PERFORMANCE ANALYSIS OF CONTRACT FOR OPTIONS

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In this dissertation I analyze performance of the contract for options in various settings.

In the second chapter, I consider a contract for options between a supplier and a manufacturer in the presence of a spot market with uncertain spot price, limited supplier capacity, and where the manufacturer must fulfill the stochastic demand of a downstream supply-chain link in full. I model the contract negotiation as a two-stage Stackelberg game in which the supplier is the leader. I derive a closed-form expression for the optimal number of options that the manufacturer should purchase, and show the (unrestrictive) conditions under which the supplier's profit is unimodal in the reservation and exercise prices. I make observations based on analytical results and numerical experimentation to assess when such a contract is incentive compatible for the players and effective in coordinating the channel.

In the third chapter, I analyze different mechanisms that lead to channel coordination. Specifically, I show channel coordination is achieved by a contract for options when the manufacturer is the leader, when a quantity discount contract is used, and when renegotiation is allowed. I demonstrate how different coordinating mechanisms affect the allocation of the profits between the supplier and the manufacturer and give some insight on when each mechanism might be appropriate. I highlight the desirability of renegotiation as a coordinating mechanism by show-

ing that it is robust – coordination is achieved despite information asymmetry – and leads to a more equitable sharing of the contract benefits than do the other mechanisms.

In the forth chapter, I evaluate capacity investment decisions of the players in the supply chain consisting of a supplier and two identical manufacturers. I compare the performance of the linear-price contract (when the supplier must use an allocation mechanism) with that of the contract for options. I demonstrate that when the supplier sets transfer prices, the contract for options performs only slightly better than the linear-price contract, which implies that the contract for options is not always an obvious choice over the linear-price contract.

BIOGRAPHICAL SKETCH

Natalia Golovachkina was born Natalia Vyuchnova in Volgograd, Russia in 1971. After a fairly typical childhood and adolescence in Russia, she started her studies in Physics in Moscow State University where she remained from 1988 to 1995. After receiving her Master's Degree in Physics, she joined the PhD program in Moscow Institute of Physical Chemistry where she remained until 1998.

In August 1999, she joined the PhD program in Management (Operations Management) at Cornell's Johnson School. She was awarded a Master of Science Degree in August 2003.

to Anastassia

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Chapter 1

Introduction

Recently, significant research has been done on the analysis of different contracts and their ability to improve and coordinate supply chain performance. It has been shown that when the transfer prices and order quantities are set properly contracts can significantly benefit the supply chain and its members. Some contracts can also let all participating parties equally share benefits and risks. In fact, many real-world companies affected by widespread globalization, unstable exchange rates, highly volatile spot market prices, and unpredictable demand fluctuations turn to contracts in order to mitigate any negative effects on the firm's performance.

One of the contracts commonly used in practice is a contract for options. In this contract, the buyer has an opportunity but not an obligation to buy a certain amount of product when he needs it. To do that, he must first reserve a certain amount of capacity at specified reservation price. When the actual demand comes, the buyer can buy product at a specified exercise price up to the amount of reserved capacity. The leader (can be either the supplier or her customer) chooses prices: reservation price and exercise price, and the follower chooses the reservation quantity.

This dissertation offers some insights on this topic. In particular, I show that the contract for options can provide significant benefits to the systems and their suppliers and manufacturers. I analyze the performance of this contract in different supply chains and for different parameters of the system.

The first two chapters of this dissertation are devoted to the analysis of contractual agreements between a single supplier and a single manufacturer in the presence of a spot market. In the first chapter, I consider a contract for options when the supplier is the leader. I show how the optimal contract parameters should be chosen. I evaluate incentive compatibility of this contract for the players and its efficiency in the supply chain. I demonstrate that while the contract for options can significantly improve supply chain performance it does not coordinate the system. I show that the contract for options is the most beneficial to the supplier and the manufacturer when both the spot market margin and the demand variability are high. At the same time, however, the whole supply chain benefits from this contract the most when the spot market margin is high and either demand or spot market price variability is low. I show that the manufacturer would always prefer a competitive supplier (one who can sell profitably to the spot market) with sufficient capacity. Only a supplier who satisfies these requirements is in a position to offer adequate contract terms.

The second chapter is devoted to the analysis of various mechanisms that lead to the channel coordination. I show that supply chain coordination can be achieved with a contract for options when the manufacturer is the leader, with a quantity-discount contract and with the contract for options when players can renegotiate contract parameters at the later stage. The two former coordinating mechanisms always give all benefits generated by the contract to the leader. A quantity-discount contract might also require an additional transfer payment that is positive only when the contracted quantity is equal to the centralized production quantity. In contrast, renegotiation allows the players to share the benefits and does not require any additional transfer payments. It turns out that with renegotiation the supplier always sets her production quantity equal to the centralized production quantity. Coordination in this case is achieved even if the manufacturer sets her reservation quantity below the centralized level. I emphasize the effectiveness of renegotiation in this case and show that it also leads to supply chain coordination even when the players have asymmetric information about demand.

In the third chapter, I consider a supply chain consisting of a single supplier and two manufacturers. All players must make their capacity investment decision prior to the demand realization. I analyze how contractual arrangements as well as allocation rules affect the players' decisions. I show that when a linear-price contract is used, the players choose the same capacity level with either proportional or linear allocation rule. I also show that the players do not change their capacity investment even if they plan to manipulate their order at the later stage. I compare the contract for options with the linear-price contract and evaluate their relative performance in this supply chain. I show that the benefits produced by the contract for options are not significantly greater than the ones produced by the linear-price contract, but that is only the case when the supplier sets transfer prices. Hence, for this particular supply chain a contract that is easier to implement should be used. Which contract is easiest to implement likely depends on the circumstances. A contract for options has the advantage that it does not require an allocation rule. It does, however, have the disadvantage that it sometimes requires that the supplier be charged a penalty cost to guarantee that it builds sufficient capacity so it can supply the reserved order quantity.

Overall, I conclude that the contract for options significantly improves the supply chain's performance, but does not coordinate the system. However, when some additional mechanisms are added to this contract, it can lead to channel coordination. Because the contract for options allows the supplier and the manufacturers to share the risks, it can have a significant practical value. However, for certain supply chains the contract for options does not produce significantly higher benefits than the linear-price contract so as to justify its administration for that sole reason. Thus, one might want to take into account other aspects of the system before making a decision about contract structure.

Chapter 2

Supplier-Manufacturer Relationships Under Forced Compliance Contracts

2.1 Introduction

Today, many retailers are adopting lean strategies to improve efficiency including Wal-Mart, The Limited, J. C. Penney, Sears Roebuck and Co. The 1993 Standard & Poor's retailing report emphasized that keeping shelves reliably stocked to minimize lost sales and "efficient warehousing, transportation, and delivery systems are among the elements of successful retailing" (see Abernathy et al. (1994)).

Manufacturers that supply lean retailers must fulfill orders accurately, rapidly, and efficiently, despite demand volatility, by appropriately structuring their production and transportation processes. The outsourcing of transportation has become common; in this case a contract might be struck that specifies how much capacity a logistics provider guarantees to the manufacturer. Given that the demand for transportation services varies day by day in the "lean retailer" context, the contract must take into account the manufacturer's risk of not fulfilling the retailer's demand if the transportation requirements exceed the agreed upon capacity and the logistics provider's risk of not using all of the committed capacity. The use of electronic spot markets for transportation has become prevalent and offers one means to mitigate these risks — the transportation provider can sell unused capacity while the manufacturer can secure additional transportation services when the logistics provider's promised capacity is insufficient. In this paper we analyze a contract in this context and its capability to mitigate the effects of demand and spot price uncertainties, as well as how each party, and the supply chain in total, benefits from the contract.

We analyze a contract for options for non-storable products or services, such as transportation, between a single supplier and a single manufacturer in the presence of a spot market, where the supplier has limited capacity and the manufacturer must fulfill stochastic demand from a downstream supply chain link. For clarity, we focus on the situation where the manufacturer must fulfill downstream demand in full, but we show later that this assumption can be relaxed.

The manufacturer minimizes her cost while negotiating supply from two sources: contracting with a supplier for options, and a spot market. The supplier maximizes profit by appropriately setting the production quantity, reservation price per option sold, and an exercise price per unit delivered. We assume that the quantity of goods desired by the manufacturer is always available on the spot market at some price, that the spot market price is exogenous, and that neither the supplier nor the buyer is of sufficient size to have a perceptible effect on it.

Modeling the presence of the spot market in conjunction with a contract for options is an important extension of the literature. Spot markets have not often been included in supply chain contracting models, although many commodities are traded on spot markets, that are used by suppliers to clear excess production and by customers for supply when contracted quantities fall short of requirements.

We show that the manufacturer's cost function is convex and provide a closed-form expression for the optimal reservation quantity. We provide nonrestrictive conditions under which the supplier's profit function is unimodal in the forward and exercise prices, and show analytical expressions for the optimal contract terms.

In our numerical tests, the supplier's (leader's) profit increased by a greater percentage than was the manufacturer's cost reduced whenever the contract was practically incentive compatible. We demonstrate, using the centralized case as a benchmark, that even though the total coordination of the channel with a contract for options is impossible, the supply chain performance can come very close to that of the centralized system performance. The contract helps to decrease the difference in profit between centralized and decentralized systems by as much as 99%, in the best case, and at least 36%, in the worst case, for the parameters that we tested. We found that the contract performs best when the margin paid in the spot market is high and either demand or spot market price variability is low.

In Section 2.2 we provide a literature review on spot markets and supply chain contracting for options. In Section 2.3 we describe our model. In Section 2.4 we formulate our model, give an analytical solution to the manufacturer's problem and demonstrate that the objective function in the supplier's problem is unimodal in forward price and in exercise price. Section 2.5 gives the main results for the centralized supply chain and decentralized supply chain without a contract. In Section 2.6 we provide results of numerical examples and comparison of the contract performance for different parameters of the system. Section 2.7 gives extensions to this problem. We present our conclusions in Section 2.8.

2.2 Literature Review

When no commitment is attached to the buyer's order the buyer tends to inflate her forecast and leave the supplier with excessive inventory. As a result, one of the questions addressed in the supply chain contract literature is how the supplier can provide the buyer with sufficient flexibility while not assuming all the risk due to demand uncertainty.

There is an extensive literature dealing with such risk sharing. Eppen and Iyer (1997) focus on "backup agreements". The buyer commits to y units, takes immediate delivery of $(1-\rho)y$ units and then, after observing sales data for the first two weeks, updates her information and decides how many of the remaining ρy units she will procure. There is a penalty cost b for each unit ordered but not actually purchased. The paper reports that a backup agreement can significantly improve the system performance for certain combinations of b and ρ.

Quantity flexibility (QF) contracts are another way to provide the buyer with some flexibility. Tsay (1999) analyzes a QF contract in which the buyer can adjust her initial order by a certain percentage. The author shows a closed-form mapping between transfer price and flexibility parameters that allows the system to perform optimally. Bassok and Anupindi (1998) consider a T-period horizon where the buyer originally forecasts her purchases and then can adjust each period's purchase one time within specified percentage bounds. The problem is very complex, and so the authors ultimately propose a heuristic policy. Milner and Rosenblatt (2002) analyze a model in which the buyer places orders for two periods and, after observing demand in the first period, may adjust her second order for which there is per-unit adjustment fee. The authors characterize the optimal adjustment strategy, but show no closed-form solution for the initial order.

Several papers analyze whether the contract for options can coordinate the channel and ensure incentive compatibility for both players, or whether some additional mechanisms are needed. Barnes-Schuster et al. (2002) investigate a two-period model with a single supplier acting as a leader and a single buyer, in which the buyer can purchase a certain number of options as well as place a firm order each period. Cachon and Lariviere (2001) consider a similar model (where the buyer acts as a leader), and analyze two different compliance regimes (voluntary compliance and forced compliance) and two different information sharing cases (full information and asymmetric information). The authors show that the contract compliance regime significantly affects the outcome of the game.

To our observation, although spot markets exist for many products and affect the decisions made by suppliers and buyers of such goods, the literature dealing with a spot market as an additional source of procurement or sale of products is not extensive. Araman et al. (2001) analyze a model with a spot market in which the supplier's predetermined pricing scheme depends on both the reserved capacity level and the amount purchased. There is no forward price in this model, and the spot market price is a function of the demand. In this model, unlike ours, the buyer must always purchase first from the supplier up to the reserved capacity, and only then can go to the spot market to satisfy the remaining demand. The authors numerically compute the optimal mix between these two channels.

Among the papers we found, Wu et al. (2002) and Spinler et al. (2002) are the closest to ours. The setting in Wu et al. is similar to ours: it includes options on the capacity, a Stackelberg game in which the supplier is a leader, and a spot market. These authors assume that the demand is deterministic and is a function of the spot price which is, in turn, state contingent. Spinler et al. analyze a model that is very similar to that in Wu et al., but with an additional assumption that both the demand and the production cost are state contingent. In contrast to these papers, we assume stochastic demand that is independent of the spot price. Also, neither Wu et al., nor Spinler et al., require the buyer to satisfy all demand as we do in this paper, but rather determine the quantity that maximizes the indirect utility of the buyer. As a consequence, some results that we present here are different. For example, we show that the exercise price is set at the level such that the probability that the spot price would exceed the exercise price is very small; Wu et al. conclude that the exercise price will be equal to the production cost. In addition, we assess contract effectiveness, and analyze incentive compatibility for the supplier and the manufacturer.

By analyzing a contract for options in a spot market setting, we simultaneously extend two streams of literature — we apply a contract for options to a new setting and assess the effect of spot markets under a new contractual setting. We also add to the research on spot markets by developing a model that is appropriate when the buyer (the manufacturer in our model) supplies a lean customer with varying demand and is able to gauge the effect of stochastic demand.

2.3 A Description of the Model

We consider a two-stage problem with a single supplier and single manufacturer. The supplier has a limited capacity of K units, and incurs a cost of b per each unit that he makes available. (In our transportation example b can be viewed as cost of keeping trucks in a working condition ready to serve the customers.) For a complete list of notation see Table 2.1.

The manufacturer faces a random demand D with cumulative distribution function $F(.)$, p.d.f. $f(.)$ and expected value $\overline{D}$. We assume that the manufacturer must always satisfy the demand in full.

We assume that the quantity of goods desired by the manufacturer is always available on the spot market at some price P_S with cumulative distribution $H(.)$, p.d.f. $h(.)$ and expected value $\overline{P_S}$. Given the structure of the problem, we assume that $D \geq 0$ and $P_S \geq 0$. The analysis presented in this paper can be easily

Table 2.1: Notation

K	-	supplier's capacity
b	-	production cost per unit
m	-	spot market margin per unit
s	-	reservation price per option
g	-	exercise price per option
Q	-	number of options purchased by manufacturer
q	-	number of options exercised
q_S	-	number of units purchased on the spot market
Q_P	-	supplier's production quantity
P_S	-	spot market price
D	-	demand random variable
$F(.)$	-	demand c.d.f.
$f(.)$	-	demand p.d.f.
$H(.)$	-	spot price c.d.f.
$h(.)$	-	spot price p.d.f.
$\widetilde{P_S}$	-	realization of the spot price variable
$\widetilde{D}$	-	realization of the demand variable
$\overline{P_S}$	-	expected spot market price
$\overline{D}$	-	expected demand

extended for the case when either distribution has finite support. The distributions of demand and spot market price are common knowledge for the buyer and the manufacturer, and are independent of one another. Practically this means that neither supplier nor buyer has any perceptible effect on the spot price because they represent just a small fraction of the whole market. We also assume that when the supplier sells to the spot market he must pay a fixed margin m per unit for clearing the market. We assume this simple form for the spot market margin for clarity of exposition although we show later that our results hold for more general forms.

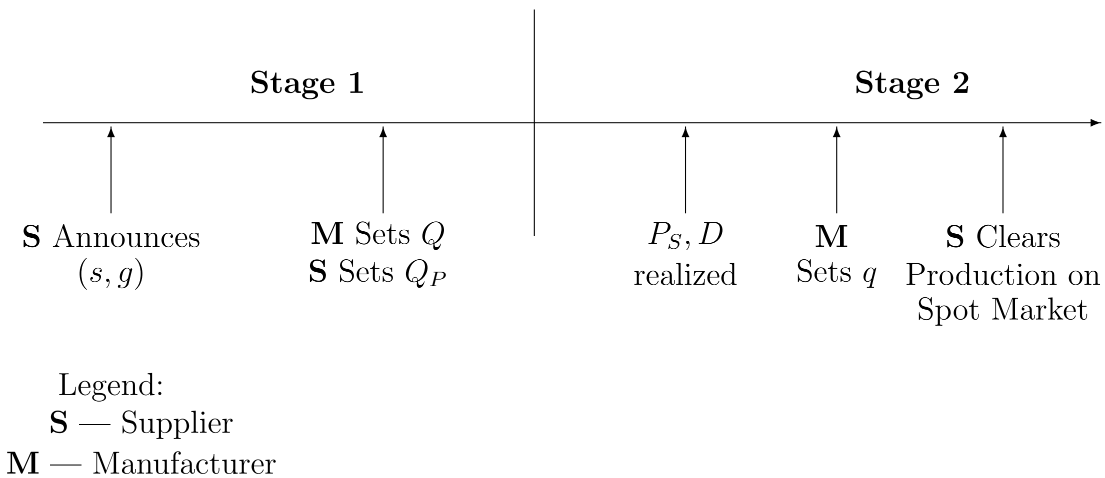

Figure 2.1: Event Timeline

We model our problem as a two-stage Stackelberg game in which the supplier is a leader (See Figure 3.1.) At stage 1, the supplier offers the manufacturer a contract with parameters s and g, where s is a price per unit of capacity reserved and g is a price per unit of the product actually purchased at stage 2. We define $P = s+g$ as the total price that the manufacturer pays for each unit delivered from the supplier. Given s and g, the manufacturer reserves a certain amount of capacity Q ($Q \leq K$) from the supplier. The supplier subsequently determines a production quantity, Q_P, which we require to be at least as great as the quantity reserved by the manufacturer, $Q_P \geq Q$. This constraint reflects the contractual requirement for the supplier to deliver up to Q units if called upon by the manufacturer and the circumstance when the supplier produces the goods rather than resells goods that he procures on the spot market. A supplier would do so when either it focuses on producing a good or a service rather than on being a broker or because the supplier is unwilling to risk its reputation selling goods that are produced by another firm and that may be inferior.

At the beginning of the stage 2, D and P_S are realized. After observing this information, the manufacturer decides how much to order from the supplier, q, and how much to buy on the spot market, q_S. The manufacturer can view the spot market as an alternative source of the product: If the spot market price is below the supplier's exercise price g, then the manufacturer buys only from the spot market; otherwise, she buys from the spot market only if the reserved capacity is insufficient to satisfy the demand in full. After the manufacturer's order is filled, we assume that the supplier can sell all his excess inventory to the spot market at some price, which may or may not be profitable.

2.4 Analysis

We solve this problem using backward induction, assuming that the decision-maker at each step acts optimally and anticipates optimal behavior from other decision-makers in each subsequent step.

2.4.1 Stage 2 Manufacturer's Problem

We start with the stage 2 manufacturer's problem. At stage 2 both the spot market price and the demand are known, and the manufacturer must decide how many options to exercise and how many units to buy on the spot market. She must solve a deterministic linear problem:

$$\textbf{(M2)} \qquad \min_{q,q_S} C_2(q, q_S) = gq + \widetilde{P_S} q_S$$

$$\text{subject to: } Q \geq q \geq 0, q_S \geq 0,$$

$$q + q_S = \widetilde{D},$$

where $\widetilde{D}$ and $\widetilde{P_S}$ are realizations of the corresponding random variables. The first inequality states that the manufacturer can order from the supplier no more than the number of options purchased, and the second inequality specifies that the manufacturer must satisfy the demand in full. The proof for the next proposition is straightforward.

Proposition 1 *The solution to the Problem M2 is given by:*

$$(q^*, q_S^*) = \begin{cases} (0, \widetilde{D}) & \text{for } g > P_S, \\ (min(Q, \widetilde{D}), (\widetilde{D} - q)) & \text{for } g \leq P_S. \end{cases}$$

2.4.2 Stage 1 Manufacturer's Problem

At stage 1 the manufacturer must decide how much capacity to reserve from the supplier. Her goal is to minimize the expected cost she incurs from using both the supplier and the spot market. The manufacturer solves the problem:

$$\textbf{(M1)} \qquad \min_{Q} C(Q) = E_{D,P_S}[P_S(D - q^*) + sQ + gq^*]$$

$$\text{subject to: } 0 \leq Q \leq K,$$

where q^* is the solution of Problem M2. The first term in the objective function is the cost associated with buying from the spot market, the second term is the cost of reserving capacity, and the third term is the cost of purchasing product from the supplier. The constraint indicates that the manufacturer cannot reserve more capacity than the supplier has available.

This is a newsvendor problem, which allows the following result. For convenience, we define the loss function

$$L_H(g) = \int_g^\infty (y - g) dH(y).$$

Proposition 2 *The solution to M1 is given by* $Q = min(Q^*, K)$, *where*

$$Q^*(s,g) = \begin{cases} F^{-1}\left(\frac{L_H(g)-s}{L_H(g)}\right) & if\ \frac{s}{L_H(g)} \leq 1, \\ 0 & otherwise \end{cases}. \tag{2.1}$$

Proof. After some elementary operations, the objective function in the problem M1 can be rewritten as

$$\begin{aligned} C(Q) = &\ \overline{P_S} \cdot \overline{D} + sQ \\ &- L_H(g)\left(\int_0^Q Df(D)dD + Q\int_Q^\infty f(D)dD\right), \end{aligned}$$

the first derivative of which is

$$\frac{\partial C}{\partial Q} = s - L_H(g)(1 - F(Q)).$$

The second derivative is

$$\frac{\partial^2 C}{\partial Q^2} = f(Q)L_H(g) > 0,$$

which implies that the objective function is strictly convex. Thus, if $s/L_H(g) \leq 1$ then the optimal amount of capacity reserved by the manufacturer, Q, is equal to the minimum of capacity K, or the solution obtained by setting the first derivative equal to zero as shown in (2.1). Otherwise, we set reservation quantity equal to 0.

■

The expected benefit from the use of an additional option is $(L_H(g) + g) - (s + g)$, and the cost of an additional option is s. The reader can verify that the critical fractile in (2.1) is indeed the ratio of the expected benefit to the sum of the expected benefit and cost of an additional option. It also follows from Proposition 2 that in a meaningful contract (i.e., a contract with $0 < Q^* < \infty$) the following inequality holds:

$$0 < s < L_H(g).$$

The intuition behind this result is that paying s to reserve supply is the manufacturer's "insurance policy" against having to pay in excess of g on the spot market. It only makes sense for the manufacturer to pay a value of s that is no greater than the expected difference between the spot market price and g. It is also true that $s \neq 0$, because $F(Q^*) = 1$ at $s = 0$ according to (2.1), and thus $Q^* = \infty$. We also notice that because $L_H(g)$ is decreasing in g, $F(Q^*)$ is decreasing in g and s, as is Q^*.

