} = 2.4$ for gold-plate patents. Suppose that the value of an economically less significant patent conditional on it being good or bad is given by

$$B_0(q) = 1.75 + 2.25q,$$

$$G_0(q) = 2 + 2q.$$

Numerically solving for the equilibrium in a two-tiered system with these functional forms, we have $\theta_* = 0.15346$, $\theta^* = 0.71983$, $q_r = 0.50822$.

Let the value of an economically significant patent for the first example be

$$B_1(q) = 2 + 2.25q,$$

$$G_1(q) = 2.25 + 2q.$$

Let the value of an economically significant patent for the second example be

$$B_2(q) = 1.1 + 3q,$$

$$G_2(q) = 2.1 + 2q.$$

Note that as assumed, $B_i(1) = G_i(1)$, $B_i(q) \leq G_i(q)$ and $G'_i(q) < B'_i(q)$ for all q . Comparing the more significant patents to the less significant ones, we have higher values $G_1(q) > G_2(q) > G_0(q)$, and a steeper slope for the value of a bad patent: $B'_1(q) = B'_2(q) > B'_0(q)$.

Numerically solving for the equilibrium in a two-tiered system for each of these examples, we find that in the first case, $\theta_* = 0.05305$, $\theta^* = 0.70012$, $q_r = 0.44611$. Therefore, in this example, more significant patents result in more

patent applications, more gold-plate patent applications, and a lower quality of regular patents. In the second case, $\theta_* = 0.21936$, $\theta^* = 0.72613$, $q_r = 0.54452$. Comparing to the less significant patents, we have less patent applications, less gold-plate patent applications and a higher quality of regular patents. ■