An Optimization Approach to Apparel Sizing

by

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SUMMARY

A novel approach for the construction of apparel sizing systems is formulated. As a first step to this process efficient sizing systems are defined based on a mathematical model of garment fit. Nonlinear optimization techniques are then used to derive a set of sizing systems using multidimensional information from anthropometric data. The method is illustrated with a sizing system designed for a dress shirt of a military uniform using anthropometric data from the U.S. Army. Results of this analysis show that endogenous size assignment and selection of disaccommodated individuals, together with relaxation of the requirement of a "stepwise" size structure result in substantial improvements in fit over existing systems. The proposed methodology enables the development of sizing systems that can either increase accommodation of the population, reduce the number of sizes in the system, or improve overall fit in accommodated individuals.

Keywords and Phrases: Apparel fit, Nelder-Mead simplex method, anthropometry, decision theory

1. Introduction

In the most general terms, the purpose of an apparel sizing system is to divide a varied population into homogeneous subgroups. Members of a subgroup are similar to each other in body size and shape so that a single garment can adequately fit each of them. Members of different subgroups are dissimilar and would therefore require different garments.

Sizing systems in the U.S. apparel industry are developed from rules that set fixed increments between sizes. The values used for these sizing systems originated from an outdated study (O'Brien and Shelton, 1941) in which regression relationships were derived from several choices of dimensions. Over time, individual companies modified their sizes based on ancedotal information generated from consumer feedback. However, most companies do not have systematic methods for developing sizes or for determining what range of consumers are being fitted by their sizing system (Hudson, 1980; McVey, 1984). Somewhat more systematic methods and proper anthropometric data are used by the armed forces. Regression analysis is used to identify two key dimensions and to derive the values of other dimensions important for the garment (Gordon and Friedl, 1994). However, the size categories are set in a heuristic manner, by plotting the key dimensions in a bivariate plot and selecting convenient categories that accommodate these two dimensions. Also, the sizes developed using these methods are often subsequently modified to more closely resemble sizing systems from the apparel industry, to accommodate the industry contractors who develop the garment patterns and manufacture the garments (Robinette, Mellian and Evin, 1990).

Sizing systems resulting from the current methods do not fully satisfy either the consumers or the providers of apparel. Consumers often have difficulty finding clothes that adequately fit them, or if the right size exists, finding it requires endless trials (LaBat and Delong, 1990; Goldsberry, 1993). Catalog companies complain about the high percentage of returned merchandise due to improper fit. The American Society for Testing and Materials has recently tried to raise funds for a new anthropometric study, but this has failed, in part due to the industry's doubts as to whether sizing problems can be solved within the current system (Amster, 1985).

The proper fit of a garment is dependent on the correspondence of several body measurements to values for which the garment is intended. Due to the number of these relevant body dimensions and the lack of high correlation amongst them (Gordon, et al, 1989, Robinette, 1986; and also see Figure 1), relevant body proportions can vary enormously. Consequently, linear size systems that range from very small to very large will not accommodate all body types. Solving this problem by splitting each size along additional body measurements is not cost effective, due to the proliferation of resulting sizes. For example in the ground-breaking experiment of Levi-Strauss (Rifkin, 1994) with mass customization, women's jeans are offered in 16 hip sizes each associated with 11 waist sizes, 4 crotch depths, and 6 lengths, resulting in 4,224 combinations, or distinctive sizes. This may be acceptable in a mass-market product like blue jeans, but many apparel products are not even made in quantities of 4,000 total units. The key question we address is how sizing systems can be designed to provide proper fit for the largest proportion of individuals without exponentially increasing the number of sizes.

There has been only a small amount of previous work on optimal sizing systems. Tryfos (1985, 1986) has suggested an integer programming approach for optimizing sales of garments. Our approach differs in that we frame it as a continuous optimization problem. This avoids the artificial division of measurements into categories and allows us to accommodate several body dimensions (Tryfos' approach would require too much data with five variables). We also allow some individuals to not be accommodated by the sizing system and we concentrate on the issue of fit rather than sales. Tryfos does not attempt to find measures of fit, as we do in Section 2.2.

Section 2 defines notation, sets up the construction of an efficient sizing system as an optimization problem, and reformulates it so as to be amenable to standard nonlinear optimization algorithms. Section 3 illustrates the approach and Section 4 offers conclusions.

