Stochastic Dominance in Mobility Analysis

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Abstract

This paper introduces a technique for mobility dominance and compares the degree of earnings mobility of men in the USA from 1970 to 1995. The highest mobility is found in the 1975–1980 or 1980–1985 periods.

1. Introduction

The purpose of this paper is to introduce dominance methods into the analysis of income mobility. As far as we are aware, the only preceding works to have used dominance methods in the income mobility field are our own unpublished and now-obsolete paper (Fields et al., 1998) and two unpublished doctoral dissertations (Leary, 1999; Freije, 2001). The goal here is to find a method that will command broad agreement on which of two mobility situations, *X* or *Y*, is more mobile.

2. Methods

2.1. Mobility distributions

In order to make mobility comparisons using dominance methods, it is necessary to define the notion of mobility distributions. The mobility of each individual will be expressed as the change in the value of some function of his or her own income and the incomes of others. For example, in the case of share movement, each individual's mobility is the change in his or her share of the total income of the population. The existence of dominance can be determined by comparing the distributions of individual mobility. To be more precise, denote the base and final incomes of individual i by y_{0i} and y_{1i}, respectively, both in real dollars, and denote the corresponding income vectors in the population by $Y_0 \equiv (Y_{01},\ldots, Y_{on})$ and $Y_1 \equiv (Y_{11},\ldots, Y_{1n})$. The expression $Y_0 \rightarrow Y_1$ means that the income distribution in a given population of size n changes from the (personalized) vector Y_0 to the vector Y_1. We will use X to represent another time period or group and define $X_0 \rightarrow X_1$ analogously.

The amount of mobility experienced by the i'th individual can be written $g(Y_{1i}, Y_1) - g(Y_{0i}, Y_0)$ or $|g(Y_{1i}, Y_1) - g(Y_{0i}, Y_0)|$, where $g(\bullet)$ is the appropriate function of the individual's

income and the incomes of the other members of the income distribution. Mobility in the economy is then an aggregation of these individual mobility experiences.

The following subsection discusses the types of g (•) functions that have been used in the mobility literature.

2.2. Concepts of mobility

Mobility *measures* are meant to represent mobility *concepts*. Several mobility measures are available for each mobility concept; it is important for the analyst to be clear on which concept is of interest and which measures represent it.

Much of the mobility literature concentrates on *positional movement*, which is the change in quintile, decile, centile, or rank of the *i*'th individual (Atkinson et al., 1992; Danziger and Gottschalk, 1995; OECD, 1996). Some of the literature is concerned with share movement, viz., the change in the income share of the *i*'th individual. A third concept is *income flux*, which is concerned with the variability or uncertainty of incomes but not their direction (Fields and Ok, 1996, 1999b). Finally, we have directional income movement, which indicates whether the income change is positive or negative and by how much incomes have changed.

For more on these different concepts, see Fields and Ok (1999a) or Fields (2001).

2.3. Stochastic dominance in mobility analysis

Dominance comparisons can be used to aggregate the individual mobility experiences gauged by one or another g (•) function in order to compare the economy-wide mobilities in different time periods or between different groups. In this paper, we use first-order stochastic dominance, the most stringent dominance criterion (Hadar and Russell, 1969; Saposnik, 1981). Consider the vectors of individual mobility for two income change situations, $X_0 \rightarrow X_1$ and $Y_0 \rightarrow Y_1$. For simplicity, call these two change vectors *X* and *Y*, and assume each has n elements, denoted X_i and Y_i. In order to make a first-order stochastic dominance comparison of *X* and *Y*, their elements are first placed in ascending order. If $X \geq Y$ for all *i* with one strict inequality somewhere, then *X* is said to first-order stochastically dominate Y. This is equivalent to the cumulative distribution function (CDF) of *X* lying someplace to the right and never to the left of the CDF of Y. The CDF comparison is useful in empirical applications, because it makes it possible to generalize to cases where the change vectors being compared are not of the same dimension.

