Mathematical and Theoretical Biology Institute at Cornell University

# Studies in Theoretical Biology: A Collection of Undergraduate Research

Interface between biology and the mathematical sciences.

Research Efforts of the 1997 Cornell-SACNAS Summer Program*

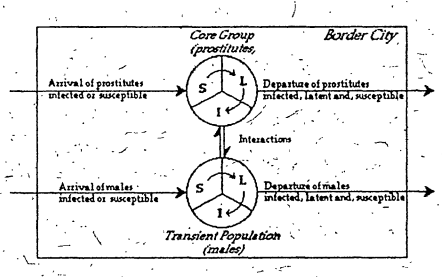
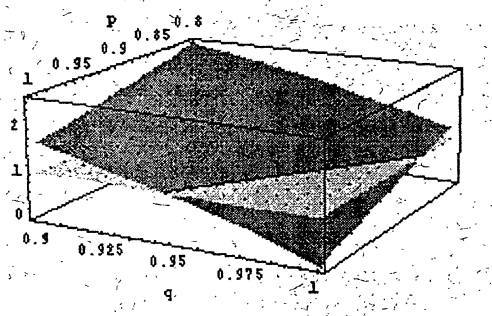



Figure 1. Conceptual model of HIV transmission through prostitution in a border town.

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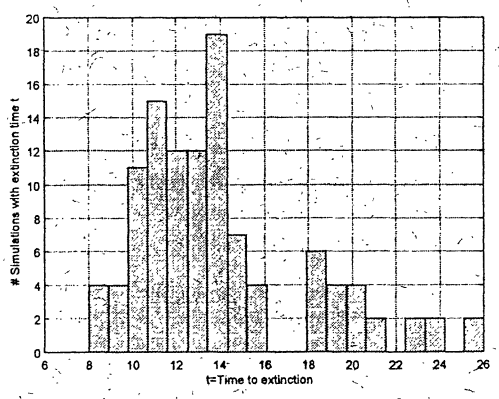


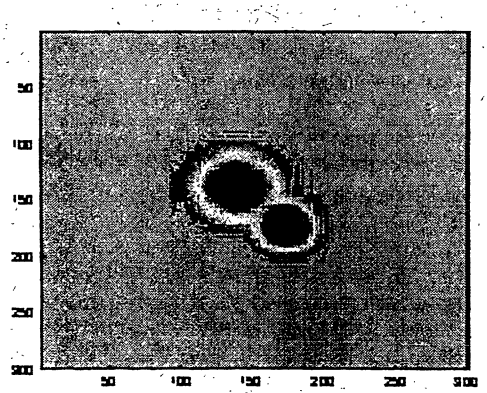

Concentration gradients of morphogens 1 and 2

Biometrics Unit Technical Report
BU-1419-M

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# Preface to the Second Year's Collection of Undergraduate Research

BU-1419-M

*Stephen Tennenbaum*

## Introduction

The following is a collection of eleven papers by thirty-four undergraduate students who participated in the Cornell-SACNAS Mathematical Sciences Summer Institute (CU-SMSSI) in 1997, part of Cornell's Mathematical and Theoretical Biological Institute (MTBI). Included in this collection are five papers by students that returned to the program for a second summer, and helped to advise the new students as well as pursue original research of their own.

As in the first year the papers presented here are the results of an intense three-week effort by all the students in the program. They are even more impressive when one considers the fact that most of the students learned much of the mathematics in just the four weeks prior to, and the biology during their research. It is no exaggeration when I say that these students put in an unparalleled amount of sustained focus and effort toward this single intellectual pursuit. How was this achieved without coercion of grades or constant cajoling? As much as we would like to attribute it to the charisma and inspiring personalities of the teachers and staff of the program, it probably had more to do with the requirement that the students study, work problems and pursue research in groups from the outset. This in concert with constantly challenging them with difficult but interesting questions not only provides the impetus of peer pressure but also enables the students to gain insight from one another, have constantly available sounding boards for ideas, and furnishes an outlet for creativity. The proof, we think, is in the papers.

The purpose of the summer institute is to provide a "first rate research experience in mathematical and theoretical biology." And so the context of these papers goes beyond just a pedantic exercise, they are valid pieces of research in their own right. The breadth of the subject matter illustrates the range of interests of the students and their imagination in applying their learning. Even though the techniques themselves are pretty much standard, many of the papers illustrate unique approaches to the particular topics investigated. Thus there are papers ranging from an analysis of advertising strategy using differential equations to spatial models of tumor growth. This year, unlike the 1996 summer program, the students were completely free to pick their own topics. And although some of the topics fell outside of problems at the interface of mathematics and biology, we think that this was a good decision.

