A MULTI-STAGED STOCHASTIC OPTIMIZATION MODEL
WITH COLUMN GENERATION BASED HEURISTICS
FOR HOSPITAL EVACUATIONS DURING HURRICANES

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ABSTRACT

Perpetual hospital evacuations caused by hurricanes require well-informed decision makings to handle uncertainty in future conditions and scarcity in various resources. Simultaneous evacuations from multiple hospitals are more challenging due to additional complexity in evacuation route choices and coordination among both evacuating and receiving hospitals. This thesis addresses above issues by developing a multi-staged stochastic optimization model and column generation based heuristics that considers uncertain future flood, wind and road traffic conditions, limited staff and vehicle resources, and effectiveness of evacuation routes generation. The model determines patient evacuation schedule and corresponding evacuation routes while trading off risk and cost. A comprehensive case study on North Carolina hospitals with Hurricane Isabel is conducted with data and model outputs of previous research. The results highlight the benefits of the model formulation and the heuristics that effectively generates complex evacuation routes and coordinates evacuating hospitals' efforts while adapting to new information.

BIOGRAPHICAL SKETCH

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Dedicated to my parents, Jianhua Zhang and Lan Xu

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CHAPTER 1

INTRODUCTION

Hurricanes occurred frequently and caused great damage to both human lives and properties in U.S. in recent years. On average there are 1.7 major hurricanes landing in U.S. coasts each year and annual fatalities related with hurricanes are 45 between 1992 and 2021. In the extreme case of 2005, there are 1016 fatalities caused by hurricanes including Hurricane Katrina (National Weather Service, 2020; NOAA's Atlantic Oceanographic and Meteorological Laboratory, 2021). To minimize both direct and collateral damages caused by hurricanes, federal, state, and local governments in U.S. have devoted numerous resources to develop robust contingency plans in past decades. Common policy tools considered in such plans are issuing voluntary/mandatory evacuation orders, identifying contraflow plans of highways, and rationing scarce resources like food and fuels. However, the evacuation plan that considers people with special need such as hospital patients are still rare and usually left solely to the hospital administrations. Yet populations such as hospital patients and nursing home residents are most vulnerable to higher rate of mortality after exposure to hurricanes (Dosa et al., 2007, 2020). National medical response programs such as the Federal Emergency Management Agency (FEMA) and the National Disaster Medical System (NDMS) was a valuable source for additional medical professionals and supplies, but they struggled to provide medical care due to poor logistic management and lack of coordination provided by regional authorities. Local medical response programs such as Medical Reserve Corps (MRC) provided vital medical support for their local communities but were unable to be coordinated and deployed at higher level (Franco et al., 2006). To properly respond to such difficult conditions, hospital administrators need full-fledged decision-making supporting tools to utilize their limited resources efficiently and to collaborate with each other to avoid making poor choices that endanger patients and cause systematic imbalances among regional hospital networks. According to Louisiana Hospital Association (LHA), there were 1749 patients and in addition 7600 people stranded in 11 New Orleans hospitals that surrounded by floodwaters caused by Hurricane Katrina. The decision to not evacuate in advance caused 8 patient deaths in Charity Hospital, 19 in Lindy Boggs Medical Center and 45 in Memorial Medical Center (Gray & Hebert, 2007). Eight patients died in a nursing home at Hollywood Hills, FL after Hurricane Irma caused power outage and four more patients died during the following weeks (Booker & Allen, 2019). On the other hand, a preemptive evacuation comes with a hefty price tag. According to one case study for a coastal Georgia hospital, the total expenditure for hurricane evacuation prior to the arrival of hurricanes is approximately $9.5 million (Desai et al., 2019). In retrospect, unnecessary preemptive evacuation could post a huge loss in hospital revenue and cause hospital administrators to prefer shelter-in-place decisions to some extent. For instance, during Hurricane Irene, hospitals in Manhattan decided to preemptively evacuate patients after the mandatory evacuation order was ordered by New York City mayor. This proved to be unnecessary and later only 2 of 5 lower Manhattan hospitals evacuated in advance during Hurricane Sandy (Ricci et al., 2015).

Hurricane caused hospital evacuations are often intricate problems across multiple agencies and regions. In a typical hurricane evacuation for hospitals, hospital administrators need to make decisions depending on partial or uncertain information such as weather conditions, road conditions and medical professionals and supplies conditions, and external decisions such as evacuation orders. The most significant characteristic of hurricanes is its trajectory uncertainty and the drastic different consequences of this uncertainty. This prompts the hurricane evacuation protocols to be different from other types of disasters such as earthquakes or hazardous materials leaking. An example of detailed efforts and obstacles faced by evacuating the University of Texas Medical Branch at Galveston under Hurricane Rita can be found in Sexton et al. (2007).

The hospitals are usually designed to be resilient and expected to function as shelters during hurricanes. In practice, many hospitals will discharge less severe patients to retain a capacity for existing critical patients and incoming patients due to the hurricanes. But they rarely transfer patients to another healthcare facility in advance during the hurricanes because extraordinary risks involved. However, shelter-in-place decisions can cause dire consequences when unexpected conditions such as flooding, power shortage, and loss of communication occur. During Hurricane Katrina, New Orleans were under 20 feet water and lost power for nearly 5 days (Franco et al., 2006). Hospitals in the region suffered from the lack of medical resources such as food, clothing, oxygen and medicine, and had to move patients around without functioning elevators (Downey et al., 2013). During Hurricane Harvey, Houston area suffered 4 feet rainfall over 48 hours and extreme flooding (Collier et al., 2020). Due to the lack of communication, state and federal officials in the region often cannot determine which hospitals were operational and which were evacuating, creating challenges in resource allocation and aid (Texas Hospital Association, 2018). A forced evacuation following a shelter-in-place failure usually faces more deteriorated conditions due to the damages caused by the hurricane landfall. The transportation options for hospitals surrounded by floodwater are often limited to boats and helicopters limited which are very expensive and hard to obtain after the landfall (Gray & Hebert, 2007). And the lack of communication makes it harder to report field conditions to coordinating authorities and to request needed personnel and supplies for evacuation (Franco et al., 2006).

Even when a hospital decides to preemptively evacuate patients in advance, there are still many risks need to be taken into consideration. One such risk factor is the uncertainty in current and future road conditions such as prolonged travel time due to road congestion and temporary detours due to flooded highway sections. During Hurricane Rita, it can take up to 20 hours driving from Houston to nearby cities such as San Antonio or Austin due to the congestion caused by 3 million Houston residents trying to evacuate around same time (Soika, 2006). In general, dedicated emergency lanes are also occupied by the regular traffic and cannot be used for patient evacuation (Sexton et al., 2007). 3500 traffic lights in NYC stopped operating during Hurricane Sandy (Gibbs & Holloway, 2013). Uncertain vehicle conditions such as vehicle-patient compatibility and availability are often put to the test. It is quite common that available vehicles are not capable of caring wheelchair patients and many ambulance providers are not stick to the contracts during the hurricane due to safety concerns or government orders (Franco et al., 2006; Dosa et al., 2007).

There are also several other logistics issues such as staffing shortage, staff preparedness, receiving hospitals identification, medical records transfer, and healthcare insurance coverage confusion. Many primary and secondary concerns of medical staffs such as family and personal safety, pet care, and basic needs of food, water, sleep, shelter and rest need to be addressed before these staffs can meet expectations and function as disaster responders effectively (French et al., 2002). Many medical staffs find themselves in conflicts between family commitment and professional obligations that often lead to a shortage of in staffing that cripples the hospital operations during the hurricane. Medical staffs who decided to continue the service during the hurricanes usually were only equipped with limited previous disaster response training or hand-on experience. In a survey for nurses in New York University Langone Medical Center, just 15% nurses identified previous disaster training or experience as a resource for them to handle the evacuation. Only 3 of 173 survey participants had any external formal experience or training in disaster preparedness before Hurricane Sandy. And most participants (80%) reported limited knowledge of hospital disaster policies and procedures (VanDevanter et al., 2017). Finding partner hospitals that has capacity, expertise, and willingness to receive evacuated patients is another hurdle for hospital administrators. Common practice between hospitals is to sign mutual aid agreements in advance. This partner hospital searching is especially hard for smaller hospitals that are not part of a larger private or public healthcare systems as HCA Healthcare or Veterans Health Administration. During Hurricane Katrina, some hospitals in Louisiana decided to not evacuate after failing to secure receiving hospitals (Gray & Hebert, 2007). And Louisiana State University Health Sciences Center had to set up a field hospital with 800 beds and 1700 volunteers that eventually treated more than 6,000 patients (Franco et al., 2006). In opposition, Greater New York City area utilized an interagency coordination mechanism called the Healthcare Facility Evacuation Center. The center was staffed with representatives from various health and medical agencies in the region. The center identified potential receiving hospitals and facilitated the communication between sending and receiving hospitals to smooth the patients transfer process. During Hurricane Irene, it provided aid to the evacuation of over 7000 patients (Adalja et al., 2014). Even those successfully transferred patients, issues like missing medical records and ambiguous healthcare insurance coverage policies at the receiving hospitals could linger long after the evacuation is over (Franco et al., 2006; Mellgard et al., 2019). Also, tracking evacuated patients during the evacuation proved to be quite nontrivial. Some evacuated patients evacuated from New Orleans hospitals cannot be tracked down after three months according to Gray and Hebert (2007).

This paper extends previous endeavors to develop a new comprehensive decision-making support tool that can help hospital administrators to coordinate simultaneous evacuations from multiple evacuating hospitals. The following list summarizes the contributions of this paper.

- Existing multi-stage stochastic programming model focused on single evacuating hospital evacuation case with simple vehicle movement choices (Rambha, Nozick, Davidson, et al., 2021a). We introduce a new stochastic program formulation that can model simultaneous evacuation operations from multiple evacuating hospitals and expanded vehicle routes beyond just a round trip between the evacuating hospital and one receiving hospital.
- We propose a column generation-based reformulation for the new model which contains exponentially more potential vehicle routes and a heuristic algorithm for solving the reformulated model's pricing subproblems. This solution approach allows hospital administrators of different evacuating hospitals to collaborate with each other in their evacuation efforts while still obtaining staged and adaptive evacuation decisions as a hurricane situation evolves. First major benefit of this model is to avoid systematic imbalances caused by each evacuating hospital spontaneously optimizing for itself and competing in scared receiving hospital capacities, medical staffs, and shared vehicle fleet. Second major benefit is to expand the accessibility of receiving hospitals for evacuating patients via the usage of more complex vehicle routes. This model can also provide insights for the long-term emergency plans development and resource allocations among hospitals in same region.
- We also illustrate the value of this new model via an intricate case study. We present a speculative optimization over two evacuating hospitals along North Carolina coastal region to determine evacuation-related decision variables.

This case study incorporates data sets from professional agencies such as the American Hospital Association (AHA) and the National Hurricane Center (NHC), and outputs from previous research on scenario-tree based stochastic programming models, risk parameters estimation, and evacuation orders prediction.

The rest of the paper is organized as follows. Section 2 provides a more detailed literature review on existing hurricane evacuation and hospital evacuation models. And we also discuss the literature of solution approaches such as column generation and heuristics, and their applications in evacuation problems. Section 3 presents more detailed description of the multi-staged stochastic programming model formulation with background information on scenario trees and complex vehicle visiting sequences. Column generation-based reformulations, pricing subproblem formulation and corresponding heuristic solution approach are presented in Section 4. In Section 5, a case study with dataset from North Carolina hospitals and Hurricane Isabel is conducted to illustrate the effectiveness of suggested modeling and solution approach in previous sections. Finally, Section 6 concludes this study's major findings and points out future research directions.

CHAPTER 2

LITERATURE REVIEW

2.1 Hurricane evacuation modeling

Hurricane evacuation involves large-scale transport of people to safer locations under constraints on risk, cost, and time. Transportation engineering plays an essential role in fulfilling this task. A long line of research has been conducted on transportation modeling during hurricane evacuations with different objectives, traffic network granularity, and traffic assignment mechanism (Wolshon, Hamilton, et al., 2005; Wolshon, Urbina, et al., 2005; Murray-Tuite & Wolshon, 2013; Bayram, 2016; Rambha et al., 2019)

To properly model traffic during evacuation, it is essential to first analyze evacuee behaviors. Evacuee behaviors reflect how people react to hurricanes, for example, how people decide when, where, and how to evacuate. Answers to these questions allow prediction of traffic demand patterns and network congestion. A common strategy to make evacuations efficient is to coordinate the issuance time of mandatory evacuation orders for different geographical areas (Sbayti & Mahmassani, 2006; Bish & Sherali, 2013; Zhang et al., 2014; X. Chen & Zhan, 2017; Siam et al., 2022). A recent work by Yi et al. (2017) proposes a scenario-tree based stochastic optimization model to improve the timing of mandatory order issuance. while considering risk factors due to flooding and high windspeeds.

Even under the mandatory evacuation orders, individuals still have a choice of whether to evacuate or not. The probability of individual evacuation is usually estimated through random utility models. These behavioral models choose independent variables from available data and identify leading factors among them. Common independent variables are personal/household socio-demographic characteristics, sources and timing of evacuation notices, hurricane characteristics (distance, category, and storm surge), and geographical locations (coastal vs. inland) (Hasan et al., 2011; Gudishala & Wilmot, 2012; Xu et al., 2016; Mongold et al., 2020; Rambha, Nozick, & Davidson, 2021). Similar models are also applied to predict evacuation destination choices (Cheng et al., 2008; Mesa-Arango et al., 2013; Bian et al., 2019). Traffic network conditions can be estimated with information from both destination choice and assumed equilibrium/system optimum route choice behaviors (Hobeika & Kim, 1998; Chiu & Zheng, 2007; Ng & Waller, 2010; Karabuk & Manzour, 2019).

Besides demand-side control, supply-side features of the road network can also be improved through strategies such as contraflow and signal control (Cova & Johnson, 2003; Wolshon, Hamilton, et al., 2005; Tuydes & Ziliaskopoulos, 2006; M. Chen et al., 2007; Y. Wang & Wang, 2019). For long-term planning, shelter locations and road network design can also indirectly influence the evacuation performance. To this end, bi-level optimization models have been proposed to solve shelter location problem (Sherali et al., 1991; Ng et al., 2010; Li et al., 2012). For the evacuation network design problem, optimization models that minimize construction costs and/or evacuation time under constraints on network capacity, demand uncertainty, and infrastructure resiliency have also been studied (Hadas & Laor, 2013; Nahum et al., 2017).

2.2 Hospital evacuation

There are relatively fewer studies that specifically look at hospital evacuation operations and modeling. Most of hospital evacuation-related research are case studies in social sciences. These studies focus on reviewing past hospital evacuation cases and experiences (McGlown, 2001; VanDevanter et al., 2017). A recent survey (Yazdani et al., 2021) summarizes the current knowledge and research gaps in the hospital evacuation modeling. Simulation- and optimization-based models dominate existing mathematical models for hospital evacuation. In the following paragraphs, some of the classical studies are reviewed.