When one CDF dominates another in the manner just described, what can we conclude about the degree of mobility in the two situations? The main theorem in stochastic dominance is this. If *X* first-order stochastically dominates *Y*, then any measure $M:R^N \rightarrow R$ that is anonymous and weakly increasing in all of its elements will give a higher value for the mobility of *X* than the mobility of *Y* (Hadar and Russell, 1969). 'Anonymity' in this context means that all of the relevant information about the constituent individuals is contained in the vectors *X* and *Y*, and all constituent individuals are treated the same. 'Increasing' means that if any individual's mobility increases, so too does the aggregate measure M.

If *X* does not first-order stochastically dominate *Y*, and *Y* does not first-order stochastically dominate *X*, then there will be some measures in this broad class that give a larger value for *X* than *Y* and others that give a larger value for *Y* than *X*. Because of this, in such cases, the mobility measure to use must be chosen with extreme care.

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It is also useful to introduce a new concept: the degree of stochastic dominance. In comparing two income change situations *X* and *Y,* we order the individual mobilities in each situation from most negative to most positive and then partition each distribution of changes into p equal-sized intervals. Let *m* be the number of times that the CDF of *X* lies below (or to the right of) that of *Y* at each of the p quantile markers. The ratio (m/p)*100 is then the degree of dominance of *X* over *Y,* which we denote $D_{m/p}$. If *X* stochastically dominates *Y,* so that the CDF of *X* is everywhere above the CDF of *Y,* then *X*D 100Y. In cases of CDF crossings, the $D_{m/p}$ index indicates how close to dominant one distribution is over another.

As a practical matter, comparisons of CDF's are often difficult to make visually, especially when more than two curves are being compared on the same graph. An equivalent but easier-to-read presentation is to plot the difference between one change situation and another. Each curve shows the difference in the share of the sample with changes at least as great as the value on the *X*-axis, so when the curve is negative a smaller share of the sample has changes at least as great as the reference situation. Therefore, whenever one difference curve is everywhere negative, it means that the situation plotted is dominated by the reference situation. Likewise, when two difference curves are plotted on the same diagram and if one (call it P) lies below another (Q), then P is mobility-dominated by Q.

3. Application: changing earnings mobility in the USA, 1970-1995

Using the Panel Study of Income Dynamics for the USA, we construct five panels of earnings at 5-year intervals: 1970-1975, 1975-1980, 1980-1985, 1985-1990 and 1990-1995. Each panel includes earnings in the first and final year of the period. For each panel the sample consists of men aged 25-55 in the base year who were not students, retired, or self-employed, and who had positive earnings (including overtime and bonuses) in both years.

The results of the mobility comparisons for the different concepts are presented in two sets of figures. In each case, the reference period is 1980-1985. In Fig. 1, data for earlier periods are compared with 1980-1985, while in Fig. 2, data for the later periods are compared with 1980-1985.

Looking first at positional movement, we gauge this by the absolute number of centiles changed[1].

