A Comparison of Different Demand Models for Joint Inventory-Pricing Decisions

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Pricing and inventory research often focuses on stylized models to illustrate pricing and ordering decision dynamics. Although decision insight is useful, the individual retailer faces tougher decisions on actually modeling demand. In an effort to understand the impact of demand modeling choices on inventory and pricing decisions, we evaluated different price-dependent demand models and the resulting profit produced through their implementation. To avoid complications created by other demand drivers, for example promotional and advertising activities, we illustrate the impacts with data from a name-your-price retailer selling a commoditized product where price is the key driver. As our data are provided by a third-party intermediary, we capture all demand requests in the marketplace versus obtaining sales only from a single retailer, enabling us to truly evaluate the profit impacts of price modeling decisions. Our choice of data set also obviates the censoring issues often associated with the evaluation of inventory and pricing decisions. A goal of the article is to provide a practitioner with helpful advice on choosing a modeling approach for joint pricing and inventory decisions.

Introduction

Focusing on a monopolist selling a homogeneous product of variable cost, c, we look to determine the optimal single period price. Let D(p) denote the random variable for the demand for the product if the price is p. The problem facing the retailer is to find the values of the inventory level, Q, and price, p, that jointly maximize the expected profit:

(1)

$$pE[\max(Q, D(p))] - cQ$$

If the price is assumed fixed, then the problem becomes the standard newsvendor problem and the optimal Q is the smallest Q that satisfies $F_p(Q) \geqslant (p-c)/p$, where $Fp(\cdot)$ is a distribution function of the random variable D(p).

The simplest approach to modeling this is to assume additive demand where $D(p)=g(p)+\varepsilon$, and $g(p)$ is a deterministic function (often linear) of price and e is a random error term. Whitin (1955) was the first to add price-dependent demand to standard inventory models. Mills (1959) extended Whitin's model to formalize demand with a price-dependent mean plus uncertainty using the additive model. The additive demand model remains a staple modeling framework in inventory research, with a sample of papers such as Karlin and Carr (1962), Zabel (1972), Lau and Lau (1988), Petruzzi and Dada (1999), Agrawal and Seshadri (2000), and Bertsimas and Perakis (2006) utilizing additive demand models. The multiplicative demand model is given by $D(p)=g(p)\varepsilon$, where, once again, $g(p)$ is a deterministic function of the price and ε is a random error term. Inventory research using multiplicative endogenous demand is more recent, with examples including Petruzzi and Dada (1999) and Agrawal and Seshadri (2000). Both multiplicative and additive demands require further assumptions on $g(p)$, with common approaches being $g(p)=a-bp$ and $g(p)=ap^{-b}$ (Petruzzi and Dada, 1999). As these are commonly assumed functional forms for the additive and multiplicative demand models, we will use these in our subsequent analysis of the joint pricing/inventory problem. Under most circumstances, errors in these endogenous demand models are assumed to be independent of price. Exceptions include Young (1978) and Federgruen and Heching (1999) where $D(p)=\alpha(p)\varepsilon$ with ε a random and independent of p. Recent work on modeling prices in a newsvendor setting includes Kocabiyikoglu and Popescu (forthcoming), Roels (2010), and Lu and Simchi-Levi (2011).

Two recent general forms of price-dependent demand include that of Raz and Porteus (2006), as well as the models used by Wilson and Sorochuk (2009) and Anderson (2009). The approach of Raz and Porteus is to use a two-stage approximation to $D(p)$. They first assume that there are N possible demand states. For each of these states, it is then assumed that the demand function can be approximated by a deterministic piecewise linear function over a partition of price points. Raz and Porteus (2006) breaks demand into a series of piecewise deterministic approximations and will subsequently be referred to as the fractile method.

Wilson and Sorochuk assume that the price a randomly selected customer is willing to pay, which we refer to as the reservation price, is given by the value of a random variable X with distribution function $F(p)$, whereas the total number of customers is given by N. If the number of customers is known, that is, $N=n$, then the normal approximation to the binomial yields the result that demand can be modeled via $D(p)=g(p)+\varepsilon(p)$. Demand uncertainty $\varepsilon(p)$ will have a mean of zero and a variance of $nF(p)(1-F(p))$. This model then approximates additive demand with the addition of price-dependent

uncertainty. If one assumes g(p)=a-bp, this implies reservation prices are uniform over *n/b((a/n)-1) to n/b((a/n)-1)+n/b*.

A more realistic assumption would be that the number of consumers is unknown, with N being a random variable; demand at the price point p is given by

$$D(p) = \sum_{i=1}^{N} I_i(p)$$

where I_1, I_2,y are independent identically distributed Bernoulli random variables with $P[I_1(p)=1]=P[X \geqslant p]$. Note that here the mean and variance of D(p) are given by $E[N]P[X \geqslant p]$, and $E[N]P[X \geqslant p]\{1-P[X \geqslant p]\}+Var[N](P[X \geqslant p])^2$, respectively.

