Modeling Heat Flows in a Hibernating Black Bear (*Ursus americanus)*

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Executive Summary

The American Black Bear (*Ursus americanus)* has the ability to sustain a high core temperature throughout the duration of its hibernation cycle, even as outside temperatures fall to -20°C. This ability is largely due the conversion of chemical energy into heat in specialized tissue known as brown fat. We demonstrate temperature variation in a hibernating black bear on a macroscopic scale, without attempting to demonstrate local temperature variation. In this first glimpse of the physical processes underlying thermoregulation in a hibernating black bear, we have incorporated heat generation within a layer of brown fat. Our model indicates that brown fat tissue is capable of providing the energy need to maintain a high temperature. However, our model also points to the importance of the thick fur layer, as well as that of the fat layer, in providing basic insulation. At steady state, a temperature drop of over 40°C occurs in these two layers, keeping the body core at a temperature high above that of the surroundings. Without the insulation provided by these essential layers, along with thermogenesis in brown fat, it is unlikely that the bear would survive a 100-day hibernation cycle.

Introduction

The American Black Bear, *Ursus americanus*, can hibernate for as long as 100 days without eating, drinking, or urinating. During the winter hibernation, the core temperature of the bear drops only by 4-5 degrees Celsius. The bear maintains this high temperature by a large amount of fat oxidation in specialized tissue known as brown fat. Metabolism in these fat cells accounts for almost 100% of energy used by the bear for the whole duration of hibernation. This metabolism is facilitated by the membrane protein thermogenin, which converts the proton gradient across the inner mitochondrial membrane directly into heat, bypassing the normal ATP synthase route. (Maxwell, 1988)

That the bear can survive environmental temperatures of -20°C is certainly dependent on much more than the metabolism of fat. A complex set of anatomical and physiological properties allow for the maintenance of a high core temperature, ranging from the thick fur layer to the nature and placement of the brown fat layer. The basic characteristics of the heat flows between these layers, and from the bear to its hibernation den, are all important in allowing for the bear's survival.

The maintenance of a high temperature during hibernation may have many practical applications. For instance, inducing organs to hibernate may allow for better preservation and more successful organ transplant methods. An improved understanding of the heat flows in a hibernating black bear will give valuable insight into the key parameters governing its thermoregulation.

Design Objectives

We aim to model the temperature profile within a hibernating black bear during a 100-day hibernation cycle. In doing so, we:

- Derive a heat source term for a hibernating bear, incorporating Kleiber's Law for basal metabolic rates
- Demonstrate that a high temperature steady state can be reached quickly.
- Perform a similar derivation for a smaller, mouse-sized mammal, and show that smaller mammals cannot effectively maintain a high temperature using the same methods as the bear.

Problem Schematic and Assumptions

The bear is assumed to be an axisymmetric solid with four main layers: an outer fur layer, followed by layers of skin and fat, and an inner lean body mass core (Figure 1). Material properties within these layers are assumed constant and isotropic. This is a valid assumption for our macroscopic model, although it may not be appropriate for a higher level model that aims to observe local temperature profiles. The ambient temperature is assumed to be constant, and radiation to the environment is considered as the main heat sink. Heat transfer due to sensible and latent losses via is assumed negligible. This assumption is reasonable because of the highly depressed respiratory rate during hibernation. Heat loss due to convection is assumed negligible, as the den is assumed to be sealed. This assumption is valid because the bear den closes due to accumulation of snow at its entrance (Maxwell, 1988). Natural convective losses are minimal due to the small air space between the bear and cave walls. Conductive losses through the bottom surface of the bear may be significant, and would require a three-dimensional geometry to model.

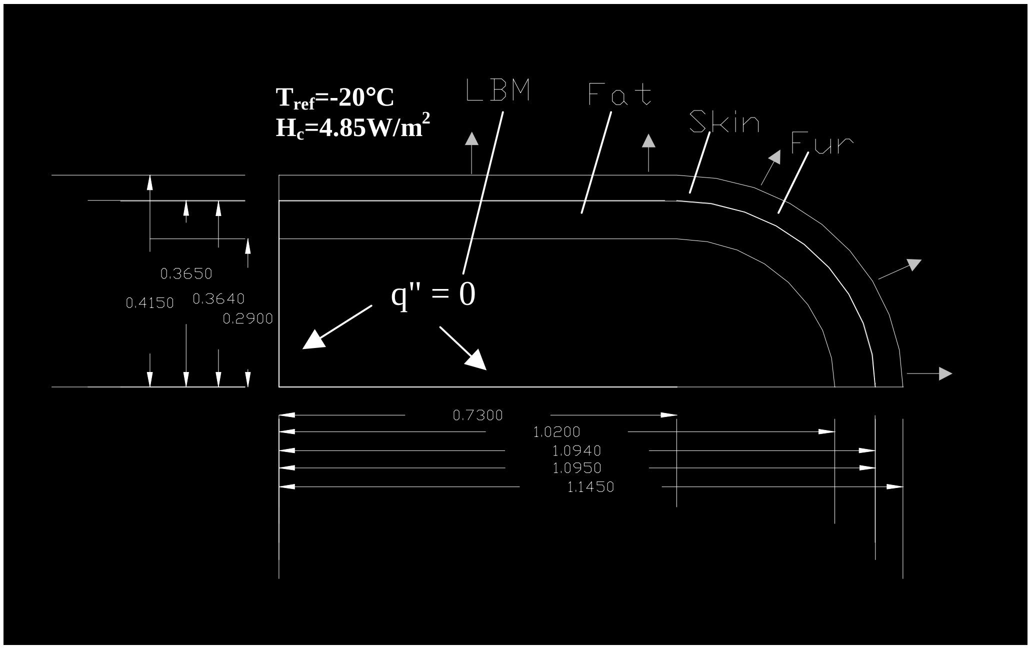

Figure 1: The hibernating mammal consists of an outer fur layer, followed by a layer of skin, fat, and the internal lean body mass. Heat generation occurs only within the fat layer, and radiative cooling occurs at the fur surface. Heat flux at the left and bottom surfaces is

