An Airline Seat Allocation Game

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We examine a seat allocation game between two airlines for flights with two fares with dependent random demands. The strategic variable of this game is each airline's booking limit for the low fare. We have shown that there exists an equilibrium booking strategy such that both airlines will protect the same number of seats for the full fare and the total number of seats available for the discount fare under competition is smaller than the total number of seats that would be available if the two airlines collude. A numerical example is used to illustrate the equilibrium solutions and to examine the impact of the capacity shares and the level of dependency between random demands.

Airline Seat Allocation

Yield Management was born with the application of overbooking with the first published work by Beckman (1958). Overbooking arose as airlines oversold capacity to mitigate losses from the arrival uncertainty of customers. In the post airline deregulation environment in the United States, airlines started to offer fares a multiple prices and in addition to dealing with arrival uncertainty, airlines focused on the allocation of capacity across price or fare classes. Littlewood (1972) is the seminal work on fare class allocation or airline seat allocation as it is more commonly called. Littlewood presented the simple decision rule for a two-fare model; continue to sell discount seats as long as $r>(1-P)R$ where r is the revenue from low-fare passengers, R revenue from full fare, and P the probability that full-fare demand exceeds remaining capacity. Indicating that an airline would continue to sell discounted seats; until such a time, the marginal revenue of full-fare seats fare times probability of selling at least that many seats (to that fare class) exceeds the revenues from selling the discounted seat.

Littlewood's rule is extended to multiple-fare classes in Belobaba (1987) with Belobaba introducing the term expected marginal seat revenue (EMSR). For more than two-fare classes, EMSR is Littlewood's rule applied sequentially in increasing fare order. The method is only optimal for two fare classes. EMSR is later refined by Belobaba into EMSRb (Belobaba and Weatherford, 1996). EMSRb works as follows: given estimates of mean demands, μ_i, and standard deviations, σ_i, for each fare class i with fare f_i, the EMSRb heuristic sets a protection level (the number of seats to reserve for this and higher yielding fare classes) θ_i so that

$$f_{i+1} = \bar{f}_i P(\bar{X}_i > \theta_i)$$

where $\overline{X_i}$i is a normal random variable with mean $\mu \equiv \sum \mu_i$ and variance $\sum {\sigma_i}^2$ and $\overline{f_i}$ i is the weighted average fare $\sum \mu_i / \mu f_i$.

EMSRb is logically the same as Littlewood's rule; continue selling discounted seats until such a time that the expected marginal revenue of future sales in higher fare classes exceeds the discounted fare. These future sales are for a weighted average fare from classes with independent demands. EMSR, EMSRb and Littlewood's rule all assume that fare classes book in increasing fare order (low before high), fare classes are mutually independent, no cancellations or no shows, single flight leg with no consideration of network effects and no batch/group bookings.

All three (EMSR, EMSRb or Littlewood's rule) may be used once per flight in a static fashion, or in an advanced static manner. The advanced static approach is conceptually the same as the static fashion, except with multiple demand distributions per fare class. The demand distributions change from total demand by class to demands by class from this point till departure. This allows the seat allocation decisions to change. The airline can become more restrictive, reducing the availability of discounted seats, or even reopen previously closed classes in the event that demand has not materialized as expected. This has the potential to cause customers to act more strategically in their purchase of seats; potentially waiting for lower fares to reopen in the future. Anderson and Wilson (2003) investigate such impacts with Wilson et al. (2006) providing optimal seat allocations in the presence of such strategic behavior. In a similar vein, Pfeifer (1989) looked at the impact of customer's willingness to purchase seats from multiple fare classes upon seat allocation approaches. The migration of customers across fare classes or in time results in demands for fare classes potentially becoming dependent versus independent as in the assumptions of the marginal rules.

Brumelle et al. (1990) was really the first explicit investigation of dependent demand upon optimal booking limits. Brumelle et al. explicitly model demand Y and B for two fare classes, which are stochastically dependent. The results are similar to Littlewood except now the probability of stocking out (or selling all remaining seats to the full fare, once discount is closed) is a conditional probability with full-fare demand conditioned on discount demand being greater or equal to the booking limit. The result is no different than that of Pfeifer (1989), in terms of booking limits, it is simply more formalized in Brumelle et al. (1990).