2.4.3 Stage 1 Supplier's Problem

At this stage the supplier must set the prices s and g to motivate the manufacturer to reserve the amount of capacity that maximizes the supplier's expected profit. Therefore, the supplier must solve this problem:

$$\textbf{(S1)} \qquad \max_{s,g,Q_P} \pi(s, g, Q_P) = E_{D,P_S}[sQ^* + (P_S - m)(Q_P - q^*) + gq^* - Q_P b]$$

$$\text{subject to: } Q^* \leq Q_P \leq K,$$

$$s \geq 0, \; g \geq 0,$$

where q^* is the solution of Problem M2, and Q^* is the solution of Problem M1.

The Optimal Q_P

Proposition 3 *Problem S1 is linear in Q_P, and thus can be separated into two cases.*

$$\text{Case I: If } \overline{P_S} - m \geq b, \text{ then } Q_P = K.$$

$$\text{Case II: If } \overline{P_S} - m < b, \text{ then } Q_P = Q^*.$$

Proof. We can rewrite the objective function as

$$\begin{aligned}\pi(s, g, Q_P) &= (\overline{P_S} - m - b)Q_P + sQ^* \\ &\quad - \int_g^\infty (P_S - m - g)h(P_S)dP_S \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^\infty Q^* f(D)dD \right).\end{aligned}$$

The problem is linear in Q_P, and thus the optimal Q_P depends on the sign of $\overline{P_S} - m - b$. The result follows immediately. ■

We will refer to the supplier in Case I as a competitive supplier, and in Case II as a noncompetitive supplier.

Optimality Conditions for s Under Unconstrained Supplier's Capacity

In this section we demonstrate how the supplier should determine the optimal price s. In order to do this we need to make certain assumptions on the demand distribution function. We say that a distribution has an increasing failure rate (IFR) if $r'(x) > 0$ for $r(x) = \frac{f(x)}{1-F(x)}$, and we say that a distribution has an increasing generalized failure rate (IGFR) if $u'(x) > 0$ for $u(x) = xr(x)$. The distribution that is IFR is also IGFR, but the converse is not always true. Normal, uniform, truncated normal, Erlang, and triangular are all examples of IFR distributions. The class of IGFR functions includes these IFR distributions as well as Weibull and Gamma distributions. So, this condition is sufficiently broad to capture most of the distributions one would use for demand and spot price.

Let us denote the failure rates of the demand and spot price distributions by $r_F(.)$ and $r_H(.)$ respectively, and the generalized failure rate of the demand distribution function by $u_F(.)$.

Proposition 4 *Case I. If $\overline{P_S} - m \geq b$, then the objective function of S1 is concave with respect to s for IFR demand distribution functions, and unimodal in s for IGFR demand distribution functions.*

Case II. If $\overline{P_S} - m < b$, then the objective function of S1 is unimodal with respect to s for IFR demand distribution functions.

Proof. Case I.

Suppose $\overline{P_S} - m \geq b$, then $Q_P = K$ and the objective function takes the following form

$$\pi(s, g, Q_P) = K(\overline{P_S} - m - b) + sQ^* \\ - \int_g^{\infty} (P_S - m - g)h(P_S)dP_S \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD \right).$$

The first derivative

$$\frac{\partial \pi}{\partial s} = Q^* - \frac{m \int_g^{\infty} h(P_S)dP_S}{L_H(g) r_F(Q^*)},$$

and the second derivative

$$\frac{\partial^2 \pi}{\partial s^2} = \frac{\partial Q^*}{\partial s} + \frac{m \int_g^{\infty} h(P_S)dP_S}{L_H(g)} \cdot \frac{r'_F(Q^*)}{(r_F(Q^*))^2} \cdot \frac{\partial Q^*}{\partial s}.$$

It is easy to see that if the demand distribution function is IFR, or $r'_F(Q^*) > 0$, then the second derivative of $\pi(s, g, Q_P)$ is negative, because

$$\frac{\partial Q^*}{\partial s} = -\frac{1}{f(F^{-1}(1 - \frac{s}{L_H}(g)))} \cdot \frac{1}{L_H(g)} < 0$$

and, therefore the objective function is strictly concave in s.

If the demand distribution function is IGFR, then we rewrite the first derivative as

$$\frac{\partial \pi}{\partial s} = Q^* \left\{ 1 - \frac{m \int_g^\infty h(P_S) dP_S}{L_H(g) u_F(Q^*)} \right\}.$$

At $s = 0, Q^* = \infty$ and $\frac{\partial \pi}{\partial s} = \infty$. As s increases, Q^* is decreases and, so does $u_F(.)$. Also, $\frac{\partial \pi}{\partial s}|_{s \to \infty} \to -\infty$. Hence, the objective function is unimodal in this case and the expression in the braces changes the sign at some point where

$$1 - \frac{m \int_g^\infty h(P_S) dP_S}{L_H(g) u_F(Q^*)} = 0.$$

Case II.

Now suppose $\overline{P_S} - m < b$, then $Q_P = Q^*$ and the objective function can be rewritten as

$$\pi(s, g, Q_P) = (\overline{P_S} - m - b + s) Q^* \\ - \int_g^\infty (P_S - m - g) h(P_S) dP_S \left(\int_0^{Q^*} D f(D) dD + \int_{Q^*}^\infty Q^* f(D) dD \right).$$

The first derivative

$$\frac{\partial \pi}{\partial s} = Q^* - \frac{1}{r_F(Q^*)} \cdot \left[\frac{m \int_g^\infty h(P_S) dP_S}{L_H(g)} - \frac{b + m - \overline{P_S}}{s} \right].$$

At $s = 0$, $Q^* = \infty$ and $\frac{\partial \pi}{\partial s} = \infty$. The expression in brackets stays negative on the interval where $s \leq \frac{b + m - \overline{P_S}}{m \int_g^\infty h(P_S) dP_S} L_H(g)$, and thus $\frac{\partial \pi}{\partial s}$ stays positive on that interval. Now consider interval where $s \geq \frac{b + m - \overline{P_S}}{m \int_g^\infty h(P_S) dP_S} L_H(g)$. The expression in the brackets is an increasing function of s, Q^* is a decreasing function of s, and $\frac{1}{r_F(Q^*)}$ is an increasing function of s (because the demand distribution function is IFR). Also, $\frac{\partial \pi}{\partial s}|_{s \to \infty} \to -\infty$. Thus, at the point where

$$Q^* - \frac{1}{r_F(Q^*)} \cdot \left[\frac{m \int_g^\infty h(P_S) dP_S}{L_H(g)} - \frac{b + m - \overline{P_S}}{s} \right] = 0$$

the sign of $\frac{\partial \pi}{\partial s}$changes, i.e., the function $\pi(s, g, Q_P = Q^*)$ is unimodal. ∎

Given that objective function is unimodal and continuously differentiable in s, the following result follows directly from Proposition 4 by setting first derivative equal to zero.

Proposition 5 *Given g, the optimal s is finite and can be determined as follows.*

1. *Case I. If $\overline{P_S} - m - b \geq 0$, s is the solution of*

$$Q^* - \frac{m \int_g^\infty h(P_S) dP_S}{L_H(g) r_F(Q^*)} = 0. \tag{2.2}$$

2. *Case II. If $\overline{P_S} - m - b < 0$, s is the solution of*

$$Q^* - \frac{1}{r_F(Q^*)} \cdot \left[\frac{m \int_g^\infty h(P_S) dP_S}{L_H(g)} - \frac{b + m - \overline{P_S}}{s} \right] = 0. \tag{2.3}$$

Optimality Conditions for g Under Unconstrained Supplier's Capacity

In this section we provide optimality conditions for g. Let us define $v(g) = \frac{1-H(g)}{L_H(g)}$ and $\widehat{g}$ as a solution of $1 - mr_H(\widehat{g}) = 0$. The function $1/\nu(g)$ is called a residual mean time and it is decreasing for IFR distributions (see, for example, Shaked and Shanthikumar (1994)).

Proposition 6 *Suppose that the spot market price and the demand distributions are IFR. Then:*

1. *For all s in Case I, and for $s \geq \frac{b - \overline{P_S} + m}{\int_{\widehat{g}}^\infty h(P_S) dP_S} L_H(\widehat{g})$ in Case II: if $\frac{\partial \pi}{\partial g}(s, g, Q_P)|_{g=0} > 0$, then the profit function $\pi(s, g, Q_P)$ is unimodal in g, otherwise the profit function is decreasing in g.*

2. *The optimal exercise price $g^* > 0$, and is finite.*

Proof. We will consider two cases separately.

Case I.

Suppose $\overline{P_S} - m \geq b$, then $Q_P = K$ and objective function takes the following form

$$\pi(s, g, Q_P = K) = K(\overline{P_S} - m - b) + sQ^* - \int_g^\infty (P_S - m - g)h(P_S)dP_S \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^\infty Q^* f(D)dD \right).$$

The first derivative

$$\frac{\partial \pi}{\partial g}(s, g, Q_P = K) = (1 - H(g))(1 - mr_H(g)) \int_0^{Q^*} xf(x)dx - \frac{(1 - H(g))s}{L_H(g)} \left(\frac{m \int_g^\infty h(P_S)dP_S}{r_F(Q^*)L_H(g)} - (1 - mr_H(g))Q^* \right).$$

We can see that $\frac{\partial \pi}{\partial g}(s, g, Q_P = K)$ is a continuous function of g. Let's define $\widehat{g}$ as a solution of $1 - mr_H(g) = 0$, and $\widetilde{g}$ as a solution of $\frac{m \int_{g_I}^\infty h(P_S)dP_S}{r_F(Q^*(s,\widetilde{g}))L_H(\widetilde{g})} - (1 - mr_H(\widetilde{g}))Q^*(s, \widetilde{g}) = 0$. Assume that $\widetilde{g} = 0$, if $\frac{m \int_g^\infty h(P_S)dP_S}{r_F(Q^*)L_H(g)} - (1 - mr_H(g))Q^*|_{g=0} > 0$.

For $g \leq \widetilde{g}$, we have $\frac{\partial \pi}{\partial g}(s, g, Q_P) > 0$

For $\widetilde{g} < g < \widehat{g}$, $\frac{\partial \pi}{\partial g}(s, g, Q_P)$ is decreasing in g. It follows directly from the assumption that both $r_F(.)$ and $r_H(.)$ are IFR, Q^* is a decreasing function of g and $\frac{1-H(g)}{L_H(g)}$ is an increasing function of g.

For $g \geq \widehat{g}$, $\frac{\partial \pi}{\partial g}(s, g, Q_P) < 0$.

Thus, if $\frac{\partial \pi}{\partial g}(s, g = 0, Q_P) > 0$, then $\frac{\partial \pi}{\partial g}(s, g, Q_P)$ changes sign only once and, $\pi(s, g, Q_P)$ is unimodal in g. Otherwise, $\pi(s, g, Q_P)$ is decreasing in g.

Now we show that optimal $g > 0$.

Suppose that it is not true, then $\frac{\partial \pi}{\partial g}(s^*(g=0), g=0, Q_P) \leq 0$, where $s^*(g=0)$ is a solution of (2.2) for $g=0$. Yet, when $g=0$ and $s=s^*(g=0)$

$$\frac{\partial \pi}{\partial g}(s,g,Q_P)|_{s=s^*(g=0),g=0} = \int_0^{Q^*(0,s^*)} Df(D)dD > 0,$$

where $Q^*(g=0,s^*) = F^{-1}(1-\frac{s^*}{\overline{P_S}}) > 0$.

Thus, optimal $g > 0$.

Case II.

Now suppose $\overline{P_S} - m < b$, then $Q_P = Q^*$ and the objective function can be rewritten as

$$\pi(s,g,Q_P=Q^*) = (\overline{P_S} - m - b + s)Q^* \\ - \int_g^{\infty}(P_S - m - g)h(P_S)dP_S\left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD\right).$$

The first derivative

$$\frac{\partial \pi}{\partial g}(s,g,Q_P=Q^*) = (1-H(g))(1-mr_H(g))\int_0^{Q^*} Df(D)dD \\ - \frac{(1-H(g))s}{r_F(Q^*)L_H(g)}\left(\frac{\int_g^{\infty} h(P_S)dP_S}{L_H(g)}m - \frac{b-\overline{P_S}+m}{s} - (1-mr_H(g))r_F(Q^*)Q^*\right).$$

is again a continuous function of g. Consider this expression for $s \geq \frac{b-\overline{P_S}+m}{\int_{\hat{g}}^{\infty} h(P_S)dP_S} L_H(\hat{g})$. $\frac{\partial \pi}{\partial g}$ is positive on the interval where $\frac{\int_g^{\infty} h(P_S)dP_S}{L_H(g)}m - \frac{b-\overline{P_S}+m}{s} - (1-mr_H(g))r_F(Q^*)Q^* \leq 0$, decreasing on the interval where $\frac{\int_g^{\infty} h(P_S)dP_S}{L_H(g)}m - \frac{b-\overline{P_S}+m}{s} - (1-mr_H(g))r_F(Q^*)Q^* > 0$ and $g \leq \hat{g}$. $\frac{\partial \pi}{\partial g}$ is negative for $g > \hat{g}$. Thus, we conclude that, for such values of s, if $\frac{\partial \pi}{\partial g}|_{g=0} > 0$, then $\pi(s,g,Q_P=Q^*)$ is unimodal, otherwise it is decreasing.

Now suppose $s < \frac{b-\overline{P_S}+m}{\int_{\hat{g}}^{\infty} h(P_S)dP_S} L_H(\hat{g})$. In this case $\frac{\partial \pi}{\partial g}|_{g=0} > 0$, also since $\frac{\partial \pi}{\partial g}|_{g\to\infty} \to -\infty$, we can say that in this case, optimal g exists, it is finite and can be determined, using necessary optimality conditions, by setting $\frac{\partial \pi}{\partial g} = 0$.

Since the optimal s^* solves (2.3), we substitute this expression in the expression for $\frac{\partial \pi}{\partial g}$. We can see that in this case $\frac{\partial \pi}{\partial g} < 0$ for $g > \widehat{g}$. Thus $\widehat{g}$ provides an upper bound on g^* in this case.

Now we show that optimal $g > 0$.

Suppose that it is not true, then $\frac{\partial \pi}{\partial g}(s^*(g=0), g=0, Q_P) \leq 0$, where $s^*(g=0)$ is a solution of (2.3) for $g=0$. But, when $g=0$ and $s = s^*(g=0)$

$$\frac{\partial \pi}{\partial g}(s, g, Q_P)|_{s=s^*, g=0} = \int_0^{Q^*(0,s^*)} Df(D)dD > 0$$

where $Q^*(g=0, s^*) = F^{-1}(1 - \frac{s^*}{E(P_S)}) > 0$.

Thus, optimal $g > 0$. ■

Proposition 6 implies that the forward contract, i.e., the contract which has only a reservation price $s > 0$ and $g = 0$, is not optimal. We also know in Case I from Proposition 6 that any local optimum is unique and globally optimal. From the proof of Proposition 6 also follows that $\frac{\partial \pi}{\partial g} < 0$ for any $g > \widehat{g}$, thus $g^* \leq \widehat{g}$.

Optimal s and g When the Supplier's Capacity is Limited

In this section we generalize our result for the case when the supplier's capacity is limited. We continue to assume that the distributions of the demand and the spot price are IFR. The results are shown in the following proposition.

Proposition 7 *The optimal contract for the limited capacity case is specified in (2.2) or (2.3), and Proposition 6 if that solution results in $Q^* \leq K$ when substituted into (2.1). Otherwise, the optimal limited capacity contract is constrained by the supplier's capacity K and the optimal contract terms (s_K, g_K) are set according to the solution of*

$$(1 - mr_H(g_K)) \left(\int_0^K Df(D)dD + \int_K^\infty Kf(D)dD \right) - K(1 - F(K)) = 0, \quad (2.4)$$

and

$$s_K = L_H(g_K)(1 - F(K)). \tag{2.5}$$

Proof. In general, the supplier does not know in advance whether the capacity is not enough to produce optimal Q^*, thus, he is faced with a general problem

$$\max_{s,g,Q_P} \pi(s,g,Q_P) = \begin{cases} (\overline{P_S} - m - b)Q_P + sQ^* - \int_g^\infty (P_S - m - g)h(P_S)dP_S \\ \times \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^\infty Q^* f(D)dD\right), \text{ if } 1 - \frac{s}{L_H(g)} \leq F(K). \\ (\overline{P_S} - m - b)K + sK - \int_g^\infty (P_S - m - g)h(P_S)dP_S \\ \times \left(\int_0^{K} Df(D)dD + \int_{K}^\infty K f(D)dD\right), \text{ if } 1 - \frac{s}{L_H(g)} \geq F(K). \end{cases}$$

$$\text{subject to} : Q^* \leq Q_P \leq K,$$

$$s \geq 0, g \geq 0.$$

Consider the first part of the profit function. In this case the optimal pair (s, g) is either a local maximum or a boundary point such that $1 - \frac{s}{L_H(g)} = F(K)$.

Now, consider the second part of the profit function. From the constraint follows that $s \leq (1 - F(K))L_H(g)$. Since, the profit function in this case is strictly increasing in s, it implies that $s_K = (1 - F(K))L_H(g)$. Again, we have a boundary solution.

We substitute the expression for s_K into the objective function and optimize it with respect to g.

The first derivative is

$$\frac{\partial \pi}{\partial g}(s_K, g, K) = (1 - H(g))\left((1 - mr_H(g))\left(\int_0^K Df(D)dD + \int_K^\infty Kf(D)dD\right)\right.$$
$$\left. - K(1 - F(K))\right)$$

Consider $\frac{\partial \pi}{\partial g}(s_K, g, K)|_{g=0} = \left(\left(\int_0^K Df(D)dD + \int_K^\infty Kf(D)dD\right) - K(1 - F(K))\right)$ $= \int_0^K Df(D)dD \geq 0$. Since $r_H(g)$ is IFR, we see that $\frac{\partial \pi}{\partial g}(s_K, g, K)$ changes the sign only once and, as a result, $\pi(s_K, g, K)$ is unimodal in g and optimal g_K is a solution of

$$(1 - mr_H(g))\left(\int_0^K Df(D)dD + \int_K^\infty Kf(D)dD\right) - K(1 - F(K)) = 0$$

while optimal $s_K = (1 - F(K))L_H(g)$. ■

2.5 The Decentralized System Without a Contract and Centralized System

The decentralized and centralized cases provide benchmarks for evaluating the performance of the contract. In this section we demonstrate how to calculate the expected profits and costs for each of this benchmarks. We will use superscript T to refer to the total system, that is, the supplier and the manufacturer together.

2.5.1 The Decentralized System without Contract

Below we present expressions used to calculate expected profit of the supplier and expected cost of the manufacturer in a decentralized system without a contract, and give an analytical expression for the additional profit brought into the total system by the contract.

Supplier. When there is no contract, the supplier sells directly to the spot market. Denote the production quantity in the decentralized system without a contract as

$$Q_{DNC} = \begin{cases} K \text{ if } \overline{P_S} - m - b \geq 0, \\ 0 \text{ if } \overline{P_S} - m - b < 0. \end{cases}$$

In this case, the supplier's expected profit is:

Case I. If $\overline{P_S} - m - b \geq 0$, then $Q_{DNC} = K$, and the expected profit is equal to $\pi_{DNC} = K(\overline{P_S} - m - b)$.

Case II. If $\overline{P_S} - m - b < 0$, then $Q_{DNC} = 0$ and the expected profit is equal to $\pi_{DNC} = 0$.

Manufacturer. In a decentralized system without a contract the manufacturer buys everything on the spot market. The expected manufacturer's cost in this case is equal to $C_{DNC} = \overline{D} \cdot \overline{P_S}$.

The value of the contract to the decentralized system. We denote the difference in profits between the decentralized system with a contract and the decentralized system without a contract as $\Delta\pi^T_{DC-DNC}$.

1. Case I. If $\overline{P_S} - m - b \geq 0$, then

$$\Delta\pi^T_{DC-DNC} = m(1 - H(g^*)) \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD \right). \tag{2.6}$$

2. Case II. If $\overline{P_S} - m - b < 0$, then

$$\Delta\pi^T_{DC-DNC} = Q^*(\overline{P_S} - m - b) + m(1 - H(g^*)) \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD \right). \tag{2.7}$$

2.5.2 The Centralized System

The centralized control case in which a single decision-maker optimizes the entire supply chain provides another benchmark for efficiency of the above contract. There are no transfer prices in the centralized system and everything that is produced by the supplier goes first to the manufacturer to satisfy demand. The remaining goods (if any) are sold to the spot market. Any unmet demand is satisfied through the spot market.

The decision-maker solves the problem:

$$\max_{Q_{CP}} \pi_C^T(Q_{CP}) = E_{D,P_S}\{(P_S - m)(Q_{CP} - D)^+ - P_S(D - Q_{CP})^+ - Q_{CP}b\} \quad \textbf{(C1)}$$

subject to: $Q_{CP} \leq K$,

where Q_{CP} is the supplier's production quantity.

The objective function of C1 can be rewritten as

$$\pi_C^T(Q_{CP}) = \overline{P_S}(Q_{CP} - \overline{D}) - m\int_0^{Q_{CP}} (Q_{CP} - D)f(D)dD - Q_{CP}b.$$

This is a newsvendor problem. Let us denote as Q_{CP}^* the solution of

$$F(Q_{CP}^*) = \frac{\overline{P_S} - b}{m}.$$

Then the optimal centralized production quantity is

$$Q_{CP} = \begin{cases} Q_{CP}^* \text{ if } \ 0 \leq \frac{\overline{P_S}-b}{m} \leq 1 \text{ and } Q_{CP}^* \leq K, \\ K \quad\ \text{if } \frac{\overline{P_S}-b}{m} > 1 \text{ or } Q_{CP}^* > K, \\ 0 \quad\ \text{if } \frac{\overline{P_S}-b}{m} < 0. \end{cases}$$

To evaluate the performance of the contract, we calculate the differences in the expected profits between the centralized and decentralized system without a contract, $\triangle\pi_{C-DNC}^T$, and between the decentralized system with the contract and decentralized system without the contract, $\triangle\pi_{DC-DNC}^T$. The ratio

$$\rho = \frac{\Delta\pi_{DC-DNC}^T}{\triangle\pi_{C-DNC}^T}$$

measures the effectiveness of the contract in closing the gap between the centralized and decentralized system without a contract, where

$$\begin{aligned} \triangle\pi_{C-DNC}^T =& (Q_{CP} - Q_{DNC})(\overline{P_S} - b - m) \\ &+ m\left(\int_0^{Q_{CP}} Df(D)dD + \int_{Q_{CP}}^{\infty} Q_{CP}f(D)dD\right), \end{aligned} \quad (2.8)$$

and $\triangle\pi^T_{DC-DNC}$ is defined by (2.6) or (2.7). The following expression represents the difference of profits of centralized system and decentralized system with the contract:

$$\triangle\pi^T_{C-DC} = (Q_{CP} - Q_P)(\overline{P_S} - b - m) - m(1 - H(g^*))\cdot \tag{2.9}$$
$$\left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD\right) + m\left(\int_0^{Q_{CP}} Df(D)dD + \int_{Q_{CP}}^{\infty} Q_{CP} f(D)dD\right)$$

2.5.3 Contract Effectiveness

To assess the value of the contract for the manufacturer and the supplier individually, we need to compare the expected cost and profit respectively in the decentralized scenarios with and without the contract. We will denote the supplier's relative profit increase as

$$\kappa = \frac{\pi - \pi_{DNC}}{\pi_{DNC}},$$

and the manufacturer's relative cost savings as

$$\theta = \frac{C - C_{DNC}}{C_{DNC}},$$

where π and C refer to the supplier's profit and the manufacturer's cost respectively in the decentralized system with the contract as defined in the previous sections.

