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THE EPICENTER OF THE NAGASAKI WEAPON:

A REANALYSIS OF AVAILABLE DATA WITH RECOMMENDED VALUES*

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ABSTRACT

A significant source of uncertainty in estimates of radiation dose for atomic-bomb survivors of Nagasaki and in correlations of radiation dose with medical effects observed in these survivors is due to unresolved discrepancies in the literature with respect to the location of the epicenter or burst point of the weapon. Available data on the epicenter of the weapon have been reanalyzed in this report using several different approaches. These appear to have resolved some of the major discrepancies found in the literature and to justify a recalculation of radiation doses for the atomic bomb survivors of Nagasaki. A recommended epicenter for recalculating the radiation doses is given in the report.

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INTRODUCTION

Techniques developed in liaison studies by the Atomic Bomb Casualty Commission (ABCC) in Japan and the Health Physics Division of the Oak Ridge National Laboratory (ORNL) are the basis of the tentative 1965 dosimetry (T65D) system of estimating radiation doses for survivors of Hiroshima and Nagasaki. This system was considered tentative mainly because of unresolved discrepancies in yields and locations of the burst point or epicenters of the two weapons. Yields used in the T65D system (Auxier et al., 1966) have since been corroborated by Lord Penny in a reanalysis of his original data (Penny et al., 1969). Based on these two independent assessments, the yield of the Hiroshima weapon has been established as being equivalent to 12.5 kilotons of TNT with a standard deviation of ± 2.5 kilotons and that of the Nagasaki weapon as 20 ± 2 kilotons of TNT.

The epicenters used in the T65D system were obtained by analyzing the extensive data of Shogo Nagaoka.* These data are from measurements of shadows, mostly on stone, resulting from the exfoliation of surfaces that were not shielded by intercepting objects against intense thermal radiation from the weapon. Nagaoka's measurements included 1178 of the shadows of vertical objects indicating the angle of azimuth toward the epicenter and 1172 of the shadows of horizontal objects indicating the elevation of the epicenter at 37 sites in Hiroshima. In Nagasaki, his thermal-shadow measurements at 23 sites included 139 indicating the angle of azimuth toward the epicenter and 201 indicating the angle of

*Some interesting sidelights on Shogo Nagaoka are given in Chapter 34 and notes to this chapter in a book by Toland (1970).

elevation of the epicenter. The analyses of data from these measurements and the results are discussed in the following ABCC technical reports: TR 12-59 (Arakawa and Nagaoka, 1959), TR 5-66 (Hubbell et al., 1966), and TR 1-68 (Milton and Shohoji, 1968). Detailed descriptions of the T65D system can be found in TR 1-68 and TR 7-67 (Noble, 1967).

Only small amounts of original data from measurements by other investigators have been located, but a number of epicenter* estimates have been found in reports written on early surveys of physical damage in the two cities by six Japanese teams, five American teams, and one British team. Since it was impossible to reconcile conflicting estimates without the original data, "best" epicenter locations were obtained by calculating weighted means from the original estimates, recalculated from original data when possible, for each of the two cities. Weighted means were used because these estimates were not of equal reliability. In some cases, the original investigators made estimates from measurements of radioactivity induced in the soil by neutrons from the weapons or from observations of the direction of fall of verticle objects knocked over by blast waves. These were less reliable than estimates of original investigators made from thermal-shadow measurements. In other cases, the results of original investigators were shown simply as points on small or inaccurate maps. Estimates based on these map locations were very unreliable in comparison to more detailed descriptions.

*Quantities often used in specifying the location of the epicenter are the hypocenter and height of burst. The hypocenter is the point on the ground beneath the epicenter, and the height of burst is the elevation of the epicenter.

The assignment of weights to the estimates collected from the literature and the techniques of calculating a weighted mean from the estimates are discussed in detail in TR 3-69 (Hubbell et al., 1969). In summarizing their results, it was stated:

> The differences in Hiroshima (15m between epicenters) do not seem to us sufficient to warrant any dose recalculations. In Nagasaki, however, the T65D coordinates were chosen on the basis of a calculation from one set of data which we feel could contain serious errors. That hypocenter is well outside the region where most other estimates of the hypocenter fall, and is unlikely to be close to the "best" point. Our epicenter point and the T65D point are 35m apart, so a recalculation of doses would seem to be justified in Nagasaki, since such a difference would make a change in estimated dose of the order of about 10 percent at 1000m from the hypocenter, and even more change closer to the hypocenter. In correlating medical effects this could be magnified to 20 percent when comparing cases in opposite directions with respect to the hypocenter.

Heights of burst of the two weapons were established very accurately in TR 3-69 with data obtained from the United States Armed Services.

