

THE IMPACT OF QUANTITATIVE EASING ON THE
TERM STRUCTURE OF INTEREST RATES AND
FOREIGN EXCHANGE RATES

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Hao Li

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THE IMPACT OF QUANTITATIVE EASING ON THE TERM STRUCTURE
OF INTEREST RATES AND FOREIGN EXCHANGE RATES

Hao Li, Ph.D.

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This dissertation estimates the impact of the Federal Reserve's 2008 - 2011 quantitative easing (QE) program on the U.S. term structure of interest rates and USD/JPY exchange rate. We estimate an arbitrage-free term structure model that explicitly includes the quantity impact of the Fed's trades on Treasury market prices. As such, we are able to estimate both the magnitude and duration of the QE price effects. We show that the Fed's QE program affected forward rates without introducing arbitrage opportunities into the Treasury security markets. Short- to medium- term forward rates were reduced (less than twelve years), but the QE had little if any impact on long-term forward rates. This is in contrast to the Fed's stated intentions for the QE program. We also extend the framework to analyze the effect of the Fed's QE program on the USD/JPY exchange rate. We find that the Fed's QE accounts for a significant portion of the depreciation of the dollar during this period. A central monetary authority can affect exchange rates directly by intervening in foreign exchange markets, or indirectly by affecting interest rates. The analysis emphasizes the importance of the latter indirect channel, especially in a period when a central bank undertakes massive bond purchasing activities.

BIOGRAPHICAL SKETCH

Hao Li was born in Jingshan in Hubei Province of China, on October 31, 1984. He grew up in Jingzhou, which is a town with more than two thousand years' history. Hao graduated from the University of Science and Technology of China with Bachelor's degrees in Physics and Computer Science in 2005. He also earned a Master's degree in Physics from the University of Maryland at College Park in 2008. Hao is married to Juan Li.

This dissertation is dedicated to my loving and supportive wife, Juan Li, and to my encouraging and faithful parents, Xiangping Kong and Aiguo Li.

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CHAPTER 1

INTRODUCTION

Following the 2007-2009 financial crisis, many central banks around the globe conducted an unconventional monetary policy tool, known as quantitative easing (QE). It is a bond market intervention where a central bank purchases very large volume of bond securities. The purpose of QE is to lower interest rates and to boost prices of risky assets, which might be beneficial for the economic recovery.

In particular, between November 2008 and March 2010 the Federal Reserve conducted massive asset purchases known as the first round of quantitative easing (QE1) to lower long-term interest rates and spur economic growth. In QE1 the Fed purchased approximately \$1.75 trillion of assets consisting of \$1.25 trillion mortgage-backed securities (MBS), \$300 billion Treasury securities, and \$200 billion federal agency debt. Between November 2010 and June 2011, another phase of quantitative easing known as QE2 was implemented, consisting of an additional purchase of \$600 billion long-term Treasuries. Because of the size of the asset purchases, the Fed's QE has important impacts in various asset markets.

One important question is whether QE was successful at lowering interest rates given a federal funds rate (Figure 1.1) that has been almost zero since the end of 2008.¹ And if effective, which Treasury rates were lowered, by how much, and for how long?

Another notable phenomenon is that the U.S. dollar has depreciated significantly against the Japanese yen during the QE periods (Figure 1.2). One may wonder whether there is any relation between the Fed's QE and the dollar depre-

¹See Bernanke and Reinhart (2004) for a discussion of monetary policies around the zero lower bound for short-term interest rates.

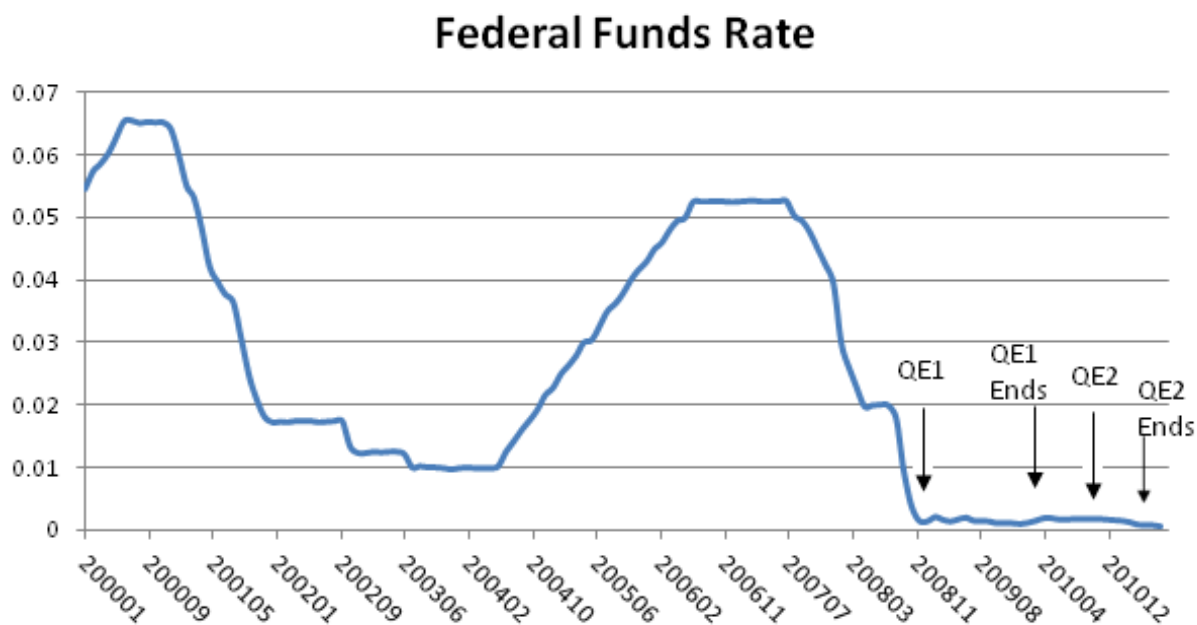


Figure 1.1: Time-series of Federal Funds Rate

ciation. If yes, how do we quantify this QE effect on the dollar value?

This dissertation tries to address the above questions. It proposes a framework to quantify the QE impact on both the interest rate market and the foreign exchange market.

The outline for the rest of this dissertation is as follows. Chapter 2 studies the QE impact on the whole term structure of U.S. Treasury rates. Chapter 3 examines the QE impact on the dollar/yen exchange rate. The conclusion is provided in Chapter 4.

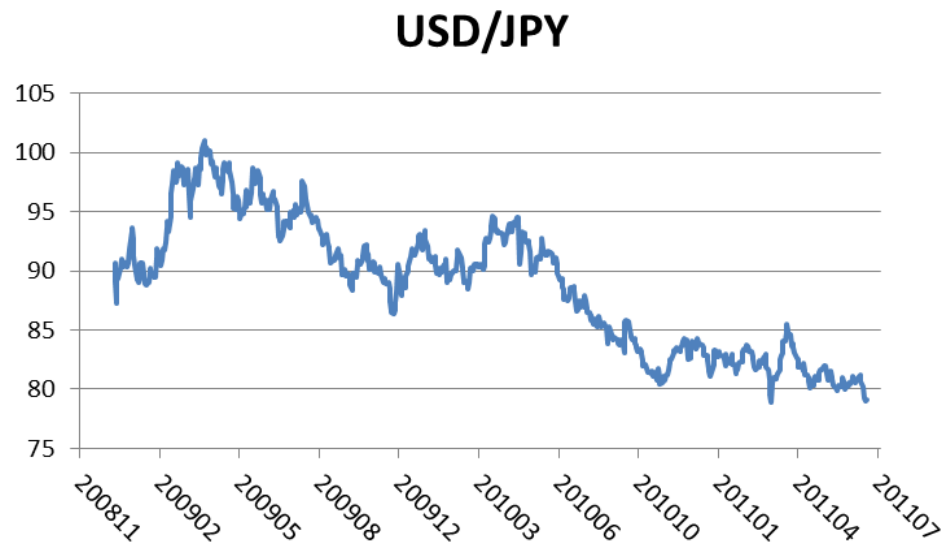


Figure 1.2: Time-series of dollar/yen exchange rate

CHAPTER 2

THE IMPACT OF QUANTITATIVE EASING ON THE TERM STRUCTURE OF U.S. TREASURY RATES

During the first and second round of quantitative easing (QE), the Federal Reserve purchased a huge volume of mortgage-backed securities and Treasury securities (see Chapter 1). Since it is a bond market intervention, one may wonder how much impact it has on interest rates.

The existing empirical literature provides strong support for the proposition that both QE1 and QE2 were effective in lowering long-term Treasury yields, but it provides less evidence with respect to how long the effects of the asset purchases lasted. Studies of the effectiveness of QE include Bernanke, Reinhart, and Sack (2004), D'Amico and King (2012), Gagnon, Raskin, Remache, and Sack (2010), Hamilton and Wu (2012), Krishnamurthy and Vissing-Jorgensen (2011), Li and Wei (2012), Meaning and Zhu (2011), and Wright (2012). Related work also includes Fuster and Willen (2010) on the QE effects of purchasing mortgages, Oda and Ueda (2005) who investigate the Bank of Japan's zero interest rate policy from 1999 to 2003, Swanson (2011) on the 1961 Operation Twist, and Joyce, Lasasoa, Stevens and Tong (2010) on the QE impacts from the Bank of England. Broadly speaking, there have been two approaches used in this literature to study the effectiveness of QE1 and QE2: performing an event study on the announcement day of a large asset purchase, or estimating a time series equilibrium or arbitrage-free model for the term structure of interest rates.

In an event study, the window after the announcement date is purposely kept small, usually one or two days, in order to minimize the confounding effect of changing macro-economic conditions on the observed change in Treasury rates.

The advantage of an event study is that it does not require the specification of a particular equilibrium or arbitrage-free model. Hence, the results are robust to model misspecification. The key disadvantage of an event study is that it only measures the impact of an asset purchase over the event window considered. Cumulating event window changes over longer time horizons is confounded by changes in macro-economic conditions. In addition, for their validity, event studies require two additional assumptions to hold. One, that the announcement was not leaked before the announcement date, and two, that any price impact is instantaneous and not lagged across time. It is an open question as to the validity of these assumptions in Treasury markets.

In a time series model, to measure the impact of QE, the usual approach is to perform a counterfactual experiment. Using a model based on no QE policy, one can estimate the expected path of the term structure of interest rates. This is the counterfactual control. This no QE policy forecast is then compared with realized rates (or conditional expectations based on the realized rates) generated under the QE policy. The difference between these two rates is due to random noise and QE. The advantage of a time series model is that it captures the changing macro-economic conditions during the estimation period. The disadvantage of this approach is that the difference between the expected and realized rates may include other components, not due to QE, if the time series model is misspecified. Unfortunately, the models implemented are simplified for analytic or econometric reasons, making this a reasonable concern.

Our contribution is three-fold. First, we estimate the impact of QE on the term structure of forward rates, and not bond yields. Forward rates correspond to the "marginal" rate for a future time interval, while yields correspond to the "average"

rate over a time horizon starting today and ending with the bond's maturity. As such, since yields are averages of forward rates, yields confound changes in short- and long- term forward rates. Our study isolates the effects of QE on the different maturity forward rates.

The second contribution is to provide a new and alternative methodology for estimating the impact of QE on the term structure of interest rates. Our methodology uses an arbitrage-free term structure evolution that explicitly includes the impact of the Fed's Treasury purchases on the price. As such, our approach is able to estimate both the magnitude and the duration of QE price impacts on the term structure of Treasuries. In addition, our methodology can be used to price interest rate derivatives (e.g. caps, floors, swaptions) given the Fed's activities. By modifying the Fed's impact parameters, our methodology can also be used to forecast/predict changes in interest rate derivative prices due to forecasted changes in the Fed's purchases. The existing approaches for estimating the impact of quantitative easing cannot be used for this purpose. The application of our approach to derivative pricing awaits subsequent research.

The idea behind our methodology is that we can decompose the observed forward rate curve into two components: one is a hypothetical market forward rate curve without the Fed's purchases, and the other is the price impact due to QE. The advantage of our approach, as contrasted with the existing time series methodology, is that we do not need to set up a counterfactual experiment. Instead, this relation is explicitly built into our parametric model for the evolution of the term structure of interest rates. Our empirical methodology is based on the literature studying the pricing of derivatives in an arbitrage-free economy with a large trader (see Jarrow (1992), Bank and Baum (2004), Jarrow, Protter, Roch (2011)). It is

most closely related to Jarrow, Protter, and Roch (2011) who study the divergence in an asset's price from fundamental value caused by trading activity. Similar to our methodology, Li and Wei (2012) add a supply factor to Treasury yields, measuring the impact of QE without a counterfactual experiment. However, both the theory and empirical methodology in Li and Wei differ from that used in this dissertation.

The third contribution is to test whether the Fed's QE purchases introduced arbitrage opportunities into the Treasury security markets. It is plausible that given such large scale purchases, the Fed could over pay for particular maturity Treasuries causing their risk premium to be distorted relative to close maturity substitutes. Although the Fed attempted to only purchase undervalued assets (see Gagnon, Raskin, Remache, and Sack (2010, p. 47)), it is an open question whether their purchases were successful in this regard.

Our estimation shows that the term structure of forward rates were affected by QE, and without the introduction of arbitrage opportunities. Short- and medium-term forward rates declined (up to 12 years) with the size of the impact decreasing in maturity. There were no discernible changes in long-term forward rates (greater than 12 years). The persistence of the price impacts increased with maturity up to 6 years then declined, with half-lives lasting approximately 4, 6, 12, 8 and 4 months for the 1, 2, 5, 10 and 12 year forwards, respectively. The Fed's QE activities did not affect long-term forward rates, contrary to the Fed's stated intentions. This is not surprising, however, given that the Fed's purchase activities were concentrated on bonds with maturities of less than 10 years (see Figure 2.3).

Since bond yields are averages of forward rates over a bond's maturity, QE did affect long-term bond yields. The average impacts on bond yields were 327, 26,

50, 70, and 76 basis points for 1, 2, 5, 10 and 30 years, respectively. These yield impacts are consistent with those estimated in the existing literature, except for the 1-year rate. Our 1-year estimated yield change is significantly greater than that in the existing literature because it includes the impact of the Fed's monetary policy - keeping short-term rates near the zero lower bound. The existing 1-year estimates come from an event study (see Krishnamurthy and Vissing-Jorgensen (2011)), which only includes the impact of QE alone. Secondly, unlike the estimates from an event study, our estimated changes in bond yields are consistent across the entire term structure. In particular, the 1-year rate change is embedded within the 5-year, 10-year, and 30-year yield changes to be consistent with an arbitrage-free term structure evolution.

These results can be best understood using the modified expectations hypothesis that always holds in an arbitrage-free term structure model (see Jarrow (2009)). The expectations hypothesis is modified for risk aversion using adjusted probabilities, instead of the actual probabilities. As in the classical expectations hypothesis, except for this modification, the time t forward rate for date T is the time t "expected" spot rate for date T . These results show that the impact of QE on the future spot rate is "expected" to disappear after 12 years. And in addition, the effect of a purchase on the future spot rate is "expected" to last longer, the longer the term of the rate up to about 6 years. Perhaps because most monetary policy activities occur on the very short-end of the curve, diminishing the lasting power of any quantity impact on the short-term forward rates.

2.1 The Model

This section constructs a Heath, Jarrow, Morton (1992) arbitrage-free term structure of interest rate model augmented to include the price impacts of a large trader, the Federal Reserve, based on the insights of Jarrow, Protter, Roch (2011). Traded are default-free zero-coupon bonds of all maturities and a money market account in a frictionless market. A frictionless market has no transaction costs, no restrictions on trade (e.g. short sale restrictions), and asset prices are perfectly divisible. All traders, except the Fed, act as price takers believing their trades have no impact on the price of the traded Treasuries. In contrast, the Fed's purchases are assumed to have a significant price impact, the details of which will be presented shortly.

2.1.1 The Term Structure Evolution

We let $P(t, T)$ denote the time t market price of a zero-coupon bond paying a dollar at time T . This price is observed at time t and reflects the presence of the Fed's purchasing activities. The time t forward rate for date T is denoted $F(t, T)$ and it is implicitly defined by

$$P(t, T) = e^{-\int_t^T F(t, s) ds}. \quad (2.1)$$

The instantaneous spot rate of interest is defined by $R(t) \equiv F(t, t)$. We note for subsequent usage in the empirical section that the spot rate of interest is a hypothetical construct that is unobservable in actual markets.¹

We let $p(t, T)$ denote the hypothetical unobserved zero-coupon bond price that would exist in the economy if the Fed did not trade. For convenience, we call

¹This is because the spot rate is defined by the limit condition: $R(t) = \lim_{\Delta \rightarrow 0} \left(\frac{1 - P(t, t + \Delta)}{P(t, t + \Delta)} \cdot \frac{1}{\Delta} \right)$.

$p(t, T)$ the "true" price. We denote the true forward and spot rate by $f(t, T)$ and $r(t)$, respectively.

We assume that the true forward rate process evolves according to an N - factor model:

$$df(t, T) = \mu(t, T)dt + \sum_{n=1}^N \sigma_n(t, T)dW_n(t) \quad (2.2)$$

where $W_i(t)$ for $i = 1, \dots, N$ are independent standard Wiener processes, $\mu(t, T)$ is the drift of the forward rate process, and $\sigma_n(t, T)$ is the volatility of the n^{th} factor. Of course, we assume the technical conditions necessary for this stochastic differential equation to exist (see Heath, Jarrow, Morton (1992) for these conditions). At this point, this evolution is quite general. Due to the Wiener processes, the only economic restriction being imposed is that the forward rates' sample path is continuous in time.

The true price process includes the impact of any expected or unexpected changes in the market's supply/demand for Treasuries caused by changes in the business cycle and normal economic activity, for example, an increase in the foreign demand for Treasuries during "flight-to-quality" episodes in the recent credit crisis. This process also reflects changes in the outstanding supply of Treasury securities as determined by the U. S. Treasury's auction activities.

2.1.2 The Fed's Price Impact

As mentioned previously, we consider the Fed as a large trader, whose purchase/sales affect the prices of Treasuries. In the large trader literature mentioned previously, a large trader's purchases/sales are private information (see Jarrow (1992), Bank and Baum (2004), Jarrow, Protter, Roch (2011)). This implies that the large

trader's price impact occurs at the time of trade, and not before. In contrast, the Fed's purchases/sales are public information, announced in advance of their trades. As such, in contrast to a large private trader, there will be an announcement effect on the price of Treasuries prior to their purchases, which needs to be incorporated into the model.

We consider a partial equilibrium model, similar to Jarrow, Protter, Roch (2011). Let $x(t, T)$ denote the time t cumulative changes in the aggregate demand for the T maturity zero-coupon bond caused by the Fed's activities, both the announcements and trades. As such, this consists of two components:

$$x(t, T) = y(t, T) + z(t, T)$$

where $y(t, T) \geq 0$ is the time t cumulative changes in the Fed's holdings of the T maturity zero-coupon bond due to their purchases, and $z(t, T)$ is the market's time t cumulative change in the demand for the T maturity zero-coupon bond due to the Fed's announcements. For example, at the announcement time in hopes of obtaining future profits, traders may purchase Treasuries in anticipation of a price rise at the time of the Fed's purchases, causing prices to react immediately.

Given the change in aggregate demand, we assume that the Fed's activities (announcements and trades) affect the evolution of the observed forward rates as follows:

$$dF(t, T) = \lambda(t, T)(f(t, T) - F(t, T))dt + df(t, T) - d\Psi(t, T) \quad (2.3)$$

where $\lambda(t, T)$ corresponds to the rate of mean reversion of the observed to the true forward rate, $g(x(t, T), T)$ corresponds to the marginal impact of the change in aggregate demand $dx(t, T)$ on the observed forward rate, and $d\Psi(t, T) \equiv g(x(t, T), T)dx(t, T)$. As noted, we assume that the forward rate decreases as aggregate demand increases.

Solving this stochastic differential equation, we can alternatively express this assumption as

$$F(t, T) = f(t, T) - \int_0^t e^{-\int_s^t \lambda(u, T) du} d\Psi(s, T). \quad (2.4)$$

In this form the motivation for this assumption is clear. The observed market forward rate $F(t, T)$ can be decomposed into two components. The first is the true forward rate $f(t, T)$, to which the observed rate mean reverts. The second component is the price impact of the Fed's activities, which depend on both the rate of mean reversion $\lambda(t, T)$ and the marginal impact of the change in aggregate demand $d\Psi(s, T)$. At this point, $\lambda(t, T)$ and $\Psi(t, T)$ can be very general stochastic processes. The only restrictions are those necessary to make expression (2.4) well-defined and exist.²

For subsequent usage, we note that in this reduced form model, the impact on Treasury prices (T near 0) due to the Fed's short-term interest rate monetary policy are included in this component as well.

Finally, it is important to note that although the Fed's holdings $y(t, T)$ are observable, the accumulated changes in aggregate demand $z(t, T)$ are not. Consequently, $x(t, T)$ is not observable, so that we will not be able to empirically decompose the marginal impact $d\Psi(s, T) = g(x(s, T), T)dx(s, T)$ into its component parts. This explains why we use the simplified notation in expression (2.4) above.

To facilitate empirical estimation, we impose the following additional structure. First, we assume that the Fed starts its activities with an announcement at time 0, and the purchases end at some known future time τ . Second, we let the Fed's price impact on the T -maturity forward rate be a deterministic function of the

²For example, for each T , $\Psi(t, T)$ needs to be a semimartingale.

rate's maturity, i.e.

$$\Psi(0, T) = 0, \quad d\Psi(t, T) = I_{\{t \leq \tau\}} \psi(T) dt \quad (2.5)$$

where $\psi(T)$ is the marginal price impact rate, per year, and $I_{\{\cdot\}}$ is an indicator function. This assumption implies that the Fed's purchases do not introduce additional randomness into the forward rate's evolution. Alternatively stated, the forward rate process can be viewed as a controlled process where the Fed chooses the marginal price impact rate.