Each of the groups was required to write up their research in a technical report and they made two presentations one at Ithaca and the other at a conference in Silver Springs Maryland, hosted by the NSA (one of the summer institutes funding agencies). In addition the students gave poster presentations of their research at the 1997 annual SACNAS conference and some of the students gave talks as well. These were the only undergraduates giving talks at this national conference.

## Project Abstracts

### A Competition Model for Advertising Companies

Patricia Fuentes, *Loyola Marymount University*
Victor A. Chacón, *Occidental College*
George F. González, Jr., *Rice University*
Mario A. Mendieta, *Universidad Autonoma Metropolitana-Izapalapa*

Abstract:
Television advertising plays a major role in influencing people to purchase a company's product. In this project, we focus on the competition between two of the top beer companies, Budweiser and Coors Light. Both companies compete for beer drinkers as well as for television advertisements. To explain this competition, we develop an S-I-S (Susceptible-Infected-Susceptible) model utilizing five differential equations in which a susceptible person is an average beer drinker who watches television, and an infected person is one who purchases one of the two major brands. Two versions of this basic model are developed. The first version of the model takes into account only competition between Budweiser and Coors Light for beer drinkers. The second version takes into account both the competition for consumers as well as for television air time. One finds that under certain conditions, one of four states occur. In one state, neither Budweiser nor Coors Light attract any beer drinkers through television advertising. In two complementary cases one beer company influences many beer drinkers to purchase its particular brand. Finally, both beer companies influence enough beer drinkers to purchase their particular brand so that they both coexist in the television advertising market.

### Mean Time to Extinction of Source-Sink Metapopulations for Different Spatial Considerations

Agustín Izquierdo-Sabido, *Universidad Nacional Autónoma de Mexico*
Jessica Lasky, *Cornell University*
Mark Muktoyuk, *Oregon State University*
Selim A. Sabillón, *Palo Alto College*

Abstract
The mean time to extinction for a single species in a stochastic metapopulation model is calculated for several landscape configurations of discrete source (high quality habitat) and sink (low quality habitat) patches.

**Can we get a head start on head lice?**

Veronica Ayala-Prado, *University of California, Irvine*
Claudia A. Catalán, *Loyola Marymount University*
Nohora Milena Londoño-Alzate, *Universidad del Quindío*
Joaquín Rivera-Cruz, *University of Puerto Rico, Cayey*

Abstract

In this research we provide a simple model to better understand the distribution of head lice among humans. A model that looks at the transmission dynamics of head lice is also studied. The basic reproductive number is given by the addition of two terms: the contributions by those with few and many lice. Conditions for eradication andpersistance of lice in humans are established. The observed properties of the distribution of lice in humans results from our "macroparasitic" lice model.

**Mathematical Model for Morphogen-Induced Cell Differentiation**

Marcia Black, *National Security Agency*
Manuel Bravo, *Skyline College*
Jose Agustin Pena Arellano, *Universidad Autonoma Metropolitana at Iztapalapa*
Olga-Nitaina-Uhuru Russi Roman, *California Institute of Technology*

Abstract

Cell differentiation is the process by which cells in their development become functionally and structurally specialized. One of the most accepted mechanisms for cell differentiation involves chemical signaling between cells. That is, molecules secreted by one cell can stimulate differentiation in another. We modeled a system with two morphogens in which each chemical induces a different cell phenotype. We developed a non-spatial deterministic model in which we use ordinary differential equations to analyze the asymptotic behavior of our cell populations. We also generated a stochastic spatial model that accounts for the diffusion rates of the morphogens and susceptibility of a cell to differentiate in each stage of its life cycle. We found a globally stable equilibrium for our system of cells. Further, we determined the expected phenotype of a given cell under different parameters.

**Dynamics of Rubella Virus in Populations with Different Vaccination Policies**

Jacqueline Flores, *Pomona College*
Cristina Garcia, *Pomona College*
Carlos W. Melgar, *University of California, Riverside*
Catalina Saenz, *Wellesley College*

Abstract:

Rubella is a mild viral disease that affects the general population. The disease becomes a serious threat to a fetus when a susceptible pregnant woman is infected. In this study we use differential equations to analyze how interaction and vaccination policies affect the dynamics of Rubella on the borders of Mexico and the United States. Interaction between bordering countries with different vaccination systems can lead to an increase of Rubella cases. We show that the United States and Mexico must develop a dual health policy to eradicate the rubella virus.