2.6 Numerical Experimentation

In this section we provide the algorithm and parameters used for numerical experiments as well as the results of these experiments and our conclusions. For notational simplicity we use subscripts D and SP when we refer to the demand and the spot price distributions respectively, and refer to Case I and Case II with the subscripts I and II respectively. We denote the smallest possible value of the demand random variable D by $\underline{D}$.

2.6.1 The Optimization Algorithm and Experiment Parameters

We chose the triangular and truncated normal distributions to model spot price and demand distributions because they are reasonable and satisfy all the necessary conditions to ensure the unimodality of the supplier's objective function (the distributions are IFR).

Table 2.2: Numerical Experiment Parameters.

	Spot Price		Demand	
Scenarios	Distribution	CV_{SP}	Distribution	CV_D
1-9	Triangular	0.05, 0.2, 0.4	Triangular	0.05, 0.2, 0.4
10-13	Triangular	0.2, 0.4	Normal	0.06, 0.2
14-19	Normal	0.02, 0.06, 0.2	Triangular	0.2,0.4
20-25	Normal	0.02, 0.2	Normal	0.02, 0.06, 0.2

We first tested the combinations of spot price and demand distributions shown in Table 2.2 (with symmetric triangular distributions) assuming the supplier's capacity is sufficient for the greatest possible demand for both Case I and Case II, while setting $m = 0.1\overline{P_S}, 0.3\overline{P_S}, 0.7\overline{P_S}$ for each of these scenarios. We set the mean demand $\overline{D}$ and spot price $\overline{P}_S$ equal to 30, $K = 300$ in all scenarios. We set the supplier's production cost $b = 0.5$ for Case I, and $b = 28$ for Case II. We also modeled both the spot price and demand with a limited number of asymmetric triangular distributions. We did not notice any difference in the general results due to skewness, and so the analysis we present here is based on the numerical experiments with symmetric distributions.

We also analyzed the case with the limited capacity using the symmetric triangular distribution for the spot price and demand. We chose three different combinations of parameters ($CV_{SP} = 0.05$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$; $CV_{SP} = 0.05$, $CV_D = 0.05$, $m = 0.1\overline{P_S}$; $CV_{SP} = 0.4$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$), and varied the capacity level ($K = 5, 10, 15, ..., 60$).

The foregoing theoretical analysis provides the basis for the following algorithm to find the optimal s^* and g^* for Case I. The algorithm for Case II is similar although we need to allow for the possibility of multiple local maxima.

1. *Calculate the globally optimal s^* and g^* for the unconstrained supplier's profit function using Newton's method.*

2. *Calculate Q^* for (s^*, g^*) found in Step 1 using (2.1) and check whether the condition $Q^* \leq K$ is satisfied. If yes, stop. If no, then go to the step 3.*

3. *Set $(s^*, g^*) = (s_K, g_K)$ using (2.4) and (2.5).*

We used Newton's method in the first step to solve

$$\nabla \pi(x) = 0,$$

where $x = (s, g)^T$. Newton's method consists of successive approximations $x(1)$, $x(2), ..., x(k+1)$, such that

$$x(k+1) = x(k) - J^{-1}(k) f(k),$$

where $f(k) = \nabla \pi(x(k))$ and $J^{-1}(k)$ is the inverse matrix of the Jacobian of vector $f(k)$. We used $|f(x)|_\infty \leq \epsilon$ as a termination criterion, and set $\epsilon = 0.0005$.

The results we obtained for different distributions were structurally the same. The CVs of the distributions and the value of $m/\overline{P_S}$ affected the results more than did which family of distributions was used. Thus the results included in the tables are from the scenarios in which both the spot price and demand distributions are triangular. The numerical results for the optimal s, g, P and Q are presented in Tables 2.3 and 2.4. Results regarding contract effectiveness are shown throughout the text.

2.6.2 Results: Case I with Unconstrained Supplier's Capacity

The supply chain with the contract does not achieve the performance of the centralized supply chain in Case I, although it provides a significant improvement over the performance of the decentralized system and benefits both the manufacturer and the supplier.

We observed that the contract effectiveness, ρ was as high as 99% (for the triangular and normal distribution with $CV_D \geq 0.2$, normal distribution with $CV_{SP} = 0.02$, and $m = 0.7\overline{P_S}$), and not less than 36% (for the triangular distribution with $CV_D = 0.4$, triangular distribution with $CV_{SP} = 0.4$, and $m = 0.1\overline{P_S}$). The contract effectiveness increased in m, decreased in CV_{SP}, and increased (decreased) in CV_D when $CV_{SP} < CV_D$ ($CV_{SP} \geq CV_D$). (See Table 2.5.) Thus the contract provides value by allowing the two parties to avoid paying the spot market margin. Furthermore, the contract is most effective when the spot market margin is high or when either the demand or the spot price variability is low.

The supplier receives a larger portion of the benefits whenever the contract is incentive compatible in a practical sense, which is expected because the supplier is the leader, but he cannot extract all the additional value. The supplier's relative profit κ increases from less than 1% to more than 20% as m increases from $0.1\overline{P_S}$

Table 2.3: Optimal contract parameters competitive versus noncompetitive suppliers with unrestricted capacity for triangular spot price and demand distributions (Part 1).

			Case I Competitive Supplier				Case II Noncompetitive Supplier			
CV_{SP}	CV_D	$m/\bar{P}_S$	s_I	g_I	P_I	Q_I^*	s_{II}	g_{II}	P_{II}	Q_{II}^*
0.05	0.05	0.1	3.94	26.03	29.97	26.45	3.97	26.02	29.99	26.3
0.05	0.05	0.3	3.71	26.09	29.79	27.28	3.98	26.01	29.99	26.3
0.05	0.05	0.7	3.12	26.14	29.26	28.47	3.99	26.00	29.99	26.2
0.05	0.20	0.1	3.14	26.50	29.64	22.64	3.56	26.25	29.81	20.8
0.05	0.20	0.3	2.28	26.55	28.82	27.67	3.78	26.07	29.85	20.2
0.05	0.20	0.7	1.18	26.47	27.64	32.50	3.86	26.02	29.89	19.7
0.05	0.40	0.1	1.88	27.25	29.13	24.44	2.64	26.74	29.38	19.2
0.05	0.40	0.3	1.12	26.94	28.06	34.07	3.42	26.15	29.57	15.2
0.05	0.40	0.7	0.44	26.61	27.05	43.70	3.70	26.04	29.74	12.1
0.20	0.05	0.1	13.56	16.54	29.99	26.14	13.48	16.52	29.99	26.0
0.20	0.05	0.3	13.33	16.60	29.93	26.41	13.49	16.51	29.99	26.0
0.20	0.05	0.7	12.93	16.72	29.65	26.92	13.49	16.50	29.99	26.0
0.20	0.20	0.1	12.51	17.32	29.82	18.76	13.04	16.88	29.92	18.0
0.20	0.20	0.3	10.84	18.12	28.96	22.15	13.30	16.62	29.92	17.9
0.20	0.20	0.7	8.73	18.42	27.15	25.98	13.38	16.55	29.03	17.9

Table 2.4: Optimal contract parameters competitive versus noncompetitive suppliers with unrestricted capacity for triangular spot price and demand distributions (Part 2).

			Case I Competitive Supplier				Case II Noncompetitive Supplier			
CV_{SP}	CV_D	$m/\bar{P}_S$	s_I	g_I	P_I	Q_I^*	s_{II}	g_{II}	P_{II}	Q_{II}^*
0.20	0.40	0.1	7.35	21.72	29.07	16.07	9.81	19.49	29.30	12.3
0.20	0.40	0.3	6.47	20.87	27.34	23.60	12.14	17.24	29.37	10.7
0.20	0.40	0.7	5.32	19.78	25.10	29.46	12.78	16.74	29.51	9.5
0.40	0.05	0.1	27.96	2.04	29.99	26.07	27.98	2.02	29.99	26.0
0.40	0.05	0.3	27.86	2.11	29.96	26.20	27.99	2.01	29.99	26.0
0.40	0.05	0.7	27.58	2.24	29.82	26.45	27.99	2.00	29.99	26.0
0.40	0.20	0.1	26.93	2.91	29.84	17.64	27.54	2.41	29.96	17.2
0.40	0.20	0.3	25.07	4.23	29.30	19.64	27.82	2.13	29.96	17.2
0.40	0.20	0.7	21.96	5.52	27.48	22.64	27.90	2.06	29.96	17.2
0.40	0.40	0.1	16.11	13.11	29.21	11.92	21.11	8.23	29.34	9.1
0.40	0.40	0.3	14.74	12.41	27.15	18.48	25.68	3.65	29.33	8.3
0.40	0.40	0.7	13.14	10.78	23.92	24.44	26.83	2.59	29.42	7.7

to $0.7\overline{P_S}$ (see Table 2.6). The value of κ is significantly affected by the changes of m, yet is relatively insensitive to the changes in CV_D and CV_{SP} mostly due to the supplier's ability to manipulate the contract prices in order to mitigate the effect of variability. The manufacturer's cost savings θ increases from less than 1% (for

Table 2.5: Case I. Contract Effectiveness ρ for triangular spot price and demand distributions.

		CV_{SP}		
		0.05	0.2	0.4
	0.05	0.88/0.91/0.94	0.87/0.88/0.90	0.87/0.87/0.88
CV_D	0.2	0.74/0.87/0.95	0.62/0.73/0.83	0.59/0.65/0.74
	0.4	0.70/0.88/0.97	0.47/0.68/0.81	0.36/0.54/0.70

Legend – $x/y/z$ where x, y, z denote contract effectiveness ρ for $m/\bar{P}_S = 0.1, 0.3, 0.7$ respectively.

$CV_D < 0.2$, and $m = 0.1\overline{P_S}, 0.3\,\overline{P_S}$) to more than 10% (for $CV_D = 0.4$, $CV_{SP} \geq 0.2$, and $m = 0.7\overline{P_S}$) as CV_D and m increase (see Table 2.7). θ increases in CV_{SP} when either CV_D or m are large, and decreases in CV_{SP} when both these parameters are small. The manufacturer's relative cost savings θ does exceed the supplier's relative profit increase κ, when $CV_{SP} < CV_D$ and the spot market margin is small ($m = 0.1\overline{P_S}$), although at that point both θ and κ are less than 1%. Thus, the contract is valuable for the supplier in avoiding the spot market margin, and the manufacturer benefits from the contract whenever m, CV_D and CV_{SP} are large.

Of all the parameters tested, m has the greatest effect on ρ, κ, and θ, which all increase in m. Although contract efficiency decreases in demand and spot price uncertainty, a significant share of the contract effectiveness gain is passed along to the manufacturer only when both demand and spot price uncertainty are high — the supplier's "control" is apparently reduced in this situation. A contract might, in fact, be most viable when CV_D, CV_{PS}, and m are simultaneously large. In that

Table 2.6: Case I. Contract Effect on κ with Triangular Spot Price and Demand Distributions.

		CV_{SP}		
		0.05	0.2	0.4
CV_D	0.05	0.01/0.04/0.23	0.01/0.04/0.22	0.01/0.04/0.22
	0.2	0.007/0.03/0.21	0.007/0.03/0.18	0.006/0.03/0.16
	0.4	0.006/0.03/0.21	0.004/0.02/0.16	0.003/0.02/0.13

Legend – $x/y/z$ where x, y, z denote κ for $m/\bar{P}_S = 0.1, 0.3, 0.7$ respectively.

case, contract effectiveness is still significant ($\rho = 70\%$ for $CV_D = CV_{PS} = 0.2$ and $m = 0.7\overline{P_S}$), and each party's gains are significant ($\theta = 12\%, \kappa = 13\%$) — both parties are thus motivated to enter into the contract.

2.6.3 Results: Case II with Unconstrained Supplier's Capacity

In Case II, the contract again was not able to coordinate the supply chain. Although it significantly improved system performance and proved beneficial for the supplier, the manufacturer's cost was reduced only slightly, and so this contract may not be incentive compatible for the manufacturer in a practical sense. Still the analysis of the case might be useful because a simple side payment to the manufacturer can create an incentive for her to enter the contract.

The contract effectiveness increases as CV_D and CV_{SP} decrease, and as the margin m increases. We observed that ρ is at least 37% in the worst case (for triangular distribution with $CV_D = 0.4$, triangular distribution with $CV_{SP} = 0.4$,

Table 2.7: Case I. Contract Effect on θ with Triangular Spot Price and Demand Distributions.

		CV_{SP} 0.05	CV_{SP} 0.2	CV_{SP} 0.4
CV_D	0.05	0.001/0.006/0.02	0.0002/0.002/0.01	0.0001/0.001/0.005
	0.2	0.008/0.03/0.07	0.004/0.02/0.07	0.001/0.01/0.06
	0.4	0.02/0.05/0.09	0.01/0.05/0.11	0.01/0.04/0.12

Legend – $x/y/z$ where x, y, z denote θ for $m/\bar{P}_S = 0.1, 0.3, 0.7$ respectively.

and $m = 0.1\overline{P_S}$), and 98% in the best case (for normal distribution $CV_{SP} = 0.02$ and $m = 0.7\overline{P_S}$). (See Table 2.8.) Among the parameters tested, demand variability CV_D had the greatest effect on contract effectiveness ρ.

Table 2.8: Case II. Contract Effectiveness ρ for triangular spot price and demand distributions.

		CV_{SP} 0.05	CV_{SP} 0.2	CV_{SP} 0.4
CV_D	0.05	0.90/0.95/0.97	0.77/0.89/0.95	0.73/0.90/0.97
	0.2	0.90/0.94/0.96	0.67/0.80/0.88	0.49/0.79/0.85
	0.4	0.90/0.94/0.96	0.64/0.77/0.84	0.37/0.55/0.72

Legend – $x/y/z$ where x, y, z denote contract effectiveness for $m/\bar{P}_S = 0.1, 0.3, 0.7$ respectively.

The manufacturer's relative cost savings θ were always below 1%. The contract always allows the noncompetitive supplier a profit, which he would not otherwise have, and it is greatest for low demand variability CV_D and low spot price margin m.

In Case II the values of s and g are always set at the level such that the critical fractile $F(Q^*) < 0.5$, which means that the supplier sets a relatively high total price P that not only bolsters his revenue from the manufacturer, but also minimizes the expected loss from selling product to the spot market.

The number of options purchased by the manufacturer, Q, which in this case equals the supplier's production quantity, is always lower than the quantity produced in the centralized supply chain, Q_{CP}, and is due to the fact that the supplier does not offer the prices that would guarantee an optimal order quantity, and thus an optimal production quantity. The difference $Q_{CP} - Q$ increases as CV_D increases and decreases with m. The probability that the reserved options will not be exercised increases with an increase in CV_D, thus the supplier is less willing to chance higher production levels. An increase in m makes selling goods to the spot market less attractive, thus the suppliers in both the centralized and decentralized systems restrict the production quantity to the amount that minimizes the loss from selling to the spot market, which leads to the decrease in $Q_{CP} - Q$.

2.6.4 Comparison between Case I and Case II, Unconstrained Capacity.

Although the contract improves system performance when the supplier is either competitive or non-competitive, there are differences between the Case I and Case II contract mechanisms and performance.

In Case I, the parameter m has the largest effect on the supply chain and the performance of both parties, whereas CV_D affects the performance of the contract most greatly in Case II. This comes as no surprise when viewed from the perspective of the supplier, who is the Stackelberg leader that controls the contract terms and who must sell into the spot market if the manufacturer does not exercise her options. The noncompetitive Case II supplier must fervently avoid the spot market because any unit sold there generates a loss. And so, demand variability, which significantly affects whether the manufacturer will exercise options at any price, significantly affects how the supplier sets the contract prices. Demand variability is much less an issue for the competitive supplier in Case I who can profitably sell in the spot market, and safely encourage the manufacturer to purchase a greater number of options, some of which are likely not to be exercised. The supplier will produce up to its capacity, and its goal is simply to sell as many units to the manufacturer as possible in order to avoid the spot market margin. Thus m most affects the supplier's actions in this case.

Both the supplier and the manufacturer benefit from the contract, although the manufacturer benefits to a lesser degree. For the manufacturer in Case I, the value of the relative cost savings θ increases with the value of m, and when combined with high demand and price variability, can be larger than 10%; in Case II, the additional gains are very low (always below 1%) so that the contract is of little value to the manufacturer. It follows that the manufacturer would prefer a competitive supplier to a noncompetitive one.

Total price P decreases in m in Case I, and increases (very slightly) in m in Case II. Reservation price s decreases in m in Case I, and increases in m in Case II, whereas the exercise price g decreases in m in both Cases I and II, although almost imperceptibly so in Case I. As a consequence, the reservation quantity Q^* increases in m in Case I and decreases in Case II. As the spot market becomes more unattractive to the Case II supplier as m increases, the supplier's actions simultaneously reduce both the number of options purchased (the supplier increases s) and the chance of selling to the spot market (the supplier decreases g). Conversely, the Case I competitive supplier encourages a greater number of options to be purchased while only marginally reducing the probability of sale to the spot market by decreasing g, which the noncompetitive supplier does more aggressively.

Both competitive and noncompetitive suppliers keep the exercise price g sufficiently low to guarantee that the probability that the spot price exceeds g is small ($H(g)$ is small), but as the demand variability increases so too does g. The reservation price s conversely decreases in CV_D in both cases. This behavior is more noticeable in Case I where the competitive supplier can more freely encourage the manufacturer to purchase a greater number of options because the downside of the spot market is not as great as for the noncompetitive supplier.

In summary, the following inequalities hold for Case I and Case II scenarios in which m, CV_{SP} and CV_D are held constant:

$$\begin{aligned} s_{II} &> s_{I,} \\ g_{II} &< g_I, \\ P_{II} &> P_I, \\ Q^*_{II} &< Q^*_I, \\ Q^*_{II} s_{II} &> Q^*_I s_I, \\ E[Q^*_{II} s_{II} + g_{II} q_{II}] &< E[Q^*_I s_I + g_I q_I], \end{aligned}$$

where the last line represents the expected revenue that the supplier receives from the manufacturer.

2.6.5 Results: Limited Supplier's Capacity in Case I

The contract again improves the performance of the decentralized system when the supplier's capacity is limited. In fact, the contract helps to attain the centralized solution when the capacity is severely constrained, although the contract is not practically incentive compatible for the manufacturer in this case.

The system profit with the contract is equal to that of the centralized system when the supplier's capacity K is below $\underline{D}$, because $Q_{CP} = Q_P = Q = K, s = \overline{P_S}$ and $H(g) = 0$, and thus according to (3.3) $\Delta\pi_{C-DC} = 0$. The supplier and the manufacturer can contract on the supplier's full capacity with certainty in this case. The supplier charges $P = \overline{P_S}$ when $K \leq D$ however, and so this contract does not provide any value for the manufacturer. The value of $\Delta\pi_{C-DC}$ increases with K until the capacity level reaches Q^*, the optimal reservation level in the non-capacitated problem, and does not change after that. Figure 2.2 shows how the contract effectiveness ρ changes depending on the value of the supplier's capacity K.

The manufacturer's relative cost savings θ is zero whenever $K \leq \underline{D}$ and increases for $\underline{D} \leq K \leq Q^*$. Thus the supplier must have sufficient capacity in order for a contract to be incentive compatible for the manufacturer. Figure 2.3 shows how θ depends on the value of the capacity K.

The supplier's additional profit increases as K increases, while P decreases and so too does the supplier's expected revenue per unit sold to the manufacturer. Thus, even though the supplier's additional profit generated by the contract is non-decreasing in K, the supplier's relative profit increase κ is non-increasing in K (see Figure 2.4).

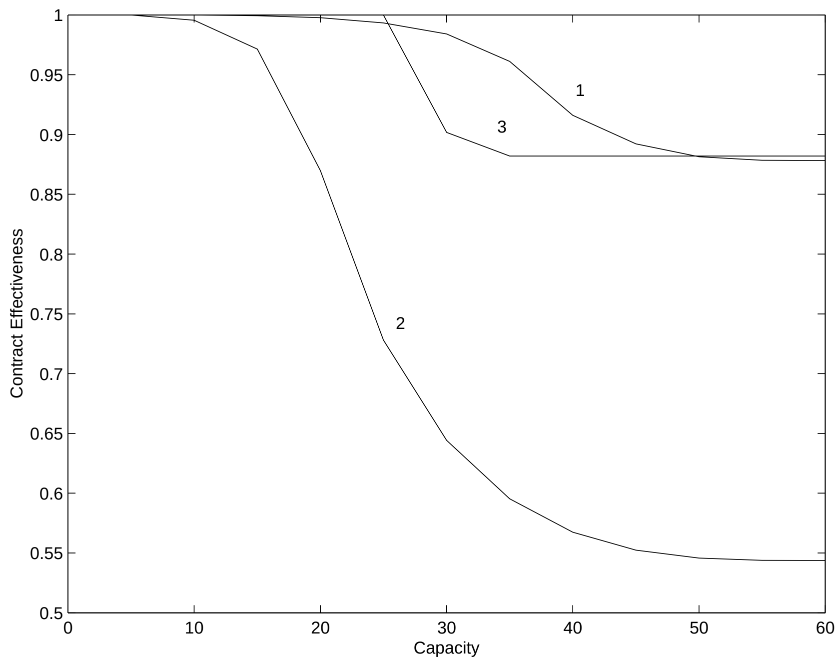

Figure 2.2: The Contract Effectiveness, ρ, [1 ($CV_{SP} = 0.05$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 2 ($CV_{SP} = 0.4$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 3 ($CV_{SP} = 0.05$, $CV_D = 0.05$, $m = 0.1\overline{P_S}$).].

