TWO PAPERS ON TRANSPORTATION AND SOCIAL WELFARE

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TWO PAPERS ON TRANSPORTATION AND SOCIAL WELFARE

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The main essay of this thesis is found in the first chapter, where I exploit the opening of a new Metro line in Los Angeles to measure the impact on surrounding highway traffic, and the impact on social welfare through different channels. I built a gravity style commuting mode choice model to predict flow between Traffic Analysis Zones in the Los Angeles County area, where mode choice is based on travel time cost, transit fare, car ownership, maintenance and insurance cost, and parking cost. This model allows me to simulate commuter mode switch after transit expansion, or after any type of cost shock in the model. Using the gravity model’s result I constructed treatment measures and estimated the actual effect on real traffic flow in the highway using traffic data from the California Department of Transportation. I find that while there are initially about 2.5% reduction in traffic going through the one highway closest to the new light rail line, it dissipates over 6 months. This also implies there will be no emission reduction from the new light rail line in the long run. I then build a general equilibrium model of the commuting market that allows agents to choose between driving and transit and calculated the marginal welfare under transit expansion. I calculated the welfare gain after opening of the light rail line to be $20 million annually.

The second essay examines transit infrastructure investment and parking subsidy in a two-part-instrument framework. I consider the parking subsidy as a tax
levied on the dirty good and transit investment as a subsidy on the clean good. I then uses the framework developed in the first chapter to examine the welfare change under different parking cost settings. The resulting welfare effect of reducing parking subsidy is positive, both through the various welfare channels according to the general equilibrium model, as well as through increased tax revenue. I found that removing $5 per day in parking subsidy for about 10% of zones located in Downtown Los Angeles creates the highest increase in welfare gain, in the presence of transit expansion. The parking subsidy reduction doubles the welfare gain from transit expansion alone, and the resulting tax revenue increase would even be greater than the welfare gain.
BIOGRAPHICAL SKETCH

Kei Fung Dennis Tai was born and raised in Hong Kong, a Chinese city in Southern China. He attended high school there and attended the Hong Kong University of Science and Technology in 2002 to 2003 majoring in Business Administration and Finance, before immigrating to the United States. After arriving the States, he worked part time while attending the City College of San Francisco, and later transferred to University of California, San Diego and completed his Bachelor degree in 2009, majored in Management Science. After undergraduate, he worked briefly at Google as a contractor. He began his PhD at Cornell in 2010 studying Environmental Economics.
To my family
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I wanted to do a Ph.D. to make myself proud, to have done something that may help make a better future, and something that many would think too difficult to do. I have learned much from and come to appreciate those I lean on and those who have made such an endeavor possible. I have also learned much about myself and life in general through this episode of my life. I leave Ithaca fulfilled, both in my skills and in my heart.

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## TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Biographical Sketch</td>
<td>iii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>viii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xii</td>
</tr>
<tr>
<td>1 On the Benefits of Public Transit Provision: Evidence from Los Angeles</td>
<td></td>
</tr>
<tr>
<td>Expo Line</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1.1 Los Angeles Metro Expo line</td>
<td>4</td>
</tr>
<tr>
<td>1.2 Theoretical Framework: Gravity Model</td>
<td>5</td>
</tr>
<tr>
<td>1.2.1 Model description</td>
<td>7</td>
</tr>
<tr>
<td>1.2.2 Data</td>
<td>9</td>
</tr>
<tr>
<td>1.2.3 Gravity model result</td>
<td>13</td>
</tr>
<tr>
<td>1.3 Empirical Analysis</td>
<td>18</td>
</tr>
<tr>
<td>1.3.1 Data</td>
<td>18</td>
</tr>
<tr>
<td>1.3.2 Empirical strategy</td>
<td>21</td>
</tr>
<tr>
<td>1.3.3 Results</td>
<td>23</td>
</tr>
<tr>
<td>1.4 Welfare Theory</td>
<td>27</td>
</tr>
<tr>
<td>1.4.1 Demand side</td>
<td>28</td>
</tr>
<tr>
<td>1.4.2 Supply side</td>
<td>30</td>
</tr>
<tr>
<td>1.4.3 Shock to infrastructure provision</td>
<td>32</td>
</tr>
<tr>
<td>1.4.4 Marginal welfare analysis</td>
<td>32</td>
</tr>
<tr>
<td>1.4.5 Welfare results</td>
<td>34</td>
</tr>
<tr>
<td>1.4.6 Cost benefit analysis</td>
<td>38</td>
</tr>
<tr>
<td>1.5 Prototype City</td>
<td>40</td>
</tr>
<tr>
<td>1.5.1 Prototype city: set-up</td>
<td>41</td>
</tr>
<tr>
<td>1.5.2 Prototype city: results</td>
<td>43</td>
</tr>
<tr>
<td>1.6 Conclusion</td>
<td>44</td>
</tr>
<tr>
<td>1.7 Tables</td>
<td>45</td>
</tr>
<tr>
<td>1.8 Figures</td>
<td>55</td>
</tr>
<tr>
<td>2 If You Cant Park, Youll Ride: Modelling the Effect of Transit Expansion</td>
<td>68</td>
</tr>
<tr>
<td>and Parking Subsidy Reduction as Two-Part Instruments</td>
<td></td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>68</td>
</tr>
<tr>
<td>2.2 Two Part Instrument</td>
<td>71</td>
</tr>
</tbody>
</table>
2.3 Los Angeles Expo Line .................................................. 73
2.4 Parking Subsidy as Tax Fringe Benefit .......................... 75
2.5 Gravity Model .............................................................. 77
   2.5.1 Model description .................................................. 80
   2.5.2 Policy representation .......................................... 84
   2.5.3 Data ................................................................. 85
2.6 Baseline Results ......................................................... 90
2.7 Welfare Theory .......................................................... 93
   2.7.1 Demand Side ...................................................... 93
   2.7.2 Supply Side ........................................................ 96
   2.7.3 Shock to infrastructure provision .......................... 97
   2.7.4 Shock to parking subsidy .................................... 98
   2.7.5 Welfare analysis ............................................... 99
2.8 Results – Transit Expansion ....................................... 101
2.9 Results – Transit Expansion with Parking Subsidy Shock .... 105
2.10 Tax implication ......................................................... 108
2.11 Conclusion .............................................................. 110
2.12 Tables ................................................................. 111
2.13 Figures ................................................................. 114

A Appendix ........................................................................... 119
   A.1 Mathematical Derivation–Gravity Model ....................... 119
      A.1.1 Model ............................................................. 119
      A.1.2 Data ............................................................. 121
      A.1.3 Calibration ...................................................... 124
      A.1.4 Prediction Procedure ....................................... 125
   A.2 Mathematical Derivation–Welfare Theory ..................... 126
      A.2.1 Demand Side .................................................. 126
      A.2.2 Supply Side .................................................. 131
      A.2.3 Welfare analysis ............................................. 136
   A.3 Detector Matching mechanism ................................... 145
   A.4 Partial equilibrium as upper bound on system-wide effects .. 148
      A.4.1 Nash equilibrium versus social optimum ................. 148
      A.4.2 Nash equilibrium versus social optimum—linear congestion function ........................................ 150
   A.5 Prototype City: Detail Description ............................... 154
      A.5.1 Prototype city setup ......................................... 154
      A.5.2 Data generating process ...................................... 155
LIST OF TABLES

1.1 Gravity Parameter Estimation ........................................ 45
1.2 Baseline Predicted Flow vs Published Boardings .................. 46
1.3 Network Externalities: Percentage change in flow after expo opening 47
1.4 Own and cross price elasticities .................................... 48
1.5 Treated vs. control group, detector statistics ...................... 48
1.6 Binary treatment, effect on logged maximum flow from covariates 49
1.7 Binary treatment, effect on logged maximum flow from covariates, biweekly aggregated ......................... 50
1.8 Binary treatment, logged maximum flow, matching methods ...... 51
1.9 Welfare Calculation: Summary ...................................... 52
1.10 Baseline hypothetical welfare calculation .......................... 53
1.11 Welfare Calculation: Total Lifetime Benefits ...................... 54
2.1 Welfare change under different parking cost settings - Before Expo Opening ........................................ 111
2.2 Welfare change under different parking cost settings - After Expo Opening ........................................ 112
2.3 Tax implication from parking subsidy reduction ................... 113
## LIST OF FIGURES

1.1 Parking setting in gravity model. Yellow dots denote zones with parking cost. ................................. 55  
1.2 Predicted impact of the Expo line on TAZ transit shares at origin . 56  
1.3 Predicted effects on station flows ........................................ 57  
1.4 Predicted impact of the Expo Line on detector level flows ...... 58  
1.5 Map of balanced panel detectors ...................................... 59  
1.6 Treatment effects ............................................................. 60  
1.7 Matched transactions ....................................................... 61  
1.8 Propensity score matching: common support .................... 62  
1.9 Prototype city: density layout ........................................... 63  
1.10 Prototype city: transit network ......................................... 64  
1.11 Prototype city: road network ........................................... 65  
1.12 Prototype city: identifying Los Angeles city centers .......... 66  
1.13 Transit market: demand and supply shift ....................... 67  

2.1 GIS network representation of Los Angeles County, public transit . 114  
2.2 GIS network representation of Los Angeles County, driving ...... 115  
2.3 Employment density by TAZ, Los Angeles County ............... 116  
2.4 Predicted baseline transit flow .......................................... 117  
2.5 Change in Station level flow on Metro network ................ 118
1.1 Introduction

This paper aims to examine the effect of transit infrastructure provision on commuting traffic, exploiting the Los Angeles Expo line as a quasi-experimentation. Existing economic research on the relationship between public transit provision and traffic congestion does not have a consensus on whether transit generates substantial reduction in traffic congestion. Provision in public transit infrastructure in the form of rail transits (light rail, heavy rail, commuter rail) create new capacity in the transit network and expand the service coverage to provide network externality benefits. By creating new routes the system would lure drivers on the margin to switch from solo driving to rail transit.

Many studies have supported this view. For example, Anderson [10] studied the LA county transit strike of 2003 and found that highway delay increase substantially during the strike where transit service was completely shut down. Ewing et al. [16] used a quasi-experiment design and estimated that with Utah’s TRAX light rail line traffic on the street decreased by 7,500 to 21,700, which saves about 362,000 gallons of gasoline and prevents about 7 million pounds of CO$_2$ from being emitted each year. Litman [25] asserted that improving travel options
through providing quality travel alternatives to driving can benefit all travelers on a corridor, both those who shift modes and those who continue to drive. Shifts to alternative modes not only reduce congestion on a particular highway, they also reduce traffic discharged onto surface streets, providing “downstream” congestion reduction benefits. Research by Winston and Langer [42] indicate that as rail transit expands in a city, motorist’s annual congestion costs drops. Baum-Snow and Khan [11] used a regional traffic model and found that commute travel times to be lower in areas near rail transit than the control group. And Nelson et al. [26] found that rail transit’s benefit through congestion reduction exceeded its subsidies.

Studies in the induced travel demand literature however would take an opposite view. The hypothesis is that any extra capacity in the highway would not change the level of congestion but would simply be refilled. For example, Duranton and Turner [15] proposes the “fundamental law of road congestion” which hypothesize that highway capacity determines the VMT and therefore level of congestion. Therefore additional highway capacity produced by public transit provision will be met by equal level of new vehicle traffic, refilling the capacity and returning congestion back to original level. Cervero and Hansen [14] and Fulton et al. [20] finds strong statistical evidence, using data from California and mid-Atlantic region, that congestion on highway and vehicle miles traveled are determined by total level of highway capacity. Some researchers would even criticize that urban rail transit investment are ineffective at reducing traffic congestion. (Stopher [34], Taylor [36])
Most of the existing economic research, however, considers the effect of congestion only on an aggregate level, and relatively little has been done to consider public transit provisions that extend existing network, how network effect affects the aggregate outcome, and where and to what extend the induced demand or refilling of new capacity would affect the final result.

In this paper, we present a gravity style choice model to predict the effects of transportation cost changes on highway traffic and metro flows. Exploiting the opening of the Metro Expo line, a new light rail line in Los Angeles, and calibrating the model with Census data, we then use our prediction to calculate treatment areas on the highway network and compare our estimated flow reduction from the model to actual traffic data. Our analysis suggest that while there is some short-run effect in traffic, some 2.5% reduction in average vehicle flow, in the long run (about 6 months) congestion would return to the same level, even though ridership increased.

The layout of this paper is as follow. Section 1.2 describe the theory and data for the gravity model, which predicts traffic flow on the detector level. This would allow us to create treatment and control set for our empirical analysis. Section 1.3 describe the data and empirical framework as well as the result of our analysis on the actual highway traffic data, using a regression discontinuity design-difference in difference framework. Section 1.4 presents a general equilibrium welfare model where we setup a GE model between commuters and transit agencies, and decompose the marginal welfare of transit expansion into different components. Finally,
1.1.1 Los Angeles Metro Expo line

The Expo Line is a new light-rail line of the Los Angeles Metro network, running westward between Downtown Los Angeles and Santa Monica. Phase one of the project, which runs for 8.6 miles from Downtown to Culver City, started construction in 2006 and opened in April 2012, with phase two expected to open in 2015. Currently there are twelve stations on the line, with vehicles running on mostly at-grade rail. The first phase cost over 900 million to build, which is mostly funded through tax increase. The Expo Line has seen increasing ridership, growing from an average 26,000 boarding on a weekday (totaling over 700,000 boarding per calendar month) in June 2013 to over 30,000 a day in July 2015. The Expo Line shares light rail vehicles with the Blue line for compatibility. The rolling stocks are Siemens P2000 and Nippon Sharyo P865, with a passenger capacity of 180 and capable of maximum speed of 55 mph. The 8.6 mile trip takes 29 minutes to complete, which averages to about 17.8 miles per hour. Trains are operated in 3-car arrangements during peak hour, with 6-minute headway.

The opening of Expo line in 2012 presents a great opportunity for quasi-experimental style analysis of infrastructure investment. Furthermore, the availability of high resolution panel data from traffic detectors in the Los Angeles area allows us to construct rich dataset for analysis. The Expo line is also in close
proximity to interstate I-10, a major highway in the area. This presents a valuable opportunity to analyze the substitution effect of transit infrastructure has on highway traffic.

### 1.2 Theoretical Framework: Gravity Model

Our ultimate goal to estimate the welfare change using a spatial approach would require knowledge of the change in number of drivers and commuters, and on what routes the changes are taking place. To do this, we have derived a spatially detailed gravity style model to provide us with the detailed pairwise origin-destination trip information.

In the international trade literature, gravity model is often used in modeling the bilateral flows of trade between countries. These models are based on the physics principle of gravity force between two masses. This force is dependent on the masses of the objects in the system, as well as the distance between each objects. In the trade literature as well as in transportation engineering literature, physical masses are replaced with economic mass, such as GDP in trade literature, or population and employment in transportation. Similarly, distance is replaced by travel costs. In addition, push and pull factors are often used in place of a single mass measure to better represent different system dynamics.

Gravity model is first used in trade as early as 1962 (Tinbergen). The model
gains in popularity several years later, after Anderson [9] formalized the gravity theory in his AER paper to estimate effects of institutional policies on international trade flows. The model has since been used to study trade patterns in Europe among the European Economic Community (EEC) and European Free Trade Association (EFTA) (Aitken [7], Sapir [30]), to study trades between OECD countries in 1970s (Geraci and Prewo [21]), international trade and flexible exchange rates (Abrams [4]), as well as trade and economic-political factors in the US (Summary [35]).

Notably, gravity model also has a strong presence in the transportation engineering literature. In transportation engineering, transportation forecasts often employ the “Four-Step models”, which includes trip generation, distribution, mode choice, and route assignment, where the gravity model plays an important role in distributing trips between zones in the system.

Although gravity model is popular in trade and engineering where it was favored for its greater degree of spatial resolution, it is less common in transportation economics, where often simpler representative agent or lower dimensional models are preferred.

In this study, we adopt a gravity model to the commuting setting to analyze changes in commuting pattern after a shock in transit infrastructure. We use population as a push factor and employment as a pull factor, with data from the census, as well as transport cost data from GIS modeling. Once the baseline model is properly calibrated, this approach allows us the freedom to model geographic
heterogeneity in the system in the layout of the transit system, where oftentimes effectiveness would depend on placement of infrastructure. Compared to a zone level discreet choice model, a gravity model would also allow mode choices to be represented in fractions.

In practice, the gravity model is first calibrated using parameters before the opening of the Expo line. This serves as the baseline case. Then, the Expo line is added to the transit network to represent the shock in infrastructure provision.

1.2.1 Model description

In general, gravity models in the trade literature take the following functional form:

$$ F_{ij} = f(Y_i, Y_j, C_{ij}) $$

where $F_{ij}$ is the value of trade between country $i$ and $j$; $Y_i, Y_j$ are the GDP of country $i, j$ respectively; and $C_{ij}$ is the resistance/enhancement factor, which is the distance between the two countries in trade.

Expanding on the trade model to include push and pull factors, we model the bilateral flow of agents $F_{ij}$ as follow:

$$ F_{ij} = GP_i^{\beta_1} E_j^{\beta_2} C_{ij}^{\beta_3} \quad (1.1) $$

and estimate the log form of the following equation:

$$ F_{ij} = \exp(\ln G + \beta_1 \ln P_i + \beta_2 \ln E_j + \beta_3 \ln C_{ij}) \epsilon_{ij} \quad (1.2) $$

7
In the above equations, $F_{ij}$ is the number of trips from $i$ to $j$, which I will refer to as origin to destination TAZ level flow. $P_i$ is the population at $i$, $E_i$ is the employment at $j$, and $C_{ij}$ is the overall trip cost traveling from $i$ to $j$. $C_{ij}$ combines both driving cost and transit cost according to transit share $S_{ij}$. The coefficients $\beta$ represent how these factors affect flow. $\beta_1$ corresponds to the elasticity of trips with respect to the population at the origin; $\beta_2$ corresponds to that of trips with respect to the employment at the destination; and $\beta_3$ corresponds to the overall cost of travel between a particular origin-destination pair.

We assume that there are two distinct transport modes available to the agents, providing different connectivity at different quasi-monetary costs: individual transportation (driving alone), and public transit. Driving cost includes the value of time spent driving, gas price, maintenance and insurance, and parking fee; transit cost includes time cost to get to transit station, transit fare, and the value of time spent on transit.

The transit share $S_{ij}$ is calculated as follow. For each of driving and transit, agents obtain utility from their relative consumption between the two modes:

$$U_{ij} = \alpha + \gamma \Delta_{ij}$$

where the cost differential $\Delta_{ij} = (C_{ij}^{PUB})/(C_{ij}^{IND})$ is the ratio between transit cost and driving cost, which includes both travel time and other costs; $\gamma$ is the cost parameter; and $\alpha$ is a constant.

The transit share for an origin destination pair from $i$ to $j$ ($S_{ij}$) is represented
as a logit equation:

$$S_{ij} = \frac{\exp(U_{ij})}{1 + \exp(U_{ij})}$$

The parameters $\alpha, \gamma$ are calibrated using actual transit share reported from Census CTPP data. Overall cost is then a linear combination of driving and transit cost:

$$C_{ij} = S_{ij}C_{ij}^{PUB} + (1 - S_{ij})C_{ij}^{IND}$$

$\varepsilon_{ij}$ is the error term following a Poisson distribution. The choice of Poisson over normal distribution is to avoid issue where OD pairs with zero flows, which is quite common especially for far away origin-destination pairs. Furthermore, Silva and Tenreyro [32] argued that OLS on the log form could lead to biases, and conversely proposed to estimate the above form using Poisson pseudo-maximum likelihood estimator, which is often used for count data.

The equation is then estimated with pre Expo line opening settings, using flow data from SCAG, census population and employment data, and the GIS cost data. A more detail discussion of the data follows in the next section. With this equation calibrated, we can predicted the bilateral flow $F_{ij}$ between any two zones, for all the zones in our model.

### 1.2.2 Data

In this section we will briefly discuss the various data used in the model.
GIS and driving/transit networks

To calculate trip cost and route using ArcGIS, we have compiled a geographically accurate transit and driving network, using publicly available shapefiles. US Census Bureau and many transportation agencies of major cities publish shapefiles that maps the actual street, bus and rail line as well as stations. These shapefiles are often very accurate and form the basis of our network dataset.

For our analysis, we use the Census bureau published TIGER/LINE shapefiles to construct the driving network in Los Angeles County. The TIGER/LINE street shapefiles contain every streets, highway, and ramps with very high precision. We assume local driving speed to average about 15 mph. \(^1\) For highway we assign each segment speed based on PeMS’ speed data. \(^2\) The speed data is used to calculate travel time which goes into the travel cost calculation.

To compile the transit network, we once again start with the TIGER/LINE road network. It is then augmented with bus and light/heavy rail shapefiles. In the Los Angeles County, the Los Angeles Metro Transit Authority (LAMTA or LA Metro) runs most of the buses and rail services. LAMTA provides shapefiles on all the transit services they provided. We assume that all agents will have to walk on the

\(^1\) Note that GIS modeling could not model waiting at stop sign, traffic lights, as well as the waiting time for pedestrian crossings. I have made the assumption of 15 mph as a “representative speed” of a trip traveling through local roads which would include all of the above nuances.

\(^2\) The Department of Transportation in California operates a Performance Management System (PeMS). Loop detectors are installed on all major highways in California to measure speed and flow of traffic on 30-second intervals. These detectors can be thought of as nodes in our driving network, similar to rail stations in the transit network.
street to take bus and/or rail. The walking speed is assumed to be 3 mph. The speed of local bus service is assumed to be 8.8 mph, and rapid bus services is 13 mph, following Anderson [10]. In Los Angeles, there are six rail lines. Red and Purple lines are underground heavy rail, and the Gold, Green, Blue, and the new Expo lines are light rails running mostly above ground. We deduce the speed of the rail lines from their length and their schedule. Speed range from 18 mph to 30 mph.

The driving and transit networks compiled using the above shapefiles is capable of point to point routing. This is essential for the construction of commuting cost matrix for both travel modes as well as generating detector level travel statistics. The statistical unit in our analysis is the Traffic Analysis Zone. It is defined by the Southern California Association of Governments to use with transportation planning. In our model there are 2241 zones, connected by the above networks.

CTPP data

As described previously, the gravity model takes population as a push factor, and employment as pull factor. These factors serve as the mass in our gravity model. Census Bureau produces a special set of tables called Census Transporta-

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3This assumption reflects a limitation in our GIS approach of not able to model park and ride. However the number of park and ride commuters is arguably very small compared to the overall number of commuters.

4The rail speeds are: Red line 29 mph, Blue line 22 mph, Green line 34 mph, Gold line 22 mph, and Expo line 17.8 mph. The Green line has a very high speed as a light rail, as it travels mostly on elevated track along a freeway with stations spread across a long distance.
tion Planning Products (CTPP), specifically for transport planners. The CTPP uses the decennial census and American Community Survey (ACS) as the data source. The CTPP is disaggregated into TAZs which is not available in the general census dataset. In addition, the CTPP tables also provide mode choice by TAZ, which is used to calibrate the gravity model.

Parking

In the previous section we defined driving cost to include value of travel time, fuel, maintenance, insurance, as well as parking cost. Parking cost is specific to agents who drive to work. In our model this is implemented to trips with certain destination TAZs where parking would not be free. During the OD cost matrix calculation, trips going to the selected zones around downtown area would incur additional parking fee. This step is done before mode choice is determined.

We select locations for non-free parking based on the report from ULTRANS center at UC Davis (ULTRANS [39]). A subset of zones around the CBD in Los Angeles is designated to have non-free parking. There are 223 zones in total with a parking cost, roughly 10% of all zones. Figure 1.1 maps all the TAZs where parking cost applies. Note that while many American commuters enjoys free parking, commuters working in downtown area of major cities such as Los Angeles often would still need to pay for at least some of their parking cost due to scarcity of parking in dense downtown areas.
1.2.3 Gravity model result

In this section we report the main results of the gravity model. Our model’s core output is flow between origin-destination pairs, and the change in flow when a shock takes place. The cost of traveling between certain origin-destination pairs changes when a new transit line opens. This leads to mode switch for certain marginal agents, reflected in a change in the transit share for the affected OD pairs. This can be observed through three channels: the TAZ level flow, detector level, and the transit station level.

TAZ level

The first channel we can observe the gravity model’s result is to consider the change in transit share at trip origins after expo line opens. As new transit line is made available, transit trip cost less, and the cost differential $\Delta_{ij}$ drops for the affected OD pairs. Transit share in these pairs would increase, and it can be observed as an increase in transit share at trip origins.

Figure 1.2 maps the predicted change in transit share, as a result in the opening of the Expo line and the subsequent drop in trip cost. The overall transit share in origin is simply the total number of transit trips divided by the total number of trips originate from one zone. Not surprisingly, the zones surrounding Expo line sees the largest increase in transit share (red is higher increase, green lower). There is also some degrees of spillover effect, evident by the slight increase in the zones.
in nearby lines (Blue line running North-South from downtown area, a short segment of Red line running towards the Northwest towards North Hollywood, and a short segment of the Gold line running Northeast).

Table 1.3, which reports the percentage change in ridership of each line after Expo line opening, shares similar result in externality. A more detail description of the table is in the next section, but for now note that the Blue and Gold line, as well as some Red line stations sees increased flow.

**Transit station level**

The second channel to observe the gravity model’s result is to consider the change in number of trips going through a specific transit station. That is, after the expo line opens, what’s the change in trips that passes through e.g. the Union station in Los Angeles. In order to do this, we have developed a mechanism that identifies what stations would be part of a particular trip between an OD pair. Detail of the mechanism is discussed in section A.3 in the appendix. With this mechanism we can match OD pairs to any transit stations in our system. The last step is to aggregate all the OD level flows that belong to each station to get the number of trips flowing through each of the stations (station level flow).

Table 1.2 reports the model predicted flow and compares it to published ridership statistics from LA Metro. The number reported in the first column is the number of flow of the station in each line which has the highest number of trips travel-
ing through (maximum flow). The second column reports the ridership statistics from LA Metro, which is the total number of boarding of each rail line during weekday. We note that this is not a 1-to-1 comparison, due to the limitation of our matching algorithm not able to report boarding and alighting information. Our gravity model predicted flow captures about 40 to 50% of the actual ridership in most of the transit lines. The Expo line flow prediction of 25,472 trips appears to be much overestimated compared to the published ridership of 13,897, which is the average weekday ridership of the line in 2012. However in subsequent years the ridership of Expo line has risen steadily, reaching over 30,000 in 2015. This trend puts the gravity predicted flow of the Expo flow in line with the rest of the system.

