Income Mobility: Concepts and Measures

Gary S. Fields
Cornell University

People's economic positions may change for a variety of reasons. The economy in which they participate may improve or deteriorate because of macroeconomic growth or contraction, employer-specific events and circumstances, business expansions and contractions, and ups and downs in local communities. Individuals may experience major life events with important economic consequences, among them completion of schooling, promotions and other movements up the career ladder, marriage and divorce, poor health, and retirement. Economic mobility studies are concerned with quantifying the movement of given recipient units through the distribution of economic well-being over time, establishing how dependent ones current economic position is on one's past position, and relating people's mobility experiences to the various influences that have been mentioned.[1]

Four methodological aspects of these studies are worth highlighting. First, mobility analysis follows given economic units through time. Consequently, longitudinal (or panel) data are required for research, which makes mobility analysis different from the measurement of poverty, inequality, or economic well-being.[2] Second, mobility analysis can be applied to a variety of recipient units. Those most commonly used are individuals and households. Third, any aspect of economic well-being can be used. Among those that are studied are the income, earnings, expenditures, or occupational attainment of the individual or household. When income and expenditures are used, the data are often per capita. Any measure in dollars should be in real dollars, adjusted for inflation. Finally, I shall limit my attention to the recipient s economic well-being in a base year versus a final year. Other mobility studies assess mobility by looking at economic position in each of *T* years, but I shall not deal with those studies or measures.[3]

Notwithstanding these points of agreement about the concept of economic mobility, there are also some fundamental disagreements. This is because the term *income mobility* conjures up very different ideas in people s minds. In much the same way that I find it helpful to reserve the term income distribution for a generic concept and to use *inequality poverty. mobility, and economic well-being* to distinguish among different specific aspects of the income distribution, so too is it helpful to reserve the term income mobility for the generic concept and to use other specific terms for particular aspects of income mobility. These five ideas—time dependence, positional movement, share movement, symmetric income movement, and directional income movement are described in the next section. The following section contrasts these approaches and shows how the choice among them makes a difference in certain illustrative examples. In the next section I look at various mobility measures and their axiomatic foundations.[4]

Five Mobility Concepts

It is said that Joseph Schumpeter likened an income distribution to a hotel.[5] The rooms at the top are luxurious, those on the middle levels are ordinary, and those in the basement are downright shabby. At any given time the occupants of the hotel experience very unequal accommodations. At a later time, if one reexamines who is living where, one finds that some have moved to higher floors, some to lower floors, and some have stayed where they were.

The difference in the quality of hotel rooms at each point in time is called inequality. The movement of hotel guests among rooms of different quality is mobility. One way in which these are linked is that the more movement of guests there is among rooms, the greater the long-term equality of accommodations.

But is the movement of guests among rooms all there is to mobility? What if the existing furnishings are redistributed from some rooms to others? Isn't there mobility then? What if the hotel is refurbished so that some of the rooms are made nicer? Don't the lucky residents of the now nicer rooms enjoy upward mobility? What about those whose rooms are not upgraded? Do they suffer downward mobility?

The hotel analogy raises some fundamental questions about what economic mobility is and by extension how it should be measured. The mobility literature is plagued by people talking past one another because one person's idea of mobility is not another's. Five concepts shall be distinguished in this chapter.

Time dependence measures the extent to which economic well-being in the past determines individuals' economic well-being at present. *Positional movement* takes place when there is a change in individuals' economic positions (ranks, centiles, deciles, or quintiles). *Share movement* occurs when individuals' shares of total income change. *Symmetric income movement* arises when individuals' incomes change and the analyst is concerned about the magnitude of these fluctuations but not their direction. Finally, in *directional income movement* income gains and income losses are treated separately. Let us now look at each of these in greater detail.

Time Dependence

Time dependence is a particular form of immobility. It arises when one's current economic position is determined by one's position in the past. Studies of time dependence arise in two contexts. In the *intergenerational* context, the question is to what extent the incomes of sons can be predicted by the incomes of their fathers.[6] In the *intragenerational* context, the problem is to what extent individuals' incomes at a later date can be predicted by their incomes at an earlier date. To be able to speak about both contexts, one can use the terminology *base income and final income.* Again, the reader is reminded that "income" is a shorthand for whichever economic or socioeconomic variable one is interested in measuring.

Data for gauging time dependence may come either in aggregated or in disaggregated form. I take up in turn how to work with each of these two types of data.

TIME DEPENDENCE IN AGGREGATED DATA. An analytical tool that facilitates measurement of time dependence is an intertemporal transition matrix. The rows of the matrix are the income classes of income recipients in the base year, and the columns are the corresponding income classes in the final year. These classes may either be income categories ($0—$ 10,000, $10,000-$20,000, and so forth) or quantiles (for example, deciles or quintiles). The entries in the transition matrix indicate what fraction of people with a given base year income end up with a given final year income, and thus each row sums to 100 percent. Quantile transition matrices are the type most commonly used, and so those are the ones I begin with here.

Let the population be divided into five income quintiles. One way there might be perfect time dependence is for each recipient s final year income quintile to be identical to his or her base year quintile. If this were the case, all entries in the transition matrix would lie along the principal diagonal running from upper left: to lower right, each element on the principal diagonal would equal 100 percent, and thus the transition matrix would be an identity matrix.

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5\text{-}1)$$

The identity matrix indicates what I shall call “perfect positive time dependence.” The closer the actual transition matrix in a country to the identity matrix, the more immobility in the sense of time dependence there is said to be.

There is, however, a very different way in which there might be perfect time dependence. Suppose that there is a complete reversal of income positions so that all those who start out rich end up poor and all those who start out poor end up rich. Though this is only a theoretical possibility that never arises in practice, what it would produce if it did happen is a transition matrix with all ones along the diagonal running from upper right to lower left.

$$P_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5\text{-}2)$$

The reverse identity matrix arises in the case of so-called perfect negative time dependence. The closer the actual transition matrix in a country to the reverse-identity matrix, the more immobility in the sense of time dependence there is said to be.

Though these two criteria may appear to be contradictory, I assure you that they are not. This is because lying in between perfect positive time dependence and perfect negative time dependence is time independence. Time independence arises when a person's final year income is independent of his or her base year income. This would produce a transition matrix in which each row is the same as every other. In the special case of a quintile mobility matrix, this would mean that every entry would be equal to 0.2.

$$P_3 = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \qquad (5\text{-}3)$$

To be able to implement the ideas of positive time dependence, negative time dependence, and time independence, one needs a way of measuring how close an actual transition matrix is to these various theoretical possibilities. Continuing with the illustration of what might be done if there is a quintile mobility matrix, take as the basis for comparison the number of people that would be observed in cell *i,j* under the null hypothesis of time independence; this would be the matrix given in 5-3 multiplied by an appropriate scaling factor such that the sum of the expected frequencies is the total sample size N.

$$P_4 = \begin{bmatrix} .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \end{bmatrix} \qquad (5\text{-}4)$$

Denoting these expected frequencies by EXP_{ij}, one can then compare them with the observed frequencies OBS_{ij} by making a standard (Pearson) chi- squared calculation

$$X^2 = \sum_i \sum_j \frac{(OBS_{ij} - EXP_{ij})^2}{EXP_{ij}}$$

The calculated chi-squared value would tell how distant an actual transition matrix is from the one that would be observed in the case of perfect time independence. And then, in comparing the chi-squared values obtained in two different mobility situations, one would be able to say that the matrix with the larger chi-squared value is more time dependent and therefore less mobile in this sense of time dependence than the other.[7]

Table 5-1 shows quintile mobility matrices for the United States for 1967—79 and 1979-91, from which chi-squared values may be calculated.

