

ESSAYS ON BANKING

A Dissertation

Presented to the Faculty of the Graduate School

of Cornell University

In Partial Fulfillment of the Requirements for the Degree of

Doctor of Philosophy

by

Yu Zhang

May 2012

© 2012 Yu Zhang

ESSAYS ON BANKING

Yu Zhang, Ph. D.

Cornell University 2012

Diamond and Dybvig (1983) provide an analytical framework of modern banking: The key role of banks is to provide risk sharing between different types of consumers, and the mismatch of short-term liabilities and long-term asset can cause bank runs. In this dissertation, I use Diamond-Dybvig framework to analyze some key issues on banking: banks and the asset market, bank runs and bailouts, and characteristics of deposit contracts.

The first chapter addresses the coexistence of banks and the asset market. Jacklin (1987) showed that banks are redundant if the asset market exists. I show that if there is aggregate liquidity shock, then asset prices will be volatile. This will make the arbitrage opportunities in the market risky. Sufficiently risk-averse depositors will not arbitrage. Hence, incentive-compatibility constraint is relaxed, leaving room for the bank to provide "insurance" to the depositors.

The second chapter addresses the relationship between the probability of bank runs and bailouts. Following Keister (2010), my model includes both a private good and a public good. The major innovation in this paper is to determine the run probability by using the global-games approach in Goldstein and Pauzner (2005), making the run probability endogenous. I show that bailouts increase the ex-ante run probability through two channels. The first channel works through the misaligned objectives of the bank and the government: Runs are less costly for banks when there are bailouts. Hence, banks take on more risk than is socially optimal. The second

channel works through the change in the depositor's incentives to run: Bailouts increase the probability that a depositor will get her money if she participates in a run, thus increasing the likelihood of a run.

The third chapter characterizes how optimal deposit contract is related to the probability of bank runs. Peck and Shell (2003) show that the optimal deposit contract can tolerate bank runs if the run probability is low. In their two-consumer example, the deposit contract is a step function of the run probability. I generalize that example and show that, for some parameters which permit bank runs, the optimal contract changes continuously with the run probability until it reaches the threshold probability level. Above that threshold, the optimal contract eliminates bank runs. Hence, the run probability affects not only whether bank runs will be tolerated (like Peck and Shell's example) but also how bank runs will be tolerated.

BIOGRAPHICAL SKETCH

Yu Zhang was born on March 25, 1982 in Tianjin, China. He graduated from Peking University (Beijing, China) in 2004 with a bachelor's degree in Economics. He joined the Ph.D. program at the Department of Economics, Cornell University in the fall of 2006. In April 2012, he defended his dissertation.

ACKNOWLEDGMENTS

I would like to thank my dissertation committee: Professor Assaf Razin (co-chair), Professor Karl Shell (co-chair) and Professor Viktor Tsyrennikov for their kindness and guidance.

TABLE OF CONTENTS

Introduction	1
Chapter 1: Banks and Markets: The Roles of Price Volatility and Risk Aversion	7
1.1 Introduction	7
1.2 The Model	11
1.3 Asset Market Equilibrium	14
1.4 The Bank And The Asset Market	17
1.5 Conclusion	27
1.6 Appendix	28
1.7 References	29
Chapter 2: Bailouts and Bank Runs	31
2.1 Introduction	31
2.2 The Model	37
2.2.1 Agents' Preferences	37
2.2.2 Endowments and Asset Returns	38
2.2.3 Agents' Information	39
2.2.4 Banks	40
2.2.5 Government	41
2.2.6 Timeline	42
2.3 Post-Deposit Game: Depositor's Withdrawal Decision in Period 1	44
2.4 Pre-Deposit Game: Choices of the Government and the Banks in Period 0	49
2.4.1 The Bank's Strategy	49
2.4.2 The Government's Strategy	54
2.5 Bailouts without Government Commitment	57
2.6 Summary	60

2.7	Appendix	61
2.8	References	76

Chapter 3: A Note on “Equilibrium Bank Runs”: Robustness of the

	Two-Consumer Example	79
3.1	Introduction	79
3.2	Necessary Conditions For Equilibrium Bank Runs	80
3.3	The Parameters	85
3.4	Concluding Remarks	91
3.5	References	92

Introduction

Banking and, more generally, financial intermediation perform essential roles for the efficient allocation of resources in modern economies, but they can also be sources of instability. One such instability is the susceptibility of banks (and other financial intermediaries) to “runs”. A bank run occurs when an abnormally large number of depositors elect to withdraw their funds, leading to the failure of the bank to meet its commitments. Bank runs can be triggered by realizations of the bank fundamentals (e.g., poor performance of the bank’s portfolio or an abnormally large fraction of depositors receiving liquidity shocks) or merely because of self-fulfilling panic of the depositors.

“Bank runs” are not observed solely at depository institutions. The current financial crisis is largely the result of the instability in the shadow banking sector: “financial firms ‘running’ on other financial firms by not renewing sale and repurchase agreements”.¹ In my dissertation, the formal model is of runs on depository banks, but the results can be loosely applied for runs generally.

Two different literatures have been developed to analyze bank runs. The first follows the seminal paper by Diamond and Dybvig (1983) [hereafter DD], which treats bank runs as panic-based: Depositors withdraw their funds only because they expect that the other depositors will do so. Hence, bank runs occur as a result of coordination failure. The other literature treats bank runs as fundamental-based. In these models (e.g., Allen and Gale (1998)), the depositors run because of unfavorable bank fundamentals, e.g. abnormally poor performance of bank assets. When a depositor decides to run, he will do so regardless of the other depositors’ actions.

In this dissertation, I will employ the DD framework, treating bank runs as purely

¹Gorton (2009).

panic-based. DD provides a workhorse for modelling not only bank runs but also the essential role of modern banking. The key role of DD banks is to provide risk sharing between the different types of consumers. The consequent mismatch of short-term liabilities and long-term assets can cause bank runs.

To be more precise, the individuals in the DD economy are subject to idiosyncratic liquidity shocks: they can be either patient or impatient. The impatient agents need to consume early and the patient agents can wait and consume later. The problem is that the long-term asset is illiquid: The return on this asset is low if harvested early. Without the bank, the impatient agents would have to consume much less than the patient agents. If an agent is risk-averse, her expected utility can be improved by smoothing consumption between when she becomes patient and when she becomes impatient. In other words, insurance against the liquidity shocks is desirable. This insurance cannot be achieved in the insurance market because of asymmetric information: An agent's type (patient or impatient) is her private information. The bank can provide some "insurance" by offering demand deposit contract: An agent makes her deposit before she knows her type. After the liquidity shocks are realized, the bank "transfers" some of the resources from the patient depositors to the impatient depositors. Although the bank cannot observe the depositors' types, the depositors can self-select since the short-term payment is lower than the long-term payment. What we have described depends on the assumption that the depositors expect that other depositors will not run on the bank. If a depositor expects that all other depositors withdraw early regardless of their types, his best response is to withdraw early. This is because the bank's assets would be exhausted in satisfying the early withdrawals. If the depositor waits, she will get nothing. Hence, a bank run occurs as a result of panic.

In this dissertation, I use the DD framework to analyze three issues in banking and finance: (1) coexistence of DD banks and asset markets; (2) bank runs and government

bailouts, and (3) the structure of the optimal deposit contract as a function of the (exogenous) run-probability.

My first chapter addresses the coexistence of banks and the asset market. DD assume that the asset market does not exist. Jacklin (1987) shows that banks are redundant if we add the asset market to the DD model. That is because the asset market provides arbitrage opportunities: If the depositors withdraw early, they can use the withdrawals to buy assets in the market. To make the patient depositors withdraw late, the late-payment has to be not only larger than the early payment but also larger than the proceeds from the asset which can be bought in the market by the early payment. Hence, the arbitrage opportunities tighten the incentive-compatibility constraint which makes the patient and impatient depositors self-select. I extend the DD model to an environment in which there is an aggregate liquidity shock and in which agents differ in their risk aversions. The aggregate liquidity shock makes asset prices volatile. Since depositors do not know the realization of the asset price while making their withdrawals, the arbitrage opportunities in the asset market are risky. Sufficiently risk-averse depositors prefer not to arbitrage in the market. Hence, the incentive-compatibility constraint is relaxed, leaving room for the bank to provide some “insurance” to these depositors. Banks are not redundant. But the less risk-averse agents will still take the risky arbitrage opportunities in the market. Hence, there is no bank targeting them as its depositors. They will make investments by themselves and obtain liquidity through the market. In this economy, there is nontrivial coexistence of banks and markets.

The second chapter addresses the relationship between the probability of bank runs and bailouts. Following Keister (2010), my model includes both a private good and a public good. The public good is introduced to represent the social cost of bank bailouts: Bailouts crowd out public good provision since the government budget must

be balanced. The major innovation I made is to determine the run probability by using the global-games approach in Goldstein and Pauzner (2005), making the run probability endogenous. I show that bailouts increase the ex-ante run probability through two channels. The first channel is that the bailouts distort the *ex-ante* incentives of banks, increasing the riskiness of their banking contracts. This is because the banks do not internalize the social cost of a bailouts, the decreased level of public good provision. Thus, bank runs are less costly for the banks than for the social planner. Anticipating government bailouts, banks promise a higher short-term deposit interest rate than is socially optimal in order to attract depositors. The higher short-term deposit interest rate makes the banking sector more fragile. The second channel is that the anticipated bailout increases the depositor's probability of getting her money from the bank when she participates in a run. With the additional resources from the bailout, banks are provided with extra withdrawal capacity during a run. Hence, if a depositor expects that a bank run will occur, the anticipated bailout increases the probability of the depositor getting her money from the bank if she participates in the run. When a depositor compares her expected utility between withdrawing early or waiting to withdraw, because of the bailout her incentive to withdraw early is increased and the *ex-ante* probability of a run is increased. Knowing the conflicts between alleviating bank runs and preventing bank runs, the government will announce, *ex ante*, a bailout policy that balances these two effects.

My third chapter is a note on Peck and Shell (2003). DD show that a bank run can be an equilibrium in the post-deposit game for the contract which supports the constrained-efficient allocation. The important question to ask is whether or not consumers will want to deposit in the bank if they expect that a bank run will occur. Peck and Shell show that under the optimal contract, the post-deposit game can have a run equilibrium. Given a propensity to run which is triggered by sunspots, the opti-

mal contract for the full pre-deposit game can be consistent with runs that occur with small, but positive probability. In Peck and Shell’s 2-consumer example, the contract is characterized by c which is the consumption received by the first depositor in line in period 1. Let $c^*(s)$ be the optimal contract offered by the bank, where s is the sunspot-driven probability of a bank run. In the numerical example of Peck and Shell, $c^*(s)$ is a step function: If the probability s is less than a critical level s_0 , the contract $c^*(s)$ tolerates runs and $c^*(s) = c^*(0)$. If the probability s is more than s_0 , the optimal contract will be immune from runs and $c^*(s) = c^{no-run}$, the highest level at which rational consumers will never run. The probability of a bank run s has only a “bang-bang” effect on the optimal contract $c^*(s)$. I generalize that example and show that, for some parameters which permit bank runs, the optimal contract changes continuously with the run probability until it reaches the threshold probability level. Above that threshold, the optimal contract eliminates bank runs. Hence, the run probability affects not only whether bank runs will be tolerated (as in Peck and Shell’s example) but also how bank runs will be tolerated.

References

- [1] Allen, Franklin and Douglas Gale. 1998. “Optimal Financial Crises.” *The Journal of Finance* 53: 1245-1284.
- [2] Diamond, Douglas W., and Philip H. Dybvig. 1983. “Bank Runs, Deposit Insurance, and Liquidity.” *Journal of Political Economy* 91: 401-419.
- [3] Goldstein, Itay and Ady Pauzner. 2005. “Demand-Deposit Contracts and the Probability of Bank Runs.” *The Journal of Finance* 60: 1293-1327.
- [4] Gorton, Gary. 2009. “Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007.” *Federal Reserve Bank of Atlanta’s 2009 Financial Markets Conference: Financial Innovation and Crisis*
- [5] Jacklin, Charles J. 1987. “Demand Deposits, Trading Restrictions, and Risk Sharing.” In *Contractual Arrangements for Intertemporal Trade*, edited by Edward J. Prescott and Neil Wallace. Minneapolis: University of Minnesota Press.
- [6] Keister, Todd. 2010. “Bailouts and Financial Fragility.” *Federal Reserve Bank of New York Staff Reports* 473.
- [7] Peck, James and Karl Shell. 2003. “Equilibrium Bank Runs.” *Journal of Political Economy* 111: 103-23.

1 Chapter 1: Banks and Markets: The Roles of Price Volatility and Risk Aversion

1.1 Introduction

There is typically a trade-off between liquidity and return. Individuals are subject to liquidity shocks, but the higher return asset is usually illiquid. Banks and the asset market² increase economic efficiency beyond autarky by improving the liquidity-return trade-off. The asset market does this by providing individuals with opportunities to sell their illiquid assets (or claims on future consumption) in the market, as in Allen and Gale (1994). Banks do this by offering demand deposit contracts as in Bryant (1980) and Diamond and Dybvig (1983). Banks and the asset market also interact with one another. This paper studies the operation of the DD-type bank in an economy with an asset market.

The model in this paper has two features: (1) The equilibrium price of the financial asset is volatile.³ Hence, the deposit contract offered by the representative bank can protect the depositors not only from idiosyncratic liquidity shocks but also from the asset price volatility. (2) There is a separating equilibrium: the agents who are more risk averse choose to be depositors in the bank and those who are less risk averse trade in the market.⁴

The first feature contributes to the debate concerning the redundancy of the DD-type bank when the financial market exists. DD analyzed banking in an economy in which the liquidity shock is purely idiosyncratic and in which there is no asset mar-

²As other papers in this literature, asset markets are ex-post security markets

³Robert J. Shiller (1981), among others, have argued the prices of financial assets such as stocks are characterized by excess volatility.

⁴In the model, for the CRRA utility function, those who have the coefficient of relative-risk-aversion greater than 1 will choose to be depositors and those who have the coefficient of relative-risk-aversion smaller than 1 will choose to participate the market.

ket. DD showed how the demand deposit contract acts as “insurance” against the liquidity shocks. They also show that the first-best allocation⁵ can be achieved as an equilibrium. The incentive-compatibility constraint, which makes depositors with different liquidity needs self-select, does not bind at the first-best allocation. The non-binding incentive-compatibility constraint relies on the assumption that there is no asset market and hence ex-post⁶ arbitrage opportunities for the depositors are limited. The viability of the DD bank in an economy with an asset market was questioned by Jacklin (1987);⁷ von Thadden (1999) elucidated Jacklin’s “critique”. If a financial market exists, out-of-bank trades will give the depositors more ex-post arbitrage opportunities and strengthen the incentive-compatibility constraint. For the case of purely idiosyncratic liquidity shocks, the prices of financial assets are stable. The incentive-compatibility constraint will be strengthened to the point that no room is left for the bank to provide the insurance against liquidity shocks and the bank can only mimic the financial market allocation. The bank is redundant. If $EU(i)$ is the representative agent’s expected utility under regime i , then we have:

$$\max EU(M) = \max EU(M\&B) < \max EU(B),$$

where M is the only-market regime, $M\&B$ is the market-and-bank regime and B is the only-bank regime. That is, the bank is redundant when there is a financial market. In my model, the price of the financial asset is volatile and I also assume realistically that asset prices are not revealed until after withdrawals are made.⁸ Hence, ex-post arbitrage

⁵First-best allocation is the allocation when there is no asymmetric information.

⁶By "ex-post", I mean "after a depositor learns his liquidity need".

⁷Haubrich and King (1990) and Hellwig (1994) also presented models that question the role of banks when there is a financial market.

⁸This assumption is realistic and reflects the intuition that individuals cannot predict the market

opportunities are risky for the depositors, which weakens the incentive-compatibility constraint. If depositors are sufficiently risk averse, the incentive-compatibility constraint will leave room for the bank to manipulate the deposit contract which can partially insure the depositors from the volatility of asset prices. In this way, banks can increase the expected utility of their depositors. Banks are not redundant.

The non-redundancy of the bank shows the advantage of banking over the pure asset market economy. Then, the next question is: Why does not everyone deposits in the bank? The second feature of the model answers this question. Agents are assumed to differ in their attitudes toward risk. Hence, even though the ex-post arbitrage opportunities are risky, depositors may still want to take those opportunities depending on their degree of risk aversion. I show that the agents with sufficiently low risk aversion⁹ will always take those arbitrage opportunities; i.e., the incentive-compatibility constraint can never be satisfied for them. Thus, there is no bank which targets those low risk-aversion agents as depositors. In equilibrium, high risk-aversion agents are depositors in banks and low risk-averse agents have to invest for themselves and obtain their liquidity in the market.¹⁰

Equilibrium asset price volatility can be explained in many different ways. In this paper, it is the uncertainty of the aggregate liquidity shock which causes the price volatility.¹¹ The relation between aggregate liquidity shock and the volatility of asset

with certainty. To achieve this, in the model I assume that the market opens after the withdrawal decisions have been made. So depositors do not know which price is going to be realized when making their withdrawal decisions.

⁹For the CRRA utility function, the agents with coefficient of relative risk aversion smaller than 1.

¹⁰Those low-risk aversion agents may want to deposit in the bank which is designed for the high risk-aversion depositors. To prevent such arbitrage activities, the bank will restrict the size of individual deposit to the level of the endowment owned by the high risk-aversion agents. If the low risk-aversion agents have much more endowments than that level, most of their endowments have to be invested by themselves and get most of their liquidity through the market. Even though the low risk-aversion agents can deposit a small fraction of their endowment, it does not cause problem for the bank if the population of the low risk-aversion agents is small relative to the high risk-aversion agents.

¹¹Aggregate liquidity shock is not new in this literature. In Diamond and Dybvig (1983), there is a section discussing why first-best allocation cannot be achieved when there is aggregate liquidity

prices has been analyzed by Allen and Gale (1994). The main issue in their paper is how limited market participation can amplify the volatility of asset prices. In my paper, I focus on how the aggregate liquidity shock can explain the coexistence of banks and markets.¹²

There are several papers which also focus on the relation between banks and markets. Wallace (1988) argued that DD model can be interpreted as a model in which no asset market exists because agents are physically separated. Diamond (1997) assumed that participation in the asset market is limited. He showed that bank is then useful because it can provide insurance among the impatient agents and the patient agents who cannot access the market. Increased participation in the market will cause the banking sector to shrink. Allen and Gale (2004) analyzed the relation between banks and inter-bank markets. Individual agents cannot trade in the inter-bank markets. Hence, markets do not strengthen the incentive-compatibility constraints. Antinolfi and Prasad (2008) assumed that the asset market is characterized by limited enforcement of contracts. If borrowers default, only a fraction of their assets can be seized. This reduces the fraction of assets that can be used as collateral and individuals hit by the liquidity shock face borrowing constraints. Antinolfi and Prasad (2008) showed that the bank can ameliorate these constraints by pooling the resources of depositors to increase the liquidity beyond that provided by the asset market. Farhi, Golosov and Tsyvinski (2009) showed that government regulation (They propose liquidity floor.) imposed on intermediaries can improve the resource allocation and the first-best can be achieved even the market exists. But in that paper, they did not explain why agents

shock. But aggregate liquidity shock has never been used to study the relation between banking and the financial market.

¹²There are other differences between Allen and Gale (1994) and my paper. For example, in Allen and Gale (1994), there is a fixed cost for agents to participate the market. If agents decide no to participate the market they have to be in autarky and they cannot invest in the illiquid asset. There is no participation cost of the market in my model. And the investment options are the same for each individual no matter whether she participate the market.

deposit in the intermediaries rather than make the investment by themselves. Peck and Shell (2009) captured the role of banks in providing checking accounts and facilitating transactions. In that paper, banks are not redundant because agents need checking account services.

