AUGMENTED EXPERIMENT DESIGNS WITH RECOVERY OF INTERBLOCK AND INTERVARIETY INFORMATION

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Abstract

The class of augmented experiment designs has been found useful for experiments involving comparisons of standard check treatments with a set of new and untried treatments. Often the new treatments have limited experimental material, or the experimenter may wish to use few experimental units and screen a large number of new treatments. The promising new treatments from this stage are then tested in replicated experiments or they can become the checks for another experiment using an augmented experiment design. By combining treatments from various stages of testing, efficiency of experimentation can be greatly increased. With regard to the new treatments, usually included only once in an experiment, every augmented design is incomplete for the new treatments. This means that interblock, interrow, and/or intercolumn information is available to use in obtaining solutions for new treatment effects. Also, since the new treatment effects can often be considered to be random effects, their distributional properties can be used to increase the efficiency of experimentation. We demonstrate the statistical procedures for recovering this information in block and row-column designed experiments.

1 Introduction

Various augmented experiment designs have been presented in the literature (Federer, 1955, 1961; Federer and Raghavarao, 1975; Federer, Nair and Raghavarao, 1975; Federer and Wright, 1988). The purpose of this paper is to present a statistical analysis for these experiment designs making use of the information obtained from the random blocking effects and from the distributional effects of the augmented (or new) treatments in the experiment. Since augmented designs are used to include treatments (varieties) for which there is little information and often limited material, these treatment or varietal effects can be considered to be distributed around some mean and with a common variance σ_τ^2. Herein we consider that each new treatment is included once in the experiment but this need not be the case as the procedure is easily extendable to take additional replication into account. There are c check or standard treatments which are used to obtain the experiment design prior to adding the augmented treatments. The check treatment yields are used to obtain solutions for blocking and check treatment effects. The former are used to adjust the new treatment effects. From the mean square for the new treatments, an estimate of the variance component σ_τ^2 is obtained and used for adjusting new treatment means for their distributional effects. Adjustment for distrinutional properties of the random effects makes use of all information from an experiment.

In Section 2, augmented block experiment designs are considered. In Section 3, augmented row-column experiment designs such as those described by Federer and Raghavarao (1975) and Federer, Nair and Raghavarao (1975) are the subject of statistical analyses. In Section 4, we present statistical analyses for resolvable row-column designs such as those presented by Federer and Wright (1988) and those which could be obtained from the row-column designs of Russell *et al.* (1981), Nguyen and Williams (1993), and John and Whitaker (1993) by a "variety cutting" procedure. Finally, some comments on the analyses are given in Section 5.

2 Augmented Block Experiment Designs

Among the experiment designs in this class are the augmented randomized complete blocks (ARCBD), augmented balanced incomplete block (ABIBD), and augmented partially balanced incomplete blocks (APBIBD). With respect to the augmented or new treatments all these designs are incomplete in that all the new treatments do not appear together in the blocks. Thus, recovery of interblock information is needed for a more efficient analysis. First consider an ARCBD with c checks and n new treatments for a total of $v = c + n$ treatments in r blocks. Let the c check treatments appear once in each of the blocks (note that the c treatments could appear in the proportions $n_1 : n_2 : \ldots : n_c$ in each of the r blocks and the design would still be an orthogonal one (See Federer, 1991, ch. 7)). An analysis of variance, ANOVA, for an ARCBD is given in Table 1. Since the n new treatments each occur once in the experiment the observation can only contribute to the new treatment estimate and nothing to block, overall mean or error estimation (Federer and Raghavarao, 1975). From an analysis on check treatment results only, the expected value of the block mean square is $\sigma_\epsilon^2 + c\sigma_\beta^2$, and an estimate of σ_β^2 is $(B - E)/c$. This is the variance component to be used to obtain adjusted new treatment means recovering interblock information. For our analysis, we use the following linear model:

$$Y_{ij} = (\mu + \rho_i + \tau_j + \epsilon_{ij})n_{ij} \tag{1}$$

where μ is a general mean effect, ρ_i is the i^{th} block effect, τ_j is the j^{th} treatment effect, $i = 1, \ldots, r, j = 1, \ldots, v, n_{ij}$ is one if the j^{th} treatment occurs in i^{th} block and zero otherwise, and ϵ_{ij} is a random error effect distributed with mean zero and variance σ_ϵ^2. The resulting normal equations are:

$$\begin{bmatrix} \mathbf{K}_{r\times r} & \mathbf{NB}_{r\times v} \\ \mathbf{NB}'_{v\times r} & \mathbf{R}_{v\times v} \end{bmatrix} \begin{bmatrix} \beta_{r\times 1} \\ \tau_{v\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{YB}_{r\times 1} \\ \mathbf{YT}_{v\times 1} \end{bmatrix} \tag{2}$$

where K is a diagonal matrix with block sizes on the diagonal, R is a diagonal matrix with replication numbers on the diagonal and is

$$\mathbf{R} = \begin{bmatrix} r\mathbf{I}_c & \emptyset_{c\times n} \\ \emptyset' & \mathbf{I}_n \end{bmatrix}, \tag{3}$$

where I_x is an identity matrix of side x, $\emptyset$ is a matrix whose elements are all zeros, and **NB** is the incidence matrix of blocks and treatments with a one appearing for the treatments which occur in block i and zero for those not appearing. The check treatments all have ones for every block. Therefore, a reasonable restriction is for the sum of the check treatment effects to be zero and likewise for the sum of the block effects. With these restrictions a solution of the normal equations is possible. Let

$$\mathbf{NB} = [\mathbf{J}_{r\times c} \quad \mathbf{NN}_{r\times n}] \tag{4}$$

and

$$\mathbf{J}\emptyset = [\mathbf{J}_{r\times c} \quad \emptyset_{r\times n}], \tag{5}$$

where **J** is a matrix whose elements are all ones. Then, the usual solutions for a randomized complete block design are

$$\hat{\beta}_i = \overline{y}_{i.} - \overline{y}_{..} \text{ and } \hat{\tau}_j = \overline{y}_{.j} - \overline{y}_{..}\ , \tag{6}$$

where the observations in the above equations are for check yields only. The new treatment solutions are:

$$\hat{\tau}_j = Y_{ij} - \overline{y}_{i.}\ , \tag{7}$$

where Y_{ij} is the yield of new treatment j in block i and $\overline{y}_{i.}$ is as in (6).

The various sums of squares in Table 1 are:

$$(n-1)N = \sum_{i=1}^{r}\sum_{j=1}^{n} \hat{\tau}_j \left(Y_{ij} - \overline{y}_{i.}\right) n_{ij}\ , \tag{8}$$

where Y_{ij} is the yield of new treatment j in block i, and the other sums of squares are from standard procedures.

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	T^*		
Correction for mean	1	$M = Y_{..}^2/T$	
Blocks (ignoring treatments)	$r-1$	block tot. squared/no.$-M$	
Treatments (eliminating blocks)	$v-1$	see text	
Among check treatments	$c-1$	see below	C
Check vs. new	1	see text	
Among new treatments	$n-1$	see text	N
Residual	$(r-1)(c-1)$	subtraction	E

ANOVA on check yields only (standard analysis)

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	rc	$\sum_{i=1}^{r}\sum_{j=1}^{c} Y_{ij}^2$	–
Correction for mean	1	$Y_{..}^2/rc$	–
Block	$(r-1)$	$\sum_{i=1}^{n} \frac{Y_{i.}^2}{c} - \frac{Y_{..}^2}{rc}$	B
Check	$(c-1)$	$\sum_{j=1}^{v_c} Y_{.j}^2/r - \frac{Y_{..}^2}{rc}$	C
Check $\times$ block	$(r-1)(c-1)$	subtraction	E

$*T = rc + n$.