The data from the thermal-shadow measurements of Nagaoka in TR 5-66 have been reanalyzed in this report using a different approach. This resolves a serious discrepancy with the hypocenter recommended by Hubbell et al. in TR 3-69 and provides justification for using a "best" epicenter in the T65D system obtained as a weighted mean of the data in TR 3-69. In recalculating a weighted mean from this data, some adjustments have been made in weights assigned to the various epicenter estimates from the literature. Reasons are discussed for making these adjustments and for eliminating several estimates from the weighted mean calculations. Recommendations are given in the report of a "best" hypocenter and height of burst for the Nagasaki weapon. This hypocenter is within 5m of the one recommended by Hubbell et al.(1969), but it is separated by more than 35m from the one currently used in the T65D system (Milton and Shohoji, 1968).

REANALYSIS OF DATA IN TR 3-69

Data used in the weighted mean calculations of a "best" epicenter for Nagasaki in TR 3-69 are shown in Table 1. The estimates of the height of burst (H) are given in meters, and the estimates of the hypocenter are specified in terms of an east-west coordinate (X) and a north-south coordinate (Y) on a U. S. Army Service map of Nagasaki from the A. M. S. L902 Series. This map is numbered 138,449 and dated 8-45. On this map, the coordinates of the hypocenter estimate from Watanabe et al. in Table 1 are X = 1293.61 and Y = 1065.95. The scale unit is 1000 yards or 914.4 meters. In practice, it is customary to omit the first two digits and to record this hypocenter estimate as X = 93.61 and Y = 65.95. This convention is followed here. As noted in Table 1, the estimates are not all independent. That is, two or more estimates of either the two hypocenter coordinates or height of burst were included from the same set of measurements or observations made by a team of original investigators. This results in excessive weighting of these sets of measurements or observations in the weighted mean calculations of TR 3-69.

Appropriate adjustments in the weights of the non-independent estimates can be made in several ways. These vary in complexity, but produce essentially the same results. The least complex is simply to average all equally plausible values from a single set of original measurements or observations. In obtaining independent estimates of

Table 1. A summary of epicenter values collected from the literature and appended weights.

SOURCE OF DATA	HYPOCENTER COORDINATES X_i	Y_i	APPENDED WEIGHT $W_i \times 10^{-2}$	HEIGHT BURST H_i	APPENDED WEIGHT $W_i \times 10^{-2}$	REFERENCES
1. Watanabe et al. (1953)	93.61	65.92	0.69	490	7.45	
2. Kimura and Tajima (1953)						
(a) *	93.59	65.89	8.00			
(b) **	93.61	65.95	8.00	490	17.90	
(c) ***	93.623	65.926	6.06			Hubbell <u>et</u> <u>al</u>. (1969)
3. Tanaka (1946)	93.638	65.942	4.48	490	11.20	Hubbell <u>et</u> <u>al</u>. (1969)
4. U.S. Strategic Bombing Survey (1947)	93.63	65.91	7.45	518	22.36	
5. British				533	4.47	U.S.Stragetic Bombing Survey (1947)
6. U.S. Navy Bureau of Yards and Docks (1945)	93.596	65.955	7.45			
7. Wright (1952)	93.61	65.92	0.75	490	8.94	
8. Nagaoka (1961)						
(a)	93.63	65.95	2.50	500	11.80	
(b)	93.66	65.92	2.50			
(c) ****	93.742	65.945	2.35	507	13.30	Shohoji (1966)
(d)	93.665	65.943	2.08	505	14.30	Shohoji (1966)
9. Weaponier's Fuse Setting				503	66.67	Hubbell <u>et</u> <u>al</u>. (1966)
10. Nakamura (1953)						
(a)	93.66	65.80	1.21	538	3.68	
(b)	93.664	65.852	1.23	571	3.68	Hubbell <u>et</u> <u>al</u>. (1969)

Table 1 (continued)

11.	Masuda, Sakata, and Nakane (1953)	93.59	65.90	1.73			
12.	Pace and Smith	93.653	66.034	1.00			
13.	Shinohara <u>et</u> <u>al</u>. (1953)	93.618	65.988	1.00			Pace and Smith (1946)
14.	Japanese Center *****	93.592	65.946	1.00			Pace and Smith (1946)
15.	Nagaoka and Hubbell						
	(a)	93.644	65.927	3.23	500	5.68	Hubbell <u>et</u> <u>al</u>. (1966)
	(b)	93.656	65.960	3.18	507	5.68	Milton and Shohoji (1968)
	(c)	93.653	65.967	3.90	497	3.41	
16.	Warren	93.64	65.95	1.15			Oughterson and Warren (1956)

* "40 east and 70m south of the intersection" description of Kimura and Tajima
** "Center of vacant lot" description of Kimura and Tajima
*** This estimate is from graphical presentation of data on a map in report by Kimura and Tajima
**** The X-coordinate is believed to be in error due to either a mistake or misprint in the memo
***** This estimate obtained from a map in Pace and Smith is possibly a reference to the results of Kimura and Tajima

Table 2 in this way, several non-equally plausible values were eliminated from the data of Table 1. These were 2(a) and 8(c). Value 8(c) was eliminated because the east-west coordinate (X) appears to be in serious error due to either a mistake or misprint in the cited reference.