Taaffe et al. (2006) proposed simulation-based hospital evacuation model to develop a hurricane evacuation plan that minimizes the total evacuation time of all patients. The discrete-event simulation contains three main areas: storm control, patient/medical staff control, and administrator/risk manager control. The model simulates evacuation of three types of patients, with 50 patients in each type, from a single evacuating hospital to three receiving hospitals with a fixed fleet of vans and ambulances. It allows admitting patients to the evacuating hospital before the evacuation starts. The travel time is stochastic due to storm conditions but does not generate actual evacuating routes for vehicles. The model also takes multi-level staff assignment into consideration. Patients who need evacuation are assigned to certain staff and a vehicle based on the patient type, staff type, and vehicle type. The simulation model was extended to a simulation-based optimization model with cost minimization as the objective to determine nurse and vehicle assignments (Tayfur & Taaffe, 2009a). The optimization portion is handled by OptQuest which is a heuristic solver in the Arena Simulator. The extended model further assumes S-shaped response curves and triangularly distributed traffic factors for generating stochastic travel times. The effect of total available evacuation time and evacuation start time on the total evacuation cost are also analyzed. Another simulation model was proposed by Chen et al. (2015) for patients of a single type evacuating from one evacuating hospital to one receiving hospital. Their model was implemented in SIMIO commercial software and mainly focuses on improving an existing French Extended White Plan.

A mixed-integer programming model similar to their previous setting was also proposed by Tayfur and Taaffe (2009b). The decision variables are the numbers of evacuated and shelter-in-place patients, staff for evacuated and shelter-in-place patients, and the number of available vehicles. The objective is to minimize the total evacuation cost of vehicles and nursing staffs under various constraints such as vehicle locations, patient/staff availability, and patient/staff assignment. Shelter-in-place behavior is discouraged by attaching a high fixed cost to such outcomes in the objective function. The exact solution using the CPLEX solver could be found only in three out of 36 cases. Thus, two heuristics based on LP rounding and local search were developed. Childers et al. (2009) proposed a Markov decision process model for patient evacuation that maximizes the chances of survival among two types of patients. The state of the system is the number of patients in each type and state transition is caused by the evacuation policy or patient deaths. The optimal decision policy is developed via dynamic programing techniques and simulation analysis is applied.

Bish et al. (2014) provided a new mixed-integer programming model that minimizes the risk instead of the time or cost associated with evacuation. Multiple types of patients are evacuated from one evacuating hospital to multiple capacitated receiving hospitals via a fixed vehicle fleet. The risk comprises of the transportation risk and the risk of staying/threat risk. The transportation risk is affected by the time taken for staging and transporting. The threat risk, on the other hand, is assumed to resemble different classes of functions such as constant, linear, and exponential. Travel times between hospitals are assumed to be time-invariant.

Rambha et al. (2021) extended this model by specifically dealing with uncertainty involved in hurricane hazards. Their objective minimized a weighted sum of expected risk and cost from the evacuation. This model recognizes time-varying risks and specifies a detailed computing framework for risk parameters that combines outputs of weather and regional-scale network loading models. Also, the uncertainty and dynamic nature of the hurricane is represented by a scenario tree so that the decision at certain time step can adapt based on the current storm-related information. This scenario-tree approach allows decision makers to revise their plan constantly along the planning horizon, thereby saving staffing and vehicle resources as well as minimizing the risk associated with patient transportation. A case study was also presented with real-world data to showcase different managerial insights.

2.3 Column generation-based heuristics

Previous research either do not specifically consider concrete evacuating routes for vehicles or only consider direct vehicle routes between one evacuating hospital and one receiving hospital. This is valid for ambulance which transfers a few severe condition patients at a time. However, for less severe patients who can be transferred via larger capacity vehicles such as vans, the usage of more complex vehicle routes with multiple stops at evacuating and receiving hospitals could be beneficial to reduce both risks and costs involved in transporting. On the other side, multiple hospitals can be affected by a hurricane and become evacuating hospitals at the same time. And many receiving hospitals may not have enough available beds to receive higher capacity vans. With the introduction of complex routes which have multiple stops at evacuating and/or receiving hospitals, the computation complexity for feasible complex routes grows drastically and can easily lead the problem to be intractable. Column generation is a common technique for mitigating such computational complexity.

Column generation was invented in 1960s and was initially a meta-algorithm for Linear Programing (LP) (Dantzig & Wolfe, 1960; Gilmore & Gomory, 1961, 1963; Fulkerson, 1971). The main idea of column generation is to start with a small number of columns of the constraint matrix and add new columns based on the reduced cost of the associated decision variables. For the past several decades, column generation has been widely applied to problems in logistics such as vehicle routing, shift scheduling, and inventory ship routing. A classic summary of the usage of column generation in mixed-integer programming and its many applications can be found in Desaulniers et al. (2005). One common example of such usage is “branch-and-price”. It is a combination of “branch-and-bound” framework and column generation. “Branch-and-price” starts with a restricted master problem which is the original master problem with a limited number of variables/columns, but gradually adds new variables/columns to the restricted master problem in each iteration via solving a pricing subproblem. The pricing subproblem is typically in the form of an optimization problem and needs to be solved to its optimality to prove the optimality of the original master problem. This is considered as a column generation-based exact method and have been widely used in many transportation problems (Christiansen, 1999; Coelho et al., 2014; Costa et al., 2019). However, column generation-based exact method is not always computational tractable. It may be hard to solve the pricing subproblem to its optimality efficiently and add feasible columns to the restricted master problem is sufficient. Therefore, column generation-based heuristics have also been studied for various problems. Some column generation heuristics are developed as a component of new solution framework for classical problems such as variants of vehicle routing problems (Beheshti & Hejazi, 2015; Guedes & Borenstein, 2015). Other heuristics are developed due to the large size of the practical problems (Zeren & Özkol, 2016; Liang et al., 2018). There are also heuristics developed for the extended variant of previous problems which are harder to solve by their structures (Moccia et al., 2009). Heuristics can be applied in different aspects of the column generation framework such as finding feasible dual subproblem solution and initializing restricted master solution. The heuristics applied in this problem is to split and only compute a partial goal of the original pricing problem. The original pricing problem requires to solve the vehicle routing and scheduling at the same time. But with the heuristics, we only consider figuring out the routes first and moving the scheduling into master problem. This paper uses a similar framework by combining the column generation approach with dual heuristics for finding feasible vehicle visiting sequences.

CHAPTER 3

PROBLEM MODELING

In this section, we provide an integer program for the multi-stage stochastic optimization model. Our formulation extends the single-hospital evacuation problem discussed in Rambha, Nozick, Davidson et al. (2021b) to the multi-hospital scenario with trip chaining. Some of the background material is, however, common but has been revisited to keep this paper self-contained.

3.1 Background

With the help of meteorological models, the possible future trajectory of a hurricane can be reduced to an ensemble which comprise of a set of a few scenarios (Skamarock et al., 2008). Each scenario may contain information on the hurricane's trajectory, central air pressure, and wind velocity. Ensembles and the scenarios related with them are typically calculated by perturbing the atmospheric condition parameters and choosing various dynamics in the Weather and Research Forecasting (WRF) model (Blanton et al., 2020). Each of these scenarios can be used to calculate the effects of the hurricane, especially wind speeds and flood depths which can be used to estimate the risk in a certain region.

While these scenarios are computed several days before landfall, as time progresses, it is possible to discard the possibility of occurrence of a subset of them. To obtain a solution that takes such uncertainty into consideration, we use a multi-stage stochastic programming model with recourse actions. The uncertainty is represented as scenario trees and at a certain time step, scenarios are partitioned into sets and each set comprise scenarios that are indistinguishable for anyone without further future knowledge (Rockafellar & Wets, 1991; Watson et al., 2010). Some indistinguishable scenarios during early time steps can become distinguishable in later time steps as more information becomes available. This type of refining of the scenario partitions corresponds to the branching of the scenario tree. The decisions made at certain time step for indistinguishable scenarios should be the same.

Let Q denote the set of scenarios, $\Pr(q)$ denote the probability of occurrence of scenario q, A_t denote the partition of Q at time step t, and let $z = \left(z_{qt}\right)_{q \in Q, t \in T}$ be a vector of recourse variables in which z_{qt} indicates actions (patient assignment, routes for vehicle fleet, and dispatch schedules) picked for scenario q at time step t. Decision-makers can utilize the hurricane information at certain time step t to identify the closest scenarios q and follow actions in z_{qt}. We can thus represent the multistage stochastic optimization in the following general form:

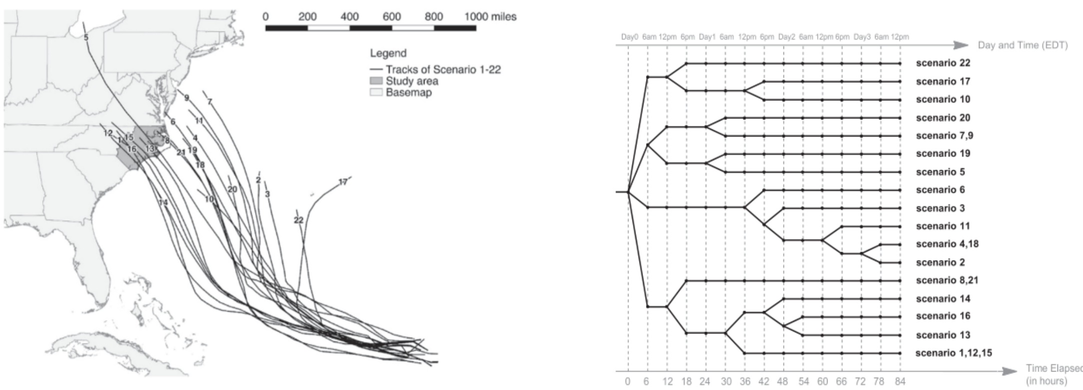
$$\min \sum_{q \in Q} \Pr(q)\, f_q(z) \tag{1}$$

$$subject\ to: z_{qt} \in Z_q, \forall q \in Q, t \in T \tag{2}$$

$$z_{qt} = z_{q't}, \forall\ t \in T, q, q' \in A, A \in A_t \tag{3}$$

In this paper, we deal with an objective function $f_q(z)$ that is bicriterion in nature and features the risk and cost of taking actions in z under the scenario q. Constraints (2) contains flow conservation and supply-demand type of constraints that should be satisfied in each scenario q along the time span of the model. Constraints (3) reflects the non-anticipativity condition. The formulation described in later sections will use decision variables for each node in the scenario tree, thus the non-anticipativity constraints are inherently satisfied.

The scenario tree used in the case study presented in this paper is constructed out of an ensemble that contains multiple hurricane trajectories obtained via a clustering-based method (Yang et al., 2017). Figure 1(a) shows an application of 22 scenarios of Hurricane Isabel (2003). Figure 1(b) shows the scenario tree generated with six-hour time steps for three-and-half days ahead of the hurricane landfall.



(a) Ensemble of 22 scenarios (b) Scenario Tree

Figure 1 Ensemble and scenario tree based on the hurricane forecasting information

3.2 Model assumptions and inputs

This section describes the assumptions and inputs for the model such as set notations and parameters for the objective function and constraints. The sets used in the formulation are described in Table 1.

Table 1 Notation of sets used in the formulation

Symbol	Description
H	Hospitals that may evacuate patients
H'	Hospitals that may receive patients
P	Patient types
V	Vehicle types
T	Time steps in the planning horizon

N	Nodes in the scenario tree
$N(n)$	Ancestor nodes of node n
R	Sample paths of the scenario tree
S	Evacuation vehicle visiting sequences
$\bar{S}$	Rebalance vehicle visiting sequences
S_h	Evacuation visiting sequences whose origin is hospital h
S^h	Evacuation visiting sequences whose destination is hospital h
$\bar{S}_h$	Rebalance visiting sequences whose origin is hospital h
$\bar{S}^h$	Rebalance visiting sequences whose destination is hospital h
$S(hh')$	Evacuation visiting sequences that cover evacuation hospital h and receiving hospital h'

Assumptions on hospitals, patients, and vehicles:

- Hospital patients are assumed to be classified into different types via a triage method based on patients' medical condition and needs.
- Number of patients of different types per evacuating hospital is known and the capacity of each patient type for all receiving hospitals is known and limited. No new patients are admitted to the hospitals during the planning horizon.
- Hospital vehicles are assumed to belong to a fixed size fleet of various types. They can be owned or rented just for evacuations. The fleet size per vehicle type is known.
- Vehicle types are advanced life support (ALS) and basic life support (BLS) ambulances, and buses/minivans. Each vehicle type can transport certain compatible patient types and has a limited passenger capacity.
- Hospitals are partitioned into two groups. One group represents evacuating hospitals, and the other group only receives patients.

ALS and BLS ambulances typically transport 1-2 patients requiring serious medical attention. Minivans and buses, on the other hand, are useful to transport patients with less severe conditions. A parameter, δ_{pv} , is introduced to represent the compatibility of transporting patient type p using vehicle type v.

Assumptions on the planning time horizon and scenario tree:

- Evacuation decisions are taken at a set of discrete time intervals denoted by $T = \{0,1,2, \dots, |T| - 1\}$.
- Each node n in the scenario tree represents a set of indistinguishable scenarios.
- Node $m \in N(n)$ is called an ancestor of node n if there exists a scenario tree sample path that contains both m and n and that m is closer to the root. In other words, the scenarios represented by node n is a subset of those represented by node m. The decision time step of node n is denoted as $t(n)$.

- The set of sample paths is represented by R. A generic sample path is represented by r and we write $n \in r$ to indicate that node n belongs to the sample path r.

To provide an example of the above notation, consider the scenario tree in Figure 1(b). Each time step here is 6 hours and the scenario tree has 22 scenarios, 17 sample paths, 15 time steps, and 165 nodes. Each sample path starts from the leftmost root node and ends at one of the rightmost leaf nodes.

Assumptions on vehicle visiting sequences:

- An evacuation visiting sequence $s \in S$ may start from one of the evacuating hospital $h_{start} \in H$ and may visit other evacuating hospitals. It is then assumed to visit one or more receiving hospitals and ends its route at one of the evacuating hospital $h_{end} \in H$. The purpose of an evacuation visiting sequence is to transfer patients from evacuating hospitals to receiving hospitals.

- A rebalance visiting sequence $\bar{s} \in \bar{S}$ moves directly from an evacuating hospital $h_{start} \in H$ to another evacuating hospital $h_{end} \in H$. These are used to rebalance vehicles between evacuating hospitals but do not carry any patients
- Both evacuation visiting sequences and rebalance visiting sequences can be grouped by their starting and ending hospitals. For example, S_h and $\bar{S}_h$ are the sets of evacuation visiting sequences and rebalance visiting sequences that start from hospital h, respectively. Likewise, S^h and $\bar{S}^h$ are the sets of evacuation visiting sequences and rebalance visiting sequences that end at hospital h, respectively.
- $S(hh')$ represents the subset of all the evacuation visiting sequences that can transfer patients from an evacuating hospital h to a receiving hospital h'.

Figure 2 illustrates a vehicle visiting sequence. A vehicle visiting sequence is the abstraction of an actual route which the vehicle takes. In Figure 2, the left panel shows the actual vehicle route and the graph on the right indicates the visiting sequence $h_{start} = A - 8 - 9 - B = h_{end}$. The sequence emphasizes the set of hospitals visited and hides routing details when assigning patients between the evacuating and receiving hospitals. The actual vehicle route is determined from the road network information and the departure time of the vehicle using time-dependent shortest paths. The vehicle visiting sequence $A - 8 - 9 - B$ is an example of "complex" vehicle visiting sequence. It means that the vehicle will visit more than one evacuating hospital or one receiving hospital before it returns to final evacuating hospital. A vehicle visiting sequence is called “direct” vehicle visiting sequence if it only visits one evacuating and one receiving hospital before it returns to final evacuating hospital. An example of such direct vehicle sequence is $A - 8 - B$.