Table 1: ANOVA for an ARCBD

To recover interblock and intervariety information formula (2.2) is changed as follows before applying the restrictions and obtaining the solutions

$$\left[\begin{array}{c|cc} \mathbf{K}+\hat{\sigma}_\epsilon^2\mathbf{I}_r/\hat{\sigma}_\beta^2 & \multicolumn{2}{c}{\mathbf{NB}} \\ \hline \mathbf{NB}' & r\mathbf{I}_c & \emptyset \\ & \emptyset & \mathbf{I}_n(1+\hat{\sigma}_\epsilon^2/\hat{\sigma}_\tau^2 \end{array}\right] \begin{bmatrix} \beta \\ \tau_c \\ \tau_n \end{bmatrix} = \begin{bmatrix} \mathbf{YB}_{r\times 1} \\ \mathbf{YC}_{c\times 1} \\ \mathbf{YN}_{n\times 1} \end{bmatrix} \tag{9}$$

where $\hat{\sigma}_\epsilon^2$ is the estimate for the error variance component σ_ϵ^2, $\hat{\sigma}_\beta^2$ is the estimated block variance component, and $\hat{\sigma}_\tau^2$ is the estimate of the variance component σ_τ^2. Here the new treatment effects are random effects distributed with mean $\tau_.$ and variance σ_τ^2. The check variety effects are fixed effects. Substituting $\mathbf{NB}-\mathbf{J}\emptyset$ for $\mathbf{NB}$ in (9) results in solutions for the τ_j effects. The treatment effects adjusted for interblock and intervarietal information are:

$$\begin{aligned} \hat{\tau}_v &= \left[\begin{pmatrix} r\mathbf{I}_c & \emptyset \\ \emptyset & \mathbf{I}_n(1+\hat{\sigma}_\epsilon^2/\hat{\sigma}_\tau) \end{pmatrix} - \mathbf{NB}'\left(\mathbf{K}+\frac{\hat{\sigma}_\epsilon^2\mathbf{I}_r}{\hat{\sigma}_\beta^2}\right)^{-1}(\mathbf{NB}-\mathbf{J}\emptyset)\right]^{-1} \\ &\quad\times\left[\begin{pmatrix} \mathbf{YC} \\ \mathbf{YN} \end{pmatrix} - \mathbf{NB}'\left(\mathbf{K}+\frac{\hat{\sigma}_\epsilon^2}{\hat{\sigma}_\beta^2}\mathbf{I}_r\right)^{-1}\mathbf{YB}\right] \\ &= \mathbf{Var}\times\left[\begin{pmatrix} \mathbf{YC} \\ \mathbf{YN} \end{pmatrix} - \mathbf{NB}'\left(\mathbf{K}+\frac{\hat{\sigma}_\epsilon^2}{\hat{\sigma}_\beta^2}\mathbf{I}_r\right)^{-1}\mathbf{YB}\right]. \end{aligned} \tag{10}$$

Variances of differences among treatment effects are obtained from the following variance-covariance matrix.

$$\hat{\sigma}_\epsilon^2 \times \mathbf{Var} \tag{11}$$

An approximate average variance of a difference between two adjusted new treatment effects may be obtained as

$$2\hat{\sigma}_\epsilon^2\|\mathbf{Var}\|^{1/n}, \tag{12}$$

where $\|\cdot\|$ denotes the determinant of a matrix and **Var** is determined for new treatments only. The value of (12) is less than or equal to the correct average variance of a difference but may be useful for a quick scrutiny of the adjusted effects (or means) in certain situations.

The statistical analysis outlined above is directly applicable to an analysis for ABIBD, APBIBD, and other augmented incomplete block designs. The **K** and **NB** matrices will need to be adjusted to take into account the experiment design for the check treatments. When the check treatments are in an incomplete block experiment design, their effects will need to be adjusted for recovery of interblock information. The remainder of the analysis proceeds as described above.

3 Augmented Row-Column Experiment Designs

A number of augmented row-column designs (ARRCD) have been presented by Federer and Raghavarao (1975) and Federer, Nair, and Raghavarao (1975). One particular row-column design for few checks and many new treatments is to have the checks repeated more than once in each row and or column. Jose Crossa, CIMMYT, personal correspondence, is using a row-column design of 15 columns and 11 rows with 2 checks which appear either once or twice in each column and twice in each row with 42 check plots and 123 new treatments. The design will be used over several sites.