The other value 2(a) eliminated from Table 1 is an estimate from a report by Kimura and Tajima (1953). Their thermal-shadow measurements were done very carefully in comparison to most other investigators, but they summarized their results as follows (see map in Figure 2):

> The hypocenter in Nagasaki was 40m east and 70m south of the intersection at the road leading up to Urakami Cathedral in Matsuyama-cho. In other words, approximately in the center of the vacant lot used for a tennis court.

As stated in TR 3-69, "the point 40m due east and 70m due south of the intersection was definitely outside the vacant lot, and was 50m away from its center." It was further stated that:

> The description "40m east and 70m south of the intersection" was inaccurate. The center of the vacant lot could be reached by going 40m from the intersection along the center of the road running in the northeasterly direction from the Matsuyama-cho intersection toward the Urakami Cathedral, then at this point turning right perpendicular to the road and going 70m southeasterly.

Estimate 2(b) based on the "center of the vacant lot" description is accurate in view of estimate 2(c) made by Hubbell _et al_. (1969) from data presented graphically on a map in the report by Kimura and Tajima and an interview of Yoshio Nishina* cited in TR 3-69. In an interview by the U. S. Strategic Bombing Survey team, Nishina quoted Kimura and Tajima as saying the epicenter in Nagasaki was "at the center of the tennis court at 170 Matsuyama-cho, at a height of 487 ± 20m." The height of burst was evidently rounded off to 490m in the report published by Kimura and Tajima (1953).

Although estimate 14 from a map in Pace and Smith (1946) is possibly a reference to the results of Kimura and Tajima, it was treated as an independent estimate in Table 2. If this is a reference to the results of Kimura and Tajima, it will not overweight their original measurements significantly due to the very small weight appended to the estimate by Hubbell et al. in TR 3-69.

Theoretical Considerations

Different criteria of optimality can be employed in seeking a "best" epicenter from the estimates in Table 2. A criterion that can be used in selecting a hypocenter is to find the value of (X,Y) which minimizes the function:

$$\Sigma W_i \left[(X - X_i)^2 + (Y - Y_i)^2 \right] . \tag{1}$$

This calls for minimizing the weighted sum of the squared Euclidean distances between the point (X,Y) and the data points (X_i,Y_i) with appended weights W_i representing the relative reliability of each of the data points. The minimizing values (X,Y) of this function are

*Extensive commentaries on Yoshio Nishina are given in books by Coffey (1970) and The Pacific War Research Society (1972). On August 8, 1945, he traveled to Hiroshima to investigate the damage at the request of General Seizo Arisue, Chief of Intelligence of the Imperial Army. Later, teams from Nishina's laboratory at the Physical and Chemical Research Institute in Tokyo were sent to investigate both the physical and biological effects of the atomic bombings in Hiroshima and Nagasaki.

Table 2. A summary of independent epicenter estimates for the Nagasaki weapon obtained by averaging equally plausible values calculated from an original set of measurements ~~of observation~~. or observations

SOURCE OF DATA	HYPOCENTER COORDINATES X_i	Y_i	APPENDED WEIGHT $W_i \times 10^{-2}$	HEIGHT OF BURST H_i	APPENDED WEIGHT $W_i \times 10^{-2}$	AVERAGED VALUES
1. Watanabe et al.	93.61	65.92	0.69	490	7.45	
2. Kimura and Tajima	93.616	65.938	7.03	490	17.90	2(b),2(c)
3. Tanaka	93.638	65.942	4.48	490	11.20	
4. U.S. Strategic Bombing Survey	93.63	65.91	7.45	518	22.36	
5. British				533	4.47	
6. U.S. Navy Bureau of Yards and Docks	93.596	65.955	7.45			
7. Wright	93.61	65.92	0.75	490	8.94	
8. Nagaoka	93.652	65.938	2.36	502	13.05	8(a),8(b),8(d)
9. Weaponier's Fuse Setting				503	66.67	
10. Nakamura	93.662	65.826	1.22	554	3.68	10(a),10(b)
11. Masuda, Sakata, and Nakane	93.59	65.90	1.73			
12. Pace and Smith	93.653	66.034	1.00			
13. Shinohara et al.	93.618	65.988	1.00			
14. Japanese Center	93.592	65.946	1.00			
15. Nagaoka and Hubbell	93.651	65.951	3.44	501	4.92	15(a),15(b),15(c)
16. Warren	93.64	65.95	1.15			

componentwise weighted means. For the east-west coordinates (X_i), the weighted mean is

$$\overline{X} = \Sigma\ W_i X_i / \Sigma\ W_i = \Sigma\ W_i^* X_i \tag{2}$$

where $W_i^* = W_i / \Sigma\ W_i$ and $\Sigma\ W_i^* = 1$. Calculation of $\overline{H}$ by eq. (2) is equivalent to finding the value of H that minimizes the function:

$$\Sigma\ W_i \left[(H - H_i)^2 \right] \tag{3}$$

where each H_i is an estimate of the height of burst with weight W_i.

