CHAPTER III.

PLANE MECHANISMS CONTAINING ONLY TURNING PAIRS.

26. Quadric Crank-chains.—If we endeavor to make a plane mechanism out of links containing only turning pairs, we find that the least number of links with which this can be done is four. A chain of *three* links so connected forms

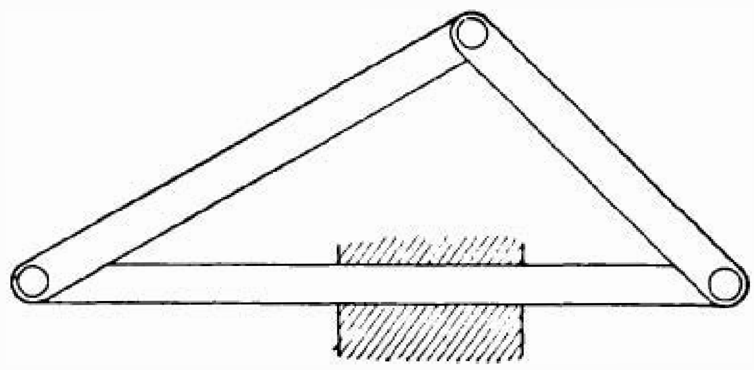


FIG. 39.

an arrangement which is of value as a *structure* (a simple triangular roof-truss), but is of no service as a mechanism, since its parts can have no relative motion.

On the other hand, a simple chain of *five* or any greater number of links connected by turning pairs is equally useless as a mechanism, since the relative motion of at least two of its links is not constrained, as has been shown in § 3.

Let us consider, therefore, a chain of four links connected by turning pairs whose axes are parallel. When the links of this chain are of unequal lengths the smallest is called the crank, and since the four links form a quadrilateral, the chain has been called by Reuleaux * the *quadric* (*cylindric*) *crank-chain*. The term 'cylindric' distinguishes this chain

* Kinematics of Mach., §§ 62–65.

from the corresponding spheric chain, in which the axes are not parallel.

In quadric crank-chains it will be convenient to distinguish between links having a swinging or partial turning movement and those which can execute complete rotations relatively to the fixed link in the chain.

The former links will be called levers, the latter cranks. It is obvious that by altering the relative lengths of the links we can obtain different relative motions, and hence different mechanisms. From these, again, other different mechanisms are produced by inversion of the chain.

27. Virtual Centres and Centrodes.—Let *abcd*, Fig. 40, represent the four links of a quadric crank-chain. Each of these links will have motion relatively to every other, and hence we shall have six virtual centres. Four of these centres are readily identified as the axes of the turning pairs; for instance, the virtual centre of *c* with regard to *d*, or of *d* with regard to *c*, is obviously the point 3, and may be

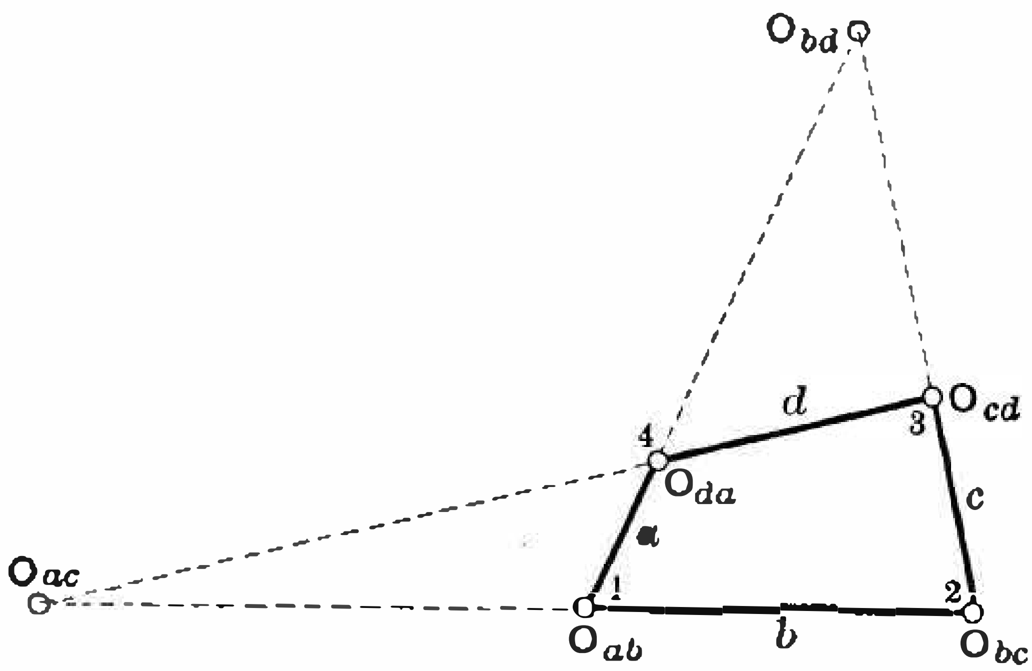


FIG. 40.

indicated as O_{cd} or O_{dc}. In the same way we have O_{ad}, O_{ab}, and O_{ac}; all these points are in fact permanent centres as regards their own pair of links. Remembering that for any three bodies having plane motion the three virtual centres lie in one straight line, it is easy to see that O_{ac} must lie at the join of the straight lines drawn through O_{ab} and O_{bc}, and through O_{ad} and O_{cd}. In the same way O_{bd} is at the intersection of the lines $O_{ab}O_{ad}$ and $O_{cb}O_{cd}$.

Supposing b to form the *frame* or fixed link, it is seen that since O_{ab} and O_{bc} are permanent centres, the centrodes of a and c with regard to b are points, namely O_{ab} and O_{bc}.

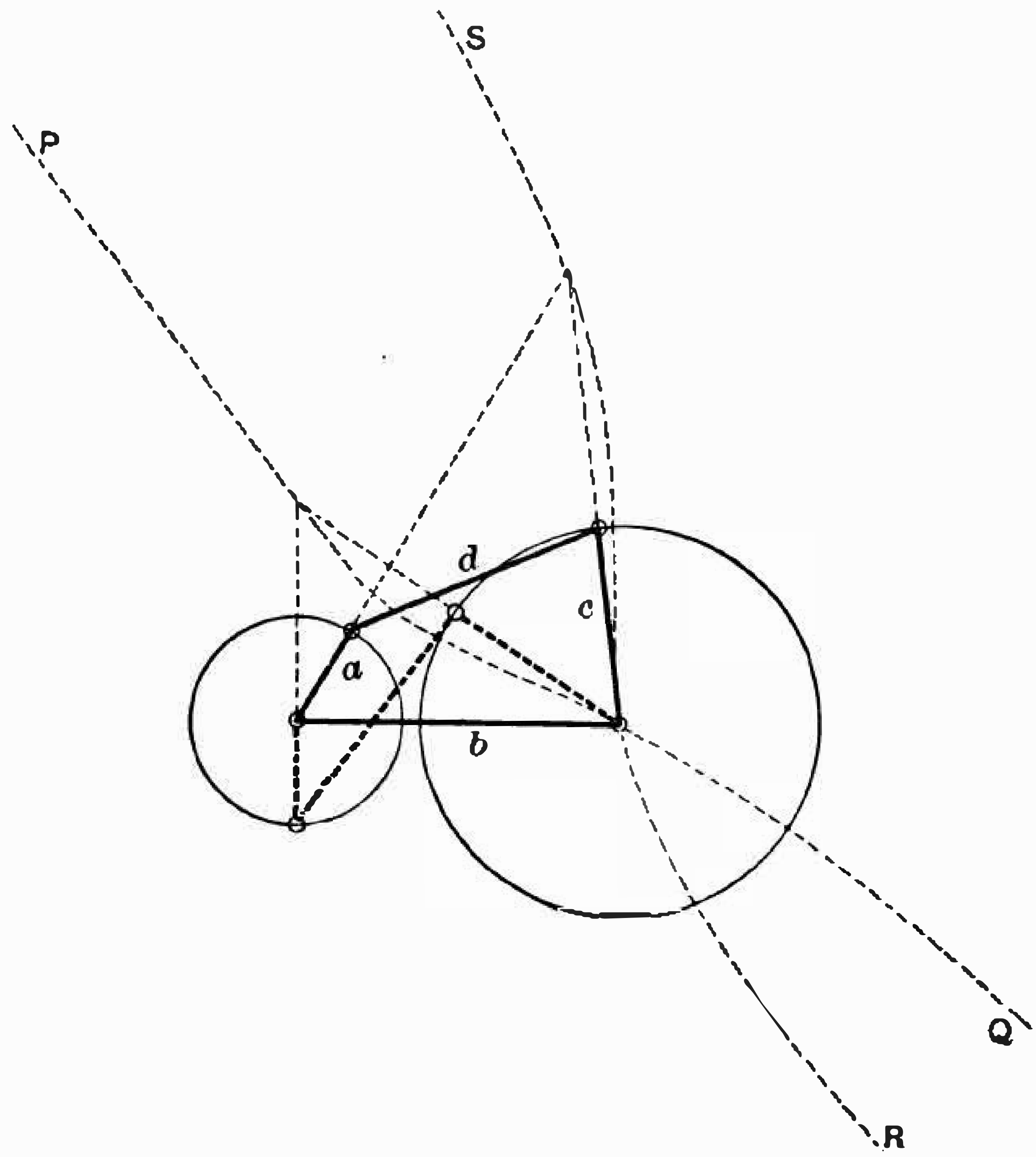


FIG. 41.

The centrode of d with regard to b is the locus of O_{db} and takes the form of a curve having four infinitely distant points; portions of it are readily drawn by finding a series of positions of O_{db} corresponding to successive positions taken up by the three links a, c, and d. In a similar way may be obtained the centrode of b with regard to d (supposing d to be the fixed link). The curve $PQRS$

in Fig. 41 represents the centrode of d with regard to b; the construction for two points on the curve is shown.

