



ESSAYS IN CREDIT PORTFOLIO MANAGEMENT

by Vikrant Tyagi

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ESSAYS IN CREDIT PORTFOLIO MANAGEMENT

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ESSAYS IN CREDIT PORTFOLIO MANAGEMENT

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The current financial crisis has lessons for three areas of credit portfolio management. First, the credit crisis has highlighted the need to manage the funding risk of a bank. Second, it has highlighted the need to manage the underwriting risk of debt syndications. Finally, it has suggested the need to understand the drivers of relationship banking. The first paper in this dissertation develops an empirically grounded model to manage the funding risk of a bank. The second paper develops an option pricing framework to manage the underwriting risk in debt syndications. The last paper in this dissertation uses a proprietary dataset to study the empirical determinants of relationship banking benefits.

BIOGRAPHICAL SKETCH

Vikrant Tyagi did his Bachelor of Electrical Engineering in India and his Masters in Finance in United Kingdom before undertaking his doctoral studies in Finance at Cornell.

To Yu, Mom and Dad

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CHAPTER 1

INTRODUCTION

The current financial crisis was primarily a credit crisis. Banks involved in originating or securitizing sub-prime loans suffered losses when the real estate bubble burst, which raised concerns about the viability of those banks. As a result, banks with significant reliance on wholesale funding had problems rolling over their existing short-term funding. Consequently, many banks perished or were taken over or came close to bankruptcy. Besides losing money on sub-prime investments, banks also lost significant amount of money on leveraged loan commitments. Banks were sitting on a huge portfolio of leveraged loan underwritings at the time of Lehman bankruptcy, and couldn't syndicate these commitments due to market meltdown. Consequently, the banks lost billions of dollars on these underwriting deals. The credit crisis also made capital scarce for the banks, which in turn made banks more cautious towards lending. The use of bank balance sheet was rationed and banks opened their tight fist only for customers with strong banking relationships.

To summarize, the current financial crisis has lessons for three areas of credit portfolio management. First, the credit crisis has highlighted the need to manage the funding risk of a bank. Second, it has highlighted the need to manage the underwriting risk of debt syndications. Finally, it has suggested the need to understand the drivers of relationship banking.

The first paper in this dissertation develops an empirically grounded model to manage the funding risk of assets with uncertain funding requirements. The model simulates the expected and unexpected funding needs of a portfolio, which can be term-funded in two ways. Static term funding is fixed over life

of the asset and is conditioned on initial information. Dynamic term funding changes over the life of the asset and is conditioned on information at future adjustment dates. Dynamic term funding has less model risk and offers more protection than static funding in most cases. The dynamic funding model is applied to the revolving credit portfolio of a large multinational investment bank between December 2007 and December 2009. The dynamic funding model performed very well in managing the funding risk of this loan portfolio during this extremely stressful period. We also use simulation to compare the expected funding cost under the short-term, static and dynamic funding frameworks. The simulation suggests that the dynamic funding method has no significant cost disadvantage as compared to the other two methods, while offering a better framework for funding risk management.

The second paper in this dissertation develops an option pricing framework to manage the underwriting risk in loan syndications. The underwriting risk is modeled as an American put option on a debt instrument. The paper argues that, due to the nature of these commitments, the price of an underwriting commitment should lie somewhere between the price of an American put option and a European put option on the debt instrument. We propose to price the commitment as an American option because it is more conservative and better protects the underwriter who aims to hedge this risk. A reduced form model is used to price commitments as American options on defaultable securities. The model is applied to two cases depending on whether or not default swaps on the borrower trade in the market. This novel framework can help banks to better manage their debt underwriting risk.

The last paper in this dissertation studies the empirical determinants of re-

lationship banking benefits. While numerous theoretical studies have identified the sources of relationship banking benefits, the empirical literature has been constrained in testing these theories by the availability of data. Due to the reliance on crude measures for relationship strength and benefit, the existing literature has provided mixed empirical evidence on various theoretical claims and has not fully identified the sources of relationship benefit. Using a proprietary dataset from a multinational bank, this paper develops comprehensive measures of relationship strength and precise measure of relationship benefit to uncover the true sources of relationship benefit. Relationship benefit is measured as the difference between the par value and the fair value of a loan where the latter captures all the relevant risks. Relationship strength is measured along three dimensions namely relationship depth, relationship breadth, and relationship potential. This paper finds that the borrowers benefit from a lending relationship through better terms on their loan contracts. The relationship benefit increases with relationship breadth and relationship potential but doesn't depend on relationship depth. While relationship potential is the most important determinant of relationship benefit for informationally opaque firms, relationship breadth is more important for the firms that are not so. This paper also identifies several problems in the empirical specifications and the corresponding results in the existing literature.

CHAPTER 2
DYNAMIC FUNDING FRAMEWORK FOR FUNDING RISK
MANAGEMENT

Until recently, the Investment Bank funded the majority of its trading assets on a short-term basis and therefore at short-term rates. . . . Now, in order to encourage more disciplined use of UBS's balance sheet, the Investment Bank will fund its positions at terms that match the liquidity of its assets as assessed by Treasury.

- UBS 2007 Letter to Shareholders

2.1 Introduction

The financial crisis of 2007-2009 has highlighted the importance of managing the funding risk in a financial institution. Prior to the crisis, most banks were funding long-term assets with short-term funding. This was partly because short-term funding was cheaper and partly because short-term funding provides a better estimate of the uncertain funding requirements in a given period. However, short-term funding is subject to funding risk because the bank may not be able to rollover its existing funding. Moreover, most banks maintained an inadequate liquidity buffer to meet their unexpected funding needs. For example, during the current crisis, banks couldn't rollover their existing funding due to concerns about their viability and their liquidity buffers were not enough to meet unexpected funding needs such as drawdowns on credit lines and increased collateral on derivative positions. Consequently, many banks perished or came close to bankruptcy during this crisis.

To better manage their funding risk, many banks, on their own accord or prompted by credit agencies, are reducing their reliance on short-term financing and increasing their liquidity buffers (for example, see the quote from UBS above). Moreover, international and national regulators are proposing rules (under Basel framework) that require banks to maintain a sizeable liquidity buffer and increase the amount of long-term funding. However, as argued below, most of the approaches being employed by banks or suggested by regulators to manage the funding risk are based on stress scenarios and rules of thumbs rather than being based on an empirically grounded predictive model.

The well known principle for mitigating funding risk is to match the maturity of assets and funding. However, this principle is difficult to practice because many assets have uncertain funding requirements. One way to avoid this funding uncertainty is to borrow long-term the maximum possible funding that could be needed for an asset. In effect, this approach implies a *variable* liquidity buffer which can meet any possible funding need for the asset. However, this approach is overly conservative (since so much funding would almost never be needed on a portfolio basis) and overly expensive (since unused funds would earn less than the opportunity cost¹ most of the times).

This paper proposes a model to overcome the above problems in managing the funding risk of assets with uncertain funding requirements. The model is developed in the HJM framework for forward spreads (e.g. [5]; [18]) and is illustrated for revolving credit lines, but it can be readily adapted to other modeling frameworks and asset classes. The model involves using historical data to estimate the statistical relationship between credit spreads and revolver utiliza-

¹For funds to be readily available when needed, unused funding has to be invested in liquid government securities which earn less return than the bank's cost of raising those funds and much less than the returns that could be generated if those funds were profitably invested.

tion; simulating credit spreads over a funding horizon; and using the simulated spreads and the estimated relationship between spreads and utilization to obtain the revolver usage distribution and hence the expected and unexpected usage (at a given confidence interval) over the funding horizon.

This model can be applied for funding risk management in two alternative ways: *static funding* and *dynamic funding*. Static funding involves simulating credit spreads and loan utilization to the loan maturity date using information as of loan inception date. In other words, the funding horizon in static funding is from the loan inception date to the loan maturity date. The expected and unexpected usage are funded at the loan inception date with maturity equal to the maturity of the loan. The unexpected funding is invested in long-term liquid securities (such as government bonds) which acts as a liquidity buffer to meet unexpected draws. Since the expected and unexpected usage is fixed through the life of loan, the cost of funding is fixed unless unexpected usage is breached during the life of the loan.

Dynamic funding involves adjusting the expected and unexpected usage at periodic adjustment dates. At each adjustment date, credit spreads and loan utilization are simulated till the next adjustment date using information as of the current adjustment date. In other words, the funding horizon in dynamic funding is from one adjustment date to the next adjustment date. The expected funding is funded for the *remaining tenor* of the loan whereas the unexpected usage is funded till the next adjustment date (a cushion period can be added to avoid funding during a prolonged liquidity crisis). The unexpected funding is invested in short-term liquid securities to serve as a liquidity cushion. If the expected or unexpected usage simulated on a given adjustment date is more

(or less) than that simulated on the previous adjustment date, the expected or unexpected funding is increased (or decreased) by the difference in the numbers simulated on these two dates. Since the dynamic funding changes over the life of the loan conditional on spreads at future adjustment dates, the cost of funding the revolver is uncertain as of the loan inception date.

Both of these funding approaches have their advantages and disadvantages. Since the horizon over which spreads are simulated is shorter in dynamic funding, the usage distribution is narrower and the unexpected funding is lower for a given confidence interval than in case of static funding. In other words, dynamic funding approach relies less on unexpected funding and depends more on conditional expected funding. Moreover, dynamic funding has less model risk and hence provides more precise estimates of funding requirements. This is because dynamic funding has smaller model errors due to shorter simulation horizon and because it partially corrects model errors due to periodic adjustments with new information. In addition, any divergence between model and reality can be more easily detected and corrected than in case of static funding. Finally, dynamic funding offers more protection against unexpected funding draws in a period of consistently high spreads and volatility. On the other hand, static funding offers more protection against large temporary spikes in usage over a short period. Static funding also locks in a cost of funding which is not the case with dynamic funding. Overall, dynamic funding appears to be a safer alternative in terms of funding risk management.

The dynamic funding model is applied to the entire revolving credit portfolio of a large multinational investment bank between December 2007 to December 2009. Since this period coincided with one of the most severe financial crisis

ever, it serves as a good test case to evaluate the performance of dynamic funding model. At the beginning of each quarter, CDS spreads and utilization were simulated for the next quarter for each borrower in the portfolio, and the simulated usage distribution of the portfolio was used to calculate the expected and unexpected funding needs for the next quarter. At the end of each quarter, the actual utilization was compared with the expected and unexpected utilization projected in the last quarter, and the new funding projections were made for the next quarter. The dynamic funding model performed very well in managing the funding risk of this loan portfolio during this extremely stressful period. In particular, the actual utilization never breached the expected plus unexpected funding during the whole financial crisis. Also, the actual utilization increased with higher CDS spreads and fell with lower CDS spreads, thus validating the basic premise of the model that utilization is a function of spreads.

We also study the expected funding cost under the three funding methods: a) short-term funding; b) static funding; and c) dynamic funding. A reduced form credit risk model (e.g. [20]) is employed to estimate the expected funding cost in each case and the effect of various model parameters is studied. The expected costs under the three funding methods are insignificantly different from each other in scenarios where the funding spreads are close to their long term values. In situations where funding spreads are far lower than their long-run means, the model suggests that short-term funding is significantly more expensive than dynamic and static funding, which is contrary to banks' preference for short-term funding in such environments (e.g during 2002-2006). Arguably, this is because banks are focused on short-term profits, rather than taking a long-term view of funding cost. On the other hand, in scenarios where funding spreads are larger than their long-run mean values, short-term funding is

cheaper than static and dynamic funding, though in economic terms, the benefit is marginal (less than 5% in our simulations). Therefore, dynamic funding seems clearly preferable to static funding and short-term funding because it provides better funding risk management framework and offers no significant cost disadvantages. Moreover, dynamic funding has lower unexpected funding than the other two methods which may yield a cost advantage.

This paper is the first to suggest the use of dynamic funding approach for funding risk management. As argued above, dynamic funding is superior to static and short-term funding in this regard. Moreover, the dynamic funding approach is superior to the approaches currently being employed by banks or suggested by regulators to manage the funding risk. For instance, the new Basel rules for liquidity risk management require banks to maintain a *liquidity coverage ratio* and a *net stable funding ratio*. The *liquidity coverage ratio* specifies the proportion of investment in various asset classes that needs to be held in liquid assets to meet the funding needs in the next 30 days. These proportions are derived based on stress scenarios which are neither bank/portfolio specific nor seem to be based on empirical experience. For instance, banks are required to hold 10% of undrawn credit line limits in liquid assets to protect against drawdown in the next 30 days. This number seems arbitrary, especially since there was no month during this severe financial crisis wherein there was a drawdown of more than 3% of undrawn limit in the revolving credit portfolio of a large investment bank studied in this paper². Moreover, in the absence of a predictive model, there is

²One possibility is that the 10 % number is a cross-bank average. However, since the investment bank studied in this paper had lower than 10% drawdown in credit lines in any month during this crisis, a cross-bank average of 10% implies that there must be some bank(s) with drawdown of more than 10% in some month. For such banks, 10% may not be enough liquidity cushion. On the other hand, 10% is too high a cushion for the bank studied in this paper. In both cases, the ratio doesn't serve its desired purpose because it is neither bank specific nor based on an predictive model.

no way to ascertain that a future financial crisis wouldn't be more severe than the current one and against which these liquidity ratios may prove ineffective. On the other hand, *net stable funding ratio* proposes the proportion of investment in various asset classes that needs to be funded with long-term liabilities. This ratio doesn't require the matching of maturity of the assets and liabilities since any funding with more than one year tenor qualifies as long-term funding. Consequently, this ratio doesn't go far enough in mitigating the funding risk. For instance, if a bank funds all its long term assets with one year liability, a bank may still face rollover risk if a severe financial crisis lasts more than one year.

Most of the literature on funding risk management is based on Asset-Liability Management (e.g. [9]) which involves techniques such as gap analysis, duration analysis, scenario analysis, stress tests and stochastic dynamic programming to manage the funding liquidity risk of a bank. To my knowledge, the dynamic funding method has not been studied in this literature. [30] proposes a static funding model wherein the revolver utilization depends on ratings. In contrast, we propose a dynamic funding model wherein utilization depends on spreads.

2.2 General Model

2.2.1 Evolution of CDS Spreads

Define hazard rate of borrower default $h(t, T)$ at time T as

$$h(t, T) = \frac{f(t, T)}{1 - F(t, T)} \quad (2.1)$$

where $F(t, T) = P(\tau \leq T | \tau > t)$ is the condition distribution of default time τ and $f(t, T) = \frac{\partial}{\partial T} F(t, T)$ is the corresponding density. If $P(t, T) = 1 - F(t, T) = P(\tau > T | \tau > t)$ is the survival probability of a credit between time t and T , the hazard rate expression can be written as

$$\begin{aligned} h(t, T) &= \frac{f(t, T)}{1 - F(t, T)} = \frac{1}{P(t, T)} \frac{\partial}{\partial T} (1 - P(t, T)) \\ &= -\frac{1}{P(t, T)} \frac{\partial}{\partial T} P(t, T) \\ &= -\frac{\partial}{\partial T} \ln P(t, T) \end{aligned} \quad (2.2)$$

$$= \widehat{f}_r(t, T) - f_r(t, T) \quad (2.3)$$

which shows that hazard rate is equal to forward credit spread $\widehat{f}_r(t, T) - f_r(t, T)$ where $\widehat{f}_r(t, T)$ and $f_r(t, T)$ are the risky and riskless forward rates (see [18]). Hence, hazard rate is analogous to forward rate in the term structure models (e.g. [5]).

The above equation implies that

$$P(t, T) = \exp\left(-\int_t^T h(t, s) ds\right) \quad (2.4)$$

which suggests that the survival probability $P(t, T)$ between current time t and any future time T can be readily calculated if hazard rate $h(t, s)$ is known for $s \in (t, T)$. On the other hand, if the survival probabilities $P(T_0, T)$ are known for all $T \in \{T_0, \dots, T_i, \dots, T_N\}$, CDS premium $C(T_0, T_I)$ at time T_0 for any maturity $T_I - T_0$ can be calculated using the following expression³

$$C(T_0, T^I) \left(\sum_{i=1}^I P(T_0, T_i) \Delta T_i d_i \right) = (1 - R) \sum_{i=1}^I (P(T_0, T_{i-1}) - P(T_0, T_i)) d_i \quad (2.5)$$

where ΔT_i is the day count fraction between CDS premium payments, $R = E(R_t)$ is the expected bond recovery rate, and d_t are the riskless discount factors.

³See section (5) for more details.

This expression relies on the fact that a CDS contract has zero value at initiation, which implies that the expected value of premiums paid on the CDS contract (left hand side of expression) must equal the expected value of losses in the event of default (right hand side of expression)⁴.

Let the dynamics of hazard rate in the empirical measure be given by

$$dh(t, T) = \alpha(t, T)dt + \sigma(t, T)dW_t + \sum_{i=1}^N \beta_i(t, T)dN_t^i \quad (2.6)$$

where W_t is a standard Brownian motion and N^i are jump processes under the empirical measure. As in [12], this jump process for forward spread must satisfy no-arbitrage conditions on drift similar to the drift restrictions on forward rates in the HJM model for interest rates (see [5]). We assume such no-arbitrage conditions are satisfied. A subsequent section contains a parametric example of this stochastic process.

Assume that hazard rates $h(T_0, T)$ at time T_0 are either exogenously given or can be implied from market data⁵. The stochastic process in *expression (2.5)* is used to simulate hazard rates from current time T_0 at a future time $T_0 + u$. These simulated hazard rates $h(T_0 + u, T)$ are used to calculate survival probabilities $P(T_0 + u, T)$ on the simulated path using *expression (2.3)*. These survival probabilities in turn are used to calculate CDS premiums $C(T_0 + u, T_I)$ on the simulated path using *expression (2.4)*. This procedure can be used to simulate the entire distribution of CDS spreads at a future point in time. As discussed in subsequent sections, the simulated CDS spreads can be used to simulate utilization and obtain funding needs for revolver loans⁶.

⁴This expression assumes that interest rates, recovery rate, and default time are independent. These assumptions are market standard in the credit derivative market to bootstrap survival probabilities.

⁵A subsequent section shows how these hazard rates can be calibrated to or implied from market data.

⁶The simulated spreads can also be used to calculate the value-at-risk (VAR) of the loan

2.2.2 Usage of a Loan Portfolio

The usage of a revolver is stochastic. The variation in loan usage is due to corporate financial decisions which are not observed by the bank. In this paper, usage is assumed to be dependent on the spread level and the expected utilization of the loan. There is an economic justification for the dependence on spreads because, at high spreads, it is cheaper to draw on the revolver than borrow using some other instrument. However, due to transaction costs and other unobservable factors, the dependence of utilization on spreads is not totally opportunistic. In particular, borrowers do not draw down on the loan as soon as their credit spread increases above the loan fees. This relation is modeled using the following expression

$$U_s^{i,j} = F^{i,j}(e_j, q_{s,s+\Delta s}^i, U_{s-\Delta s}^{i,j}, \vec{x}_t) + \varepsilon_s \quad (2.7)$$

where $U_s^{i,j}$ is the utilization of loan $j \in \{1, 2, \dots, J_i\}$ to borrower $i \in \{1, 2, \dots, I\}$ at a time s , e^j is the expected utilization of loan j , $q_{s,s+\Delta s}^i$ is the survival probability of borrower i from time s to $s + \Delta s$, \vec{x}_t is a vector of variables that determine loan utilization, and $F^{i,j}(\cdot, \cdot)$ is a deterministic function, and ε_s is an error term which is identically and independently distributed across time, borrowers and loans. The function $F^{i,j}$ may be non-linear.

The assumption that the utilization depends on default probability till next period rather than later periods is made because the firm can always increase its utilization when the default probability increases in the future. This expression can also be written as

$$\begin{aligned} U_s^{i,j} &= F^i(e_j, f^{-1}(C_{s,s+\Delta s}^i), U_{s-\Delta s}^{i,j}, \vec{x}_t) + \varepsilon_s \\ &= G^i(e_j, C_{s,s+\Delta s}^i, U_{s-\Delta s}^{i,j}, \vec{x}_t) + \varepsilon_s \end{aligned} \quad (2.8)$$

portfolio.

where $C_{s,s+\Delta s}^i$ is the CDS spread of borrower i at time s for maturity Δs and f^{-1} is the inverse function obtained from *expression (2.7)*. Historical data can be used to estimate this relation if a specific functional form is assumed for $G^i(\cdot, \cdot)$. If there is limited data, the above expression can be assumed to have the same form for all firms

$$U_s^{i,j} = G(e^j, s_{s,s+\Delta s}^i, U_{s-\Delta s}^{i,j}, \vec{x}_t) + \varepsilon_s \quad (2.9)$$

A subsequent section employs a few functional forms for $G()$ and estimates panel data regressions of the form:

$$U_t^{i,j} = \alpha + \beta e^j + \gamma s_{t,t+\Delta t}^i + \delta U_{t-\Delta t}^{i,j} + \theta \vec{x}_t + \varepsilon_t \quad (2.10)$$

The CDS spreads $C_{s,s+\Delta s}^i$ for borrower i for maturity Δs at a future time s is simulated $k \in \{1, 2, \dots, K\}$ times at current time t using the model described in previous section. The expected utilization e_j and the simulated spreads $s_{s,s+\Delta s}^{i,k}$ can be used to obtain the usage $U_s^{i,j}(e_j, s_{s,s+\Delta s}^{i,k})$ at future time s for each loan for each simulation using *expression (2.7)*. The portfolio utilization $U_p^k(s)$ at future time s is calculated for simulation k as follows.

$$U_p^k(s) = \sum_{i=1}^I \sum_{j=1}^{J_i} U_t^{i,j}(e_j, s_{s,s+\Delta s}^{i,k}) \quad (2.11)$$

The distribution of simulated portfolio usage $U_p^k(s)$, $k \in \{1, 2, \dots, K\}$, can be used to obtain the estimate of expected utilization $U_p^{exp}(s)$ and the utilization $U_p^{CL-\alpha}(s)$ at α confidence interval.

$$U_p^{exp}(s) = \frac{1}{K} \sum_{k=1}^K U_p^k(s) \quad (2.12)$$

$$U_p^{CL-\alpha}(s) = \inf(u \in \mathcal{R} : \Pr(U_p^k(s) > u) < 1 - \alpha)$$

The unexpected utilization $U_p^{unexp-\alpha-Cl}$ at α confidence interval is given by

$$U_p^{unexp-\alpha-Cl}(s) = U_p^{Cl-\alpha}(s) - U_p^{exp}(s)$$

The expected and unexpected utilization obtained above are used in the subsequent section to decide how to fund the portfolio.

2.2.3 Funding of the Loan Portfolio

The expected and unexpected usage of a loan portfolio can be term funded in two ways namely static and dynamic funding. To show the difference between these two alternatives, we modify the notation introduced in previous section. Let $U_p^{exp}(s|t)$ and $U_p^{unexp-\alpha-Cl}(s|t)$ be the expected usage and unexpected usage at α confidence interval respectively for portfolio p at a future time s obtained from simulation at time t . Consider a portfolio of L loans containing loans with loan maturity l^i where $i \in \{1, 2, \dots, L\}$. Consider n maturity time points $\{l^0, l^1, l^2, \dots, l^n\}$ where $l^0 < l^1 < \dots < l^n$ and $l^0 = 0$. Divide the loan portfolio into sub-portfolios such that all loans with $l^i \in (l_{k-1}, l_k]$ are categorized into the sub-portfolio denoted as L_k .

Static Funding

In static funding, the expected funding $F_{L_k}^{exp}(s|t)$ and the unexpected funding and $F_{L_k}^{unexp-\alpha-Cl}(s|t)$ at α confidence interval for sub-portfolio L_k at a future time s obtained from simulation at time t are given by

$$\begin{aligned} F_{L_k}^{exp}(s|t) &= U_{L_k}^{exp}(l_k|0) \quad \text{if } 0 \leq s < l_k \\ &= 0 \quad \text{otherwise} \end{aligned} \tag{2.13}$$

$$\begin{aligned}
F_{L_k}^{unexp-\alpha-CI}(s|t) &= U_{L_k}^{unexp-\alpha-CI}(l_k|0) \quad \text{if } 0 \leq s < l_k \\
&= 0 \quad \text{otherwise}
\end{aligned} \tag{2.14}$$

Note that these expressions imply $t = 0$ and $s = l_k$ which means that the expected and unexpected funding are fixed on the loan issuance date and do not change subsequently till all the loans in that sub-portfolio mature (assuming no new loans enter that portfolio). In other words, credit spreads and utilization are simulated till the loan maturity date using information as of the loan inception date. The expected and unexpected utilization from the simulation is funded for the maturity of the loan and these funded amounts remain fixed through the life of the loan. A graphical illustration is included in Figure 2.1 for a loan with three year maturity. At time 0, spreads and usage are simulated till time 3 and the expected and unexpected usage are funded for three years maturity.