In the present paper, the essential role of the bank is not based on its checking account service. Furthermore, there is no restriction on market participation and the asset market is free from the enforcement problems leading to collateral contracts. The coexistence of banks and markets comes from the market price volatility and heterogeneity in agents' risk aversion.

In section 1.2, I describe the setup of the model. In section 1.3, I analyze equilibrium for the case in which agents can only trade in the asset market. This will serve as a benchmark. Section 1.4 is on the coexistence of banks and markets. I will show how banks improve the expected utility of the high risk aversion agents and why there is no bank for the low risk-aversion agents and thus they have to use the market for re-adjusting their portfolios. Section 1.5 is the conclusion.

1.2 The Model

There are three periods: 0, 1 and 2. In each period, there is one good which can be used for either investment or consumption. There are two available assets. The first asset is liquid; it is the storage technology: every unit invested in period t ($t = 0, 1$) returns one unit of the good in period $t + 1$. The other asset is illiquid but offers a higher return: every unit of investment in period 0 can generate nothing in period 1 but it generates $R > 1$ units of the good in period 2. Table 1.1 describes the returns of the two assets.

	Period 0	Period 1	Period 2
Liquid Asset	-1	1	
		-1	1
Illiquid Asset	-1	0	R

Table 1.1: Asset Return

There is a continuum of agents. They can be divided into two groups. We label them as group D and group M . It is useful later to think of group D as potential depositors and group M as the potential market participants. (In the next section, when we introduce the banking industry, we will show that group D will be the depositors and group M will be the market participants.) The measure of the groups is 1 and m respectively. The endowments at period 0 of each individual in the two groups are 1 and e respectively. There are no new endowments in later periods.

Both groups of agents have the DD preferences: In period 0, agents of a given group are identical. In period 1, they receive a liquidity shock which turns some of them into patient consumers and the rest into impatient consumers. A patient consumer values only period-2 consumption and an impatient consumer values only period-1 consumption. The liquidity shock to group- D agents is purely idiosyncratic: The fraction of group- D agents who turn to be impatient in period 1 is λ_D and λ_D is a constant. group- M agents, however, face aggregate liquidity shocks: The fraction of the group- M agents becoming impatient in period 1 can be either λ_H or λ_L , where $\lambda_H > \lambda_L$. The probabilities of having high aggregate liquidity demand λ_H and low aggregate liquidity demand λ_L are q and $(1 - q)$ respectively. The aggregate liquidity shock will be shown to make the equilibrium asset price volatile. Jacklin's critique is based on

the polar version of this model in which $\lambda_H = \lambda_L$ or $me = 0$, that is, if either there is no aggregate liquidity shock for the M agents or the endowment adjusted measure of M agents is zero.

If an agent from group- i consumes the consumption in the period of her liquidity need, her utility is given by the standard *CRRA* utility function:

$$U_i(C) = (C^{1-\alpha_i} - 1)/(1 - \alpha_i)$$

where $\alpha_i > 0$ for $i = D, M$. group- D agents are more risk averse than the group- M agents:

$$\alpha_M < 1 < \alpha_D.$$

The degree of risk aversion determines whether or not there is going to a bank that targets that group for its depositors. We will show in section 1.4 that since $\alpha_M < 1$, a bank can never make the agents with different liquidity needs in that group self-select given the arbitrage opportunities in the market. Group- M agents might deposit at the bank designed for the D group if there is arbitrage opportunity in doing so. But the deposit contract for group- D agents will take this into account and prevent such arbitrage activities. In equilibrium, most or all of the endowments of the M group has to be invested by themselves and they obtain the liquidity from the market.

An agents' group (D or M) and consumption type (patient or impatient) are her private information. Aggregate liquidity demand is revealed to agents only through observing asset prices.

In the next section, we will analyze the equilibrium when there is an asset market in period 1 in which agents can trade the illiquid asset after they learn their liquidity

need and in which there is no bank.

1.3 Asset Market Equilibrium

The asset market is the place where agents trade the illiquid asset for the consumption good (the return from the liquid asset). For this section, we assume that there is no bank. The sequence of the events is listed in the timeline in Figure 1.1.

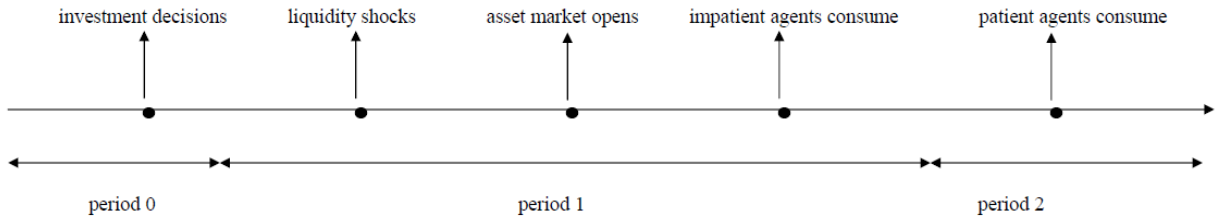


Figure 1.1: Timeline 1

Each agent decides how to allocate her endowment at the beginning of period 0 given her expectation of the asset price of the illiquid asset in terms of consumption. In equilibrium, expectation must be correct. Let p_S be the price in period 1 of the illiquid asset when the aggregate liquidity shock to the M agents is λ_S ($S = H, L$). The state S is not revealed directly. Agents learn S from the asset price. At any positive price, the impatient agents would like to sell all of their illiquid asset. For the patient agents, if $p_S > R$, they also want to sell the illiquid asset. If $p_S = R$, they are indifferent between selling and buying illiquid asset. If $p_S < R$, they strictly prefer to buy the illiquid asset with the proceeds from their liquid asset.

In the equilibrium, we must have $p_S \leq R$. If $p_S > R$, no one wants to buy the illiquid asset in state S and the market would be in equilibrium only if all of the endowment was invested in the liquid asset. But this means that the asset price will be larger

than R in each state. Then agents would invest only in the illiquid asset, which is a contradiction. Thus in equilibrium we must have $p_S \leq R$.

Equilibrium in the asset market is characterized by the following three conditions:

Condition (1) : Given asset prices in the different states, a group- D agent will choose to invest the fraction l_D of her endowment in the liquid asset to maximize her expected utility. That is, she chooses $l_D \in [0, 1]$ to maximize

$$q\{\lambda U_D[l_D + (1 - l_D)p_H] + (1 - \lambda)U_D[l_D R/p_H + (1 - l_D)R]\} \\ + (1 - q)\{\lambda U_D[l_D + (1 - l_D)p_L] + (1 - \lambda)U_D[l_D R/p_L + (1 - l_D)R]\}.$$

Condition (2) : Given asset prices in the different states, a group- M agent will choose to invest the fraction l_M of her endowment in the liquid asset to maximize her expected utility. That is, she chooses $l_M \in [0, 1]$ to maximize

$$q\{\lambda_H U_M[(l_M + (1 - l_M)p_H)e] + (1 - \lambda_H)U_M[(l_M R/p_H + (1 - l_M)R)e]\} \\ + (1 - q)\{\lambda_L U_M[(l_M + (1 - l_M)p_L)e] + (1 - \lambda_L)U_M[(l_M R/p_L + (1 - l_M)R)e]\}.$$

Condition (3) : The asset market clears in each state:

$$p_S = \min\left\{R, \frac{(1 - \lambda_D)l_D + m(1 - \lambda_S)l_M e}{\lambda_D(1 - l_D) + m\lambda_S(1 - l_M)e}\right\}$$

for $S = H, L$.

Definition 1.1 *An asset market equilibrium consists of $\{l_D, l_M, p_H, p_L\}$ satisfying conditions (1), (2) and (3).*

The existence of the asset market equilibrium can be established with a standard fixed-point argument. The details are given in the appendix.

Proposition 1.1 *There exists an asset market equilibrium of this model.*

One property of the equilibrium is $p_H < 1 < p_L$.¹³ It means when the aggregate liquidity need is high, the illiquid asset can only be sold at a loss. This leads to volatile consumption. Let C_{PS}^j and C_{IS}^j be the consumption of a patient agent (subscript “P”) and an impatient agent (subscript “I”) of group j ($j = D, M$) when the aggregate liquidity need of the M agents is λ_S . We have $C_{IH}^j < C_{IL}^j \leq C_{PL}^j < C_{PH}^j$. ($C_{IL}^j = C_{PL}^j$ if and only if $p_L = R$). So the expected utility of agents can be increased if one can provide a “smoother” consumption bundle. This makes the bank useful. The numerical example below illustrates the market equilibrium.

¹³It can be seen by the following argument. By Condition (3), we have $p_H \leq p_L$. If $p_L \geq 1$, then $1 \leq p_H \leq p_L$. Condition (1) and (2) imply $l_D = l_M = 0$. No liquid asset investment and Condition (3) then imply that $p_H = p_L = 0$. A contradiction. Hence we have $p_H < 1$. Now we can prove that $p_L > 1$. If otherwise, then Condition (1) and (2) imply $l_D = l_M = 1$. No illiquid asset investment and Condition (3) imply that $p_H = p_L = R$. A contradiction. Hence, we must have $p_H < 1 < p_L$.

Example 1.1

<i>Parameters</i>	<i>group-D agents</i>	<i>group-M agents</i>
<i>fraction who are impatient</i>	$\lambda_D = 0.4$	$\begin{cases} \lambda_H = 0.45 \text{ w.p. } q = 0.5 \\ \lambda_L = 0.35 \text{ w.p. } 1 - q = 0.5 \end{cases}$
<i>coefficient of risk aversion</i>	$\alpha_D = 3$	$\alpha_M = 0.5$
<i>measure</i>	1	$m = 1$
<i>endowment</i>	1	$e = 1$
<i>asset return</i>	$R = 1.2$	

The equilibrium is characterized by : (fraction of group-D agents' endowment invested in the liquid asset) $l_D = 0.4604$, (fraction of group-M agents' endowment invested in the liquid asset) $l_M = 0.3393$, (price of the illiquid asset when λ_H is realized) $p_H = 0.9019$ and (price of the illiquid asset when λ_L is realized) $p_L = 1.1111$

In the above example, we can see that group-M agents invest less of their endowment than the D agents do. This is because group-M agents are less risk averse. The equilibrium consumptions of a group-D agent are: $C_{IH}^D = 0.9471$, $C_{PH}^D = 1.260$, $C_{IL}^D = 1.060$ and $C_{PL}^D = 1.145$. The equilibrium consumptions of a group-M agent are: $C_{IH}^M = 0.9343$, $C_{PH}^M = 1.2430$, $C_{IL}^M = 1.0723$ and $C_{PL}^M = 1.1581$.

1.4 The Bank And The Asset Market

Now we introduce a banking industry into the model. As other papers in the literature, we assume that there is free entry in banking. Hence, a representative bank earns zero profits and the equilibrium deposit contract is designed to maximize the depositors' welfares.

The timeline will is described in Figure 1.2. (“deposit contract announced” and “withdrawal decisions” are new compared to Figure 1.1)

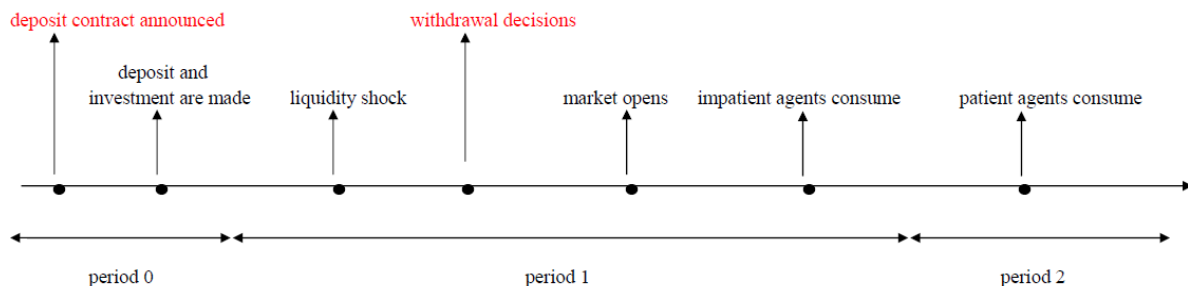


Figure 1.2: Timeline 2

Consider the case in which group- D agents have the opportunity to deposit in the bank in period 0. To make it simple, we assume that group- D agents can choose to deposit either 1 unit of the good or nothing. If an agent has deposited her entire endowment, she can withdraw either D_1 units of good in period 1 or D_2 units of good in period 2. The withdrawal decision has to be made before the asset market opens. Since the aggregate liquidity shock is not observable at withdrawal, so D_1 and D_2 cannot depend on the aggregate liquidity need. The deposit contract (the values of D_1 and D_2) is announced by the bank at the beginning of period 0. The timeline (with two events added) is shown in Figure 1.2. As in the original Diamond-Dybvig model, the bank faces the resource constraint

$$\lambda_D D_1 + (1 - \lambda_D) D_2 / R = 1$$

The new, important feature here is that given the asset market, bank will face different incentive-compatibility constraints. Incentive-compatibility constraints are used to allow depositors with differing liquidity needs self-select correctly. If a depositor is patient and demands withdrawal early, she gets D_1 . She can either store the proceeds

and consume D_1 in period 2 or go to the market, buy the illiquid asset and consume the proceeds of the illiquid asset in period 2. Preventing the former deviation requires the usual incentive constraint $U_D(D_1) \leq U_D(D_2)$. To prevent the latter deviation, we need

$$qU_D(D_1R/p_H) + (1 - q)U_D(D_1R/p_L) \leq u(D_2). \quad (1)$$

Since both p_H and p_L are no larger than R , we know D_1R/p_H and D_1R/p_L are not less than D_1 . So condition (1) implies that $U_D(D_1) \leq U_D(D_2)$ holds.

For the impatient depositor, she can sell her right to withdraw D_2 in exchange for the period-1 good. Since the price of the illiquid asset is p_S , the price of one unit of period-2 good is p_S/R . By selling D_2 units of period-2 good, the impatient depositor can get $D_2(p_S/R)$ units of period-1 good. Her expected utility from this deviation is $qU_D(D_2p_H/R) + (1 - q)U_D(D_2p_L/R)$. The incentive constraint which prevents this deviation is

$$U_D(D_1) \geq qU_D(D_2p_H/R) + (1 - q)U_D(D_2p_L/R). \quad (2)$$

Since $U_D(D) = (C^{1-\alpha_D} - 1)/(1 - \alpha_D)$, we have that (1) holds if and only if

$$\frac{D_2}{D_1} \geq b(\alpha_D, p_H, p_L)R,$$

where $b(\alpha_D, p_H, p_L) = 1/[q(p_H)^{\alpha_D-1} + (1 - q)(p_L)^{\alpha_D-1}]^{1/(\alpha_D-1)}$. Inequality (2) holds if and only if

$$a(\alpha_D, p_H, p_L)R \geq \frac{D_2}{D_1},$$

where $a(\alpha_D, p_H, p_L) = [q/(p_H)^{\alpha_D-1} + (1 - q)/(p_L)^{\alpha_D-1}]^{1/(\alpha_D-1)}$. It can be seen that

$a(\alpha_D, p_H, p_L) > b(\alpha_D, p_H, p_L)$ since $\alpha_D > 1$. So we have the incentive constraints faced by the bank are

$$a(\alpha_D, p_H, p_L)R \geq \frac{D_2}{D_1} \geq b(\alpha_D, p_H, p_L)R \quad (3)$$

Given the incentive constraints and the resource constraint, the bank will choose (D_1, D_2) to maximize

$$\lambda_D U_D(D_1) + (1 - \lambda_D) U_D(D_2),$$

such that $\lambda_D D_1 + (1 - \lambda_D) D_2 / R = 1$ and

$$a(\alpha_D, p_H, p_L)R \geq \frac{D_2}{D_1} \geq b(\alpha_D, p_H, p_L)R.$$

Let $W(p_H, p_L)$ denote the maximum value¹⁴ of the expected utility.

Given the optimal contract (D_1, D_2) , group- D agents will strictly prefer to deposit in the bank if and only if (D_1, D_2) provides higher expected utility than the asset market (meaning that the bank is not redundant). That is,

$$W(p_H, p_L) > V(p_H, p_L)$$

¹⁴Bank runs are ruled out since the illiquid asset cannot generate any return in the short run and it will not be liquidated. Hence, the patient depositors can be guaranteed that they will get their consumption in the final period from the illiquid asset.

If we change the assumption on the asset returns so that the less liquid asset is not completely illiquid then bank runs are possible. But the results of the model still hold as long as the probability of bank runs is small. Peck and Shell (2003) study how bank runs can be tolerated when the run probability is small.

where $V(p_H, p_L)$ is the expected utility given the market price p_H and p_L . That is,

$$V(p_H, p_L) = \underset{0 \leq l_D \leq 1}{Max} \{ q\{\lambda_D U_D[l_D + (1 - l_D)p_H] + (1 - \lambda_D)U_D[l_D R/p_H + (1 - l_D)R]\} \\ + (1 - q)\{\lambda_D U_D[l_D + (1 - l_D)p_L] + (1 - \lambda_D)U_D[l_D R/p_L + (1 - l_D)R]\} \}$$

Banks targeting M agents as depositors will face the same incentive-compatibility constraint as in (3). However, since group- M agents are less risk averse than the group- D agents and their coefficient of relative risk aversion, α_M , is less than 1. Then we have,

$$a(\alpha_M, p_H, p_L) < b(\alpha_M, p_H, p_L).$$

So the incentive-compatibility constraint cannot be satisfied and there is no banks which targeting M agents as depositors.

Lemma 1.1 *Banks targeting M agents as depositors are not available.*

M agents may want to deposit in the bank for D agents. To prevent such arbitrage activities, the D agents' bank will restrict the individual deposit size to 1 unit of goods. If each M agent has endowment $e \gg 1$, most of their endowments cannot be deposited in the bank. The total deposit from the M agents, which is equal to the measure of that group, is m . It does not cause problem for the bank if m is sufficiently small relative to measure of D agents, which is 1.

Lemma 1.2 *If m is sufficiently smaller than 1 and e is sufficiently larger than 1, most of the M agents' endowment cannot be deposited and they have to obtain liquidity from the market. The bank for the D agents will not be affected much.*

Hence a group- M agent uses only the market and maximizes her expected utility by choosing $l_M \in [0, 1]$ to maximize¹⁵

$$q\{\lambda_H U_M[(l_M + (1 - l_M)p_H)e] + (1 - \lambda_H)U_M[(l_M R/p_H + (1 - l_M)R)e]\} \\ + (1 - q)\{\lambda_L U_M[(l_M + (1 - l_M)p_L)e] + (1 - \lambda_L)U_M[(l_M R/p_L + (1 - l_M)R)e]\}.$$

The market-clearing condition must be satisfied. If the group- D agents decide to deposit in the bank, only the group- M agents will trade on the asset market and hence we have the market clearing condition:¹⁶

$$p_S = \min\left\{R, \frac{(1 - \lambda_S)l_M}{\lambda_S(1 - l_M)}\right\}$$

for $S = H, L$.