28. Angular Velocities.—It is frequently of importance, having given the angular velocity, say, of the link a, to find

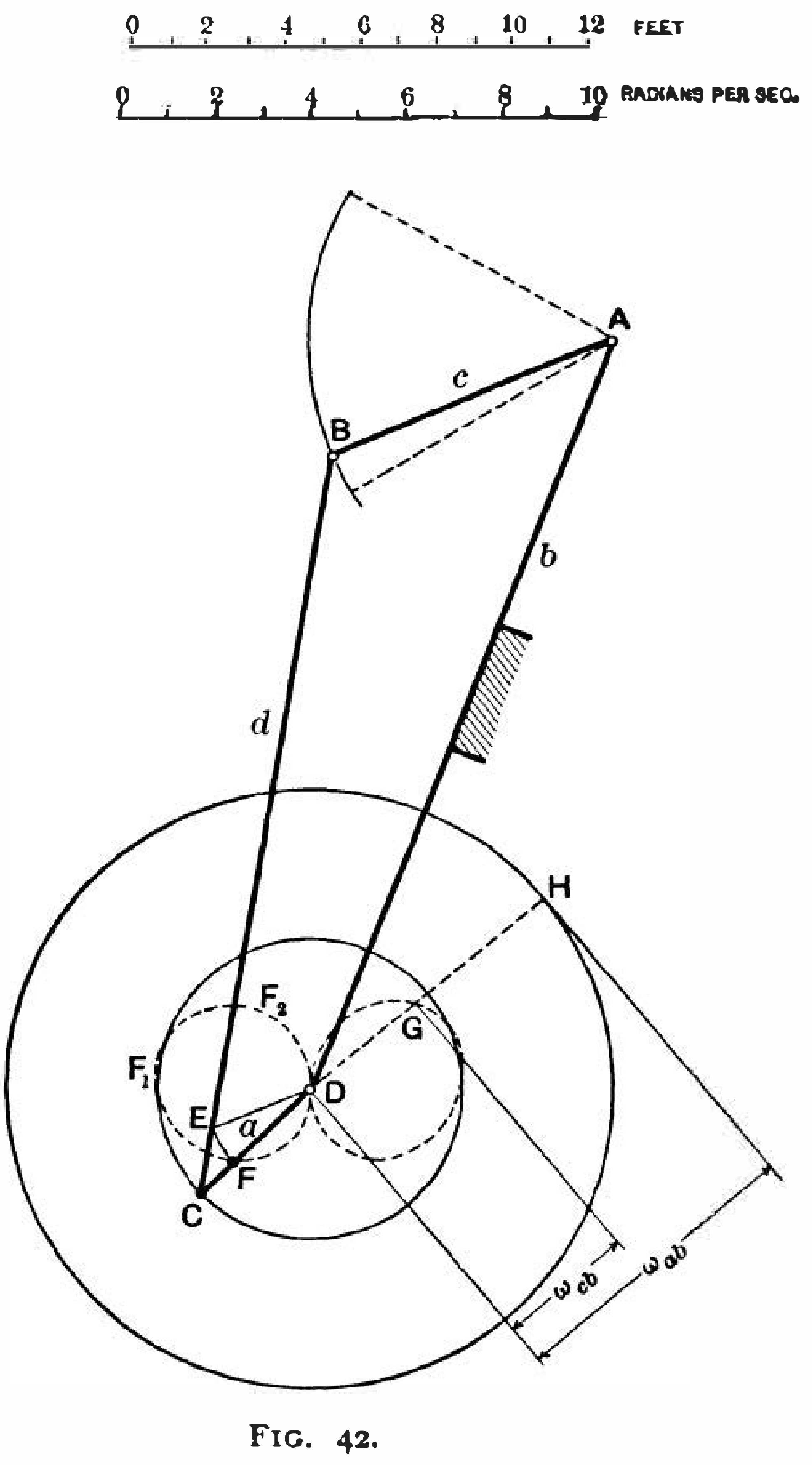


FIG. 42.

that of any other link, say c, or, in other words, to determine the angular velocity ratio of the chain. This can be very simply done by construction.

In Fig. 42 let $ABCD$ represent the mechanism, AD being the fixed link, and the uniform angular velocity of CD being known. It is required to determine the angular velocity of AB for any position of the mechanism.

Draw DE parallel to AB, and cutting BC, or BC produced, in E. With centre D and radius DE mark off DF

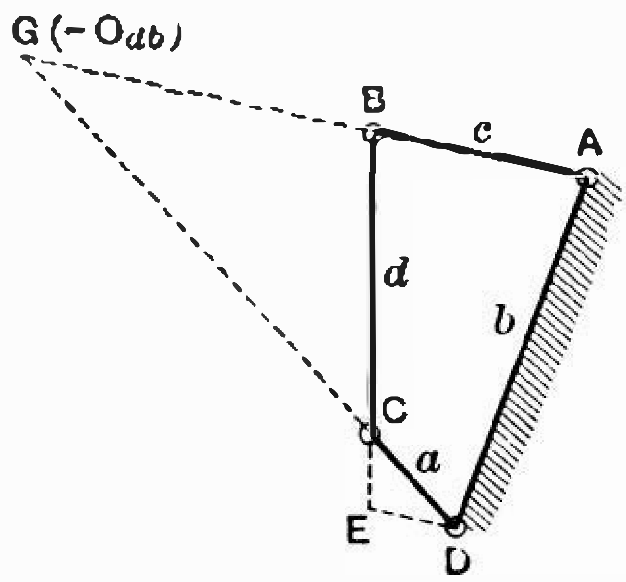


FIG. 43.

along DC. Then DF represents the angular velocity of c on the same scale as that on which AB represents the angular velocity of a, and if a series of points such as F be obtained, the curve FF_1F_2G . . . drawn through them will form a polar diagram of angular velocities for c and a.

To prove this construction, let ω_{cb}, ω_{ab} be the angular velocities of c and a respectively with regard to b. In Fig. 43 find O_{db}, the intersection of AB and DC at G, and draw DE parallel to AB, meeting BC in E.

Since the link d is turning for the instant about G, we must have

$$\frac{\text{linear velocity of } B}{\text{linear velocity of } C}=\frac{GB}{GC}.$$

Now $$\omega_{cb}=\frac{\text{linear velocity of } B}{AB},$$

and $$\omega_{ab}=\frac{\text{linear velocity of } C}{CD};$$

hence
$$\frac{\omega_{cb}}{\omega_{ab}}=\frac{\text{linear velocity of } B}{\text{linear velocity of } C}\cdot\frac{CD}{AB}$$

$$=\frac{CD.GB}{AB.GC}.$$

But by construction the triangle BGC is similar to the triangle EDC; hence
$$\frac{GB}{GC}=\frac{ED}{DC}$$

Therefore
$$\frac{\omega_{cb}}{\omega_{ab}}=\frac{CD.ED}{AB.DC}=\frac{ED}{AB}$$

Thus if AB represents the angular velocity of a with regard to b, ED represents on the same scale that of c with regard to b.

Fig. 42 gives such a velocity diagram, drawn to scale, for the beam of a beam-engine when the crank rotates uniformly. For comparison the circle of radius $DH=AB$ has been drawn, so that for any radius DGH the intercept DG represents ω_{cb}, just as DH represents ω_{ab}. The polar curve of velocity is shown by a dotted line.

The distances taken are:

$$AB=8 \text{ feet}=DH;$$
$$BC=20 \text{ feet};$$
$$CD=4 \text{ feet};$$
$$DA=21.5 \text{ feet}.$$

When the crank is in the position DH the angular velocity ratio is
$$\frac{DG}{DH}=\frac{3.5}{8}=0.438,$$

or at that particular instant the beam is swinging with 0.438 the angular velocity of the crank. If the crank rotates uniformly at 60 revolutions per minute or 6.28 radians per second, in the position AB the beam is moving with an angular velocity of $6.28 \times 0.438 = 2.75$ radians per second.

From the curve of angular velocity thus obtained we might draw the curve of angular acceleration by the construction described in § 22. Notice that the construction just described can still be applied in positions of the mechanism where O_{bd} is inaccessible, i.e., when AB and CD are nearly parallel, and when the relative angular velocities, therefore, could not be found from the position of the virtual centres.

29. Inversions of the Quadric Crank-chain.—In the particular example of the quadric crank-chain just examined, the lengths of the links are such that while the link a executes complete rotations with reference to b or d, c only swings. a is then a crank, c a lever, and if the link b is the fixed one, the resulting mechanism is called the lever crank-chain.

In order that a may execute complete rotations with regard to b it is necessary that $a+b \leqq c+d$, while also $a+d \leqq c+b$, a being the smallest of the links.

With these proportions let us see the result of inversion of the chain. On considering the relative motions of the links we find that the motion of a relatively to b or d is that of complete rotation, while with regard to either of the

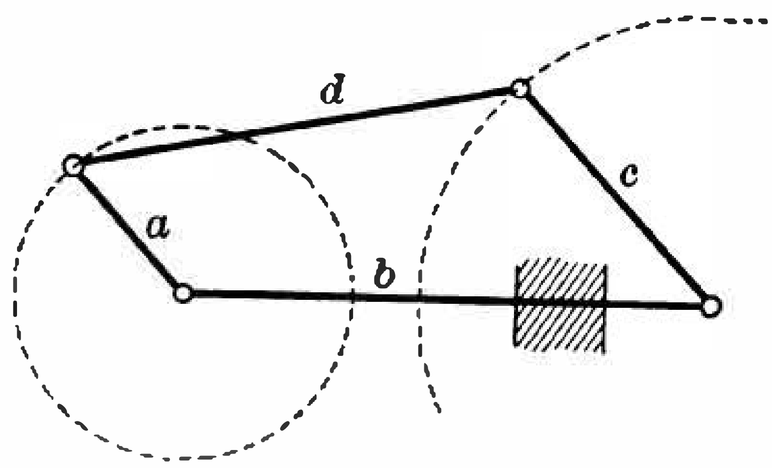


FIG. 44.

same links c only swings or performs partial revolutions. As has been already pointed out, inversion can make no change in the *relative motions* of the links, and hence the mechanism will remain a *lever crank-chain* whether b or d be the frame (or fixed link).