Dynamic Funding

To illustrate dynamic funding, assume that there are $n + 1$ funding adjustment time points $\{0, t_a, 2t_a, \dots, nt_a\}$ such that $t_a > 0$ and $nt_a < l_n < (n+1)t_a$. In dynamic funding, the expected and unexpected funding are given by

$$\begin{aligned}
F_{L_k}^{exp}(s|t) &= U_{L_k}^{exp}(mt_a|(m-1)t_a) \quad \text{if } s, t \in [(m-1)t_a, mt_a] \\
&= 0 \quad \text{otherwise}
\end{aligned} \tag{2.15}$$

$$\begin{aligned}
F_{L_k}^{unexp-\alpha-CI}(s|t) &= U_{L_k}^{unexp-\alpha-CI}(mt_a|(m-1)t_a) \quad \text{if } s, t \in [(m-1)t_a, mt_a] \\
&= 0 \quad \text{otherwise}
\end{aligned} \tag{2.16}$$

where m is any integer such that $m \leq n$. The expected funding $F_{L_k}^{exp}(mt_a|(m-1)t_a)$ is funded for maturity $l_k - (m-1)t_a$, which is the maximum remaining

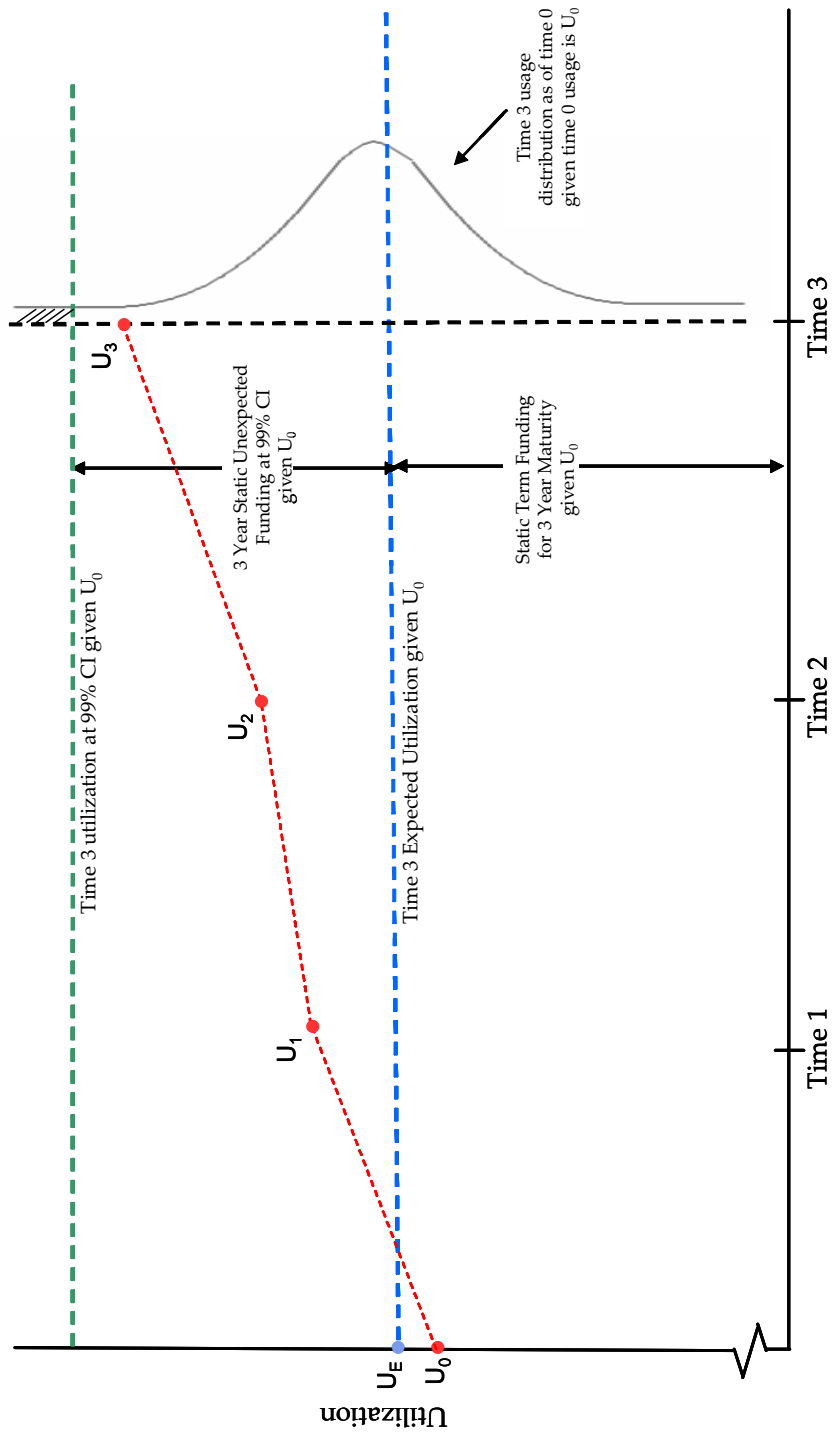


Figure 2.1 Graphical illustration of Static Funding

maturity for the sub-portfolio L_k . If the expected funding $F_{L_k}^{exp}((m + 1)t_a|mt_a)$ differs from that in the previous period $F_{L_k}^{exp}(mt_a|(m - 1)t_a)$, the difference is either funded (if the difference is positive) or unwound (if the difference is negative) for maturity $l_k - mt_a$. The unexpected funding $F_{L_k}^{unexp-\alpha-CI}(mt_a|(m - 1)t_a)$ is funded for maturity t_a or $t_a + l_c$ where l_c is a cushion period to avoid funding during liquidity crisis.

In words, dynamic funding involves adjusting the expected and unexpected usage at periodic adjustment dates. At each adjustment date, credit spreads and loan utilization are simulated till the next adjustment date using information as of the current adjustment date. The expected utilization from the simulation is funded for the remaining maturity of the loan whereas the unexpected usage is funded till the next adjustment date (a cushion period can be added to avoid funding during a future liquidity crisis). The unexpected funding is invested in short-term liquid securities. If the expected or unexpected usage simulated on a given adjustment date is more (less) than that simulated on the previous adjustment date, the expected or unexpected funding is increased (decreased) by the difference in the numbers simulated on these two dates.

Dynamic funding is illustrated graphically in the next two figures. Assume there is a 3 year loan and the funding adjustment period is one year. At time 0, spreads and usage are simulated till time 1 to obtain the expected and unexpected usage. The expected funding is funded for three year maturity whereas unexpected funding is funded for one year period⁷. Figure 2.2 illustrates this case.

At time 1, assume that the spreads have risen higher and the utilization has

⁷If liquidity crises need to be avoided and the maximum time a liquidity crisis may last is one year, unexpected funding could have two years maturity.

reached U_1 . Spreads and usage are simulated till time 2 given the spreads at time 1 to obtain the expected and unexpected usage at time 2. Since the spreads at time 1 are higher, the expected usage at time 2 would be higher than that at time 1. The difference between these numbers is the incremental expected funding required and it is funded with 2 year maturity. Likewise any difference in unexpected funding is funded for one year maturity. Figure 2.3 illustrates this case.

Comparison between Static and Dynamic Funding

The law of iterated expectations implies that

$$\begin{aligned}
 U_{L_n}^{exp}(nt_a|0) &= E_0(\dots E_{(n-3)t_a}(E_{(n-2)t_a}(U_{L_n}^{exp}(nt_a|(n-1)t_a)))) \\
 &= E_0(U_{L_n}^{exp}(nt_a|(n-1)t_a))
 \end{aligned} \tag{2.17}$$

where we have assumed $l_n = nt_a$. In words, the static funding for a loan with maturity $l_n = nt_a$ is equal to the iterated expectation at time 0 of the dynamic funding for that loan at time nt_a obtained at time $(n-1)t_a$. So the static and dynamic funding are inextricably linked.

However, these two alternatives differ significantly from the viewpoint of funding risk management. Since the horizon over which spreads are simulated is shorter in dynamic funding, the usage distribution is narrower and the unexpected funding is lower for a given confidence interval than in case of static funding. In other words, dynamic funding approach relies less on unexpected funding and depends more on conditional expected funding.

The two models also differ in their performance because of model risk. If the model used for simulating spreads is not a complete representation of reality,

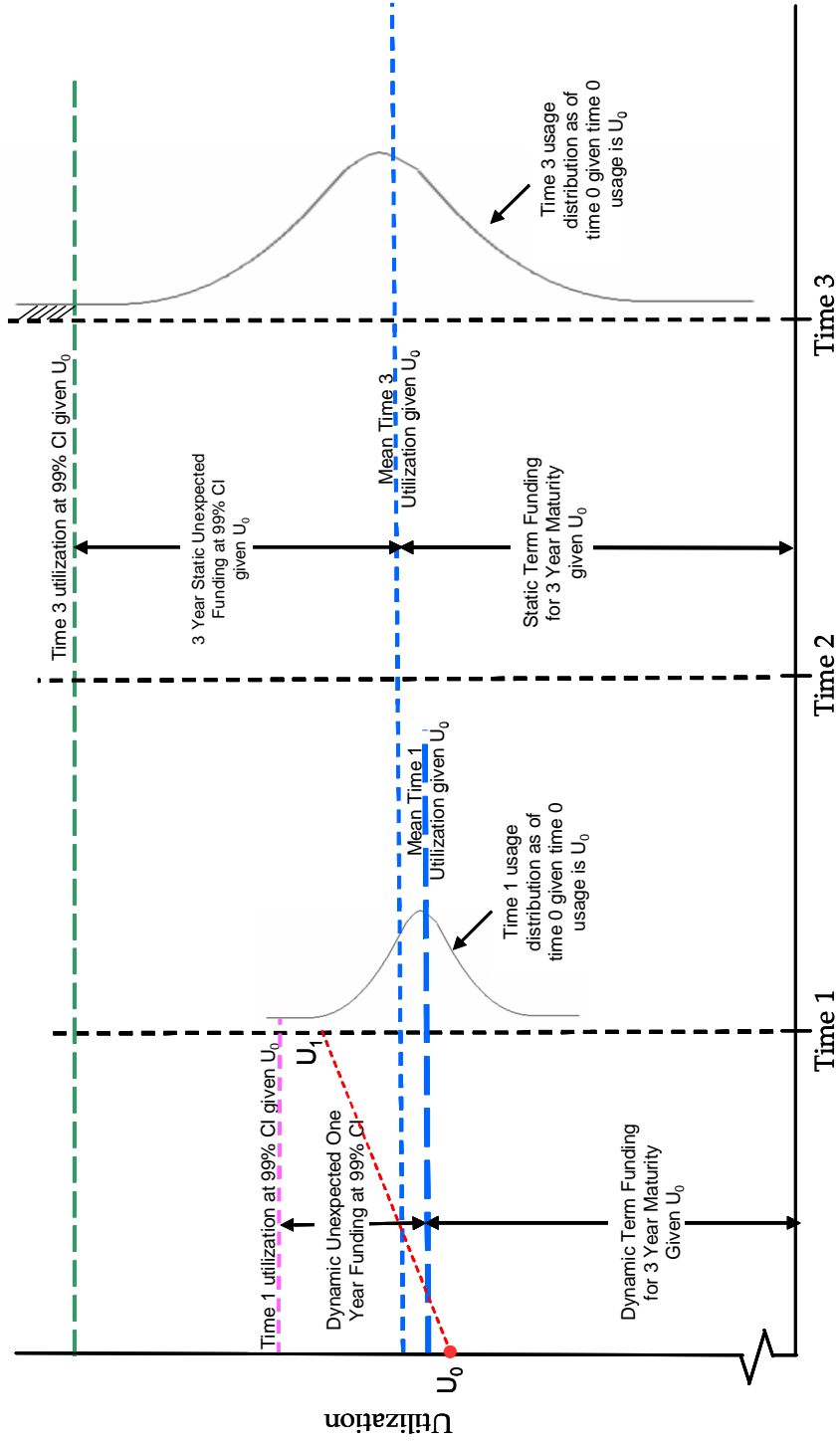


Figure 2.2 Graphical illustration of Dynamic Funding - I

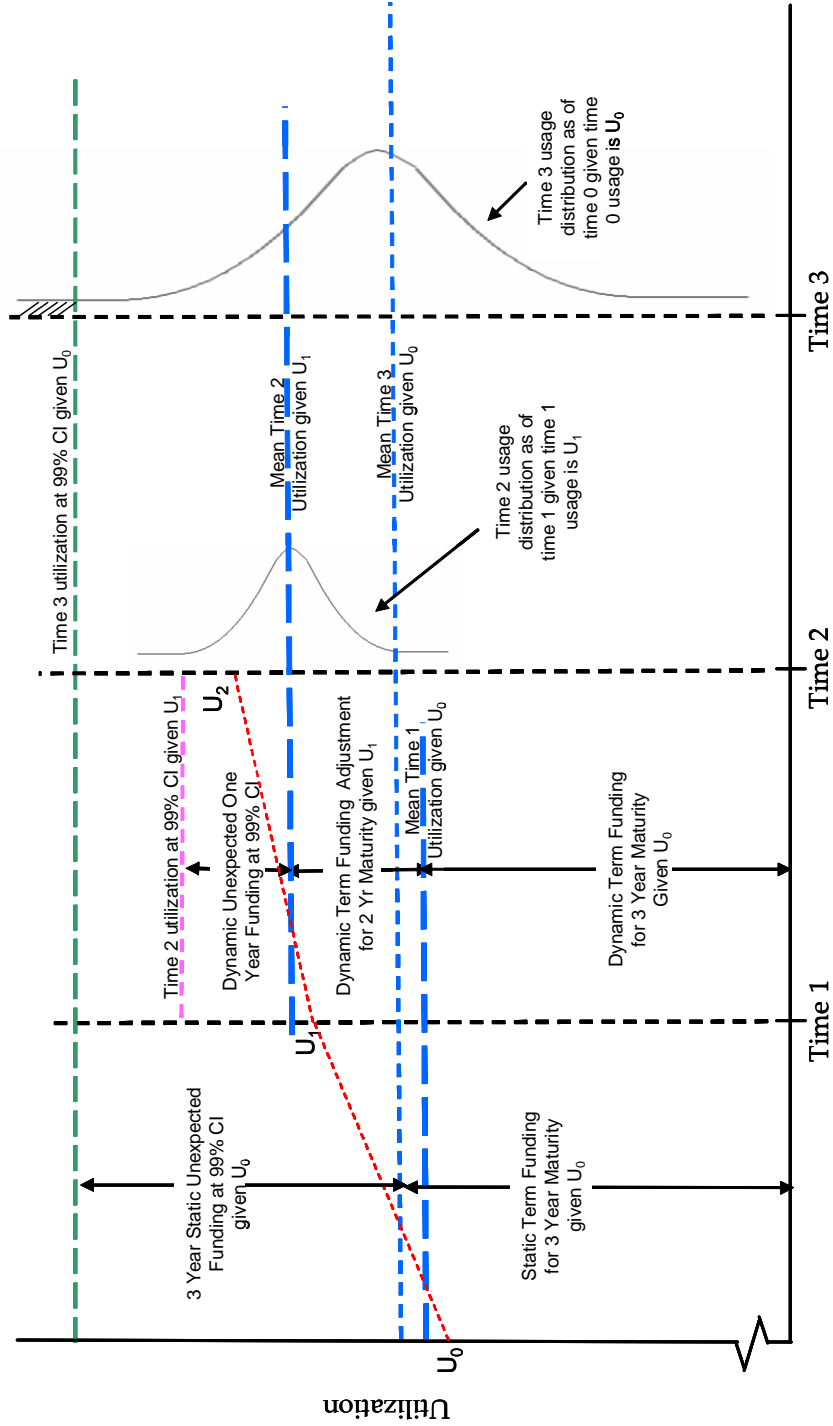


Figure 2.3 Graphical illustration of Dynamic Funding - II

the static funding is more likely to diverge from reality than dynamic funding because it simulates the data over a longer horizon and more unmodeled events can occur over a longer horizon than over shorter horizon. On the other hand, dynamic funding partially corrects itself periodically using the information at the adjustment dates. Hence the model risk is much higher with static funding. Moreover, because of the shorter simulation horizon in dynamic funding, any divergence between model and reality can be more easily detected and corrected using subjective views than in case of static funding. Figure 2.4 illustrates graphically how model risk exacerbates the funding shortfall in the case of static funding as compared to that in the case of dynamic funding.

For a given confidence interval, dynamic funding offers more protection against unexpected funding draws in a period of consistently high spreads and volatility such as during the credit crunch in 2007-2009. To see this, consider a simple example which is *not* meant to be rigorous but illustrative. Assume that the usage distribution is i.i.d normal over successive periods⁸. Assume that the model estimated expected funding is 50% at both the 3 month horizon and the 4 year horizon and the model estimated standard deviation is 1% for the 3 month horizon and 4% for the 4 year horizon. These numbers ensure that the variance in utilization over four year static funding period is equal to the variance in utilization over sixteen successive 3 month dynamic funding periods. These numbers imply that, at 97.5% confidence interval, the unexpected funding is 2% for 3 month horizon and 8% for 4 year horizon. If the usage increases by 2% in each 3 month period, the unexpected usage will always be met by the

⁸The i.i.d. assumption is not valid in this model if spreads are mean reverting. The normal distribution assumption can lead to usage that is less than 0% or greater than 100% which is impossible. A beta distribution can be used to restrict the usage values between 0 and 1 but the additivity of beta variates is complex. This simple example is just for illustration and is not meant to be rigorous.

dynamic unexpected funding of 2% but the unexpected usage of 32% at the end of 4 years will be much higher than the static unexpected funding of 8%. Figure 2.5 illustrates this case graphically.

On the other hand, static funding offers more protection against large temporary spikes in usage over a short period. In the above example, if usage jumps by 5% in a given adjustment period but falls subsequently, the dynamic unexpected funding cannot meet this temporary surge whereas static funding can. However, most credit crises involve long periods of high spreads and volatility rather than short-term spikes in spreads across borrowers.

Overall, dynamic funding appears to be a safer alternative than static funding in terms of liquidity risk management. In particular, static funding is likely to have much larger model risk than dynamic funding and it provides much less flexibility in incorporating current market information and subjective views in the analysis. Dynamic funding also offers more protection than static funding in most stress cases.

In terms of costs, static funding cost is fixed over the life of the loan unless the unexpected funding amount is breached. On the other hand, dynamic funding cost varies because the funded amount and the cost of purchasing incremental funding change over the life of the loan. However, as noted earlier, unexpected funding cost for dynamic funding is cheaper than that for static funding because the former has lower unexpected funding. A subsequent section uses simulation to compare the funding cost in these two cases.

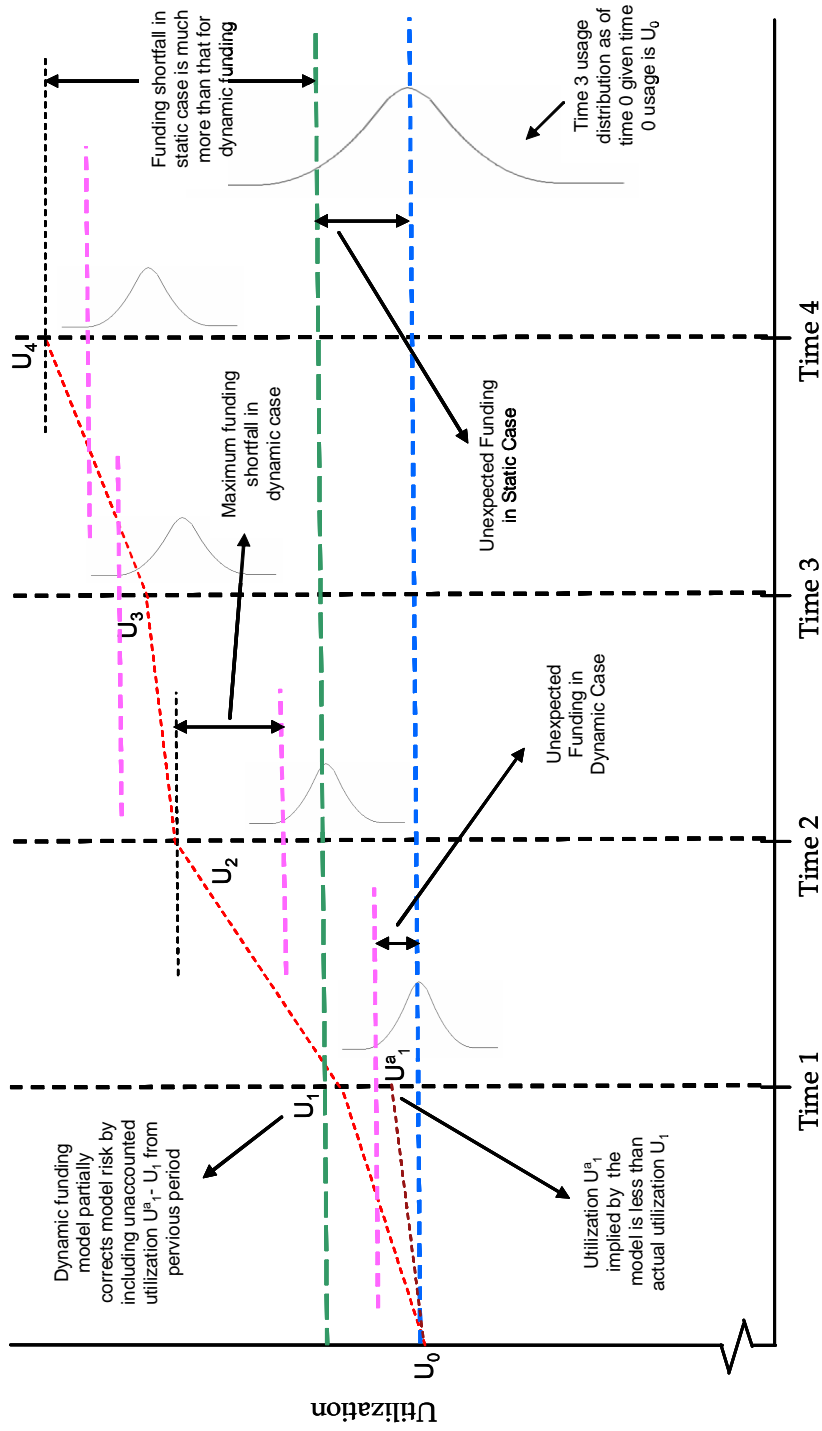


Figure 2.4 Model Risk with Static and Dynamic Funding

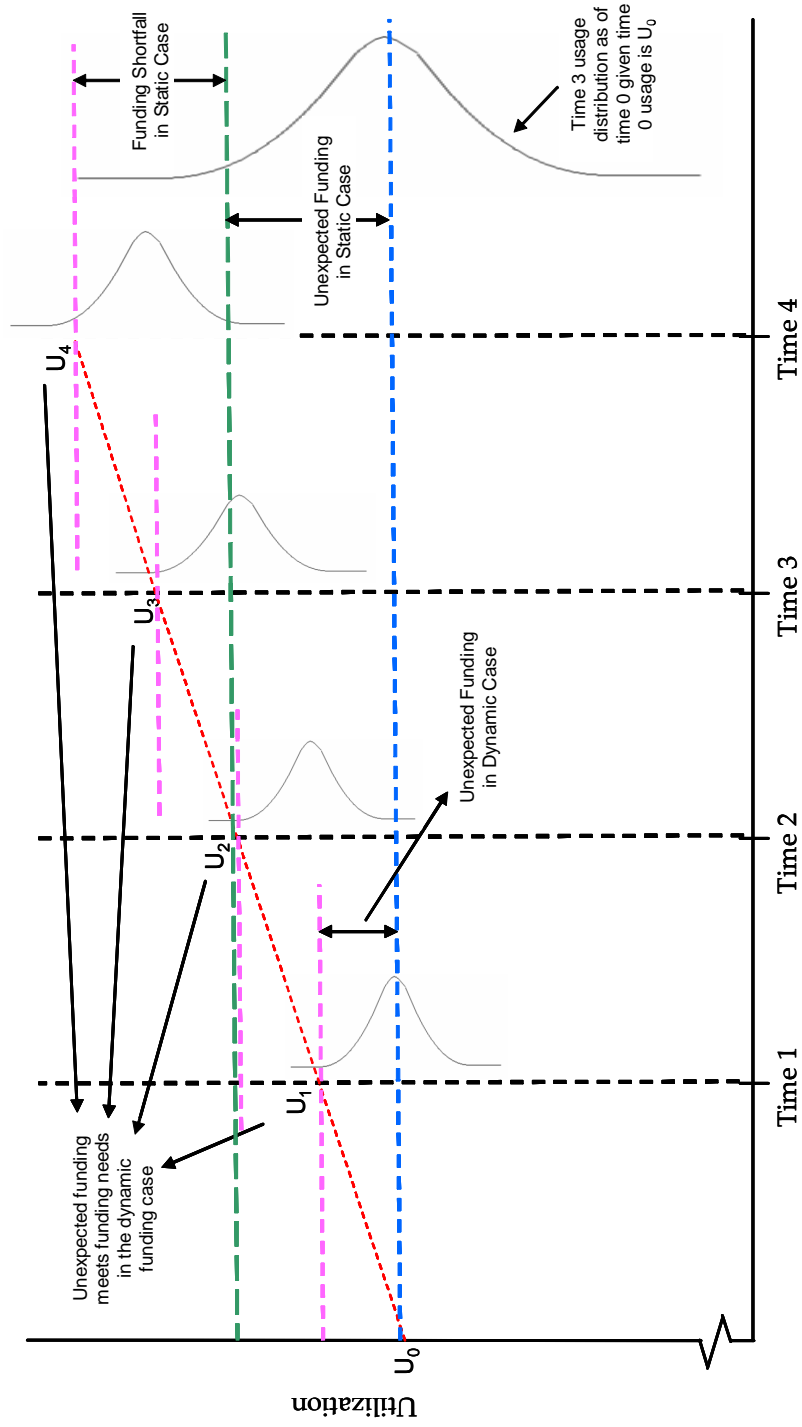


Figure 2.5 Static and Dynamic Funding with High Spreads and Volatility

2.3 Application of the Dynamic Funding Model

This section illustrates the application of dynamic funding model using data from a large multinational investment bank.

