Covariance Analysis for Split Plot and Split Block Designs and Computer Packages

By

Walter T. Federer and Michael P. Meredith
Mathematical Sciences Institute
Cornell University, Ithaca, N.Y. 14853

ABSTRACT. Covariance analysis for data from experiments designed in a split plot or split block design is mostly ignored in statistical literature. When it is considered, it is often done incorrectly and/or incompletely. This is especially true for computer packages. A discussion of what should be done, what is or can be done with computer packages, and a possible solution to the problems is given. The proposed solution is to obtain computer output for a particular package such as SAS, GENSTAT, BMDP, etc. and to annotate the output explaining which computations have been performed, which have not, and which are still needed. If an incorrect or useless procedure has been given, it is so stated. A short description of annotated computer outputs prepared to date is given. Annotated computer outputs for five packages for principal component analyses, and for three packages for covariance in a split plot design have been prepared. Two technical reports and an annotated computer output have been written for cluster analysis. Copies of these reports are available from the Mathematical Sciences Institute.

COVARIANCE ANALYSIS FOR SPLIT PLOT AND SPLIT BLOCK DESIGNS AND COMPUTER PACKAGES

By

Walter T. Federer and Michael P. Meredith
337 Warren Hall, Biometrics Unit
Cornell University
Ithaca, NY 14853

BU-974-M*

May, 1988

1. INTRODUCTION. Split plot and split block designs appear to be rather mystifying to many individuals. They apparently are not cognizant of the many and varied forms these designs may take, the philosophical nature, concepts, and usage of the several error mean squares that are required, and the nature and use of covariance analyses for these designs. Since the computational procedure for an analysis of variance (ANOVA) for orthogonal split plot and split block designs are trivial, many individuals feel that the concepts are also simple. Computational procedures for an ANOVA do not explain concepts contrary to some opinions.

Yates (1937) described *one* type of split plot design as an example of a *class* of designs. Unfortunately this one type of split plot design is described as THE split plot design in almost all of statistical literature, especially in textbooks. Federer (1955, 1975, 1977) described some variations, some misconceptions, and possible population structures for these designs. With regard to the last point, a glaring omission in statistics textbooks is the failure to include any discussion of population structure for even the simplest of experiment designs. This necessarily raises the question about meaningful inferences when the population is undefined and undescribed.

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* In the Technical Report Series of the Biometrics Unit, Cornell University, Ithaca, N.Y. 14853.

When analyses of covariance (ANCOVA) are attempted, the confusion continues. This becomes strikingly evident in outputs for computer packages purporting to give such analyses for any but the simplest of experiment designs (See, e.g., Federer, 1955, Federer *et al.* 1979, 1987a, 1987b, 1987c, and Searle *et al.* 1982a, 1982b, 1982c). The concept of a separate regression for each error mean square is lacking in a number of computer packages. Hence, if a package does supply output for means adjusted for a covariate, the adjusted means given are often incorrect. The fact that there may be as many regression coefficients as there are error mean squares appears not to be understood. Since many regression coefficients can be and are computed in an ANCOVA, it is important to understand which ones are to be used for adjusting means for covariates and why.

Herein we shall discuss only ANCOVA for three specific designs, i.e.

(i) the standard split plot design where the whole plot treatments are in a randomized complete block design and split plot treatments are randomized within each whole plot,

(ii) a split-split plot design which is the one in (i) except that the split plot is further split to have whole plot treatments, split plot treatments, and split-split plot treatments, and

(iii) a split block design or two-way whole plot design where each set of treatments are in a randodmized complete block design arrangement.

In addition, a list of available annotated computer outputs (ACOs) is given in the last section.

2. Split Plot Experiment Designs. The almost universal split plot experiment design discussed in statistics textbooks is the one wherein the whole plot treatments are in a randomized complete block design and the split plots are completely randomized within each whole plot. Denote this as the standard design. However, Federer (1955, 1975) has pointed out that there is a vast variety of split plot experiment designs which are used in practice. There are many different experiment designs for whole plot treatments as well as for split plot treatments. Also, almost all statistics textbooks confine their discussion to an ANOVA for the standard split plot design with no discussion of an ANCOVA or of an ANOVA for nonorthogonal situations. Computer packages such as SAS, GENSTAT, BMDP,

and others are set up to provide computations for nonorthogonal situations but a full description and use of computer output computations is lacking, resulting in a need for annotating computer output (ACO). S. R. Searle and several co-workers have been very active in this area. A list of ACOs prepared by this group is given later in the paper. It should be noted that Searle is currently updating a number of previously prepared ACOs.

In order to keep this paper relatively short, only the standard (or usual) split plot experiment design will be considered in detail. Many response models may be used for the vast variety of experiments designed as a split plot but we shall confine ourselves to the linear model in Federer (1955). Let the ijk*th* observation Y_{ijk} with an associated covariate Z_{ijk} be represented as follows:

$$Y_{ijk} = \mu + \rho_j + \tau_i + \delta_{ij} + \alpha_k + \alpha\tau_{ik} + \beta_1(\bar{Z}_{ij.} - \bar{Z}_{...}) + \beta_2(Z_{ijk} - \bar{Z}_{ij.}) + \varepsilon_{ijk}, \quad (1)$$

