PART I.

GEOMETRY OF MACHINERY.

CHAPTER I.

ELEMENTARY RULES IN DESCRIPTIVE GEOMETRY.

SECTION I.—*General Explanations—Projection of Points and Lines.*

6. **Descriptive Geometry** is the art of representing solid figures upon a plane surface. In the present chapter are given some general elementary rules in that art, whose application is of very frequent occurrence in designing mechanism. The more special and complex rules will be given in the ensuing chapters, in treating of the particular kinds of mechanism to which those rules belong.*

7. By the **Projection of a Point** upon a given plane is meant the foot of a perpendicular let fall from the point on the plane. For example, in fig. 1, X Z Z X represents a plane (called a plane of projection), A a point, and A B a perpendicular let fall from the point on the plane; the foot, B, of that perpendicular is the projection of the point A on the plane X Z Z X.

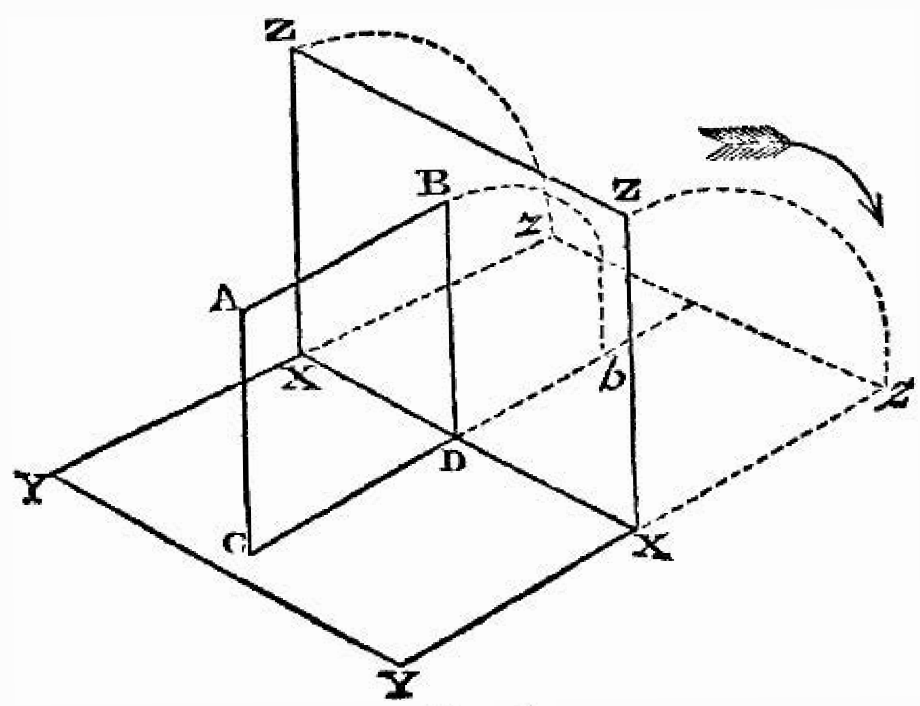

Fig. 1.

8. The **Position of a Point** is completely determined when its projections upon two planes not parallel to each other are known. In descriptive geometry a pair of planes of projection at right angles to each other are used; and in general one of these is vertical and the other horizontal. Thus, in fig. 1,

* For complete information on the subject of descriptive geometry, reference may be made to the works of Monge and Hachette in French, and of Dr. Woolley in English.

X Z Z X is the vertical plane of projection, and X Y Y X the horizontal plane of projection; B is the vertical projection, and C the horizontal projection of the point A; and those two projections completely determine the position of the point A; for no other point can have the same pair of projections.

9. The **Axis of Projection** is the line X X, in which the two planes of projection cut each other.

10. **Rabatment.**—When the two projections of an object are shown in one drawing, it is convenient to represent to the mind that the following process has been performed:—Suppose that the vertical plane of projection is hinged to the horizontal plane at the axis X X, and that after the projection of the object on the vertical plane has been made, that plane is turned about that axis until it lies flat in the position X *z z* X, so as to be continuous with the horizontal plane: thus bringing down the projection B to *b*. This process is called the *rabatment* of the vertical plane upon the horizontal plane (to use a term borrowed from the French "*rabattement*" by Dr. Woolley). The two points C and *b* are in one straight line perpendicular to X X. The process of rabatment may be conceived also to be performed upon a plane in any position when a figure contained in that plane is shown in its true dimensions on one of the planes of projection.

11. **Projections of Lines.**—The projection of a line is a line containing the projections of all the points of the projected line. The projection of a straight line perpendicular to the plane of projection is a point; for example, the projection on the vertical plane, X Z Z X (fig. 1), of the straight line A B, perpendicular to that plane, is the point B. The projection of a straight line in any other position relatively to the plane of projection is a straight line. If the projected line is parallel to the plane of projection, its projection is parallel and equal to the projected line itself; thus the projection on the horizontal plane, X Y Y X, of the horizontal straight line A B, is the parallel and equal line C D. If the projected line is oblique to the plane of projection, the projection is shorter than the original line.

The projections, on the same plane, of parallel and equal straight lines are parallel and equal. The projections, on the same plane, of parallel lines bearing given proportions to each other are parallel lines bearing the same proportions to each other. When the plane of a plane curved line is perpendicular to a plane of projection, the projection of the curve on this plane is a straight line, being the intersection of the plane of the curve with the plane of projection. When the plane of the projected curve is parallel to a plane of projection, the projection of the curve on this plane is similar and equal to the original curve. In all other cases, it follows from the preservation of the proportions of a set of parallel ordinates amongst their projections, that the projections of a plane curve of a given algebraical order are curves of the same algebraical order. The projections of a circle are ellipses; the projections of a parabola of a given order are parabolas of the same order. The projections of a straight tangent to a plane curve are straight tangents to the projections of that curve. The projections of a point of contrary flexure in a plane curve are points of contrary flexure in its projections.

12. **Drawings of a Machine.**—A third plane of projection, perpendicular to the first two, is often employed, not as being mathematically necessary, but as being more convenient for the representation of certain lines. Thus, for example, the drawings of a machine usually consist of three projections on three planes at right angles to each other; one horizontal (*the plan*), and the other two vertical (*the elevations*). Any two of those projections are mathematically sufficient to show the whole dimensions and figure of the machine; and from any two the third can be constructed; but it is convenient, for purposes of measurement, calculation, and construction, to have the whole three projections.

In the application of the rules about to be stated in the sequel of this Section, the two planes of projection may be held to represent any two of the three views of a machine; and the axis of projection will then have the directions stated in the following table:—

Views Represented by the Planes of Projection.	Direction of the Axis of Projection.
Longitudinal Elevation and Plan,........	Longitudinal.
Longitudinal and Transverse Elevations,....	Vertical.
Plan and Transverse Elevation,..............	Transverse.

Projections of figures upon planes oblique to the principal planes of projection may be used for special purposes.

SECTION II.—*Traces of Lines and Surfaces.*

13. By a **Trace** is meant the intersection of a line with a surface, or of one surface with another. The trace of a line upon a surface is a point; the trace of one surface upon another is a line.

In descriptive geometry the term *traces* is specially employed, when not otherwise specified, to denote the intersections of a line or surface with the planes of projection.