Definition 1.2 *An equilibrium for the asset market and the bank in which the bank is not redundant consists of $\{D_1, D_2, l_M, p_H, p_L\}$ which satisfies (1) (D_1, D_2) solves the bank's maximization problem, (2) $W(p_H, p_L) > V(p_H, p_L)$, (3) l_M solves group- M agents' maximization problem and (4) the asset market clears in each of the two states.*

That conditions (3) and (4) in Definition 1.2 are satisfied can be established by a standard fix-point argument as in the proof of Proposition 1.1. Hence, the equilibrium exists if and only if $W(p_H, p_L) > V(p_H, p_L)$. The price volatility in the market makes the participation in the market too risky for a group- D agent. If she needs liquidity in

¹⁵The value of l_M is independent of the endowment because of the *CRRA* utility function. Hence, whether or not the M agents deposit 1 unit of resource in the bank will not change their choice of l_M .

¹⁶Since only the M agents use the market to get the liquidity, the size of the group M agents (m) and their endowment (e) cancel. Only l_m affects the market clearing price.

period 1 (being impatient) and the aggregate liquidity shock is high (λ_H is realized), she has to sell her illiquid asset at a low price p_H . This will make her consumption lower than that of the patient agents. Although when the aggregate liquidity need is low the gap is smaller, a sufficiently risk averse group- D agent would like to make the worst scenario (being impatient in a high aggregate liquidity need market) less bad. By depositing in the bank, a group- D agent is protected from price volatility. So we have the following proposition.

Proposition 1.2 *If the group- D agents are sufficiently risk-averse, then there is an equilibrium in which both the asset market and the bank are active and in which neither is redundant.*

Proof. As we have seen, it is sufficient to prove that we have $W(p_H, p_L) > V(p_H, p_L)$. Let us first look at $V(p_H, p_L)$ which is defined by:

$$V = \underset{0 \leq l_D \leq 1}{Max} q\{\lambda_D U_D[l_D + (1 - l_D)p_H] + (1 - \lambda_D)U_D[l_D R/p_H + (1 - l_D)R]\} \\ + (1 - q)\{\lambda_D U_D[l_D + (1 - l_D)p_L] + (1 - \lambda_D)U_D[l_D R/p_L + (1 - l_D)R]\}.$$

This is a standard maximization problem. Given the utility function $U_D(C) = (C^{1-\alpha_D} - 1)/(1 - \alpha_D)$, the first-order interior condition becomes:

$$q(1 - p_H)\left\{\frac{\lambda_D}{(l_D + (1 - l_D)p_H)^{\alpha_D}} + \frac{(1 - \lambda_D)R/p_H}{[l_D R/p_H + (1 - l_D)R]^{\alpha_D}}\right\} \\ + (1 - q)(1 - p_L)\left\{\frac{\lambda_D}{(l_D + (1 - l_D)p_L)^{\alpha_D}} + \frac{(1 - \lambda_D)R/p_L}{[l_D R/p_L + (1 - l_D)R]^{\alpha_D}}\right\} = 0.$$

Since $p_H < 1$, $(l_D + (1 - l_D)p_H)^{\alpha_D} \rightarrow 0$ as $\alpha_D \rightarrow \infty$. So the left-hand side of the above

equation must be positive for $0 \leq l_D \leq 1$. So the corner solution $l_D = 1$ is optimal. A sufficiently risk averse group- D agent would invests all of her endowment in the liquid asset to protect herself against the worst scenario i.e., being impatient when aggregate liquidity shock is high. So we have

$$V(p_H, p_L) = \lambda_D U_D(1) + (1 - \lambda_D)[qU_D(R/p_H) + (1 - q)U_D(R/p_L)].$$

To find $W(p_H, p_L)$, solve the bank's problem. As $\alpha_D \rightarrow \infty$, $a(\alpha_D, p_H, p_L) \rightarrow 1/p_H$ and $b(\alpha_D, p_H, p_L) \rightarrow 1/p_L$. So the incentive constraints become $R/p_H \geq D_2/D_1 \geq R/p_L$. It is easy to see that as $\alpha_D \rightarrow \infty$, the lower bound of the incentive constraint must bind and the optimal solution satisfies

$$\lambda_D D_1 + (1 - \lambda_D) D_2/R = 1$$

and

$$D_2 = D_1 R/p_L.$$

Since $(1, R)$ satisfies both the resource constraint and the incentive-compatibility constraint, we have

$$W(p_H, p_L) > \lambda_D U_D(1) + (1 - \lambda_D) U_D(R). \quad (4)$$

Also, for sufficiently large α_D , we must have $U_D(1)(R) > qU_D(1)(R/p_H) + (1 - q)U_D(1)(R/p_L)$. This is because given the *CRRA* utility function, $U_D(1)(R) > qU_D(1)(R/p_H) +$

$(1 - q)U_D(1)(R/p_L)$ is true if and only if $1 < q(p_H)^{\alpha_D-1} + (1 - q)(p_L)^{\alpha_D-1}$. Since $p_L > 1$, the last inequality must hold for sufficiently large α_D . Given $U_D(R) > qU_D(R/p_H) + (1 - q)U_D(R/p_L)$, we have

$$\begin{aligned} & \lambda_D U_D(1) + (1 - \lambda_D) U_D(R) \\ & > \lambda_D U_D(1) + (1 - \lambda_D) [q U_D(R/p_H) + (1 - q) U_D(R/p_L)] \\ & = V(p_H, p_L). \end{aligned} \tag{5}$$

(4) and (5) imply that when α_D is sufficiently large, we have

$$W(p_H, p_L) > V(p_H, p_L).$$

So there is an equilibrium in which both the asset market and the bank are active and in which neither is redundant. This assumes that the group- D agent is sufficiently risk-averse. ■

In Example 1.2, I use the same parameters as Example 1.1 except that $m = 0.001$ and $e = 1000$. These two changes are made to decrease the effect of the deposits from M agents in the D agents' bank.¹⁷ Since $m \times e = 1$ which is the same as Example 1.1, the pure market equilibrium will be the same as Example 1.1.¹⁸

Example 1.2 *Using the same parameters as in Example 1.1, we have $D_1 = 1.0189$, $D_2 = 1.1849$, $l_M = 0.4056$, $p_H = 0.8302$, $p_L = 1.2$. $W = 0.0938 > V = 0.0884$.*

¹⁷In this example, M agents strictly prefer to deposit in D agents' bank and withdraw in period 1 no matter what their liquidity needs are. The D agents' bank will not be affected much if it restricts the size of the deposit to 1.

¹⁸Of course the equilibrium individual consumption of the M agents will be 1000 times the value of the individual consumption in Example 1.1 because of the increased endowment for each individual.

By depositing in the bank, a group- D agent will receive higher expected utility than by participating in the market. The increase in a group- D agent's welfare through the bank (compared to the market) will be the equivalent of an increase in the endowment of 0.66%.¹⁹ The bank will invest 0.4083 of the endowment in the liquid asset. This is much lower than the fraction of endowment invested in liquid asset by D agents in the pure market result in Example 1.1 ($l_D = 0.4604$). This is because now there is mutual insurance among the depositors. Hence, there is no need to keep as much of the liquid asset to prepare for the liquidity need. The asset price is more volatile compared to the pure market equilibrium since the D agents are depositors who will not participate in the market where they absorb some of the aggregate liquidity shock caused by the M agents. M agents will invest $l_M = 0.4056$ of their endowment in the liquid asset. This is more than in the pure market equilibrium of Example 1.1 ($l_M = 0.3393$) because of great asset price volatility.

When there is no aggregate uncertainty in the market, that is, when $\lambda_H = \lambda_L$ or $m = 0$, things are different. The price of the illiquid asset is stable. In equilibrium, we must have $p_H = p_L = 1$. The incentive constraints (3) collapse to $D_2/D_1 = R$. Combined with the resource constraint the bank faces, we have that $D_1 = 1$ and $D_2 = R$. Hence, in this case the bank is redundant in equilibrium: $W = V = \lambda_D U_D(1) + (1 - \lambda_D) U_D(R)$. Jacklin's critique applies in this case.

Proposition 1.3 *If liquidity shocks are purely idiosyncratic, the bank is redundant.*

In the present model, uncertainty of aggregate liquidity shocks, which generates asset-price volatility, is necessary for non-redundancy of the bank. It is easier to coax

¹⁹If there is no market, that is, if we go back to the original Diamond-Dybvig model, the increase in a group- D agent's welfare due to the bank (compared to autarky) will be the equivalent to an increase of the endowment by 1.53%

depositors to "tell the truth" because they do not like the uncertainty in the asset market. The bank provides the depositors with some "insurance" against idiosyncratic liquidity shocks of the depositors and against the volatility of asset prices. Although this "insurance" cannot be at the same level as in the case of the original DD model (where no stock market exists), it nonetheless increases welfare over that of the pure asset-market allocation.

1.5 Conclusion

This paper studies a DD-type bank in an economy with a financial asset market. Previous studies have shown that the opportunity trade in the asset market strengthens the incentive-compatibility constraint for the bank, making the bank redundant. Two new factors are introduced in the present paper: (1) The aggregate liquidity shock is assumed to be uncertain (hence the market price of the illiquid asset is uncertain). (2) Agents differ in their risk-aversion. Since depositors do not know the asset price when making withdrawals, the first factor makes arbitrage opportunities in the market risky. For sufficiently risk-averse agents, they will not arbitrage in the market, and the incentive-compatibility constraint leaves room for the bank to provide insurance. That is, banks are not redundant. But for less risk-averse agents, they will still take the risk arbitrage opportunity. Hence, there is no bank targeting them as the depositors. They will make investments by themselves and obtain liquidity through the market. In this economy, there is non-trivial coexistence of banks and markets.

1.6 Appendix

Proof of Proposition 1.1

For any $(l_D, l_M) \in [0, 1] \times [0, 1]$, there exists a unique price function $p_S(l_D, l_M)$ defined by the market clearing condition (Condition 3):

$$p_S(l_D, l_M) = \min\left\{R, \frac{(1 - \lambda_D)l_D + m(1 - \lambda_S)l_M}{\lambda_D(1 - l_D) + m\lambda_S(1 - l_M)}\right\}$$

for $S = H, L$.

Given the asset price, a group- D agent maximizes her expected utility by choosing liquid asset investment (Condition 1). Since the expected utility function is continuous and strictly concave, there is a unique solution. Let $F(l_D, l_M) = \max_{\hat{l} \in [0, 1]} EU(\hat{l}; p_H(l_D, l_M), p_L(l_D, l_M))$. By the Theorem of Maximum, $F(l_D, l_M)$ is a contiguous function of (l_D, l_M) .

Similarly, a group- M agent maximizes her expected utility by choosing liquid asset investment (Condition 2). We denote the solution as $G(l_D, l_M)$. $G(l_D, l_M)$ is a contiguous function of (l_D, l_M) .

It is easy to see that $F \times G$ maps $[0, 1] \times [0, 1]$ into itself and satisfies the conditions of Kakunati's fixed-point theorem. Hence, the fixed point exists and it is the equilibrium.

1.7 References

- [1] Allen, Franklin and Douglas Gale. 1994. “Limited Market Participation and Volatility of Asset Prices.” *American Economic Review* 84: 933–955.
- [2] Allen, Franklin and Douglas Gale. 2004. “Financial Intermediaries and Markets.” *Econometrica* 72: 1023–1061.
- [3] Antinolfia, Gaetano and Suraj Prasad. 2008. “Commitment, banks and markets.” *Journal of Monetary Economics* 55: 265–277.
- [4] Bryant, John. 1980. “A Model of Reserves, Bank Runs, and Deposit Insurance.” *Journal Banking and Finance* 4: 335–344.
- [5] Diamond, Douglas W. 1997. “Liquidity, Banks and Markets.” *Journal of Political Economy* 105: 928–956.
- [6] Diamond, Douglas W. and Philip H. Dybvig. 1983. “Bank Runs, Deposit Insurance, and Liquidity.” *Journal of Political Economy* 91: 401–419.
- [7] Farhi, Emmanuel, Mikhail Golosov and Aleh Tsyvinski. 2009. “A Theory of Liquidity and Regulation of Financial Intermediation.” *Review of Economic Studies* 76: 973–992.
- [8] Haubrich, Joseph G. and Robert G. King. 1990. “Banking and Insurance.” *Journal of Monetary Economics* 26: 361–386.
- [9] Hellwig, Martin. 1994. “Liquidity Provision, Banking, and the Allocation of Interest Rate Risk.” *European Economic Review* 38: 1363–1389.
- [10] Jacklin, Charles J. 1987. “Demand Deposits, Trading Restrictions, and Risk Sharing.” In *Contractual Arrangements for Intertemporal Trade*, edited by Edward J. Prescott and Neil Wallace. Minneapolis: University of Minnesota Press.
- [11] Peck, James and Shell, Karl. 2003. “Equilibrium Bank Runs”, *Journal of Political Economy* 111: 103–123.
- [12] Peck, James and Shell, Karl. 2010. “Could Making Banks Hold Only Liquid

Assets Induce Bank Runs?” *Journal of Monetary Economics* 57: 420–427.

[13] Shiller, Robert J. 1981. “Do Stock Prices Move Too Much To Be Justified by Subsequent Changes in Dividends?” *American Economic Review* 71: 421-436.

[14] von Thadden, Ernst-Ludwig. 1997. “Liquidity creation through banks and markets: Multiple insurance and limited market access.” *European Economic Review* 43: 991-1006.

[15] Wallace, Neil. 1988. “Another Attempt to Explain an Illiquid Banking System: The Diamond-Dybvig Model with Sequential Service Taken Seriously.” *Quarterly Review of the Federal Reserve Bank of Minneapolis* 12: 3-16.

2 Chapter 2: Bailouts and Bank Runs

2.1 Introduction

Most countries, if not all, have suffered from bank runs. According to Laeven and Valencia (2008), 124 episodes of systemic bank runs have occurred in 93 countries since the late 1970s.²⁰ Bank runs are not merely historical events nor are all “bank runs” runs on depository institutions. The recent financial crisis, which involved “financial firms ‘running’ on other financial firms by not renewing sale and repurchase agreements”,²¹ is suggestive of a panic-based bank run. As former Federal Reserve Chairman Paul Volcker said, “The psychological panic—a classic run on banks—gives rise to a problem for the whole system”.²² Understanding panic-based bank runs is important.²³ When a bank run occurs, government bailouts of banks (among other types of government interventions²⁴) often follow. In some cases, including the recent financial meltdown, the magnitude of the government bailout is substantial. But bailouts are controversial. A systematic framework to analyze the relationship between runs and bailouts is called for.

In thinking about bailouts, two questions need to be addressed. First, why would the government want to intervene and bail out banks *ex-post*, i.e. after a run? In other words, what is the crisis-management role²⁵ of bailouts? Second, how does the bailout

²⁰For other empirical studies, see e.g. Caprio, Klingebiel, Laeven, and Noguera (2005), Demirguc-Kunt, Detragiache, and Gupta (2000), Lindgren, Garcia, and Saal (1996) and Martinez-Peria and Schmukler (2001).

²¹Gorton (2009). Gorton (2009) showed that: (1) The recent financial crisis is a bank run in the shadow banking system. (2) The shadow banking system, including the combination of repo and securitized debt, is a kind of bank. (3) The runs on the shadow banking system had common features with the classical runs on commercial banks.

²²Scott, Kenneth, George P. Shultz and John B. Taylor eds. (2010).

²³Gorton (2009) pointed out how important it is to understand classic panic-based bank runs for understanding the recent crisis and for thinking about future financial regulation policy.

²⁴Laeven and Valencia (2008) collected data on crisis containment and resolution policies using a variety of sources in the 42 systemic banking crises episodes (in 37 countries) that are well documented.

²⁵Acharya and Yorulmazer (2007)

policy change the *ex-ante* probability of runs? That is, what is the crisis-prevention role of bailouts?

In this paper, I answer these two questions by using a modified DD model.²⁶ I show that these two roles of government bailouts are in conflict with each other. In my model, a benevolent government has an incentive to bail out banks when a run occurs since runs cause a misallocation of resources in the banking sector: Some depositors lose their deposits in a run since the bank's assets are exhausted during the run. Bailouts, which transfer some resources from the government to the banking sector, can alleviate that misallocation by enabling more depositors to withdraw.

However, anticipated bailouts increase the *ex-ante* probability of runs through two channels. The first channel is that the bailouts distort the *ex-ante* incentives of banks, increasing the riskiness of their banking contracts. Anticipating government bailouts, banks promise a higher short-term deposit interest rate in order to attract depositors. The higher short-term deposit interest rate makes the banking sector more fragile and increases the probability of a run. Regulation of the deposit-rate eliminates this distortion. The second channel is that the anticipated bailout increases the depositor's probability of getting her money from the bank when she participates in the run.²⁷ With the additional resources from the bailout, banks are provided with extra withdrawal capacity during a run. Hence, if a depositor expects that a bank run will occur,

²⁶Diamond and Dybvig (1983) (also Bryant (1980)) modeled the essential function of banks as providing maturity transformation. The maturity transformation can improve the *ex-ante* welfares of the agents subject to idiosyncratic liquidity shocks. They also show that bank runs, in which some agents "misrepresent" their liquidity needs and withdraw early, may occur as a result of panic. That is, those agents who don't need liquidity may decide to withdraw early if they expect that a large number of depositors will do so. As a result, the bank cannot satisfy all the withdrawal demand and indeed fails.

²⁷In the model, the government does not have enough resources to bailout the whole banking sector and satisfy all the withdrawal demand if a system-wide bank run occurs. The reason for this is that holding so much resources will distort the resources allocation in normal times and we assume the cost of that inefficiency outweighs the benefit of completely ruling out bank runs.

This assumption is realistic given the size of the banking sector in our economy. In early 2007, total assets of the entire banking system of the United States were about \$10 trillion (Geithner, 2008).

the anticipated bailout increases the probability of the depositor getting her money from the bank if she participates in the run. When a depositor compares her expected utility between withdrawing early or waiting to withdraw, because of the bailout her incentive to withdraw early is increased and the *ex-ante* probability of a run is increased. Knowing the conflicts between alleviating bank runs and preventing bank runs, the government will announce, *ex ante*, a bailout policy that balances these two effects. At first, I assume that the government can commit to this bailout policy. In Section 2.5, I analyze the case of no-commitment and I show how the distortion of resources due to bailouts is exacerbated with lack of commitment.

In a seminal paper, Keister (2010) introduced public good consumption (funded by lump-sum tax revenue) into the DD model, providing a convenient way to model the social cost of government bailouts. Bailouts are transfers from the government²⁸ to the banking sector in the event of a bank run. Both bailouts and the public good provision are funded by taxes collected *ex-ante*. Hence, bailouts crowd out the public good provision.

My main innovation is to make the run probability endogenous. To do this, I use a similar setup to Goldstein and Pauzner (2005), who applied the global-games approach²⁹ to DD bank-run model.

The misalignment of objectives of the social planner and the banks is the key to explaining the distortion of resources due to bailouts.³⁰ The government's objective is to maximize social welfare: An individual agent's welfare depends on her private

²⁸This view (bailouts are transfers from the government, made to firms or banks) was also raised by Green (2010).

²⁹Carlsson and van Damme (1993) and Morris and Shin (1998) showed that the introduction of noisy signals to multiple-equilibria games may lead to a unique equilibrium. Goldstein and Pauzner (2005) first used the global-games approach to study the Diamond-Dybvig-type banks. They focus on how the probability of bank runs are determined as a function of short-run deposit rate. They do not discuss government policies which are the focus of this paper.