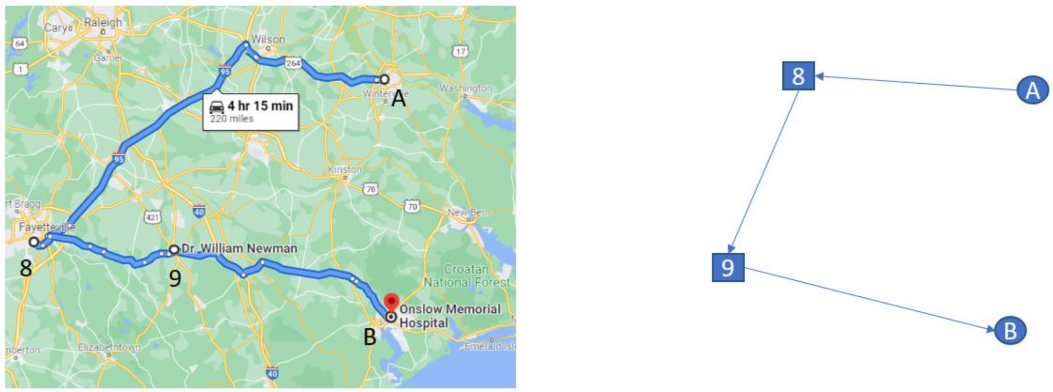


Figure 2 Vehicle route and vehicle visiting sequence

In calculating these shortest paths, we use outputs from a dynamic traffic assignment (DTA) model where network loading uses predictions of evacuating demand for different time periods based on optimal mandatory evacuation orders (Yi et al., 2017). This DTA model is based on earlier formulations by Janson(1991) and Li et al. (2012). Based on the outputs, dynamic link travel times are used to compute time-dependent shortest paths (TDSP) between any pair of hospitals (Chabini, 1998). The shortest path for a sequence s is derived by concatenating the time-dependent shortest paths between individual segments. These results are then used to define a binary vehicle availability parameter ζ_s^{mn} which is set to 1 if a vehicle starts sequence s at time $t(m)$ and can finish the route at or before time $t(n)$. Constraints related parameters are shown in Table 2.

Table 2 Constraint parameters

Symbol	Description
α_{ph}	Number of patients of type p in evacuating hospital h considered for evacuation
$\kappa_{ph'}$	Number of available beds of type p in receiving hospital h'
ζ_s^{mn}	Vehicle availability parameter which is 1 if a vehicle that starts sequence s at $t(m)$ can finish sequence s at or before $t(n)$ and is 0 otherwise.
β_v	Total number of vehicles of type v available for evacuation
β_{vh}	Number of vehicles of type v in hospital h available for evacuation at the start of the time horizon
χ_v	Passenger capacity of a type v vehicle
δ_{pv}	Vehicle-patient compatibility parameter which is 1 if type p patient can be transported in type v vehicle and is 0 otherwise

The objective function of the evacuation problem is to minimize a weighted sum of the expected risk and expected cost of the evacuation operations. The expectation is calculated using π^n, which is defined as the probability of observing node n at time step $t(n)$ and is calculated using the sum of probabilities of indistinguishable scenarios belonging to node n. Each sample path r is assumed to occur with a probability ρ^r. For instance, in Figure 1(b), if each scenario occurs with the same probability, at time step $t = 6$, the four scenario tree nodes from top to bottom can be observed with probabilities 0.136, 0.227, 0.273, and 0.364. This is because the first node contains 3 out of 22 scenarios, and hence π^n of the first node at time step $t = 6$ is $\frac{3}{22} = 0.136$. Other probabilities are estimated similarly.

Two types of risk parameters are introduced for patient related risk. $\lambda_{phh'}^{nvs}$ represents the risk involved in evacuating a patient to a receiving hospital and μ_{ph}^r represents the risk of sheltering a patient and not evacuating. These risk parameters are probability measurements of occurrence of adverse events that lead to deterioration of patient condition. Patient related risk can be influenced by several aspects such as patient and vehicle types, flooding events, and chosen vehicle visiting sequences. More detailed explanation can be found in Section 5.1. The cost of evacuation is assumed to comprise of vehicle operation and staffing expenditures. Vehicle operation costs contain fixed cost of renting/leasing vehicles and per-mile expenditures. Staffing costs are approximately estimated using overtime wage rates in case the zone containing the evacuating hospital receives a mandatory evacuation order. Table 3 provides a full list of objective function related parameters.

Table 3 Objective function parameters

Symbol	Description
π^n	Probability of observing node n
ρ^r	Probability of observing sample path r (observing the leaf node of r)
ψ	Weight factor of risk components in the objective function
$\lambda_{phh'}^{nvs}$	Risk of transferring patient of type p from evacuating hospital h to receiving hospital h' in vehicle type v via vehicle sequence s with departure time $t(n)$
μ_{ph}^r	Risk of stranding patients of type p at evacuating hospital h when sample path r is observed
ω_{vs}^n	Cost of a type v vehicle that transfers patients along vehicle sequence s starting in time step $t(n)$
σ_p^n	Cost of providing care to a patient of type p up to time step $t(n)$
σ_p	Cost of providing care to a stranded patient

3.3 Optimization model (MIP-1)

Below is the formulation MIP-1 for the optimization model. The decision variables of the model as shown in Table 4 include $x_{phh'}^{nvs}$, the number of type p patients that are transferred from evacuating hospital h to receiving hospital h' using vehicle type v and sequence s when at scenario tree node n, and y_{vs}^n, the number of vehicles of type

v that follow sequence s at scenario tree node n. Vehicles are allowed to start on a new sequence to transfer more patients after they complete their previous sequence. The objective function minimizes a weighted sum of the expected risk and the expected cost of the evacuation operations. The risk portion of the objective function is weighted by $\psi \in [0,1]$ and contains two components. The first component $\sum_{n \in N} \pi^n \left(\sum_{p \in P} \sum_{h \in H} \sum_{h' \in H'} \sum_{v \in V} \sum_{s \in S(hh')} \lambda_{phh'}^{nvs} x_{phh'}^{nvs} \right)$ denotes the expected number of adverse events that caused by transferring patients.

$$\min \left\{ \sum_{n \in N} \pi^n \left(\sum_{p \in P} \sum_{h \in H} \sum_{h' \in H'} \sum_{v \in V} \sum_{s \in S(hh')} \lambda_{phh'}^{nvs} x_{phh'}^{nvs} \right) + \sum_{r \in R} \rho^r \left(\sum_{p \in P} \sum_{h \in H} \mu_{ph}^r \left(\alpha_{ph} - \sum_{h' \in H'} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \right) \right) \right\}$$
$$+(1-\psi) \left\{ \sum_{n \in N} \pi^n \left(\sum_{p \in P} \sum_{h \in H} \sum_{h' \in H'} \sum_{v \in V} \sum_{s \in S(hh')} \sigma_p^n x_{phh'}^{nvs} + \sum_{v \in V} \sum_{s \in S \cup \bar{S}} \omega_{vs}^n y_{vs}^n \right) + \sum_{r \in R} \rho^r \left(\sum_{p \in P} \sum_{h \in H} \sigma_p \left(\alpha_{ph} - \sum_{h' \in H'} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \right) \right) \right\} \quad (4)$$

s.t.

$$\sum_{h' \in H'} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \leq \alpha_{ph}, \forall p \in P, h \in H, r \in R \quad (5)$$

$$\sum_{h \in H} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \leq \kappa_{ph'}, \forall p \in P, h' \in H', r \in R \quad (6)$$

$$\sum_{p \in P} \sum_{hh' : hh' \subseteq s} \delta_{pv} x_{phh'}^{nvs} \leq \chi_v y_{vs}^n, \forall v \in V, s \in S, n \in N \quad (7)$$

$$\sum_{s \in S_h \cup \bar{S}_h} y_{vs}^n \leq \beta_{vh} - \sum_{s \in S_h \cup \bar{S}_h} \sum_{m \in N(n)} y_{vs}^m + \sum_{s \in S^h \cup \bar{S}^h} \sum_{m \in N(n)} \zeta_s^{mn} y_{vs}^m, \forall h \in H, v \in V, n \in N \quad (8)$$

$$x_{phh'}^{nvs} \leq M \delta_{pv}, \forall p \in P, n \in N, v \in V, s \in S, hh' \subseteq s \quad (9)$$

$$x_{phh'}^{nvs} \in Z_+, \forall p \in P, n \in N, v \in V, s \in S, hh' \subseteq s \quad (10)$$

$$y_{vs}^n \in Z_+, \forall n \in N, v \in V, s \in S \quad (11)$$

The second component, $\sum_{r \in R} \rho^r \left(\sum_{p \in P} \sum_{h \in H} \mu_{ph}^r \left(\alpha_{ph} - \sum_{h' \in H'} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \right) \right)$, denotes the number of expected number of adverse events that caused by non-evacuating patients. The risk of sheltering in place is computed per sample path and weighted by the probability of the corresponding sample path. Similarly, the expected cost is the combination of vehicle operating costs and staffing costs. The expected cost part is weighted by $1 - \psi$.

Constraints (5) guarantees that the total number of type p patients in hospital h who are transferred in different time steps is less or equal than the total number of available type p patients in hospital h. Constraints (6) guarantees that a receiving hospital h' has enough capacity for all patient types transferred to it along each sample path. In Constraints (7), the maximum number of patients in type v vehicles using sequence s starting at node n should be less than the total number of type v vehicles using sequence s starting at node n times its capacity. The maximum number of patients along a sequence s are present when the vehicle travels from its last pick-up hospital to first drop-off hospital. This maximum value is achieved by adding up the number of all patients transported from one evacuating hospital h to one receiving hospital h' along the sequence s departing at time node n. Constraint (8) implies that the number of type v vehicles at each evacuating hospital h available at node n is less than or equal to the original fleet size of type v vehicles minus the number of vehicles that departed in previous time steps via sequences starting from hospital h plus the number of vehicles that returned via sequences ending in hospital h at or before the current time step. Constraint (9) is used to guarantee that each patient type can only be transported by compatible vehicle types. M is set to a sufficiently large constant. Integrality constraints are represented using (10) and (11). By using one variable per scenario tree node, we automatically guarantee that all indistinguishable scenarios share similar decisions, thus ensuring non-anticipativity.

Table 4 Decision variables

Symbol	Description
$x_{phh'}^{nvs}$	Number of type p patients transferred from evacuating hospital h to receiving hospital h' in vehicle type v using sequence s starting in time step $t(n)$ at scenario tree node n
y_{vs}^{n}	Number of type v vehicles using sequence s departing in time step $t(n)$ in scenario tree node n

CHAPTER 4

SOLUTION ALGORITHM

4.1 Limitation of MIP-1 formulation and column generation

MIP-1 formulation has one major computation bottleneck caused by the size of feasible vehicle visiting sequences which grows drastically with respect to the number of hospitals allowed to be visited by one sequence. For example, if a vehicle visiting sequence is allowed to visit m evacuating hospitals and n receiving hospitals, then in total there are $O(m^3n^2)$ feasible vehicle visiting sequences. The soaring size of feasible vehicle visiting sequences lead to a surge in the total computational time and costs of vehicle availability parameter ζ for each sequence. Values of ζ are derived from running time-dependent shortest path algorithm repetitively for adjacent hospitals in each vehicle visiting sequence with different departing time since road conditions change quickly during hurricane landing. Although computational efforts for each vehicle visiting sequence's vehicle availability parameter ζ is moderate, MIP-1 formulation contains all feasible vehicle visiting sequences and thus requires precomputation of ζ for all feasible vehicle visiting sequences which is time consuming and expensive.

To mitigate this bottleneck caused by precomputing ζ, it is ideal to only compute vehicle visiting sequences with the potential to reduce the objective function value and to be utilized in the final solution. Optimal solutions from solving a small size MIP-1 example also showed that only a fraction of all possible vehicle visiting sequences was used. Thus, we propose to apply column generation technique on MIP-1 formulation to solve it efficiently. Column generation has been studied in mixed integer programming for a long time and has a wide range of applications in transportation problems (Desaulniers et al., 2005). It is usually suitable for the model that contains large number of feasible variables but fewer number of constraints. Generally, applying column generation contains three steps. The first step is to recognize a subproblem that can generate an elemental solution building block called “pattern” that can be used in the master problem’s solution and reformulate the master problem into a restricted master problem corresponding to the subproblem. The second step is to solve the subproblem efficiently to provide feasible patterns as column variables to the master problem or prove that no such building blocks exists anymore. Final step is to initialize with a starting set of patterns for the restricted master problem, iteratively solve the pricing subproblem to generate more patterns which can be added into the restricted master problem and resolve the updated restricted master problem to generate dual variables to update the pricing subproblem. The process stops when there are no more feasible “patterns” can be generated or certain pre-determined criteria have been met.

4.2 Column generation reformulation MIP-CG

Based on the nature of patients transfers and vehicle usages, we can construct a pattern g called “route assignment pattern” which describes a vehicle trip conducted by a single vehicle visiting hospitals according to a predetermined vehicle visiting sequence and transferring corresponding patients en route. There is a multiplicity of feasible route assignment patterns due to variations of departure time, vehicle types and vehicle visiting sequences. We denote G_{nv}^{s} as the set of route assignment patterns that departs at time corresponding to a scenario tree node n, using vehicle type v and vehicle visiting sequence s. And we further denote g_{nv}^{s} as one concrete route assignment pattern from G_{nv}^{s}. This implies $g_{nv}^{s} = (x_{p_0 h_0 h_0'}^{nvs}, \cdots, x_{p_i h_j h_k'}^{nvs}, \cdots, x_{p_{|P|} h_{|H_S|} h'_{|H_S'|}}^{nvs})$ where each element $x_{p_i h_j h_k'}^{nvs}$ represents the number of type p_i patients that transferred from evacuation hospital h_j to receiving hospital h_k' and all elements in g_{nv}^{s} forms an ordered permutation of all possible patients transfers indexed by patient type p_i, evacuating hospital h_j and receiving hospital h_k'. H_s represents the set of evacuating hospitals along vehicle visiting sequence s . Similarly, H_s' represents the set of receiving hospitals along vehicle visiting sequence s. Regarding to the master problem, g_{nv}^{s} is a parameter vector instead of a decision variable vector and each element in g_{nv}^{s} is a parameter that should be computed via pricing subproblems. Each concrete route assignment pattern g_{nv}^{s} can only be assigned to a single vehicle. In other words, g_{nv}^{s} is applied to vehicles at individual vehicle level instead of at vehicle type level. Thus, $G_{nv}^{s} = \{g_{nv}^{s}(i),\ i = 1, \ldots, |G_{nv}^{s}| \,\big|\, \mathbf{1}^T g_{nv}^{s}(i) \leq \chi_v, e_{ph}^T g_{nv}^{s}(i) \leq \alpha_{ph}\ \forall p \in P, h \in H_s\,, e_{ph'}^T g_{nv}^{s}(i) \leq \kappa_{ph'}\ \forall p \in P, h' \in H_s'\}$ where i is the index for one concrete vector g_{nv}^{s} in G_{nv}^{s} and $|G_{nv}^{s}|$ is the cardinality of G_{nv}^{s}. Each $g_{nv}^{s}(i)$ vector should satisfy the vehicle capacity constraints for vehicle type v which is $\mathbf{1}^T g_{nv}^{s}(i) \leq \chi_v$, evacuating hospital patient availability constraints $e_{ph}^T g_{nv}^{s}(i) \leq \alpha_{ph}\ \forall p \in P, h \in H_s$, and receiving hospital patient capacity constraints $e_{ph'}^T g_{nv}^{s}(i) \leq \kappa_{ph'}\ \forall p \in P, h' \in H_s'$. Here e_{ph} is an indicator column vector for elements in $g_{nv}^{s}(i)$ that corresponds to patient type p and evacuating hospital type h involved in vehicle visiting sequence s. Similarly, $e_{ph'}$ is an indicator column vector for elements in $g_{nv}^{s}(i)$ that corresponds to patient type p and receiving hospital h' involved in vehicle visiting sequence s.