14. **Traces of a Straight Line.**—The position of a *straight line* is completely determined when its traces are known. For example, the straight line A C, in fig. 2, has its position completely determined by its traces, A and C, being the points where it cuts the

two planes of projection. The *rabatment* of the trace C is represented by *c*.

A straight line parallel to one of the planes of projection has only one trace, being the point where it cuts the other plane of projection.

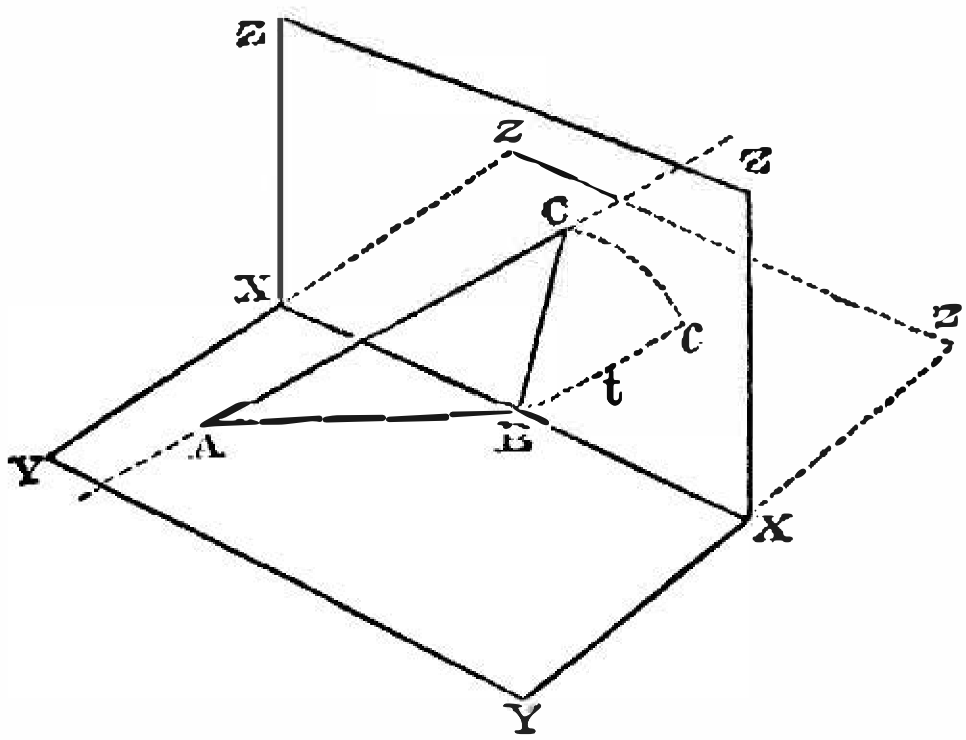

Fig. 2.

A straight line parallel to the axis of projection has no traces.

15. The **Traces of a Plane** are straight lines which (unless they are both parallel to the axis of projection) meet that axis in one point. The position of a plane is completely determined when its traces are known. For example, the plane A B C, in fig. 2, has its position completely determined by its traces, B A and B C.

A plane perpendicular to one of the planes of projection has its trace on the other plane of projection perpendicular to the axis of projection. A plane perpendicular to both planes of projection has for its traces two lines perpendicular to the axis. Thus, in fig. 1, page 3, the traces of the plane A B C D are D C and D B, both perpendicular to X X.

A plane parallel to one of the planes of projection has a trace on the other plane of projection only, being a straight line parallel to X X.

If a plane traverses a straight line, the traces of the plane traverse the traces of the line.

Section III.—*Rules Relating to Straight Lines.*

16. **General Explanations.**—In each of the figures illustrating the following rules the axis of projection is represented by X X; and in general the part of the figure above that line represents the rabatment of the vertical plane of projection, and the part below, the horizontal plane of projection. The projections of points on the horizontal plane are in general marked with capital letters, and the projections on the vertical plane with small letters.

17. **Given** (in fig. 3), **the Traces, A, *b*, of a Straight Line, to Draw its Projections.**—From A and *b* let fall A *a* and *b* B perpendicular to X X. Then *a* will be the vertical projection of the

trace A, and B the horizontal projection of the trace *b*. Join *a b*, A B; these will be the projections required.

(It may here be remarked, that *a* A and *a b* are the traces of a plane traversing the given line, and perpendicular to the vertical plane of projection; and that B A and B *b* are the traces of a plane traversing the given line, and perpendicular to the horizontal plane of projection.)

18. **Given** (in fig. 3), **the Projections**, A B, *a b*, **of a Straight Line, to Find its Traces.**—From *a* and B, where the given projections meet the axis, draw *a* A and B *b* perpendicular to X X, cutting the given projections in A and *b* respectively. These points will be the required traces.

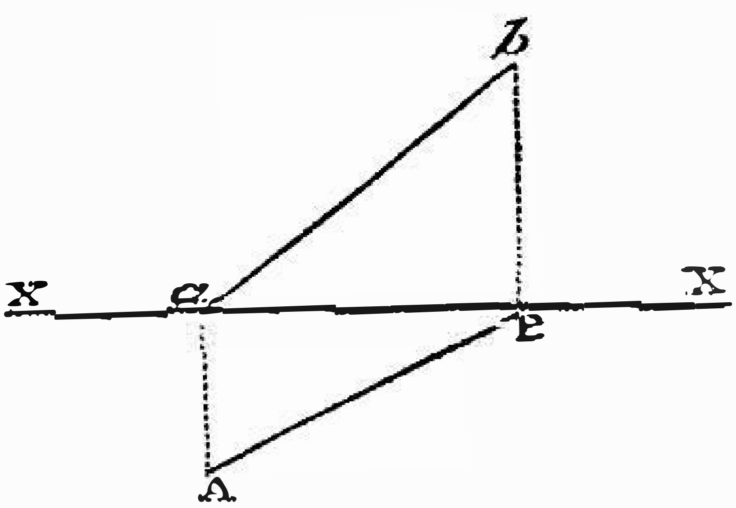

Fig. 3.

19. **Given, the Projections of two Points**, A, *a*, B, *b* (fig. 4), **to Measure the Distance between them.**—Join *a b*, A B; these will be the projections of the straight line to be measured. Through either end of either of those projections (as *b*) draw *d b e* parallel to X X; through the other end, *a*, of the same projection, draw *a d* perpendicular to X X, cutting *d b e* in *d*; make *d e* = the other projection, A B; join *a e*; this will be the length required.

The same operation may be performed on the other plane of projection.