Heat generation occurs mainly within the layer of brown fat, and is assumed to be constant and uniform. It is important to note that the bear has a complex biological thermoregulatory system that is active even during hibernation. This system will ensure heat generation within the brown fat layer that maintains a nearly constant body temperature. Given constant environmental conditions, as we have assumed, this source term should be nearly constant.

Heat generation elsewhere is assumed negligible. Again, the error due to this assumption is minimal because of the depressed metabolic state of the hibernating bear. There is a minimal amount of muscle contraction, and almost no heat transfer due to perfusion.

The geometry and mesh are created in GAMBIT v.1.3, and the solution is determined using the finite-element technique in FIDAP v.8.6. A more detailed problem description and mathematical formulation are provided in Appendices A and B.

Results

Figure 2 shows the temperature distributions in the various layers of the bear at the completion of the hibernation period (100 days). Layer boundaries have been superimposed on the contours.

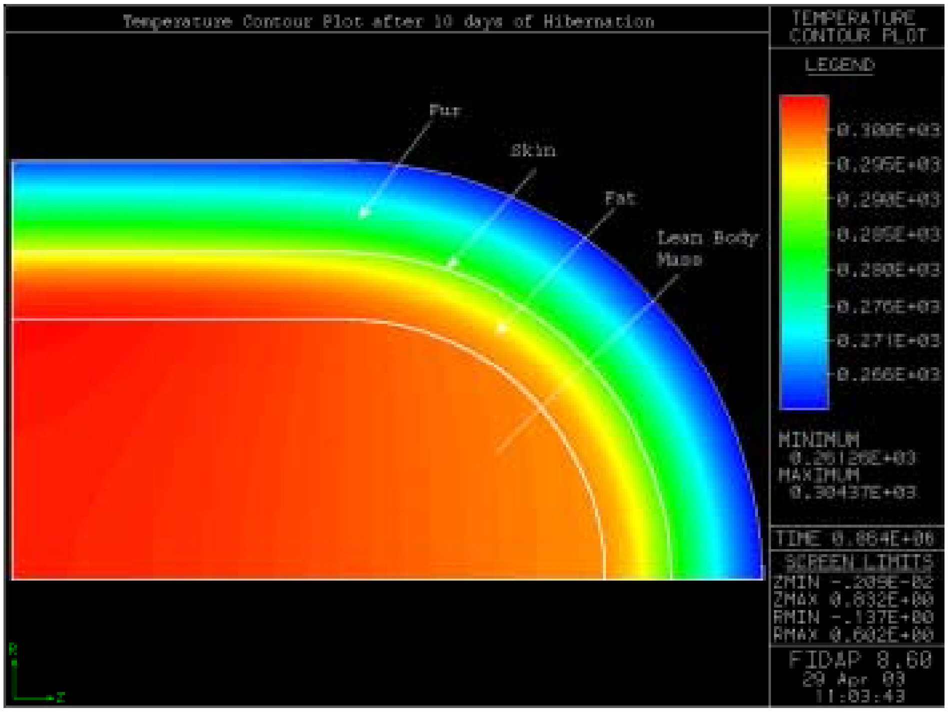

Figure 2. Temperature contour plot after 100 days of hibernation.

Interestingly, sharp changes in temperature do not correspond directly to layer boundaries. Additionally, the steady state temperature profile (Figure 2) demonstrates that the core of the bear (LBM) is at least 25°C throughout, with the temperature being warmer at the center. A history plot of the center node (node 117) shows that the center of the bear does not fall below 30°C.

The full 100 day simulation was unnecessary, as the temperature profile actually reaches steady state in just under a day. Temperature history plots for each layer are shown in Figure 3. The nodes chosen were representative nodes from the middle of each layer. The transient profile within each layer has the same time constant because the solution for each layer is coupled. A perturbation away from the steady state temperature within one layer will be reflected by perturbations in the profiles within each of the other layers. The maximum temperatures at each of these representative nodes, is given in Table 3.