More recently, attention has focused on the fact that information (for the consumer) is becoming more transparent, and these seat allocations need to take into account that customers can book on multiple airlines, in essence looking at seat allocation in competitive environments. While research on competitive airline seat allocation is sparse, there does exist similar areas of study in the inventory literature. Littlewood's rule is simply a version of the newsvendor problem with the critical fractile determining the protection level for full fares. Parlar (1988) and Karjalainen (1992) extended the classic newsvendor in a setting where the newsvendors compete with each other in the sale of undifferentiated products.

Lippman and McCardle (1997) generalize the early results and formalize the existence of solutions. In Lippman and McCardle (1997), total market demand is random and common to all vendors. They describe four rules used to allocate random demand to each of the firms. The rules include a simple deterministic splitting of demand, random block splitting where all demand initially chooses the same firm, random individual splitting where each customer randomly chooses a firm and finally an independent demand model that simply splits aggregate demand into two (or more) separate pseudo sets of random demands (not really random as they are conditioned on the original aggregate demand). While Lippman and McCardle (1997) prove the existence of equilibrium, the nature of inventory decisions by firms depends largely on the demand allocation rules as their splitting rules encompass the range of independent to perfectly dependent firm demand.

Until recently, for example, Netessine and Shumsky (2005), there has been very little attention given competitive issues in yield management research, while clearly firms must be cognizant of competition when making decisions. Consider a city pair served by two airlines. Given information on market demands and each airline's seating capacity, the strategic interaction between two airlines involves two main decisions: how many discounts seats to make available, and what should each airline do when the other stops selling discount tickets? Unlike Lippman and McCardle (1997) who start with an aggregate market demand, Netessine and Shumsky (2005) assume a firm-specific demand which if unmet overflows into a market demand serviceable by all firms. Netessine and Shumsky (2005) go on to show that the monopolists booking limits (allowable seats at the discount fare) is never smaller than the sum of allowable discount sales of competing airlines.

We discuss seat allocation under competition when each of two airlines is operating a single-leg flight with two fare classes (full and discount). Similar to Lippman and McCardle (1997), we assume both airlines are competing for a share of a common market, in essence commoditized seats. Our model mimics the industry trend of migration to products with less restrictions with itineraries simply a series of one-way legs resulting in prices of competing airlines often simply price matched or within a few dollars. We add an additional layer of uncertainty to Lippman and McCardle (1997) as we model demand for two product classes. Unlike Netessine and Shumsky (2005) we assume no firm-specific demand, while one could argue that frequent flyer programs create firm-specific demand. Most firm-specific demand is probably at unrestricted or higher fare classes not the lower more-competitive fare classes. One could look at our approach as a seat allocation targeted at today's less-differentiated fare class products in competitive markets. While this paper limits to discrete decision variables, Li et al. (2007) addresses the same issue by assuming continuous decision variables.

In the following section, we develop the model setting that specifies the seat allocation game between two competing airlines. The next section formally introduces the seat allocation game and provides a characterization of the equilibrium booking limits. The following section presents a numerical example for equilibrium solutions under different capacities and levels of positive correlations between the random demands for two fare classes that follow bivariate normal distributions. The last section summarizes.

The Model Setting

Consider a competitive city pair served by two airlines. Given (full) information on market demands and each airline's seating capacity, the strategic interaction between two airlines, given that prices are market driven, involves two main decisions: how many discounts seats to make available, and what should each airline do when the other stops selling discount tickets. We use the same notation as in Brumelle et al. (1990). The discounted fare is ρ_B and the full fare is ρ_Y. Airline k's capacity is C_k for k=1, 2. The market demand for the discount fare is given by B and the market demand for full fare is given by Y. Random variables B and Y are not assumed to be independent. Each airline chooses a booking limit, the maximum seats sold at the discounted fare, as its decision variable and its objective is to maximize the total expected revenue.

Similar to most seat allocation research (Littlewood, 1972; Belobaba, 1987; Netessine and Shumsky, 2005), we develop a static formulation where the airline sets a booking limit, and once sales are closed to discounted classes they are never reopened. An additional standard assumption is that discount-seeking customers arrive first with full-fare customers arriving after all discount demand is met or rejected.