where μ is an overall mean effect, τ_i is the effect of the i*th* whole plot treatment, α_k is the effect of the k*th* subplot treatment, $\alpha\tau_{ik}$ is the interaction effect for the ik*th* combination of whole plot treatment i and split plot treatment k, ρ_j is a random block effect distributed with mean zero and variance σ^2_ρ, δ_{ij} is a random whole plot error effect distributed with mean zero and variance σ^2_δ, ε_{ijk} is a random split plot error effect distributed with mean zero and variance σ^2_ε, $\bar{Z}_{ij.}$ is the mean of the covariate for the ij*th* whole plot, $\bar{Z}_{...}$ is the over-all mean of the covariate (i.e., the usual dot and bar notation), i = 1,..., a, j = 1, ..., r, k = 1, ..., s, β_1 is a whole plot linear regression coeficient of the Y whole plot residuals on the Z whole plot residuals, and β_2 is a split plot linear regression of the Y split plot residuals on the Z split plot residuals. Note that using estimates of β_1 and β_2, i.e, $\hat{\beta}_1$ and $\hat{\beta}_2$, to adjust means is the correct thing to do. The purpose of using covariates is to reduce the variation in observed Y variable means by measuring and using an associated covariate. The reduction must then occur in the error or residual line in the ANOVA. We have encountered individuals who did not use this regression to adjust treatment means but used another regression, e.g., on the total line in the ANCOVA. This is incorrect and possible with present computer packages by eliminating the effect of the covariate first.

In some situations, the formulation of the response model as in (1) is inappropriate. Although (1) could be appropriate for one variable or for one investigation it may not be for another. Also, as formulated (1) has two error effects, the δ_{ij} and ε_{ijk}. When the whole plot treatments, e.g., represent a random sample of treatments from a population, then the τ_i are distributed with mean zero and variance σ_τ^2. An appropriate error term for the fixed split plot treatment effects α_k would be the whole plot by split plot treatment interaction mean square. The $\alpha\tau_{ik}$ would have $E_{i,k}\,\alpha\tau_{ik} = 0$ and variance $\sigma_{\alpha\tau}^2$. Likewise in an ANCOVA, the appropriate regression for split plot treatment means would be computed from the interaction line rather than the error (b) line (see Table 1). In other situations, the split plot treatments or both split plot and whole plot treatments could be considered as a random sample of treatments and the effects would be random rather than fixed effects. Appropriate modifications in ANOVA and ANCOVA would be required for both situations.

A response model for variable Y is formulated and then an ANCOVA as in Table 1 is appropriate for a single covariate Z related to the variable Y in a linear manner. Note that the relation between Y and Z could be polynomial or nonlinear in nature. The number of covariates, say c, may exceed one. This situation may be handled as a straight-forward extension but we shall not consider these additional complexities. For response model equation (1), the ANCOVA is given in Table 1. The sums of products are computed in the usual manner. For example, $T_{yz} = \sum\sum\sum_{ijk} Y_{ijk} Z_{ijk}$, $A_{yz} = \sum_i \sum_j \hat{\delta}_{yij}\, \hat{\delta}_{zij}$, where $\hat{\delta}_{yij}$ is the residual for the variable Y alone and $\hat{\delta}_{zij}$ is the residual for the variable Z alone, and $B_{yz} = \sum_i \sum_j \sum_k \hat{\varepsilon}_{yijk}\, \hat{\varepsilon}_{zijk}$, where the $\hat{\varepsilon}_{hijk}$ are the computed split plot residuals for variable h = y, z. The above computations would still hold even for non-orthogonal experiment designs. The mean squares in ANCOVA are obtained by dividing by the appropriate degrees of freedom. If, in addition to an ANCOVA, it is desired to obtain F-statistics, the ratios $W'_{yy}(ar-r-a) \,/\, A'_{yy}(a-1)$, $S'_{yy}[a(r-1)(s-1)-1] \,/\, B'_{yy}(s-1)$, and $I'_{yy}[a(r-1)(s-1)-1] \,/\, B'_{yy}(a-1)(s-1)$ may be computed. Given that the δ_{ij} and ε_{ijk} are NIID, the probability of obtaining a larger F-statistic may be obtained from prepared tables or computer programs. Even if normality does not hold, the probabilities will be approximately correct for most situations.

Table 1. ANCOVA for equation (1) for a split plot experiment design [1]

Source of Variation	Degrees of Freedom (df)	Sums of Products YY	YZ	ZZ	df	Adjusted Sums Of Squares
Total	ars	T_{yy}	T_{yz}	T_{zz}		
Correction for Mean	1	M_{yy}	M_{yz}	M_{zz}		
Block	(r-1)	R_{yy}	R_{yz}	R_{zz}		
Whole Plot = W	(a-1)	W_{yy}	W_{yz}	W_{zz}		
Error (a)	(a-1)(r-1)	A_{yy}	A_{yz}	A_{zz}	(ar-a-r)	$A_{yy}-A^2_{yz} / A_{zz} = A'_{yy}$
Split Plot = S	(s-1)	S_{yy}	S_{yz}	S_{zz}		
S X W	(a-1)(s-1)	I_{yy}	I_{yz}	I_{zz}	(as-a-s)	$I_{yy}-I^2_{yz} / I_{zz} = I'_{yy}$
Error (b)	a(r-1)(s-1)	B_{yy}	B_{yz}	B_{zz}	a(r-1)(s-1)-1	$B_{yy}-B^2_{yz} / B_{zz} = B'_{yy}$

Whole Plot (adj. for $\hat{\beta}_1$) (a-1)
$$W_{yy} - \frac{(W_{yz} + A_{yz})^2}{W_{zz} + A_{zz}} + \frac{A^2_{yz}}{A_{zz}} = W'_{yy}$$

Split Plot (Adj. for $\hat{\beta}_2$) (s-1)
$$S_{yy} - \frac{(S_{yz} + B_{yz})^2}{S_{zz} + B_{zz}} + \frac{B^2_{yz}}{B_{zz}} = S'_{yy}$$

S X W (Adj. for $\hat{\beta}_2$) (a-1)(s-1)
$$I_{yy} - \frac{(I_{yz} + B_{yz})^2}{I_{zz} + B_{zz}} + \frac{B^2_{yz}}{B_{zz}} = I'_{yy}$$

[1] The various mean squares may be obtained by dividing by the appropriate degrees of freedom.