Last, we let the mean reversion rate also be a deterministic function of the forward rate's maturity, i.e.

$$\lambda(t, T) = \lambda(T). \quad (2.6)$$

Under these additional assumptions, we can rewrite expression (2.4) as:

$$F(t, T) = \begin{cases} f(0, T) \\ f(t, T) - \frac{\psi(T)}{\lambda(T)}(1 - e^{-\lambda(T)t}), & \text{if } 0 < t \leq \tau \\ f(t, T) - \frac{\psi(T)}{\lambda(T)}(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t}, & \text{if } t > \tau. \end{cases} \quad (2.7)$$

From expression (2.7), we can easily derive the evolution of the observed zero-coupon bond price.

For $0 < t \leq \tau$ we have

$$\begin{aligned} P(t, T) &= e^{-\int_t^T F(t, s) ds} \\ &= p(t, T) \exp\left\{\int_t^T \frac{\psi(s)}{\lambda(s)}(1 - e^{-\lambda(s)t}) ds\right\}. \end{aligned} \quad (2.8)$$

For $t > \tau$, we have

$$P(t, T) = p(t, T) \exp\left\{\int_t^T \frac{\psi(s)}{\lambda(s)}(e^{\lambda(s)\tau} - 1)e^{-\lambda(s)t} ds\right\} \quad (2.9)$$

As indicated, the Fed's purchases increase the true price $p(t, T)$ by the proportionality factor given in expressions (2.8) and (2.9). Defining the price impact as $\delta(t, T) \equiv \ln P(t, T) - \ln p(t, T)$, expressions (2.8) and (2.9) imply that

$$\delta(t, T) = \begin{cases} \int_t^T \frac{\psi(s)}{\lambda(s)} (1 - e^{-\lambda(s)t}) ds, & \text{if } 0 < t \leq \tau \\ \int_t^T \frac{\psi(s)}{\lambda(s)} (e^{\lambda(s)\tau} - 1) e^{-\lambda(s)t} ds, & \text{if } t > \tau. \end{cases} \quad (2.10)$$

We see that the price impact increases as time t increases, then decreases as the bond approaches its maturity. The price impact is zero at both $t = 0$ and $t = T$. The maximum distortion is achieved at some time $t^* \in (0, \tau]$. Of course, the objective of the empirical section is to estimate the magnitude of this quantity for the different maturity Treasury securities during the Fed's QE.

2.1.3 The Arbitrage-free Restrictions

We assume that the observed forward rate evolution, before and after the Fed's purchases, is arbitrage-free. This section studies the restrictions that this no arbitrage assumption imposes. To obtain these restrictions, we start with expression (2.7), rewritten in differential form:

$$dF(t, T) = \begin{cases} df(t, T) - \psi(T) e^{-\lambda(T)t} dt, & \text{if } t \leq \tau \\ df(t, T) + \psi(T) (e^{\lambda(T)\tau} - 1) e^{-\lambda(T)t} dt, & \text{if } t > \tau. \end{cases} \quad (2.11)$$

Here it is seen that the Fed's buying activity is deterministic and only affects the drift of the observed forward rate's evolution. Otherwise, the evolution of the true forward rate process is unaffected. Thus, one can directly apply the HJM no arbitrage drift conditions (HJM (1992)) to obtain the following theorem. The proof is in the appendix.

Theorem 1 *No Arbitrage Conditions*

Given $(\sigma_n(t, T), \phi_n(t))$ for all n , and $(\lambda(T), \psi(T))$ for all T , the observed forward rate evolution is arbitrage free if and only if there exist $(\Phi_n(t))$ for all n such that

$$\sum_{n=1}^N \sigma_n(t, T) \Phi_n(t) = \sum_{n=1}^N \sigma_n(t, T) \phi_n(t) + \begin{cases} \psi(T) e^{-\lambda(T)t}, & \text{if } 0 < t \leq \tau \\ \psi(T) (1 - e^{\lambda(T)\tau}) e^{-\lambda(T)t}, & \text{if } t > \tau \end{cases} \quad (2.12)$$

where $\Phi_n(t)$ ($\phi_n(t)$) are the market prices of risk for factor n with (without) the Fed's price impact.

This theorem shows that in an economy whose term structure evolution is arbitrage-free, the Fed's purchases necessarily change the market prices of risk in the economy (from $\phi_n(t)$ to $\Phi_n(t)$ for all n). This makes intuitive sense because the Fed's purchases, changing aggregate demand, causes a shifting in the economy's equilibrium. To reduce the traders' aggregate demands, to meet the decreased available supply, equilibrium risk premium must adjust and expression (2.12) shows exactly how.

This shift in risk premium can be better understood by studying a one-factor model. In this case, the HJM no-arbitrage drift restriction is

$$\Phi(t) = \phi(t) + \begin{cases} \frac{\psi(T) e^{-\lambda(T)t}}{\sigma(t, T)}, & \text{if } 0 < t \leq \tau \\ \frac{\psi(T) (1 - e^{\lambda(T)\tau}) e^{-\lambda(T)t}}{\sigma(t, T)}, & \text{if } t > \tau. \end{cases} \quad (2.13)$$

Expression (2.13) shows that when the Fed is buying, risk premium must increase by the positive quantity on the right side of this expression to keep the term structure evolution arbitrage-free.

We will test below to see if expression (2.12) holds during the QE program. As mentioned in the introduction, although the Fed attempted to only purchase

undervalued assets (see Gagnon, Raskin, Remache, and Sack (2010, p. 47)), it is plausible that given such large scale purchases the Fed could have over paid for particular maturity Treasuries causing their risk premium to be distorted relative to close maturity substitutes.

2.2 Estimation

This section estimates the Fed's QE price impact on the term structure of interest rates using the arbitrage-free term structure model developed in the previous section. Included in this estimation is the Fed's price impact on the spot rate of interest (the difference between $R(t)$ and $r(t)$). Because the spot rate of interest is unobservable and important to the model's formulation, we necessarily estimate the impact parameters using a Kalman filter.³ We start with estimating a one-factor affine model,⁴ and then generalize to more realistic two- and three-factor models.

2.2.1 The Data

Since QE1 was officially announced on November 25, 2008 and QE2 was completed on June 30, 2011, we choose the sample period spanning from November 24, 2008 to June 30, 2011 to estimate the impact of the Fed's QE activities.

The term structure of interest rate data is the daily instantaneous forward rates time series constructed by Gürkaynak, Sack, and Wright (GSW (2007)) and

³Bolder (2001) provides a good technical guide on implementing a Kalman filter.

⁴This is sometimes called a Vasicek (1977) model.

available on the Federal Reserve website.⁵ This data set contains 30 forward rates with maturities ranging from 1 year to 30 years. The GSW data is based on the forward rate smoothing procedure described in Svensson (1994), which assumes a parametric form with six parameters, and it is chosen for easy comparison with the existing literature.⁶

Figure 1.1 graphs the Federal funds rate before and during the QE1 and QE2 estimation period. As seen, the Fed funds rate drop to near zero corresponds with the start of the QE time period. This implies that our estimates of the Fed's impact on bond prices will also reflect the impact of the Fed's short-term interest rate monetary policy activities as well. As discussed in the introduction chapter, the Fed funds rate being maintained close to the zero lower bound is an important reason for determining the additional effectiveness of both QE1 and QE2 in lowering long-term interest rates.

Table 2.1 provides summary statistics for the different maturity forward rates over the sample period. This table provides a benchmark for the level of forward rates and their standard deviations over the estimation period. Figure 2.1 plots the forward rates' time series evolutions. Interestingly, one can see a decline in the observed forward rates between 2008 and 2011, most pronounced for the 1, 2, 3, and 5 year forward rates.

Figure 2.2 graphs the evolution of the Fed's balance sheet over the sample period.⁷ The Fed's purchases mainly focused on mortgage-backed securities (MBS) during QE1 and Treasury securities during QE2. This difference in the types of

⁵<https://www.federalreserve.gov/econresdata/researchdata.htm>

⁶We also explored the estimation using forward rates based on a polynomial spline smoothing procedure yielding similar results. For brevity these results are not reported in the subsequent text.

⁷Data source: <http://www.federalreserve.gov/econresdata/>

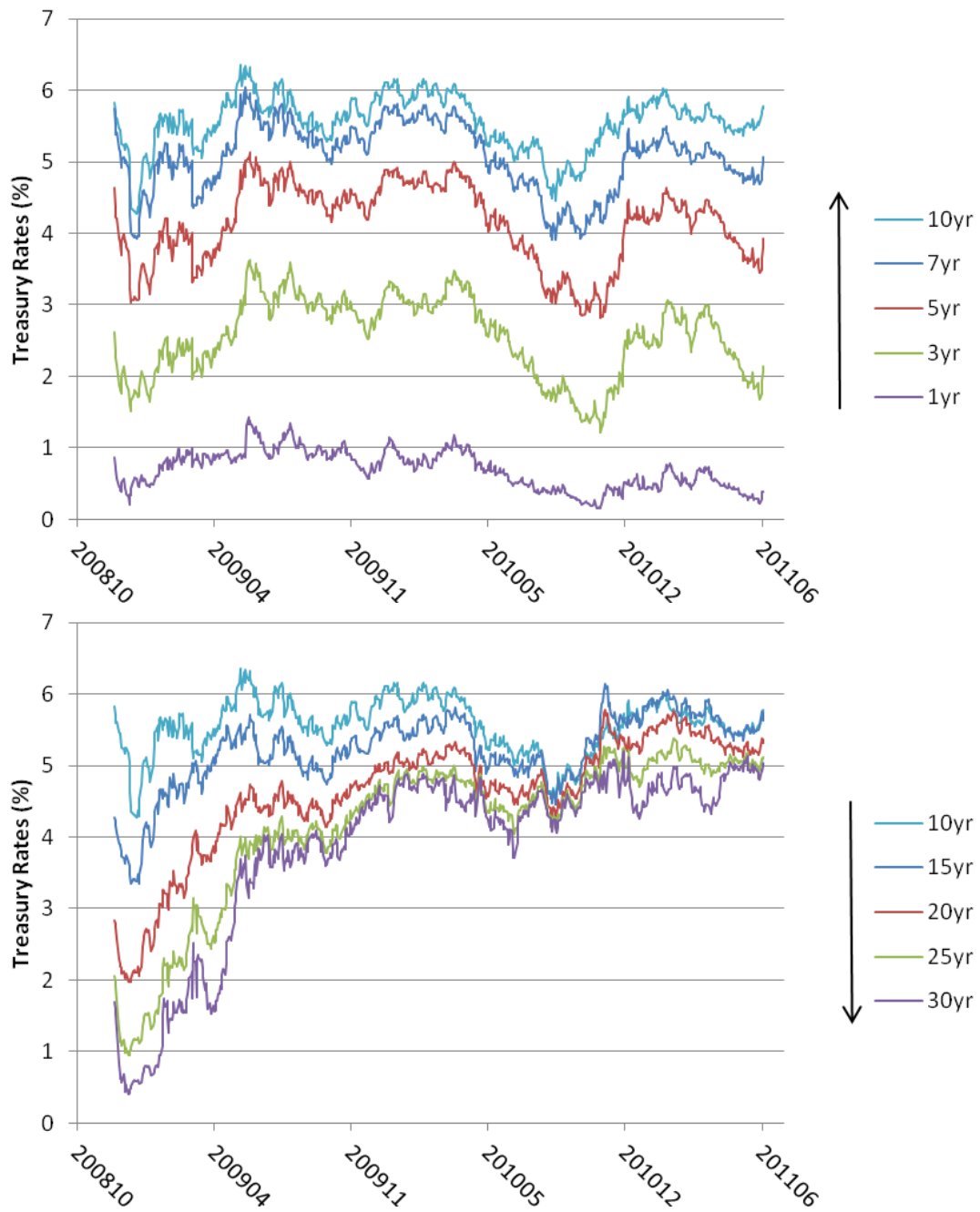
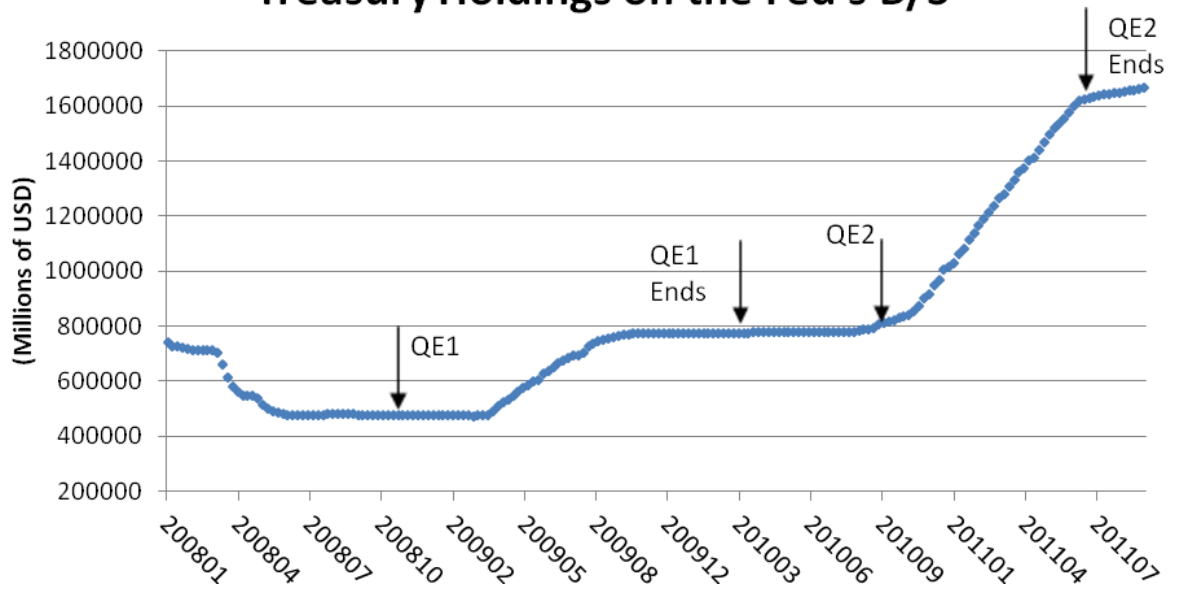


Figure 2.1: Time-series of GSW Forward Rates

Treasury Holdings on the Fed's B/S



MBS and Agency Debt Holdings on the Fed's B/S

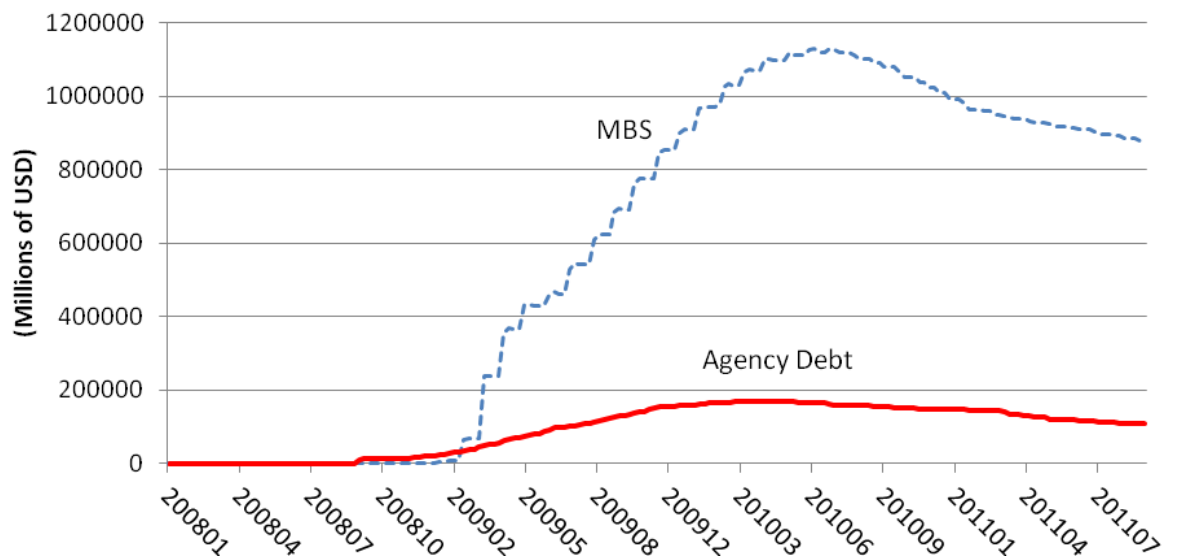


Figure 2.2: Federal Reserve's Holdings of Treasuries, MBS and Agency Debt

Maturity (year)	Mean (%)	Std	Skewness	Kurtosis
1	0.6925	0.2651	0.1080	2.2363
2	1.5845	0.4574	-0.1564	2.2357
3	2.5465	0.5495	-0.3282	2.1799
4	3.4148	0.5733	-0.4328	2.2141
5	4.1280	0.5566	-0.4907	2.3009
6	4.6753	0.5206	-0.5112	2.4173
7	5.0688	0.4783	-0.5192	2.5881
8	5.3303	0.4379	-0.5528	2.8596
9	5.4838	0.4046	-0.6407	3.2392
10	5.5523	0.3831	-0.7762	3.6518
11	5.5559	0.3775	-0.9166	3.9903
12	5.5119	0.3902	-1.0248	4.2518
13	5.4342	0.4203	-1.1017	4.5078
14	5.3340	0.4645	-1.1650	4.7551
15	5.2200	0.5187	-1.2232	4.9368
20	4.6211	0.8293	-1.4136	4.7804
25	4.1592	1.0822	-1.4696	4.2606
30	3.8604	1.2457	-1.4651	3.9361

Table 2.1: Summary Statistics

asset purchased across QE1 and QE2 suggests that there may be differing price impacts. We explore this possibility in our estimation below.

Figure 2.3 provides a breakdown of the Fed’s Treasury holdings by maturity over our sample period. For their Treasury security purchases, the Fed’s activities are mostly concentrated on securities with maturities between one and ten years. These holdings will be relevant when discussing the QE’s impact on bond price yields in subsequent sections.

Relevant to the Fed’s Treasury purchases and their impact on forward rates is the outstanding supply of Treasury securities during the QE period. As mentioned earlier, we do not explicitly adjust our estimates of the Fed’s QE impact on forward rates for changes in the outstanding supply of Treasuries. In our methodology, this

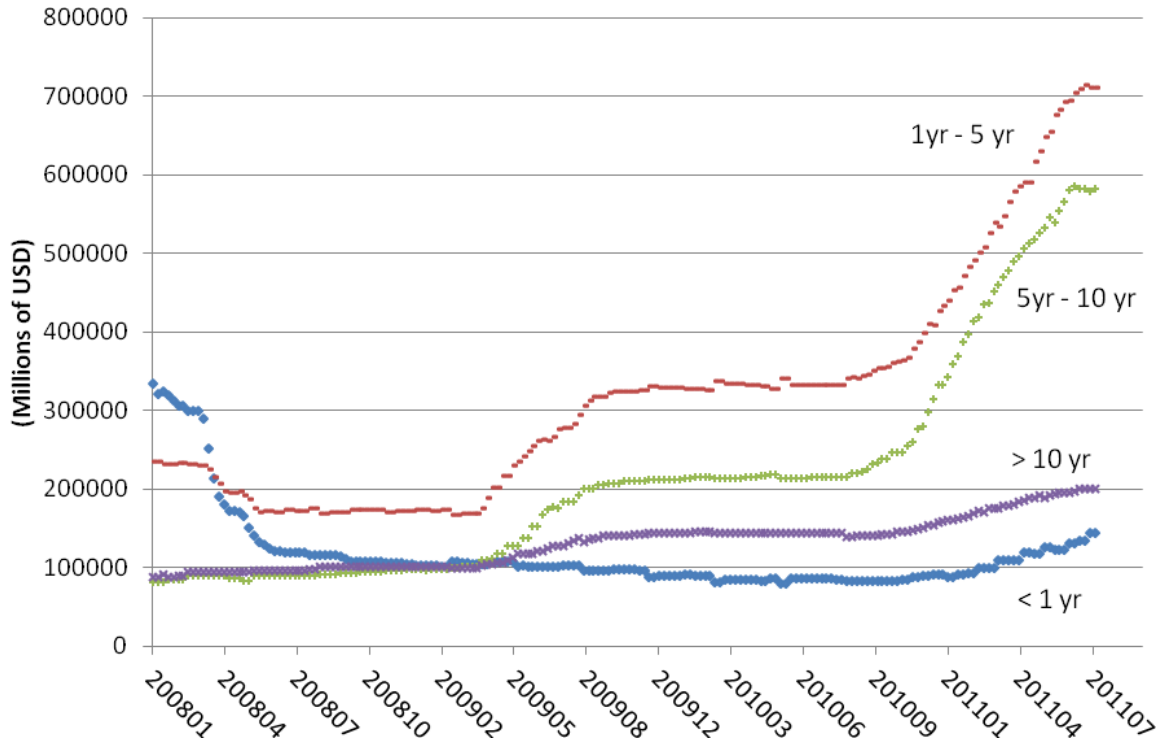


Figure 2.3: Breakdown of Treasury Holdings by Maturity

supply adjustment is implicitly captured through its impact on the estimated true forward rate process (the drift and volatilities) over this time period. A potential concern with our methodology, therefore, is that if the U. S. Treasury purposely increased its auction of Treasuries to take advantage of the Fed’s QE activities, then our estimated price impacts would be biased low. To investigate this potential bias, Figure 2.4 shows the time series of newly auctioned Treasury securities over the QE period.⁸ As seen, the newly auctioned Treasuries are quite stable and only slightly increasing across time, the upward trend reflecting an increase in the size of the Federal budget deficit over this same time period. It does not appear that the U.S. Treasury’s auction process was directly influenced by the Fed’s QE activities, minimizing this potential bias in our estimation methodology.