Table 5-1. *Quintile Mobility Rates for Equivalent Family Income*
Percent

1967–79 transition matrix					
	Quintile in 1979				
Quintile in 1967	*Bottom quintile*	*Second quintile*	*Third quintile*	*Fourth quintile*	*Top quintile*
Bottom quintile	51.3	25.0	15.3	5.9	2.4
Second quintile	21.8	27.0	24.3	19.1	7.8
Third quintile	12.3	21.3	22.7	24.5	19.3
Fourth quintile	8.1	15.0	19.7	26.5	30.7
Top quintile	6.4	11.7	17.8	24.1	40.0
N = 3,277					

1979–91 transition matrix					
	Quintile in 1991				
Quintile in 1979	*Bottom quintile*	*Second quintile*	*Third quintile*	*Fourth quintile*	*Top quintile*
Bottom quintile	47.8	25.6	13.1	10.4	3.2
Second quintile	22.1	26.7	24.8	18.3	8.1
Third quintile	12.2	18.9	25.6	21.6	21.8
Fourth quintile	12.3	19.5	18.2	23.1	26.9
Top quintile	5.7	9.0	17.7	27.6	40.0
N = 3,322					

Source: Gittleman and Joyce (1998).

The calculated values are 1.153 for 1967-79 and 1.025 for 1979-91. This indicates that the United States had less mobility in the sense of time dependence in 1967-79 than in 1979-91.

Consider now how you would treat aggregated data in which you are given a transition matrix among income classes that contain unequal numbers of people. In similar fashion, you would need to calculate the distance between the observed frequency distribution and the theoretically expected one, but now the calculation would need to be made in a slightly different way. Under time independence, all rows would have identical conditional probabilities, but because the marginal frequencies are different across income classes in this case (whereas they are identical in the case of quintiles or deciles), the expected frequencies would differ proportionately. Again, in comparing two mobility situations, the one with the larger calculated chi-squared statistic could be said to exhibit more mobility in the sense of time dependence than the other.

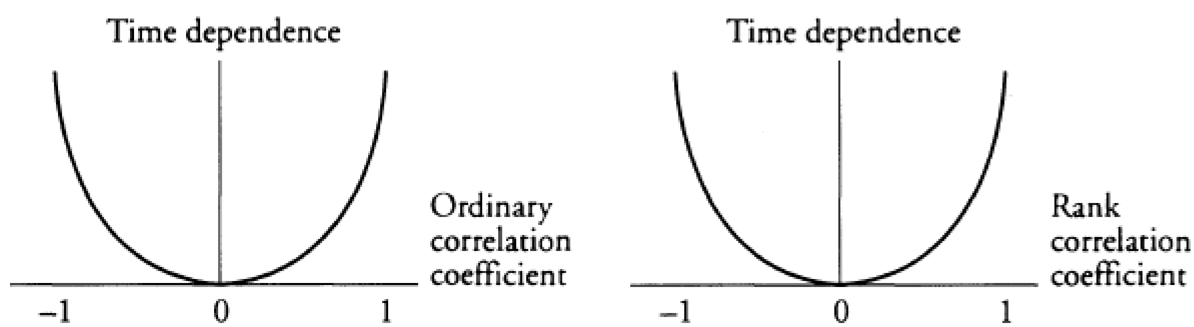
Finally, it bears mention that the chi-squared statistic is not the only one that might be calculated as a measure of time dependence. Standard statistical packages contain a contingency table procedure that produces a number of such statistics. For instance, the Stata statistical package will tell you, in addition to the standard (Pearson) chi-squared, the likelihood-ratio chi-squared, Cramers V, gamma, and Kendalls tau-b—all of which are measures of time dependence in the sense of gauging deviations from randomness. Using the data shown in table 5-1, all of these statistics show higher values for the United States in 1967-79 than in 1979—91, and therefore less mobility in the sense of time dependence in the 1970s than in the 1980s. But Thomas Hungerford calculated the lambda asymmetric statistic, Cramer s V, and the contingency coefficient using a different extract from the U.S. Panel Study of Income Dynamics (PSID) and found that each of these produced essentially identical values in the United States in 1969-76 and 1979-86.[8] From this he concluded that income mobility was unchanged between these two seven-year periods.

The preceding methods give practical ways of measuring time dependence in aggregated data. We turn now to the case of disaggregated data.

TIME DEPENDENCE IN DISAGGREGATED DATA. Increasingly researchers are working with microeconomic data rather than published data. Such data sets may contain observations on many thousands or even tens of thousands of income recipients. If you have such disaggregated data on the base year and final year incomes for each income recipient, you could create your own intertemporal transition matrix. However, you would lose a great deal of information by doing this, so you might prefer another option: calculating a measure of time dependence using the disaggregated data directly.

A commonly used measure of income mobility is the ordinary (Pearson) coefficient of correlation between base year income and final year income.[9]

Figure 5-1. *Time Dependence as a Function of the Ordinary and Rank Correlation Coefficients*



The correlation coefficient can be used to gauge income mobility precisely in the sense of time dependence:[10]

- The closer the value of the correlation coefficient to +1, the more positive time dependence there is.
- The closer the value of the correlation coefficient to -1, the more negative time dependence there is.
- The closer the value of the correlation coefficient to 0, the more time independence there is.

Figure 5-1 shows the relationship between time dependence and the ordinary correlation coefficient.[11]

A related measure of income mobility is the rank correlation coefficient. Denote the poorest individual by 1 and the richest by N in both the base year and final year distributions and calculate the correlation among income ranks. Exactly the same three points as in the previous paragraph apply to the rank correlation coefficient, and thus the graph between time dependence and the rank correlation coefficient has the same U shape as above (figure 5-1).

When these measures have been used to make mobility comparisons, several patterns emerge. First, the correlation coefficient is always positive, so in practice the distinction between measures of time dependence and income movement is more theoretical than practical. Second, as would be expected, the longer the time elapsed between base year and final year, the lower the correlation between incomes in the two years.[12] And third, the variations across countries are fairly large.[13]

Let us turn now to measures that are explicitly movement measures.