A possible objection to using weighted and squared Euclidean distances in seeking a "best" epicenter is that remote outlying observations can have a significant effect on the results. This suggests minimizing the functions

$$\Sigma\ W_i \left[|X - X_i| + |Y - Y_i| \right]$$

and

$$\Sigma\ W_i \left[|H - H_i| \right] \tag{4}$$

as an alternative criterion. Minimizing values of X, Y, and H in this case are componentwise "weighted medians." The weighted median of the east-west coordinates (X_i) will be denoted as $\tilde{X}$. It can be calculated as follows: relabel each observation and its normalized weight W_i^* so that $X_1 \leq X_2 \leq \ldots \leq X_n$. Then calculate the partial sums of the normalized weights, W_1^*, $W_1^* + W_2^*$, $W_1^* + W_2^* + W_3^* \ldots$, until the sum first exceeds one half. If X_m is the observation for which the sum first exceeds one half, then $\tilde{X} = X_m$. If the weights are such that for X_m, the sum equals exactly one half, then every real number between X_m and X_{m+1} can serve as a weighted median. An example of the calculation of a weighted median is given in Appendix A.

Results

A summary is given in Table 3 of weighted mean and weighted median calculations using the data of Table 2. Some of the small differences between the weighted means and medians of Table 3 can be attributed to the fact that the median values, but not the mean values, must fall on a data point. Some may also be due to outlying points, but they do not have a significant effect on the results. Standard deviations σ of the weighted means in Table 3 were estimated by relationships of the type shown below for $\overline{X}$:

$$\hat{\sigma}^2_{\overline{X}} = \frac{\Sigma W_i^2 (\overline{X} - X_i)^2}{(\Sigma W_i)^2 - \Sigma W_i^2} = \frac{\Sigma W_i^{*2} (X - X_i)^2}{1 - \Sigma W_i^{*2}} . \qquad (5)$$

Justification for using this estimator of variance ($\hat{\sigma}^2$) of a weighted mean is discussed in Appendix B. No attempt was made at estimating the variance of weighted medians.

An alternative technique of calculating weighted means from the data in Table 2 is also considered in Appendix B. This technique was suggested by Charles Land (1969) of the ABCC. Results of these calculations are not significantly different from those given in Table 3. In particular, the agreement between the north-south ($\overline{Y}$) coordinates is excellent. This is fortunate since most survivors in Nagasaki were

Table 3. Hypocenter coordinates and heights of burst for the Nagasaki weapon calculated as a weighted median and weighted mean of the data in Table 2.

CRITERION	HYPOCENTER COORDINATES X ± σ	Y ± σ	HEIGHT OF BURST H ± σ
Weighted Median	93.630	65.938	503m
Weighted Mean	93.624 ± 0.007	65.936 ± 0.008	503m ± 4m

clustered north and south of the hypocenter as shown in Figure 5 of the report by Noble (1967) and Figure 14 of the report by Milton and Shohoji (1968). The accuracy in this coordinate is, therefore, of the utmost importance in correlating medical effects observed in these survivors with radiation dose.

REANALYSIS OF DATA IN TR 5-66

Data from the thermal-shadow measurements of Nagaoka in Nagasaki are available in Appendix 1 of TR 5-66. The hypocenter data can be represented by 23 triplets of numbers (X_i, Y_i, β_i) and the height of burst data by 27 triplets of numbers (X_i, Y_i, T_i). In these triplets of numbers, the X_i and Y_i are the east-west and north-south coordinates, respectively, of the i^{th} site of a set of thermal-shadow measurements on the U. S. Army Map Service map of Nagasaki discussed in the previous section. The coordinates of each measurement site were located very accurately on this map by Hubbell and Nagaoka using aerial photographs. Each β_i is the azimuthal angle from the measurement site toward the apparent epicenter measured from grid north on the map, and each T_i is the angle of elevation above the horizon at the measurement site to the apparent epicenter. The β_i's and T_i's of these triplets of numbers are the simple average of several thermal-shadow measurements at each site.

