POSTDATA PROBABILITY THAT THE NUMERATOR OF F IS SIGNIFICANTLY LARGE

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Abstract

The postdata H_o-probability that the numerator chi-square of an $F_{\nu,r}$-ratio exceeds its 5 percent critical value ($\chi^2_{\nu;.05}$) can be read from a chi-square table using the observed value (F) of the $F_{\nu,r}$-ratio. The answer is read as the probability that a chi-square variable on $r+\nu$ degrees of freedom will exceed the value $[1+r/(\nu F)]\chi^2_{\nu;.05}$.

Keywords and phrases: Conditional probability; Analysis of variance.

The F-statistic is assumed to be H_o-distributed as a ratio of independent chi-squares over degrees of freedom,

$$F_{\nu,r} = \frac{\chi^2_\nu/\nu}{\chi^2_r/r} \quad .$$

Conditional on a realized value of this F-statistic the numerator χ^2_ν is seen to be H_o-distributed as

$$P\left\{\chi^2_\nu < \frac{\nu F}{r+\nu F}\, y \,\middle|\, F_{\nu,r} = F\right\} = F_{\chi^2_{\nu+r}}(y) \quad .$$

Setting

$$y_\alpha = \frac{r+\nu F}{\nu F}\, F^{-1}_{\chi^2_\nu}(1-\alpha) = \left(\frac{r}{\nu F} + 1\right)\chi^2_{\nu;\alpha} \tag{1}$$

and calculating

$$P\left\{\chi^2_\nu > \chi^2_{\nu;\alpha} \,|\, F_{\nu,r}=F\right\} = 1-F_{\chi^2_{\nu+r}}(y_\alpha) \tag{2}$$

we obtain the H_o-conditional probability that the numerator chi-square exceeds the critical value $\chi^2_{\nu;\alpha}$.

If our realized $F_{\nu,r}$ does exceed its critical value then in an analysis of variance setting we would hope that this might reasonably be attributed to an excessively large numerator, as is implicitly inferred. Note, however, that if $F_{\nu,r}$ only marginally exceeds the critical value $F_{\nu,r;\alpha}$, then for moderately large r we would find from (1) that y_α is no larger than $r+\chi^2_{\nu;\alpha}$, and (2) would then yield a disappointingly intermediate value. A marginally significant analysis of variance test is thus not very compelling evidence against the null hypothesis, and becomes weaker as numerator degrees of freedom (ν) increase. This is illustrated in Table 1, where 30 degrees of freedom in a two-tailed t-test are seen to be sufficient to provide comforting assurance that a nominal 1 percent P-value does impute statistical significance to the numerator at level $P < .05$.

Table 1. H_o-Probability that the numerator chi-square of a significant $F_{\nu,r}$-ratio exceeds $\chi^2_{\nu;.05}$.

Numerator d.f. ν	Denominator d.f. r	$P_{H_o}\left(\text{numerator } \chi^2 > \chi^2_{\nu;.05} \middle| F\right)$ $F=F_{\nu,r;.05}$	$F=F_{\nu,r;.01}$
1	10	.40	.74
	20	.43	.90
	30	.44	.95
2	10	.35	.62
	20	.39	.80
	30	.42	.89
3	10	.32	.55
	20	.37	.74
	30	.39	.84