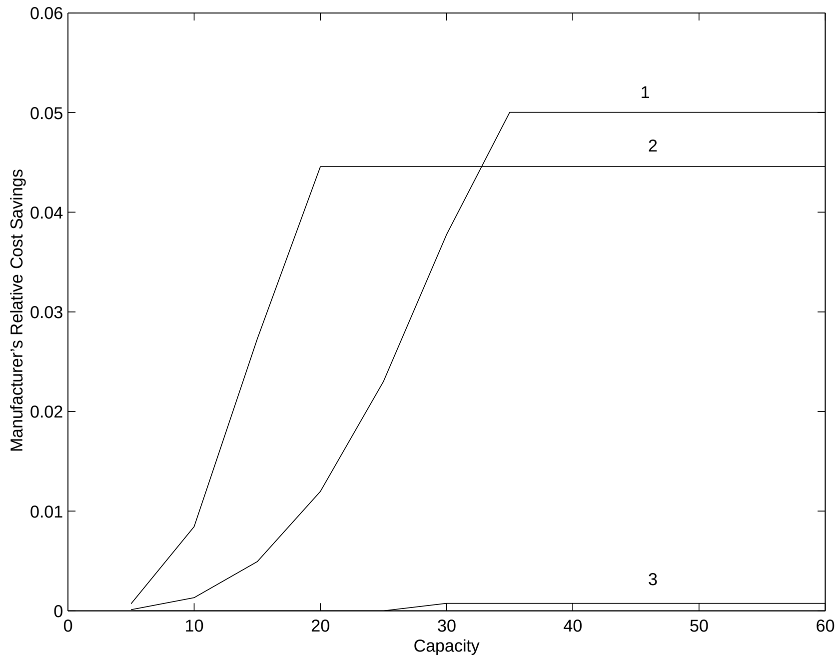

Figure 2.3: Manufacturer's relative cost savings, θ, [1 ($CV_{SP} = 0.05$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$);2 ($CV_{SP} = 0.4$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 3 ($CV_{SP} = 0.05$, $CV_D = 0.05$, $m = 0.1\overline{P_S}$).].

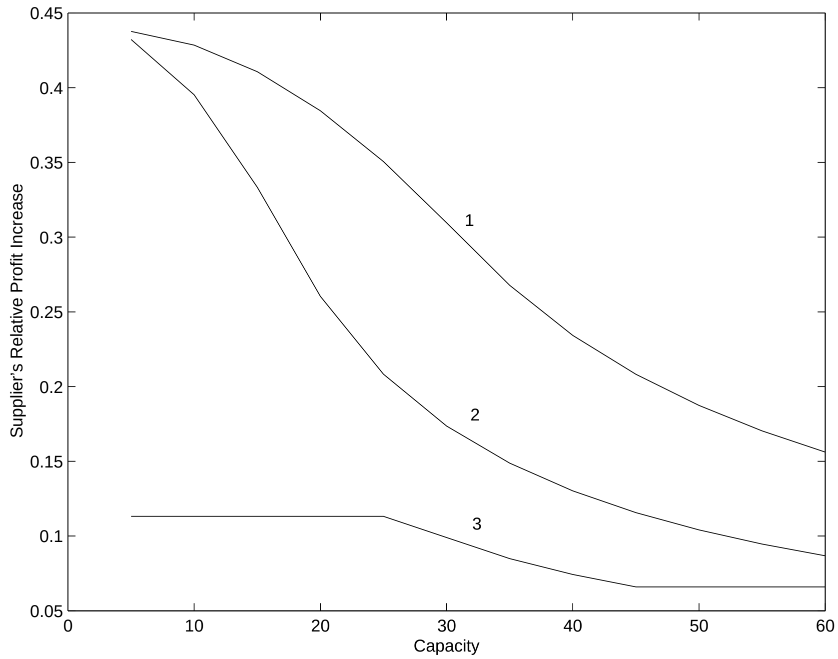

Figure 2.4: Supplier's relative profit increase, κ, [1 ($CV_{SP} = 0.05$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 2 ($CV_{SP} = 0.4$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 3 ($CV_{SP} = 0.05$, $CV_D = 0.05$, $m = 0.1\overline{P_S}$).].

2.7 Extensions

2.7.1 Penalty Cost

In this section we relax the assumption that the manufacturer must satisfy all demand in full. We assume that the manufacturer pays a penalty p for each unit that her delivered quantity falls short of the quantity ordered by her customer and receives a revenue a for each unit delivered. (Forced compliance can be viewed as the case with $p = \infty$.) The main difference from the previous analysis is that the supplier will only buy the product or service if the exercise price does not exceed $(p + a)$, the loss for each unit she is short from the amount demanded by her customer.

In this case, the optimal quantity reserved by the manufacturer is

$$F(Q^*) = 1 - \frac{s}{L_H(g) - \int_{p+a}^{\infty}(P_S - (p+a))h(P_S)dP_S}, \tag{2.10}$$

if $g < p + a$, otherwise there is no contract.

Note the additional term in the denominator of (2.10) that is not present in the forced compliance case that represents lost revenue when the spot market price is high. It follows that the optimal reservation quantity is increasing in p:

$$\frac{\partial Q^*}{\partial p} = \frac{s(1 - H(p+a))}{f(Q^*)\left(L_H(g) - \int_{p+a}^{\infty}(P_S - (p+a))h(P_S)dP_S\right)^2} > 0.$$

It also follows from (2.10) that the contract quantity $0 < Q < \infty$ if

$$\int_{g}^{p+a} P_S h(P_S)dP_S + \int_{p+a}^{\infty}(p+a)dH - g(1 - H(g)) > s.$$

Thus the reservation price should be smaller than the expected savings from using contract rather than buying directly from the spot market.

It is easy to verify in this setting that all the previous results in Propositions 3, 4 and 6 hold, and for the optimal reservation price we have the following expressions:

1. Case I. If $\overline{P_S} - m - b \geq 0$, s is the solution of

$$Q^* - \frac{m\int_g^\infty h(P_S)dP_S - \int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S}{\left(L_H(g) - \int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S\right) r_F(Q^*)} = 0. \tag{2.11}$$

2. Case II. If $\overline{P_S} - m - b < 0$, s is the solution of

$$Q^* - \frac{1}{r_F(Q^*)} \cdot \left(\frac{m\int_g^\infty h(P_S)dP_S - \int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S}{L_H(g) - \int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S} \right.$$
$$\left. - \frac{b + m - \overline{P_S}}{s} \right) = 0. \tag{2.12}$$

In this setting, for certain (low) values of $p + a$ the supplier cannot offer the manufacturer a contract that is incentive compatible for both players. For example, from (2.11) or (2.12) it follows that there is no contract if

$$m\int_g^\infty h(P_S)dP_S - \int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S < 0 \Rightarrow$$
$$\int_{p+a}^\infty (P_S - (p+a))h(P_S)dP_S > m. \tag{2.13}$$

This provides us with a lower bound on the value of $p+a$, below which a contract for options is not possible. Rewrite (2.13) as

$$\overline{P}_S - m > \int_0^{p+a} P_S h(P_S)dP_S + \int_{p+a}^\infty (p+a)h(P_S)dP_S.$$

Thus the supplier does not offer a contract if the expected profit he can receive form selling directly to the spot market exceeds the expected price manufacturer pays when she buys on the spot market. The right-hand side of the inquality represents the highest price the manufacturer is willing to pay to the supplier, thus the supplier can motivate the manufacturer to sign the contract only if he offers a lower price, but supplier will never do that since selling directly to the spot market is more profitable.

If globally optimal contract parameters exist, then the results of Proposition 7 (restricted supplier's capacity) hold again in this case. When the manufacturer is unable to reserve a globally optimal quantity due to the insufficient capacity, then the supplier will offer her (s, g) as defined by

$$(1 - mr_H(g_{K'})) \left(\int_0^K Df(D)dD + \int_K^\infty Kf(D)dD \right) - K(1 - F(K)) = 0, \quad (2.14)$$

and

$$s_{K'} = L_H(g_K)(1 - F(K)) - \int_{p+a}^\infty (P_S - (p + a))h(P_S)dP_S. \quad (2.15)$$

2.7.2 Spot Market Margin

It is straightforward to show that the results of Section 2.4 hold if m is a non-decreasing function of P_S. For example, if m is a fixed percentage of the current spot price P_S then our analytical results are still valid.

2.8 Conclusions

In this paper, we analyzed the design of an optimal contract for options in the presence of a spot market. We modeled our problem as a two-stage Stackelberg game in which a supplier is the leader. We showed that the manufacturer's cost function is convex and provided a closed-form expression for the optimal reservation quantity. We provided nonrestrictive conditions under which the supplier's profit function is unimodal in the forward and exercise prices.

We demonstrated, using centralized and decentralized cases as benchmarks, that while the contract significantly improved the system overall performance, it could not guarantee total channel coordination. We showed nonetheless that the contract always brings value into the supply chain and can decrease the gap between centralized and decentralized system profit by as much as 99%. The supplier, who is the leader, benefits most from the contract, but is unable to extract all the additional profit from the channel. A supplier must be competitive and have sufficient capacity in order for a contract to be incentive compatible for the manufacturer. A noncompetitive supplier sets prices that minimize its exposure to the unattractive spot market, and thus offers a high price and insufficient quantity to the manufacturer. Even if competitive, a supplier with insufficient capacity will not offer the manufacturer an attractive price.

With a competitive supplier of sufficient capacity, the contract is most attractive when the spot market margin is large, and allows the spot market to be circumvented. In this circumstance the supplier (the leader) improves his profits by a greater percentage than does the manufacturer reduce her cost. The manufacturer can still reduce her cost significantly when spot price variability and demand uncertainty are high. In fact, the percentage improvement in each party's position is comparable under high CV of both spot price and demand and large m — these are perhaps the most viable contract conditions when both parties benefit greatly and are motivated to enter into a contract.

We can succinctly summarize the optimal strategy for the competitive supplier, who is in a position to offer an incentive compatible contract to the manufacturer. That is, the exercise price should be set sufficiently low to virtually guarantee that the manufacturer will exercise the options and the reservation prices should be set to balance the trade-off between immediate revenues (sQ) and the future revenues ($gq + (Q_P - q)(P_S - m)$). If the spot price margin is small, then immediate revenue dominates the decision. If the spot market margin is large, then the reservation price is set low to encourage a large number of options to be purchased and the opportunity to avoid the spot market and its requisite margin maximized.

One possible extension to the problem is designing a contract mechanism that leads to the system efficiency. Another extension is to derive an optimal contract for a similar problem with multiple periods in which either the supplier or the buyer can store goods between periods for which a holding cost would be incurred. A capacity-constrained supplier might find it beneficial to store goods not sold to the manufacturer in one period for sale in the next period. A manufacturer might find some inventory holding cost preferable to possibly high contract or spot prices in the next period.

Chapter 3

Channel Coordination with a Spot Market

3.1 Introduction

Spot markets often exist for commodity goods and can help to mitigate the effect of uncertain demand and supply. For example, Shell purchased styrene on the spot market to mitigate the effect of an unplanned production outage in February 2002 (Cage 2002). Relying exclusively on spot markets rather than contractual relationships, however, may reduce profits, especially when market prices fluctuate significantly. For example, the profitability of many U.S. steel-consuming companies suffered from a 40% jump in steel prices that resulted from a new tariff on steel imports that was imposed in March 2002 (see knowledge.wharton.upenn.edu 2003a). Similarly, the margins of many small trucking companies were squeezed this past year by a 23% increase in retail diesel prices combined with stagnant transportation prices — many of these companies were forced out of business altogether (see knowledge.wharton.upenn.edu 2003b). Long term contracts, however, provide protection from volatile spot prices. For example, many large automobile companies were protected from the recent surge in spot steel prices by long-term contracts with steelmakers. In this paper we study the optimal balance of contractual relationships and use of the spot market and, in particular, how contracts and the spot market can be used together to coordinate the supply chain.

We consider a simple one-period model with a supply chain that consists of a supplier and a manufacturer who engage in a contract for options for an intermediate good. The supplier produces the intermediate good, which the manufacturer transforms into another product that is used downstream in the supply chain. We assume that the supplier can sell its excess production of the intermediate good to the spot market and that the manufacturer can buy the intermediate good from the spot market if the contracted quantity is insufficient or if the spot market price is preferable to the exercise price offered by the supplier. For clarity of exposition we will assume that the manufacturer must satisfy all the demand from his customer in full. This assumption can be relaxed to the case when the manufacturer pays a penalty cost for unsatisfied demand, in which case the intuition that we draw from the analysis of this model applies.

We consider three different mechanisms that coordinate the channel in this setting. In the first, we allow the manufacturer to choose the contract prices. In the second mechanism, we consider the contract for options where the reservation price is a function of the contracted capacity, i.e., a quantity discount contract. In the third mechanism, we allow for partial renegotiation of the contract parameters after the demand and the spot price are realized. When the manufacturer is the leader, we show an equivalence between the optimal contract parameters in our model and a classic result by Hirshleifer (1956) in a deterministic setting. We also show how each coordinating mechanism affects the allocation of the profits between the supplier and the manufacturer and give some insight into when each mechanism is most appropriate. Most notably though, we find that the renegotiation dominates the other two coordination mechanisms. First, renegotiation is robust — we show that it coordinates the channel even under information asymmetry. Second, renegotiation induces a more equitable sharing of the contract benefits than the other mechanisms.

There is a large literature on contracting and supply chain coordination, which is reviewed by Cachon (2002). Of this literature, a number of papers analyze contracts between suppliers and buyers in the presence of spot markets. One stream of research has been mainly concerned with finding a Nash Equilibrium strategy of the players and has not, in general, focused on the channel coordination. Wu et al. (2002) and Spinler et al. (2002) consider an optimal contract for options in the presence of a spot market when the supplier is the leader. Both papers assume that demand is price dependent and endogenous. Wu et al. assume deterministic demand while Spinler et al. assume stochastic demand and production costs. Golovachkina and Bradley (2002) analyze a similar model with exogenous price-independent demand. They show that in this case when the supplier is a Stackelberg leader the contract for options does not coordinate the channel. The basic model we present here is similar the model in Golovachkina and Bradley (2002), but we concentrate primarily on the mechanisms that lead to the channel coordination while Golovachkina and Bradley (2002) were mostly concerned with finding optimal parameters for the contract for options that in many cases do not coordinate the channel.

Another stream of research demonstrates how a spot market can complement the contract between a supplier and a manufacturer and improve the performance in a decentralized supply chain, but again does not deal with achieving of channel coordination. Araman et al. (2001) analyze a model with a spot market in which the supplier's predetermined pricing scheme depends on both the amount of capacity reserved by the buyer and the quantity of goods it ultimately purchases. The spot market price is a function of the demand in this model. Contrary to our model, the buyer must always purchase first from the supplier up to the reserved capacity, and only then can go to the spot market to satisfy the remaining demand. The authors numerically compute the optimal mix between these two channels. Erhun et al. (2000) consider a stylized model in a deterministic environment. They show that the supplier and the manufacturer can use spot markets to strategic advantage even when there is no uncertainty.

One of the issues we deal with in this paper is contract renegotiation, which is closely related to incomplete contracting. There are number of papers in economics literature devoted to this subject. Hart and Moore (1988) consider a model in which two parties are forced to sign an incomplete contract due to their inability to describe all possible future states of the world and later renegotiate upon realization of the future state. The authors show that the inability to prevent renegotiation results in under-investment by the agents. Laffont and Tirole (1990) consider renegotiation when the buyer and supplier have asymmetric information. Noldeke and Schmidt (1995) show that no renegotiation is necessary to achieve coordination when the agents sign a simple option contract, which gives the seller the right but not the obligation to deliver the good at the certain price. Aghion et al. (1994) show that coordination can be achieved when the initial contract specifies default options if renegotiation breaks down, and assigns all bargaining power in renegotiation to either buyer or the supplier. We add to this literature by considering a model with a spot market and showing how renegotiation improves the performance of the contract for options in this case.

Renegotiation is very important in practice. A recent survey of logistics managers, for example, revealed that renegotiating shipping rates was a dominant strategy in companies efforts to reduce costs in 2002 (see The Controller's Report (Jan. 2003)). Yet, to the best of our knowledge Plambeck and Taylor (2002) is the only other paper in the supply chain management literature that considers contract renegotiation. Plambeck and Taylor consider a model with a single supplier and two buyers in which the buyers are allowed to renegotiate the contracted quantity after demand is observed. Benefit is derived in this system because the buyers can effectively trade goods when one's demand is high and other's is low. In contrast, we show that the system can benefit from renegotiation even when the supplier has just one customer. Specifically, when information asymmetry is present, anticipating renegotiation allows the supplier to screen the manufacturer for his true type without sacrificing system efficiency.

In a similar vein to renegotiation, Van Mieghem (1999) studies an incomplete contract between a supplier (subcontractor) and a manufacturer in a two-stage game where rather than revising contract terms set in the first stage, as is done in renegotiation, the negotiation of some contract terms is left for the second stage. The channel is coordinated when the subcontractor and manufacturer set a fixed per-unit transfer price and leave to the second stage the bargaining over the division of the surplus generated by the contract. Among the differences in Van Mieghem's and our model is that he draws in the capacity decision, whereas we consider only the production and exchange decisions. In that sense our model takes a shorter-term perspective because we fix production capacity and analyze the production and exchange decisions in finer granularity — i.e., whereas production and exchange decisions are made simultaneously after demand is observed in Van Mieghem's model, production decisions are made in the first stage in our model before demand is realized and exchange takes place in the second stage. Moreover, we assume that all contract parameters are specified before uncertainty of demand is resolved. Finally, a key feature of our model is the presence of the spot market, which is absent in Van Mieghem's model.

In Section 4.3 we describe our general model. In Section 3.3 we describe a centralized system and a decentralized system without a contract that we use as benchmarks. In Section 3.4 we show how the channel can be coordinated when the manufacturer is the leader. In Section 3.5 we show that the supply chain can be coordinated with a quantity discount contract. In Section 3.6 we show how coordination can be achieved with renegotiation and demonstrate how renegotiation coordinates the channel even if the manufacturer and the supplier have asymmetric information about demand. We present our conclusions in Section 3.7.

3.2 The Model

We consider a single-period, two-stage problem with a single supplier and single manufacturer. The supplier provides a commodity to the manufacturer that it uses to produce a good for use in downstream supply chain links. Without loss of generality, we assume that one unit of the intermediate commodity is required for each unit of the manufacturer's product. We assume that the supplier has a capacity K and pays b for each unit she produces. (See Table 3.1 for a complete list of notation.) The manufacturer faces a stochastic demand D, with cumulative distribution function $F(.)$, p.d.f. $f(.)$ and expected value $\overline{D}$. We assume that the price paid by the manufacturer for the commodity is independent of the demand for the good he produces. This is the case, for example, when the manufacturer produces a subassembly or if the manufacturer sells to a more powerful customer that can limit the extent to which the manufacturer can pass forward its costs. We also assume that neither the supplier nor the buyer has any perceptible effect on the spot price for the commodity because they represent just a small fraction of the whole market. We assume that the quantity of goods desired by the manufacturer is always available on the spot market at some stochastic price P_S with cumulative distribution $H(.)$, p.d.f. $h(.)$ and expected value $\overline{P}_S$. Given the structure of the problem, we assume that $D \geq 0$ and $P_S \geq 0$. We assume that there is a fixed margin m per unit that the spot market extracts in order to clear the market. We assume this simple form for the spot market margin for clarity of exposition although we can easily extend our results to more general forms.

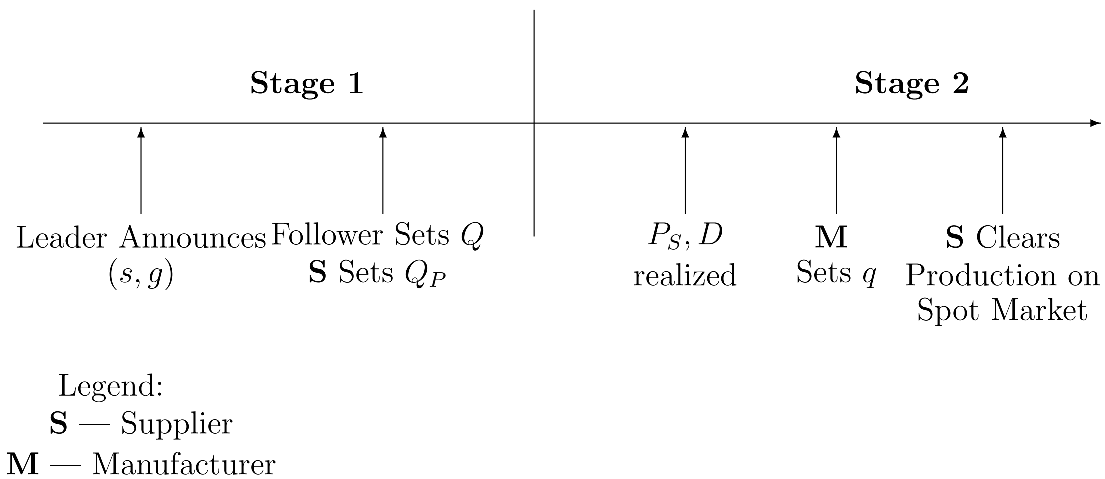

Figure 3.1: Event Timeline

We model our problem as a two-stage Stackelberg game (see Figure 4.1), where either party (supplier or manufacturer) can be the leader. At the beginning of the first stage, the leader announces the contract parameters (s, g), where s is a reservation price per unit the manufacturer pays to have the product available to him and g is a per-unit exercise price that the manufacturer pays for each unit delivered in stage 2. The follower then decides how much capacity Q is reserved

Table 3.1: Notation

K	-	supplier's capacity
b	-	production cost per unit
m	-	spot market margin per unit
s	-	reservation price per option
g	-	exercise price per option
Q	-	number of options purchased by manufacturer
q	-	number of options exercised
q_S	-	number of units purchased on the spot market
Q_P	-	supplier's production quantity
P_S	-	spot market price
D	-	demand random variable
$F(.)$	-	demand c.d.f.
$f(.)$	-	demand p.d.f.
$H(.)$	-	spot price c.d.f.
$h(.)$	-	spot price p.d.f.
$\widetilde{P_S}$	-	realization of the spot price random variable
$\widetilde{D}$	-	realization of the demand random variable
$\overline{P_S}$	-	expected spot market price
$\overline{D}$	-	expected demand

for the manufacturer at the given prices s and g. After the contract parameters s, g, and Q are set, the supplier chooses her production quantity Q_P assuming that the supplier must produce at least as much as the manufacturer has reserved $(Q_P \geq Q)$.

At the beginning of the second stage the demand D and the spot price P_S are realized, after which the manufacturer decides how much to order from the supplier q, and how much to buy on the spot market q_S. The manufacturer goes to the spot market if either the contracted quantity is insufficient to satisfy his demand or if the spot market price is lower than the exercise price charged by the supplier, i.e., if $\widetilde{P}_S < g$. After the manufacturer's order is filled, we assume that the supplier can sell all her excess inventory to the spot market at the current price $\widetilde{P}_S$ minus commission m, which may or may not be profitable.

3.3 Centralized and Decentralized Systems

In order to demonstrate how to coordinate the channel we need to introduce a centralized system in which a single decision-maker optimizes the entire supply chain. There are no transfer prices in the centralized system and everything that the supplier produces goes first to the manufacturer to satisfy demand. The remaining goods (if any) are sold to the spot market. Any unmet demand is satisfied through the spot market.