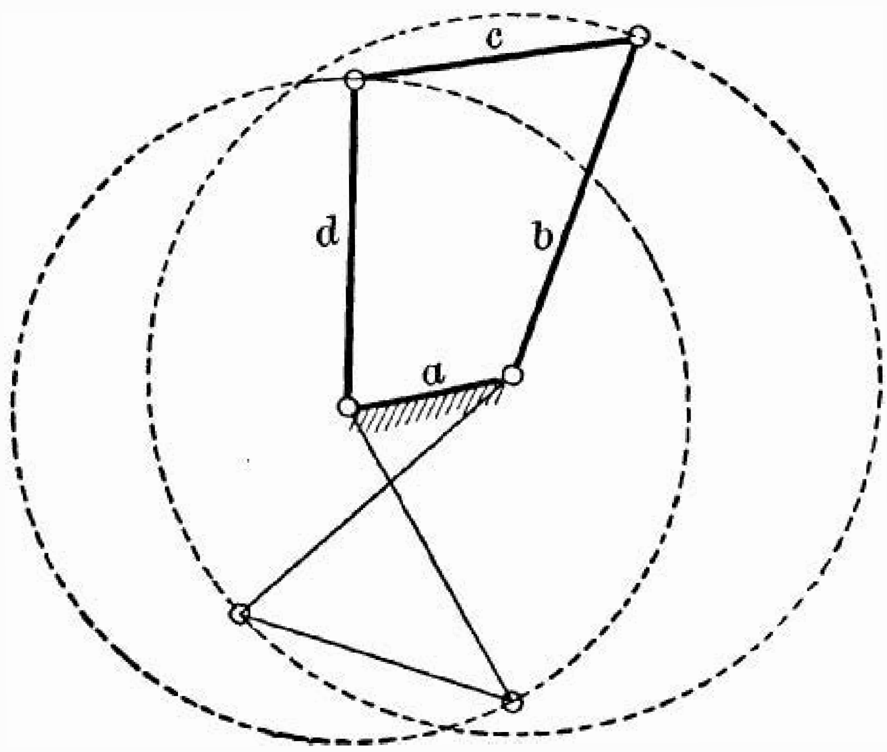


FIG. 45.

On fixing a, however, as in Fig. 45, a new mechanism is obtained which may be called the *double crank*, inasmuch as both b and d can now execute complete rotations about the axes of the pairs ba and da.

This mechanism is used in practice as a drag-link coupling, b and d being represented by the discs keyed on to the two shafts, a by the frame containing the bearings, and c by the drag-link connecting the pin on d with that on b. The mechanism is also employed in the construction of feathering paddle-wheels.

When used for this purpose the object is to cause the floats to enter and leave the water edgewise, (so as to avoid splashing,) while remaining vertical at the bottom of their travel.

Thus suppose A, B, C, Fig. 46, to be points on the path of the outer edge of a float, AC being the water-line. The steamer has a certain speed relatively to the water, so that a point A on the wheel is moving horizontally with a velocity

AD in common with every other point on the vessel, the length of AD thus representing the speed of the ship to any convenient scale. In virtue of the rotation of the wheel, A has also a linear velocity (represented by AE to the same scale), relatively to the ship; therefore the real direction in which

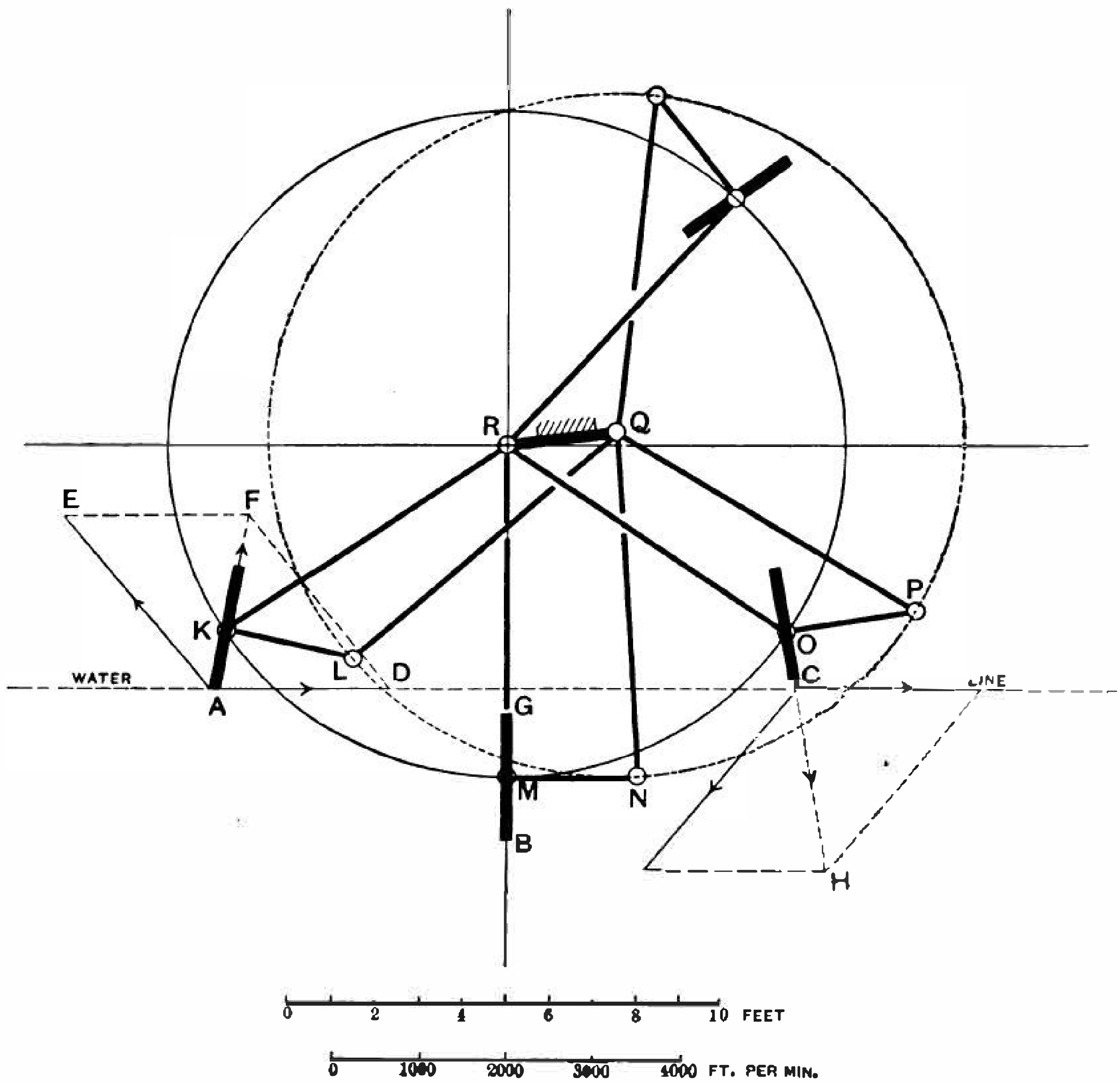


FIG. 46.

A moves relatively to the water is AF, the diagonal of the parallelogram $AEFD$. The floats then at entrance and exit should lie in the positions AF and CH, while the float in its lowest position should evidently be vertical, as at BG. The floats are pivoted at their centres to the framework of the wheel and have attached float-levers KL, MN, OP. The ends L, N, and P are all connected by radius-rods to an eccentric-pin, generally fixed on the sponson-beam of the paddle-box. The centre of this pin is of course at Q, the centre of the circle passing through N, P, and L, while R is the centre of the wheel itself. It will be seen that the paddle-wheel arm, the float-lever, the radius-rod, and part of the ship's structure form a double-crank mechanism, thus giving the floats the desired movement. Fig. 46 is drawn to scale from the following datan

Speed of ship 21 knots = 2026 feet per minute.
Diameter of wheel (ext.) 18.5 feet.
Revolutions per minute 48.
Breadth of floats 3′ 0″.
Immersion of lower edge 3′ 6″.
Length of float-levers 3′ 0″.
Speed of outer edge of float 2790 feet per minute.

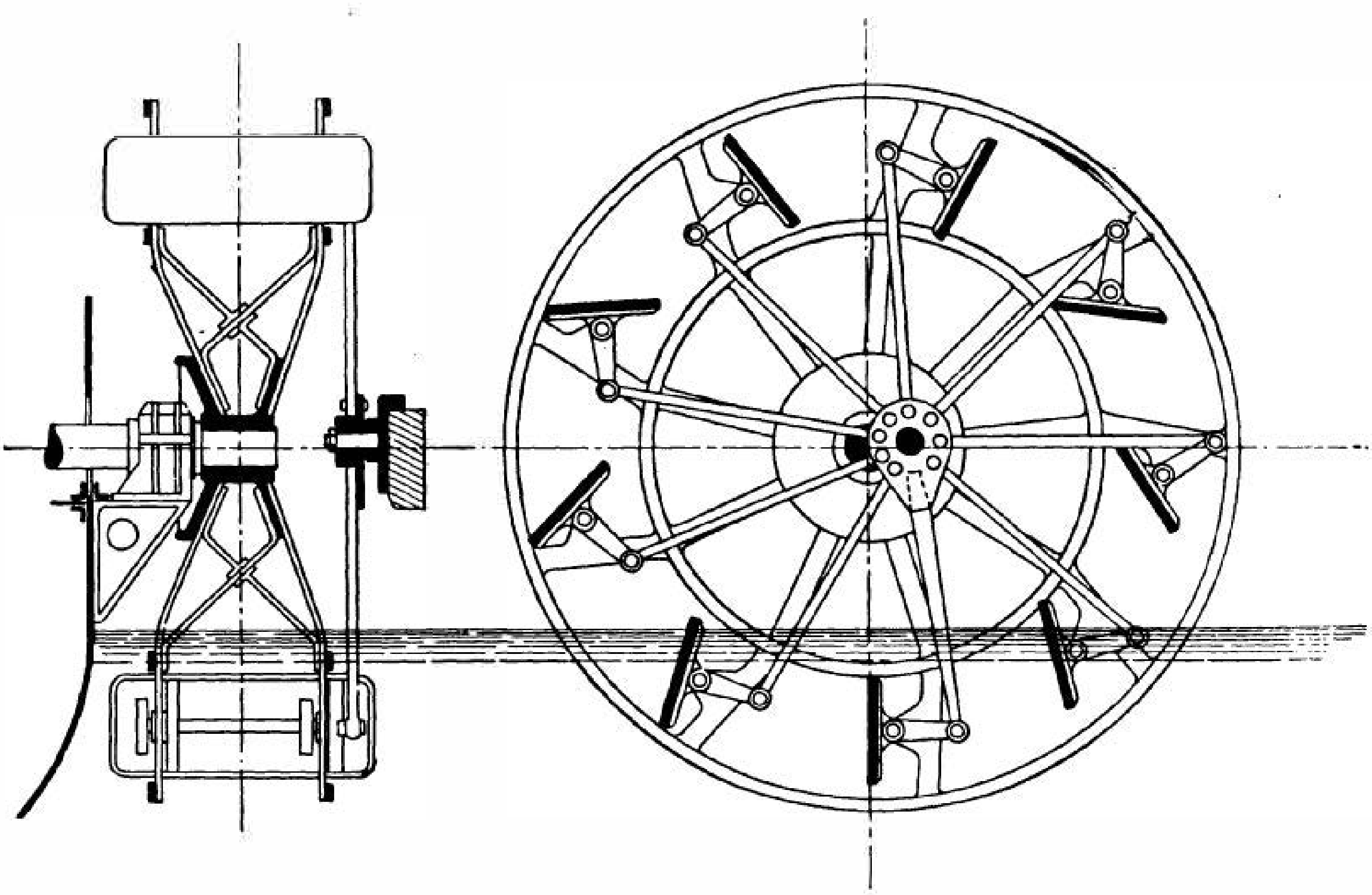


FIG. 46*a*.