³⁰In the original DD model, there is no public good consumption and the bank's objective is the same as a social planner.

good consumption and its timing and on the level of the public good. Since each agent consumes the same amount of the public good, an individual agent’s objective is to maximize her welfare from the private good consumption given the level of public good provision. In the free-entry banking sector,³¹ banks compete for depositors and make zero profits in equilibrium. Hence, the deposit contract offered by the representative bank is the same as the one that maximizes the depositor’s objectives.³² That is, the deposit contract maximizes the depositor’s welfare solely from private good consumption. The representative bank, acting rationally and independently, does not internalize the fiscal cost of bailouts to the aggregate economy, which is reduced public good provision. Hence, bank runs are “less costly” to the bank than to society (or the benevolent government). The deposit contract offered by banks provides greater maturity transformation (i.e., a higher short-term deposit rate of interest) than the socially optimal level. If the run probability is an increasing function of the short-term deposit rate, this distortion results in an increased probability of bank runs.

To solve for the relationship between the short-term deposit rate and the probability of a bank run, I make the major innovation in the model. I show that the probability of a bank run is endogenous, and is a function of the government bailout policy and the short-term deposit rate promised by the representative bank. I use a similar setup to that of Goldstein and Pauzner (2005), who applied the global-games approach to endogenize the *ex-ante* bank-run probability:³³ The fundamentals of the economy are stochastic. Agents do not have common knowledge of the realization of the fundamentals, but rather obtain slightly noisy private signals. There is a unique

³¹The free-entry assumption is standard in the DD literature.

³²In the DD literature which assumes free-entry competitive banking sector, the deposit contract offered by the bank is the same as the one that will maximize the depositor’s objective. Hence, without the wedge between the objectives of a representative agent and the government, the deposit contract offered by a representative bank is socially optimal.

³³Goldstein and Pauzner (2005) did not analyze government policy.

Bayesian-Nash equilibrium, in which a bank run occurs if and only if the realized state of the fundamentals is below a critical value. The critical value, which determines the probability of a bank run, is a function of the government bailout policy and the short-term deposit interest rate. I show, in this paper, that the run probability is an increasing function of the bailout level and short-deposit rate. It needs to be emphasized that runs in this model are largely panic based.³⁴ In most scenarios, agents run on the banks because they fear that others will do so. The realization of the fundamentals determines whether or not a run occurs through changing agents' expectations. Without the change in agents' expectations, the fundamentals themselves can only make bank runs occur when the realization is extremely bad, which I assume occurs with only low probability.

There are several papers which analyzed government bailouts and bank runs. Keister (2010) introduced the public good into DD model and showed that government bailouts can increase the “financial system fragility”. My paper is different from Keister (2010) in the three aspects: *Firstly*, the probability of bank runs is exogenous in Keister (2010).³⁵ Increased “financial system fragility” means for Keister that the set of parameters which tolerates the bank runs is larger. In my paper, the probability is

³⁴Both the panic-based bank runs and the recent financial meltdown have their roots in expectations—depositors or lenders are afraid that banks or other counter-parties cannot honor their obligations.

Federal Reserve Chairman Ben Bernanke expressed the same view in his testimony (September 2, 2010) to the Congress by saying that “Lenders in the commercial paper market and other short-term money markets, like depositors in a bank, place the highest value on safety and liquidity. Should the safety of their investments come into question, it is easier and safer to withdraw funds—run on the bank—than to invest time and resources to evaluate in detail whether their investment is, in fact, safe.”

³⁵In Keister (2010), bank runs are driven by a sunspot variable. Peck and Shell (2003) first used this method and showed that if the probability is below a certain range, the optimal deposit contract will tolerate bank runs. Zhang (2011) analyzed the two-consumer example in Peck and Shell (2003) with different parameter values. Zhang (2011) found that for some range of the parameters the run probability determines not only whether the bank runs should be tolerated but also how bank runs should be tolerated—The deposit rate is a strictly decreasing function of the bank run probability up to a critical probability and above that critical probability the optimal deposit contract is run-proof.

endogenous. Hence, I can analyze how the run probability changes with the bailout policy. *Secondly*, Keister (2010) assumes that once the bank run occurs, the deposit contract is re-scheduled. In my paper, as in Goldstein and Pauzner (2005), the deposit contract is not re-scheduled. The bank will keep paying the promised payment to the depositors until it runs out of resources. *Thirdly*, in my paper, the “wedge” (between the bank’s objective and the government’s objective) is different from the wedge in Keister (2010). In my paper, the wedge is *solely* the public good consumption. The wedge causes distortion because: (1) the bank does not “see” the decreased public good consumption during bank runs. (2) the run probability increases as the bank increases the short-term deposit interest rate. Hence, the “expected cost” of increasing the deposit interest rate is lower for the bank than the for the planner. In Keister (2010), the “wedge” includes two parts: the public good consumption in a run, and the private good consumption when the government starts the bailout. The latter part is necessary for the wedge to cause distortion in Keister (2010). This is because the run probability is assumed to be exogenous by Keister and the run probability is independent of the short-term deposit interest rate. Hence, if the wedge is merely the public good consumption (as in my paper), the “expected cost” of increasing the deposit interest rate is the same for the bank as for the planner. If, as in my paper, the only thing the representative bank cannot “see” is its effect on public goods, then in the Keister set-up in which the probability of run is exogenous, the bank’s choice of deposit interest rate would be the same as if it were chosen by the planner.

Acharya and Yorulmazer (2007), Chari and Kehoe (2010), Farhi and Tirole (2009) analyzed government bailouts and banking crises without modelling the behavior of depositors and they focus on moral hazard caused by bailouts.

The next section introduces the formal model. I will describe the agents’ preferences, asset returns, information structure, and the timing. The model is analyzed in

two parts: the “post-deposit game” among the depositors (Section 2.3) and the “pre-deposit game” (Section 2.4) between the government and the bank. In the post-deposit game I analyze the depositor’s withdrawal decision in period 1, while the depositors take the government policy and the deposit contract as given. In the pre-deposit game I analyze the government’s and the bank’s choices in period 0, when they anticipate the effects of their choices on the depositors in the post-deposit game. In section 2.5, I analyze the case in which the government is unable to make a commit to a bailout policy. Section 2.6 is the summary.

2.2 The Model

2.2.1 Agents’ Preferences

There are three periods, $t = 0, 1$ and 2. And there is a continuum of agents with measure 1 and indexed by $i \in [0, 1]$. The preferences of the agents are the same as Keister (2010): The agents consume both the private good and the public good. The public good is consumed in period 1. For the private good consumption, there is an idiosyncratic preference shock³⁶ to each agent in period 1 : the ex-ante identical agents become either patient or impatient. An impatient agent values only the private good in period 1. A patient agent values the private good in both period 1 and period 2, and they are perfect substitutes. If we denote c_t as the consumption of the private good in period t and g as the consumption of the public good in period 1, the utility function of an impatient agent is

$$u(c_1) + v(g) \tag{6}$$

³⁶The preference shock can also be interpreted as a liquidity shock. The impatient agents need liquidity to consume.

The utility function of a patient agent is

$$u(c_1 + c_2) + v(g) \tag{7}$$

u and v are continuously differentiable, strictly increasing and strictly concave.

$$-cu''(c)/u'(c) > 1,$$

for $c > 0$. That is, for the private good consumption, the coefficient of relative risk aversion is larger than 1.³⁷ We normalize $u(0)$ to be 0.³⁸

With probability λ the agent becomes impatient and with probability $(1 - \lambda)$ she becomes patient. Agents' types are i.i.d. Hence, the proportion of impatient and patient agents in period 1 are λ and $(1 - \lambda)$ respectively.

2.2.2 Endowments and Asset Returns

Each agent is endowed with one unit of resources in period 0. There are no endowments in later periods. The endowments in period 0 can be invested in an asset. The return of the investment³⁹ depends on θ , which is the state of fundamentals and is uniformly distributed over the interval $[0, 1]$. Higher realizations of θ corresponds to stronger fundamentals.

For $\theta \in [0, \bar{\theta}]$, if the asset is liquidated in period 1, one unit of the asset yields one

³⁷As Diamond-Dybvig, this assumption makes the maturity transformation desirable.

³⁸This normalization is just for simplification of the algebra. All the results still hold as long as $u(0) > -\infty$. If $u(0) = -\infty$, then bank runs are devastating and banks will not provide any maturity transformation.

³⁹The assumption of the asset return is the same as Goldstein and Pauzner (2005).

unit of output. If the asset is liquidated in period 2, one unit of the asset can generate R units of output with probability $p(\theta)$, or 0 units with probability $1 - p(\theta)$. R is strictly larger than 1 and $p(\theta)$ is strictly increasing in θ . We assume $E_\theta[p(\theta)]u(R) > u(1)$. This implies that for patient agents the expected long-term return is superior to the short-term return.

For $\theta \in [\bar{\theta}, 1]$, we will have a state at which the economic fundamentals are extremely strong. Extremely strong fundamentals make the the high return R on the long-term asset to be certain: $p(\theta) = 1$ for $\theta \in [\bar{\theta}, 1]$. Since this is just an extreme case, we assume that $\bar{\theta} \rightarrow 1$. The short-term asset return also improves because of the extremely strong fundamentals: if liquidated in period 1, one unit of asset can also yield R units of output. We will see in the next section that in the range of states at which fundamentals are extremely strong, a patient agent withdraws late no matter what others do. Hence, there are no panic runs. The assumption⁴⁰ that this range of strong fundamentals exists reflects the intuition that sufficiently strong fundamentals can prevent the depositors from panicking.⁴¹

Both the private good and public good can be produced directly from the proceeds of the asset. One unit of the proceeds can produce one unit of public good or one unit of private good.

2.2.3 Agents' Information

In period 0, agents are identical. Each agent i knows that she will become impatient with probability λ and patient with probability $(1 - \lambda)$ in period 1. She also knows

⁴⁰This assumption is also used by other models which apply the global-games method. Morris and Shin (1998), the authors assume that when the fundamentals are extremely strong, the speculators' profits from attacking the currency are outweighed by the transaction cost of doing so. Hence, there is no currency attack when the fundamentals are extremely strong.

⁴¹Some empirical studies, like Demirguc-Kunt and Detragiache (1998), show that the probability of a banking crisis is decreasing when the macroeconomic performance is strong.

that the state of fundamentals, θ , is uniformly distributed over the interval $[0, 1]$.

In period 1, each agent i knows her own type (patient or impatient), which is her private information. The state of the fundamentals θ is determined at the beginning of period 1, but the realization is not revealed. Agent i gets a private noisy signal θ_i concerning the state of the fundamentals.

$$\theta_i = \theta + \varepsilon_i,$$

where the noise ε_i is *i.i.d* uniformly distributed over $[-\varepsilon, \varepsilon]$. The scalar ε is assumed to be small. Agent i uses her signal to update her belief about θ . Conditional on θ_i , the posterior distribution of θ for agent i is a uniform distribution on the interval:

$$[\max\{0, \theta_i - \varepsilon\}, \min\{1, \theta_i + \varepsilon\}].$$

2.2.4 Banks

Agents suffer from preferences shocks. Banks can improve the agents' welfares by offering a demand-deposit contract, which works as an "insurance" against these shocks. I assume banks offer a DD-like deposit contract: An agent deposits her after-tax endowment $(1 - \tau)$ in period 0 (the tax will be discussed later). If she demands to withdraw in period 1, she gets a fixed payment c_1 until the bank runs out of resources. (The bank serves the depositors sequentially⁴² and the position of the depositor in the queue is random.) If she waits until period 2, she shares the remaining resources in the bank

⁴²The sequential service constraint (Wallace 1988) implies that the bank can't observe how many agents would like to withdraw in period 1. The bank keeps satisfying the withdrawal demand until it runs out of resources.

with the other depositors who have not withdrawn in period 1. Hence, the deposit contract can be characterized by the short-term payment c_1 .

As in the DD literature, the banking industry is subject to free-entry. Banks compete for depositors and make zero profits in equilibrium. Hence, the deposit contract offered by a bank is the same as the one that maximizes an individual depositor's objectives.

Banks cannot observe agents' types (patient or impatient) nor receive up-dated information on the fundamentals.

2.2.5 Government

The (benevolent) government provides the public good and maximizes the agents' welfares. To finance the public good provision (in period 1), the government imposes a lump-sum tax τ on each agent's endowment in period 0.⁴³ We use g to denote the level of public good provision.

If a bank run occurs in period 1 and the withdrawals have exhausted the banks' assets, the government can transfer some of the tax revenue to the banks, allowing the banks to satisfy more withdrawal demand. These transfers are the bailouts.⁴⁴ The government's bailout policy is characterized by the maximum transfers it can make, \bar{b} . The government will announce \bar{b} in period 0. We assume that the government can commit to \bar{b} . (We will discuss the case of no commitment in Section 2.5).

The actual level of bailouts, b , depends on the proportion of depositors who demand early withdrawal. Let us use n to denote the proportion of depositors who want to withdraw in period 1. If the total withdrawal demand, nc_1 , is lower than the bank's

⁴³Since agents start to consume at the beginning of period 1, the government cannot collect more tax in period 1.

⁴⁴One thing needs to be emphasized is that the government bailout is not a guarantee of deposit return.

total available resource $(1 - \tau)$, then the bank can stand alone and no bailouts will be granted. If $nc_1 > (1 - \tau)$, government bailouts will cover the withdrawal demand beyond $(1 - \tau)$ until the bailouts reach the maximum level \bar{b} . To summarize, we have:

$$b = \begin{cases} 0 & \text{if } nc_1 \leq (1 - \tau) \\ nc_1 - (1 - \tau) & \text{if } (1 - \tau) < nc_1 \leq \bar{b} + (1 - \tau) \\ \bar{b} & \text{if } \bar{b} + (1 - \tau) < nc_1 \end{cases}$$

The budget of the government must be balanced.⁴⁵ Hence, the tax revenue of the government equals the sum of the expenditures on bailouts and public goods:

$$\tau = b + g.$$

The government does not observe agents' types (patient or impatient) nor receive updated information on the fundamentals.

2.2.6 Timeline

In period 0 of the pre-deposit game:



Figure 2.1: Pre-deposit Game

⁴⁵This is a very simple case of Ricardian equivalence.

(1) The government collects the lump-sum tax τ and announces the bailout limit \bar{b} . Banks announce the short-term payment c_1 . The government and the bank move simultaneously.

(2) Agents make their deposit decisions. In equilibrium, agents will deposit their after-tax endowment $(1 - \tau)$ in the banks.

In period 1 of the post-deposit game:

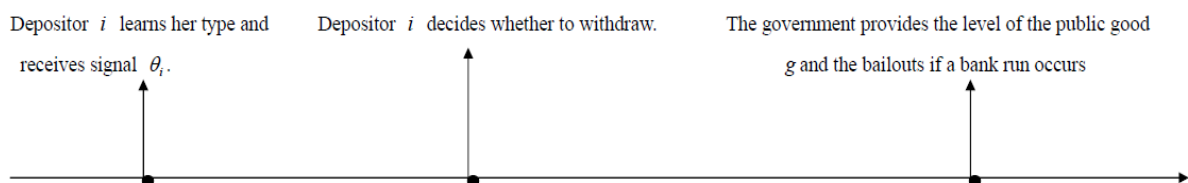


Figure 2.2: Post-deposit Game

(1) Depositor i learns her type and receives her signal θ_i .

(2) Depositor i makes her withdrawal decision according to her type and her signal θ_i . If she attempts to withdraw, she gets a random position in the queue of depositors who seek to withdraw.

(3) The government provides public goods at the level g . If a bank run occurs and banks runs out of resources, the government will provide bailouts at level b .

When the government and the bank make their choices in period 0, they consider depositors' responses in period 1. Hence, we need to analyze the model backward. In the next section, I analyze the depositors' withdrawal decisions in period 1 taking τ , \bar{b} and c_1 as given.

2.3 Post-Deposit Game: Depositor’s Withdrawal Decision in Period 1

In period 1, after the depositors learn their types, the impatient depositors withdraw since they value only the period-1 consumption. The only thing that needs to be analyzed is the patient depositors’ withdrawal decisions. Each patient depositor i can choose an action a_i between two possibilities: $a_i = 1$ (withdraw), or $a_i = 0$ (wait). This is a game among the patient depositors (the post-deposit game) since individual payoffs also depend on others’ choices. Each patient depositor forms her expectation about others’ choices from her private signal on the state of the fundamentals. The strategy of a patient depositor is a function specifying her action for each possible private signal θ_i .

Definition 2.1 *In the post deposit game of period 1, a strategy for a patient depositor i is a function $s_i : [0 - \varepsilon, 1 + \varepsilon] \rightarrow \{0, 1\}$.*

Throughout the paper, I focus on “switching strategy”⁴⁶ and “threshold equilibrium”. A “switching strategy” is characterized by a cutoff point k such that a patient depositor withdraws if and only if her private signal is below k :

$$s_i = \begin{cases} 1, & \text{if } \theta_i < k \\ 0, & \text{if } \theta_i \geq k \end{cases}$$

Definition 2.2 *A threshold equilibrium of the post deposit game, is a Bayesian-Nash equilibrium in which each patient depositor i finds it optimal to use the switching strategy with cutoff point k , given that all other patient depositors use the same strategy.*

⁴⁶Morris and Shin (2001)

Suppose the realized state of the fundamentals is θ . The signals will be uniformly distributed over $[\theta - \varepsilon, \theta + \varepsilon]$. Hence, if all patient depositors use the switching strategy with cutoff point k , the proportion of patient depositors who withdraw equals 1 if $\theta < k - \varepsilon$; It equals $[k - (\theta - \varepsilon)]/(2\varepsilon)$ if $k - \varepsilon \leq \theta \leq k + \varepsilon$; It equals 0 if $\theta > k + \varepsilon$. Since the impatient depositors always withdraw and the proportion of impatient depositors is λ , the total proportion of depositors who withdraw is:

$$n(\theta, k) = \begin{cases} 1, & \text{if } \theta < k - \varepsilon \\ \lambda + (1 - \lambda)[\frac{1}{2} + \frac{k - \theta}{2\varepsilon}], & \text{if } k - \varepsilon \leq \theta \leq k + \varepsilon \\ \lambda, & \text{if } \theta > k + \varepsilon. \end{cases} \quad (8)$$

When $n(\theta, k)c_1 \leq (1 - \tau)$, the bank can satisfy all the withdrawal demand in period 1 by liquidating its own asset. No government bailouts are granted. If a patient depositor i waits until period 2 to withdraw, she can share the proceeds from the remaining asset with others who have not withdrawn. When the state of the fundamentals is θ , the probabilities of high long-term asset return, R , and low asset return, 0, are $p(\theta)$ and $1 - p(\theta)$ respectively. Hence, the patient depositor's expected utility from withdrawing in period 2 is

$$p(\theta)u\left(\frac{1 - \tau - nc_1}{1 - n}R\right) + [1 - p(\theta)]u(0) = p(\theta)u\left(\frac{1 - \tau - nc_1}{1 - n}R\right).$$

If she withdraws in period 1, she can be served by the bank since the total period-1 withdrawal demand is smaller than the liquidation value of the bank's asset, and her expected utility is $u(c_1)$. We use $\delta(\theta, k)$ to denote the difference in the expected utility

for a patient depositor between withdrawing in period 2 and withdrawing in period 1. Hence, when $n(\theta, k)c_1 \leq (1 - \tau)$,

$$\delta(\theta, k) = p(\theta)u\left(\frac{1 - \tau - nc_1}{1 - n}R\right) - u(c_1).$$

When $(1 - \tau) < n(\theta, k)c_1 \leq \bar{b} + (1 - \tau)$, the liquidation value of the bank is smaller than the total withdrawal demand in period 1. The bank has to liquidate all its asset to satisfy the early withdrawal demand. If a patient depositor i waits until period 2 to withdraw, she cannot get anything. But with the additional resources from the government bailout, all the period-1 withdrawal demand can be satisfied, and if the patient depositor chooses to withdraw in period 1, her utility is $u(c_1)$. Hence, when $(1 - \tau) < n(\theta, k)c_1 \leq \bar{b} + (1 - \tau)$,

$$\delta(\theta, k) = -u(c_1).$$

When $n(\theta; k)c_1 > \bar{b} + (1 - \tau)$, the total period-1 withdrawal demand is larger than the sum of maximum bailout level and the bank's liquidation value. If the patient depositor i withdraws in period 2, she still cannot get anything as the previous case. If she withdraws in period 1, the probability of being served is $\frac{\bar{b} + (1 - \tau)}{nc_1}$ and the expected utility is $\frac{\bar{b} + (1 - \tau)}{nc_1}u(c_1)$. Hence, when $n(\theta, k)c_1 > \bar{b} + (1 - \tau)$,

$$\delta(\theta, k) = -\frac{\bar{b} + (1 - \tau)}{nc_1}u(c_1).$$

However, a patient depositor i cannot observe the realized state of the fundamentals directly. She has to calculate the expectation of $\delta(\theta, k)$ from the posterior distribution over the states conditional on her signal θ_i . Denoting that expectation by $\Delta(\theta_i, k)$, we have:

$$\Delta(\theta_i, k) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k) d\theta$$

Lemma 2.1 states the continuity property of $\Delta(\theta_i, k)$, which is useful in establishing the existence of the threshold equilibrium.