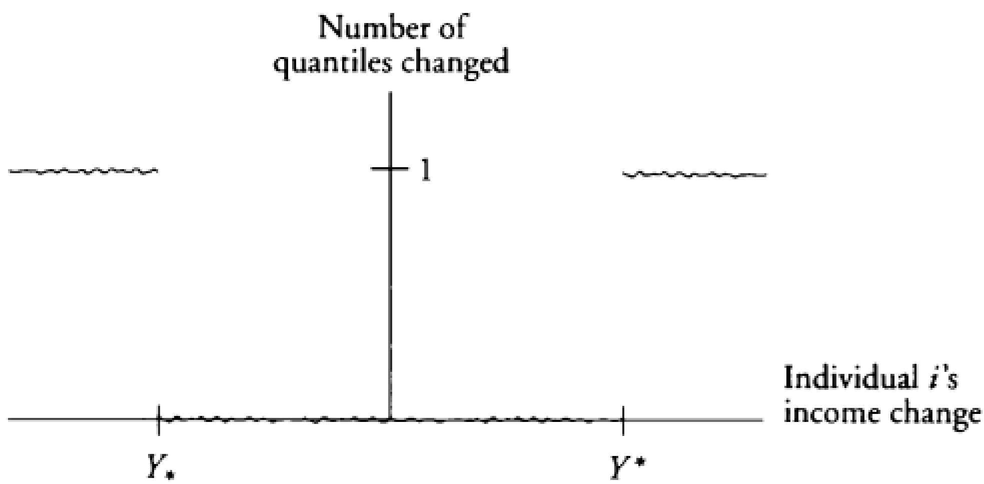
Positional Movement

In the study of positional movement, the measure of economic wellbeing is the individuals position in the income distribution.[14] Although the most commonly used measures of economic position are individuals' quintiles or deciles in the income distribution, there is no reason that one could not work with ventiles, centiles, or even ranks.

The main reason that positional analysis is so popular in income mobility studies is that movement among positions is the way most analysts have become accustomed to thinking about mobility.[15] History plays an important role in this. When income mobility studies were first done, it would have been difficult with the available technology to measure and evaluate income changes person by person. The masses of individual information had to be summarized somehow. Analyzing decile or quintile mobility matrices was a convenient and comprehensible way of doing this. Researchers therefore became accustomed to working with such matrices and simply have continued to do so for reasons of hysteresis.

Some observers give a more substantive justification. To them, small income movements are negligible; income changes become important only when the change is large enough that the income recipient crosses a decile or quintile boundary. Figure 5-2 depicts the movement function implied by such measures. (In this figure, Y* is the income level where the next higher quantile begins and Y is the corresponding amount for the next lower quantile.) If such a discontinuous movement function seems odd, positional mobility may not be the right mobility concept for you.

Figure 5-2. *Quantiles Changed as a Function of Income Change*



But supposing that positional mobility is what you want to look at. How might it be measured? A natural benchmark is perfect positional immobility. In this case everybody keeps his or her previous position. (Of course, you will need to have decided whether you are measuring positions in terms of deciles, quintiles, or whatever. The results will be sensitive to that choice.) Perfect positional immobility means that the quantile transition matrix is an identity matrix, as in 5-1.

To gauge how far an actual quantile mobility matrix is from perfect positional immobility, some sort of metric is needed. The most frequently used is the "trace," more commonly called the "immobility ratio." This is the fraction of income recipients who remain in the same quantile as before.[16]

The immobility ratio varies with a number of factors. First, the longer the time period, the smaller the immobility ratio: for example, Richard Burkhauser and colleagues found immobility ratios of 67.6 percent and 69.3 percent comparing quintiles in a given year with those one year later for the United States and Germany, respectively, but when the base year and final year were five years apart, the corresponding immobility ratios were 50.4 percent and 53.4 percent.[17] Second, immobility ratios vary across countries. International comparisons across OECD countries show five- year immobility ratios that range from 0.43 in France to 0.53 in Denmark.[18] Finally, and obviously, the immobility ratio varies inversely with the number of quantiles.

The immobility ratio indicates the fraction of people who remain in or change quantiles. However, it gives no indication of how many quantiles the movers move. It seems reasonable to have a measure that is sensitive to that. One such is the mean number of quantiles moved, where zero is assigned to those who do not move at all. (Here too, the results may be sensitive to whether movement is being measured across quintiles, across deciles, or across ranks.)

A great many studies have been done using positional movement measures.[19] In evaluating the studies, you should bear in mind that they are thoroughly relative: a person can experience relative income mobility even if his or her own income does not change, provided that others' incomes change by enough that the person in question experiences a change in position.[20] In positional movement analysis, what matters is ones income position vis-a-vis others. But relative considerations might enter in a different kind of way, and that is by looking at ones income share vis-ä-vis others. I turn now to the concept and measurement of share movement.

Share Movement

Some people, even those who are thoroughgoing relativists, may not care which quintile, decile, or centile of the income distribution they are in—in fact, they may not even know. To the extent that people are relativists in their thinking, what they are much more likely to care about is their income as it compares with that of others. If your income rises by 50 percent but everyone else's rises by 100 percent, you may feel that you have lost ground. Share-movement measures would say that you have experienced downward income mobility, precisely because your share of the total has fallen.

Now that technology allows virtually instantaneous calculations of changes in income shares for samples of thousands or tens of thousands of income recipients, the practical advantage of using quantile mobility matrices rather than share-movement measures is gone. So the choice between these two types of relative mobility approaches is better made on conceptual grounds. Here is a simple self-test.

Suppose you start out in a given quantile of the income distribution. Let s say that your income remains stationary while incomes around you are rising, but the fall in position is sufficiently small that you remain in the same quantile that you were in. What is your mobility experience?

If you are a positional movement adherent, you find no income mobility: you were in the *x*th quantile before and you are in the *x*th quantile afterwards. "Stop," you say. "That's not me. I do experience mobility. I have moved down." If this is your answer, then there surely is a relativist element to your concept of mobility, and it reflects your changing income share, not your unchanged quantile.

If you think in this way, what should you do to measure mobility? Your own mobility is readily gauged by the change in your income share. The population analogue is an aggregation of the changes in income shares experienced by all the people in the population.

What is it about these changes that you might want to measure? Of course, you would not want to measure the mean change in income shares, because this is identically zero: all the gains in share enjoyed by some must be counterbalanced by losses of share suffered by others. Instead you might look at the standard deviation of changes in income shares, larger values signifying greater share movement. Or you might take the absolute value of the change in each person's income share and average these; here too, a larger value signifies greater share movement.

Thus far, in my review of the literature, I have not seen anyone who has actually measured the change in income shares. However, as noted earlier, the correlation between base income and final income is commonly used as a measure of immobility, and it can easily be shown that the correlation between incomes is the same as the correlation between income shares. So perhaps inadvertently, share movement has been measured.

Note that what share movement measures is flux, how much variation there is from base year to final year. Here, the aspect of flux that is being measured is income shares, whereas in the previous section, it was positions in the income distribution. If you are interested in flux but are more concerned about incomes than income shares or income positions, you might find the class of measures presented next more interesting.