20. **Given** (in fig. 4), **the Projections**, A, *a*, **of a Point, and the Projections**, A B, *a b*, **of a Straight Line through that Point, to Lay off a given Distance from the Point along the Line.** — In any convenient position, draw a straight line, B *b*, perpendicular to X X, meeting the projections of the given straight line in two points, B, *b*, which are the projections of one point; then perform the construction described in Article 19, so as to find *a e*. From the point *a*, in the line *a e*, lay off the given distance, *a f*. Through *f* draw *f h* parallel to X X, cutting *a b* in *g*; *a g* will be one of the projections of the given distance. Then draw *g* G perpen-

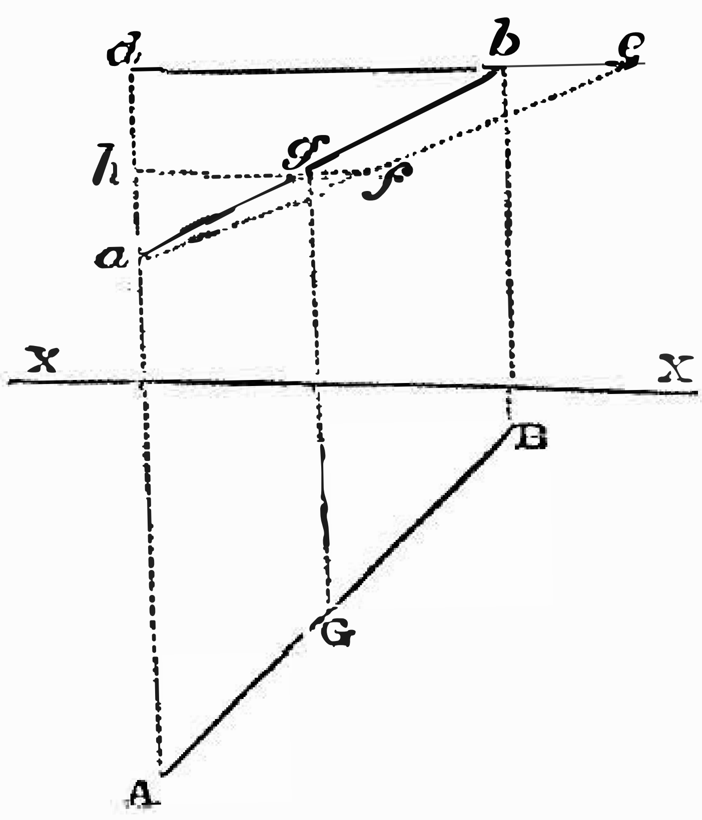

Fig. 4.

dicular to X X, cutting A B in G; A G will be the other projection of the given distance.

Another method of finding G is to lay off A G = *h f*.

21. **Given** (in fig. 4), **the Projections**, *a b*, A B, **of a Straight Line, to Find the Angle which it makes with One of the Planes of Projection** (for example, the horizontal plane).—Perform the construction described in Article 19; then *d e a* is the angle made by the given line with the horizontal plane. The same construction performed in the horizontal plane of projection will give the angle made by the given line with the vertical plane of projection.

22. **Given** (in fig. 5), **the Projections**, *a b* **and** A B, *a c* **and** A C, **of a Pair of Straight Lines which Intersect each other in the Point whose Projections are** *a*, A, **to find the Angle between those Lines.**—In either of the planes of projection (for example, the vertical

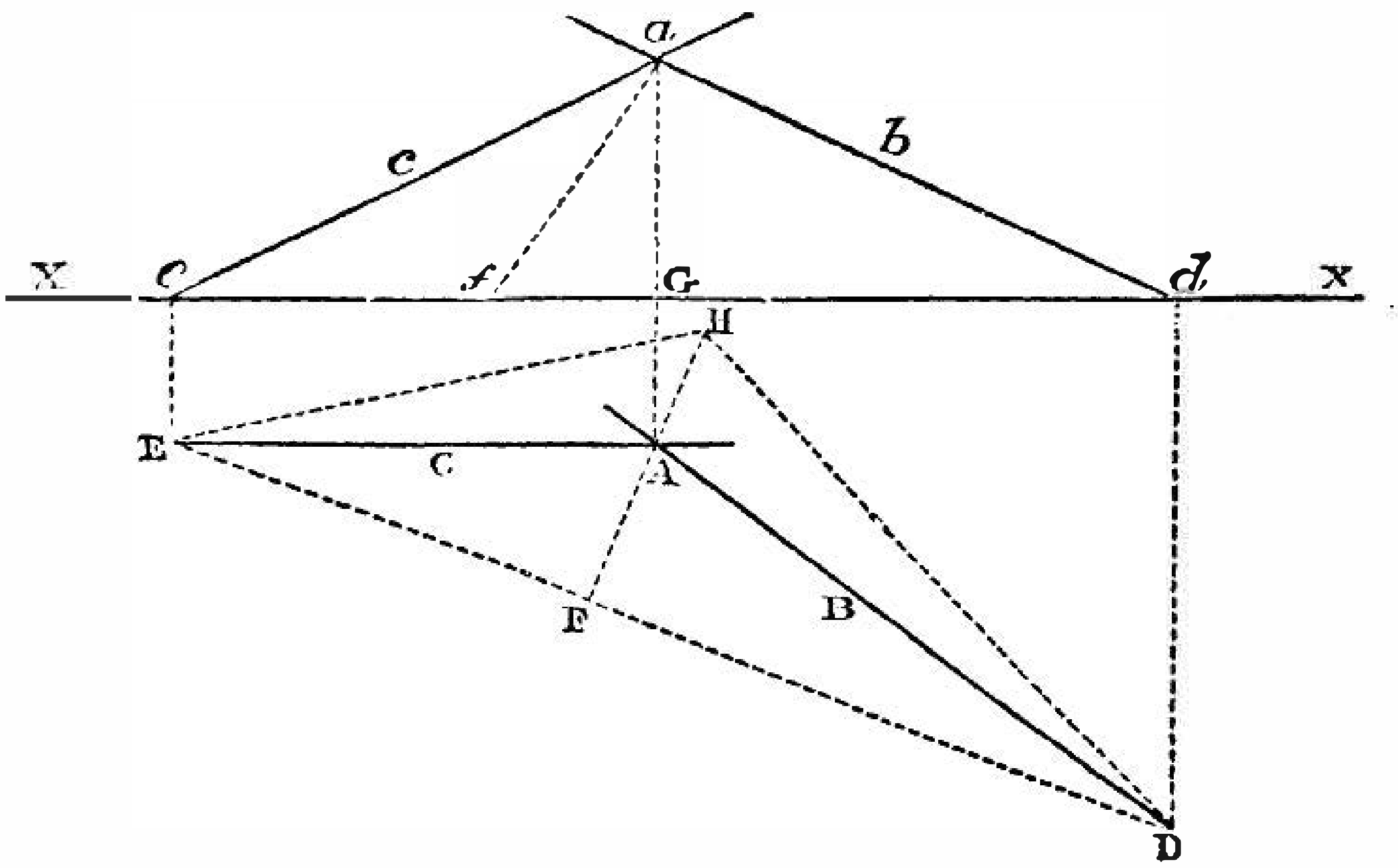

Fig. 5.

plane) find the points, *d*, *e*, where the projections of the given line cut the axis X X; these will be also the vertical projections of the horizontal traces of the lines. Through *e* and *d* draw *e* E, *d* D, perpendicular to X X, cutting A C and A B in E and D respectively; these points will be the horizontal traces of the lines. Join D E (which will be the horizontal trace of the plane containing the lines), and on it let fall the perpendicular F A. Join A *a* (which of course is perpendicular to X X); let it cut X X in G. Make G *f* = A F, and join *a f*. In F A produced, take F H = *a f*; join H E, H D; E H D will be the angle required.