Lemma 2.1 $\Delta(\theta_i, k)$ is continuous in θ_i and k .

Proof. It is easy to see that $\delta(\theta, k)$ is bounded. Continuity of $\Delta(\theta_i, k)$ with respect to θ_i holds because a change in θ_i only shifts the limits of integration $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ in the computation of $\Delta(\theta_i, k)$, and $\delta(\theta, k)$ is bounded. Continuity of $\Delta(\theta_i, k)$ with respect to k holds since $\Delta(\theta_i, k)$ is an integral over a segment of θ 's and $\delta(\theta, k)$ is bounded. ■

Given the definition of the threshold equilibrium, we know that the switching strategy with cutoff point k is an equilibrium strategy if and only if the following two conditions hold:⁴⁷ (1) $\Delta(\theta_i, k) < 0$, for $\theta_i < k$ and (2) $\Delta(\theta_i, k) \geq 0$, for $\theta_i \geq k$. We prove the existence and uniqueness of the threshold equilibrium by the following two steps: first, in Lemma 2.2, we prove there is a unique k^* such that if all others use k^* as a cutoff point, patient depositor i is indifferent between waiting and withdrawing if she gets a signal equal to k^* . Then, in Lemma 2.3, we prove that depositor i strictly prefers to withdraw if she gets a signal below k^* and strictly prefers to wait if she gets a signal above k^* . The proofs are in the appendix.

Lemma 2.2 There is a unique k^* such that $\Delta(k^*, k^*) = 0$

⁴⁷We assume that a patient depositor does not withdraw if she is indifferent between the period-1 and period-2 withdrawals.

Lemma 2.3 $\Delta(\theta_i, k^*) < 0$ for $\theta_i < k^*$ and $\Delta(\theta_i, k^*) > 0$ for $\theta_i > k^*$

Proposition 2.1 follows directly from Lemma 2.2 and Lemma 2.3.

Proposition 2.1 *In the post-deposit game, there is a unique threshold equilibrium in which each patient depositor uses the switching strategy with the cutoff point k^* . k^* is the unique solution which solves $\Delta(k, k) = 0$.*

The uniqueness of the threshold equilibrium is central to the analysis of the impacts of government policy and the deposit contract. We can see how the maximum bailout level (\bar{b}) and the short-term deposit rate of interest ($c_1/(1 - \tau)$) affect the value of k^* . Since a bank run is defined as occurring when a positive measure of patient depositors withdraw in period 1, it follows that k^* is the ex-ante probability of runs.⁴⁸

Corollary 2.1 *The ex-ante probability of runs k^* is non-decreasing in \bar{b} . It is strictly increasing in \bar{b} if $\bar{b} < c_1 - (1 - \tau)$.*

The proof of Corollary 2.1 is in the appendix. The anticipated bailouts can change the run probability by affecting the depositors' incentives to run. If $\bar{b} + (1 - \tau) < c_1$, only a proportion of depositors can be served when the withdrawal demand nc_1 is larger than $\bar{b} + (1 - \tau)$. Because of sequential service, the probability of being served is $\frac{\bar{b} + (1 - \tau)}{nc_1}$. A higher level of bailouts implies a higher probability of being served, which makes withdrawing more attractive. Hence, a patient depositor is indifferent between “wait” and “withdraw” ($\Delta(k^*, k^*) = 0$) only when she expects a stronger state of fundamentals. That is why the cutoff point k^* is strictly higher. If $\bar{b} + (1 - \tau) \geq c_1$, all the withdrawal demand can be satisfied even when everyone withdraws ($n = 1$). Hence,

⁴⁸The patient depositor withdraws only when she receives a signal lower than k^* . The signal of a depositor is uniformly distributed in the interval $[\theta - \varepsilon, \theta + \varepsilon]$. Hence, a bank run occurs if $\theta < k^* + \varepsilon$. Since the prior distribution of the fundamental θ is the uniform distribution over the interval $[0, 1]$, as ε approaches 0 the probability of bank runs approaches k^* .

a first-order change in the bailout does not affect the patient depositors' decision and hence does not affect k^* .

Corollary 2.2 states the relationship between the run probability and the short-term deposit rate of interest.

Corollary 2.2 *The ex-ante probability of runs k^* is increasing in τ and c_1 .*

The proof can be found in the appendix. A higher short-term payment c_1 makes the bank more vulnerable to runs. The intuition is simple: If the period-1 payment is increased, then the payment in period 2 is decreased and the incentive of patient agents to withdraw in period 1 is higher. Furthermore, knowing that other patient agents are more likely to withdraw in period 1, the agent assigns a higher probability to the event of a bank run. This will increase the run probability further.

2.4 Pre-Deposit Game: Choices of the Government and the Banks in Period 0

In Section 2.3, there are only partial-equilibrium results: depositors' choices were analyzed taking government policy and the deposit contract as given. This section shows how the government policy and the deposit contract are determined in period 0, given the depositors' responses in the post-deposit game. In this pre-deposit game, the government chooses τ and \bar{b} , and the bank chooses c_1 simultaneously. The equilibrium is Nash. In this section, I assume that the government can commit to the bailout policy. The no-commitment case is analyzed in the next section.

2.4.1 The Bank's Strategy

Each depositor enjoys the same level of the public good g as the other depositors. Given g , the depositor maximizes her welfare from private good consumption. In

the free-entry banking sector, banks compete for depositors and make zero profits in equilibrium. The deposit contract offered by the bank maximizes the depositor's objective. That is, given τ and \bar{b} , the bank chooses c_1 to maximize:⁴⁹

$$\int_0^{k^*(\tau, \bar{b}, c_1)} \min\left\{1, \frac{1-\tau+\bar{b}}{c_1}\right\} u(c_1) d\theta$$

$$+ \int_{k^*(\tau, \bar{b}, c_1)}^1 [\lambda u(c_1) + (1-\lambda)p(\theta)u\left(\frac{(1-\tau)-\lambda c_1}{1-\lambda}R\right)] d\theta. \quad (9)$$

where $k^*(\tau, \bar{b}, c_1)$ is the cutoff point of the depositor's switching strategy, which has been analyzed in the previous section. The objective of the representative bank includes two parts: The first integral is the depositor's expected utility from private good consumption when there is a bank run. When a bank run occurs, the proportion of depositors who can be served is $\min\left\{1, \frac{1-\tau+\bar{b}}{c_1}\right\}$. The second integral is the depositor's expected utility from private good consumption when there is no bank run.

If the bank chooses c_1 which is larger than $1 - \tau + \bar{b}$, the following first-order condition must be satisfied (To simplify the notation, k^* is not written explicitly as a function):

$$k^* \frac{1-\tau+\bar{b}}{c_1} [u'(c_1) - u(c_1)/c_1]$$

$$+ (1 - k^*) \left\{ \lambda u'(c_1) - \lambda u'\left(\frac{(1-\tau)-\lambda c_1}{1-\lambda}R\right) RE[p(\theta) | \theta > k^*] \right\} \quad (10)$$

$$= 0$$

The first term is the misallocation effect due to c_1 when a bank run occurs: higher c_1 implies fewer depositors can be served and a larger consumption disparity between

⁴⁹This expression is under the assumption that $\varepsilon \rightarrow 0$ and $\bar{\theta} \rightarrow 1$.

the depositors who are served and those who are not. The second term is the risk-sharing effect of c_1 in normal times: higher c_1 implies more “insurance” provided to the depositors against the preference shocks. The bank balances these two effects.

If the bank chooses c_1 which is smaller than $1 - \tau + \bar{b}$, the following first-order condition must be satisfied:

$$\begin{aligned}
& k^* u'(c_1) \\
& + (1 - k^*) \left\{ \lambda u'(c_1) - \lambda u' \left(\frac{(1-\tau) - \lambda c_1}{1-\lambda} R \right) RE[p(\theta) | \theta > k^*] \right\} \\
& = 0
\end{aligned} \tag{11}$$

Now, even if a bank run occurs, all depositors can be served with the additional resources from the bailout. The misallocation effect of c_1 no longer exists; on the contrary, higher c_1 can increase the expected utility when a bank run occurs since the depositors can get more resources from the bailout. The risk-sharing effect of c_1 in normal time remains the same. Hence, the bank chooses $c_1 \leq 1 - \tau + \bar{b}$ only if the risk-sharing effect requires the bank to do so. Otherwise, increasing c_1 always increases the depositor’s expected utility.

The risk-sharing effect of c_1 depends on the risk aversion of the depositors, the asset return and the size of the deposit $(1 - \tau)$. Define $\hat{c}_1(\tau)$ as the optimal short-term payment when the deposit is $(1 - \tau)$ and the bank run does not occur. That is,

$$\hat{c}_1(\tau) = \arg \max_{c_1} \lambda u(c_1) + (1 - \lambda) E[p(\theta)] u \left(\frac{(1 - \tau) - \lambda c_1}{1 - \lambda} R \right).$$

Lemma 2.4 $\hat{c}_1(\tau)$ is a decreasing function of τ and there exists a unique $\hat{\tau}$ such that

$$\widehat{c}_1(\widehat{\tau}) = 1.$$

Proof. $\widehat{c}_1(\tau)$ is characterized by the first-order condition:

$$u'(c_1) = E[p(\theta)]u'\left(\frac{(1-\tau) - \lambda c_1}{1-\lambda}R\right)R. \quad (12)$$

Since $-cu''(c)/u'(c) > 1$ for $c > 0$, we have

$$1 - \tau < \widehat{c}_1(\tau) < (1 - \tau)R. \quad (13)$$

By the Implicit Function Theorem, we have

$$\widehat{c}_1'(\tau) = -1/\lambda - E[p(\theta)]u''\left(\frac{(1-\tau) - \lambda \widehat{c}_1}{1-\lambda}R\right)R^2 / [(1-\lambda)u''(\widehat{c}_1)] < 0. \quad (14)$$

Hence, $\widehat{c}_1(\tau)$ is an decreasing function of τ . Since $\widehat{c}_1(0) > 1$ and $\widehat{c}_1(1 - 1/R) < 1$, we know there exists a unique $\widehat{\tau}$ such that $\widehat{c}_1(\widehat{\tau}) = 1$. ■

$1 - \widehat{\tau}$ is the value of deposits above which the risk-sharing effect requires that the short-term payment be strictly greater than 1. Let $p(\widehat{\theta}) = u(1)/u\left(\frac{(1-\widehat{\tau})-\lambda}{1-\lambda}R\right)$, where $\widehat{\theta}$ is the state of the fundamentals below which the patient depositor always withdraws in period 1, when the short-term payment is equal to 1. If the short-term payment is less than 1, the run probability k^* is less than $\widehat{\theta}$. Proposition 2.2 says that if $\widehat{\theta}$ is sufficiently small (i.e., the expected return of the asset is sufficiently favorable) and the deposit is sufficiently large (larger than $1 - \widehat{\tau}$), the bank provides liquidity via the deposit contract and takes on the risk that in a bank run not all depositors who want

to withdraw can do so.

Proposition 2.2 *If $\hat{\theta}$ is sufficiently small and $\tau \leq \hat{\tau}$, the bank chooses a short-term payment c_1 which is strictly larger than the sum of the bailouts and the liquidation value of the bank's asset ($1 - \tau + \bar{b}$). Hence, if a bank run occurs, some depositors cannot be served even with the bailout.*

The proof can be found in the appendix. The intuition for this proposition is the following. When $\tau \leq \hat{\tau}$, the risk-sharing effect requires c_1 to be larger than 1. Since the bailout is restricted by the government budget, we have $\bar{b} \leq \tau$ and $1 - \tau + \bar{b} < 1$. (11) implies that if $c_1 < 1 - \tau + \bar{b}$, the expected utility of the depositors can be improved by increasing c_1 . If $c_1 = 1 - \tau + \bar{b}$, the risk-sharing effect still requires c_1 to be increased. But (10) shows that the bank also considers the misallocation effect. However, $\hat{\theta}$ is the upper bound of the run probability k^* . If $\hat{\theta}$ is sufficiently small, the misallocation effect is small and is dominated by the risk-sharing effect. Hence, the bank chooses c_1 which is larger than $1 - \tau + \bar{b}$.

A corollary of this theorem is that, agents always choose to deposit in period 0.

Corollary 2.3 *If $\hat{\theta}$ is sufficiently small and $\tau \leq \hat{\tau}$, agents choose to deposit in period 0.*

Proof. The bank can always mimic the autarky allocation by setting $c_1 = 1 - \tau$. Proposition 2.2 says that the bank will not choose the autarky allocation, which means that the expected utility of an individual is higher by using the deposit contract than autarky. An agent always chooses to deposit to get a higher expected utility from the private good consumption than she would in autarky, taking the public good as given.

■

2.4.2 The Government's Strategy

Now we can analyze the government's policy. As discussed before, the government and the bank have different objectives. The government's objective is to maximize the depositor's expected utility not only from the private good consumption but also from the level of the public good consumption. Given c_1 , the government chooses τ and \bar{b} to maximize

$$\begin{aligned}
& \int_0^{k^*(\tau, \bar{b}, c_1)} \min\left\{1, \frac{1-\tau+\bar{b}}{c_1}\right\} u(c_1) d\theta \\
& + \int_{k^*(\tau, \bar{b}, c_1)}^1 [\lambda u(c_1) + (1-\lambda)p(\theta)u(\frac{(1-\tau)-\lambda c_1}{1-\lambda}R)] d\theta \\
& + \int_0^{k^*(\tau, \bar{b}, c_1)} v(\tau - \bar{b}) d\theta \\
& + \int_{k^*(\tau, \bar{b}, c_1)}^1 v(\tau) d\theta.
\end{aligned} \tag{15}$$

The sum of the last two integrals is the wedge between the government's and the bank's objectives. The third integral is the utility from the public good consumption when a bank run occurs. Some tax revenue is used to provide bailouts. Hence, the public good provision is reduced to $(\tau - \bar{b})$. The last integral is for the case when there is no bank run and hence all the tax revenue is used to finance the public good.

Since the government considers the allocation of the resources between the public and private sectors, we focus on the case in which $\lim_{\tau \rightarrow 0} v'(\tau) = +\infty$ and $\lim_{\tau \rightarrow \hat{\tau}} v'(\tau) = 0$. Imposing a high level of taxation ($\tau > \hat{\tau}$) is costly since it distorts the resources allocation between the public and private sectors. Hence, the optimal tax chosen by the government will be in the range $(0, \hat{\tau})$. According to Proposition 2.2, in equilibrium we must have $c_1 > 1 - \tau + \bar{b}$.

I will first analyze the distortion of resources caused by the bailout: the bank takes

on more risk than the socially optimal level. The socially optimal c_1 is characterized by the first order condition of (15) with respect to c_1 :⁵⁰

$$\begin{aligned}
& k^* \frac{1-\tau+\bar{b}}{c_1} [u'(c_1) - u(c_1)/c_1] \\
& + (1 - k^*) \{ \lambda u'(c_1) - \lambda u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R) RE[p(\theta) | \theta > k^*] \} \\
& - [v(\tau) - v(\tau - \bar{b})] \times \partial k^* / \partial c_1 \\
& = 0.
\end{aligned} \tag{16}$$

The first and second terms are the misallocation effect and risk-sharing effect of c_1 , which have been analyzed in (10). The third term is the bailout effect of c_1 : higher c_1 increases the expected cost of the bailout since higher c_1 increases the run probability k^* (Corollary 2.1). The expected cost of the bailout is measured by the run probability multiplied by the loss of utility from the reduced level of the public good due to the bailout. Since the bank does not consider the public good provision, the third effect does not appear in (10). The missing third term implies that the bank does not “see” the complete social cost of a bank run, which also includes the cost of the bailout. Therefore, the bank offers a higher short-term payment c_1 and takes on more risk than is socially optimal.

Proposition 2.3 *Given the bailouts, the bank offers a higher short-term payment c_1 and takes on more risk than the socially optimal level. This distortion increases the ex-ante run probability.*

If the government cannot directly implement the deposit-rate controls, its best

⁵⁰Since $\lim_{\tau \rightarrow \bar{\tau}} v'(\tau) = 0$, we know that $\frac{1-\tau+\bar{b}}{c_1} < 1$ from Proposition 2.2.

strategy on choosing τ and \bar{b} is characterized by the first-order conditions for given c_1 . The first-order conditions of (15) with respect to \bar{b} is:

$$\begin{aligned}
& k^*[u(c_1)/c_1 - v'(\tau - \bar{b})] \\
& -\partial k^*/\partial \bar{b} \times [v(\tau) - v(\tau - \bar{b})] \\
& = 0.
\end{aligned} \tag{17}$$

The first term in (17) reflects the net benefit of the bailout when a bank run occurs: providing one more unit of resource to the bank can make $(1/c_1)$ more depositors get their money and crowd out 1 unit of the public good. This increases the expected utility of a depositor by $u(c_1)/c_1 - v'(\tau - \bar{b})$. The second term captures the destabilizing cost of the bailout: a larger bailout increases the run probability through increasing the depositor's incentive to run in the post-deposit game. The optimal bailout balance these two effects.

The first-order conditions of (15) with respect to τ is:

$$\begin{aligned}
& k^*[v'(\tau - \bar{b}) - u(c_1)/c_1] \\
& +(1 - k^*)[v'(\tau) - u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R)RE[p(\theta) | \theta > k^*]] \\
& -\partial k^*/\partial \tau \times [v(\tau) - v(\tau - \bar{b})] \\
& = 0.
\end{aligned} \tag{18}$$

The first term in (18) reflects the fact that if the bank run occurs, a higher tax-rate

gives the government more room to implement the bailout. The second term states that the higher tax, in normal times, distorts the resource allocation between the public good and private good. The last term states that higher tax increases the ex-ante run probability, thus increasing the expected cost of the bailout.

Definition 2.3 *An equilibrium of the pre-deposit game, is a Nash equilibrium in which the bank chooses c_1 by (10) and the government chooses τ and \bar{b} by (17) and (18).*

Proposition 2.4 states that the equilibrium level of bailouts must be strictly positive.