The various Y means adjusted for the covariate Z are:

$$\bar{Y}_{i..}(adj.) = \bar{Y}_{i..} - \hat{\beta}_1(\bar{Z}_{i..} - \bar{Z}_{...}) = \bar{Y}'_{i..} ,$$

$$\bar{Y}_{..k}(adj.) = \bar{Y}_{..k} - \hat{\beta}_2(\bar{Z}_{..k} - \bar{Z}_{...}) = \bar{Y}'_{..k},$$

and

$$\bar{Y}_{i.k}(adj.) = \bar{Y}_{i.k} - \hat{\beta}_1(\bar{Z}_{i..} - \bar{Z}_{...}) - \hat{\beta}_2(\bar{Z}_{i.k} - \bar{Z}_{i..}) = \bar{Y}'_{i.k} ,$$

where $\hat{\beta}_1 = A_{yz} / A_{zz}$, $\hat{\beta}_2 = B_{yz} / B_{zz}$, and the usual dot notation is used for the various means.

Estimated variances of a difference between two adjusted means for $i \neq i'$ and $k = k'$ are:

Variance of a difference between two adjusted whole plot treatment means

$$V\,(\bar{Y}'_{i..} - \bar{Y}'_{i'..}) = (\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta}) \left[\frac{2}{sr} + \frac{(\bar{Z}_{i..} - \bar{Z}_{i'..})^2}{A_{zz}} \right] .$$

Variance of a difference between two adjusted split plot treatment means

$$V\,(\bar{Y}'_{..k} - Y'_{..k'}) = \hat{\sigma}^2_{\varepsilon} \left[\frac{2}{ar} + \frac{(\bar{Z}_{..k} - \bar{Z}_{..k'})^2}{B_{zz}} \right] .$$

Variance of a difference between two adjusted split plot treatment means for the same whole plot treatment

$$V\,(\bar{Y}'_{i.k} - \bar{Y}'_{i.k'}) = \hat{\sigma}^2_{\varepsilon} \left[\frac{2}{r} + \frac{(\bar{Z}_{i.k} - \bar{Z}_{i.k'})^2}{B_{zz}} \right] .$$

Variance of a difference between two adjusted whole plot treatment means for the same split plot treatment

$$V\,(\bar{Y}'_{i.k} - \bar{Y}'_{i'.k}) = \frac{2}{r}\,(\hat{\sigma}^2_{\varepsilon} + \hat{\sigma}^2_{\delta}) + (\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta})\,\frac{(\bar{Z}_{i..} - \bar{Z}_{i'..})^2}{A_{zz}}$$

$$+\ \hat{\sigma}^2_{\varepsilon}\,\frac{(\bar{Z}_{i.k} - \bar{Z}_{i'.k} - \bar{Z}_{i..} + \bar{Z}_{i'..})^2}{B_{zz}} .$$

$$(\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta}) = A'_{yy} \,/\, (ar-a-r),\quad \hat{\sigma}^2_{\varepsilon} = B'_{YY} \,/\, [a(r-)(s-1)-1] ,$$

$$\text{and } \hat{\sigma}^2_{\delta} = [\ (\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta}) - \hat{\sigma}^2_{\varepsilon}\ ] \,/\, s .$$

$\hat{\sigma}^2_{\varepsilon}$ is associated with a(r-1)(s-1)-1 degrees of freedom, $(\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta})$ is associated with ar-r-a degrees of freedom, and the degrees of freedom for the last variance above are approximated as the degrees of freedom f associated with

$$t_{\alpha}(f) = \frac{(s-1)(\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta})\ t_{\alpha}\ (ar-r-a) + \hat{\sigma}^2_{\varepsilon} t_{\alpha}\ [a(r-1)(s-1)-1]}{(s-1)\ (\hat{\sigma}^2_{\varepsilon} + s\hat{\sigma}^2_{\delta}) + \hat{\sigma}^2_{\varepsilon}}$$

where t_α (f) is the tabulated value of the t-statistic at the α percentage level for f degrees of freedom. This approximation underestimates the degrees of freedom for this variance (see Cochran and Cox, 1950, and Grimes and Federer, 1984).

Given the above variances, one may now use a multiple range procedure to compare individual pairs of means. Some authors (e.g. Cochran and Cox, 1950) consider that there is a correlation between the split plot experimental units. Hence, the whole plot expected error mean square would be given as σ^2 and the split plot error would be written as $\sigma^2(1-\rho) = \sigma^2_\varepsilon$ where the correlation ρ is equal to $s\sigma^2_\delta / \sigma^2$. Although this formulation is useful for many situations it is not of universal application; e.g. when measurement error or competition exists between split plot experimental units but not between whole plot experimental units. Statistical modeling for any investigation should be carefully considered.