If we let l_k be the booking limit for the discount fare set by airline k, k=1, 2, then sales or bookings of discount (and full) fares to each airline will be a function of their booking limit and given our common market approach, also a function of the competing airlines booking limit. The dependency of firm 2's booking limit upon firm 1's bookings will depend upon the mechanism used to split demand among the service providers.

Since the two airlines face the same market demands (for a common market of both the full and discount fares), the specification of the revenue functions are critically related to how the two airlines share market demands. There are two common specifications for a market-splitting rule: the proportional rationing rule and the efficient rationing rule (cf. Tirole (1988, pp. 212–214) for more discussion). For the proportional rationing rule (also known as the randomized rationing rule), it means that in the event that the total commitment for a certain fare class from both airlines exceeds the market demand, then the two airlines will split the market demand according to their proportions of the total commitment. For the efficient rationing rule (also known as the parallel rationing rule), it assumes that in the event that the total commitment for a certain fare class from both airlines exceeds the market demand, each airline will get a half of the market demand or reach its commitment level, whichever is less. As long as the customers arrive in random order and cannot be sorted by their willingness to pay and there are no resale opportunities, it is appropriate to use proportional rationing rule.

As bookings arrive at random and tickets cannot be resold, we will apply the randomized rationing rule to model the demand-splitting process; this is analogous to Rule 3 (Incremental Random Splitting) in Lippman and McCardle (1997). For random-realized discount market demand B, given a pair of booking limits (l_k, l_j), airline k's share of the demand for the discount fare is given by

$$B_k(l_k,\, l_j) = \frac{l_k}{l_k + l_j} B, \quad \text{for } k = 1, 2$$

The actual sales for airline k's discount fare will be the minimum of its booking limit and its demand share, that is, $s_k(l_k, l_j)$¼ $B_k(l_k, l_j) \wedge l_k = l_k(1 \wedge B/l_k + l_j)$, where $x \wedge y \equiv \min(x, y)$. After the bookings for the discount fare are closed, airline k has a residual capacity of $l_k = C_k - s_k(l_k, l_j)$. Consequently, airline k's demand share for the full fare, for random full-fare demand Y, using the proportional rationing rule, now becomes

$$Y_k(l_k,\, l_j) = \frac{L_k}{L_k + L_j} Y$$

We now turn our attention to the revenue functions r_1 and r_2. Since airline k's expected revenue is determined by the joint decision (l_k, l_j), we write airline k's expected revenue function as $r_k(l_k, l_j)$ for k=1, 2 and k≠j. More specifically, if we let $B_k(l_k, l_j)$ be the airline k's demand share for the discount fare and $Y_k(l_k, l_j)$ be the airline k's demand share for the full fare, then the expected revenue function $r_k(l_k, l_j)$ is given by

$$\begin{aligned} r_k(l_k, l_j) =& \rho_B E[B_k(l_k, l_j) \wedge l_k] \\ & + \rho_Y E[Y_k(l_k, l_j) \wedge \\ & (C_k - (B_k(l_k, l_j) \wedge l_k)))], \\ r_k(l_k, l_j) =& \rho_B E(B_k \wedge l_k) + \rho_Y E(Y_k \wedge L_k) \\ \equiv& E(\tilde{r}_k(l_k, l_j)), \end{aligned}$$

where $\tilde{r}_k(l_k, l_j)$ is the random revenue associated with the pair of booking limits (l_k, l_j) and for clarity of presentation, we will use B_k and Y_k to denote $B_k(l_k, l_j)$ and $Y_k(l_k, l_j)$, respectively. It is now straightforward to verify that $S_k=l_k(1\wedge(B/l))$, $s_k+s_j=B\wedge l$, and $L_k+L_j=C-(l\wedge B)$, where $C\equiv C_1+C_2$ and $l\equiv l_1+l_2$.

This leads to a simple two-person nonzero-sum game with payoff functions r1 and r2. With this basic setting, we aim to characterize a pure-strategy Nash equilibrium for the game, which is a pair of booking limits (l_1^*, l_2^*) such that for k=1, 2, $r_k(l_k^*,l_j^*)>r_k(l_k, l_j^*)$ for all other l_k. It is well known that Nash equilibrium in pure strategies may not exist.