Proposition 2.4 *In equilibrium, the government chooses a strictly positive bailout level.*

The proof is in the appendix. The intuition for this proposition is that when the bailout level is low, the destabilizing effect is negligible. Hence, the government chooses zero bailout only if the utility loss due to the crowding out of the public good outweighs the utility gain from allowing more depositors to consume the private good. That is, $u(c_1)/c_1 < v'(\tau)$. Since the private good consumption is valued even less in normal times than the private good consumption when a bank run occurs, increasing τ and providing more of the public good can increase the depositor's welfare in both the normal times and when a bank occurs. Hence, in equilibrium we cannot have $u(c_1)/c_1 < v'(\tau)$, which implies that $\bar{b} > 0$ in equilibrium.

2.5 Bailouts without Government Commitment

The previous analysis assumes that the government can commit to \bar{b} which is announced *ex-ante*. A natural interpretation of that assumption is that \bar{b} is established by legislation or even a constitution. This section analyzes the case in which the government cannot make a prior commitment to its future bailout decision.

Without commitment, once the bank run occurs, the government will choose the *ex-post* efficient bailouts. If we use the superscript NC to denote the case of no commitment, we have that the government will choose \bar{b}^{NC} to equate the marginal utility from the public good with the marginal utility from private good consumption.

$$v'(\tau - \bar{b}^{NC}) = \frac{1}{c_1}u(c_1). \quad (19)$$

The reason is that once a bank run occurs, the government is no longer concerned with its effect on the run probability.⁵¹ Hence, the *ex-post* efficient level of the bailout is set to equalize the marginal utilities of the public good and the private good consumptions.

When the government cannot make a commitment to the bailout level, the structure of the post-deposit game remains the same as the one which is analyzed in Section 2.3. The reason is that when a depositor makes her withdrawal decision, she takes the level of the bailout as given. It does not matter whether that given bailout level is the *ex-ante* committed level or the *ex-post* efficient level. Hence, in the post-deposit game there is a unique threshold equilibrium. The next proposition is similar to Proposition 2.1.

Proposition 2.5 *When the government cannot make a commitment to the bailout level, there is a unique threshold equilibrium in the post-deposit game. In equilibrium, each patient depositor uses a switching strategy with the cutoff point k^{*NC} . k^{*NC} is an increasing function of τ and c_1 .*

In the pre-deposit game, the bank chooses c_1 to maximize (9), the welfare from private good consumption. Now the government has only one ex-ante choice variable,

⁵¹Since this is not a repeated game, the government does not take into account its reputation or the impact of bailout on the future actions of the bank.

τ . The ex-post bailout is a function of τ and c_1 given by (19). The bank's strategy is given by:

$$\begin{aligned}
& k^{*NC} \frac{1-\tau+\bar{b}^{NC}}{c_1} [u'(c_1) - u(c_1)/c_1] \\
& + (1 - k^{*NC}) \{ \lambda u'(c_1) - \lambda u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R) RE[p(\theta) | \theta > k^{*NC}] \} \\
& + k^{*NC} \times (\partial \bar{b}^{NC} / \partial c_1) \times u(c_1)/c_1 \\
& = 0,
\end{aligned} \tag{20}$$

where to simplify the notation, k^{*NC} and \bar{b}^{NC} are not written explicitly as functions.

The government's strategy is given by the first-order condition:

$$\begin{aligned}
& (1 - k^{*NC}) [v'(\tau) - u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R) RE[p(\theta) | \theta > k^{*NC}]] \\
& - \partial k^{*NC} / \partial \tau \times [v(\tau) - v(\tau - \bar{b}^{NC})] \\
& = 0.
\end{aligned} \tag{21}$$

Proposition 2.6 *When the government cannot commit to a bailout policy, in the equilibrium the depositor's expected utility is lower than in the commitment case. The ex-post bailout level and the run probability are higher than the commitment case.*

The proof is in the appendix. The intuition for this proposition is that when the government cannot make a commitment, the bailout cannot be chosen by the government ex ante. In period 0, there are many feasible bailout levels. The government

that is able to commit, chooses the best of these $b's$. Hence, welfare in the no commitment case is lower than in the commitment case. For given τ and c_1 , (17) implies that the ex-post bailout level is larger than the ex-ante efficient level. Hence, without commitment, the government provides a bigger bailout. The anticipated larger bailout increases the run probability both through changing the depositor's incentives to run in the post-deposit game and through increasing the deposit-rate of interest because of the distortion of resources due to bailouts.

2.6 Summary

In this chapter, I analyzed bailouts and bank runs in a modified DD model. Following Keister (2010), my model includes both a private good and a public good. The major innovation in this paper is to determine the run probability by using the global-games approach in Goldstein and Pauzner (2005), making the run probability endogenous. I show that bailouts increase the ex-ante run probability through two channels. The first channel works through the misaligned objectives of the bank and the government: Runs are less costly for banks when there are bailouts. Hence, banks take on more risk than is socially optimal. Regulation of the short-term deposit-rate can eliminate this distortion, but such regulation might not be so simple in shadow banking. At any rate, an analysis of the effects of short rate regulation though very important is beyond the scope of this paper. The second channel works through the change in the depositor's incentives to run: Bailouts increase the probability that a depositor will get her money if she participates in a run, thus increasing the likelihood of a run.

2.7 Appendix

Proof of Lemma 2.2

Proof. Let $a(k) = \Delta(k, k)$. $a(k)$ is the expected difference in the utilities of a patient depositor i between withdrawing in period 1 and withdrawing in period 2, when she receives a signal equal to the cutoff point used by other patient depositors.

$$a(k) = \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{k+\varepsilon} \delta(\theta, k) d\theta.$$

$a(k)$ is increasing in k . This is because when we increase k , we shift the interval of integration upwards. The distribution of n over the new interval of integration does not change (n is distributed uniformly over $[\lambda, 1]$). That is because the both the state of the fundamentals and the cutoff point increase by the same amount. But over the new interval of integration, we have higher $p(\theta)$. The higher probability of high asset return increases the expected utility of withdrawing in period 2. Hence, $a(k)$ is increasing in k . If $p(\theta)$ is strictly increasing over some part of the interval of integration, then $a(k)$ is strictly increasing.

If $\theta \geq \bar{\theta}$, the short-term liquidation value of the asset is R and $p(\theta) = 1$. Hence, $\delta(\theta, k) = u(\frac{(1-\tau)R - nc_1}{1-n}) - u(c_1)$. Since $c_1 \leq (1-\tau)R$ ⁵², we have that $\delta(\theta, k) > 0$ for $\theta \geq \bar{\theta}$ and $a(\bar{\theta} + \varepsilon) > 0$. Let $\underline{\theta}$ be defined by $p(\underline{\theta}) = u(c_1)/u(\frac{(1-\tau) - \lambda c_1}{1-\lambda} R)$. We can get $\delta(\theta, k) < 0$ for $\theta < \underline{\theta}$. Hence, $a(\underline{\theta} - \varepsilon) < 0$. From Lemma 2.1, we also know that $a(k)$ is continuous in k . Hence, by the Intermediate Value Theorem, we know that there is a unique k^* such that $\Delta(k^*, k^*) = 0$. ■

⁵²Consumption smoothing implies that c_1 should never be larger than $(1-\tau)R$ which is the maximum possible consumption of a patient depositor.

Proof of Lemma 2.3

Proof. Let $A_1(k^*)$ be the set of the states of the fundamentals at which the bank can satisfy all the withdrawal demand in period 1 by its own asset and no government bailout is granted. We have $A_1(k^*) = \{\theta | n(\theta, k^*)c_1 \leq (1 - \tau)\}$. Let $A_2(k^*)$ be the set of the states of the fundamentals at which the bank cannot satisfy all of the withdrawal demand in period 1 by itself. But with the bailout from the government, all the withdrawal demand can be satisfied. We have $A_2(k^*) = \{\theta | (1 - \tau) < n(\theta, k^*)c_1 \leq \bar{b} + (1 - \tau)\}$.

We know that $\delta(\theta, k^*) < 0$ for $\theta \in [A_1(k^*)]^C$ and $\delta(\theta, k^*)$ is strictly increasing in θ for $\theta \in A_1(k^*)$. Since $\Delta(k^*, k^*) = 0$, we must have that (1) $A_1(k^*) \neq \emptyset$. (2) $k^* + \varepsilon \in A_1(k^*)$. (3) $\delta(k^* + \varepsilon, k^*) > 0$. (4) $\delta(k^* - \varepsilon, k^*) < 0$.

(1) First, we prove that $\Delta(\theta_i, k^*) > 0$ for $\theta_i > k^*$.

When $k^* + 2\varepsilon > \theta_i > k^*$,

$$\begin{aligned} \Delta(\theta_i, k^*) &= \Delta(\theta_i, k^*) - \Delta(k^*, k^*) \\ &= \frac{1}{2\varepsilon} \int_{k^* + \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta - \frac{1}{2\varepsilon} \int_{k^* - \varepsilon}^{\theta_i - \varepsilon} \delta(\theta, k^*) d\theta. \end{aligned}$$

For $\theta \in [k^* + \varepsilon, \theta_i + \varepsilon]$, $n(\theta, k^*) = \lambda$. Hence, $[k^* + \varepsilon, \theta_i + \varepsilon] \subset A_1(k^*)$. Furthermore, since $\delta(\theta, k^*)$ is strictly increasing in θ when $\theta \in A_1(k^*)$ and $\delta(k^* + \varepsilon, k^*) > 0$, we have that $\frac{1}{2\varepsilon} \int_{k^* + \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta > 0$. For $\theta \in [k^* - \varepsilon, \theta_i - \varepsilon]$, $\delta(\theta, k^*)$ is either negative (when $\theta \in [A_1(k^*)]^C$) or lower than $\delta(k^* + \varepsilon, k^*)$ (when $\theta \in A_1(k^*)$). So we must have $\Delta(\theta_i, k^*) = \frac{1}{2\varepsilon} \int_{k^* + \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta - \frac{1}{2\varepsilon} \int_{k^* - \varepsilon}^{\theta_i - \varepsilon} \delta(\theta, k^*) d\theta > 0$.

When $\theta_i \geq k^* + 2\varepsilon$, we have that $n(\theta, k^*) = \lambda$ for $\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. Hence, $[\theta_i - \varepsilon, \theta_i + \varepsilon] \subset A_1(k^*)$. Since $\delta(\theta, k^*)$ is strictly increasing in θ for $\theta \in A_1(k^*)$, we have $\delta(\theta, k^*) > \delta(k^* + \varepsilon; k^*) > 0$. Hence, $\Delta(\theta_i, k) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta > 0$.

(2) Second, we prove that $\Delta(\theta_i, k^*) < 0$ for $\theta_i < k^*$

When $k^* > \theta_i > k^* - 2\varepsilon$,

$$\begin{aligned} & \Delta(\theta_i, k^*) \\ &= \Delta(\theta_i, k^*) - \Delta(k^*, k^*) \\ &= \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{k^* - \varepsilon} \delta(\theta, k^*) d\theta - \frac{1}{2\varepsilon} \int_{\theta_i + \varepsilon}^{k^* + \varepsilon} \delta(\theta, k^*) d\theta. \end{aligned}$$

For $\theta \in [\theta_i - \varepsilon, k^* - \varepsilon]$, $\delta(\theta, k^*) < 0$. This is because, if $\theta \in [A_1(k^*)]^C$, we have that $\delta(\theta, k^*) < 0$. If $\theta \in A_1(k^*)$, $\delta(\theta, k^*) < \delta(k^* - \varepsilon; k^*) < 0$. Thus, we have $\frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{k^* - \varepsilon} \delta(\theta, k^*) d\theta < 0$. Furthermore, we must have that $\frac{1}{2\varepsilon} \int_{\theta_i + \varepsilon}^{k^* + \varepsilon} \delta(\theta, k^*) d\theta \geq 0$. This can be proved by contradiction. If $\frac{1}{2\varepsilon} \int_{\theta_i + \varepsilon}^{k^* + \varepsilon} \delta(\theta, k^*) d\theta < 0$, $\Delta(k^*, k^*) = 0$ requires that $\frac{1}{2\varepsilon} \int_{k^* - \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta > 0$, which means that $\theta_i + \varepsilon \in A_1(k^*)$ and $\delta(\theta_i + \varepsilon, k^*) > 0$. This implies that $[\theta_i + \varepsilon, k^* + \varepsilon] \subset A_1(k^*)$ and $\delta(\theta, k^*) > \delta(\theta_i + \varepsilon, k^*) > 0$ for $\theta \in [\theta_i + \varepsilon, k^* + \varepsilon]$, which is a contradiction to $\frac{1}{2\varepsilon} \int_{\theta_i + \varepsilon}^{k^* + \varepsilon} \delta(\theta, k^*) d\theta < 0$.

When $\theta_i \leq k^* - 2\varepsilon$, we have that $\delta(\theta, k^*) < 0$ for $\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. This is because, If $\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [A_1(k^*)]^C$, we have that $\delta(\theta, k^*) < 0$. If $\theta \in [\theta_i - \varepsilon, \theta_i + \varepsilon] \cap A_1(k^*)$ we have that $\delta(\theta, k^*) < \delta(k^* - \varepsilon, k^*) < 0$. Thus, $\Delta(\theta_i, k^*) = \frac{1}{2\varepsilon} \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} \delta(\theta, k^*) d\theta < 0$. ■

Proof of Corollary 2.1

Proof. We know the relation between θ and n from (8). It is convenient to write $\Delta(k^*, k^*)$ as an integration over n rather than θ . If $1 - \tau + \bar{b} \geq c_1$, $\Delta(k^*, k^*)$ is:

$$\int_{\lambda}^1 [p(\theta(k^*, n))u(\frac{1 - \tau - nc_1}{1 - n}R) - u(c_1)]dn$$

\bar{b} does not appear in the expression. Hence, \bar{b} does not affect k^* .

If $1 - \tau + \bar{b} < c_1$, $\Delta(k^*, k^*)$ is equal to $f(k^*; \tau, c_1, \bar{b})$:

$$\begin{aligned} f(k^*; \tau, c_1, \bar{b}) &= \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R) - u(c_1)]dn \\ &\quad - \int_{\frac{1-\tau}{c_1}}^{\frac{1-\tau+\bar{b}}{c_1}} u(c_1)dn - \int_{\frac{1-\tau+\bar{b}}{c_1}}^1 \frac{1-\tau+\bar{b}}{nc_1}u(c_1)dn \end{aligned}$$

Since $\frac{\partial f}{\partial k^*} > 0$, to prove that $\frac{\partial k^*}{\partial \bar{b}} > 0$, we only need to prove $\frac{\partial f}{\partial \bar{b}} < 0$.

$$\frac{\partial f}{\partial \bar{b}} = [-\frac{1}{c_1} + \frac{1}{c_1} \ln \frac{1 - \tau + \bar{b}}{c_1} + \frac{1}{c_1}]u(c_1) < 0$$

if $\frac{1-\tau+\bar{b}}{c_1} < 1$. ■

Proof of Corollary 2.2

Proof. We prove the case of $1 - \tau + \bar{b} < c_1$. The case of $1 - \tau + \bar{b} \geq c_1$ can be proved in a similar way, which is omitted here. $\Delta(k^*, k^*)$ can be written as $f(k^*; \tau, c_1, \bar{b})$:

$$\begin{aligned} f(k^*; \tau, c_1, \bar{b}) &= \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R) - u(c_1)]dn \\ &\quad - \int_{\frac{1-\tau}{c_1}}^{\frac{1-\tau+\bar{b}}{c_1}} u(c_1)dn - \int_{\frac{1-\tau+\bar{b}}{c_1}}^1 \frac{1-\tau+\bar{b}}{nc_1}u(c_1)dn \end{aligned}$$

Since $\frac{\partial f}{\partial k^*} > 0$, we only have to prove $\frac{\partial f}{\partial c_1} < 0$ and $\frac{\partial f}{\partial \tau} < 0$.

(1) Proof of $\frac{\partial f}{\partial c_1} < 0$:

Let $\widehat{f}(k^*, \tau, c_1, \bar{b}) = c_1 f(k^*, \tau, c_1, \bar{b})$. Since $f(k^*, \tau, c_1, \bar{b}) = 0$, $\frac{\partial f}{\partial c_1} < 0$ is equivalent to $\frac{\partial \widehat{f}}{\partial c_1} < 0$.

$$\begin{aligned} \frac{\partial \widehat{f}}{\partial c_1} &= \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\ &\quad + c_1 \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u'(\frac{1-\tau-nc_1}{1-n}R)(-\frac{n}{1-n})R]dn \\ &\quad - [-\lambda + \frac{1-\tau+\bar{b}}{c_1}]u(c_1) \\ &\quad - [1 - \tau + \bar{b} - \lambda c_1 - (1 - \tau + \bar{b}) \ln \frac{1-\tau+\bar{b}}{c_1}]u'(c_1) \end{aligned}$$

Since the last two terms are negative, it is sufficient to prove that the sum of the first two terms is negative.

$$\begin{aligned}
& \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& +c_1 \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u'(\frac{1-\tau-nc_1}{1-n}R)(-\frac{n}{1-n})R]dn \\
& = \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& +c_1 \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))\frac{\partial u(\frac{1-\tau-nc_1}{1-n}R)}{\partial n}\frac{(1-n)n}{c_1+\tau-1}]dn \\
& = \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& -c_1 p(\theta(k^*, \lambda))u(\frac{1-\tau-\lambda c_1}{1-\lambda}R)\frac{(1-\lambda)\lambda}{c_1+\tau-1} \\
& -c_1 \int_{\lambda}^{\frac{1-\tau}{c_1}} u(\frac{1-\tau-nc_1}{1-n}R)[p'(\theta(k^*, n))\frac{-2\varepsilon}{1-\lambda}\frac{(1-n)n}{c_1+\tau-1}] \\
& +p(\theta(k^*, n))\frac{1-2n}{c_1+\tau-1}]dn
\end{aligned}$$

$$\begin{aligned}
&= -c_1 p(\theta(k^*, \lambda)) u\left(\frac{1-\tau-\lambda c_1}{1-\lambda} R\right) \frac{(1-\lambda)\lambda}{c_1+\tau-1} \\
&\quad - \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n)) u\left(\frac{1-\tau-nc_1}{1-n} R\right) \frac{1-\tau-2nc_1}{c_1+\tau-1}] dn \\
&\quad + 2\varepsilon \int_{\lambda}^{\frac{1-\tau}{c_1}} u\left(\frac{1-\tau-nc_1}{1-n} R\right) [p'(\theta(k^*, n)) \frac{c_1}{1-\lambda} \frac{(1-n)n}{c_1+\tau-1}] dn \\
&= -c_1 p(\theta(k^*, \lambda)) u\left(\frac{1-\tau-\lambda c_1}{1-\lambda} R\right) \frac{(1-\lambda)\lambda}{c_1+\tau-1} \\
&\quad - p(\theta(k^*, \lambda)) \int_{\lambda}^{\frac{1-\tau}{c_1}} [u\left(\frac{1-\tau-nc_1}{1-n} R\right) \frac{1-\tau-2nc_1}{c_1+\tau-1}] dn \\
&\quad + 2\varepsilon \int_{\lambda}^{\frac{1-\tau}{c_1}} u\left(\frac{1-\tau-nc_1}{1-n} R\right) [p'(\theta(k^*, n)) \frac{c_1}{1-\lambda} \frac{(1-n)n}{c_1+\tau-1} \\
&\quad + \int_{\lambda}^n p'(\theta(k^*, x)) dx \frac{1-\tau-2nc_1}{c_1+\tau-1}] dn
\end{aligned}$$

The upper bound of the first two term is $-\lambda p(\theta(k^*, \lambda)) u\left(\frac{1-\tau-\lambda c_1}{1-\lambda} R\right)$. Hence, as $\varepsilon \rightarrow 0$, $\frac{\partial f}{\partial c_1} < 0$.