2. Formulation of the Sizing Problem

2.1 The optimization problem

An effective and economical sizing system must satisfy multiple objectives. The most important of these are the following:

a. Accommodate as large a percentage of the population as possible with ready-made garments,

b. For accommodated individuals, provide as good a fit as possible, and

c. Use as few sizes as possible.

A sizing system is improved if its performance on any of these criteria is improved without its performance on any of the other criteria being compromised. We define a system to be *efficient* if it is on the boundary where improvements on any objective come necessarily at the expense of deterioration of other objectives.

Clearly it is not possible to simultaneously optimize for all three of these criteria. A single optimal sizing system does not exist. The relative importance of the criteria will vary from one problem to the next depending on the garment being sized and the population being fitted. The choice between the set of efficient systems is dependent on factors which are independent of the anthropometric data and is outside the scope of this paper. Our goal, therefore, is to describe and illustrate an approach for finding efficient systems. We proceed by treating the number of sizes (objective c.) and the accommodation rate (objective a.) as fixed parameters while optimizing fit (objective b.). We can carry out several optimizations, each time fixing the number of sizes and accommodation rate at different values to explore the frontier of efficient sizing systems.

We will use the following notation. We assume we have sample of N individuals from a population for which we wish to create a sizing system. The nth individual is represented by a vector of their body measurements, x_n. The sizing system will divide the population into $S+1$ groups (S size groups and a group of individuals not accommodated by the sizing system). We will denote the set of those individuals accommodated by the size system by A and the proportion of disaccommodated individuals as α. We represent size group s as a prototype body form with measurements y_s, which could serve as a dress form or fitting model.

We begin by defining a distance function as a way of mathematically capturing the idea of garment fit. Specifically we assume that a dissimilarity measure, $d(x_n, y_s)$, exists which relates x_n to y_s. Therefore $d(x_n, y_s)$ represents how far a garment made for prototype s would be from the measurements for the nth individual.

Since each subject must be accommodated by only a single size, the rest of the sizes which would not be worn by this individual do not affect the quality of the fit. Therefore, the loss from imperfect fit to individual n is only dependent on n's distance to the closest prototype. Accordingly define $p(x_n) = p(x_n, y_1,\ldots,y_S) = \min_s \{ d(x_n, y_s)\}$. Next, we define a loss function that puts an appropriate penalty on poor fit for an individual and aggregates these penalties into an overall measure of loss. For simplicity we assume that the loss is additively separable, i.e., loss= $L(p(x_1), p(x_2),\ldots, p(x_N)) = \sum_{n=1}^{N} l(p(x_n))$, but this is not necessary for the development which follows. Here, $l(p)$ is assumed to be monotonically non-decreasing in p. We are now in a position to formally state our optimization problem.

Optimization formulation 1: For a given number of sizes, S, and a given disaccommodation rate, α, select $y_1, y_2, \ldots, y_S$ so as to

$$\text{minimize } \sum_{x_n \in A} l(p(x_n, y_1, \ldots, y_S))$$

$$\text{subject to } |A|/N > (1-\alpha). \tag{1}$$

In its current format, the optimization problem is not amenable to standard optimization techniques. So we reformulate it by introducing a modified loss function.

In formulation 1, the disaccommodated individuals are clearly those with the largest values of $l(p(x_n))$. Suppose $l(p(x_k))$ is the largest loss for the accommodated individuals and $l(p(x_m))$ is the smallest loss for the disaccommodated individuals. Define c_α by $l(p(x_k)) \le l(c_\alpha) \le l(p(x_m))$. Notice that minimizing values of the objective function in (1) are the same as those which minimize

$$\sum_{x_n \in A} l(p(x_n)) + \sum_{x_n \notin A} l(c_\alpha) \tag{2}$$

since the second term is constant in $y_1, y_2, \ldots, y_N$. Upon defining

$$l^*(p) = \begin{cases} l(p), & \text{if } p < c_\alpha \\ l(c_\alpha), & \text{if } p \ge c_\alpha \end{cases}, \tag{3}$$

we can rewrite (2) as

$$\sum_{x_n \in A} l^*(p(x_n)) + \sum_{x_n \notin A} l^*(p(x_n)) = \sum_{n=1}^{N} l^*(p(x_n)),$$

and create the following equivalent problem.