Table 1.3 reports the percentage change in ridership of the transit lines after Expo line opens. The column reports the mean, minimum, median, and maximum station level flow change because of the infrastructure shock. For example, the first row reports that among the stations in the Blue line, on average flow through blue line stations increased by 5.17%. At least one station reports no change in flow, and the station that sees the largest change reports a 26.41% increase in flow. The results reported in this table demonstrate our model’s ability to represent network externality. Similar to the results observed in TAZ transit share change, we observe that the Blue, Red and Gold line has increase flow after the Expo line opened. Figure 1.3 maps the flow change by station, and the network externality effect becomes apparent. Segments of the Blue, Red and Gold line sees an increase in flow as Expo line enables transit riders to reach new destinations. Also notable
is that the effect seems to fade after a certain distance on all the lines, which is around 1 hour of travel time from the end of Expo line.

For a few stations in the system, negative externality is observed. This takes place along the Purple line stations (the two red stations North of Expo line) and the western segments of the Green line. This also matches with the intuition that Expo line provides new travel option for agents traveling to and from the West part of the county, which was previously served mostly by the Green line, Purple line, with transfer to local bus service.

As we have full control of every aspect of the cost of travel, we also compute the own and cross fare elasticity of transit ride. Table 1.4 reports the fare elasticity based on the predicted change in flow under shock from our model. Our model reports an own fare elasticity of -0.3283, and a cross fare elasticity on driving to be 0.0407, and the result is consistent with a fare increase and decrease. The reported elasticity is also very much in line with the literature, with own price elasticity in the range of -0.2 to -0.5, and automobile elasticity of 0.03 to 0.1. (Litman [24])

**Highway detector level**

The third channel to observe the gravity model’s result is through the number of trips going through a specific traffic detector on the highway. The California Department of Transportation operates and maintains a network of traffic detectors on all major highways across California. A more detail discussion of the system
is presented in the empirical section data discussion. For our gravity model, we include these traffic detectors as features in our transportation network in order to obtain “detector level” flow statistics of driving agents in the system. The same mechanism mentioned in the previous section is used to match detectors to OD trip, and individual OD trips are aggregated by detectors to obtain detector level flow statistics. The benefit of having this channel is twofold. The first benefit is the ability to create visualization of traffic relief of the transit policy similar to the map that we presented in the previous section. The second benefit is that this approach would allow us to identify segments of highway as treatment group of the policy in non-arbitrary fashion.

Using the same procedure as outline in previous section, we plot the change of driving flow by detector in figure 1.4. The pattern is intuitive and also similar to the transit case. The detector level flow change is most pronounced along the I-10 highway segment running parallel to the Expo line. The I-10 segment running from downtown Los Angeles to Santa Monica is one of the busiest highways in the region, and also happens to run in parallel to the entire length of the Expo line including Phase 2. As the highway is less than 2 miles from the Expo line at the farthest point, it is expected that the majority of the mode switch would happen to OD trips with origin or destination that is within this area. The figure also shows positive spillover effect of flow on connecting highway segments, similar to the case of station level flow.

The highway detector level flow change provide us with a clear and non-
arbitrary way to define treatment group for our empirical analysis in the next section, where we study the actual effect of transit infrastructure provision on traffic and congestion.

1.3 Empirical Analysis

So far we have considered the hypothetical effect of the Expo line on highway traffic. Based on the gravity model prediction, the segment of highway closest to the Expo line would see about 5% reduction in traffic flow. This theoretical implication is testable using empirical analysis on real world traffic data.

1.3.1 Data

The California Department of Transportation operates and maintains a system called the Performance Management System (PeMS), which covers most major highways in the state. This system collects and archives real time data in a wide variety of formats to facilitate research in transportation. PeMS draw data from loop detectors and weight-in-motion sensors installed in highway vehicle detector stations (VDS) throughout California. Loop detectors are inductive coils embedded on highways, and can keep count of the number of cars moving over the coil. There are two main types of loop detectors. Single loop detectors can record the number of cars driven over it over 30 seconds (the smallest time unit of mea-
sure in the system); double loop detectors can record both the number of cars and also speed by tracking the time difference between each coil’s activation. Weight-in-motion sensors work on pressure instead of induction and produces similar records. The network of detectors is extensive, covering both regular highway lanes as well as HOV lanes, on and off-ramps, and highway-to-highway ramps. In addition to the extensive loop detector data, the PeMS system can also access data from California Highway Patrol (CHP) Incident data, as well as lane closure information from the Caltrans Lane Closure System. Today the PeMS system has over 35,000 detectors reporting data at 30 second intervals. There are 4,353 detectors in the PeMS system for the Los Angeles County, of which 1,692 are detectors on the main lanes, and 760 are detectors on HOV lanes. For our empirical analysis of traffic congestion, we will use the main lane detectors.

The detector level count data of traffic flow from PeMS is the most comparable data we have to the prediction from our gravity model. To ensure comparability with our settings of the commuting gravity model, only detector counts recording on week-days are used in the analysis. The data is aggregated to temporal bins defined based on the time relative to the treatment. In our analysis we used both bi-weekly bins and monthly bins. Afterwards, a balanced panel is generated where we included only detectors with observations in all period bins. The observation period is restricted to a symmetric time window with the same number of periods before and after the Expo line opening.

We define a binary treatment variable according to the level of predicted flow
from our gravity model. We identify a detector as treated if the predicted change in driving flow is in the upper 5 percentiles of all detectors, and identify detector as control if predicted changes in flow is below the 10th percentile. The buffer of 5 percentile is assigned for the following reason. Highway and roads are inherently connected, and traffic level changes in any segment of the freeway will unavoidably spillover to connected segments. The buffer would help to identify the effect more precisely. In addition, by including a buffer in our treatment and control definition we can identify with a higher confidence that the control group detectors are not treated. The treatment and control group are mapped in figure 1.5.

We also define, as an initial step, a spatial window around the Expo line that’s roughly 12 miles to the north and south of the Expo line, and 10 miles to the east of the Culver City station. The window is represented as the grey rectangle in figure 1.5. We define this spatial as an initial attempt to create better matching between treatment and control detectors, as an untreated road segment that is located on the other side of the county has little chance to share similar transport characteristics as the treated segment. This is especially true for large county such as the Los Angeles County. The East-West highways included within the spatial window are often considered alternative routes for commuters traveling in this part of the county. Later on we also implement other types of treatment and control matching algorithms.

Table 1.5 reports the average log point change in traffic flow for the treatment
and control group. We observe that the 5 percentile buffer to separate the two groups achieve the intended purpose of ensuring that control groups can reasonably be expected to have remain untreated, while at the same time allows for control observations in reasonable proximity to the treated, thereby ensuring some similarities in the location’s characteristics.

From table 1.5, we can see that the 5 percentile buffer introduced to separate the two groups achieves the intended purposes of ensuring that the control groups can reasonably expected to have remained untreated, while at the same time allows for control observations in reasonable proximity to the treated, thereby ensuring some similarities in the location’s characteristics.

Next we describe the empirical strategy for this analysis.

1.3.2 Empirical strategy

We start with a standard quasi-experimental strongly balanced panel setting for our empirical exercise. Our specification can be viewed as a hybrid model that combines features of Difference-in-Differences and Regression Discontinuity Design specifications (Similar to [5]). We borrow the counterfactual aspect from the Diff-in-Diff (i.e. the control group) and the way we treat time as a running variable from the regression discontinuity design. By removing shocks unrelated to the treatment via the control group, we are able to not only identify short-run effect (from standard RDD), but also long-run trend.
We estimate a linear regression of the form:

$$Y_{it} = \alpha_1 T_i \times TREND_t + \beta_1 T_i \times POST_t + \beta_2 T_i \times TREND_t \times POST_t + \varphi_t + \omega_i + \epsilon_{it} \quad (1.3)$$

where $i$ and $t$ index detectors and periods (months after opening). $Y_{it}$ is the outcome, logged total flows per lane. $T_i$ is the binary identifier for treated group as defined previously. $TREND_t$ is a monthly running variable scaled to be zero in the 30 days before the expo line opening and 1 during the 30 days after. $POST_t$ is a binary variable taking the value of 1 if $TREND > 0$, that is after Expo line opens. $\varphi_t$ and $\omega_i$ are time and detector fixed effect that absorb all time-invariant spatial heterogeneity and macroeconomic shocks common to the study area. The parameters of interest in this specification are $\beta_1$, which gives the short-run adjustment at the time of the intervention as in a standard Diff-in-Diff, and $\beta_2$, which captures long-run effect on trends after the line’s opening.

One critical identifying assumption in a Diff-in-Diff model is that the control group would have to respond in exactly the same way to macroeconomic shocks as the treated group. In reality this assumption is often difficult to satisfy, especially with spatial data. In our empirical setting we allow for heterogeneous pre-trends via the $T_i \times TREND_t$ term. By controlling for individual pre-trends we are able to slightly relax the identifying assumption.

Identification also rests on the assumption that the heterogeneity in trends between the two groups can be extrapolated from the 10 month before the intervention to the ten months after. It is notable that in some empirical settings the pre-treatment trends may capture the effects of the treatment, i.e. if there are an-
ticipation effects (e.g. in capitalization effects). However in our setting the possibility of anticipation effect is low, since travel pattern should arguably not change before the actual implementation or availability of the transport innovation has taken place.

Our baseline specification imposes a relatively restrictive linear function on the trend. Note that given the difference-in-difference part of our specification, the trend need not accommodate macroeconomic shocks as in typical Regression Discontinuity design. All that’s needed is for the functional form to be flexible enough to capture possible pre-trends and to allow for a decomposition of the treatment effect into a short-run and a long-run effect.

1.3.3 Results

Baseline models

Table 1.6 shows results from our regression using the binary treatment definition. Column 1 to 3 uses all control detectors, while column 4 to 6 uses the subset of control group based on the spatial window defined previously. Result from column 1 shows that without controlling for pre-trend and post-Expo trend adjustment, there is about 0.01 log points of reduction when Expo line opens. The sign of the short run adjustment matches with our intuition that there is traffic reduction on the highway when Expo line first opens. Controlling for both pre-trend
and post-Expo trend our regression estimates a statistically slightly less significant reduction of the same magnitude and post-Expo adjustment of 0.00235 per month, implying the effect of traffic reduction dissipates in around 4.5 months. Restricting the control set to within the spatial window results in more efficient predictions of 0.0263 log point reduction in traffic, and post-Expo adjustment of 0.00433 per month, and effect dissipates around 6 months. Figure 1.6 provides a graphical illustration of the estimated pre-trend and the treatment effects (Table 1.6, column 6).

Table 1.7 repeats the same regression but uses bi-weekly aggregated data. Significance of the short run effect mostly disappears if we use every detector in the control set. Once we start restricting to the spatial window subset we are able to obtain very consistent result for the short term adjustment as we see in the previous table. The regression predicted a 0.0228 log point reduction of traffic in the short run and post-Expo adjustment of 0.00253 per 2 weeks, which implies roughly 18 weeks or 4.5 months of adjustment before highway capacity is refilled. While the adjustment period varies a bit, the overall message is clear: in the short run there’s about 0.025 log points reduction in traffic, and the reduction disappears within half a year. The results are in general consistent and become more statistically significant once the restricted spatial window sample is used. These results suggest that after Expo line’s opening, the relatively lower congestion at the treated highway segment attract additional drivers until the previous level of congestion is restored. These results aligns almost perfectly with the fundamental law of congestion [15], which suggests that the increase in highway capacity
will be met by an equal increase in induced demand. Thus it is difficult to tackle congestion through increasing supply of public transit capacity alone.

**Matched models**

As previously discussed, one critical identifying assumption in quasi-experimental research, conditional on individual and time effects, is that the treated and control subjects respond to macroeconomic shocks in the same way. That is, in the absence of the intervention both groups would have followed exactly the same trend. In another words, this assumption requires the treated and control group to be very similar other than receiving treatment. In a spatial setting with geographic and demographic heterogeneity this condition is difficult to satisfy credibly, because road segments at different location are understandably different and serves different sets of drivers on various parts of their trips. Such differences across the treated and the control group may lead to heterogeneous responses to macroeconomic shocks, which will lead to different pre-trends. Thus in our baseline specification we try to deal with this heterogeneity by allowing for heterogeneous pre-trends, and making the identifying assumption that without the intervention these pre-trends would have continued.

To the extent that similarities in observable characteristics make it more likely that subjects respond homogeneously to the macroeconomic shocks, we can strengthen identification in quasi-experimental designs by choosing the control group that resemble the treated using observable characteristics. In the base-
line models discussed above, we have matched control detectors to the treated based on proximity using a spatial window, implicitly making the assumption that nearby freeway segments are more likely to be similar.

Table 1.8 provides a series of estimates as we compare various matching methods for defining the control group. Column 1 and 2 comes from table 1.6 column 3 and 6 as a baseline comparison. Other columns in the table reports regression result using 1 to 1 nearest neighbor matching of one or more of the following characteristics: pre-trend, detector’s distance to CBD, detector coordinates in lat/lon, direction of highway segment of the detector, and the pre-Expo flow level. Notice that in most cases the story is the same: a short run reduction which disappears within 6 months. The short run traffic reduction effect ranges from 0.02 log points to 0.0385 log points.

While the magnitude of traffic reduction varies, the overall story that the short run effect disappears in a short time contributes the most to our welfare analysis in the next section. The idea is that, regardless of how many cars the Expo line takes out from the nearby highway, the effect goes away in the long run, which nullify any welfare benefit in the form of emission reduction.

In addition to the Nearest Neighbor matching as shown here, we also considered adopting the increasingly popular propensity matching technique [13, 29]. However, we note that reweighing observation with the propensity score is problematic in our application, as it is the nature of the treatment that detectors with similar spatial characteristics that predict treatment will also be close to the treated.
Therefore it is not possible to properly find an untreated highway segment that is within the range of propensity scores of the treated areas, and cannot satisfy the common support requirement with propensity score matching. For example, figure 1.8 illustrate the resulting propensity distribution when we perform matching on pre-trend, distance to CBD, and detector coordinates, and notice how the graph has little common support compared with other cases where propensity score can be implemented.

1.4 Welfare Theory

To study the social welfare change from the shock of transit infrastructure, we devise a general equilibrium model with households as demand side and transit authority as supply side. The model would allow us to decompose marginal welfare into various components, and provides the framework to analyze welfare change.

Our model follows the overall structure of Parry and Small [27]. We augment the model with an extra component representing transit service coverage, to model transit infrastructure investment. A more detail explanation of variable and equations is available in the mathematical derivation section A.2 in the appendix. The basic model structure is as follow.
1.4.1 Demand side

There are two regions in the economy, A and B, with $N^A$ agents in A and $N^B$ agents in B. We define location A as the location where transit expansion takes place, without loss of generality. There are total $N = \sum_{i,l} N^i_l = N^A + N^B$ agents in the economy indexed by $i$ and superscript $l$ for location. Each agent in each location choose to consume numeraire good $X^i_l$ and commute $M^i_l$ miles on either car or public transit at peak or off peak period, $M^i_l = \{M^i_{jl}; j = C(\text{car}), T(\text{transit}), k = P(\text{peak}), O(\text{offpeak}), l = A, B\}$.

Each agent is endowed with exogenous income $I^i_l \in [I^i_l, \bar{I}^i_l]$. In addition, agent’s utility also depends on $\Gamma$, a composite travel time cost function, and $Z$, the pollution/Accident externalities. Thus, the agent’s utility maximization problem is:

$$\max_{X^i_l, M^i_l} u^i_l(X^i_l, M^i_l, \Gamma^i_l) - Z$$

subject to:

$$X^i_l + \sum_{k,j} P^{jl}M^i_{jl} = I^i_l - T A X^i_l$$

The travel time cost function $\Gamma^i_l$ is defined as follow:

$$\Gamma^i_l = \Gamma(T^i_l, W^i_l, A^i_l, C^i_l, E^i_l).$$

Time cost $\Gamma^i_l$ is increasing on four components: $T$, $W$, $A$, $C$ and decreasing on $E$. The components in $\Gamma$ are defined as:

- $T^i_l$: travel time cost $= \sum_{i,l} t^{i,j}_l M^{i,j}_l \times VOT_i$
• W: waiting cost = $\sum_k w^{kT} M_i^{kT}$
• A: accessing cost = $\sum_k a^{kT} M_i^{kT}$
• C: crowding cost = $\sum_k c^{kT} M_i^{kT}$
• E: network externality = $-\sum_k e^{kT} M_i^{kT}$

Travel time cost ($T$) captures time spent in the vehicle, on car or public transit. $t_{lkj}^i$ is the per-mile travel time defined as:

$$t_{lkj}^i = \begin{cases} 
\sum_k t_{lkC}^i (V_{lkC}^i + \alpha \sum_{h \neq i} V_{lkC}^h) \\
\sum_k t_{lkT}^i V_{lkT}^i 
\end{cases}$$

where $t_{lkC}^i, t_{lkT}^i$ are performance functions and $t_{lkj}^i = \frac{1}{v_i^k}$, the inverse of mode travel speed at peak or off peak. To model congestion, we consider an added penalty term in the time cost as other agent’s choice of vehicle miles consumed $V_{lkC}^j$ contributes to overall congestion linearly at a coefficient $\alpha$. This approach follows Parry and Small [27] where agents are assumed to ignore their own contribution to congestion and only consider vehicle miles contributed by others. We also assume that public transit system (the metro network) is not affected by congestion, so travel time on transit is independent of other agent’s choice. The term $VOT_i^l = \phi I_i^l$ represents the value of time of agent and is a fraction $\phi$ of income. $p_{Cki}, p_{Tkl}$ are the per mile price of driving and transit respectively.

The per mile price for car is $p_{lkC} = p_{0lkC}^i + \tau_{lkC} + P_i$ where $\tau_{lkC}$ is fuel tax, and $p_{0lkC}$ is pre-tax car operating cost per mile, including fuel, maintenance, and insur-
ance cost. Each agent also have to pay parking cost $P_i$. Transit trips do not incur parking cost, and not all driving trips incur parking cost.

Each agent has to pay lump sum tax $TAX_i^l$ such that $\sum_i TAX_i^l = TAX$. The lump sum tax and fuel tax is used to finance operation of public transit.

The first order condition for the agent would be:

$$\frac{U_{M,kl}}{U_X} = q_{ikl} = p_{ikl} + \rho^T l^{kl} VOT_i^l + \rho^W w_{ikl} + \rho^A a_{ikl} + \rho^C c_{ikl} - \rho^E e_{ikl}$$

where $\rho^K = -\frac{u_K}{u_i}$ for $K = T, W, A, C, E$.

### 1.4.2 Supply side

The supply side of our model is the transit agency. Each location $l = A, B$ has a total number of OD pairs $\{\Xi^l\}$ where agents will need to travel to/from. Define $\{\xi^l : \xi^l \subset \Xi^l\}$ as pairs in the universe on which the agency offer public transit service. For each route $i$ in $\{\xi^l\}$ the agency choose $n_i^l$ (vehicle size), $D_i^l$ (density of service / number of vehicles servicing each line), and $V_{i,kl}^T$. Speed $s_i^l$ is assumed to be exogenous, and $s_i^{lT} < s_i^{lC}$ driving speed is faster than transit speed. The number of vehicle miles offered is $V_{i,kl}^T = \text{length of line} \times \# \text{ vehicles}$ and $M = V \times \text{occupancy} = V \times o$.

While each route can be considered a separate market, it is more manageable to consider the system in aggregate. Define the variable $\xi^l = \frac{\text{number of paris in } \{\xi^l\}}{\text{number of paris in } \{\Xi^l\}}$ to represent the extent of public transit coverage in the system. The agency chooses
an overall level of $n, D, V_T$, and resources are subsequently allocated to each route in $\{\xi\}$ optimally. The four variables are:

- **Vehicle size** $n^{jkl} = \sum_{i \in (\xi)} n_i^{jkl}$
- **Density of service** $D^{jkl} = \frac{\sum_{i \in (\xi)} D_i^{jkl}}{\text{number of pairs in } (\Xi)}$
- **Vehicle miles provided** $V^{jkl} = \sum_{i \in (\xi)} V_{i}^{jkl}$
- **Speed** is assumed to be exogenous, and the average speed agents will be travelling on each route in $\{\Xi\}$ if they opt to take public transit is $s^{jkl} = \frac{\sum_{i \in (\Xi)} s_i^{jkl}}{\text{number of pairs in } (\Xi)}$

The agency must balance its budget:

$$TAX + \sum_{jkl} \tau^{jkl} V_i^{jkl} = \sum_{ik} \sum_{j \not\in C} (OC^{k,j} - p^{jkl} \sum_i M_i^{jkl})$$

where operational cost for mode $j$, period $k$ is

$$OC^{jkl} = F^{jkl} + k^{jkl} \sum_i V_i^{jkl}$$

and

$$K^{jkl} = k_1^{jkl} + k_2^{jkl} n^{jkl} + k_3^{jkl} s^{jkl}$$

Here $n^{k,j}$ is the vehicle size and $s^{k,j}$ the transit speed defined above. Notice that we assume $k_1^{k,j} > 0$. As a result, increasing transit vehicle size $n^{k,j}$ and speed $s^{k,j}$ incurs constant marginal cost and decreasing average cost.
1.4.3 Shock to infrastructure provision

In our model, transit investment or a shock in transit infrastructure is modeled as an increase in the routes being covered by the transit agency in location A, $\{\xi^A\}$. As a result the fraction $\xi^A$ also increase, and so does the overall network size $\xi = \xi^A + \xi^B$. The effect is different between the two locations. In location A, where the infrastructure expansion happens, $\xi^A$ increase, and as a result transit services provided in the location also increased. In location B, the effect of an expansion of transit service of A translates to an increase in overall connectivity $\xi = \xi^A + \xi^B$, which increase the network externality bonus for agents in B. As a result, ridership in B increase and in response, the agency will provide more transit capacity through larger cars or higher frequency, leading to the Mohring effect.

1.4.4 Marginal welfare analysis

A detail explanation of the effects for all the travel characteristics is available in the mathematical derivation section A.2 in the appendix. Here we provide a condensed version of the welfare implication from transit expansion. First consider
the total welfare function:

\[
U = \sum_{il} \left[ \tilde{u} \left( \{ p^{jkl}, t^{jkl}, w^{jkl}, a^{jkl}, c^{jkl}, e^{jkl} \}, I, TAX \right) - Z \right]
\]

\[
= \max_{X, \{ M^{jkl} \}, \lambda} \sum_{l} \left\{ u \left( X, M(\{ M^{jkl} \}) \right),
\Gamma \left[ \sum_{kj} t^{jkl} M^{jkl}, \sum_{kj} w^{jkl} M^{jkl}, \sum_{kj} a^{jkl} M^{jkl}, \sum_{kj} c^{jkl} M^{jkl}, \sum_{kj} e^{jkl} M^{jkl} \right]
- \sum_{kj} z^{jkl} V^{jkl} + \lambda \left[ I - TAX - \sum_{kj} p^{jkl} M^{jkl} - X \right] \right\}
\]

(1.6)

Partially differentiating (1.6) with respect to the level of transit coverage \( \xi^A \), and regroup, we have marginal welfare from infrastructure shock as:

\[
MW_{\xi^A} = \left( \sum_{jkl} MB^{jkl}_{TT} M^{jkl}_{\xi^A} - \left( MC^{AKT}_{\text{supply}} - p^{AKT} \right) M^{AKT}_{\xi^A} \right) + \sum_{lk} \left( MC^{Ckl}_{\text{ext}} - MC^{Ckl}_{m} \right) M^{Ckl}_{\xi^A}
+ \sum_{lk} \left( MB^{Tkl}_{\text{scale}} - MC^{Tkl}_{\text{occ}} \right) M^{Tkl}_{\xi^A} - \sum_{k} \left( MC^{BKT}_{\text{supply}} - p^{BKT} \right) M^{BKT}_{\xi^A}
\]

(1.7)

The marginal welfare change outlined above represents different channels through which the increase in transit coverage (infrastructure provision) affects overall social welfare. Note that the marginal welfare terms can be grouped into five components: (a) Driving market wedge–change in market externalities less fuel tax (This includes congestion relief, as well as emission reduction if there’s any); (b) Travel time savings net of marginal cost of supplying new route less
revenue; (c) Transit demand wedge—Scale economies vs cost from increased occupancy; and (d) Transit supply wedge—P/MC gap.

1.4.5 Welfare results

In this section we will discuss the welfare implication of Expo line expansion following the theoretical framework outlined previously. Table 1.9 summarizes the calculation of welfare change after Expo line opens, using predictions from the gravity model.

Table 1.10 presents a detail analysis of each of the components listed in table 1.9. The calculations here follow closely the four components outlined in the welfare theory in the previous section. The four components are as follow.

Panel A reports the welfare benefits from congestion relief. Several steps are required to calculate this component. First we have to calculate the baseline vehicle miles traveled by all the drive-alone trips in the model. This is done by summing up the distance traveled of all the trips at the origin-destination level in the baseline model before Expo line opening. Next, we calculate a “change in delay” measure as follow. We take the baseline flow and calculate the difference in travel time given the baseline flow and speed, compared to the free-flowing speed which is set as the speed limit. This would give us the baseline “delay” measure as “additional minutes per mile compared to travel at speed limit”. We then calculate another “delay” measure for the case with the Expo line. We take
the difference between the delays as the change, which tells us how many minutes are shaved from the travel time per mile driving on a congested road after the Expo line opens. The time saving can be attributed to the substitution effect by the Expo line as it removes marginal drivers from the highway. The time saving also represent congestion relief experienced by drivers. The total value of congestion relief is the total delay saved in minutes times the value of time. This amounts to about $14 million annually in congestion benefits.