⁸Data source: <http://www.treasurydirect.gov/instit/annceresult/press/preanre/preanre.htm>

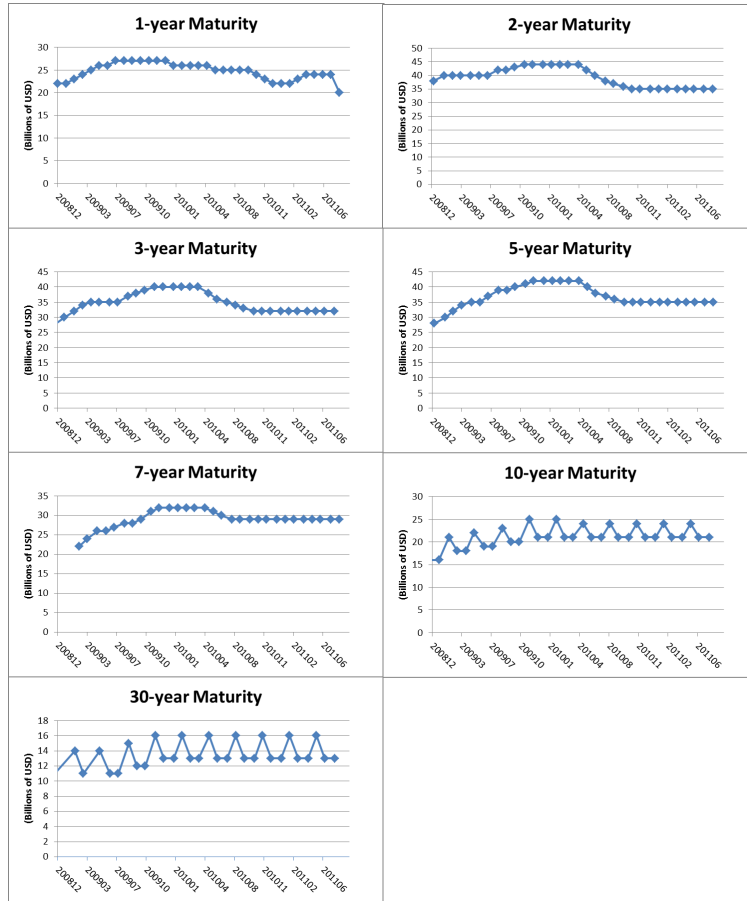


Figure 2.4: Amount of Newly Auctioned Treasury Securities

2.2.2 A One-Factor Model

This section estimates a one-factor affine model for the evolution of the term structure of interest rates. We start with the one-factor model to both illustrate the methodology and to provide a benchmark for comparing the results for two- and three-factor models.

The Methodology

In the one-factor affine model, the true forward rates evolution given by expression (2.2) can be written as:

$$f(t, T) = (1 - e^{-k(T-t)}) \left(\theta - \frac{\sigma_r^2}{2k^2} (1 - e^{-k(T-t)}) \right) + e^{-k(T-t)} r_t \quad (2.14)$$

where the state variable r_t is the true instantaneous spot rate. Substitution into expression (2.7) gives the observed forward rate process, including the Fed's price impact:

$$F(t, T) \quad (2.15)$$

$$= \begin{cases} (1 - e^{-k(T-t)}) \left(\theta - \frac{\sigma_r^2}{2k^2} (1 - e^{-k(T-t)}) \right) + e^{-k(T-t)} r_t - \frac{\psi(T)}{\lambda(T)} (1 - e^{-\lambda(T)t}), \\ \text{if } 0 < t \leq \tau \\ (1 - e^{-k(T-t)}) \left(\theta - \frac{\sigma_r^2}{2k^2} (1 - e^{-k(T-t)}) \right) + e^{-k(T-t)} r_t - \frac{\psi(T)}{\lambda(T)} (e^{\lambda(T)\tau} - 1) e^{-\lambda(T)t}, \\ \text{if } t > \tau. \end{cases}$$

As mentioned previously, since the spot rate is unobservable⁹, to estimate our system we use a Kalman filter. In our Kalman filter, the time-discretized state transition equation for the spot rate is given by

$$r_{t+\Delta t} = \theta(1 - e^{-k\Delta t}) + e^{-k\Delta t} r_t + \sigma_r \varepsilon_t \quad (2.16)$$

where ε_t follows a standard normal distribution.

As indicated, this evolution allows the spot rate to be negative with positive probability. Although alternative evolutions could be used that preclude negative

⁹Instead, one could obtain estimated spot rates using the intercept of the smoothed GSW forward rate curve with the y-axis. We choose not to use these estimates because the intercept with the y-axis explicitly depends on the functional form of the smoothing function, which in turn, is greatly influenced by the prices of the long-term Treasuries. In reality, short-term Treasury rates (less than one year) are influenced more by the impact of the Fed's short-term interest rate policies than the assumed shape of a smoothing function. Our estimation methodology avoids this potential bias.

rates, both economic theory and the empirical evidence are more consistent with evolutions that allow negative (nominal) rates with positive probability. Indeed, from a theoretical perspective, large financial institutions cannot store currency, they can only invest it in either deposits or securities; and consequently, negative rates are possible. Empirically, negative rates on Treasuries were observed in each of November 2009, June 2011, and August 2011;¹⁰ and the Bank of New York Mellon paid negative deposit rates in August 2011.¹¹

For the Kalman filter, the measurement equation is given by the evolution of the observed forward rate process:

$$F(t, \Lambda_i) = A_i + B_i r_t + u_t(\Lambda_i) \quad (2.17)$$

where

$$A_i = \begin{cases} (1 - e^{-k\Lambda_i}) \left(\theta - \frac{\sigma_r^2}{2k^2} (1 - e^{-k\Lambda_i}) \right) - \frac{\psi_i}{\lambda_i} (1 - e^{-\lambda_i t}), & \text{if } 0 < t \leq \tau \\ (1 - e^{-k\Lambda_i}) \left(\theta - \frac{\sigma_r^2}{2k^2} (1 - e^{-k\Lambda_i}) \right) - \frac{\psi_i}{\lambda_i} (e^{\lambda_i \tau} - 1) e^{-\lambda_i t}, & \text{if } t > \tau \end{cases} \quad (2.18)$$

$$B_i = e^{-k\Lambda_i}, \text{ and } \Lambda_i = T_i - t.$$

For simplicity, we assume $u_t(\Lambda_i)$ follows an independent normal distribution.

We estimate the parameters using three forward rate series ($\Lambda_i = 1yr, 2yr, 3yr$).

The parameters to be estimated are $(k, \theta, \sigma_r, \psi_1, \lambda_1, \psi_2, \lambda_2, \psi_3, \lambda_3)$.

¹⁰See WSJ Blog, Market Beat, November 20, 2009, "Some Treasury Bill Rates Negative Again Friday;" Bloomberg, November 19, 2009, "U.S. 3-month Bills Turn Negative on Concern Risk Rally Overdone;" Bloomberg, June 27, 2011, "Treasury 4-week Bill Rates Negative for First Time since 2010;" WSJ Blog, Market Beat, August 4, 2011, "From One Crisis to Another: One Month T-Bill Yields go Negative Again."

¹¹See Bloomberg.com/news, August 5, 2011, "BNY Mellon Makes Clients Pay for Deposits as Investors Seek Safety in Cash;" Online WSJ, August 5, 2011, "New Fee to Bank Cash."

Parameter	Estimate	Std
θ	0.037	0.003
k	0.234	0.017
σ_r	0.013	0.001
λ_1	2.20	0.29
λ_2	1.28	0.21
λ_3	0.0001	0.13
ψ_1	0.053	0.008
ψ_2	0.023	0.004
ψ_3	0.0037	0.001

Table 2.2: One-Factor Parameter Estimates

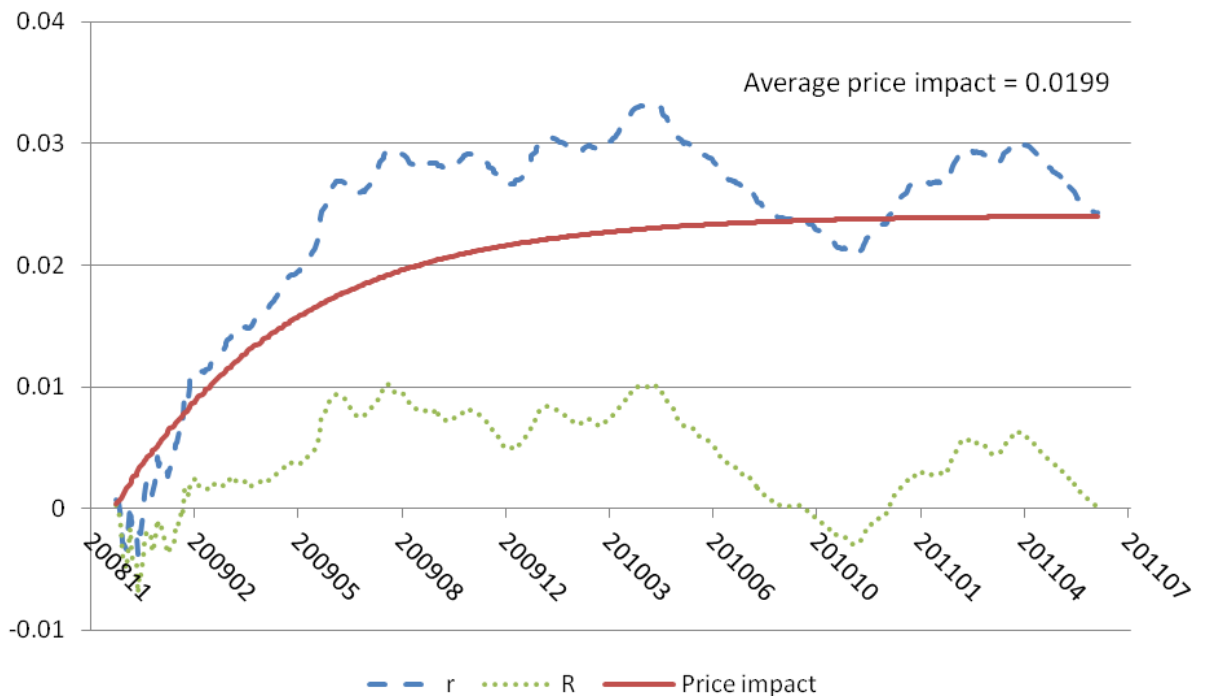


Figure 2.5: One-factor Estimation of the Instantaneous Short Rate

The Results

The parameter estimates are shown in Table 2.2 and the evolution of the true spot rate is plotted as the dashed curve in Figure 2.5.

The spot rate, with the Fed's impact included, is the limit of expression (2.15) as $T \rightarrow t$, i.e.

$$R_t = r_t - \begin{cases} \frac{\psi(0)}{\lambda(0)}(1 - e^{-\lambda(0)t}), & \text{if } 0 < t \leq \tau \\ \frac{\psi(0)}{\lambda(0)}(e^{\lambda(0)\tau} - 1)e^{-\lambda(0)t}, & \text{if } t > \tau. \end{cases}$$

To obtain an estimate of this spot rate, we use the estimates of $(\lambda(1), \psi(1))$ instead of $(\lambda(0), \psi(0))$. The corresponding estimated short rate (denoted by R_t) is plotted as the dotted curve in Figure 2.5. The difference $(r_t - R_t)$ is the Fed's price impact, which is plotted as the solid curve. A positive and upward trending price impact curve in Figure 2.5 is consistent with the facts that the Fed's monetary policy was targeting near zero short-term rates, and the Fed had been continuously purchasing Treasury securities over the estimation period. It shows that, under the one-factor model, the Fed's price impact on the short rate has been increasing since QE started, and stayed in the range of 2.3% \sim 2.4% until the end of June 2011.

Table 2.2 presents the estimated price impact parameters for the one-, two-, and three- year forward rates (λ_i, ψ_i) for $i = 1, 2, 3$. The marginal impact parameter is decreasing in maturity, i.e. $\psi_1 > \psi_2 > \psi_3$. In contrast, the mean reversion parameter is increasing with maturity, i.e. $\lambda_1 > \lambda_2 > \lambda_3$. This implies that the duration of the price impact increases with maturity. Defining the half-life of the price impact as the time $t_0^i = \ln(2)/\lambda_i$ for $i = 1, 2, 3$, then $t_0^1 = 0.32 \approx 3.8$ months and $t_0^2 = 0.54 \approx 6.5$ months. The half life of the price impact of the 3-year forward rate is not defined since λ_3 is insignificantly different from zero.

These results can be best understood using the modified expectations hypothesis that always holds in an arbitrage-free term structure model (see Jarrow (2009)). The expectations hypothesis is modified for risk aversion using adjusted probabilities, instead of the actual probabilities.¹² As in the classical expectations hypoth-

¹²These adjusted probabilities are called the forward price martingale probability measures,

esis, except for this modification, the time t forward rate for date T is the time t "expected" spot rate for date T . These results show that the impact of QE on the future spot rate is "expected" to decline as time progresses. And in addition, the effect of a purchase on the future spot rate is "expected" to last longer, the longer the term of the rate. Perhaps because most monetary policy activities occur on the very short-end of the curve, diminishing the lasting power of any quantity impact on the short-term forward rates.

2.2.3 An N-Factor Model

The above estimation procedure can be extended to a N - factor affine model, where the short rate is a sum of N factors

$$r(t) = \sum_{n=1}^N z_n(t). \quad (2.19)$$

Each factor $z_n(t)$ evolves as

$$dz_n(t) = k_n(\theta_n - z_n(t))dt + \sigma_n dW_n(t) \quad (2.20)$$

where $W_i(t)$ for $i = 1, \dots, N$ are independent standard Wiener processes.

Under this framework, one can show that the zero-coupon bond price is¹³

$$P(t, T) = \exp \left\{ C(t, T) - \sum_{n=1}^N D_n(t, T) z_n(t) \right\} \quad (2.21)$$

where

$$D_n(t, T) = \frac{1 - e^{-k_n(T-t)}}{k_n}$$

$$C(t, T) = - \sum_{n=1}^N \theta_n \left[T - t + \frac{e^{-k_n(T-t)} - 1}{k_n} \right].$$

see Jarrow (2009).

¹³For the technical details, see Chapter 4 of Brigo and Mercurio (2006), Chapter 2 of Jeanblanc, Yor and Chesney (2009), and Bolder (2001).

The corresponding forward rates are

$$\begin{aligned}
F(t, T) &= -\frac{\partial \ln P(t, T)}{\partial T} \\
&= \sum_{n=1}^N \frac{\partial D_n(t, T)}{\partial T} z_n(t) - \frac{\partial C(t, T)}{\partial T} \\
&= \sum_{n=1}^N e^{-k_n(T-t)} z_n(t) + \sum_{n=1}^N \theta_n [1 - e^{-k_n(T-t)}]. \tag{2.22}
\end{aligned}$$

Therefore, the time-discretized state transition equation can be written as

$$z_n(t + \Delta t) = \theta_n(1 - e^{-k_n \Delta t}) + e^{-k_n \Delta t} z_n(t) + \varepsilon_n(t) \quad n = 1, \dots, N \tag{2.23}$$

where $\varepsilon_n(t)$ follow zero-mean normal distributions with the following variance and covariance

$$\begin{aligned}
\text{Var}[\varepsilon_n(t) | \mathcal{F}_{t-\Delta t}] &= \frac{\sigma_n^2}{2k_n} (1 - e^{-2k_n \Delta t}) \\
\text{Cov}[\varepsilon_n(t), \varepsilon_m(t) | \mathcal{F}_{t-\Delta t}] &= 0 \quad n \neq m
\end{aligned}$$

where \mathcal{F}_t is the natural filtration generated by the state variables process up to time t .

Recall that expression (2.7) describes the relation between the unobserved forward rates without the Fed's impact ($f(t, T)$) and the observed forward rates with the Fed's impact ($F(t, T)$). Combining expressions (2.7) and (2.22), we obtain the measurement equation:

$$F(t, \Lambda_i) = A_i + \sum_{n=1}^N B_{i,n} z_n(t) + u_t(\Lambda_i) \tag{2.24}$$

where $u_t(\Lambda_i)$ are assumed to follow independent normal distributions,

$$B_{i,n} = e^{-k_n \Lambda_i}, \text{ and } \Lambda_i = T_i - t.$$

For $0 < t \leq \tau$,

$$A_i = \sum_{n=1}^N \theta_n [1 - e^{-k_n \Lambda_i}] - \frac{\psi_i}{\lambda_i} (1 - e^{-\lambda_i t}).$$

Parameter	Estimate	Std
θ_1	0.002	1.647
θ_2	0.058	1.646
k_1	0.145	0.009
k_2	0.479	0.046
σ_1	0.0121	0.0005
σ_2	0.0001	0.001
λ_1	2.03	0.22
λ_2	1.53	0.21
λ_3	0.91	0.18
λ_4	0.16	0.21
ψ_1	0.061	0.009
ψ_2	0.039	0.007
ψ_3	0.018	0.004
ψ_4	0.006	0.002

Table 2.3: Two-Factor Parameter Estimates

For $t > \tau$,

$$A_i = \sum_{n=1}^N \theta_n [1 - e^{-k_n \Lambda_i}] - \frac{\psi_i}{\lambda_i} (e^{\lambda_i \tau} - 1) e^{-\lambda_i t}.$$

The Results

This section estimates both two- and three-factor models. We estimate the parameters using four forward rates ($\Lambda_i = 1, 2, 3, 4$ years). For the two-factor model, the parameters to be estimated are $(k_i, \theta_i, \sigma_i, (\psi_j, \lambda_j)_{j=1,2,3,4})_{i=1,2}$ and the results are shown in Table 2.3. For the three-factor model, the parameters to be estimated are $(k_i, \theta_i, \sigma_i, (\psi_j, \lambda_j)_{j=1,2,3,4})_{i=1,2,3}$ and the results are shown in Table 2.4. For comparison with the existing literature estimating affine models without Fed purchases, Table 2.4 provides the estimates for this model as well. These estimates without the Fed purchases included are consistent with those found in the existing

	With Impact		Without Impact	
Parameter	Estimate	Std	Estimate	Std
θ_1	0.003	1.30	0.025	0.803
θ_2	0.039	1.44	0.046	0.818
θ_3	0.031	0.95	0.018	0.797
k_1	0.104	0.008	0.134	0.006
k_2	0.446	0.072	0.134	0.108
k_3	0.104	0.672	0.588	0.071
σ_1	0.0106	0.0005	0.0107	0.0003
σ_2	0.0039	0.0007	0.0008	0.0009
σ_3	0.0002	0.0012	0.0001	0.0011
λ_1	2.01	0.18		
λ_2	1.51	0.17		
λ_3	1.14	0.16		
λ_4	0.73	0.18		
ψ_1	0.081	0.012		
ψ_2	0.057	0.010		
ψ_3	0.036	0.008		
ψ_4	0.02	0.006		
lnL	11171		11065	

Table 2.4: Three-factor Parameter Estimates

literature (see Babbs and Nowman (1999)).

The estimates of θ_n ($n = 1, 2, 3$) have large standard errors because the expression for A_i reveals that the model has poor identification for the individual θ_n . However, λ_i and ψ_i can be estimated with much higher precision.

Consistent with the results from the one-factor model, we find that the magnitude of the impact on the i - year forward rate becomes smaller as i gets larger ($\psi_j > \psi_i$ for $j < i$), while the impact on the i - year forward rate lasts longer for larger i ($1/\lambda_j < 1/\lambda_i$ for $j < i$). The half-lives of the impact for the two-factor model is $t_0^1 = 0.34 \approx 4.1$ months, $t_0^2 = 0.45 \approx 5.4$ months, $t_0^3 = 0.76 \approx 9.1$ months,

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	2.01	0.18	4.1	ψ_1	0.081	0.012
λ_2	1.51	0.17	5.5	ψ_2	0.057	0.01
λ_3	1.14	0.16	7.3	ψ_3	0.036	0.008
λ_4	0.73	0.18	11.4	ψ_4	0.020	0.006
λ_5	0.69	0.10	12.1	ψ_5	0.015	0.0013
λ_6	0.48	0.11	17.3	ψ_6	0.010	0.0011
λ_7	0.52	0.11	16.0	ψ_7	0.010	0.0012
λ_8	0.62	0.14	13.4	ψ_8	0.010	0.0017
λ_9	0.85	0.11	9.8	ψ_9	0.012	0.0015
λ_{10}	1.06	0.16	7.8	ψ_{10}	0.012	0.0019
λ_{11}	1.70	0.14	4.9	ψ_{11}	0.015	0.003
λ_{12}	2.14	0.23	3.9	ψ_{12}	0.016	0.0031
λ_{13}	2.59	2.91	-	ψ_{13}	0.007	0.009
λ_{14}	3.16	3.87	-	ψ_{14}	0.008	0.017

Table 2.5: Term Structure of the Fed's Impact

and the half-lives of the three-factor model is $t_0^1 = 0.34 \approx 4.1$ months, $t_0^2 = 0.46 \approx 5.5$ months, $t_0^3 = 0.61 \approx 7.3$ months, $t_0^4 = 0.95 \approx 11.4$ months. The fact that the two-factor and three-factor models give similar results shows the robustness of the estimation procedure.

To determine the impact of the Fed's QE program on long-term rates, Table 2.5 presents the Fed's impact parameters estimated using a three-factor model for all maturity forward rates ranging from one to thirty years. This is the key Table of this chapter. These parameter estimates are obtained by fitting a three-factor model using the forward rates ($\Lambda = 1, 2, 3, i$ years) for $i = 5, 6, \dots, 30$ where the parameters $(\psi_j, \lambda_j)_{j=1,2,3}$ are fixed at their values given in Table 2.4. Hence, only the parameters $(k_i, \theta_i, \sigma_i)_{i=1,2,3}$ and (ψ_i, λ_i) are reestimated where i corresponds to the longest term forward rate used in the estimation. This two-step procedure is invoked because there are too many parameters to estimate in the larger system

of equations, given the size of our data set.

As seen in Table 2.5, only the first 12 year forward rates' marginal price impact parameters (ψ , λ) are significantly different from zero. Because the impacts on rates beyond twelve years are all insignificantly different from zero, only the results for maturities less than 14 years are shown. A graphic representation of these estimates is given in Figure 2.6. The top panel plots the half-life of the impact for each maturity forward rate. These results show that the half-life increases as the maturity increases up to about 6 years, then declines thereafter. The lower panel plots the magnitudes of the marginal price impacts. They decrease monotonically as the maturity increases.

These results show that the Fed's QE program affects only short- and medium-term forward rates, up to about 12 years. After 12 years, the Fed's QE program has no discernible effect on forward rates. This is in contrast to the Fed's stated intention of QE to affect long-term rates. The absence of any impact on long-term forward rates is not surprising given that the Fed concentrated its Treasury purchases on maturities of less than 10 years (see Figure 2.3). Although there is a spill-over effect on the 11- and 12- year maturity Treasuries, there is little if any spill-over on the 20 and 30 year bonds. If the Fed hopes to affect the long-term forward rates, the evidence suggests that they need to purchase the long-term bonds directly.