The Airline Seat Allocation Game and Its Solution

Establishing the existence of a pure-strategy Nash equilibrium for the seat allocation game is critical for practical reasons because implementing mixed strategies can be hardly operational. Lippman and McCardle (1997) use the notion of supermodularity (Topkis, 1978, 1979) to establish the existence of pure-strategy Nash equilibrium (Theorem 1, p. 58) for the one-class newsboy game. By showing that the underlying game under horizontal competition is not submodular, Netessine and Shumsky (2005) must rely on the assumption of multivariate total positivity of order 2 among the random demands to assure the existence of a pure-strategy Nash equilibrium. Unfortunately, we cannot apply the same technique as in Lippman and McCardle (1997) because of two sources of uncertainty in our model. We also cannot use the technique as in Netessine and Shumsky (2005) because our demand structures differ from theirs and our decision variables are discrete.

We decide to use a similar technique to Brumelle et al. (1990) to examine the equilibrium pair of booking limits. Given that airline *j*'s booking limit is fixed at lj and that airline k has accepted l_k *-1* requests for its discount fare and there is an additional request for its discount fare, then airline *k* must decide to accept or to reject this particular request for the discount ticket. If airline k decides to reject this booking request, its expected revenue is given by $E(\tilde{r}_k(l_k - 1, l_j)|B_k > l_k)$; and if airline k decides to accept the request, its expected revenue becomes $E(\tilde{r}_k(l_k - 1, l_j)|B_k > l_k)$. The primary consideration is the revenue difference between these two decisions. For this, we define the direct incremental gain function $G_k(l_k, l_j)$ for airline k as follows:

$$G_k(l_k, l_j) = E[\tilde{r}_k(l_k, l_j)|B_k \geqslant l_k] - E[\tilde{r}_k(l_k - 1, l_j)|B_k \geqslant l_k]$$

The following lemma gives a simple expression for the direct incremental gain functions.

Lemma 1 For k=1, 2,

(1)

$$\begin{aligned} G_k(l_k, l_j) = &\rho_B - \rho_Y P(Y > C - l|B \geqslant l) \\ &- \rho_Y \frac{C_j - l_j}{(C - l)(C - l + 1)} \\ &\times E(Y|Y \leqslant C - l, B \geqslant l) P(Y \leqslant C - l|B \geqslant l) \end{aligned}$$

Proof: First of all, it is easy to check that, from the definition of proportional rationing rule, $B_k > l_k$ if and only if B>l, which is equivalent to $B_j > l_j$. Therefore,

$$
\begin{aligned}
G_k(l_k, l_j) =&\rho_B + \rho_Y \frac{C_k - l_k}{C - l} E(Y \wedge (C - l) | B \geqslant l) \\
&- \rho_Y \frac{C_k - l_k + 1}{C - l + 1} E(Y \wedge (C - l + 1) | B \geqslant l) \\
=&\rho_B + \rho_Y \frac{C_k - l_k}{C - l} \times \{E(Y \wedge (C - l) | Y > C - l,\ B \geqslant l) \\
&\times P(Y > C - l | B \geqslant l) \\
&+ E(Y \wedge (C - l) | Y \leqslant C - l,\ B \geqslant l) \\
&\times P(Y \leqslant C - l | B \geqslant l)\} - \rho_Y \frac{C_k - l_k + 1}{C - l + 1} \\
&\times \{E(Y \wedge (C - l + 1) | Y > C - l,\ B \geqslant l) \\
&\times P(Y > C - l | B \geq l) \\
&+ E(Y \wedge (C - l + 1) | Y \leqslant C - l,\ B \geqslant l) \\
&\times P(Y \leqslant C - l | B \geqslant l)\} \\
=&\rho_B - \rho_Y P(Y > C - l | B \geqslant l) \\
&- \rho_Y \frac{C_j - l_j}{(C - l)(C - l + 1)} \\
&\times E(Y | Y \leqslant C - l,\ B \geqslant l) P(Y \leqslant C - l | B \geqslant l)
\end{aligned}
$$

This proves the lemma.