Note that the estimated congestion benefit is based on the hypothesis that solo drivers would move to public transit when the Expo line is open (or when there’s a shock to the parking subsidy), and there’s no induced demand on the highway. Recall in section 1.3 our empirical analysis shows that the traffic reduction created by the Expo line dissipates after about 6 months, indicating existence of induced demand. We maintain that the calculated congestion relief benefit provides an upper bound to the welfare gain in the presence of induced demand. In essence, this is the case where agents can obtain higher welfare under partial equilibrium compared to a complete equilibrium, and we have provided a mathematical proof in section A.4 in the appendix.

Panel B reports emission reduction benefits. We first estimate the hypothetical reduction in total vehicle miles traveled, using the total VMT output provided by the gravity model from the baseline and the case when Expo line opens. Emission reduction is then calculated as the change in VMT times the total social cost of emission per mile. Following Bento et al. [12], we obtain the average emission
amount of CO$_2$, NO$_x$, and VOCs per mile traveled, as well as their corresponding marginal social damage in dollar value. Combining these gives us the total emission benefit. This amounts to about $636,000 annually, which is much smaller in magnitude compared to the other components.

Similar to panel A, the emission benefit component of the welfare change is calculated using results from the Gravity model, which presents the hypothetical welfare gain when there is no induced demand on driving. Once the induced demand that refills any extra capacity created by the policy change is considered, there would be no change on the total flow of the driving network. This corresponds to findings in Ahlfeldt, Bento, and Tai [6] where there is virtually no change to the highway level of traffic after five months. As a result, the emission component of welfare gain would be zero.

Panel C reports travel time savings. Following the calculation from our welfare theory, the travel time saving is calculated as the change in travel time times the change in commuters moving to transit. We first estimate the number of commuters switching to transit by calculating for each origin-destination pair the difference in number of transit trips before and after the Expo line opens. The flow change is then multiplied by the transit time change, which is the reduction in transit travel time by each OD pair. This travel time saving figure is then aggregated system-wide and aggregated to find the annual travel time saving in hours. Note that according to the welfare theory, the travel time component of the wel-
fare is the triangular wedge below the demand curve. The dollar amount of travel time saving amounts to $5.5 million annually.

Panel D reports other components in the welfare calculation. The components are as follow.

The scale economy/occupancy cost gap captures the effect of increased transit ridership has on the transit service frequency and capacity. In other words, this component of the welfare tries to capture the magnitude of the Mohring effect. Following Parry and Small [27], we use their parameters on the marginal scale economy per additional passenger mile, and their marginal cost of occupancy per additional passenger mile. We calculate the change in total number of miles traveled on transit across all OD pairs, then multiplied by \((M \text{ scale economy} - MC \text{ occupancy})\). The Mohring effect comes to about $1.5 million annually.

The price/marginal cost gap captures the wedge between transit fare and the operational cost. Most, if not all, public transit system in the US has a farebox recover ratio of less than 70%. This means the fare collected covers less than 70% of the operating cost of providing the transit service. For example, Amtrak is at 85% (Amtrak [8]), San Francisco BART at 40%, and LACMTA is at only 30% (Lindquist et al. [23]). Operation of public transit system thus often requires subsidy from the government. This component in the welfare calculation tries to subtract this subsidy from the total welfare gain. Here, I refer to Parry and Small [27] once again for their parameters on the per mile fare and per mile marginal supply cost.

---

5See figure 1.13
of transit service. This amounts to roughly $11 million per year. Note that this component is negative.

Combining the four components, the total welfare change for the Expo line shock amounts to almost $10 million per year ($9.7 million). We note that our gravity model simulation outputs an annual vehicle miles traveled number that is about 40% of the total vehicle miles traveled commonly cited. This corresponds to our transit flow result as well. As a back of the envelop style correction we consider doubling the trips and miles traveled and as expected the welfare gain will double to just under $20 million per year ($19.45 million).

1.4.6 Cost benefit analysis

Here we consider a basic cost and benefit analysis. Consider the results we have so far. The Expo line at its current speed is estimated to generate slightly less than $10 million in social welfare per year. If we assume a 4% discount rate and perpetual service of transit infrastructure, the present value of the welfare gain would amount to $243 million. If we follow the usual assumption of 40 years of service life for the transit infrastructure, the present value would drop to $192.5 million. Table 1.11 reports the calculations in this section. Now consider that the total construction cost of the first phase of Expo line stands at $930 million dollars as of 2015, we could conclude that the new transit line is being heavily subsidized and could not break even taking into account the welfare gain. Even if we take
into account the ad-hoc correction to our model’s underestimating total flow of the system and double the welfare, the total present value is still less than half of the construction cost with the 40 year lifetime.

What if the Expo line was constructed as a heavy rail instead of a light rail project? This counterfactual question is very intriguing and very tricky to answer at the same time. We compute a counterfactual output where we double the Expo line speed from 17.9 mph to 37.8 mph, to mimic the speed of e.g. an underground heavy rail project. We follow the same procedure as above and estimated the total present value of welfare benefit to be $713 million with the 40 year lifetime. For the construction cost of the project there’s little we can do but speculate, since there has not been any engineering estimates for the “Expo line as heavy rail” scenario. There’s no consensus as to how much more expensive a heavy rail project would be compared to light rail, even in back of the envelop fashion. Various sources would point the cost difference from 5 times the cost (if considering the building cost of at-grade vs underground tracks/stations) to around 2.5 times the cost (if use meta-analysis of built projects).

If we make a conservative estimation that building the Expo line as heavy rail would cost about 2.5 times more, that would put the construction cost to about $2.3 billion. In this case, the $713 million estimated lifetime welfare is still a ways off from the construction cost, but the gap between cost and benefit is shrinking. If we take into consideration the ad-hoc adjustment of model underestimation, the Expo line as heavy rail would have generated over $1.4 billion in welfare for
$2.3 billion in cost, and the project’s benefit seems more justified than the light rail case. However this estimation completely hinges on the cost of infrastructure, which is still the biggest uncertainty.

1.5 Prototype City

The result of our Expo line cost and benefit study is meaningful and shed light on the welfare channel and mechanism. However because of the high degree of spatial resolution and its highly geographical nature, the result presented is very location and case specific. To use the above framework to analyze a project in another city, an entirely new network needs to be constructed and numbers recalculated using GIS data from another city.

In an attempt to generalize the result from this framework, we have constructed a prototype city with a stylized layout of roads and rail network, where the population and employment density, as well as the speed of rail can be fully controlled. The goal of this exercise is to take a simulation approach to see whether different demographic densities would affect the choice of transportation forms. The hypothesis is that as density of city increase, rail system with higher speed would become more favorable (such as underground heavy rail).
1.5.1 Prototype city: set-up

The basic description of the city is described as follow. We provide an in depth description of the city’s set-up in the appendix section A.5.

We set up the prototype city in a 15.5 by 15.5 mile square. The city is divided into 961 TAZs (31 by 31 zones). We define the zone in the center of the square to be the main center of the city, and define four sub-centers towards the North, East, South, and West. Figure 1.9 is a density map where the five centers can be seen clearly. The employment an population are spread over the entire city following this density function:

\[ y_i = a_i + b_{1i}D_{main} + b_{2i}D_{sub} \]  

(1.8)

where \( i \) denotes employment or population, \( y_i \) is logged employment or logged population, \( a_i \) is a constant term, \( D_{main} \) is logged distance to main center, \( D_{sub} \) is logged distance to the closest sub-center. The baseline parameters are calibrated to the Los Angeles demographic distribution. Details of the calibration is described in the appendix section A.5.

The city has a grid style road system that connects each zone’s center, which is the first connection of any zone to the rest of the city. The city also has a rail system in the shape of a ring and a radial cross, which passes through all major centers in the city, and a two-ring-and-cross highway system for driving. Figure 1.10 and 1.11 plots the road and rail network for better visualization.

To get a sense of the population and density of the prototype city, we calcu-
late an index to measure spatial centrality of the city called the UCI (Pereira et al. [28]). The UCI consists of two coefficients, the location coefficient (LC) and a proximity index (PI), using the employment distribution of the city ($s_i$). The UCI is defined as follow:

$$UCI = LC \times PI,$$

$$LC = \frac{1}{2} \sum_{i=1}^{n} |s_i - \frac{1}{n}|,$$

$$PI = 1 - \frac{V}{V_{max}},$$

$$V = S' \times D \times S$$ \hspace{0.5cm} (1.9)$$

where: $S$ is a column vector of $s_i$; $D$ is a distance matrix for all TAZs. $s_i = \frac{E_i}{E}$ is each zone’s employment as a percentage of total employment in the city; $n$ is the total number of zones in the city. The range of $LC$ is zero to $1 - \frac{1}{n}$. $V_{max}$ is the hypothetical maximum value $V$ can attain. In the basic case of a square city, such as that in our prototype city, $V_{max}$ can be calculated when employment is equally distributed to the four zones at the four corners of the city.

Using the parameters calibrated to the Los Angeles distribution, we calculated the UCI of the prototype city to be 0.16. Note that Pereira et al. [28] table 3 reports Los Angeles’ UCI to be 0.045, Pittsburgh (US) to be 0.11, and Sao Paulo to be 0.201, and our calculation of 0.16 seems to be off. However the UCI calculated in Pereira et al. [28] considers all of Los Angeles County, which is nearly 20 times bigger in land mass (4752 square miles) as the prototype city. One can consider that our prototype city is trying to mimic the 15-by-15 mile square around the Los Angeles CBD.

Once the prototype city is properly setup and population and employment properly distributed, we follow the gravity model-based welfare framework as outlined in previous sections to calculate flow and welfare numbers. We then
tweak the distribution parameter to alter the population or employment density and repeat the process for a range of values. By tweaking the parameters we can shift the population or employment to concentrate towards the main center or the sub-centers. Calculating and obtaining welfare values under the range of scenarios should provide us with some insight into the correlation between centrality of a city and transit arrangement.

1.5.2 Prototype city: results

Similar to our main Expo line analysis the quantitative result from our prototype city has little external validity. However the qualitative result is still interesting to explore. To do this, for each of the density setting we consider the marginal cost and marginal benefits as transit speed increases, and denotes the point where the two curves cross as the speed where welfare is maximized.

We found that when we increase the parameters to push higher concentration of population or employment towards the city center, the speed at which welfare is maximized appears to increase. Similar effect is observed when we tweak the parameters to shift population towards sub-centers, but the rate of change of the effect is much smaller.

Further analysis is still needed to ascertain a conclusion. However the observation do seem to reinforce a commonly held idea that cities with higher population and/or employment density would favor higher speed rail transit such as
underground or above-ground heavy rail.

1.6 Conclusion

In this paper, we present a gravity style choice model to predict effects of changes in transport cost on highway and metro flows. We predict highway segments in close proximity to the new Metro Expo line would see the highest reduction in traffic and used this result to define treatment and control groups. Using the prediction result we generate treatment variables and estimate the real-world effect using highway traffic data, we estimate some 0.025 log points reduction in vehicle flow at detectors along the treated segment in the short run. The effect gradually dissipates over 6 months, after which the traffic level converges back to the pre-opening level. We also use our developed framework to construct a prototype city attempt meta-analysis on the relationship between a city’s centrality and its choice of rail transit configuration, and preliminary results seems to agree with the long held idea that cities with higher density would favor heavy rail.
1.7 Tables

Table 1.1: Gravity Parameter Estimation

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$: Log origin population, driving</td>
<td>0.535***</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\beta_2$: Log destination employment, driving</td>
<td>0.589***</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
</tr>
<tr>
<td>$\beta_3$: Log OD travel cost</td>
<td>-2.077***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>$\ln(G)$: constant</td>
<td>-3.289***</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
</tr>
<tr>
<td>N</td>
<td>5021824</td>
</tr>
</tbody>
</table>

Note: Table reports the parameter estimation of the gravity equation using Poisson regression. Dependent variable is number of trip flow for each OD pair.
Table 1.2: Baseline Predicted Flow vs Published Boardings

<table>
<thead>
<tr>
<th>Line</th>
<th>Maximum station flow (daily)</th>
<th>Published ridership</th>
<th>Flow captured (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>25472</td>
<td>82212</td>
<td>31.0%</td>
</tr>
<tr>
<td>Red/Purple</td>
<td>64420</td>
<td>151727</td>
<td>42.5%</td>
</tr>
<tr>
<td>Green</td>
<td>17356</td>
<td>43402</td>
<td>40.0%</td>
</tr>
<tr>
<td>Gold</td>
<td>20530</td>
<td>41078</td>
<td>50.0%</td>
</tr>
<tr>
<td>Expo (post)*</td>
<td>25472</td>
<td>13897</td>
<td>183.3%</td>
</tr>
<tr>
<td>Orange</td>
<td>14003</td>
<td>25105</td>
<td>55.8%</td>
</tr>
<tr>
<td>Silver</td>
<td>18904</td>
<td>10611</td>
<td>178.2%</td>
</tr>
</tbody>
</table>

Note: Table reports the number of flows of the corresponding station in each line that has the most flow across all stations in that line. These are stations that are not towards the edge of the line, where because of the nature of the simulation these stations will be travelled through the least and possibly does not represent the typical usage of the respective line. Published ridership taken from LA Metro’s ridership archive at http://isotp.metro.net/MetroRidership/IndexRail.aspx. Percentage flow captured is calculated as the ratio between two times of the typical station flow and the published ridership boarding figure from 2012. Two times the flow was taken as the average predicted daily flow since our gravity model models home to work trips.

Note*: Expo published average weekday ridership in 2012 is 13897, subsequent years the ridership figure rises to 23315 in 2013, 28237 in 2014, and 30671 in 2015. This in turn means the percentage of flow captured is roughly 80%.

Note**: The Orange and Silver lines are bus rapid transit lines. Silver line serves the San Gabriel valley area from Downtown Los Angeles to El Monte terminal. There are a few local municipalities in the valley that provides bus service to the downtown area, such as the Foothill Transit agency serving all the cities in the valley. However, no shapefile is available for these lines. As a result the Silver line ridership is over estimated as it possibly includes riderships for those other lines.
Table 1.3: Network Externalities: Percentage change in flow after expo opening

<table>
<thead>
<tr>
<th>Metro Line</th>
<th>Mean</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>5.17%</td>
<td>0.00%</td>
<td>2.95%</td>
<td>26.41%</td>
</tr>
<tr>
<td>Red/Purple</td>
<td>-0.20%</td>
<td>-12.80%</td>
<td>1.08%</td>
<td>4.09%</td>
</tr>
<tr>
<td>Green</td>
<td>-0.76%</td>
<td>-10.13%</td>
<td>0.22%</td>
<td>2.02%</td>
</tr>
<tr>
<td>Gold</td>
<td>3.18%</td>
<td>2.10%</td>
<td>3.12%</td>
<td>4.17%</td>
</tr>
<tr>
<td>Orange</td>
<td>0.50%</td>
<td>0.06%</td>
<td>0.50%</td>
<td>1.38%</td>
</tr>
<tr>
<td>Silver</td>
<td>-2.81%</td>
<td>-18.02%</td>
<td>2.76%</td>
<td>3.96%</td>
</tr>
</tbody>
</table>

*Note:* Table reports the percentage change per station by metro lines. For example, on average stations on the Blue line sees an increase of 2.01% in flow after the expo opening, there is one station on the Blue line with 12.23% increase in flow, and one station with 7.62% decrease in flow after expo opening. In general, the effect of opening of Expo line seems logical: increases on the lines which extends the Expo line’s reach (Blue, Gold, and most of Red line), and decreases on the lines which runs parallel to the Expo’s service area (Green line). Reader should also refer to the map to see the spatial distribution of the change in flows. Of note here is there’s two stations on the Red/Purple line which sees 15% reduction in flow. These two stations belongs to the purple line segment running East-West direction parallel to the Expo line and in close proximity, while the rest of the Red/Purple line goes North and extends to the San Fernando Valley area, of which the stations along that segment sees an increase in flow. There’s also one station on the Blue line with 12% increase in flow. Said station is one that is shared with the Expo line.
Table 1.4: Own and cross price elasticities

Panel A: Fare hike of 10%

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>1 to 3 miles</th>
<th>3 to 5 miles</th>
<th>5+ miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transit</td>
<td>-0.3283</td>
<td>-0.3551</td>
<td>-0.2926</td>
<td>-0.1594</td>
</tr>
<tr>
<td>Driving</td>
<td>0.0407</td>
<td>0.0664</td>
<td>0.0232</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

Panel B: Fare subsidy of 10%

<table>
<thead>
<tr>
<th></th>
<th>Overall</th>
<th>1 to 3 miles</th>
<th>3 to 5 miles</th>
<th>5+ miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transit</td>
<td>-0.3425</td>
<td>-0.3647</td>
<td>-0.2987</td>
<td>-0.1612</td>
</tr>
<tr>
<td>Driving</td>
<td>0.0408</td>
<td>0.0670</td>
<td>0.0234</td>
<td>0.0054</td>
</tr>
</tbody>
</table>

Note: Table reports fare elasticity according to model prediction. Each column represents a geographic location. The first column reports system wide elasticities. Subsequent columns report elasticities for origin-destination pairs within certain distance to public transit. In the first scenario (panel A) fare is increased by 3%, and in the second, there’s a fare subsidy of 10%. Elasticity is calculated as percentage change in flow in Transit or Driving divided by percentage change in fare. As expected the two case gives roughly consistent results. Transit own price elasticity is always negative as expected and driving is positive. This is very much in line with the literature which quotes a transit own price elasticity of -0.2 to -0.5 and automobile elasticity of 0.03 to 0.1 (Litman 2004)

Table 1.5: Treated vs. control group, detector statistics

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Window</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>-.0508094</td>
<td>-.0508094</td>
</tr>
<tr>
<td>Control</td>
<td>-.0024137</td>
<td>-.0046298</td>
</tr>
</tbody>
</table>

Note: Table reports average log point change in flow by detector within different overall or within the “window” spatial area defined in the main text.
Table 1.6: Binary treatment, effect on logged maximum flow from covariates

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$: Short-run Adjustment</td>
<td>-0.0107**</td>
<td>-0.00771</td>
<td>-0.0101*</td>
<td>-0.0103*</td>
<td>-0.0218***</td>
<td>-0.0263***</td>
</tr>
<tr>
<td></td>
<td>(0.00538)</td>
<td>(0.00566)</td>
<td>(0.00559)</td>
<td>(0.00597)</td>
<td>(0.00626)</td>
<td>(0.00628)</td>
</tr>
<tr>
<td>$\alpha$: Pre Trend</td>
<td>-0.000318</td>
<td>-0.00130*</td>
<td>0.00122*</td>
<td>-0.000595</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000594)</td>
<td>(0.00067)</td>
<td>(0.000709)</td>
<td>(0.000796)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_2$: Long-run Trend</td>
<td></td>
<td></td>
<td></td>
<td>0.00235***</td>
<td>0.00433***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.000798)</td>
<td>(0.00121)</td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Period Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Detector Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Window Matching</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N$</td>
<td>541838</td>
<td>541838</td>
<td>541838</td>
<td>117163</td>
<td>117163</td>
<td>117163</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.837</td>
<td>0.837</td>
<td>0.837</td>
<td>0.851</td>
<td>0.851</td>
<td>0.851</td>
</tr>
<tr>
<td>A.I.C.</td>
<td>-644338.3</td>
<td>-644337.5</td>
<td>-644351.6</td>
<td>-183419.6</td>
<td>-183440.5</td>
<td>-183503.7</td>
</tr>
</tbody>
</table>

Notes: Table reports regression result of logged detector level maximum flow on dependent variables. Coefficients for $\alpha$ (Pre Trend) and $\beta_2$ (Long-run Trend) re-scaled from daily value to monthly increments.
### Table 1.7: Binary treatment, effect on logged maximum flow from covariates, bi-weekly aggregated

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$: Short-run Adjustment</td>
<td>-0.00506</td>
<td>-0.00593</td>
<td>-0.00748</td>
<td>-0.00656</td>
<td>-0.0200***</td>
<td>-0.0228***</td>
</tr>
<tr>
<td></td>
<td>(0.00536)</td>
<td>(0.00580)</td>
<td>(0.00577)</td>
<td>(0.00621)</td>
<td>(0.00661)</td>
<td>(0.00657)</td>
</tr>
<tr>
<td>$\alpha$: Pre Trend</td>
<td>0.0000426</td>
<td>-0.000613*</td>
<td>0.000661*</td>
<td>0.000661*</td>
<td>-0.000508</td>
<td>-0.000508</td>
</tr>
<tr>
<td></td>
<td>(0.000282)</td>
<td>(0.000326)</td>
<td>(0.000359)</td>
<td>(0.000359)</td>
<td>(0.000439)</td>
<td></td>
</tr>
<tr>
<td>$\beta_2$: Long-run Trend</td>
<td>0.00142***</td>
<td>0.00253***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000394)</td>
<td>(0.000394)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
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<td>59158</td>
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<td>$R^2$</td>
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<td>0.886</td>
<td>0.886</td>
<td>0.935</td>
<td>0.935</td>
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<td>-97602.3</td>
<td>-97604.9</td>
<td>-32314.1</td>
<td>-32319.9</td>
<td>-32346.3</td>
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</table>

Standard errors in parentheses, clustered on detectors.

$^* p < 0.1, \quad ^{**} p < 0.05, \quad ^{***} p < 0.01$

**Notes:** Table reports regression result of logged detector level maximum flow on dependent variables. Coefficients for $\beta_2$ (Long-run Trend) is reported as is, which is in bi-weekly increments. Note that results in this table matches with the previous table 1.6 where the effect of traffic reduction in the short run dissipates over about 5 to 6 months.
Table 1.8: Binary treatment, logged maximum flow, matching methods

<table>
<thead>
<tr>
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<th>(1)</th>
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<th>(4)</th>
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<tbody>
<tr>
<td>( \beta_1 ): Short-run Adjustment</td>
<td>-0.0101*</td>
<td>-0.0263***</td>
<td>0.0066</td>
<td>-0.0201**</td>
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<tr>
<td></td>
<td>(0.00559)</td>
<td>(0.00628)</td>
<td>(0.0114)</td>
<td>(0.00793)</td>
</tr>
<tr>
<td>( \alpha ): Pre Trend</td>
<td>-0.00130*</td>
<td>-0.000595</td>
<td>-0.0000837</td>
<td>-0.00174</td>
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<tr>
<td></td>
<td>(0.00067)</td>
<td>(0.000796)</td>
<td>(0.00105)</td>
<td>(0.00116)</td>
</tr>
<tr>
<td>( \beta_2 ): Long-run Trend</td>
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<td>0.00433***</td>
<td>0.00135</td>
<td>0.00467**</td>
</tr>
<tr>
<td></td>
<td>(0.000798)</td>
<td>(0.00121)</td>
<td>(0.00152)</td>
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Matching method

<table>
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<th>Window</th>
<th>Pre Trend</th>
<th>Distance to CBD</th>
<th>Coordinates</th>
<th>Direction</th>
<th>Pre Flow</th>
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<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Pre Trend</td>
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<td>Coordinates</td>
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<tr>
<td>Direction</td>
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<td>Pre Flow</td>
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</table>

<table>
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<th></th>
<th>N</th>
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<td>117163</td>
<td>57293</td>
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<td>Pre Trend</td>
<td>57352</td>
<td>57291</td>
<td>57392</td>
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<tr>
<td>Distance to CBD</td>
<td>57752</td>
<td>57291</td>
<td>57392</td>
<td></td>
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<tr>
<td>Coordinates</td>
<td>0.76</td>
<td>0.806</td>
<td>0.784</td>
<td></td>
</tr>
<tr>
<td>Direction</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Pre Flow</td>
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<table>
<thead>
<tr>
<th></th>
<th>A.I.C.</th>
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</thead>
<tbody>
<tr>
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<td>-183503.7</td>
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<td>-83374.1</td>
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<tr>
<td>Pre Trend</td>
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<td>-103413.2</td>
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<tr>
<td>Distance to CBD</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coordinates</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses, clustered on detectors.

∗ \( p < 0.1 \), ∗∗ \( p < 0.05 \), ∗∗∗ \( p < 0.01 \)

Notes: Table reports robustness check using nearest neighbor matching on various characteristics. Coefficients for trends reported in monthly increments. All regressions includes period and detector fixed effects.
### Table 1.9: Welfare Calculation: Summary

#### Panel D: Total

<table>
<thead>
<tr>
<th>Total benefits</th>
<th>Annual</th>
<th>Annual Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congestion relief</td>
<td>$14,297,574.37</td>
<td>$28,595,148.74</td>
</tr>
<tr>
<td>Travel time saving (transit riders)</td>
<td>$5,548,678.20</td>
<td>$11,097,356.40</td>
</tr>
<tr>
<td>Scale economy/occupancy cost gap</td>
<td>$1,529,727.17</td>
<td>$3,059,454.34</td>
</tr>
<tr>
<td>Price/MC gap</td>
<td>-$11,651,037.30</td>
<td>-$23,302,074.59</td>
</tr>
<tr>
<td>Total</td>
<td>$9,724,942.44</td>
<td>$19,449,884.88</td>
</tr>
</tbody>
</table>

**Note:** Table reports hypothetical calculation of total welfare change based on gravity model prediction. Detail breakdown of each component is reported in table 1.10. Emission benefits is set to zero following our empirical analysis.
Table 1.10: Baseline hypothetical welfare calculation

<table>
<thead>
<tr>
<th>Panel A: Congestion Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Congestion relief</td>
</tr>
<tr>
<td>Delay reduced (min/mile)</td>
</tr>
<tr>
<td>Vehicle mile</td>
</tr>
<tr>
<td>VOT ($/min)</td>
</tr>
<tr>
<td>Total congestion relief ($)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Emission Benefits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emission Parameters</td>
</tr>
<tr>
<td>CO2 grams per mile</td>
</tr>
<tr>
<td>NOX grams per mile</td>
</tr>
<tr>
<td>VOC grams per mile</td>
</tr>
<tr>
<td>Marginal social damage of GHG emissions</td>
</tr>
<tr>
<td>Marginal social damage of NOX emissions</td>
</tr>
<tr>
<td>Marginal social damage of hydrocarbon emissions</td>
</tr>
<tr>
<td>Emission cost per mile (Fleet average)</td>
</tr>
<tr>
<td>Emission Relief</td>
</tr>
<tr>
<td>Removed vehicle miles</td>
</tr>
<tr>
<td>Total emission benefits</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Travel time savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transit riders (hrs)</td>
</tr>
<tr>
<td>Transit riders ($)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel D: Other components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Marginal supply cost, $/pass-mi</td>
</tr>
<tr>
<td>Fare, $/pass-mi</td>
</tr>
<tr>
<td>Marginal scale economy, $/pass-mi</td>
</tr>
<tr>
<td>MC of occupancy, $/pass-mi</td>
</tr>
<tr>
<td>Added transit miles</td>
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<tr>
<td>Scale economy/occupancy cost gap</td>
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<tr>
<td>Price/MC gap</td>
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</tbody>
</table>

Note: Emission parameters cited from Bento et al. [12] taking the parameters for fleet average. Parameters in panel D cited from Parry and Small [27]. We reported the hypothetical emission relief here for reference.

