Should Poverty and Inequality Measures Be Combined?

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Introduction

It is a great pleasure for me to present a paper at a conference in Erik Thorbecke's honor. Erik was instrumental in attracting me to Cornell. In the years during which we have been colleagues, Erik and I offered the graduate economic development sequence together. We have had many stimulating discussions, none more so than when Foster, Greer, and Thorbecke's P_α poverty index was being developed in Uris Hall. Erik and I have talked often about poverty, not only how to measure it but how to combat it.

As the title of my paper indicates, the purpose of today's presentation is to analyze whether poverty and inequality measures should be blended into a single index. This question was first raised to me by Erik, so it is very fitting that an answer be presented at a conference honoring his distinguished career.

The first step in answering Erik's question is to be clear on what one means by "poverty" and "inequality." For this conference, there can be only one way of measuring poverty: the P_α (or *FGT)* class of indices (Foster, Greer, and Thorbecke, 1984). "Inequality" will be measured by Lorenz curves and a particular Lorenz-consistent inequality index.

The next step is to ask what purpose would be served by blending poverty and inequality measures. Researchers who work with poverty lines and poverty measures judge it valuable to assess deprivation vis-a-vis a poverty line. Deprivation profiles and deprivation indices have been analyzed systematically by Jenkins and Lambert (1997) and Shorrocks (1998). As with many other poverty measures, the P_α class gauges deprivation by fixing a poverty line in real terms and calculating poverty based only on the incomes of people below that line. This is what Amartya Sen dubbed the "focus axiom" in justifying his own poverty measure (Sen, 1976). One critique of this exclusive focus on those at or below a fixed real poverty line is that it is *too* focused, because poverty has a relative as well as an absolute component. That is, *i's* poverty can change not because *i's* real income changes but because *others'* real incomes change. Such "relative poverty" notions are described in Fields (2001). This motivation for combining poverty and inequality may be called the "relative poverty" rationale.

With this as motivation, I now proceed to analyze whether poverty and inequality should be blended into a single index. The first step is to ask, can it be done, to which the answer is "yes, it is possible." The second step is to ask, is it desirable to do it? My answer is, "for the most part, no."

Notation and Terminology

The P_α poverty measure applies to deficits of any variable among any units of observation. For ease of exposition, I will refer here to "incomes" received by "persons." Incomes among the n persons in the population are denoted by $z_1,...,z_n$ and are assumed to be ordered from lowest income to highest.

The P_α measure was designed to be an index of absolute poverty and is usually thought of in that way. Absolute poverty holds a poverty line constant in real dollars, pesos, or rupees. The poverty line therefore varies with inflation and only with inflation; it is therefore invariant with respect to other changes in the economy, in particular, to economic growth or economic decline. This fixed real poverty line will be denoted here by $\underline{z}$. (It is straightforward to extend the analysis to allow for persons to be of different types and for poverty lines to vary with needs, but I shall not pursue that extension here.)

The P_α index is a function of the normalized poverty deficits of individuals. The normalized poverty deficit of the i'th individual is

1. $$x_i = \frac{z-z_i}{z} \; if \; \frac{z-z_i}{z} \geq 0,$$

$$= 0 \; if \; \frac{z-z_i}{z} < 0.$$

The P_α index takes each of the normalized deficits, raises them to some power α, and then averages. Specifically, the P_α index, is

2. $$P_\alpha \equiv \frac{1}{n}\sum_i x_i^\alpha .$$

Blending Poverty and Inequality

The natural way of combining poverty and inequality would be to combine measures of each into a blended index. The *BLEND* concept is an amalgam of poverty and inequality, and as such can be thought of as an indicator of economic ill-being. Accordingly, any b(.) function is assumed to be increasing in both its poverty and its inequality arguments. Assuming differentiability, such a blend function would be of the form

3. $$BLEND = b(POV, INEQ), b_1 >0, b_2 >0.$$

An example of such a function is

4. $$BLEND = [w^* POVJ + [(I -w)^* INEQ],$$

with the weight *w* being chosen by the analyst. For the poverty measure, I shall use the P_α index, and for the inequality measure, the ratio of high incomes to low incomes.

Functions such as (3) and (4) show that it is *possible* to combine poverty and inequality by blending them into a single index. The next question we turn to is whether we *want* to.

Four Stylized Growth Types

In order to be able to gauge the suitability of blend measures of the form *BLEND = b(POV, INEQ),* it is useful to choose some stylized types of economic growth and ask how this class of measures behaves in each. I do this based on stylized dualistic development models of the type first formulated in Fields (1979). To sharpen the arguments, the analysis is carried out in a world of two incomes, z_p for the poorer persons and z_r $(>Z_p)$ for the richer ones. In all cases, the poverty line z is assumed to be between both the original and the final values of z and z_r.

Four cases of economic growth are considered:

Case I: Uniform percentage enrichment of everyone, raising both Z and z_r by the same multiple $m > 1$.

Case II: Poorer sector enrichment, which involves raising z_p only, holding z_r constant.

Case III: Richer sector enrichment, which involves raising z_r only, holding Z_p constant.

Case IV: Disequalizing growth, which involves raising both z_p and Z_r and raising z_p by a smaller percentage than z_r .

Let us now see how the *BLEND* class performs in each of these stylized growth types.

Applying *BLEND* To stylized Types of Economic Growth

Consider an index belonging to the class of *BLEND* measures *BLEND = b(POV, INEQ),* $b_1,>0$, $b_2>0$. Any such index i) rises when *POV* and *INEQ* both rise or when one rises and the other stays the same, ii) falls when *POV* and *INEQ* both fall or when one falls and the other stays the same, and iii) changes ambiguously when *POV* changes in one direction and *INEQ* changes in the other.