The fractile model is quite general and can include the other models as special cases. It is a macro model in that no assumptions are made about individual consumer behavior. The Wilson and Sorochuck model assumes that information is available on individual reservation prices. It also holds when $P[X \geqslant p]$ is replaced by an individual purchase probability from a discrete choice model in a competitive setting or one with substitutes. The model is also easily approximated with a normal approximation, with $g(p)=E[N]P[X \geqslant p]$ and the error term e is normal with a mean of zero and variance as given above. In the fractile model, demand is assumed to be deterministic once the number of customers is known, whereas it depends on customer's individual purchase probabilities for the Wilson and Sorochuck model. Thus, we will refer to the Wilson and Sorochuk model as the customer-driven demand model.

It is possible to think of these models in an integrative framework where demand is given by the general equation

$$D(p, N, \varepsilon) = \gamma_N(p) + \varepsilon\phi_N(p)$$

where N and ε are random variables independent of p.

(Given the data set that will be analyzed and the models employed, it can be helpful to think of N as representing the size of the population interested in buying at some price.) The functions $\gamma_N(p)$ and $\phi_N(p)$ depend on the particular model being employed.

The Wilson and Sorochuk (2009) model (using the normal approximation to the binomial random variable) is put into this form by using the definitions: $\gamma_N(p)=N(1-F(p))$ and

$$\phi_N(p) = \sqrt{\begin{matrix} E[N]P[X \geqslant p]\{1-P[X \geqslant p]\} \\ +Var[N](P[X \geqslant p])^2 \end{matrix}}$$

where N is the number of people interested in buying at some price and e is the standard normal random variable. If N is deterministic and equals n, then the appropriate functions become $\gamma_n(p)=n(1-F(p))$ and $\phi\downarrow N(p)=O\sqrt{}(nF(p)(1-F(p).))$

In the Wilson Sorochuk model (2009), bivariate uncertainty is explicitly modeled. The additive, multiplicative and fractile-based approach of Raz and Porteus (2006) considers univariate uncertainty. One way to model this using the general equation is to set $\varepsilon=0$ with probability one. The fractile-based approach would now estimate $\gamma_{n_1}(p), \gamma_{n_2}(p)$, …, using piecewise linear approximations at a set of price points, where the quantities n_1, n_2,… are the possible values for N. Now the integrative model would take the form $D(p,N, \varepsilon)=\gamma_N(p)$, where $\gamma_N(p)= \gamma_{n_i}(p)$ with probability equal to $P[N=n_i]$. If N is deterministic and equals n, then the model becomes the deterministic one: $D(p, n, \varepsilon)=\gamma_n(p)$. Multiplicative and additive approaches can be seen as a special case of this by setting the function $\gamma_N(p)$ equal to $ap^{-b}N$ or $a+bp+N$, respectively. For N known to equal n, this model gives the deterministic values $ap^{-b}n$ and $ab+bp+n$, respectively. This approach is more restrictive than the traditional approach of estimating additive demand in that the error term is specially assumed to be related to the number of possible customers.

In general, there is a serious practical issue in evaluating joint inventory and pricing decisions – that of censored observations. For instance, suppose a retailer allocates a certain amount of inventory at a certain price. If this inventory sells out, all the retailer knows is that demand at that price exceeded available inventory. This makes comparison of the various approaches to determining inventories and prices problematic. We sought out a data set that obviates some of these issues. The data set comes from an online name-your-own-=price (NYOP) retailer. Specifically, on each day, customers place their bid for a hotel room at a specific star level in a specific location in the United States. Only one bid is allowed. A customer's credit card details are obtained when the bid is placed. A successful bid results in a charge being placed on the customer's credit card. Hotels that contract with the NYOP retailer face the issue of how many rooms to assign and at what price. This must be done before the bids are observed. In this situation, one can retrospectively see how different strategies would have played out for the hotel chain. It should be noted that as all of the above models can be considered as local approximations within the observed data range of a more general demand model, although of unknown functional form, all optimizations were restricted to the observed price set.

Data

Data were collected from the NYOP retailer for a period of 13 weeks. Consumers would place a bid for a hotel room on the day of their hoped-for arrival. For instance, Table 1 contains the actual bid prices for Wednesday, 29 July 2009 and Wednesday, 5 August 2009. There were 10 bids on 29 July and nine bids on 5 August. Therefore, based on the data for Wednesday, 29 July, a price of US$46 or less, which was the lowest bid price, would have resulted in 10 rooms being sold. A price greater than $46 but equal to or lower than $48, which is the next lowest bid, would have resulted in exactly nine rooms having been sold, that is, demand would be nine for $46<p\leqslant 48$; exactly eight rooms would have been sold if $48<p\leqslant 52$ and so on. Similarly for the bids on 5 August, a price of $35 would result in nine rooms being sold, whereas for $35<p\leqslant 39$ would result in six rooms being sold as three bids were less than that price range. Figures 1 and 2 display demand curves for our 13 weeks with the data separated into weekend and weekday prices. Data for week one are overlaid in both figures as a solid line. We have segmented data into weekend (Friday and Saturday night stays) and weekdays (Sunday through Thursday night stays) as the data are from a city core hotel that caters to business travelers at higher rates during the week and leisure travelers on weekends. Therefore, it makes sense to think of weekdays and weekends as being two different processes in terms of the number of arrivals and customer reservation prices. We are using the data to illustrate joint pricing/inventory decisions where non-censored data and data at different price points are available – and thus our focus is not really on hotel rooms. If the focus were on this particular situation, the weekday decision facing the retailer would be to decide on the total number of rooms to make available for all of Sunday through Thursday and at what price these rooms should be sold. The weekend decision would be to decide on the total number of rooms to make available for Friday and Saturday and the price at which to sell them.