The centralized decision-maker solves the problem:

$$\max_{Q_{CP}} \pi_C^T = E_{D,P_S}\{(P_S - m)(Q_{CP} - D)^+ - P_S(D - Q_{CP})^+ - Q_{CP}b\}$$

subject to: $Q_{CP} \leq K$,

where Q_{CP} is the supplier's production quantity in the centralized system. Here we use subscript C to refer to the centralized system and superscript T to refer to the whole system.

The objective function can be rewritten as

$$\pi_C^T = (\overline{P_S} - m - b)Q_{CP} - \overline{P}_S\overline{D} + m\left(\int_0^{Q_{CP}} Df(D)dD + \int_{Q_{CP}}^{\infty} Q_{CP}f(D)dD\right), \quad (3.1)$$

which is a newsvendor problem. Let us denote as $\widehat{Q}_{CP}$ the solution of

$$F(\widehat{Q}_{CP}) = \frac{\overline{P_S} - b}{m}.$$

Then the optimal centralized production quantity is

$$Q^*_{CP} = \begin{cases} \widehat{Q}_{CP} \text{ if } \; 0 \leq \frac{\overline{P_S}-b}{m} \leq 1 \text{ and } \widehat{Q}_{CP} \leq K, \\ K \qquad \text{if } \frac{\overline{P_S}-b}{m} > 1 \text{ or } \widehat{Q}_{CP} > K, \\ 0 \qquad \text{if } \frac{\overline{P_S}-b}{m} < 0. \end{cases} \tag{3.2}$$

The following expression represents the difference between the profits in the centralized system and the decentralized system with the contract for options (specified by s, g, and Q) (see Golovachkina and Bradley 2002):

$$\triangle\pi^T_{C-DC} = (Q^*_{CP} - Q_P)(\overline{P_S} - b - m) - m(1 - H(g^*)) \tag{3.3}$$

$$\left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD\right) + m\left(\int_0^{Q^*_{CP}} Df(D)dD + \int_{Q^*_{CP}}^{\infty} Q^*_{CP} f(D)dD\right).$$

Thus, to coordinate the supply chain we want to find mechanisms that result in $\Delta\pi^T_{C-DC} = 0$.

In order to assess the value of the contract for each player we introduce the decentralized system without a contract (we will use subscript DNC to denote this system). In this case both the supplier and the manufacturer go directly to the spot market and do not engage in any transactions with each other.

The manufacturer's expected costs in this case are

$$C_{DNC} = \overline{P}_S \overline{D}.$$

The supplier's expected profit is

$$\begin{aligned} \pi_{DNC} &= K(\overline{P}_S - m - b), && \text{when } \overline{P}_S - m - b \geq 0, \\ \pi_{DNC} &= 0, && \text{when } \overline{P}_S - m - b < 0. \end{aligned}$$

Thus, the contract will be incentive compatible for the players if the manufacturer's costs with a contract, C, satisfies $C \leq C_{DNC}$ and the supplier's profit with a contract, π, satisfies $\pi \geq \pi_{DNC}$.

3.4 Channel Coordination when the Manufacturer is the Leader

In this section we show how the supply chain can be coordinated when the manufacturer is the leader and chooses contract prices (s, g). The supplier in this case decides how many units Q_P she should produce and how many options Q the manufacturer can reserve at the announced prices. The supplier's first-stage problem is therefore

$$\max_{Q,Q_P} \pi(Q, Q_P) = E_{D,P_S}[-Q_P b + sQ + (P_S - m)(Q_P - q) + gq] \tag{S1}$$
$$\text{subject to: } 0 \leq Q \leq Q_P \leq K.$$

In anticipation of the supplier's solution to Problem $S1$, the manufacturer selects a pair (s, g) in the first stage that minimizes his expected costs:

$$\min_{s,g} C(s, g) = E_{D,P_S}[P_S(D - q) + sQ + gq] \tag{M1}$$
$$\text{subject to: } s \geq 0, g \geq 0.$$

Proposition 8 *Suppose $\overline{P}_S - m - b \geq 0$, then*

1. *The manufacturer offers*

$$\begin{aligned} s &= (\overline{P_S} - m)\left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right), \qquad (3.4) \\ g &= 0, \end{aligned}$$

and the supplier sets

$$Q = K.$$

2. *The supply chain is coordinated.*

3. *The supplier receives no benefit from this contract and the manufacturer's cost savings are*

$$\Delta C = C_{DNC} - C = m\left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right). \tag{3.5}$$

Proof. We rewrite the supplier's objective function as

$$\pi(Q, Q_P) = (\overline{P_S} - m - b)Q_P + sQ \\ - \int_g^\infty (P_S - m - g)h(P_S)dP_S\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right).$$

It is straightforward to see that when $\overline{P_S} - m - b \geq 0$, then $Q_P = K$.

The first derivative of $\pi(Q, Q_P)$ is

$$\frac{\partial \pi}{\partial Q} = s - (1 - F(Q))\int_g^\infty (P_S - g - m)h(P_S)dP_S,$$

and the second derivative is

$$\frac{\partial^2 \pi}{\partial Q^2} = f(Q)\int_g^\infty (P_S - g - m)h(P_S)dP_S.$$

We see that if $\int_g^\infty (P_S - g - m)h(P_S)dP_S \geq 0$, then the supplier's objective function is convex in Q and thus the optimal Q^* is equal to either K or 0.

If $\int_g^\infty (P_S - g - m)h(P_S)dP_S < 0$, the supplier's objective function is concave. But in this case $\frac{\partial \pi}{\partial Q} > 0$ and thus the optimal $Q^* = K$.

Now consider the manufacturer's problem.

$$\min_{s,g} C(s,g) = \overline{P_S} \cdot \overline{D} + sK - \int_g^\infty (P_S - g)h(P_S)dP_S \cdot \left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right)$$

$$\text{s.t.} \quad \pi(K,K) \geq \pi(0,K)$$

$$s \geq 0, g \geq 0,$$

where the first constraint is an incentive constraint for the supplier to set the order quantity above zero.

The above problem can be rewritten as

$$\min_{s,g} C(s,g) = \overline{P_S} \cdot \overline{D} + sK - \int_g^\infty (P_S - g)h(P_S)dP_S \left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right)$$

$$\text{s.t.} \ s - \int_g^\infty (P_S - m - g)h(P_S)dP_S \left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right) \geq 0$$

$$s \geq 0, g \geq 0.$$

After taking derivatives we observe that

$$\begin{aligned} \frac{\partial C}{\partial s} &= K > 0, \\ \frac{\partial C}{\partial g} &= (1 - H(g))\left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right) > 0. \end{aligned}$$

The objective function is strictly increasing in both s and g, and thus the optimal point is a boundary point. The feasibility region for this problem is defined by

$$\begin{aligned} s &\geq \int_g^\infty (P_S - m - g)h(P_S)dP_S \left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right), \quad (3.6) \\ s &\geq 0, g \geq 0. \end{aligned}$$

If $\int_g^\infty (P_S - g - m)h(P_S)dP_S < 0$, then from (3.6) follows that $s = 0$ and $g = 0$ which is equivalent to no contract case.

If $\int_g^\infty (P_S - g - m)h(P_S)dP_S \geq 0$, then inequality $s \geq 0$ is redundant and we have

$$\begin{aligned} s &\geq \int_g^\infty (P_S - m - g)h(P_S)dP_S \left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right), \quad (3.7) \\ g &\geq 0, \int_g^\infty (P_S - g - m)h(P_S)dP_S \geq 0, \end{aligned}$$

so that there are two possible combinations for the optimal solution, which we denote with the appropriate subscripts.

Suppose $s_1 = 0$. Then from (3.7) follows that $g_1 > 0$ and is such that

$$\int_{g_1}^\infty (P_S - m - g_1)h(P_S)dP_S = 0.$$

Suppose $g_2 = 0$. Then from (3.7) follows that

$$s_2 = (\overline{P_S} - m)\left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right).$$

We determine the optimal solution by comparing the values of the objective function in each one.

$$C_1(s_1, g_1) = \overline{P_S} \cdot \overline{D} - m\int_{g_1}^\infty h(P_S)dP_S\left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right),$$

and

$$C_2(s_2, g_2) = \overline{P_S} \cdot \overline{D} - m\left(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD\right).$$

Because $g_1 > 0$, we conclude that $C_1(s_1, g_1) > C_2(s_2, g_2)$ and thus

$$\begin{aligned} s^* &= (\overline{P_S} - m)\left(\frac{1}{K}\int_0^K Df(D)dD + \int_K^\infty f(D)dD\right), \\ g^* &= 0, \\ Q^* &= Q_P^* = K. \end{aligned}$$

The corresponding manufacturer's cost savings under (s^*, g^*) in comparison with no-contract situation in this case is

$$\Delta C = C_{DNC} - C = m(\int_0^K Df(D)dD + K\int_K^\infty f(D)dD) > 0,$$

which implies that the contact is always incentive compatible for the manufacturer.

In this case the difference of the profits between centralized and decentralized systems is

$$\Delta\pi^T_{C-DC} = 0.$$

Thus, the channel coordination is achieved. ■

This contract prices in (3.4) are identical to the classic result by Hirshleifer (1956), who shows in a deterministic setting that system coordination is attained only when the transfer price for an intermediate good is equal to the competitive market price. In our stochastic setting, Hirshleifer's result holds in expectation. That is, the expected unit price paid by the manufacturer is equal to the supplier's expected unit revenue from selling to the spot market, $\overline{P}_S - m$. Thus, although the supply chain is coordinated when the manufacturer is the leader, the supplier's expected additional profit is zero so that a risk neutral supplier has no incentive to sign this contract. However, a risk averse supplier would consider this contract because it eliminates profit variability due to the fluctuating spot market price for the quantity of goods sold to the manufacturer.

The comparison of this result with the one presented in Golovachkina and Bradley (2002) shows that whether the channel is coordinated and whether the contract benefits are distributed equitably depend on who is the leader. While the channel is not coordinated when the supplier is the leader, the channel is coordinated when the manufacturer is the leader. We conclude that when supply chain coordination is important, the leadership should be given to the downstream player because he is better able to set the transfer prices at the optimal level. Coordinating the channel by giving leadership to the manufacturer, however, results in a less equitable sharing of the contract benefits. When the supplier leads, it receives a majority of the contract benefits, but still the manufacturer receives some benefit. When the manufacturer leads, it receives all the contract benefits because it is capable of extracting all the welfare from the system and in doing so giving the supplier (in expectation) a price no greater than she would have received from the spot market, $\overline{P}_S - m$. While unfortunate for the supplier this transfer price is efficient for the supply chain. Of course, barring a risk averse supplier, the practical implementation when the manufacturer leads will require a transfer payment from the manufacturer to motivate the supplier to sign this contract.

For our next result we use the following definition (Shaked and Shanthikumar 1994).

Definition. For two random variables D_1 and D_2, we say that D_1 is smaller than D_2 in convex order, and write $D_1 \leq_{cx} D_2$, if $E[\phi(D_1)] \leq E[\phi(D_2)]$ for all convex functions ϕ, provided the expectations exist.

Practically, $D_1 \leq_{cx} D_2$ means that the random variable D_2 is more variable than D_1.

Proposition 9 *The following results hold:*

1. *The manufacturer's expected cost is decreasing in m and in K.*

2. *The type of the spot price distribution does not affect the manufacturer's expected cost (only the expected spot price matters).*

3. *Suppose D_i, for $i = 1, 2$ are two random variables such that $D_1 \leq_{cx} D_2$. If C_i are corresponding expected costs of the manufacturer, then $C_1 \leq C_2$.*

Proof. The manufacturer's expected costs with the contract are equal to

$$C = \overline{P}_S \overline{D} - m \left(\int_0^K D f(D) dD + K \int_K^\infty f(D) dD \right).$$

The first part of the Proposition follow directly from differentiating C with respect to m and K. The second part follows directly from (3.5).

To show that the third part holds we rewrite the manufacturer's expected costs

$$C = E_D[\overline{P}_S D - m \min(D, K)].$$

Define $\phi(D) = \overline{P}_S D - m \min(D, K)$, which is convex in D. Thus,

$$C_1 = E_D[\overline{P}_S D_1 - m \min(D_1, K)] \leq E_D[\overline{P}_S D_2 - m \min(D_2, K)] = C_2.$$

■

As the margin increases, the supplier is less willing to sell to the spot market and, thus, the manufacturer is able to strike a better deal with her. The manufacturer's costs decrease in m as a result. When the supplier's capacity increases, the manufacturer, who according to this contract reserves all K units, pays a lower price per each unit. At the same time, a higher reservation quantity implies a lower probability that the manufacturer will also buy from the spot market. Thus, the manufacturer wins twice from a larger K, which results in lower costs. The second part of proposition says that the manufacturer is on average not affected by the changes in the spot price distributions as long as the expected spot price remains the same. The third part of the proposition implies that the manufacturer with a less variable demand has lower costs with this contract. Figure 3.2 demonstrates the results presented in Proposition 9.

3.5 Channel Coordination with Quantity Discounts

In this section we show how the supply chain can be coordinated using the quantity discount mechanism. It is known that the quantity discount contract is often able

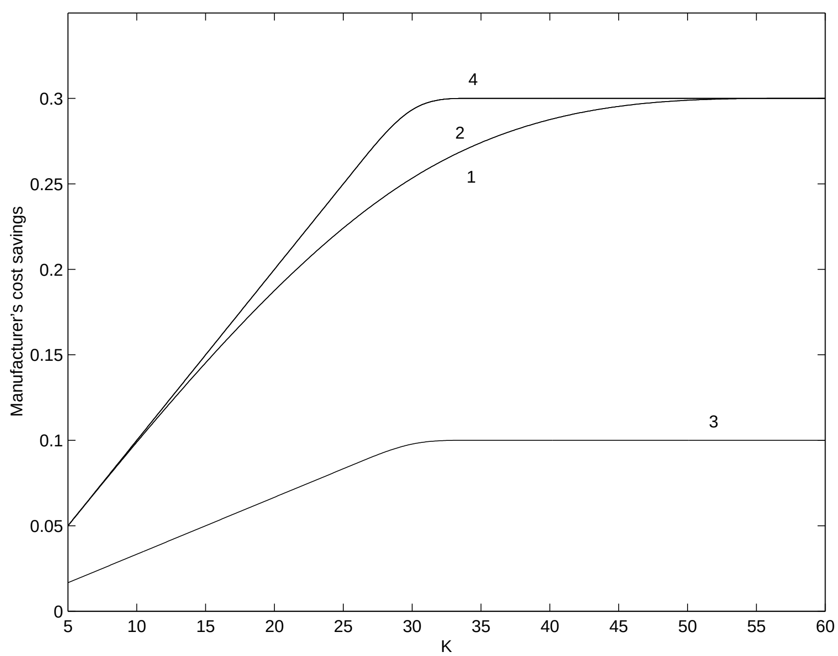

Figure 3.2: Manufacturer's cost savings, [1 ($CV_{SP} = 0.05$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 2 ($CV_{SP} = 0.4$, $CV_D = 0.4$, $m = 0.3\overline{P_S}$); 3 ($CV_{SP} = 0.05$, $CV_D = 0.05$, $m = 0.1\overline{P_S}$).4. ($CV_{SP} = 0.4$, $CV_D = 0.05$, $m = 0.3\overline{P_S}$)].

to coordinate the channel when other contracts fail (see, for example, Cachon 2002). We first consider the case when the supplier is the leader.

Proposition 10 *If the supplier is the leader, the following contract with s decreasing in Q coordinates the channel:*

$$\begin{aligned} s(Q) &= \frac{\overline{P}_S}{Q}\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right), \\ g &= 0. \end{aligned}$$

The supplier will extract all benefits from this contract and the manufacturer will have zero cost savings.

Proof. The result can be easily verified. We substitute given (s, g) in the manufacturer's cost function. The resulting

$$C = \overline{P}_S \overline{D}$$

thus the manufacturer's cost savings are zero for any quantity he reserves from the supplier.

The supplier's profit in this case is

$$\begin{aligned} \pi &= (\overline{P}_S - m - b)Q_P + m\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right) \\ &= \pi_C^T + \overline{P}_S\overline{D}, \text{ when } Q = Q_P = Q_{CP}. \end{aligned}$$

Since the centralized system is the best the supplier can do, we conclude that the supplier wants the manufacturer to reserve $Q = Q_{CP}$, which the manufacturer does.

The resulting system's profit is

$$\pi^T = \pi_C^T \ ,$$

and, thus, the system is coordinated. ■

This contract allows for the supply channel coordination but leaves the manufacturer with zero benefits.

Now we assume that the manufacturer is the leader and sets the prices.

Proposition 11 *Suppose the manufacturer is the leader. Then the following contract with s decreasing in Q coordinates the channel.*

If $\overline{P}_S - m - b \geq 0$, then

$$\begin{aligned} s(Q) &= \frac{(\overline{P}_S - m)}{Q}\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right), \\ g &= 0. \end{aligned}$$

If $\overline{P}_S - m - b < 0$, then

$$\begin{aligned} s(Q) &= \frac{(\overline{P}_S - m)}{Q}\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right) + b + m - \overline{P}_S, \\ g &= 0. \end{aligned}$$

The manufacturer extracts all benefits from this contract and the supplier is left with zero benefit.

Proof. We substitute given values of (s, g) into the supplier's profit function. The resulting profit is

$$\begin{aligned} \text{if } \overline{P}_S - m - b &\geq 0, \text{ then } \pi = Q_P(\overline{P}_S - m - b), \\ \text{if } \overline{P}_S - m - b &< 0, \text{ then } \pi = 0. \end{aligned}$$

Thus, the supplier's profit does not depend on the quantity reserved by the manufacturer.

The manufacturer's costs are

$$C = \overline{P}_S\overline{D} - m\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right) - (\overline{P_S} - m - b)Q \cdot 1_{\{\overline{P_S} - m - b < 0\}}$$

where $1_{\{\overline{P}_S - m - b < 0\}}$ is an indicator function, which is equal to one if the subscript is true and zero otherwise. Comparison of this expression with (3.1) and (3.2) shows that the manufacturer's costs are minimized when $Q = Q_{CP}$. The resulting system's profit is

$$\pi^T = \pi_C^T$$

and the channel is coordinated. ■

It follows from Propositions 10 and 11 that the quantity discount contract coordinates the channel but gives the follower no benefit. Moreover, the leader offers a price schedule such that the follower's benefit is invariant of the reservation quantity chosen. Thus, besides the inequitable sharing of benefits, another practical implementation issue arises. The leader must signal to the follower what reservation quantity to choose by offering an additional (very small) transfer payment when $Q = Q_{CP}^*$.

When the manufacturer is the leader and the supplier can sell profitably to the spot market, $\overline{P}_S - m - b \geq 0$, the actual prices that the manufacturer pays to the supplier are the same as with the contract for options when the manufacturer is the leader in (3.4) because the supplier is induced to choose a reservation quantity equal to its capacity, $Q = K$.

It is interesting to note from Proposition 11 that when $\overline{P}_S - m - b < 0$ and the supplier cannot sell profitably to the spot market, the price that the manufacturer pays, s, compensates the supplier for expected loss from selling to the spot market:

$$\begin{aligned} s &= \frac{(\overline{P}_S - m)}{Q}\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right) + b + m - \overline{P}_S \\ &\geq b + m - \overline{P}_S. \end{aligned}$$

At the same time, the price that the manufacturer pays to the supplier for each unit can be below the supplier's production cost,

$$\begin{aligned} s &= \frac{(\overline{P}_S - m)}{Q}\left(\int_0^Q Df(D)dD + \int_Q^\infty Qf(D)dD\right) + b + m - \overline{P}_S \\ &\leq \frac{(\overline{P}_S - m)}{Q}Q + b + m - \overline{P}_S \\ &= b. \end{aligned}$$

Therefore, with this contract, the manufacturer compensates the supplier for the loss she will incur if she sells goods to the spot market, but he does not pay the supplier enough to compensate her for the production costs. Thus, this contract arrangement is not tenable for the supplier.

3.6 Channel Coordination with Renegotiation

In this section we show how channel coordination can be achieved if the manufacturer and the supplier are allowed to renegotiate the parameters of the contract after the demand and the spot price are observed. We, first, use the contract for options when the supplier is the leader as a basis for this analysis because both the supplier and manufacturer benefit from that contract, although it does not coordinate the channel. With renegotiation we assume that if the spot market price falls below the exercise price, then the supplier and the manufacturer can renegotiate the exercise price to be equal to the spot price, $g = \widetilde{P_S}$, so that the manufacturer is still motivated to purchase from the supplier, and the supplier can avoid paying m per unit to the spot market to sell its goods. If manufacturer's demand exceeds the quantity reserved from the supplier, then we allow the manufacturer to buy an additional quantity at the current spot price $\widetilde{P_S}$ up to the quantity of goods that the supplier has left over. Again, the supplier can avoid the costs of selling to the spot market.

In this case the supplier's expected profit is

$$\pi(Q_P, s, g) = E_{D,P_S}[sQ^* + P_S \cdot \min(Q_P - q^*, D - q^*) + gq^* + (P_S - m)(Q_P - D)^+ - Q_P b], \tag{3.8}$$

and the manufacturer's expected costs are

$$C(Q) = E_{D,P_S}[P_S(D - q^*) + sQ + gq^*], \tag{3.9}$$

so that the total system profit from (3.8) and (3.9) after taking the expectation is

$$\pi_{DC}^T = Q_P(\overline{P_S} - m - b) - \overline{D} \cdot \overline{P_S} + m \left(\int_0^{Q_P} Df(D)dD + \int_{Q_P}^{\infty} Q_P f(D)dD \right). \tag{3.10}$$

We see that the expected supply chain profit equals the centralized profit $\pi_{DC}^T = \pi_C^T$ if the supplier's production quantity equals the centralized optimal, $Q_P = Q_{CP}^*$. Notice from (3.10) that the values of s and g do not affect the total system profit in this case as long as they induce the supplier's production quantity to equal the optimal centralized solution, $Q_P = Q_{CP}^*$.

Now we will show that the supplier sets the production quantity optimally (i.e., $Q_P = Q_{CP}^*$). The supplier solves:

$$\begin{aligned} \max\ \pi(Q_P, s, g) = E_{D,P_S}[&sQ^* + P_S \cdot \min(Q_P - q^*, D - q^*) \\ &+ gq^* + (P_S - m)(Q_P - D)^+ - Q_P b] \\ \text{s.t. } : &\ s \geq 0, g \geq 0, Q_P \geq Q^*. \end{aligned}$$

The profit function can be rewritten as

$$\begin{aligned} \pi(Q_P, s, g) = sQ^* + Q_P(\overline{P_S} - m - b) + m \left(\int_0^{Q_P} Df(D)dD + \int_{Q_P}^{\infty} Q_P f(D)dD \right) \\ - L_H(g) \left(\int_0^{Q^*} Df(D)dD + \int_{Q^*}^{\infty} Q^* f(D)dD \right). \end{aligned}$$

Differentiating with respect to Q_P yields

$$\frac{\partial \pi}{\partial Q_P} = (\overline{P_S} - m - b) + m(1 - F(Q_P))$$

and

$$\frac{\partial^2 \pi}{\partial Q_P^2} = -mf(Q_P) < 0.$$

Thus, the function is strictly concave in Q_P, and the optimal production quantity $Q_P^* = \max(Q, Q_{CP}^*)$. Thus, if the reservation quantity is less or equal to the centralized production quantity, $Q \leq Q_{CP}^*$, then the supplier will produce optimally for the supply chain, i.e. $Q_P = Q_{CP}^*$. Thus, $Q \leq Q_{CP}^*$ is a sufficient condition for supply chain coordination when renegotiation is allowed.