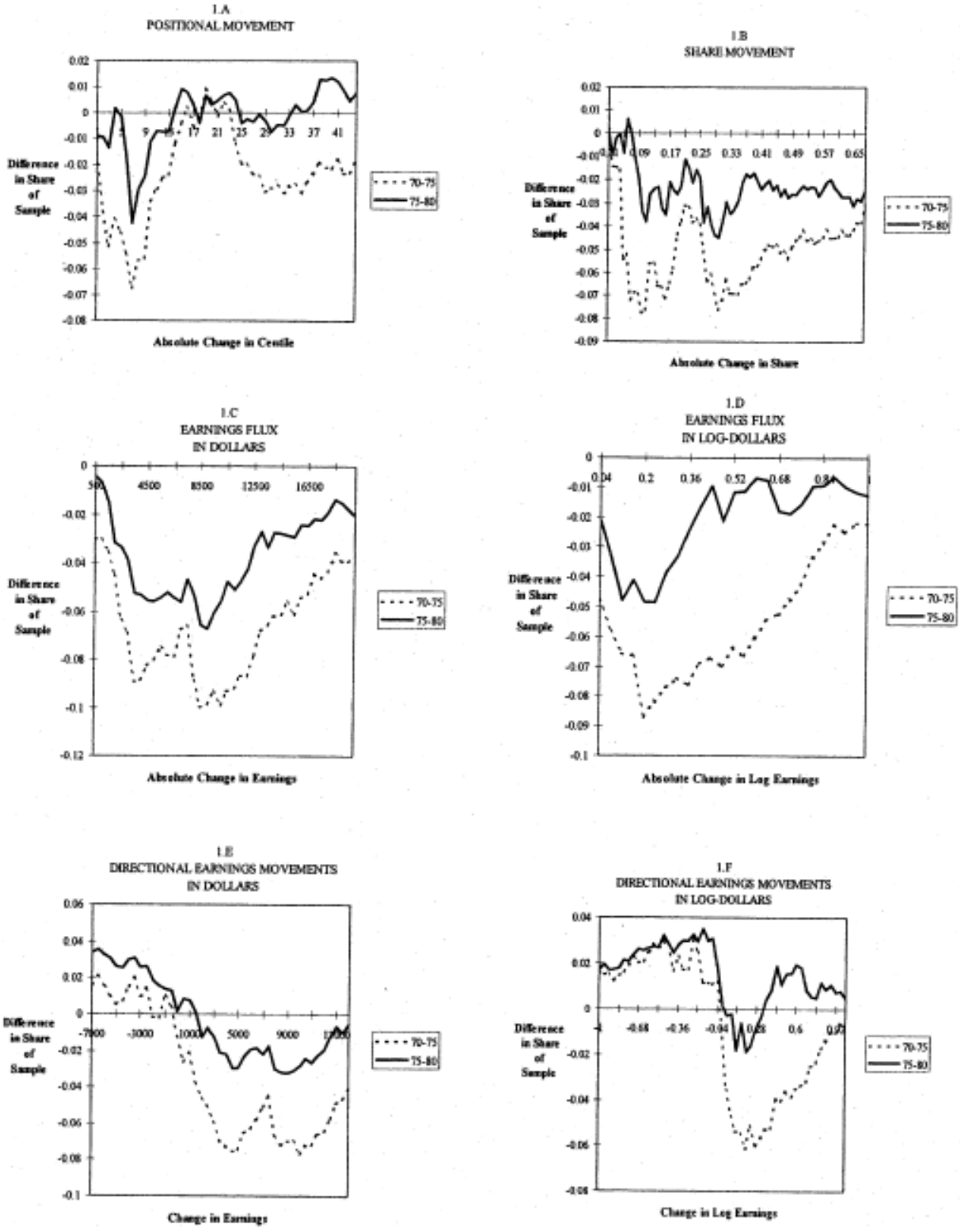

Fig. 1. Six different diagrams relative to 1980–1985 for 1970–1975 and 1975–1980.