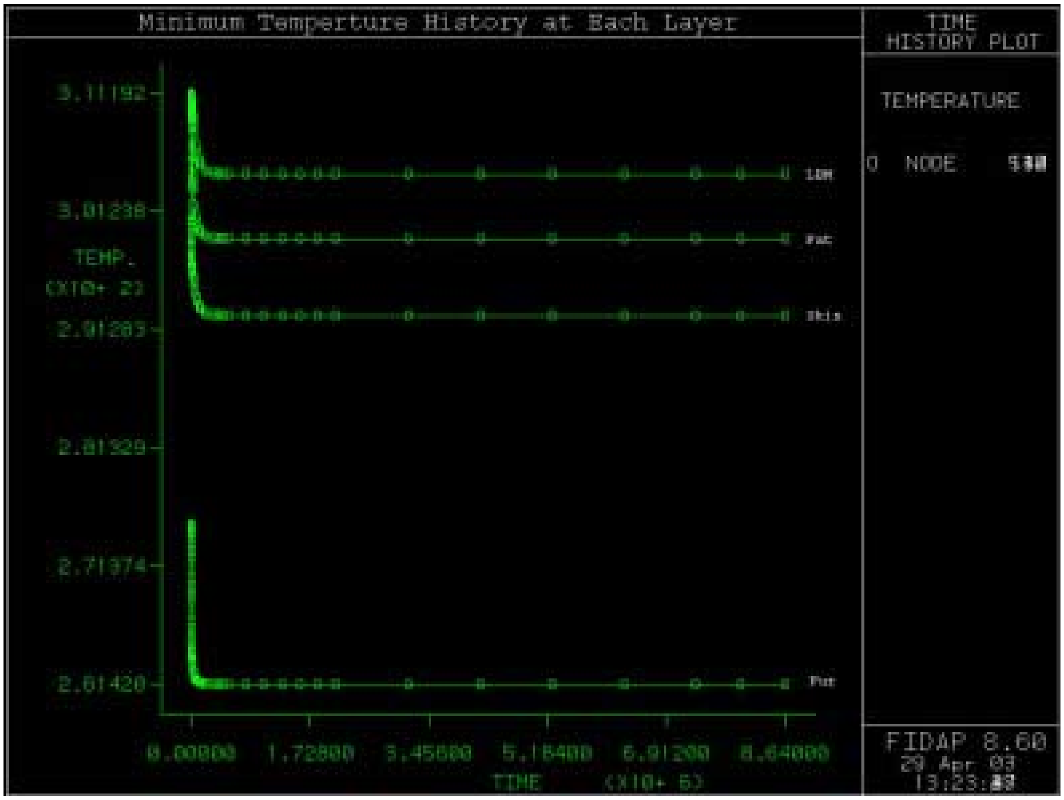

Figure 3. Temperature history plots for representative nodes of the respective layers (from top to bottom: lean body mass, fat, skin, and fur layers)

Table 3. Steady-state and maximum temperatures for representative layer nodes during the course of the 100-day hibernation period.

Layer	**Node**	**Steady State Temp (°C)**	**Max Temp (°C)**
LBM	117	29.706	36.927
Fat	179	29.048	37.951
Skin	387	17.97	26.69
Pelt	663	2.7	6.119

The relative insulative importance of each layer can be seen by calculating the maximum and minimum temperatures in each layer along a radial path from the center of the bear to the outside. These data were calculated and are given below in Table 4.

Table 4. Overall steady state temperature range within each layer.

	Minimum S.S. Temp.		**Max. S.S. Temp.**		
Layer	*Node*	*Steady State Temp(℃)*	*Node*	*Steady State Temp (℃)*	*Temp. drop (℃*
LBM	2	25.607	29	30.213	4.6
Fat	17	12.546	148	31.068	18.5
Skin	134	11.934	226	12.158	0.224
Pelt	546	-11.88	245	10.186	22.1

Sensitivity analysis:

For our sensitivity analysis we looked at changes in fat conductivity and fat density. In analyzing fat conductivity we increased and decreased the conductivity in order to see the effects on the center of body and fat steady state temperatures. Figure 4, shows temperature history plot for each value of conductivity at the center node. The shape of the curve is similar for each value of conductivity, however the maximum and steady state temperatures decrease as conductivity increases. As the value of conductivity increases, the steady state temperatures begin to converge.

From Table 5 we see that the temperature changes in both the fat and at the center of the body change similarly with a change in fat conductivity. Again in both the fat layer and at the center, as the fat conductivity increases there is much less of an effect on steady state temperature as they begin to converge to their respective temperatures at each layer.

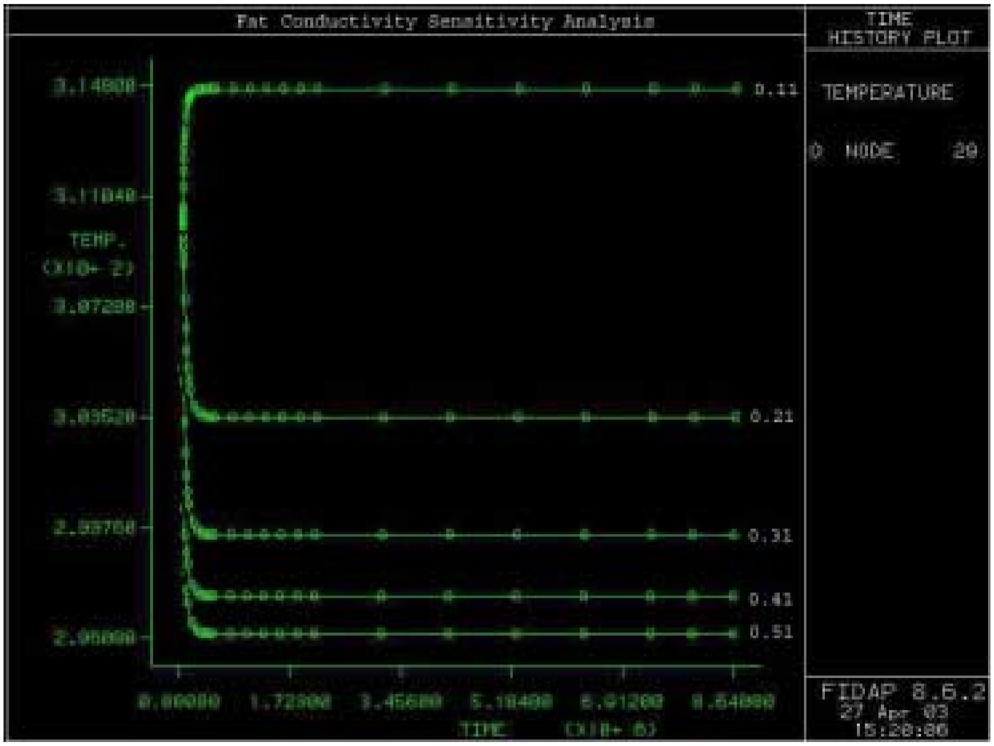

Figure 4: Sensitivity analysis for fat conductivity. Each plot represents a different value for fat conductivity, 0.11, 0.21, 0.31, 0.41 from bottom to top

Table 5: Temperatures at steady state from center of body node and node within the fat at different values of fat conductivity (final value in bold).