2.3.1 Simulation of Spreads

Credit spreads are simulated using a stochastic model of hazard rates which is a special case of the general model discussed in *Section 2.2*. The model is set in discrete time with time steps $\{1, 2, \dots, t, \dots, s, \dots, s_1, \dots, s_2, \dots, T\}$. The discrete time hazard rate $h_{s_1, s_2}(t)$ at time t for maturity interval $\{s_1, s_2\}$ is given by the discrete time equivalent of *expression (2.2)*

$$h_{s_1, s_2}(t) = \frac{-\ln\left(\frac{q_{s_1}(t)}{q_{s_2}(t)}\right)}{s_2 - s_1} \quad (2.18)$$

where $q_s(t) = P(t, s)$ is the survival probability till time s as of time t . This expression implies that discrete time survival probability $q_s(t)$ is given by the discrete time equivalent of *expression (2.3)*

$$q_{s_{n+1}}(s_1) = P(s_1, s_{n+1}) = \exp\left(-\sum_{i=1}^n h_{s_i, s_{i+1}}(s_1)\right) \quad (2.19)$$

The survival probabilities $q_s(t)$ can be obtained from CDS spreads observed in the market. Since the value of CDS contract is zero at initiation, the expected value of premiums paid on the CDS contract must equal the expected value of losses in the event of default

$$E\left[\sum_{i=t+1}^n 1_{\{\tau>i\}} C_n(t)\right] = E\left[\sum_{i=t+1}^n 1_{\{\tau=i\}} (1 - R_i) \exp\left(-\sum_{j=t+1}^i r_j\right)\right] \quad (2.20)$$

where $C_n(t)$ is the CDS spread at time t for default protection till time $n \in \{t + 1, t + 2, \dots, T\}$, and R_n is the bond recovery rate. If interest rates, recovery rate and default time are independent, *expression (2.19)* simplifies to

$$C_n(t) \left(\sum_{i=t+1}^n q_i(t) d_i(t) \right) = (1 - R) \sum_{i=1}^n (q_{i-1}(t) - q_i(t)) d_i(t) \quad (2.21)$$

where $R = E(R_t)$ is the expected bond recovery rate. Noting that $q_t(t) = d_t(t) = 1$, the above expression can be used to bootstrap $q_i(t)$, $i \in \{t + 1, t + 2, \dots, T\}$, from the T equations corresponding to CDS spreads at T maturities.

Using the survival probabilities $q_s(t)$ implied from CDS spreads at time t , hazard rates $h_{s_1, s_2}(t)$ at time t for $s_1, s_2 \in \{t + 1, t + 2, \dots, T\}$ can be calculated using *expression (2.17)*. Given these hazard rates at time t , we now specify the model to simulate hazard rates at time $t + n$. The general framework for credit spread simulation discussed in *Section 2.2* had two components: a diffusion term and a jump term. Here, we model the diffusion term using the HJM framework for hazard rates ([5];[18]) and model jumps in hazard rates using structural credit risk modeling approach ([8];[21]). The structural credit risk approach is used to model defaults and rating migrations and the reduced form credit risk approach is used to model changes in spread from rating migrations and otherwise. In other words, this is a hybrid jump-diffusion model where the diffusion is driven by a reduced form model and the jumps are driven by a structural model.

Assume that asset price A follows geometric Brownian motion which implies that, in discrete time, the return r from a firm's assets is given by

$$r_{t+\Delta t} = \frac{A_{t+\Delta t} - A_t}{A_t} = \mu\Delta t + \sigma\sqrt{\Delta t}\varepsilon_{t+\Delta t} \quad (2.22)$$

which can be normalized as

$$\tilde{r}_{t+\Delta t} = \frac{r_{t+\Delta t} - \mu\Delta t}{\sigma\sqrt{\Delta t}} = \varepsilon_{t+\Delta t} \quad (2.23)$$

where $\varepsilon_{t+\Delta t}$ is an i.i.d. standard normal random variable.

The normalized return $\tilde{r}_{t+\Delta t}$ is assumed to be given by

$$\tilde{r}_{t+\Delta t} = \varepsilon_{t+\Delta t} = \lambda \sum_{i=1}^N w_i x_i + \sqrt{1 - \rho^2} \tilde{\varepsilon} \quad (2.24)$$

where x_i are N systematic factors and $\tilde{\varepsilon}$ is a standard normally distributed idiosyncratic factor. The systematic factors, which could be any macroeconomic or financial factors that explain the asset return, are assumed to be normally distributed with covariance matrix Σ . The ρ^2 term is the proportion of the normalized return explained by systematic factors and w_i are the weights on systematic factors. The λ term ensures that \tilde{r}_{t+1} has standard normal distribution.

Assume there are $M + 1$ ratings $\{1, 2, \dots, m, \dots, M, d\}$ with default rating d . For the time horizon Δt , let p_{ij} be the probability of migrating from rating i to rating j calculated using historical data. Then the cumulative probability of a credit with rating i being between rating 1 and rating k in Δt time period is given by

$$\pi_{i,k} = \sum_{j=1}^k p_{ij} \quad (2.25)$$

Estimate parameters for each credit and simulate the independent normally distributed idiosyncratic factor and simulate the systematic (industry and country) factors from a normal distribution with covariance matrix Σ to obtain the residual term \tilde{r}_{t+1} as per the expression above. A credit with rating i migrates to rating k if

$$\pi_{i,k-1} < N(\tilde{r}_{t+\Delta t}) \leq \pi_{i,k} \quad (2.26)$$

where $N(\cdot)$ represents the cumulative standard normal distribution.

The effect of rating migration on CDS spreads is modeled using generic CDS spread curves. Roughly speaking, generic CDS spread for a given rating for a

given maturity is the median CDS spreads for that maturity across borrowers with that rating (see next section for more details). Using these generic CDS spread curves, generic survival probabilities $q_s(t, r)$ for time s at time t for rating r are obtained using *expression (2.20)*. Then the generic survival probabilities $q_s(t, r)$ are used to obtain generic hazard rates $h_{s_1, s_2}(t, r)$ at time t for rating r for maturity interval (s_1, s_2) using *expression (2.17)*.

The percentage hazard rate change during time interval Δt can be written as

$$\ln \left(\frac{h_{s_1, s_2}^i(t + \Delta t, m(t + \Delta t))}{h_{s_1, s_2}^i(t, m(t))} \right) = \ln \left(\frac{h_{s_1, s_2}^i(t, m(t + \Delta t))}{h_{s_1, s_2}^i(t, m(t))} \right) + \ln \left(\frac{h_{s_1, s_2}^i(t + \Delta t, m(t + \Delta t))}{h_{s_1, s_2}^i(t, m(t + \Delta t))} \right) \quad (2.27)$$

where $h_{s_1, s_2}^i(t, m(t))$ is the hazard rate at time t for borrower i with rating $m(t)$ at time t for maturity interval (s_1, s_2) . This expression just states the fact that percentage hazard rate change during time interval Δt is due to ratings changes and non-rating changes.

We assume that hazard rate changes for borrower i due to rating migration are given by

$$\ln \left(\frac{h_{s_1, s_2}^i(t, m(t + \Delta t))}{h_{s_1, s_2}^i(t, m(t))} \right) = \ln \left(\frac{h_{s_1, s_2}^g(t, m(t + \Delta t))}{h_{s_1, s_2}^g(t, m(t))} \right) \quad (2.28)$$

where $h_{s_1, s_2}^g(t, m(t))$ is the hazard rate at time t for generic curve with rating $m(t)$ for maturity interval (s_1, s_2) .

We also assume that hazard rate changes for borrower i due to reasons other than rating migrations are given by

$$\begin{aligned} \ln \left(\frac{h_{s_1, s_2}^i(t + \Delta t, m(t + \Delta t))}{h_{s_1, s_2}^i(t, m(t + \Delta t))} \right) &= a^i(s_1, s_2)(b^i(s_1, s_2) \\ &\quad - \ln(h_{s_1, s_2}^i(t, m(t + \Delta t))) \Delta t \\ &\quad + \sigma^i(s_1, s_2)\varepsilon_{t+\Delta t}^h \end{aligned} \quad (2.29)$$

where $a^i(s_1, s_2)$, $b^i(s_1, s_2)$ and $\sigma^i(s_1, s_2)$ are parameters that can be estimated from *historical* hazard rates (implied from historical CDS spreads) of the borrower and ε^h is given by

$$\varepsilon_{t+\Delta t}^h = \beta\omega_{t+\Delta t} + \sqrt{1 - \beta^2}\varepsilon_{t+\Delta t}^i \quad (2.30)$$

where ω and ε^i are macro and firm-specific factors respectively with a standard normal distribution and β^2 is the proportion of variance explained by the systematic factor. The systematic factor could be a credit index or any other relevant factor. The β term can be estimated as the correlation between hazard rates of the borrower and the systematic factor⁹.

The last four expressions state that the percent hazard rate change has two components. The first component is the percent hazard rate change due to any change in ratings and it is assumed to be equal to the percent difference in hazard rates of the generic curve of new and old ratings. The second component is the percent hazard rate change due to reasons unrelated to rating change and it is assumed to have a mean reverting component and a stochastic component.

Using the simulated hazard rates $h_{s_1, s_2}(t + s)$ at a future time $t + s$, the risk-neutral survival probabilities $q_{s_i}(t + s)$ at time $t + s$ are calculated using *expression (2.18)*. The risk neutral probabilities $q_{s_i}(t + s)$ in turn are used to calculate the CDS spreads $C_n(t + s)$ at time $t + s$ using *expression (2.20)*. The CDS spreads simulated in this section are used in the subsequent sections to simulate usage and obtain the funding needs for the revolver portfolio¹⁰.

⁹A correlation between the systematic factor ω and the systematic factors x_i can be introduced if necessary.

¹⁰The simulated spreads can also be used to calculate the value-at-risk (VAR) of the loan portfolio which includes not only realized losses from defaults but also unrealized losses from spread movements.

Implementation of the Model

Expression (2.23) is estimated using country and industry factors as systematic factors x_i . The estimation involves regressing the normalized stock index returns for these countries and industries on normalized stock returns of the borrower. The coefficients of this regression represent the factor weights w_i and the R-square of this regression represents the proportion ρ of borrower returns explained by systematic factors. Since there are several hundred borrowers in the loan portfolio of the bank whose data is used here, the estimation results of these hundreds of regressions are not reported to conserve space. The rating transition matrix used in *expression (2.24)* is standard in credit models and are regularly published by rating agencies such as Moody's.

Generic CDS spread curves used in *expression (27)* are created for each rating for a given day by grouping borrower CDS spread curves of that rating on that day, removing outlier spreads, and identifying the median spread for each maturity. These median spread curves for various ratings are then smoothed to remove kinks and eliminate cross-over of curves across ratings. The resulting curves are generic curves for each rating on that day.

The estimation of *expression (2.28)* is complicated by the fact that many borrowers either do not have traded CDS or do not have long enough history of observed CDS spreads. Therefore, we assume that parameters $a^i(s_1, s_2)$, $b^i(s_1, s_2)$ and $\sigma^i(s_1, s_2)$ in *expression (2.28)* are same for all borrowers i with the same rating. To further simplify the implementation, we assume that these parameters do not depend on hazard rate maturity (s_1, s_2) . These assumptions imply

$$a^i(s_1, s_2) = a^r \tag{2.31}$$

Table 2.1:**Estimation of Parameters for Hazard Rate Diffusion**

This table provides the panel data estimates for expression (2.34). The dependent variable is generic bond spread for each rating. The independent variables are a mean reversion term a^r , a long term mean term b^r , and a volatility term σ^r . All coefficients are significant at 1% level.

Variable	AAA	AA	A	BBB	BB	B
a^r	0.37	0.24	0.19	0.44	0.49	0.51
b^r	4.61	5.07	5.22	5.32	5.93	6.28
σ^r	0.37	0.34	0.33	0.36	0.44	0.43

$$b^i(s_1, s_2) = b^r \quad (2.32)$$

$$\sigma^i(s_1, s_2) = \sigma^r \quad (2.33)$$

where a^r , b^r and σ^r are the common parameters for all borrowers with rating r .

These parameters are estimated using the following expression

$$\ln \left(\frac{s^r(t+1)}{s^r(t)} \right) = a^r(b^r - \ln(s^r(t))) + \sigma_h^r \varepsilon_{t+\Delta t}^h \quad (2.34)$$

where $s^r(t)$ are the generic bond spreads for rating r at time t . Note that the above expression modifies *expression (2.28)* by using *expressions (30)-(32)* and replacing $h_{s_1, s_2}^i(t)$ with $s^r(t)$. The hazard rate $h_{s_1, s_2}^i(t)$ is replaced by generic bond spread $s^r(t)$ because of the above assumption that parameters b_0^r , b_1^r and σ_h^r do not depend on hazard rate maturity (s_1, s_2) . The generic bond spread data used for estimation is obtained for each rating between 1991 to 2007 from Bloomberg. The estimation results for each rating are included in *Table 2.1*.

Table 2.2
Summary Statistics For Utilization and Spread Data

Variable Name	Mean	StdDev	Median
utilization	0.18	0.32	0
expecteddrawn	0.28	0.32	0.13
1y_spread (bps)	205	3493	38

2.3.2 Utilization

This section estimates the empirical behavior of loan utilization. We use three sets of data for this purpose. The first set of data contains the details of long-term investment grade loans issued by a large multinational bank between October 2003 and June 2008. The second set of data contains the utilization on these loans at monthly frequency from October 2003 to July 2008. The third set of data consists of the CDS spreads of the borrowers of these loans. The number of loans in the first data set varies from 200 to 1000 in any given month.

The summary statistics for the sample is included in *Table 2.2*. The median utilization is 0 implying that most facilities in the sample are undrawn most of the time. The expected utilization is higher than actual utilization on average indicating that the expected utilization of the loan indicated at the time of loan issuance is biased upward on average¹¹. The 1 year CDS spreads is about 205 bps on average though its median value is only 38 bps.

Table 2.3 contains the estimation results. Model 1 includes expected utilization and current period 1 year CDS spread as explanatory variables. This model

¹¹Expected utilization is an indicative variable provided at loan issuance by the relationship manager who maintains the bank's relationship with the borrower.

Table 2.3**Empirical Determinants of Utilization**

This table provides the panel data estimates for equation (14). The dependent variable is current period utilization. AIC and BIC are functions based on the log likelihood of the estimation with lower values representing better model fit. Standard errors are included in parenthesis. (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

	Model_1	Model 2	Model 3
AIC	33953	-95764	-59227
BIC	33973	-95745	-59208
30 day prior usage		0.8539*** (0.001728)	
90 day prior usage			0.7743*** (0.002231)
expecteddrawn	0.5685*** (0.002)	0.0891*** (0.0014)	0.138*** (0.0019)
current 1yr spread (bps)	0.000154*** (0.000002)	0.000043*** (0.0000016)	0.000094*** (0.0000017)

suggests that a 100 bps increase in CDS spread results in a 1.54% absolute increase in utilization rate. There could be two reasons for this result. First, banks may be utilizing loans depending on whether it is costlier to draw on the revolver or raise money in the bond market. This is the opportunistic behavior hypothesis. Second, an increase in spread may correspond to lower credit worthiness of the borrower which may make it more difficult to raise money elsewhere. This is the credit constraint hypothesis. The data doesn't allow us to differentiate between these two hypothesis.

The other two models in *Table 3* include lagged utilization as an independent

variable in addition to expected utilization and CDS spread. In Model 2, the lagged variable is 1 month prior utilization and in Model 3, the lagged variable is 3 month prior utilization. First note that including lagged utilization reduces the sensitivity on expected utilization and CDS spread as compared to the case where there is no lagged utilization. About 70-80% of the current period utilization is explained by the previous period utilization. Comparing the results of Model 2 and Model 3 shows that, as the lag period increases, the coefficient on lagged utilization falls, and the sensitivity to expected utilization and CDS spread increases. The pattern of increasing sensitivity to spreads and decreasing sensitivity to lagged utilization as the lag period increases suggests that borrowers either respond to spreads with a lag or respond only to permanent changes in spreads.

2.3.3 Dynamic Funding

The revolving credit portfolio of a large investment bank is subjected to the dynamic funding model with quarterly adjustment periods between December 2007 and December 2009. Since this period coincided with one of the most severe financial crisis ever, it is a good test case to evaluate the performance of dynamic funding model.

At the beginning of each quarter, one year CDS spreads for all borrowers in this portfolio were simulated one quarter into the future using the hazard rate simulation model discussed in *Section 2.3.1*. The simulated spreads, current revolver utilization, and expected revolver utilization are plugged into Model 3 estimated in *Section 2.3.2* to obtain the one quarter ahead utilization for each re-

volver in each simulation. The revolvers are divided into different tenor buckets based on the remaining tenor of each loan. The portfolio utilization is calculated for each tenor in each simulation, which results in the simulated usage distribution for that tenor. The usage distribution for each tenor is used to obtain the expected utilization for each tenor and these expected amounts are term funded to match the tenor of that bucket. The simulated usage distribution is also obtained for the whole portfolio (without segregating by tenor). This distribution is used to obtain the expected utilization and unexpected utilization (at 99% confidence interval) for the whole portfolio. The unexpected utilization is funded in liquid securities. At the end of each quarter, the actual utilization of the whole portfolio is recorded and the expected and unexpected funding for the next quarter are calculated using the above steps.

The results of this exercise are included in *Figure 2.6*. Actual utilization was about 15% in December 2007 and the model projected the expected funding to be about 15% and the unexpected funding to be about 3% for March 2008. The actual utilization in March 2008 of about 16% was higher than expected utilization, which was presumably because of the high CDS spreads during the Bear Sterns rescue period. The high spreads and utilization in March 2008 resulted in higher expected funding projection of about 17% for June 2008. However, actual utilization fell to 15% in June 2008, which coincided with tighter spreads in June 2008. Due to lower spreads and utilization in June 2008, the model projected lower expected utilization of 16% for September 2008. However, actual utilization increased to about 17% in September 2008, which coincided with high CDS spreads during the Lehman bankruptcy. Due to such high spreads and utilization in September 2008, the model projected higher expected utilization of about 18% for December 2008. Actual utilization increased more than the expected

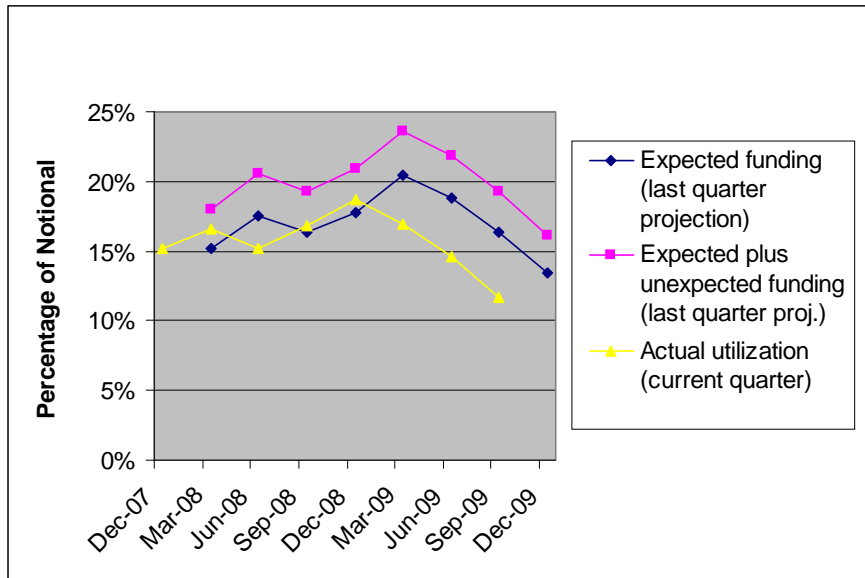


Figure 2.6 Application of Dynamic Funding Model on Real Data

funding to about 19% in December 2008, which was the peak of the financial crisis with CDS spreads at their peak. Based on the high utilization and spreads in December 2008, the model projected expected utilization of about 20%, but the actual utilization fell in March to about 17%. Since then, both actual and expected utilization have been falling each quarter coinciding with falling CDS spreads and general improvement in the economy.

Note that the actual utilization never breached the expected plus unexpected funding during the whole financial crisis. This suggests that the dynamic funding model performed well in managing the funding risk of loan portfolio during this extremely stressful period. Also, the actual utilization increased with higher CDS spreads and fell with lower CDS spreads, thus validating the basic premise of the model that utilization is a function of spreads. Finally, observe that the expected utilization for the next quarter is slightly higher than the actual utilization of the current quarter (by about 1-2%). There are two reasons for the

higher expected utilization. First, actual utilization doesn't include expected or actual utilization during default. Second, the multivariate model above involving spreads and utilization uses information from facilities with high spreads to give richer predictions of expected utilization.

2.4 Expected Funding Costs

In this section, we study how the expected funding cost varies depending on the funding method. We look at three funding methods: a) short-term stochastic funding; b) long-term static funding; and c) long-term dynamic funding.

2.4.1 Model

To study the expected funding cost under various methods, we use the reduced form credit risk modeling approach (e.g. [20]). Let r_t be the riskless short rate and λ_t^b be the intensity of the Cox process governing default of the borrower. Assume that the risk-neutral dynamics of the intensity process λ_t^b is given by the square root process

$$\begin{aligned} d\lambda_t^b &= (\alpha^b - \beta^b \lambda_t^b)dt + \sigma^b \sqrt{\lambda_t^b} dW_t^b \\ &= \beta^b (\bar{\lambda}^b - \lambda_t^b)dt + \sigma^b \sqrt{\lambda_t^b} dW_t^b \end{aligned} \quad (2.35)$$

where α^b , β^b and σ^b are positive constants, ω^b is the fractional loss given default, and W_t^b is a standard Brownian motion. This process allows for mean reversion with long-run mean $\bar{\lambda}^b = \frac{\alpha^b}{\beta^b}$ and guarantees that the intensity is always nonnegative.

Let $s(t, T)$ denote the CDS premium at time t for buying default protection on the borrower for maturity T . Equating the present value of the premium leg of CDS and the present value of the protection leg of a CDS, we can solve for the CDS premium and apply the results in [6] to obtain:

$$s^b(t, T) = \frac{E_t \left[\omega \left(\int_t^{t+T} \lambda_u^Q \exp \left(- \int_t^u (r_s + \lambda_s^Q) ds \right) du \right) \right]}{E_t \left[\left(\int_t^{t+T} \exp \left(- \int_t^u (r_s + \lambda_s^Q) ds \right) du \right) \right]} \quad (2.36)$$

where ω is constant fractional loss given default.