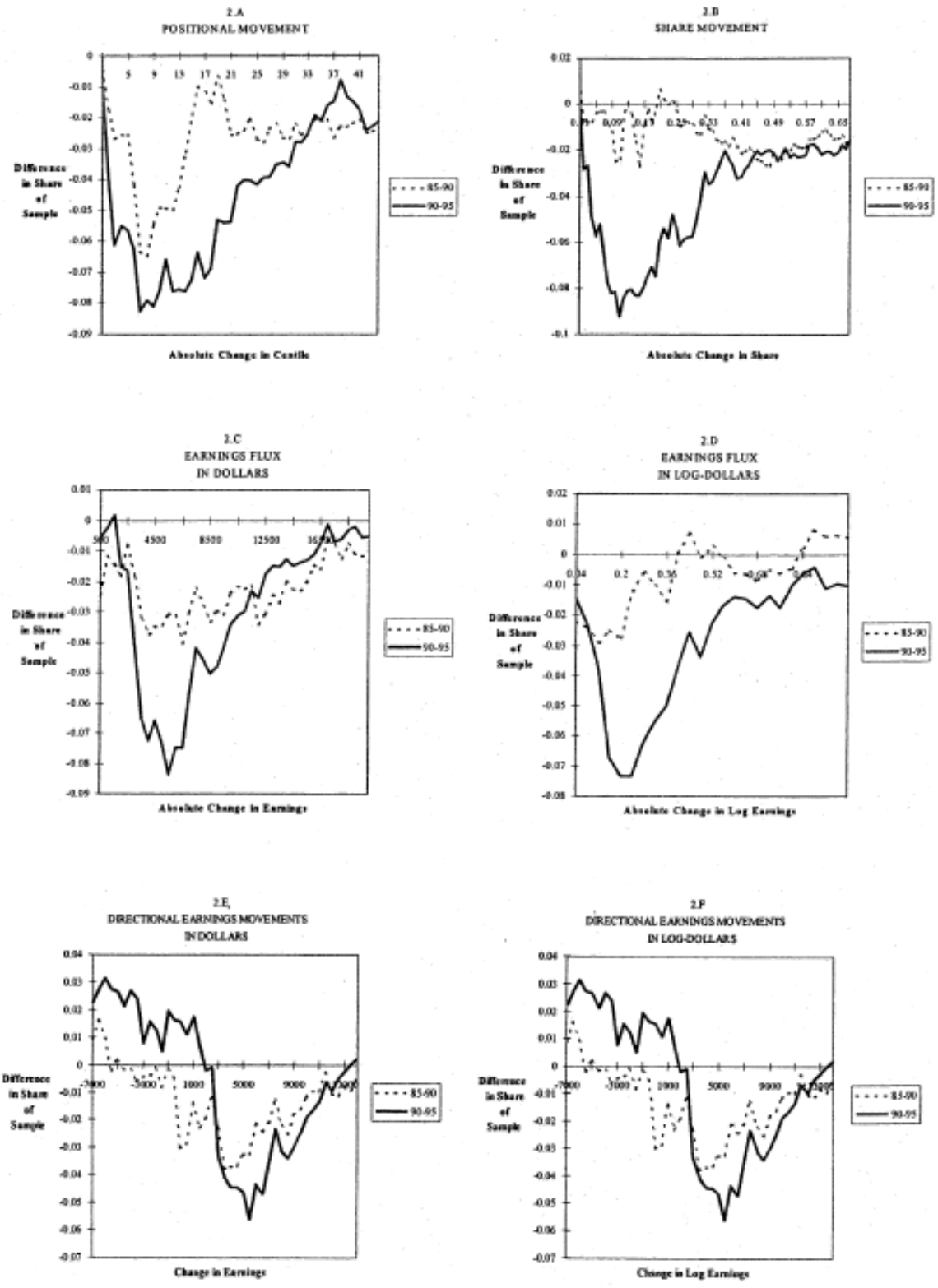

Fig. 2. Six different diagrams relative to 1980–1985 for 1985–1990 and 1990–1995.

Panel A in Fig. 2 shows the cumulative distributions of centile changes in 1985-1990 and 1990-1995 relative to 1980-1985. Here, there is a clear dominance result: 1980-1985 had more positional movement than 1985-1990 and 1990-1995. However, the 1985-1990 and 1990-1995 curves cross one another, which means that 1985-1990 had neither unambiguously more positional movement nor unambiguously less compared with 1990-1995. As for the earlier period (Fig. 1A), 1970-1975 is almost dominated by 1980-1985 (the degree of dominance is D_{96}) and by 1975-1980 (D_{99}).