Table 1: Bid prices and resulting demand for 29 July and 5 August

(29/07/2009)			*(05/08/2009)*		
Bid prices	*Price*	*Demand*	*Bid prices*	*Price*	*Demand*
$46	46	10	$35	35	9
$48	48	9	$35	39	6
$52	52	8	$35	44	4
$56	56	7	$39	48	3
$57	57	6	$39	52	2
$60	60	5	$44	—	—
$61	61	4	$48	—	—
$66	66	3	$52	—	—
$72	72	2	$52	—	—
$102	102	1	—	—	—

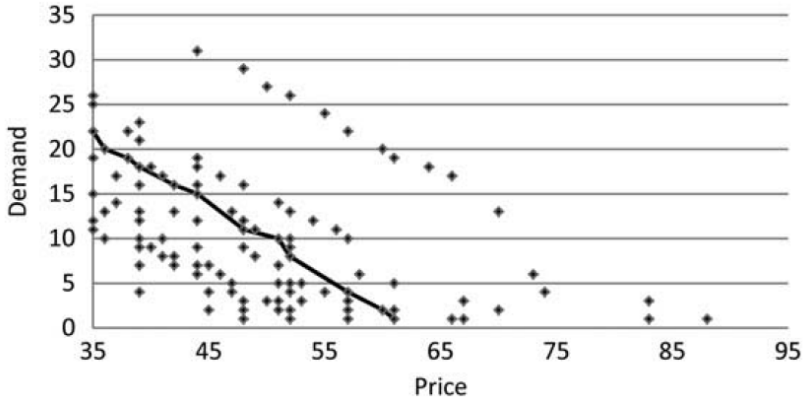

Figure 1 Weekend Demand Curve

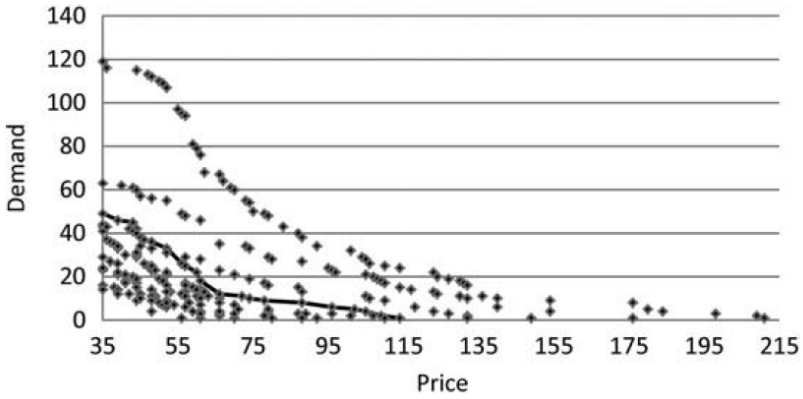

Figure 2 Weekday Demand Curve

Price-Dependent Demand Modeling

In the following section, we discuss the fitting of the various demand models and their application to inventory and price decisions. To test the different models, we estimated a series of models using n-1 weeks of data to fit models that were then applied to the nth (holdout) week of data resulting in n (13 in our case) profit/revenue results. Joint price-inventory optimization was performed, assuming a range of per unit inventory costs from 0 to 30.

Additive Demand

To evaluate the additive demand model, we use the standard form of the linear demand model given as $D(p)=a+bp+\varepsilon$, where ε is distributed as a normal random variable with mean zero and standard deviation s. An appealing aspect of this model is the ease with which estimates of the parameters can be made via simple ordinary least squares regression (OLS) and is the method used here (Talluri and van Ryzin, 2004, p. 323).

For the joint quantity-price optimization of expected profit (1), assuming a normal distribution on demand, Winkler et al (1972) have shown that the resulting partial expectation is given as $E[\min(Q, d(p))]=\mu F(z)-\sigma f(z)+Q(1-F(z))$, where F(z) and f(z) are the standard normal cumulative and probability density function, and $z=(Q-\mu)/\sigma$, with μ-d(p) or the mean expected demand at a given price and s estimated using OLS. Thus, we can reduce the problem to a one-dimensional search over p to find the optimal price-quantity pair (P*,Q*). The price for the revenue management optimization was determined similarly via a search over prices for a fixed inventory position.

Multiplicative Demand

For the multiplicative model, we assumed that the deterministic component of demand is the iso-elastic or constant elastic demand function $ap^{-b}\varepsilon$, where it is assumed that a, b>0 and e is a random variable with an expected value of 1. As the model is readily linearized by taking the natural logarithm, parameter estimates are performed via OLS on the linearized model.