Theoretical Considerations

The perpendicular distance from an arbitary point (X,Y) to a line extending at an angle β_i from the site (X_i, Y_i) toward the apparent epicenter as shown in Figure 1 is

$$D_i = \overline{AC} - \overline{AB}$$

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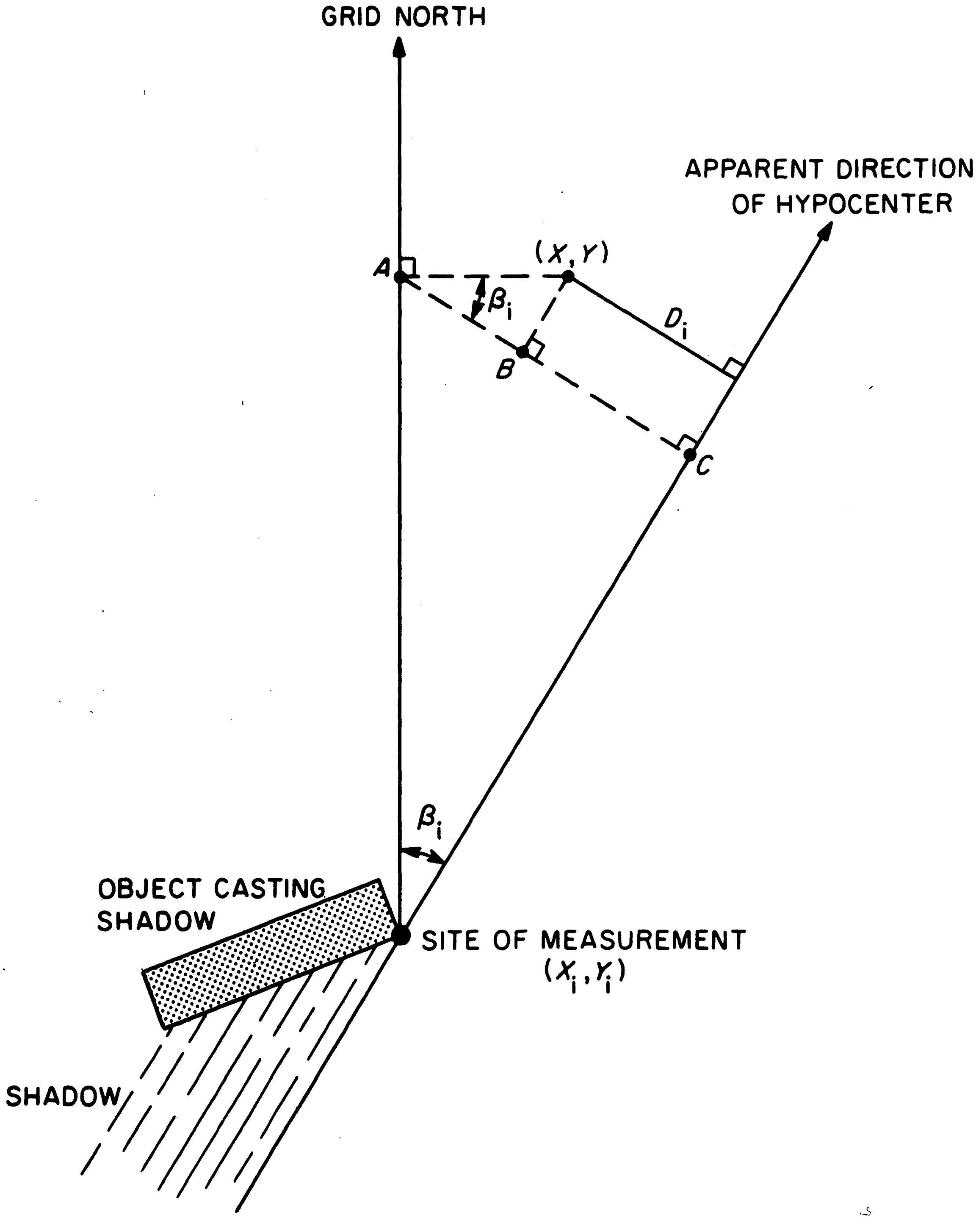

Figure 1. Diagram showing the calculation of the perpendicular distance D_i from an arbitary point (X,Y) to a line extending at an azimuthal angle β_i from the site (X_i,Y_i) toward the apparent epicenter.

or

$$D_i = (Y - Y_i) \sin \beta_i - (X - X_i) \cos \beta_i \quad . \tag{6}$$

In TR 5-66 (Hubbell et al., 1966), the hypocenter was selected as the point (X,Y) minimizing the function ΣD_i^2, and in TR 3-69 (Hubbell et al., 1969), it was selected as the point minimizing the sum of the squares of the perpendicular distances to lines extending from individual, rather than site averaged, thermal-shadow measurements. In TR 1-68 (Milton and Shohoji, 1968), $\Sigma n_i D_i^2$ was minimized where n_i is the number of measurements made at the i^{th} site. This hypocenter is currently being used in the T65D dose calculations for survivors of Nagasaki.