Fig. 46*a* is a drawing of the arrangement of an actual feathering paddle-wheel. The reader will have no difficulty in recognizing various links in the double-crank mechanism.

Next suppose c (Fig. 44) to be the fixed link. Remembering that the relative motions of b and c and d and c are partial and not complete rotations, we see that a third mechanism, the *double lever*, is the result of this inversion.

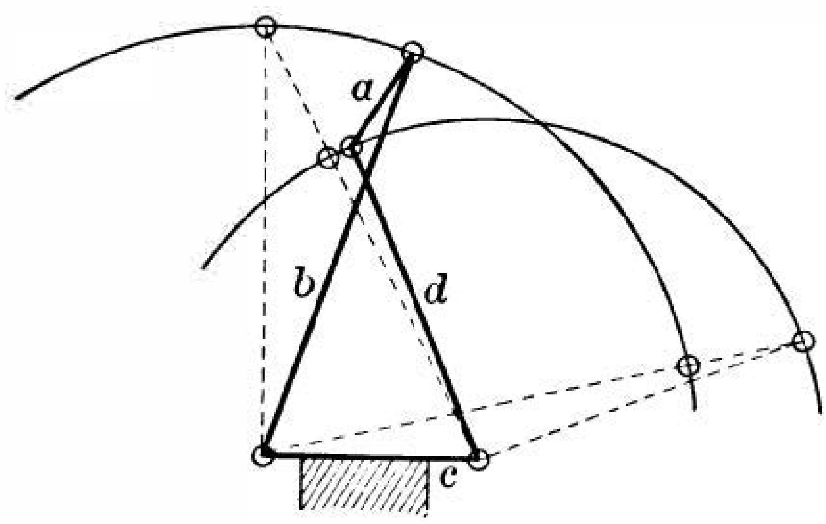


FIG. 47.

The double-lever mechanism is shown in Fig. 47, and such an arrangement finds an application in certain approximate straight-line motions (compare Fig. 56b).

The result of the various inversions of the quadric crank-chain may be summarized as follows:

Fixed Link.	Mechanism.
a	Double crank.
b	Lever-crank.
c	Double lever.
d	Lever-crank.

The four inversions have thus given us three different mechanisms.

Certain special cases of the quadric crank-chain have peculiarities which are of interest. Suppose that the lengths of the links are such that either $a+b=c+d$ or $a+d=c+b$, b being the fixed link. This condition is shown in Fig. 48.

As we have seen, it is still possible for a to execute complete rotations, but it will now be found that c can swing on either side of the fixed link *b*. We obtain one position of the chain in which all the links are in a line, and the angle through which *c* can swing is then doubled. This condition is expressed by saying that the mechanism passes through

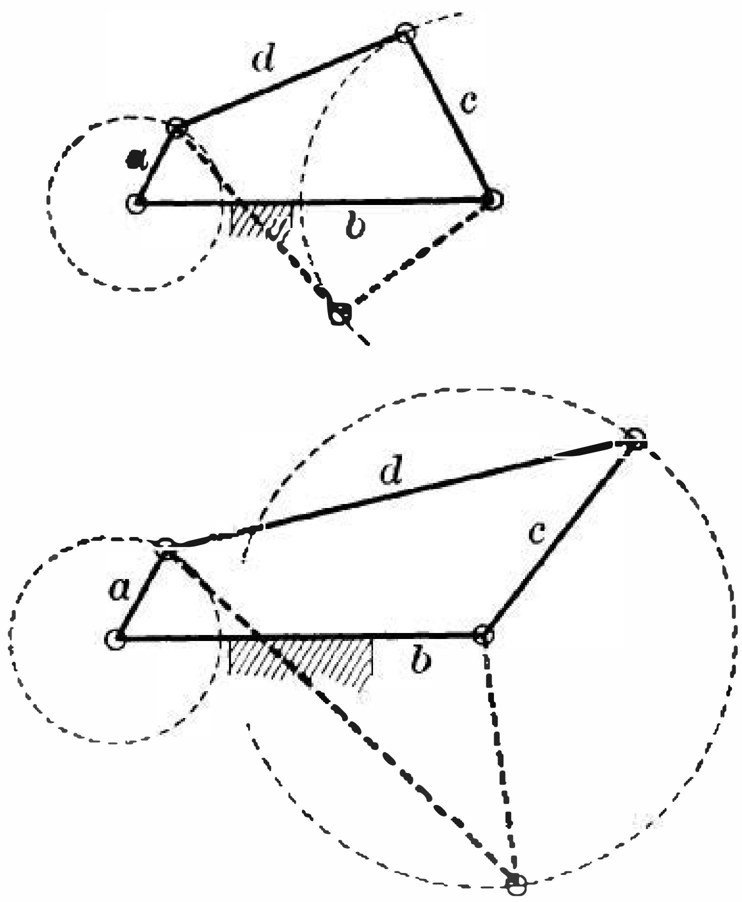


FIG. 48.

a *change-point* when all the links are in line, and for any given position of the link *a* the mechanism can assume either the position shown by the full lines or that shown by the dotted lines in the figure.

30. Change-points and Dead-points. — A *change-point* may be defined as a position of a mechanism in which such a want of constrainment occurs that it is possible for the arrangement to transform itself into another mechanism, or, in some cases, into a pair of elements.

A very familiar instance of such a change-point occurs in the quadric crank-chain when of the form known as *parallel cranks;* that is, when *a* and *c* are equal in length. and considerably shorter than *b* and *d*, which are also equal This is of course a particular case of the condition illustrated in Fig. 48, and is shown in Fig. 49.

At the instant when all the links are in a line it becomes possible for the mechanism *abcd* either to take up such a form as *abc'd'* or to continue its motion in its original form. The mechanism in question is of common occurrence in locomotive engines having coupled driving-wheels, and the necessary constraint at the change-points is provided by duplicating the chain, namely, by arranging another pair of cranks and a coupling-rod on the other side of the engine, so placed that the change-points of the two chains do not occur at the same time. Other methods of obtaining a similar object will be found discussed in a later chapter.

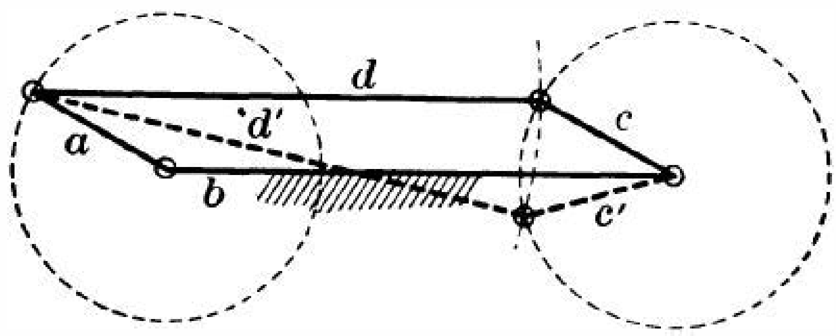


FIG. 49.

By the term *dead-point* in a mechanism is meant a position of the various links such that one of them directly opposes itself to the action of the forces tending to produce motion. The term was first applied by Watt to those positions of the crank and connecting-rod in a steam-engine in which the axes of three turning pairs lie in one plane, so that a force applied to the piston is not able to cause any torque on the shaft. It is plain that, in the absence of some means of overcoming this difficulty, the further motion of the chain becomes impossible, and the chain may be regarded as incomplete. The occurrence of dead-points must not be confused with that of change-points, although they may, and often do, occur together.

It is important to note an essential difference between dead-points and change-points. The occurrence of a dead-point depends on the particular link to which the driving force is applied, and on the manner of its application. For instance, in the lever-crank mechanism of Fig. 42, if the crank be turned by the application of a continuous torque to its shaft, no dead-point exists. In the very same mechanism, however, if the driving effort be applied to the lever c (as in a beam-engine), dead-points occur twice in each revolution of the crank.

A change-point, on the other hand, is caused by the configuration of the chain itself, and is present whichever link is fixed, so long as the chain is the same.