This does not mean, however, that the Fed's QE program does not affect long-term bond yields. It does because long-term bond yields are an average of the forward rates over the bond's life, and the Fed's QE program has a large impact on short-term forward rates. The impact of the Fed's QE program on bond yields is presented in a subsequent section.

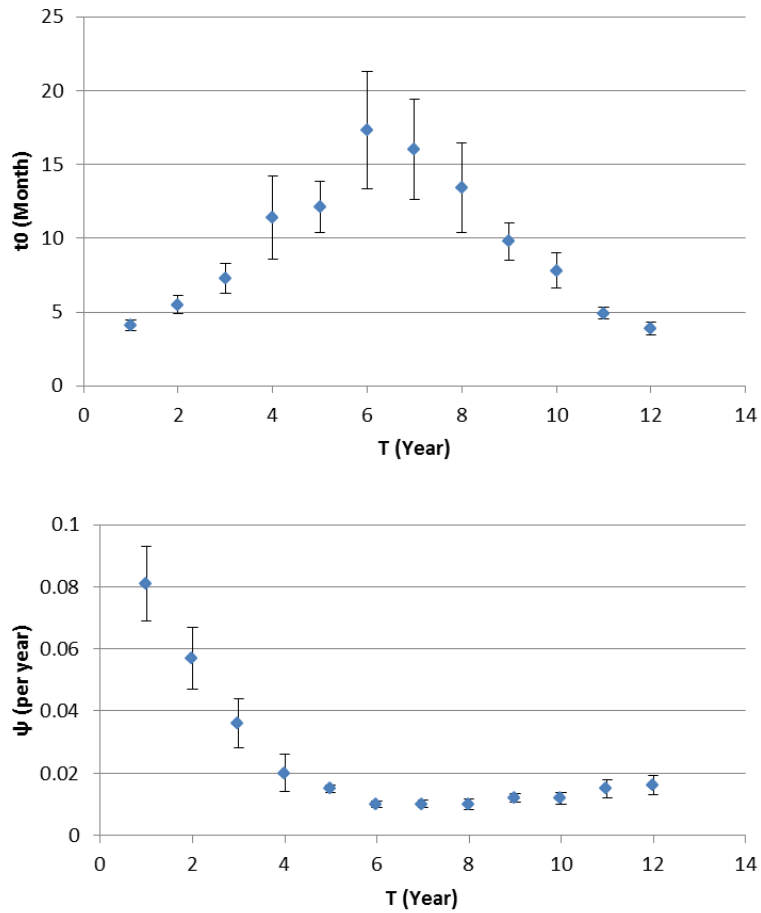


Figure 2.6: Term Structure of the QE Impact

2.2.4 Separating QE1 and QE2

As mentioned earlier and shown in Figure 2.2, the Fed’s asset purchases differed across QE1 and QE2. For this reason, it is likely that the price impact on Treasuries differed across these two periods. This section addresses this possibility by estimating the model’s parameters separately for each of the two time periods. To capture any information leakage, we choose the estimation periods for both QE events to start one day ahead of the official announcement, i.e., the QE1 estimation period spans from November 24, 2008 to March 31, 2010, and the QE2 estimation period ranges from November 2, 2010 to June 30, 2011.

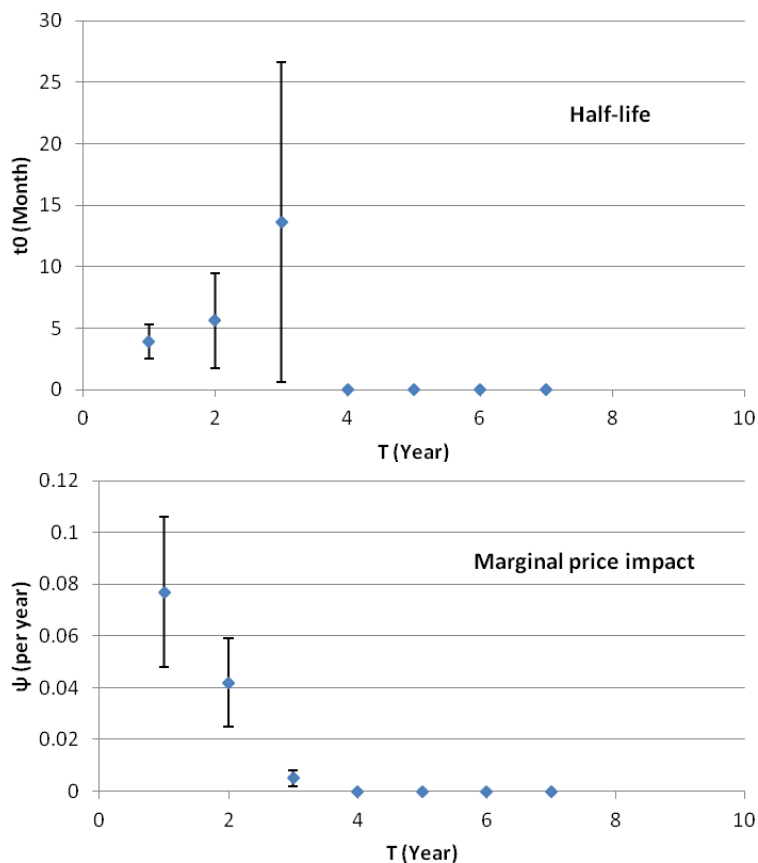


Figure 2.7: Impact of QE1

Due to the small sample size, estimating a three-factor affine model for each sub-period generates too large a set of sample errors and inconclusive results. Therefore, in order to get more reliable estimates, we fit a one-factor model. To justify this simplification, we performed the analysis on only the short- to medium-section of the term structure, up to 8 years. A principal component analysis (PCA) using forward rates with maturities of less than eight years confirms that the first principal component accounts for 93% of the variation, showing that a one-factor model provides a good approximation for this section of the term structure.

The estimation results for QE1 and QE2 are shown in Table 2.6 (Figure 2.7 and Figure 2.8 provide graphic views). Similar to the previous results obtained using the whole sample period (Table 2.5 and Figure 2.6), we find that the impacts

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	2.13	0.76	3.9	ψ_1	0.077	0.029
λ_2	1.48	1.02	5.6	ψ_2	0.042	0.017
λ_3	0.61	0.58	13.6	ψ_3	0.005	0.003
λ_4	Large	-	0	ψ_4	0	-
λ_5	Large	-	0	ψ_5	0	-
λ_6	Large	-	0	ψ_6	0	-
λ_7	Large	-	0	ψ_7	0	-

Panel A: QE1 Period

Parameter	Estimate	Std	Half-life (months)	Parameter	Estimate	Std
λ_1	3.25	0.81	2.6	ψ_1	0.11	0.031
λ_2	2.17	0.55	3.8	ψ_2	0.065	0.011
λ_3	0.65	0.54	12.8	ψ_3	0.018	0.005
λ_4	0.91	0.88	9.5	ψ_4	0.003	0.001
λ_5	Large	-	0	ψ_5	0	-
λ_6	Large	-	0	ψ_6	0	-
λ_7	Large	-	0	ψ_7	0	-

Panel B: QE2 Period

Table 2.6: Term Structure of the Fed's Impact

of both QE's are limited to the one- to four- year forward rates. For maturities longer than four years, the estimates of the mean reversion parameter λ are denoted "Large" because in the numerical convergence procedure use for optimizing the likelihood function, the estimates always reach the pre-set upper bound, even when the upper bound is set at very large values (> 50). Since the half-life is the inverse of λ , a large λ means that the price impact lasts for only a very short period.

Perhaps not surprising given that QE2's purchases concentrated on Treasuries

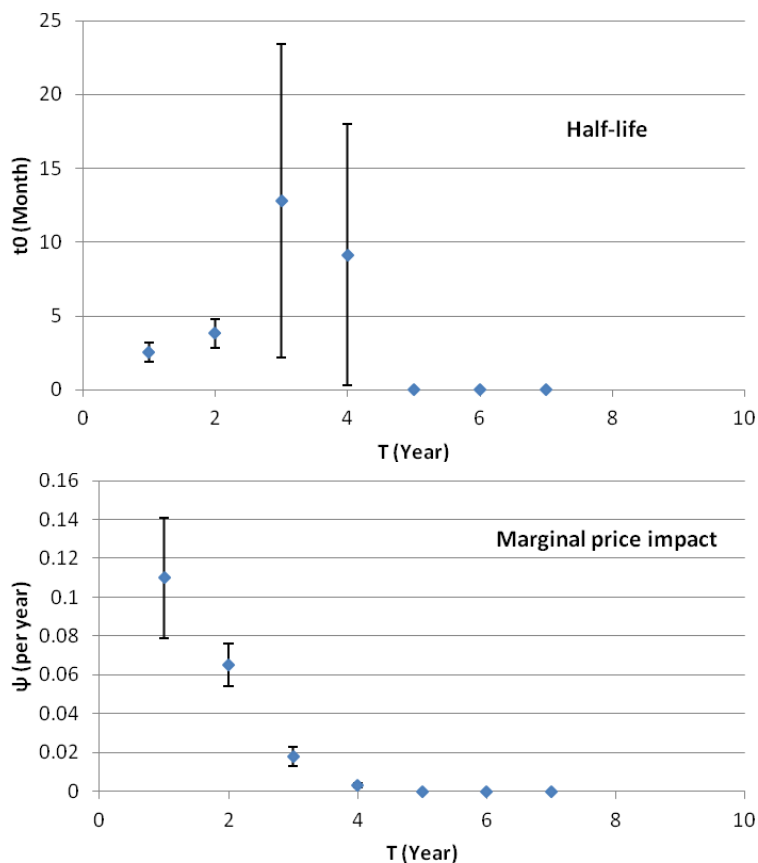


Figure 2.8: Impact of QE2

instead of MBS and agencies as in QE1 (see Figure 2.2), we find that for maturities less than four years, QE2’s price impact on Treasury rates is larger than that of QE1. However, the duration of the impact lasts longer in QE1 than in QE2. For instance, QE1’s impact on the two-year rate lasts for 5.6 months with magnitude of 4.2% per year, while QE2’s impact on the same rate lasts for 3.8 months with a magnitude of 6.5% per year. It is important to note that although the direct purchases of Treasury securities have a larger price impact on Treasury rates than do the purchases of MBS and agencies, the price impact on Treasuries of purchasing these alternative assets is significant. These results confirm the belief that asset substitution is an important effect of Fed purchases in fixed income security markets (see Bernanke and Reinhart (2004)).

2.2.5 Test for Arbitrage

Given the parameter estimates from Table 2.5, we can test for the satisfaction of expression (2.12) to see whether the Fed's QE Treasury purchases distorted risk premium and introduced arbitrage opportunities into the economy. Since the QE purchases only affected forward rate maturities of less than or equal to 12 years, we only use these rates to test this proposition.

For the three-factor model, from expression (2.12), we have that

$$\begin{aligned} & \sigma_1(t, \Lambda_j)\Delta\phi_1(t) + \sigma_2(t, \Lambda_j)\Delta\phi_2(t) + \sigma_3(t, \Lambda_j)\Delta\phi_3(t) \\ &= \psi(\Lambda_j)e^{-\lambda(\Lambda_j)t} \text{ for } 0 < t \leq \tau \end{aligned} \quad (2.25)$$

where $\Delta\phi_n(t) = \Phi_n(t) - \phi_n(t)$ and $(\Lambda_j = 1, \dots, 12 \text{ years})$.

To understand the intuition underlying our testing procedure, consider solving expression (2.25) for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ using any three-tuple of distinct maturity forward rates. In general, the solution for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ will depend upon the particular forward rate maturities selected. Theorem 1 states that the evolution is arbitrage-free if and only if this is not the case, i.e. no matter which three-tuple of forward rates is selected, the same solution for $(\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$ must occur. We test this observation below.

To formulate our test statistic, fix a time t in the QE time period, and let $y_{jt} = \psi(\Lambda_j)e^{-\lambda(\Lambda_j)t}$, $x_{jt} = (\sigma_1(t, \Lambda_j), \sigma_2(t, \Lambda_j), \sigma_3(t, \Lambda_j))'$, and $\beta_t = (\Delta\phi_1(t), \Delta\phi_2(t), \Delta\phi_3(t))$. Note that in this notation, we are assuming that β_t does not depend on the forward rate's maturity. First, we estimate β_t using a simple linear regression

$$y_{jt} = \beta_t x_{jt} + \varepsilon_{jt} \text{ for } j = 1, \dots, 12 \quad (2.26)$$

where ε_{jt} are assumed to be *i.i.d.* normal distributions with zero mean, representing

observational noise in the data.

If expression (2.25) is true, the null hypothesis, excluding the noise in the data we would expect to see $\varepsilon_{jt} \equiv 0$ for all j . However, given noise in the data, we would expect to see $\text{var}(\varepsilon_{jt})$ small relative to $\text{var}(y_{jt})$. To test this expectation, we form the test statistic $s_t = \sum_{j=1}^{12} (y_{jt} - x_{jt}\hat{\beta}_t)^2 / \text{var}(y_{jt})$ where $\hat{\beta}_t$ represents the regression estimate from expression (2.26). The test statistic s_t has a χ^2 distribution with 9 degrees of freedom (12 data points are used to estimate 3 parameters). If s_t is large, we can reject the null hypothesis of no arbitrage.

Estimating expression (2.26) for each day (t) over the sample period we obtain a time-series of the estimated market prices of risk $\hat{\beta}_t$, which are plotted in Panel A of Figure 2.9, and the test statistic, which is graphed in Panel B of Figure 2.9. As seen, the test statistic is well below the 5% significance threshold. We can not reject the null hypothesis that there is no arbitrage over the QE period. As intended, the Fed's QE program appears to have been successful in not introducing arbitrage opportunities into the economy. A qualification of our results needs to be noted. Since our parameters are estimated with smoothed Treasury price data, the smoothing procedure could itself remove arbitrage opportunities, providing only a weak test of our hypothesis. A better test would involve using unsmoothed Treasury prices directly.

2.3 Model Specification Tests

This section provides various model specification tests that support the model's validity.

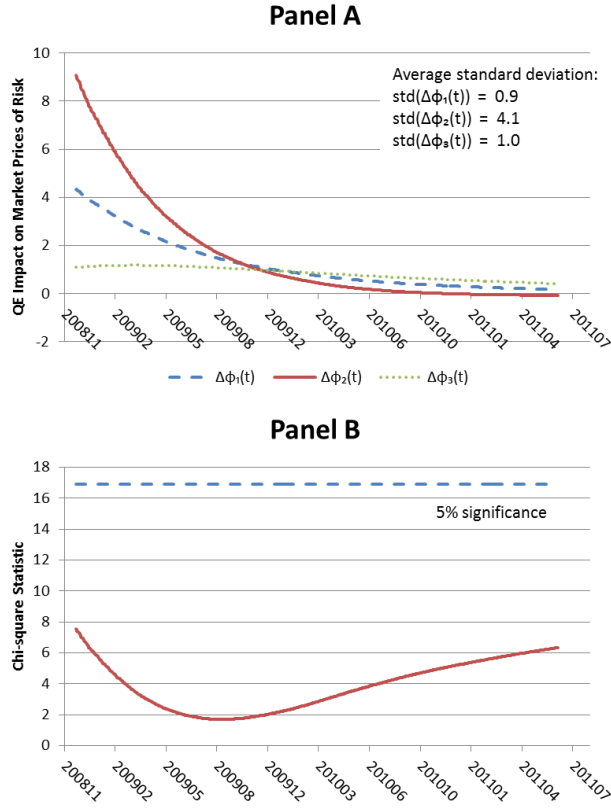


Figure 2.9: Test for Arbitrage

2.3.1 A Comparison Pre- QE

To test the model’s specification, we estimated the three-factor model for two time periods before the onset of QE1. One is from January 2, 2001 to August 1, 2003, when the Fed lowered interest rates, and the second from January 2, 2004 to August 1, 2006, when the Fed increased interest rates. If the additional structure in our model captures the Fed’s QE activities, one would expect to see the mean reversion and marginal impact parameters (λ, ψ) insignificantly different from zero during these time periods. The parameters are estimated using four forward rates ($\Lambda_i = 1, 2, 3, 4$ years) and the results are presented in Tables 2.7 and 2.8.

For the period when the Fed was lowering interest rates, all of the impact para-

Parameter	With Impact		Without Impact	
	Estimate	Std	Estimate	Std
θ_1	0.021	0.82	0.023	0.82
θ_2	0.02	0.82	0.029	0.82
θ_3	0.022	0.82	0.024	0.82
k_1	0.59	0.31	0.26	0.15
k_2	0.28	0.06	0.25	0.02
k_3	0.28	0.12	0.25	0.03
σ_1	0.0001	0.015	0.005	0.009
σ_2	0.032	0.009	0.026	0.005
σ_3	0.016	0.007	0.021	0.005
λ_1	1.35	0.62		
λ_2	0.75	0.64		
λ_3	0.35	0.71		
λ_4	0	0.9		
ψ_1	0.071	0.023		
ψ_2	0.035	0.015		
ψ_3	0.017	0.011		
ψ_4	0.008	0.008		
lnL	9563		9508	

Table 2.7: Robustness Check: Jan. 2, 2001 - Aug. 1, 2003

parameters (λ , ψ) are insignificantly different from zero, except for the price impacts of the one- and two- year rates. Although the one-year rate impact is significant, its magnitude (0.071) is less than its magnitude (0.081) in the QE period (see Table 2.4). The same is true for the two-year rate. These impacts on the shortest term forward rates are consistent with the Fed's direct monetary policy activities having a spill over effect on the one- and two- year rates.

For the period when the Fed was increasing interest rates, all of the marginal impact parameters (ψ) are insignificantly different from zero, except for the four-year rate. The mean reversion parameters (λ) are significant for years two through

Parameter	With Impact		Without Impact	
	Estimate	Std	Estimate	Std
θ_1	0.006	0.94	0.02	0.82
θ_2	0.071	1.25	0.022	0.82
θ_3	0.009	0.95	0.018	0.82
k_1	0.45	0.08	0.98	0.06
k_2	0.21	0.02	0.2	0.03
k_3	0.21	0.04	0.2	0.03
σ_1	0.0001	0.005	0.069	0.007
σ_2	0.019	0.003	0.015	0.002
σ_3	0.012	0.005	0.013	0.003
λ_1	0.87	0.38		
λ_2	1.59	0.31		
λ_3	1.11	0.21		
λ_4	1.2	0.16		
ψ_1	-0.01	0.007		
ψ_2	0.0006	0.007		
ψ_3	0.01	0.005		
ψ_4	0.018	0.004		
lnL	10738		10698	

Table 2.8: Robustness Check: Jan. 2, 2004 - Aug. 1, 2006

four. The significance for years two and three are irrelevant, since the market impact parameter is not different from zero. The significance of both of the price impact parameters (λ , ψ) for the four-year rate is probably due to noise in the data, but it could be due to the simplicity of the model being estimated. A resolution of these two possibilities awaits the estimation of more complex models in subsequent research.

2.3.2 Likelihood Ratio Tests, Pre- and Post- QE

This section provides likelihood ratio tests for the model with and without the Fed's impact function for all three sample periods, both pre- and post- QE. These results are given in Tables 2.4, 2.7, and 2.8. The test statistic is $2(\ln(L_1) - \ln(L_2))$, where L_1 (L_2) is the maximized likelihood value with (without) the price impact term.

At the 5% significance level, the likelihood ratio test rejects the model without the price impact for all three sample periods. This is to be expected since this is an in sample test, and the price impact model has more parameters. More insightful is a comparison of the magnitudes of the changes in the likelihood values over the different sample periods. For the 1/2/2001-8/1/2003 sample (no QE), the log-likelihood increases by 0.6% after adding the price impact term. For the 1/2/2004-8/1/2006 (no QE) sample period, the increase is only 0.4%. In contrast, for the QE period, the log-likelihood increases the most, by 1%, after adding the price impact term. These relative changes in the likelihood ratio tests are consistent with the validity of the model.

2.3.3 Pricing Errors

Another way to test the model's specification is to study the model's pricing errors in matching the observed forward rates. Table 2.9 presents the statistical properties of the forward rate errors for our three factor model (whose parameters are given in Table 2.4). Panel A shows the result for the model with the price impact term (call it the "adjusted model") and Panel B shows the result without the price impact term (call it the "conventional model").

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	$\rho(1)$	$\rho(10)$	$\rho(20)$
1	-2.10	0.75	-2.80	0.14	2.32	0.93	0.75	0.60
2	-3.97	0.73	-5.45	0.34	2.62	0.88	0.52	0.25
3	-2.30	1.07	-2.16	-0.24	2.61	0.94	0.71	0.50
4	-1.86	1.29	-1.44	-0.53	2.90	0.95	0.76	0.56
5	-1.59	1.44	-1.10	-0.55	2.88	0.97	0.76	0.56
6	0.49	1.47	0.34	-0.51	3.01	0.97	0.74	0.55
7	1.43	1.37	1.04	-0.32	3.19	0.96	0.71	0.50
8	1.29	1.31	0.98	-0.37	3.11	0.96	0.68	0.47
9	3.40	1.24	2.74	-0.47	3.02	0.96	0.67	0.45
10	1.01	1.23	0.82	-0.58	3.13	0.96	0.68	0.46
11	0.37	1.14	0.32	-0.57	2.92	0.96	0.66	0.43
12	-1.08	1.24	-0.87	-0.56	3.12	0.96	0.71	0.49
13	0.64	1.25	0.51	-0.51	2.71	0.96	0.71	0.51
14	-1.36	1.27	-1.07	-0.31	2.45	0.97	0.73	0.54

Panel A (With the price impact term)

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	$\rho(1)$	$\rho(10)$	$\rho(20)$
1	-3.84	0.71	-5.43	0.30	2.29	0.93	0.69	0.50
2	-15.08	0.83	-18.16	0.38	3.07	0.91	0.56	0.28
3	-6.14	1.28	-4.79	-0.13	2.31	0.96	0.76	0.56
4	4.84	1.50	3.24	-0.35	2.28	0.97	0.80	0.63
5	2.58	1.52	1.70	-0.43	2.41	0.97	0.79	0.62
6	-0.36	1.47	-0.24	-0.45	2.59	0.97	0.76	0.59
7	-1.96	1.40	-1.40	-0.45	2.81	0.97	0.73	0.55
8	-3.92	1.35	-2.91	-0.50	3.09	0.97	0.72	0.52
9	-3.00	1.32	-2.28	-0.60	3.40	0.97	0.71	0.51
10	-1.41	1.33	-1.06	-0.73	3.66	0.97	0.72	0.51
11	0.60	1.26	0.47	-0.66	3.24	0.96	0.73	0.53
12	-1.62	1.47	-1.10	-0.94	4.10	0.97	0.78	0.59
13	-0.89	1.61	-0.55	-1.03	4.37	0.98	0.82	0.66
14	-1.06	1.76	-0.60	-1.11	4.62	0.98	0.85	0.71

Panel B (Without the price impact term)

Table 2.9: Summary Statistics of Pricing Errors

The pricing errors for the adjusted model (Panel A) are quite small, on the order of 2 basis points. Compared to Panel B, one can see that for maturities within five years, the average pricing errors estimated from the conventional model are significantly larger than those from the adjusted model. For maturities longer than five years, the two models generate pricing errors with similar magnitudes. This evidence is consistent with the Fed's price impact on long-term forward rates being small. The pricing errors for both models exhibit autocorrelations, perhaps indicating that a more complex model may provide a better fit.