We now use a small example to illustrate route assignment patterns more concretely. Assuming that there are two evacuating hospital A and B, and two receiving hospital 1 and 2. A type v_0 vehicle departs at a time denoted by scenario tree node n_0 to transfer two types of patients (p_0 and p_1) using a vehicle visiting sequence $s_0 = A - B - 1 - 2 - B$. The set of all possible route assignment patterns corresponding to above assumptions is denoted by $G_{n_0 v_0}^{s_0}$ and one specific route assignment pattern is represented as $g_{n_0 v_0}^{s_0}$.

We explicitly represent $g_{n_0 v_0}^{s_0} =$

$\left(x_{p_0A1}^{n_0v_0s_0}, x_{p_0A2}^{n_0v_0s_0}, x_{p_0B1}^{n_0v_0s_0}, x_{p_0B2}^{n_0v_0s_0}, x_{p_1A1}^{n_0v_0s_0}, x_{p_1A2}^{n_0v_0s_0}, x_{p_1B1}^{n_0v_0s_0}, x_{p_1B2}^{n_0v_0s_0}\right)$ in the example since only four possible patient transfers exist, i.e. $A - 1, B - 1, A - 2$, and $B - 2$, for both patient types with given s_0. Each element $x_{p_0hh'}^{n_0v_0s_0}$ represents the number of type p_0 patients who transfer from evacuating hospital h to receiving hospital h'. Since $g_{n_0 v_0}^{s_0}$ is a parameter vector for the master problem, $x_{p_0hh'}^{n_0v_0s_0}$ is a parameter instead of decision variable for the master problem. The dimension of column vector g_{nv}^{s} is $1 \times |H||H'||P|$ where $|H|$ is the number of evacuating hospitals and $|H'|$ is the number of receiving hospitals, and $|P|$ is the number of patient types. Thus, in the example, the dimension of g_{nv}^{s} is $2 \times 2 \times 2 = 8$. If an evacuating hospital h^* or a receiving hospital pair $h^{*\prime}$ is not visited by vehicle visiting sequence s, then corresponding $x_{ph^*h'}^{nvs}$ or $x_{phh^*}^{nvs}$, values are set to be 0 for all patients types. If type v vehicle cannot accommodate type p^* patients, then $x_{p^*hh'}^{nvs}$ values are set to 0 for all evacuating and receiving hospital pair h, h' visited during vehicle visiting sequence s. Detailed explanations and visualizations of the above example can be found in Figure 3.

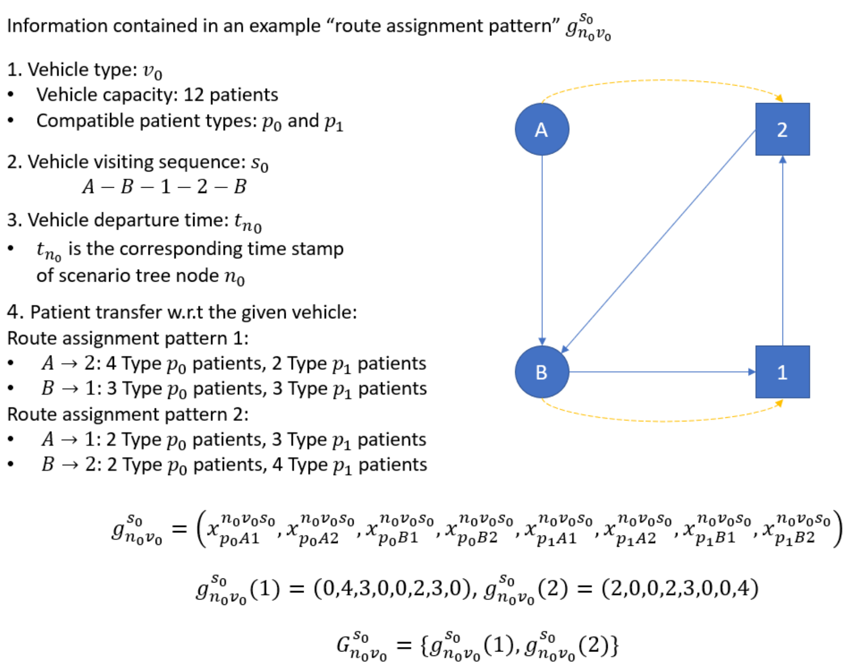


Figure 3 Illustration of one route assignment pattern. Blue arrow represents vehicle movement and yellow arrow represents patients transfer.

With the introduction of route assignment pattern g, we start to explain other symbols used in MIP-CG reformulation. We denote θ_g as the number of times when a route assignment pattern g is carried out. θ_g is the decision variable of the master problem of MIP-CG. Subscriptions and superscriptions of g_{nv}^{s} will be dropped when the context is clear and there is no need to express further details related with g. Set G_n represents all route assignment patterns which depart at the time corresponding to scenario tree node n regardless of the vehicle type or vehicle visiting sequence. Thus, $G_n = \{\cup\, G_{nv}^s, \forall s \in S, v \in V\}$.

The evacuating hospital patient availability constraints (5) can be reformulated with constraint matrix A^H after regrouping them by scenario tree sample path r. The dimension of A^H is $|H||P| \times |H||H'||P|$ where $|H|$ is the size of evacuating hospitals, $|H'|$ is the size of receiving hospitals, and $|P|$ is the size of patient types. In the above example, A^H dimension is 4×8 since both $|H|$ and $|H'|$ are 2, and $|P|$ is 2. We use a_{h^*,p^*}^H to denote the row vector in constraint matrix A^H that corresponds to the evacuating hospital h^* and patient type p^*. Row vector a_{h^*,p^*}^H is an indicator vector to select patients transfers in route assignment pattern g that is related with evacuating hospital h^* and patient type p^* which means the element in a_{h^*,p^*}^H is set to 1 if it corresponds to the element $x_{p^*h^*h'}^{nvs}$ in the route assignment pattern g, and 0 otherwise. A concrete example of a_{h^*,p^*}^H with given example $g_{nv}^s = (0{,}4{,}3{,}0{,}0{,}2{,}3{,}0)$, is the following. If h^* is evacuating hospital A and p^* is p_0, then corresponding row vector $a_{Ap_0}^H = (1{,}1{,}0{,}0{,}0{,}0{,}0{,}0)$. If h^* is evacuating hospital B and p^* is p_1, then corresponding row vector $a_{Bp_1}^H = (0{,}0{,}0{,}0{,}0{,}0{,}1{,}1)$.

The receiving hospital patient capacity constraints (6) can also be reformulated similarly with constraint matrix $A^{H'}$ after regrouping. $A^{H'}$shares a similar structure as A^H with dimension $|H'||P| \times |H||H'||P|$. In above example, $A^{H'}$ also has a dimension of 4×8. We use $a_{h'^*p^*}^{H'}$ to denote the row vector in constraint matrix $A^{H'}$ that corresponds to the receiving hospital h'^*'s and patient type p^*. Row vector $a_{h'^*,p^*}^H$ is an indicator vector to select patients transfers in route assignment pattern g that is related with receiving hospital h'^* and patient type p^* which means the element in $a^{H'}_{h'^*p^*}$ is set to 1 if it corresponds to the element $x^{nvs}_{p^*hh'^*}$ in the route assignment pattern g and 0 otherwise. If h'^* is receiving hospital 1 and p^* is p_0, then corresponding row vector is $a^{H'}_{1p_0} = (1,0,1,0,0,0,0,0)$. If h'^* is receiving hospital 2 and p^* is p_1, then corresponding row vector is $a^{H'}_{2p_1} = (0,0,0,0,0,1,0,1)$.

Vehicle availability constraints in MIP-CG shares similar form as constraints (8) in MIP-1. y^n_{vs} is replaced with $\sum_{g \in G^s_{nv}} \theta_g$ since the number of type v vehicles used for vehicle visiting sequence s that departs at time corresponding to scenario tree node n is indeed the total number of times that a route assignment pattern g in G^s_{nv} is carried out. Table 5 below summarizes symbols used in MIP-CG formulation.

Table 5 Notations in MIP-CG formulation

Symbols	Description
α	Column vector with each row corresponds to available patients in one evacuating hospital
κ	Column vector with each row corresponds to patient capacity in one receiving hospital
β_{vh}	Scalar corresponds to the number of available type v vehicles in hospital h
g	Column vector corresponds to a route assignment pattern
g^s_{nv}	A route assignment pattern g indexed by scenario tree node n, vehicle type v and vehicle visiting sequence s
c_g	A combined coefficient represents risk and cost that invoked by one route assignment pattern g
θ_g	Decision variable indicates how many times route assignment pattern g is carried out
G_n	Set of route assignment patterns with departing time same as scenario tree time node n's time stamp
G^s_{nv}	Set of route assignment patterns g^s_{nv} that indexed by scenario tree node n, vehicle type v and vehicle visiting sequence s

A^H	Constraint parameter matrix for patient availability of evacuating hospitals
$A^{H'}$	Constraint parameter matrix for patient capacity of receiving hospitals
a_{hp}^H	Row vector in A^H that represents evacuating hospital h's type p patient availability
$a_{h'p}^{H'}$	Row vector in $A^{H'}$ that represents receiving hospital h''s type p patient capacity
h_s^{ori}	The beginning evacuating hospital where vehicle visiting sequence s starts
h_s^{dest}	The ending evacuating hospital where vehicle visiting sequence s ends
H_s	Set of evacuating hospitals along vehicle visiting sequence s (Not include destination)
H_s'	Set of receiving hospitals along vehicle visiting sequence s

With above symbols, we present the reformulated MIP-CG formulation as below:

$$\min_{\theta} \sum_{n\in N} \sum_{g\in G_n} c_g \theta_g \tag{12}$$

$$s.t. \sum_{n\in r} \sum_{g\in G_n} \theta_g \cdot A^H g \leq \alpha, \forall r \in R \tag{13}$$

$$\sum_{n\in r} \sum_{g\in G_n} \theta_g \cdot A^{H'} g \leq \kappa, \forall r \in R \tag{14}$$

$$\sum_{n'\preccurlyeq n} \sum_{s\in S_h \cup \bar{S}_h} \sum_{g\in G_{n'v}^s} \theta_g - \sum_{n'\prec n} \sum_{s\in S^h \cup \bar{S}^h} \sum_{g\in G_{n'v}^s} \theta_g \zeta_s^{n'n} \leq \beta_{vh}, \forall n \in N, \forall v \in V, \forall h \in H \tag{15}$$

When a new route assignment pattern $g^* \in G_{nv}^s$ is added into the MIP-CG formulation, new cost term $c_{g^*}\theta_{g^*}$ will be added into the objective function (12). Notation G_n in objective function (12) represents the union of G_{nv}^s for any vehicle type v and vehicle visiting sequence s, which means $G_n = \cup\, G_{nv}^s, \forall v \in V, \forall s \in S$. For evacuating hospital patient availability constraint (13), if $n \in r$ for some scenario tree sample path r, then the constraint corresponding to sample path r will add a term $\theta_{g^*}(A^H g^*)$ on the left-hand side of the inequality. Similarly, for receiving hospital patient capacity constraint (14), if $n \in r$ for some scenario tree sample path r, then the constraint corresponding to sample path r will add a term $\theta_{g^*}(A^{H'} g^*)$ on the left-hand side of the inequality.

In vehicle availability constraint (15), g^* will provide vehicle type v and vehicle visiting sequence s with origin h_s^{ori} and destination h_s^{dest}. Intuitively, the usage of g^* will have potential effects in this constraint for future scenario tree nodes. In the constraint, we introduce two notations to represent preceding relationship between two scenario tree nodes. $n' \prec n$ means scenario tree node n' precedes the scenario tree node n. Similarly, $n' \preccurlyeq n$ means scenario tree node n' either precedes or equals to scenario tree node n. For any scenario tree node n that is the descendant of the current scenario tree node n', vehicle type v, and begins at evacuating hospital h_s^{ori}, the constraint (15) that corresponds to n, v, h_s^{ori} will add term θ_{g^*} in the first term at the left hand side of the inequality. On the other hand, for any scenario tree node n that is the descent of the current scenario tree node n', vehicle type v, and ending at evacuating hospital h_s^{dest}, the constraint (15) that corresponds to n, v, h_s^{dest} will add term $-\theta_{g^*}\zeta_s^{n'n}$ in the second term at the left hand side of the inequality. Here $\zeta_s^{n'n}$ represents whether a vehicle departing at scenario tree node n' has finished the vehicle visiting sequence at the timestamp of scenario tree node n.

4.3 Pricing subproblem of MIP-CG formulation

The restricted master problem requires a pricing subproblem to generate potential route assignment patterns. In this case, the pricing subproblem is essentially how to simultaneously find best vehicle visiting sequences and corresponding patients transferring plan, given a certain scenario tree node n. In theory, there should be one pricing subproblem per scenario tree node. But in the practice, we can skip certain scenario tree nodes to save computational resources since adjacent scenario tree node on the same sample path will not change drastically.

For the illustration purpose, the pricing subproblem will be conceptually split into two levels. Outer level is finding the shortest vehicle visiting sequence and the inner level is finding the best patient transfer assignment. We also introduce a new graph called "Abstract Hospital Graph" (AHG) to simply the constraints on certain properties of the vehicle visiting sequence.

Here is an example on AHG with three evacuating hospitals and two receiving hospitals. There are five types of vertices and six types of edges in the AHG. Vertex types consist of artificial source node, evacuating hospitals, receiving hospitals, duplicated evacuating hospitals, and artificial sink node. Edge types consist of 1) artificial source node to the origin of vehicle visiting sequence evacuating hospital, 2) evacuating hospital to another evacuating hospital, 3) evacuating hospital to receiving hospital, 4) receiving hospital to receiving hospital, 5) receiving hospital to the evacuating hospital as the destination of the vehicle visiting sequence, 6) the destination of the vehicle visiting sequence to the artificial sink node.

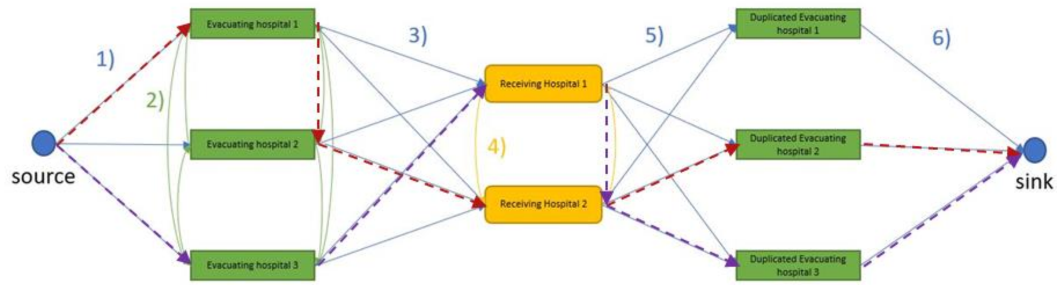


Figure 4 An example of abstract hospital graph (AHG). Red and purple dash lines represent two vehicle visiting sequences.