REMARK.—The triangle E H D is the *rabatment* upon the horizontal plane of the triangle whose projections are E A D and *e a d*.

22 A. **Given** (in fig. 5), **the Projections,** *a b* **and** A B, **of a Straight Line, and One Trace** (say D E) **of a Plane Traversing that Line, to Find the Projections of a Straight Line which shall, at a given Point,** *a*, A, **make a given Angle in the given Plane with the given Straight Line.**—Join A *a*, which will be perpendicular to X X. On D E let fall the perpendicular A F. In X X take G *f* = A F; join *a f*. In F A produced take F H = *a f*. Join H D; and draw H E, making D H E = the given angle, and cutting D E in E. From E let fall E *e* perpendicular to X X; join A E, *a e*; these will be the projections of the line required.

SECTION IV.—*Rules Relating to Planes.*

23. **Given, the Projections of Three Points, to draw the Traces of a Plane Passing through them.**—Draw straight lines from one of the points to the two others; find, by Article 18, the traces of those straight lines; through those traces, on the two planes of projection respectively, draw two straight lines; these will be the traces required.

23 A. **Given, the Projections of Two Points and of a Straight Line, to Draw the Traces of a Plane Traversing the Points and Parallel to the Line.**—Through the projections of either of the given points draw straight lines parallel respectively to the corresponding projections of the given line; these will be the projections of a straight line through the given point, parallel to the given straight line; then, by Article 23, find the traces of a plane traversing the new straight line and the other given point.

24 **Given** (in fig. 6), **the Traces of a Plane,** B A, B C, **to Find the Angle which it makes with one of the Planes of Projection** (for example, the vertical plane).—From any convenient point, A, in the horizontal trace let fall A D perpendicular to X X. From D let fall D *e* perpendicular to B C. In D B lay off D *f* = D *e*. Join *f* A (this will represent the perpendicular distance from B C of the point whose projections are D and A). A *f* D will be the angle required.

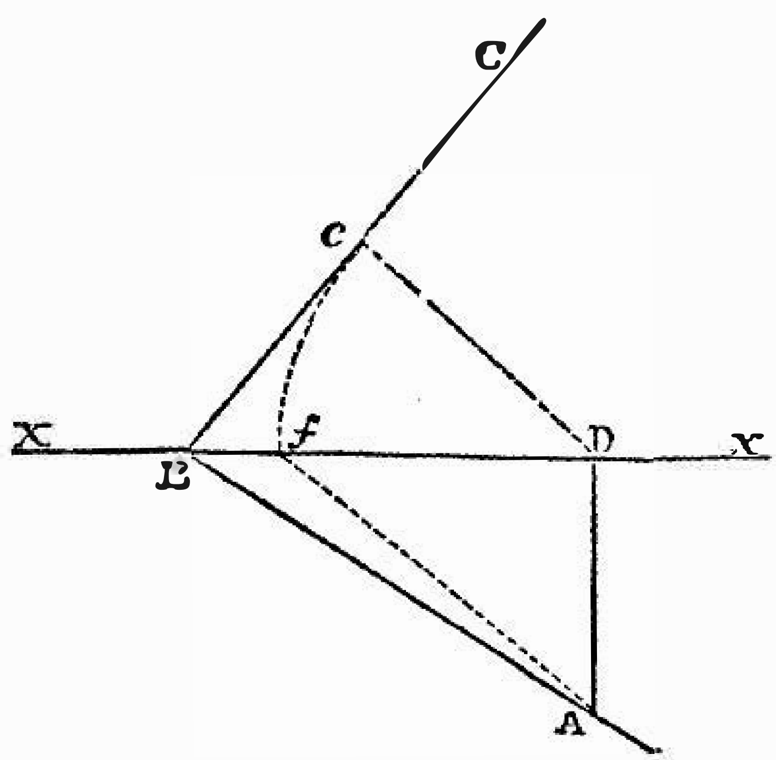

Fig. 6.

25. **Given** (in fig. 7), **the Traces of a Plane,** B A, B C, **to Find the Angle which it makes with the Axis of Projection,** X X.—In either of the two traces (for

example, B A) take any convenient point, A, from which let fall A D perpendicular to X X; and on B D as a diameter describe a circle. From D let fall perpendiculars, D *e*, D F, on the two given traces. From the point *e*, thus found on the opposite trace to that on which the point A was assumed, let fall *e* E perpendicular to X X; join E A, cutting D F in G. From G draw G H perpendicular to X X, cutting the circle in H; D B H will be the required angle.

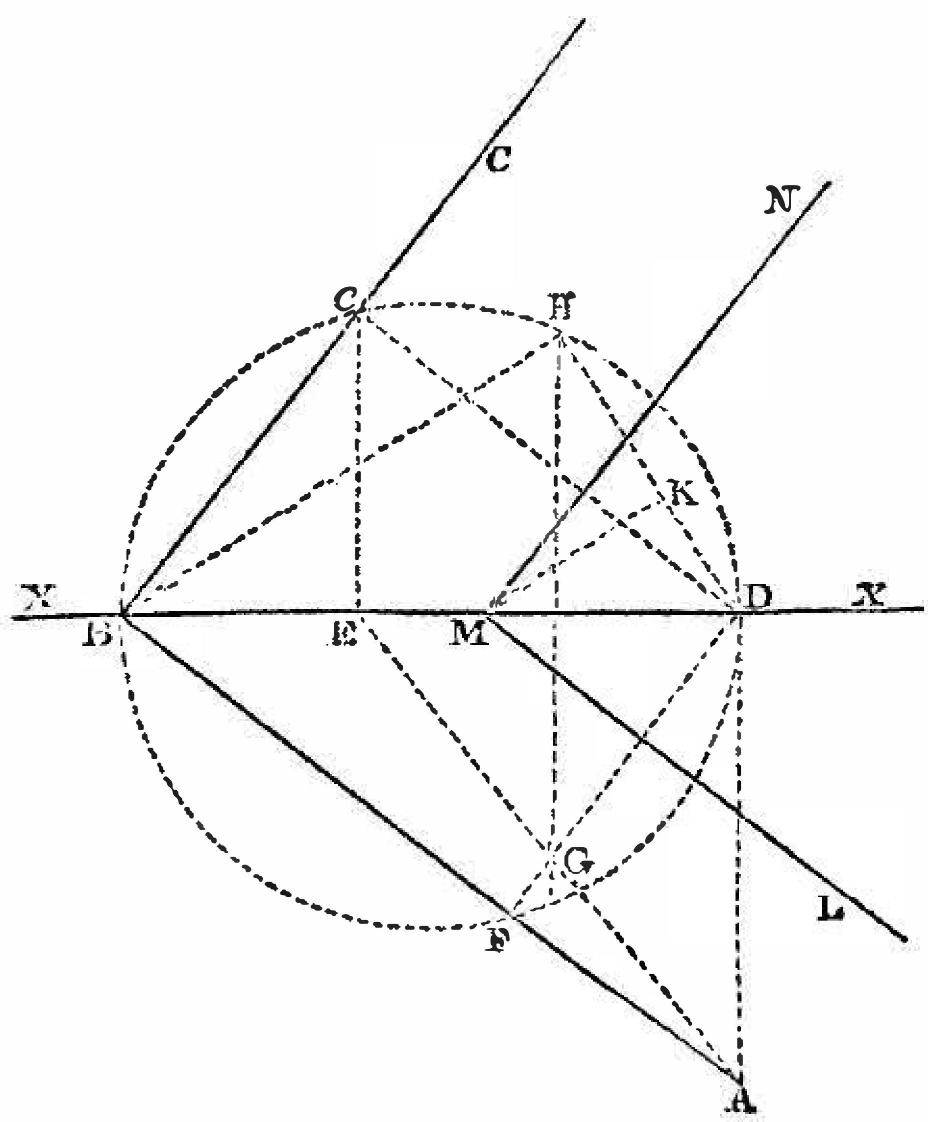

Fig. 7.