It is important to note that these results are similar in magnitude to the pricing errors obtained in the 4-factor affine model estimated by Adrian, Crump and Moench (2012, Table 4), where instead of adding the Fed's deterministic price impact component, one adds an additional Brownian motion random shock to the forward rate's evolution. The ability of the deterministic price impact component to match the performance of an additional random factor lends credence to the validity of the model.

2.4 Comparison to Existing Literature

This section compares our price impact estimates with those in the existing empirical literature. The estimates in the existing empirical literature are summarized in Table 2.10, Panel A for five studies: D'Amico and King (2011), Gagnon, Raskin, Remache, and Sack (2010), Krishnamurthy and Vissing-Jorgensen (2011), Li and Wei (2012), and Meaning and Zhu (2011). The existing literature studies the price impact on bond yields for maturities ranging between 1 - 30 years. As seen, the estimated price impact is around 40 basis points for short-term rates (less than 5

Paper	Event	Methodology	Treasury Yield Changes (bp)					
			1yr	2yr	3yr	5yr	10yr	30yr
Gagnon, Raskin, Remache, and Sack (2010)	QE1	Event study (Cumulative response)		-34			-91	
		Time-series regression					-52	
Krishnamurthy and Vissing-Jorgensen (2011)	QE1	Event study (Cumulative response)	-25		-39	-74	-107	-73
	QE2	Event study (Cumulative response)	-2		-8	-20	-30	-21
D'Amico and King (2012)	QE1	Stock effect					-30	
		Flow effect	-3.5bp on the sector purchased					
Li and Wei (2012)	QE1&2	Time-series estimation					-100	
Meaning and Zhu (2011)	QE2	Panel regression	-21bp on the whole yield curve on average					

Panel A: Other Papers' Results

Event	Treasury Yield Changes (bp)				
	1yr	2yr	5yr	10yr	30yr
QE1 and QE2	-327	-26	-50	-70	-76

Panel B: Our Results

Table 2.10: Comparison with Other Papers' Results

years), and 75 - 100 basis points for long-term yields (greater than 5 years). To make this comparison, we need to transform our estimated impacts on forward rates from the three-factor model in Table 5 to changes in bond yields.

This transformation is a multi-step process. First, we compute the changes in the true and observed constant maturity zero-coupon bond prices using expression

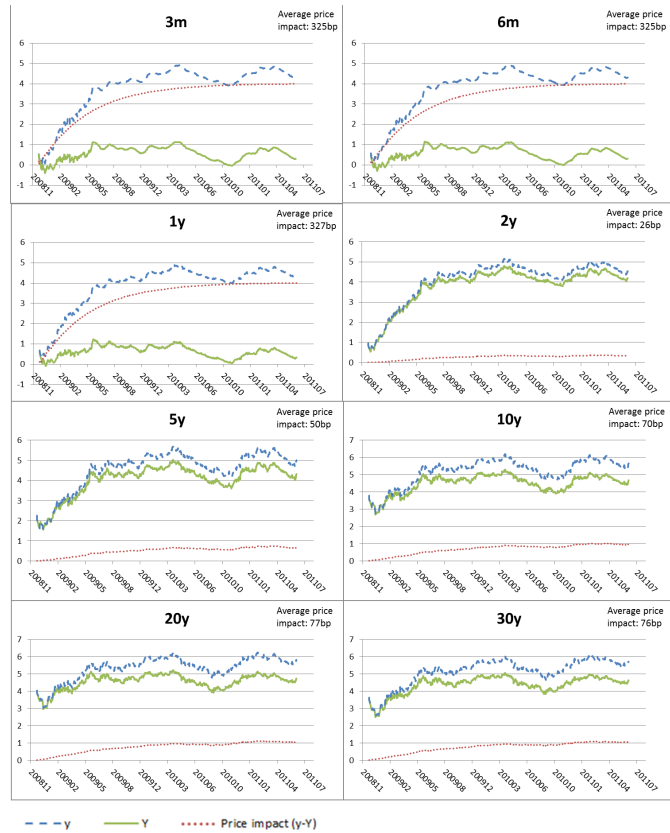


Figure 2.10: QE Impacts on Bond Yields

(2.10). Then, given these true and observed constant maturity zero-coupon bond prices, we compute the true constant maturity par-bond yields for bonds with maturities 2 - 30 years.¹⁴ These true par-bond yields give the coupon payments to use for computing the prices of the observed bonds, using the observed zero-coupon bond prices. Finally, from these observed bond prices, we can compute the observed yields. A comparison of the true par-bond yields with the observed yields generates the desired change in the Treasury yields due to the Fed’s QE activities. These yield changes are contained in Table 2.10, Panel B and graphed in Figure 2.10.

¹⁴A par bond yield is that coupon payment that makes a bond’s current price equal its face value (\$100). We compute the true coupon bond’s par-bond yield using the true zero-coupon bond prices.

As seen, the average yield changes are 327, 26, 50, 70, and 76 basis points for the 1, 2, 5, 10, and 30 year bond yields. Except for the 1-year rate, our numbers are similar in magnitude to those in the previous literature. Our estimate for the 1-year rate is significantly larger. As discussed previously, this difference is due to the fact that our estimates include the impact of the Fed's short-term interest rate monetary policy activity during the QE period.

2.5 Final Comments

This chapter proposes a new framework to analyze the price impact on the Treasury yield curve due to a central bank's bond trading activities. To test the theory, we estimated an arbitrage-free affine term-structure model over the time period when the Federal Reserve conducted QE program from late 2008 to the middle of 2011. Our findings suggest that the QE program has generated significant price impacts on the short to mid-term Treasury forward rates of up to 12 years. However, the impacts on long-term rates (beyond 12 years) appear to be insignificant. Long-term yield can still be affected because yield is an geometric average of all the future forward rates.

Our model is simplified to facilitate an analytic representation. It can be generalized to incorporate a more complex process for the large trader impact and the term structure of interest rates. These extensions could be topics for future research.

CHAPTER 3
**THE IMPACT OF QUANTITATIVE EASING ON FOREIGN
EXCHANGE RATES**

The Federal Reserve's quantitative easing (QE) program between 2008 and 2011 has generated significant impact across financial markets. Not only the bond markets but also the currency markets have been affected. In this period, the U.S. dollar has depreciated significantly against the Japanese yen (Figure 3.1). An important question here is how much of the dollar depreciation can be ascribed to QE. We attempt to address this question in this chapter.

A central monetary authority can impact the foreign exchange (FX) rates directly by trading in the currency market or indirectly by affecting interest rates. Previous studies have found that direct currency market interventions have limited impact (see overview in Dominguez and Frankel (1993)). However, there have been few studies to examine the indirect channel, i.e., affecting FX rates by affecting interest rates. The Federal Reserve's 2008-2011 QE program provides an ideal event to study this impact channel. Over this period, the U.S. dollar depreciated significantly against the Japanese yen (Figure 3.1). The size of the Fed's balance sheet increased dramatically. However, the size of the Bank of Japan's (BOJ) balance sheet remained relatively stable since it had been cutting back from its quantitative easing from 2000 to 2005 (Figures 3.2 and 3.3). Due to this, one can focus on the impact caused by the Fed's activities without worrying about those of the BOJ.

In addition to the Fed's QE, there are several other possible explanations for the significant appreciation of the yen. The first is a safe-haven effect. There have been persistent current account surpluses in Japan. When the financial crisis hit the

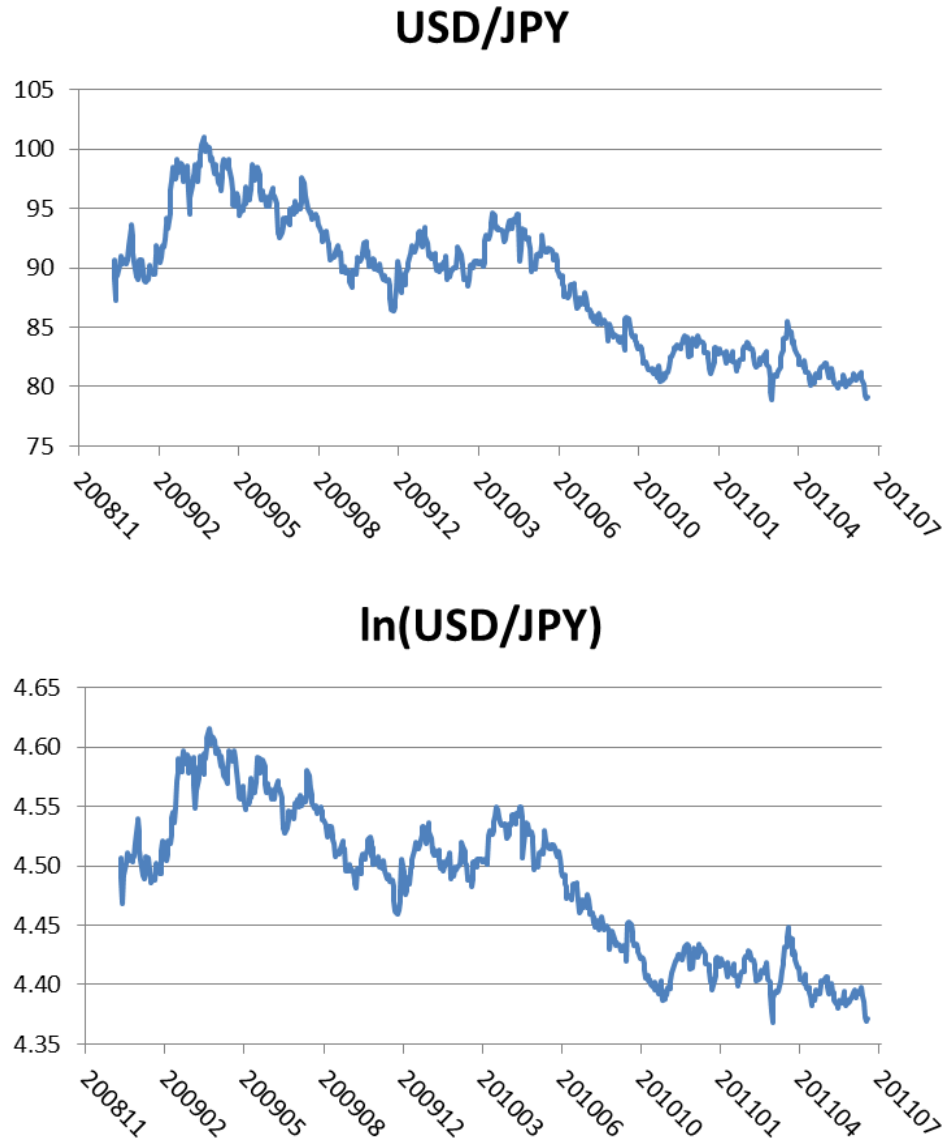
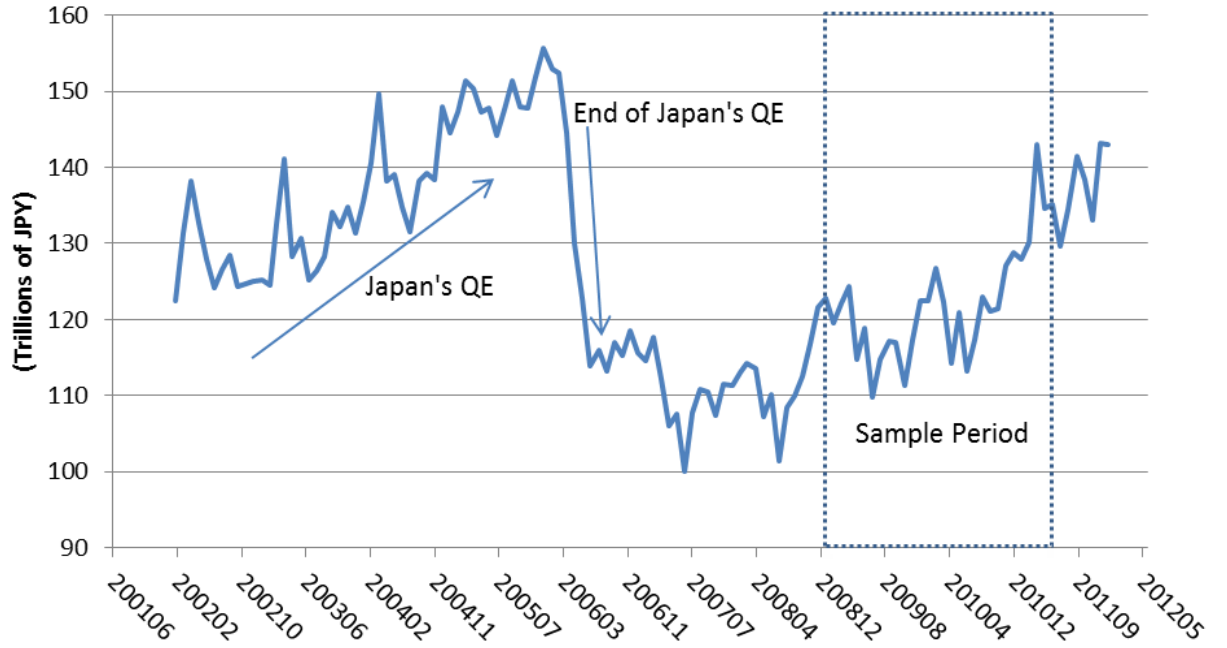


Figure 3.1: Evolution of USD/JPY

global economy, risk-averse investors flocked to yen-denominated assets and that increased demand for the yen. The second argument for the yen appreciation is that it is due to the unwinding of carry trades. At the end of 2008, the Fed lowered the target federal funds rate to almost zero, making the yen carry trade unprofitable. As a result, investors unwound their positions, pushing up the demand for the yen. Third, Japan's economy has been significantly affected by the triple disaster

Bank of Japan's Balance Sheet Size



Federal Reserve's Balance Sheet Size

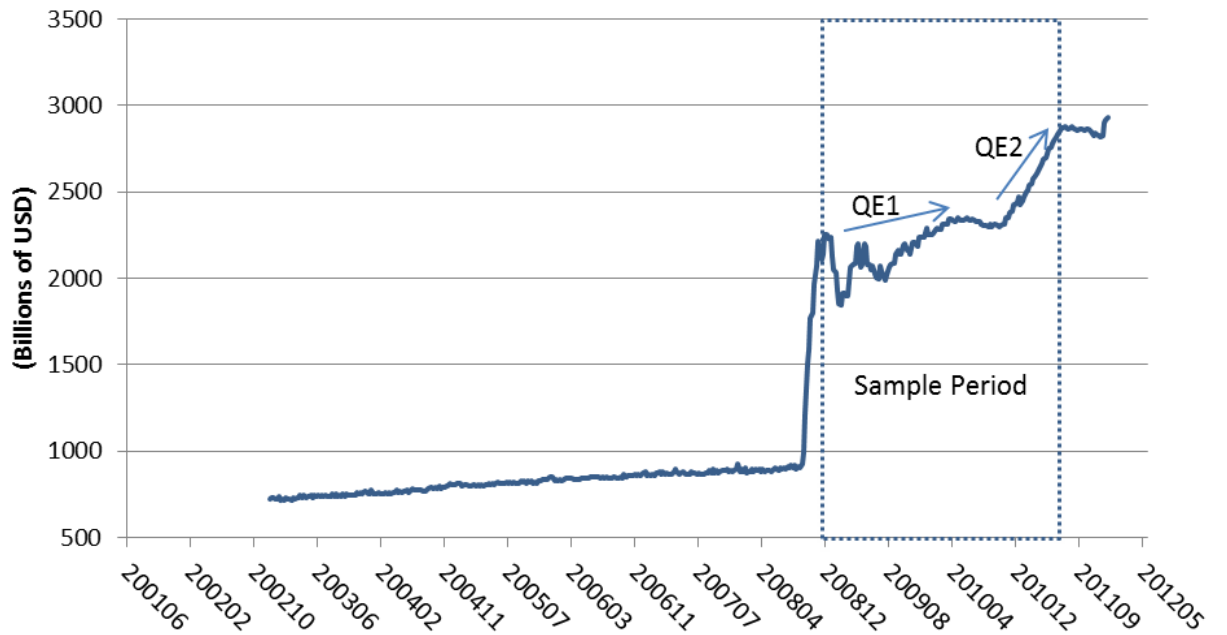
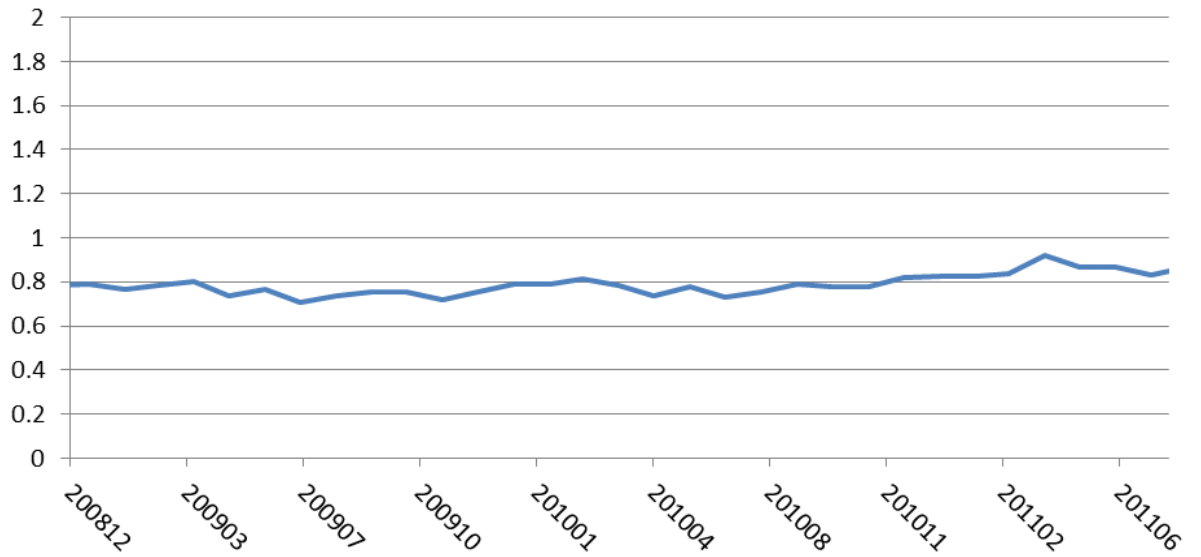


Figure 3.2: Central Banks' Balance Sheet Sizes

Normalized BOJ's Balance Sheet Size



Normalized Fed's Balance Sheet Size

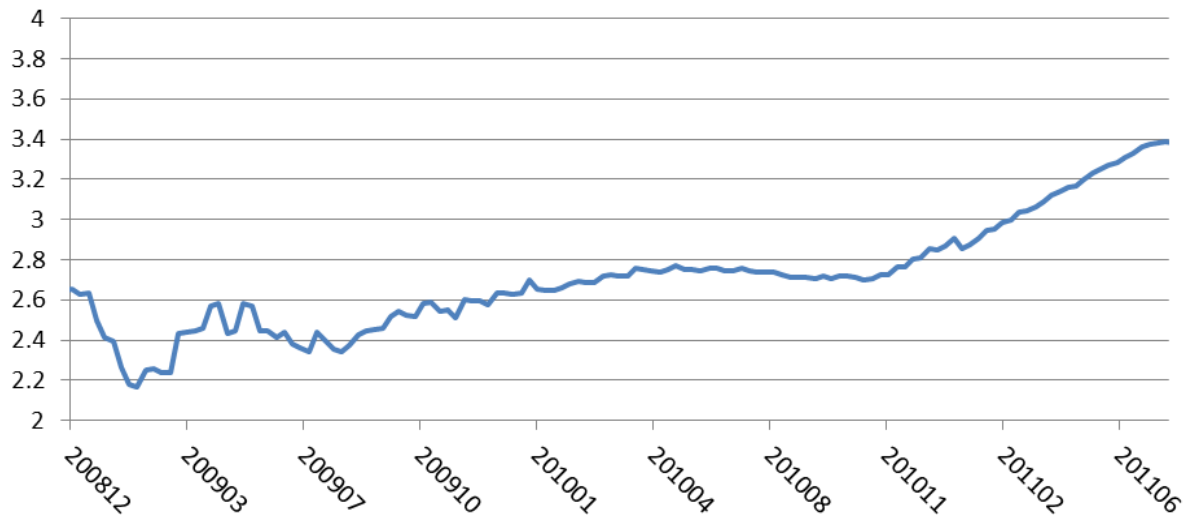


Figure 3.3: Normalized Central Banks' Balance Sheet Sizes

of the earthquake that occurred on March 11, 2011, and the subsequent tsunami and nuclear plant failures. Following this three-way catastrophe, the yen began to appreciate with the expectation of Japanese firms' repatriation back home.