Note: Travel time savings is calculated as follow. According to the welfare theory, the travel time saving component is the change in travel time times the change in commuters moving to transit. We calculate the commuters moving to transit by calculating the change in transit flow after the expo line opens. This is multiplied by the transit time saving, the OD level transit time change after the Expo line opens. Afterwards we aggregate to the daily morning peak period, and multiply by 2 trips a day 260 workdays a year. Travel time savings in dollar amount is simply the number of hours saved times value of time.
Table 1.11: Welfare Calculation: Total Lifetime Benefits

<table>
<thead>
<tr>
<th>Total benefits</th>
<th>Current Expo Speed</th>
<th>2x Expo Speed</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Annual</td>
<td>Adjusted</td>
</tr>
<tr>
<td>Congestion relief</td>
<td>$14,298</td>
<td>$28,595</td>
</tr>
<tr>
<td>Travel time saving</td>
<td>$5,549</td>
<td>$11,097</td>
</tr>
<tr>
<td>Scale economy/occ gap</td>
<td>$1,530</td>
<td>$3,059</td>
</tr>
<tr>
<td>Price/MC gap</td>
<td>-$11,651</td>
<td>-$23,302</td>
</tr>
<tr>
<td>Total</td>
<td>$9,725</td>
<td>$19,450</td>
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<tr>
<td>Total PV (perpetual)</td>
<td>$243,124</td>
<td>$486,247</td>
</tr>
<tr>
<td>Total PV (40 years)</td>
<td>$192,484</td>
<td>$384,967</td>
</tr>
</tbody>
</table>

Note: Table reports hypothetical calculation of total welfare change based on gravity model prediction. Emission benefits is set to zero following our empirical analysis. All figures in thousand dollars ($000s). Present value perpetual is calculated assuming a 4% discount rate and perpetual service of the transit infrastructure. Present value 40 years is similarly calculated but assume a service lifetime of 40 years for the transit infrastructure. Actual Expo speed is 17.9 mph based on calculation from rail schedule, and 2x Expo speed is about 36 mph, which is closer to the average speed of heavy rail in Los Angeles.
1.8 Figures

Figure 1.1: Parking setting in gravity model. Yellow dots denote zones with parking cost.
Figure 1.2: Predicted impact of the Expo line on TAZ transit shares at origin

Notes: This figure maps the predicted transit shares at trip origins (living place i).
Notes: Flows are first predicted at the origin-destination TAZ pairs level. The flow is then aggregated by stations based on whether a station is on the least cost route by public transit connecting the TAZ pair, using a matching algorithm.
Figure 1.4: Predicted impact of the Expo Line on detector level flows

Notes: Flows are first predicted at the origin-destination TAZ pairs level. The flow is then aggregated by highway detector based on whether a detector is on the least cost driving route connecting the TAZ pair, using a matching algorithm.
Figure 1.5: Map of balanced panel detectors

Notes: Data are aggregated to monthly bins before forming a balanced panel. Red dots indicate detectors identified as treatment group, and blue dots are detectors in the control group. The grey rectangle represent the spatial window described in the main text.
Figure 1.6: Treatment effects

Notes: Solid thick lines illustrate the parametric treatment estimates from table 1.6, column (6). The dashed vertical lines indicate the period when the cumulative treatment effect (long run + short run) becomes zero.
Figure 1.7: Matched transactions

Notes: Data are aggregated to monthly bins before forming a balanced panel.
Figure 1.8: Propensity score matching: common support

Notes: Estimated propensity score based on matching of pre-trend, distance to CBD, and detector coordinates. Notice that there is very little common support between the two extremes, and thus the propensity score matching requirement cannot be satisfied.
Figure 1.9: Prototype city: density layout

Notes: The figure is the density plot of the population density generated by the baseline density function, to illustrate the main and sub-center locations and demonstrate the way our density function generates smooth density across the city zones.
Notes: The figure plots the circle and cross transit rail system overlaying over the grid style road network. Agents would have to first walk on the grid roads from the trip origin to the transit line then take rail for the rest of their trip. We allow agents to get onto the rail network wherever the rail and grid network crosses.
Figure 1.11: Prototype city: road network

Notes: The figure plots the circle and cross highway system in red, overlaying over the grid road network. Agents would drive on the grid road network at local road speed (15 mph) and change to highway network wherever the highway and grid network crosses.
Figure 1.12: Prototype city: identifying Los Angeles city centers

Notes: This figure shows the LA roads map overlaid on top of a map identifying city centers and sub-centers in the Los Angeles county. We use the centers and sub-centers map from Pereira et al. [28] and identify one main center (Downtown Los Angeles CBD) and sub-centers (Burbank, Hollywood, Santa Monica, Culver City, and Long Beach). Using this data we calculate the distance from all TAZs in the Los Angeles county to the main center and the closest sub-center and calibrate the density function parameters for the prototype city.
Notes: This figure concerns the transit market and the dynamics when transit expansion takes place. As a result of increased coverage, the travel time decreases creating the wedge in welfare as shaded.
CHAPTER 2

IF YOU CANT PARK, YOU'LL RIDE: MODELLING THE EFFECT OF TRANSIT EXPANSION AND PARKING SUBSIDY REDUCTION AS TWO-PART INSTRUMENTS

2.1 Introduction

This paper aims at examining the effect of combining two policy interventions—transit investment and parking subsidy—in a two part instrument framework, using the Los Angeles Expo line as the background. The value of rail transit in easing traffic congestion has been a long standing interest among transportation planner and environmental economists. Building new public transit infrastructure, such as light rail, heavy rail, or even bus rapid transit lines, increases transit network’s capacity, expands the service coverage, and reduces travel time for transit commuters. The hope is that reduction in travel time would move marginal commuters from driving solo to taking public transit. Many researches have been done to support this view. For example, [10] found that highway delay increase substantially if transit service is stopped completely. Ewing et al. [16] used a quasi-experiment design and estimated that with Utah’s TRAX light rail line traffic on the street decreased by 7,500 to 21,700, which saves about 362,000 gallons of gasoline and prevents about 7 million pounds of CO2 from being emitted each year. Litman [25] asserted that improving travel options through providing quality travel alternatives to driving can benefit all travelers on a corridor, both those
who shift modes and those who continue to drive. Shifts to alternative modes not only reduce congestion on a particular highway, they also reduce traffic discharged onto surface streets, providing “downstream” congestion reduction benefits. On the other hand, studies involving induced travel demand hypothesizes that expansion in highway capacity would not alter the level of congestion on the roads, due to induced travel demand (“refilling”). Examples include the “fundamental law of road congestion” from [15], and empirical analysis such as Cervero and Hansen [14] and Fulton et al. [20] which finds strong statistical evidence from California and mid-Atlantic region. In this school of thought, congestion on highway and VMT are determined by highway capacity. Thus the extra capacities created by those solo drivers moving to transit would soon be refilled, restoring congestion.

While the transit investment alone would be ineffective towards congestion relief, from a two-part instrument standpoint, transit provision can potentially be coupled with another policy instrument to achieve the desired result. In this paper, I look at parking subsidy as a potential partner to transit investment. In the United State, most commuters do not take into consideration the issue of parking when making their travel mode choice. The reason is usually because of employer provided parking space or parking subsidy. Many studies believe that this is a major contributor to the nation’s congestion problem. Gillen [22] showed that parking subsidy significantly increase solo driving. Gillen showed that the price elasticity of parking ranges from -0.2 to -1.69, and lower income commuters are more price sensitive. Feeney [17] surveyed nineteen discrete choice models and concluded
that parking price and subsidies play a significant role in mode choice. Willson and Shoup [40] surveyed several case studies and found that paying for parking would reduce auto commuting consistently. Wilson [41] uses a multinomial logit model to assess the effect of employer-paid parking on commuting mode choice, and found that paying for parking would cut driving commuters by 25 to 34 percent. Shoup et al. [31] argues that the overabundance of free parking is subsidizing driving as a commuting mode, skewing the travel choice and contributing to congestion.

In this study, I try to use population and employment data from census and GIS resources to construct a gravity style model to examine the effect of coupling transit provision investment with parking subsidy reduction. From the two-part instrument perspective, the reduction in parking subsidy would be the tax (deposit) versus the subsidy which is provision on public transit (refund).

I will first describe the basics of the two-part instrument framework, and provide some backgrounds on parking subsidy and a real world example of transit expansion–the Los Angeles Expo line. Next I will describe the gravity style model used to predict changes in mode choice in Los Angeles County. Finally I will present a general equilibrium framework based on Parry and Small [27] to study the components in the social welfare change by these instruments.
2.2 Two Part Instrument

To understand the rationale of combining transit expansion instrument with a tax instrument such as changing parking subsidy benefit, I consider the two-part instrument literature. The theory of two part instrument (2PI) rises from the standard Pigouvian tax theory. In the standard Pigouvian framework, the solution to curb activities that creates social disutility would be to tax such activity directly. The issue with this approach is that it would require the target activity or goods to involve in market transactions in order for taxes to be applied. Oftentimes this is hard to achieve. For one, not all undesirable activities involve market transactions. Others are simply very difficult to implement both technologically and politically. For example, in the case of trying to tax drivers for congestion, it is difficult to implement system wide congestion tax on every driver on the road.

The two part instrument theory tries to tackle this deadlock by showing that the same effect of a Pigovian tax can be achieved by using two separate policy instruments: combining a general tax with a subsidy for clean alternative to the pollution. In the context of commuting and congestion, the Pigouvian tax would be a direct tax on driving, while the two part instrument would be to tax inner city workers and subsidy public transportation. A policy that changes the tax benefit in providing subsidy for commuter’s parking would affect every agent in the society, including commuters whos commute does not go through the city center and thus not contributing to the central city traffic congestion. On the other hand the expansion of the transit network can be seen as providing more transit service
and increase network connectivity, service area, and thus benefits the commuters or the consumers of the “clean” good.

Following the notation in Fullerton and Wolverton [19], the idea of the two part instrument is as follow. Consider the case where there are n households, each with 1 unit of resources where household consume on either a household good $h$, clean good $t$ which is transit to work, and dirty good $d$ which is driving to work. Without loss of generality assume production function to be linear, and marginal rate of substitution to be 1. The total pollution in the society is defined as $D = nd$ Each household’s utility is proportional to the consumption goods $h, c, d,$ and negatively on total pollution $D$. In the first best world, the social planner’s maximization problem would be to maximize the representative household’s utility:

$$\max u(t, d, h, D) + \gamma(1 - h - t - d)$$

Meanwhile, each household maximizes their own utility subject to their after tax budget constraint:

$$\max u(t, d, h, D) + \lambda[(1 - h)(1 - t_r) - t(1 + t_r) - d(1 + t_d)]$$

Social optimum in this setting implies the first order conditions of the household match those of the social planner’s.

Under this setting, a Pigouvian tax would be a tax $t_d$ that applies directly on the dirty good $d$, in this case driving. Solving the first order conditions would yield $t_d = \frac{\mu_d}{\lambda} - 1 = \frac{-\text{max}}{\lambda}$. This is the marginal external damage per agent, i.e. the
total change in disutility from an additional unit of \( d \) consumed normalized to the numeraire good. In the case where the dirty good cannot be directly taxed \((t_d = 0)\), solving the equations would yield

\[
\begin{align*}
t_t &= \frac{\gamma}{\lambda} - 1 = \frac{nu_D}{\lambda} \\
t_r &= 1 - \frac{\gamma}{\lambda} = \frac{-nu_D}{\lambda}
\end{align*}
\]

This shows if a direct tax on driving cannot be imposed, social optimal can still be reached using a tax on everyone and a subsidy on the consumption of the clean good–public transit \((t)\). Fullerton and Wolverton (2005) further shows that even in the second-best world, using two part instrument can still achieve social optimal with the following tax rates:

\[
\begin{align*}
\frac{t_t}{(1 + t_t)} &= -\frac{\tau}{\mu/\lambda} \\
\frac{t_r}{(1 - t_r)} &= \frac{\tau}{\mu/\lambda} + \frac{(1 - \gamma)}{\epsilon}
\end{align*}
\]

This framework provides a great workaround to problems such as congestion where a Pigouvian tax would be difficult to implement.

### 2.3 Los Angeles Expo Line

Before considering the welfare model it’s best to understand better the background of the two policy instruments that I am considering.

The first instrument I am considering is an infrastructure expansion to transit network. The Expo line in Los Angeles provides us with a valuable real world
quasi-experimental setup to study this kind of instruments. Expo line is a new light rail line of the Los Angeles Metro Rail network, getting its name from the Exposition Boulevard corridor that the line basically runs along. The line is built in two phases. Phase 1, which opened April 2012, services 8.6 miles between Downtown Los Angeles and Culver City, with Phase 2 extending the line 7.7 miles Westward towards Santa Monica scheduled to open in 2016. For our study I would be focusing on phase 1 of the project. There are 12 stations on the Expo line phase 1 project. Most of the rail line runs at-grade rail in private right-of-way, and there are some street-running, elevated, and trenched segments. The line crosses several major arterial road intersections, sharing the intersection with road traffic near the Downtown area, while elevated segments are used to cross the intersection towards the Culver City area. The line runs through mostly residential areas, with commercial areas towards each of its two ends.

Phase 1 of the project was originally planned to open in 2009 and budgeted at $640 million. However after multiple delays the total built costs had ballooned to over $900 million in 2012 when it finally opens. The project is funded mostly through leftover funding from the Red Line, a heavy rail underground subway in Los Angeles, and sales tax increase. Ridership on the Expo line phase 1 segment has seen continual increase after its opening. Average weekday ridership has grown steadily from around 18,000 per day in July 2012 (three months after opening) to over 30,000 per day in July 2015.

The opening of Expo line presents great opportunity for quasi experimental
analysis of infrastructure investment. Specifically, since the Expo line is situated within close proximity and running in parallel to an existing highway (I-10), it presents a great opportunity to study the real life effect of transit expansion on surrounding automobile traffic.

2.4 Parking Subsidy as Tax Fringe Benefit

The second policy instrument I consider is parking subsidy. Ever since the rising popularity of the car, commuters who drive, regardless of solo or carpooled, have been receiving employer provided parking as a tax free benefit. Many employees in America can park their car at their workplace, and for cities with higher density it is common for employers to provide parking either through reserved spots or through subsidy. For a long period in history parking subsidy has been an expected part of the employment package and little has been done about it in the tax code. To many workers, particularly those living in cities with sprawling suburbs, driving to work is a necessity and so does parking at work sites. Many US cities simply have so much land that allocating land to parking is not an issue. On the other hand, parking becomes a scarce and pricey good in the downtown areas of major cities, and in those locations, employer provided parking represents a valuable transfer of benefits to the employee. As a result, in 1976, the IRS starts to formalize parking subsidy as a tax benefits out of the concern that more employee compensations are escaping taxation in the form of fringe benefit like
parking subsidies. In 1984, congress codifies parking exemptions from taxable income, and at the same time adding transit subsidy into the benefits.

Currently, the federal government provides parking and transit subsidy through tax code for employer provided and employer paid parking, transit passes, and other commuter expenses. Under the current tax code as of 2014, employers may offer their employees commuting benefits that are exempt from employee’s taxable income in several ways: 1) employer can provide or pay for parking at or near the work site, with a value up to $250 per month; 2) employer can provide transit passes or van-pool benefits of up to $130 per month; 3) employer can also provide reimbursements for expenses related to commuting on bicycle up to $20 a month. These benefits can be provided either as a supplement to salary, or in place of it.

Commuter parking benefit is by no means a small value, considering the fact that in Los Angeles County alone there are over 4 million commuters a day and most of them driving. Some research estimates the parking tax benefit at around $7.3 billion per year (Transit Center [38] 2014), representing billions of dollars in forgone tax revenue. To make matters worse, this subsidy is putting an estimated 820,000 cars on the roads, adding 4.6 billion miles in additional VMT per year nationwide. Moreover, the tax benefit actually has the most effect to those contributing to the most congested areas, as parking is pricier in dense downtown areas in major cities, and that the subsidy only affects those commuting to work. Thus those working in downtown and other high density employment centers benefits
the most from this subsidy, and consequently the subsidy would promote driving during peak commuting hours, further contributing to the already high congestion level. A reduction in parking subsidy would have a direct impact on most solo driving commuters. This makes such policy an interesting choice to consider together with transit provision.

In the next section I will discuss the main model in this study that would enable us to calculate welfare change across the system.

2.5 Gravity Model

In order to pinpoint the source of welfare change, I would need detailed information regarding the number of commuters that would change their commuting mode with the policy shocks, as well as changes in vehicle miles for both driving and public transit commuters given the shock in transit infrastructure and parking subsidy. To accomplish this, I employ a gravity style model to simulate the flow of commuters in our study area. I first establish the model for the baseline scenario, of Los Angeles before the opening of the Expo line phase 1. After the model is established for the baseline, I can change the setting of our model to represent an expansion to public transit, in this case the opening of Expo line phase 1, and a change to parking subsidy. This allows us to predict hypothetically what would happen to driving flow and transit ridership.
Let’s discuss briefly the idea of a gravity model. In the trade literature, gravity model is often used in modeling and predicting bilateral flows of trade between countries. This class of models uses the idea of the attractive gravity force between two mass to predict bilateral flows. The force is dependent on the masses of the objects in the system as well as the distance between each object. Similar to the physics theory, in economics and international trade, prediction of bilateral flow is based on economic masses and cost of travel between any two units in the model. Extensions to the gravity model is also used in the migration literature, where other factors are used to augment the model to include push and pull factors to model factors attracting immigrants and others that pushes people away.

The idea of using gravity theory for flow prediction has been used in the trade literature as early as 1962 (Tinbergen). After Anderson [9] formalized the gravity theory in his AER paper, the model has been widely used to estimate effects of institutional policies on international trade flows. Aitken [7] used the gravity model to analyze effect from the European Economic Community (EEC) and European Free Trade Association (EFTA) on trade of their member countries. Sapir [30] also used a gravity style model to estimate trade creation effect of EEC. Abrams [4] analyzed the effect of flexible exchange rates on international trade flows using the gravity framework. Geraci and Prewo [21] uses transport costs versus distance in a gravity framework to analyze trades between OECD countries in 1970. Summary [35] developed a gravity type model to explore the interaction between political and economic factors affecting United States bilateral trade. While this class of models are used far and wide in the trade literature, in the transportation
economics literature it has not been used as commonly, mostly because most theories tries to tackle the issue either at a more individual or more aggregate level.

We adopt this model to the transportation and especially commuting setting to examine and predict travel and commuting pattern both before and after the shocks. Using population (push factor) and employment (pull factor) data from census, and transportation cost (distance) data from GIS modeling, the gravity model would allow us to predict bilateral flow between any two zones in our model. The advantage of this approach is that, once the baseline model is properly calibrated, bilateral flow can be calculated relatively quickly without the need for transportation engineering models like the four step travel demand model, which is computationally intensive and time consuming. Another reason for a gravity model is that, in reality almost all origin zones have some level of both driving and public transit commuting trips. Whereas a zone level discreet choice model would give an all or nothing mode choice, the gravity model is more flexible in allowing mode choices to be modeled in fractions.

To begin, the gravity model is calibrated using parameters from before the opening of the Expo line. This would serve as the baseline scenario for other cases. After the baseline is calculated, the model is changed to add in the Expo line and different levels of parking cost is set to mimic the effect of a reduction of parking subsidy at different levels. For each of these counterfactual cases predictions are made for the flows of both driving and transit commuters.

The rest of this section will be organized as follow. First I will introduce and
briefly outline the mechanisms of the gravity model. Then I will describe how the policy interventions are represented. Finally I’ll discuss briefly the various data used in the model. A more detailed discussion of the model and equations are available in the mathematical derivation section (section A.1).

2.5.1 Model description

In general, gravity models in the trade literature take the following functional form:

\[ F_{ij} = f(Y_i, Y_j, C_{ij}) \]

where \( F_{ij} \) is the value of trade between country \( i \) and \( j \); \( Y_i, Y_j \) are the GDP of country \( i, j \) respectively; and \( C_{ij} \) is the resistance/enhancement factor, which is the distance between the two countries in trade. In this paper, I adopt the gravity model in trade to estimate the flow or trip of agents between Traffic Analysis Zones (TAZ). The gravity equation is specified as:

\[ F_{ij} = GP_i^\beta_1 E_j^\beta_2 C_{ij}^\beta_3 \]  (2.1)

and I estimate the following equation:

\[ F_{ij} = \exp(ln G + \beta_1 \ln P_i + \beta_2 \ln E_j + \beta_3 \ln C_{ij}) \varepsilon_{ij} \]  (2.2)

The variables in equation 1.1 and 1.2 are as follow. \( F_{ij} \) is the number of trips from \( i \) to \( j \), which I will refer to as origin to destination TAZ level flow. \( P_i \) is the
population at \( i \), \( E_i \) is the employment at \( j \), and \( C_{ij} \) is the overall trip cost traveling from \( i \) to \( j \). Overall cost combines both driving cost and transit cost as a linear combination using the transit share \( S_{ij} \). Driving cost includes the value of time spent driving, gas price, and parking fee; transit cost includes access (walking) time cost, and the value of time spent on transit. A detailed discussion of the calculation of driving and transit time is discussed in the GIS data section. The coefficients \( \beta \) represent how these factors affect bilateral flow. \( \beta_1 \) is elasticity of number of trips with respect to the population at the origin; \( \beta_2 \) is elasticity of number of trips with respect to the employment at the destination; and \( \beta_3 \) is elasticity of number of trips to overall cost of travel between a particular origin-destination pair.

The transit share \( S_{ij} \) is calculated as follows. For each agent in the model, there are two modes of commuting: driving alone (IND) or taking public transit (PUB). For each choice, agents obtain utility from their relative consumption between the two modes:

\[
U_{ij} = \alpha + \gamma \Delta_{ij}
\]

where \( \Delta_{ij} = (C_{ij}^{PUB})/(C_{ij}^{IND}) \) is the cost differential, the ratio between transit cost and driving cost, and as previously mentioned, these cost includes both travel time and other costs; \( \gamma \) is the cost parameter; and \( \alpha \) is a constant.

The transit share for an origin-destination pair from \( i \) to \( j \) (\( S_{ij} \)) is given by the following logit equation:

\[
S_{ij} = \frac{\exp(U_{ij})}{1 + \exp(U_{ij})}
\]
where the parameters $\alpha, \gamma$ are calibrated using actual transit share reported from Census CTPP data. Note that this logit style transit share is a discrete choice model. This function is also used in traditional four-step travel demand model to estimate modal split. Overall cost is then a linear combination of driving and transit cost:

$$C_{ij} = S_{ij}C_{ij}^{PUB} + (1 - S_{ij})C_{ij}^{IND}$$

Finally, $\epsilon_{ij}$ is the error term following a Poisson distribution. The equation could have been estimated in log form with OLS, however in cases where there are zero flows the estimation would fail. In regional travel models such as this one, it is actually possible and likely for origin-destination pairs that are sufficiently far away to have zero flows between them. Furthermore, Silva and Tenreyro [32] argued that OLS on the log form could lead to biases, and conversely proposed to estimate the above form using Poisson pseudo-maximum likelihood estimator, which is often used for count data.

The equation is then estimated using Poisson regression on the baseline model with pre Expo line opening settings, using flow data from SCAG, census population and employment data, and the GIS cost data. A more detail discussion of the data follows in the next section. With this equation calibrated, we can predicted the bilateral flow $F_{ij}$ between any two zones, for all the zones in our model.

Now that the baseline flow estimated, these flows between zones need to be assigned to routes in order to see what kind of flow we will get on highways (for driving flow) and transit stations (for transit flow). This would allow calculation
of highway detector level and station level flow, which measures how many trips or agents pass through a specific point on the highway or the transit network. In order to do this, the model utilizes Dijkstra’s algorithm solver from ArcGIS to solve for the route and actual travel cost base on route, and matrix manipulation in Python.

The Dijkstra’s solver in ArcGIS calculates a number of useful outputs for the model. The solver would find the least cost route between any two zones $i$ and $j$ both with driving and taking public transit, and the solver would then calculate the total cost of traveling between the two points $C_{i,j}^{IND}$ and $C_{i,j}^{PUB}$. In order to assign the TAZ level flow to routes, the program is also set up to calculate costs from each zone to each highway detector or transit station. By matching the total cost with the sum of cost traveling between zones and detectors we can match OD flows to individual nodes on the driving or transit network.  

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1Detectors here refers to the traffic detectors on the California highway system installed and maintained by the Department of Transportation Performance Management System (PeMS). Loop detectors are installed on all major highways in California to measure speed and flow of traffic on 30-second intervals. These detectors can be thought of as nodes in our driving network, similar to rail stations in the transit network. Here I'll present an example of the route matching. In ArcGIS I first calculate the cost of traveling from zone A to B to be $10. I then calculate the cost of travelling from zone A to all detectors and stations, and the cost from all detectors and stations to zone B. Detectors or stations where the sum of the costs matches $10 would then be assigned to the route between A and B. Since the model used in ArcGIS is geographically accurate (more discussion in the next section), this matching is very accurate.
2.5.2 Policy representation

In the previous section I have discussed how the coefficients are estimated in the model using pre-expo level driving flow, population, employment, and cost data. This forms the baseline model in our analysis. The next step here would be to model the two policy interventions: transit infrastructure provision and parking subsidy shock.