In general, it is understood that the iso-elastic demand function is ill-behaved for pricing only optimization (Talluri and van Ryzin, 2004, p. 325), or equivalently where inventory costs are zero, with optimal prices being zero or infinity for demand-price elasticity, b, that are greater or less than one, respectively. For cases where the elasticity is exactly one, all prices result in the equivalent revenue. However, joint price-quantity optimization values can be obtained when the elasticity values are strictly greater than one and inventory costs are greater than zero, although low costs and high elasticities, as is the case with the current data, often result in unreasonably low optimal prices and correspondingly high inventory levels. However, as the decisions were restricted to the observed historical data as discussed above, this resulted in pricing decisions that were bound at the minimum, that is, the lowest bid price. Price and inventory optimization then proceeded in a manner similar to that given above for the additive demand model, using the log transformed data. However, it should be noted that most models we fit using this were not statistically significant. In addition, restricting prices as above usually resulted in the optimal price as the lower bound. Therefore, the results for this model are overly optimistic.

The Fractile Approach

The fractile approach assumes no structural properties about D(p). Instead, two approximation techniques are utilized. The first is to assume that demand can be any of N states. For instance, a demand state could be the size of the possible market. Conditioned on the demand state, the actual demand at price p is assumed to be a deterministic continuous piecewise linear function over a finite

partition of possible price points. In order to implement this approach, we first empirically estimated the size of the customer demand for the period of interest, that is, weekend or weekday. For example, consider the case where we wished to make decisions for weekend arrivals in week 1 and are using all of the data from weeks 2–13 for demand estimation. The raw data are displayed in Table 2. This then gives the empirical distribution for the number of bids presented in Table 3. On weekend number 2, for instance, 12 people were interested in a room. A price of $35 dollars would have resulted in 12 sales, whereas a price of $40 would have resulted in only seven sales.

Table 2: Demand and price for weeks 2 through 13 (weekend arrivals)

Price	*Demand by week*											
	2	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
35	12	19	15	19	11	7	25	12	4	26	18	31
40	7	13	10	9	8	6	18	9	2	19	18	31
45	4	7	9	2	5	2	17	4	2	16	13	29
50	3	5	7	0	3	0	14	2	1	10	10	27
55	0	3	2	0	1	0	11	0	1	4	4	24
60	0	2	0	0	0	0	5	0	1	0	1	20
65	0	1	0	0	0	0	3	0	0	0	1	17
70	0	0	0	0	0	0	2	0	0	0	0	13
75	0	0	0	0	0	0	1	0	0	0	0	3
80	0	0	0	0	0	0	1	0	0	0	0	3
85	0	0	0	0	0	0	0	0	0	0	0	1
90	0	0	0	0	0	0	0	0	0	0	0	0

From Table 2, there were two weekends out of 12 where the number of bids was 12. Thus, (empirical) probability that 12 bids will be received on any given weekend is 2/12. Most of the bid numbers occurred once in the data set and thus were assigned a probability of 1/12 (see Table 3).

Conditioned on the number of bids, this approach assumes that demand at a given price is deterministic and known. We implement this assumption by using the average demand. For instance, consider weekends 2 and 9. On both of these weekends the demand is 12. However, a price of $40 would have resulted in seven sales in weekend 2 and nine sales in weekend 9. Therefore, on our piecewise linear function number 1 in Table 4, we have inserted 8, the average of 7 and 9, as the approximating deterministic demand if the number of bids is 12 and the price is set at $40. At the initial price points, there would have been equivalent demands depending on the particular week (over the 12 weeks) that is considered. Figure 3 contains the 12 piecewise linear functions corresponding to the 12 columns given in Table 4. Because the number of sales at a given price is assumed to be known and

deterministic, we can determine the optimal price-quantity for each piecewise fractile function and then determine the overall optimal demand-price pair and then determine the overall optimal pair.

Table 3: Empirical distribution for number of arrivals

Number of arrivals	*Probability*
4	0.083
7	0.083
11	0.083
12	0.167
15	0.083
18	0.083
19	0.167
25	0.083
26	0.083
31	0.083

The Customer-Driven Approach

Assume that the size of the market can be represented by a random variable N. The price a customer is willing to pay can be represented by a random variable X. Thus, if the price is set at p and if the size of the market equals n (that is N=n), then the number of people who will purchase (assuming inventory is available) is the value of a binomial random variable with parameters n and P[X⩾p]. In this scenario, the expected profit when inventory is set equal to Q and price is set equal to p is given by

$$\sum_n \left\{ \sum_{x=0}^{Q} pxP[D(p) = x|n] + Q \sum_{x=Q+1}^{n} pP[D(p) = x|n] \right\} P[N = n] - cQ$$

Table 4: Piecewise linear demand functions for weeks 2–13 (weekend arrivals)

Price	*Piecewise linear functions*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
35	12	19	15	11	7	25	4	26	18	31
40	8	11	10	8	6	18	2	19	18	31
45	4	4.5	9	5	2	17	2	16	13	29
50	2.5	2.5	7	3	0	14	1	10	10	27
55	0	1.5	2	1	0	11	1	4	4	24
60	0	1	0	0	0	5	1	0	1	20
65	0	0.5	0	0	0	3	0	0	1	17
70	0	0	0	0	0	2	0	0	0	13
75	0	0	0	0	0	1	0	0	0	3
80	0	0	0	0	0	1	0	0	0	3
85	0	0	0	0	0	0	0	0	0	1
90	0	0	0	0	0	0	0	0	0	0
Probability	0.167	0.167	0.083	0.083	0.083	0.083	0.083	0.083	0.083	0.083

where

$$P[D(p) = x|n] = \binom{n}{x} \{P[X \geqslant p]\}^x \{p[X < p]\}^{n-x}$$

The procedure for calculating the distribution of N was the same as that used for implementing the fractile approach (see Tables 2 and 3). In order to implement the customer-driven approach, it is necessary to fit a distribution for the random variable X. We decided to estimate this empirically. It is also possible to use other approaches. For instance, as in Anderson (2009), the authors found that the gamma distribution provided a reasonable fit.