3. <u>Split-Split Plot Experiment Designs</u>. For this class of designs, various experiment designs may be used for whole plot treatments, for split plot treatments, and for split-split plot treatments. However, we shall confine our remarks to a single member of this class, i.e., the whole plot treatments are arranged in a randomized complete blocks design, the split plot treatments are randomly allocated to the split plot experimental units within each whole plot unit, and the split-split plot treatments are randomly assigned to the split-split plot experimental units within each split plot experimental unit. There will be r randomizations for the a whole plot treatments, ra randomizations for the s split plot treatments, and ras randomizations for the p split-split plot treatments. The

treatment design considered here is a three factor factorial with asp combinations, but it should be noted that other treatment designs are possible. The factors are assumed to be fixed effects to simplify presentation.

One possible response model for the above experiment and treatment design for a variable Y with a covariate Z is:

$$Y_{hijk} = \mu + \rho_h + \tau_i + \delta_{hi} + \beta_1 (\bar{Z}_{hi..} - \bar{Z}_{....}) + \alpha_j + \alpha\tau_{ij} + \varepsilon_{hij}$$
$$+ \beta_2 (\bar{Z}_{hij.} - \bar{Z}_{hi..}) + \gamma_k + \gamma\tau_{ik} + \alpha\gamma_{jk} + \alpha\gamma\tau_{ijk} + \pi_{hijk}$$
$$+ \beta_3 (Z_{hijk} - \bar{Z}_{hij.}) \quad , \qquad (2)$$

where the first nine effects are as defined for equation (1), γ_k is the effect of the *kth* split-split plot treatment, $\gamma\tau_{ik}$ is a two-factor interaction effect for combination ik, $\alpha\gamma_{jk}$ is a two-factor interaction effect for combination jk, $\alpha\gamma\tau_{ijk}$ is a three-factor interaction effect for combination ijk, π_{hijk} is a random error effect associated with split-split plot experimental unit hijk and distributed with mean zero and variance σ^2_π, β_3 is a linear regression coefficient of the split-split plot Y residuals on the corresponding Z residuals, h = 1, ..., r, i = 1, ..., a, j = 1, ..., s, and k = 1, ..., p. An ANCOVA for this design and response model is given in Table 2.

The various adjusted means are computed as:

$$\bar{Y}_{.i..}(adj.) = \bar{Y}_{.i..} - \hat{\beta}_1(\bar{Z}_{.i..} - \bar{Z}_{....}) = \bar{Y}'_{.i..} \quad ,$$

$$\bar{Y}_{..j.}(adj.) = \bar{Y}_{..j.} - \hat{\beta}_2(\bar{Z}_{..j.} - \bar{Z}_{....}) = \bar{Y}'_{..j.} \quad ,$$

$$\bar{Y}_{...k}(adj.) = \bar{Y}_{...k} - \hat{\beta}_3(\bar{Z}_{...k} - \bar{Z}_{....}) = \bar{Y}'_{...k} \quad ,$$

$$\bar{Y}_{.ij.}(adj.) = \bar{Y}_{.ij.} - \hat{\beta}_1(\bar{Z}_{.i..} - \bar{Z}_{....}) - \hat{\beta}_2(\bar{Z}_{.ij.} - \bar{Z}_{.i..}) = \bar{Y}'_{.ij.} \quad ,$$

$$\bar{Y}_{.i.k}(adj.) = \bar{Y}_{.i.k} - \hat{\beta}_1(\bar{Z}_{.i..} - \bar{Z}_{....}) - \hat{\beta}_3(\bar{Z}_{.i.k} - \bar{Z}_{.i..}) = \bar{Y}'_{.i.k} \quad ,$$

$$\bar{Y}_{..jk}(adj.) = \bar{Y}_{..jk} - \hat{\beta}_2(\bar{Z}_{..j.} - \bar{Z}_{....}) - \hat{\beta}_3(\bar{Z}_{..jk} - \bar{Z}_{.j..}) = \bar{Y}'_{..jk} \quad ,$$

Table 2. ANCOVA for equation (2) for a split-split plot experiment design [1]

Source of Variation	df	Sums of Products yy	yz	zz	df	Adjusted Sums Of Squares
Total	rasp	T_{yy}	T_{yz}	T_{zz}		
Correction for Mean	1	M_{yy}	M_{yz}	M_{zz}		
Block	(r-1)	R_{yy}	R_{yz}	R_{zz}		
Whole Plot = W	(a-1)	W_{yy}	W_{yz}	W_{zz}		
Error (a)	(a-1)(r-1)	A_{yy}	A_{yz}	A_{zz}	(ar-r-a)	$A_{yy}-A^2_{yz} / A_{zz} = A'_{yy}$
Split Plot = S	(s-1)	S_{yy}	S_{yz}	S_{zz}		
S X W	(a-1)(s-1)	I_{yy}	I_{yz}	I_{zz}		
Error (b)	a(r-1)(s-1)	B_{yy}	B_{yz}	B_{zz}	a(r-1)(s-1)-1	$B_{yy}-B^2_{yz} / B_{zz} = B'_{yy}$
Split-Split Plot = P	(p-1)	P_{yy}	P_{yz}	P_{zz}		
W X P	(a-1)(p-1)	Q_{yy}	Q_{yz}	Q_{zz}		
S X P	(p-1)(s-1)	U_{yy}	U_{yz}	U_{zz}		
W X S X P	(a-1)(p-1)·(s-1)	V_{yy}	V_{xy}	V_{zz}		
Error (c)	as(r-1)(p-1)	C_{yy}	C_{yz}	C_{zz}	as(r-1)(p-1)-1	$C_{yy}-C^2_{yz} / C_{zz} = C'_{yy}$