31. **Special Forms of Quadric Crank-chain.**—Suppose in the quadric crank-chain that we make $a = b$ and $c = d$; we then obtain a chain called by Professor Sylvester the *kite*, from its form. When one of the links is fixed, and the motion examined, it will be found that there are two change-

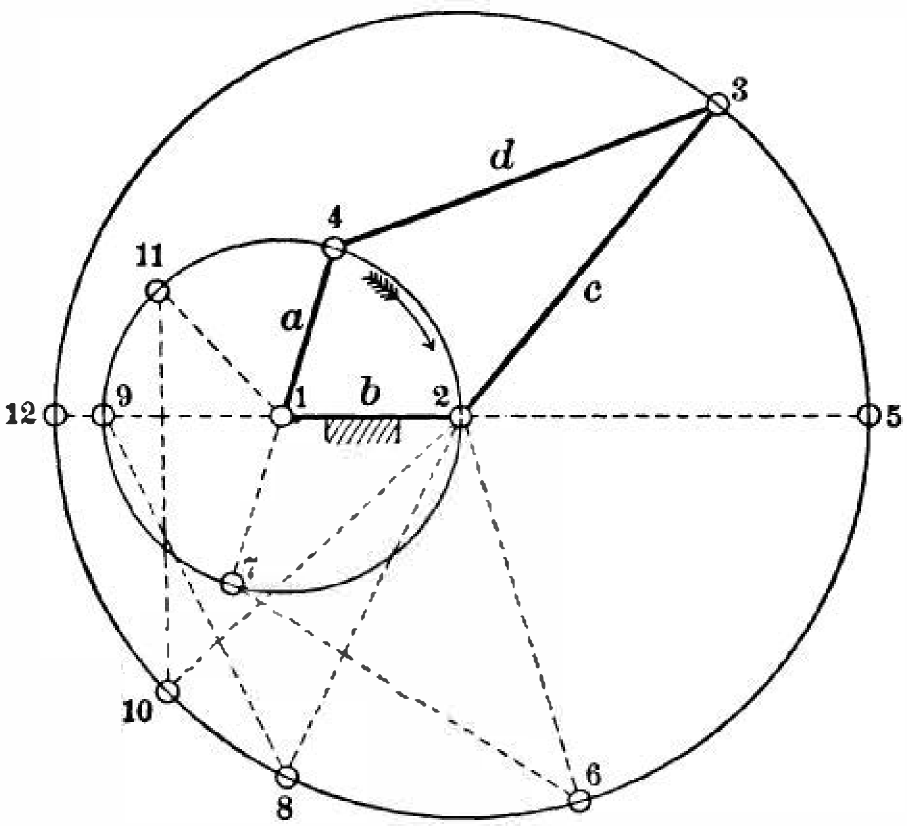


FIG. 50.

points. Thus, in Fig. 50, imagine that a rotates in the direction of the arrow, b being fixed. When the joint 4 coincides with 2 the chain becomes, for the instant, a turning pair, having its centre at 2, c and d rotating together. If a continues its motion for another complete rotation, another change-point occurs, the chain having passed through the positions 1, 2, 6, 7; 1, 2, 8, 9; 1, 2, 10, 11; and so on, until the position 1, 2, 12 is reached. If proper restraint is applied at the change-points, so as to prevent *c* and *d* rotating together, we see that for one complete rotation of *c*, *a* will have made two revolutions.

The kite finds its principal application in certain straight-line motions,* some of which will be presently discussed.

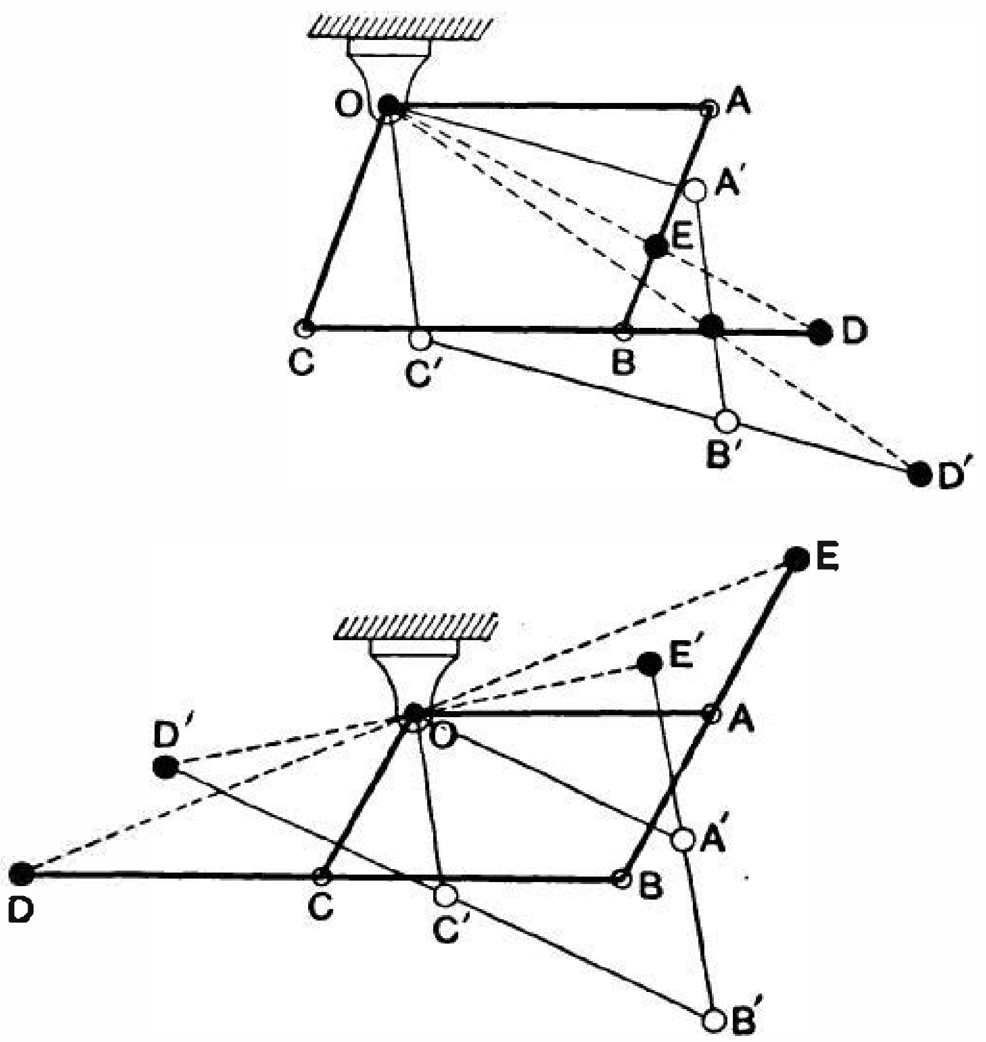


FIG. 51

A quadric crank-chain in which opposite links are of equal lengths is employed (with the addition of a fifth fixed link) for copying purposes under the name of a *pantagraph*.

Let *OABC*, in Fig. 51, represent such a chain attached to a point on a fixed link at *O*, and let *E* and *D* be any two points fixed on *AB* and *CB*, so that *OED* is originally a straight line.

* Kempe, How to Draw a Straight Line. Macmillan, Nature Series, 1877

The figure $OABC$ is a parallelogram whatever the position of the mechanism, hence the angles OAE, EBD are always equal. The lengths of the links are invariable, hence $\frac{OA}{AE} = \frac{BD}{BE}$ for any position, so that the triangles OAE, DBE are always similar, the angle OEA = the angle BED, and OED remains a straight line whatever the position of the mechanism, as at $OA'B'C'$.

If we suppose a pencil to be attached to E, any figure it describes will therefore be the polar projection of the figure described by D; the two curves being similar and similarly placed with regard to the pole O. The ratio of reduction is of course $\frac{OE}{OD}$. The fixed point O need not be at the join of the two links, but may be anywhere on any link, so long as O, E, and D are taken in one straight line.

If in the ordinary parallelogram we replace the bars AB, BC by wider pieces of material, and then choose on these pieces points P and Q such that the triangles PAB, BCQ are similar, we obtain a mechanism called by its inventor, Professor Sylvester, the *skew pantagraph* (Fig. 52).

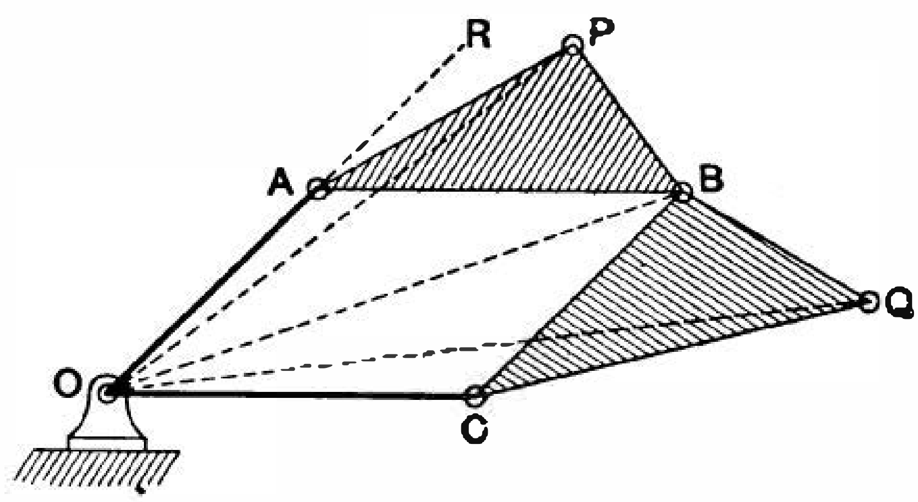


FIG. 52.

Join OP, OB, OQ. Then

$$\frac{PA}{AB} = \frac{BC}{CQ} \quad \text{and} \quad \frac{PA}{OC} = \frac{OA}{CQ}.$$

But the angles PAO, QCO are seen to be equal, therefore the triangles PAO, OCQ are similar and

$$\frac{OP}{OQ}=\frac{OA}{CQ}=\frac{AP}{OC}=\text{constant}.$$

Further, the angle $POQ = AOC - AOP - QOC$

$$= AOC - (AOP + APO).$$

Produce OA to R; then

$$\text{angle } POQ = RAB - RAP = PAB = \text{constant}.$$

From the facts that OP and OQ are in constant ratio and include a constant angle, it is evident that the paths of P and Q are similar but of different sizes, and one is turned through the angle POQ with regard to the other.