Results

In the following, we present a series of results using the 13 weeks of consumer data. Results are averages of 13 situations where under each situation 12 weeks of data are used to estimate model parameters (and determine optimal prices and inventory), with the holdout week used to evaluate resulting revenues and profits for each of the models.

The total profits from the joint optimization for each model is presented in Figures 4 and 5 for weekend and weekday arrivals, respectively, at per unit inventory costs from 0 to $30, along with the profits that would result from perfect information. Tables 5–10 include both optimal price and order quantities for each model for inventory costs of $1, $10, and $30, respectively, along with demand and

resultant profits over each of the 13 weeks, as well as the total and average profits. Although the results are somewhat mixed, in general, the additive model performs poorly over all inventory costs, whereas the fractile approach consistently performs the best, with results from the customer-driven following closely but generally lagging the fractile model profits. However, all models are still significantly short of the profits determined for the optimal price-quantity pair under perfect information. Overall, we tend to see more variation in the results for weekday compared with weekend, most likely a result of the higher variance observed in both bid-prices and demand during weekday arrivals. Further, it should be noted that the performance of the additive and the multiplicative demand models may be partly due to the specific functional form of the price-demand curves (respectively, linear and isoelastic), and that other demand specifications could potentially perform better.

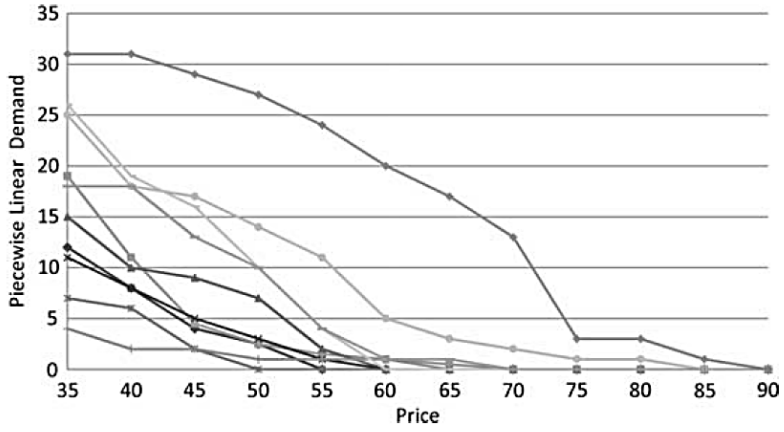

Figure 3 Piecewise Linear Demand Functions

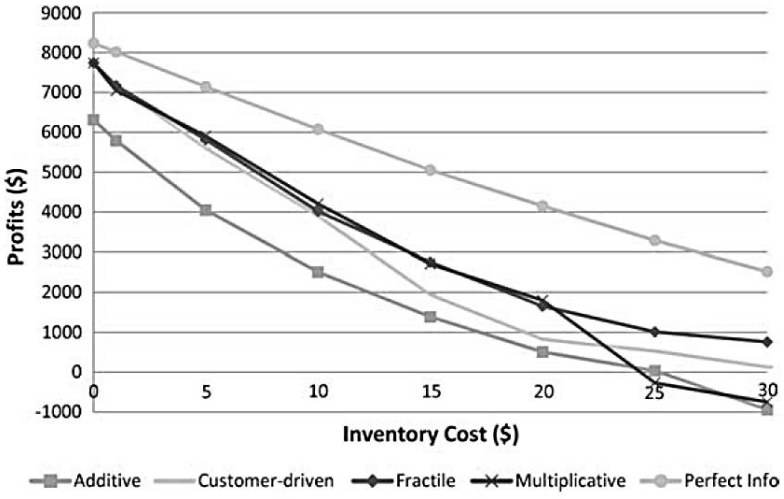

Figure 4 Profit as a Function of Inventory Cost for Weekend Arrivals

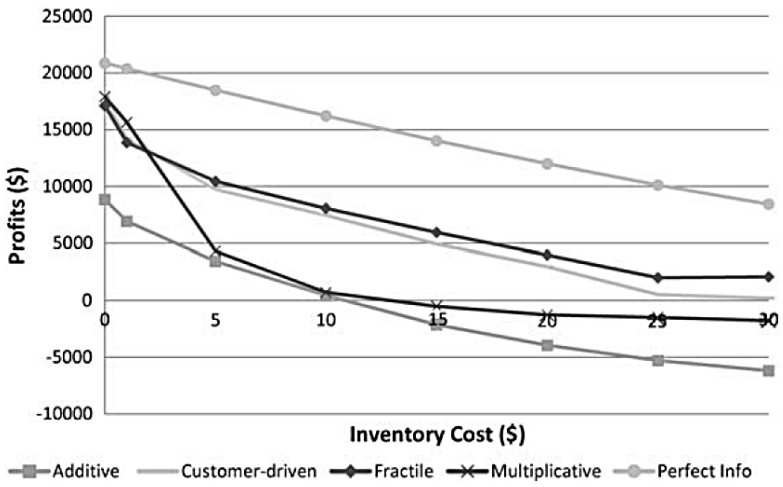

Figure 5 Profit as a Function of Inventory Cost for Weekday Arrivals

The results of the multiplicative model are somewhat interesting, with results varying dramatically over costs and by weekday/weekend arrivals, with the model often performing well at lower costs, particularly for weekend arrivals. This can at least partly be explained by the optimal pricing of the model. As previously described, the multiplicative model fit from the data often resulted in high elasticity estimates, which in turn gave very low optimal prices (often in single digits). In order to use the model, we had bound the optimal prices to be within the historical prices, and thus the optimal price was reset to the minimum price with low inventory costs and the optimal inventory then determined. As can be seen in Tables 4 and 5 for inventory costs of $1, the optimal price for all 13 weeks was the minimum historical price of $35. This often was the true optimal price, as observed under perfect information. However, without the bounds, the multiplicative model performs quite poorly.