To recover interrow and intercolumn information on both check treatments and new treatments, as well as intervariety information on new treatments, we proceed as follows for an ANOVA as presented in Table 2. The linear model used here is

$$Y_{ghi} = (\mu + \rho_g + \gamma_h + \tau_i + \epsilon_{ghi})\, n_{ghi} \quad , \tag{13}$$

where μ is a general mean effect, ρ_g is the effect of the g^{th} row distributed with mean zero and variance σ^2_ρ , γ is the effect of the h^{th} column distributed with mean zero and variance σ^2_γ , τ_i is the effect of the i^{th} treatment where the check treatments are fixed effects and the new treatments are random effects distribution with mean $\tau_.$ and variance σ^2_τ, and n_{ghi} is one if treatment i occurs in row g and column h and zero otherwise.

The normal equations for check plot yields only are:

$$\begin{bmatrix} \mathbf{K}_a + \frac{\hat{\sigma}^2_\epsilon \mathbf{I}_a}{\hat{\sigma}^2_\rho} & \mathbf{RC}_{a\times b} & \mathbf{NR}_{a\times c} \\ \mathbf{RC}' & \mathbf{K}_b + \frac{\hat{\sigma}^2_\epsilon \mathbf{I}_b}{\hat{\sigma}^2_\gamma} & \mathbf{NC}_{b\times c} \\ \mathbf{NR}' & \mathbf{NC}' & \mathbf{V}_c \end{bmatrix} \begin{bmatrix} \rho_{a\times 1} \\ \gamma_{b\times 1} \\ \tau_{c\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{YR}_{a\times 1} \\ \mathbf{YC}_{b\times 1} \\ \mathbf{YT}_{c\times 1} \end{bmatrix} \tag{14}$$

where K_a is a diagonal matrix with the number of check plots occurring in each row, K_b is a comparable diagonal matrix for columns, V_c is a diagonal matrix of replication numbers for the check treatments and is rI_c for equal replication of checks, **RC** is the $a \times b$ row-column design matrix of n_{ghi} values, **NR** is the row-check treatment design matrix, **NC** is the column-check treatment design matrix, **YR** is a column vector of row totals, **YC** is a column vector of column totals, and **YT** is a column vector of check treatment totals. When there are an equal number of check plot yields for each row, column, and check treatment, $\overline{y}\ldots$ is a solution for μ and **YR**, **YC**, and **YT** are totals of $Y_{ghi} - \overline{y}_{...}$ check plot yields. Otherwise $\overline{y}_{...}$ will not be a solution for $\hat{\mu}$ and $\hat{\mu}$ will need to be evaluated. In this case, $Y_{ghi} - \hat{\mu}$ values will be used to obtain totals for **YR**, **YC**, and **YT**. The various sums of squares in Table 2 are computed as follows.

Columns (ignoring treatments and eliminating rows):

$$\left[\left[\mathbf{K}_b - \mathbf{RC}'\,\mathbf{K}_a^{-1}\mathbf{RC} + \mathbf{J}_{b\times b}/k\right]^{-1} \left[\mathbf{YC} - \mathbf{RC}'\,\mathbf{K}_a^{-1}\,\mathbf{YR}\right]\right]' \times \left[\mathbf{YC} - \mathbf{RC}'\,\mathbf{K}_a^{-1}\mathbf{YR}\right] \tag{15}$$

(k an appropriate scalar, **J** a matrix of ones).

Treatment (eliminating rows and columns)

Let

$$\hat{\tau} = \left[\mathbf{V}_c - (\mathbf{NR}'\;\mathbf{NC}')\mathbf{M}\begin{bmatrix}\mathbf{NR}\\ \mathbf{NC}\end{bmatrix} + \mathbf{J}/k\right]^{-1}$$

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	T	$\sum_{g=1}^{a}\sum_{h=1}^{b}\sum_{i=1}^{c} n_{ghi}Y_{ghi}^2$	–
Correction for mean	1	$Y_{...}^2/T$	–
Row (ignor. col. and treat.)*	$a-1$	$\sum_{g=1}^{a} Y_{g..}^2/b_g - Y_{...}^2/T$	–
Column (ignor. treat. and elim. rows)	$b-1$	see text (15)	–
Treatment (elim. rows and col.)	$c-1$	see text (17)	V
Remainder	$T-a-b-c+2$	subtraction	E
Row (elim. col. and treat.)*	$a-1$	see text (19)	R
Column (elim. rows and treat.)	$b-1$	see text (21)	C
Augmented treatment (elim. rows and col.)	$n-1$	see text (23)	N

*b_g =number of check treatment yields in row g.