Since λ_t^b has a square root process, the CDS premium $s^b(t, T)$ in the above expression has a closed form solution given by (see [10] for a proof):

$$s^b(t, T) = \frac{\omega^b \int_t^{t+T} \exp(B_u^b \lambda_u^b) D_t(G_u^b + H_u^b \lambda_u^b) du}{\int_0^T A_u^b \exp(B_u^b \lambda_u^b) D_u du} \quad (2.37)$$

where

$$\begin{aligned} A_u^b &= \exp \left(\frac{\alpha^b (\beta^b + \phi^b)}{(\sigma^b)^2} u \right) \left(\frac{1 - \kappa^b}{1 - \kappa^b e^{\phi^b u}} \right)^{\frac{2\alpha^b}{(\sigma^b)^2}} \\ B_u^b &= \left(\frac{\beta^b - \phi^b}{(\sigma^b)^2} \right) \left(\frac{2\phi^b}{(\sigma^b)^2 (1 - \kappa^b e^{\phi^b u})} \right) \\ G_u^b &= \frac{\alpha^b}{\phi^b} (e^{\phi^b u} - 1) \exp \left(\frac{\alpha^b (\beta^b + \phi^b)}{(\sigma^b)^2} u \right) \left(\frac{1 - \kappa^b}{1 - \kappa^b e^{\phi^b u}} \right)^{\frac{2\alpha^b}{(\sigma^b)^2} + 1} \\ H_u^b &= \exp \left(\frac{\alpha^b (\beta^b + \phi^b) + \phi^b (\sigma^b)^2}{(\sigma^b)^2} u \right) \left(\frac{1 - \kappa^b}{1 - \kappa^b e^{\phi^b u}} \right)^{\frac{2\alpha^b}{(\sigma^b)^2} + 2} \\ \phi^b &= \sqrt{2(\sigma^b)^2 + (\beta^b)^2} \end{aligned}$$

and

$$\kappa^b = \frac{\beta^b + \phi^b}{\beta^b - \phi^b}$$

Assume that the intensity λ_t^f of the Poisson process governing default of the bank has risk-neutral dynamics given by a square root process

$$\begin{aligned} d\lambda_t^f &= (\alpha^f - \beta^f \lambda_t) dt + \sigma^f \sqrt{\lambda_t^f} dW_t^f \\ &= \beta^f (\bar{\lambda}^f - \lambda_t^f) dt + \sigma^f \sqrt{\lambda_t^f} dW_t^f \end{aligned} \quad (2.38)$$

where α^f, β^f and σ^f are positive constants with $\bar{\lambda}^f = \frac{\alpha^f}{\beta^f}$, ω^f is the fractional loss given default, and W_t^f is a standard Brownian motion such that $\text{corr}(W^b, W_t^f) = \theta$.

The CDS spread for the bank $s^f(t, T)$ at time t for maturity T , is given by equations (22-27) with $\lambda_t^b, \alpha^b, \beta^b, \sigma^b$ and ω^b replaced by $\lambda_t^f, \alpha^f, \beta^f, \sigma^f$ and ω^f respectively. Given the correlation θ between the two Brownian motions, the funding spread $s^f(t, T)$ and the borrower CDS spreads $s^b(t, T)$ are correlated as well.

2.4.2 Expected Cost of Alternative Funding Methods

Now consider a loan with notional limit M and remaining maturity T with a drawn portion $u_t M$ and an undrawn portion $(1 - u_t)M$ at time t . The borrower pays commitment fee c on the undrawn amount and a base rate L_t plus margin m on the drawn amount. The bank funds the drawn portion of the loans at the base rate L_t plus the funding spread f_t^s at time t . Assume that the current time is t_0 and the loan runs from t_0 to $t_N = t_0 + T$. Also, assume that the loan is drawn and the fees on the loan are paid at discrete times $t \in \{t_0, t_1, \dots, t_n, \dots, t_N\}$.

The expected funding cost of a loan $V_F(t_n)$ at time t_n represents the present value of funding cost incurred by the bank in the form of funding spread payments f_t^s for the remaining life of the loan. We consider three specifications of the funding leg. The first specification involves *long-term static funding* whereby the bank purchases a fixed funding amount $uM = E_0(u_t M)$ at a funding spread $f_t^s = s^f(t_n, T)$. In other words, the static funding amount is equal to the expected utilization on the loan and the static funding spread is equal to the CDS

spread on the bank at time t for maturity T . In this case, the value of funding leg $V_F(t_n)$ at time t_n is given by:

$$V_F(t_n) = \sum_{i=n+1}^N E_{t_n} \left[\exp \left(- \int_{t_n}^{t_i} \lambda_s^b ds \right) \exp \left(- \int_{t_n}^{t_{i+1}} r_s ds \right) \right] uM s^f(t_n, T) \Delta t_i \quad (2.39)$$

Note that this expression assumes that, if the borrower defaults between two coupon payment dates, the bank has to pay the funding cost $s^f(t_n, T)$ for that period but can stop paying the funding cost thereafter. In other words, this expression assumes that, in the event of default, funding for the loan can be sold at $s^f(t_n, T)$. This is obviously a simplification since bank's funding spreads may not be $s^f(t_n, T)$ when the borrower defaults. However, since this specification has constant funding amount (uM), constant funding spread ($s^f(t_n, T)$) and constant funding cancellation spread ($s^f(t_n, T)$), it contrasts and helps illustrate the effect of variable funding amount, variable funding spread and variable funding cancellation spread in the other two specifications.

The second specification involves *short-term stochastic funding* wherein the bank purchases a fixed funding amount $uM = E_0(u_t M)$ and rolls over the funding each period at the prevailing next period funding spread $s^f(t_i, t_{i+1})$, which is random. If the borrower defaults between two periods, there is no need to cancel funding in this case because the existing funding need not be rolled over to the next period. Most banks were funding their assets this way prior to the current financial crisis. The value of funding leg in this case is given by:

$$V_F(t_n) = \sum_{i=n+1}^N E_{t_n} \left[\exp \left(- \int_{t_n}^{t_i} \lambda_s^b ds \right) \exp \left(- \int_{t_n}^{t_{i+1}} r_s ds \right) s^f(t_{i-1}, t_i) \Delta t_i \right] u_i M \quad (2.40)$$

The third specification involves *long-term dynamic funding* wherein the funding amount $u_t M$ is variable and the incremental funding in each period $u_i - u_{i-1}$ is purchased or sold at the random funding spread $s^f(t_i, T)$ for the remaining loan tenor.

$$V_F(t_n) = \sum_{i=n+1}^N E_{t_n} \left[\begin{array}{c} \exp \left(- \int_{t_n}^{t_i} \lambda_s^b ds \right) * \\ \exp \left(- \int_{t_n}^{t_{i+1}} r_s ds \right) (u_i - u_{i-1}) s^f(t_{i-1}, T) \Delta t_i \end{array} \right] M \quad (2.41)$$

Comparing the last three expressions, note that calculating funding cost is more complicated for dynamic funding than for static funding and short-term stochastic funding. For example, if there is a positive correlation between borrower spread and funding spread, if the CDS spread of borrower falls next period resulting in lower funding need for a loan, the funding cost of the loan increases since the incremental funding amount is likely to be unwound at lower funding spread. On the other hand, if there is a negative correlation between the two spreads, funding cost is likely to fall when borrower spreads fall. The opposite holds if borrower spreads are likely to rise. Therefore, the funding cost depends on the interaction between utilization, borrower spread and the correlation between funding and borrower spread.

2.4.3 Simulation Procedure

Since the last two expressions have no closed form solution, we use simulation to calculate the funding cost for the three different ways of funding a loan. For simplicity interest rate r_t is assumed to be zero¹². We consider a one year loan and assume that utilization changes can be done once a quarter. For the default intensity of borrower and funding spreads

$$d\lambda_t = \beta(\bar{\lambda} - \lambda_t)dt + \sigma\sqrt{\lambda_t}dW_t \quad (2.42)$$

with $d = 4\bar{\lambda}\beta/\sigma^2 > 1$ ¹³, we simulate from the exact transition density of the square root process on the time grid $t_n < t_{n+\frac{1}{365}} < t_{n+\frac{2}{365}} < \dots < t_{n+1}$ as follows (see [17] for a proof)

$$\lambda_{t_{i+\frac{1}{365}}} = c[(Z + \sqrt{Y})^2 + X] \quad (2.43)$$

where

$$c = \sigma^2(1 - e^{-\beta(t_{i+\frac{1}{365}} - t_i)})/4\beta \quad (2.44)$$

$$Y = \lambda_{t_i}e^{-\beta(t_{i+\frac{1}{365}} - t_i)}/c \quad (2.45)$$

$$Z \sim N(0, 1) \quad (2.46)$$

$$X \sim \chi_{d-1}^2 \quad (2.47)$$

To simulate the time to default τ for the square root process, we simulate a uniform random variable $0 \leq U \leq 1$ and find $\tau = t_{n+\frac{\theta}{365}}$ where θ is given by

$$\sum_{i=1}^{\theta-1} \lambda_{t_{n+\frac{i}{365}}} < -\ln(U) \leq \sum_{i=1}^{\theta} \lambda_{t_{n+\frac{i}{365}}} \quad (2.48)$$

¹²Assuming constant or stochastic interest rates that are independent of default intensity does not affect the qualitative results in this section.

¹³For interest rate and credit models, d is typically larger than 2 (see Glasserman 2003, pp 125). Therefore, we do not consider the case where $d < 1$ which requires a different simulation procedure.

where λ_s is a default intensity path from time t_n onwards simulated using the above simulation procedure.

Quarterly changes in utilization u_t are assumed to be given by the following expression which is estimated using data referred to in *Section 2.3.2*.

$$u_{t+0.25} = \min\{1, u_t + 0.0087[s^b(t + 0.25, T) - s^b(t, T)]\} \quad (2.49)$$

where $s^b(t, T)$ is estimated using *expression (2.36)* based on λ_s simulated using the above simulation procedure.

So the simulation involves: using *expressions (34) and (37)* respectively to draw paths of borrower default intensity λ_s^b and funding spread intensity λ_s^f with correlation $\text{corr}(W^b, W_t^f) = \theta$ on the time grid $t_n < t_{n+\frac{1}{365}} < t_{n+\frac{2}{365}} < \dots < t_{n+1}$ given initial intensities $\lambda_{t_n}^b$ and $\lambda_{t_n}^f$; using *expression (2.47)* to calculate the time to default τ on each path; using *expression (2.36)* to calculate the borrower spread $s^b(t_n + 0.25n, T)$ and funding spreads $s^f(t_n + 0.25n, T)$ in each quarter $n \in \{0, 1, 2, 3, 4\}$ where there is no default; using *expression (2.48)* to calculate the utilization in each quarter; and calculating the funding cost in the three cases as per *expressions (38), (39) and (40)*.

The base parameter values for the simulation are: $\lambda_{t_n}^b = 1.67\%$ and $\lambda_{t_n}^f = 0.83\%$, $\text{corr}(W^b, W_t^f) = \theta = 0.5$, $\sigma^b = 50\%$, $\sigma^f = 40\%$, $\omega^b = \omega^f = 40\%$, $\beta^b = 7.5$, $\beta^f = 9.6$, $\bar{\lambda}^b = 1.67\%$, $\bar{\lambda}^f = 0.83\%$. These base parameter values imply initial 1 year spreads of $s^b(t_n, t_{n+1}) = 1\%$ and $s^f(t_n, t_{n+1}) = 0.5\%$. In addition to the simulation with base parameter values, multiple simulations were run involving a change in one of the base parameters while other parameters were left unchanged.

2.4.4 Simulation Results

Table 2.4 contains the simulation results with 250,000 simulations for the base parameter case (wherein initial borrower and funding spreads coincide with corresponding long-term mean values). In terms of point estimates, stochastic funding is the cheapest followed by static funding and dynamic funding. However, in terms of statistical significance, the expected funding cost under the three methods are not significantly different from each other¹⁴. Even if we can statistically estimate the ranking of these funding costs, the economic difference between these funding costs is negligible¹⁵ (less than 0.25% even after factoring three standard errors from the point estimates). So from a cost perspective, the banks should be indifferent between these three funding methods especially in the scenarios where the current borrower and funding spreads are close to their long term values.

Next, we study the effect of various parameters on the expected funding cost under the three methods. Since the purpose of this exercise is study how the funding cost under a given funding method changes for different parameter values rather than compare the funding cost across the three funding methods (which is difficult as discussed above), we run 10,000 simulations in each case with the same random numbers in each simulation. This simulation strategy reduces the computational time while allowing (qualitative rather than statisti-

¹⁴Several million simulations are needed to reliably estimate the relative ranking of these funding costs. As discussed in Schonbucher (2003, pp 215-216), the direct simulation of default times is difficult because the low probability of default requires more than a million simulations to reliably estimate. Therefore, he recommends simulation using the branching to default method (Schonbucher 2003, pp 217-218). We implemented the branching to default method with 50,000 simulations and the three funding costs were still not statistically different from each other.

¹⁵While correlation between default intensity and interest rate may result in statistically significant difference in the three funding costs, the empirical correlation between these variables is not large enough to make the difference economically significant.

Table 2.4

Simulation Results for Base Parameter Values

This table provides the simulation results for the expected funding cost under various funding methods for the base parameter case.

(*** represents significance at one percent level)

Funding Method	Expected Funding Cost	Std. Error
Stochastic Funding	0.1986047***	0.00008
Static Funding	0.1986644***	0.00003
Dynamic Funding	0.1986972***	0.00003

cal) comparison of the funding costs under each method for various parameter values. *Figures 7-17* contain the results of these simulations. The first point to note from these figures is that, for the base parameter case, the expected funding costs between the three methods are not significantly different from each other in statistical terms, even though stochastic funding is more expensive than dynamic and static funding in terms of point estimates. The other observations from these figures are noted below.

Spread correlation: The effect of spread correlation θ on funding cost under the three methods is illustrated in *Figure 2.7*. Correlation has no effect on static funding (*expressions 38*) costs because both the price and amount of funding is fixed upfront. In the case of stochastic funding (*expressions 39*), the value of funding leg decreases with correlation. This is because, with higher correlation, borrower intensity is on average higher when funding intensity is high, which implies higher probability of borrower default. Therefore in environments with high funding spreads, funding costs are lower because the borrower defaults

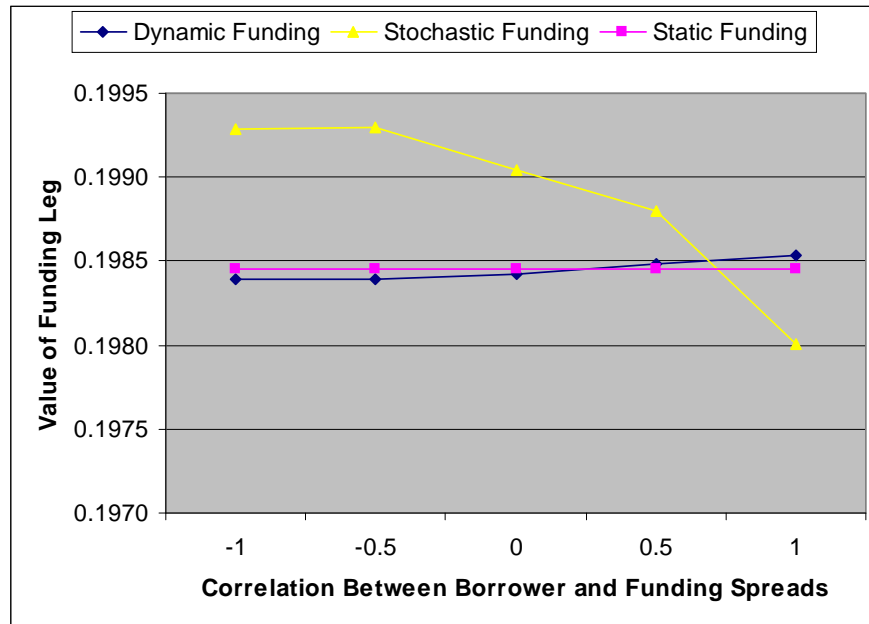


Figure 2.7 Effect of Borrower and Funding Spread Correlation

more often and hence funding costs are incurred less often. On the other hand, in scenarios with low funding spreads, borrower defaults less often and hence funding cost is incurred more often when correlation is higher. However, because the long run funding spread is low in the base parameter case and since funding spread is bounded below by zero, the scenarios with higher funding spread have a larger impact on the average funding cost. Therefore, for the base parameter case, the net effect is that funding cost falls with higher correlation.

In contrast, the funding cost increases with correlation in the case of dynamic funding (*expressions 40*). This is because higher correlation implies that when funding spread is high, borrower spreads are high which implies more utilization at a time when funding cost is high and low utilization when funding cost is low. The effect of higher utilization dominates the effect of lower borrower default probability resulting in higher funding cost. Of course, the opposite holds

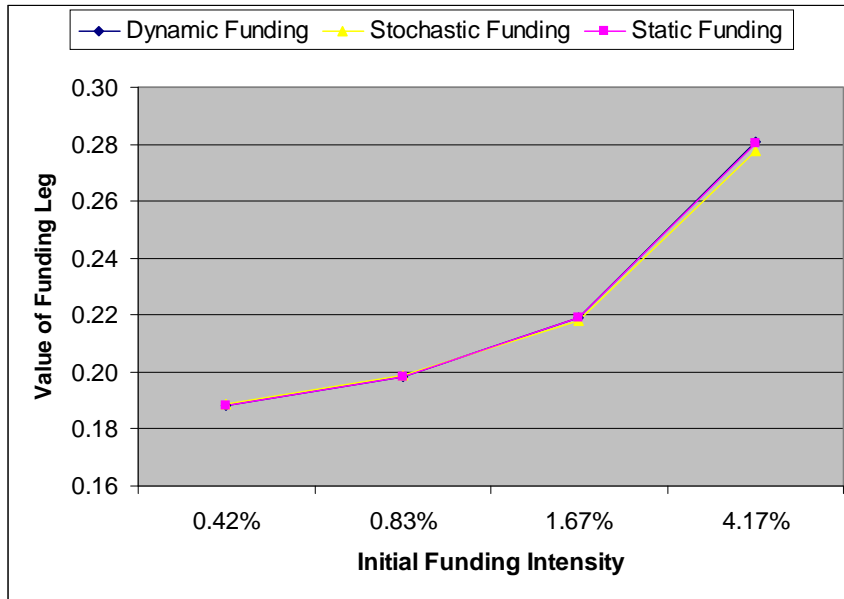


Figure 2.8 Effect of Initial Funding Intensity on Funding Costs

in scenarios where funding spreads are low, but as discussed above, the effect of high spread environments is higher on average funding cost, thus resulting in higher dynamic funding cost with higher correlation.

Initial funding intensity: The effect of initial funding intensity $\lambda_{t_n}^f$ on funding cost under the three methods is illustrated in *Figure 2.8*. When funding spread intensity at time 0 is high, the funding spread at time 0 is high. Therefore, funding cost increases with higher funding intensity at time 0 for all the three methods. When the initial funding spread is above the long run mean funding spread, the increase in funding cost is higher in the case of static and dynamic funding because most or all of the funding in these cases is purchased at time 0 when the intensity is high whereas in the case of stochastic funding the initial funding intensity has less impact because funding is rolled over every quarter and mean reversion pulls down the subsequent funding spreads on average. The opposite holds when initial funding spread is below long run mean

funding spread because dynamic and static funding help lock in cheap funding in such a case whereas stochastic funding requires purchasing more expensive funding in future due to mean reversion.

Initial borrower default intensity: The effect of initial borrower default intensity $\lambda_{t_n}^b$ on funding cost under the three methods is illustrated in *Figure 2.9*. In all the three methods, the funding cost goes down with higher initial borrower default intensity. This is because initial higher default intensity implies higher default probability and hence the loan has to be funded for a shorter duration on average. If the initial borrower default intensity is higher than the long-run mean borrower default intensity, mean reversion implies that the expected future borrower spread is lower than the case where the initial and the long-run value of borrower default intensity are about the same. Therefore, in such a scenario, less funding is required and hence the funding cost falls more with dynamic funding than with static and stochastic funding because utilization falls with lower future borrower spreads. The opposite holds if initial borrower default intensity is lower than the long run mean borrower default intensity.

Funding intensity volatility: *Figure 2.10* shows that the loan funding cost under the three methods converges to the same value as the funding intensity volatility σ^f goes to zero. This is because if the funding volatility is zero (with everything else same as in the base parameter case), the funding intensity is always equal to the initial and long-run mean funding intensity of 0.83%, which implies a flat and constant term structure of funding spreads. In such a case, loan funding cost is obviously same under the three methods.

Borrower default intensity volatility: *Figure 2.11* suggest that the loan funding cost decreases with borrower default intensity volatility σ^b under all the three

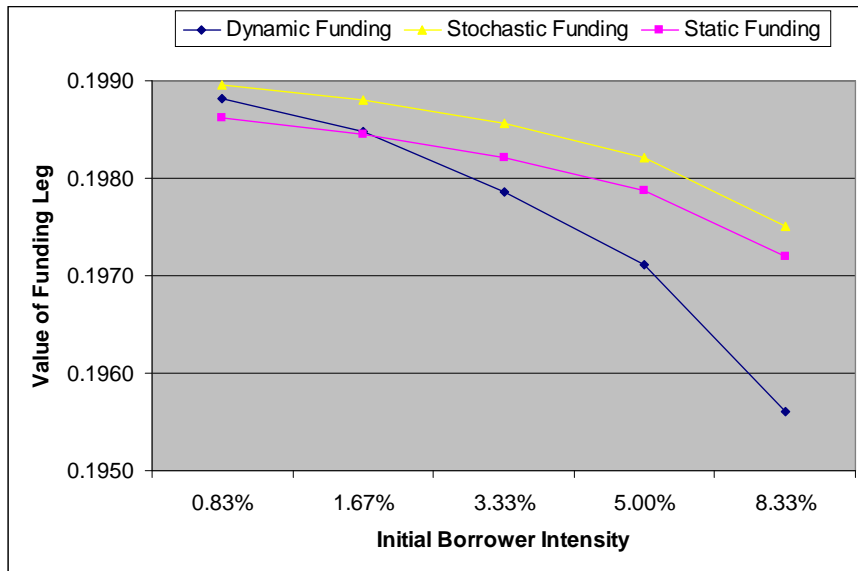


Figure 2.9 Effect of Initial Borrower Intensity on Funding Costs

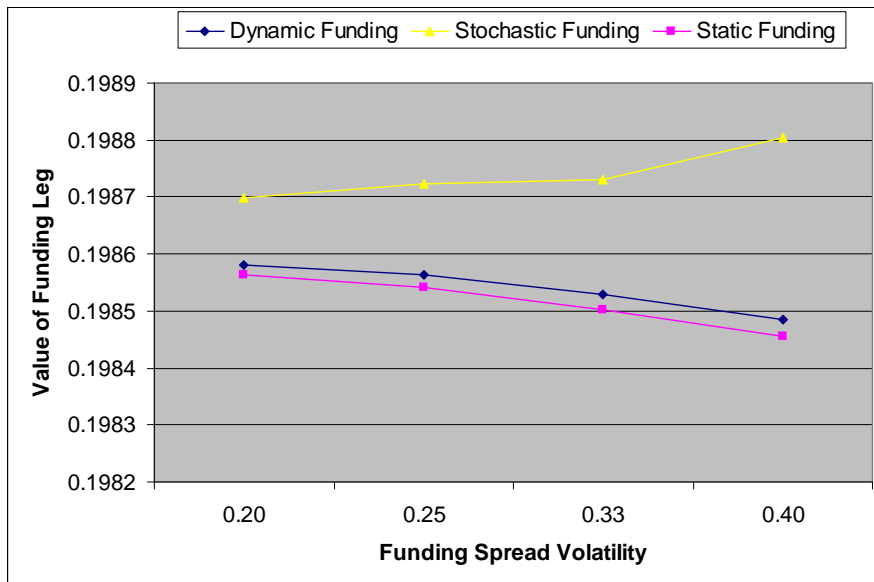


Figure 2.10 Effect of Funding Spread Volatility on Funding Costs

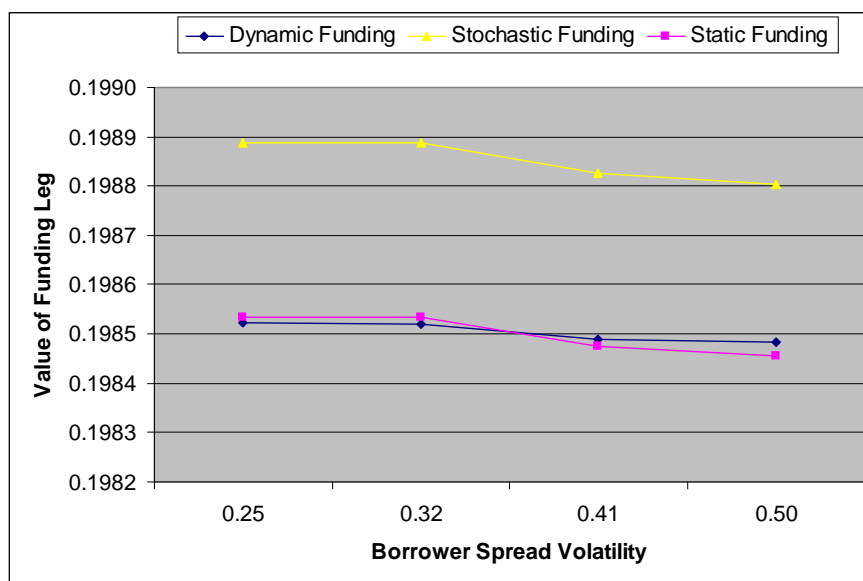


Figure 2.11 Effect of Borrower Spread Volatility on Funding Costs

methods. However, as is well known, higher volatility of default intensity reduces the default probabilities under the square root model due to the exponential effect, which in turn should imply a higher funding cost. The opposite result here is due to small sample bias.