On the other hand, if airline k declines a request for the discount fare, the passenger will make the same request to airline j, which may accept or reject the booking request. The direct incremental gain is calculated based on the assumption that airline j's decision is fixed at l_j. If airline j accepts the request, that is, airline j changes its initial booking limit for the discount fare from lj to l_j+1, then airline k must take into account the impact of this action by airline j on its revenue. Because of this, we introduce the notion of indirect incremental gain function

$$
\begin{aligned}
g_k(l_k, l_j) =&E[\tilde{r}_k(l_k, l_j) | B_k \geqslant l_k] \\
&- E[\tilde{r}_k(l_k - 1, l_j + 1) | B_k \geqslant l_k]
\end{aligned}
$$

The following lemma gives us a simple formula for the indirect incremental gain function.

Lemma 2 For k=1, 2,

$$g_k(l_k, l_j) = \rho_B - \rho_Y E(Y/C-l \wedge 1 \mid B \geqslant l).$$

Proof: By the definition of indirect incremental gain function, it follows that

$$\begin{aligned} g_k(l_k, l_j) =& E[\tilde{r}_k(l_k, l_j)|B_k \geqslant l_k] \\ & - E[\tilde{r}_k(l_k-1, l_j+1)|B_k \geqslant l_k] \\ =& \left[l_k \times \rho_B + \rho_Y \frac{C_k - l_k}{C-l} . E(Y \wedge (C-l)|B \geqslant l) \right] \\ & - \left[(l_k - 1) \times \rho_B + \rho_Y \frac{C_k - l_k + 1}{C-l} \right. \\ & \left. \times E(Y \wedge (C-l)|B \geqslant l) \right] \\ =& \rho_b - \rho_Y E\left(\frac{Y}{C-l} \wedge 1 | B \geqslant l \right) \end{aligned}$$

which proves the lemma.

From Lemma 2, it follows that $g_1(l_1, l_2)$=$g_2(l_2, l_1)$ and $g_k(l_k, l_j)$ is decreasing in $l=l_1+l_2$, and consequently, is decreasing l_k for any given l_j, and vice versa. On the other hand, it is easy to check that

$$\begin{aligned} g_k(l_k, l_j) =& \rho_B - \rho_Y P(Y > C-l|B \geqslant l) \\ & - \frac{\rho_Y}{C-l} E(Y|Y \leqslant C-l,\ B \geqslant l) \\ & \times P(Y \leqslant C-l|B \geqslant l) \end{aligned}$$

This, together with (1) and the fact that $(C_j-l_j)/(C-l+1)<1$, leads to $G_k(l_k, l_j)>g_k(l_k, l_j)$, which implies that neither player will be passive. Because of this inequality, each airline will only focus on its direct incremental gain function. But it is not clear whether or not the direct incremental gain function G_k is decreasing in l_k for any given l_j. If G_k is indeed decreasing in l_k for any given l_j, then we can use each airline's response function to characterize an equilibrium pair of booking limits. In fact, we have the following main result of this paper.

Theorem 1 If for k=1 and 2, the gain function $G_k(l_k, l_j)$ is decreasing in l_k for any given l_j, there exists an equilibrium pair of booking limits (l^*_1, l^*_2) such that

(2)

$$l_k^* = \max\{l_k : G_k(l_k, l_j^*) \geqslant 0\}$$

Proof: Since $G_k(l_k, l_j)$ is decreasing in l_k for any given l_j, we can define airline k's response function to airline j's choice l_j as follows:

$$\begin{aligned}\eta_k(l_j) &= \max\{l_k : G_k(l_k, l_j) \geqslant 0\} \\ &= \max\left\{l_k : P(Y > C - l|B \geqslant l) + \frac{C_j - l_j}{(C-l)(C-l+1)}\right. \\ &\quad \left. \times E(Y|Y \leqslant C - l,\ B \geqslant l)P(Y \leqslant C - l|B \geqslant l) \leqslant \frac{\rho_B}{\rho_Y}\right\}\end{aligned}$$

First, it is clear that η_k is well defined. To finish the proof, it suffices to show that, for l^*_k and l^*_j given by (2), we will have $\eta_k(l^*_j)=l^*_k$ for k=1, 2. Note that (1) and (2) imply that each airline will protect the same number of seats for the high fare, that is, $p^*=C1-l^*_1=C_2-l^*_2$. Upon agreeing on this, both airlines will face the exactly same decision on how to choose their booking limit for the discount fare so that there is no further possible revenue gain by allocating additional seats for the discount fare. This is in fact captured by the response function. Since in the end neither airline can unilaterally improve its revenue, the pair of booking limits (l_{1*}, l_{2*}) must be an equilibrium.