Our approach is not to try to disentangle the different impact channels stated above. Instead, we analyze a simple model where one can pin down how impacts on interest rates affect exchange rates. The basic intuition is a no-arbitrage relation between the interest rate markets and FX markets. For this reason, one can estimate how QE's impact on interest rates affects the exchange rate, without concern about the other channels. Once this impact is quantified, the residual difference can be ascribed to the other factors. Our analysis emphasizes the importance of the indirect channel for a central authority to affect exchange rates, not its direct intervention.

Since QE is a monetary policy tool, this dissertation expands the literature that analyzes the impact of a central bank's monetary policy on exchange rates. In this literature, the difficult aspect is the endogeneity issue. For example, a country facing currency depreciation pressure might raise interest rates. As a result, a negative relation between interest and exchange rate movements is observed. However, the rise in interest rates could have prevented the exchange rate falling further; this is a positive relation. This makes it difficult to assess the "true" exchange rate response to monetary policy.

Based on the method of handling the endogeneity issue, previous works in this body of literature can be separated into two categories. The first adopts an event regression approach and measures the announcement effect over a very short time window. For example, Zettelmeyer (2004) studies the impact of monetary policy on exchange rates for Australia, Canada, and New Zealand during the 1990s. To

mitigate the endogeneity issue, he analyzes rate changes over a one-day announcement time window. He finds that a 1% increase in the 3-month interest rate will appreciate the exchange rate by 2~3%. The major limitation of this approach is that it cannot examine the cumulative impact over a longer period beyond the announcement time window.

The other category estimates a time-series model and uses lagged variables to mitigate the endogeneity issue. Eichenbaum and Evans (1995) study the effect of U.S. monetary policy shocks on exchange rates. They apply a vector autoregression (VAR) approach and measure the effect by impulse response functions. They find that a contractionary shock to U.S. monetary policy leads to appreciation in the U.S. dollar. Gould and Kamin (2001) examine the 1997 Asian financial crisis and analyze the effect of monetary policy on exchange rates for related Asian countries. They employ an error correction model. To overcome the endogeneity, they use international credit spreads and domestic stock prices as instrument variables. They do not find significant interest rate impacts on exchange rates.

Our approach is similar to the time-series model (the latter category). We propose a reduced-form model by assuming a parameterized functional form for the price impact; this can be estimated without introducing control variables.

This chapter also relates with existing studies that focus on the exchange rate impact of a central authority's direct currency market interventions. Dominguez and Frankel (1993) provide a comprehensive overview. Iwata and Wu (2006) study the effectiveness of FX intervention in a zero-interest-rate environment using Japanese data in the 1990s. They find that FX intervention can affect the exchange rate, but the effect is greatly reduced by the zero interest rate bound. Neely (2011) studies the G7 coordinated FX intervention following Japan's earthquake in March

2011. Related work also includes Chaboud and LeBaron (2001) on the relation between trading volume in currency futures markets and the Federal Reserve's FX intervention. All the findings tend to suggest that a central authority's direct FX interventions may have a strong impact in the very short term, but a long-term effect is hard to prove. Our work indicates that a long-term effect does exist, but that the impact may originate in the interest rate channel.

This chapter also complements research on the impact of QE on interest rates. Bernanke, Reinhart, and Sack (2004) provide a good overview on monetary policy tools in a zero-interest-rate environment. Jarrow and Li (2012) estimate the QE impact on the term structure of U.S. Treasury rates. Gagnon, Raskin, Remache, and Sack (2010) and Amico and King (2011) document that the announcements of the Fed's large-scale asset purchasing decisions lead to reductions in interest rates. Krishnamurthy and Vissing-Jorgensen (2011) examine different channels through which QE affects interest rates. Other related studies also include Fuster and Willen (2010) on the QE effects of purchasing mortgages; Swanson (2011) on the 1961 Operation Twist; Joyce, Lasasosa, Stevens and Tong (2010) on the QE impacts from the Bank of England; and Hamilton and Wu (2012) on the impact on interest rates of changing the debt maturity structure.

The studies mentioned above isolate bond markets from FX markets and none have quantified the impact of QE on exchange rates. This dissertation fills this gap. Our main contribution is to provide a framework to analyze the impact of a central bank's bond market intervention on the FX rate. With this framework, we estimate that the Fed's bond market intervention may have resulted in a dollar depreciation of 7.29% against the yen during the QE period. As mentioned above, central authorities can affect exchange rates through both direct (intervene in FX

markets) and indirect (affect interest rates) channels. Our results demonstrate that the indirect channel is very important, especially in a period when a central bank conducts massive bond purchases.

Krugman (1991) proposes a valuable model for target-zone exchange rate management. In his paper, the government's intervention is modeled as boundary conditions for exchange rate dynamics. This assumption has limitations in currency pairs such as USD/JPY because there are no specific policy bands. Therefore, we assume a different approach and introduce a hypothetical exchange rate in the absence of intervention. The impact of a government's intervention is modeled as the difference between the hypothetical and the observed rate.

This work is also related with Jarrow and Protter (2005) who study potential arbitrage opportunities caused by large traders' impact on the price process. In the QE case, when the Fed (the larger trader) announces the commencement of QE, there can be potential arbitrage opportunities for small traders. Jarrow and Li (2012) show that QE does not generate arbitrage opportunities in the Treasury markets. This chapter does not discuss whether QE generates arbitrage opportunities in the currency markets. Instead, we assume no-arbitrage holds and rely on this to link the impact on interest rates with that on FX rates.

3.1 The Model

Central authorities can affect exchange rates through two channels. They can intervene in currency markets directly, or generate the impact by affecting interest rates. In this section, we derive relations between the impacts on interest and foreign exchange rates, where the interest rate impact is a result of the central bank's trading in government bonds. The theory consists of two parts. The first

one studies the impact of QE on interest rates while the second studies how the impact on interest rates is reflected in foreign exchange rates.

3.1.1 QE's Impact on the U.S. Treasury Rates

Consider an economy where traded assets are default-free zero-coupon bonds of all maturities and a money market account. The asset market is frictionless (no transaction costs, no restrictions on trade, asset prices are perfectly divisible). The Fed is a large trader and its trades have price impacts. In addition, the Fed's announcement of QE affects other participants in the economy and could change the aggregate demand for different securities.

The time- t cumulative changes in the aggregate demand for the T -maturity zero-coupon bond can be written as

$$x(t, T) = y(t, T) + z(t, T)$$

where $y(t, T)$ is the time- t cumulative changes in the Fed's holdings of the T -maturity zero-coupon bond due to its purchases, and $z(t, T)$ is the time- t cumulative changes in the market demand for the T -maturity zero-coupon bond due to the Fed's QE announcements.

Since knowing the price of a zero-coupon bond is equivalent to knowing the forward rate of the same maturity, we assume the QE announcement and the Fed's trading afterwards affect the observed forward rates as follows:

$$df(t, T) = df'(t, T) + \lambda(t, T)(f'(t, T) - f(t, T))dt - d\Psi(t, T) \quad (3.1)$$

where $f(t, T)$ ($f'(t, T)$) denotes the forward rate with (without) the QE impact, and $d\Psi(t, T)$ represents the price impact due to changes in aggregate demand $x(t, T)$.

In particular, $d\Psi(t, T) \equiv g(x(t, T), T)dx(t, T)$, where $g(x(t, T), T)$ corresponds to the marginal impact of the change in aggregate demand $dx(t, T)$ on the observed forward rate.

The motivation for Eqn. (3.1) is that one can decompose the observed forward rate movement $df(t, T)$ into three components: the first is the movement of the true forward rate $df'(t, T)$ in the absence of QE, the second is a mean-reversion around the true rate, and the third is the price impact due to QE. $\lambda(t, T)$ measures the speed of mean reversion and $d\Psi(t, T)$ measures the marginal price impact.

Solving the differential equation (3.1) leads to

$$f(t, T) = f'(t, T) - \int_0^t e^{-\int_s^t \lambda(u, T) du} d\Psi(s, T). \quad (3.2)$$

The intuition now becomes more clear. The observed forward rate $f(t, T)$ can be decomposed into two components: one is the true forward rate $f'(t, T)$ to which the observed rate mean reverts. The other is the price impact due to the Fed's activities, which depends on the rate of mean reversion $\lambda(t, T)$ and the marginal impact $d\Psi(t, T)$ due to changes in aggregate demand. Up to this point, $\lambda(t, T)$ and $\Psi(t, T)$ can be very general stochastic processes.

Although the Fed's holdings $y(t, T)$ are observable, the cumulative changes in market demand $z(t, T)$ are not. Consequently, $x(t, T)$ is not observable. Because of this, we will not be able to estimate $g(x(s, T), T)$ and $x(s, T)$ separately. This explains why the simplified notation in expression (2.4) is used above.

For tractability, we assume that the Fed buys the securities only up to time τ . The rate of mean reversion and the marginal impact on forward rates are both deterministic functions that only depend on the rate's maturity T . Analytically, we assume that $d\Psi(t, T) = I_{\{t \leq \tau\}} \psi(T) dt$, $\lambda(t, T) = \lambda(T)$, where $I_{\{\cdot\}}$ is an indicator

function. Then the forward rates can be written as:

$$f(t, T) = \begin{cases} f(0, T), & \text{if } t = 0 \\ f'(t, T) - \frac{\psi(T)}{\lambda(T)}(1 - e^{-\lambda(T)t}), & \text{if } 0 < t \leq \tau \\ f'(t, T) - \frac{\psi(T)}{\lambda(T)}(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t}, & \text{if } t > \tau. \end{cases} \quad (3.3)$$

In particular, the spot rate can be written as:

$$r_t = \begin{cases} r_0, & \text{if } t = 0 \\ r'_t - \frac{\psi(0)}{\lambda(0)}(1 - e^{-\lambda(0)t}), & \text{if } 0 < t \leq \tau \\ r'_t - \frac{\psi(0)}{\lambda(0)}(e^{\lambda(0)\tau} - 1)e^{-\lambda(0)t}, & \text{if } t > \tau. \end{cases} \quad (3.4)$$

3.1.2 Simultaneous Impacts on Interest and Foreign Exchange Rates

Consider a world where both the Federal Reserve and the Bank of Japan affect the spot interest rates of the U.S. dollar and the Japanese yen by buying respective government bonds. Traded assets are default-free zero-coupon bonds of all maturities and money market accounts (mma) denominated in the dollar and yen. The Fed keeps buying until time τ_d , while the BOJ keeps buying until time τ_y . The other traders act as price takers.

From Eqn. (3.4), the relation for U.S. dollar spot rates can be written as:

$$r_{d,t} = \begin{cases} r'_{d,t} - \frac{\psi_d}{\lambda_d}(1 - e^{-\lambda_d t}), & \text{if } t \leq \tau_d \\ r'_{d,t} - \frac{\psi_d}{\lambda_d}(e^{\lambda_d \tau} - 1)e^{-\lambda_d t}, & \text{if } t > \tau_d \end{cases}$$

where the first term $r'_{d,t}$ is the hypothetical spot rate if the Fed did not conduct QE and the second term is the price impact due to QE. $r_{d,t}$ is the realized spot rate in the presence of QE.

Similarly, for yen spot rates:

$$r_{y,t} = \begin{cases} r'_{y,t} - \frac{\psi_y}{\lambda_y}(1 - e^{-\lambda_y t}), & \text{if } t \leq \tau_y \\ r'_{y,t} - \frac{\psi_y}{\lambda_y}(e^{\lambda_y \tau} - 1)e^{-\lambda_y t}, & \text{if } t > \tau_y. \end{cases}$$

Therefore, the value of a dollar denominated mma evolves as

$$\begin{aligned} B_t &= \exp\left(\int_0^t r_{d,s} ds\right) \\ &= \begin{cases} B'_t \exp\left\{-\frac{\psi_d}{\lambda_d}t + \frac{\psi_d}{\lambda_d^2}(1 - e^{-\lambda_d t})\right\}, & \text{if } t \leq \tau_d \\ B'_t \exp\left\{-\frac{\psi_d}{\lambda_d}\tau_d + \frac{\psi_d}{\lambda_d^2}(e^{\lambda_d \tau_d} - 1)e^{-\lambda_d t}\right\}, & \text{if } t > \tau_d \end{cases} \end{aligned}$$

where B'_t is the mma value without the central bank's impact.

Similarly, the value of a yen denominated mma evolves as

$$\begin{aligned} \widehat{B}_t &= \exp\left(\int_0^t r_{y,s} ds\right) \\ &= \begin{cases} \widehat{B}'_t \exp\left\{-\frac{\psi_y}{\lambda_y}t + \frac{\psi_y}{\lambda_y^2}(1 - e^{-\lambda_y t})\right\}, & \text{if } t \leq \tau_y \\ \widehat{B}'_t \exp\left\{-\frac{\psi_y}{\lambda_y}\tau_y + \frac{\psi_y}{\lambda_y^2}(e^{\lambda_y \tau_y} - 1)e^{-\lambda_y t}\right\}, & \text{if } t > \tau_y. \end{cases} \end{aligned}$$

Since $\lim_{t \rightarrow \infty} B_t = B'_t \exp\left\{-\frac{\psi_d}{\lambda_d}\tau_d\right\}$, $\lim_{t \rightarrow \infty} \widehat{B}_t = \widehat{B}'_t \exp\left\{-\frac{\psi_y}{\lambda_y}\tau_y\right\}$, the impact on the price of mma does not decay to zero eventually.

For convenience, define

$$Z(t; \psi, \lambda, \tau) \equiv \begin{cases} \exp\left\{-\frac{\psi}{\lambda}t + \frac{\psi}{\lambda^2}(1 - e^{-\lambda t})\right\}, & \text{if } t \leq \tau \\ \exp\left\{-\frac{\psi}{\lambda}\tau + \frac{\psi}{\lambda^2}(e^{\lambda \tau} - 1)e^{-\lambda t}\right\}, & \text{if } t > \tau. \end{cases}$$

Denote Y_t as the spot exchange rate of yen per dollar. We assume there are no arbitrage opportunities from trading the mma, hence there exists an equivalent local martingale measure Q under which $\frac{\widehat{B}_t/Y_t}{B_t}$ is a Q -local martingale. The no-arbitrage relation implies

$$\frac{\widehat{B}_t/Y_t}{B_t} = E_t^Q \left(\frac{\widehat{B}_T/Y_T}{B_T} \middle| \mathcal{F}_t \right) \quad (3.5)$$

where T is a very long time period from time t , \mathcal{F}_t is the natural filtration up to time t . With the central banks' impact, the "distorted" price of the dollar value of the yen mma is

$$\begin{aligned}
\frac{\widehat{B}_t}{Y_t} &= E_t^Q \left(\frac{\widehat{B}_T/Y_T}{B_T} \middle| \mathcal{F}_t \right) B_t \\
&= E_t^Q \left(\frac{\widehat{B}_T Z(T; \psi_y, \lambda_y, \tau_y)/Y_T}{B_T' Z(T; \psi_d, \lambda_d, \tau_d)} \middle| \mathcal{F}_t \right) B_t' Z(t; \psi_d, \lambda_d, \tau_d) \\
&= E_t^Q \left(\frac{\widehat{B}_T'/Y_T}{B_T'} \middle| \mathcal{F}_t \right) B_t' Z(t; \psi_d, \lambda_d, \tau_d) \cdot \frac{Z(T; \psi_y, \lambda_y, \tau_y)}{Z(T; \psi_d, \lambda_d, \tau_d)} \quad (3.6)
\end{aligned}$$

Eqn. (3.6) holds because the $Z(\cdot)$ functions are deterministic and can be moved out of the expectation.

Theorem 2 *Under the above framework, the central banks' impact on the exchange rate is*

$$\frac{Y_t'}{Y_t} = \frac{Z(t; \psi_d, \lambda_d, \tau_d)}{Z(t; \psi_y, \lambda_y, \tau_y)} \exp \left\{ \frac{\psi_d}{\lambda_d} \tau_d - \frac{\psi_y}{\lambda_y} \tau_y \right\} e^{-u(t)} \quad (3.7)$$

where $Y_t(Y_t')$ is the spot exchange rate of yen per dollar with (without) the central banks' impact; ψ_d, λ_d (ψ_y, λ_y) are impact parameters on the dollar (yen) interest rates; $e^{-u(t)}$ represents the exchange rate impact through other channels.

The proof is offered in the appendix. The no-arbitrage relation (Eqn. (3.5)) is a key assumption in this chapter. Considering the large trading volume in the fixed income and exchange rate markets, it is reasonable to assume that there are no long-lasting arbitrage opportunities in these markets. The no-arbitrage relation is related to the uncovered interest-rate parity (UIP), but is weaker. It is weaker

because UIP, as distinct from IP, involves equilibrium risk premium. There are numerous studies testing UIP (see, for instance, Fama (1984); Bekaert and Hodrick (1993); Engel (1996); Flood and Rose (1996); Bansal (1997); Bakshi and Naka (1997); Huisman, Koedijk, Kool, and Nissen (1998); Backus, Foresi, and Telmer (2001); Chinn and Meredith (2005); Brennan and Xia (2006); Bekaert, Wei, and Xing (2007); Lothian and Wu (2011)). The evidence is mixed. Violations of UIP could be due to actual violations, small sample bias, or the poor estimation of risk premium.

Define $y_t = \ln Y_t$, $y'_t = \ln Y'_t$. Eqn. (3.7) can be written as

$$y_t - y'_t = \ln Z(t; \psi_y, \lambda_y, \tau_y) - \ln Z(t; \psi_d, \lambda_d, \tau_d) + \frac{\psi_y}{\lambda_y} \tau_y - \frac{\psi_d}{\lambda_d} \tau_d + u(t) \quad (3.8)$$

Eqn. (3.7) shows that the central banks' impact on exchange rates can be decomposed into two parts. One is an impact through interest rates:

$$\frac{Z(t; \psi_d, \lambda_d, \tau_d)}{Z(t; \psi_y, \lambda_y, \tau_y)} \exp \left\{ \frac{\psi_d}{\lambda_d} \tau_d - \frac{\psi_y}{\lambda_y} \tau_y \right\},$$

and the rest is represented by $e^{-u(t)}$. $u(t)$ includes the impact from other factors such as the direct foreign exchange intervention and flight to quality during the sample period. In particular, we assume

$$du(t) = dq(t) + \lambda_t(y'_t - y_t)dt$$

where the first term $dq(t)$ represents the impact from direct FX market intervention, and the second term $\lambda_t(y'_t - y_t)dt$ represents other effects such as flight to quality. The other effects are measured as a proportion of the total impact. A detailed analysis about $u(t)$ is given later in the chapter.

As a reality check, one can see that when the Fed's U.S. bond purchasing activities dominate ($Z(t; \psi_d, \lambda_d, \tau_d) > Z(t; \psi_y, \lambda_y, \tau_y)$ or $\frac{\psi_d}{\lambda_d} \tau_d > \frac{\psi_y}{\lambda_y} \tau_y$), the dollar

depreciates ($Y_t < Y'_t$) relative to the case when there is no central banks' impact. When the BOJ's yen bond purchasing activities dominates, the dollar appreciates ($Y_t > Y'_t$). The degree of depreciation (appreciation) depends on respective impact functions.

Substituting the expressions for $Z(t; \psi_d, \lambda_d, \tau_d)$ and $Z(t; \psi_y, \lambda_y, \tau_y)$ into Eqn. (3.8) , one obtains

$$y_t - y'_t = \left(\frac{\psi_d}{\lambda_d} - \frac{\psi_y}{\lambda_y} \right) t + \left[\frac{\psi_y}{\lambda_y^2} (1 - e^{-\lambda_y t}) - \frac{\psi_d}{\lambda_d^2} (1 - e^{-\lambda_d t}) \right] + \frac{\psi_y}{\lambda_y} \tau_y - \frac{\psi_d}{\lambda_d} \tau_d + u(t), \text{ if } t \leq \min(\tau_d, \tau_y) \quad (3.9)$$

$$y_t - y'_t = \left[\frac{\psi_y}{\lambda_y^2} (e^{\lambda_y \tau} - 1) e^{-\lambda_y t} - \frac{\psi_d}{\lambda_d^2} (e^{\lambda_d \tau} - 1) e^{-\lambda_d t} \right] + u(t), \text{ if } t \geq \max(\tau_d, \tau_y).$$

3.2 Data

We examine the USD/JPY currency pair during the Federal Reserve's QE period. The exchange rate data is the daily USD/JPY rate obtained from Bloomberg. The sample period spans from December 15, 2008 to July 15, 2011. It starts at the beginning of QE1 and ends at the termination of QE2. Panel A of Table 3.1 presents summary statistics of the exchange rates.

The interest rate data are the daily instantaneous U.S. Treasury forward rates constructed by Gürkaynak, Sack, and Wright (GSW (2007)) and are available on the Federal Reserve website.¹ We choose this dataset for easy comparison with the existing literature. In the following sections, we estimate a three-factor term structure model using a Kalman filter. Therefore, four time-series of forward rates

¹<https://www.federalreserve.gov/econresdata/researchdata.htm>

Mean	Std	Skewness	Kurtosis
-0.00017	0.007	0.13	5.23

Panel A: USD/JPY

Maturity (year)	Mean (%)	Std	Skewness	Kurtosis
1	0.6905	0.2681	0.0960	2.2091
2	1.5868	0.4574	-0.1643	2.2404
3	2.5479	0.5509	-0.3361	2.1781
4	3.4133	0.5764	-0.4297	2.1876

Panel B: GSW Forward Rates

Table 3.1: Summary Statistics

are enough to estimate a three-factor model. The input data are the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning from December 15, 2008 to July 15, 2011. The forward rates are plotted in Figure 3.4 and the summary statistics are presented in Panel B of Table 3.1.

The balance sheet data for BOJ and the Fed are downloaded from their respective websites.² Japan's Ministry of Finance website provides data about their direct currency market interventions. The quarterly reports of Treasury and Federal Reserve foreign exchange operations provide data on direct currency interventions by the U.S. monetary authorities.