Now we have to find (s, g) such that corresponding reservation quantity Q satisfies $Q \leq Q_{CP}^*$ and the manufacturer has an incentive to choose the contract with renegotiation rather than a contract without renegotiation. As shown in Golovachkina and Bradley (2002), the prices (s_o, g_o) in the optimal contract for options without renegotiation (here we use o to indicate those *original* optimal contract parameters without renegotiation) when the supplier is the leader satisfy this condition because the corresponding reservation quantity satisfies $Q_o \leq Q_{CP}^*$ due to double marginalization. However, the supplier is the only party who benefits from renegotiation and the manufacturer is indifferent between renegotiating and buying directly on the spot market. Therefore, to motivate the manufacturer to accept this agreement the supplier can, for example, increase the manufacturer's cost savings by decreasing either s or g, from s_o and g_o, so that the manufacturer would prefer renegotiation and the corresponding manufacturer's order quantity would still satisfy $Q \leq Q_{CP}^*$ and lie within the interval $[Q_o, Q_{CP}]$.

Notice that in this case coordination is achieved under voluntary compliance from the supplier because her production quantity, Q_P, is set independently of the manufacturer's reservation quantity. However, coordination can only be achieved if both the supplier and the manufacturer comply with the terms of the contract which includes renegotiating at the stage 2. The manufacturer gains nothing from renegotiating the price or quantity at the later stage unless, contrary to our assumption of a nondifferentiated homogenous commodity, the supplier's goods are somehow superior to those on the spot market. (This can, in fact, be the case, for example, where steel on the spot market might meet the technical content requirements, but be more prone to quality defects such as surface imperfections.) Barring such differentiation, and assuming a greedy manufacturer, the supplier should not expect the manufacturer to renegotiate. Yet, research in organizational theory shows that such behavior does sometimes occur due to "embeddedness" in inter-firm relationships. Uzzi (1997) shows that owners and managers in the position of our manufacturer are, in fact, willing to help owners and managers in the position of our supplier, even at a cost. Embeddedness causes owners and agents of firms to occasionally provide consideration in addition to previously agreed upon terms when unforeseen circumstances compromise the position of the other party, and is due to interpersonal relationships, trust, and the tacit expectation that such consideration will flow in both directions. In a word, Uzzi's work shows that selfish behavior should not be expected in all instances, and especially where transactions are at less than arm's length.

Using (3.10), we can demonstrate numerically that when renegotiation is expected, coordination can be achieved even if we restrict our attention to a single-price contract (i.e., a contract with a reservation price $s > 0$ only and $g = 0$). As our numerical examples suggest, the corresponding $Q_s \leq Q^*_{CP}$ (we use subscript s for parameters corresponding to the optimal *single-price* contract), which is sufficient for the coordination of the channel. Tables 3.2 and 3.3 demonstrates the absolute improvement in system profit, $\Delta = \pi_C^T - \pi_s^T$, and the relative improvement, $\rho = \frac{\pi_C^T - \pi_S^T}{\pi_C^T - \pi_{DNC}^T}$, that renegotiation brings to the system when this contract is used. Renegotiation is particularly effective when the contract for options is not, specifically when either the coefficient of variation of demand or the spot price is high.

3.6.1 Coordination under Information Asymmetry

In this section we show how the renegotiation allows coordination of the channel when the supplier and the manufacturer have asymmetric information about the demand that the manufacturer faces. Furthermore, in this section we assume that the supplier is the leader, in which case coordination is not achieved without renegotiation. We assume that a demand signal $\theta \in [\underline{\theta}, \overline{\theta}]$ can be observed by the manufacturer but not by the supplier. We assume that θ has c.d.f. $G(\theta)$, which is common knowledge to both the supplier and the manufacturer. The demand distribution c.d.f. given θ is $F(D|\theta)$, again is common knowledge to both players, and it is stochastically increasing in θ.

As we saw before, when renegotiation is allowed, the channel is coordinated when the supplier sets the production quantity such that $Q_P = Q_{CP}^*$ and the manufacturer sets the reservation quantity such that $Q \leq Q_{CP}^*$. Consider the supplier's problem first. If the supplier can profitably sell to the spot market, $\overline{P}_S - m - b \geq 0$, then for any value of θ the supplier sets $Q_P = K = Q_{CP}^*$. If the supplier sells to the spot market at a loss, $\overline{P}_S - m - b < 0$, then the supplier sets Q_P according to the newsvendor solution:

$$E_\theta[F(Q_P|\theta)] = \frac{\overline{P}_S - b}{m}.$$

Table 3.2: A single-price contract with unrestricted capacity for triangular spot price and demand distributions (Part 1).

			$\bar{P}_S - m - b \geq 0$				$\bar{P}_S - m - b < 0$			
CV_{SP}	CV_D	$m/\bar{P}_S$	Q_{CP}	Q_S	Δ	ρ	Q_{CP}	Q_S	Δ	ρ
0.05	0.05	0.1	300	26.06	11.81	0.131	30.73	26.04	6.09	0.105
0.05	0.05	0.3	300	26.18	34.38	0.127	28.67	26.04	3.47	0.062
0.05	0.05	0.7	300	26.42	75.12	0.119	27.75	26.04	2.25	0.041
0.05	0.20	0.1	300	17.54	37.38	0.415	32.48	17.2	19.44	0.361
0.05	0.20	0.3	300	19.28	96.66	0.358	25.5	17.2	10.59	0.236
0.05	0.20	0.7	300	21.87	173.72	0.276	22.38	17.2	6.47	0.158
0.05	0.40	0.1	300	9.74	61.09	0.679	35.14	8.17	31.06	0.657
0.05	0.40	0.3	300	15.46	135.58	0.502	20.67	7.93	13.43	0.465
0.05	0.40	0.7	300	21.44	212.61	0.337	14.22	7.53	5.99	0.295
0.20	0.05	0.1	300	26.06	11.81	0.131	30.73	26.04	6.09	0.105
0.20	0.05	0.3	300	26.18	34.38	0.127	28.67	26.04	3.47	0.063
0.20	0.05	0.7	300	26.42	75.12	0.119	27.75	26.04	2.25	0.041
0.20	0.20	0.1	300	17.54	37.38	0.415	32.48	17.2	19.44	0.361
0.20	0.20	0.3	300	19.28	96.66	0.358	25.5	17.2	10.59	0.236
0.20	0.20	0.7	300	21.87	173.72	0.276	22.38	17.2	6.47	0.158

Thus, the supplier needs to know the true value of θ in this case in order to choose the right production quantity.

Now consider the manufacturer's problem in this case. From (3.9) it follows

Table 3.3: A single-price contract with unrestricted capacity for triangular spot price and demand distributions (Part 2).

			$\bar{P}_S - m - b \geq 0$				$\bar{P}_S - m - b < 0$			
CV_{SP}	CV_D	$m/\bar{P}_S$	Q_{CP}	Q_S	Δ	ρ	Q_{CP}	Q_S	Δ	ρ
0.20	0.40	0.1	300	9.74	61.09	0.679	35.14	8.17	31.06	0.657
0.20	0.40	0.3	300	15.46	135.58	0.502	20.67	7.93	13.43	0.465
0.20	0.40	0.7	300	21.44	212.61	0.337	14.22	7.53	5.99	0.295
0.40	0.05	0.1	300	26.06	11.81	0.131	30.73	26.04	6.09	0.105
0.40	0.05	0.3	300	26.18	34.38	0.127	28.67	26.04	3.47	0.063
0.40	0.05	0.7	300	26.42	75.12	0.119	27.75	26.04	2.25	0.041
0.40	0.20	0.1	300	17.54	37.38	0.415	32.48	17.2	19.44	0.361
0.40	0.20	0.3	300	19.28	96.66	0.358	25.5	17.2	10.59	0.236
0.40	0.20	0.7	300	21.87	173.72	0.276	22.39	17.2	6.47	0.158
0.40	0.40	0.1	300	9.74	61.09	0.679	35.14	8.17	31.06	0.657
0.40	0.40	0.3	300	15.46	135.58	0.502	20.67	7.93	13.43	0.465
0.40	0.40	0.7	300	21.44	212.61	0.337	14.22	7.53	5.99	0.295

that he sets his reservation quantity according to

$$1 - F(Q|\theta) = \frac{s}{\int_g^\infty (P_S - g)h(P_S)dP_S}. \qquad (3.11)$$

Thus, given s and g, the manufacturer's order quantity is determined by θ, and the supplier can infer the true value of θ from the reservation quantity Q chosen by the manufacturer. After observing the signal $\theta_{observed}$ indirectly through the manufacturer's reservation quantity the supplier can set the production order quantity optimally, so that

$$F(Q_P|\theta_{observed}) = \frac{\overline{P}_S - b}{m}, \tag{3.12}$$

which results in $Q_P = Q^*_{CP}$, because $\theta_{observed} = \theta$.

To achieve channel coordination we also need $Q \leq Q^*_{CP}$. This condition is always satisfied if $\overline{P_S} - m - b \geq 0$, because in this case the supplier produces up to her capacity, $Q_P = K$ and the manufacturer chooses the reservation quantity which is the smallest of his unconstrained optimal quantity and the supplier's capacity, $Q = \min(Q^*, K)$. Thus, no matter what s and g the supplier chooses, the manufacturer sets $Q \leq Q^*_{CP} = K$. If $\overline{P_S} - m - b < 0$, then the optimal supply chain performance is guaranteed by the condition $Q \leq Q^*_{CP}$, which is equivalent to

$$F(Q|\theta) \leq F(Q^*_{CP}|\theta)$$

which, as follows from (3.11) and (3.12), is in turn equivalent to

$$1 - \frac{s}{\int_g^\infty (P_S - g)h(P_S)dP_S} \leq \frac{\overline{P}_S - b}{m}. \tag{3.13}$$

Therefore, the supplier needs to choose s and g that satisfy (3.13). We now show that when the supplier chooses prices (s, g) as in the *original* contract for options, then condition (3.13) is satisfied.

Indeed, when $\overline{P_S} - m - b < 0$, the supplier solves the following equation to find the optimal s:

$$Q^* - \frac{1}{E_\theta r_F(Q^*|\theta)} \cdot \left[\frac{m \int_g^\infty h(P_S)dP_S}{\int_g^\infty (P_S - g)h(P_S)dP_S} - \frac{b + m - \overline{P_S}}{s} \right] = 0.$$

Since $Q \geq 0$, it follows that we must have

$$\frac{m \int_g^\infty h(P_S)dP_S}{\int_g^\infty (P_S - g)h(P_S)dP_S} - \frac{b + m - \overline{P_S}}{s} \geq 0,$$

which is equivalent to

$$\frac{s}{\int_g^{\infty}(P_S - g)h(P_S)dP_S} \geq \frac{b + m - \overline{P_S}}{m\int_g^{\infty} h(P_S)dP_S}. \tag{3.14}$$

¿From (3.14) follows that

$$1 - \frac{s}{\int_g^{\infty}(P_S - g)h(P_S)dP_S} \leq 1 - \frac{b + m - \overline{P_S}}{m\int_g^{\infty} h(P_S)dP_S} \leq 1 - \frac{b + m - \overline{P_S}}{m} = \frac{\overline{P_S} - b}{m}$$

Thus, (s, g) that are chosen in this case satisfy (3.13).

Thus, renegotiation is a powerful tool that helps to improve system performance even when the parties have asymmetric information about the demand distribution. Because the supplier's production quantity is independent of the manufacturer's reservation quantity when the parties agree to renegotiate at the later stage, the supplier can screen the manufacturer for his true demand type without sacrificing system efficiency. As follows from (3.10) the total system profit in this case is independent of the values of the transfer prices that the supplier charges to the manufacturer. Thus, the supplier's choice of the reservation and exercise price does not affect the performance of the system in this case as long as these prices satisfy (3.13).

Numerical Example

In this section we provide a simple example of how information asymmetry affects the distribution of the profits in the system. We consider a case when $\theta = \overline{D}$. We use a triangular distribution with $Var = 130.67$ to model the demand distribution, and set $\theta_1 = 30$ and $\theta_2 = 60$, with $\Pr(\theta_1) = 0.5, \Pr(\theta_2) = 0.5$. To model the spot price, we use a triangular distribution with $\overline{P}_S = 30$ and $Var = 30.38$. We also set $m = 21$, $b = 0.5$ and $K = 300$. Table 4.2 shows the contract parameters and profit distribution in each case. The first two lines in the table, for comparison, represent the full information cases when the demand is either type 1 or type 2. The next two lines show the results for the cases when either type 1 or 2 demand is observed by the manufacturer but not by the supplier. The profit π_R is the supplier's profit in this case, π^T is the profit of the whole system, and C is the associated manufacturer's cost. The supplier's profit π_{NR} without renegotiation is given for comparison. As we can see the manufacturer benefits from having sole knowledge of the demand signal θ.

Table 3.4: Optimal contract parameters for triangular spot price and demand distributions with information asymmetry and renegotiation.

$Pr[\theta_1]$	$Pr[\theta_2]$	$\theta_{observed}$	s	g	Q	C	π_R	π_{NR}	π^T
1	0	θ_1	5.32	19.78	29.46	799.89	3079.9	2960.6	2280
0	1	θ_2	8.54	18.49	52.13	1664.9	3674.9	3461.6	2010
0.5	0.5	θ_1	3.83	20.52	32.91	771.91	3051.9	2956.6	2280
0.5	0.5	θ_2	3.83	20.52	62.91	1500.7	3510.7	3387.4	2010

3.7 Conclusions

In this paper we analyzed three different mechanisms that lead to supply chain coordination when a spot market is present. We demonstrated that the channel can be coordinated with a contract for options when the manufacturer is the leader, with a quantity discount contract and with a contract for options when renegotiation is allowed.

The supply chain is coordinated with a contract for options when the manufacturer is the leader and the supplier can sell profitably to the spot market. In this case the manufacturer extracts all benefits that the contract brings into the system and leaves nothing to the supplier, but the supplier's risk due to spot price uncertainty is decreased. The practical implementation probably requires the manufacturer to give some transfer payment to the supplier in order to motivate her to use this contract. We showed the optimal contract to be equivalent to Hirshleifer's classic result in the case of a competitive market for an intermediate good, which states that the channel is coordinated in this case if the transfer price is equal to the competitive market price.

Another way to coordinate the channel is with a quantity discount contract. However, the leader, who proposes the contract price, is the only one who benefits from the contract. Thus, again, a transfer payment might be required in order to implement this mechanism. Specifically, a transfer payment should be made contingent upon the reservation quantity chosen by the follower in this case, and should be positive only if the reservation quantity Q satisfies $Q = Q^*_{CP}$.

We showed that the contract for options coordinates the supply chain if the parties can renegotiate the exercise price and quantity after the demand and the spot price are observed. Renegotiation also coordinates the channel with a single-price contract and when the supplier and the manufacturer have asymmetric information about the manufacturer's demand. When the supplier anticipates renegotiation, she can screen the manufacturer for his true demand type and then set her production quantity using the information that she infers about demand from the manufacturer's reservation quantity. For the types of the contracts that we considered the supplier is the only one who benefits from renegotiation and, thus, she might find it useful to motivate the manufacturer to renegotiate by offering better terms of trade. Fortunately, that flexibility is available by adjustment of the contract parameters with either a single-price contract or contract for options. Thus, renegotiation appears to be a robust mechanism that coordinates the channel under a wide variety of circumstances. Moreover, unlike the other mechanisms, the supplier and the manufacturer share the contract benefits.

One possible extension to this problem is to consider the effect of renegotiation on a similar system with multiple manufacturers to assess whether renegotiation between the supplier and a manufacturer or renegotiation between a manufacturer and another manufacturer is more effective.

Chapter 4

Allocation Rules and Supply Chain Coordination

4.1 Introduction

In this paper we analyze a model in which two manufacturers and a single supplier invest in capacity prior to the realization of demand. All players make their capacity investment decisions at the same time in a first stage and their capacity choice constrains their ability to satisfy demand at a later stage. The supplier must also choose an allocation rule that determines how the supplier's capacity is divided between the manufacturers when it is insufficient to satisfy their total demand. We consider two different manipulable allocation rules (proportional and linear) in order to assess the effect that the allocation rule has on the supplier's and manufacturers' capacity choice. We also evaluate how transfer prices influence the players' capacity choices. We compare the performance of a linear-price contract with that of a contract for options.

We add to the existing literature by modeling capacity constraints at the manufacturers' and the supplier's sites. We show that with a linear-price contract the manufacturers' capacity investment decisions are independent of the allocation rule. In fact, the allocation rule is important to the manufacturer only when he places his actual order and not when he makes his capacity investment.

We show that with linear-price contract the players' profit functions are concave in their respective capacity levels. We show that with the contract for options the each manufacturer's profit function is concave in both its reservation quantity and its capacity level. We also show that the supplier's profit function is concave in reservation price and in exercise price.

Our numerical results show that with both contracts the supply chain's profit can be equal to at least 92% of the centralized profit, but with a linear-price contract that is only possible if the supplier is the one who sets the prices. The administration of either contract presents a certain level of complexity. With a linear-price contract the supplier must use an allocation rule. With the contract for options the manufacturer might need to select an appropriate penalty cost that assures that the supplier indeed builds sufficient capacity to fulfill the contract.

In Section 4.2 we provide a literature review. We describe our model and provide analytical and numerical analysis in Section 4.3. We present our conclusions in Section 4.4.

4.2 Literature Review

In the recent years much attention has been given to the performance of contracts in decentralized supply chains. Cachon (2003) gives a very good review on the most recent literature in that area. One of the contracts that he analyzes, and that we analyze in this paper is the contract for options. Several papers consider the contract for options, its ability to coordinate the supply chain and ensure incentive compatibility for both players. Barnes-Schuster et al. (2002) investigate a two-period model with a single supplier acting as a leader and a single buyer, in which the buyer can purchase a certain number of options as well as place a firm order each period. Cachon and Lariviere (2001) consider a similar model (where the buyer acts as a leader), and analyze two different compliance regimes (voluntary compliance and forced compliance) and two different information sharing cases (full information and asymmetric information). The authors show that the contract compliance regime significantly affects the outcome of the game. One of the conclusions that both papers make is that the contract for options while significantly improving performance of the decentralized supply chain does not lead to the channel coordination. Barnes-Schuster et al. also show that the supplier is able to extract all benefits generated by the contract for options.

The problem where all the players have to make their capacity investments prior to the demand realization is considered in Tomlin (2003). The author compares the performance of piecewise-linear, quantity-discount and quantity-premium contracts with linear-price contract in the supply chain with a single supplier and a single manufacturer. The author shows the existence of coordinating price-only contracts that arbitrarily allocate the supply chain profit and demonstrates that for certain values of the supplier's reservation profit the manufacturer prefers a simple piecewise-linear, quantity-premium contract.

The supplier's allocation problem was considered in Cachon and Lariviere (1999). The authors analyze the behavior of the multiple retailers under different allocation mechanisms: truth-inducing and manipulable. They also consider the supplier's capacity decision under different allocation schemes. The authors show that under manipulable allocation mechanisms the profits of the supplier, the retailers, and the whole supply chain are higher than under truth-inducing mechanisms. Deshpande and Schwarz (2002) is another paper that considers different allocation rules in the supply chain with multiple buyers. The authors show that if manipulable mechanisms are replaced by a certain optimal truth-telling allocation mechanism then the supplier's and the supply chain's profits can be significantly increased.

We add to this literature by considering the effect of two manipulable allocation mechanisms on the players' capacity investment decisions. We also show that the contract for options is able to significantly improve the profits of the supplier and the supply chain eventhough no allocation rule is needed. This result is similar to the one of Plambeck and Taylor (2002). The authors consider how the possibility of renegotiation affects the supply chain with one supplier and two buyers, where players engage in quantity flexibility contracts. The supplier is the only one who must invest in the capacity. The authors demonstrate that in many cases the contract for options provides a better alternative to the quantity flexibility contract with renegotiation and significantly improves the supply chain performance. We show that the contract for options outperforms linear-price contracts in the system with two manufacturers but gives the manufacturers no incentive to use it.

4.3 The Model

In this chapter we consider a supply chain in which two identical manufacturers contract with a single supplier for available resources. Both the supplier and the two manufacturers must invest in their capacity before the demand for the end product is known. The players incur a marginal capacity cost that is c_M for the manufacturer and c_S for the supplier. After demand is realized the manufacturers order and pay for the supplier's component, which is produced at the marginal cost p_S, and use it to produce their final product at marginal production cost p_M. They sell this product to their customer at a fixed retail price r with $r > c_M+c_S+p_M+p_S$. We use x_1 and x_2 to denote the respective demands of manufacturers 1 and 2. We use $f(x)$ and $F(x)$ to denote pdf and cdf of the manufacturers' demand. The sequence of events is depicted in Figure 4.1.