Symmetric Income Movement

Imagine that you and I constitute a two-person society and that both of us experience a change in income. You experience a $ 1,000 income gain and I experience a $1,000 income loss. How much income movement has taken place?

If your answer is $2,000 total or $1,000 per capita, you have revealed much about the concept of mobility that you have in mind. For one, your concept of mobility is symmetric in the sense that gains and losses are both being treated nondirectionally. For another, your concept of mobility is dollar based in that you regard your gain and my loss as being $1,000 regardless of our respective base incomes.

Looking at mobility in this way has been justified formally by Gary Fields and Efe Ok.[21] Denoting base income and final income by x_I and Y_i respectively, their measure of total dollar movement in a population of size *n* is the sum of the absolute values of income changes ($2,000 in the above example):

$$(5-5a) \qquad d_n^{(1)}(x,y) = \sum_{j=1}^{n} |x_i - y_i|$$

Corresponding to this is a measure of per capita dollar movement in a population of size n ($1,000 in the above example):

$$(5-5b) \qquad m_n^{(1)}(x,y) = \frac{1}{n}\sum_{j=1}^{n} |x_i - y_i|$$

Finally, to gauge whether $1,000 per capita is a large or small amount of income change, $m^{(1)}{}_n$ can be expressed as a percentage of the mean base year income:

$$(5-5c) \qquad p_n^{(1)}(x,y) = \sum_{j=1}^{n} |x_i - y_i| / \sum_{i=1}^{n} x_i$$

These are measures of total symmetric dollar income movement ($d^{(1)}n$), per capita symmetric dollar income movement ($m^{(1)}n$), and percentage symmetric dollar income movement ($p^{(1)}n$), respectively. Together, these shall be denoted the F-O 1 set of measures.

To give some idea of the magnitudes involved, Gary Fields and colleagues calculated these measures using data from the U.S. Panel Study of Income Dynamics (PSID).[22] They found that in the United States between 1979 and 1986, m{l)n = $16,506 (in real 1982-84 dollars) and the mean in that year was $33,943 (in the same real dollars). Thus the average income change was 49 percent of the mean base year income. While this is a matter of interpretation, to me, a $p^{(1)}n$ value of 49 percent is a sign of considerable income flux.

Now, you may object to the perspective taken here by saying, "You haven't taken adequate account of base year income. A $ 1,000 income change for me is very important if my base year income is $1,000. It is much less important if my base year income is $1,000,000. What matters is by what percentage my income has changed." Gary Fields and Efe Ok have dealt with this concern by formulating measures of income movement that are sensitive to base year incomes.[23] This is achieved by working with the logs of base year and final year income rather than the incomes themselves.[24] The resultant measure of per capita relative income movement is

$$(5-6) \qquad m_n^{(2)}(x,y) = \tfrac{1}{n}\textstyle\sum_{j=1}^{n} |log x_i - log y_i|$$

The total measures $d^{(2)}n$ and percentage measures $p^{(2)}n$ are defined analogously. Together, these are called the F-O 2 set.

When this was applied empirically to the same PSID data as reported for 1979-86, Fields and colleagues found that $m^{(2)}n$ = .528. That is, the mean percentage income changes between these two years in the United States was approximately 52.8 percent. Again, it is a matter of perception, but I would say that this is indicative of a high degree of income flux.

One feature of F-O 1 and F-O 2 is that they are exactly decomposable into two parts, one that reflects income changes due to economic growth and the other that reflects income changes due to movements up and down, holding the mean constant.[25] In a growing economy, the breakdown of

$$d_n^{(1)}(x,y) = \sum_{j=1}^{n} |x_i - y_i|$$

into a growth component and a transfer component is given by

$$(5-7a) \qquad (x,y) = \sum_{j=1}^{n} |x_i - y_i| = G_n^{(1)}(x,y) + T_n^{(1)}(x,y),$$

where

$$(5-7b) \qquad G_n^{(1)}(x,y) = \sum_{i-1}^{n} y_i - \sum_{i=1}^{n} x_{i,}$$

and

$$(5-7c) \qquad T_n^{(1)}(x,y) = 2\left(\sum_{i \in L_N(x,y)}^{n} (x_i - y_i \right),$$

in which $L_n(x,y)$ denotes the set of people who lost income over time.[26]

Among the panel individuals in the PSID sample, the mean growth of family income was $1,121 (3.3 percent of average base year income) between 1979 and 1986. But the average income change $(m^{(1)}n)$ was $16,506 (in absolute value) between 1979 and 1986. From these figures, it would be expected that only a small fraction of total income mobility was due to income growth in the economy, and indeed that is what the decomposition in equation 5-7 shows: only 6.8 percent of symmetric income movement as gauged by $(m^{(1)}n)$ was due to growth in the 1980s. This is because an overwhelming portion of U.S. income mobility (in the sense of symmetric income movement) is accounted for by people moving up or down within the income distribution.

These decompositions tell us, in an accounting sense, why incomes change: but to what extent is it because the economy grew and individuals 'incomes grew along with it and to what extent is it because people moved up or down within a given structure? What has been measured and decomposed is income flux.

Now, it may be that you are not as interested in income flux as you are in the direction of change, in particular how many people are experiencing income gains and losses of what magnitude and which people in the population are the gainers and the losers. Once the distinction between income gains and income losses becomes important to you, you would do better to consider directional income movements.

Directional Income Movement

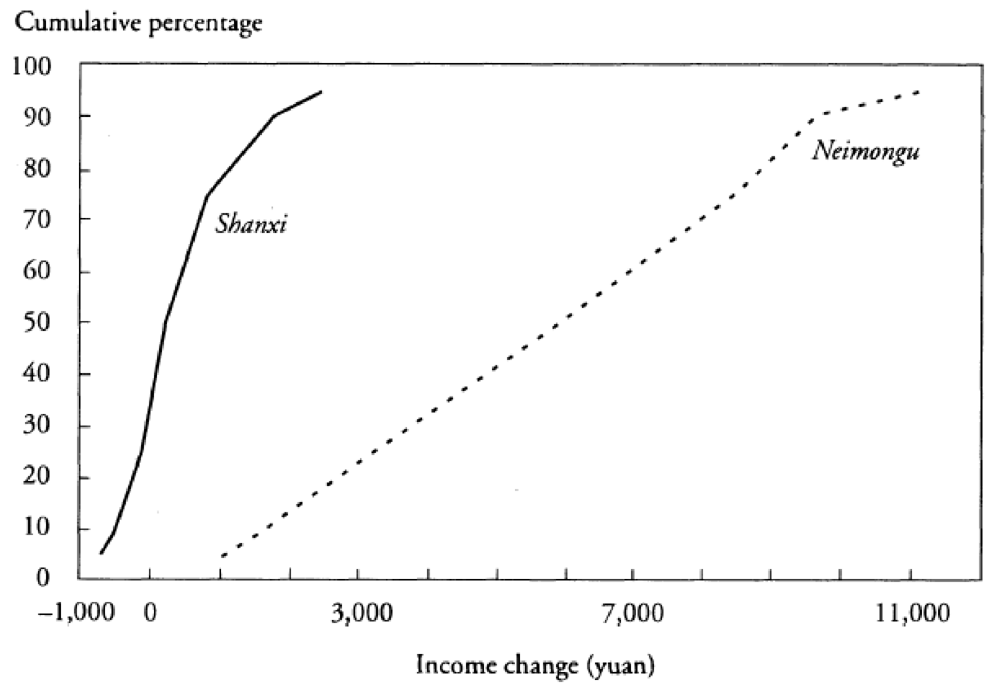
The three previous subsections were concerned with different aspects of income flux: positional movement, share movement, and nondirectional income movement, respectively. If directional income movements are of concern, you may find the measures discussed in this subsection to be of interest.