²The Bank of Japan's balance sheet: <http://www.boj.or.jp/en/statistics/boj/other/acmai/index.htm/>
The Federal Reserve's balance sheet: <http://www.federalreserve.gov/releases/h41/>

GSW Forward Rates

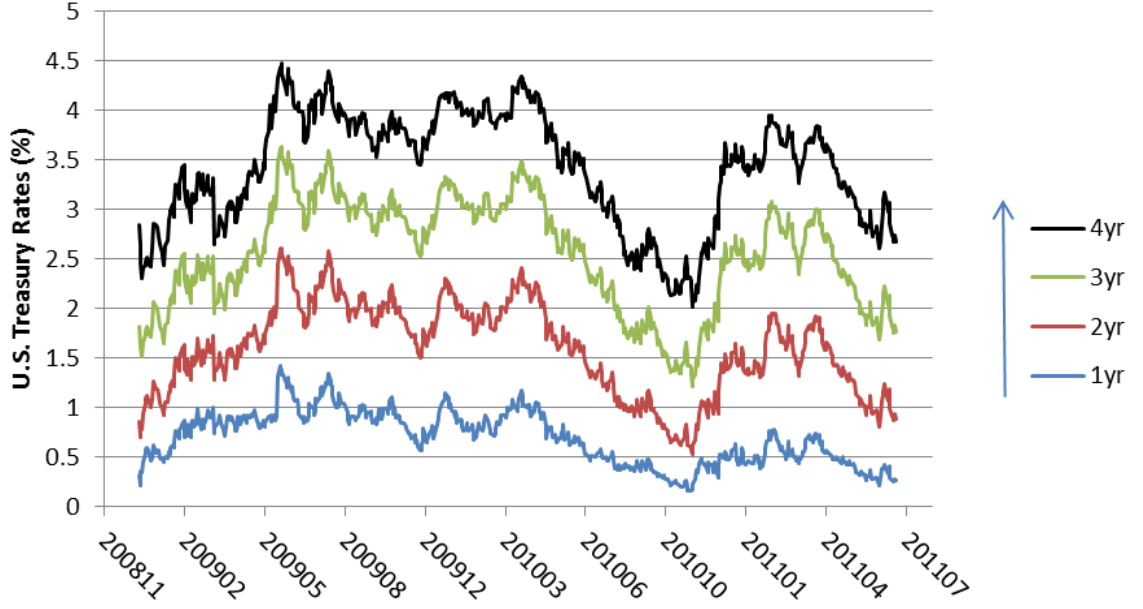


Figure 3.4: GSW Forward Rates

3.3 Estimation

Since the BOJ did not conduct massive asset purchases over this period (see Figures 3.2 and 3.3), we believe the assumption $\psi_y = 0$ is a good approximation and expression (3.9) can be simplified as

$$\begin{aligned}
 y_t - y'_t &= \frac{\psi_d}{\lambda_d} t - \frac{\psi_d}{\lambda_d^2} (1 - e^{-\lambda_d t}) - \frac{\psi_d}{\lambda_d} \tau_d + u(t), \quad \text{if } t \leq \tau_d \\
 y_t - y'_t &= -\frac{\psi_d}{\lambda_d^2} (e^{\lambda_d \tau_d} - 1) e^{-\lambda_d t} + u(t), \quad \text{if } t > \tau_d.
 \end{aligned} \tag{3.10}$$

Define

$$v(t) = \frac{\psi_d}{\lambda_d} t - \frac{\psi_d}{\lambda_d^2} (1 - e^{-\lambda_d t}) - \frac{\psi_d}{\lambda_d} \tau_d \tag{3.11}$$

$v(t)$ measures the impact due to QE.

Following Jarrow and Li (2012), we estimate the impact parameters ψ_d and λ_d using a three-factor affine term structure model (Vasicek (1977)). In a three-factor affine model, the short rate is the sum of three factors.

$$r(t) = \sum_{n=1}^3 z_n(t).$$

Each factor $z_n(t)$ evolves as

$$dz_n(t) = k_n(\theta_n - z_n(t))dt + \sigma_n dW_n(t)$$

where $W_n(t)$ for $n = 1, 2, 3$ are independent standard Wiener processes.

This evolution allows the spot rate to be negative with positive probability. Although alternative evolutions could be used to preclude negative rates, both economic theory and the empirical evidence are more consistent with evolutions that allow negative (nominal) rates with positive probability. Indeed, from a theoretical perspective, large financial institutions cannot store currency; they can only invest it in either deposits or securities; and consequently, negative rates are possible. Empirically, negative rates on Treasuries were observed in each of November 2009, June 2011, and August 2011,³ negative yield on German government bond was observed in July 2012,⁴ and the Bank of New York Mellon paid negative deposit rates in August 2011.⁵

Since the spot rate is unobservable, we estimate the model using a Kalman filter. The measurement equation is

$$f(t, \Lambda_i) = A_i + \sum_{n=1}^3 B_{i,n} z_n(t) + w(t, \Lambda_i) \quad (3.12)$$

³See WSJ Blog, Market Beat, November 20, 2009, "Some Treasury Bill Rates Negative Again Friday;" Bloomberg, November 19, 2009, "U.S. 3-month Bills Turn Negative on Concern Risk Rally Overdone;" Bloomberg, June 27, 2011, "Treasury 4-week Bill Rates Negative for First Time since 2010;" WSJ Blog, Market Beat, August 4, 2011, "From One Crisis to Another: One Month T-Bill Yields go Negative Again."

⁴See Online WSJ, July 18, 2012, "Negative Yield on German 2-Year Note."

⁵See Bloomberg.com/news, August 5, 2011, "BNY Mellon Makes Clients Pay for Deposits as Investors Seek Safety in Cash;" Online WSJ, August 5, 2011, "New Fee to Bank Cash."

where $w(t, \Lambda_i)$ are assumed to follow independent normal distributions.

$$B_{i,n} = e^{-k_n \Lambda_i}, \quad \Lambda_i = T_i - t.$$

For $0 < t \leq \tau_d$,

$$A_i = \sum_{n=1}^3 \theta_n [1 - e^{-k_n \Lambda_i}] - \frac{\psi_i}{\lambda_i} (1 - e^{-\lambda_i t}).$$

The time-discretized state transition equation can be written as

$$z_n(t + \Delta t) = \theta_n (1 - e^{-k_n \Delta t}) + e^{-k_n \Delta t} z_n(t) + \varepsilon_n(t) \quad n = 1, 2, 3 \quad (3.13)$$

where $\varepsilon_n(t)$ follow zero-mean normal distributions with the following variance and covariance

$$\begin{aligned} Var[\varepsilon_n(t) | \mathcal{F}_{t-\Delta t}] &= \frac{\sigma_n^2}{2k_n} (1 - e^{-2k_n \Delta t}) \\ Cov[\varepsilon_n(t), \varepsilon_m(t) | \mathcal{F}_{t-\Delta t}] &= 0 \quad n \neq m \end{aligned}$$

where \mathcal{F}_t is the natural filtration generated by the state variables process up to time t .

Since it is a three-factor model, it can be identified as long as there are more than three time-series of input forward rates. Thus, we use the 1-year, 2-year, 3-year and 4-year GSW forward rates spanning the QE period. The estimation results are listed in Table 3.2. Since the instantaneous spot rate is unobservable, we use the impact parameters on the one-year rate as an approximation: $\lambda_d = \lambda_1 = 2.22$, $\psi_d = \psi_1 = 0.1$. Plugging the numbers into the right hand side of Expression (3.11), one obtains the time series of $v(t)$, which is the impact of QE and is graphed as the dashed curve in Figure 3.5. The dotted curve plots the observed exchange rate evolution (y_t).

Figure 3.5 shows that QE leads to a depreciation of the dollar. If QE was not conducted, the dollar value could have been higher. Over the sample period, the

Parameter	With Impact		Without Impact	
	Estimate	Std	Estimate	Std
θ_1	0.02	0.82	0.024	14.4
θ_2	0.022	0.81	0.052	4.4
θ_3	0.025	0.81	0.009	10.3
k_1	0.11	0.009	0.14	0.02
k_2	0.11	0.038	0.14	0.09
k_3	0.59	0.067	0.10	1.69
σ_1	0.0121	0.0006	0.0114	0.0023
σ_2	0.0036	0.0008	0.0021	0.0072
σ_3	0.0001	0.0017	0.0001	0.0014
λ_1	2.22	0.21		
λ_2	1.77	0.19		
λ_3	1.30	0.17		
λ_4	0.82	0.20		
ψ_1	0.10	0.01		
ψ_2	0.07	0.01		
ψ_3	0.04	0.008		
ψ_4	0.02	0.007		
lnL	11235		11127	

Table 3.2: Three-factor Parameter Estimates

average value of $v(t)$ is -7.29% . Our interpretation is that the Fed's QE resulted in a 7.29% depreciation of the dollar on average.

3.4 Impact Due to Other Factors

The Fed's QE is not the only factor to affect the dollar/yen exchange rate over the sample period. For instance, during the 2007-2009 global financial crisis, many risk-averse investors flocked to U.S. assets. This safe-haven effect can cause the dollar to appreciate. In addition, direct FX intervention by the central authority

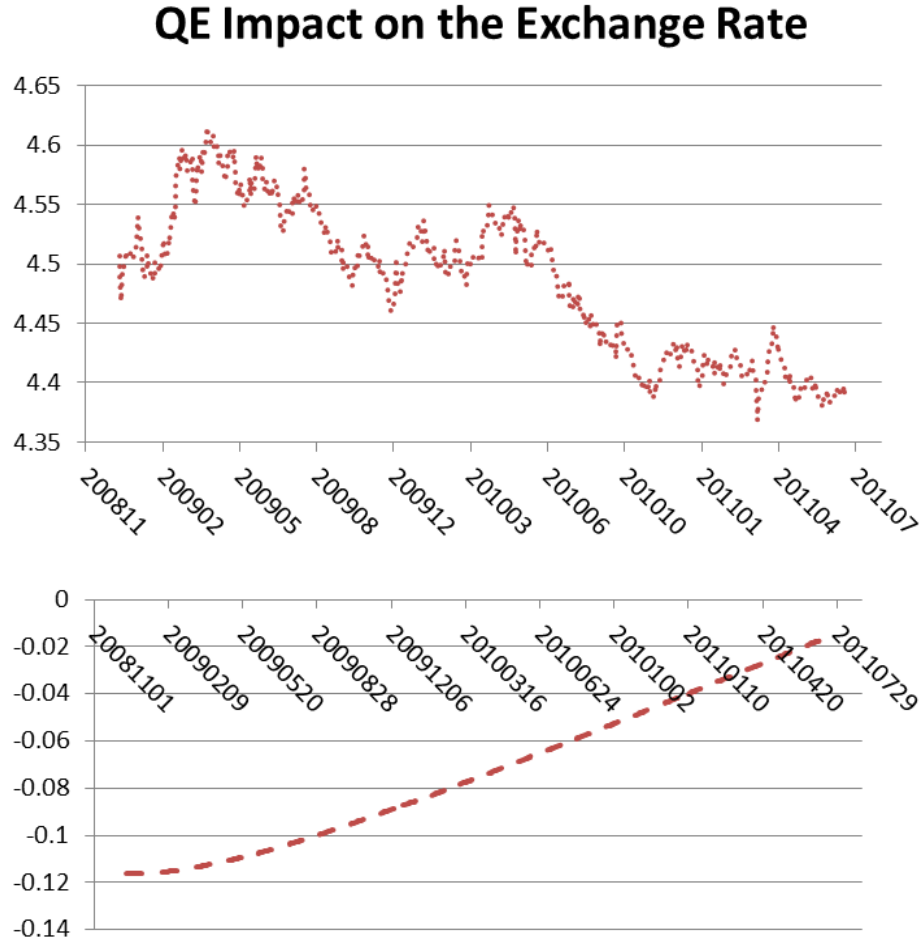


Figure 3.5: QE Impact on USD/JPY

could also have an impact. In this section, we propose a reduced-form approach to estimate the net effect due to factors other than QE. It has been denoted as $u(t)$ in the theory model.

3.4.1 The Derivation

Eqn. (3.10) leads to

$$dy_t - dy'_t = \frac{\psi_d}{\lambda_d}(1 - e^{-\lambda_d t})dt + du(t), \text{ if } t \leq \tau_d. \quad (3.14)$$

As discussed in Section 2, we assume

$$du(t) = \lambda_t(y'_t - y_t)dt + dq(t)$$

where $dq(t)$ represents the impact from direct FX market intervention. $\lambda_t(y'_t - y_t)dt$ represents the impact from other factors such as the safe-haven effect mentioned above. Eqn. (3.14) can be written as

$$dy_t = dy'_t + \frac{\psi_d}{\lambda_d}(1 - e^{-\lambda_d t})dt + \lambda_t(y'_t - y_t)dt + dq(t). \quad (3.15)$$

Previous studies have found that direct currency market interventions have a short-lived impact, especially sterilized interventions (see Dominguez and Frankel (1993)). As an approximation, we assume $dq(t) = 0$.

Assuming $y_0 = y'_0$, the solution to Eqn. (3.15) is:

$$y_t = y'_t + \int_0^t e^{-\int_s^t \lambda_u du} \frac{\psi_d}{\lambda_d} (1 - e^{-\lambda_d s}) ds.$$

For simplicity, assuming the parameters to be constant: $\lambda_t = \lambda$, $\psi_t = \psi$, one has

$$y_t = y'_t + \frac{\psi_d/\lambda_d}{\lambda} (1 - e^{-\lambda t}) - \frac{\psi_d/\lambda_d}{\lambda - \lambda_d} (e^{-\lambda_d t} - e^{-\lambda t}), \quad t \leq \tau_d \quad (3.16)$$

It is worth noting that Eqn. (3.16) is consistent with the assumption that $y_0 = y'_0$.

Combining Eqn. (3.10) and (3.16) leads to

$$u(t) = \frac{\psi_d/\lambda_d}{\lambda} (1 - e^{-\lambda t}) - \frac{\psi_d/\lambda_d}{\lambda - \lambda_d} (e^{-\lambda_d t} - e^{-\lambda t}) - \left(\frac{\psi_d}{\lambda_d} t - \frac{\psi_d}{\lambda_d^2} (1 - e^{-\lambda_d t}) - \frac{\psi_d}{\lambda_d} \tau_d \right) \quad (3.17)$$

for $t \leq \tau_d$.

3.4.2 Estimation

To estimate the impact parameter λ , one needs to assume an underlying process for the "true" exchange rate y'_t . We assume y'_t follows a geometric Brownian motion (GBM):

$$dy'_t = \mu dt + \sigma dW_t$$

where W_t is a Wiener process $W_t \sim N(0, t)$. Then

$$y'_t = y'_0 + \mu t + \sigma W_t$$

The GBM assumption is made for two reasons. First, it is easy to estimate and avoids the problem of overfitting. Second, it is a widely-adopted assumption in studying currency options (for empirical work see Shastri and Tandon (1986), Hilliard, Madura and Tucker (1991); for theoretical work see Biger and Hull (1983), Amin and Jarrow (1991)).

Since $y'_0 = y_0$, Eqn. (3.16) can be written as

$$y_t = y_0 + \mu t + \sigma W_t + \frac{\psi_d/\lambda_d}{\lambda}(1 - e^{-\lambda t}) - \frac{\psi_d/\lambda_d}{\lambda - \lambda_d}(e^{-\lambda_d t} - e^{-\lambda t}), \text{ if } t \leq \tau_d \quad (3.18)$$

which leads to the following distribution for y_t :

$$y_t \sim N\left(y_0 + \mu t + \frac{\psi_d/\lambda_d}{\lambda}(1 - e^{-\lambda t}) - \frac{\psi_d/\lambda_d}{\lambda - \lambda_d}(e^{-\lambda_d t} - e^{-\lambda t}), \sigma^2 t\right)$$

The impact parameter λ can be estimated by the maximum likelihood method. The estimation results are listed in Panel B of Table 3.3: $\mu = -0.0229$, $\lambda = 3.94$.

The negative sign of μ implies that the true exchange rate y'_t has a depreciating trend over the sample period. A positive and significant λ suggests that there are other factors supporting the dollar. It is noteworthy that $\lambda > \lambda_d$. In the

	Estimate	Std
Panel A		
λ_d	2.22	0.21
ψ_d	0.1	0.01
Panel B		
μ	-0.0229	0.0002
λ	3.94	0.07
$\ln L_1 = -53907$		
$BIC = 107827$		

Table 3.3: Impact Model Estimation

absence of these factors, the dollar could have depreciated more over the sample period. One possible explanation could be a safe-haven effect in the dollar, which refers to the phenomenon that risk-averse investors flocked to U.S. assets during the 2007-2009 global financial crisis. The total impact $(y_t - y'_t)$ (Eqn. (3.16)) and the safe-haven effect $u(t)$ (Eqn. (3.17)) can be computed using the estimated parameters. Figure 3.6 provides a decomposition of the total impact. The average total impact $(y_t - y'_t)$ is 0.84%. This slightly positive value can be interpreted as the result of a combination of a negative QE impact (-7.29% on average) and a positive safe-haven effect (8.13% on average).

To further understand the intuition behind the results, one can write

$$y_t - y_0 = v(t) + u(t) + (y'_t - y'_0) \quad (3.19)$$

where $y_t - y_0$ is the the observed dollar depreciation, $v(t)$ is the QE impact, $u(t)$ is the safe-haven effect and $y'_t - y'_0$ is the unobserved true rate depreciation.

Rewriting Eqn. (3.19) gives

$$1 = \frac{v(t)}{y_t - y_0} + \frac{u(t)}{y_t - y_0} + \frac{y'_t - y'_0}{y_t - y_0} \quad (3.20)$$

With the help of Eqn. (3.20), the interpretation becomes more clear. The esti-

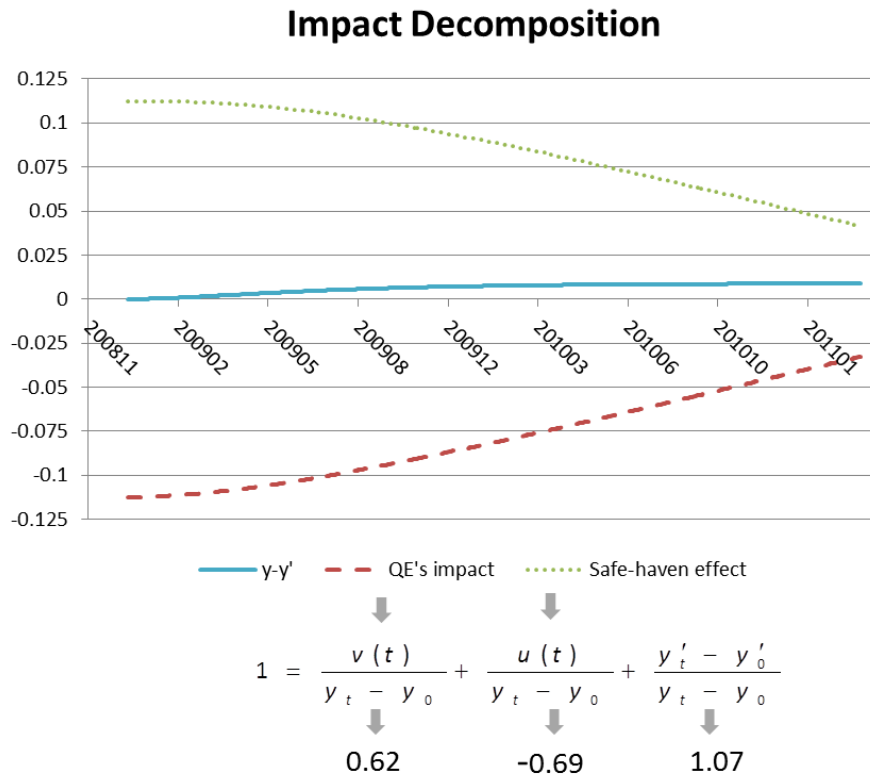


Figure 3.6: Impact Decomposition

mation results show that, the QE impact (the first term) can explain 62% of the observed total dollar depreciation. Meanwhile, there is a safe-haven effect (the second term) that causes a dollar appreciation. The magnitude is about 69% of the total dollar depreciation but the sign is the opposite. In addition, the unobserved true dollar value depreciated 7% more than the observed dollar depreciation (the last term). This decomposition is also shown in Figure 3.6.

3.5 Robustness Checks

To show that the model described by Eqn. (3.18) is able to measure the impact of QE, we conduct two parts in the robustness check. The first part is related with interest rate dynamics and the second is related with exchange rate dynamics.

3.5.1 Interest Rate Dynamics

In this section, we try to show that Eqn. (3.12) and (3.13) is able to capture QE's impact on interest rates. The endogeneity issue mentioned in the introduction largely resides here. Once people are convinced that the model is able to capture the QE impact on interest rates, then the impact on the exchange rate is an equilibrium result derived from a no-arbitrage condition.

To show that the impact parameters indeed do measure the QE impact, we estimate the model using two other sample periods when QE was absent. One ranges from January 2, 2001 to August 1, 2003 when the Fed lowered interest rates. The other period spans from January 2, 2004 to August 1, 2006 when the Fed raised interest rates. The input data are the 1-year, 2-year, 3-year and 4-year GSW(2007) forward rates. Tables 3.4 and 3.5 list the estimation results. In Table 3.4, only the impact parameters on the one-year forward rate are significant. In Table 3.5, even though the duration parameter (λ) is significant for the two- and three-year rates, the magnitude parameter (ψ) is not. Also, the impact parameters on the one-year rate are not significant. These results suggest that the model does not provide a good fit for periods when there is no QE. In contrast, all the impact parameters are strongly significant for the QE period (see Table 3.2). These results support that the model is able to measure the impact of QE on interest rates.

Parameter	With Impact		Without Impact	
	Estimate	Std	Estimate	Std
θ_1	0.021	0.82	0.023	0.82
θ_2	0.020	0.82	0.029	0.82
θ_3	0.022	0.82	0.024	0.82
k_1	0.59	0.31	0.26	0.15
k_2	0.28	0.06	0.25	0.02
k_3	0.28	0.12	0.25	0.03
σ_1	0.0001	0.015	0.005	0.009
σ_2	0.032	0.009	0.026	0.005
σ_3	0.016	0.007	0.021	0.005
λ_1	1.35	0.62		
λ_2	0.75	0.64		
λ_3	0.35	0.71		
λ_4	0.00	0.90		
ψ_1	0.071	0.023		
ψ_2	0.035	0.015		
ψ_3	0.017	0.011		
ψ_4	0.008	0.008		
lnL	9563		9508	

Table 3.4: Robustness Check: Jan. 2, 2001 - Aug. 1, 2003

As a comparison, Tables 3.2, 3.4 and 3.5 also list the estimation results for a model without the price impact term. The bottom row provides the maximized log-likelihood value and a likelihood ratio test is conducted. The test statistic is $2(\ln(L_1) - \ln(L_2))$, where L_1 (L_2) is the maximized likelihood value with (without) the price impact term.