(2) Proof of $\frac{\partial f}{\partial \tau} < 0$:

Let $\tilde{f} = \frac{f}{1-\tau+\bar{b}}$. $\frac{\partial \tilde{f}}{\partial \tau} = \frac{\frac{\partial f}{\partial \tau}(1-\tau+\bar{b})+f}{(1-\tau+\bar{b})^2}$. Since $f(k^*, \tau, c_1, \bar{b}) = 0$, to prove $\frac{\partial f}{\partial \tau} < 0$, it is sufficient to prove that $\frac{\partial \tilde{f}}{\partial \tau} < 0$.

$$\begin{aligned}
&(1-\tau+\bar{b})^2 \frac{\partial \tilde{f}}{\partial \tau} \\
&= \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n)) u\left(\frac{1-\tau-nc_1}{1-n} R\right)] dn \\
&\quad + (1-\tau+\bar{b}) \left[\int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n)) u'\left(\frac{1-\tau-nc_1}{1-n} R\right) \frac{-1}{1-n} R] dn \right. \\
&\quad \left. + \left[\frac{\lambda}{1-\tau+\bar{b}} - \frac{1}{c_1} \right] \frac{u(c_1)}{1-\tau+\bar{b}} \right]
\end{aligned}$$

Since the last term is negative, it is sufficient to prove that the sum of the first two terms is negative.

$$\begin{aligned}
& \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& + (1-\tau+\bar{b})[\int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u'(\frac{1-\tau-nc_1}{1-n}R)\frac{-1}{1-n}R]dn \\
& = \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& + (1-\tau+\bar{b})[\int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))\frac{\partial u(\frac{1-\tau-nc_1}{1-n}R)}{\partial n}\frac{(1-n)}{c_1+\tau-1}]dn \\
& = \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)]dn \\
& - (1-\tau+\bar{b})p(\theta(k^*, \lambda))u(\frac{1-\tau-\lambda c_1}{1-\lambda}R)\frac{(1-\lambda)}{c_1+\tau-1} \\
& - (1-\tau+\bar{b})\int_{\lambda}^{\frac{1-\tau}{c_1}} u(\frac{1-\tau-nc_1}{1-n}R)[p'(\theta(k^*, n))\frac{-2\varepsilon}{1-\lambda}\frac{(1-n)}{c_1+\tau-1}] \\
& - p(\theta(k^*, n))\frac{1}{c_1+\tau-1}]dn \\
& = \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R)\frac{c_1+\bar{b}}{\tau+c_1-1}]dn \\
& - (1-\tau+\bar{b})p(\theta(k^*, \lambda))u(\frac{1-\tau-\lambda c_1}{1-\lambda}R)\frac{(1-\lambda)}{c_1+\tau-1} \\
& + (1-\tau+\bar{b})\int_{\lambda}^{\frac{1-\tau}{c_1}} u(\frac{1-\tau-nc_1}{1-n}R)p'(\theta(k^*, n))\frac{2\varepsilon}{1-\lambda}\frac{(1-n)}{c_1+\tau-1}dn
\end{aligned}$$

The first term is strictly lower than

$$\left(\frac{1-\tau}{c_1} - \lambda\right)p(\theta(k^*, \lambda))u\left(\frac{1-\tau-\lambda c_1}{1-\lambda}R\right)\frac{c_1+\bar{b}}{\tau+c_1-1}$$

Since $(c_1 + \bar{b})(\frac{1-\tau}{c_1} - \lambda) < (1 - \tau + \bar{b})(1 - \lambda)$, we have that the sum of the first two terms is negative. As $\varepsilon \rightarrow 0$, the third term $\rightarrow 0$. Hence, as $\varepsilon \rightarrow 0$, $\frac{\partial f}{\partial \tau} < 0$. ■

Proof of Proposition 2.2

Proof. We prove the general case when ε is small but not equal to zero. Let us denote the bank's objective function as $EU_B(c_1)$:

$$\begin{aligned}
 & EU_B(c_1) \\
 &= \int_0^{\tilde{\theta}} \left[\frac{1-\tau+\bar{b}}{c_1} u(c_1) \right] d\theta \\
 &+ \int_{\tilde{\theta}}^{\bar{\theta}} \left[\frac{1-\tau+\bar{b}}{c_1} u(c_1) \right] d\theta \\
 &+ \int_{\bar{\theta}}^{k^*+\varepsilon} \left[nu(c_1) + (1-n)p(\theta)u\left[\frac{1-\tau-nc_1}{1-n}R\right] \right] d\theta \\
 &+ \int_{k^*+\varepsilon}^{\bar{\theta}} \left[\lambda u(c_1) + (1-\lambda)p(\theta)u\left[\frac{1-\tau-\lambda c_1}{1-\lambda}R\right] \right] d\theta \\
 &+ \int_{\bar{\theta}}^1 \left[\lambda u(c_1) + (1-\lambda)u\left[\frac{(1-\tau)R-\lambda c_1}{1-\lambda}\right] \right] d\theta
 \end{aligned}$$

where $\tilde{\theta}$ is the state of the fundamentals at which the total withdrawal demand equals the liquidation value of the bank in period 1. That is, $n(\tilde{\theta}, k^*)c_1 = 1 - \tau$. $\bar{\theta}$ is the state of the fundamentals at which the total withdrawal demand equals the sum of the liquidation value of the bank in period 1 and bailouts. That is $n(\bar{\theta}, k^*)c_1 = 1 - \tau + \bar{b}$.

$$\partial EU_B / \partial c_1$$

$$\begin{aligned}
&= \int_0^{\tilde{\theta}} \frac{1-\tau+\bar{b}}{c_1} [u'(c_1) - u(c_1)/c_1] d\theta \\
&+ \int_{\tilde{\theta}}^{\bar{\theta}} \frac{1-\tau+b}{c_1} [u'(c_1) - u(c_1)/c_1] d\theta \\
&+ \int_{\tilde{\theta}}^{k^*+\varepsilon} [nu'(c_1) + (-n)p(\theta)u'[\frac{1-\tau-nc_1}{1-n}R]R] d\theta \\
&+ \frac{1-\lambda}{2\varepsilon} \frac{\partial k^*}{\partial c_1} \int_{\tilde{\theta}}^{k^*+\varepsilon} [u(c_1) - p(\theta)u[\frac{1-\tau-nc_1}{1-n}R]] \\
&- \frac{-1+\tau+c_1}{(1-n)} Rp(\theta)u'[\frac{1-\tau-nc_1}{1-n}R]] d\theta \\
&+ \int_{k^*+\varepsilon}^{\bar{\theta}} [\lambda u'(c_1) + (-\lambda)p(\theta)u'[\frac{1-\tau-\lambda c_1}{1-\lambda}R]R] d\theta \\
&+ \int_{\tilde{\theta}}^1 [\lambda u'(c_1) + (-\lambda)u'[\frac{(1-\tau)R-\lambda c_1}{1-\lambda}]] d\theta
\end{aligned}$$

By the definition of k^* , when $c_1 = 1 - \tau + \bar{b}$ we have $\tilde{\theta} = k^* - \varepsilon$. Hence,

$$\begin{aligned}
& \partial EU_B / \partial c_1 \Big|_{c_1=1-\tau+\bar{b}} \\
&= \int_0^{k^*-\varepsilon} [u'(c_1) - u(c_1)/c_1] d\theta \\
&+ \int_{k^*-\varepsilon}^{k^*+\varepsilon} [nu'(c_1) + (-n)p(\theta)Ru'(c_1R)] d\theta \\
&+ \frac{1-\lambda}{2\varepsilon} \frac{\partial k^*}{\partial c_1} \int_{k^*-\varepsilon}^{k^*+\varepsilon} [u(c_1) - p(\theta)u(c_1R)] d\theta \\
&+ \int_{k^*+\varepsilon}^{\bar{\theta}} [\lambda u'(c_1) + (-\lambda)p(\theta)Ru'(c_1R)] d\theta \\
&+ \int_{\bar{\theta}}^1 [\lambda u'(c_1) + (-\lambda)u'(\frac{R-\lambda}{1-\lambda}c_1)] d\theta
\end{aligned}$$

When $\tau \leq \hat{\tau}$, the second and fourth terms are positive since

$$c_1 = 1 - \tau + \bar{b} < 1$$

and

$$1 < \hat{c}_1(\hat{\tau}) \leq \hat{c}_1(\tau).$$

The third term is zero by the definition of k^* . The last term is positive since $\frac{R-\lambda}{1-\lambda} > 1$.

As $\varepsilon \rightarrow 0$, $p(k^*) \rightarrow u(c_1)/u(\frac{(1-\tau)-\lambda c_1}{1-\lambda}R) < u(1)/u(\frac{(1-\tau)-\lambda}{1-\lambda}R)$. The last inequality comes from the $c_1 = 1 - \tau + \bar{b} < 1$. Hence,

$$k^* < \hat{\theta}.$$

If $\widehat{\theta}$ is sufficiently small, the first term will be small and we have

$$\left. \frac{\partial EU_B}{\partial c_1} \right|_{c_1 = 1 - \tau + \bar{b}} > 0.$$

So the bank chooses a short-term payment c_1 which is strictly larger than the sum of the bailouts and the liquidation value of the bank's asset ($1 - \tau + \bar{b}$). ■

Proof of Proposition 2.4

Proof. If $\bar{b} = 0$, (17) implies that we must have $u(c_1)/c_1 < v'(\tau)$. By the concavity of u and $u(0) = 0$, we know $u'(c_1) < u(c_1)/c_1$. From (10), we know that the deposit-rate offered by the bank will satisfy $u'(c_1) \geq \lambda u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R) RE[p(\theta) | \theta > k^*]$. Hence, we must have

$$\lambda u'(\frac{(1-\tau)-\lambda c_1}{1-\lambda} R) RE[p(\theta) | \theta > k^*] \leq u'(c_1) < v'(\tau), \quad (22)$$

which implies that (18) is negative. Thus, the government will choose zero bailout only when $\tau = 0$. But since $\lim_{\tau \rightarrow 0} v'(\tau) = +\infty$, $\tau = 0$ cannot be equilibrium. ■

Proof of Proposition 2.6

Proof. I use τ^{*NC} and c_1^{*NC} to denote the equilibrium choices of the government and the bank when there is no commitment to the bailout level. The *ex-post* bailout satisfies

$$v'(\tau^{*NC} - \bar{b}^{NC}) = \frac{1}{c_1^{*NC}} u(c_1^{*NC}).$$

Suppose the government and the bank still choose τ^{*NC} and c_1^{*NC} , but the government can commit ex-ante to a certain bailout level \bar{b}^C . According to (17), we have $\bar{b}^{NC} > \bar{b}^C$. In the post-deposit game, the run probability k^* is determined by

$$\begin{aligned} & \int_{\lambda}^{\frac{1-\tau}{c_1}} [p(\theta(k^*, n))u(\frac{1-\tau-nc_1}{1-n}R) - u(c_1)]dn \\ & - \int_{\frac{1-\tau}{c_1}}^{\frac{1-\tau+\bar{b}}{c_1}} u(c_1)dn - \int_{\frac{1-\tau+\bar{b}}{c_1}}^1 \frac{1-\tau+\bar{b}}{nc_1} u(c_1)dn = 0 \end{aligned}$$

Since $\bar{b}^{NC} > \bar{b}^C$, we have $k^{*NC} > k^{*C}$. That is, the ex-ante run probability is lower in the commitment case than the no commitment case if the government and the bank choose the same tax and short-term payment. Since the depositor's welfare is given by

$$\begin{aligned} & \int_0^{k^*} \frac{1-\tau+\bar{b}}{c_1} u(c_1)d\theta + \int_{k^*}^1 [\lambda u(c_1) + (1-\lambda)p(\theta)u(\frac{(1-\tau)-\lambda c_1}{1-\lambda}R)]d\theta \\ & + \int_0^{k^*} v(\tau - \bar{b})d\theta + \int_{k^*}^1 v(\tau)d\theta. \end{aligned}$$

Higher run probability implies lower expected welfare. Hence, commitment can strictly improve the ex-ante welfare even if the government chooses the same strategy. In the commitment case, since the committed bailout level is strictly lower than the no commitment case, the bank chooses a lower short-term deposit-rate according to (10).

This will make the equilibrium bailout level and the run probability even lower in the commitment case. ■

2.8 References

- [1] Acharya, Viral V. and Tanju Yorulmazer. 2007. “Too many to fail—An analysis of time-inconsistency in bank closure policies.” *Journal of Financial Intermediation* 16: 1–31.
- [2] Bryant, John. 1980. “A Model of Reserves, Bank Runs, and Deposit Insurance.” *Journal of Banking and Finance* 4: 335–344.
- [3] Bernanke, Ben S. 2010. Statement by Ben S. Bernanke, Chairman, Board of Governors of the Federal Reserve System, before the Financial Crisis Inquiry Commission. September 2, Washington D.C.
- [4] Caprio, Gerard, Daniela Klingebiel, Luc Laeven, and Guillermo Noguera. 2005. “Appendix:Banking Crisis Database.” in Patrick Honohan and Luc Laeven (eds.), *Systemic Financial Crises: Containment and Resolution*. Cambridge, U.K.: Cambridge University Press.
- [5] Caprio, Gerard, and Lawrence Summers. 1993. “Finance and its Reform: Beyond Laissez-Faire.” *Policy Research Working Paper 1171*.
- [6] Carlsson, Hans, and Eric van Damme. 1993. “Global games and equilibrium selection.” *Econometrica* 61: 989-1018.
- [7] Chari, Varadarajan V., and Patrick J. Kehoe. 2010. “Bailouts, Time-Inconsistency, and Optimal Regulation.” *Mimeo Minneapolis Fed and Princeton*.
- [8] Demirguc-Kunt, Asli, and Enrica Detragiache. 1998. “The determinants of banking crises: Evidence from developed and developing countries.” *IMF Staff Papers* 45: 81-109.
- [9] Demirguc-Kunt, Asli, Enrica Detragiache, and Poonam Gupta. 2000. “Inside the crisis: An empirical analysis of banking systems in distress.” *World Bank Working paper 2431*.
- [10] Diamond, Douglas W., and Philip H. Dybvig. 1983. “Bank Runs, Deposit

Insurance, and Liquidity.” *Journal of Political Economy* 91: 401-19.

[11] Ennis, Huberto and Todd Keister. 2009. “Bank Runs and Institutions: The Perils of Intervention”, *American Economic Review* 99: 1588-1607.

[12] Farhi, Emmanuel and Jean Tirole. 2009. “Collective moral hazard, maturity mismatch and systemic bailouts.” *NBER Working Paper 15138*.

[13] Geithner, Timothy F. 2008. “Reducing Systemic Risk in a Dynamic Financial System.” *Remarks at The Economic Club of New York, New York City*.

[14] Goldstein, Itay and Ady Pauzner. 2005. “Demand-Deposit Contracts and the Probability of Bank Runs.” *The Journal of Finance* 60: 1293-1327.

[15] Gorton, Gary. 2009. “Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007.” *Federal Reserve Bank of Atlanta’s 2009 Financial Markets Conference: Financial Innovation and Crisis*.

[16] Green, Edward J. 2010. “Bailouts” *Federal Reserve Bank of Richmond Economic Quarterly* 96:11-32.

[17] Hellmann, Thomas F., Kevin C. Murdock and Joseph E. Stiglitz. 2000. “Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?” *American Economic Review* 90: 147–165.

[18] Judd, Kenneth L. 1985. “The Law of Large Numbers With A Continuum of i.i.d. Random Variables.” *Journal of Economic Theory* 35: 19-25.

[19] Keister, Todd. 2010. “Bailouts and Financial Fragility.” *Federal Reserve Bank of New York Staff Reports* 473.

[20] Laeven, Luc and Fabian Valencia. 2008. “Systemic Banking Crises: A New Database.” *IMF Working Paper* WP/08/224.

[21] Martinez-Peria, Maria S., and Sergio L. Schmukler. 2001. “Do Depositors Punish Banks For Bad Behavior? Market Discipline, Deposit Insurance, and Banking Crises.” *Journal of Finance* 56: 1029-1051.

- [22] Morris, Stephen, and Hyun S. Shin. 1998. “Unique equilibrium in a model of self-fulfilling currency attacks.” *American Economic Review* 88: 587-597.
- [23] Morris, Stephen, and Hyun S. Shin. 2001. “Global Games: Theory and Applications.” *Cowles Foundation Discussion Paper NO. 1275R*.
- [24] Peck, James and Karl Shell. 2003. “Equilibrium Bank Runs.” *Journal of Political Economy* 111: 103-123.
- [25] Scott, Kenneth, George P. Shultz and John B. Taylor eds. 2010. “Ending Government Bailouts as We Know Them.” *Hoover Press, Stanford, California*.
- [26] Suzuki, Yoshio. 1987. “The Japanese Financial System.” Oxford: Clarendon.
- [27] Wallace, Neil. 1988. “Another Attempt to Explain an Illiquid Banking System: the Diamond and Dybvig Model with Sequential Service Taken Seriously.” *Quarterly Review Federal Reserve Bank of Minneapolis*, issue Fall, pages 3-16.
- [28] Zhang, Yu. 2010. “A Note on ‘Equilibrium Bank Runs’: Robustness of the Two-Consumer Example” Working Paper.

3 Chapter 3: A Note on “Equilibrium Bank Runs”: Robustness of the Two-Consumer Example

3.1 Introduction

Diamond and Dybvig (1983) showed that a bank run can be an equilibrium to the contract which supports the constrained-efficient allocation in the post-deposit game.⁵³ The important question to ask is whether or not consumers will want to deposit in the bank if they expect bank runs occur. Peck and Shell (2003) answered this question by analyzing the optimal contract in the full pre-deposit game in which runs are triggered by sunspots.⁵⁴ In Peck and Shell’s 2-consumer example, the contract will be characterized by c which is the consumption received by the first depositor in line in period 1.⁵⁵ Let $c^*(s)$ be the optimal contract offered by the bank, where s is the probability of a bank run. In the numerical example of Peck and Shell, $c^*(s)$ is a step function: If the probability s is less than a critical level s_0 , the contract $c^*(s)$ tolerates runs and $c^*(s) = c^*(0)$. If the probability s is more than s_0 , the optimal contract will be immune from runs and $c^*(s) = c^{no-run}$, the level at which rational consumers will never run. The probability of a bank run s has only a “bang-bang” effect on the optimal contract $c^*(s)$.

One might think that if tolerating runs is still optimal, as the probability of a bank run increases the optimal contract should give less to the first depositor in the line and reserve more for the second in line since a bank run is more likely.⁵⁶ That is, one might

⁵³To be more precise, it is the constrained-efficient allocation in the post-deposit game when pre-supposing no runs.

⁵⁴The sunspot variable, δ , is distributed uniformly on the interval $[0, 1]$. For $\delta < s$, the run-equilibrium in the post-deposit game is chosen. For $\delta \geq s$, the non-run-equilibrium in the post-deposit game is chosen.

⁵⁵We use the same notation as Peck and Shell (2003).

⁵⁶Dividing the resource equally is optimal when a bank run occurs.

have conjectured that $c^*(s)$ should be a decreasing function of s when $s \leq s_0$. The reason why the optimal contract is not always like this is that $c^*(s)$ must satisfy the incentive-compatibility constraint which induces a patient consumer to choose period 2 when other patient consumers do so. This constraint does not change with s . For small s , that incentive-compatibility constraint may bind so $c^*(s)$ is independent of s . The numerical example in Peck and Shell (2003) has this property.