Justification given in TR 5-66 for minimizing the sum of the squares of the D_i's was mathematical tractability. As pointed out on pages 8 to 10 of ~~this~~ that report, the optimality of this technique is based on large sample (Gaussian) normal theory, and the underlying assumptions are not likely to be met by this data. It was pointed out in TR 5-66 that the technique "... gives excessive weight to observations with large errors..." and in TR 3-69 that Nagaoka's data for Nagasaki "...are very inaccurate, probably because the shadows measured were old (the stones were weathered before they were measured), and because the compass and clinometer were not accurate enough." This evidence suggests that a criterion which treats the measurements more uniformly would be appropriate. Such a criterion is to minimize $\Sigma|D_i|$.

Although a value of (X,Y) minimizing $\Sigma|D_i|$ cannot be calculated as easily as one for ΣD_i^2, it can be shown that the minimum must occur at an intersection of lines extending from two measurement sites. Thus,

$\Sigma |D_i|$ need only be evaluated at each point of intersection of two lines and the minimizing one selected as the hypocenter. This renders the problem feasible.

Results

The 23 triplets of numbers (X_i, Y_i, β_i) representing Nagaoka's hypocenter data for Nagasaki are shown graphically in Figure 2. For reasons given in TR 5-66, measurements at sites or points 14 and 15 were omitted from the hypocenter calculation. This leaves 21 measurements, and thus 210 points of intersection. The minimizing value of $\Sigma |D_i|$ from these 21 measurements occurs at the intersection of the lines extending from sites 10 and 21. Coordinates of this point of intersection indicated by the ⊗ on Figure 2 are X = 93.618 and Y = 65.943. These hypocenter coordinates and eq. (9) of TR 5-66 were used to calculate an apparent height of burst H_i indicated by each of the 27 triplets of numbers (X_i, Y_i, T_i) representing Nagaoka's angle of elevation measurements in Nagasaki. The height of burst calculated as a mean of these H_i's for purposes of comparison with other reports is 492m with a standard deviation of ± 75m. This height of burst is in good agreement with others obtained from the same data in TR 5-66 (Hubbell *et al.*, 1966), TR 1-68 (Milton and Shohoji, 1968), and TR 3-69 (Hubbell *et al.*, 1969). The hypocenters of these reports are, however, separated by large distances from those derived from the same data in this report. These distances range from 28m for the coordinates obtained in TR 5-66 by minimizing ΣD_i^2 to 39m for those obtained in TR 1-68 by minimizing $\Sigma n_i D_i^2$ where n_i is the number of measurements made at each of the different sites. It is felt for reasons discussed previously that