With above preparations, we present the formulation of a pricing subproblem corresponds to a certain scenario tree node n and a certain vehicle type v as the following:

$$SubP(n,v) \coloneqq c_v + \min_{s} \sum_{(i,j)\in E_{AHG}} c_{ij}s_{ij} + Q(s,v,n) \tag{16}$$

$$s.t. \quad \sum_{i\in V_{AHG}} s_{source,i} = 1 \tag{17}$$

$$\sum_{i\in V_{AHG}} s_{i,sink} = 1 \tag{18}$$

$$\sum_{(i,j)\in\delta^{+}_{AHG}(\{i\})} s_{ij} = \sum_{(j,i)\in\delta^{-}_{AHG}(\{i\})} s_{ji} = z_i, \qquad \forall i \in V_{AHG} \tag{19}$$

$$\sum_{(i,j)\in\delta^{+}_{AHG}(V'_{AHG})} s_{ij} \geq z_i, \qquad \forall\, V'_{AHG} \subseteq V_{AHG}, V'_{AHG} \neq \emptyset, i \in V'_{AHG} \tag{20}$$

$$\sum_{(i,j)\in E_{AHG}} s_{ij} \leq L_{max} \tag{21}$$

$$z_i \in \{0,1\}, \forall i \in V_{AHG} \tag{22}$$

$$s_{ij} \in \{0,1\}, \forall i,j \in V_{AHG} \tag{23}$$

where inner level minimization over feasible patient assignment is defined as the following:

$$Q(s,v,n) \coloneqq \min_{g_{nv}^{s}} \; c_{nvs}^{T} g_{nv}^{s} - \sum_{r\in R, n\in r} u_{H}^{r}\left(A^{H} g_{nv}^{s}\right) - \sum_{r\in R, n\in r} u_{H'}^{r}\left(A^{H'} g_{nv}^{s}\right) \qquad (24)$$

$$s.t. \qquad \mathbf{1}^{T} g_{nv}^{s} \le \chi_{v} \qquad (25)$$

$$e_{ph}^{T} g_{nv}^{s} \le \alpha_{ph} \quad \forall p \in P, h \in H_{s} \qquad (26)$$

$$e_{ph'}^{T} g_{nv}^{s} \le \kappa_{ph'} \quad \forall p \in P, h' \in H_{s}' \qquad (27)$$

$$g_{nv}^{s} \in \mathbf{Z}_{+}^{|H|\times|H'|\times|P|} \qquad (28)$$

For the outer level, dual variable corresponding to vehicle availability constraints (15) is integrated into the edge cost in the AHG. As mentioned before, the edge cost of edge type that goes from the artificial source vertex to an evacuating hospital h_{s}^{ori} as the origin of the vehicle visiting sequence s is $-\sum_{n':n\preccurlyeq n'} u_{vh_{s}^{ori}}^{n'}$. The edge cost of edge type that goes from an evacuating hospital h_{s}^{dest} as the destination of the vehicle visiting sequence s to the artificial sink vertex is $\sum_{n':n\prec n'} u_{vh_{s}^{dest}}^{n'} \zeta_{s}^{nn'}$. u_{vh}^{n} represents the dual variable corresponds to vehicle availability constraints (15) indexed with n, v, h. The reason why the edge costs are formed as such need further explanations. A compact formulation for the pricing subproblem essentially tries to find one route assignment pattern g that violates the constraint of $p^{T} A_{g} \le c_{g}$ where c_{g} are the compact cost coefficient of g and A_{g} is the corresponding column vector in the compact constraint matrix. It is not easy to write MIP-CG master problem into a compact form, but we can identify which terms are added into the constraints when a new route assignment pattern g enters the restricted master problem. For the vehicle availability constraints (15), when a new pattern g_{nv}^{s} enters the master problem, assuming h_s^{ori} is the origin evacuating hospital of vehicle visiting sequence and h_s^{dest} is the destination evacuating hospital of vehicle visiting sequence, the departure of a vehicle caused by using this route assignment pattern g_{nv}^{s} only affects the first term in the constraints (15) indexed by n', v, h_s^{ori} where $n \preccurlyeq n'$ and the arrival of the vehicle caused by using this route assignment pattern g_{nv}^{s} only affects the second term in the constraints (15) indexed by n', v, h_s^{dest} where $n \prec n'$. Therefore, only the corresponding dual variables associated with these constraints will be considered in the pricing subproblem's objective function for the route assignment pattern g_{nv}^{s}. $\theta_{g_{nv}^{s}}$ in the first term of each constraint indexed by n', v, h_s^{ori} contributes $-u_{vh_s^{ori}}^{n'}$ to the pricing subproblem's objective function. And $\theta_{g_{nv}^{s}}$ in the second term of each constraint indexed by n', v, h_s^{dest} contributes $u_{vh_s^{dest}}^{n'}\zeta_s^{nn'}$ to the pricing subproblem's objective function. Summing these dual variables along the sample path after scenario tree node n yields the following results. The first term in constraints (15) will contribute $-\sum_{n':n\preccurlyeq n'} u_{vh_s^{ori}}^{n'}$ and the second term in constraints (15) will contribute $\sum_{n':n\prec n'} u_{vh_s^{dest}}^{n'}\zeta_s^{nn'}$ in the pricing subproblem's objective function.

For the inner level, u_H^r is the dual variable vector corresponding to evacuating hospital availability constraint (13) that represents sample path r, $u_{H'}^r$ is the dual variable vector corresponding to receiving hospital capacity constraint (14) that represents sample path r.

Table 6 summarizes symbols used in formulating the pricing subproblem.

Table 6 Notations in MIP-CG pricing subproblem

Symbol	Description
AHG	Abstract hospital graph (AHG) is a graph constructed to enforce vehicle visiting sequence requirements and simplify road networks between hospitals.
V_{AHG}	Vertices in the abstract hospital graph (AHG)
E_{AHG}	Directed edges in the abstract hospital graph (AHG)
s_{ij}	0,1 variable indicating whether a vehicle visiting sequence s will travel from vertex i to vertex j in the abstract hospital graph (AHG).
c_{ij}	1) Vehicle traveling cost invoked by traveling directly from hospital i to hospital j, 2) Dual variable value corresponding from artificial source node to the origin of the vehicle visiting sequence, 3) Dual variable value corresponding from the destination of the vehicle visiting sequence to the artificial sink node.
z_i	0,1 variable indicating whether vertex i is visited or not.
$\delta^{+}_{AHG}(V)$	Set of outgoing edges in AHG from vertices set V
$\delta^{-}_{AHG}(V)$	Set of incoming edges in AHG to vertices set V
c_v	Vehicle initial cost invoked by choosing vehicle type v
c_{nvs}	Combined risk and cost coefficient vector for transporting a patient given a vehicle visiting sequence s, departing time denoted by time of scenario tree node n and vehicle type v
L_{max}	Maximum hops the vehicle visiting sequence is allowed

4.4 Heuristic solution approach

In the column generation approach formulation MIP-CG, the subproblem decomposes into $|N| \times |V|$ separate problems, each one minimizing a combined cost and risk by finding a vehicle visiting sequence s and patients transferred along the given vehicle visiting sequence over all feasible vehicle visiting sequences and their corresponding patient transferring assignment.

This coupled pricing subproblem is computationally intensive to solve. There are several difficulties involved to solve it. First, we cannot formulate the pricing subproblem explicitly as an IP since we don’t have and don’t want to precompute the set of all feasible vehicle visiting sequences. Second, the pricing problem cannot be transformed exactly into a resource constrained shortest path problem framework since the objective of the inner level problem, which is the combined risk and cost of transferring patients, is not additive. The total risk of transferring a patient from evacuating hospital h to receiving hospital h' along a vehicle visiting sequence s is not same as the summation of the risk of transferring the patient between adjacent hospitals along the vehicle visiting sequence. For example, if the patient is transferred via $h - h_1 - h'$, the risk of transferring from h to h' via h_1 does not equal to the summation of the risk of transferring from h to h_1 and the risk of transferring from h_1 to h'. This property of the inner level objective makes it difficult to frame the inner level problem into a shortest path problem framework and combine with the outer level problem. Third, the vehicle availability parameter $\zeta_s^{nn'}$ in the pricing subproblem's objective still need a precomputation of the corresponding vehicle visiting sequence s.

4.4.1 Reformulation MIP-HCG based on vehicle visiting sequences

We introduce a new column generation formulation MIP-HCG with only vehicle visiting sequence s as the column variable to resolve the difficulties encountered in solving MIP-CG formulation's pricing subproblem. Comparing to MIP-CG, MIP-HCG's pricing subproblem only needs to search for vehicle visiting sequence s instead of a route assignment pattern g. Patients transfer related decision variables are left to the restricted master problem to decide. Following is the MIP-HCG formulation:

$$\min \sum_{s\in S'} \left(\sum_{r\in R}\sum_{n\in r}\sum_{p\in P}\sum_{v\in V}\sum_{hh'\subseteq s} \rho^r \left(\psi\left(\lambda_{phh'}^{nvs} - \mu_{ph}^{r}\right) + (1-\psi)(\sigma_p^n - \sigma_p) \right) x_{phh'}^{nvs} \right)$$

$$+ \sum_{s\in S'\cup\bar{S}} \left(\sum_{r\in R}\sum_{n\in r}\sum_{v\in V} \rho^r (1-\psi) w_{vs}^n y_{vs}^n \right) \tag{29}$$

$$s.t. \sum_{s\in S'(hh')} \left(\sum_{h'\in H'}\sum_{n\in r}\sum_{v\in V} x_{phh'}^{nvs} \right) \le \alpha_{ph}, \qquad \forall p\in P, h\in H, r\in R \tag{30}$$

$$\sum_{s\in S'(hh')} \left(\sum_{h\in H}\sum_{n\in r}\sum_{v\in V} x_{phh'}^{nvs} \right) \le \kappa_{ph'}, \qquad \forall p\in P, h'\in H', r\in R \tag{31}$$

$$\sum_{p\in P}\sum_{hh'\subseteq s} \delta_{pv} x_{phh'}^{nvs} \le \chi_v y_{vs}^n, \qquad \forall v\in V, s\in S', n\in N \tag{32}$$

$$\sum_{s\in S_h'\cup\bar{S}_h} y_{vs}^n \le \beta_{vh} - \sum_{s\in S_h'\cup\bar{S}_h}\sum_{m\in N(n)} y_{vs}^m + \sum_{s\in S'^h\cup\bar{S}^h}\sum_{m\in N(n)} \zeta_s^{mn} y_{vs}^m, \quad \forall h\in H, v\in V, n\in N \tag{33}$$

$$x_{phh'}^{nvs} \le M\delta_{pv}, \qquad \forall p\in P, n\in N, v\in V, s\in S', hh'\subseteq s \tag{34}$$

$$x_{phh'}^{nvs} \in Z_+, y_{vs}^n \in Z_+ \tag{35}$$

MIP-HCG formulation shares a great similarity with MIP-1 formulation but focuses more on the vehicle visiting sequences. Only a subset of vehicle visiting sequences are used in the MIP-HCG formulation. Therefore, the objective function (29), hospital patient availability constraints (30), and hospital patient capacity constraints (31) are all rearranged using vehicle visiting sequence s as a column variable. Most symbols displayed in MIP-HCG can be referred to Section 3. Below we only describe the meanings of new symbols. S' denotes a subset of all feasible vehicle visiting sequences that generated via pricing subproblems. Similarly, S_h' represents the set of vehicle visiting sequences that starts at hospital h and S'^h the set of vehicle visiting sequences that ends at hospital h. Rebalancing vehicle visiting sequences are precomputed and added into the MIP-HCG formulation in the beginning since the number of these is limited. The solution approach emphasizes on generating evacuating vehicle visiting sequences.

4.4.2 Heuristic algorithm

In the following we illustrate the detailed algorithm for solving MIP-HCG formulation described above. From now on, MIP-HCG will be referred to as the master problem. On a high level, heuristics are applied on two parts of the solution approach. For the restricted master problem, we applied a restricted master heuristic which solves the restricted master problem as a static IP at the root node without branching. Linear relaxation of the restricted master problem will be solved iteratively to provide dual variables to update pricing subproblem parameters. On the other hand, in the pricing subproblem, we utilized a heuristic objective function to form it as finding shortest path over the abstract hospital graph (AHG) with limited number of hops. Such shortest paths found are potential feasible vehicle visiting sequences which can be added into the restricted master problem as column variables. The pseudo code of the column generation is described in Algorithm 1.

Algorithm 1 Column generation algorithm for the solution of MIP-HCG

Step 1. Initialize MIP-HCG with initial subset $S' \cup \bar{S}$.
Step 2. Solve the linear relaxation of MIP-HCG.
Step 3. Solve the pricing problem heuristically.
Let $\tilde{S}$ be the set of feasible vehicle visiting sequences found.
Step 4. If $\tilde{S}$ is not empty and the iteration limit is not reached,
$S' \leftarrow S' \cup \tilde{S}$ and go to Step 2.
Step 5. Apply the restricted master heuristic.

The pricing subproblem at the root node can be modeled as the following with the usage of abstract hospital graph (AHG) and dual variables $u^r_{ph}, u^r_{ph'}$, and u^h_{nv} associated with constraints (30), (31), and (33).

$$SubP_{heuristic}(n,v) \coloneqq \sum_{(i,j)\in E_{AHG}} \tilde{c}_{ij} s_{ij} \tag{36}$$

$$s.t. \sum_{i\in V_{AHG}} s_{source,i} = 1 \tag{37}$$

$$\sum_{i\in V_{AHG}} s_{i,sink} = 1 \tag{38}$$

$$\sum_{(i,j)\in\delta^+_{AHG}(\{i\})} s_{ij} = \sum_{(j,i)\in\delta^-_{AHG}(\{i\})} s_{ji} = z_i, \qquad \forall i \in V_{AHG} \tag{39}$$

$$\sum_{(i,j)\in\delta^+_{AHG}(V'_{AHG})} s_{ij} \geq z_i, \qquad \forall\, V'_{AHG} \subseteq V_{AHG}, V'_{AHG} \neq \emptyset, i \in V'_{AHG} \tag{40}$$

$$\sum_{(i,j)\in E_{AHG}} s_{ij} \leq L_{max} \tag{41}$$

$$z_i \in \{0,1\}, \qquad \forall i \in V_{AHG} \tag{42}$$

$$s_{ij} \in \{0,1\}, \qquad \forall (i,j) \in E_{AHG} \tag{43}$$

Here $\tilde{c}_{ij}$ represents the reduced cost of arc $(i,j) \in E_{AHG}$. This pricing subproblem resembles the outer level formulation of the pricing subproblem (16) ~ (28) of MIP-CG. The abstract hospital graph (AHG) structure is the same as Figure 4 mentioned in previous subsection 4.3. However, edge costs of AHG are chosen heuristically to simplify the computation. Detailed explanations of $\tilde{c}_{ij}$ on different edges in AHG is listed in Table 7.