**Tumor Growth Dynamics:A Deterministic and Stochastic Analysis of the Interaction between Normal and Abnormal Cells**

Brendaliz Acosta, *University of Puerto Rico, Cayey*
Jaime H. Barrera, *Texas A&M*
Ernesto S. Clarke II, *Pitzer College*
Nicolas Davidenko, *Harvard University*
Derek Ting, *Cornell University*

Abstract:
In this project, we study the interactions between normal and abnormal cell populations as they occur in a tumorous growth. The purpose of our research is to find conditions under which the spatial arrangement of abnormal cells in a tissue is a significant factor governing the spread of the tumor. To this end, we model how normal and abnormal cells compete for nutrients using a deterministic model and a spatial stochastic model. We vary nutrient competition rates as well as drug treatment effects for the two cell populations. The deterministic model indicates how the cell populations interact without consideration of spatial arrangement, while the stochastic model includes this factor. Our results show that different spatial arrangements of cells may cause significant differences in the growth dynamics of the cells, even if the population sizes are the same. These differences are most pronounced when we increase the death rate of normal cells due to competition. We have found that the spatial model reveals some growth dynamics that the deterministic model overlooks. Therefore it is of interest to obtain more realistic spatial models. For this, we need to focus research on the most distinctive factor of the spatial model: how cells on the boundary of a tumor compete for nutrients.

**Models of the Transmission Dynamics of Gonorrhea in a Homosexually-Active Population**

Sharon K. Lima, *Purdue University*
Mabel Torres, *University of Miami*

Abstract
The purpose of this project is to study two questions associated with the transmission dynamics of gonorrhea in a homosexually-active population. First, we look at the role of a partially effective vaccine on gonorrhea dynamics. Second, we look at the role of antibiotic-driven mutation on the survival and spread of resistant gonorrhea strains. A model that combines vacciantions and multiple strains of gonorrhea is also studied. The role of heterogeneity, age-structure, is explored on both questions. A partial mathematical analysis of the system is given including the computation of the basic reproductive number.

**Dynamics of HIV/AIDS in Core Groups in the Presence of a Transient Population**

Moira Zellner, *Universidad de Belgrano, Argentina*
Dámaris Santana-Morant, *University of Puerto Rico*

Abstract
We investigate the role of a transient population of males individuals who interact with a core group of prostitutes in the dynamics of a sexually transmitted disease. We use a stochastic model to follow the evolution of the individuals of each population, in terms of the disease. The analysis is concentrated in the equilibrium of the process. We adapt this

model to a city of the U.S.-Mexico border, where some of the parameters have been estimated. In this way, we seek to provide a method of evaluating the effect of U.S. border policy in the transmission of the disease to U.S.

**Three models for Measles Control**

Julio Villarreal, *Cornell University*
Carlos Castillo-Garsow, *Cornell University*

Abstract

We study the dynamics of three models consisting of infant vaccination and booster shots to control the spread of measles. Varying the rates of infant vaccination and administration of booster shots, we alter the characteristics of the basic reproductive number to study the effectiveness of both infant vaccination and booster shot control methods. In one model, we find that the total effort on birth vaccination rather than on booster shots lowers cases of recovered and infected individuals to possibly eradicate measles.

**Macrophage Activating and Tissue Damaging Immune Responses to *M. tuberculosis***

Guarionex Jordan Salivia, *Universidad de Puerto Rico recinto de Rio Piedras*
Ariel Rodriguez Herrera, *Stanford University*

Abstract

The two principal immune responses as the TB bacilli enter the body are the Macrophage Activating Response (MAR) and the Tissue Damaging Response (TDR). The TDR is the responsible for the caseous necrosis in the lung tissue and it prevents the bacterial growth. In the MAR the macrophages are activated by the T-cells in order to kill the bacilli while inside the macrophages cytoplasm. With the use of a mathematical model we intend to find a proper interplay of both mechanisms to obtain an effective immune response where the amount of necrotic tissue is reduced to its minimum and still the disease is eradicated.

**Analysis of an age-structured epidemic model with a chronic state**

Ricardo Saenz, *University of Texas at El Paso*

Abstract

In this work we study an age-structured epidemic model for cytomegalovirus infection. We compute the basic reproductive number, determine the stability of the disease-free state and show the existence of an endemic equilibrium point. Conclusions on disease dynamics are discussed.

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