Borrower Loss Given Default: The effect of borrower LGD ω^b on funding cost under the three methods is illustrated in *Figure 2.12*. If borrower loss given default increases and the initial and long-run mean borrower default intensity remain unchanged, borrower spread increases which implies higher funding cost in the case of dynamic funding because funding amount increases with higher spread under this funding method. For the other two methods, the funding cost remains the same, because the funding amount and funding spread remains the same.

Funding Loss Given Default: The effect of funding LGD ω^f on funding cost under the three methods is illustrated in *Figure 2.13*. If funding loss given de-

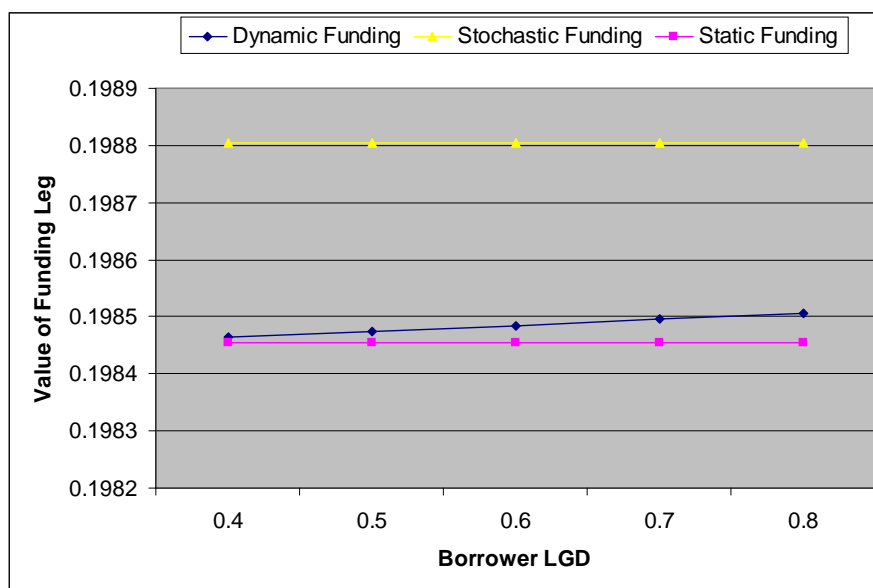


Figure 2.12 Effect of Borrower Loss Given Default on Funding Costs

fault increases and the initial and long-run mean funding intensity remain unchanged, funding spread increases and hence funding cost is higher in all the three methods.

Borrower default intensity mean reversion: Figure 2.14 suggests that the expected funding cost increases with the borrower default intensity mean reversion β^b under all the three methods. However, as is well known, higher mean reversion of default intensity increases the default probabilities under the square root model, which in turn should imply a lower funding cost. The opposite result here is due to small sample bias.

Funding intensity mean reversion: The effect of funding intensity mean reversion β^f on funding cost under the three methods is illustrated in Figure 2.15. As funding intensity mean reversion increases to infinity, loan funding cost under the three methods converges to the same value. This is because any ran-

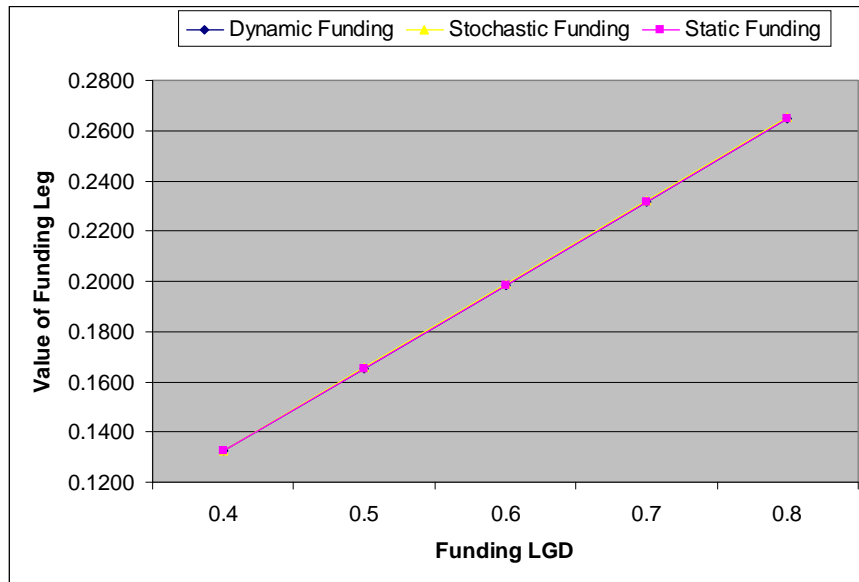


Figure 2.13 Effect of Funding Loss Given Default on Funding Costs

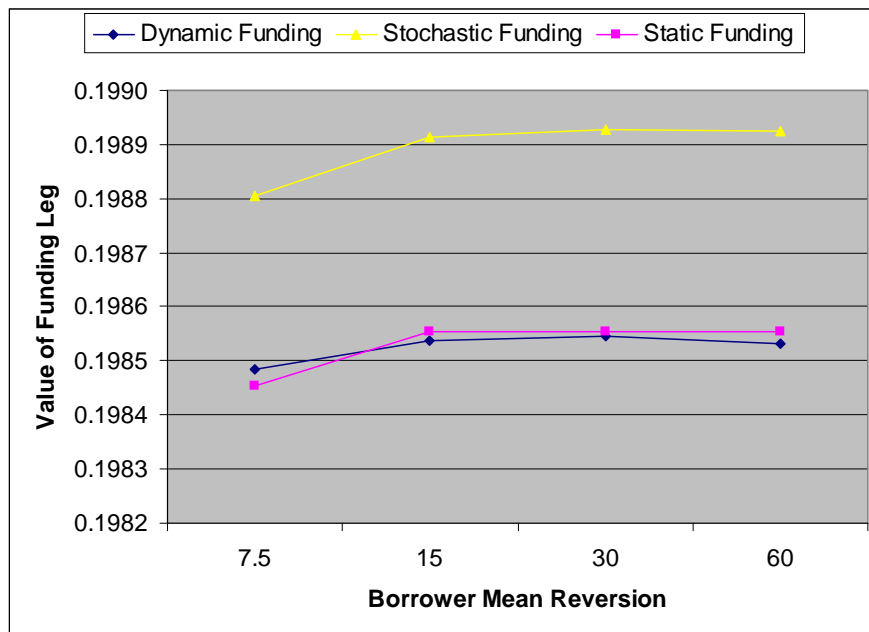


Figure 2.14 Effect of Borrower Mean Reversion on Funding Costs

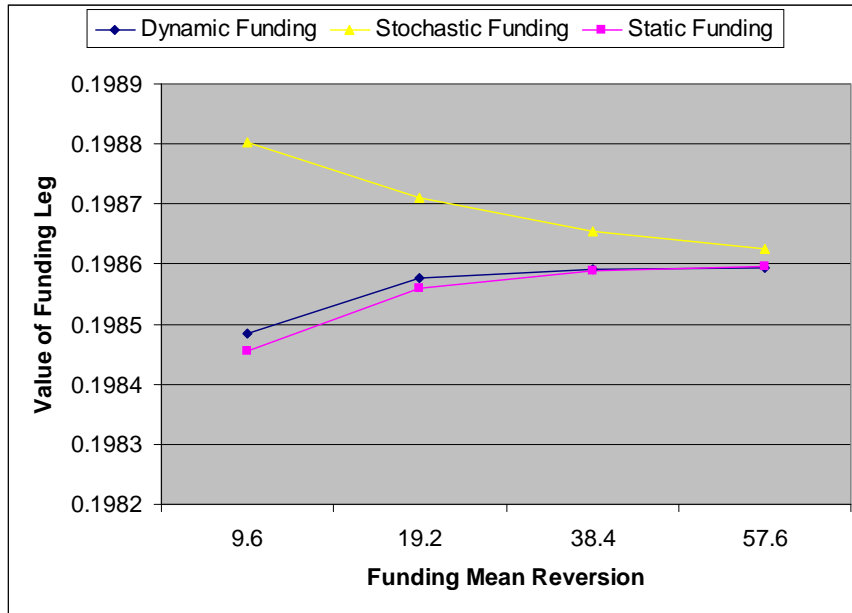


Figure 2.15 Effect of Funding Mean Reversion on Funding Costs

dom change in funding intensity in a given period is reversed (on average) in the next period due to infinite mean reversion, which implies that the expected funding intensity is the same in each period and hence the term structure of funding spreads is flat and constant. Therefore, with infinite mean reversion, the expected funding cost is the same irrespective of which funding method is used.

Long run mean borrower default intensity: Figure 2.16 shows that, under all the three funding methods, funding cost is lower if the long run mean borrower default intensity $\bar{\lambda}^b$ is higher (with the same initial borrower default intensity). This is because a higher long run mean of borrower default intensity implies a higher probability of borrower default, which in turn implies lower funding cost because the loan is likely to be alive for a shorter duration. If the long-run mean borrower default intensity is higher than the initial borrower default

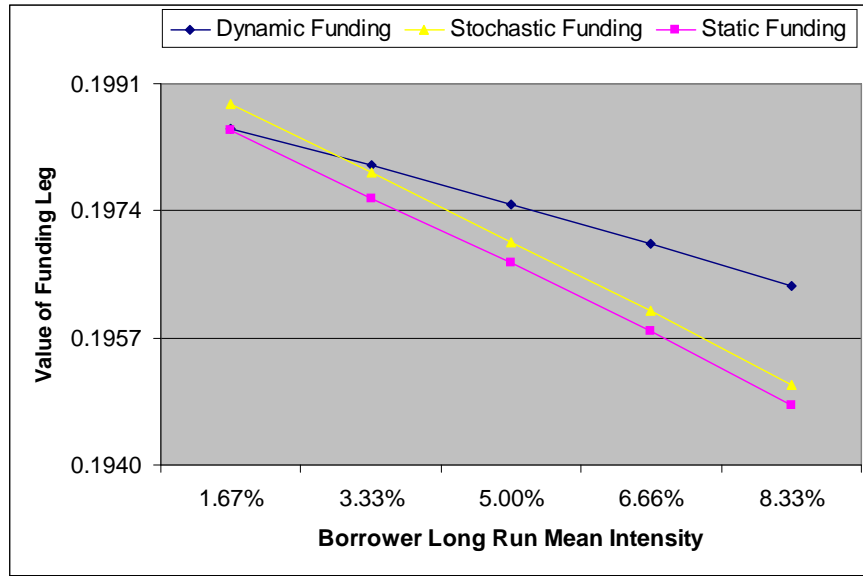


Figure 2.16 Effect of Borrower Long Run Mean Intensity on Funding

intensity, mean reversion implies that the expected future borrower spread is higher than the case where the long-run mean and the initial value of borrower default intensity are about the same. Therefore, in such a scenario, more funding is required and hence funding cost is higher with dynamic funding than with static and stochastic funding because utilization increases with higher future borrower spreads. The opposite holds if long-run mean borrower default intensity is lower than the initial borrower default intensity.

Long run mean funding intensity: The effect of long run mean funding intensity $\bar{\lambda}^f$ on funding cost under the three methods is illustrated in *Figure 2.17*. When long run mean of funding intensity is high (for the same initial funding intensity), the funding spread is high. Therefore, under all the three methods, funding cost of a loan is high when the long run mean of funding intensity is high. If the long run mean funding intensity is higher than the initial funding intensity, the initial funding spread is lower than the expected future funding

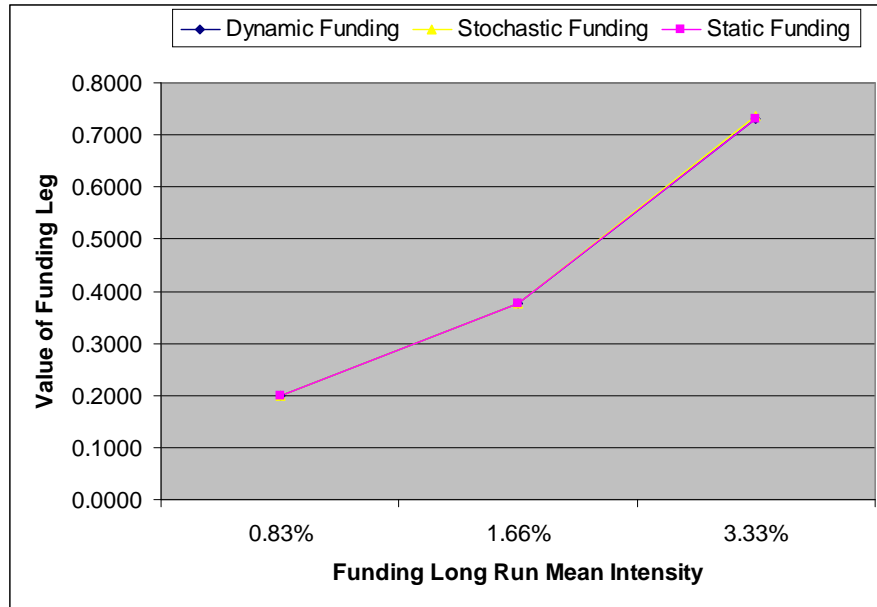


Figure 2.17 Effect of Funding Long Run Mean Intensity on Funding

spread due to mean reversion. Therefore, it is cheaper to purchase (most of or all) funding upfront in such a case, which implies that dynamic and static funding are cheaper than stochastic funding when long run mean funding intensity is higher than initial funding intensity. The opposite holds when long run mean funding intensity is lower than initial funding intensity.

Discussion

As discussed above, in the base parameter case, the expected funding costs under the three funding methods is not significantly different from each other. Moreover, except for the cases where the initial funding intensity and the long-run mean funding intensity are significantly different from each other, none of the parameter changes causes these expected costs significantly different from each other. Therefore, from a cost perspective, the banks should be indiffer-

ent between these three funding methods, especially in the scenarios where the funding spreads are close to their long term values. However, as discussed in previous sections, dynamic funding is better than static funding which in turn is better than stochastic funding from the perspective of funding risk management. Therefore, banks should prefer dynamic funding especially in the cases where the funding spreads are close to their long-term values.

In environments where the current funding spreads are significantly lower than the long-term funding spreads, stochastic funding is significantly more expensive than dynamic and static funding. This suggests that, from a cost perspective, banks have an incentive to use long-term funding in low spread environments. However, in reality, banks tend to prefer short-term funding in low spread environments such as 2002-2006. This suggests that, in low spread environments, banks take a short-term view of saving funding cost in the short-term, while ignoring the long-term view that they may need to purchase future short-term funding at high costs due to mean reversion. A long-term view would suggest that banks should fund long-term in low spread environments.

On the other hand, in high spread environments, stochastic funding is cheaper than static and dynamic funding. However, in economic terms, the benefit of short-term funding is marginal (less than 5% in our simulations). Moreover, as is clear from the current financial crisis, long-term funding is far superior to short-term funding in terms of funding risk management. Therefore, even in high spread environments, it is prudent for banks not to use short-term funding.

The expected costs under static and dynamic funding are not significantly different from each other in any of the scenarios simulated above. Therefore,

dynamic funding seems clearly preferable to static funding since the former method provides better funding risk management framework and offers no significant cost disadvantages as compared to the latter method.

2.5 Conclusion

The current financial crisis has highlighted the need to manage the liquidity risk of a bank. Prior to the crisis, most banks were funding long-term assets with short-term funding. However, during the current crisis, banks couldn't rollover their existing funding due to concerns about their viability. Consequently, many banks perished or came close to bankruptcy during this crisis.

To better manage their funding risk, many banks are reducing their reliance on short-term financing and increasing their liquidity buffers. Moreover, international and national regulators are proposing thumb rules and stress scenarios (under Basel framework) that require banks to maintain a sizeable liquidity buffer and increase the amount of long-term funding.

This paper proposes an empirically grounded predictive model to manage the funding risk of assets with uncertain funding requirements. The model is estimated for revolving credit lines, but can be readily adapted to other asset classes. The model involves simulating CDS spreads and revolver usage over the funding horizon based on an empirically estimated model. The simulated usage distribution is used to obtain expected and unexpected usage of the revolver, which can be term funded in two alternative ways: static funding and dynamic funding.

Static term funding is fixed over life of the loan and is conditioned on initial spreads. Dynamic term funding changes over the life of the loan and is conditioned on spreads at future adjustment dates. Dynamic term funding has less model risk and offers more protection against unexpected draws in high spread environment whereas static funding offers more protection from large temporary spikes in utilization.

The dynamic funding model is applied to the revolving credit portfolio of a large multinational investment bank between December 2007 and December 2009. The dynamic funding model performed very well in managing the funding risk of this loan portfolio during this extremely stressful period. The actual utilization never breached the expected plus unexpected funding during the whole financial crisis.

We also use simulation to compare the expected funding cost under the short-term, static and dynamic funding frameworks. The simulation suggests that the dynamic funding method has no significant cost disadvantage as compared to the other two methods, while offering better framework for funding risk management.

CHAPTER 3

MANAGING THE UNDERWRITING RISK IN DEBT SYNDICATIONS

3.1 Introduction

During the current financial crisis, investment banks lost a significant amount of money on leveraged loan commitments. Banks were sitting on a huge portfolio of leveraged loan underwritings at the time of Lehman bankruptcy, and couldn't syndicate these commitments due to market meltdown. As a result many banks had to issue debt to leveraged borrowers at below market prices, resulting in billions of dollars in losses. Since banks did not properly hedge these commitments prior to the crisis, their losses on commitments were not offset by gains on hedges.

This paper proposes an option pricing framework for pricing and hedging such underwriting commitments. The underwriting risk is modeled as an American put option on a debt instrument. We argue that, given the nature of these commitments, the price of an underwriting commitment should lie somewhere between the price of an American put option and a European put option on the debt instrument. We propose to price the commitment as an American option because it is more conservative and better protects the underwriter who aims to hedge this risk.

A reduced form model (e.g. [20]; [18]) is used to price these commitments as American options on defaultable securities. The model is applied to two cases depending on whether or not default swaps on the borrower trade in the market. Examples are presented for both cases to illustrate the model.

To my knowledge, there has been no research in the literature on pricing and managing underwriting commitment risk. This framework can help banks manage such commitment risk conservatively and systematically as opposed to their arbitrary and often sporadic hedging prior to the financial crisis.

3.2 Model

Assume that the debt being syndicated has indicative pricing features given by the set I . The set I could include features such as notional, currency, fixed or floating loan, coupon, recovery rate, amortization schedule, call protection, Libor floors etc. In other words, any feature that affects pricing. Let the indicative original issue discount be given by O_I ¹. The indicative issue price of the commitment $100 - O_I$ is the underwriter's best estimate of where the debt with indicative features I would clear if it were issued today.

Assume that the bank has signed an underwriting commitment at time t on this debt for period T years. If the debt is part of an acquisition, the borrower has to satisfy the terms of the acquisition (such as regulatory approvals etc) during the period T before the loan can be effective or funded. If the debt is not part of an acquisition, period T is the syndication period during which the underwriter attempts to syndicate the debt.

Assume that the underwriter has the ability to flex² some of the features of the debt during the syndication. Let the flex features be given by the set F . Basi-

¹Original issue discount is the amount paid below par at issuance. For example, if the loan is issued at 98, the original issue discount is 2.

²The word flex derives from the flexibility an underwriter has in syndicating the commitment.

cally, the set F includes how much of each pricing feature the underwriter may change to syndicate the loan. For example, the underwriter may increase the coupon, change the amortization schedule, shorten the maturity etc. Assume further that the original issue discount can be flexed to O_F .

Let the price of this debt at time $s \in (t, T)$ for pricing set I be given by $V_s(I)$. Likewise, the debt price for pricing set F at time s be given by $V_s(F)$. Assume that the debt becomes effective at time s ³. The loss $P_s(F)$ to the underwriter on the debt effective date s is given by

$$P_s(F) = \max(100 - O_F - V_s(F), 0)N \quad (3.1)$$

where N is the underwritten notional. The model assumes that if the syndication fails the underwriter is left with all the debt⁴. Since the underwriter can flex to pricing set F , this formula says that the underwriter can lose money only if the debt price with maximum flex $V_s(F)$ on effective date s has fallen below the lowest debt price $100 - O_F$ at which the debt can issued.

Note that $P_s(F)$ in expression (3.1) is the payoff of a put option on debt $V_s(F)$ with strike $100 - O_F$. If the debt effective date s were known with certainty, the underwriting commitment with payoff $P_s(F)$ could be priced as a European put option. However, in reality, the effective date s is uncertain. For acquisition related underwritings, this is partly because there is uncertainty in meeting the conditions for closing the acquisition and partly because the borrower may not

³In the case of debt underwritings related to acquisitions, the debt becomes effective when the borrower satisfies the conditions for closing the acquisition and decides to fund the debt. For other debt underwritings, the debt becomes effective when the syndication is completed and the loan is issued (either at the end or during the syndication period).

⁴This implies that there is a market clearing price for the syndication and, at this clearing price, the underwriter can either sell down all the debt it expects to syndicate or cannot sell any of the debt. Note that the underwriter may have an expected hold amount which it doesn't want to sell due to relationship reasons.

immediately fund the debt after the acquisition has closed. For other underwritings, this is partly because there is uncertainty about how long the syndication will last and partly because the borrower may not immediately fund the debt after the syndication is complete. This uncertainty suggests that the underwriting commitment could be priced as an American put option. While the exercise date s is not fully in control of the borrowers, they do have some control over the process and can exercise this option rationally in some of the cases⁵. These observations suggest that the price of an underwriting commitment should lie somewhere between the price of an American put option and a European put option with payoff as per expression (3.1).

We price the commitment as an American option because it is more conservative and better protects the underwriter who aims to hedge this risk⁶. In cases where the borrower exercises the option *irrationally*, there is a windfall gain to the underwriter who uses American option price to hedge the risk. On the other hand, if the borrower exercises the option *rationally*, there is a loss to the underwriter who uses European option price to hedge the risk.

Following [20], let r_t be the riskless short rate and λ_t be the intensity of the Cox process governing default of the borrower. Let ω be the fractional loss given default of the borrower. Assume for simplicity that the riskless rate, default intensity and loss given default are independent of each other. Assume that the risk-neutral dynamics of the intensity process λ_t is given by the following

⁵Most underwriting commitments have ticking fees that are paid during the commitment period, which may incentivize the borrower to do a rational early exercise of the option.

⁶Another alternative is to price the commitment as a put option with random exercise time. While this technique may better capture the price of the commitment, we choose a conservative pricing approach because the commitment is usually originated and hedged by different business divisions, which creates incentives for conservative transfer pricing between the two divisions.

process

$$d\lambda_t = \alpha(t, \lambda_t)dt + \sigma(t, \lambda_t)dW_t^Q \quad (3.2)$$

where $\alpha(t, \lambda_t)$ and $\sigma(t, \lambda_t)$ are the risk neutral drift and volatility of the intensity process, and W_t^Q is a standard Brownian motion under the risk neutral measure. Assume that $\alpha(t, \lambda_t)$ and $\sigma(t, \lambda_t)$ are such that the intensity is always nonnegative.