Table 7 Edge costs of AHG in MIP-HCG's pricing subproblem

Edge Type	Values included in $\tilde{c}_{ij}$
1) Source node to evacuating hospital	Dual variable u_{nv}^{h} for vehicle availability at starting evacuating hospital, and dual variable u_{ph}^{r} for patient availability.
2) Evacuating hospital to evacuating hospital	Variable cost of operating vehicle type v, estimated combined risks and costs for patients c_{nvs}, and dual variable u_{ph}^{r} for evacuating hospital patient availability.
3) Evacuating hospital to receiving hospital	Variable cost of operating vehicle type v, estimated combined risks and costs for patients c_{nvs}, and dual variable $u_{ph'}^{r}$ for receiving hospital patient capacity.
4) Receiving hospital to receiving hospital	Variable cost of operating vehicle type v, estimated combined risks and costs for patients c_{nvs}, and dual variable $u_{ph'}^{r}$ for receiving hospital patient capacity.
5) Receiving hospital to destination evacuating hospital	Dual variable u_{nv}^{h} for vehicle availability at the destination evacuating hospital.
6) Destination evacuating hospital to sink node	Fixed cost of operating vehicle type v.

This pricing subproblem is a special form of the classic Elementary Shortest Path Problem with Resource Constraint (ESPPRC). ESPPRC itself is NP-hard in the strong sense. Common practice is to relax subtour elimination constraints and solve the Shortest Path Problem with Resource Constraints with a label setting dynamic programing algorithm. However, the restricted resource in this special case pricing subproblem is the maximum hop limit of the shortest path. Shortest path problem with limited number of hops can be solved by applying Bellman-Ford algorithm with the number of internal iterations sets to the maximum hop limit of the shortest path. Meanwhile, to improve the efficiency and speed up convergence, we search for k-shortest paths instead of just the best one. The implemented algorithm is a variant of Bellman-Ford algorithm that maintains k-shortest paths with a limited number of hops. The output of the algorithm is a set of potential vehicle visiting sequences that can be added into the restricted master problem. In practice, we don't need to solve all $|N| \times |V|$ pricing subproblems. Only a set of scenario tree nodes are sampled since pricing subproblems of adjacent scenario tree nodes usually yield similar vehicle visiting sequences.

New vehicle visiting sequences generated from solving the pricing subproblem is added into the restricted master problem. The linear relaxation of the updated restricted master problem will be solved again to generate new dual values to update the pricing subproblem's parameters. Since the pricing subproblem is solved heuristically with no optimality guarantee, we also set a limit on the number of times the pricing problem is solved to terminate the process when no new feasible evacuating vehicle visiting sequences are generated from solving the pricing subproblem. In the end, the restricted master heuristic is applied to solve MIP-HCG in the root node as a static IP without branching via a commercial solver. All pricing subproblem generated evacuating vehicle visiting sequences are added into the final restricted master problem of MIP-HCG.

CHAPTER 5

NUMERICAL EXPERIMENT

MILP models and solution algorithms described previous section were implemented in Java with the CPLEX 12.8 solver. The experiment was run on University of Delaware's high performance computing clusters with a single computing node equipped with Intel®E5-2695 v4 18 cores CPU and 128 GB DDR4 memory. We study a hypothetical evacuation of two hospitals along North Carolina's coastal line with Hurricane Isabella which made landfall on September 18th, 2003.

5.1 Data

Figure 5 shows the two evacuating hospitals (orange markers) and nine receiving hospitals (blue markers). The nine receiving hospitals are chosen based on their proximity to the evacuating hospitals and the optimal mandatory evacuation orders computed from previous research (Yi et al., 2017). Evacuating hospitals are chosen in areas where potential mandatory evacuation are given in some scenarios and receiving hospitals are chosen in areas where no mandatory evacuation orders are issued and are hence under no threat. Also, the issue time of mandatory evacuation orders is different for each evacuating hospital. Evacuation operations were allowed in 60-hour time windows from time step 20 to time step 80. The decision update interval is one hour. The scenario clusters are generated every six hours with algorithms in previous research (Yi et al., 2017). Six-hour granularity is a common choice for weather forecasting services such as NHC and NWS.

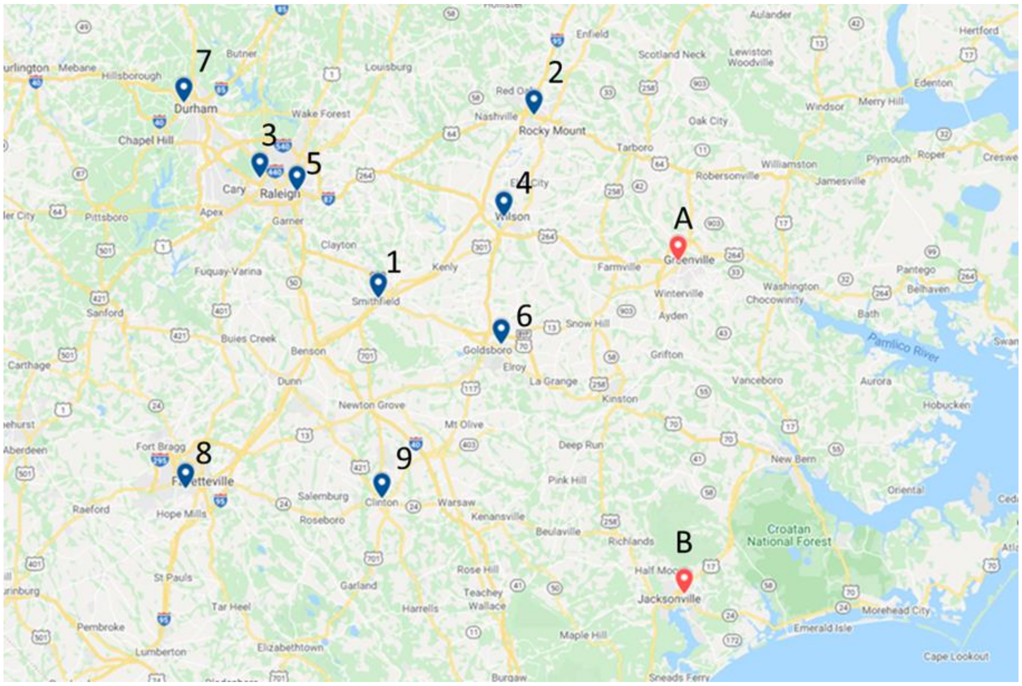


Figure 5 Case study hospital locations

The number of available beds of each receiving hospital is set to 34 which is estimated to be around 10% of the average hospital bed size in the region. Correspondingly, the number of patients in each evacuating hospital is set to 112 to match the total capacity of receiving hospitals.

Each evacuating hospital is assumed to have a vehicle fleet that consists of two ALS-I ambulances, one ALS-II ambulance, three BLS ambulances, and one medical transporting van. Detailed specifications of the four types of vehicles are listed in Table 8. ALS-II provides more advanced health care services and thus has a higher price. In general, vehicle operation costs are assumed to have two components: per-trip costs for drivers and emergency personnel compensation and per-mile costs of $16.36 for fuel consumption. Also, the vehicle operation costs are computed per vehicle visiting sequence. Mileage costs are approximated by travel mileages which are determined via the lengths of TDSPs between adjacent hospitals along the vehicle visiting sequence given the departure time. The rates of ambulances are assumed based on average costs of emergency medical service (EMS) from one county in North Carolina region (http://www.wakegov.com/ems/patient/Pages/feeandpatientcost.aspx). Medical transporting van rate is assumed to be same as ALS-II because of the lack of accurate price information. Rates used in the case study are multiplied by a factor of 1.5 compared to the website price to reflect price surges during hurricanes. Patient treatment costs only occur when the evacuating hospital's zone is under mandatory evacuation orders. The evacuation order predictions used are from the previous research (Yi et al., 2017). According to the census by AHA, the average expenditure for caring one patient per day is $1837.51. Taking the overtime wages into consideration for the hurricane emergency, we assume an extra cost of 50% when a patient is sheltered in place. Ideally the expenditure value should depend on the patient type, but the average is used here due to the lack of such information.

Table 8 Fleet characteristics

S. No.	Vehicle Type	Number available	Capacity	Rate
1	BLS	6	2	773.95
2	ALS I	4	1	919.06
3	ALS II	2	1	1330.23
4	Medical Van	2	32	1330.23

The optimal travel time of a vehicle visiting sequence is assumed to be the cumulative sum of optimal travel time of each adjacent hospital pair along the sequence. The reason for this assumption is that, to the best of our knowledge, there is no known algorithm to compute time-dependent shortest paths while visiting certain nodes in given order. The optimal travel times between adjacent hospitals among various scenarios are computed via TDSP algorithms over an interstate/state highway network in North Carolina. This road network contains 3,596 nodes and 9,986 links. Each link is one segment of an interstate highway or state highway. The travel time of each link is time-dependent and changes every 15 min and are also predicted by previous research (Yi et al., 2017). Nodes that are closest to hospitals are considered as a proxy for origins and destinations for vehicles. These optimal travel times can later be used to estimate transporting risks associated with vehicle visiting sequence and vehicle availability parameter ζ_s^{mn}. For each pair of adjacent hospitals in a vehicle visiting sequence, a one-to-one TDSP problem is solved for all possible departure times via a label-correcting algorithm. Sequentially solving above TDSP problem between adjacent hospital pairs along the vehicle visiting sequence yields the above-mentioned risks and parameters. This turns into a major computational bottleneck when the number of possible vehicle visiting sequences increases.

Hospitals are assumed to implement a triage system for patients to classify them based on their conditions. Triage system is a common practice in both day-to-day medical care and urgent conditions such as mass casualty disasters to prioritize limited resources for patients that need them the most. Patient triage system usually contains 3 to 5 levels. Patients are classified based on scores computed from predetermined protocols that consider comorbidity of patients, nature of the injury, and clinical descriptions (Champion et al., 1989; Iserson & Moskop, 2007). There are several variations of triage systems such as Emergency Severity Index (ESI) developed by Agency for Healthcare Research and Quality, Canadian Tirage and Acuity Scale

(CTAS) developed by Canadian Association of Emergency Physicians, and Abbreviated Injury Scale (AIS) developed by Association for Advancement of Automotive Medicine (Lerner et al., 2008). In this case study, we chose a 5-level triage system with range from 0 to 4. Patient type 0 is the least severe condition, and patient type 4 is the most severe condition. The proportions of each patient are derived from emergency department patient admission statistics of U.S. in 2009 (Hing & Bhuiya, 2009). Receiving hospital capacity is also distributed according to the same patient type distribution. The detailed distribution is listed in Table 9.

Table 9 Tirage classification distribution

S. No.	Patient Type	Percentage	Evacuation Hospital A	Evacuation Hospital B	Receiving Hospitals
1	Minimum	7	14	13	3
2	Low	35	54	54	12
3	Moderate	41	68	67	15
4	High	10	14	13	3
5	Very High	2	5	4	1

Details of several parameters of the model in this case study are listed below. Binary parameter δ_{pv} is the patient vehicle compatibility parameter which indicates whether a type-p patient can be transported via a type-v vehicle. For example, a medical transport van can only transport low risk patients at "minimum" or "low" level while an ambulance can transport all types of patients from the highest level to the lowest. Details of δ_{pv} values are in Table 10.

The evacuating risk parameter $\lambda_{phh'}^{nvs}$ is associated with the risk of transporting a type-p patient from evacuating hospital h to receiving hospital h' via type-v vehicle along vehicle visiting sequence s that corresponds to scenario tree node n. This evacuating risk is assumed to be caused by four sources: 1) scattered medical resources, 2) flooding at the evacuating hospital prior to the evacuation of the patient, 3) limited medical care in ambulances, and 4) flooding along road segments during the evacuation. These four risk sources are assumed to be mutually independent for this case study due to limited data. A binary variable ϕ_h^n equals 1 if the zone of evacuating hospital h is under a mandatory evacuation order at time $t(n)$, and 0 otherwise. Since an issued order will not be cancelled, ϕ values of all descendant scenario tree nodes of a node n equal 1 if $\phi_h^n = 1$. The values of ϕ are derived from previous work (Yi et al., 2017).

Table 10 Patient vehicle compatibility and capacity

	BLS	ALS I	ALS II	Medical Van
Minimum	1	1	1	1
Low	1	1	1	1
Moderate	1	1	1	0
High	0	1	1	0
Very High	0	0	1	0

Risk variable γ_{ph}^n represents the probability that an adverse event will occur for type-p patient in an evacuating hospital h at scenario tree node n. These adverse events can be caused due to the lack of medical resources such as power, staff, or equipment. This risk variable is only considered after the mandatory evacuation order was issued and its value is associated with the effective time periods of the order. For this case study, γ_{ph}^n is modeled as a linear function (Bish et al., 2014). If more past data is available, one can try to replace the predefined function with complex and advanced risk measures. Let l denote the ancestor node of node n when the mandatory order was first issued and b_h denotes the base risk for evacuating hospital h.We assume $\gamma_{min,h}^n =$

$b_h(t(n) - t(l))/|T|$ for each evacuating hospitals, where $t(n) - t(l)$ is the time periods between the issuance of the mandatory evacuation order and the current time. In this case study, we set $b_h = 0.02$ and $\gamma^n_{low,h} = 2\gamma^n_{min,h}$, $\gamma^n_{moderate,h} = 3\gamma^n_{min,h}$ and so on. Thus, the probability of no adverse events happening prior to the evacuation of one type-p patient at node n a time $t(n)$ is $\prod_{m\in N(n)}\left(1 - \gamma^m_{ph}\right)^{\phi^m_h}$.

Second risk variable ξ^n_h represents the probability of an adverse event caused by the flooding of the evacuating hospital h. These values are calculated in three steps. First, we obtain the outputs from a hydrological model called Coupled Routing and Excess Storage (CREST) (J. Wang et al., 2011) and a hurricane storm surge model Advanced CirCulation (ADCIRC) (Westerink et al., 2008; Dietrich et al., 2011). Then, we derive the corresponding risks associated with the flood depth and wind speed using the chart in Figure 6 respectively (Apivatanagul et al., 2012). Finally, these two risks are combined to generate ξ^n_h. With above information, the probability of no adverse events happening due to flooding of evacuating hospital h before time $t(n)$ is $\prod_{m\in N(n)}(1 - \xi^m_h)$. The risk values for 'shelter' are used to calculate ξ^n_h for the shelter-in-place patients.

A third risk variable $\varphi^{nvs}_{phh'}$ represents the risk related to the deterioration of medical service level in an ambulance/van during the evacuation. This risk is also modeled as a predefined linear function (Bish et al., 2014). $\varphi^{nvs}_{phh'}$ denotes the transportation risk of evacuating a type-p patient from hospital h to hospital h' via type-v vehicle using a vehicle visiting sequence s at time node n at time $t(n)$. It is modeled as a function taking the travel time during the evacuation as input. Let κ_{pv} to be the probability of an adverse event happening during a 15 min interval when type-v vehicle is used to transport type-p patient and let $\tau_{hh'}^{ns}$ be the number of such 15 min intervals needed to travel from evacuating hospital h to the receiving hospital h' when the vehicle visiting sequence starts at node n. Thus, $\varphi_{phh'}^{nvs} = 1 - \left(1 - \kappa_{pv}\right)^{\tau_{hh'}^{ns}}$. The values of κ_{pv} for the 'minimum' type level patient are set to 4E-05, 2E-05, 2E-06, and 2E-04 for BLS, ALS I, ALS II, and medical van, respectively. For each type of vehicle, κ_{pv} increases with γ which implies $\kappa_{low,v} = 2\kappa_{min,v}$, $\kappa_{moderate,v} = 3\kappa_{min,v}$ and so on.