For the fractile approach, as estimates of the deterministic piecewise linear demand functions are restricted to the current data, only prices observed are considered. Resultant optimal inventories then follow the same pattern as observed in the estimation data at that price point. The potential importance of using an empirical approach can also be observed in the stochastic approach. Figure 6 shows the profits results from the stochastic approach described above with that using an empirical estimation of arrivals for weekdays. The profits using the empirical distribution are at least equal to, and most often greater than, that observed from using a fitted Gamma distribution.

Table 11 shows a comparison of optimal price and quantity pairs for the four models and that obtained under perfect information for the 13 weeks using weekday arrivals at a per unit cost of $10. We immediately see the weakness of the additive model as it often overestimates both optimal price and quantities for the most of the 13 weeks. The multiplicative demand also tends to inflate prices, but

then underestimates the proper order quantity. The stochastic approach also has higher prices and lower order quantities, but only slightly so. Finally, the fractile approach generally does a good, although conservative, job of determining the pricing, but tends to slightly overestimate the inventory, thus resulting in lower overall profits.

Table 5: Weekend optimal prices and order quantities for item cost of $1

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
25	44.56	11	465	31	35.00	22	739	31	35.00	22	739	50	35.00	22	720
26	42.09	7	269	31	35.00	12	389	31	35.00	12	389	57	35.00	12	363
26	43.97	12	502	31	35.00	19	634	31	35.00	19	634	53	35.00	19	612
26	42.84	9	360	31	35.00	15	494	31	35.00	15	494	55	35.00	15	470
26	42.16	6	227	31	35.00	19	634	31	35.00	19	634	56	35.00	19	609
26	42.51	7	272	31	35.00	11	354	31	35.00	11	354	55	35.00	11	330
26	42.07	6	226	31	35.00	7	214	31	35.00	7	214	55	35.00	7	190
25	43.59	18	760	31	35.00	25	844	31	35.00	25	844	50	35.00	25	825
26	42.67	9	358	31	35.00	12	389	31	35.00	12	389	54	35.00	12	366
26	42.41	2	59	31	35.00	4	109	31	35.00	4	109	54	35.00	4	86
25	43.81	19	807	31	35.00	26	879	31	35.00	26	879	52	35.00	26	858
26	43.20	16	665	31	35.00	18	599	31	35.00	18	599	51	35.00	18	579
25	33.81	31	820	26	35.00	31	884	26	35.00	31	884	51	35.00	31	1034
Total	—	—	5790	—	—	—	7162	—	—	—	7162	—	—	—	7042
Average	—	—	445	—	—	—	551	—	—	—	551	—	—	—	542

Table 6: Weekday optimal prices and order quantities for item cost of $1

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
77	88.63	6	455	105	40.00	45	1695	117	37.50	46	1608	136	35.00	49	1579
77	86.28	3	182	105	40.00	30	1095	115	40.00	30	1085	142	35.00	43	1363
78	86.49	3	181	105	40.00	30	1095	115	40.00	30	1085	143	35.00	41	1292
77	84.65	1	8	105	40.00	18	615	116	38.75	21	698	148	35.00	23	657
76	83.64	0	−76	105	40.00	12	375	116	38.75	14	427	150	35.00	16	410
76	83.98	0	−76	105	40.00	9	255	116	38.75	12	349	148	35.00	14	342
76	84.19	0	−76	105	40.00	21	735	115	40.00	21	725	146	35.00	29	869
77	85.52	27	2232	105	40.00	62	2375	118	36.25	62	2129	133	35.00	63	2072
78	88.77	11	898	105	40.00	42	1575	118	36.33	42	1408	135	35.00	44	1405
77	83.76	0	−77	105	40.00	17	575	116	38.86	22	739	153	35.00	24	687
46	69.75	60	3162	56	40.00	115	2184	63	35.00	119	2142	107	35.00	119	3638
76	84.97	0	−76	105	40.00	15	495	117	37.50	15	445	147	35.00	15	378
77	86.22	3	182	105	40.00	31	1135	118	36.25	31	1006	140	35.00	31	945
Total	—	—	6919	—	—	—	14 204	—	—	—	13 846	—	—	—	15 637
Average	—	—	532	—	—	—	1093	—	—	—	1065	—	—	—	1203