The price of the underwriting commitment $V_F(t_n)$ expressed as an American option is given by

$$V_F(t_n) = \sup_{t \leq \eta \leq t+T} E_t^Q \left[1_{\{\tau > \eta\}} \exp \left(- \int_t^\eta r_s ds \right) P_\eta(F) \right] \quad (3.3)$$

where τ is the default time of the borrower. Following the results in Lando (1998), the above expression simplifies to

$$V_F(t_n) = \sup_{t \leq \eta \leq t+T} E_t^Q \left[\exp \left(- \int_t^\eta \lambda_s ds \right) \exp \left(- \int_t^\eta r_s ds \right) P_\eta(F) \right] \quad (3.4)$$

3.3 Application of the Model with Examples

In this section, we parameterize the general model in previous section. We apply the model to two cases where the default swaps on the borrower trade or do not trade in the market. Examples are presented for both cases to illustrate the model.

3.3.1 Parametric Model

We employ Schonbucher's ([18]) extension of Hull and White's ([13]) reduced model of interest rates that incorporates default risk. The risk-neutral dynamics

of the risk-free short rate r_t and the default intensity process λ_t are given by the Gaussian models:

$$dr_t = (\beta_t - \alpha r_t)dt + \sigma dW_t^Q \quad (3.5)$$

$$d\lambda_t = (\bar{\beta}_t - \bar{\alpha}\lambda_t)dt + \bar{\sigma}d\bar{W}_t^Q \quad (3.6)$$

where α and $\bar{\alpha}$ are the mean reversion coefficients, β_t and $\bar{\beta}_t$ are time-dependent parameters, σ and $\bar{\sigma}$ are the local volatilities, and W_t and \bar{W}_t are the standard brownian motions for the two stochastic processes respectively. Assume further that the recovery rate in the case of default is a constant R . As discussed below, the time dependent parameters k_t and \bar{k}_t are used to calibrate the initial term structure of default-free and defaultable interest rate. For simplicity, we assume that the correlation between the two Brownian motions $\rho(W_t, \bar{W}_t) = 0$.

A recombining trinomial tree is implemented for risk free interest rate as proposed by Hull and White in [13]. A trinomial tree with branch to default is implemented for default intensity as proposed by Schonbucher in [18]. The first step towards building these trees is to define the auxiliary processes r_t^* and λ_t^* given by

$$dr_t^* = -\alpha r_t^* dt + \sigma dW_t^Q \quad (3.7)$$

$$d\lambda_t^* = -\bar{\alpha}\lambda_t^* dt + \bar{\sigma}d\bar{W}_t^Q \quad (3.8)$$

which imply that

$$r_t = r_t^* + \delta_t \quad (3.9)$$

$$\lambda_t = \lambda_t^* + \eta_t \quad (3.10)$$

where δ_t and η_t are non-stochastic and time-dependent.

The next step is to build the trees for these auxiliary processes r_t^* and λ_t^* as depicted in *Figure 3.1*. Given the auxiliary risk-free rate r_t^* at time t , the auxiliary

riskless rate at time $t + \Delta t$ rises to $r_t^* + \Delta r_t$ on the up node with probability p_u , stays at r_t^* on the middle node with probability p_m , and falls to $r_t^* - \Delta r_t$ on the down node with probability p_d . Along the tree, the auxiliary short interest rate $r^*(n, j)$ at time $n\Delta t$ and node j is given by $r_t^* + j\Delta r_t$. The tree for auxiliary default intensity λ_t^* has a branch for default with probability p and a branch for survival with probability $1 - p$. On the survival branch, this tree has up, middle and down nodes similar to the auxiliary riskless rate tree but with jump $\Delta\lambda_t$ rather than Δr_t and with probabilities $p'_u, p'_m,$ and p'_d rather than $p_u, p_m,$ and p_d .

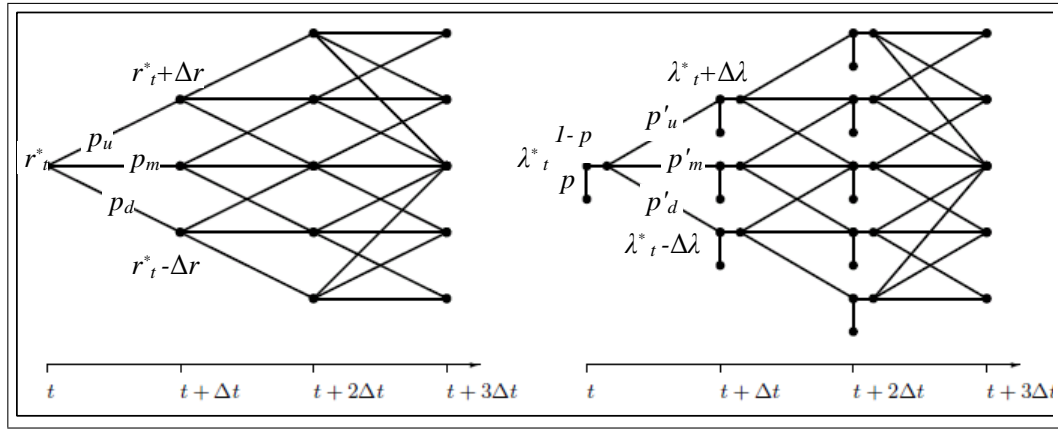


Figure 3.1 Trees for Auxilliary Riskless Rate and Default Intensity

Assuming $\Delta r_t = \bar{\sigma}\sqrt{3}\sqrt{\Delta t}$, the probabilities $p_u, p_m,$ and p_d can be calculated by matching the first and second moments of the auxiliary short rate tree to the first and second moments of the continuous time process of r^* (see [13] for details)⁷. Likewise, $p'_u, p'_m,$ and p'_d can be calculated using $\Delta\lambda_t = \bar{\sigma}\sqrt{3}\sqrt{\Delta t}$.

The final step is to combine the two trees and use *forward induction* to calibrate the model to match the term structure of default free and defaultable bond

⁷Note that the two trees are flat at the top and the bottom. This branching method was proposed by Hull and White ([13]) to match the first moments at very high and low levels of the tree where the mean reversion effect is very high. The probabilities of three branches originating from such nodes are different for those on other nodes (see [13]).

prices using δ_t in Expression 3.9 and η_t in Expression 3.10. The details of the calibration can be found in [18]. Using the calibrated tree, options on defaultable loans such as the one in Expression 3.4 can be priced using *backward induction*.

3.3.2 Investment Grade Bridge Commitments for Acquisition

Finance

Loans to investment grade borrowers generally do not trade in the market. However, most of these borrowers have liquid bonds and CDS traded in the market. Using these bond prices or CDS spreads, we can imply the default probabilities and calibrate the default intensity process for these borrowers

Assume that the term structure of interest rates is flat at 2% with a local volatility σ of 10% and zero mean reversion. Assume that a borrower has a 1% bps flat term structure of bond spreads. Assume that the borrower wants to finance an acquisition and requests its bank to issue a bridge commitment of \$100 which provides funding if the acquisition closes successfully during a fixed commitment period, say three months. Assume that the requested bridge has a tenor of one year and the borrower would pay Libor plus 2% if the bridge is funded. Assume for simplicity that the bridge is non-cancellable. Finally assume that the borrower pays an upfront fee of 100 bps on the bridge funding date and a signing fee of 200 bps on the commitment signing date.

Clearly, if the bridge were to fund today, it would have a value above par since the bridge spread of 2% above libor is more than the bond spread of 1%. In such a case, the bank would willingly enter into the bridge. However, this

bridge is a contingent security that can fund any time during the next three months. If the borrower's bond spreads are much more than 2% when the bridge is funded, resulting in loan value below 99 (par minus upfront fee of 100 bps), the bank would incur a loss while funding the bridge. The expected value of all such potential losses is captured by the option price in Expression 4. The option price represents the expected losses to the bank from committing to the bridge. If the option price is less than the bridge signing fee of 200 bps, the bank still gains by making the commitment. If the option price is more than 200bps, the bank may still commit to the bridge to the borrower if it has a past relationship or expects future revenues from the borrower (the next chapter on relationship banking provides evidence to support this view).

Assume that the local volatility for the borrower is a constant 25% and the mean reversion coefficient is 0.2. The term structure of interest rates and bond spreads can be used to obtain the default-free and defaultable bond prices, which along with the volatilities of interest rate and default intensity assumed above, can be used to calibrate the tree model discussed in the previous section. The forward price of the bridge including upfront fee is slightly below 102⁸ and the strike price is 99 (par less upfront fee). The commitment price (without the bridge signing fee) is 216 basis points.

Figure 3.2 contains a sensitivity analysis of the commitment price to various model parameters. As expected from option pricing theory, the commitment price increases with commitment tenor and volatility of default intensity and decreases with interest rates and mean reversion of default intensity.

⁸Loan margin of 2% and bond spread of 1% imply a surplus of 1% over 1 year tenor, which along with upfront fee of 1% and discounting implies a loan price slightly below 102.

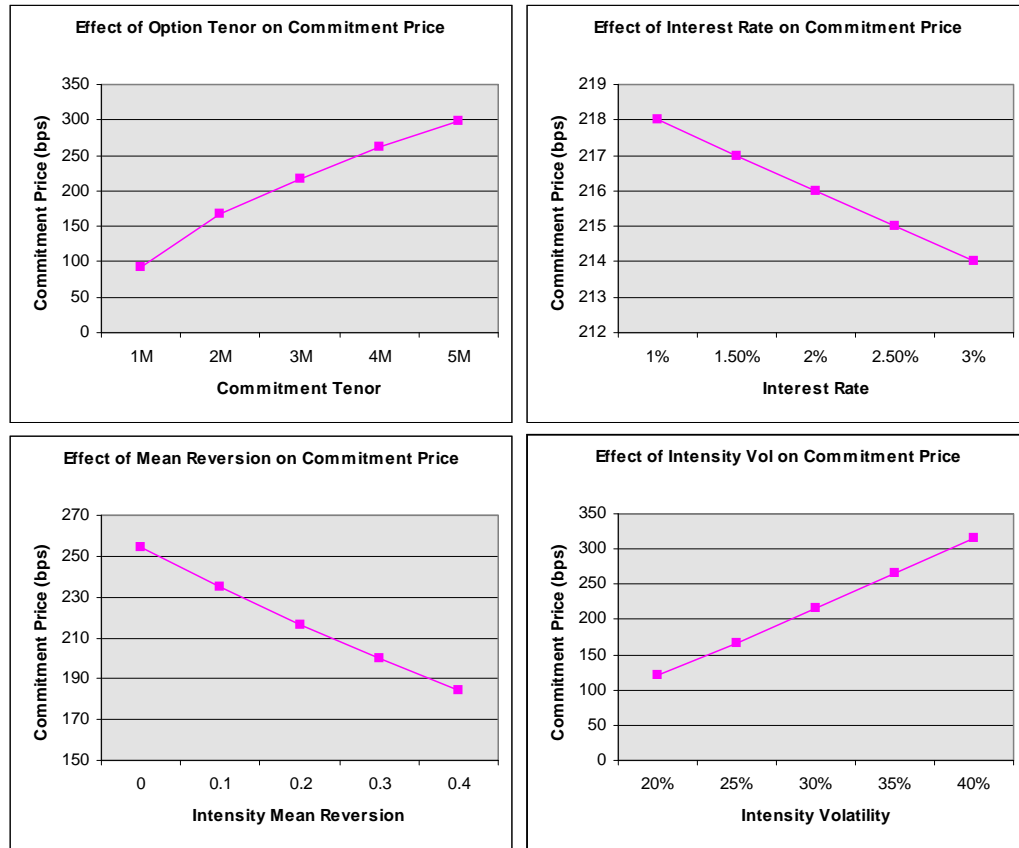


Figure 3.2 Effect of Model Parameters on Commitment Price

3.3.3 Leveraged Loan Underwriting Commitments

While leveraged loans trade in the market, in most cases, there are not enough traded loans for a borrower to extract the term structure of loan spreads. In some cases, there may be no traded loan for a borrower in the market. Therefore, we consider the case where the borrower has one loan or no loan traded in the market.

If there is no traded loan for a borrower, the starting point of the calculation is to obtain the indicative issue price of the commitment. As discussed in Section 3.2, given the indicative original issue discount O_I , the indicative issue

price of the commitment $100 - O_I$ is the underwriter's best estimate of where the loan with indicative features I would clear if it were issued today. The underwriter does comparative fundamental analysis of the borrower against its peers to come up with the indicative price. In absence of any other information, the indicative price $100 - O_I$ for indicative pricing set I is the best available information to extract the term structure of loan spreads. Thus the case with no traded loan is equivalent to the case with one traded loan since both cases involve starting with the price of one loan.

Of course, one loan doesn't contain enough information to extract the term structure of loan spreads. To resolve this problem, we identify the relevant index for the borrower. For example, if the borrower is from North America, the index could be LCDX for a loan commitment and CDX High Yield index for bond commitment. As discussed below, the relevant index is used to obtain the slope of the term structure of loan spreads.

The calibration involves equating the borrower curve to the index curve and making parallel shifts to this curve till the loan price under consideration is matched. Specifically, the initial index curve implies a term structure of defaultable debt prices which is compared to the bond price under consideration. If the implied bond price for the required maturity doesn't match the traded or indicative bond price, the initial curve is shifted in a parallel manner and the corresponding term structure is compared to the traded or indicative bond price. This process is continued till the two prices match. The calibrated loan spread curve has the same slope as the underlying index and is able to match the traded or indicative bond price. The interest rate curve, calibrated loan spread curve along with assumptions on volatilities of interest rate and default inten-

sity are used to calibrate the tree model discussed in the previous section.

Assume that the interest rate parameters are same as in previous section. Assume that the leveraged loan commitment is non-cancellable, has an indicative coupon of 2%, indicative loan tenor of 1 year, indicative loan price of 101 and commitment period of 3 months. Assume that the relevant index (with same recovery as the indicative loan) is flat at 4%. Since the relevant index is flat, and there is no market information about the borrower, we assume a flat loan spread curve for the borrower. A flat 1% curve matches the indicative loan price of 101⁹.

Leveraged loan commitments usually have flex (flexible) terms. For example, the borrower may allow the bank to syndicate at a margin of up to 3% and an issue price of up to 98 even though the indicative margin is 2% and the indicative issue price is 101. In some cases, the bank may also have the ability to reduce the tenor of issued loan. Figure 3.3 illustrates the effect of flex parameters on commitment price. As one would expect, the commitment price decreases with more flexibility in increasing margin and issue discount as well as more flexibility in decreasing loan tenor. Figure 3.3 also illustrates the obvious result that the commitment price increases as the current loan spread increases (or indicative loan price decreases) because there is less flexibility left to syndicate.

⁹Loan margin of 2% and loan spread of 1% imply a surplus of 1% over 1 year tenor which on discounting implies a loan price slightly below 101.

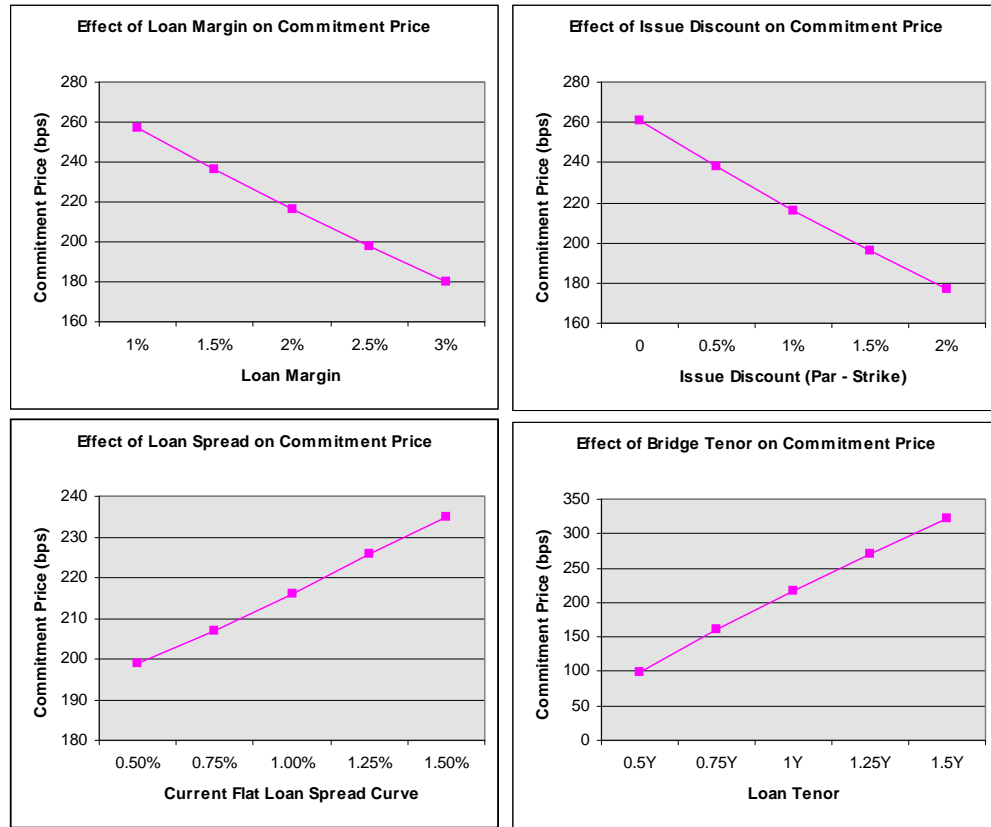


Figure 3.3 Effect of Flex Parameters on Commitment Price

3.4 Conclusion

The current financial crisis has highlighted the need for banks to better hedge their underwriting risk since banks lost billions of dollars on such underwritings without offsetting gains on hedges. This paper proposes an option pricing framework for pricing and hedging this risk. The underwriting risk is modeled as an American put option on the underlying debt. Using reduced form pricing, the model is applied to two cases depending on whether or not default swaps on the borrower trade in the market. This framework can help banks manage commitment risk conservatively and systematically as opposed to their arbitrary and often sporadic hedging prior to the financial crisis.

CHAPTER 4

THE SOURCES OF RELATIONSHIP BANKING BENEFITS

4.1 Introduction

Relationship banking has been studied extensively as a means to resolve the information asymmetry between borrowers and lenders (see [2] for a comprehensive review). The financial intermediation theory suggests that banks can avoid the adverse selection of poor quality borrowers by screening borrowers through costly information production and can overcome the borrower moral hazard of sub-optimal investment by monitoring the borrower's investment decisions ([7], [23]). Besides increasing the availability of credit to the borrower by removing information asymmetries, a banking relationship can also lead to better loan contract terms for the borrower ([15],[14], [3]) due to the production of borrower-specific durable and reusable information ([2]). On the other hand, relationship banking can be costly to the borrower if the bank may "holdup" the borrower and extract informational rents due to information asymmetries between the borrower and *other* lenders ([26], [22]).

Empirical literature has tried to test these theories but has been constrained by the availability of data (see [25] for a review). While event studies such as [4] and [27] show an increase in a firm's stock price on the announcement of a bank loan, these studies show the *existence* of relationship benefit not the *sources* of this benefit. To determine the sources of this benefit, empirical researchers have relied on crude measures for relationship strength and benefit since the true values of these variables are not publicly observed. As discussed in the following paragraphs, the use of these crude measures has yielded *mixed* empirical sup-

port for the claims made in the theoretical literature and has not fully identified the sources of relationship benefit.

This paper uses precise measures of relationship benefit and strength rather than crude proxies to uncover the true sources and extent of relationship benefit to borrowers. We overcome the limitations of the extant empirical literature by using a proprietary dataset, obtained from a multinational universal bank, that contains detailed relationship information.

The existing literature measures relationship benefit as lower loan fees (e.g. [16]), or lower collateral requirement (e.g. [29]), or more credit availability (e.g. [1]). However, none of these measures by themselves or taken together imply a relationship benefit unless the borrower's default risk and recovery rate has been controlled for and all the options in the loan contract are accounted for. Since the borrower's default risk is not observed by researchers, the extant literature uses crude measures such as bank prime rate and generic default premium to control for default risk whereas loan recovery rates and embedded options are ignored due to lack of data. Therefore, the relationship benefit claimed in these studies may not be the actual relationship cost viewed by the bank either because the researchers have not controlled for all risks and loan features (giving an illusion of relationship benefit where none exists) or because the banks are irrational (banks may be mis-pricing the loan without realizing that they are giving benefit to borrowers)¹.

In contrast, this paper measures relationship benefit as the difference between the par value and the fair value of a loan where the fair value is calculated using the market implied default probabilities, collateral-dependent recovery

¹Many banks hold their loans at par on the accrual book, which may obscure the true cost of providing the loan.

rates, and loan features including any embedded options. In other words, we use a dollar measure of relationship benefit after controlling for all risks and loan features rather than any of the crude measures used in the literature. This dollar measure is the actual relationship benefit to the borrower because it is a real cost incurred by the bank with an explicit understanding that it is a cost for the bank².

We find that the fair value of loans in our dataset are generally lower than the par value of loans, suggesting that borrowers obtain the relationship benefit through better terms on the loan contract. Specifically, the loan rate is much lower than the borrower credit spread after controlling for recovery rates and embedded options in the loan contract.

Relationship strength has been measured in the existing literature using one or more of the following measures: the length of the relationship (e.g. [1]); the use of bank's other financial services besides loans (e.g. [16]); and the proportion of loans provided by the bank to the borrower in the recent past (e.g. [29]). However, none of these measures precisely capture the strength of banking relationship. Length of a relationship doesn't measure the depth or breadth of a relationship. Dummy variables for the use of bank's other financial services don't measure the extent of the use of those financial services. Finally, the proportion of loans provided by the bank to the borrower in the recent past neither captures the breadth of the banking relationship nor perfectly measures the depth of the lending relationship because it ignores the internal and/or regulatory single borrower concentration limits which affect this proportion. Moreover, none of these measures capture the future potential of a relationship.

²The bank that provided the data follows the practice of fair valuing its loans. Hence any value below par is recognized by the bank as a loss.

In contrast, this paper measures relationship strength using relationship depth, relationship breadth, and relationship potential. Relationship depth is measured as the annual revenue generated from all existing loans issued to the borrower by the bank. Relationship breadth is measured as the annual revenue generated from all non-loan products and services purchased by the borrower from the bank. Relationship potential is measured as the revenue expected to be generated in the next one year across all products and services provided to the borrower by the bank. The first variable measures the depth of the lending relationship, the second variable measures the *scope* of the banking relationship, and the third variable measures the future potential of the relationship. These dollar measures capture the extent of the relationship along various dimensions not just the presence or length of a relationship as is captured by the dummy variables used in the existing literature.

Why should relationship potential matter? [28] argue that repeated interaction of the bank with the borrower produces reusable and proprietary information about the borrower that can help the bank win future loan business and other fee generating business from the relationship borrower. [24] and [28] show empirically that the probability of winning future loan business increases significantly and the probability of winning other fee generating business increases marginally (in economic terms) if the bank has a lending relationship with the borrower. Based on this evidence, [28] conclude that lending relationship is beneficial to the bank. However, one cannot definitively say from these results whether the bank benefits from the relationship without weighing the probabilities of winning the future business with the revenues generated from these mandates. In particular, loans are generally loss making products as shown in this paper and the fact that a past relationship significantly increases the chance

of winning a future loan mandate only shows that the expected value to the bank from future loan business is significantly negative. Since the increase in probability of winning non-loan business is marginal, the fee generated from non-loan business must be very large to offset the expected negative value from future loan business. Moreover, the causation is not clear: maybe the banks provide higher relationship benefit in anticipation of higher future revenues rather than the higher relationship benefit causing higher future revenues. This paper addresses the narrower question of whether there is a correlation between relationship benefit and expected future revenues; the issue of causation between these variables cannot be addressed with our data.

We find that the relationship benefit increases with relationship breadth and relationship potential but doesn't depend on relationship depth. In other words, the benefit of relationship banking is higher for relationships with broader scope and larger future potential. Moreover, relationship potential is more important than relationship breadth in determining the relationship benefit provided by borrowers. The lack of significance of relationship depth in this paper contrasts with some of the existing studies (e.g. [29]) which find a positive effect. The paper finds some evidence which suggests that the significance of relationship depth in some of these studies could be because these studies do not fully control for credit risk and do not include more comprehensive relationship variables such as relationship breadth and relationship potential. The significant effect of relationship breadth on shortfall benefit suggests that the lack of such an effect in some of the existing papers (e.g. [16]) could be due to the use of simple dummy variables to measure relationship breadth in these papers. The novel result on the effect of relationship potential on relationship benefit suggests that banks are forward looking in terms of passing on the relationship

benefit to borrowers, though the causation cannot be clearly established.