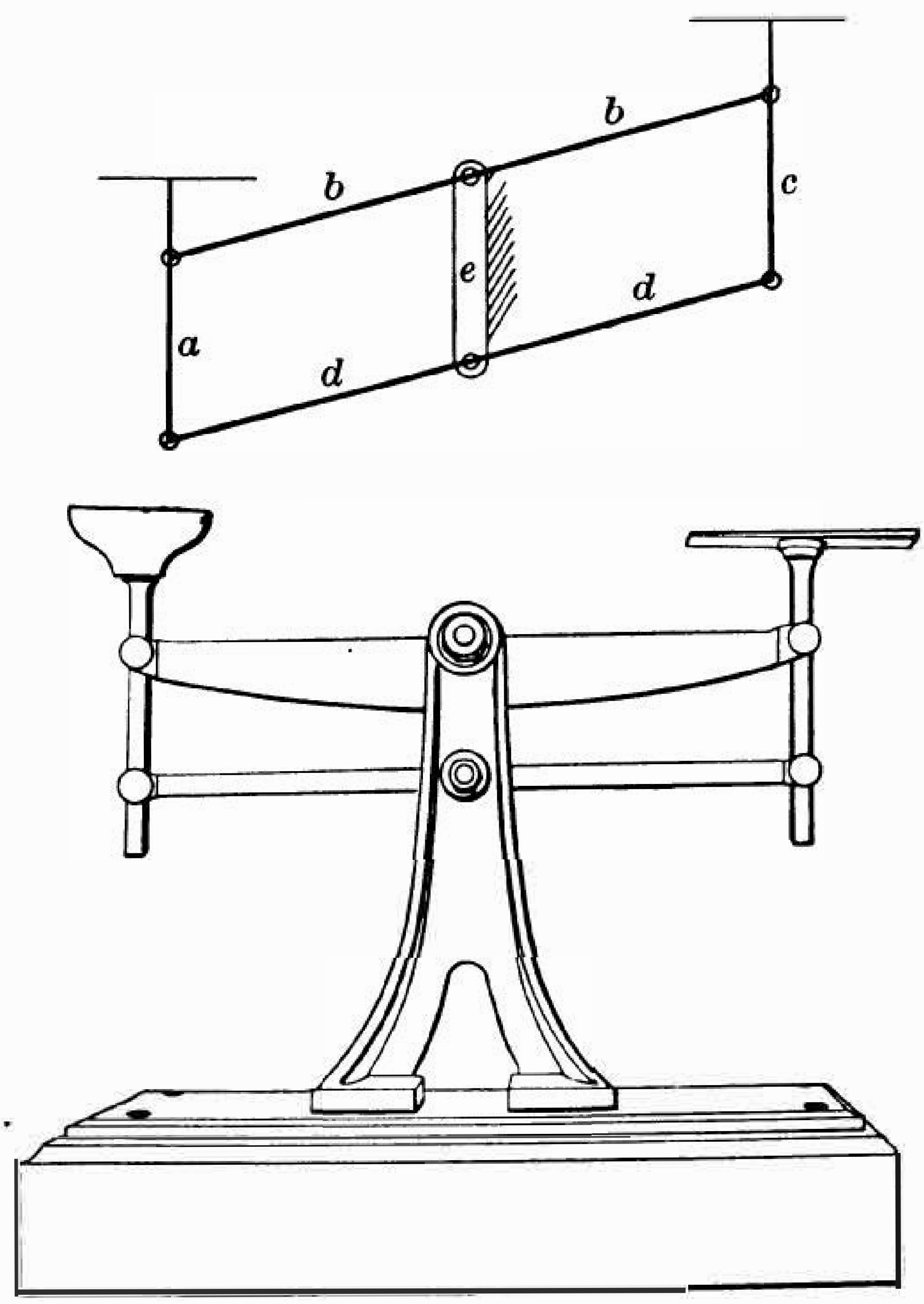


Fig. 53.

The parallelogram linkage is also applied in other ways. Fig. 53 represents the Roberval Balance in which a fifth link is added and the parallelogram is thus doubled. During the movement of the mechanism the distance moved by a point on c is equal to that moved in the opposite sense by any point on a. Accordingly if used as a balance it does not matter where the weights are placed on the scales, and the latter remain always horizontal.

32. Straight-line Motions. — Under the name *straight-line motions* may be considered mechanisms whose characteristic feature is that one or more points in them travel exactly or approximately in straight lines, without being directly guided. Those contrivances which contain turning pairs only will be discussed here; nearly all of them are formed from the quadric crank-chain.

In the earlier days of mechanical engineering plane surfaces were not so readily produced as at the present time, and straight-line motions, or parallel motions as they were called, played an important part as substitutes for straight-line guides. The linkage employed by Watt is shown in its simplest form in Fig. 54, and is seen to be a quadric crank-chain so proportioned that one point on the shortest link

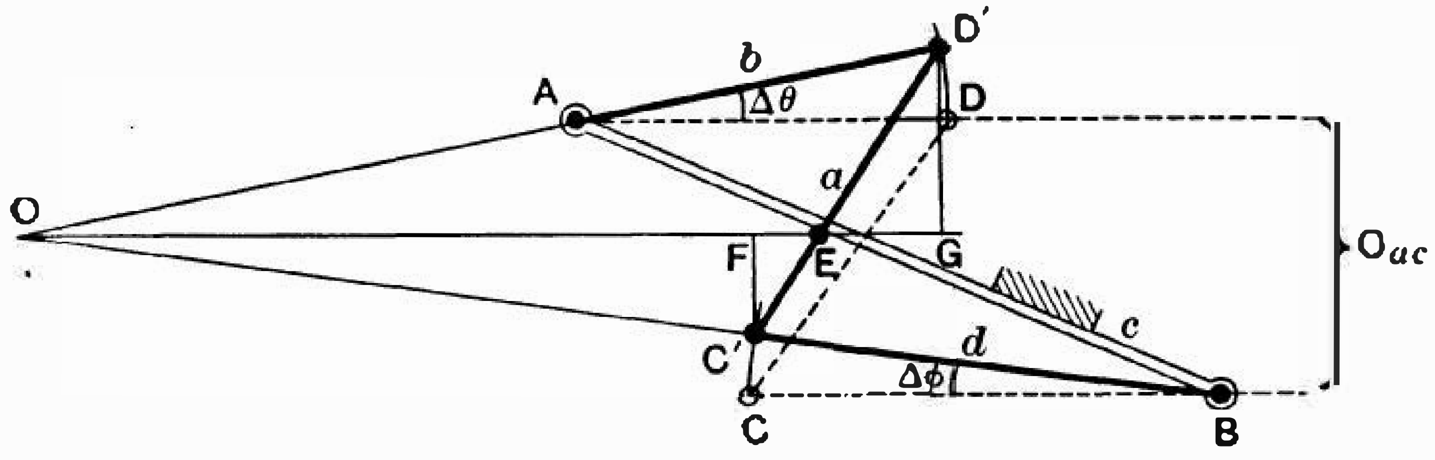


FIG. 54.

describes a path of which certain parts are approximately straight, the whole path being a kind of lemniscoid, or figure of eight. Such a mechanism is now used as a guide for the motion of the pencil of a Richards steam-engine indicator.

The mechanism consists of four links, the fixed link c being usually the longest, while b and d are generally of equal length.

This condition is not, however, necessary. Suppose the proportions to be those shown in the figure. When the mechanism is in such a position that b and d are parallel, as shown by dotted lines in the figure, the virtual centre of a with regard to c is at an infinite distance away, in a direction parallel to b and d. At that instant every point on a is moving in a direction perpendicular to its virtual radius and therefore at right angles to AD and CB. We wish to find a point on a which will describe a straight line (approximately) during small displacements of the mechanism from its dotted position. Such a straight line must be at right angles to the parallel position of b and d, as we have just seen.

Imagine that the mechanism is moved into such a position as that shown by the full lines $AD'C'B$, b describing a small angle $\Delta\theta$, while d describes a small angle $\Delta\varphi$. The position of the virtual centre O_{ac} being found at O, it is evident that the point required on $C'D'$ must be such that its virtual radius is parallel to AD or CB, for if not, it would be describing a line not perpendicular to those lines, and therefore not in a straight line with its original direction of motion. If, then, we draw a line OE parallel to AD and cutting $C'D'$ in E, E is the point required. Draw $C'F$, $D'G$, perpendicular to OE. For an indefinitely small displacement of the mechanism the link a may be considered as remaining sensibly parallel to itself, so that $DD' = CC'$ and

$$\frac{AD}{BC} = \frac{\Delta\varphi}{\Delta\theta};$$

also for such a displacement O is at so great a distance that we may write $OG = OF$ without sensible error; hence for very small displacements

$$\frac{\Delta\varphi}{\Delta\theta} = \frac{C'F}{D'G} = \frac{C'E}{D'E}.$$

Finally,

$$\frac{AD}{BC} = \frac{C'E}{D'E}.$$

so that E must divide $C'D'$ in this proportion if its path is to be a straight line for small displacements. For larger displacements the path departs considerably from such a line.

It is better to proportion the lengths of the links so that DC is vertical when AD and BC are parallel; in that case the path of E will be approximately straight over a larger range of movement.

The links b and d may be placed both on the same side of a, as in Fig. 55. In this case E will be found to lie out-