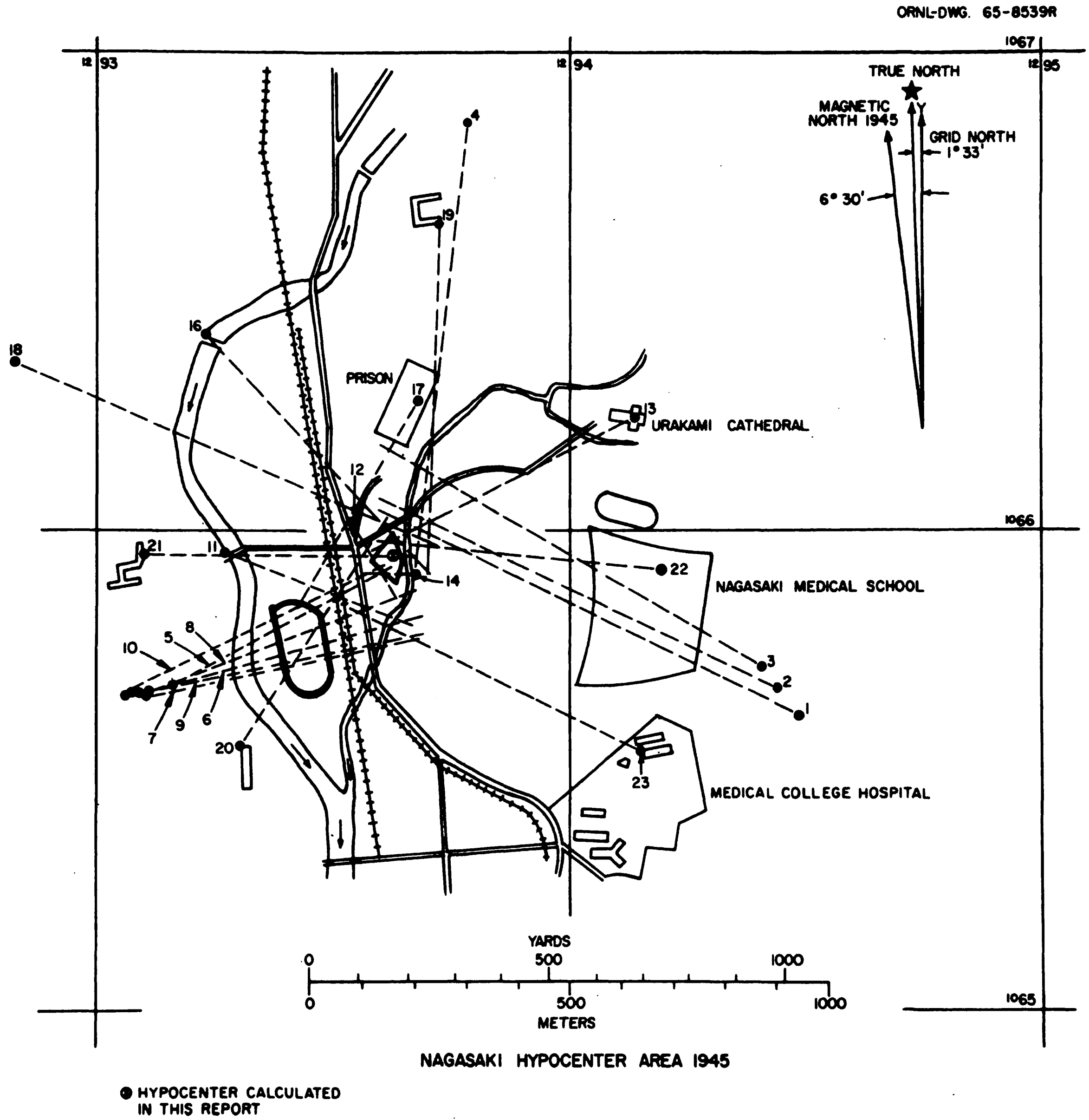

Figure 2. Schematic showing directions toward the apparent hypocenter from shadow measurements made at 23 sites in Nagasaki by Shogo Nagaoka. Point 13 on this map is at the Urakami Cathedral, the intersection of the road in Matsuyama-cho leading up to this cathedral is near point 12 on the map, and the irregular shaped lot used as a tennis court is near point 14.

minimizing $\Sigma|D_i|$ is better treatment of this hypocenter data. The hypocenter derived from the data of Nagaoka in this manner is within 10m of that calculated as a weighted mean of all hypocenter values collected from the literature (see Table 3).

CONCLUSIONS AND RECOMMENDATIONS

Based on our reanalysis of data in TR 5-66 of TR 3-69, a recalculation of T65D estimates of radiation dose to atomic-bomb survivors of Nagasaki would clearly be justified. Our recommended hypocenter coordinates and height of burst for the Nagasaki weapon are X = 93.624, Y = 65.936, and H = 503m. These were obtained as weighted means (see Table 3) in our reanalysis of the data in TR 3-69. Estimates of the uncertainty in this hypocenter location and height of burst are ± 22m and ± 10m, respectively, at the 99 percent confidence level. This recommended hypocenter is within 5m of the one recommended by Hubbell _et al_. (1969) in TR 3-69 and within 10m of the one obtained in our reanalysis of the thermal-shadow measurements of Nagaoka in TR 5-66, but is separated by 37m from the one currently being used in the T65D system (Milton and Shohoji, 1968). A recalculation would allow the T65D estimates for atomic-bomb survivors of both Hiroshima and Nagasaki exposed in the open or in typical Japanese residential structures and other lightly shielding structures to be finalized. The radiation transmission factors of Japanese-type house can be calculated very accurately with the "9-parameter formula" developed at ORNL (Cheka _et al_., 1965). A review of the transmission factors for concrete and other heavily shielding structures is recommended before finalizing the T65D estimates for all survivors.

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APPENDIX A

As examples of the calculation of a weighted median, consider the X_i and Y_i estimates of the coordinates of the hypocenter in Nagasaki and appended weights W_i given in Table 2. To calculate the weighted median of these estimates, they are ordered from smallest to largest with their associated weights as shown in Tables A1 and A2. The weights are normalized and partial sums of the normalized weights, W_1^*, $W_1^* + W_2^*$, $W_1^* + W_2^* + W_3^*$,... are then calculated. As shown in Table 1A, the partial sums of the normalized weights of the X_i's first exceeds one-half with the addition of the weight associated with the estimate from the U. S. Strategic Bombing Survey. Hence, the weighted median of these estimates is $\tilde{X} = 96.630$. The weighted median of the Y_i estimates in Table 2A is $\tilde{Y} = 65.938$.

TABLE A1. Calculation of a weighted median from the values for the east-west coordinate of the hypocenter in Nagasaki and the appended weights.

