Discussion of

"Interim Analyses: The Repeated Confidence Interval Approach"

by C. Jennison and B. W. Turnbull

George Casella

Cornell University

BU-1003-M December, 1988

Summary

It is pointed out that the repeated confidence interval approach is important because it can guarantee nominal confidence levels for inferences made conditional on stopping. Thus, it provides a method for constructing conditionally acceptable sequential procedures.

Professors Jennison and Turnbull are to be congratulated for so clearly presenting such an important advance in the topic of sequential analysis. One of the major strengths of their approach is that their interim analyses produce acceptable interim *inferences*, that is, acceptable inferences conditional on stopping.

In many sequential procedures the overall confidence level is not maintained conditionally. It can happen that the conditional confidence level, conditional on stopping early, is uniformly less than the nominal level. (For example, Stein's two-stage confidence interval for a normal mean does not maintain $1 - \alpha$ coverage if the procedure stops at the first stage, as shown in Casella, 1988.) A strength of the Jennison-Turnbull approach is that the nominal confidence level can be maintained no matter when the procedure stops, a property guaranteed by construction. Although this property may result in intervals that are wider than those of other sequential constructions, these other methods cannot maintain the nominal confidence level for all interim inferences.

The development in Section 2.2 is most important. In particular, the condition expressed in equation (2.6) is crucial, as it can guarantee that $c_k > z_\alpha$ for every k and, hence, that any interim inference has confidence (at least) $1 - 2\alpha$. This effectively eliminates *negatively biased relevant subsets*, sets in the sample space on which the actual confidence level can be bounded below the nominal level. (The idea of acceptable conditional inferences were a concern of Fisher, 1959. A formalization of many conditional inference properties can be found in Robinson, 1979.)

The real strength of repeated confidence intervals, however, is not just that interim inferences can be guaranteed to be at the nominal level, but the somewhat more subtle property that the confidence level of each interim inference can be controlled. (This is not only atypical for a sequential procedure, but for many procedures it is impossible to make any conditional guarantees.) Thus, when choosing the constants $\pi_1, \cdots, \pi_k$ of equation (2.6) the conditional level of each interim inference may dictate the pattern. For example, as Jennison and Turnbull point out in Section 3.3, it may be desirable to have the earlier intervals narrower. In fact we may want them so narrow that the interim inferences are below the nominal level. This is not only reasonable, it is conditionally acceptable as we are able to specify the level of the conditional inference.

Interim inferences from repeated confidence intervals possess many desirable properties. In particular they are independent of the stopping rule and are conditionally sound. Jennison and Turnbull have shown that conditionally valid frequentist inferences are possible in a practical sequential setting.

References

Casella, G. (1988). *Conditionally Acceptable Frequentist Solutions.* In *Statistical Decision Theory and Related Topics IV*, Volume I (S.S. Gupta and J.O. Berger, eds.), 73-117.

Fisher, R. A. (1959). *Statistical Methods and Scientific Inference*, second edition. New York: Hafner.

Robinson, G. K. (1979). Conditional Properties of Statistical Procedures. *Ann. Statist.* 7, 742-755.