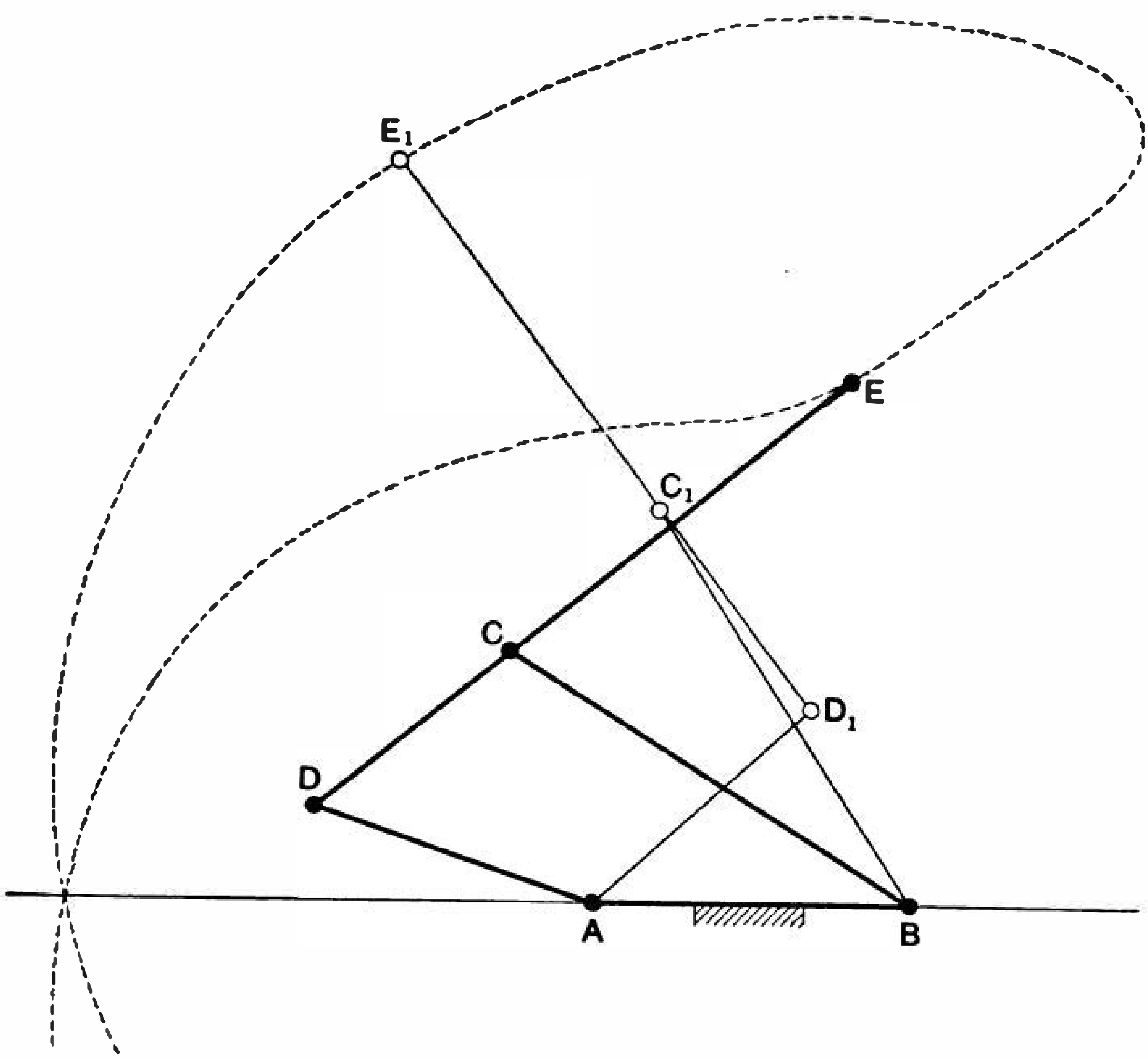


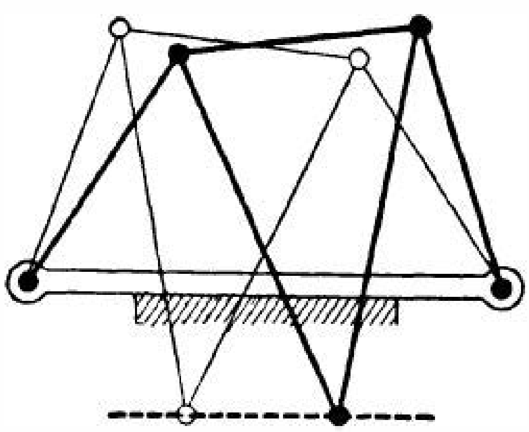
FIG. 55.

side CD, but as before $\frac{DE}{CE}=\frac{BC}{AD}$. The figure shows one half of the complete path of the point E, as well as two positions of the mechanism.

This arrangement was also used by Watt.

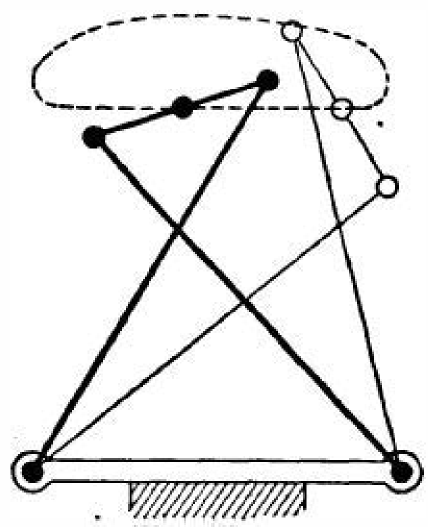
It may be shown that the part of the path of E used for a straight line is in reality wavy. Rules for designing Watt straight-line motions, as well as other forms, are given in Rankine's "Machinery and Millwork," § 253 *et seq.*

A number of other approximate straight-line motions* are derived from the quadric crank-chain. Among these the most interesting are those of Roberts and of Tchebicheff,† shown in Figs. 56*a* and 56*b*.



ROBERTS

FIG. 56*a*.



TCHEBICHEFF

FIG. 56*b*.

33. Accurate Straight-line Motions.—The first geometrically correct straight-line motion was devised in 1864 by M. Peaucellier. It is shown in Fig. 57 and is a compound chain of eight links of which

$$a = b = c = d;$$
$$e = f;$$
$$g = h.$$

The links a, b, c, d, e, f form a kite-shaped figure known as the Peaucellier cell. It has the following properties:

(1) A, D, and C must always be in one straight line.

(2) $AB^2 - BC^2 = AG^2 - GC^2$ (since AC and BE are at right angles).

Hence $AB^2 - BC^2 = (AG + GC)(AG - GC) = AC \,.\, AD$.

Therefore $AC \,.\, AD$ is constant.

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* Burmester's Lehrbuch der Kinematik, § 255 et seq.

† See *Engineering*, Vol. XVI, p. 284.

Suppose the chain moved until the line AC coincides with AF. Let J be the position of C, while H is the position of D. We have

$$HA \,.\, JA = DA \,.\, CA,$$

or

$$\frac{HA}{DA} = \frac{CA}{JA}.$$

Also, $FH = FD = FA$, so that H, D, and A lie on a semicircle.

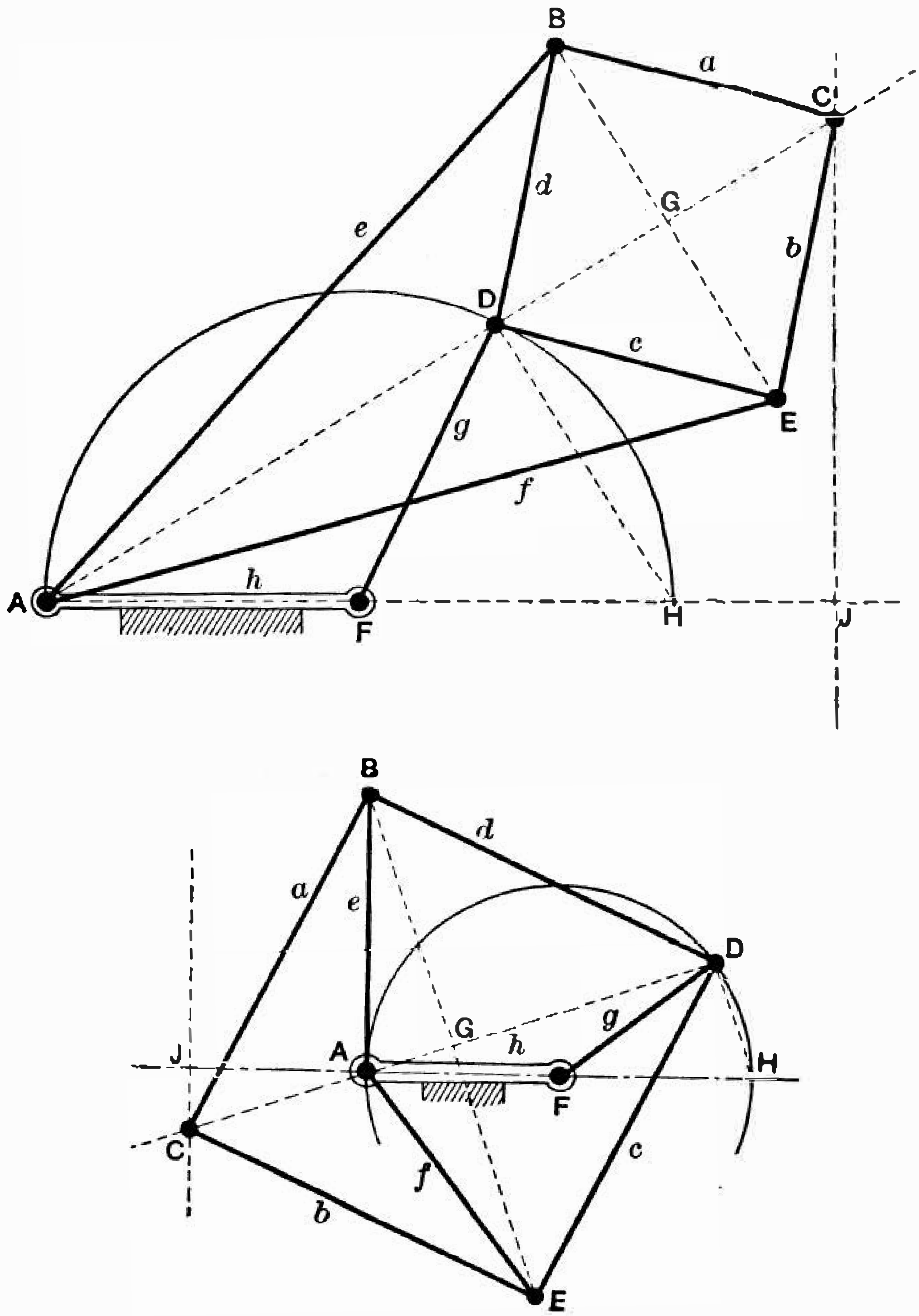

FIG. 57.

But in the triangles HAD, CAJ the angle CAJ is common, so that the triangles are similar, and the angle CJA is equal to HDA, which is the angle in a semicircle. Therefore CJA is a right angle. In the same way it may be proved that for any other position of C the line drawn from C to J is at right angles to AJ.

Hence all possible positions of C lie on a line drawn through J perpendicular to AJ, and therefore C describes a straight line.