Optimization formulation 2: For a given number of sizes, S, and a given loss cutoff, c_α, select $y_1, y_2, \ldots, y_S$ so as to

$$\text{minimize} \sum_{n=1}^{N} l^*(p(x_n, y_1, \ldots, y_S)) \tag{4}$$

Based on this equivalence, we can adopt the following strategy to solve (1). Instead of α, treat c_α as our second parameter in addition to S. This can be interpreted as substituting distance beyond which fit is unacceptable for the proportion of accommodation to be the parameter representing objective a. We then solve the transformed optimization problem using $l^*(p)$ to find the optimal prototypes. Once the prototypes are derived, the disaccommodated set can be found by calculating each individual's distance to the closest prototype. Those closer to a prototype than c_α are accommodated and those farther away are disaccommodated. By varying c_α we can achieve the desired disaccommodation rate α.

The main advantage of this procedure is its ability to identify the optimal set of disaccommodated individuals simultaneously with selection of the prototype body sizes. Traditionally, a fraction of individuals are identified as outliers and discarded *before* size groups are derived. Another advantage of our approach is that the distance measure which is the basis of the optimization routine automatically assigns individuals to their proper sizes. By contrast, heuristic methods of choosing prototypes leave the problem of size assignment unresolved.

2.2 Specification of the distance and loss functions

The quantification of fit as a function of body measurements of the wearer would ideally be based upon empirical information. However, such direct information will not usually be available. In the absence of empirical information, the approach which we take is to specify a set of reasonable functions for our specific problem. We make no claim that the specified functions will be appropriate for sizing systems in general.

We apply the following criteria in generating the loss and distance functions:

a. The more the individual's measurements differ from the prototype, the worse the fit.

b. Fit is better predicted by proportional rather than absolute differences between individual and prototype measurements. This assumption is supported by some empirical evidence (Mellian, Ervin and Robinette, 1990). One way to meet this requirement is to log transform the measurements.

c. Small differences between the wearer and the prototype may not influence the quality of the fit, i.e., perfect fit may occur in a range of values around the prototype.

d. A garment which is too small may not affect fit in the same way as one which is too large.

e. Discrepancies in certain dimensions are more critical to fit than others.

It is important to recognize that there are a wide variety of functional forms which satisfy the above requirements. Functional forms other than the ones we chose can be used in the same manner as long as they result in a continuous objective function.

Keeping these reservations in mind, we assume *d(x,y)*, can be written as a sum of squared discrepancies over each of the *I* measurements:

$$d(x,y) = \sum_{i=1}^{I}[d_i(x,y)]^2 = \sum_{i=1}^{I}[d_i(x_i,y_i)]^2 . \qquad (5)$$

The discrepencies in each measurement are given, in turn, by

$$d_i(x_i,y_i) = \begin{cases} a_i^l(\ln(y_i) - b_i^l - \ln(x_i)), & \text{if } \ln(x_i) < \ln(y_i) - b_i^l \\ 0, & \text{if } \ln(y_i) - b_i^l \le \ln(x_i) \le \ln(y_i) + b_i^h . \\ a_i^h(\ln(x_i) - b_i^h - \ln(y_i)), & \text{if } \ln(y_i) + b_i^h < \ln(x_i) \end{cases} \qquad (6)$$

This specification, illustrated in Figure 3, satisfies criteria a.-e., while allowing a great deal of flexibility (Paal, 1996) through the choice of parameter values.

3. Illustration

In this section we use data from a 1988 survey of 2,208 observations on women's body measurements for the purpose of sizing the dress shirt of a woman's military uniform. The sizing system chosen for comparison was developed for a population of Navy women, based on an anthropometric study of 906 women (Robinette et al, 1990) and reflects the traditional industry sizing practice. In addition, this three dimensional system (based on chest circumference, hip circumference, and height) designed for a wide variety of styles. It includes sizes that differ from one another only in lower body measurements. Therefore, for our purposes, only 27 sizes of the original are relevant and form our benchmark for comparison.

3.1 Specification of distance and loss functions

A preliminary regression analysis (Ashdown and Paal, in press) identified five variables which seemed to control the variation in all the relevant measurements which were available. These were: chest circumference, neck circumference at base, shoulder circumference, sleeve outseam, and neck to buttock length. These five variables were therefore used in the optimization. All variables were log transformed to meet criterion b.

The constants required to specify the distance function (6) are given in Table 1. These values were chosen so that:

a. A person being larger than the prototype was penalized three times more than being smaller ($b_i^l = 3b_i^h$, and $a_i^l = 3a_i^h$, for $i = 1, 2, ..., 5$).

b. The discrepancy still consistent with a perfect fit (a_i^l) was based on generally accepted apparel design values selected by the authors for a person with the population average value of that variable. The result was then transformed to the log scale.

c. The relative values of b_i^h (and therefore the b_i^l) across measurements were chosen to reflect our judgment about the relative rate at which increasing discrepancies in these measurements deteriorate fit.