26. **Given** (in fig. 7), **the Traces of a Plane,** B A, B C, **to Draw the Traces of another Plane which shall be Parallel to the given Plane, and at a given Perpendicular Distance from it in either Direction.**—Complete the construction described in Article 25. Join D H (this represents the perpendicular distance of the point D in the axis from the given plane); then from H, along H D (or along D H produced, according to the direction in which the new plane is to lie), lay off the given perpendicular distance between the planes, H K. From K draw K M parallel to H B, cutting X X in M. From M draw M N parallel to B C, and M L parallel to B A; these will be the traces of the plane required.

Or otherwise:—Complete the construction described in Article 24 (see fig. 8). A *f* is the rabatment of the intersection of the given plane with a plane, A D *e*, perpendicular to the vertical trace B C. Through A draw A M perpendicular to A *f*, and make A M equal to the given distance between the planes; draw M N parallel to A *f*, cutting X X in N. In D *e* produced take D O equal to D N. O is a point in the trace of the plane required. Through O draw O P parallel to B C, cutting X X in P; and through P draw P Q parallel to B A. O P Q is the plane required.

27. **Given** (in fig. 9), **the Traces of Two Planes,** C A *d* **and** C B *d*, **to Draw the Projections of their Line of Intersection.**—The traces of

the required line are C and *d*, where the traces of the given planes intersect. From those points respectively let fall C *c* and

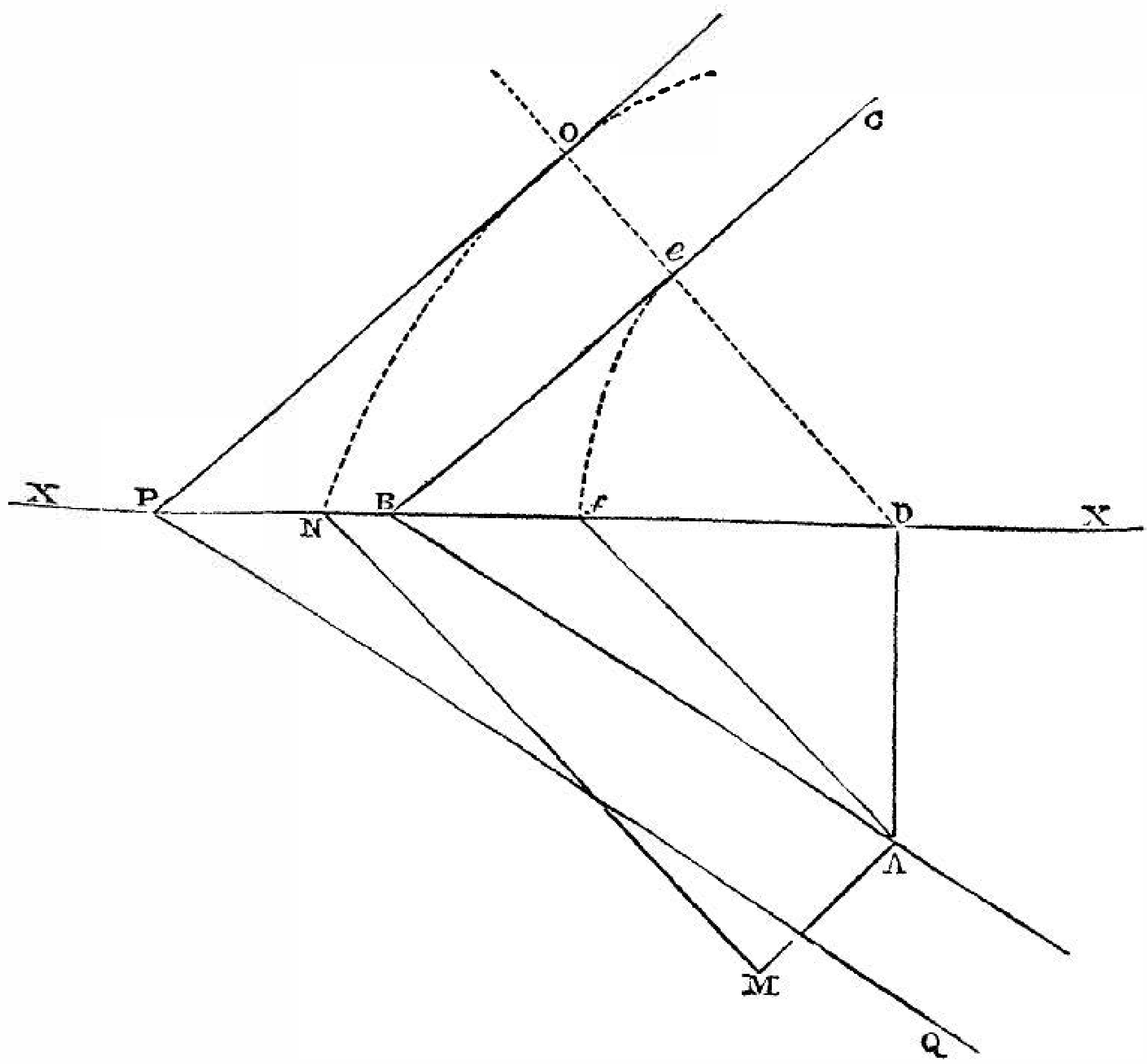

Fig. 8.

d D perpendicular to X X; join C D, *c d*; these will be the projections required.

28. **To Find the Projections of the Point where a Straight Line Intersects a Plane** (the traces of the line and of the plane being given), it is only necessary to draw the traces of two planes traversing the given line in convenient directions, and find the projections of the lines in which those two planes cut the given plane; the intersections of those projections will be the projections of the point required.

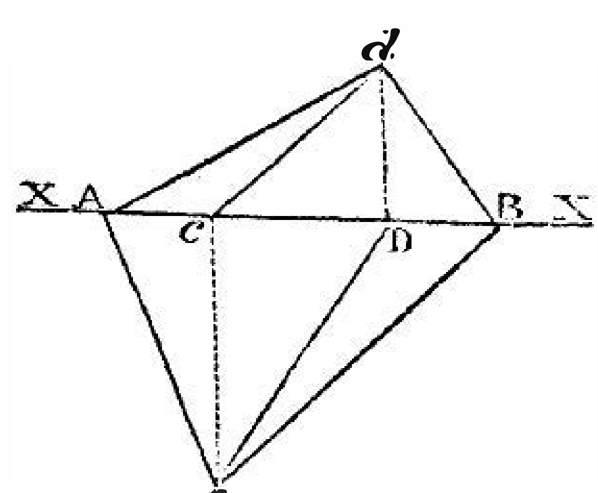

Fig. 9.