While it is difficult to prove that the incremental gain function is decreasing, all numerical experiments indicate this assumption is valid. Figure 1 illustrates a typical example, with parameters from Brumelle et al. (1990), for competitors each operating a 40-seat plane. The incremental gain is plotted for increasing booking limits for four separate competitor booking limit (l_2) levels (l_2=1, 10, 20 and 40).

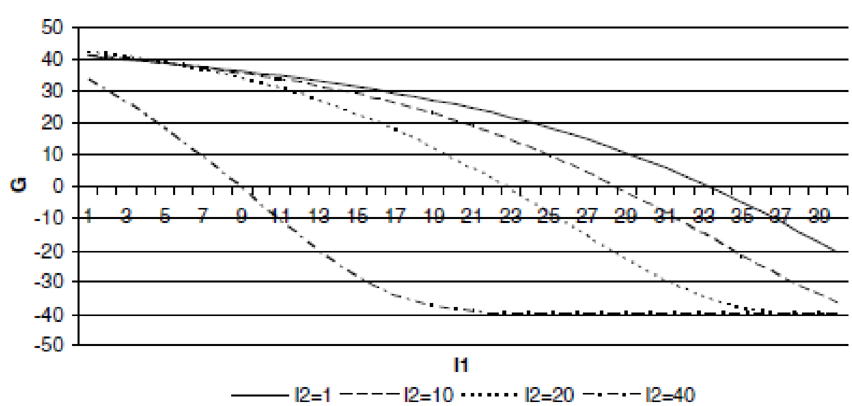

Figure 1: Incremental gain (G) as a function of booking limit (l_1)

From the proof of Theorem 1, we get the following interesting corollary:

Corollary 1: Under the same assumption as in Theorem 1, each airline will protect the same number of seats for the high fare at equilibrium.

The following result indicates that competition will collectively reduce the total number of seats that will be made available for the discount fare.

Theorem 2: Under the same assumption as in Theorem 1, at the equilibrium, the total number of seats available for the low-fare class is less than the total number of seats that will be available for the low fare if the airlines completely cooperate.

Proof: If the airlines completely cooperate, they together will act as a monopoly. Then it follows from Brumelle et al. (1990) that the optimal booking limit η* for the low fare is given by:

$$\eta^* = \max\left\{0 \leqslant \eta \leqslant C : P(Y > C - \eta | B \geqslant \eta) \leqslant \frac{\rho_B}{\rho_Y}\right\}$$

So it suffices to show that

(3)

$$l^* \equiv l_1^* + l_2^* < \eta^*$$

By Theorem 1, we know that

$$l_k^* = \max\Bigg\{ l_k \in S_k : P(Y > C - l_k - l_j^* | B \geqslant l_k + l_j^*) + \frac{C_j - l_j^*}{(C - l_k - l_j^*)(C - l_k - l_j^* + 1)} \times E(Y | Y \leqslant C - l_k - l_j^*, B \geqslant l_k + l_j^*) \times P(Y \leqslant C - l_k - l_j^* | B \geqslant l_k + l_j^*) \leqslant \frac{\rho_B}{\rho_Y} \Bigg\}$$

which implies that

$$P(Y > C - l^* | B \geqslant l^*) + \frac{C_j - l_j^*}{(C - l^*)(C - l^* + 1)} \times E(Y | Y \leqslant C - l^*, B \geqslant l^*) \times P(Y \leqslant C - l^* | B \geqslant l^*) \leqslant \frac{\rho_B}{\rho_Y}$$

Therefore, we must have P(Y>C-l*|B>l*)<ρ_B/ρ_Y. Then, by definition of η*, (3) must be true. This proves the result.