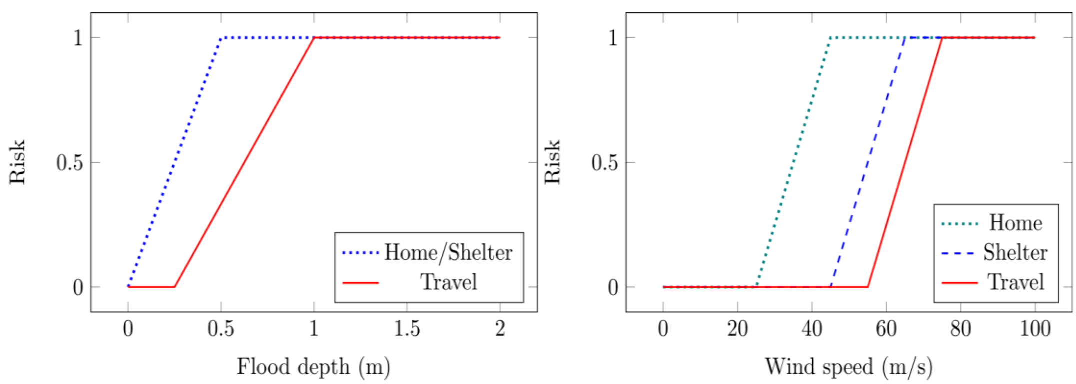


Figure 6 Risk factors as a function of flood depth (left) and wind speed (right)

The last risk variable $\vartheta_{hh'}^{ns}$ represents the risk related to highway flooding during the evacuation. Using previously computed flooding risk for each road network link and the time dependent shortest path information, we can write $\vartheta_{hh'}^{ns} = 1 - \prod_{e \in P_{hh'}^{s}} (1 - \varsigma_e)$, where e is a highway link in the road network, $P_{hh'}^{s}$ is the actual path taken by a vehicle to travel from h to h' along the vehicle visiting sequence s that starts at node n at time $t(n)$, and ς_e is the risk of flooding for the highway road link e.

With above mentioned risk variables, we can model the evacuating risk $\lambda_{phh'}^{nvs}$ and shelter in place risk μ_{ph}^{r} as follows.

$$\lambda_{phh'}^{nvs} = 1 - \left(\prod_{m \in N(n)} \left(1 - \gamma_{ph}^{m}\right)^{\phi_h^m} \prod_{m \in N(n)} (1 - \xi_h^m) \left(1 - \varphi_{phh'}^{nvs}\right)\left(1 - \vartheta_{hh'}^{ns}\right) \right)$$

$$\mu_{ph}^{r} = 1 - \left(\prod_{n \in r} \left(1 - \gamma_{ph}^{n}\right)^{\phi_h^n} \prod_{n \in r} (1 - \xi_h^n) \right)$$

5.2 Results

5.2.1 Efficiency frontier

Relaxed linear programming (LP) models with different weights ψ are solved to generate an efficient frontier. The results are presented in the Figure 7. The horizontal axis represents the risk component, and the vertical axis represents the cost component. The value of ψ is between 0 and 1. Specific weights ψ are chosen using the following bisection method. First, the LP model with $\psi = 0$ and $\psi = 1$ are solved. After LPs corresponding to two endpoints are solved, we resolve LP with a weight equal to the middle point value of two endpoint weights, in this case $\psi = 0.5$. If the objective components corresponding to $\psi = 0.5$ does not coincide with existing solutions' objective components, then two new regions for $\psi \in [0, 0.5]$ and $\psi \in [0.5, 1]$ are created. We repeat the above procedure in each region $\psi \in [0, 0.5]$ and $\psi \in [0.5, 1]$, recursively. If the middle point solution is same as the endpoint solution, then no further subregions are be generated from the current region. For example, if $\psi = 0.25$ yields the same LP solution as that $\psi = 0.5$, then the region $[0, 0.5]$ is omitted. We terminated this process after about 100 solutions were found.

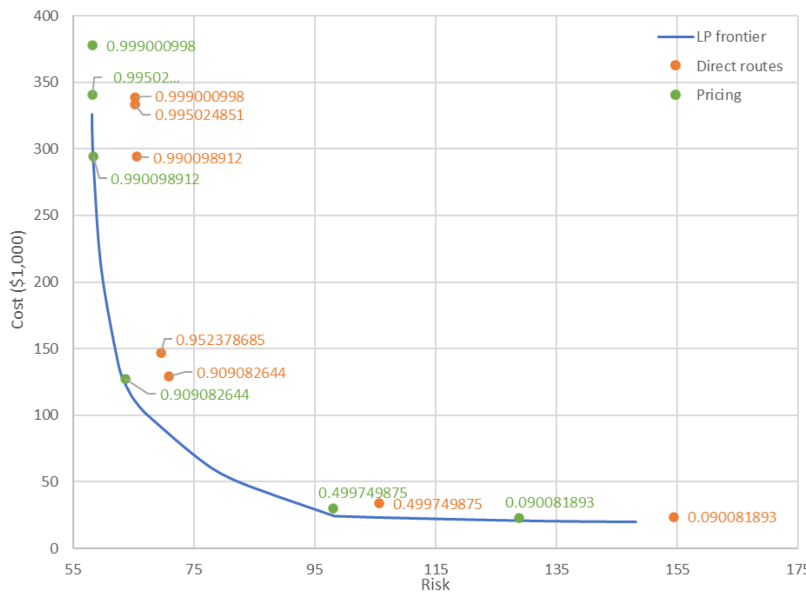


Figure 7 Efficient frontier of LP solutions and certain IP solutions

LP solution frontier is preferable due to the speed of solving an LP model over an integer programming (IP) model. This LP frontier can provide an approximation of the evacuation risks and costs for hospital administrators. Thus, they can make decisions on which ψ weights are appropriate for further analysis with an IP model. For illustration purposes, we chose 6 different ψ weights for solving an IP model with direct vehicle visiting sequences only and another one involving complex vehicle visiting sequences. The orange dots in Figure 7 represent IP solutions with only direct vehicle visiting sequences allowed. The green dots in Figure 7 represents IP solutions that allow complex vehicle visiting sequences. The values of ψ in IP solutions are determined from ψ values of LP solutions on the efficient frontier. When ψ is close to 0, the problem instance emphasizes cost minimization and when ψ is close to 1, the problem instance emphasizes minimizing risk. The percentage gap of the IP objective with the corresponding LP relaxation is around 0.5% for IP solutions when complex vehicle visiting sequences are allowed. For IP solutions allowing direct vehicle visiting sequences, the gap between IP objective and LP relaxation is about 4% when ψ is greater than 0.5 and starts to drop to 0.4% as ψ decreases. Even when the integrality gap is small, the LP relaxations provide many fractional values in the optimal decision variables and simple rounding could introduce infeasibilities. Therefore, solving IPs with CPLEX is still necessary. Every IP run is performed with a threshold of 2 hours.

The IP solution results indicate that, when $\psi \approx 91\%$, the difference between models with direct vehicle visiting sequences and complex vehicle visiting sequences is most significant. At $\psi = 90.9\%$, the IP objective for complex vehicle visiting sequence is about 9% less than that of with direct vehicle visiting sequences. In the following subsections, our analysis focuses on IP solutions of two cases: Case (I) $\psi = 90.9\%$, both direct and complex vehicle visiting sequences are allowed, and Case (II) $\psi = 90.9\%$, only direct vehicle visiting sequences are allowed.

Table 11 shows a detailed breakdown of risk and cost components in the optimal IP solutions corresponding to Case (I) and (II). The two columns on the left represent expected risk associated with evacuating $\sum_{n \in N} \pi^n \left(\sum_{p \in P} \sum_{h \in H} \sum_{h' \in H'} \sum_{v \in V} \sum_{s \in S(hh')} \lambda_{phh'}^{nvs} x_{phh'}^{nvs} \right)$ and with sheltering-in-place $\sum_{r \in R} \rho^r \left(\sum_{p \in P} \sum_{h \in H} \mu_{ph}^r \left(\alpha_{ph} - \sum_{h' \in H'} \sum_{n \in r} \sum_{v \in V} \sum_{s \in S(hh')} x_{phh'}^{nvs} \right) \right)$.

The two columns on the right represent expected transportation costs

$\sum_{n\in N}\pi^n\left(\sum_{p\in P}\sum_{h\in H}\sum_{h'\in H'}\sum_{v\in V}\sum_{s\in S\left(hh'\right)}\sigma_p^n x_{phh'}^{nvs}+\sum_{v\in V}\sum_{s\in S\cup\bar{S}}\omega_{vs}^n y_{vs}^n\right)$ and

staffing cost $\sum_{r\in R}\rho^r\left(\sum_{p\in P}\sum_{h\in H}\sigma_p\left(\alpha_{ph}-\sum_{h'\in H'}\sum_{n\in r}\sum_{v\in V}\sum_{s\in S\left(hh'\right)}x_{phh'}^{nvs}\right)\right)$.

Table 11 Risk and cost in dollars component of Case (I) and Case (II) optimal IP solutions

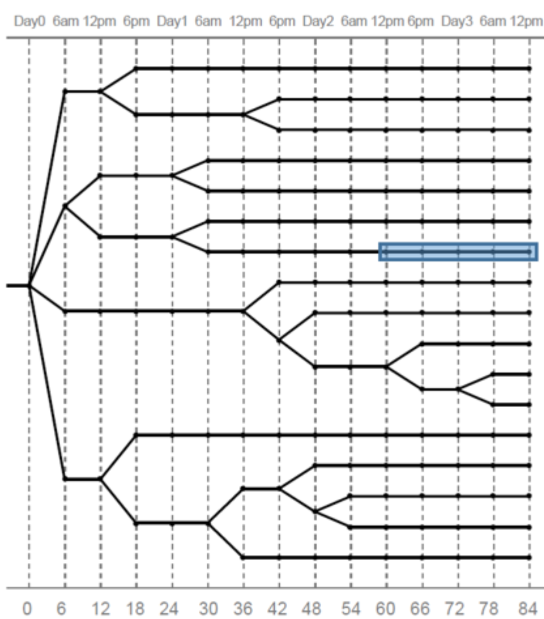
$\psi = 90.9\%$	Evacuation Risk	Shelter-in-Place Risk	Transportation Cost ($1,000)	Staffing Cost ($1,000)
Case (I)	22.899	40.804	118.683	8.519
Case (II)	29.836	40.995	119.936	9.187

At $\psi = 90.9\%$, introducing complex vehicle visiting sequences in the IP model could significantly reduce the evacuation risk and staffing costs without sacrificing shelter-in-place risk and transportation costs.

5.2.2 Model performance under different scenarios

The number of patients evacuated for each evacuating hospital along each sample path for the two cases are listed in Figure 8. The highlighted portion (along the sample path ending with Scenario 5) is the nodes where evacuating hospitals were given evacuation orders.

Evacuating hospital B always evacuated along all sample paths but evacuating hospital A only evacuated along sample paths ending with Scenario 5. This difference was due to the locations of these two evacuating hospitals. Evacuating hospital B is in a zone that has a higher shelter-in-place risk. Thus, evacuating hospital B evacuated patients more preemptively than evacuating hospital A. This led to a higher number of evacuated patients from Hospital B along all sample paths. On the other hand, staffing cost weighs more in the evacuation decision for evacuation hospital A since its zone has a lower shelter-in-place risk. As observed, evacuating hospital A only started to evacuate when staffing costs enter the objective at the highlighted nodes. Thus, evacuating hospital A only evacuated patients at certain time stamps along sample paths that were indistinguishable with the sample path ending at Scenario 5. Also, notice that not all patients from evacuating hospitals A and B were evacuated due to limited vehicle capacity and higher evacuation risk in later time periods.



Scenario	Case (I)		Case(II)	
	Evacuation Hospital A	Evacuation Hospital B	Evacuation Hospital A	Evacuation Hospital B
22	0	112	0	112
17	0	112	0	112
10	0	112	0	112
20	92	112	84	112
7,9	92	112	84	112
19	92	112	84	112
5	92	112	84	112
6	0	112	0	112
3	0	112	0	112
11	0	112	0	112
4,18	0	112	0	112
2	0	112	0	112
8,21	0	112	0	112
14	0	112	0	112
16	0	112	0	112
13	0	112	0	112
1,12,15	0	112	0	112

Figure 8 Number of patients in each evacuating evacuated along different sample paths for Case (I) and Case (II)

The risk and cost of the objective function components per sample path among two cases are listed in Table 12. The values in Case (I) comes from optimal IP solution for $\psi = 90.9\%$ with complex vehicle visiting sequences allowed and in Case (II), it comes from an optimal IP solution for $\psi = 90.9\%$ with direct vehicle visiting sequences only. Both cases showed that the sample path ending with Scenario 5 has the highest risk and cost components. However, the optimal solutions differ in risk and cost components between evacuating hospital A and evacuating hospital B for other sample paths. For sample paths ending with Scenarios such as 19, {7,9} and 20, hospital A has a higher level of risk and cost compared to all remaining sample paths while hospital B has almost same level of risk and cost.

It is worth noticing that for certain sample paths, the cost value for hospital A is positive when the risk for hospital A is 0. These are caused by rebalancing vehicles from hospital A to hospital B.

Table 12 Objective function components along sample paths for two cases

Scenario	Case (I)				Case (II)			
	Risk		Cost ('000)		Risk		Cost ('000)	
	Hospital A	Hospital B	Hospital A	Hospital B	Hospital A	Hospital B	Hospital A	Hospital B
22	0	61.87	30.65	79.23	0	68.81	0	110.69
17	0	61.87	30.65	79.23	0	68.81	0	110.69
10	0	61.87	30.65	79.23	0	68.81	0	110.69
20	0.13	61.87	69.37	79.23	0.09	68.81	40.7	110.69
7,9	0.13	61.87	69.37	79.23	0.09	68.81	40.7	110.69
19	0.13	61.87	69.37	79.23	0.09	68.81	40.7	110.69
5	39.83	61.86	185.14	150.9	44	68.8	171.17	182.35
6	0	61.87	30.65	79.23	0	68.81	0	110.69
3	0	61.87	30.65	79.23	0	68.81	0	110.69
11	0	61.87	30.65	79.23	0	68.81	0	110.69
4,18	0	61.87	30.65	79.23	0	68.81	0	110.69
2	0	61.87	30.65	79.23	0	68.81	0	110.69
8,21	0	61.87	30.65	79.23	0	68.81	0	110.69
14	0	61.87	30.65	79.23	0	68.81	0	110.69
16	0	61.87	30.65	79.23	0	68.81	0	110.69
13	0	61.87	30.65	79.23	0	68.81	0	110.69
1,12,15	0	61.87	30.65	79.23	0	68.81	0	110.69

Expected Value	1.83	61.87	44.71	82.49	2.02	68.81	15.18	113.94

When comparing the two cases, the benefits of complex vehicle visiting sequences is clear. For the same weight $\psi = 90.9\%$, complex vehicle visiting sequences allow the risk values to decrease without increasing the cost significantly. The overall risk value decreases by around 10% with a similar overall cost value.