Modeling the Expo line expansion is relatively simple. In the baseline model the Expo line is “disabled” in the GIS network, meaning that when Dijkstra’s solver is trying to solve for routes, it cannot route through the Expo line. To model the shock, I simply “enable” the Expo line shape in the route calculation. This would allow the routing algorithm to direct flows through the Expo line instead of through local roads. Afterwards, routes, costs, and flows are recalculated. To model the parking subsidy shock, I changed the parking price assigned to selected zones to different values. In the baseline model, certain selected areas have parking price set at $10 a day. A reduction in the parking subsidy means that each agent would need to pay more out of pocket per day for parking. This can be thought of as an increase in the parking price. For the model I simply tweak the parking price across all the non-zero parking price zones to different values to represent different level of parking subsidy reduction. Finally the transport cost of driving and transit commuters are recalculated, and the new cost is used to update the new transit share for each OD pairs. The total flow between any two zone is fixed, as I assume that the spatial distribution of population and employment to
be exogenous. With the new transit share I update the driving and transit flow to calculate changes in commuting modes and changes in VMT in both driving and public transit network. This would provide the necessary data and figures for our welfare calculations.

2.5.3 Data

In this section we will briefly discuss the various data used in the model.

GIS and driving/transit networks

In order to calculate trip cost and find proper routing, I have constructed a geographically rich transit and driving network in ArcGIS, using publicly available cartographic data known as shapefiles. The US Census Bureau as well as many transportation agencies in major US cities across the country publish shapefiles containing geographically accurate lines representing real life geographic features such as streets and highways, bus lines, rail lines, transit stations, utility piping schemes, districting information, among others. These shapefiles are valuable resources for planners as well as transportation engineers for many purposes.

\footnote{Spatial distribution of population and employment are assumed to be fixed for the following reason. Expo line is a small intervention when we look at the regional transportation as a whole. In fact, the Expo line only accounts for roughly 7\% of the total Los Angeles Metro rail ridership (note that there’s only six rail lines total), and even at 30,000 ridership per weekday, or roughly 15,000 commuters at most per weekday, it’s only a drop in the bucket compared to the 4 million commuters in the whole Los Angeles County. It can be argued that in the short to medium run, population and employment is likely to stay constant.}
For this project, we construct the driving network using Census bureau published TIGER/LINE database. The TIGER/LINE street shapefiles contain every streets, highway, and ramps nationwide, and is very high precision. In the model I have used the street and highway lines for the whole Los Angeles County. As ultimately travel cost would need to be calculated, I assign speed for each segment of local roads using the assumption of 15 mph for average driving speed on local roads. ³ For highways, I assign each highway segment’s speed based on PeMS reported highway speed reported by detectors.⁴ With these speeds I calculate the travel time for each segment of local road or highway, to be used later for travel cost calculation.

To model the transit network I augmented the TIGER/LINE road network with shapefiles representing the bus network and light and heavy rail network. Los Angeles Metro Transit Authority (LAMTA or LA Metro) operates most of the buses and rail services in the Los Angeles County. LA Metro also runs the developer GIS data portal, a website providing GIS resources for transit developers. I included both the bus network and the light and heavy rail network. For the transit network speed, I assumed that each agent can only walk to take bus and/or rail. ⁵ I assume a walking speed of 3 mph on the street, a 8.8 mph effective speed

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³Note that our GIS modeling could not model actions such as waiting at stop sign, traffic lights, or the waiting time for pedestrian crossings. I have made the assumption of 15 mph as a “representative speed” of a trip traveling through local roads which would include all of the above nuances.
⁴See footnote 1.
⁵The assumption of only walking to bus and/or rail essentially excludes the cases of park and rides. This is due to the limitation of our GIS program. However I also argue that the number of park and ride commuters is minuscule compared to the overall number of commuters.
for local bus services, and a 13 mph effective speed for rapid bus services. For the four light rail lines and two heavy rail lines we deduce the speed of each rail line from their length and scheduled arrival time at each station. Light rail lines run at around 20 mph, and heavy rail (the underground Red line) runs at around 30 mph.

With these shapefiles, I created detail driving and transit networks capable of performing point to point routing. This allows for the construction of detail commuting cost matrix for both driving and public transit, as well as predicting flow of commuters through highway detector and subway stations. Furthermore, each of these networks is connected to the centroid of each Traffic Analysis Zones. The TAZ is our primary geographical unit, which is defined by the Southern California Association of Governments to use with transportation planning. In the model there are 2241 zones covering the whole Los Angeles County.

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6Following Anderson [10]

7There are six rail lines operated by LA Metro. Red and Purple lines are heavy rail running underground, and the Gold, Green, Blue, and Expo lines are light rails running mostly above ground.

8The rail speeds are: Red line 29 mph, Blue line 22 mph, Green line 34 mph, Gold line 22 mph, and Expo line 17.8 mph. Note that even though Green line is a light rail, it travels mostly on elevated track that runs in between the 710 Freeway with stations spread across a much larger distance. Whereas the Red line stations are much closer together as it serves the downtown area and the only long segment is between Hollywood and Universal City across the hill. The distance difference leads to the reverse in speed between light and heavy rail.
CTPP data

The Census Transportation Planning Products (CTPP) is a set of special tables specifically compiled for transportation planners. The CTPP tables uses data from the decennial census as well as American Community Survey (ACS). There are three advantages to using the CTPP dataset compared to the census dataset. First, CTPP data provide residence based and work based tabulation on population and employment. Second, these data are available disaggregated into Traffic Analysis Zones (TAZ), which provides much higher spatial resolution that even a zip code based disaggregation. Lastly, the CTPP dataset also provide tabulation disaggregated into mode choices by TAZ. This is used as the calibration target for the gravity model.

The gravity model takes the population and employment numbers as inputs for the push and pull factor. Each zone’s employment represents how many commuters would need to travel to that zone, and similarly the population represents how many commuters would need to travel from the zone. The population and employment numbers serves as the mass in the model.

Parking

In the gravity model, travel cost consists of the value of time traveling for both driving and transit, fuel and maintenance cost for driving commuters, and bus and rail fare for transit commuters. Parking cost is an add-on cost to agents who
drives to work. In the model this is implemented as follows. First, certain TAZs are selected where parking would incur a parking cost. During the calculation of the OD cost matrix, if a trip from zone A involves going into zone B which is in our parking cost selection, a parking fee will be added to the total trip cost. Conversely, a trip from A to zone C which is not in our parking cost selection will not be added the parking fee. This calculation is done at the cost calculation step before the mode choice is determined.

The selection of locations where parking would incur cost is based on the report from ULTRANS center at UC Davis (ULTRANS [39]). I identified zones in the Los Angeles central business districts that would charge a parking fee. In total, there are 223 zones with a parking cost in our model, or roughly 10% of all TAZs. For the baseline model, a $10 parking cost is assumed for all zones that would incur a parking cost. Figure 1.1 maps all the TAZs where parking cost applies. As seen on the map, the areas which we’ve applied parking costs are (from East to West) downtown Los Angeles, Wilshire corridor, and downtown Santa Monica. This setting corresponds well to the corridor of high employment density in the area. Figure 2.3 maps the employment density in the Los Angeles County. Comparing the two maps it is apparent that the parking cost corresponds roughly to the “ridge” that represents high employment density. Note that while many American commuters enjoys free parking, commuters working in downtown area of major cities such as Los Angeles often would still need to pay for at least some of their parking cost due to scarcity of parking in dense downtowns.
The parking cost in the gravity model and GIS program represents each agent’s “out of pocket” parking cost \((P_i - S_i)\). When subsidy is reduced, the out of pocket parking cost increases. In the analysis, I gradually reduce the parking subsidy, thus increase the parking cost, in 1$ increments.

The gravity model is set up in a way such that it is possible to change the amount of subsidy by geographic locations. However, I have opted to alter the subsidy amount uniformly across all the zones where parking cost applies in our analysis. The reasoning is that, this approach would more closely mimic a change in the tax code regarding parking fringe benefit, which would affect all agents in the system. Another possible extension to this model would be to implement parking subsidy shock that varies geographically corresponding to the employment level, which correlates with the scarcity of parking in each zone.

2.6 Baseline Results

In this section I will report the baseline results from the gravity model. This serves to provide a sense of how the model is doing in terms of predicting flows for driving and transit users.

Table 1.2 reports the gravity model predicted maximum station level flow by metro lines. LA Metro published ridership data is used as a comparison. Because of the limitation in output of the GIS and gravity model we are only able to cal-
culate trips flowing through each station instead of line level boarding figure. In this table we report the maximum station level flow by line. Figure 2.4 maps the predicted baseline station level flow. On average, the modeled flow is about half of the daily ridership on the transit lines. Remember that these measurements are the maximum flow that any station in each line gets in the model, and possibly do not directly correspond to the ridership figure. For example, a trip that travels on say the Red line that does not pass through the station with the highest flow (in this case it’s the downtown Los Angeles station) would not be counted in the maximum station flow. It is likely that if it is possible to calculate ridership from the gravity model the percentage flow captured would improve.

To illustrate that the gravity model provides a sensible representation of commuting mode choice, I examined the fare elasticity of the modelled transit and driving system. Table 1.4 reports the fare elasticity. I tweak the bus and rail fare and calculated the change in transit and driving trips caused by the fare change. After adjusting the fare amount, I recalculate the transit share and finally the corresponding new number of trips on each mode, and subsequently the point fare elasticity. I also try to calculate elasticity for subsets of trips within a certain distance from the transit system. For example in column 2 we restrict the sample to all OD pairs within 1 to 3 miles of metro service area and report the fare elastic-

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9 As of 2013 LAMTA charges each rider of bus or rail a flat fare of $1.50 per ride, which is good for any length of travel on the transit network. Transfers within the system are free as long as the trip is not a round trip and within two hours from initial entering of the fare gate. LAMTA also provide the option of a monthly pass for $75 a month. In late 2014 LAMTA raises the price to $1.75 per trip and $100 for a monthly pass. For the model I used the 2013 setting since Expo line opens in 2012.
ity within this subsample of trips. The overall elasticity of -0.3 is in line with the literature of transit own price elasticity of -0.2 to -0.5 (Litman 2004).

Figure 2.5 maps the predicted change in station level flow after Expo line opens. The changes represented in the map is condensed into table 1.3 which summarizes the changes into line level statistics. After expo line opens, metro lines that serves to extend the reach of Expo line is predicted to increase in flow, while lines that runs parallel to the Expo line is predicted to drop in flow, possibly due to competition. For example, both the Blue and the Gold line observe an increase in flow (5% and 3% respectively). Since both of these lines run in directions that extend the reach of the Expo line, all of their stations observe non-negative changes in flow after Expo line opens. For the Red/Purple line, the stations where it runs parallel to the Expo line (which is the purple line along the Wilshire corridor) is predicted to drop in flow, while the other stations sees and increase in flow. Another segment of line that also sees predicted drop in flow is the Green line running West of the Blue line, as it also runs parallel to the Expo line. On the other hand, the segment of the Green line to the East of Blue line is predicted to have increased flow. The interactions represented here matches with general expectation in terms of network externalities.
2.7 Welfare Theory

In order to explore the sources and interactions of welfare changes, instead of the basic model in section 2.2, I devise a general equilibrium model to decompose the source of welfare into subcomponents. This provides the basis for examining the source of welfare change.

This model follows the overall structure of Parry and Small (2009). Two extra components are added: 1) transit service coverage, to model transit infrastructure provision; and 2) parking subsidy. A more detailed explanation of variable and equations is available in the mathematical derivation section A.2. The basic model structure is as follow.

2.7.1 Demand Side

There are two regions in the economy, A and B. There are $N_1$ agents in A and $N_2$ agents in B, thus total $N = \sum_{i,l} N^l_i = N_1 + N_2$ agents in the economy indexed by $i$ and superscript $l$ for location. Each agent in each location choose to consume amount of numeraire good $X^l_i$ and commute $M^l_i$ miles on either car or public transit at peak or off peak period, $M^l_i = \{M^{ijkl}_i; j = C(\text{car}), T(\text{transit}), k = P(\text{peak}), O(\text{offpeak}), l = A(\text{primary}), B(\text{secondary})\}$.

Each agent is endowed with exogenous income $I_i \in [L, \bar{I}]$. In addition, agent’s utility also depends on $\Gamma$, a composite travel time cost function, and $Z$, the pollu-
tion/Accident externalities. Thus, the agent’s utility maximization problem is:

$$\max_{X_i, M_i} u_i'(X_i, M_i, \Gamma_i') - Z$$

subject to:

$$X_i' + \sum_{kj} p^{jk} M_{ij} = I_i' - TAX_i'$$

The travel time cost function $\Gamma_i$ is defined as follow:

$$\Gamma_i' = \Gamma(T_i', W_i', A_i', C_i', E_i').$$

Time cost $\Gamma_i$ is increasing on four components: $T$, $W$, $A$, $C$ and decreasing on $E$. The components in $\Gamma$ are defined as:

- $T_i$: travel time cost = $\sum_{lkj} t^{lkj}_{ik} M^{lkj}_i \times VOT_i$
- $W$: waiting cost = $\sum_k w^{lk} M^{lk}_i$
- $A$: accessing cost = $\sum_k a^{lk} M^{lk}_i$
- $C$: crowding cost = $\sum_k c^{lk} M^{lk}_i$
- $E$: network externality = $-\sum_k e^{lk} M^{lk}_i$

Travel time cost ($T$) captures time spent in the vehicle, on car or public transit. $t^{lkj}$ is the per-mile travel time defined as:

$$t^{lkj}_{ij} = \begin{cases} 
\sum_k t^{lk}_{ik} (V^{lk}_i + \alpha \sum_{h \neq i} V^{lk}_h) \\
\sum_k t^{RT}_{ij} V^{lk}_i 
\end{cases}$$
where $t^{kc}_s$, $t^{kt}_s$ are performance functions and $t^{kj}_s = \frac{1}{x^j_s}$, the inverse of mode travel speed at peak or off peak. To model congestion, I consider other agent’s choice of vehicle miles consumed $V^{kc}_j$ contributes to overall congestion linearly at a coefficient $\alpha$. This is consistent with Parry and Small [27] where they assume agents consider other agents’ contribution to congestion as constant and ignore self-contribution to congestion. I also assume that public transit system (the metro network) is not affected by congestion, so travel time on transit is independent of other agent’s choice. The term $VOT^i_l = \phi I^i_l$ represents the value of time of agent and is a fraction $\phi$ of income. $p^{ckl}_i, p^{tkl}_i$ are the per mile price of driving and transit respectively.

The per mile price for car is $p^{kc}_l = p^{kc}_0 + \tau^{kc}_l + (P_i - S_i)$ where $\tau^{kc}_l$ is fuel tax, and $p^{kc}_0$ is pre-tax fuel cost per mile. Each agent also have to pay a “true parking cost” $P_i$ and receive a certain amount of subsidy $S_i$. Transit trips do not incur parking cost, and not all driving trips incur parking cost. Agents could receive parking subsidy as little as nothing and as much as their true parking cost ($P_i \geq S_i \geq 0$). Both the true parking cost and subsidy are exogenous to the agent’s choice.

Each agent has to pay lump sum tax $TAX^i_l$ such that $\Sigma_{il} TAX^i_l = TAX$. The lump sum tax and fuel tax is used to finance operation of public transit.

The first order condition for the agent would be:

$$\frac{U_{M,kl}^{jkl}}{U_X} = q^{jkl}_i = p^{jkl}_i + \rho^T t^{jkl} VOT^i_l + \rho^W w^{jkl}_i + \rho^A q^{jkl}_i + \rho^C c^{jkl}_i - \rho^E e^{jkl}_i$$

(2.3)

where $\rho^K = \frac{-m_i^K}{u_i}$ for $K = T, W, A, C, E$. 95
2.7.2 Supply Side

The supply side of the model is the transit agency. Each location \( l = A, B \) has a total number of OD pairs \( \{ \xi^l \} \) where agents will need to travel to/from. Define \( \{ \xi^l : \xi^l \subset \Xi^l \} \) as pairs in the universe on which the agency offer public transit service. For each route \( i \) in \( \{ \xi^l \} \) the agency choose \( n_i^l \) (vehicle size), \( D_i^l \) (density of service / number of vehicles servicing each line), and \( V_i^{RT} \). I assume speed \( s_i^l \) is exogenous, and that \( s_i^T < s_i^C \), driving speed is faster than transit speed. The number of vehicle miles offered is \( V_i^{RT} = \) length of line \( \times \# \) vehicles and \( M = V \times \text{occupancy} = V \times o \).

Since the model modeling each route as each market, the system can be considered in aggregate. Define the variable \( \xi^l = \frac{\text{number of pairs in } \{ \xi^l \}}{\text{number of pairs in } \{ \Xi \}} \) to represent the extend of public transit coverage in the system. The agency chooses an overall level of \( n, D, V^T \), and that subsequently the resources are allocated to each route in \( \{ \xi^l \} \) optimally. The four variables are:

- Vehicle size \( n^{kl} = \sum_{i \in \{ \xi \}} n_i^{kl} \)
- Density of service \( D^{kl} = \frac{\sum_{i \in \{ \xi \}} D_i^{kl}}{\text{number of pairs in } \{ \Xi \}} \)
- Vehicle miles provided \( V^{RT} = \sum_{i \in \{ \xi \}} V_i^{Tkl} \)
- Speed is assumed to be exogenous, and the average speed agents will be travelling on each route in \( \{ \Xi \} \) if they opt to take public transit is \( s^{RT} = \frac{\sum_{i \in \{ \Xi \}} s_i^{RT}}{\text{number of pairs in } \{ \Xi \}} \)
The agency must balance its budget:

\[
TAX + \sum_{ijkl} r_{ik}^l v_{i}^{lk} C_{kl} = \sum_{lk} \sum_{j \neq c} (OC_{k j} - p_{jkl} \sum_{i} M_{i}^{jkl}) \tag{2.4}
\]

where operational cost for mode \( j \), period \( k \) is

\[
OC_{k j} = F_{jkl} + K_{jkl} t_{jkl} \sum_{i} v_{i}^{jkl}
\]

and

\[
K_{jkl} = k_{1}^{jkl} + k_{2}^{jkl} n_{jkl} + k_{3}^{jkl} s_{jkl}
\]

Here \( n_{k j} \) is the vehicle size and \( s_{k j} \) the transit speed defined above. Notice that we assume \( k_{1}^{jkl} > 0 \). As a result, increasing transit vehicle size \( n_{k j} \) and speed \( s_{k j} \) incurs constant marginal cost and decreasing average cost.

### 2.7.3 Shock to infrastructure provision

In our model, a shock in infrastructure is modeled as an increase in the routes being covered by the transit agency in location A, \( \{ \xi^A : \xi^A \in \Xi^A \} \). As a result the fraction \( \xi^A \) also increase, and so does the overall network size \( \xi = \xi^A + \xi^B \). The effect is different between the two locations. In location A, where the infrastructure expansion happens, \( \xi^A \) increase, and as a result transit services provided in the location also increased. In location B, the effect of an expansion of transit service of A translates to an increase in overall connectivity \( \xi = \xi^A + \xi^B \), which increase the network externality bonus for agents in B. As a result, ridership in B increase and
in response, the agency will provide more transit capacity through larger cars or higher frequency, leading to the Mohring effect.

A detail explanation of the effects for all the travel characteristics is available in the mathematical derivation section A.2.

2.7.4 Shock to parking subsidy

Recall that the per mile cost of driving is defined as $p^{lkC} = p_0^{lkC} + \tau^{lkC} + (P_i - S_i)$.

A shock in the parking subsidy amount is represented as a shock to the cost of travel, and it is the same as in Parry and Small (2009) as a change in $p^{Ckl}$, the price of commuting by car at period $k$ location $l$. Later on we shall see that the final decomposition of welfare change by the parking cost shock would match that of the transit coverage shock.
2.7.5 Welfare analysis

The welfare implication from the shocks is as follow. First consider the total welfare function:

\[
\tilde{U} = \sum_{il} \left[ \tilde{u} \left( \{ p_{jkl}^{ijkl}, t_{jkl}^{ijkl}, w_{jkl}^{ijkl}, a_{jkl}^{ijkl}, c_{jkl}^{ijkl}, e_{jkl}^{ijkl} \}, I, TAX \right) - Z \right]
\]

\[
= \max_{X, \{ M^{ijkl} \}, A} \sum_{l} \left\{ u \left[ X, M(\{ M^{ijkl} \}) \right], \Gamma \left( \sum_{kj} t_{ijkl}^{ijkl} M^{ijkl}, \sum_{kj} w_{ijkl}^{ijkl} M^{ijkl}, \sum_{kj} a_{ijkl}^{ijkl} M^{ijkl}, \sum_{kj} c_{ijkl}^{ijkl} M^{ijkl}, \sum_{kj} e_{ijkl}^{ijkl} M^{ijkl} \right] \right\}
\]

\[
- \sum_{kj} z_{ijkl}^{ijkl} V_{ijkl} + \lambda \left( I - TAX - \sum_{kj} p_{ijkl}^{ijkl} M^{ijkl} - X \right)
\]

(2.5)

Partially differentiating (2.5) derivate with respect to the two shocks, and regroup, we have marginal welfare as:

1. Marginal welfare from infrastructure shock

\[
MW_{\xi}^{A} = \begin{cases}
\text{Travel time saving} & \sum_{ijkl} MB_{ijkl}^{ijkl} T_{ijkl}^{ijkl} \\
\text{MC supply expo net price} & - \left( MC_{\text{supply}}^{A} - p^{A} \right) M_{\xi}^{A} \\
\text{Driving market externality} & \sum_{lk} C_{ijkl}^{ijkl} (\tau - MC_{\text{ext}}^{ijkl}) M^{ijkl}_{\xi} \\
\text{Net scale economy} & \sum_{lk} B_{ijkl}^{ijkl} - MC_{\text{scale}}^{ijkl} M^{ijkl}_{\xi} \\
\text{P/MC gap - Related mkt} & - \sum_{k} \left( MC_{\text{supply}}^{Bk} - p^{Bk} \right) M^{Bk}_{\xi}
\end{cases}
\]

(2.6)
2. Marginal welfare from parking cost shock

\[
MW^p = \left( \sum_{jkl} MB_{TT}^{jkl} M^p_{\rho^C} \right) \left( MC_{\text{supply}}^{AkT} - p^{AkT} \right) M^p_{\rho^C} + \sum_{lk} \left( MC_{\text{ext}}^{Ckl} - MC_{\text{supply}}^{Ckl} \right) M^p_{\rho^C} + \sum_{lk} \left( \sum_{m} \tau_{Ckl}^{Ckl} V_{Ckl}^{m} \right) M^p_{\rho^C} + \sum_{lk} \left( MB_{\text{scale}}^{Tkli} - MC_{\text{occ}}^{Tkli} \right) M^p_{\rho^C} - \sum_{k} \left( MC_{\text{supply}}^{BkT} - p^{BkT} \right) M^p_{\rho^C} \right)
\]

(2.7)

The marginal welfare change outlined above represents different channels through which the increase in transit coverage (infrastructure provision) and shock in parking price affects overall social welfare. Note that marginal welfare from both shocks can be grouped into the same five components: (a) Travel time savings net of marginal cost of supplying new route less revenue; (b) Driving market wedge–change in market externalities less fuel tax; (c) Transit demand wedge–Scale economies vs cost from increased occupancy; and (d) Transit supply wedge–P/MC gap.

As parking cost increase, the benefit in welfare will be overwhelmed by the increasing marginal cost. This is due to the low farebox recovery rate. 10 In the United States, almost all the transit agencies charges a fare that’s lower than the fare. For example, the farebox recovery ratio of Amtrak is at 71% (Amtrak [8]), and it’s already among the highest in the United States. As the usage of transit system increases, the gap between price and marginal cost eats away at the welfare.

In the gravity model, detail geographical zones is used instead of the two zone simplified settings in the theory, which allow for more flexibility in setting the area

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10Farebox recovery rate is the fraction of operational costs met by the passenger fares paid.
of effect for the price shock. However a true optimal parking subsidy would not be solvable analytically and would also heavily depend on the actual geographical setting of the parking price.

2.8 Results – Transit Expansion

In this section we will discuss the results with just the Expo line expansion. This would set up the basic framework for analyzing the different components of welfare change. Table 1.9 summarizes the calculation of welfare change after Expo line opens, using predictions from the gravity model. I shall use this opportunity to explain in detail each of the welfare categories and components. In the next part we will add in the parking subsidy shock.

Table 1.10 presents a detail analysis of each of the components listed in table 1.9. The calculations here follow closely the four components outlined in the welfare theory in the previous section. The four components are as follow.

Panel A reports the welfare benefits from congestion relief. In order to calculate the congestion relief benefits, consider the following steps. First I need to calculate the baseline vehicle miles traveled by all the drive-alone trips in the model. This can be obtained by summing up the distance traveled of all the OD level driving trip in the baseline model before Expo line opening. For the next step, a “change in delay” measure after the Expo line opens needs to be calculated. To do
this I first take the baseline flow and calculate the difference in travel time given the baseline flow and speed, compared to the free-flowing speed which is set as the speed limit. This would give us the baseline “delay” measure in per mile basis. Next, using flows predicted with the Expo line enabled, I calculated an updated “delay” measure. Finally the change in delay is calculated as the difference of delay between the two cases. This change in delay can be thought of as the amount of time saved for each vehicle mile driven, as the Expo line removes commuters from the highway thus increasing speed. This travel time saving component represents the congestion relief experienced by drivers. Finally, I calculate the total congestion relief as the total delay saved (change in delay times VMT) times the value of time. This amounts to about $14 million annually in congestion benefits. Note that this congestion benefit figure is based on the hypothesis that solo drivers would move to public transit, when the Expo line is open (or when there’s a shock to the parking subsidy) and there’s no induced demand on the highway. Later on we will show that this actually provides an upper bound to the welfare gain even if we take induced demand into consideration.

Panel B reports emission reduction benefits. The calculation is as follow. The gravity model provides predictions for both number of trips as well as VMT for both baseline (pre Expo line opening) and after Expo line opens. First I calculate the hypothetical number of VMT removed. Emission reduction is then simply the change in VMT times the total social cost of emission per mile. Following Bento et al. [12], we obtain the average emission amount of CO2, NOX, and VOCs per mile travelled, as well as their corresponding marginal social damage in dol-
lar value. Combining these gives us the total emission benefit. This amounts to about $636k annually, which is much smaller in magnitude compared to the other components.