Fat Conductivity (W/m-K)	Center Steady-State (°C)	Fat Steady-State (°C)
0.11	41.488	30.584
0.21	**30.251**	**22.423**
0.31	26.228	19.183
0.41	24.137	18.103
0.51	22.842	17.23

We also performed a sensitivity analysis on fat density. In our problem, as the bear metabolizes fat, the volume of fat will decrease. In order to account for this in our model, we would need to remesh our geometry as the solution was created. To simplify this event we could view the problem as the fat decreasing in density as it was being metabolized over time. This still requires extra programming, so we ran a sensitivity analysis to see if this was necessary. As seen in Figure 5, in our model, there is little if no difference in maximum or steady state temperature at the center node. In reality there should be a change in temperature as the density of fat decreases, however we account our lack of change to the bear reaching equilibrium temperature quite rapidly. In this short period of time, the change in fat density has little or no effect. From this sensitivity analysis we were able to predict little effect in our solution if we were to incorporate a changing density to our problem.

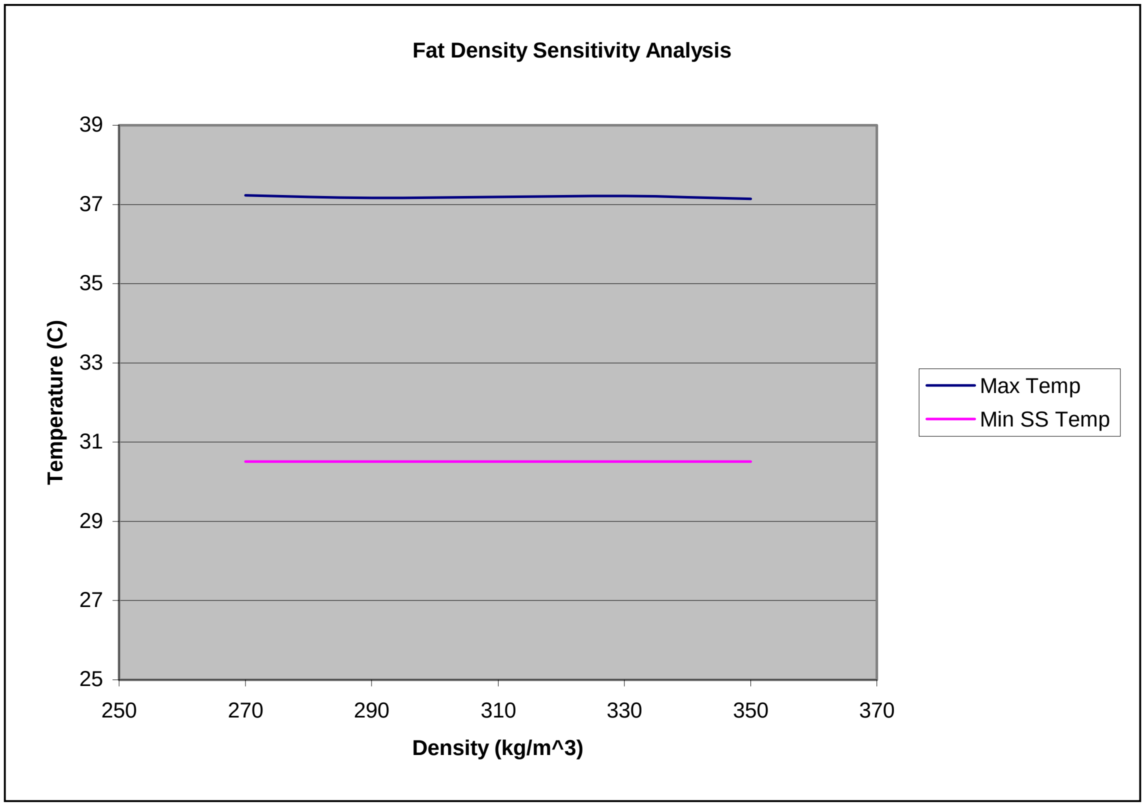

Figure 5: Temperature vs fat density at center of body for maximum and steady state temperature.

Conclusions and Discussion:

This model provides a first glimpse of the physical processes underlying thermoregulation in a hibernating black bear. The axisymmetric geometry, while a simplified model of an actual bear, does represent the key biological layers, incorporating necessary thermal properties. Our goal has only been to demonstrate temperature variation on a macroscopic scale, and we have made no attempt to demonstrate local temperature variation.

In simplifying the geometry, however, we have been especially careful in modeling tissue layers outside of the body core, i.e. fur, skin, and subcutaneous fat. It is in these regions that temperature changes are most drastic. In contrast, within the core lean body mass, we have lumped together properties of musculoskeletal and organ systems. A look at our results and sensitivity analysis demonstrates that property variation within this core has a negligible effect on body temperature results.