W(adj. for $\hat{\beta}_1$)	(a-1)	$W_{yy} - (W_{yz} + A_{yz})^2 / (W_{zz} + A_{zz}) + A^2_{yz} / A_{zz} = W'_{yy}$
S(adj. for $\hat{\beta}_2$)	(s-1)	$S_{yy} - (S_{yz} + B_{yz})^2 / (S_{zz} + B_{zz}) + B^2_{yz} / B_{zz} = S'_{yy}$
SXW(adj. for $\hat{\beta}_2$)	(a-1)(s-1)	$I_{yy} - (I_{yz} + B_{yz})^2 / (I_{zz} + B_{zz}) + B^2_{yz} / B_{zz} = I'_{yy}$
P(adj. for $\hat{\beta}_3$)	(p-1)	$P_{yy} - (P_{yz} + C_{yz})^2 / (P_{zz} + C_{zz}) + C^2_{yz} / C_{zz} = P'_{yy}$
WXP(adj. for $\hat{\beta}_3$)	(a-1)(p-1)	$Q_{yy} - (Q_{yz} + C_{yz})^2 / (Q_{zz} + C_{zz}) + C^2_{yz} / C_{zz} = Q'_{yy}$
SXP(adj. for $\hat{\beta}_3$)	(p-1)(s-1)	$U_{yy} - (U_{yz} + C_{yz})^2 / (U_{zz} + C_{zz}) + C^2_{yz} / C_{zz} = U'_{yy}$
WXSXP(adj. for $\hat{\beta}_3$)	(a-1)(p-1)(s-1)	$V_{yy} - (V_{yz} + C_{yz})^2 / (V_{zz} + C_{zz}) + C^2_{yz} / C_{zz} = V'_{yy}$

[1] The various mean squares may be obtained by dividing by the appropriate degrees of freedom.

and

$$\bar{Y}_{.ijk}\,(adj.) = \bar{Y}_{.ijk} - \hat{\beta}_1(\bar{Z}_{.i..} - \bar{Z}_{....}) - \hat{\beta}_2(\bar{Z}_{.ij.} - \bar{Z}_{.i..})$$

$$- \hat{\beta}_3(\bar{Z}_{.ijk} - \bar{Z}_{.ij.}) = \bar{Y}'_{.ijk} \quad ,$$

where $\hat{\beta}_1 = A_{yz} / A_{zz}$, $\hat{\beta}_2 = B_{yz} / B_{zz}$, and $\hat{\beta}_3 = C_{yz} / C_{zz}$.

Estimated variances of a difference between two means adjusted for a covariate for $i \neq i'$, $j \neq j'$, $E_a = A'_{yy} / (ar-r-a)$, $E_b = B'_{yy} / [a(r-1)(s-1)-1]$, and $\hat{\sigma}^2_\pi = C'_{yy} / [as(r-1)(p-1)-1]$ are given below:

Variance of a difference between two whole plot treatment adjusted means

$$V(\bar{Y}'_{.i..} - \bar{Y}'_{.i'..}) = E_a \left[\frac{2}{rsp} + \frac{(\bar{Z}_{.i..} - \bar{Z}_{.i'..})^2}{A_{zz}} \right] .$$

Variance of a difference between two split plot treatment adjusted means

$$V(\bar{Y}'_{..j.} - \bar{Y}'_{..j'.}) = E_b \left[\frac{2}{arp} + \frac{(\bar{Z}_{..j.} - \bar{Z}_{..j'.})^2}{B_{zz}} \right] .$$

Variance of a difference between two split-split plot treatment adjusted means

$$V(\bar{Y}'_{...k} - \bar{Y}'_{...k'}) = \hat{\sigma}^2_\pi \left[\frac{2}{ars} + \frac{(\bar{Z}_{...k} - \bar{Z}_{...k'})^2}{C_{zz}} \right] .$$

Variance of a difference between two adjusted means for combinations ij and ij'

$$V(\bar{Y}'_{.ij.} - \bar{Y}'_{.ij'.}) = E_b \left[\frac{2}{rp} + \frac{(\bar{Z}_{.ij.} - \bar{Z}_{.ij'.})^2}{B_{zz}} \right] .$$

Variance of a difference between two adjusted means for combinations ij and i'j

$$V(\bar{Y}'_{.ij.} - \bar{Y}'_{.i'j.}) = \frac{2}{rp} [\hat{\sigma}^2_\delta + \hat{\sigma}^2_\varepsilon] + E_a \frac{(\bar{Z}_{.i..} - \bar{Z}_{.i'..})^2}{A_{zz}}$$

$$+ E_b \frac{(\bar{Z}_{.ij.} - \bar{Z}_{.i..} - \bar{Z}_{.i'j.} + \bar{Z}_{.i'..})^2}{B_{zz}} .$$

Variance of a difference between two adjusted means for combinations ik and ik'

$$V(\bar{Y}'_{.i.k} - \bar{Y}'_{.i.k'}) = \hat{\sigma}^2_\pi \left[\frac{2}{rs} + \frac{(\bar{Z}_{.i.k} - \bar{Z}_{.i.k'})^2}{C_{zz}} \right] .$$

Variance of a difference between two adjusted means for combinations ik and i'k

$$V(\bar{Y}'_{.i.k} - \bar{Y}'_{.i'.k}) = \frac{2(s\hat{\sigma}^2_\delta + \hat{\sigma}^2_\varepsilon + \hat{\sigma}^2_\pi)}{rs} + E_a \frac{(\bar{Z}_{.i..} - \bar{Z}_{.i'..})^2}{A_{zz}}$$

$$+ \frac{\hat{\sigma}^2_\pi (\bar{Z}_{.i.k} - \bar{Z}_{.i..} - \bar{Z}_{.i'.k} + \bar{Z}_{.i'..})^2}{C_{zz}} \quad .$$