5.2.3 Distribution of patients across hospitals

Table 13 shows the patient distribution between evacuating hospitals and receiving hospitals for the sample path that ends in Scenario 5.

In Case (I), with complex vehicle visiting sequences, receiving hospitals further away from the evacuating hospital have a better chance of admitting patients. For example, receiving hospital 8 and 9 can admit patients from evacuating hospital A in case (I) even though they are relatively farthest from it. Similarly, receiving hospital 4 and 5 can increase patient admission from evacuating hospital B by allowing complex vehicle visiting sequences. This allows rural area hospital patients to have access to higher quality hospitals in further urban regions.

In Case (II), evacuating hospital A chose receiving hospitals mainly in the northern region of North Carolina while evacuating hospital B chose receiving hospitals mainly in the southern region. The choice of receiving hospitals for an evacuating hospital relates not only to the distance and its associated risks, but also the interference from other evacuating hospitals. For example, evacuating hospital A did not choose to evacuate to receiving hospital 6 even though the distance between them is short since its capacity was utilized by evacuating hospital B.

The patient distribution information is very useful since it provides insights for hospital administrators to coordinate with receiving hospitals by providing the number of estimated patients they would receive. Receiving hospitals can take actions to reserve enough medical resources and create sufficient in-patient capacity accordingly if such information provided ahead of time. This patient distribution information also provides a glimpse of how evacuating hospitals interfere with each other's receiving hospital choices during the evacuation. This would further help hospital administrators from different hospitals to coordinate their evacuation efforts.

Table 13 Patient distribution during sample path ending with Scenario 5

	Dist.		Case (I)		Case (II)	
	Evacuating Hospital A	Evacuating Hospital B	Evacuating Hospital A	Evacuating Hospital B	Evacuating Hospital A	Evacuating Hospital B
Receiving Hospital 1	63.7	97	0	15	2	15
Receiving Hospital 2	44.6	119	30	0	29	0
Receiving Hospital 3	84.8	132	15	0	15	0
Receiving Hospital 4	38	90	9	22	23	7
Receiving Hospital 5	77.7	125	0	15	0	7
Receiving Hospital 6	39	70	0	34	0	34
Receiving Hospital 7	102	151	15	0	15	0
Receiving Hospital 8	112	107	15	0	0	15
Receiving Hospital 9	78	66	8	26	0	34
Total			92	112	84	112

5.2.4 Average cost per patient type

One major measure to distinguish the differences between complex vehicle visiting sequences and direct vehicle visiting sequence is the average cost per patient type. The average costs for Case (I) and (II) are shown in Table 14 and Table 15.

For the sample path terminating at Scenario 5, we observe that average cost for patient type decreases drastically when complex vehicle visiting sequences are allowed and the reduced costs are associated with the vehicle cost component. Both evacuating hospitals have a reduced average vehicle cost for all patient types except “Very High”. For ‘Minimum’ type patients, average vehicle cost reduces by nearly 60%. For ‘Low’, ‘Moderate’, and ‘High’ type patients, average vehicle costs also reduce by 20%~40%. “N/A” represents the case where no such type of patient is transported and thus no average cost is computed. The efficiency provided with complex vehicle visiting sequences and larger capacity vehicles reduces the total transportation cost and also improves the number of patients to be transported under a certain transportation budget.

Table 14 Average cost per patient type in Case (I) in sample path ending with Scenario 5

		Minimum	Low	Moderate	High	Very High
Avg. staff cost	Evacuating Hospital A	-1837.51	-1837.51	-1837.51	N/A	N/A
	Evacuating Hospital B	-1837.51	-1837.51	-1837.51	-1837.51	-1837.51
Avg. vehicle cost	Evacuating Hospital A	232.27	258.03	731.40	N/A	N/A
	Evacuating Hospital B	125.83	121.17	568.36	1787.04	3302.21

Table 15 Average cost per patient type in Case (II) in sample path ending with Scenario 5

		Minimum	Low	Moderate	High	Very High
Avg. staff cost	Evacuating Hospital A	-1837.51	-1837.51	-1837.51	N/A	N/A
	Evacuating Hospital B	-1837.51	-1837.51	-1837.51	-1837.51	-1837.51
Avg. vehicle cost	Evacuating Hospital A	592.55	324.86	1285.35	N/A	N/A
	Evacuating Hospital B	305.39	318.67	1738.45	2389.55	3302.21

5.2.5 Complex vehicle visiting sequences usages

Four complex vehicle visiting sequences $A-3-7-A$, $A-8-9-B$, $B-6-4-A$, and $B-1-5-A$ were utilized in the final solution when MIP-HCG was solved in Case (I). Figure 9 to Figure 12 illustrate actual routes of these complex vehicle visiting sequences.

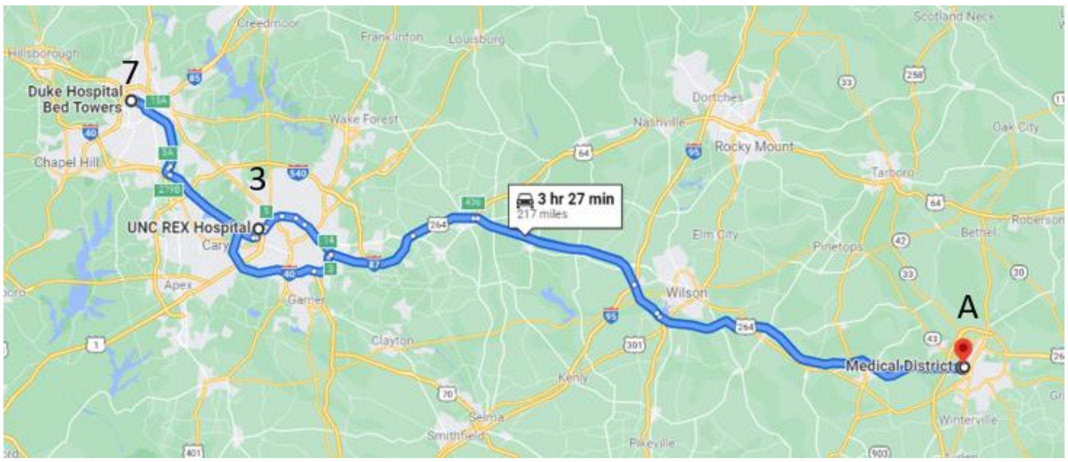


Figure 9 Complex vehicle visiting sequence A-3-7-A

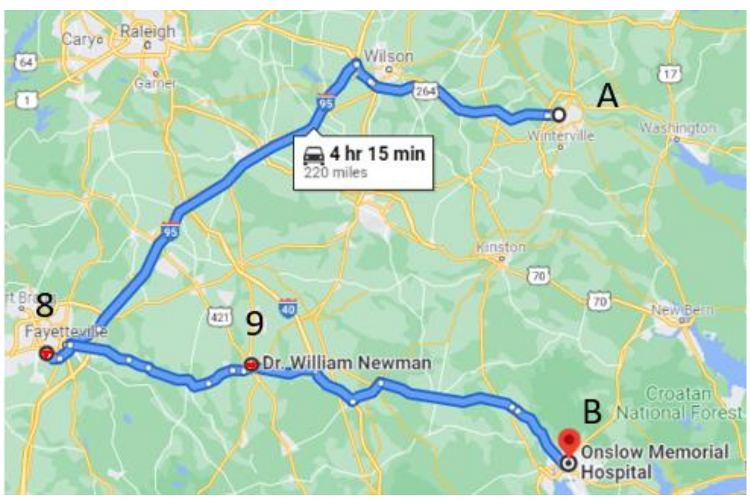


Figure 10 Complex vehicle visiting sequence A-8-9-B

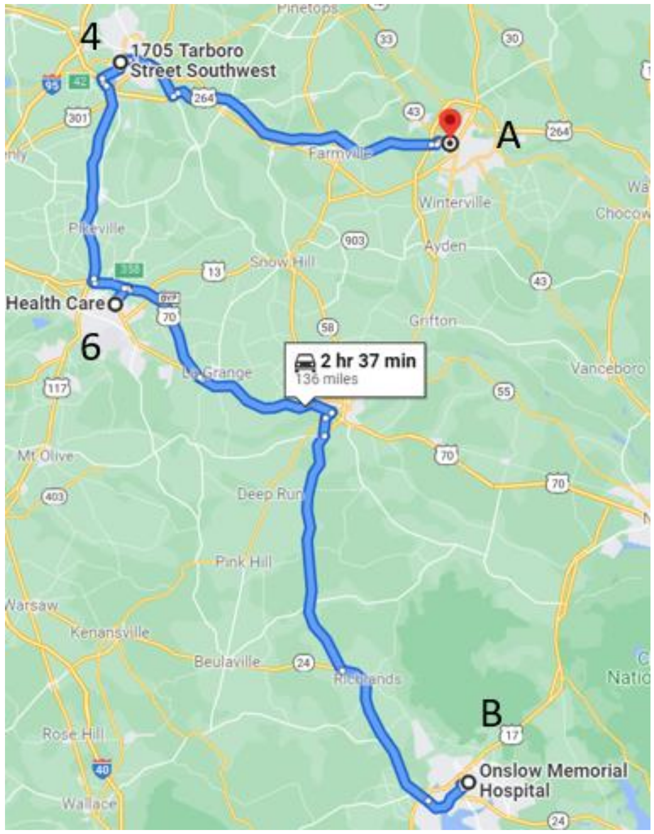


Figure 11 Complex vehicle visiting sequence B-6-4-A

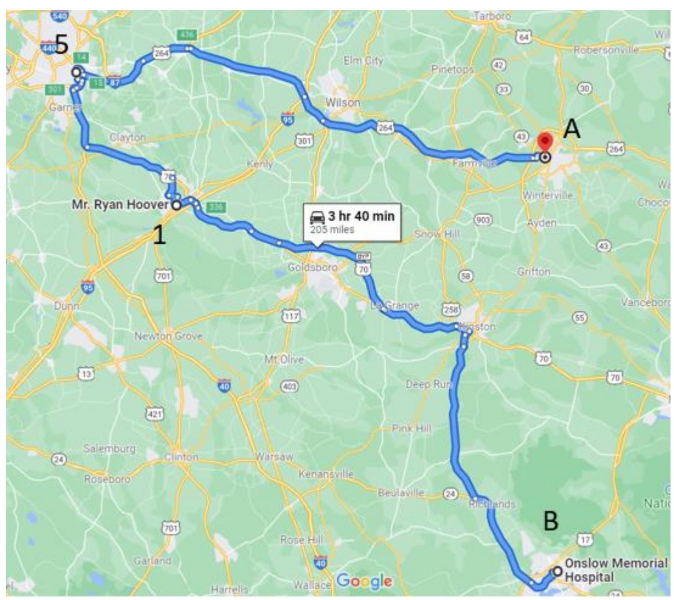


Figure 12 Complex vehicle visiting sequence B-1-5-A

On average, a single van can transport 23~30 patients and travel 150~250 miles using complex vehicle visiting sequence in one run. In total, 113 out of all 204 patients evacuated under the scenario tree sample path ending with Scenario 5 using complex vehicle visiting sequences. Among the different patient types, the least severe patients (Type 0 and Type 1) were transferred via complex vehicle visiting sequences.

There are also interesting differences among these complex vehicle visiting sequences. Complex vehicle visiting sequence $A - 3 - 7 - A$ is a round trip that can be interpreted as a combination of a direct round trip $A - 3 - A$ and an extra short trip from receiving hospital 3 to a nearby receiving hospital 7. Such round trip-like vehicle visiting sequences improved the efficiency by merging trips and reducing overlapping parts of two round trips. For instance, when the vehicles are restricted to direct visiting sequences only, two round trips $A - 3 - A$ and $A - 7 - A$ must be utilized to fulfill the same goal.

On the other hand, the remaining three complex vehicle visiting sequences are paths from one origin evacuating hospital to another destination evacuating hospital. These path-like complex vehicle visiting sequences extended the reachability of evacuating hospital patients by allowing patients to be transferred to farther receiving hospitals. For example, when the vehicles are restricted to direct vehicle visiting sequences, only severe patients (Type 2) from evacuating hospital B will be transferred to receiving hospital 4 with ambulances. But when complex vehicle visiting sequences are allowed, with the help of the sequence $B - 6 - 4 - A$, even less severe patients (Type 0 and Type 1) can be transferred to the farther receiving hospital 4. Such path-like sequences also help naturally rebalance vehicles between evacuating hospitals without deliberating using rebalancing vehicle visiting sequences.

CHAPTER 6

CONCLUSION

6.1 Summary

In this paper, we studied the problem of evacuating patients from multiple hospitals under the uncertainty of hurricanes. A multi-stage stochastic program was developed with column generation-based formulation and heuristic pricing subproblem to identify the optimal number of patients of different types in each evacuating hospital who need to be evacuated to their corresponding receiving hospitals at different time periods, using different vehicle types, and different vehicle visiting sequences.

The objective was a combined risk and cost related measure. The risk component contained evacuating risk involved in transferring patients and shelter-in-place risk involved in sheltering patients at the original hospital. The cost component contained transportation cost of both vehicle and patients, and staffing cost for patients.

Uncertainty in the process of a hurricane was represented using a scenario tree that was generated from predicted hurricane trajectories and formed the basis for a stochastic optimization problem.

This new stochastic program introduced complex vehicle visiting sequences that allows a vehicle to visit multiple evacuating and receiving hospitals in one dispatch. Due to the size of the feasible complex vehicle visiting sequences, the stochastic program was reformulated as a column generation formulation with respect to "route assignment variable" as a compound variable. The corresponding pricing subproblems were formulated to find route assignment variables that can improve the master problem's objective value. The difficulties in solving such pricing subproblems led to a heuristic solution method and reformulation of the model based on decomposing vehicle visiting sequences. In the final heuristic formulation, pricing subproblems were solved to dynamically generated vehicle visiting sequences that was later added into the restricted master problem and the master problem was solved to generate decisions on patient transfer plan and vehicle usages.

A case study with Hurricane Isabel and hospitals from North Carolina was used to provide a concrete demonstration on the benefits of using complex vehicle visiting sequences. The usage of complex vehicle visiting sequences reduced the average transportation costs among low-risk patient types and extended the reachable range of receiving hospitals for an evacuating hospital. Overall, the complex vehicle visiting sequence allowed a reduction in total risk without causing a huge burden in the cost objective.

This model also provided more systematic and quantitative information on detailed operational issues for hospital administrators who are evacuating patients from multiple evacuating hospitals. Especially, the model results could help hospital administrators from different evacuating hospitals to avoid unnecessary competition with each other and coordinate patients' transportation arrangements and shared vehicle fleet. With the information from this model, hospital administrators could work together to mitigate systematic imbalances that would otherwise arise from spontaneous optimization by each evacuating hospital.

6.2 Limitations and Future Research

Further research is needed in the following areas. First, new algorithms could be developed for the currently computationally intense pricing subproblem. Second, other decomposition methods such as progressive hedging could be applied on the stochastic program alongside column generation to further simplify the pricing subproblem structure. Third, it is worth exploring how this solution approach scales as the hospital network size grows and how the resulting computational bottleneck can be overcome.

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