Along with our careful simplification of the model geometry, we have created a mesh that allows for minimal computation time without solution distortion. Alternative meshing schemes, as well as mesh refinement, have provided identical results while significantly increasing computation time. Our geometry and mesh has allowed for efficient analysis of the general effects of varying input parameters on body core temperature.

Our source term provides a reasonable first approximation of constant, uniform volumetric heat generation within the fat layer. It is important to note that the bear has a complex biological thermoregulatory system that is active even during hibernation. This system will ensure heat generation within the brown fat layer that maintains a nearly uniform body temperature. Given constant environmental conditions, as we have assumed, this source term should be nearly constant.

We have not modified the source term to incorporate heat fluxes through the vasculature, as done in the bio-heat equation. We have assumed that any convective heat transfer due to blood flow would be negligible, as blood flow rates in the hibernating bear are highly depressed from those measured in an active bear. In fact, flow rates are so slow that the assumptions that go into the bio-heat equation may not be valid.

We have found that the bear will have a core body temperature near 30°C with a volumetric heat generation term of $1050.369 W/m^3$. This value is completely reasonable if a macroscopic energy balance is used for the hibernating bear. We show this by computing a BMR based on Kleiber's Law, incorporating a 40% efficiency of energy conversion in the BMR. Adding energy needs to maintain the BMR along with that necessary to maintain a high body core temperature, our bear has a suitable amount of energy stores to survive the winter. In fact, field studies show weight losses of up to 45%, mostly fat, in hibernating bears, while our model demonstrates a decrease in body mass by ~40%, all in fat.

Some of the most valuable results concern the relative insulative importance of the various tissue layers. Our analysis demonstrates a dramatic temperature change in both the fat and fur layers, as would be expected. The steady state temperature profile shows temperatures below freezing within the fur layer, and temperatures as low as 12.5°C within the fat layer, while the minimum temperature of the body core is 26°C. This is an important result, graphically demonstrating how the bear is able to maintain a high core temperature in seemingly deadly temperatures. As expected, variation of thermal properties in these layers produces significant effects.

Another interesting observation is that a change ΔdT_e in environmental temp is mirrored by a shift in the entire steady state temperature profile of the bear, $\Delta T_{bear} = \Delta T_{environment}$. An energy balance will show that at steady state, the net heat flux due to radiation equals the net heat generation within the fat tissue. Radiative flux is a function of the temperature difference between surface temperature and outside temperature. Because the heat generation term is constant, the temperature difference must be constant. As a result, a change in the external temperature is mirrored by a shift in the bear temperature profile.

We have provided further insight from our mouse model, which essentially incorporates an identical problem formulation for a smaller, yet proportional, geometry. It is not accidental that the hibernating Black Bear, a large mammal that relies on its fat stores during the winter sleep, is able to withstand 100 days of sub-zero temperatures without freezing to death. Smaller mammals, such as various kinds of rodents, could not possibly maintain constant body temperatures for such prolonged periods without employing other methods of thermogenesis, such as shivering. Field studies have demonstrated that these smaller mammals do indeed shiver during hibernation, and, additionally, often do experience large drops in core body temperature. As Figure 3 clearly shows, a "mouse" would be unable to maintain a body temperature that would enable it to keep from freezing to death as the temperature dips to 254 K (-19°C) very rapidly.

Given the results from this simplified model, we have identified several areas for improvement in a higher-level model. The first major change would be to adopt a three-dimensional geometry. The main advantage of this model is that it would allow for greater manipulation of boundary conditions. For example, with our axisymmetric model, we have essentially assumed that the bear is suspended, surrounded by a uniform layer of air. In reality, one side of bear must be matted down due to contact with cave surface, and conduction through this surface would be significant. Fur loses much of its insulating properties because porosity is decreased when matted down.

Another major limitation of our model is that we have not incorporated any model for the changing geometry due to fat loss. As can be seen in the sensitivity analysis, changes in the fat layer thickness can have drastic effects. Because the bear loses a substantial fraction of its subcutaneous fat during the hibernation cycle, a more complex analysis should incorporate a model for the decreasing fat layer thickness. We have made some attempts to incorporate this by using a variable fat density model, but we ran into difficulties.

Various other minor changes to our model could be to incorporate changes in environmental conditions, and variables such as the size of the bear, percentage fat, etc. Having provided this first generation model, however, all such modifications would be fairly simple.

Appendix A: Problem Definition

Schematic

An axisymmetric model is used, with the bear modeled as a cylinder with hemispherical caps on either end. Figure 6, below, shows a schematic of the bear, with boundary conditions and proportions indicated.