Variance of a difference between two adjusted means for combinations ijk and ijk'

$$V(\bar{Y}'_{.ijk} - \bar{Y}'_{.ijk'}) = \frac{2}{r}(\hat{\sigma}^2_\pi) + \hat{\sigma}^2_\pi \frac{(\bar{Z}_{.ijk} - \bar{Z}_{.ijk'})^2}{C_{zz}} \quad .$$

Variance of a difference between two adjusted means for combinations ijk and ij'k

$$V(\bar{Y}'_{.ijk} - \bar{Y}'_{ij'k}) = \frac{2}{r}(\hat{\sigma}^2_\varepsilon + \hat{\sigma}^2_\pi) + \frac{E_b}{B_{zz}} \frac{(\bar{Z}_{.ij.} - \bar{Z}_{.ij'.})^2}{}$$

$$+ \frac{\hat{\sigma}^2_\pi}{C_{zz}} \frac{(\bar{Z}_{.ijk} - \bar{Z}_{.ij.} - \bar{Z}_{.ij'k} + \bar{Z}_{.ij'.})^2}{} \quad .$$

Variance of a difference between two adjusted means for combinations ijk and i'jk

$$V(\bar{Y}'_{.ijk} - \bar{Y}'_{.i'jk}) = \frac{2}{r}(\hat{\sigma}^2_\delta + \hat{\sigma}^2_\varepsilon + \hat{\sigma}^2_\pi) + \frac{E_a}{A_{zz}} \frac{(\bar{Z}_{.i..} - \bar{Z}_{.i'..})^2}{}$$

$$+ \frac{E_b}{B_{zz}} (\bar{Z}_{.ij.} - \bar{Z}_{.i..} - \bar{Z}_{.i'j.} + \bar{Z}_{.i'..})^2$$

$$+ \frac{\hat{\sigma}^2_\pi}{C_{zz}} (\bar{Z}_{.ijk} - \bar{Z}_{.ij.} - \bar{Z}_{.i'jk} + \bar{Z}_{.i'j.})^2 \quad .$$

Note that $V(\bar{Y}'_{.ijk} - \bar{Y}_{.i'jk}) = V(\bar{Y}'_{.ijk} - \bar{Y}'_{.i'j'k}) = V(\bar{Y}'_{.ijk} - \bar{Y}'_{.i'jk'})$

$= V(\bar{Y}'_{.ijk} - \bar{Y}'_{.i'j'k'})$ and that $V(\bar{Y}'_{.ijk} - \bar{Y}'_{.ij'k}) = V(\bar{Y}'_{.ijk} - \bar{Y}_{.ij'k'})$.

Most variances above without the covariate were given by Federer (1955). Also, the expected values of E_a and E_b are $\sigma^2_\pi + p\sigma^2_\varepsilon + ps\sigma^2_\delta$ and $\sigma^2_\pi + p\sigma^2_\varepsilon$, respectively. Estimates of variance components σ^2_δ, σ^2_ε, and $E(\hat{\sigma}^2_\pi) = \sigma^2_\pi$ are needed to compute the fifth, seventh, ninth, and tenth variances above.

The degrees of freedom for these variances need to be approximated as they were in the previous section. Also note that $ps(\hat{\sigma}^2_\pi + \hat{\sigma}^2_\varepsilon + \hat{\sigma}^2_\delta) = s(p-1)\hat{\sigma}^2_\pi + (s-1)E_b + E_a$ and $p(\hat{\sigma}^2_\pi + \hat{\sigma}^2_\varepsilon) = (p-1)\hat{\sigma}^2_\pi + E_b$.

4. Split Block Experiment Design. The experiment design considered here is denoted as a split block design. It has also been called a two-way whole plot and a strip trial design. This design has received no attention in statistical textbooks with an exception being Federer (1955). It does occur frequently in practice but sometimes is not analyzed correctly as a split block design. The member of this class of designs we shall discuss will be for a two-factor factorial treatment design with the levels of one factor being applied perpendicularly across all levels of the second factor within each replicate or complete block. The levels of each factor will have the same design for our example, that is a randomized complete block design. (The levels of one factor could be in a randomized complete block design and the levels of the second factor could be in a latin square, balanced incomplete block, or other experiment design.) Note that there will be r separate randomizations for the levels of each of the factors. The number of levels of factor one is a and the number of levels of the second factor is b, resulting in an a × b factorial treatment design.

A response model equation as given in Federer (1955) for a variable Y and a covariate Z is:

$$Y_{hij} = \mu + \rho_h + \alpha_i + \delta_{hi} + \gamma_j + \pi_{kj} + \alpha\gamma_{ij} + \varepsilon_{hij} + \beta_1(\bar{Z}_{hi\cdot} - \bar{Z}_{\cdots}) + \beta_2(\bar{Z}_{h\cdot j} - \bar{Z}_{\cdots}) + \beta_3(Z_{hij} - \bar{Z}_{hi\cdot} - \bar{Z}_{h\cdot j} + \bar{Z}_{\cdots}) , \quad (3)$$

where μ is a general mean effect, ρ_h is the h*th* block effect, which has mean zero and variance σ^2_ρ, α_i is the effect of the i*th* level of factor one, say A, γ_j is the effect of the j*th* level of factor two, say B, δ_{hi} is a random error effect for the hi*th* whole plot for factor A and has mean zero and variance σ^2_δ, π_{hi} is a random error effect for the hj*th* whole plot for factor B and has mean zero and variance σ^2_π, $\alpha\gamma_{ij}$ is the interaction effect for the ij*th* combination of levels of factors A and B, ε_{hij} is a random error effect associated with the hij*th* subplot for the

$A \times B$ interaction and has mean zero and variance σ_ε^2, β_1 is the linear regression of Y whole plot residuals on the Z whole plot residuals for factor A, β_2 is the linear regression of the Y whole plot residuals on the Z whole plot residuals for factor B, and β_3 is the linear regression of Y subplot residuals on Z subplot residuals.