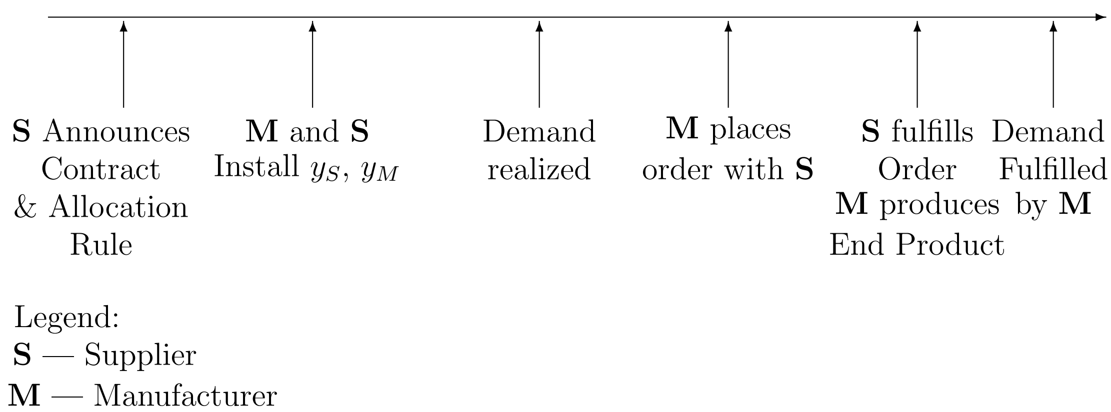

Figure 4.1: Event Timeline

4.3.1 Centralized System

In this section we consider how the players set their capacity levels in the centralized system. This is the benchmark by which we will judge the other contracts in a decentralized supply chain. A single decision-maker chooses the capacity levels of the supplier y_S and each manufacturer y_M that maximize the expected profit of the supply chain, $\pi_C(y_S, y_M)$,

$$
\begin{aligned}
\pi_C(y_S, y_M) = E_{x_1,x_2} &\left[-c_S y_S - 2c_M y_M \right. \\
&\left. + (r - p_M - p_S)\left(\min[y_S, \min(y_M, x_1), \min(y_M, x_2)]\right)\right].
\end{aligned}
$$

After taking the expectations, we have

$$\begin{aligned}\pi_C(y_S, y_M) = -c_S y_S - 2c_M y_M + (r - p_M - p_S)\\
\Bigg(\int_0^{y_S-y_M} f(x_1)\int_0^{y_M}(x_1+x_2)f(x_2)dx_2dx_1\\
+\int_0^{y_S-y_M} f(x_1)\int_{y_M}^{\infty}(x_1+y_M)f(x_2)dx_2dx_1\\
+\int_{y_S-y_M}^{y_M} f(x_1)\int_0^{y_S-x_1}(x_1+x_2)f(x_2)dx_2dx_1\\
+\int_{y_S-y_M}^{y_M} f(x_1)\int_{y_S-x_1}^{\infty} y_S f(x_2)dx_2dx_1\\
+\int_{y_M}^{\infty} f(x_1)\int_0^{y_S-y_M}(x_2+y_M)f(x_2)dx_2dx_1\\
+\int_{y_M}^{\infty} f(x_1)\int_{y_S-y_M}^{\infty} y_S f(x_2)dx_2dx_1\Bigg).\end{aligned}$$

We can show that this function is concave in y_S and $y_{M.}$ Indeed,

$$\begin{aligned}\frac{\partial\pi_C}{\partial y_M} &= 2\left(-c_M + (r-p_M-p_S)\int_0^{y_S-y_M} f(x_1)\int_{y_M}^{\infty} f(x_2)dx_2dx_1\right),\\
\frac{\partial\pi_C}{\partial y_S} &= -c_S + (r-p_M-p_S)\cdot\\
&\left(\int_{y_S-y_M}^{y_M} f(x_1)\int_{y_S-x_1}^{\infty} f(x_2)dx_2dx_1 + \int_{y_M}^{\infty} f(x_1)\int_{y_S-y_M}^{\infty} f(x_2)dx_2dx_1\right),\\
\frac{\partial^2\pi_C}{\partial y_M^2} &= 2(r-p_M-p_S)\left(-f(y_M)F(y_S-y_M) - f(y_S-y_M)(1-F(y_M))\right) < 0,\\
\frac{\partial^2\pi_C}{\partial y_S^2} &= (r-p_M-p_S)\cdot\\
&\left(-2f(y_S-y_M)(1-F(y_M)) - \int_{y_S-y_M}^{y_M} f(y_S-x)f(x)dx\right) < 0,\\
\frac{\partial^2\pi_C}{\partial y_S\partial y_M} &= 2(r-p_M-p_S)f(y_S-y_M)(1-F(y_M)),\end{aligned}$$

from which the concavity of function $\pi_C(y_S, y_M)$ follows. Thus, we can find the optimal capacity y_S^C and y_M^C levels from:

$$\int_0^{y_S^C-y_M^C} f(x_1)\int_{y_M^C}^{\infty} f(x_2)dx_2dx_1 = \frac{c_M}{(r-p_M-p_S)}, \tag{4.1}$$

$$\left(\int_{y_S^C-y_M^C}^{y_M^C} f(x_1)\int_{y_S^C-x_1}^{\infty} f(x_2)dx_2dx_1 + (1-F(y_M^C))\int_{y_S^C-y_M^C}^{\infty} f(x_2)dx_2\right.$$
$$= \frac{c_S}{(r-p_M-p_S)}. \tag{4.2}$$

It follows that the amount of capacity reserved by the manufacturer increases in the level of the supplier's capacity y_S^C:

$$\frac{\partial y_M^C}{\partial y_S^C} = \frac{f(y_S^C-y_M^C)(1-F(y_M^C))}{f(y_S^C-y_M^C)(1-F(y_M^C))+f(y_M^C)F(y_S^C-y_M^C)} > 0.$$

4.3.2 Linear-Price Contract

Now we consider the players' capacity investments when system is decentralized. In this case, the capacity level chosen by the player maximizes its private profit function. Since the supplier might not build enough capacity to satisfy the manufacturers' orders, she must also choose an allocation rule that she will use when the total quantity ordered by the manufacturers exceeds the amount that she can produce given her capacity constraint. We want to see how the allocation rules affects the capacity decisions of the players. We consider two different allocation schemes: linear and proportional allocation. We assume that the supplier charges the manufacturers a transfer price p_T for each unit purchased.

We start with the supplier's problem, which does not depend on the allocation rule and can be stated as:

$$
\begin{aligned}
\max_{y_S} \pi_S(y_S) &= -c_S y_S + (p_T - p_S) \\
&\Bigg(\int_0^{y_S - y_M} f(x_1) \int_0^{y_M} (x_1 + x_2) f(x_2) dx_2 dx_1 \\
&+ \int_0^{y_S - y_M} f(x_1) \int_{y_M}^{\infty} (x_1 + y_M) f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_M} f(x_1) \int_0^{y_S - x_1} (x_1 + x_2) f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_M} f(x_1) \int_{y_S - x_1}^{\infty} y_S f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_0^{y_S - y_M} (x_2 + y_M) f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_{y_S - y_M}^{\infty} y_S f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}
$$

Since

$$
\frac{\partial^2 \pi_S}{\partial y_S^2} = (p_T - p_S) \left(-2 f(y_S - y_M)(1 - F(y_M)) - \int_{y_S - y_M}^{y_M} f(y_S - x) f(x) dx \right) < 0,
$$

the function is concave in y_S and we can find optimal supplier's capacity y_S by solving $\frac{\partial \pi_S}{\partial y_S} = 0$, i.e.,

$$
\begin{aligned}
(p_T - p_S) \Bigg(\int_{y_S - y_M}^{y_M} f(x_1) \int_{y_S - x_1}^{\infty} f(x_2) dx_2 dx_1 + \int_{y_M}^{\infty} f(x_1) \int_{y_S - y_M}^{\infty} f(x_2) dx_2 dx_1 \\
= c_S. \qquad (4.3)
\end{aligned}
$$

Now we consider each manufacturer's problem with different allocation schemes. We assume for now that due to information symmetry players are not able to manipulate their orders in order to increase their profits. We will relax this assumption later on in order to evaluate the effect of information asymmetry on the manufacturers' decisions.

Proportional Allocation

With the proportional allocation mechanism, given manufacturer $i's$ order m_i, he is allocated

$$g_i(m) = \min\left(m_i, \frac{y_S m_i}{\sum_j m_j}\right).$$

Thus, each manufacturer's expected profit in this case is

$$\begin{aligned}
\pi_M(y_S) &= -c_M y_M + (r - p_M - p_T) \\
&\Bigg(\int_0^{y_S - y_M} x_1 f(x_1) dx_1 + \int_{y_S - y_M}^{y_M} f(x_1) \int_0^{y_S - x_1} x_1 f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_M} f(x_1) \int_{y_S - x_1}^{y_M} \frac{x_1 y_S}{x_1 + x_2} f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_M} f(x_1) \int_{y_M}^{\infty} \frac{x_1 y_S}{x_1 + y_M} f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_{y_M}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_0^{y_S - y_M} y_M f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_{y_S - y_M}^{y_M} \frac{y_M y_S}{x_2 + y_M} f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}$$

The function is concave in y_M since

$$\frac{\partial^2 \pi_M}{\partial y_M^2} = (r - p_M - p_T)\left(-f(y_M)F(y_S - y_M) - f(y_S - y_M)(1 - F(y_M))\right) < 0,$$

and the optimal y_M is a solution of

$$(r - p_M - p_T) \int_0^{y_S - y_M} f(x_1) \int_{y_M}^{\infty} f(x_2) dx_2 dx_1 = c_M. \tag{4.4}$$

Linear Allocation

If we index N manufacturers in the decreasing order of their order quantities, i.e., $(m_1 \geq m_2 \geq ... \geq m_N)$, then the quantity allocated to the manufacturer i with linear allocation is

$$g_i(m,\widetilde{n}) = \begin{cases} m_i - \frac{1}{\widetilde{n}}\max(0, \sum_j m_j - K), & i \le \widetilde{n} \\ 0, & i > \widetilde{n} \end{cases}$$

where $\widetilde{n}$ is the largest integer less than or equal to N such that $g_i(m,\widetilde{n}) \ge 0$.

Each manufacturer's expected profit in this case is

$$\begin{aligned}
\pi_M(y_S) \quad = \quad & -c_M y_M + (r - p_M - p_T) \\
& \Bigg(\int_0^{y_S - y_M} x_1 f(x_1) dx_1 + \int_{y_S-y_M}^{y_M} f(x_1) \int_0^{y_S - x_1} x_1 f(x_2) dx_2 dx_1 \\
& + \int_{y_S-y_M}^{y_M} f(x_1) \int_{y_S-x_1}^{y_M} \frac{x_1 - x_2 + y_S}{2} f(x_2) dx_2 dx_1 \\
& + \int_{y_S-y_M}^{y_M} f(x_1) \int_{y_M}^{\infty} \frac{x_1 - y_M + y_S}{2} f(x_2) dx_2 dx_1 \\
& + \int_{y_M}^{\infty} f(x_1) \int_{y_M}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
& + \int_{y_M}^{\infty} f(x_1) \int_0^{y_S-y_M} y_M f(x_2) dx_2 dx_1 \\
& + \int_{y_M}^{\infty} f(x_1) \int_{y_S-y_M}^{y_M} \frac{y_M + y_S - x_2}{2} f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}$$

The function is concave in y_M since

$$\frac{\partial^2 \pi_M}{\partial y_M^2} = (r - p_M - p_T) \left(-f(y_M) F(y_S - y_M) - f(y_S - y_M)(1 - F(y_M)) \right) < 0,$$

and the optimal y_M is a solution of

$$(r - p_M - p_T) \int_0^{y_S - y_M} f(x_1) \int_{y_M}^{\infty} f(x_2) dx_2 dx_1 = c_M. \tag{4.5}$$

¿From (4.1), (4.2), (4.3), (4.4) and (4.5), we can see that with linear or proportional allocation there does not exist p_T such that the centralized solution is achieved. Still, concavity of the supplier's and the manufacturers' profit functions guarantees the existence of the Nash Equilibrium (NE) in this game. Now we will show that this equilibrium is unique.

Suppose that $R_i(a_j)$ is a reaction function of the player i to the action chosen by the player j. The sufficient condition for the NE to be unique is

$$\left|\frac{\partial R_i(a_j)}{\partial a_j}\right| = \left|-\frac{\frac{\partial^2 \pi(R_i(a_j))}{\partial a_i \partial a_j}}{\frac{\partial^2 \pi(R_i(a_j))}{\partial a_i^2}}\right| < 1.$$

This condition holds for our problem because

$$\begin{aligned}
\left|\frac{\partial R_M(a_S)}{\partial a_S}\right| &= \frac{f(y_S - y_M)(1 - F(y_M))}{f(y_M)F(y_S - y_M) + f(y_S - y_M)(1 - F(y_M))} < 1,\\
\left|\frac{\partial R_S(a_M)}{\partial a_M}\right| &= \frac{2f(y_S - y_M)(1 - F(y_M))}{\int_{y_S - y_M}^{y_M} f(y_S - x)f(x)dx + 2f(y_S - y_M)(1 - F(y_M))} < 1.
\end{aligned}$$

Information Asymmetry

In this section we evaluate the effect that information asymmetry has on the manufacturers' capacity choices. When manufacturers possess private information about their actual demand they might choose to inflate their orders in order to maximize their profits. Both linear and proportional allocation rules give the manufacturers an opportunity to manipulate their order quantities in this manner. However, the manufacturer expects that the other manufacturer is also rational and manipulates his order quantity as well.

We start with proportional allocation. Each manufacturer's profit function in this case is

$$\begin{aligned}
\pi_M(y_S) &= -c_M y_M + (r - p_M - p_T) \\
&\Bigg(\int_0^{y_S - y_M} x_1 f(x_1) dx_1 + \int_{y_S - y_M}^{y_M} f(x_1) \int_0^{y_S - x_1} x_1 f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_S/2} f(x_1) \int_{y_S - x_1}^{y_M} x_1 f(x_2) dx_2 dx_1 \\
&+ \int_{y_S/2}^{y_M} f(x_1) \int_{y_S - x_1}^{y_S/2} (y_S - x_2) f(x_2) dx_2 dx_1 \\
&+ \int_{y_S/2}^{y_M} f(x_1) \int_{y_S/2}^{y_M} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&+ \int_{y_S - y_M}^{y_S/2} f(x_1) \int_{y_M}^{\infty} x_1 f(x_2) dx_2 dx_1 \\
&+ \int_{y_S/2}^{y_M} f(x_1) \int_{y_M}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_{y_S/2}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_0^{y_S - y_M} y_M f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^{\infty} f(x_1) \int_{y_S - y_M}^{y_S/2} (y_S - x_2) f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}$$

The function is concave in y_M since

$$\frac{\partial^2 \pi_M}{\partial y_M^2} = (r - p_M - p_T)\left(-f(y_M)F(y_S - y_M) - f(y_S - y_M)(1 - F(y_M))\right) < 0,$$

and the optimal y_M is a solution of

$$(r - p_M - p_T) \int_0^{y_S - y_M} f(x_1) \int_{y_M}^{\infty} f(x_2) dx_2 dx_1 = c_M. \tag{4.6}$$

In the same way, when the supplier uses linear allocation, each manufacturer's profit function is

$$
\begin{aligned}
\pi_M(y_S) &= -c_M y_M + (r - p_M - p_T) \\
&\quad \Bigg(\int_0^{y_S - y_M} x_1 f(x_1) dx_1 + \int_{y_S - y_M}^{y_M} f(x_1) \int_0^{y_S - x_1} x_1 f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_S - y_M}^{y_S/2} f(x_1) \int_{y_S - x_1}^{y_M} x_1 f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_S/2}^{y_M} f(x_1) \int_{y_S - x_1}^{y_S/2} (y_S - x_2) f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_S/2}^{y_M} f(x_1) \int_{y_S/2}^{y_M} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_S - y_M}^{y_S/2} f(x_1) \int_{y_M}^{\infty} x_1 f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_S/2}^{y_M} f(x_1) \int_{y_M}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_M}^{\infty} f(x_1) \int_{y_S/2}^{\infty} \frac{y_S}{2} f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_M}^{\infty} f(x_1) \int_0^{y_S - y_M} y_M f(x_2) dx_2 dx_1 \\
&\quad + \int_{y_M}^{\infty} f(x_1) \int_{y_S - y_M}^{y_S/2} (y_S - x_2) f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}
$$

This function is also concave in y_M because

$$
\frac{\partial^2 \pi_M}{\partial y_M^2} = (r - p_M - p_T)\left(-f(y_M)F(y_S - y_M) - f(y_S - y_M)(1 - F(y_M))\right) < 0,
$$

and the optimal y_M is a solution of

$$
(r - p_M - p_T) \int_0^{y_S - y_M} f(x_1) \int_{y_M}^{\infty} f(x_2) dx_2 dx_1 = c_M. \tag{4.7}
$$

¿From (4.5), (4.4), (4.6) and (4.7) it follows that the manufacturer builds the same amount of capacity even when he plans to manipulate his order at the next stage. Thus, the ability to manipulate the order quantity important for the manufacturers only at the second stage when the actual product is ordered.

4.3.3 Contract for Options

In this section we analyze whether the contract for options induces the players to build the optimal amount of capacity, or at least provides some improvement over the decentralized case with linear pricing.

The supplier offers the manufacturers a set of prices (s, g), where s is a reservation price per unit and g is an exercise price per unit. Each manufacturer reserves Q units of capacity. We assume that the supplier indeed builds $2Q$ units of capacity in order to satisfy manufacturers' orders at the next stage.

Since the supplier builds $2Q$, she need not use an allocation rule to distribute her capacity. Rather, if one of the players has demand less than Q and another one has demand greater than Q but smaller than y_M, the supplier can sell some additional quantity to the latter manufacturer at some transfer price p_T. For convenience we assume that $p_T = g$.

We start with the manufacturers' problem. Given (s, g), each manufacturer decides how much capacity to reserve from the supplier and how much capacity to build at his site. Consider the first manufacturer. His problem can be formulated as

$$\begin{aligned}\pi_M^1(y_M, Q) = E_x[&-sQ - c_M y_M + (r - p_M - g)\min(Q, y_M, x_1) \\ &+ (r - p_M - p_T)\min([\min(x_1, y_M) - Q]^+, [Q - \min(x_2, y_M)]^+)],\end{aligned}$$

or after taking the expectation

$$\begin{aligned}
\pi_M^1(y_M, Q) &= -sQ - c_M y_M + (r - p_M - g) \\
&\Bigg(\int_0^Q x_1 f(x_1) dx_1 + \int_Q^\infty Q f(x_1) dx_1 \\
&+ \int_Q^{y_M} (x_1 - Q) f(x_1) \int_0^{2Q - x_1} f(x_2) dx_2 dx_1 \\
&+ \int_Q^{y_M} f(x_1) \int_{2Q - x_1}^{Q} (Q - x_2) f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^\infty f(x_1) \int_0^{2Q - y_M} (y_M - Q) f(x_2) dx_2 dx_1 \\
&+ \int_{y_M}^\infty f(x_1) \int_{2Q - y_M}^{Q} (Q - x_2) f(x_2) dx_2 dx_1 \Bigg).
\end{aligned}$$

The second manufacturer faces an identical problem. The concavity of each manufacturer's profit function follows from

$$\begin{aligned}
\frac{\partial^2 \pi_M}{\partial y_M^2} &= -2(r - p_M - g)\left(f(y_M) F(2Q - y_M)\right. \\
&+ \left. f(2Q - y_M)(1 - F(y_M))\right) < 0, \\
\frac{\partial^2 \pi_M}{\partial Q^2} &= -4(r - p_M - g) f(2Q - y_M) \int_Q^\infty f(x) dx < 0, \\
\frac{\partial^2 \pi_M}{\partial Q \partial y_M} &= 2(r - p_M - g) f(2Q - y_M)(1 - F(y_M)).
\end{aligned}$$

Thus, we can calculate values of optimal y_M and Q from the following equations

$$(r - p_M - g) \int_0^{2Q - y_M} f(x_1) \int_{y_M}^\infty f(x_2) dx_2 dx_1 = c_M, \tag{4.8}$$

$$\begin{aligned}
&\Bigg((1 - F(Q))^2 + 2 \int_Q^{y_M} f(x_1) \int_{2Q - x_1}^{Q} f(x_2) dx_2 dx_1 \\
&+ 2(1 - F(y_M)) \int_{2Q - y_M}^{Q} f(x_2) dx_2 \Bigg) = \frac{s}{(r - p_M - g)}.
\end{aligned} \tag{4.9}$$

The supply chain is coordinated if $y_S^C = 2Q$ and $y_M^C = y_M$. ¿From (4.1), (4.2), (4.8) and (4.9) we conclude that this is possible only if $g = p_S$ and $s = c_S - (r -$

$p_M - p_S)(1 - F(y_S^C))$. But that is not incentive compatible for the supplier whose expected profit is negative in this case:

$$\pi_S(s, g) = -y_S^C(r - p_M - p_S)(1 - F(y_S^C) < 0.$$

Now we consider the supplier's problem. The supplier's expected profit function is

$$\begin{aligned}\pi_S(s,g) &= -2c_S Q + 2Qs + 2(g - p_S)\\ &\quad \Bigg(\int_0^Q x_1 f(x_1)dx_1 + \int_Q^\infty Q f(x_1)dx_1\\ &\quad + \int_Q^{y_M} (x_1 - Q) f(x_1) \int_0^{2Q-x_1} f(x_2)dx_2dx_1\\ &\quad + \int_Q^{y_M} f(x_1) \int_{2Q-x_1}^{Q} (Q - x_2) f(x_2)dx_2dx_1\\ &\quad + \int_{y_M}^\infty f(x_1) \int_0^{2Q-y_M} (y_M - Q) f(x_2)dx_2dx_1\\ &\quad + \int_{y_M}^\infty f(x_1) \int_{2Q-y_M}^{Q} (Q - x_2) f(x_2)dx_2dx_1\Bigg).\end{aligned}$$

Proposition 12 *If the demand distribution function is IFR, then the supplier's profit function $\pi_S(y_S)$ is concave in s for a fixed g, and concave in g for a fixed s .*

Proof. Assume that the demand distribution function is IFR. The second derivative of the supplier's profit function with respect to s is

$$\frac{\partial^2 \pi_S}{\partial s^2} = \frac{\partial Q}{\partial s}\left(\frac{r - p_M - p_S}{r - p_M - g} + 1\right) + \frac{\partial^2 Q}{\partial s^2}\left(\frac{r - p_M - p_S}{r - p_M - g}\right) \cdot s,$$

where

$$\frac{\partial Q}{\partial s} = -\frac{1}{4(r - p_M - g)\left[\int_Q^{y_M} f(2Q - x) f(x)dx + f(2Q - y_M)\int_{y_M}^\infty f(x)dx\right]} < 0$$

and

$$\frac{\partial^2 Q}{\partial s^2} = \frac{-f^2(Q) + 2\int_Q^{y_M} f'(2Q - x) f(x)dx + 2\int_{y_M}^\infty f'(2Q - y_M) f(x)dx}{4(r - p_M - g)\left[\int_Q^{y_M} f(2Q - x) f(x)dx + f(2Q - y_M)\int_{y_M}^\infty f(x)dx\right]^2} \cdot \frac{\partial Q}{\partial s}. \tag{4.10}$$

$\frac{\partial^2 Q}{\partial s^2} < 0$, because $\frac{\partial Q}{\partial s} < 0$ and the numerator of the first factor in (4.10) is positive because the demand distribution function is IFR. Thus, $\frac{\partial^2 \pi_S}{\partial s^2} < 0$ and π_S is concave in s.

The second derivative of the supplier's profit function with respect to g is

$$\frac{\partial^2 \pi_S}{\partial g^2} = 2\frac{\partial Q}{\partial g}\left(\frac{s}{(r-p_M-g)^2} + \frac{s}{r-p_M-g}\right) + 2\frac{\partial^2 Q}{\partial g^2}\left(\frac{r-p_M-p_S}{r-p_M-g}s - c_S\right),$$

where

$$\frac{\partial Q}{\partial g} = -\frac{s}{4(r-p_M-g)^2\left[\int_Q^{y_M} f(2Q-x)f(x)dx + f(2Q-y_M)\int_{y_M}^{\infty} f(x)dx\right]} < 0,$$

and

$$\frac{\partial^2 Q}{\partial g^2} = \frac{\left(-f^2(Q) + 2\int_Q^{y_M} f'(2Q-x)f(x)dx + 2\int_{y_M}^{\infty} f'(2Q-y_M)f(x)dx\right)s}{4(r-p_M-g)^2\left[\int_Q^{y_M} f(2Q-x)f(x)dx + f(2Q-y_M)\int_{y_M}^{\infty} f(x)dx\right]^2} \cdot \frac{\partial Q}{\partial g}$$
$$- \frac{s}{2(r-p_M-g)^3\left[\int_Q^{y_M} f(2Q-x)f(x)dx + f(2Q-y_M)\int_{y_M}^{\infty} f(x)dx\right]}.$$

$\frac{\partial^2 Q}{\partial g^2} < 0$, because $\frac{\partial Q}{\partial g} < 0$, and $2\int_Q^{y_M} f'(2Q-x)f(x)dx + 2\int_{y_M}^{\infty} f'(2Q-y_M)f(x)dx - f^2(Q) < 0$ for IFR distribution functions. Thus, $\frac{\partial^2 \pi_S}{\partial g^2} < 0$ and π_S is concave in g.