Similar to panel A, the emission benefit component of the welfare change is calculated using results from the Gravity model, which presents the hypothetical welfare gain when there is no induced demand on driving. If we consider the induced demand that refills any extra capacity created by the policy change, there would essentially be no change on the flow of the driving network. This corresponds to findings in Ahlfeldt, Bento, and Tai [6] where there is virtually no change to the highway level of traffic after five months. Thus in reality, this part of the welfare would be zero.

Panel C reports travel time savings. According to the welfare theory, the travel time saving part of the welfare change is calculated as the change in travel time times the change in commuters moving to transit. I calculate the commuters moving to transit by calculating the increase in transit flow after the expo line opens. This flow figure is multiplied by the transit time saving, which is the OD level transit time change after the Expo line opens. Finally, this travel time saving is aggregated system wide, and multiply by 2 trips a day, and 260 workdays a year. Travel time savings in dollar amount is simply the number of hours saved times value of time. This amounts to roughly $5.5 million annually.

Panel D reports other components in the welfare calculation. The components are as follow.
The scale economy/occupancy cost gap tries to capture the effect of increased transit ridership has on the transit service frequency and capacity. In other words, this component of the welfare tries to capture the magnitude of the Mohring effect. In order to calculate this component, I follow Parry and Small [27] for their parameters on the marginal scale economy per additional passenger mile, and their marginal cost of occupancy per additional passenger mile. Using the gravity model, I calculate the change in total number of miles travelled on transit across all OD pairs. This figure is then multiplied by \((M \text{ scale economy } MC \text{ occupancy})\). The Mohring effect comes to about $1.5 million annually.

The price/marginal cost gap tries to capture the wedge between transit fare and the operational cost. Most, if not all, public transit system in the US has a farebox recover ratio less than 70%. This means the fare collected covers less than 70% of the operating cost of providing the transit service. For example, Amtrak is at 85% (Amtrak [8]), San Francisco BART at 40%, and LACMTA is at only 30% (Lindquist et al. [23]). Operation of public transit system thus often requires subsidy from the government. This component in the welfare calculation tries to subtract this subsidy from the total welfare gain. Here, I refer to Parry and Small [27] once again for their parameters on the per mile fare and per mile marginal supply cost of transit service. This amounts to roughly $11 million per year. Note that this component is negative.

Combining the four components, the total welfare change for the Expo line shock amounts to roughly $10 million per year.
2.9 Results – Transit Expansion with Parking Subsidy Shock

In the previous section I have established a structure to calculate social welfare change for the expansion of the transit system, in this case the opening of Expo line. In this section we consider the change when parking subsidy is reduced to various levels.

In section 2.7 I have shown that the overall welfare change with respect to parking subsidy shock can be grouped into the same terms as the transit coverage shock. This means the previous structure in calculating welfare introduced in the last section can be used directly here.

Table 2.1 and table 2.2 summarizes welfare change under various scenarios. Table 2.1 and 2.2 reports the scenarios before and after Expo line opening. In each of the tables, the parking price column reports the OOP parking price. So for example a parking price of $12 corresponds to a parking subsidy reduction of $2. The second column reports the number of driving trips that has moved to transit under each scenario. For example, in table 2.1 line 2, for the case without the Expo line but reducing the parking subsidy by $1 (essentially increasing the OOP parking price to $11), the gravity model would predict a reduction of 2.67 million driving trips. Similarly, in table able 2.2 line 2, reducing parking subsidy by $1 moves 6.47 million driving trips to the transit system when combined with the effect of the Expo line. Each subsequent column corresponds to a panel in table 1.10. For example, the Congestion relief column reports (in millions) the
total welfare gain from congestion relief as in table 1.10 Panel A and so on.

Table 2.1 reveals how the welfare first increases as parking subsidy is reduced from the original level, reaching a peak at around $5 reduction in subsidy, and then gradually decreasing. The same effect is observed in the case with Expo line available. This indicates an optimal level of a parking subsidy reduction of about $5. The result actually is dependent on a few factors.

First, note that the main mechanism of welfare change comes from how much trips are moved between modes by the shocks. The gravity model predicts that the Expo line would shift almost 4 million trips to transit annually. For the parking subsidy, the initial $1 reduction in subsidy moves 2.67 million trips to transit without the Expo line, and 2.61 million trips with the Expo line. The number of trips moved to transit drops as the parking subsidy is further reduced. For example, the next $1 reduction in subsidy moves 2.27 million trips to transit without Expo line, and the next $1 reduction moves 1.94 million. The optimal level of reduction is achieved at $5, where the policy generates $10.41 million without and $20.44 million with Expo line, in terms of annual welfare gain.

In all cases the effect on trips by the reduction of subsidy is smaller in the with Expo line case than without. This seems counter-intuitive as one might expect that an increased parking cost would drive more people to transit when the transit network coverage is higher. On the other hand, consider the case where the order of event is such that the parking subsidy is reduced before the Expo line opens. From this perspective, it’s logical to expect that a good portion of the marginal
commuters would want to switch to public transit because of the high parking cost. At this point when the Expo line open it would be expected that fewer “new” transit converter would happen, as some of them have already converted when the parking subsidy is reduced.

Second, it is important to note that the two major components in the welfare change at different rates with respect to the change in driving trips. Both the congestion relief and the P/MC gap changes at a decreasing rate when subsidy is reduced, reflecting the decreasing change in number of trips moved to transit. The P/MC gap change is a linear relationship to the number of VMT added to the transit network (which is roughly the VMT removed from the driving network, but not exactly because of different routing). On the other hand, the level of congestion relief does not change linearly with the trip or VMT removed from the driving network. It is easier to understand with an example: on a congested road, the effect of removing the 10 cars is much stronger than the last 10 cars before free flow speed is achieved. As a result the congestion relief benefit drops much faster than the P/MC gap. This difference in rate of change leads to the existence of an optimal level of parking subsidy.

It is also interesting to note that, a $2 reduction in parking subsidy in the 10% of all the TAZs in our model would move nearly 5 million trips to the transit network per year, compared with the 3.8 million trips moved by the Expo line. While the large magnitude of the effect is definitely interesting, this should not be taken as an indication that reducing parking subsidy is the silver bullet to conges-
tion problem for several reasons. First, remember that the parking costs that the model imposes concentrates in the high employment density areas around downtown area. This implies the additional effect would be smaller when the parking subsidy shock is spread to more zones in the area, as it is already set in the highest impact area. Second, the effect of the parking subsidy shock also highly depends on the quality and coverage of the transit system. Without quality alternative options to driving, regardless of how high the parking cost is people are still going to need to drive to work if there’s no way to take transit for commuting. Similarly if the transit service quality is bad, such as very low frequency or unpleasant riding experience, then it would require a much higher parking subsidy shock to achieve the same result in mode shift. In addition, note that even though the number of trips shifting mode is large, it would still take $5 reduction in parking subsidy to reach the same level of welfare change in dollar amount compared to the Expo line.

2.10 Tax implication

In addition to the welfare gain through the various channels described in the general equilibrium model, reduction in transit subsidy fringe benefit would also made a sizable impact on tax revenue, given the predicted change in flow.

To calculate the tax impact, I made a few assumptions. First, I assume that companies offer the same amount of total pre-tax income to each employee, and in
the presence of parking subsidy part of the income becomes tax exempt. There is no systematic survey regarding how companies uses parking subsidy as incentive to employees to our best knowledge. Base on this assumption, the reduction in parking fringe benefit reduces the amount of income that is tax exempt. Next, as the gravity model represent home to work trips, I assume that tax benefit in the form of parking subsidy would only affect work trips. I assume 20 workdays in a month. For drivers that still drive after the reduction in subsidy, the $5 daily subsidy would amount to $100 a month and $1,200 a year. For drivers that has switched to transit, I assume the full subsidy of $250 a month ($3,000 a year) is now taxable. I also assume that not everyone would be affected by this shock, and only 10% of the persistent drivers would be affected.

Table 2.3 reports the calculation of additional tax revenue using model predicted flow change after implementation of parking subsidy reduction. The annual additional tax revenue would amount to nearly $53 million a year. The additional tax revenue is even larger than the annual welfare gain we have estimated previously. Even if we assume transit switchers to receive transit subsidy in form of tax exemption ($150 a month), the additional tax revenue would still be over $49 million a year.
2.11 Conclusion

In this study I have constructed a gravity style model to predict the change in flow and subsequently change in welfare from a transit investment and parking subsidy shock in a two-part instrument framework. I found that parking subsidy, which affects out of pocket parking cost, contribute significantly to the agent’s mode choice decision. Based on model prediction I found that removing $5 in parking subsidy (increasing out of pocket parking cost by $5) leads to an optimal level of welfare where the welfare gain is maximized in the presence of a transit infrastructure expansion. In terms of parking policy implication, this study found that even a relatively local policy of reducing parking subsidy at the top 10% zones with highest employment density would have a sizable effect. Furthermore, the additional tax government can collect from the reduction in subsidy would amount to $50 million a year, making the policy even more enticing from a budget standpoint.
2.12 Tables

Table 2.1: Welfare change under different parking cost settings - Before Expo Opening

<table>
<thead>
<tr>
<th>Parking Price</th>
<th>Driving trips change</th>
<th>Total welfare change</th>
<th>Congestion relief</th>
<th>Emission relief</th>
<th>Travel time saving</th>
<th>Scale economy</th>
<th>P/MC gap</th>
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<td>$0.00</td>
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*Note: Table reports annual welfare change $ and number of trips in Millions.
*Note*: This is the baseline case before Expo line opening.
Table 2.2: Welfare change under different parking cost settings - After Expo Opening

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<th>Parking Price</th>
<th>Driving trips change</th>
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<td>$6.85</td>
<td>$8.56</td>
<td>-$65.19</td>
</tr>
</tbody>
</table>

*Note: Table reports annual welfare change $ and number of trips in Millions.*
Table 2.3: Tax implication from parking subsidy reduction

<table>
<thead>
<tr>
<th>Trip characteristics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline number of driving trips</td>
<td>3,772,980</td>
</tr>
<tr>
<td>Trips switched to transit</td>
<td>26,288</td>
</tr>
<tr>
<td>Persistent drivers affected by subsidy (assume 20%)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Change in annual tax exempt amount</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistent drivers</td>
<td>449,602,985</td>
</tr>
<tr>
<td>Transit switchers</td>
<td>78,865,385</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Additional tax revenue (annual, assuming 10% tax rate)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistent drivers</td>
<td>$44,960,298</td>
</tr>
<tr>
<td>Transit switchers</td>
<td>$7,886,538</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$52,846,836</strong></td>
</tr>
</tbody>
</table>

*Note: Table reports the calculation of additional tax revenue using model predicted flow change after implementation of parking subsidy reduction. I assume that companies offer the same amount of total pre-tax income to each individual, and in the presence of parking subsidy, part of the income becomes tax exempt. The reduction in parking fringe benefit reduces the amount of income that is tax exempt. As the gravity model represent home to work trips, I assume 20 workdays in a month, and for drivers that still driver after the $5 daily subsidy reduction, $1,200 per year in income is no longer tax exempt. For transit switchers the full $250 per month ($3,000 per year) is now exempt. I also assume out of the remaining 3,746,692 persistent drivers (3,772,980-26,288) only 10% would be receiving parking subsidy, and the rest can get free parking.*
2.13 Figures

Figure 2.1: GIS network representation of Los Angeles County, public transit
Figure 2.2: GIS network representation of Los Angeles County, driving
Figure 2.3: Employment density by TAZ, Los Angeles County
LA Metro Transit Services

Figure 2.4: Predicted baseline transit flow
Figure 2.5: Change in Station level flow on Metro network
A.1 Mathematical Derivation–Gravity Model

A.1.1 Model

In order to model traffic flow on freeways, we start with a gravity model which is commonly used in the trade literature. We model the bilateral flow of agents $F$ based on the population $P$ living at the origin $i$, the employment opportunities $E$ available at destination $j$, as well as the bilateral transport cost $C$. $G$ is a constant and $\varepsilon$ captures random shocks. $\beta_1 > 0, \beta_2 > 0, \tau < 0$ are the parameters of the standard model. The total flow takes the following form:

$$F_{ij} = GP_i^{\beta_1}E_j^{\beta_2}C_{ij}^\tau e^{\varepsilon_{ij}}$$  \hspace{1cm} (A.1)

We assume that there are two distinct transport modes available to the agents, providing different connectivity at different quasi-monetary costs: individual transportation (driving alone), and public transit. We further assume the cost of either transportation mode takes the following linear functional form:

$$C^{IND} = VOT \times \frac{\text{road distance}}{\text{road speed}} + \text{gas price} \times \frac{\text{road distance}}{\text{average mpg}}$$  \hspace{1cm} (A.2)

$$C^{PUB} = VOT \times \frac{\text{walking distance}}{\text{walking speed}} + VOT \times \frac{\text{rail distance}}{\text{rail speed}} + \text{fare}$$  \hspace{1cm} (A.3)
where $C^{IND}$ is the cost of individual transport, $C^{PUB}$ the cost of public transit, $VOT$ the value of time. Following common practice in the economic geography literature the internal distances $ij$ are defined as *internal distance* $= \frac{2}{3} \sqrt{\frac{\text{AREA}}{\pi}}$

In addition to the monetary costs and monetized values of time described in (A.2) and (A.3), there are delay penalties and inconvenience costs that factors into agent’s transport mode choice, which depends on idiosyncratic preferences. The mode decision is therefore not strictly based on minimization of the cost described in (A.2) and (A.3). We also assume that on aggregate for a given origin-destination connection, the share of public transit $S$ at total flow is described as a logit function of the cost differential $\Delta C$:

$$\Delta C = \frac{C^{PUB}}{C^{IND}}$$

$$S = \frac{e^{c_1+e_2\Delta C}}{1 + e^{c_1+e_2\Delta C}}$$

(A.4)

The effective cost on an origin-destination connection can therefore be described as:

$$C = S \times C^{PUB} + (1 - S) \times C^{IND}$$

(A.5)

With the share of public transit $S$ the total flow $F$ between origin-destination pair can be broken down into flows by public transit and individual transport:

$$F^{PUB}_{ij} = S_{ij} \times F_{ij}$$

(A.6a)

$$F^{IND}_{ij} = (1 - S_{ij}) \times F_{ij}$$

(A.6b)
Under the above assumptions, the total flow becomes a function of the push and pull factors, the costs of the two transport modes, as well as their cost differential. By substituting (A.4) and (A.5) into (A.1), we obtain the following functional form of total flow:

\[
F_{ij} = GP_i^{\beta_1} E_j^{\beta_2} \left[ \frac{e^{c_1+c_2\Delta C}}{1 + e^{c_1+c_2\Delta C}} \times C^{PUB} + \frac{1}{1 + e^{c_1+c_2\Delta C}} \times C^{IND} \right]^T e^{\epsilon_{ij}}
\]  

(A.7)

A.1.2 Data

Here we discuss in greater detail each of the data sources.

The first source comes from Southern California Association of Governments (SCAG), which describes for each origin-destination pair of Traffic Analysis Zones (TAZs) the number of trips there is for the morning commute. Trips are broken down into various transportation modes: solo drivers, shared-rides, light trucks, medium Trucks, Heavy Trucks, shared-ride 2 person non-HOV, shared ride 3 person non-HOV. Trucks refers to commercial trucks, not pick-ups or SUVs, which go into the estimates of solo or shared rides. The number of trips comes from SCAG’s traffic simulation model, which is based on the four-step trip generation estimation method common in transportation engineering. Various data sources go into this simulation model in order to adjust the result to match observed transit travel patterns. These includes: household travel survey, which provide the share of trip modes, time of trip, and other information; and on-board survey conducted on various public transits, which provides a target for estimating number of trips.
SCAG’s simulation result is further validated by comparing the trips generated to those from travel survey data scaled up from sampling weights.

The second source is population and employment numbers by TAZ from census. We used the Census Transportation Planning Package of 2000 (CTPP 2000) dataset, which contains population at place of residence and population at place of work by census tract. The data is also disaggregated by transportation modes: drive alone, carpool (2-7 people), bus, street car, subway, railroad, ferry, bike, walk, taxi, motorcycle, and others. This information gives us a population and employment estimate from each location and a share of transportation modes. We then translate the population and employment data from census tract to TAZ by overlaying the tract boundary over TAZ boundary, followed by a spatial interpolation.

The third source comes from an extensive GIS modeling of the highway system, Metro rail and bus system, and all the side streets. We start with Census Bureau’s TIGER/Line shapefiles for all the streets, roads, and highways, and subsequently adding shapefiles from LA Metro for bus lines, bus stops, rail lines, and rail stations. We compiled two different road network models: one that simulate agents driving in individual transport (containing the streets and highway network), and one that simulate agents walking to take public transit (bus and Metro). We then calculate, for each origin-destination TAZ pair, the least cost route with either transport mode and collect the corresponding cost ($C^{IND}$ and $C^{PUB}$).
The following parameters were used in the individual transport model: regular road speed of 15 mph, highway speed using PeMS highway detector reported average weekday driving speed, gas price of $3.752 obtained from weekly data of regular reformulated gas from the Energy Information Administration, fleet average fuel efficiency of 20 mpg (EPA, [12]), value of time taken to be $15. we also assign a parking cost of $10 at locations around the CBD [39].

For the public transit model, the following parameters were used: we assume agents walk to take the nearest bus/transit and make the corresponding transfers, walking speed on regular road of 3 mph, regular bus lines travel at 8.8 mph, and BRT lines travel at 11.6 mph [10]. For the various Metro lines we calculate the effective speed by dividing the length (in miles) of each Metro line by the corresponding travel time base on their posted schedule. The following are the effective speed of various lines: red/purple line 29.3 mph, blue line 22.1 mph, green line 34.3 mph, gold line 22.3 mph, and Expo line 17.9 mph. Value of time is again taken to be $15.

Using the above parameters, we calculate three cost matrices: individual transport, public transit pre opening of Expo line, and public transit post opening, thus giving us the cost of choosing individual transportation or public transit both before and after the opening of the Expo line. The costs are then used to predict transit shares.
A.1.3 Calibration

We recover our parameters in several steps. First, the standard gravity model parameters $G, \beta_1, \beta_2, \tau$ are recovered from an auxiliary standard poisson regression of (log) individual transport flows on the (log) number of individual transport users at origin $i$ and destination $j$ and a (log) measure of individual transport cost $C^{IND}$. The remaining critical parameters are $c_1$ and $c_2$, which is used to determine the bilateral flows $F_{ij}$ and the transit share $S_{ij}$ for each origin-destination pair. We solve for these parameters in a grid-search over the parameter space defined by all combinations of $c_1 = \{-2, -1.9, \ldots, 2\}$ and $c_2 = \{-2, -1.9, \ldots, 2\}$. We choose our preferred parameter combination by minimizing the root sum of square deviations between predicted transit shares at origin TAZs ($\hat{S}_i = \sum_j S_{ij} F_{ij}$) and the actual transit share at TAZs that we observe in the data ($S_i$). Our objective is, thus, to minimize the following selection selection criterion $R$:

$$R = \sqrt{\sum_i \left( \sum_j \left( \frac{e^{c_1 + c_2 \Delta C_{ij}}}{1 + e^{c_1 + c_2 \Delta C_{ij}}} P_i^{\beta_1} E_j^{\beta_2} \left[ \frac{e^{c_1 + c_2 \Delta C_{ij}}}{1 + e^{c_1 + c_2 \Delta C_{ij}}} \times C^{PUB} + \frac{1}{1 + e^{c_1 + c_2 \Delta C_{ij}}} \times C^{IND} \right]^T - S_j \right)^2}$$

(A.8)

The best parameter combination is $c_1 = 1.1, c_2 = -1.1$ from our grid search. These values are in line with decreasing transit shares as the relative advantage of public transit decreases.
A.1.4 Prediction Procedure

With the model calibrated, we can predict the impact on the bilateral flows and transit shares of changes in our transportation system, such as a change in bilateral transport cost when a new public transit line is made available. The predicted change in transit share is defined as:

$$\Delta S_{ij} = \Delta \left( \frac{e^{c_1+c_2(C_{ij}^{PUB}/C_{ij}^{IND})}}{1 + e^{c_1+c_2(C_{ij}^{PUB}/C_{ij}^{IND})}} \right)$$ (A.9)

Taking logs and differences equation (A.7) gives the predicted change in total flow:

$$\Delta \log F_{ij} = \tau \Delta \log \left[ \frac{e^{c_1+c_2\Delta C}}{1 + e^{c_1+c_2\Delta C}} \times C_{ij}^{PUB} + \frac{1}{1 + e^{c_1+c_2\Delta C}} \times C_{ij}^{IND} \right]$$ (A.10)

The predicted impacts on public transit and individual transport flows are, thus, given as:

$$\Delta \log F_{ij}^{PUB} = \Delta \log \left( \frac{e^{c_1+c_2\Delta C}}{1 + e^{c_1+c_2\Delta C}} \right) + \Delta \log F_{ij}$$ (A.11)

$$\Delta \log F_{ij}^{IND} = \Delta \log \left( 1 - \frac{e^{c_1+c_2\Delta C}}{1 + e^{c_1+c_2\Delta C}} \right) + \Delta \log F_{ij}$$ (A.12)


A.2 Mathematical Derivation–Welfare Theory

This section outlines the general equilibrium model used to analyze the source of welfare under a two-part instrument scenario, when the transit system experience infrastructure expansion coupled with a reduction in parking subsidy. We describe a model based on Parry and Small [27] with some modifications. Specifically, there are two geographic areas, named A and B. In our context A would be the Expo line service area. There are $N_1 + N_2$ agents with varying level of income to choose their level of consumption of transit modes, and a transit agency which charge fares for public transit and provide transit service. We also added a parking cost and parking subsidy component in the demand side budget constraint to model the parking cost an agent would face.

A.2.1 Demand Side

Agent’s utility and travel characteristics

There are $N_1$ agents in A and $N_2$ agents in B, thus total $N_1 + N_2$ agents in the economy indexed by $i$ and superscript $l$ for location. Each agent in each location choose to consume amount of numeraire good $X^l_i$ and commute $M^l_i$ miles on either car or public transit at peak or off peak period, $M^l_i = \{M^{jkl^l}_i; j = C(car), T(transit), k = P(peak), O(offpeak), l = A(primary), B(secondary))\$. 

126
Each agent is endowed with exogenous income \( I_i \in [I_\text{L}, I_\text{U}] \). In addition, agent’s utility also depends on \( \Gamma_i \), a composite travel time cost function, and \( Z \), the pollution/Accident externalities. Thus, the agent’s utility is:

\[
U_i = u_i(X_i^l, M_i^l, \Gamma_i^l) - Z
\]

Agent’s utility is increasing on \( X_i, M_i \) and decreasing on \( \Gamma_i \). Pollution/accident externalities are correlated with the total amount of car travel:

\[
Z_i^l = \sum_{jk} z_{jk}^l V_i^j
\]

The travel time cost function \( \Gamma_i \) is defined as follow:

\[
\Gamma_i^l = \Gamma(T_i^l, W_i^l, A_i^l, C_i^l, E_i^l).
\]

Time cost \( \Gamma_i \) is increasing on four components: \( T, W, A, C \) and decreasing on \( E \). The components in \( \Gamma \) are defined as:

- \( T_i \): travel time cost = \( \sum_{jkl} t_{ikl}^l M_{ikl}^l \times VOT_i \)
- \( W_i \): waiting cost = \( \sum_k w_{kT}^l M_{ikl}^l \)
- \( A_i \): accessing cost = \( \sum_k a_{kT}^l M_{ikl}^l \)
- \( C_i \): crowding cost = \( \sum_k c_{kT}^l M_{ikl}^l \)
- \( E_i \): network externality = \( -\sum_k e_{kT}^l M_{ikl}^l \)

Travel time cost \( T_i \) captures time spent in the vehicle, on car or public transit. \( t_{ikl}^l \) is the per-mile travel time defined as:

\[
t_{ikl}^l = \begin{cases} 
  \sum_k r_{kC}^l (V_i^Ck) + \alpha \sum_{h \neq i} V_{kC}^l \\
  \sum_k r_{kT}^l V_{ikl}^T
\end{cases}
\]
$t^C_s, t^T_s$ are performance functions where $t^k_s = \frac{1}{v^k_s}$, the inverse of mode travel speed at peak or off peak. We assume that public transit moves a lot slower than cars, so that $t^C < t^T$ (for most conditions). To model congestion, we consider other agent’s choice of vehicle miles consumed $V^k_j$ contributes to overall congestion linearly at a coefficient $\alpha$. This is consistent with Small et al. [33] where they assume agents consider other agents’ contribution to congestion as constant and ignore self contribution to congestion. We also assume that public transit system (the metro network) is not affected by congestion, so travel time on transit is independent of other agent’s choice. The term $VOT^l_i = \phi I^l_i$ represents the value of time of agent and is a fraction $\phi$ of income.

Waiting cost ($W$) captures time spent waiting for trains to arrive. As frequency of transit increase, less time is spent waiting for vehicle arrival and waiting cost decreases. Improvement on frequency reduce waiting cost at a decreasing rate.

Accessing cost ($A$) captures the cost associated with walking to a station. As density of service increase, agents would need to walk shorter to arrive at transit stations. Thus this cost is decreasing with higher service density, and similarly improvement on density reduce accessing cost at a decreasing rate.

Crowding cost ($C$) captures the disutility from a crowded train. This cost is increasing with higher crowdedness in transit vehicle.

Network bonus ($E$) captures the network externality from improved connectivity when the transit service expands. Note that in our setting we do not ex-
Explicitly model routes connecting between the two locations. Rather we model network externality as a utility bonus term in the composite cost function based on the overall network size \( \xi = \xi_A + \xi_B \) and level of transit service \( V^T \), and thus it has a negative sign in \( \Gamma \).