Table 2: ANOVA for check treatment yields in an ARCD

$$\times \left[\mathbf{YT} - [\mathbf{NR}'\ \mathbf{NC}']\mathbf{M}\begin{bmatrix}\mathbf{YR}\\ \mathbf{YC}\end{bmatrix}\right] \tag{16}$$

where $\mathbf{M} = \begin{bmatrix}\mathbf{K}_a & \mathbf{RC}-\mathbf{J}\\ \mathbf{RC}' & K_b\end{bmatrix}^{-1}$.

Then, the sum of squares is

$$\hat{\tau}'\left[\mathbf{YT} - (\mathbf{NR}'\ \mathbf{NC}')\begin{bmatrix}\mathbf{K}_a & \mathbf{RC}-\mathbf{J}\\ \mathbf{RC}' & \mathbf{K}_b\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{YR}\\ \mathbf{YC}\end{bmatrix}\right]. \tag{17}$$

Row (eliminating columns and treatments)

Let

$$\hat{\rho} = \left[\mathbf{K}_a - (\mathbf{RC}\ \mathbf{NR})\begin{bmatrix}\mathbf{K}_b & \mathbf{NC}-\mathbf{J}\\ \mathbf{NC}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{RC}'\\ \mathbf{NR}'\end{bmatrix} + \mathbf{J}/k\right]^{-1}$$

$$\times \left[\mathbf{YRT} - (\mathbf{RC}\ \mathbf{NR})\begin{bmatrix}\mathbf{K}_b & \mathbf{NC}-\mathbf{J}\\ \mathbf{NC}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{YC}\\ \mathbf{YT}\end{bmatrix}\right]. \tag{18}$$

Then the sum of squares is

$$\hat{\rho}' \times \left[\mathbf{YR} - (\mathbf{RC}\ \mathbf{NR})\begin{bmatrix}\mathbf{K}_b & \mathbf{NC}-\mathbf{J}\\ \mathbf{NC}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{YC}\\ \mathbf{YT}\end{bmatrix}\right]. \tag{19}$$

Column (eliminating treatments and rows):

Let

$$\hat{\gamma} = \left[\mathbf{K}_b - (\mathbf{RC}'\ \mathbf{NC})\begin{bmatrix}\mathbf{K}_a & \mathbf{NR}-\mathbf{J}\\ \mathbf{NR}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{RC}\\ \mathbf{NC}'\end{bmatrix} + \mathbf{J}/k\right]^{-1}$$

$$\times \left[\mathbf{YC} - (\mathbf{RC}'\ \mathbf{NC})\begin{bmatrix}\mathbf{K}_a & \mathbf{NR}-\mathbf{J}\\ \mathbf{NR}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{YR}\\ \mathbf{YT}\end{bmatrix}\right]. \tag{20}$$

Then, the sum of squares is

$$\hat{\gamma}' \times \left[\mathbf{YC} - (\mathbf{RC}'\ \mathbf{NC})\begin{bmatrix}\mathbf{K}_a & \mathbf{NR}-\mathbf{J}\\ \mathbf{NR}' & \mathbf{V}_c\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{YR}\\ \mathbf{YT}\end{bmatrix}\right]. \tag{21}$$

The intra-row-column solutions for new treatments are

$$\hat{\tau}_i = Y_{ghi} - \hat{\mu} - \hat{\rho}_g - \hat{\gamma}_h \ . \tag{22}$$

The sum of squares among new treatments is

$$\sum_{i=1}^{n} \hat{\tau}_i \left(Y_{ghi} - \hat{\mu} - \hat{\rho}_g - \hat{\gamma}_h\right) = \sum_{i=1}^{n} \hat{\tau}_i^2 \ . \tag{23}$$

The expected values for R, C, and N in Table 2 may be evaluated using various computer software. Then estimates of the variances $\sigma_\rho^2, \sigma_\gamma^2$, and $\hat{\sigma}_\epsilon^2$ may be obtained. These solutions are then substituted in (14) and the matrices are expanded to include the new treatments. With these changes the new treatment effects with recovery of interrow, intercolumn, and intervariety information are obtained. The variances of difference may be obtained as described in Section 2.