■

We can find optimal s and g from

$$\frac{\partial Q}{\partial s}\left(s - c_S + (g - p_S)\left(1 - F(Q)\right)^2 + 2\int_Q^{y_M} f(x_1)\int_{2Q-x_1}^{Q} f(x_2)dx_2dx_1 \right.$$
$$\left. + 2\int_{y_M}^{\infty} f(x_1)\int_{2Q-y_M}^{Q} f(x_2)dx_2dx_1 + Q = 0, \text{and}\right.$$
$$2\frac{\partial Q}{\partial g}\left(s - c_S + (g - p_S)\left((1 - F(Q))^2 + 2\int_Q^{y_M} f(x_1)\int_{2Q-x_1}^{Q} f(x_2)dx_2dx_1\right.\right.$$
$$+ 2\int_{y_M}^{\infty} f(x_1)\int_{2Q-y_M}^{Q} f(x_2)dx_2dx_1$$
$$+2\left(\int_0^Q x_1 f(x_1)dx_1 + \int_Q^{\infty} Qf(x_1)dx_1\right.$$
$$+ \int_Q^{y_M}(x_1 - Q)f(x_1)\int_0^{2Q-x_1} f(x_2)dx_2dx_1$$
$$+ \int_Q^{y_M} f(x_1)\int_{2Q-x_1}^{Q}(Q - x_2)f(x_2)dx_2dx_1$$
$$+ \int_{y_M}^{\infty} f(x_1)\int_0^{2Q-y_M}(y_M - Q)f(x_2)dx_2dx_1$$
$$\left.\left. + \int_{y_M}^{\infty} f(x_1)\int_{2Q-y_M}^{Q}(Q - x_2)f(x_2)dx_2dx_1\right) = 0,\right.$$

where

$$\frac{\partial Q}{\partial s} = -\frac{1}{4(r - p_M - g)\left[\int_Q^{y_M} f(2Q - x)f(x)dx + f(2Q - y_M)\int_{y_M}^{\infty} f(x)dx\right]}, \text{ and}$$

$$-\frac{\partial Q}{\partial g} = \frac{(1 - F(Q))^2 + 2\int_Q^{y_M} f(x_1)dx_1\int_{2Q-x_1}^{Q} f(x_2)dx_2 + 2\int_{y_M}^{\infty} f(x_1)dx_1\int_{2Q-y_M}^{Q} f(x_2)dx_2}{4(r - p_M - g)\left[\int_Q^{y_M} f(2Q - x)f(x)dx + f(2Q - y_M)\int_{y_M}^{\infty} f(x)dx\right]}.$$

We have assumed so far that the supplier builds $2Q$ units of capacity. We now find a penalty cost that, if charged by the manufacturer, would guarantee such capacity investment. Assume that the supplier builds y_S^o units of capacity and that the manufacturer can charge a penalty cost b for each unit that the supplier fails to deliver due to $y_S^o < 2Q$. The supplier's expected profit function in this case is

$$\begin{aligned}
\pi_S(y_S) \;=\; & -c_S y_S^o + 2Qs + (g - p_S) \\
& \left(\int_0^{y_S^o - Q} f(x_1) \int_0^Q (x_1 + x_2)\, f(x_2) dx_2 dx_1 \right. \\
& +2 \int_0^{y_S^o - Q} f(x_1) \int_Q^\infty (x_1 + Q) f(x_2) dx_2 dx_1 \\
& + \int_{y_S^o - Q}^{Q} f(x_1) \int_0^{y_S^o - x_1} (x_1 + x_2) f(x_2) dx_2 dx_1 \\
& + \int_{y_S^o - Q}^{Q} f(x_1) \int_{y_S^o - x_1}^\infty y_S^o f(x_2) dx_2 dx_1 \\
& + \int_Q^\infty f(x_1) \int_{y_S^o - Q}^\infty y_S^o f(x_2) dx_2 dx_1 \\
& + \int_0^{y_S^o - y_M} f(x_1) \int_Q^{y_M} (x_2 - Q) f(x_2) dx_2 dx_1 \\
& +2 \int_0^{y_S^o - y_M} f(x_1) \int_{y_M}^\infty (y_M - Q) f(x_2) dx_2 dx_1 \\
& +2 \int_{y_S^o - y_M}^{y_S^o - Q} f(x_1) \int_{y_S^o - x_1}^\infty (y_S^o - Q - x_1) f(x_2) dx_2 dx_1 \\
& + \int_{y_S^o - y_M}^{y_S^o - Q} f(x_1) \int_Q^{y_S^o - x_1} (x_2 - Q) f(x_2) dx_2 dx_1 \\
& + \int_Q^{y_M} f(x_1) \int_{y_S^o - x_1}^{y_S^o - Q} (y_S^o - Q - x_2) f(x_2) dx_2 dx_1 \\
& \left. + \int_Q^{y_M} f(x_1) \int_0^{y_S^o - x_1} (x_1 - Q) f(x_2) dx_2 dx_1 \right) \\
& -b \left(2 \int_{y_S^o - Q}^{Q} f(x_1) \int_Q^\infty (x_1 + Q - y_S^o) f(x_2) dx_2 dx_1 \right. \\
& + \int_{y_S^o - Q}^{Q} f(x_1) \int_{y_S^o - x_1}^{Q} (x_1 + x_2 - y_S^o) f(x_2) dx_2 dx_1 \\
& \left. + \int_Q^\infty f(x_1) \int_Q^\infty (2Q - y_S^o) f(x_2) dx_2 dx_1 \right).
\end{aligned}$$

The function is concave in y_S^o, because

$$\frac{\partial^2 \pi}{\partial y_S^2} = -b\left(2f(y_S^o - Q)(1-F(Q)) + \int_{y_S^o - Q}^{Q} f(y_S^o - x)f(x)dx\right)$$
$$-(g-p_S)\left(2f(y_S^o - y_M)(1-F(y_M)) + \int_{y_S^o - y_M}^{y_S^o} f(y_S^o - x)f(x)dx\right)$$

and

$$\frac{\partial^2 \pi}{\partial y_S^2} < 0.$$

Thus, optimal y_S^o is calculated from

$$0 = -c_S + b\left(\int_{y_S^o - Q}^{Q} f(x_1)\int_{y_S^o - x_1}^{\infty} f(x_2)dx_2dx_1 + \int_{Q}^{\infty} f(x_1)\int_{y_S^o - Q}^{\infty} f(x_2)dx_2dx_1\right) \tag{4.11}$$
$$+(g-p_S)\left(\int_{y_M}^{\infty} f(x_1)\int_{y_S^o - y_M}^{\infty} f(x_2)dx_2dx_1 + \int_{y_S^o - y_M}^{y_M} f(x_1)\int_{y_S^o - x_1}^{\infty} f(x_2)dx_2dx_1\right).$$

It follows from (4.11) that to assure that the supplier indeed builds $2Q$ units of the capacity, the manufacturer must set b at the following level

$$b = \frac{c_S - (g-p_S)\left(\int_{y_M}^{\infty} f(x_1)\int_{2Q-y_M}^{\infty} f(x_2)dx_2dx_1 + \int_{2Q-y_M}^{y_M} f(x_1)\int_{2Q-x_1}^{\infty} f(x_2)dx_2dx_1\right)}{\left(\int_Q^{\infty} f(x)dx\right)^2}.$$

4.3.4 Numerical Results

In this section we numerically evaluate the linear-price contract and contract for options. The following numerical results are presented for the uniform demand distribution $[0, a]$ and unless noted otherwise the system parameters are set at the following values: $r = 10, c_S = 1, c_M = 1, p_S = 1, p_M = 1$.

The numerical results demonstrate that neither the linear-price contract nor the contract for options coordinates the supply chain. Some of the results are shown in Tables 4.1 and 4.2. The contract for options performs better and guarantees at least 93% of the centralized system profit, but each manufacturer's profit is always zero in this case. With a linear-price contract, the manufacturers sometimes build extra capacity in anticipation of a lower than expected demand at the other manufacturer's site. On the other hand, with a contract for options the manufacturer builds as much capacity as he reserves from the supplier. Thus, he does not expect to benefit from low demand at the other manufacturer. However, the manufacturers as well as the supplier build more capacity with the contract for options than with linear-price contract, but less than they would build in the centralized system.

Table 4.1: Optimal capacity levels and corresponding expected profits in the centralized system.

a	y_S^C	y_M^C	π_C
100	143	80	455
110	157	88	499
120	171	96	545
130	185	104	590
140	200	112	636

As Figures 4.2 and 4.3 demonstrate, the supplier benefits most from a linear-price contract when the transfer price is the highest and the manufacturers' expected profits are approaching zero. The supply chain profit is also greater with a greater transfer price and represents 92% of the centralized system profit at the highest transfer price (for our parameter values) that is still incentive compatible

Table 4.2: Optimal contract parameters in linear-price contract and contract for options

a	p_T	y_S^L	y_M^L	π_S^L	π_M^L	s	g	Q	y_M^O	π_S^O	π_M^O	b
100	7.6	110	55	418	0.4	1.92	5.4	59	59	430	0	1.7
110	7.6	121	60.5	459	0.8	1.94	5.4	65	65	472	0	1.6
120	7.6	132	66	500	0.9	1.96	5.4	71	71	515	0	1.5
130	7.6	142	71	542	0.99	1.97	5.4	75	75	554	0	1.2
140	7.6	154	77	584	1.1	1.98	5.4	80	80	595	0	1.0

for the manufacturers. As the transfer price increases it becomes more profitable for the supplier to sell to the manufacturers, thus, she is willing to invest in more capacity. Higher transfer price makes the manufacturers more risk averse and less willing to invest in capacity. Thus, there is a value of p_T above which the manufacturers' capacity investments are equal exactly to the supplier's capacity investment.

We tested the effect that changes in that the supplier's and manufacturers' capacity investments and production costs had on the performance of the contract for options. We can see from Figure 4.4 that with a contract for options the supplier's and the supply chain's profits are decreasing in the values of c_M, c_S, p_M and p_S. The higher the values of those parameters the less willing the players are to invest in extra capacity and thus less capable of satisfying higher demand, ultimately sacrificing some portion of the supply chain's profit. Because the value of c_M has the highest effect on the manufacturers' capacity investment decisions it also has the greatest effect on the change in the profits. At the same time, the

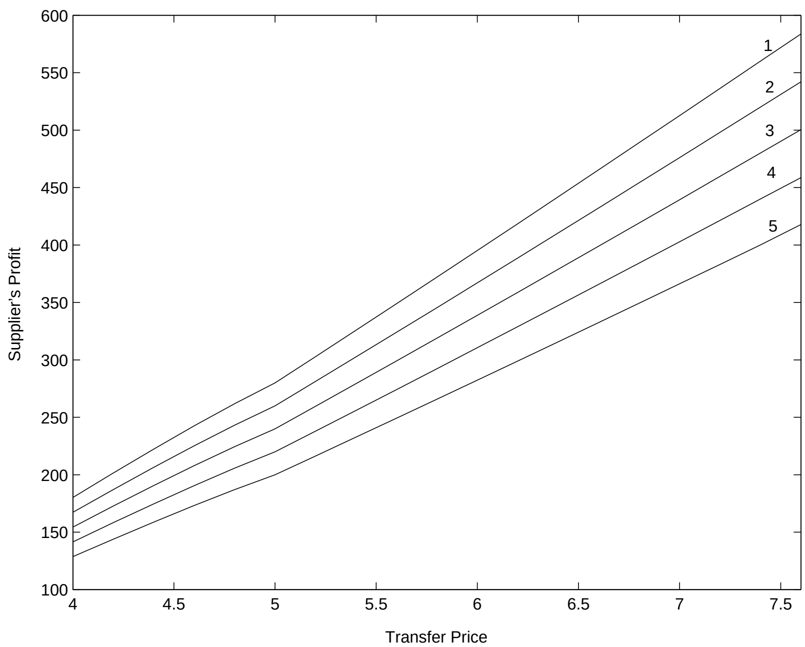

Figure 4.2: Supplier's Profit with Linear-Price Contract ($1.a = 140; 2.a = 130; 3.a = 120; 4.a = 110; 5.a = 100.$)

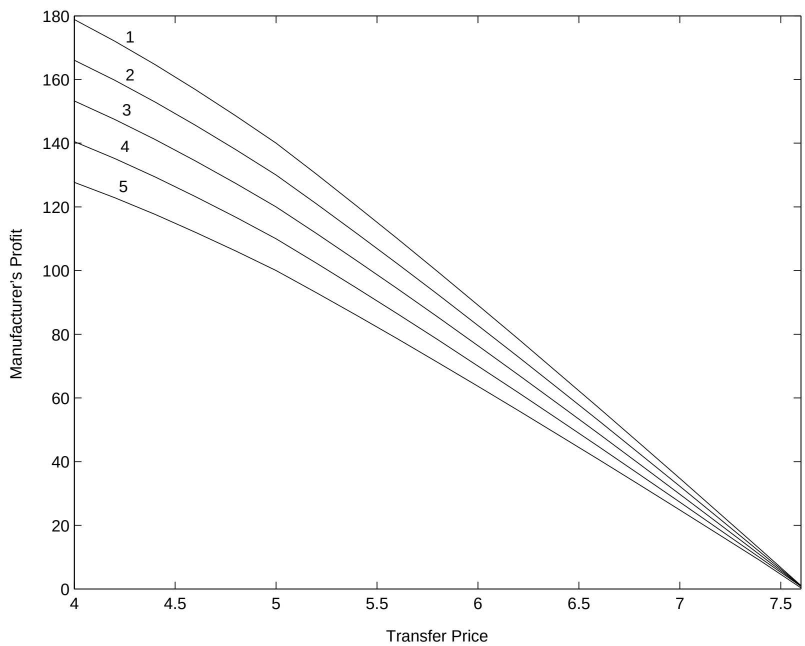

Figure 4.3: Manufacturer's Profit with Linear-Price Contract ($1.a = 140; 2.a = 130; 3.a = 120; 4.a = 110; 5.a = 100.$)

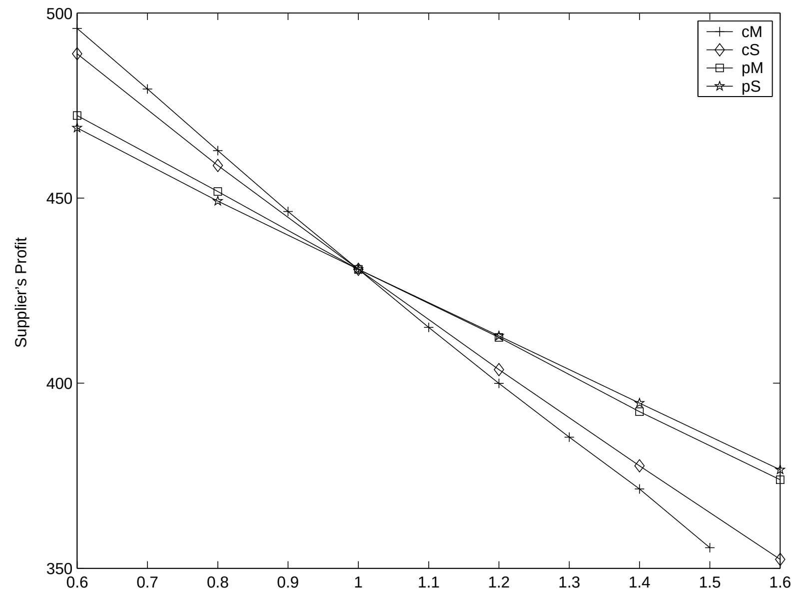

Figure 4.4: Supplier's Profit with Contract for Options

value of p_S has the smallest effect, mostly because this cost is incurred by the supplier and after the demand is observed.

4.4 Conclusion

In this paper we consider a supply chain comprised of a single supplier and two manufacturers. All players make their capacity investment decisions prior to the demand realization. When a linear-price contract is used, the supplier must also choose an allocation rule that is used when the manufacturers' orders exceed the available amount of goods.

We proved that the manufacturers' and the supplier's profit functions are concave in their capacity levels. We also showed that with a linear-price contract the manufacturers' capacity investments do not depend on whether the allocation rule is linear or proportional. We showed that the manufacturers would set transfer price very low, maximize their own profits and leave nothing to the supplier. The supplier would choose a very high transfer price that would maximize both her and the supply chain's profits but leave nothing to the manufacturers. We showed that the manufacturers' decisions about the capacity investment do not depend on their decision to manipulate their order at the next stage.

We prove that with a contract for options the supplier's profit function is concave in the reservation price and in the exercise price. We also proved that each manufacturer's profit function is concave in the reservation quantity as well as in each manufacturer's capacity investment decision. Again, as with a linear-price contract, the supplier, who is in a position to choose prices, leaves the manufacturers with nothing.

Our numerical results show that both contracts can bring the supply chain profit up to 92% of the centralized profit but with linear-price contract that is only possible if the supplier sets the prices. When the manufacturers set the prices with the linear-price contract the supply chain's profit represents only 71% of the centralized system's profit.

The contract for options might be preferred in the supply chain because it does not involve any allocation rules. However, to guarantee that the supplier builds enough capacity, a certain penalty cost must be charged. Unfortunately, that might make this contract as complex as a liner price contract with allocation rules.

Some further work can be done in order to understand what happens in the system when truth-telling allocation mechanisms are used. It might also be interesting to see whether a contract with non-linear prices can improve the supply chain performance and allow for arbitrary allocation of profits.

BIBLIOGRAPHY

[1] http://knowledge.wharton.upenn.edu. February 12, 2003a. "U.S. Steel Users Claim Tariffs "Protect a Few at the Expense of the Majority."

[2] http://knowledge.wharton.upenn.edu. January 29, 2003b. "The Outlook for Oil: What Lies Ahead."

[3] http://www.ioma.com. January 2003. The Controller's Report.

[4] F. H. Abernathy, J. T. Dunlop, J. H. Hammond, and D. Weil, A Stitch in time: lean retailing and the transformation of manufacturing - Lessons from the apparel and textile industries, Oxford University Press, New York, NY, 1999.

[5] Aghion, P., M. Dewatripont, and P. Rey. 1994. Renegotiation Design with Unverifiable Information, *Econometrica*, **62**(2) 257-282.

[6] Araman, V., J. Kleinknecht, and R. Akella. 2001. Supplier and procurement risk management in e-business: Optimal long-term and spot market mix, Working paper, Department of Management Science and Engineering, Stanford University, Stanford.

[7] D. Barnes-Schuster, Y. Bassok, and R. Anupindi, Coordination and flexibility in supply contracts with option,. *Manufacturing & Service Operations Management*, 4(3) (Summer 2002), 171-207.

[8] Y. Bassok, and R. Anupindi, Analysis of supply contracts with commitments and flexibility, Working Paper, J. L.. Kellogg Graduate School of Management, Northwestern University, Evanston, 1998.

[9] Cachon, G. Supply Chain Coordination with Contracts. 2002. Working paper, University of Pennsylvania, Philadelphia, PA.

[10] G. P. Cachon, and M. A. Lariviere, Capacity Choice and Allocation: Strategic Behavior and Supply Chain Performance, *Management Sci*, 45(8) (1999), 1091-1108.

[11] G. P. Cachon, and M. A. Lariviere, Contracting to assure supply: How to share demand forecasts in a supply chain, *Management Sci*, 47(5) (2001), 629-646.

[12] Cage, S. 2002. European Styrene Spot Advances, *Chemical Week*, **164** (12) 45.

[13] V. Deshpande, and L. B. Schwarz, Optimal Capacity Choice and Allocation in Decentralized Supply Chains, Working Paper, Krannert School of Business, Purdue University, W. Lafayette, IN, 2002.

[14] G. D. Eppen, and A. V. Iyer, Backup agreements in fashion buying- The value of upstream flexibility, *Management Sci,* 43(11) (1997), 1469-1484.

[15] Erhun, F., P. Keskinocak, and S. Tayur. 2000. Spot Markets for Capacity and Supply Chain Coordination, Working Paper, Carnegie Mellon University, Pittsburgh, PA.

[16] Golovachkina, N., J. R. Bradley. 2002. Supplier-Manufacturer Relationships under Forced Compliance Contracts, Working Paper, Cornell University, Ithaca, NY.

[17] Hart, O., J. Moore. 1988. Incomplete Contracts and Renegotiation, *Econometrica*, **56**(4) 755-785.

[18] Hirshleifer, J. 1956. On the Economics of Transfer Prices, *The Journal of Business*, **29**(3) 172-184.

[19] J. M. Milner, and M. J. Rosenblatt, Flexible supply contracts for short life-cycle goods: The buyer's perspective, *Naval Res Logistics*, 49(1) (2002), 25-45.

[20] Laffont, J., J. Tirole. 1990. Adverse Selection and Renegotiation in Procurement, *The Review of Economic Studies*, **57**(4) 597-625.

[21] Noldeke, G., K. M. Schmidt. 1995. Option Contracts and Renegotiation: A Solution to the Hold-up Problem, *The RAND Journal of Economics*, **26**(2) 163-179.

[22] Plambeck, E. L., T. A. Taylor. 2002. Renegotiation of Supply Contracts, Working Paper, Stanford University, Stanford, CA.

[23] Shaked, M., J. G. Shanthikumar. 1994. Stochastic orders and their applications, Academic Press, San Diego, CA.

[24] Spinler, S., A. Huchzermeier, and P. R. Kleindorfer. 2002. The Valuation of Options on Capacity, Working paper, Otto-Beisheim Graduate School of Management, WHU, Germany.

[25] B. Tomlin, Capacity Investment in Supply Chains: Sharing the Gain Rather than Sharing the Pain, *Manufacturing & Service Operations Management,*5(4) (2003), 317-333.

[26] A. Tsay, The quantity flexibility contract and supplier-customer incentives, *Management Sci*, 45(10) (1999) 1339-1358.

[27] B. Uzzi, Social structure and competition in inter-firm networks: the paradox of embeddedness. 1997. *Administration Science Quarterly*, **42** 35-67.

[28] Van Mieghem, J. A. 1999. Coordinating Investment, Production, and Subcontracting, *Management Science,* **45**(7) 954-971.

[29] Wu, D. J., P. R. Kleindorfer, and J. E. Zhang. 2002. Optimal bidding and contracting strategies for capital-intensive goods, *European Journal of Operational Research*, **137** 657-676.