The value of c_α was chosen to be 1.75^2 based on exploration of the relationship between α and c_α in order to achieve an accommodation rate of approximately 95% (Paal, 1996). 20 sizes were chosen initially, based on preliminary work (Ashdown and Paal, in press) which suggested that 20 sizes might perform as well or better than the 27 sizes currently in use for this garment. Specification of the loss function in (4) was completed by using $l^*(p)$ equal to the identity function. That is, the loss for an individual was equal to the smaller of the distance to the nearest size and c_α.

3.2 Finding the efficient sizing system

We expected that the nonlinear optimization in (4) would be difficult for several reasons: the surface is multimodal since interchanges of the sizes leads to same value of the loss, the problem is high-dimensional, with 100 parameters (20 sizes by 5 variables) to determine, and the distance function is not a differentiable function of the sizes. Accordingly we took several precautions. We used the Nelder-Mead simplex method (Press, et al, 1986) for optimization since it does not require any derivative information. We also used a quasi-Newton method (Aptech Systems, 1991) which used numerical (difference-based) derivatives. We tried a number of different starting values and we restarted the Nelder-Mead method several times after convergence in order to verify that it had indeed converged. The Nelder-Mead method requires 101 starting values for each of the 100 parameters, so to begin the optimization we obtained the *k*th initial starting values by using the five actual measurements for the *k*th through (*k*+19)th subjects. This guaranteed that each size would have at least one individual at the start. In restarting the Nelder-Mead algorithm, we used a random restart method where we kept the (supposed) optimal value from the previous optimization and generated new starting values around that value. The new values were generated by adding random normal noise with standard deviation equal to 1/5 of the standard deviation across sizes.

3.3 Results

We reserved one-quarter (552) of the individuals for model assessment and used the remaining three-quarters (1,656) to fit an optimal sizing system. Our optimization converged to a sizing system with an aggregate loss across the 1,656 individuals of 1128.0. Figure 1 shows two of the measurements, chest circumference and sleeve outseam, and plots the accommodated and disaccommodated individuals from the reserved set along with the optimal sizes. Figure 2 shows the same information but for the Navy sizing system. Several things are clear: the optimal sizing system does not have the structure often associated with current sizing systems (e.g., a stepwise structure based on the two variables) but it does a much better job of covering the reserved individuals. For either sizing system our approach enables us to identify the disaccommodated individuals. The percent of accommodated individuals was 96% for the optimal system while the Navy system only accommodates 49% of the individuals to the degree of fit we specified. The aggregate fit for the 552 individuals was 422.6 for the optimal system, almost three times better than the 1274.3 achieved by the Navy system. In summary, the optimal sizing system greatly outperforms the Navy sizing system while using 7 fewer sizes (20 versus 27).

4. Conclusions

The core of our approach to deriving a sizing system is to fix the number of sizes and the disaccommodation rate and optimize the quality of the fit. This naturally leads to the optimization problems in Section 2 and exploration of a frontier of efficient sizing systems corresponding to different values of S and α. This approach has several advantages over currently used systems. It creates a formal link between sizing goals and the methodology by which sizes are created. This makes simultaneous the selection of disaccommodated individuals, the derivation of prototypes, and the assignment of individuals to size classes. Our empirical illustration shows that it can be a great improvement over a currently used sizing system.

A possible disadvantage of the sizing systems derived by our method is the sizes generated do not possess a clear structure, i.e., a grid like structure. A structured system facilitates creation of patterns for the production of garments and for the assignment of sizes to individuals, e.g., in a retail store. This complaint can be overcome within our framework by imposing such structure on the sizing system prior to the optimization. For example, the prototypes could be restricted to have even increments between measurements. We would then be searching for an optimal linear sizing system with even increments between sizes.

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Table 1. Constants required for the specification of the distance function given by equation (6) in the text.

Measurement	Constant			
	a_i^l	a_i^h	b_i^l	b_i^h
Chest circumference	7.559	22.678	0.027	0.009
Neck circumference	23.068	69.204	0.009	0.003
Shoulder circumference	8.550	25.648	0.023	0.008
Sleeve outseam	18.235	54.705	0.011	0.004
Neck to buttock length	9.483	28.450	0.021	0.007

Figure 1: Plot of chest circumference and sleeve outseam showing accommodated (+) and disaccommodated (•) individuals and the optimal sizing system (□) for the 552 individuals reserved for model assessment. Disaccommodated individuals who seem to be within the range of one of the sizes on this plot are outliers in one of the three body dimensions not shown.