Table 7: Weekend optimal prices and order quantities for item cost of $10

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
16	47.71	11	365	18	35.00	22	450	19	35.00	22	475	17	35.00	22	425
17	45.38	3	−34	19	35.00	12	230	22	35.00	12	200	20	35.00	12	220
17	46.93	5	65	18	35.00	19	450	22	35.00	19	445	18	35.00	19	450
17	45.98	9	244	19	35.00	15	335	22	35.00	15	305	19	35.00	15	335
17	45.37	2	−79	15	40.00	9	210	22	35.00	19	445	19	35.00	19	475
17	45.77	5	59	19	35.00	11	195	22	35.00	11	165	19	35.00	11	195
17	45.31	1	−125	19	35.00	7	55	22	35.00	7	25	20	35.00	7	45
16	46.64	16	586	18	35.00	25	450	19	35.00	25	475	17	35.00	25	425
17	45.84	4	13	19	35.00	12	230	22	35.00	12	200	19	35.00	12	230
17	45.56	2	−79	19	35.00	4	−50	22	35.00	4	−80	19	35.00	4	−50
16	47.01	16	592	18	35.00	26	450	19	35.00	26	475	18	35.00	26	450
17	46.11	13	429	19	35.00	18	440	22	35.00	18	410	18	35.00	18	450
17	37.66	31	470	18	35.00	31	450	19	35.00	31	475	22	35.00	31	550
Total	—	—	2507	—	—	—	3895	—	—	—	4015	—	—	—	4200
Average	—	—	193	—	—	—	300	—	—	—	309	—	—	—	323

Table 8: Weekday optimal prices and order quantities for item cost of $10

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
51	90.74	6	34	28	50.00	33	1120	41	35.77	46	1057	11	211.00	0	−130
51	88.49	1	−422	30	50.00	21	750	43	39.29	30	749	14	169.31	0	−140
52	88.64	3	−254	30	50.00	19	650	43	38.41	34	876	14	178.94	0	−140
51	86.94	1	−423	31	50.00	13	340	43	37.50	21	357	18	118.71	0	−180
50	86.00	0	−500	31	50.00	9	140	43	37.50	15	132	20	97.75	0	−200
50	86.30	0	−500	31	50.00	1	−260	43	37.50	12	20	19	102.41	0	−190
51	86.34	0	−510	31	50.00	13	340	43	37.71	26	551	17	120.07	0	−170
50	87.45	27	1861	28	50.00	55	1120	41	35.77	62	1057	17	99.16	21	1516
52	90.75	11	478	29	50.00	31	1160	41	35.77	43	1057	11	211.00	0	−120
51	86.09	0	−510	31	50.00	8	90	43	37.55	22	396	20	99.33	0	−200
31	72.71	55	1944	28	50.00	110	1120	41	35.77	116	1057	25	49.38	110	985
51	87.21	0	−510	31	50.00	8	90	44	35.55	15	93	18	117.05	0	−180
51	88.43	3	−245	31	50.00	22	790	44	35.00	31	645	14	175.19	0	−140
Total	—	—	443	—	—	—	7450	—	—	—	8047	—	—	—	710
Average	—	—	34	—	—	—	573	—	—	—	619	—	—	—	55

Table 9: Weekend optimal prices and order quantities for item cost of $30

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
8	52.79	4	−29	2	55.00	4	50	7	41.25	16	79	2	80.39	0	−60
9	50.93	3	−117	2	55.00	0	−60	7	42.00	7	84	3	63.30	0	−90
8	52.73	3	−82	2	55.00	3	50	7	41.25	13	79	2	80.77	0	−60
9	51.27	5	−14	2	55.00	2	50	7	41.25	9	79	2	75.60	0	−60
9	50.80	0	−270	2	55.00	0	−60	7	41.25	7	79	3	63.25	0	−90
9	51.34	1	−219	2	55.00	1	−5	7	41.25	8	79	3	64.94	0	−90
9	50.97	0	−270	0	35.00	7	0	7	41.67	6	40	3	63.67	0	−90
8	51.57	13	173	1	55.00	11	25	7	41.25	18	79	2	75.38	1	15
9	51.43	1	−219	2	55.00	0	−60	7	41.67	9	82	3	65.16	0	−90
9	51.16	1	−219	2	55.00	1	−5	8	40.00	2	−160	3	64.74	0	−90
8	52.30	5	22	2	55.00	4	50	7	41.25	19	79	2	76.18	0	−60
8	51.85	10	175	2	55.00	4	50	7	41.25	16	79	2	77.80	0	−60
8	45.60	29	125	2	50.00	27	40	7	41.25	31	79	4	48.23	27	73
Total			−944				125				754				−752
Average			−73				10				58				−58