We also find that relationship potential is the most important determinant of relationship benefit for firms that are more informationally opaque (firms with high market-to-book ratio and small size) whereas relationship scope is more important for less informationally opaque firms. Since small firms and growth firms usually have low relationship breadth and high relationship potential, the higher sensitivity of relationship benefit to relationship potential than to relationship breadth for these firms implies that relationship benefit is passed on to the such borrowers much before the relationship breadth increases. On the other hand, since large firms and value firms usually have high relationship breadth and stagnant relationship potential, the higher sensitivity of relationship benefit to relationship breadth than to relationship potential for these firms implies that the relationship benefit tapers gradually if the borrower's relationship potential is declining.

This paper makes several contributions to the literature. First, it shows definitively that borrowers benefit from a lending relationship through better terms on their loan contracts since the fair value of loans is generally lower than the par value of loans. Existing studies cannot say this for sure because their measures of relationship benefit are not precise. These studies regress relationship variables on loan fees or collateral requirements or credit availability to determine whether stronger relationships result in better loan terms. However, as argued earlier, lower loan rates or lower collateral requirements or better credit availability by themselves or together do not imply a relationship benefit unless all the risks are controlled for. This paper finds that relationship borrowers get a dollar benefit on their loan contracts after controlling for all risks.

Second, this paper uses comprehensive measures of relationship strength and a precise measure of relationship benefit to uncover the true sources of relationship benefit. The crude measures of relationship strength used in the literature have resulted in mixed empirical evidence on the sources of relationship benefit. For instance, using relationship length as the measure of relationship depth, some studies find that relationship depth results in lower loan rates and lower collateral requirements (e.g. [1]) whereas some studies find no effect on loan rates (e.g. [16]) and no effect on credit availability ([19]). Likewise, measuring relationship breadth using dummy variables to denote the use of non-loan financial services, some studies find that relationship breadth results in lower loan rates (e.g. [11]) and some studies find no effect on loan rates (e.g. [16]). This paper uses a precise dollar measure of relationship depth to show that relationship depth does not affect relationship benefit. On the other hand, a precise dollar measure of relationship breadth is used to show that relationship breadth positively affects relationship benefit. Finally, this paper is the first to show that relationship benefit is driven by relationship potential, measured using a precise dollar measure of the future revenue potential.

Third, it identifies several problems in the empirical specifications and the corresponding results in the existing literature. The existing literature does not fully control for the borrower's credit risk in the regressions due to which one cannot be sure if the borrower benefits from the lending relationship³. This paper finds evidence that suggests that the positive effect of relationship depth on relationship benefit in some of the existing studies may be because these studies

³Bharath et al (2008) use an instrument variable approach and a treatment effects model to overcome the problem of unobserved credit quality. However, they acknowledge that their instrument variable may be correlated with the error term contrary to the assumption of instrument variable model. Their treatment effects model on the other hand allows for correlation but assumes that the error term is bi-variate normal. They acknowledge that neither of these assumptions can be empirically tested.

do not fully control for credit risk and do not include more comprehensive relationship variables such as relationship breadth and relationship potential. The significant effect of relationship breadth on shortfall in this paper suggests that the lack of such an effect in some of the existing papers could be due to their crude measures for relationship breadth that do not capture the extent of use of non-loan products and services. The existing literature uses the correlation between relationship strength and the choice of borrower's future mandates to argue that banks benefit from the relationship through higher probability of winning future business, an assertion which need not be true due to the loss-making nature of loans and due to the difficulty in proving causation from such correlation.

4.2 Loan Pricing

A loan with maximum committed limit L and maturity T comprises a drawn portion $u_{t-1}L$ and an undrawn portion $(1 - u_{t-1})L$. The borrower pays commitment fee c on the undrawn amount and base rate b plus margin m on the drawn amount. The bank funds the drawn portion of the loans at the base rate b plus the funding cost f .

If the loan can be drawn on or defaulted on *only* at discrete times $t \in \{0, 1, 2, \dots, T\}$ ⁴. The time 0 value V_L of the loan is given by the formula:

$$V_L = E \left[\sum_{t=1}^T \left(\begin{array}{l} 1_{\{\tau > t\}} [(b + m)u_{t-1}L - (b - f)u_{t-1}L] \\ + c(1 - u_{t-1})L - 1_{\{\tau = t\}}L(1 - RR_t) \end{array} \right) \exp \left(- \sum_{j=1}^t r_j \right) \right] \quad (4.1)$$

⁴The extension of this expression to the general case wherein the firm can draw or default any time during the maturity of the loan is straightforward but makes the notation more complex without adding any new economic insights.

where RR_t is the loan recovery rate, τ is the default time, and r_t is the short interest rate⁵. The above expression assumes that the loan is fully drawn in the event of default. If we assume that r_t is independent of τ , expression (4.2) simplifies to

$$V_L = \sum_{t=1}^T E \left[(1_{\{\tau > t\}} L[(m - f)u_{t-1} + c(1 - u_{t-1})] - L(1 - RR_t)1_{\{\tau = t\}}) \right] d_t \quad (4.2)$$

where d_t is the risk-free discount rate for maturity t . If we assume that the utilization u_t is independent of default probability with $U = E(u_{t-1})$ and the loan recovery rate RR is independent of interest rate and default time with $R_L = E(RR_t)$ ⁶, expression (4.2) simplifies to

$$V_L = \sum_{t=1}^T [q_t[(m - f)UL + c(1 - U)L] - L(1 - R_L)(q_{t-1} - q_t)] d_t \quad (4.3)$$

where $q_t = P(\tau > t)$ is the survival probability till time t and $(q_{t-1} - q_t) = P(\tau = t)$ is the default probability at time t .

The survival probabilities q_t are bootstrapped from CDS spreads. Since the value of CDS contract is zero at initiation, the expected value of premiums paid on the CDS contract must equal the expected value of losses in the event of default

$$E \left[\sum_{t=1}^n 1_{\{\tau > t\}} C_n \right] = E \left[\sum_{t=1}^n 1_{\{\tau = t\}} (1 - R_t) \exp \left(- \sum_{j=1}^t r_j \right) \right] \quad (4.4)$$

⁵The loan usually has many embedded options such as cancellation option, variable usage option, extension option, multiple currency borrowing option, term-out option etc. Due to these options, the cashflows from the loan are different from those mentioned in expression 1. However, these embedded options mainly introduce non-linearity in the loan price as a function of credit spreads. Since this study uses linear regressions, we abstract away from these non-linearities.

⁶In general, utilization and recovery rate are not independent of credit spreads which in turn are not independent of interest rate. However, the correlations between these variables has little effect on pricing. Assuming independence between these variables simplifies the notation and provides more economic insights.

where C_n is the CDS spread at maturity $n \in \{1, 2, \dots, T\}$ and R_t is the bond recovery rate. If interest rates, recovery rate and default time are independent⁷, expression (4.4) simplifies to

$$C_n \left(\sum_{i=1}^n q_i d_i \right) = (1 - R_C) \sum_{i=1}^n (q_{i-1} - q_i) d_i \quad (4.5)$$

where $R_C = E(R_t)$ is the expected bond recovery rate. Noting that $q_0 = d_0 = 1$, the above expression can be used to bootstrap q_i , $i \in \{1, 2, \dots, T\}$, from the T equations corresponding to CDS spreads at T maturities.

These survival probabilities are used in the loan pricing equation to get the net present value of loan. If the expected bond recovery rate R_C and the expected loan recovery rate R_L are equal⁸, the loan pricing equation (3) and CDS pricing equation (5) can be combined to obtain

$$V_L = \sum_{t=1}^T L(l_f - C_T) q_t d_t \quad (4.6)$$

where $l_f = (m - f)U + c(1 - U)$ is the *net loan fees* (fees received on the loan net of funding cost).

Expression (4.6) states that the value of the loan is equal to the discounted⁹ value of the difference between the revenues received on the loan and the cost of hedging and funding the loan. In particular, Ll_f is the annual revenue on the loan net of funding, LC_T is the annual cost of hedging the loan and the difference between these two numbers when discounted and summed over the maturity of the loan gives the value of the loan.

⁷These assumptions are market standard in the credit derivative market to bootstrap survival probabilities. Moreover, as mentioned in the previous footnote, the correlations between these variables has little effect on pricing.

⁸In general, these two recovery rates are different but we make this assumption to simplify the notation and obtain more economic insights.

⁹The discounting is done using the risky discount rate which is equal to the riskless discount rate multiplied by the survival probability.

The loan shortfall S_L is the value of loan below par. It is given by

$$\begin{aligned} S_L &= \max(-V_L, 0) \\ &= \max\left(\sum_{t=1}^T L(C_T - l_f)q_t d_t, 0\right) \end{aligned} \quad (4.7)$$

$$= \max\left(\sum_{t=1}^T L((1 - R_L)(q_{t-1} - q_t) - l_f q_t) d_t, 0\right) \quad (4.8)$$

4.3 Data

The data comes from four sources. The first data source is the Loan Database which contains all the investment grade loans issued by the bank between January 2004 and June 2008. This dataset contains all loan specific details such as committed limit, currency, issuance date, maturity date, expected recovery rate, funding cost, expected utilization, and loan fees such as margin, commitment fee, and facility fee. It also contains the loan *lifetime shortfall* which is the difference between the par value and the fair value of the loan. A total of 1,971 investment grade loans were issued to North American firms during this period. These loans include new loans, and extensions or refinancing of existing loans. The types of loan include term loans, loan commitments, letters of credit and guarantees.

The second data source is the Customer Relationship Management (CRM) database. It contains the yearly revenues generated for the bank by each customer and the yearly shortfall incurred by the bank for each client between 2003 and 2007¹⁰. The number of customers ranges between 3048 in 2003 and 5234 in

¹⁰The revenue data in the CRM database is recorded for each customer group rather than for each customer. A customer group is a group of companies that belong to a legal group company. For the purpose of this paper, we use the terms client, customer, and customer group interchangeably.

2007. The revenues generated by each customer are divided by product types. The total revenue earned in a year from all existing loans to a client is referred to as *loan revenues* or *relationship depth*. It is our measure of the strength of the lending relationship as it depends on the number and size of loans issued to a client in the last several years¹¹. The annual revenue generated by a client from all *other* products and services is referred to as *non-loan revenues* or *relationship breadth*. It is our measure of the scope of the relationship since it depends on the number and size of all non-loan products and services purchased by the client. The yearly shortfall is the annual amortized shortfall over the loan maturity. If the riskfree rate is zero, the yearly shortfall on a loan is equal to lifetime shortfall divided by the loan tenor.

Note that the CRM database contains the yearly shortfall paid across all outstanding loans to a customer whereas the loan database contains the lifetime shortfall paid on each loan. For example, assume that as per the loan database a customer took a 3 year loan in 2003 with a lifetime shortfall of \$9mn, a 1 year loan in 2003 with a shortfall of \$1mn, a 2 year loan in 2004 with a lifetime shortfall of \$4mn, a 1 year loan in 2005 with a shortfall of \$1mn. Then, if the risk-less rate is zero, the CRM database would show the shortfall for this customer as \$4mn in 2003, \$5mn in 2004 and \$6mn in 2006. In short, the CRM database divides the lifetime shortfall into annual installments and aggregates these installments across all outstanding loans to the customer.

The third data source is the minutes of the Loan Screening Committee (LSC) meetings. Every loan must be approved by the LSC before it can be issued to the borrower. During the LSC, the economics of issuing the loan are dis-

¹¹Since a loan may have a tenor of several years (generally not more than 5 years), the loan revenues in a given year may have been contributed by a loan issued several years ago.

cussed including the shortfall on the loan, the past relationship with the borrower, and expected future revenues from the borrower over the next twelve months. Based on this information, LSC decides whether to extend the loan¹². We use the minutes of these LSC meetings to obtain the expected loan and non-loan revenues from the borrower over the next one year. The sum of the expected loan and non-loan revenues from the borrower over the next one year is referred to as *expected revenues* or *relationship potential*. It is our measure of the future potential of the relationship since it depends on the number and size of all products and services that are expected to be procured by the client. The LSC minutes are in the form of PDF files stored on the server and are organized by the dates of LSC meetings. Around 300 of these files were manually searched to get the expected revenues for 374 deals over the 2003-2008 period.

The fourth data source is the Compustat database which contains the yearly accounting data and fiscal year-end share price for all North American firms. In addition, the founding date of the firm is obtained from Capital IQ whereas default spread and prime rate are obtained from the Federal Reserve website.

4.4 Empirical Specifications

It is easy to see from the shortfall expressions (7) and (8) that, if all else is equal, shortfall is increasing in committed limit L , decreasing in loan fees l_f , increasing in loan tenor T , decreasing in expected loan recovery rate R_L , and increasing in the CDS spread C_T for maturity T . The following linear expression can be empirically estimated to see if the shortfall is related to the above variables as

¹²The LSC decision to extend a loan is subject to approval by the Credit Risk Management group which evaluates the loan based on its credit risk.

suggested by expressions (7) and (8).

$$S_L = \alpha + \beta L + \theta T + \delta R_L + \eta C_n + \gamma l_f \quad (4.9)$$

Of course, this specification just captures the mechanistic relationship between shortfall and these variables and doesn't add to the economic understanding of how the shortfall was determined. Therefore, to gain economic insights, we use relevant economic variables as proxies for the independent variables in expression (4.9)

Usually, a bank can control the amount of shortfall by varying either the loan fees or the committed limit, depending on the type of loan. In a bilateral loan, usually the bank can vary only the loan fees to control the shortfall because the borrower generally desires a specific committed limit and tenor. In the case of a syndicate loan, a participating bank generally cannot influence the loan fees or tenor but can choose the loan amount it commits to whereas the lead bank can influence the loan fees as well. Therefore, the relationship benefit provided by a bank should be reflected in the committed limit and loan fees agreed with the customer: a more valued customer should get higher committed limits and/or lower loan fees. Hence we use relationship variables as proxies for committed limit and loan fees.

Our main relationship variables are loan revenues (relationship depth) and non-loan revenues (relationship breadth) in the year *prior* to the year a loan is issued, and expected revenues (relationship potential) in the year *after* a loan is issued. In contrast, the extant literature has mainly used relationship length to measure the strength of a relationship. Since our relationship data is from 2003 onwards, we cannot measure the length of each relationship. Instead, we use a new relationship indicator which indicates whether a new relationship was

initiated when a loan was issued. We also include firm age as a control variable since [16] find that firm age is a better predictor of loan rate than relationship length. We use book leverage, interest coverage ratio, default spread and prime rate¹³ as proxies for credit spread and tangible asset percentage as a proxy for loan recovery rate. We also include log sales, log market value, market-to-book ratio and EBITDA by assets in the regressions since these variables could be proxies for either relationship strength or credit spreads. In short, we estimate the following modification of expression (4.9)

$$\frac{S_L}{T} = \alpha + \sum_{i=1}^l \delta_i p_i(R_L) + \sum_{j=1}^m \eta_j p_j(C_n) + \sum_{k=1}^n \gamma_k p_k(l_f, L) \quad (4.10)$$

where $\frac{S_L}{T}$ is the annual shortfall, $p_i(R_L)$ are proxies for loan recovery rate R_L , $p_j(C_n)$ are proxies for credit spread C_n , and $p_k(l_f, L)$ are proxies for loan fees l_f and committed limit L . *Annual* shortfall $\frac{S_L}{T}$ is used in the above regression rather than *lifetime* shortfall S_L because our relationship variables are *yearly* revenues. However, for robustness, we also estimate the following specification using lifetime shortfall

$$S_L = \alpha + \theta T + \sum_{i=1}^l \delta_i p_i(R_L) + \sum_{j=1}^m \eta_j p_j(C_n) + \sum_{k=1}^n \gamma_k p_k(l_f, L) \quad (4.11)$$

to ensure that the use of annual shortfall is not driving the results. To account for the possibility that committed limit is independent of relationship variables, we also estimate the following specification

$$S_L = \alpha + \beta L + \theta T + \sum_{i=1}^l \delta_i p_i(R_L) + \sum_{j=1}^m \eta_j p_j(C_n) + \sum_{k=1}^n \gamma_k p_k(l_f) \quad (4.12)$$

which assumes that relationship variables effect loan fee only.

¹³Default spread is the difference between the average yield on BAA rated corporate bonds and the average yield on 10 year government bonds. Prime rate is the average prime rate for USD lending offered by banks in US to their best customers. Both the default spread and prime rate are published daily by Federal Reserve.

As mentioned in the previous section, the CRM database also has shortfall data but in the form of yearly shortfall paid across all outstanding loans to a customer. So as an alternative test, we use the shortfall data in CRM database to estimate the following specification

$$S_C = \alpha + \sum_{i=1}^l \delta_i p_i(R_{LC}) + \sum_{j=1}^m \eta_j p_j(C_{nC}) + \sum_{k=1}^n \gamma_k p_k(l_{fC}, L_C) \quad (4.13)$$

where S_C is the CRM yearly shortfall across all outstanding loans to a customer, $p_j(C_{nC})$ are proxies for default probability of the customer, $p_i(R_{LC})$ are proxies for loan recovery for the customer, and $p_k(l_{fC}, L_C)$ are relationship variables that proxy for loan amounts and fees agreed with the customer.

The advantage of specification (13) over (10) is that the former specification uses the aggregate shortfall S_C paid across all loans to a customer which is more directly related to the annual revenue data than the annual shortfall $\frac{S_L}{T}$ for each individual loan. For example, suppose a customer generated a total revenue of \$10mn last year and requests two loans during the year with annual shortfall of \$1mn and \$2mn respectively. In such a case, specification (10) would attribute both the shortfalls separately to the \$10mn revenue resulting in double counting of revenue whereas specification (13) would more reasonably attribute the combined shortfall of \$3mn to the \$10mn revenue¹⁴.

However, S_C in specification (13) aggregates not only the annual shortfall from loans issued within a given year but also the annual shortfall from all outstanding loans issued in prior years. For example, if a 5 year loan is issued to a customer, the annual shortfall from that loan would be included in S_C for the next 5 years along with annual shortfalls from all other loans whose life overlaps

¹⁴Another advantage of specification (13) is that both the shortfall and revenue data in the CRM database pertain to a customer group. In contrast, specifications (10)-(12) have shortfall at the customer level and revenues at the customer group level.

with those years. Therefore, specification (13) attributes combined yearly shortfall from all outstanding loans to the past year and expected next year yearly revenues, which may seem problematic. However, since a customer can have multiple loans across different years with different tenors, the combined annual shortfall from all outstanding loans is one potential measure of the annual relationship benefit provided by the bank for a given level of annual revenues.

Since all the above specifications have their own strengths and shortcomings, we estimate all the specifications to ensure that the main results of the paper are not driven by the shortcomings of one specification.

4.5 Summary Statistics

As mentioned in the previous section, we test two alternative specifications using shortfall from two different databases. For the first specification, each North American loan in the loan database is linked to the firm's accounting data from Compustat corresponding to the last fiscal year prior to loan issuance. The resulting sample of 433 loans is linked to the expected revenue data to obtain a sample of 164 loans which is hereafter referred to as *Loan Database Sample*. The summary statistics of this sample are included in Table 4.1. The median committed limit is around EUR 52mn and the median loan tenor is five years. The average expected utilization is quite low at 16% since most facilities are liquidity lines or commercial paper back-up lines with 0% expected utilization. The expected loan recovery rate varies very little around 40% with a low standard deviation of 5%. The CDS spread is higher than the net loan fees by 33bps on average, which results in an average shortfall of around EUR 0.8mn. The me-

dian default spread for BAA bonds is 173 bps which is much larger than the median CDS spread of 30bps for the loans in the sample. The average prior year loan revenues is about EUR 0.4mn which is much smaller than the average prior year non-loan revenues of EUR 2.8mn. The median expected revenues of EUR 1.7mn is about twice the median prior year loan plus non-loan revenues. The borrowers in the sample are large investment grade firms with an average age of 75 years, median market value of EUR 18bn and average sales of EUR 36bn. About 2.4% of the borrowers have a new relationship with the bank.

Table 4.1 shows that the net loan fee is much lower than the CDS spread both in terms of the mean and median. This result might suggest that borrowers benefit from the relationship through better terms on the loan contract. However, one cannot arrive at this conclusion if the loan recovery rate is much higher than the bond recovery rate. Moreover, the loan has embedded options which need to be accounted for to reach this conclusion. However, since the fair value of a loan takes into account recovery rates and embedded options, a positive shortfall reflects a relationship benefit to borrowers. Besides positive mean and median lifetime shortfall in our sample, more than 95% of the loans in our sample have a positive lifetime shortfall suggesting that most borrowers benefit from the relationship in the form of a better than fair loan rate that doesn't compensate for all the risks associated with the loan.

To obtain data for the second specification, we first filter out all the customers with zero shortfall in all the years 2003-2007 in the CRM database. This filtering is done because majority of the customers have no loans and hence no shortfalls. For the remaining customers, the current yearly shortfall and past year revenues in the CRM database are linked to the past fiscal accounting data

Table 4.1
Summary Statistics for the *Loan Database Sample*

Panel A: Loan characteristics

Variable	Mean	StdDev	Median
lifetime_shortfall (EUR)	806,943	1,443,453	346,664
facility_limit (EUR)	89,346,852	144,315,427	52,501,991
expected_utilisation (%)	16	28	0
loan_recovery_rate (%)	41	5	40
margin (bps)	44	29	38
commitment_fee (bps)	9	6	8
funding_cost (bps)	9	9	8
net_loan_fees (bps)	16	17	10
loan_tenor (years)	4	2	5

Panel B: Relationship variables

Variable	Mean	StdDev	Median
loan_revenues (EUR)	382,395	602,079	181,943
non_loan_revenue (EUR)	2,817,792	7,287,856	856,630
expected_revenues (EUR)	4,328,448	8,899,153	1,748,500
new_relationship_indicator	2.4%	15.5%	0.0%

Panel C: Company characteristics

Variable	Mean	StdDev	Median
market_value (mn EUR)	45,341	69,816	18,343
sales (mn EUR)	36,782	64,327	12,586
tangible_assets_pct	65%	19%	67%
debt_by_assets	35%	21%	29%
interest_coverage_ratio	11	15	6
ebitda_by_assets	13%	5%	11%
MKT_TO_BOOK	3.43	7.14	2.49
firm_age (years)	75	45	77

Panel D: Market variables

Variable	Mean	StdDev	Median
prime_rate (bps)	698	139	750
default_spread for BAA (bps)	190	46	173
cds_spread (bps)	49	51	30

from Compustat. The resulting sample of 679 records are combined with the expected revenue data as follows. Since the expected revenue data is loan specific whereas the CRM database is customer-specific, to link these two databases, the expected revenue is *averaged* if any company has taken multiple loans in a given year. The *averaged* expected revenue data is linked to the CRM data such that the year of the loan issuance in the former database matches the year of shortfall in the latter database. This results in a sample of 162 records, which is hereafter referred to as *CRM Database Sample*. The summary statistics of this sample are included in Table 4.2. The customer characteristics in the *CRM Database Sample* such as accounting variables, market value and firm age are not much different from those in the *Loan Database Sample* in terms of the median values but the average market value and sales are much lower in the former sample than in the latter sample. The median prior year *loan* revenues in the *CRM Database Sample* is slightly less than that in the *Loan Database Sample*. The opposite holds for median prior year *non-loan* revenue. The median *yearly* shortfall of 0.14mn EUR is about one third of the median *lifetime* shortfall in the *Loan Database Sample*. The median expected revenue of EUR 2mn is about twice the median prior year loan plus non-loan revenue, although in terms of average the difference between these variables is much less.

4.6 Empirical Results

Table 4.3 contains the results when specification (9) is estimated. All the regression coefficients are statistically significant at 99% confidence and the signs on the coefficients are same as those suggested by the shortfall equation. In particular, CDS spread, loan tenor and facility limit increase the shortfall whereas

Table 4.2
Summary Statistics for the CRM Database Sample

Panel A: Shortfall and relationship variables

Variable	Mean	StdDev	Median
shortfall (EUR)	412,552	1,106,355	141,080
loan_revenues (EUR)	328,386	573,242	171,212
non_loan_revenue (EUR)	3,141,822	8,132,369	943,099
expected_revenues (EUR)	3,689,955	5,947,444	2,080,000
new_firm	2.5%	15.6%	0.0%

Panel B: Company characteristics

Variable	Mean	StdDev	Median
sales (mn)	25,932	43,617	11,711
market_value (mn)	34,927	51,219	17,045
tangible_assets_pct	63%	21%	67%
debt_by_assets	31%	20%	26%
interest_coverage_ratio	13	17	8
MKT_TO_BOOK	3.3	2.5	2.6
ebitda_by_assets	13%	6%	12%
firm_age (years)	72	47	69

recovery rate and loan fees reduce the shortfall.