Figure 1: Plot of chest circumference and sleeve outseam
showing optimal sizes and accommodated and disaccommodated individuals

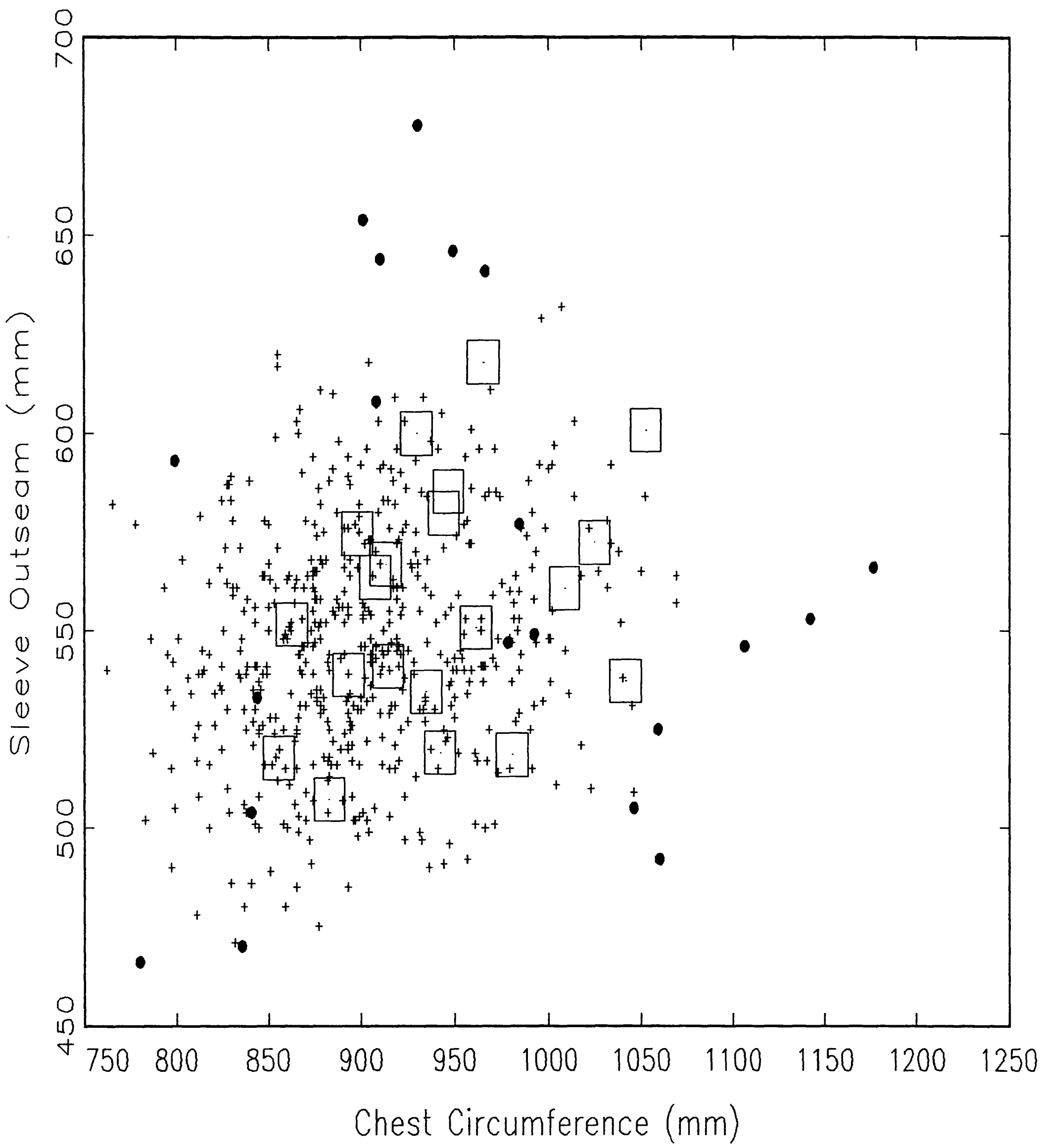

Figure 2: Plot of chest circumference and sleeve outseam showing accommodated (+) and disaccommodated (•) individuals and the Navy's sizing system (O) for the 552 individuals reserved for model assessment. Disaccommodated individuals who seem to be within the range of one of the sizes on this plot are outliers in one of the three body dimensions not shown.