Several ad hoc directional indexes are in use, such as the fraction of upward or downward movers and the average amount gained by the winners or lost by the losers. Accompanying the use of these measures is a strong normative judgment, rarely stated explicitly: one income mobility situation is better than another when there is a larger fraction of upward movers, when the upward movers gain more on average, and when the downward movers lose less on average.

Whenever an index is used, one has (or should have) a nagging doubt about the robustness of the finding. Just as different poverty and inequality measures can give opposing ordinal judgments, the conclusion that one situation has better mobility than another may hinge on the choice of a particular mobility measure. In what follows, I will work solely with directional income movement. But even given that restriction, if I switch from one measure of directional income movement to another, there are conditions under which different measures will give very different conclusions. However, in other circumstances, one can determine that the same qualitative conclusion holds for a broad class of specific directional movement measures.

The technique I shall use is the familiar criterion of stochastic dominance, applied to directional income movements.[27] Suppose for now that

Figure 5-3. *Directional Income Movement in Two Chinese Provinces, 1978–89*

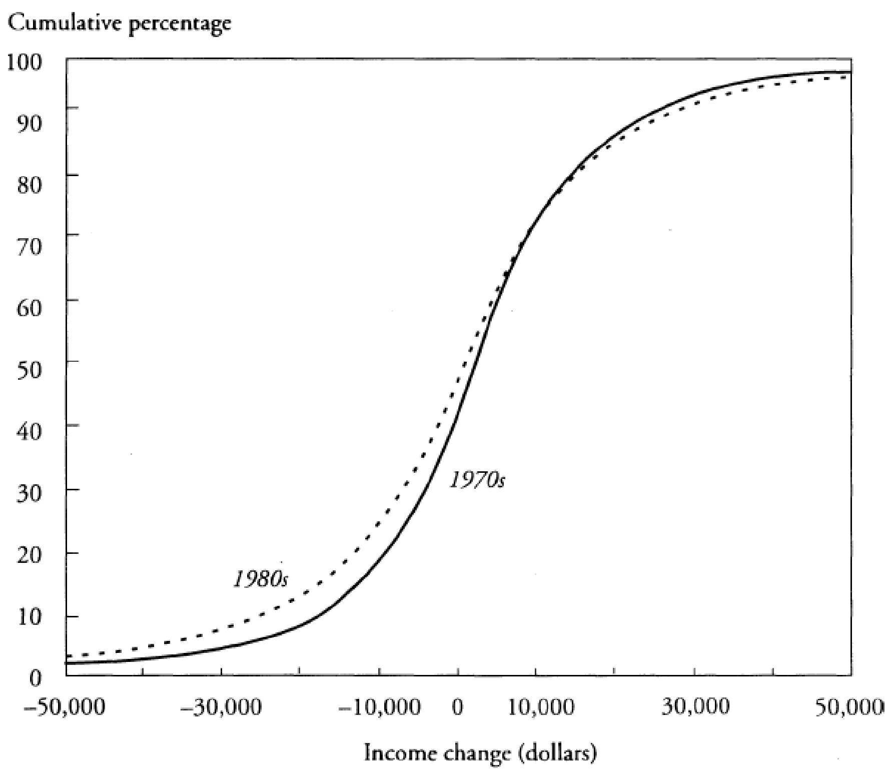


Source: Survey conducted by Chinese Academy of Preventive Medicine.

the measure of income movement is the individuals income change (measured, as always, in terms of real dollars). The population is then arrayed from most negative income change to most positive. What is needed for stochastic dominance analysis of income changes is the fraction of people experiencing income changes less than each possible amount. One distribution of income changes is said to stochastically dominate another if the percentage of people below any given income change amount is smaller in the first situation than in the second, or equivalently, if the income change cutoff for each given percentage grouping is higher in one distribution than another. Graphically, this means that a better distribution (better in the sense of stochastic dominance) is one that lies everywhere below or to the right of another. (One can look at it either way.)

Figure 5-3 shows the distribution of income changes in two provinces of China over an eleven-year period.[28] Neimongu registered rapid economic

Figure 5-4. *Distribution of Directional Income Movement in the United States, 1970s and 1980s*



Source: Author's calculations based on data from U.S. Panel Study of Income Dynamics.

growth, while Shanxi suffered an economic decline. The distribution of income changes in Neimongu stochastically dominates the income changes in Shanxi. In this sense economic growth may be said to have brought about a distribution of income changes that was better in Neimongu than in Shanxi.

When two directional income movement distributions are plotted, it is possible that neither dominates the other. This possibility is shown to be a reality in the crossing curves in figure 5-4. These data pertain to directional movement distributions for the United States in the 1970s and the 1980s using the same PSID data described earlier. The 1970s distribution dominates at the lower end of the change distribution, while the 1980s distribution dominates at the upper end.

This means that in the 1980s more people lost more dollars than in the 1970s, but those who gained the most in fact gained more dollars in the 1980s than in the 1970s.

Included in the preceding calculation is the judgment that income changes are most appropriately gauged in dollars of income movement. You may not like this practice of measuring income changes independently of base income amounts. For example, suppose that you gain $1,000 of income and I lose $1,000, but your income was twice as high as mine to start with. Your $1,000 income gain is only half as large in percentage terms as my $1,000 income loss. To assign a smaller absolute value change to your income gain as compared with my income loss, we might do as above and measure not changes in income but rather changes in log income for each person in the sample and then test for stochastic dominance between the log-income differences.

This is done in figure 5-5 for the PSID data. When log dollars are used rather than dollars, it is still the case that the distribution of income changes for the 1970s dominates the 1980s distribution for the smallest 90 percent of income changes. However, for the largest 10 percent of income changes, the two curves come together and effectively coincide the rest of the way. No crossing arises, and there is therefore a dominance result. Thus, when directional income movements are measured in log-dollar changes rather than dollar changes, the 1980s distribution of income movements is found to be dominated by the 1970s distribution.

This completes the presentation of the five different classes of mobility measures. We turn now to a comparison of them. First, we shall see how they differ in certain prototypic situations. Then, with that as the base we shall explore the axiomatic foundations of the different approaches.