At the 5% significance level, the likelihood ratio test rejects the model without the price impact for all three sample periods. This is to be expected since it is an in sample test, and the price impact model has more parameters. More insightful is a comparison of the magnitudes of the changes in the likelihood values over

Parameter	With Impact		Without Impact	
	Estimate	Std	Estimate	Std
θ_1	0.006	0.94	0.020	0.82
θ_2	0.071	1.25	0.022	0.82
θ_3	0.009	0.95	0.018	0.82
k_1	0.45	0.08	0.98	0.06
k_2	0.21	0.02	0.20	0.03
k_3	0.21	0.04	0.20	0.03
σ_1	0.0001	0.005	0.069	0.007
σ_2	0.019	0.003	0.015	0.002
σ_3	0.012	0.005	0.013	0.003
λ_1	0.87	0.38		
λ_2	1.59	0.31		
λ_3	1.11	0.21		
λ_4	1.20	0.16		
ψ_1	-0.010	0.007		
ψ_2	0.0006	0.007		
ψ_3	0.010	0.005		
ψ_4	0.018	0.004		
lnL	10738		10698	

Table 3.5: Robustness Check: Jan. 2, 2004 - Aug. 1, 2006

the different sample periods. For the 1/2/2001-8/1/2003 sample (no QE), the log-likelihood increases by 0.6% after adding the price impact term. For the 1/2/2004-8/1/2006 (no QE) sample period, the increase is only 0.4%. In contrast, for the QE period, the log-likelihood increases the most, by 1%, after adding the price impact term. These relative changes in the likelihood ratio tests are consistent with the validity of the model.

We also compute pricing errors for an affine model with and without the Fed's price impact term. The results are listed in Table 3.6. It shows that the model with a price impact term generates much smaller pricing errors. This lends additional

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	$\rho(1)$	$\rho(10)$	$\rho(20)$
1	-1.61	0.56	-2.87	0.23	2.71	0.88	0.57	0.31
2	-2.17	0.73	-2.99	0.37	2.62	0.88	0.51	0.25
3	-1.84	1.07	-1.71	-0.23	2.64	0.94	0.71	0.50
4	-0.51	1.29	-0.40	-0.63	2.88	0.95	0.77	0.58

Panel A (With the price impact term)

Maturity (year)	Mean (bp)	SE	t	Skew	Kurt	$\rho(1)$	$\rho(10)$	$\rho(20)$
1	-4.34	0.71	-6.14	0.32	2.33	0.93	0.69	0.49
2	-15.34	0.84	-18.19	0.36	3.07	0.91	0.56	0.29
3	-5.90	1.31	-4.51	-0.16	2.32	0.96	0.76	0.57
4	5.96	1.53	3.88	-0.35	2.25	0.97	0.81	0.64

Panel B (Without the price impact term)

Table 3.6: Summary Statistics of Pricing Errors

support to the methodology.

Another concern is that the GSW forward rates are generated by a smoothing method that assumes a parametric model with six parameters (see Svensson (1994)). This special smoothing method may cause biases. To address this issue, we estimate the model using forward rates based on a polynomial spline smoothing procedure. The results are very similar.

3.5.2 Foreign Exchange Dynamics

The following subsections address two issues. First, over the QE period, one should expect Eqn. (3.18) (call it the "impact model") to outperform a simple model without the price impact term. Second, one needs to rule out the effect of direct

	Estimate	Std
μ	-0.017	0.0002
$\ln L_2 = -55970$		
$BIC = 111947$		

Table 3.7: GBM Model Estimation

FX market interventions.

QE v.s. Non-QE Period

In this subsection, we first show that the impact model provides a better fit than a simple geometric Brownian motion (GBM) model for the QE period. Second, we show that the simple GBM model performs better in a non-QE period than in a QE one.

If one imposes $\psi_d = 0$, then the impact model becomes the GBM model. We estimate the GBM model using maximum likelihood for the QE period. The results are listed in Table 3.7. The new drift parameter $\mu = -0.017$ is significantly different from the one estimated with the impact model.

The impact model has two free parameters (μ and λ) while the GBM model has one free parameter (μ). To discover which one provides a better fit for the QE period, we conduct a likelihood-ratio test to compare them. The test statistic is $2(\ln(L_1) - \ln(L_2))$, where L_1 and L_2 are the corresponding likelihood functions. Since the difference in free parameters is 1, the test statistic should follow a χ^2 distribution with 1 degree of freedom. The values of $\ln(L_1)$ and $\ln(L_2)$ are listed in Table 3.3 and 3.7. For the $\chi^2(1)$ distribution, the 1% p-value corresponds to a critical value of 6.6. The calculation shows that $2(\ln(L_1) - \ln(L_2)) = 4126$, which is far greater than the critical value. Therefore, the likelihood ratio test suggests

that the impact model fits the data much better than the GBM model does for the QE period.

To provide further support, we also adopt the Bayesian information criterion (BIC) (Schwarz (1978)) to compare the two models. The BIC criterion is defined as

$$BIC = -2\ln(L) + k\ln(N)$$

where L is the maximized likelihood function, k is the number of free parameters and N is the sample size. The smaller the BIC value, the better the model.

The BIC values are listed in Tables 3.3 and 3.7. The BIC increases by about 4% as one moves from the GBM to the impact model. This is additional evidence to favor the impact model.

Furthermore, one should expect the simple GBM model to perform better in a non-QE period than in a QE period. Therefore, we conduct the estimation using another time-series with no QE. The time period spans from June 30, 2006 to July 15, 2008 and includes 533 business days. Call it "control period". In this period, both the Fed and the BOJ did not conduct QE. To match the control period, we truncate the main sample period to range from December 15, 2008 to December 29, 2010 so that it also contains 533 business days. We estimate the GBM model for both periods by minimizing the following objective function:

$$f(\mu) = \sum_{t=1}^{533} \frac{[y_t - (y_0 + \mu t\Delta)]^2}{\sigma^2 t\Delta}$$

where $\Delta = 1/250$.

The results are listed in Table 3.8. One can see that the minimized objective function for the control period is $f_C(\mu) = 41939$ while for the QE period is

	Estimate	Std
μ	-0.0031	0.0002
Obj. function	41939	

Jun. 30, 2006 – Jul. 15, 2008

	Estimate	Std
μ	-0.0009	0.0002
Obj. function	50856	

Dec. 15, 2008 – Dec. 29, 2010

Table 3.8: QE v.s. Non-QE Period

$f_{QE}(\mu) = 50856$. The fact that $f_C(\mu)$ is about 18% lower than $f_{QE}(\mu)$ suggests that the simple GBM model is a better fit for a non-QE period.

The Impact of Direct Interventions

In Figure 3.7, one can see that there are two direct intervention days during this sample period. On September 15, 2010, Japan’s monetary authorities sold yen and bought dollars to weaken the yen and protect its exporters. On March 18, 2011, both the Japanese and U.S. monetary authorities intervened in the FX markets following Japan’s triple disasters. This intervention successfully resulted in a depreciation of the yen by 3~4% within hours. Neely (2011) studies the latter episode and argues that the direct intervention has an immediate short-term impact, while a long-term effect is hard to identify.

As another robustness check, we show that the exchange rate impact measured above cannot be ascribed to direct FX interventions. To achieve this, we exclude the direct intervention days over the QE period and re-estimate the model. The

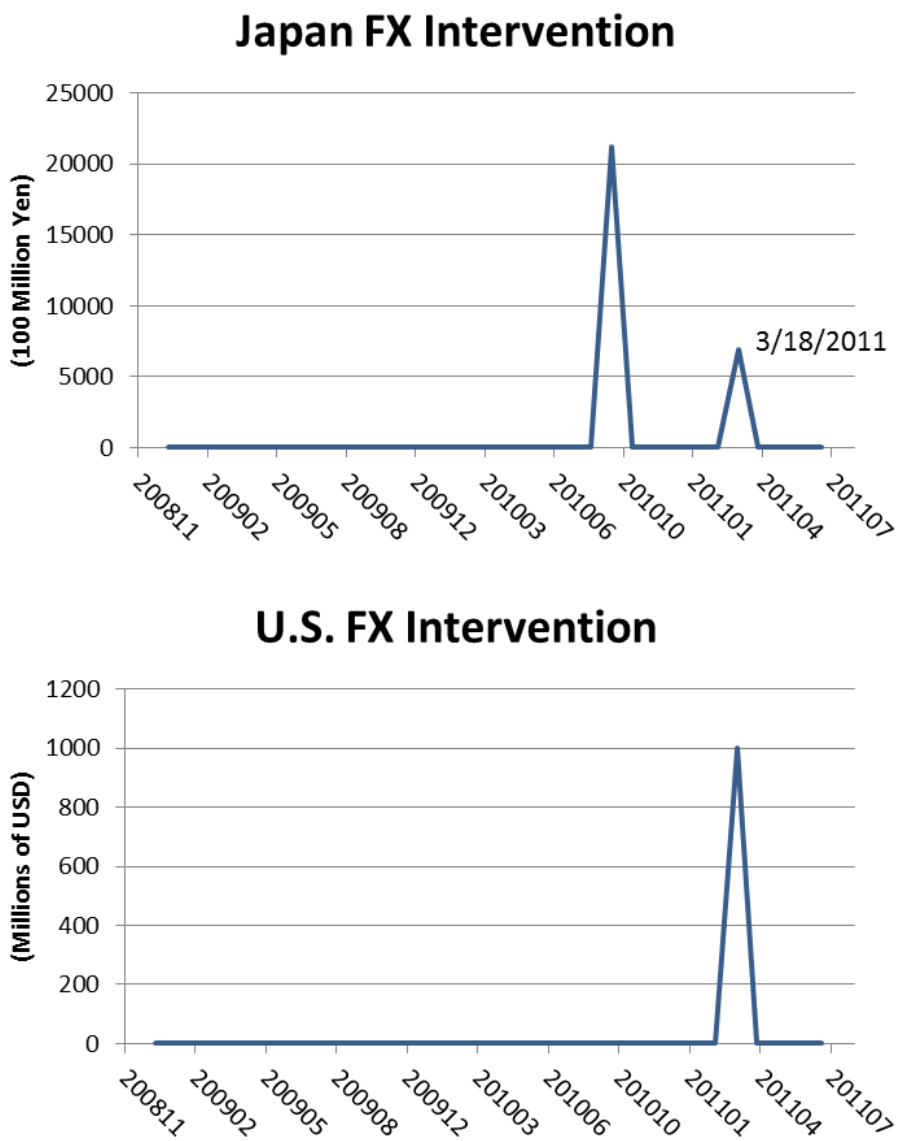


Figure 3.7: Direct FX Interventions

results are listed in Table 9. The top panel is estimated with the full sample; the middle panel is estimated with data excluding direct intervention days; and the bottom panel is estimated with data excluding three days around the intervention days. After accounting for estimation errors, the parameter estimates are almost the same. This suggests that the model does indeed measure the effect due to QE instead of direct exchange interventions, hence the assumption $dq(t) = 0$ presented

	Estimate	Std
Full sample		
μ	-0.0229	0.0002
λ	3.94	0.07
Excluding intervention days		
μ	-0.0228	0.0002
λ	3.95	0.07
Excluding 3 days around intervention days		
μ	-0.0227	0.0002
λ	3.97	0.07

Table 3.9: Impact Model Estimation Excluding Direct Intervention Days

in Section 5 is a good approximation.

To summarize, the above robustness checks demonstrate that the impact model is able to capture the effect of the Fed's QE on the USD/JPY exchange rate.

3.6 Final Remarks

This chapter provides a framework to study the impact of a central bank's bond market intervention on foreign exchange rates. The key assumption is a no-arbitrage relationship between the interest rates and foreign exchange markets. With this framework, we estimate the impact of the Federal Reserve's 2008 - 2011 QE program on the USD/JPY exchange rate. We find that the Fed's QE can explain about 62% of the total dollar depreciation during this period. The implication is that a central bank's bond market intervention might have significant impact on foreign exchange rates.

CHAPTER 4

CONCLUSIONS

This dissertation provides a new framework for analyzing the impact of the Federal Reserve's quantitative easing (QE) on the Treasury yield curve and foreign exchange rates. To test our theory, we estimated an arbitrage-free affine model that includes the price impact of a large trader, the Fed, over the time period when the Fed conducted its QE program: late 2008 to the middle of 2011. Our findings indicate that the QE program generated significant price impacts on short- and medium- term Treasury forward rates of up to 12 years without introducing arbitrage opportunities into the markets. In contrast to the Fed's stated intentions, however, the impact on long-term forward rates appears to have been insignificant. The half-life of the forward rate impacts increased with the maturity of the forward rate up to approximately 6 years, and then declined thereafter. The largest half-life estimated is approximately 1.4 years in duration. Since yields are averages of forward rates, QE did have an impact on long-term bond yields. Our estimates of the magnitude of the QE yield changes are similar to those that appear in the existing literature.

The model estimated herein was simplified in order to facilitate an analytic representation and the use of maximum likelihood estimation procedures. As such, the model can and should be generalized to explore its empirical validity. Two immediate extensions are to have a more complex large trader impact process and a more complex evolution for the term structure of interest rates.

We also extend the framework to analyze the impact of a central bank's bond market intervention on foreign exchange rates. With this framework, we estimate the impact of the Federal Reserve's 2008 - 2011 QE program on the USD/JPY

exchange rate. The advantage of studying this event is that both the U.S. and Japanese government did not conduct much direct intervention in the currency markets at that time, and only the Federal Reserve conducted significant bond purchasing activities. Over this sample period, the observed cumulative dollar depreciation is about 11.70%. We find that the Fed's QE may have resulted in a depreciation of the dollar by 7.29% during this period. That accounts for 62% of the total depreciation. The policy implication is that, when a central bank intervenes in the interest rate market by trading a large volume of bonds, it should consider the impact on the exchange rate, which could be very large.

Since the Fed's QE is not the sole factor to affect the exchange rate, we propose a reduced-form approach to estimate the net effect due to other factors. Our estimation results suggest that some other factors supported the dollar over the sample period, offsetting the QE impact. Interestingly, we find that the net impact is slightly positive after combining all factors. The average net impact is about a 0.84% appreciation of the dollar. Our explanation of this result is that a flight-to-quality safe-haven effect was very strong during the sample period.

Broadly speaking, there are two channels through which a central monetary authority can affect exchange rates: one is direct intervention in the FX markets; the other is to affect exchange rates through interest rates. This dissertation shows that the second channel lasts longer than the first one, and that the impact magnitude can be very large.

Some extensions would be valuable following our studies. One extension is to apply this framework to different episodes and different currency pairs. It can be used to study the case in which two central banks conduct QE simultaneously. Another extension is to introduce more complex models for exchange rates move-

ments. These extensions await subsequent research.

APPENDIX A

APPENDIX

Proof of Theorem 1:

From expression (2.11), for $t \leq \tau$, we have

$$dF(t, T) = (\mu(t, T) - \psi(T)e^{-\lambda(T)t})dt + \sum_{n=1}^N \sigma_n(t, T)dW_n(t)$$

The HJM condition on $f(t, T)$ implies that

$$\mu(t, T) = - \sum_{n=1}^N \sigma_n(t, T) \left[\phi_n(t) - \int_t^T \sigma_n(t, s)ds \right] \quad (\text{A.1})$$

The HJM condition on $F(t, T)$ implies that

$$\mu(t, T) - \psi(T)e^{-\lambda(T)t} = - \sum_{n=1}^N \sigma_n(t, T) \left[\Phi_n(t) - \int_t^T \sigma_n(t, s)ds \right] \quad (\text{A.2})$$

where $\Phi_i(t)$ ($\phi_i(t)$) is the price of risk for factor i with (without) the Fed's price impact.

From expression. (A.1) and (A.2), we obtain the difference in risk premium:

$$\sum_{n=1}^N \sigma_n(t, T)[\Phi_n(t) - \phi_n(t)] = \psi(T)e^{-\lambda(T)t} > 0$$

From expression (2.11), for $t > \tau$, we have

$$dF(t, T) = [\mu(t, T) + \psi(T)(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t}]dt + \sum_{n=1}^N \sigma_n(t, T)dW_n(t)$$

The HJM condition on $F(t, T)$ implies that

$$\mu(t, T) + \psi(T)(e^{\lambda(T)\tau} - 1)e^{-\lambda(T)t} = - \sum_{n=1}^N \sigma_n(t, T) \left[\Phi_n(t) - \int_t^T \sigma_n(t, s)ds \right] \quad (\text{A.3})$$

From expressions (A.1) and (A.3), we obtain the difference in risk premium:

$$\sum_{n=1}^N \sigma_n(t, T)[\Phi_n(t) - \phi_n(t)] = \psi(T)(1 - e^{\lambda(T)\tau})e^{-\lambda(T)t} < 0$$

To sum up, the Fed's impact on the risk premium is

$$\sum_{n=1}^N \sigma_n(t, T) [\Phi_n(t) - \phi_n(t)] = \begin{cases} \psi(T) e^{-\lambda(T)t}, & \text{if } t \leq \tau \\ \psi(T) (1 - e^{\lambda(T)\tau}) e^{-\lambda(T)t}, & \text{if } t > \tau \end{cases}$$

In the special case of a one-factor model, we have

$$\Phi(t) - \phi(t) = \begin{cases} \frac{\psi(T) e^{-\lambda(T)t}}{\sigma(t, T)}, & \text{if } t \leq \tau \\ \frac{\psi(T) (1 - e^{\lambda(T)\tau}) e^{-\lambda(T)t}}{\sigma(t, T)}, & \text{if } t > \tau \end{cases}$$

Proof of Theorem 2:

Let T be the stopping time when either the U.S. or Japan "matures". We assume there are no arbitrage opportunities from trading the mma, hence there exists an equivalent local martingale measure Q under which $\frac{\widehat{B}_t/Y_t}{B_t}$ is a Q -local martingale. With the central banks' impact, the "distorted" price of the dollar value of the yen mma is

$$\begin{aligned} \frac{\widehat{B}_t}{Y_t} &= E_t^Q \left(\frac{\widehat{B}_T/Y_T}{B_T} \middle| \mathcal{F}_t \right) B_t \\ &= E_t^Q \left(\frac{\widehat{B}'_T Z(T; \psi_y, \lambda_y, \tau_y)/Y_T}{B'_T Z(T; \psi_d, \lambda_d, \tau_d)} \middle| \mathcal{F}_t \right) B'_t Z(t; \psi_d, \lambda_d, \tau_d) \\ &= E_t^Q \left(\frac{\widehat{B}'_T/Y_T}{B'_T} \middle| \mathcal{F}_t \right) B'_t Z(t; \psi_d, \lambda_d, \tau_d) \cdot \frac{Z(T; \psi_y, \lambda_y, \tau_y)}{Z(T; \psi_d, \lambda_d, \tau_d)} \end{aligned} \quad (\text{A.4})$$

Eqn. (A.4) holds because the $Z(\cdot)$ functions are deterministic and can be moved out of the expectation. Since T is an extremely long time, one can assume T to be close to infinity and write

$$Z(T; \psi_y, \lambda_y, \tau_y) = \exp \left\{ -\frac{\psi_y}{\lambda_y} \tau_y \right\}, \quad Z(T; \psi_d, \lambda_d, \tau_d) = \exp \left\{ -\frac{\psi_d}{\lambda_d} \tau_d \right\}$$

then Eqn. (A.4) becomes

$$\frac{\widehat{B}_t}{Y_t} = E_t^Q \left(\frac{\widehat{B}'_T/Y_T}{B'_T} \middle| \mathcal{F}_t \right) B'_t Z(t; \psi_d, \lambda_d, \tau_d) \exp \left\{ \frac{\psi_d}{\lambda_d} \tau_d - \frac{\psi_y}{\lambda_y} \tau_y \right\} \quad (\text{A.5})$$

The price of the dollar value of the yen mma without the central banks' impact is

$$\frac{\widehat{B}'_t}{Y'_t} = E_t^{Q'} \left(\frac{\widehat{B}'_T/Y'_T}{B'_T} \middle| \mathcal{F}'_t \right) B'_t \quad (\text{A.6})$$

Define the price impact as

$$\frac{\widehat{B}_t/Y_t}{\widehat{B}'_t/Y'_t} = \frac{Y'_t}{Y_t} Z(t; \psi_y, \lambda_y, \tau_y) \quad (\text{A.7})$$

From Eqn. (A.5) and (A.6),

$$\frac{\widehat{B}_t/Y_t}{\widehat{B}'_t/Y'_t} = \frac{E_t^Q \left(\frac{\widehat{B}'_T/Y'_T}{B'_T} \middle| \mathcal{F}_t \right) B'_t Z(t; \psi_d, \lambda_d, \tau_d) \exp \left\{ \frac{\psi_d}{\lambda_d} \tau_d - \frac{\psi_y}{\lambda_y} \tau_y \right\}}{E_t^{Q'} \left(\frac{\widehat{B}'_T/Y'_T}{B'_T} \middle| \mathcal{F}'_t \right) B'_t} \quad (\text{A.8})$$

Define

$$e^{-u(t)} \equiv E_t^Q \left(\frac{\widehat{B}'_T/Y'_T}{B'_T} \middle| \mathcal{F}_t \right) / E_t^{Q'} \left(\frac{\widehat{B}'_T/Y'_T}{B'_T} \middle| \mathcal{F}'_t \right)$$

$e^{-u(t)}$ represents the central banks' impact on exchange rates through channels other than affecting interest rates.

From Eqn. (A.7) and (A.8), we derive the central banks' impact on exchange rates:

$$\frac{Y'_t}{Y_t} = \frac{Z(t; \psi_d, \lambda_d, \tau_d)}{Z(t; \psi_y, \lambda_y, \tau_y)} \exp \left\{ \frac{\psi_d}{\lambda_d} \tau_d - \frac{\psi_y}{\lambda_y} \tau_y \right\} e^{-u(t)}$$

where Y_t (Y'_t) is the spot exchange rate of yen per dollar with (without) the central banks' price impact.

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