29. **Given** (in fig. 10), **the Traces of Two Planes,** C A *d*, C B *d*,

to Find the Angle between them.—From either of the intersections of the traces (say *d*) let fall *d* D perpendicular to X X; draw D C, joining D with the other intersection of the traces. Through any convenient point, I, in D C, draw G I H perpendicular to D C,

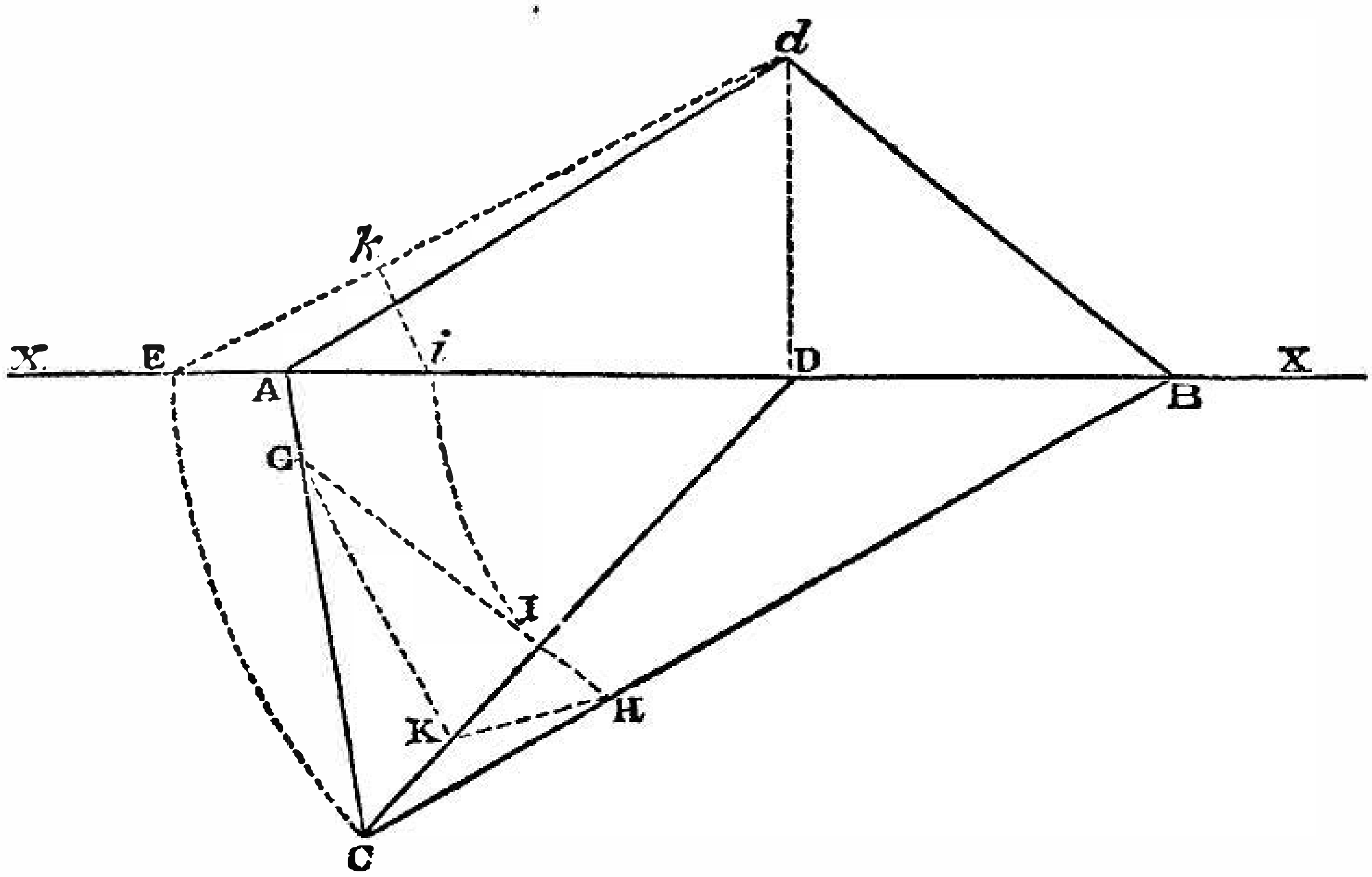

Fig. 10.

cutting A C in G and B C in H. Along X X lay off D E = D C, and D *i* = D I; join *d* E (this will be the length of the line of intersection of the planes). From *i* let fall *i k* perpendicular to *d* E; in I C take I K = *i k*; join K G, K H; G K H will be the angle required.

When the traces of the two given planes are inconveniently placed for the completion of the figure, we may substitute for either pair of traces another pair of traces parallel to them, and more conveniently placed.

30. **Given** (in fig. 10), **the Traces, A *d* and A C, of a Plane; also the Traces, *d* and C, of a Straight Line in that Plane; to Draw the Traces of a Plane which shall Cut the given Plane in that Line at a given Angle.**—From either of the traces of the straight line, as *d*, let fall *d* D perpendicular to X X; draw the straight line D C, joining D with the other trace, C, of the straight line. Through any convenient point, I, in D C, draw I G perpendicular to D C, cutting C A in G. In X X lay off D E = D C and D *i* = D I; join *d* E, and on it let fall the perpendicular *i k*. In I C take I K = *i k*; join K G. Then draw K H, making G K H = the given angle, and cutting G I, produced if necessary, in H. Draw C H, cutting X X in B, and join B *d*; these will be the traces of the plane required.

31. **Given** (in fig. 11), **the Traces of a Plane,** A B C, **and the Projections of a Point,** G, *g*, **to Draw the Traces of a Plane Traversing the given Point, and Parallel to the given Plane.**—Through either of the projections of the given point (say G) draw G H parallel to the corresponding trace of the given plane, and cutting X X in H. (This will be one of the projections of a line through the given point, parallel to the trace A B of the given plane.)