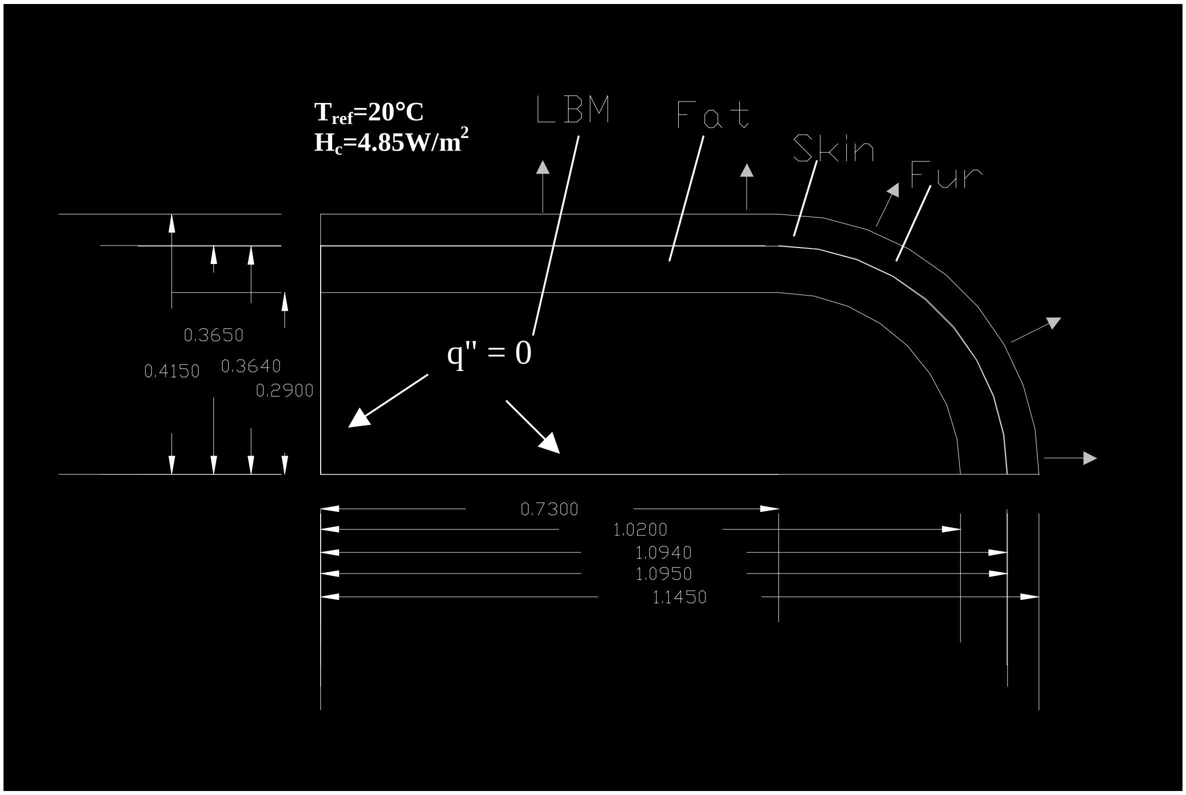

Figure 6: The hibernating mammal consists of an outer fur layer, followed by a layer of skin, fat, and the internal lean body mass. Heat generation occurs only within the fat layer, and radiative cooling occurs at the fur surface. Heat flux at the left and bottom surfaces is set to zero, due to symmetry.

Along with a bear model, a small mammal model, which will henceforth be called ‘the mouse model,’ has also been developed. Both models utilize the geometry shown in Figure 6. Table 6, below, gives the linear dimensions, masses, and fat volumes of both the bear model and the mouse model. The bear model is scaled down by a factor of 1/5 to provide the mouse model.

Table 6: Linear dimensions, total body mass, fat mass, and fat volume of the bear and mouse models.

Dimensional Quantity	***Bear***	***Mouse***
Lean Body Mass Layer, hemisphere and cylinder radius	.29m	.058m
Fat Layer, hemisphere and cylinder radius	.364m	0728m
Skin Layer, hemisphere and cylinder radius	.365m	.073m
Pelt Layer, hemisphere and cylinder radius	.415m	.083m

Body Cylinder length	.73m	.146m
Fat Mass	65.363kg	.523kg
Fat Vol	.211m^3	.00169m^3
Total Body Mass (Fat+Lean)	160.499kg	1.284kg

Mathematical Formulation

A transient, linear form, of the heat equation was used (Eq. 1) with a source term included for the fat layer (Eq. 2):

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right), \qquad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right)+Q\text{'''} \qquad (2)$$

where ρ is the density of a given layer, c_p is the layer specific heat, k is the layer thermal conductivity, T is the layer temperature and Q''' is the volumetric heat generation (fat layer only).

The source term was derived by assuming that the bear would lose 40% of its body mass, all in fat, due to brown fat metabolism. This is reasonable assumption, as bears typically lose up to 45% of their body weight, primarily in fat, during the hibernation period (Biel, 1996). Assuming a constant rate of metabolism, the total rate of energy metabolized within the brown fat layer is:

$$E' = a*(FAT)/t \qquad (3),$$

where E' is the rate of energy metabolism, a is a proportionality constant for the conversion of fat to energy, FAT is the initial amount of fat in the bear, and t is the elapsed time in the hibernation cycle (100 days). However, some of the energy metabolized must go towards maintenance, and can be computed using Kleiber's Law for the Basal Metabolic Rate of the bear. Kleiber's Law describes the relationship between body weight and basal metabolic rate (BMR) as:

$$BMR = 3.546227(BM)^{3/4} \qquad (4),$$

where BMR is reported in joules per second and BM is the total bear body mass in kilograms. However, because biochemical pathways are not 100% efficient, some of the basal metabolic energy is also released as heat. For the purposes of this analysis, we will assume a biochemical efficiency of 40% (60% of BMR acts as a heat source).

Energy that does not go towards maintenance (basal metabolic rate) is released as heat by the bear. This gives a total heat source term for the bear,

$$Q = 4.35FAT - 0.4*(3.546227BM^{3/4}) \qquad (5),$$

where Q is the total amount of heat produced by the bear in Joules/second. Almost all heat produced by the bear is produced in the brown fat layer. Dividing the total heat produced by the volume of fat cells in the bear gives a final volumetric source term:

$$Q''' = \frac{4.35FAT - 1.498491BM^{3/4}}{V_F} \qquad (6),$$

where Q''' is the volumetric source term for the bear in Joules per second per cubic meter, and V_F is the volume of the fat layer in cubic meters.