SOURCE OF DATA	EAST-WEST COORDINATE X_i	APPENDED WEIGHT $W_i \times 10^{-2}$	NORMALIZED WEIGHT W_i^*	PARTIAL SUMS OF NORMALIZED WEIGHTS
1. Masuda, Sakata, and Nakane	93.59	1.73	0.0425	0.0425
2. Japanese Center	93.592	1.00	0.0246	0.0671
3. U.S. Navy Bureau of Yards and Docks	93.596	7.45	0.1828	0.2499
4. Watanabe *et al*.	93.61	0.69	0.0169	0.2668
5. Wright	93.61	0.75	0.0184	0.2852
6. Kimura and Tajima	93.616	7.03	0.1725	0.4577
7. Shinohara *et al*.	93.618	1.00	0.0246	0.4823
8. U.S. Strategic Bombing Survey	93.63	7.45	0.1828	0.6651
9. Tanaka	93.638	4.48	0.1099	0.7750
10. Warren	93.64	1.15	0.0282	0.8032
11. Nagaoka and Hubbell	93.651	3.44	0.0844	0.8876
12. Nagaoka	93.652	2.36	0.0579	0.9455
13. Pace and Smith	93.653	1.00	0.0246	0.9701
14. Nakamura	93.662	1.22	0.0299	1.0000

TABLE A2. Calculation of a weighted median from the values for the north-south coordinate of the hypocenter in Nagasaki and the appended weights.

SOURCE OF DATA	NORTH-SOUTH COORDINATE Y_i	APPENDED WEIGHT $W_i \times 10^{-2}$	NORMALIZED WEIGHT W_i^*	PARTIAL SUMS OF NORMALIZED WEIGHTS
1. Nakamura	65.826	1.22	0.0299	0.0299
2. Masuda, Sakata, and Nakane	65.90	1.73	0.0425	0.0724
3. U.S. Strategic Bombing Survey	65.91	7.45	0.1828	0.2552
4. Watanabe et al.	65.92	0.69	0.0169	0.2721
5. Wright	65.92	0.75	0.0184	0.2905
6. Nagaoka	65.938	2.36	0.0579	0.3484
7. Kimura and Tajima	65.938	7.03	0.1725	0.5209
8. Tanaka	65.942	4.48	0.1099	0.6308
9. Japanese Center	65.946	1.00	0.0246	0.6554
10. Warren	65.95	1.15	0.0282	0.6836
11. Nagaoka and Hubbell	65.951	3.44	0.0844	0.7680
12. U.S. Navy Bureau of Yards and Docks	65.955	7.45	0.1828	0.9508
13. Shinohara et al.	65.988	1.00	0.0246	0.9754
14. Pace and Smith	66.034	1.00	0.0246	1.0000

APPENDIX B

The following discussion is restricted to the east-west (X) coordinates of the hypocenter, but identical considerations apply to the north-south (Y) coordinates of the hypocenter and the heights of burst (H). If the X_i have a common mean (i.e., the true hypocenter) and if there is a constant σ^2/U_i for i = 1,2,..., n, then the estimator which has a minimum variance among all unbiased linear functions of the X_i is

$$\overline{X} = \Sigma\, U_i X_i / \Sigma\, U_i \ . \qquad \text{(B1)}$$

That is, the optimal weighting of the observations is inversely proportional to their variances (Beers, 1957 and Land, 1969). With these assumptions, it can be shown that an unbiased estimator of the variance $\hat{\sigma}^2_{\overline{X}}$ of the weighted mean $\overline{X}$ is

$$\hat{\sigma}^2_{\overline{X}} = \frac{\Sigma\, U_i^2\, (\overline{X} - X_i)^2}{(\Sigma\, U_i)^2 - \Sigma\, U_i^2} \ . \qquad \text{(B2)}$$

This estimator of variance differs from the one given by Beers (1957) and Land (1969) by the small subtraction term $\Sigma\, U_i^2$ in the denominator.

In the text, the U_i of eqs. (1B) and (2B) were set equal to the appended weights W_i of Table 2. An alternative method of weighting the data in Table 2 has been suggested by Charles Land (1969) of the ABCC. In this method, the U_i in the above equations are replaced by the squares of the appended weights W_i^2. This is equivalent to choosing the hypocenter as the (X,Y) which minimizes the function

$$\Sigma\, W_i^2 \left[(X - X_i)^2 + (Y - Y_i)^2 \right] \qquad \text{(B3)}$$

and the height of burst as the H which minimizes the function

$$\Sigma\ W_i^2 \left[(H - H_i)^2 \right]. \qquad \text{(B4)}$$

The appropriate choice of weights $U_i = W_i$ or $U_i = W_i^2$ can be viewed as a decision as to which can be more reasonably interpreted as being inversely proportional to the variance of X_i. As seen in Table B1, the choice does not significantly influence either the estimates of the hypocenter coordinates and height of burst or their estimated variances.

Table B1. Hypocenter coordinates and heights of burst for the Nagasaki weapon calculated as a weighted mean using linear and squared appended weights.

METHOD OF WEIGHTING	HYPOCENTER COORDINATES $X \pm \sigma$	$Y \pm \sigma$	HEIGHT OF BURST $H \pm \sigma$
Linear Weights ($U_i = W_i$)	93.624 ± 0.007	65.936 ± 0.008	503m ± 4m
Squared Weights ($U_i = W_i^2$)	93.620 ± 0.008	65.936 ± 0.010	503m ± 3m

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