Taking the P_α index

$$P_\alpha \equiv \frac{1}{n}\sum_i x_i^a.$$

as our measure of *POV,* we find that P_α falls when some or all of the incomes below the poverty line rise; this happens in Cases I, II, and IV. Taking the rich/poor income ratio z_r / Z as our measure of inequality, we find that *INEQ* is constant in Case I, falls in Case II, and rises in Cases III and IV.

Looking now at the four growth cases one by one:

Case I involves a uniform percentage enrichment of everyone, raising Z_p and z_r by the same multiple $m > 1$. We would expect that economic ill-being would fall if all incomes rise by the same percentage, and indeed both approaches agree with this expectation.

Case II entails poorer sector enrichment, in which only z_p is raised, holding z_r constant. Increases in incomes of the poorer persons, holding incomes of the richer persons constant, lowers poverty and lowers inequality. Because *BLEND* is increasing in both of these arguments, the *BLEND* approach therefore concludes that economic ill-being falls when poorer sector enrichment takes place.

In Case III, the richer people get richer. Because they are above the poverty line to begin with, *POV* does not change. However, inequality as measured by z_r / Z_p increases. Constant *POV* and rising *INEQ* imply that economic ill-being *increases* with this type of growth.

Finally, in Case IV, everyone gets richer but those with z_r enjoy larger percentage gains than those with z_p do. Because the poor are getting richer, *POV* falls. And because the richer are getting larger percentage gains, *INEQ* as measured by the ratio z_r / Z_p increases. *BLEND* is therefore ambiguous.

In summary, ill-being as gauged by *BLEND* falls in Cases I and II, rises in Case III, and changes ambiguously in Case IV. These results are summarized in Table 5-1.

Discussion of Results

Discrepancy between *BLEND* results and *POV* results

The discrepancies that arise between the *BLEND* results and the *POV* results in Cases HI and IV come about for the following reason. *BLEND* assigns a *positive* change in ill-being to an increase in *INEQ.* On the other hand, the *POV* approach in general and the P_α approach in particular are concerned only with the incomes of the poor, and so give no weight, positive or negative, to the income gains of the non-poor. When z_P rises, *POV* falls and so too does ill-being.

Which is the "better" way to look at distributional change? The choice is best made axiomatically. The interested reader is invited to contrast the POV axioms in Chakraborty, Pattanaik, and Xu (2002) and Foster (this volume) with the *INEQ* axioms in Fields and Fei (1978) and Foster and Sen (1997). You might also try to axiomatize *BLEND.* I have been singularly unsuccessful in getting anywhere with it; I think this is because it is unclear to me what the primitive concept of *BLEND* is.

Contrast with welfare dominance results

The *BLEND* and *POV* results differ from the results that would be obtained by comparing the distributions using dominance methods. In all four of these cases, applying the methods devised for welfare dominance (Hadar and Russell, 1969; Saposnik, 1981) and for poverty dominance (Atkinson, 1987; Ravallion, 1994), we find that the new income-distribution first-order-dominates the old. Therefore, the class of economic well-being functions of the form

$$W=W(z_1, Z_2,\ldots, z_n),\ \text{W(.) increasing in all } z_i$$

shows *higher* economic well-being when any of these types of economic growth takes place. The unambiguous improvements recorded in Cases III and IV using dominance methods are at odds with the results using *BLEND.* As is well-known (e.g., Shorrocks, 1983) the different judgments come about because of the way the different methods evaluate the income gains of the richer people.

How does using *BLEND* compare with using its components but not blending them?

Let us now evaluate the usefulness of extending the analysis beyond P_α by using *BLEND.* The advantage is that which Erik Thorbecke first suggested: that although the P_α and other *POV* measures pay no attention to incomes of people above the poverty line, *BLEND* does. In particular, *BLEND* takes account of the incomes of the non-poor by looking at inequality of the income distribution and then combining *POV* and *INEQ* into a single number. An example of how to do this is to choose a particular *BLEND* function - for example,

4. $$BLEND = [w * POV] + [(1 - w) * INEQ],$$

with the weight *w* being chosen by the analyst - and calculating just *BLEND.*

Anyone who only calculates *BLEND* will miss entirely what happens with each of the components. To me, this is a serious omission, because I would most assuredly want to know what happens to *POV* and *INEQ separately,* not just the blend of the two.

There is, however, one distinct advantage of using a particular *BLEND* function such as (4) with particular weights *w.* It is that the analyst may wish to use the calculated values to *decide* what s/he thinks of the overall change in the income distribution - in particular, whether the changes that take place have made things better or worse. Of course, for *BLEND* calculations to be useful, the analyst must be extremely careful about which *POV* and *INEQ* measures to use and which weights to choose.

Conclusion

This paper set out to answer two questions. The first was whether poverty in general and the P_α measure in particular can be extended to include inequality considerations. The answer has been "yes." The second question was, given that the answer to the first question is "yes," is it desirable to extend the *POV* measure in this way? The analysis was carried out using the P_α measure and also *INEQ* and applying them to four stylized growth types. My summary judgment is that poverty and inequality measures can be combined but anyone who does so should be very aware of the limitations involved.

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Table 5-1. [Changes in Four Dualistic Growth Typologies]

Growth Type	*BLEND* and its Components			Dominance
	POV	*INEQ*	*BLEND* ≡ *b(POV, INEQ)*	*First order dominance*
Case I. Uniform percentage increases.	↓	→	↓	↑
Case II. Increase in incomes only of the poorer.	↓	↓	↓	↑
Case III. Increase in incomes only of the richer.	→	↑	↑	↑
Case IV. Increase in all incomes with larger percentage increases for the richer.	↓	↑	↑ or ↓	↑

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