4 Resolvable Augmented Row-Column Experiment Designs

Lattice square experiment designs are resolvable row-column experiment designs and with "variety-cutting" may be used to construct augmented resolvable row-column designs, ARRCD (Federer and Wright, 1988). For $v = k^2$ there are $k - 1$ suitable arangements from a balanced lattice square design to construct resolvable row-column designs and a computer package has been prepared for constructing such designs (Nam-Ky Nguyen, 1994, personal communication). These designs may be used to construct ARRCDs by using "variety cutting" in the manner described by Federer and Wright, (1988). To illustrate, consider the following arrangements from a balanced lattice square plan with $v = 9$

Replicate 1	*Replicate* 2	*Replicate* 3	*Replicate* 4
1 2 3	1 4 7	1 5 9	1 6 8
4 5 6	2 5 8	6 7 2	9 2 4
7 8 9	3 6 9	8 3 4	5 7 3

Then "cut-varieties" 7, 8, and 9 and fill these spots with new treatments a, b, c, d, e and f. Replicates 1 and 2 are not suitable as all new treatments would appear in a row or column and solutions for these rows or columns would not be possible with the usual restrictions for an analysis. With $2k$ check plots in $k - 1$ replicates, $k(k - 1)(k - 2)$ new treatments each replicated once could be included in an ARRCD. Note that the $2k$ checks could include duplicated checks in each replicate. An ANOVA for an ARRCD is given in Table 3, for the following linear model

$$Y_{ghi} = \left(\mu + \beta_g + \rho_{gh} + \gamma_{gi} + \tau_j + \epsilon_{ghij}\right) n_{ghij}. \tag{24}$$

μ is a general mean effect, β_g is a g^{th} replicate effect, ρ_{gh} is the k^{th} row effect in replicate g and is distributed with mean zero and variance σ_ρ^2, γ_{gh} is the i^{th} column effect in replicate

g and is distributed with mean zero and variance σ_γ^2, τ_j is the j^{th} treatment effect, ϵ_{ghi}, is a random error effect distributed with mean zero and variance σ_ϵ^2, and η_{ghij} is an indicator variable which is one when treatment j occurs in row h and column i in replicate g and zero otherwise.

The normal equations recovering interrow and intercolumn information are:

$$\begin{bmatrix} \mathbf{I}_{ra}\left(b+\hat{\sigma}_\epsilon^2/\hat{\sigma}_\rho^2\right) & \mathbf{RC} & \mathbf{NR} \\ \mathbf{RC}' & \mathbf{I}_{rb}\left(a+\hat{\sigma}_\epsilon^2/\hat{\sigma}_\gamma^2\right) & \mathbf{NC} \\ \mathbf{NR}' & \mathbf{NC}' & \mathbf{V} \end{bmatrix} \begin{bmatrix} \rho_{ra\times 1} \\ \gamma_{rb\times 1} \\ \tau_{c\times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{YR}_{ra\times 1} \\ \mathbf{YC}_{rb\times 1} \\ \mathbf{YT}_{c\times 1} \end{bmatrix} \tag{25}$$

where $\mathbf{YR}$, $\mathbf{YC}$, and $\mathbf{YT}$ are totals of $Y_{ghij}-\left(\hat{\mu}+\hat{\beta}_g\right)$ values to remove effect of general mean μ and replicate (complete block) effects when the usual restrictions that sums of effects are zero are imposed. When μ and β_g effects are orthogonal to the remaining effects $\hat{\mu}+\hat{\beta}_g=\overline{y}_{g...}$.