Different Mobility Concepts in Certain Stylized Examples

Let x. represent the base-year income of the i th individual, and let x be the vector of x_i's s arrayed in some order, which without loss of generality, we will take to be in increasing order. Let y_i be the final-year income of the i th individual and y the vector of y_i's with individuals arrayed in the same order as they were in the x distribution. Then let $x \rightarrow$ y denote individual l's income going from x_l to y_l , individual 2's income going from x_2 to y_2, and so on.

Let us now examine how mobility changes in some stylized situations.

Example One: Constant Incomes

Suppose that over time, income remains constant for every single person in society. All of the mobility concepts considered above agree on one thing: when $x \rightarrow x$, there is no mobility.

Actually, this is the only case in which all of the different approaches agree. The differences between them in other cases can be elucidated by means of some other equally simple examples.

Example Two: A Rank-Preserving Equalization

Starting with a given income distribution *x*, let all incomes but two remain the same between base year and final year. For the two that change, let one person transfer a given sum of money to the other, such that their positions remain the same. (Whichever aspect of position is of interest— rank, decile, or quintile—hold that constant when the transfer is made.) What can be said about mobility?

- By the positional movement approach, there is no mobility, because nobody's position has changed.
- By the share movement approach, there is mobility, because the donor's income share shrank and the recipient's income share expanded.
- For the time-dependence approach, it matters what you measure. If you use the correlation coefficient on disaggregated data, you will get a less than perfect correlation between base year income and final year income, and therefore mobility takes place. However, using a quantile mobility matrix, both the donor and the recipient experience no change in quantile, and therefore no mobility takes place.

These differing judgments rendered in this example show two things. One, the positional movement approach and the share movement approach differ from each other. Two, the time-dependence approach implemented on aggregated data may give a different answer from the time-dependence approach implemented on disaggregated data.

Example Three: A Change in Ranks within Given Quantiles

In both of the previous examples the positional movement approach and the aggregate time-dependence approach gave the same answer. Can these ever disagree? The following example shows that the answer is yes.

As in example two, suppose that everyone's income remains the same except for two individuals, one of whom transfers a given sum of money to the other. But now, unlike example two, suppose that the amount transferred is large enough that the donor or the recipient or both change ranks in the income distribution (and of course, those whom they overtake change ranks as well). However, let these changes be small enough so that both donor and recipient remain in the same quantile (decile, quintile, and so on) of the income distribution.

Because the ranks have changed, there is positive positional movement. Yet because everyone remains in the same quantile as before, the quantile mobility matrix is an identity matrix, and therefore mobility is zero.

In this example the discrepancy arises for practical reasons as opposed to conceptual ones. If we had constructed an N *x* N transition matrix rather than a 10 *x* 10 or a 5 *x* 5 matrix, we would have seen positive mobility. It is because we do not do this in practice that no mobility is observed.

Example Four: Proportionate Income Change

Consider two different situations. In situation one, everyone's income is unchanged, so $x \rightarrow x$. In situation two, everyone's income becomes the same nonzero multiple of base year income λ, $\lambda > 0$, which is written as $x \rightarrow \lambda x$. How do the mobilities in $x \rightarrow x$ and $x \rightarrow \lambda x$, $\lambda \neq 1$ compare with each other?

It turns out to make a stunning difference which approach is adopted.

- By the time-dependence approach, $x \rightarrow x$ and $x \rightarrow \lambda x$ have the same mobility as each other—none.
- By the positional movement approach, $x \rightarrow x$ and $x \rightarrow \lambda x$ have the same mobility as each other—none.
- By the share movement approach, $x \rightarrow x$ and $x \rightarrow \lambda x$ have the same mobility as each other—none.
- By the symmetric income movement approach, $x \rightarrow x$ and $x \rightarrow \lambda x$ have different mobilities. The first, $x \rightarrow x$, has no mobility. The second, $x \rightarrow \lambda x$, has positive mobility. Furthermore, the more different λ is from 1, the more mobility there is.
- By the directional income movement approach, $x \rightarrow x$ and $x \rightarrow \lambda x$ have different mobilities. The first, $x \rightarrow x$, has no mobility. The second, $x \rightarrow \lambda x$, has nonzero mobility. The mobility that there is positive if $\lambda > 1$ and negative if $\lambda < 1$. Furthermore, the further λ is from 1, the more mobility there is.

These examples show clearly that the income movement approaches are fundamentally different from the others. This is because the income movement approaches say that income mobility takes place whenever someone's real income changes. Furthermore, the larger these income changes in absolute value, the more income movement there has been. The other approaches characterize movement differently.

Example Five: All Incomes Change by a Constant Dollar Amount

Suppose that everyone experiences an income change of $\$\alpha, \alpha \neq 0$, which is written as $x \rightarrow x + \alpha$. As was the case in example four, the different mobility approaches give very different answers.

- By the time-dependence approach, $x \rightarrow x + \alpha$ may or may not exhibit mobility: it depends on how time dependence is measured.
- Because no one's position in the income distribution changes, by the positional movement approach, $x \rightarrow x + \alpha$ has no mobility.
- Because the income shares of those below the mean expand and those above the mean contract, by the share movement approach, $x \rightarrow x + \alpha$ has positive mobility.
- Income movement approaches, whether directional or non-directional, will say that $x \rightarrow x + \alpha$ exhibits positive mobility. Furthermore, the larger α is, the more mobility there is.

Example Six: Manna from Heaven

According to the biblical story, when the Israelites were starving in the desert, a food called manna dropped from the sky. When manna drops from heaven on some people's houses but not on others', those who get the manna enjoy a gain in income, a gain in income share, and possibly a gain in position. Those who do not get the manna have no change in income, a loss in income share, and possibly a loss in position. What do you want to say about the mobility experiences of non-recipients in such a case? If your view is that they experienced downward mobility, you are a thoroughgoing relativist. At minimum you are a share-movement adherent, and you may be a positional movement adherent as well. But if, in your judgment, those who received no manna had no mobility, the share-movement and positional movement approaches are not for you, and the income movement approach may be more suitable.

Mobility Measures and Axioms

The previous section showed that it makes a difference in theory which approach is used to gauge mobility because the different approaches make fundamentally different judgments about certain key aspects of mobility. This implies that when you choose a mobility concept and a measure of it to take to data, you will want to be sure that it captures what for you is the right approach. We now study the crucial distinctions among the various measures.

Some Mobility Measures

Formally, an income mobility measure is a function *m(x,y)* defined on the domain of vectors $x = (x_1\ x_2 ...\ x_n)$ and final year incomes $y = (y_1 y_2 ....\ y_n)$ such that the ith income recipient occupies the same position in the two distributions.[29] There are a great many mobility measures. Among them are the following.

- The correlation coefficient between *x* and *y*.
- The rank correlation coefficient, that is, the correlation between income recipients' ranks in distribution *x* and distribution y.
- The quantile immobility ratio, defined as the fraction of income recipients who remain in the same quintile, decile, or ventile of the income distribution (also called the trace).
- Shorrocks's rigidity index, defined in the two-period case as

$$R = \frac{I(x+y)}{\frac{\left[I(x) + u_y I(y)\right]}{u_x + u_y}}$$

where I (.) is a particular scale-invariant inequality index.