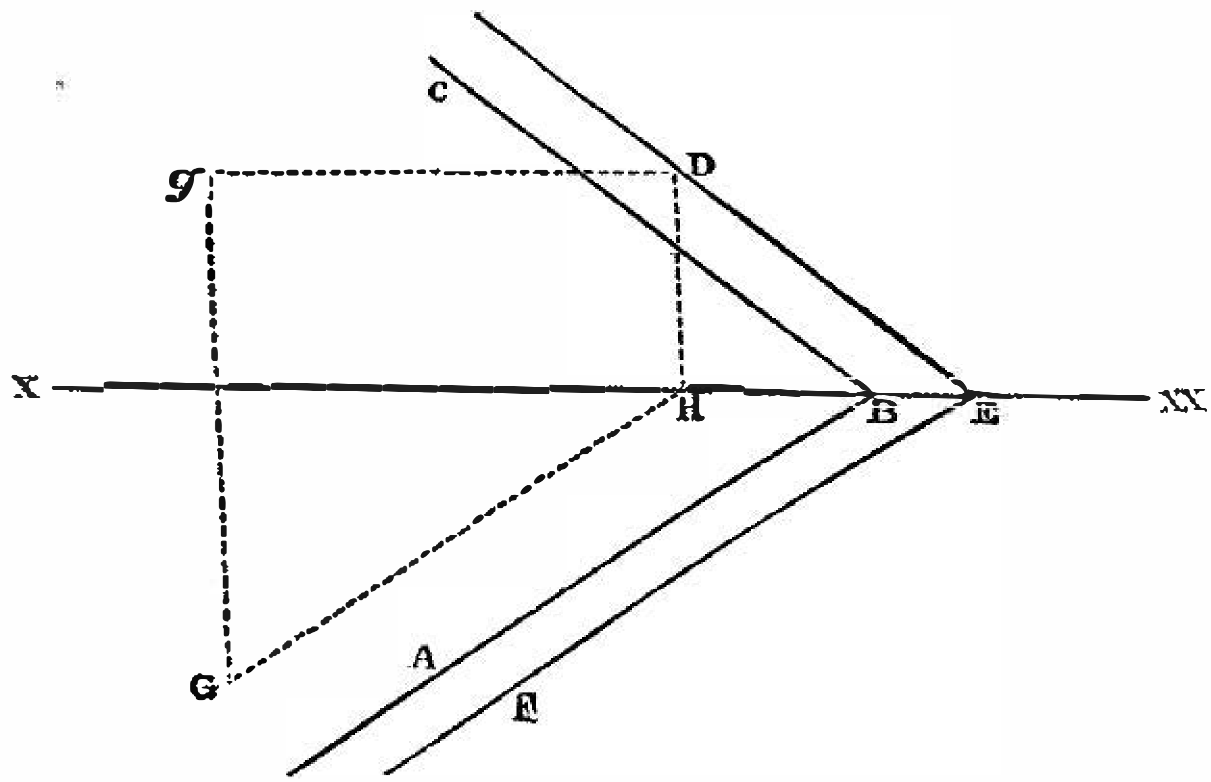

Fig. 11.

Through H draw H D perpendicular to X X; and through *g* draw *g* D parallel to X X, cutting H D in D (*g* D will be the projection and D one of the traces of the line before mentioned). Through D draw D E parallel to C B, cutting X X in E; and through E draw E F parallel to B A; D E F will be the traces of the required plane.

32. **Given, the Traces of a Plane,** E F, E D (in fig. 11), **and One Projection of a Point in that Plane, to Find the other Projection of that Point.**—Suppose *g*, the vertical projection of the point, to be given. Draw *g* D parallel to X X, cutting E D in D. From D let fall D H perpendicular to X X. From *g* draw *g* G perpendicular to X X, and from H draw H G parallel to E F; the intersection of those lines, G, will be the required horizontal projection of the given point.

33. **Given** (in fig. 12), **the Traces,** A B C, **of a Plane, and the Projections,** D, *d*, **of a Point, to Draw the Projections of a Perpendicular let Fall from the Point on the Plane.**—From one of the projections of the given point (say D) draw D E F perpendicular to the corresponding trace, B A, of the given plane, and cutting B A in E, and X X in F. From E let fall E *e* perpendicular to

X X; from F draw F *f* perpendicular to X X, cutting the trace B C in *f*; join *f e*; from *d* draw *d g* perpendicular to B C, cutting *f e* in *g*; and from *g* draw *g* G perpendicular to X X, cutting D F in G. D G and *d g* will be the projections of the perpendicular required.

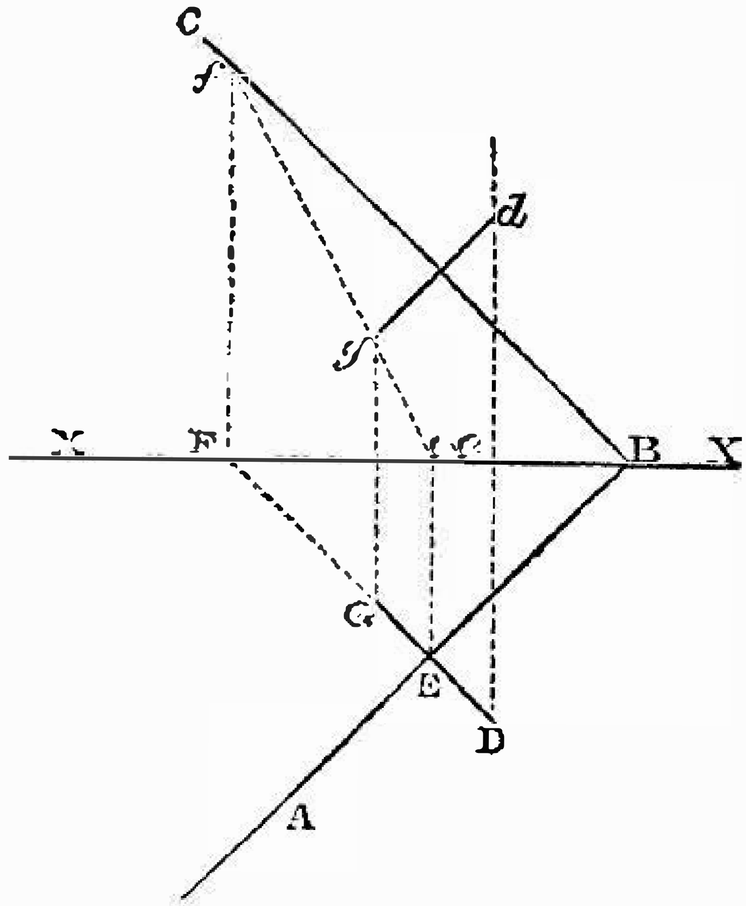

Fig. 12.

34. **Given (in fig. 13), the Projections of a Point, D, *d*, and those of a Straight Line, A B, *a b*, to Draw the Traces of a Plane which shall Traverse the Point, and be Perpendicular to the Line.**—Through one of the projections of the given point (say D) draw D G perpendicular to A B (the corresponding projection of the given line), cutting X X in G. Through G draw G *g* perpendicular to X X; through *d*, the other projection of the point, draw *d g* parallel to X X, cutting G *g* in *g*; through *g* draw E C perpendicular to *a b*, cutting X X in C; and through C draw C F perpendicular to A B. E C F will be the traces of the required plane.

35. **Given, the Projections of a Point and of a Straight Line, to Draw the Projections of a Perpendicular let Fall from the Point upon the Straight Line.**—Find by the preceding rule the traces of a plane traversing the given point, and perpendicular to the given line; then, by Article 23, find the traces of a plane traversing the given point and line; and finally, by Article 27, find the projection of the line of intersection of those two planes.