Let us inquire what is the result of varying the lengths of the links g and h. It is plain that by making $h=0$ the points A and F would coincide and C would describe a circle of radius AC. From this it would appear probable

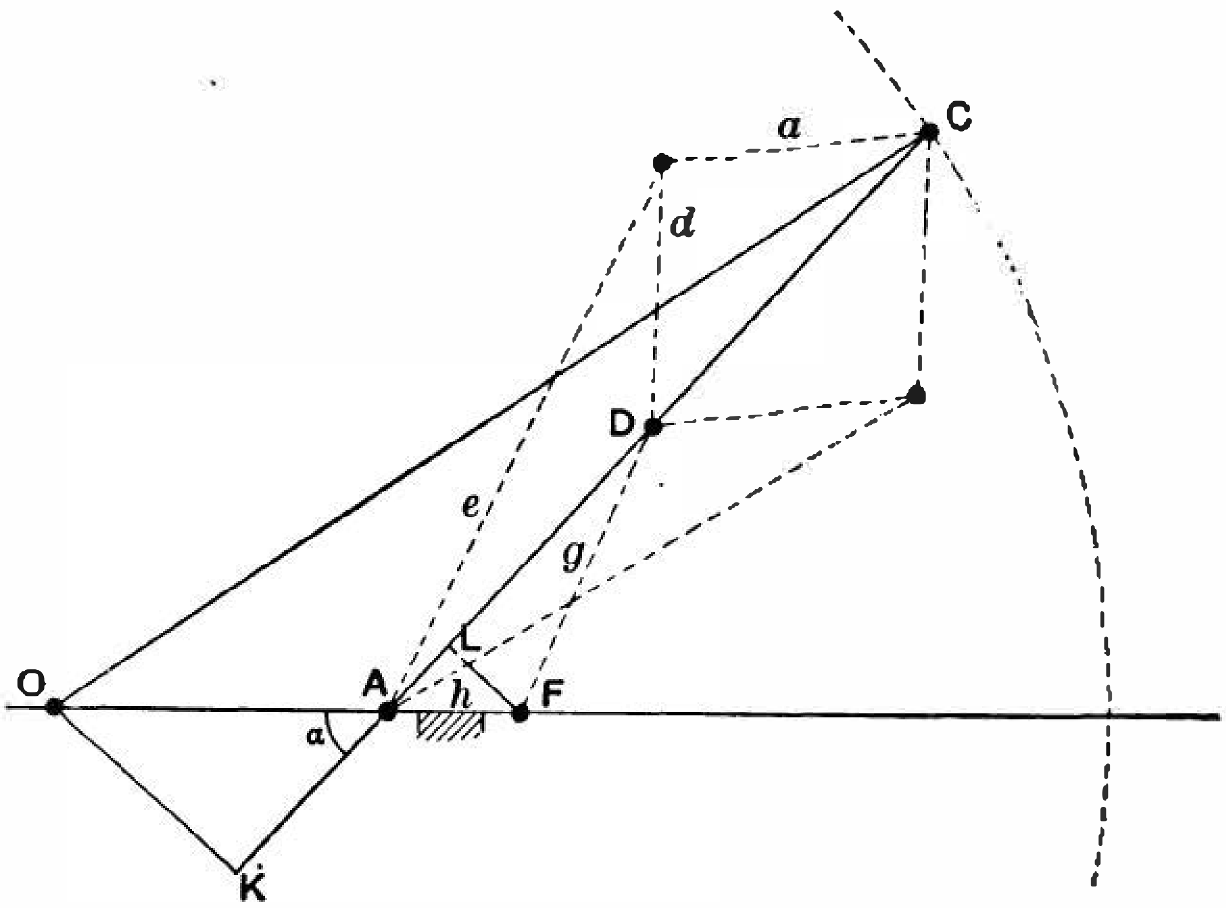

FIG. 58.

that by giving h suitable lengths less than that of g, we might describe circular arcs of any radius from AC up to infinity. If h be made greater than g, c will still describe a circular arc, but it will be convex towards A and the radius continually diminishes as g diminishes.

The locus of C may be proved to be a circular arc as follows:

In Fig. 58 we have, as before, $e^2-a^2=AC \cdot AD$. On FA produced mark off $AO=k$, where

$$k=\frac{-h(e^2-a^2)}{h^2-g^2},$$

and

$$k(h^2-g^2)=-h(e^2-a^2).$$

Join OC. Draw OK, FL perpendicular to AC. Let the angle $CAF=\alpha$. Then

$$AD=h\cos\alpha+\sqrt{g^2-h^2\sin^2\alpha},$$

from which

$$\cos\alpha=\frac{AD^2+h^2-g^2}{2h\,.\,AD}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again,

$$AC=\sqrt{OC^2-k^2\sin^2\alpha}-k\cos\alpha,$$

from which

$$\cos\alpha=\frac{OC^2-k^2-AC^2}{2k\,.\,AC}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2),

$$\frac{AD}{h}+\frac{h^2-g^2}{h\,.\,AD}=\frac{OC^2}{k\,.\,AC}-\frac{k}{AC}-\frac{AC}{k}.$$

But $$\frac{h^2-g^2}{h\,.\,AD}=-\frac{AC}{k}.$$

Hence $$\frac{AD}{h}=\frac{OC^2-k^2}{k\,.\,AC}$$

and $$OC^2=k^2-\frac{k^2}{h^2}(h^2-g^2)=\frac{g^2k^2}{h^2}.$$

But $$\frac{k}{h}=\frac{e^2-a^2}{h^2-g^2}.$$

Thus $OC=-\frac{g(e^2-a^2)}{h^2-g^2}$, which is constant.

The locus of C is therefore a circle of radius

$$R=-\frac{g(e^2-a^2)}{h^2-g^2},$$

whose centre is at a distance k from A, such that

$$k=-\frac{h(e^2-a^2)}{h^2-g^2}.$$

Note that if $h=g$, $k=\infty$ and $R=\infty$; the locus of C is then a straight line as in the Peaucellier straight-line motion. If $h<g$ and $e>a$, R is positive; while if $h>g$, R has a negative value, and the circle is convex towards A. The mechanism may thus be used conveniently for describing arcs of large circles. A graphic method of determining the proper lengths of links for this purpose has been devised by Professor Elliott.*

Another compound chain containing only turning pairs, and giving a geometrically correct straight-line motion, is that of Bricard.†

It consists of six links, arranged as in Fig. 59, such that the lengths

$$AB=AC=a;$$
$$FB=GC=b;$$
$$FG=c;$$
$$BD=CE=\frac{a^2}{b}.$$

* MacLay, Mechanical Drawing, § XLII.

† See Comptes Rendus, Vol. 120.

If the point A is to describe the straight line AK, bisecting FG at right angles, the length DE must be constant and equal to $\frac{ac}{b}$.

This is seen to be the case from the following considerations: If AF, AG, AD, AE, be joined, it is evident from

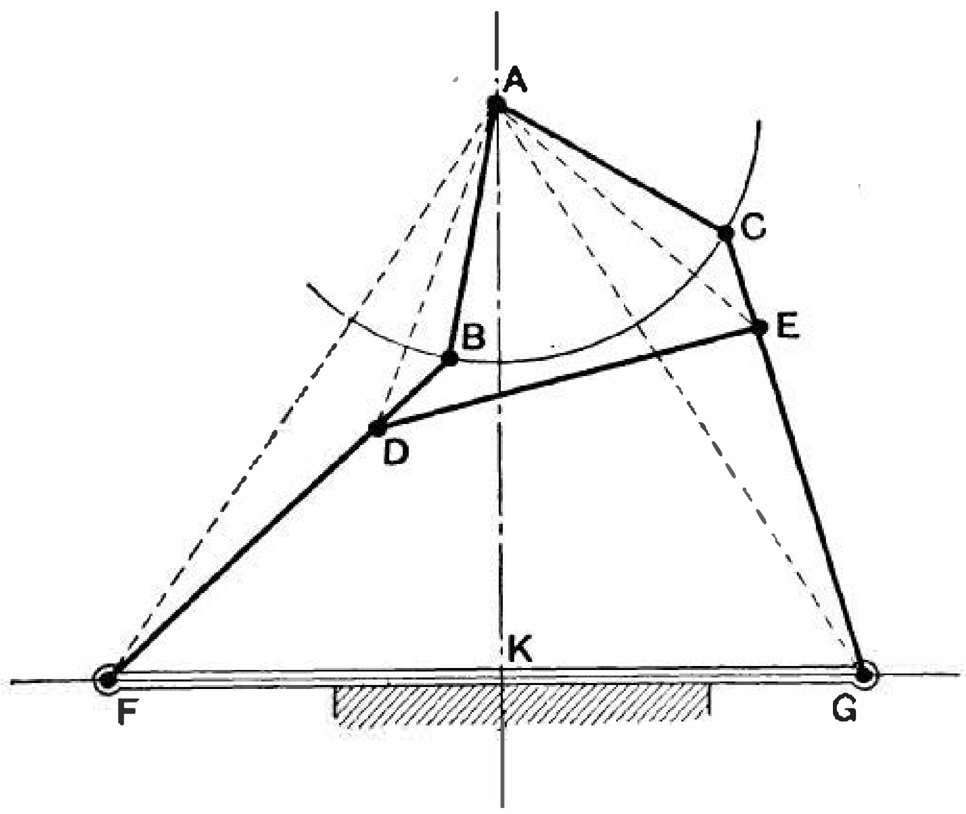

FIG. 59.

symmetry that the triangles ABF, ACG are equal in all respects, so that the angles DBA, ECA are equal. The sides AB, BD are equal to the sides AC, CE; thus $AD=AE$. Then the triangles ADF, AEG are equal and the angles DAF and EAG are equal. Add to each the angle DAG, then the angles FAG, DAE are equal. But $DA=EA$ and $FA=GA$, therefore the triangles FAG, DAE are similar and $\frac{DE}{FG}=\frac{AD}{AF}$, or $DE=FG.\frac{AD}{AF}$. But in the triangles AFB, BAD we have the angle FBA common, while by construction $\frac{FB}{AB}=\frac{b}{a}=\frac{AB}{BD}$; thus

$$\frac{FA}{AD}=\frac{FB}{AB} \quad \text{and} \quad DE=FG.\frac{AB}{FB}=\frac{ac}{b}.$$

For the required path of A, therefore, DE is constant and equal to $\frac{a}{b} \times c$.

Further information on the subject of straight-line motions will be found in the books and papers to which references have been given.