- Fields and Oks log-dollar per capita measure,

$$(5-6a) \qquad m_n^{(2)}(x,y) = \frac{1}{n}\sum_{j=1}^{n} |log x_i - log y_i|$$

- Fields and Oks dollar per capita measure,

$$(5-6b) \qquad m_n^{(1)}(x,y) = \frac{1}{n}\sum_{j=1}^{n}|x_i - y_i|$$

Table 5-2 summarizes the performance of these six measures in the examples. Some noteworthy differences appear. First, when a rank-preserving equalization of income takes place, the rank correlation coefficient is unchanged. Thus, the rank correlation is a measure of positional movement. Second, when people change ranks within given quantiles, the quantile immobility ratio is unchanged. This too is a measure of positional movement, but the positions here are broad groups rather than individual ranks. Third, when all incomes change proportionately, only the Fields-Ok measures declare that mobility has taken place. In this sense the Fields-Ok indices measure income movement in ways that the other indices do not. Fourth, when all incomes change by a constant dollar amount, the Fields-Ok indices capture that movement. Shorrocks s rigidity index changes too, because relative inequality is changed by a uniform increase or decrease in everybody's income. Finally, when manna drops from heaven, the various measures behave very differently. Because incomes change, the two Fields-Ok measures indicate that there was mobility. The rank correlation and quantile immobility ratio register mobility if and only if people change ranks or

Table 5-2. *Mobility in Some Prototypic Situations*[a]

	Mobility if all incomes are unchanged	*Mobility if a rank-preserving equalization takes place*	*Mobility if some people change ranks within given quantiles*	*Mobility if all incomes change proportionately*	*Mobility if all incomes change by a constant dollar amount*	*Mobility if some people gain income, others do not*
Correlation coefficient between income in year *t* and *t* + 1	No	Yes	Yes	No	No	Yes
Rank correlation coefficient between income ranks in year *t* and *t* + 1	No	No	Yes	No	No	Yes, if and only if someone changes rank
Quantile immobility ratio (quintile, decile, or ventile)	No	Yes, if and only if someone changes quantile	No	No	No	Yes, if and only if someone changes quantile
Shorrocks's rigidity index, two-period case	No	Yes	Yes	No	Yes	Yes
Fields and Ok's log-dollar measure	No	Yes	Yes	Yes	Yes	Yes
Fields and Ok's dollar measure	No	Yes	Yes	Yes	Yes	Yes

a. All measures are defined in the text.

quantiles. The correlation coefficient and Shorrocks's rigidity index will signal mobility because of the nonuniformity of the income changes.

These differences in behavior among the measures highlight their fundamental differences. The two Fields-Ok measures are income-movement measures. The rank correlation coefficient and quantile immobility ratio are measures of positional movement. Shorrocks's R is a measure of share movement. Finally, the correlation coefficient is a measure both of share movement and origin independence. These distinctions will be useful to keep in mind when you decide how best to measure mobility given your conception of what income mobility is.

Axiomatic Foundations

The axiomatic approach to mobility measurement has been developed by a number of authors.[30] The axiomatic foundations of the different measures may be compared and contrasted.

The earlier discussion exhibited one element of commonality: when everyone's income is unchanged, there is no mobility. Anthony Shorrocks (1993) suggested a normalization axiom whereby mobility is at a minimum when all incomes are unchanged, and earlier suggested that a mobility measure should range from zero to one. Combined, these imply that when all incomes are unchanged, mobility is zero, which can be written as the normalization axiom: $m(x, x) = 0$. This axiom is hardly controversial: all the mobility measures presented in this chapter satisfy it, as do all others that I know about.

It is also essential to specify how mobility changes when people's incomes change. I consider four concepts.

The first concept is level sensitivity. By the normalization axiom, when everyone's income remains the same, there is no mobility. Such a situation can be thought of as fulfilling two conditions: income shares are maintained and the income level is maintained. Now keep just one of these: keep everyone's income share the same but change the income level. In some mobility conceptions, there is no mobility in such a case. Such a conception can be defined formally as the

normalized level-insensitivity axiom: $m\{x, \lambda x) = m(x, x) =0$ *for all* $\lambda > 0$.

Note carefully what this implies: not only is there no mobility in (1, 2, 3) → (1, 2, 3) but there is no mobility either in (1, 2, 3) → (2,4, 6). Let us call the measures that fulfill this axiom the (normalized) level-insensitive measures.

A second concept is relativity. A mobility concept is defined to be relative if multiplying everyone's base year and final year income by the same positive scalar leaves mobility unchanged:

relativity axiom: $m(\lambda x, \lambda y) = m(x, y)$ *for all* $\lambda, > 0$.

If you accept the relativity axiom, you are obligated to say that there is the same degree of mobility in the situation (1, 2, 3) → (2, 4, 6) as there is in the situation (2, 4, 6) → (4, 8, 12), and likewise the situation (1, 2, 3) → (6, 4, 2) has the same mobility as the situation (2, 4, 6) → (12, 8, 4).

The third concept is what has been called intertemporal scale invariance, although I prefer to call it strong relativity. If you think of mobility as a function of income shares, you can convert the vectors x and y to their corresponding share equivalents by multiplying x by $1/u_x$ and y by $1/u_y$, obtaining $s_x = x/u_x$ and $s_y = y/u_y$. You can then define your mobility measure as a function of these shares: $m(s_x , s_y)$. Now, if two different x, y pairs x^1, y^1 and x^2, y^2 have the same

s_x, s_y vectors, then m (x^1, y^1) = m (x^2,y^2). More generally, if you choose any multiple y > 0 of base year income and any other multiple λ > 0 of final year income and judge that mobility is necessarily the same in going from y *x* to λ y as in going from *x* to y, you have a strongly relative mobility notion, m{yx, λ y) = m(x, y) for all y, λ > 0. Combining strong relativity with the normalization axiom produces:

normalized strong relativity: m(yx, λ *y) = m(x, y) = 0 for all Y,* λ *> 0.*

Measures possessing the normalized strong relativity property work in a particular way. To illustrate, take the following mobility situations: (1, 2, 3) → (2, 4, 6) and (1, 2, 3) → (3, 6, 9). Strongly relative measures would say that these have the same mobility as each other; normalized strongly relative measures would say that the common amount of mobility they both have is zero.

The final concept is translation invariance. If a given amount α is added to or subtracted from everybody's base year and final year income, a translation-invariant mobility measure would declare the new situation to be as mobile as the original one:

translation-invariance axiom: m(x + α*, y +* α*) = m(x, y) for all* $\alpha > 0$*.*

These examples bear careful examination. They will help you decide whether you want to use a measure that is level insensitive, relative, strongly relative, or translation invariant. The choice depends, of course, on what you yourself understand the very concept of mobility to be.

Which Mobility Measures Satisfy Which Axioms?

Before choosing a mobility measure to take to data, you will want to know which measures satisfy which properties. Table 5-3 shows this. You are free to choose measures satisfying all four of the preceding normalized axioms, just three, just two, or just one.