Note that the above components only apply to trips on public transit, and we assume there’s no waiting/accessing/crowding disutility from driving in a car. The coefficients \( w^{Tkl}, a^{Tkl}, c^{Tkl}, e^{kT} \) are further defined as follow:

- \( w^{Tkl} = w(f^{Tkl}), w'(f^{Tkl}) < 0, w''(f^{Tkl}) < 0 \)
- \( a^{Tkl} = a(D^{Tkl}), a'(D^{Tkl}) < 0, a''(D^{Tkl}) < 0 \)
- \( c^{Tkl} = c(I^{Tkl}), c'(I^{Tkl}) > 0 \)
- \( o^{jkl} = M^{jkl}/V^{jkl} \)
- \( e^{kT} = e(\xi, V^T), e' > 0 \)

\( f^{Tkl} = V^{Tkl}/(h^kD^{Tkl}) \) represents the frequency of transit service of period \( k \), and is calculated from the total vehicle miles at period \( k \) provided by transit \( V^{Tkl} = \sum_i V^{iTkl} \) divided by the duration \( h^k \) and density \( D^{Tkl} \) of transit service; \( I^{Tkl} = o^{Tkl}/n^{Tkl} \) represents load factor, and is calculated from the occupancy \( o \) divided by vehicle size \( n \); occupancy \( o^{jkl} \) on mode \( j \) is simply the number of miles traveled per mode \( M^{jkl} \) over the actual number of vehicle miles \( V^{jkl} \). Waiting cost \( w^{Tkl} \) is defined as decreasing on frequency at a decreasing rate. Access cost \( a^{Tkl} \) is defined as decreasing on density at a decreasing rate.
Budget constraint

The agent faces budget constraint as follow:

\[ X_i^l + \sum_{kj} p_{jkl} M_{jkl}^l = I_i^l - TAX_i^l \]

where \( p^{Tkl} \), \( p^{Ckl} \) are the per mile prices of transit and driving respectively. Note that the price of transit per mile is assumed to be lower than driving \( p^{Tkl} < p^{Ckl} \). The per mile price for car is \( p^{Ckl} = p_{0}^{Ckl} + \tau^{Ckl} + (P_i - S_i) \) where \( \tau^{Ckl} \) is fuel tax, and \( p_{0}^{Ckl} \) is pre-tax fuel cost per mile. Each agent also have to pay a “true parking cost” \( P_i \), and receive a certain amount of subsidy \( S_i \). Transit trips do not incur parking cost, and not all driving trips incur parking cost. Agents could receive parking subsidy as little as nothing and as much as their true parking cost \( (P_i \geq S_i \geq 0) \). Both the true parking cost and subsidy are exogenous to the agent’s choice.

Each agent has to pay lump sum tax \( TAX_i^l \) such that \( \sum_{il} TAX_i^l = TAX \). The lump sum tax and fuel tax is used to finance operation of public transit.

Maximization problem

Agent \( i \) thus optimize to following:

\[
\max_{X_i^l, M_i^l} u_i^l(X_i, M_i^l, \Gamma_i^l) - Z
\]

subject to:

\[ X_i^l + \sum_{kj} p_{jkl} M_{jkl}^l = I_i^l - TAX_i^l \]
From the first order conditions:

\[
\frac{U_{M^i}}{U_X} = q_{i}^{jkl} = p^{jkl} + \rho^T t^{jkl} V O T_i + \rho^W w^{jkl} + \rho^A a^{jkl} + \rho^C c^{jkl} - \rho^E e^{jkl}
\]  

(A.13)

where \(\rho^K = -\frac{m_{\Gamma^K}}{u_i}\) for \(K = T, W, A, C, E\).

Thus we have the indirect utilities and demand function as follow:

\[
\hat{U}_i = \hat{u}([q_{i}^{jkl}], I_i - T A X_i, P_i, S_i) - Z
\]

\[
X_i = X_i([q_{i}^{jkl}], I_i - T A X_i, P_i, S_i)
\]

\[
M_i^i = M_i^i([q_{i}^{jkl}], I_i - T A X_i, P_i, S_i)
\]

where \([q_{i}^{jkl}] = \{p^{jkl}, t^{jkl}, w^{jkl}, a^{jkl}, c^{jkl}, e^{jkl}\}\), and \(l = A, B; k = P, O; j = C, T\)

A.2.2 Supply Side

Service and travel characteristics

The supply side of our model is the transit agency. Each location \(l = A, B\) has a total number of OD pairs \([\Xi^l]\) where agents will need to travel to/from. Define \([\xi^l : \xi^l \subset \Xi^l]\) as pairs in the universe on which the agency offer public transit service. For each route \(i\) in \([\xi^l]\) the agency choose \(n_i^l\) (vehicle size), \(D_i^l\) (density of service / number of vehicles servicing each line), and \(V_i^{Tkl}\). We assume for now speed \(s_i^l\) is exogenous, and that \(s_i^{lT} < s_i^{lC}\), driving speed is faster than transit speed. The number of vehicle miles offered is \(V_i^{Tkl} = \) length of line x # vehicles and \(M = V \times \) occupancy = \(V \times o\).
Note that in this setup there’s no way to match individual agents $i$ to the route in $\{\xi^l\}$. Consequently we can consider the aggregate level, where we consider the agency chooses an overall level of $n, D, V^T$, and that subsequently the resources are allocated to each route in $\{\xi^l\}$ optimally. We define the variable $\xi^l = \frac{\text{number of paris in } \{\xi^l\}}{\text{number of paris in } \{\Xi\}}$ to represent the extend of public transit coverage in the system. The overall level of $n^{jkl}, D^{jkl}, s^{jkl}, V^{jkl}$ correlates to the route level as follow:

\begin{itemize}
  \item $n^{jkl} = \sum_{i \in \{\xi\}} n^{jkl}_i = n(\xi, l, k, j), \frac{\partial n^{jkl}}{\partial \xi} > 0^1$
  \item $D^{jkl} = \sum_{i \in \{\xi\}} D^{jkl}_i = D(\xi, l, k, j), \frac{\partial D^{jkl}}{\partial \xi} > 0^2$
  \item $s^{jkl} = \sum_{i \in \{\xi\}} s^{jkl}_i = s(\xi, l, k), \frac{\partial s^{jkl}}{\partial \xi} > 0^3$
  \item $V^{jkl} = \sum_{i \in \{\xi\}} V^{jkl}_i = V^T(\xi, l, k), \frac{\partial V^{jkl}}{\partial \xi} > 0$
\end{itemize}

**Budget Constraint**

Transit agency operates all public transit services with following operational cost for mode $j$, period $k$:

$$OC^{jkl} = F^{jkl} + K^{jkl} \sum_i V^{jkl}_i$$

\begin{itemize}
  \item $^1$Since we assume $n_i$ is chosen to maximize $\tilde{U}$, adding new line (i.e. $\uparrow \xi$) shouldn’t lead to change in pre-existing $n_i$.
  \item $^2$Similar to argument in footnote 1, we assume there’s a constant $D$ that is optimal to all route in $\{\xi\}$. So when service expands (i.e. $\uparrow \xi$), overall $D$ increases.
  \item $^3$s is the average speed agents will be traveling on each route in $\{\Xi\}$ if they opt to take public transit, regardless of its availability. Therefore when transit agency increase public transit coverage ($\uparrow \xi$), the average speed for all routes increase.
\end{itemize}
where $F^{kj}$ represent fixed costs in public transit operations such as operation cost of stations and terminals. $K^{kj}$ captures the variable cost, and is defined as:

$$K^{jkl} = k_1^{jkl} + k_2^{jkl} n^{jkl} + k_3^{jkl} s^{jkl}$$

where $n^{kj}$ is the vehicle size and $s^{kj}$ the transit speed defined above. Notice that we assume $k_1^{kj} > 0$. As a result, increasing transit vehicle size $n^{kj}$ and speed $s^{kj}$ incurs constant marginal cost and decreasing average cost.

The agency’s income comes from lump sum and fuel tax from driving agents, as well as the price charged from the public transit. The agency must balance its budget:

$$TAX + \sum_{ikl} r^{Ckl} v_i^{Ckl} = \sum_{ik} \sum_{j \in C} (Oc^{kj} - p^{jkl} \sum_{i} M_i^{jkl}) \quad (A.14)$$

Market clears when the vehicle miles supplied by the agency at $l$ equals to the miles demanded by the $N_l$ agents.
Density, vehicle size

Furthermore, we assume the agency will optimize the route setup $D, n, s$ to maximize utility $\bar{U}$. Consider the indirect utility function:

$$\bar{U} = \sum_{il} \left[ \tilde{u} \left( \{ p_{jkl}, t_{jkl}, w_{jkl}, a_{jkl}, c_{jkl}, e_{jkl} \}, I, TAX, P_i, S_i \right) - Z \right]$$

$$= \max_{X, M(M^{jl}), \lambda} \sum_{l} \left\{ u \left( X, M(M^{jl}) \right), \Gamma \left( \sum_{kj} t_{jkl} M^{jkl}, \sum_{kj} w_{jkl} M^{jkl}, \sum_{kj} a_{jkl} M^{jkl}, \sum_{kj} c_{jkl} M^{jkl}, \sum_{kj} e_{jkl} M^{jkl} \right) \right\}$$

$$- \sum_{kj} \zeta_{jkl} V_{jkl} + \lambda \left[ I - TAX - X - \sum_{kj} p_{jkl} M^{jkl} \right]$$

Notice that the above is a function of $V, M, D, n$. Given $V, M$, to optimize $D, n$ we consider the first order conditions:

$$0 = \frac{\partial \bar{U}}{\partial D} = \bar{U}_w \frac{\partial w}{\partial D} + \bar{U}_a \frac{\partial a}{\partial D} = -\lambda \rho^M M \frac{\partial f}{\partial D} - \lambda \rho A M \quad \Rightarrow \quad \rho^W w_\eta = \rho^A a_\eta$$

$$0 = \frac{\partial \bar{U}}{\partial n} = \bar{U}_c \frac{\partial c}{\partial n} + \bar{U}_{OC} \frac{\partial OC}{\partial n} = -\lambda \rho^C M c_t \frac{\partial l}{\partial n} - \lambda Vt \frac{dK}{dn} \quad \Rightarrow \quad \rho^C c_\eta o = tk_2$$

$$0 = \frac{\partial \bar{U}}{\partial s} = \bar{U}_t \frac{\partial t}{\partial s} + \bar{U}_{OC} \frac{\partial OC}{\partial s} = -\lambda \rho^T M \frac{dt}{ds} - \lambda Vt \frac{dK}{ds} \quad \Rightarrow \quad \rho^T o = tk_3 s^2$$

The equations above set $D, n, s$ for a given $M, V$. The elasticity terms $\eta_{(\cdot)}$ are described in section A.21.

The above conditions states that route density is increased until the incremental cost of extra waiting, resulting from less frequent service, equals the incremental reduction in access cost; and that transit vehicle size is increased until the incremental reduction in crowding costs to its occupants equals the incremental cost to the agency from operating a larger vehicle.
Note that the above determines the level of $D, n, f$ for a given $M, V$. The agency must meet demand of transit $M^T$ with enough capacity. Capacity is the frequency of service times vehicle capacity, thus $M^T = fn$ for each route that the agency serves.

With the above setup, everything in the transit agency’s problem is pin to the vehicle miles $V$ and $M$. To further simplify the problem we can assume transit occupancy $o = \frac{M}{V}$ is fixed.

Shock to infrastructure provision

In our model, a shock in infrastructure is modeled as an increase in the routes being covered by the transit agency in location $A$, $\{\xi^A : \xi^A \in \Xi^A\}$. As a result the fraction $\xi^A$ also increase, and so does the overall network size $\xi = \xi^A + \xi^B$.

The effect is different between the two locations. In location $A$, where the infrastructure expansion happens, $\xi^A$ increase, and as a result transit services provided in the location also increased. By the definition in section A.2.2 the agency related service characteristics will change as follow:

$$\frac{dn^A}{d\xi^A} > 0, \quad \frac{dD^A}{d\xi^A} > 0, \quad \frac{ds^{AT}}{d\xi^A} > 0, \quad \frac{dV^{AT}}{d\xi^A} > 0$$ (A.15)

The consumer facing travel characteristics, as defined in section A.2.1 will change as follow:

$$\frac{\partial o^{AT}}{\partial \xi^A} < 0, \quad \frac{\partial f^{AT}}{\partial \xi^A} = 0, \quad \frac{\partial l^{AT}}{\partial \xi^A} < 0$$
\[ \frac{\partial r^{AT}}{\partial \xi^A} < 0, \quad \frac{\partial w^{AT}}{\partial \xi^A} = 0, \quad \frac{\partial a^{AT}}{\partial \xi^A} < 0, \quad \frac{\partial c^{AT}}{\partial \xi^A} < 0 \] (A.16)

the shock will translate into travel time, access time, and occupancy decrease in location A. We assume for now frequency of service remain the same.

In location B, the effect of an expansion of transit service of A translates to an increase in overall connectivity \( \xi = \xi^A + \xi^B \), which increase the network externality bonus for agents in B. As a result, ridership in B increase and in response, the agency will provide more transit capacity through larger cars or higher frequency, leading to the Mohring effect.

Recall the agency’s operational cost is defined as: \( OC^{jkl} = F^{jkl} + (k_1^{jkl} + k_2^{jkl} n^{jkl} k_3^{jkl} s^{jkl}) t^{jkl} V^{jkl} \). The shock would lead to the following change to the agency’s operational cost,

\[
\frac{\partial OC^{jkl}}{\partial \xi^A} = \frac{\partial OC^{jkl}}{\partial n^{jkl}} \frac{dn^{jkl}}{d\xi^A} + \frac{\partial OC^{jkl}}{\partial s^{jkl}} \frac{ds^{jkl}}{d\xi^A} + \frac{\partial OC^{jkl}}{\partial t^{jkl}} \frac{dt^{jkl}}{d\xi^A} + \frac{\partial OC^{jkl}}{\partial V^{jkl}} \frac{dV^{jkl}}{d\xi^A}
\]
\[
= k_2^{jkl} t^{jkl} V^{jkl} \frac{dn^{jkl}}{d\xi^A} + k_3^{jkl} t^{jkl} V^{jkl} \frac{ds^{jkl}}{d\xi^A} + K^{jkl} V^{jkl} \frac{dt^{jkl}}{d\xi^A} + K^{jkl} t^{jkl} V^{jkl} M^{jkl} \xi^A \] (A.17)

A.2.3 Welfare analysis

One of the main goal of setting up the general equilibrium model is to shed light on welfare change when the system experience a shock. In our analysis we want to see how welfare is changed under increase in transit infrastructure provision.
Welfare change under infrastructure shock

The welfare implication from an infrastructure shock is as follow. First consider the total welfare function:

\[
\tilde{U} = \sum_{it} \left[ \tilde{u} \left( \{p^{ijl}, t^{ijl}, w^{ijl}, a^{ijl}, c^{ijl}, e^{ijl} \}, I, TAX \} - Z \right) \right]
\]

\[
= \max_{X, \{M^{ijl}\}, \lambda} \left\{ \sum_{l} \left[ u \left( X, M((M^{ijl}) \right), \right. \right.
\]

\[
\left. \Gamma \left( \sum_{kj} t^{ijl} M^{ijl}, \sum_{kj} w^{ijl} M^{ijl}, \sum_{kj} a^{ijl} M^{ijl}, \sum_{kj} c^{ijl} M^{ijl}, \sum_{kj} e^{ijl} M^{ijl} \right) \right] \]

\[
- \sum_{kj} z^{ijl} v^{ijl} + \lambda \left[ I - TAX - \sum_{kj} p^{ijl} M^{ijl} - X \right] \right\}
\]

(A.18)

Partially differentiating (A.18) with respect to \( \{q_{ij}^{xyz} \} = \{p^{xyz}, t^{xyz}, w^{xyz}, a^{xyz}, c^{xyz}, e^{xyz} \}, TAX \) with the definition of \( \rho^K \) in (A.13) gives the following:

\[
\tilde{U}_p^{ijl} = -\lambda M^{ijl}, \quad \tilde{U}_t^{ijl} = -\lambda \rho^T M^{ijl}, \quad \tilde{U}_w^{ijl} = -\lambda \rho^W M^{ijl},
\]

\[
\tilde{U}_a^{ijl} = -\lambda \rho^A M^{ijl}, \quad \tilde{U}_c^{ijl} = -\lambda \rho^C M^{ijl}, \quad \tilde{U}_e^{ijl} = -\lambda \rho^E M^{ijl}, \quad \tilde{U}_{TAX} = -\lambda = -u_X
\]

(A.19)

Totally differentiating (A.18) with respect to \( \xi^A \), substituting (A.19) yields:

\[
\frac{d\tilde{U}}{d\xi^A} = \sum_{l} \left\{ \sum_{kj} \tilde{U}_{p^{ijl}} \frac{dp^{ijl}}{d\xi^A} + \tilde{U}_{TAX} \frac{dTAX}{d\xi^A} - \sum_{kj} \tilde{U}_{v^{ijl}} \frac{dv^{ijl}}{d\xi^A} \right\}
\]

\[
+ \sum_{kj} \left\{ \tilde{U}_{t^{ijl}} \frac{dt^{ijl}}{d\xi^A} + \tilde{U}_{w^{ijl}} \frac{dw^{ijl}}{d\xi^A} + \tilde{U}_{a^{ijl}} \frac{da^{ijl}}{d\xi^A} + \tilde{U}_{c^{ijl}} \frac{dc^{ijl}}{d\xi^A} + \tilde{U}_{e^{ijl}} \frac{de^{ijl}}{d\xi^A} \right\}
\]

137
Base on the above equation, we define the marginal welfare effect in consumption unit as $MW^{\xi_A} = \frac{dU}{d\xi_A} - \frac{1}{\lambda} \frac{dU}{d\xi_A}$.

$$MW^{\xi_A} = -\sum_l \left\{ \sum_{kj} M_{jkl} d p_{jkl} + \frac{dTAX}{d\xi_A} - \sum_{kj} z_{jkl} dV_{jkl} \right\}$$

For $\frac{dTAX}{d\xi_A}$, we use the agency’s budget constraint in equation (A.14):

$$\frac{dTAX}{d\xi_A} = \sum_l \left\{ \sum_{kj} M_{jkl} d p_{jkl} - \sum_k \tau_{Ckl} V_{Ckl} M_{\xi_A}^{Ckl} - \sum_k p^{Tkl} M_{\xi_A}^{Tkl} \right\}$$

and the equation become:

$$MW^{\xi_A} = -\sum_l \left\{ \sum_{kj} z_{jkl} V_{jkl} M_{\xi_A}^{Mkl} - \sum_k \tau_{Ckl} V_{Ckl} M_{\xi_A}^{Ckl} - \sum_k p^{Tkl} M_{\xi_A}^{Tkl} \right\}$$

$^4$Recall $\lambda = \lambda$ from consumer FOC
The above expression can be interpreted as follow:

\[
MW^{\xi A} = - \sum_l \left\{ \text{pollution reduction} \right. \\
- \frac{1}{\lambda} \sum_{kj} z_{jkl}^v M_{M}^k dM_{\xi A}^k \\
\left. + \text{Fuel Revenue} \right. \\
- \sum_k \tau_{Ckl} V_{M}^k M_{Ckl}^k \\
\left. - \text{Transit Revenue} \right. \\
+ \sum_k \rho_{\xi A}^T M_{Tkl}^k \\
\right\}

We define the following travel characteristics elasticities:

\[
\eta_{w}^{jkl} = - f_{w}^{jkl} w_{f}^{jkl} \\
\eta_{a}^{jkl} = - D_{a}^{jkl} a_{D}^{jkl} \\
\eta_{c}^{jkl} = l_{c}^{jkl} c_{l}^{jkl} \\
\eta_{e}^{jkl} = V_{jkl} e_{V}^{jkl}
\]

\[
\epsilon_{V} = M_{V}^{jkl} V_{M}^{jkl} = o_{V}^{jkl} V_{M}^{jkl} \\
1 - \epsilon_{V} = M_{o}^{jkl} o_{M}^{jkl} = V_{o}^{jkl} o_{V}^{jkl}
\]

\[
\epsilon_{f} = V_{f}^{jkl} f_{V}^{jkl} = D_{f}^{jkl} f_{V}^{jkl} \\
1 - \epsilon_{f} = V_{D}^{jkl} D_{V}^{jkl} = f_{D}^{jkl} D_{V}^{jkl}
\]

\[
\epsilon_{n}^{jkl} = l_{n}^{jkl} n_{l}^{jkl} = t_{n}^{jkl} t_{l}^{jkl} \\
1 - \epsilon_{n}^{jkl} = l_{t}^{jkl} n_{t}^{jkl} = t_{t}^{jkl} t_{t}^{jkl}
\]

Using the above definitions and definitions of travel characteristics, we compute several derivatives from equation (A.20)
\[
\frac{dw_{ijkl}}{d\xi^A} = w_{ij}^{jkl} \frac{df_{ijkl}}{dM_{ijkl}} M_{\xi^A}^{ijkl} = -w_{ij}^{jkl} \eta_{ij}^{jkl} \epsilon_{f}^{jkl} \epsilon_{V}^{ijkl} M_{\xi^A}^{ijkl} / M_{ijkl}^{ijkl}
\]
\[
\frac{da_{ijkl}}{d\xi^A} = d_{ijkl}^{ijkl} \frac{dD_{ijkl}}{dM_{ijkl}} M_{\xi^A}^{ijkl} = -a_{ijkl}^{ijkl} \eta_{ijkl}^{ijkl} (1 - \epsilon_{f}^{ijkl}) \epsilon_{V}^{ijkl} M_{\xi^A}^{ijkl} / M_{ijkl}^{ijkl}
\]
\[
\frac{dc_{ijkl}}{d\xi^A} = c_{ijkl}^{ijkl} \frac{dI_{ijkl}}{dM_{ijkl}} M_{\xi^A}^{ijkl} = c_{ijkl}^{ijkl} \eta_{ijkl}^{ijkl} (1 - \epsilon_{f}^{ijkl})(1 - \epsilon_{V}^{ijkl}) M_{\xi^A}^{ijkl} / M_{ijkl}^{ijkl}
\]
\[
\frac{dt_{ijkl}}{d\xi^A} = (t_{ijkl}^{ijkl} / o^{C}) M_{\xi^A}^{ijkl} C_{ijkl}^{ijkl} + (t_{ijkl}^{ijkl} / o^{T}) \epsilon_{V}^{ijkl} M_{\xi^A}^{ijkl}
\]
\[
\frac{de_{ijkl}}{d\xi^A} = e \epsilon_{ijkl}^{ijkl} \epsilon_{V}^{ijkl} M_{\xi^A}^{ijkl} / M_{ijkl}^{ijkl}
\]

Now we can consider the terms in equation (A.20). Using equations 11-17:

1. Wait, access, and connectivity

\[-(\rho^{w} \frac{dw_{ijkl}}{d\xi^A} + \rho^{A} \frac{da_{ijkl}}{d\xi^A} - \rho^{E} \frac{de_{ijkl}}{d\xi^A}) M_{ijkl}^{ijkl} = -\left(\rho^{w} w_{ijkl}^{ijkl} \eta_{ijkl}^{ijkl} \epsilon_{f}^{ijkl} - \rho^{A} a_{ijkl}^{ijkl} \eta_{ijkl}^{ijkl} (1 - \epsilon_{f}^{ijkl}) + e \epsilon_{ijkl}^{ijkl} \right) \epsilon_{V}^{ijkl} M_{\xi^A}^{ijkl}
\]

\[= MB_{scale}^{ijkl} M_{\xi^A}^{ijkl} \tag{A.25}\]

2. OPSUPPLY

\[ -K^{ijkl} t^{ijkl} V_{M}^{ijkl} M_{\xi^A}^{ijkl} = -K^{ijkl} t^{ijkl} \frac{\epsilon_{V}^{ijkl}}{M_{ijkl}^{ijkl} / V_{ijkl}^{ijkl}} = -K^{ijkl} \frac{\epsilon_{V}^{ijkl}}{o^{ijkl} t^{ijkl} M_{\xi^A}^{ijkl}} = -MC_{supply}^{ijkl} M_{\xi^A}^{ijkl} \tag{A.26}\]

140
3. Travel time

\[ - \sum_{jkl} M_{ijkl} \left\{ \rho T \frac{dt_{ijkl}}{dA} \right\} \]

\[ = - \sum_{k} (M_{ijkl} \rho T) \left[ (t_{C}^{ijkl} / o_{Cijkl}) M_{Cijkl} \xi_{A} + (t_{T}^{ijkl} / o_{Tijkl}) \epsilon_{V} M_{Tijkl} \xi_{A} \right] \]

\[ = - \sum_{k} (M_{ijkl} \rho T) \left[ (t_{s}^{Cijkl} / o_{cijkl}) M_{Cijkl} \xi_{A} + (t_{s}^{Tijkl} / o_{Tijkl}) \epsilon_{V} M_{Tijkl} \xi_{A} \right] - \sum_{k} (\alpha t_{s}^{Cijkl} / o_{Cijkl}) M_{Cijkl} \xi_{A} \]

\[ = \sum_{jkl} MB_{ijkl} M_{ijkl} \xi_{A} - \sum_{k} (\alpha t_{s}^{Cijkl} / o_{cijkl}) M_{Cijkl} \xi_{A} \]  \hspace{1cm} (A.27)

4. OPCONG

\[ - \left( \sum_{k} K^{ijkl} \lambda T_{ijkl} \frac{dt_{ijkl}}{dA} + \sum_{k} (\alpha t_{s}^{Cijkl} / o_{cijkl}) M_{Cijkl} \xi_{A} \right) \]

\[ = - \left( \sum_{k} (K^{ijkl} \lambda T_{ijkl} (t_{C}^{ijkl} / o_{Cijkl}) \epsilon_{V} M_{Cijkl} \xi_{A} + \sum_{k} (\alpha t_{s}^{Cijkl} / o_{cijkl}) M_{Cijkl} \xi_{A} \right) \]

\[ = -(MC_{cong}^{C} / o_{C}) M_{Cijkl} \xi_{A} - (MC_{cong}^{T} / o_{T}) \epsilon_{V} M_{Tijkl} \xi_{A} \]  \hspace{1cm} (A.28)

5. Pollution reduction

\[ \frac{1}{\lambda} \sum_{jkl} z^{ijkl} V_{ijkl} M_{ijkl} \xi_{A} \]

\[ = \sum_{k} z^{ijkl} \frac{1}{u_{x} o_{ijkl} M_{ijkl}} \]  \hspace{1cm} (A.29)