Share movement takes place when an individual's share of total income changes. For the individual, this change is simply $(y_{1_i}/\sum y_1) - (y_{0_i}/\sum y_0)$. Because the income shares must sum to one, positive changes are necessarily exactly offset by negative changes. As in the case of rank changes, it does not make sense to compare the directional share changes that take place in two different income change situations, and therefore, we consider only the magnitude of the changes that take place, using the absolute values of the share changes.

For share-movement, we find that 1980-1985 dominates 1990-1995 and 1970-1975 and almost dominates the other years (1980-1985 D_{94} 1975-1980; 1980-1985 D_{91} 1985-1990). Thus, share movement peaked in the 1980-1985 period and was lower in other years.

Income flux is measured by the absolute value of changes in people's incomes. These fluctuations, measured in real dollars, are of interest in their own right. On the other hand, the simple change in income may not capture the full meaning of an income change, because we may want to consider a given dollar change differently depending on how rich or poor the person was initially. In order to address this, we also measure income changes in logarithmic terms, which has been justified in previous work by Fields and Ok (1999a,b).

Figs. 1C and 2C show the dominance relationships of changes in absolute value of earnings for the five periods, relative to 1980-1985. As with the preceding mobility concepts, 1980-1985 was the period of peak income flux. Also, 1975-1980 dominates 1970-1975, but the periods 1985-1990 and 1990-1995 cannot be ranked. Looking at absolute changes in log earnings we find too that 1980-1985 dominates 1970-1975, 1975-1980 and 1990-1995, and that 1980-1985 almost dominates 1985-1990 (D_{88}). The dominance results for absolute changes in log earnings are shown in Figs. 1D and 2D.

Finally, looking at directional income changes, these too can be measured in dollars or in log-dollars. As shown in the bottom panels of Figs. 1 and 2, there is no pattern to the dominance results, either for directional earnings changes or directional log earnings changes. The only dominance result is that 1975-1980 dominates 1970-1975.

4. Conclusions

This paper has introduced a technique for mobility dominance, which we applied to earnings mobility of men in the USA from 1970 to 1995. We found many cases in which one mobility situation dominated another, but also many in which they did not. For positional movement, share movement, and income flux, the peak mobility was in 1980-1985, while for directional movement, dominance methods proved inconclusive. The inverted-U pattern of the

positional movement, share movement, and income flux aspects of earnings mobility among men in the USA is a new discovery.

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References

Atkinson, A., Bourguignon, F., Morrisson, C., *Empirical Studies of Earnings Mobility*. (Chur, Switzerland: Harwood, 1992).

Danziger, S., Gottschalk, P., 1995. *America Unequal*. Russell Sage Foundation, New York.

Fields, G., *Distribution and Development: A New Look at the Developing World*. (Cambridge, MA: MIT Press and the Russell Sage Foundation, 2001).

Fields, G.S., Leary, J.B., Ok, E.A., 1998. *Income Movement in the USA in the 1970-1980s*. Cornell University, USA.

Fields, G.S., Ok, E.A., 1996. *The meaning and measurement of income mobility*. Journal of Economic Theory 71, 349-377.

Fields, G.S., Ok, E.A., 1999a. The measurement of income mobility: an introduction to the literature. In: Silber, J. (Ed.), *Handbook of Income Distribution Measurement*. Kluwer Academic Publishing, Boston.

Fields, G.S., Ok, E.A., 1999b. *Measurement of income movement*. Economica 66, 455-472.

Freije, S., *Household Income Dynamics in Venezuela*. Unpublished Ph.D. Dissertation, Cornell University, 2001.

Hadar, J., Russell, W.R., 1969. Rules for ordering uncertain prospects. *American Economic Review 59*, 25-34.

Leary, J., *Evaluating Longitudinal Income Changes: The United States, 1969-1995*. Unpublished Ph.D. Dissertation, Cornell University, 2001.

Organisation for Economic Co-Operation and Development, 1996. Employment Outlook.

Saposnik, R., 1981. Rank Dominance in Income Distributions, *Public Choice, 36*.