An ANCOVA for response model (3) is given in Table 3. For this design and for fixed effects for the $a \times b$ factorial, there are three error variances and three error regressions. Given that the error effects are NIID, the usual F statistics may be used if desired. The adjusted means are given by:

$$\bar{Y}_{\cdot i \cdot}(\text{adjusted}) = \bar{Y}_{\cdot i \cdot} - \hat{\beta}_1(\bar{Z}_{\cdot i \cdot} - \bar{Z}_{\cdot\cdot\cdot}) = \bar{Y}'_{\cdot i \cdot} \quad ,$$

$$\bar{Y}_{\cdot\cdot j}(\text{adjusted}) = \bar{Y}_{\cdot\cdot j} - \hat{\beta}_2(\bar{Z}_{\cdot\cdot j} - \bar{Z}_{\cdot\cdot\cdot}) = \bar{Y}'_{\cdot\cdot j} \quad ,$$

and

$$\bar{Y}_{\cdot ij}(\text{adjusted}) = \bar{Y}_{\cdot ij} - \beta_1(\bar{Z}_{\cdot i \cdot} - \bar{Z}_{\cdot\cdot\cdot}) - \beta_2(\bar{Z}_{\cdot\cdot j} - \bar{Z}_{\cdot\cdot\cdot}) -$$
$$\hat{\beta}_3(\bar{Z}_{\cdot ij} - \bar{Z}_{\cdot i \cdot} - \bar{Z}_{\cdot\cdot j} + \bar{Z}_{\cdot\cdot\cdot}) = \bar{Y}'_{\cdot ij} \quad ,$$

where the $\hat{\beta}$s are defined in Table 3.

Estimated variances of a difference between adjusted means are given below for $i \neq i'$, $j \neq j'$:

$$V(\bar{Y}'_{\cdot i \cdot} - \bar{Y}'_{\cdot i' \cdot}) = E_a\left[\frac{2}{rb} + \frac{(\bar{Z}_{\cdot i \cdot} - \bar{Z}_{\cdot i' \cdot})^2}{A_{zz}}\right] \quad ,$$

$$V(\bar{Y}'_{\cdot\cdot j} - \bar{Y}'_{\cdot\cdot j'}) = E_b\left[\frac{2}{ab} + \frac{(\bar{Z}_{\cdot\cdot j} - \bar{Z}_{\cdot\cdot j'})^2}{B_{zz}}\right] \quad ,$$

$$V(\bar{Y}'_{\cdot ij} - \bar{Y}'_{\cdot ij'}) = \frac{1}{r}(\hat{\sigma}_\pi^2 + \hat{\sigma}_\varepsilon^2) + \frac{E_b}{B_{zz}}(\bar{Z}_{\cdot\cdot j} - \bar{Z}_{\cdot\cdot j'})^2 + \frac{E_c}{C_{zz}}(\bar{Z}_{\cdot ij} - \bar{Z}_{\cdot ij'} - \bar{Z}_{\cdot\cdot j} + \bar{Z}_{\cdot\cdot j'})^2 \quad ,$$

$$V(\bar{Y}'_{\cdot ij} - \bar{Y}'_{\cdot i'j}) = \frac{1}{r}(\hat{\sigma}_\delta^2 + \hat{\sigma}_\varepsilon^2) + \frac{E_a}{A_{zz}}(\bar{Z}_{\cdot i \cdot} - \bar{Z}_{\cdot i' \cdot})^2$$
$$+ \frac{E_c}{C_{zz}}(\bar{Z}_{\cdot ij} - \bar{Z}_{\cdot i'j} - \bar{Z}_{\cdot i \cdot} + \bar{Z}_{\cdot i' \cdot})^2 \quad ,$$

and

$$V(\bar{Y}'_{\cdot ij} - \bar{Y}'_{\cdot i'j'}) = \frac{1}{r}(\hat{\sigma}^2_\delta + \hat{\sigma}^2_\pi + \hat{\sigma}^2_\varepsilon) + \frac{E_a}{A_{zz}}(\bar{Z}_{\cdot i\cdot} - \bar{Z}_{\cdot i'\cdot})^2 + \frac{E_b}{B_{zz}}(\bar{Z}_{\cdot\cdot j} - \bar{Z}_{\cdot\cdot j'})^2$$

$$+ \frac{E_c}{C_{zz}}(\bar{Z}_{\cdot ij} - \bar{Z}_{\cdot i'j'} - \bar{Z}_{\cdot i\cdot} + \bar{Z}_{\cdot i'\cdot} - \bar{Z}_{\cdot\cdot j} + \bar{Z}_{\cdot\cdot j'})^2 ,$$

where $E_a = A'_{yy}/(ar-a-r) = \hat{\sigma}^2_\varepsilon + b\hat{\sigma}^2_\delta$, $E_b = B'_{yy}/(br-b-r) = \hat{\sigma}^2_\varepsilon + a\hat{\sigma}^2_\pi$,

and $E_c = C'_{yy}/[(a-1)(b-1)(r-1)-1] = \hat{\sigma}^2_\varepsilon$.