A Numerical Example

To illustrate the impact of competition on seat allocation we extend a simple example taken from Brumelle et al. (1990), which, to our knowledge, is the only paper in the literature that addresses the dependent demands with a numerical example. The following parameters were used in Brumelle et al. (1990): ρ_B/ρ_Y=0.6, that is, the discount fare is 60 per cent of the full fare; the full-fare demand Y follows a normal distribution with a mean 30 and a standard deviation 11.5; the discount-fare demand B follows a normal distribution with a mean 70 and a standard deviation 26.5 and the joint distribution of (Y, B) follows a bivariate normal distribution with a correlation coefficient ρ=0.9. We will use these parameters in our calculation. In addition, we will add two other cases with decreasing correlations (ρ=0.45 and 0.1), to capture the intensity of competition in relation to the dependency level between the two random demands. The results are summarized in Table 1.

From Table 1, we can make the following observations about the equilibrium outcomes of the seat allocation game. First, it is evident that the collective protection level for the full fare is higher than the full-fare protection level under monopoly. Secondly, we notice that the capacity shares between the two airlines do not affect the equilibrium protection level for the full fare, which is a strong indication that both airlines are determined to equally split the full-fare market whenever possible. As a result of this strategic focus on the full-fare market, the equilibrium solution in terms of protection levels is in fact symmetric regardless of the capacity combinations so long as each airline has the capacity to satisfy half of the full-fare market. Thirdly, as the random demands for the two fare classes become less (positively) dependent, the equilibrium protection level will decrease.

Table 1: Equilibrium solutions for the numerical example

	Total system capacity			
	80	*100*	*120*	*140*
(1) Littlewood (1972): ρ=0, single airline				
Protection level	27	27	27	27
Booking limit	53	73	93	113
(2) Brumelle et al. (1990):				
(a) ρ=0.9, single airline				
Protection level	31	35	39	43
Booking limit	49	65	81	97
(b) ρ=0.45, single airline				
Protection level	29	31	34	37
Booking limit	51	69	86	103
(c) ρ=0.1, single airline				
Protection level	28	29	29	30
Booking limit	52	71	91	110

	Airline		*Airline*		*Airline*		*Airline*	
	1	*2*	*1*	*2*	*1*	*2*	*1*	*2*
(3) Duopoly model								
(a) Case 1: ρ=0.9; Capacity split: 50% (airline 1) versus 50% (airline 2)								
Protection level	18	18	20	20	22	22	24	24
Booking limit	22	22	30	30	38	38	46	46
(b) Case 2: ρ=0.9; Capacity split: 40% (airline 1) versus 60% (airline 2)								
Protection level	18	18	20	20	22	22	24	24
Booking limit	14	30	20	40	26	50	32	60
(c) Case 3: ρ=0.45; Capacity split: 50% (airline 1) versus 50% (airline 2)								
Protection level	18	18	19	19	20	20	21	21
Booking limit	22	22	31	31	40	40	49	49
(d) Case 4: ρ=0.45; Capacity split: 40% (airline 1) versus 60% (airline 2)								
Protection level	18	18	19	19	20	20	21	21
Booking limit	14	30	21	41	28	52	35	63
(e) Case 5: ρ=0.1; Capacity split: 50% (airline 1) versus 50% (airline 2)								
Protection level	17	17	17	17	18	18	18	18
Booking limit	23	23	33	33	42	42	42	42
(f) Case 6: ρ=0.1; Capacity split: 40% (airline 1) versus 60% (airline 2)								
Protection level	17	17	17	17	18	18	18	18
Booking limit	15	15	23	43	30	54	38	66

Summary

In this paper, we have discussed a competitive two-fare seat allocation problem between two airlines. We have shown that there exists an equilibrium booking policy such that each airline will protect the same number of seats for the full fare. It is further demonstrated that at equilibrium the total number of seats that are available for the discount fare is smaller than the total number of seats that would be available if the two airlines cooperate. These findings are validated by a numerical example, which further illustrates the impact of capacity shares between two airlines and the level of positive dependency between two fare classes.

This note contributes to the literature by extending Lippman and McCardle (1997)'s model to two sources of uncertainty and complements that of Netessine and Shumsky (2005) by investigating the competitive allocation issues between two airlines confronting a common market. Additional research is needed to address the general multiple-fare competition and joint pricing and allocation problems.

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