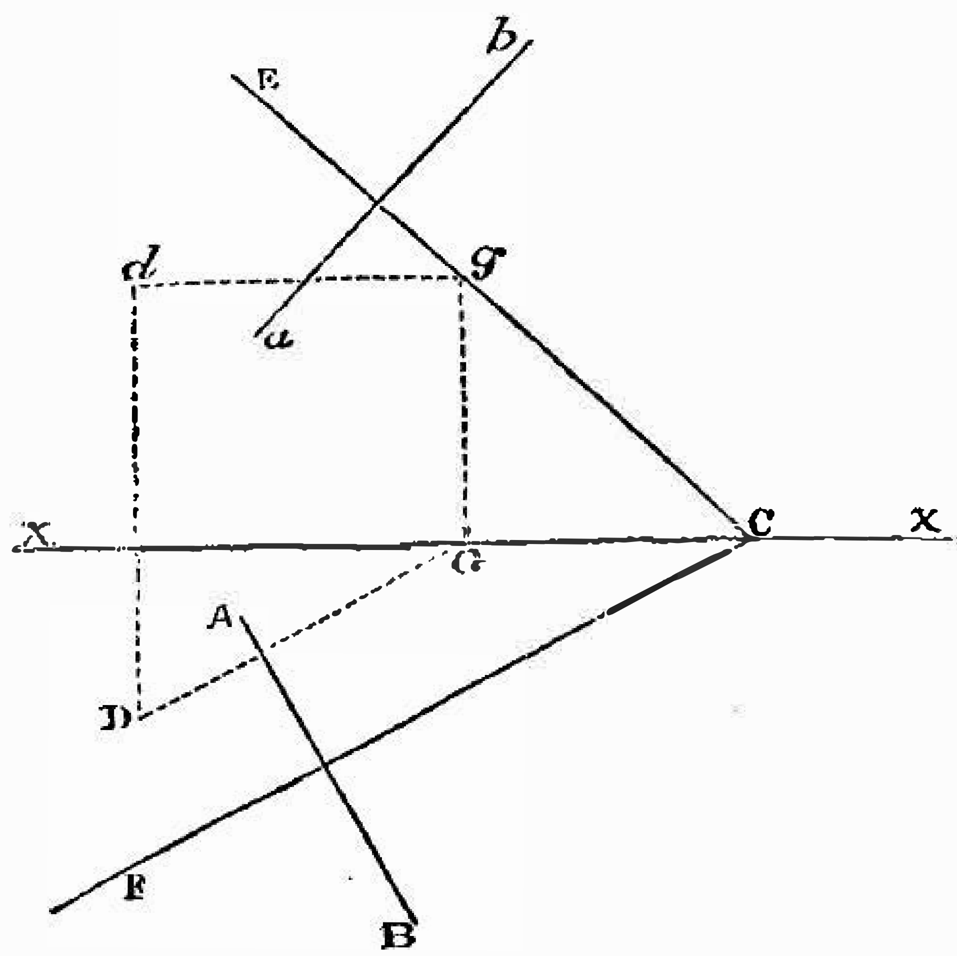

Fig. 13.

36. **Given, the Projections of Two Straight Lines that are neither**

Parallel nor Intersecting, to Find the Projections of their Common Perpendicular.—By Article 23 A, find the traces of a plane traversing one of the lines and parallel to the other. Then, by Article 33, find the projections of a perpendicular let fall on that plane from any convenient point in the second line. Then through the projections of the foot of that perpendicular draw the projections of a straight line parallel to the second straight line; these will cut the projections of the first straight line at one end of the common perpendicular, whose projections will be parallel and equal to those of the perpendicular already found.

37. **Projections of a Circle.**—When an instrument which draws ellipses *accurately* is at hand, it may be used for the purpose of drawing the projections of a circle of a given radius, described about a given point in a given plane, and may thus facilitate much the solution of various problems. The following is the process for obtaining the projections of a circlec—

Given (in fig. 14), **the Traces of a Plane,** A B C, **and the Projections of a Point in that Plane,** D, *d*, **to Draw the Projections of a Circle of a given Radius, described in the given Plane and about the given Point.**—For the vertical projection, describe about *d* a circle of the given radius, *d f* = *d e*, and draw the diameter *e f* parallel to the trace C B; *e f* will itself be the vertical projection of one diameter of the circle. Draw *d g* perpendicular to *e f*. Find, by Article 24, the angle which the given plane makes with the vertical plane of projection, and lay off *g d h* equal to the angle so found. From *h*, in the circle, draw *h k* parallel to *f e*, and cutting *d g* in *k*; then *d k* will be the vertical projection of a radius of the circle perpendicular to *e f*. Then on the major axis, *e f*, and minor semi-axis, *d k*, describe an ellipse; that ellipse will be the required vertical projection of the circle.

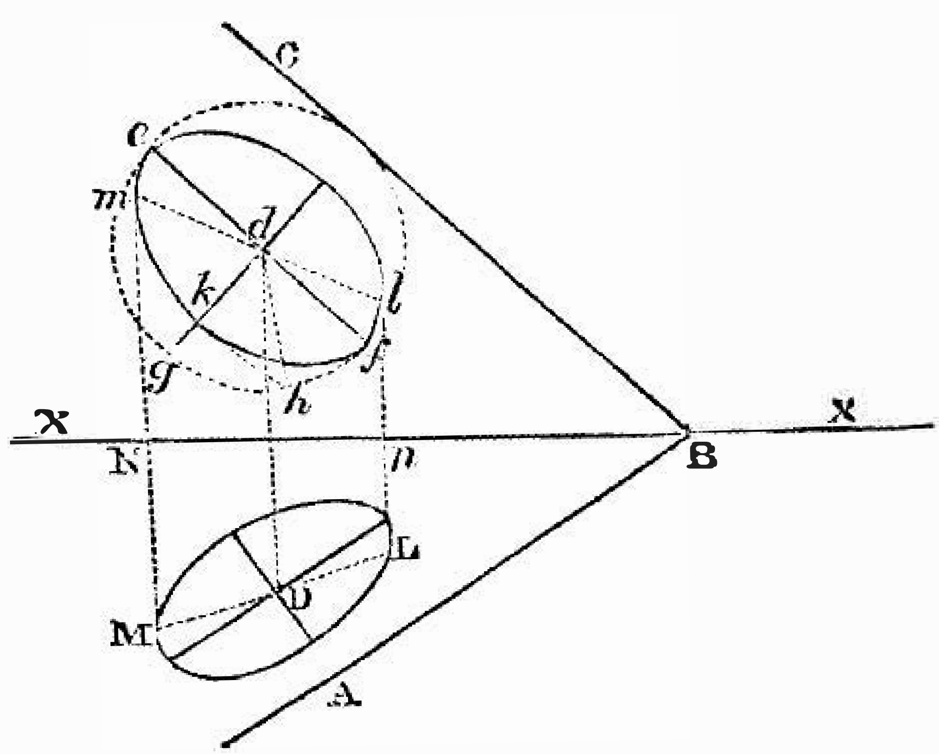

Fig. 14.

The horizontal projection is obtained by a precisely similar process, the rule of Article 24 being now used to find the angle which the given plane makes with the horizontal plane of projection.

The two ellipses are both touched by a pair of tangents, M *m*, L *l*, perpendicular to X X; and the diameters, *l m*, L M, are the projections of one diameter of the circle—viz., that diameter in which the plane A B C is cut at right angles by a plane parallel to X X. The perpendicular distance, N *n*, between the two tangents is equal to the diameter of the circle multiplied by the cosine of the angle which the given plane makes with X X, and is bisected by the line D *d*.