Table 5-3 also deals with a technical point. It is obvious that (normalized) level insensitivity, m(λ x, *x*) = m(x, *x*) = 0 for all λ > 0, is a special case of (normalized) strong relativity, m (Yx, λ y) = m(x, y) = 0 for all y, λ > 0. For this axiom to be interesting, there must exist a reasonable mobility measure that is level insensitive but is not strongly relative. As shown in table 5-3, Shorrocks's R is precisely such a measure. This means that one is free to make independent choices among these different concepts and measures.

Finally, note that several mobility measures are both relative (in the sense of both relativity and strong relativity) and absolute (in the sense of translation invariance). This contrasts with the case of inequality measures, which may be relative or absolute but not both.[31]

Conclusions

This chapter began by emphasizing that income mobility is a generic concept connoting a wide range of ideas. Five such concepts were time dependence, positional movement, share movement, nondirectional income movement, and directional income movement.

I then went on to consider what these different mobility concepts would produce in certain simple examples. Only when everyone's income stays the same do they all give the same answer. In other cases they give different answers because they are measuring in light of fundamentally different concepts.

The next section compared measures and axioms. The correlation coefficient, it showed, is a measure of both share movement and time independence, Shorrocks's R is a measure of share movement, the rank correlation coefficient and quantile immobility ratio are measures of positional movement, and the Fields-Ok indices are measures of income movement. I also considered several possible axioms for income mobility, including axioms for normalization, level insensitivity, relativity, strong relativity, and translation invariance and showed that different mobility measures satisfy these different axioms.

Table 5-3. *Properties of Some Two-Period Mobility Measures*[a]

Properties	*Normalization* $m(x, x) = 0$	*Level insensitivity* $m(x, \lambda x) = m(x, x)$	*Relativity* $m(\lambda x, \lambda y) = m(x, y)$	*Strong relativity* $m(\gamma x, \lambda y) = m(x, y)$	*Translation invariance* $m(x + \alpha, y + \alpha) = m(x, y)$
Correlation coefficient between income in year *t* and *t* + 1	Yes	Yes	Yes	Yes	Yes
Rank correlation coefficient between income ranks in year *t* and *t* + 1	Yes	Yes	Yes	Yes	Yes
Quantile immobility ratio (quintile, decile, or ventile)	Yes	Yes	Yes	Yes	Yes
Shorrocks's rigidity index, two-period case	Yes	Yes	Yes	No	No
Fields and Ok's log-dollar measure	Yes	No	Yes	No	No
Fields and Ok's dollar measure	Yes	No	No	No	Yes

a. All measures are defined in the text.

The results of this chapter show that how mobility is conceived and how it is measured make an important difference.

References

Atkinson, Anthony B., Francis Bourguignon, and Christian Morrisson. 1992. *Empirical Studies of Earnings Mobility.* Chur, Switzerland: Harwood Academic.

Bartholomew, David J. 1982. *Stochastic Models for Social Processes.* Wiley.

Burkhauser, Richard V., Douglas Holtz-Eakin, and Stephen E. Rhody. 1997. "Labor Earnings Mobility and Inequality in the United States and Germany during the Growth Years of the 1980s." *International Economic Review* 38 (4): 775-94.

Chakravarty, John Satya, Bhaskar Dutta, and John A. Weymark. 1985. "Ethical Indices of Income Mobility." *Social Choice and Welfare* 2 (1): 1-21.

Cowell, Frank A. 1985. "Measures of Distributional Change: An Axiomatic Approach." *Review of Economic Studies* 52 (1): 135-51.

Danziger, Sheldon, and Peter Gottschalk. 1995. *America Unequal*. Russell Sage Foundation.

Eichhorn, W, and W. Gehrig. 1982. "Measurement of Inequality in Economics." In Bernhard H. Korte, ed., *Modem Applied Mathematics: Optimization and Operations Research*, pp. 658-93. Amsterdam: North Holland.

Fields, Gary S., and Efe A. Ok. 1996. "The Meaning and Measurement of Income Mobility." *Journal of Economic Theory* 71 (2): 349-77.

Fields, Gary S., and Efe A. Ok. Forthcoming. "Measuring Movement of Incomes." *Economica*.

Fields, Gary S., Jesse B. Leary, and Efe A. Ok. 1998. "Income Movement in the United States in the Seventies and Eighties." Working Paper. Cornell University and New York University.

Friedman, Milton, and Simon Kuznets. 1954. *Income for Independent Professional Practice*. New York: National Bureau of Economic Research.

Gottschalk, Peter, and Robert Moffitt. 1994. "The Growth of Earnings Instability in the U.S. Labor Market." *Brookings Papers on Economic Activity* 2: 217-72.

Hadar, Josef, and William R. Russell. 1969. "Rules for Ordering Uncertain Prospects." *American Economic Review* 59 (March): 25-34.

Hungerford, Thomas L. 1993. "U.S. Income Mobility in the Seventies and Eighties." *Review of Income and Wealth* 39 (4): 403-17.

Jarvis, Sarah, and Stephen P. Jenkins. 1996. "Changing Places: Income Mobility and Poverty Dynamics in Britain." Working Paper 96-19. Colchester: ESCR Research Centre on Micro- Social Change, University of Essex.

---------.1998. "How Much Income Mobility Is There in Britain?" *Economic Journal* 108 (447): 428-43.

King, Mervyn A. 1983. “An Index of Inequality: With Applications to Horizontal Equity and Social Mobility.” *Econometrica* 51 (1): 99-115.

McMurrer, Daniel P, and Isabel V. Sawhill. 1998. *Getting Ahead: Economic and Social Mobility in America.* Washington: Urban Institute Press.

Markandya, Anil. 1984. “The Welfare Measurement of Changes in Economic Mobility.” *Economica* 51 (204): 457-71.

Mitra, Tapan, and Efe A. Ok. 1998. “The Measurement of Income Mobility: A Partial Ordering Approach.” *Economic Theory* 12 (1): 77-102.

Moffitt, Robert A., and Peter Gottschalk. 1995. “Trends in the Autocovariance Structure of Earnings in the U.S.: 1969-1987.” Johns Hopkins University and Boston University.

Organization for Economic Cooperation and Development. 1996. “Earnings Inequality, Low-Paid Employment and Earnings Mobility.” *Employment Outlook* (July): 59-108.

---------. 1997. “Earnings Mobility: Taking a Longer Run View.” *Employment Outlook* (July): 27-62.

Sawhill, Isabel V, and Mark Condon. 1992. “Is U.S. Income Inequality Really Growing?” *Policy Bites* 13. Urban Institute.

Shorrocks, Anthony F. 1978. “The Measurement of Mobility.” *Econometrica* 46 (5): 1013-24.

---------. 1993. “On the Hart Measure of Income Mobility.” In M. Casson and J. Creedy, eds., *Industrial Concentration and Economic Inequality*, pp. 3-21. Brookfield, Vt.: Edward Elgar.