The various sums of squares are computed in the manner described in the previous section taking into account the nature of the various matrices involved. The intra-row-column solution for a new treatment effect is

$$\hat{\tau}_j = Y_{ghij} - \hat{\mu} - \hat{\beta}_g - \hat{\rho}_{gh} - \hat{\gamma}_{gi}, \tag{26}$$

and is

$$\hat{\tau}_j = Y_{ghij} - \overline{y}_{g...} - \hat{\rho}_{gh} - \hat{\gamma}_{gi}, \tag{27}$$

when $Y_{....}$ on the checks contains an equal number of each of the effects ρ_{gh}, γ_{gi}, and τ_j. That is, when the restrictions on the solutions is that

$$\sum_{h=1}^{a} \hat{\rho}_{gh} = \sum_{i=1}^{b} \hat{\gamma}_{gi} = \sum_{j=1}^{c} \hat{\tau}_g = 0, \tag{28}$$

these effects disappear in the total $Y_{....}$ from the check yields. A sum of squares for new treatments is

$$\sum_{j=1}^{n} \hat{\tau}_g\left(Y_{ghij} - \hat{\mu} - \hat{\beta}_g - \hat{\rho}_{gh} - \hat{\gamma}_{gi}\right) = \sum_{j=1}^{n} \hat{\tau}_j^2. \tag{29}$$

The expected value for (29) needs to be evaluated to obtain the coefficients of σ_ϵ^2 and σ_τ^2. With estimates of σ_ϵ^2, σ_ρ^2, σ_γ^2, and σ_τ^2, new treatment effects recovering interrow, intercolumn, and intervariety information are obtained from (10) with the appropriate interpretation of the various matrices. Likewise, variances for differences of these adjusted effects may be obtained as explained for equation (11).

5 Comments and Discussion

Computation of sums of squares and adjusted effects for the proposed analyses poses little difficulty owing to the availability of computer software packages such as GAUSS. Some other standard statistical packages such as SAS and GENSTAT can be used to recover interblock and intervariety information. Some packages such as MATHEMATICA may be used to obtain expected values for mean squares. Then, solutions for treatment effects

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Total	rc^*	$\sum_{g=1}^{r}\sum_{h=1}^{a}\sum_{i=1}^{b}\sum_{j=1}^{c} Y_{ghij}^2\, n_{ghij}$	–
Correction for mean	1	$Y_{....}^2/rc$	–
Replicate = B	$r-1$	$\sum_{g=1}^{r} Y_{g...}^2/c - Y_{....}^2/rc$	–
Row in B (ignor col. and checks)	$r(a-1)^+$	$\sum_{g=1}^{r}\sum_{h=1}^{a}\frac{Y_{gh..}^2}{b_{gh}} - \sum_{g=1}^{r}\frac{Y_{g...}^2}{c}$	–
Column in B (ignor. treat., elim. rows)	$r(b-1)$	see text	–
Check (elim. rows and columns)	$c-1$	see text	T
Error	$rc - ra - rb$ $+ r - c + 1$	subtraction	E E
Row (elim. col. and checks)	$r(a-1)$	see text	R
Column (elim. rows and checks)	$r(b-1)$	see text	C
New treatment (elim. B, rows, and col.)	$(n-1)$	see text	N

*for c check treatments included once in each replicate. Otherwise, the total here is the total number of check treatment yields and c needs to be changed as required.

+ b_{gh} equals the number of check plot yields in row gh.

Table 3: ANOVA for an ARRCD with rc check plot yields.

recovering interblock, interrow, intercolumn, and intervariety information are possible. These computations pose little difficulty using GAUSS and concatanating submatrices or multiples thereof.

Using only intrablock or intrarow-column information leads to inefficient statistical analyses for a set of data. Since other information such as interblock, interrow, intercolumn and/or intervariety information is available in experimental data it should be used to obtain solutions for treatment effects. As demonstrated by Federer and Speed (1987), the efficiency of an experiment design can change considerably when using a measure of design efficiency that incorporates interblock information as compared to an intrablock measure.

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