For specification (10), several models are estimated in Table 4.4. As a starting point, model 1 includes all the non-relationship variables, which explain 34% of the variation in shortfall. The generic default spread and prime rate are positively significant as is expected from these measures of credit spread. The tangible asset percentage is negatively significant as is expected from this measure of loan recovery rate. None of the other variables are significant. Model 2 extends model 1 by including firm age and new relationship indicator. None of these commonly used measures of relationship are significant and the explanatory power reduces slightly. Model 3 includes relationship depth (prior year loan revenues) along with the other variables in Model 2. The explanatory power is unchanged and the relationship depth is an insignificant predictor of

Table 4.3**Effect of Loan Characteristics on Shortfall in *Loan Database Sample***

This table provides the OLS estimates for equation (9). The dependent variable is lifetime loan shortfall. Standard errors are in parenthesis (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

Intercept	2351004** (1134765)
facility_limit	0.006*** (0.0006)
net_loan_fees	-49705*** (9132)
loan_tenor	1590*** (232)
loan_recovery_rate	-137155*** (26683)
cds_spread	39908*** (3171)
Adj. R-square	0.4628

shortfall. Model 4 extends the previous model by adding relationship breadth (prior year non-loan revenues) which is positively significant and the explanatory power increases significantly to 40%¹⁵. Model 5 further adds relationship potential (expected next year revenues) which increases the explanatory power by another 7%. Both relationship breadth and relationship potential have positive coefficients which are significant at 99% confidence level and relationship depth remains insignificant. Finally, model 6 extends the previous model by

¹⁵Since Model 4 doesn't have expected revenues as an independent variable, we can estimate this model on the larger sample of 433 loans which when linked with the expected revenue data resulted in the loan database sample of 164 loans. The main results from the smaller sample hold in the larger sample.

adding CDS spread, which is positively significant and the explanatory power increases to 51%. Nonetheless, relationship breadth and potential remain positively significant at 99% confidence.

The insignificant effect of relationship depth on shortfall in this paper contrasts with the results in [29] who find that their measure of relationship depth significantly explains the loan rate offered to borrowers. These contrasting results could be due to the fact that they do not fully control for credit spread and because they do not include measures for relationship breadth and relationship potential. In particular, their independent variable is not risk-adjusted and their control for default risk using default spread is imperfect. In contrast, the independent variable here is risk adjusted since shortfall takes into account default risk. Note also that a comparison of models 3-6 shows that the coefficient on relationship depth reduces successively from small positive value to large negative value as the other two relationship variables and CDS spread are added one by one.

The significant effect of relationship breadth on shortfall in this paper clarifies the mixed results in the literature on whether the scope of a relationship is a source of relationship benefit. Measuring relationship breadth using dummy variables to denote the use of non-loan financial services, some existing studies find that relationship breadth results in lower loan rates (e.g. [11]) and some studies find no effect on loan rates (e.g. [16]). Since this paper uses a dollar measure of relationship breadth which captures both the use and the extent of use of other financial services, the results in this paper suggest that the lack of significant effect of relationship breadth in some of the existing papers could be due to the crude dummy variables used to measure relationship breadth.

Table 4.4**Effect of Relationship Variables on Shortfall in *Loan Database Sample***

This table provides OLS estimates for equation (10) for the *loan* database sample. The dependent variable is *annual loan* shortfall. Standard errors are included in paranthesis (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level).

	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-1292189*** (456727)	-1308697*** (462068)	-1344752*** (468677)	-1118051** (445792)	-950473** (424783)	-1864702*** (688302)
interest_coverage_ratio	-24 (2559)	-287 (2625)	-425 (2648)	846 (2518)	1424 (2389)	878 (2541)
debt_by_assets	68777 (337811)	39443 (351916)	-7346 (363827)	37791 (342748)	287868 (332663)	593015 (395736)
prime_rate	671** (298)	685** (301)	729** (313)	732** (295)	539* (285)	712 (604)
default_spread	5154*** (791)	5077*** (805)	5150*** (820)	4952*** (774)	4852*** (733)	3987*** (1212)
cds_spread						4427*** (1511)
tangible_assets_pct	-517044*** (176300)	-530351*** (178096)	-537413*** (179269)	-378344** (174580)	-399516** (165340)	-330912* (187547)
relationship_depth			0.04 (0.08)	0.02 (0.08)	-0.06 (0.08)	-0.2 (0.2)
relationship_breadth				0.03*** (0.008)	0.02*** (0.008)	0.02*** (0.009)
relationship_potential					0.04*** (0.01)	0.04*** (0.01)
new_relationship		155794 (290784)	156922 (291921)	121048 (275005)	190075 (261059)	98256 (272051)
firm_age		585 (724)	535 (733)	104 (700)	-108 (666)	151 (755)
log_sales	26853 (42775)	18851 (44362)	15541 (44957)	12994 (42329)	22261 (40154)	15800 (46254)
log_market_value	1479 (51097)	9603 (52140)	12014 (52534)	-13814 (49986)	-37767 (47832)	37055 (60759)
MKT_TO_BOOK	3572 (3574)	2347 (4905)	2303 (4925)	1906 (4638)	-1447 (4499)	383 (4768)
ebitda_by_assets	457599 (786021)	444520 (791181)	511862 (804053)	117546 (765010)	265861 (725323)	376382 (816736)
Adj. R-square	0.34	0.33	0.33	0.40	0.47	0.51

The result that relationship potential affects shortfall is novel since none of the existing studies have used relationship potential to study relationship benefit. It suggests the possibility that the bank is forward looking in terms of passing on the relationship benefit to borrowers. However, this correlation doesn't necessarily imply a causation from expected revenue to shortfall. The issue of causation cannot be addressed with this data. The coefficient on relationship potential is about twice the coefficient on relationship breadth, suggesting that the bank puts more emphasis on relationship potential than relationship breadth while deciding the shortfall.

Tangible assets percentage is negatively significant in all the models which suggests that it is a good proxy for loan recovery rate. The generic default spread and prime rate are positively significant in almost all the models which suggests that these variables proxy well for credit spread. However, the fact that CDS spread is significant in the last model suggest that these variables do not fully control for credit spread. The fact that default spread and prime rate are significant in the last model despite including CDS spread may be due to the non-linear dependence of shortfall on CDS spread.

The results for specifications (11) and (12) are included in Table 4.5. Both these specifications have lifetime shortfall as the dependent variable and control for loan tenor as an independent variable but differ in the assumption whether facility limit is affected by relationship variables or not. In specification (11) where facility limit is assumed to be affected by relationship factors, loan tenor is positively significant but the relationship variables have the same effect as in specification (10) estimated in Table 4.4. Specification (12) includes facility limit as an additional independent variable which turns out to be positively sig-

Table 4.5**Other Specifications on the Effect of Relationship Variables on Shortfall**

This table provides the OLS estimates for equation (11) and (12) for the *loan* database sample. The dependent variable is *lifetime loan* shortfall. Standard errors are in paranthesis. (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level).

	Spec 11	Spec 12
Intercept	-3376930* (1955400)	-2418904 (1909561)
facility_limit		0.003*** (0.001)
loan_tenor	635*** (234)	855*** (238)
debt_by_assets	-2777 (10431)	-18807 (11486)
interest_coverage_ratio	468053 (1449455)	-471030 (1431652)
prime_rate	1850 (1261)	1373 (1224)
default_spread	14837*** (3192)	13240*** (3120)
tangible_assets_pct	-1199325* (720409)	-794278 (706935)
relationship_depth	-0.3 (0.3)	-0.3 (0.3)
relationship_breadth	0.11*** (0.04)	0.09*** (0.03)
relationship_potential	0.16*** (0.05)	0.1** (0.05)
new_relationship	594635 (1137474)	452269 (1094754)
firm_age	-872 (2903)	-1928 (2815)
log_sales	55950 (175010)	36626 (168398)
log_market_value	-122761 (213790)	-166627 (206119)
MKT_TO_BOOK	-3203 (19611)	-566 (18877)
ebitda_by_assets	-273218 (3164852)	-311932 (3042877)
Adj. R-square	0.37	0.42

nificant but the relationship variables have the same significant effect as in the previous model. Prime rate and tangible asset percentage become insignificant but generic default spread remains significant in both these specifications¹⁶.

Table 4.6 includes the estimation results for various models with specification (13). Since this specification uses the yearly aggregate shortfall (from *CRM Database Sample*) comprising shortfall from all existing loans issued at various points in time, we cannot use generic default spread or prime rate to capture the default risk. The remaining variables used in this specification are the same as those used in specification (10). Model 1 includes only accounting variables, model 2 adds firm age and new relationship indicator, and models 3-5 successively adds the three relationship variables. The results with these models are similar to those with specification (10) in Table 4.4¹⁷. Firm age, new relationship indicator and relationship depth have an insignificant effect on shortfall. Relationship breadth and relationship potential are positively significant and together these variables triple the explanatory power. This specification too suggests that the bank places more emphasis on relationship potential than relationship breadth.

To summarize, across all the specifications and models, the shortfall increases with relationship breadth and potential, suggesting that the benefit of relationship banking is larger for relationships with broader scope and better future potential. Also, it seems that relationship potential is more important than relationship breadth in determining the relationship benefit obtained by

¹⁶Several other models were estimated with specifications (11) and (12) but the main results are same as those presented in Table 5.

¹⁷Since Model 4 doesn't have expected revenue as an independent variable, we can estimate this model on the larger sample of 679 records which when linked with the expected revenue data resulted in the CRM database sample of 162 records. The main results from the smaller sample hold in this larger sample.

Table 4.6**Effect of Relationship Variables on Shortfall in CRM Database Sample**

This table provides the OLS estimates for equation (13) for the CRM Database Sample. The dependent variable is *annual customer shortfall*.

Standard errors are in paranthesis.(*** Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

	(1)	(2)	(3)	(4)	(5)
Intercept	-491757 (305713)	-529089* (308156)	-527272* (310476)	-464534 (301330)	-307222 (281309)
interest_coverage_ratio	599 (2088)	509 (2092)	505 (2103)	609 (2036)	-138 (1892)
debt_by_assets	523306* (270763)	529772* (271373)	524354* (280623)	429995 (273699)	491743* (253665)
tangible_assets_pct	-146397 (142920)	-126494 (144098)	-124982 (145984)	-47073 (144029)	-162610 (136071)
relationship_depth			0.006 (0.08)	0.02 (0.07)	-0.03 (0.07)
relationship_breadth				0.02*** (0.006)	0.01** (0.006)
relationship_potential					0.05*** (12114)
new_firm		-43815 (196433)	-44200 (197461)	-44053 (191108)	-73122 (176958)
firm_age		-802 (623)	-803 (627)	-1021* (612)	-770 (569)
log_sales	5873 (38538)	12769 (38974)	12140 (39911)	16186 (38654)	14080 (35768)
log_mkt_value	74669* (44130)	75733* (44310)	75990* (44640)	61542 (43514)	34858 (40760)
MKT_TO_BOOK	-10952 (11873)	-8138 (12111)	-8036 (12234)	-3464 (11954)	-5427 (11070)
ebitda_by_assets	-809519 (631620)	-820520 (633561)	-818203 (637327)	-965271 (619078)	-410352 (587838)
Adj. R-square	0.09	0.09	0.08	0.14	0.26

borrowers. Unlike the past literature, relationship depth, new relationship and firm age do not explain the relationship benefit provided to borrowers. The significance of these variables in previous studies could be because these studies do not fully control for credit risk and do not include more comprehensive relationship variables such as relationship breadth and relationship potential.

4.7 Additional Tests

While expected future revenue (relationship potential) is a significant predictor of relationship benefit, this relation is economically meaningful only if expected future revenue is a significant predictor of actual future revenues. Testing this hypothesis is complicated by the fact that the revenue data in *CRM Database Sample* is measured as of year-end whereas expected revenue data is measured as of the loan issuance date, which could be anytime during the year. To overcome this problem, we estimate whether next year total revenue (loan plus non-loan revenue) can be explained by current year total revenue, past year total revenue and expected revenue as of loan issuance date in the current year. For example if the expected revenue data is as of May 2006, we investigate whether the total revenue in 2007 can be explained by the total revenues in 2005 and 2006 and the expected revenues as of May 2006. The estimation results are reported in Table 4.7. All the three independent variables are positively significant with an explanatory power of 55%. Expected revenues has the highest sensitivity followed by current total revenues and past total revenues. Therefore, expected revenue is an economically meaningful variable that can predict future revenue beyond what can be predicted with historical revenue data. This test also justifies the use of expected revenue as a measure of relationship potential. Note

Table 4.7

Relationship Between Expected Revenues and Future Revenues

This table provides the OLS estimates of a regression wherein the dependent variable is next period revenue and the independent variables are previous period, current period and expected revenues. Standard errors are in parenthesis. (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

Intercept	-483208 (413053)
expected_revenues	0.42*** (0.09)
current_revenues	0.41*** (0.13)
past_revenues	0.23** (0.11)

Adj. R-square	0.53
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that this result shows a correlation not causation between expected future revenues and future revenues. Therefore, this result combined with the results in the previous section imply a correlation not causation between shortfall and future revenues. The issue of causation cannot be addressed with this data set.

Panel A of Table 4.8 studies the time variation of parameter estimates by splitting the *Loan Database Sample* by date. The first half sample broadly corresponds to the period 2004-2005 and the second half sample broadly corresponds to the period 2006-2007. The explanatory power is higher in the second half sample but both relationship breadth and relationship potential are significant in both half samples. As compared to the first half sample, the sensitivity of

shortfall to relationship breadth has increased and its sensitivity to relationship potential has decreased in the second half sample. However, in absolute terms, relationship potential is more important than relationship breadth in both half samples. These results suggest that the criteria for providing relationship benefit has changed in the second half sample. The time variation in these parameter estimates could potentially be explained by the fact that LSC meetings were initiated in 2003. It is likely that this time variation is the result of the learning process by which the bank became accustomed to the new LSC process.

4.8 Effect of Informational Opacity

In this section, we investigate how our results are affected by differences in informational opacity across borrowers. Since the main premise of relationship banking is that it resolves information asymmetry between the bank and the borrower, the effect of relationship banking should be different depending on whether the borrower is more or less informationally opaque. The commonly used measures of informational opacity are market-to-book ratio, asset size, inclusion in S&P500, existence of credit ratings, and trading in CDS markets ([29]). We use the first two measures of informationally opacity for our analysis because the remaining three measures result in a very small sample¹⁸.

Panel B of Table 4.8 studies the effect of the splitting the sample into two subsets based on the market-to-book ratio. Both the samples have about the same explanatory power and relationship depth is insignificant in both the samples. The growth firms have high positively significant sensitivity to relationship po-

¹⁸Only four of the 164 loans in our sample correspond to borrowers who have no credit rating, or no traded CDS, or are not included in S&P 500.

tential and insignificant sensitivity to relationship breadth. On the other hand, value firms have insignificant sensitivity to relationship potential and high positively significant sensitivity to relationship breadth. Since growth firms usually have low past non-loan revenues and high expected revenues, the higher sensitivity of shortfall to relationship potential than relationship breadth for these firms implies that the relationship benefit is passed on to the borrower much before the non-loan revenues increase. On the other hand, since value firms usually have high non-loan revenues and stagnant expected revenues, the higher sensitivity of shortfall to non-loan revenues than expected revenues for these firms implies that the relationship benefit tapers gradually if the borrower's expected revenues are declining.

Finally, Panel C of Table 4.8 studies the effect of splitting the sample in two based on the book value of assets. The sample with large firms has significantly higher explanatory power than the sample with small firms. Here too, the two samples have different sensitivities to relationship breadth and potential whereas relationship depth is insignificant in both the samples. Large firms have insignificant sensitivity to relationship potential and positively significant sensitivity to relationship breadth. Small firms have the exact opposite effect. As in the case of growth and value firms, these results imply that small firms get a sizeable relationship benefit despite low past non-loan revenues (in anticipation of future business) and large firms get a sizeable relationship benefit despite declining expected revenues (as reward for current business).

Together, the results in Table 4.8 suggest that relationship potential is the most important determinant of relationship benefit for firms that are more informationally opaque (firms with high market-to-book and small size) whereas

Table 4.8**Effect of Splitting *Loan Database Sample* by Date, M/B and Asset Size**

This table provides the OLS estimates for equation (10) when the *Loan Database Sample* is split by date, market-to-book and asset size. The dependent variable is *annual loan* shortfall. The sample is split in to two exact halves by date or M/B or asset size. Standard errors are included in paranthesis. (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

	Panel A		Panel B		Panel C	
	Sample Split by Date		Sample Split by M/B		Sample Split by Asset Size	
	First Half	Second Half	Low M/B	High M/B	Small Firms	Large Firms
Adj. R-square	0.34	0.63	0.75	0.74	0.30	0.64
Intercept	-212478 (1155818)	-5539497*** (1606278)	646331 (662952)	-2003555*** (387133)	-70980 (740527)	-2106973** (1033168)
interest_coverage_ratio	-2451 (4062)	3932 (3314)	-36434* (17864)	-57 (1834)	-5656 (4822)	6855* (3769)
debt_by_assets	-623828 (503427)	934851* (494805)	-917611 (692752)	123904 (293508)	-169894 (256874)	1752984** (785043)
prime_rate	506 (547)	4529*** (1420)	1162*** (365)	753*** (252)	272 (239)	419 (600)
default_spread	2638 (4226)	12106*** (2374)	4850*** (808)	8809*** (812)	2970 (2168)	5646*** (1118)
tangible_assets_pct	-201873 (247493)	-79808 (240554)	-777098*** (209134)	-117312 (188837)	-176250 (116896)	-1284137*** (330784)
relationship_depth	-0.03 (0.09)	-0.5 (0.3)	0.2 (0.2)	0.004 (0.06)	-0.06 (0.05)	-0.4 (0.3)
relationship_breadth	0.02** (0.009)	0.03* (0.02)	0.1*** (0.02)	-0.004 (0.006)	-0.0004 (0.005)	0.05*** (0.01)
relationship_potential	0.07*** (0.02)	0.05*** (0.01)	0.01 (0.01)	0.04*** (0.01)	0.03*** (0.009)	0.01 (0.02)
firm_age	70306 (254105)	-5695940* (2992629)	0 (0)	198330 (172020)	80798 (124915)	0 (0)
new_firm	-666 (935)	948 (937)	1054 (834)	-227 (755)	101 (455)	746 (1376)
log_sales	-39355 (66625)	58590 (55867)	33649 (52239)	1766 (40949)	-26790 (24632)	67769 (107481)
log_mkt_value	26754 (66394)	-111517 (71913)	-205328*** (72742)	-7273 (42640)	-20398 (30575)	65479 (169727)
MKT_TO_BOOK	-2631 (17548)	67644* (34880)	-48582 (118675)	-1497 (3236)	537 (2282)	-59395 (61414)
ebitda_by_assets	-801567 (1175667)	-985600 (1129212)	2783310** (1257588)	277858 (936815)	546439 (543681)	-732535 (1422269)

relationship breadth is more important for less informationally opaque firms.

Table 4.9 includes the same tests as in Table 4.8 but with the *CRM Database Sample* rather than the *Loan Database Sample*. Panels A, B and C of Table 4.9 study the effect of splitting the *CRM Database Sample* by date, market-to-book ratio and asset size respectively. The main results and conclusions are same as in Table 4.8. So the results from sample splitting are robust to alternative specifications.

4.9 Conclusion

While numerous theoretical studies have identified the sources of relationship banking benefits, empirical studies have been constrained in testing these theories by the availability of data. The crude measures for relationship strength and benefit used in the existing literature has yielded mixed empirical support to the claims made in the theoretical literature and has not fully identified the sources of relationship benefit. This paper uses a proprietary dataset from a multinational bank to precisely measure relationship strength and uncover the true sources of relationship banking benefits to borrowers. Relationship benefit is measured as the difference between the par value and the fair value of a loan where the fair value is calculated using the market implied default probabilities, collateral-dependent recovery rates, and loan features including any embedded options. Relationship strength is measured along three dimensions: relationship depth, relationship breadth, and relationship potential. Relationship depth is measured as the annual revenue generated from all existing loans issued to the borrower by the bank. Relationship breadth is measured as the an-

Table 4.9**Effect of Splitting CRM Database Sample by Date, M/B and Asset Size**

This table provides the OLS estimates for equation (10) when the CRM Database Sample is split by date, market-to-book and asset size. The dependent variable is *annual customer shortfall*. The sample is split into two halves by date or M/B or asset size. Standard errors are included in parenthesis. (***) Significant at one percent level, ** Significant at five percent level, * Significant at 10 percent level)

	Panel A		Panel B		Panel C	
	Sample Split by Date		Sample Split by M/B		Sample Split by Asset Size	
	First Half	Second Half	Low M/B	High M/B	Small Firms	Large Firms
Adj. R-square	0.27	0.31	0.24	0.38	0.13	0.46
Intercept	-48812 (427978)	-113209 (436208)	600382 (701874)	-200804 (308025)	-339781 (501344)	-624559 (624692)
interest_coverage_ratio	-136 (2108)	-2614 (3869)	-11504 (12055)	742 (1932)	1108 (2122)	-1151 (3779)
debt_by_assets	271841 (332048)	365253 (381172)	-468084 (780095)	609138** (286464)	300286 (288460)	875174* (480911)
tangible_assets_pct	-222506 (201254)	-222063 (190679)	-392509* (223067)	123438 (179342)	81630 (171655)	-512267** (219131)
relationship_depth	-0.01 (0.06)	-0.1 (0.3)	0.09 (0.1)	-0.1 (0.09)	-0.08 (0.09)	0.05 (0.1)
relationship_breadth	-0.0002 (0.006)	0.04*** (0.01)	0.04** (0.02)	0.01* (0.006)	0.008 (0.006)	0.06*** (0.02)
relationship_potential	0.08*** (0.02)	0.043** (0.02)	0.02 (0.02)	0.1*** (0.02)	0.06** (0.02)	0.02 (0.02)
firm_age	0 (0)	-80293 (202438)	10903 (274023)	-499709** (246343)	-84394 (165650)	0 (0)
new_firm	260 (813)	-1175 (890)	-388 (993)	-471 (784)	-703 (779)	1027 (1058)
log_sales	-2298 (45321)	32854 (62277)	13043 (62484)	60820 (44166)	23499 (40753)	13902 (68035)
log_mkt_value	25057 (48776)	-2010 (74366)	-11546 (81756)	-48575 (52007)	23870 (56617)	55437 (90802)
MKT_TO_BOOK	-2909 (13052)	-18751 (20433)	-112131 (135733)	2229 (11660)	2354 (11906)	-13696 (34227)
ebitda_by_assets	-574742 (718109)	665764 (1074009)	1476576 (1196150)	-894446 (726371)	-1029972 (704182)	91182 (1018953)

nual revenue generated from all non-loan products and services purchased by the borrower from the bank. Relationship potential is measured as the revenue expected to be generated in the next one year across all products and services provided to the borrower by the bank.

This paper finds that the borrowers benefit from a lending relationship through better terms on their loan contracts. The relationship benefit increases with relationship breadth and relationship potential but does not depend on relationship depth. Moreover, relationship potential is more important than relationship breadth in determining the relationship benefit provided by borrowers. The paper also finds some evidence which suggests that the significance of relationship depth in some of these studies could be because these studies do not fully control for credit risk and do not include more comprehensive relationship variables such as relationship breadth and relationship potential. The significant effect of relationship breadth on shortfall benefit suggests that the lack of such an effect in some of the existing papers could be due to their crude measures for relationship breadth that do not capture the extent of use of non-loan products and services. The novel result about the effect of relationship potential on relationship benefit suggests that banks are forward looking in terms of passing on the relationship benefit to borrowers, though the causation cannot be clearly established. The paper also suggests that the assertion in the existing literature that banks benefit from a relationship through higher probability of winning future loan and fee business need not be true due to the loss-making nature of loans and because the causation between relationship benefit and future revenue *cannot* be clearly established.

We also find that relationship potential is the most important determinant of

relationship benefit for firms that are very informationally opaque (firms with high market-to-book ratio and small size) whereas relationship breadth is more important for less informationally opaque firms. These results imply that relationship benefit is passed on to small firms and growth firms much before their relationship breadth increases. These results also imply that relationship benefit tapers gradually for large firms and value firms if their relationship potential is declining.

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