The present note provides a generalization of the Peck and Shell's example. I derive conditions on the parameters that are necessary for equilibrium bank runs. For the parameters that permit tolerating bank runs, I derive the range in which we have the Peck and Shell's step-function result and the range in which the optimal contract tolerating runs changes continuously in the run probability until it reaches the threshold s_0 , at which the optimal contract switches to the best run-proof contract. The implication of the latter case is that the run probability has greater influence on the deposit contract and resource allocation: it affects not only *whether* bank runs will be tolerated (like Peck and Shell's example) but also *how* bank runs will be tolerated.

The next section analyzes the necessary conditions on the parameters to allow for tolerating bank runs. Section 3.3 analyzes the range of these parameters for which $c^*(s)$ is a decreasing function of s for small s . Section 3.4 is for the concluding remarks.

3.2 Necessary Conditions For Equilibrium Bank Runs

We first review the notation in the example of Peck and Shell (2003). The utility function of the patient and impatient consumers are respectively

$$v(x) = \frac{x^{1-b}}{1-b} \tag{23}$$

and

$$u(x) = A \frac{x^{1-a}}{1-a}, \text{ where } A \geq 1. \quad (24)$$

A reflects the importance of “impulse demand” by the impatient consumers. We will discuss this parameter in detail in the next section. $a > 1$ and $b > 1$ are the coefficients of risk aversion of the impatient and patient consumers respectively. Each consumer is impatient with probability p and patient with probability $(1 - p)$. Types are uncorrelated and private information. Investing one unit of period 0 consumption yields $R > 1$ units if harvested in period 2 and yields one unit if harvested in period 1. c is the consumption received by the first depositor in line in period 1.

We next define “run equilibrium” in the post-deposit game.

Definition 3.1 (*Peck-Shell (2003) Definition 1*) *Given a mechanism $m \in M$,⁵⁷ the post-deposit game is said to have a run equilibrium if there is a Bayes-Nash equilibrium in which all consumers choose to withdraw in period 1, independent of the realization of their types.*

In the 2-consumer example, m is characterized by c . In the post-deposit game, the impatient consumer always chooses to withdraw in period 1 since she values only the period-1 consumption. For the patient consumer, given the expectation that the other consumer will withdraw in period 1, her expected utility from withdrawing early is $[v(c) + v(2y - c)]/2$. This is because she may be the first or the second in the arrival position. The probability of each case is $1/2$. If she waits until period 2, she will get $v((2y - c)R)$. The run equilibrium exists in the post-deposit game if and only if the patient consumer strictly prefers to withdraw in period 1. That is, if and only if we

⁵⁷ M is the set of the mechanisms which satisfy the resource constraint. The resource constraint is equation (2) in Peck and Shell (2003).

have

$$[v(c) + v(2y - c)]/2 - v((2y - c)R) > 0. \quad (25)$$

Withdrawing early is “risky” for the patient consumer: If she is the last in the queue, she can only get $(2y - c)$, which is less than what she can get if she waits, $(2y - c)R$. Given $v(c) = (c^{1-b} - 1)/(1 - b)$, the patient consumer will never take the risk if she is sufficiently risk averse.

Lemma 3.1 *If $b \geq 1 + \ln 2 / \ln R$, there is no run equilibrium in the post-deposit game.*

Proof.

$$\begin{aligned} & [v(c) + v(2y - c)]/2 - v((2y - c)R) \\ = & \frac{[c^{1-b} + (2y - c)^{1-b}]/2 - [(2y - c)R]^{1-b}}{(1 - b)} \\ = & \frac{-(c^{1-b})/2 + (2y - c)^{1-b}(R^{1-b} - 1/2)}{(b - 1)} \end{aligned}$$

If $(R^{1-b} - 1/2) \leq 0$, then we always have $[v(c) + v(2y - c)]/2 - v((2y - c)R) \leq 0$, which implies there is no run equilibrium in the post-deposit game. The inequality $b \geq 1 + \ln 2 / \ln R$ is equivalent to the inequality $(R^{1-b} - 1/2) \leq 0$. ■

Lemma 3.1 implies that a necessary condition for the existence bank run in the post-deposit game is

$$b < 1 + \ln 2 / \ln R. \quad (26)$$

If (26) is satisfied, it can be seen that (25) is equivalent to

$$c > c^{no-run} = 2y/[(2/R^{b-1} - 1)^{1/(b-1)} + 1]. \quad (27)$$

Of course, the more important equilibrium concept is equilibrium in the pre-deposit game.

Definition 3.2 (*Peck-Shell (2003) Definition 2*) *Given a mechanism $m \in M$, the pre-deposit game is said to have a run equilibrium if there is a subgame-perfect Nash equilibrium in which (i) consumers are willing to deposit, and (ii) for some set of realizations of δ occurring with positive probability, all consumers choose to withdraw in period 1, independent of the realization of their type.*

The pre-deposit game has a run equilibrium only if there is a non-run equilibrium in the post-deposit game. Otherwise, no consumer would deposit in the bank. The non-run equilibrium in the post-deposit game is defined as the following.

Definition 3.3 *Given a mechanism $m \in M$, the post-deposit game is said to have a non-run equilibrium if there is a Bayes-Nash equilibrium in which only the impatient depositors choose to withdraw in period 1.*

The impatient consumer always chooses to withdraw in period 1. For the patient consumer, given the expectation that only the impatient consumer will withdraw in period 1, her expected utility from withdrawal in period 1 is $p[v(c) + v(2y - c)]/2 + (1 - p)v(c)$. If she waits until period 2, she will get $pv((2y - c)R) + (1 - p)v(yR)$. The non-run equilibrium exists in the post deposit game if and only if the patient consumer prefers to withdraw in period 1. That is, the following incentive-compatibility constraint is satisfied.

$$pv((2y - c)R) + (1 - p)v(yR) - p[v(c) + v(2y - c)]/2 - (1 - p)v(c) \geq 0. \quad (28)$$

The LHS of (28) is a continuous function of c . From (26), it is also decreasing in c . When $c = 0$, the LHS of (28) is equal to $+\infty > 0$. When $c = 2y$, the LHS of (28) is equal to $-\infty < 0$. Hence there is a unique c , $0 < c < 2y$, such that inequality (28) holds with equality. Denote that c by c^{IC} . So the non-run equilibrium in the post-deposit game exists if and only if we have

$$c \leq c^{IC}. \quad (29)$$

The superscript IC stands for the “incentive compatibility” constraint in the post-deposit game which makes the patient consumer wait hence allows the non-run equilibrium to exist. For there to be a run equilibrium in the pre-deposit game, we need both the run equilibrium and the non-run equilibrium to exist in the post-deposit game. Given (27) and (29), this is possible only if

$$c^{IC} > c^{no-run}. \quad (30)$$

It is equivalent to say the (28) is satisfied when $c = c^{no-run}$, that is, when we have

$$\frac{2/R}{(2/R^{b-1} - 1)^{1/(b-1)} + 1} < 1. \quad (31)$$

Inequalities (26) and (31) are necessary conditions for the existence of a bank-run equilibrium in the pre-deposit game. Each of them is a restriction on the values of the parameters R and b . The parameters which we have not discussed so far are a , y , p and A . As the example in Peck and Shell, we set $a = b$. That is, the patient and impatient consumer are the same in risk aversion. y is not essential for the optimal contract since it will only change the scale of the economy. In the next section, we will discuss how varying the parameters of A and p affects the qualitative nature of the

optimal contract.

3.3 The Parameters

In this section, we assume that the two necessary conditions (26) and (31) are satisfied and $a = b$. We will discuss ranges of the other parameters, A and p , that can affect the nature of the optimal contract. We derive the ranges of A and p in which there is bank-run equilibrium to the pre-deposit game and which can make $c^*(s)$ a decreasing function of s for small s when $c^*(s)$ tolerates bank runs.

Using the same notation as Peck and Shell (2003), the ex ante consumer welfare, if the non-run equilibrium is realized, is

$$\widehat{W}(c; A, p) = p^2[u(c) + u(2y - c)] + 2p(1 - p)[u(c) + v((2y - c)R)] + 2(1 - p)^2v(yR). \quad (32)$$

$\widehat{W}(c; A, p)$ is a concave function of c and it has a unique maximum point. Let $\bar{c}(A, p) = \arg \max \widehat{W}(c; A, p)$. $\bar{c}(A, p)$ is the first-best solution for the deposit contract (i.e. the optimal solution if consumers' types are observed in period 1). Given the *CRRA* utility function form, we have that

$$\bar{c}(A, p) = \frac{2y}{\{p/(2 - p) + 2(1 - p)/[(2 - p)AR^{a-1}]\}^{1/a} + 1}. \quad (33)$$

Hence \bar{c} is an increasing function of A . This is because ex-ante the impatient consumer is “more important” compared to the patient consumer as A increases. So the bank would like to give the first consumer in period 1 (must be impatient) more. \bar{c} is also an decreasing function of p . This is because as p increases it is more likely that both consumers are impatient. When both consumers are impatient, the optimal resource

allocation is to divide the resources equally among the two consumers.

Of course, consumer types are not observable by the bank, so we need to check whether the incentive-compatibility constraint (29) is satisfied at \bar{c} . That is, whether the patient consumer wants to wait until period 2 given the expectation that others will not run. And we also need to check whether a run equilibrium is possible.

When the -run equilibrium is realized, the ex ante consumer welfare is

$$\begin{aligned}
 W^{run}(c) = & p^2[u(c) + u(2y - c)] + p(1 - p)[u(c) + v(2y - c) + v(c) + u(2y - c)] \\
 & + (1 - p)^2[v(c) + v(2y - c)].
 \end{aligned} \tag{34}$$

$W^{run}(c)$ is a concave function and the unique unconstrained optimal solution is y .

If the propensity to run is s , ex ante welfare for the pre-deposit game is

$$W(c; s, A, p) = \begin{cases} (1 - s)\widehat{W}(c; A, p) + sW^{run}(c; A, p) & \text{if } c^{no-run} < c < c^{IC} \\ \widehat{W}(c; A, p) & \text{if } c \leq c^{no-run}. \end{cases} \tag{35}$$

Let c^* denote the optimal contract in the pre-deposit game,

$$c^* = \arg \max W(c; s, A, p).$$

For different values of A and p , $c^*(s; A, p)$ has qualitatively different properties as a function of s .

Case 3.1 $\bar{c}(A, p) \leq c^{no-run}$.

$\bar{c}(1, p) < c^{no-run}$ and $\bar{c}(A, 1) < c^{no-run}$. Hence we will have Case 3.1 when either A is close to 1 or p is close to 1. That is, we will be in this case if the patient and impatient are becoming “equally important” or it is becoming more probable that both consumers are impatient. In this case, the incentive compatibility constraint will never bind at the first-best solution \bar{c} . Furthermore, $\bar{c}(A, p) \leq c^{no-run}$ implies that there is no run equilibrium in the post-deposit game. So we have that $c^*(s; A, p) = \bar{c}(A, p)$. There is no run-equilibrium in the pre-deposit game. c^* does not depend s .

Example 3.1

$$a = b = 1.01; y = 3; R = 1.5;$$

$$p = 0.5; A = 1.$$

The values of b and R satisfy the necessary conditions in Section 3.2: $c^{no-run} = 4.1560 < c^{IC} = 4.2809$. And we have $\bar{c} = 3 < c^{no-run} < c^{IC}$. So $c^ = \bar{c} = 3$*

Case 3.2 $c^{no-run} < \bar{c}(A, p) \leq c^{IC}$

As the impatient consumer gets more “important” (i.e. as A becomes larger) or it is more likely to have at least one patient consumer (i.e. as p becomes smaller), the first best solution requires more “insurance”. Hence \bar{c} will increase.

In this case, $\bar{c}(A, p)$ still satisfies the incentive compatibility constraint. But a run equilibrium exists in the post deposit game at $\bar{c}(A, p)$. So the bank should compare the contract which tolerates runs and the contract which is run-proof. The optimal contract is the one which gives the higher expected utility.

The bank can rule out the run equilibrium by choosing $c \leq c^{no-run}$. Since $\widehat{W}(c)$ is strictly increasing when $c \leq \bar{c}(A, p)$, the optimal contract to rule out runs is $c = c^{no-run}$.

Let the optimal contract which tolerates runs should be denoted by \widehat{c} . Hence we have

$$\widehat{c} = \arg \max_{c \in [c^{no-run}, c^{IC}]} (1 - s)\widehat{W}(c; A, p) + sW^{run}(c; A, p). \widehat{c}(s; A, p). \quad (36)$$

$\widehat{c}(s; A, p)$ is a decreasing function of s . And when $s = 0$, $\widehat{c}(0; A, p) = \bar{c}(A, p)$. Clearly, $W(\widehat{c}()); s, A, p$ is a decreasing function of s .

$$W(\widehat{c}()); 0, A, p) = \widehat{W}(\bar{c}; A, p) > \widehat{W}(c^{no-run}; A, p) \quad (37)$$

and

$$W(\widehat{c}()); 1, A, p) = W^{run}(c^{no-run}; A, p) < \widehat{W}(c^{no-run}; A, p). \quad (38)$$

So there exists a unique $0 < s < 1$ such that $W(\widehat{c}()); s, A, p) = \widehat{W}(c^{no-run}; A, p)$. Denote this s by s_0 .

If $s > s_0$, $c^* = c^{no-run}$ and there is no run equilibrium in the pre-deposit game. If $s < s_0$, $c^*(s; A, p) = \widehat{c}(s; A, p)$; for $s < s_0$, there is a run equilibrium in the pre-deposit game and c^* is a decreasing function of s . For $s = s_0$, both the run-proof contract, c^{no-run} , and the contract tolerating runs $\widehat{c}(s; A, p)$ are optimal.

Example 3.2 *We use the same parameter values as in Example 3.1 except that A has been increased to $A = 9$. The values of c^{no-run} and c^{IC} are unchanged, since they don't depend on A . We have $c^{no-run} < \bar{c} = 4.2531 < c^{IC}$, $s_0 = 0.0027$, and $c^*(s) = c^{no-run} = 4.1560$ for $s \geq s_0$, and $c^*(s) > c^{no-run}$ and strictly decreasing in s for $s < s_0$. Figure 3.1 shows $c^*(s)$.*

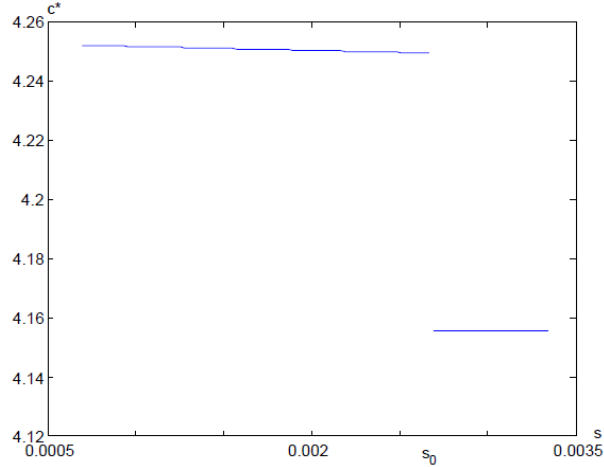


Figure 3.1: $c^*(s)$ in Case 2

Case 3.3 $c^{IC} < \bar{c}(A, p)$

$\bar{c}(\infty, 0) > c^{IC}$. Hence we have this case for sufficiently large A and sufficiently low p . The first-best solution $\bar{c}(A, p)$ does not satisfy the incentive compatibility constraint. So as $s \rightarrow 0$, the incentive compatibility constraint will bind and $\hat{c}(s; A, p) = c^{IC}$. Let $s_1 = \max\{s : \hat{c}(s; A, p) = c^{IC}\}$. That is, if $s \leq s_1$ the incentive constraint will bind and the optimal contract which tolerates bank runs equals c^{IC} . If $s > s_1$, the incentive constraint will not bind and the optimal contract which tolerates bank runs is lower than c^{IC} and is a decreasing function of s .

As in Case 3.2, bank runs can be ruled out by setting $c = c^{no-run}$. The bank will compare the best run-proof contract with the best run-tolerating contract and choose the one that gives the greater ex-ante utility. Let s_0 be the cut-off point between the best run-proof contract and the best run-tolerating contract $W(\hat{c}(); s_0, A, p) = \widehat{W}(c^{no-run}; A, p)$.

$$\begin{aligned} \text{If } s_1 \geq s_0, \text{ we have that } c^*(s; A, p) &= \begin{cases} c^{IC} & \text{if } s \leq s_0 \\ \widehat{c}(s; A, p) < c^{IC} & \text{if } s_0 < s \leq s_1 \\ c^{no-run} & \text{if } s_1 < s. \end{cases} \\ \text{If } s_1 < s_0, \text{ we have that } c^*(s; A, p) &= \begin{cases} c^{IC} & \text{if } s \leq s_1 \\ c^{no-run} & \text{if } s_1 < s. \end{cases} \end{aligned}$$

Example 3.3 We use the same parameter values as Example 3.1 except that A is further increased to $A = 10.5$. The values of c^{no-run} and c^{IC} do not change since they do not depend on A . We have $c^{no-run} < c^{IC} < \bar{c} = 4.2851$ and $s_1 = 0.0031 < s_0 = 0.0047$. For $s \leq s_1$, we have $c^*(s) = c^{IC} = 4.2809$. For $s_1 < s < s_0$, we have $c^{no-run} < c^*(s) < c^{IC}$ and that c^* is strictly decreasing in s . For $s \geq s_0$, we have $c^*(s) = c^{no-run} = 4.1560$. Figure 3.2 displays the optimal contract as a function of s .

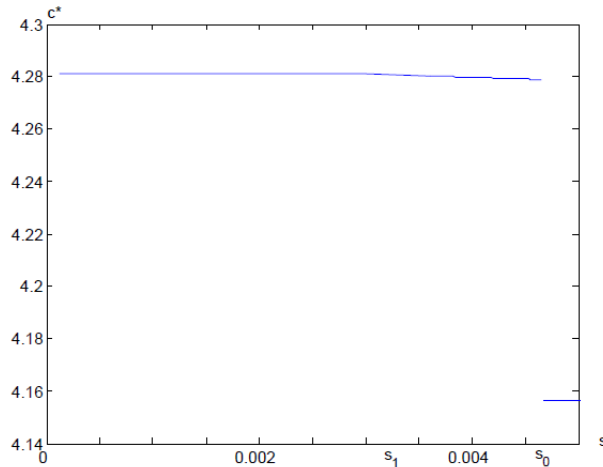


Figure 3.2: $c^*(s)$ in Case 3

Example 3.4 We use the same parameter values as Example 3.1 except that A is increased even further to $A = 12$. The values of c^{no-run} and c^{IC} are unchanged. We have $c^{no-run} < c^{IC} < \bar{c} = 4.3094$ and $s_1 = 0.0209 > s_0 = 0.0064$. For $s \leq s_0$, we have $c^*(s) = c^{IC} = 4.2809$. For $s \geq s_0$, we have $c^*(s) = c^{no-run} = 4.1560$.

3.4 Concluding Remarks

This note generalizes the 2-consumer example in Peck and Shell (2003). I derive necessary conditions for equilibrium bank runs. For the parameters that permit tolerating bank runs, I derive the range in which we have the Peck and Shell's step-function result and the range in which the optimal contract tolerating runs changes continuously in the run probability until it reaches the threshold at which the optimal contract switches to the best run-proof contract. Hence, the run probability affects not only *whether* bank runs will be tolerated but also *how* bank runs will be tolerated.

3.5 References

[1] Diamond, Douglas W. and Philip H. Dybvig. 1983. “Bank Runs, Deposit Insurance, and Liquidity.” *Journal of Political Economy* 91: 401-419.

[2] Peck, James and Karl Shell. 2003. “Equilibrium Bank Runs” *Journal of Political Economy*, 111: 103-123