Figure 2: Plot of chest circumference and sleeve outseam showing Navy sizes and accommodated and disaccommodated individuals

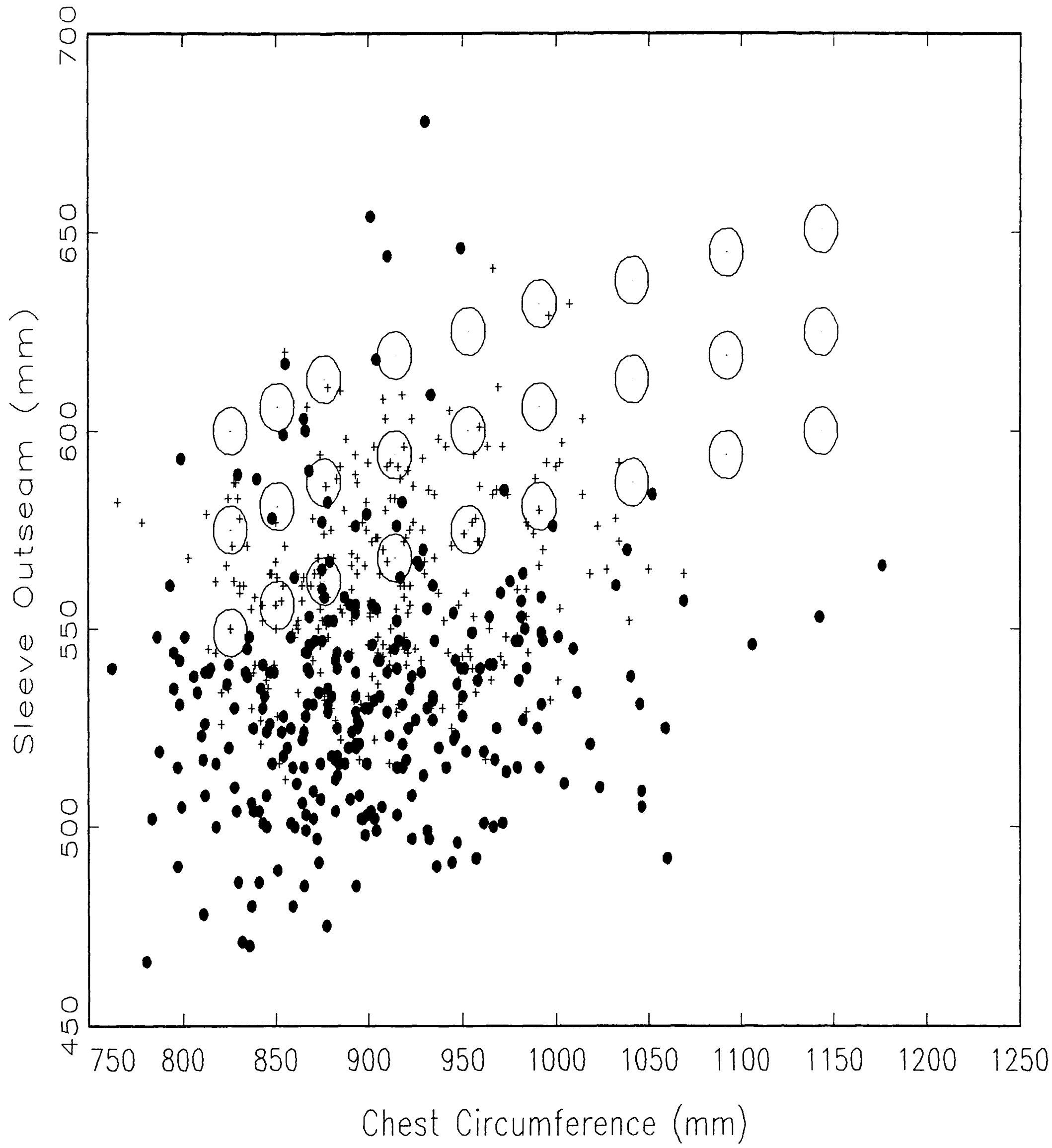

Figure 3: The function $d_i(x_i, y_i)$ which measures the degree of misfit between the prototype and an individual for the *ith* variable.