The degrees of freedom for the last three variances need to be approximated by the method previously given or by some other appropriate approximation (See e.g., Grimes and Federer, 1984).

Table 3. ANCOVA for equation (3) for a split block experiment design.

Source of variation	df	Sum of products	df Adjusted sums of squares
Total	rab	T_{yy} T_{yz} T_{zz}	
Correction for mean	1	M_{yy} M_{yz} M_{zz}	
Replicate = R	(r-1)	R_{yy} R_{yz} R_{zz}	
Whole plot A	(a-1)	W_{yy} W_{yz} W_{zz}	
Error (a)	(r-1)(a-1)	A_{yy} A_{yz} A_{zz}	$(ra-a-r)$ $A_{yy} - \frac{A^2_{yz}}{A_{zz}} = A'_{yy}$
Whole A adjusted for $\hat{\beta}_1 = A_{yz}/A_{zz}$			$(a-1)$ $W_{yy} - \frac{(W_{yz}+A_{yz})^2}{W_{zz}+A_{zz}} + \frac{A^2_{yz}}{A_{zz}} = W'_{yy}$
Whole plot B	(b-1)	S_{yy} S_{yz} S_{zz}	
Error (b)	(b-1)(r-1)	B_{yy} B_{yz} B_{zz}	$(rb-b-r)$ $B_{yy} - \frac{B^2_{yz}}{B_{zz}} = B'_{yy}$
Whole plot B adjusted for $\hat{\beta}_2 = B_{yz}/B_{zz}$			$(b-1)$ $S_{yy} - \frac{(S_{yz}+B_{yz})^2}{S_{zz}+B_{zz}} + \frac{B^2_{yz}}{B_{zz}} = S'_{yy}$
A X B	(a-1)(b-1)	I_{yy} I_{yz} I_{zz}	
Error (ab)	(r-1)(a-1)(b-1)	C_{yy} C_{yz} C_{zz}	$(r-1)(a-1)(b-1)-1$ $C_{yy} - \frac{C^2_{yz}}{C_{zz}} = C'_{yy}$
Interaction adjusted for $\hat{\beta}_3 = C_{yz}/C_{zz}$			$(a-1)(b-1)$ $I_{yy} - \frac{(I_{yz}+C_{yz})^2}{I_{zz}+C_{zz}} + \frac{C^2_{yz}}{C_{zz}} = I'_{yy}$

5. Some Comments. Since formulas for many of the above adjusted means and variances do not appear in statistical literature, it was deemed appropriate to include them here. As can be seen from the analyses for relatively simple designs from each of the three classes, there are a variety of formulas for adjusted means and variances of differences between two adjusted means. The more complex members of each class may have 5, 10, 15, or 20 error mean squares and the same number of regression coefficients. Experiments are conducted wherein some of the factors are arranged in split blocks and others in split plot arrangements. Many different designs may be used for the different factors (See e.g., Federer, 1955, 1975). The most complex experiment design encountered is described by Federer and Farden (1955), where there are several split plot and several split block arrangements with a total of 75 error mean squares and 203 lines in the ANOVA.

One method of aiding investigators with ANOVAs and ANCOVAs of complexly designed experiments is to ascertain how much of a statistical analysis can be obtained with computer packages such as SAS, BMDP, GENSTAT, SPSS, and others. Then, the output can be annotated, i.e. an explanation is appended to the computer output describing what has been computed and how to use the results. Annotated computer outputs for two different split plot designs with a covariate have been completed for SAS, BMDP, and GENSTAT (see Federer *et al.* 1987a, 1987b, 1987c). In addition to these covariance analyses, annotated computer outputs have been prepared for principal component analysis from five computer packages and the mixture method of cluster analysis on SAS. A listing of these is given in Appendix A. A second list of material available from the Biometrics Unit is given in Appendix B.

The analyses have been described for a single covariate. Noting that $A_{yy} - A^2_{yz}/A_{zz} = A_{yy}(1-r^2_{yz}) = A'_{yy}$, one may simply use $A_{yy}(1-R^2) = A'_{yy}$ when there are several covariates and where R^2 is the squared multiple correlation coefficient computed on the error line. If the relationship between a covariate Z and Y is curvilinear, it may be possible to use some function of Z, e.g. log Z, $\sqrt{Z}$, 1/Z, which makes the relation linear. If this can be accomplished both computations and interpretations are simplified.

A simplification of the estimated variances for differences of means has been given by Yates (1934) and Finney (1946). Instead of computing the quantities $(\bar{Z}_{.i.} - \bar{Z}_{.i'.})^2 / A_{zz}$ and $(\bar{Z}_{..j} - \bar{Z}_{..j'})^2 / B_{zz}$, e.g, for each pair of means, one may compute a single variance by using $W_{xx}/(a-1)A_{zz}$ and $S_{xx}/(s-1)B_{zz}$, respectively. The quantity $W_{xx}/(a-1)$ is an average of all pairs ii' of $(\bar{Z}_{.i.} - \bar{Z}_{.i'.})^2$. This simplication and approximation considerably reduces the number of computations for large a and/or s. For the quantities $(\bar{Z}_{.ij} - Z_{.ij'} - \bar{Z}_{.i.} + \bar{Z}_{.i'.})^2$ and $(\bar{Z}_{.ij} - \bar{Z}_{.i'j} - \bar{Z}_{..j} + \bar{Z}_{..j.})^2$ it is suggested that $I_{xx} / (a-1)(s-1)B_{zz}$ be used if it is desired to compute only a single variance.

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