Table 10: Weekday optimal prices and order quantities for item cost of $30

Additive				*Customer-driven*				*Fractile*				*Multiplicative*			
Q	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*	*Q*	*Price*	*Demand*	*Profit*
32	93.08	6	−402	5	105.00	4	270	13	50.00	33	260	7	211.00	0	−210
32	91.18	1	−869	5	105.00	0	−150	13	50.00	21	260	7	211.00	0	−210
32	91.49	3	−686	5	105.00	2	60	13	50.00	19	260	7	211.00	0	−210
32	89.93	0	−960	5	105.00	0	−150	13	50.00	13	260	6	211.00	0	−180
32	88.85	0	−960	5	105.00	0	−150	13	50.00	9	60	6	211.00	0	−180
31	89.24	0	−930	5	105.00	0	−150	13	50.00	1	−340	6	211.00	0	−180
31	89.25	0	−930	5	105.00	0	−150	13	50.00	13	260	6	211.00	0	−180
30	89.05	24	1237	4	105.00	21	300	13	50.00	55	260	5	211.00	0	−150
32	92.97	11	63	5	105.00	11	375	13	50.00	31	260	7	211.00	0	−210
32	89.24	0	−960	5	105.00	0	−150	13	50.00	8	10	6	211.00	0	−180
20	77.53	49	951	4	105.00	28	300	13	50.00	110	260	4	157.67	8	511
32	90.27	0	−960	5	105.00	0	−150	13	50.00	8	10	6	211.00	0	−180
32	91.14	2	−778	5	105.00	1	−45	13	50.00	22	260	7	211.00	0	−210
Total			−6183				210				2080				−1769
Average			−476				16				160				−136

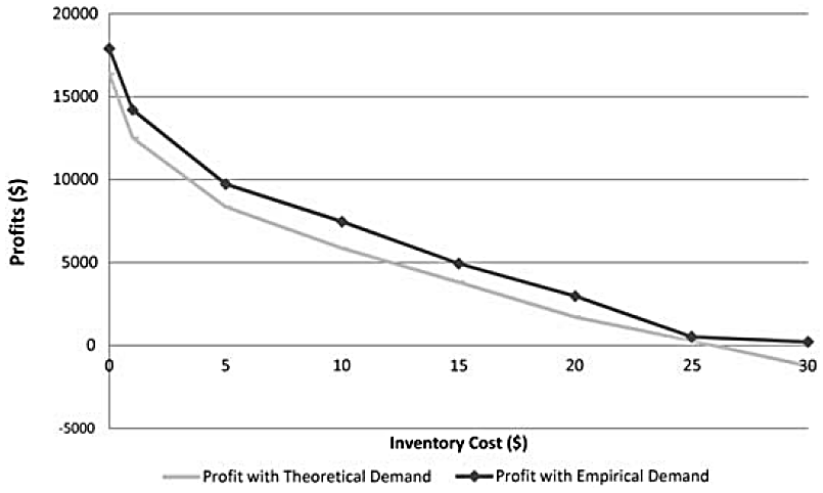

Figure 6 Comparison of Profit for Customer-Driven Model with Theoretical and Empirical Demand

Table 11: Comparison of weekday prices and order quantities to perfect information (cost of $10)

Perfect info		*Additive*		*Customer-driven*		*Fractile*		*Multiplicative*	
Q	*Price*	*Q*	*Price*	*Q*	*Price*	*Q*	*Price*	*Q*	*Price*
45	43.00	51	90.74	31	55.09	41	35.77	11	211.00
43	35.00	51	88.49	34	55.50	43	39.29	14	169.31
41	35.00	52	88.64	34	55.35	43	38.41	14	178.94
21	39.00	51	86.94	34	54.65	43	37.50	18	118.71
11	49.00	50	86.00	34	54.51	43	37.50	20	97.75
14	35.00	50	86.30	34	54.49	43	37.50	19	102.41
26	39.00	51	86.34	34	55.17	43	37.71	17	120.07
46	61.00	50	87.45	30	51.70	41	35.77	17	99.16
28	61.00	52	90.75	33	54.04	41	35.77	11	211.00
22	39.00	51	86.09	34	54.91	43	37.55	20	99.33
107	52.00	31	72.71	31	49.15	41	35.77	25	49.38
15	44.00	51	87.21	34	54.29	44	35.55	18	117.05
31	44.00	51	88.43	33	54.86	44	35.00	14	175.19

Summary

We have provided an illustration of revenue and profit implications resulting from different demand modeling assumptions. The unique nature of our data set – uncensored price dependent demand data – provides a unique opportunity to evaluate different price-dependent demand formulations, the first such illustration to our knowledge. A general takeaway from our data set is that assumptions of any form tend to hinder model performance. The deterministic approximation as developed in Raz and Porteus (2006), while requiring numerous linear approximations, performs well under all circumstances even though it is deterministic in nature. The customer-driven approach as utilized in Wilson and Sorochuk (2009) also performs well and, while stochastic, requires fewer approximations than that of Raz and Porteus (2006). Generally speaking, assuming linear additive or multiplicative demand often results in poor performance with their required assumptions perhaps too restrictive or unrealistic with the given price-demand specification (at least for this sample data set).

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