All properties have been assumed constant for the range of temperatures the bear and mouse experience during the hibernation period.. Pelt layer properties were approximated using the properties of stagnant air with an increased conductivity, density and specific heat due to the presence of hairs. All other properties come from the sources cited in Table 7, below.

Table 7: Property values for the mouse and bear pelt, skin, fat and lean body mass (Ng, 2001). The volumetric heating term in both the fat layer of the bear and mouse are calculated by equation above.

Tissue	Thermal Conductivity W/m-K	Specific Heat J/kg-K	Density kg/m_3
Pelt	0.209	500	250
Skin	0.22	3660	1100
Fat	0.22	975	310
Lean Body Mass	0.5	975	310
Source Term-Bear	1050.369W/m^3		
Source Term-Mouse	351.660W/m^3		

Boundary conditions are limited by the use of an axi-symmetric geometry. During an actual hibernation period, a bear will experience temperature loss due to conductive losses to a poorly insulated cave, convective losses through small wind currents, radiative losses, and evaporative cooling due to moisture loss in respiration. It has been assumed that most air surrounding the bear is stagnant, thus convection is neglected. This is a reasonable assumption, given that the cave is usually isolated from the external environment due to snow accumulation at the entrance. Though natural convection would still be present, this also should be minimal due to small air space in the cave. The left and bottom surfaces in our schematic have a heat flux set to zero, due to symmetry.

$$q'' = 0 \qquad (7),$$

Conduction has been neglected in favor of a radiative boundary condition on all sides. To model radiation, a convection analogue has been utilized in the following manner:

$$\sigma\varepsilon\left(T_{pelt}^{\ 4} - T_{outer}^{4}\right) = h_r\left(T_{pelt} - T_{outer}\right) \qquad (8),$$

where T_{pelt} is the outer pelt temperature (at the boundary), T_{outer} is the outer cave temperature, σ is the boltzmann constant, ε is the pelt emmissivity, and h_r is the radiation coefficient modeled using convection (HTRANSFER in FIDAP). For a pelt emmissivity of .95, an outer temperature of -20°C and an average pelt temperature of 2°C, h_r is roughly equal to 4.85W/m^2.

The lean body mass, fat, and skin are all given an initial temperature of 37°C, which is a typical internal temperature for an active bear prior to hibernation. The pelt has an initial temperature of 275K, which roughly half way between the initial core temperature and the -20°C cave walls. While a discontinuous initial temperature distribution is a physical non-reality, it does provide a good approximation for a bear in a pre-hibernating stage surrounded by an atmospheric temperature of $-20°C$.

Appendix B: GAMBIT and FIDAP

The mesh was generated using triangular elements in GAMBIT with the geometry specified above, and finite element analysis was implemented using FIDAP v.8.6 (Fluent, Inc.).

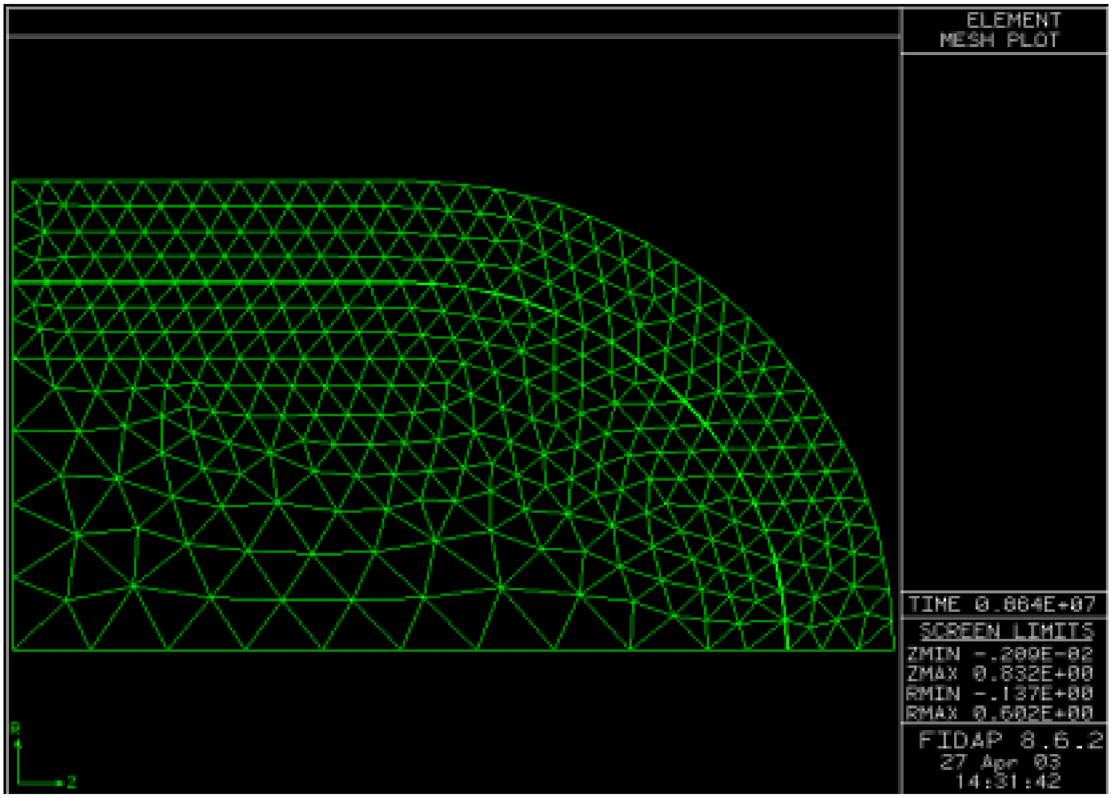

Figure 7. Optimized mesh showing larger element size toward the center (1147 three-node triangles; computing time: 20 sec.)