6. FUELREV

\[ \sum_{k} \tau^{k} V_{ijkl} M_{ijkl} \]

\[ (A.30) \]

\[ \text{Travel time term will be negative because of shorten travel time, so } MB_{T}T \text{ is defined as the negative to reflect the benefit.} \]
7. Crowding, VEHSIZE, VEHSPEED

\[- \rho_c d c^{ikl} M^{Tkl} - k_2^{Tkl} t^{Tkl} V^{Tkl} \frac{d n^{Tkl}}{d \xi^A} - k_3^{Tkl} t^{Tkl} V^{Tkl} s^{Tkl} \frac{d e^A}{d \xi^A} \]

\[= - \left[ \rho_c^c M^{Tkl} T_k (1 - \epsilon_n^{Tkl})(1 - \epsilon_V) M^{Tkl}_{\xi^i} + k_2^{Tkl} t^{Tkl} V^{Tkl} \frac{d n^{Tkl}}{d \xi^A} + k_3^{Tkl} t^{Tkl} V^{Tkl} s^{Tkl} \frac{d e^A}{d \xi^A} \right] \]

\[= - \left[ \rho_c^c M^{Tkl} T_k (1 - \epsilon_n^{Tkl})(1 - \epsilon_V) M^{Tkl}_{\xi^i} + k_2^{Tkl} t^{Tkl} V^{Tkl} \frac{d n^{Tkl}}{d \xi^A} + k_3^{Tkl} t^{Tkl} V^{Tkl} s^{Tkl} M^{Tkl}_{\xi^i} \right] \]

\[= - M C_{\text{occ}}^{Tkl} M^{Tkl}_{\xi^A} \quad (A.31) \]

Finally, we combine equation (A.28) - (A.29) to become

\[= - \sum_k M C_{\text{ext}}^{kC} M^{kC}_{\xi^A} \]

and equation (A.20) can be expressed as:

\[M W_{\xi^A} = \left( \sum_{jkl} M B_{jkl}^{Tkl} M_{\xi^i}^{jkl} - \left( M C_{\text{supply}}^{Tkl} - p^{Tkl} \right) M_{\xi^A}^{Tkl} \right) + \sum_{lk} \left( M C_{\text{occ}}^{Tkl} \right) M_{\xi^A}^{Tkl} \]

\[+ \sum_{lk} \left( M B_{\text{scale}}^{Tkl} - M C_{\text{occ}}^{Tkl} \right) M_{\xi^A}^{Tkl} - \sum_k \left( M C_{\text{supply}}^{Bkl} - p^{Bkl} \right) M_{\xi^A}^{Bkl} \]

\[= (A.32) \]

Equation (A.32) implies that sources of welfare in the system can be broken down into:

a) Travel time changes net of marginal cost of supplying new route less revenue

b) Wedge in driving market: change in externality less fuel tax

142
c) Wedge in transit supply: AC/MC gap – welfare decrease to the extend of how much more marginal cost is to the price

d) Wedge in transit demand: Scale economies (Mohring effect) v.s. cost from increased occupancy

Welfare change under parking subsidy shock

The welfare analysis for parking subsidy shock closely follows that of the infrastructure shock. We once again start with the total welfare function (A.18) and consider it’s total differential with respect to $p_C$, substituting (A.19) yields:

$$
\frac{d\tilde{U}}{dp_C} = \sum_l \left\{ \sum_{kj} \tilde{U}_{p_C} dTAX - \sum_{kj} z_{jkl} \frac{dV_{jkl}}{dp_C} \right\} + \sum_l \left\{ \tilde{TAX} + \tilde{U}_{t_{ijkl}} \frac{dT_{ijkl}}{dp_C} + \tilde{U}_{w_{ijkl}} \frac{dW_{ijkl}}{dp_C} + \tilde{U}_{a_{ijkl}} \frac{dA_{ijkl}}{dp_C} + \tilde{U}_{c_{ijkl}} \frac{dC_{ijkl}}{dp_C} + \tilde{U}_{e_{ijkl}} \frac{dE_{ijkl}}{dp_C} \right\}
$$

Similar to the infrastructure shock, the marginal welfare effect in consumption unit is defined as $MW_{p_C} = \frac{d\tilde{U}}{dp_C} \frac{1}{\lambda x} = \frac{1}{\lambda} \frac{d\tilde{U}}{dp_C}$

For $dTAX$, we use the agency’s budget constraint in equation (A.14):

$$
\frac{dTAX}{dp_C} = \sum_l \left\{ - \sum_{kj} M_{ijkl} \frac{dp_{ijkl}}{dp_C} - \sum_k T_{ijkl} V_{ijkl} M_{ijkl} - \sum_k p_{ijkl} M_{ijkl}^T \right\} + \sum_k \left\{ k_2 T_{ijkl} V_{ijkl} \frac{dn_{ijkl}}{dp_C} + k_3 T_{ijkl} V_{ijkl} \frac{ds_{ijkl}}{dp_C} + K_{ijkl} V_{ijkl} \frac{dT_{ijkl}}{dp_C} + K_{ijkl} T_{ijkl} V_{ijkl} M_{ijkl}^T \right\}
$$

143
And once again, similar to the infrastructure shock case, the marginal welfare equation takes the form of the following:

\[
MW^p_C = - \sum_{j} \left( -\frac{1}{\lambda} \sum_{kl} z^{ijkl} V_{M}^{ijkl} dM^{ijkl} - \sum_{k} r^{Ckl} V_{M}^{Ckl} M_{pC}^{Ckl} - \sum_{k} p^{Tkl} M_{pC}^{Tkl} \right) + \sum_{k} \left[ k_2^{Tkl} T_{Tkl} ^{dC} + k_3^{Tkl} T_{Tkl} ^{dC} M_{Tkl}^{Ckl} \right] + \sum_{kl} \left( \rho^{Tkl} \frac{dC_{Tkl}}{dp} + \rho^{W} \frac{dW_{ijkl}}{dp} + \rho^{A} \frac{dA_{ijkl}}{dp} + \rho^{C} \frac{dc_{ijkl}}{dp} - \rho^{E} \frac{de_{ijkl}}{dp} \right)
\]

Notice that the marginal welfare formula in both cases contains similar terms, using the derivations from (A.21) to (A.31), we once again group the marginal welfare terms into the same categories as (A.32)

\[
MW^p_C = \left( \sum_{ijkl} MB^{ijkl}_{TT} M_{pC}^{ijkl} - \left( MC^{Akl}_{supply} - p^{Akl} \right) M_{pC}^{Akl} \right) + \sum_{kl} \left( r^{Ckl} V_{m}^{Ckl} - MC^{Ckl}_{ext} \right) M_{pC}^{Ckl} \]

\[
+ \sum_{lk} \left( MB^{Tkl}_{scale} - MC^{Tkl}_{occ} \right) M_{pC}^{Tkl} - \sum_{k} \left( MC^{Bkl}_{supply} - p^{Bkl} \right) M_{pC}^{Bkl} \] (A.33)

Both the shocks from infrastructure expansion and parking subsidy reduction works through the same welfare channels.
A.3 Detector Matching mechanism

In our gravity model we calculate the origin-destination cost matrix using ArcGIS’s OD cost matrix function within the network analyst tool box. This function provides a relatively more efficient way to calculate the least cost to travel for all OD pairs within the study area, and reports other necessary trip statistics such as trip length in miles, and trip time. The OD cost matrix function uses the same Dijkstra’s algorithm as the regular routing function to seek the least cost route within the network.

The route function in ArcGIS output a route for any given number of waypoints, and the route can be exported as an independent line shape. This line shape can be used to determine whether a particular stations or highway detector lies on the route. Because of all the extra information accumulated by the algorithm, trying to loop through all the OD pairs in our model using the regular route function makes the whole process much inefficient and processing time exponentially longer. It takes the OD cost matrix function about five hours to generate the OD cost for all the OD pairs between the 2241 TAZs, or about 1 million routes per hour. Within the same time frame, the route function would only complete 5% of all the trip pairs. Ultimately we decided to use the OD cost matrix function for it’s better performance.

We instead chose to create a matching mechanism to carryout detector/station to OD route matching. To do so, we calculate two additional matrices after the
OD cost matrix: a matrix that calculates the cost from any TAZ to any detector (TD), and by symmetry the detector to TAZ matrix (same case for station in transit matrix). One pass of the TD calculation is sufficient to obtain both TD and DT matrix in the transit case, since speed are independent of direction and there’s no parking cost involved. For the driving matrices TD and DT are calculated separately. Because the number of detectors are smaller than the number of TAZs, and only the total TD/DT cost is required, the calculation is much faster than the full OD cost matrix. The TAZ to station matrix can be computed in less than an hour.

Once the OD, TD, and DT matrix are computed, we import them into python and carry out the matching. We identify detector/station $d$ to be on the route between $i$ and $j$ when the cost match:

$$C_{id} + C_{dj} = C_{ij}$$

where $C_{xy}$ is the cost of traveling from $x$ to $y$, which can be the origin $i$, destination $j$, or detector (or station) $d$. We chose to carry out this matching in Python instead of Stata for two reasons. The Pandas package in Python has much better reading and writing speed when reading in large matrices, and we can attain much higher precision when importing the CSV output from GIS into Python than to STATA. We turned to Python after attempting to do the same procedure in STATA and discovered that many detectors/stations cannot be matched to any OD pairs.

The matching result of which OD pair a detector belongs to is stored in a matrix of binary values (e.g. $m^d_{ij} = 1$, else 0, if traveling from $i$ to $j$ needs to go through
detector $d$), and the collection of all stations are stored in an array of matrices. The array is applied when we aggregate the OD flow result into detector level or station level flow statistics.
A.4 Partial equilibrium as upper bound on system-wide effects

Here we try to demonstrate why the partial equilibrium congestion interaction effect can be an upper bound to the system wide congestion interaction effect.

Consider a scenario where we have a fast and direct route \((m)\) such as the treated highway segment (I-10) between two points A and B. There exists a number \((n)\) of alternative options \((o)\) that are either longer in distance, slower to travel, or less preferred time of day. Assume also in the beginning the metro network is not available to the agents.

A.4.1 Nash equilibrium versus social optimum

We first compare, in general, the allocation of highway commuters under Nash equilibrium versus the social optimum. Suppose that travel time on route \(i\) is given by the following form:

\[
TT_i = \alpha_i + f(C_i) \tag{A.34}
\]

where \(f(C_i)\) represents the convex congestion function such that \(f'(C_i) > 0, f''(C_i) > 0\). We will assume that all \(o\) routes have identical free-flow and congestion functional forms, and that the direct route is fastest in free-flow, such that \(\alpha_m < \alpha_o\). We will assume that there are \(\tilde{C}\) people wishing to travel from point A to point B, such that \(C_m + nC_o = \tilde{C}\).
Nash Equilibrium requires that travel time be at an equilibrium across all routes, such that no driver could improve his well-being by switching route, and all drivers are allocated. Thus:

$$\alpha_m + f(C_m) = \alpha_0 + f(C_0) \quad \text{and} \quad C_m + nC_0 = \bar{C} \quad (A.35)$$

Rearranging the above yields: $$(\alpha_m - \alpha_0) + [f(C_m) - f(C_0)] = 0.$$ Since $\alpha_m < \alpha_o$ it follows that $f(C_m) > f(C_0)$ and thus the Nash equilibrium $C^*_m > C^*_o$.

Next we consider the social planner’s case. A social planner would choose $C_m, C_o$ in order to minimize the aggregate travel time:

$$\min C_m(\alpha_m + f(C_m)) + nC_o(\alpha_o + f(C_o)) \quad \text{s.t.} \quad C_m + nC_o = \bar{C} \quad (A.36)$$

substituting $C_o = \frac{\bar{C} - C_m}{n}$, differentiating with respect to $C_m$, and then substituting back yields the first order condition as follow:

$$(\alpha_m - \alpha_0) + [f(C_m) - f(C_0)] + C_m f'(C_m) - C_o f'(C_o) = 0 \quad (A.37)$$

we know that in Nash equilibrium, $C^*_m > C^*_o \Rightarrow f'(C^*_m) > f'(C^*_o)$, the first two terms are zero, while the term $C_m f'(C_m) - C_o f'(C_o) > 0$. As a result the social planner would want to remove vehicles, relative to the Nash allocation, from the over-utilized fast, direct route on to other routes such that $C^*_m > C^S_m$ and $C^*_o < C^S_o$.

As shown above, number of vehicles in the fast, direct route exceeds the socially optimal level in the Nash equilibrium. Suppose now a new route through the metro system is available. Consider a partial equilibrium scenario where a
fraction of agents are removed from the direct highway route \( m \) and put into the newly available metro, and drivers do not re-optimize their travel choices. This would clearly constitute a welfare gain for the remaining drivers as travel times improve on the direct route while other routes remained the same level of occupancy. Now consider a general equilibrium scenario where this improved travel time caused drivers to re-optimize. Cars from the slower other routes \( o \) would move to the direct route. This effectively increase \( C_m \) and decrease \( C_o \), pushing the equilibrium away from the social optimum. This erosion of welfare from re-optimizing means that the partial equilibrium welfare gain is the upper bound of the general equilibrium welfare gain.

Note that since travel cost is the value of travel time, i.e. \( TT \times VOT \), the above framework applies exactly the same way if agents optimize based on travel cost.

### A.4.2 Nash equilibrium versus social optimum—linear congestion function

To visualize the above argument, consider the case where the congestion component in the travel time function takes a linear form:

\[
f(C_i) = b_i C_i \quad \Rightarrow \quad TT_i = \alpha_i + b_i C_i
\]  

(A.38)

where \( \alpha_i \) is free-flow speed, and \( b_i C_i \) is the increase in travel time due to congestion from cars \( C_i \) using route \( i \).
Nash Equilibrium requires that travel time be equilibrated across all routes, such that no driver could improve his well-being by utilizing a different route, and all drivers are allocated. Thus:

\[ \alpha_m + b_mC_m = \alpha_o + b_oC_o \quad \text{and} \quad C_m + nC_o = \tilde{C} \]  

(A.39)

Solving for the equilibrium values for the number of cars on each route types, we have:

\[ C_m^* = \frac{\tilde{C}b_o + (\alpha_o - \alpha_m)n}{b_o + bn} \quad \text{and} \quad C_o^* = \frac{\tilde{C}b_m + (\alpha_m - \alpha_o)}{b_o + bn} \]  

(A.40)

Notice that if \( \alpha_m < \alpha_o \), \( C_m^* > C_o^* \). Now let’s consider the social optimal case. Again, social planner optimize according to the following:

\[
\min C_m(\alpha_m + b_mC_m) + nC_o(\alpha_o + b_oC_o) \quad \text{s.t.} \quad C_m + nC_o = \tilde{C}
\]  

(A.41)

Solving the optimization problem yields:

\[ C_m^S = \frac{2\tilde{C}b_o + (\alpha_o - \alpha_m)n}{2(b_o + bn)} \quad \text{and} \quad C_o^S = \frac{2\tilde{C}b_m + (\alpha_m - \alpha_o)}{2(b_o + bn)} \]  

(A.42)

In comparison to the Nash equilibrium allocations:

\[ C_m^* - C_m^S = \frac{(\alpha_o - \alpha_m)n}{2(b_o + bn)} > 0 \quad \text{and} \quad C_o^* - C_o^S = \frac{(\alpha_m - \alpha_o)}{2(b_o + bn)n} < 0 \]  

(A.43)

This confirms our previous assertion that the most direct route \( m \) is over-utilized and \( o \) is under-utilized in the Nash equilibrium.

We now consider two scenarios: First, a partial equilibrium where drivers leave \( m \), no other drivers adjust, and the remaining drivers on \( m \) get congestion relief. Second, a general equilibrium where drivers leave \( m \), all other drivers adjust, moving the system to a new equilibrium.
**Scenario 1: partial equilibrium with no adjustment**

Suppose a fraction $\varphi$ of $C_m^*$ drivers is removed from the highway into the metro system. Remaining drivers get a boost in travel times. The welfare gain is measured by the change in travel time multiply by the number of drivers:

$$
\Delta W_{PECI} = (1 - \varphi)C_m^*[(\alpha_m + b_mC_m^*) - (\alpha_m + b_m(1 - \varphi)C_m^*)]
$$

(A.44)

$$
= \varphi(1 - \varphi)b_m[\bar{C}b_o + (\alpha_o - \alpha_m)n]^2
$$

$$
= \frac{\varphi(1 - \varphi)b_m[\bar{C}b_o + (\alpha_o - \alpha_m)n]^2}{(b_o + b_mn)^2}
$$

**Scenario 2: general equilibrium**

Suppose after the fraction $\varphi$ of $C_m^*$ drivers is removed from the highway into the metro system, the rest of drivers re-optimize, moving the system to a new equilibrium. The new equilibrium in the highway is given by:

$$
\alpha_m + b_mC_m = \alpha_o + b_oC_o \quad \text{and} \quad C_m + nC_o = \bar{C} - \varphi C_m^*
$$

(A.45)

where $C_m^*$ is the Nash equilibrium allocation of drivers in equation A.40. The new general equilibrium allocation is:

$$
C_{m}^{GE} = \frac{\bar{C}b_o + (\alpha_o - \alpha_m)n}{(1 + \varphi)b_o + b_mn}
$$

$$
C_{o}^{GE} = \frac{\bar{C}b_m + (1 + \varphi)(\alpha_m - \alpha_o)}{(1 + \varphi)b_o + b_mn}
$$

(A.46)
It is trivial that $C_{m}^{GE} > (1 - \varphi)C_{m}$ \footnote{6} and $C_{o}^{GE} < C_{o}$ \footnote{7}, the new equilibrium is moving away from the social optimum, thus completing the proof.

\footnote{6}{(1 - \varphi)\frac{1}{(a+b)} \times \frac{(1+\varphi)a+b}{1} = \frac{(1-\varphi)(1+\varphi)a+(1-\varphi)b}{a+b} = \frac{(1-\varphi^2)a+(1-\varphi)b}{a+b} < 1 \ \forall \varphi \in (0, 1)}

\footnote{7}{(1 - \varphi)\frac{1}{(a+b)} < \frac{1}{(1+\varphi)a+b} }^{7}\text{Footnote 6 and } (a_{m} - a_{o}) < 0
A.5 Prototype City: Detail Description

In this section we outline the steps that generate the prototype city counterfactual dataset. In order to make the Expo line analysis appeal to a more general audience, we construct a grid style city with roads, highway network, and rail transit infrastructure. Then, we alter the city’s employment and population distribution to see what effect it would have on the “ideal” speed of rail transit. This would give us a variety of data points to examine the correlation between demographic distributions and transit choice (light or heavy rail).

A.5.1 Prototype city setup

The prototype city is set up in GIS as a grid. We assign the coordinate system to be FIPS 1983 California Feet, for easy interpolation with existing data. The city is set up as a 15.5-by-15.5 mile square, in a 31-by-31 grid, with a total of 961 TAZs (traffic analysis zone). Each zone is 0.5-by-0.5 mile. We chose 31 for symmetry and that the center of the city can be assigned to one zone. The city center is zone # 481. There are four sub-centers in the city, each located midway from the main center to each of the city’s edge. The zone IDs for the sub-centers are: 264, 474, 488, and 698.

We create a grid style road system that connects the centroid of every zone. This serves as the primary access roads from any zone to reach either the job cen-
ters or rail transit/highway. For driving trips, the grid roads represent driving on
local roads. For rail transit, the grid roads represent road access to rail stations.

The city has a rail transit system in the shape of a ring and a radial cross. The ring
passes through all four surrounding sub-centers, and the radial cross passes through
the main center and the four sub-centers. Figure 1.10 maps the grid road with transit
line overlays. In this meta-analysis, the speed of the transit varies from 10 mph to
50 mph to mimic a variety of rail transit configurations. We assume a higher rail
speed would represent preference towards heavy rail. In addition, the city also has
a ring and radial cross shaped “highway” system that offers driver higher speed
travel. The shape of the ring and radial highway is overlaid on top of the grid
network as mapped in figure 1.11.

A.5.2 Data generating process

The data generating process consists of multiple steps. We first generate the proto-
type city population and employment densities and corresponding statistics, then
we carry out gravity calculation to predict flow for each of the density scenarios,
and finally we calculate welfare changes.
Generate prototype city densities

We base the prototype city population and employment densities on actual demographic data of Los Angeles County. We continue to use TAZ as our geographic unit in this regression. We calibrate the following equation using the LA data, once for each of population and employment:

\[ y_i = a_i + b_1 D_{main} + b_2 D_{sub} + \varepsilon_i \]  

(A.47)

Where \( i \) denotes employment or population, \( y \) is log(employment) or log(population), \( a \) is the constant term, \( D_{main} \) is log(distance to main center), and \( D_{sub} \) is log(distance to closest sub center). \( \varepsilon \) is standard OLS error term. We designate the downtown LA area as the main employment center, and designate other sub-centers according to employment density. The centers designated in Los Angeles are mapped in Figure 1.12. Distances of prototype city’s zones to main and sub-centers are calculated using GIS and imported to Stata. Figure 1.9 shows a density plot of the prototype city’s population distribution using the baseline parameters.

We alter the prototype city’s population and employment density by adjusting the coefficients \( b_1 \) and \( b_2 \). \( a \) is used to adjust the overall density level which also adjusts the overall population/employment level. We vary each of the parameters from 10\% of the calibrated value to 250\%. Higher values were omitted as they produce obscure concentrations which leads to non-sensible flow for some origin-destination pairs.
Gravity flow calculation

Once population and employment densities are generated, we can calculate bilateral flow between any origin-destination pair using the gravity equation from our main model. The structural formulation is included in the gravity theory handouts.

We first designate a baseline setup for the prototype city. In this case, we use the unaltered prototype city density coefficients and set the rail transit speed to 10 mph. Using this initial setup we load the transit and driving cost to calculate mode split \(S_{ij}\) and subsequently the total and mode specific commuter flow following our main model for the Expo line. For the baseline distribution parameters, as well as all other sets of counterfactual parameters, we repeat the process with a range of rail transit speed from 10 mph to 50 mph (increment of 1 mph) and calculate the following necessary statistics: the change in total and mode specific flow, change in delay, change in driving VMT, change in transit VMT, and change in travel time saving.

At this stage we have all the ingredients required to calculate welfare change following our welfare theory. The next step is to calculate the welfare change under each of the speed settings.
Calculating welfare changes

We follow the formulas used in our main specification to calculate the total welfare change and its various components. Following closely the tables and formula in section 1.4.5 we calculate marginal welfare by its four components: congestion relief, emission relief, travel time savings, and others. We also calculate the present value of lifetime benefit by calculating (total benefits per year / 4%).

Calculating projected construction cost

In addition to total benefits, we also want to have an idea what kind of cost or investment it would take to create the transit infrastructure in the prototype city, for cost and benefit analysis. To do so, we compiled a dataset of rail transit projects around the country, drawing data from various sources (Freemark [18], FTA [3], Ped [2], FTA [1]). We opt to include only projects that took place in the past 20 years and with opening date no later than the year 2023\(^8\). We also select only projects in the US to minimize unobserved variation in construction cost. There are 71 projects in our sample.

We compiled the year and construction cost data from these sources, and collected length from each project’s website. We also estimated density measures for each of the project, which would serve as a proxy for land price, difficulty of

\(^8\)We included LA proposed purple line which is projected to open in 2023, but all other projects opens before 2020
construction, and other unobserved variables. We create the density measure by first obtaining the project’s shapefile or digitize the project shape if not shapefile is available. Then we do a spatial merge with Census bureau population and employment data to find the TAZs that is within a 0.5 mile buffer from each project. We then calculate the average, min, and max population and employment density within said buffer for each line.

Using this sample dataset, we estimate a reduced form function for construction cost of rail infrastructure as follow:

\[
y_i = a + b_1 S_i + b_2 L_i + b_3 D_j^i + b_4 Y_i + \epsilon_i
\]  \hspace{1cm} (A.48)

Where \( y_i \) is \( \log(\text{construction cost}) \), \( S_i \) is \( \log(\text{rail speed}) \), \( L_i \) is \( \log(\text{rail length}) \), \( Y_i \) is the year of line opening. \( D_j^i \) is the log of average population or employment densities \( (j \in \{\text{Pop}, \text{Emp}\}) \).

Calculating spatial statistics

Part of the meta-analysis of this prototype city requires us to be able to represent the city distribution using simple and commonly used statistics. Thus in addition to the coefficients from the city distribution equation, we also calculate some additional statistics to report employment and population concentration. Following Pereira et al. [28] we calculate an index to measure the spatial centrality of the prototype city, called the UCI. The UCI consists of two coefficients, the location
coefficient (LC) and a proximity index (PI).

\[
UCI = LC \times PI, \quad LC = \frac{1}{2} \sum_{i=1}^{n} |s_i - \frac{1}{n}|, \quad PI = 1 - \frac{V}{V_{max}}, \quad V = S' \times D \times S \quad (A.49)
\]

where: \( S \) is a column vector of \( s_i \); \( D \) is a distance matrix for all TAZs. \( s_i = \frac{E_i}{E} \) is each zone’s employment as a percentage of total employment in the city; \( n \) is the total number of zones in the city. The range of \( LC \) is zero to \( 1 - \frac{1}{n} \). \( V_{max} \) is the hypothetical maximum value \( V \) can attain. In the basic case of a square city, such as that in our prototype city, \( V_{max} \) can be calculated when employment is equally distributed to the four zones at the four corners of the city.

One of the feature of UCI is that it takes into account only employment distribution of the city. As a result, we also estimated two other version of the UCI, one using the population distribution, and one using the sum of employment and population. This gives us a better range of dimensions to measure each prototype city’s distribution.

One observation from the UCI calculation is that, using the unaltered baseline coefficient for density prediction ([10, 10, 10, 10]), the UCI of this city is estimated to be 0.16. At first sight this seems to be a far cry to the 0.045 value estimated by Pereira et al (2013) Table 3, rather much closer to Pittsburgh US (0.11) or Sao Paulo (0.201). However readers should keep in mind that while the prototype city is set up to try to mimic certain aspects of Los Angeles it was based on, there are still some differences. While we constructed the baseline density regression so that the average density in employment and population is close to Los Angeles, we are restricting the prototype city to be a square city of 15.5 by 15.5 mile (240.25
square miles) while the estimation in Pereira et al (2013) is considering all of Los Angeles county, which is nearly 20 times bigger (4752 square miles). So in a sense we are almost looking at the subset of the actual Los Angeles city proper when calculating this UCI.
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167