Mesh Refinement

In order to avoid excessive computing time, we conducted an element-size study. While holding the

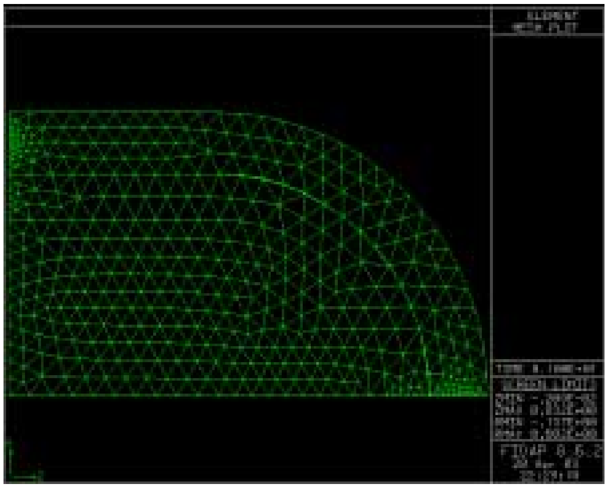

Figure 8. Original mesh composed of 1012 three-node triangles (838 faces; computing time: 20 sec.)

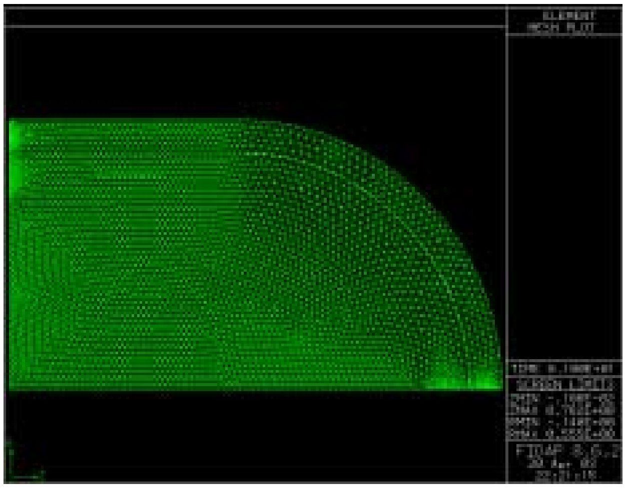

Figure 9. Refined mesh showing a 975% increase in the number of elements (9863 elements; 9291 faces; computing time: 90 sec.)

geometry and all properties and input parameters constant, we subjected our original mesh (see Fig. 8) to an h-type refinement, increasing the element density by a factor of approximately 9.75. Figure 9 shows the post-refinement mesh.

Upon examination of the temperature profiles within the hibernating Black Bear produced by the coarse and fine mesh, respectively, we found a 4.5-fold increase in computation time with a negligible difference in the solution. The finer mesh provides no additional accuracy and decreases the time efficiency significantly. In addition, to make minimize computing time further, we increased the element size in the center where temperature does not change much. Our final mesh is shown in Fig 7.

FIDAP Problem Statement

>PROB (AXI-, INCO, TRAN, LAMI, LINE, NEWT, NOMO, ENER, FIXE, NOST,
>NORE, SING)

Geometry type	Axisymmetric	The bear is modeled as a cylindrical object with spherical caps, with an axis of rotational symmetry.
Flow regime	Incompressible	No fluids in this problem (incompressible is default)
Simulation type	Transient	We solve for temperature as a function of time.
Flow type	Laminar	No flow in this problem (laminar is default)
Convective term	Linear	No convection was considered in the problem
Fluid type	Newtonian	No fluids in this problem (Newtonian is default)
Momentum equation	No momentum	No flow in this problem.
Temperature dependence	Energy	We are modeling the temperature profiles in the bear, as dependent upon the energy equation.
Surface type	Fixed	All surfaces are fixed (geometry of the bear does not change in our model)
Structural solver	No structural	No structural solver was used in the model
Elasticity remeshing	No remeshing	FIDAP did not remesh the geometry during the problem solution
Number of phases	Single phase	Only one phase modeled in this problem

Solution statement:

```
>SOLU (S.S. = 10, ACCF = 0.000000000000E+00)
```

Solution method	Successive substitution = 10	The solution used successive substitution iterations to solve the problem, with a maximum of 10 iterations for any step to attain convergence
Relaxation factor	ACCF = 0	The relaxation factor at any time was set at 0.

Time Integration:

```
> TIME (BACK, NSTE = 86400, TSTA = 0.000000000000E+00, TEND = 8640000.0,
> DT = 1.0, VARI = 0.1, NOFI = 2, INCM = 4.0)
```

Time integration	Backward	The backwards Euler method (implicit) is used in the simulation
No. time steps	Nsteps = 86400	A maximum of 86400 time steps is set
Starting time	Tstart = 0	Begins simulation at t=0
Ending time	Tend = 8640000	Ends simulation at t=864000
Time increment	dt = 1.0	The time increment is set at 1.0
Time stepping algorithm	Variable = 0.01	The time steps varied, and were determined by adherence to a tolerance level of 0.01 to the truncation errors. (0.01 is set as the default value)
No. fixed steps	Nofixed = 2	The number of fixed time increment steps at the beginning of the simulation is set to 2.
Max Increase Factor	Incmax = 4.0	The maximum factor by which successive time steps could be increased is 4.

Appendix C: References

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