On-The-Job Search in a Labor Market Model
Ex Ante Choices and Ex Post Outcomes

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This paper builds a multi-sector labor market model including wage dualism, open unemployment, underemployment, on-the-job search, and expected wage equalization. The innovative feature of this model is the distinction between the ex-ante allocation of the labor force among search strategies and the ex post allocation of the labor force among labor market outcomes. Among the findings are: more efficient on-the-job search *lowers* the equilibrium unemployment rate; in a rational expectations equilibrium, the average rural and urban wages will not be equal; modern sector enlargement may leave labor market conditions in one of the sectors *unchanged,* even when wages and employment in that sector are fully flexible.

1. Introduction

The purposes of this paper are to formulate a theoretical model of some of the pertinent features of labor markets and to indicate how this model might be used to analyze patterns of employment, unemployment, and underemployment and policies to affect them. The model is set in the context of a multi-sector economy, the sectors being defined by product, location, or method of wage determination. The model builds on previous contributions by Harris and Todaro (1970), Fields (1975), and McDonald and Solow (1985). The most important new feature is the distinction between the ex-ante allocation of the labor force among search strategies on the one hand and, on the other, the ex post allocation of the labor force among employment in different sectors and unemployment.

This model is motivated by a number of empirical observations about employment and wages in various countries' labor markets. One is that certain geographical areas tend to have persistently higher real wages than others [Hanushek (1983), Squire (1981)]. This suggests that wage rigidity rather than market-clearing wages may typify certain segments of the labor market. Another is the finding that high wage labor markets are characterized by high unemployment rates [Hall (1970), Harris and Todaro (1970)]. This may be interpreted as reflecting the influx of job-seekers in pursuit of jobs in high-paying industries or localities. Another is the observation [Fields (1976), Todaro (1976)] that patterns of labor force migration can be substantially explained by the rates of hiring, quits, and layoffs prevailing in different labor markets. One may infer from this that workers on the margin will indeed seek to reallocate themselves toward areas where job opportunities are more plentiful. And finally, an important feature of labor markets is the fact that many workers take up low-paying jobs from which they search (not necessarily successfully) for better ones [Tobin (1972), Matilla (1974), Merrick (1976), Banerjee (1983)].

We need a model which captures these features. The textbook characterization of labor markets does not suffice. In that model, different firms pay the same wage to comparable workers, the wage being set by supply and demand to clear the market. No unemployment emerges, save possibly for frictional unemployment. As a general characterization of labor markets, the textbook model is at odds with two observed facts: the existence of unemployment, apparently above frictional levels; and wage diversity for observationally-equivalent workers. Other models also assume away these facts - among them, international trade models, dualistic economic development models, and even some minimum wage models.

Some existing models incorporate both unemployment and wage diversity. Foremost among these are the works of Harris and Todaro (1970), Harberger (1971), Grämlich (1976), Mincer (1976), Stiglitz (1982), and McDonald and Solow (1985). Wage diversity arises in these and kindred models due to partial applicability of minimum wage laws and limited enforcement of them, strong trade unions in some sectors of the economy but not others, etc. and/or efficiency wages being paid in some economic sectors but not in others. Open unemployment results from the purposeful movement of labor between sectors on the basis of the *expected* wages (or utility) in each. In these models, the equilibrium tendency is toward equalization of wages or utility adjusted for probability of employment; it is an increase in unemployment in the high wage sectors rather than a fall in wages there that ultimately brings the supply side of the labor market into balance.

The model developed below incorporates wage dualism, open unemployment, underemployment, on-the-job search, and expected wage equalization among search strategies. By 'wage dualism', I mean that a wage floor above market-clearing levels applies to one economic sector but not others. 'Open unemployment' is distinguished from 'underemployment', whereby workers are engaged in labor market activity and thus classified as employed, yet because this activity pays wages below what they might earn elsewhere in the economy, the workers may be said to be underemployed. 'On-the-job search' means that workers employed in the lower-paying economic sectors devote some fraction of their time to obtaining better positions elsewhere in the economy and have some non-zero probability of obtaining such positions. 'Expected wage equalization' is the rule by which the labor force allocates itself among jobs and search opportunities.

Some of the components for this model were first presented in Fields (1975) and later appeared, apparently independently, in the work of McDonald and Solow (1985). The model formulated here moves beyond those earlier works, both by drawing the distinction between the ex-ante allocation of the labor force among job search strategies and the ex post allocation of the labor force among employment in the various sectors and unemployment and by deriving new results based on that distinction.

Why does the ex ante/ex post distinction matter? Consider first a multisector model *without* on-the-job search, such as Harris-Todaro (1970) or Mincer (1976). In such a model, all workers end up in the same sector in which they began. Think about the number who choose a search strategy ex ante (e.g., those who migrate to the urban economy and seek a job there). Then consider that sector's labor force ex post (e.g., those who are in the urban economy, employed or unemployed) after the jobs are allocated. Without on-the-job search, it is not necessary to distinguish between the number of ex ante searchers and the ex post labor force - the two numbers are the same, because people get jobs only in the sectors where they are located. But *with* the possibility of searching on-the-job and getting a job in one sector while living in another, the two numbers will differ. Take the example where wages are higher in the urban economy than in the rural economy and rural residents have a positive (i.e., strictly non-zero) probability of obtaining a high-paying urban job. In this case, some urban job- getters will be rural residents. Consequently, the rural labor force ex post will be *smaller* than the number of rural searchers ex ante; and likewise, the urban labor force ex post will be *larger* than the number of urban searchers ex ante. Unemployment rates will be affected as a result.

It bears mention that this model is characterized by rational expectations. That is, workers allocate themselves among search strategies ex ante knowing what those decisions imply about ex post outcomes, and those ex post outcomes are in fact realized. The ex ante/ex post distinction is made here to reflect the predictable movement of workers across sectors when some get jobs and some do not. It does *not* reflect intervening shocks, because there are no intervening shocks in the model; the only stochastic element is *which* individuals will get jobs and which will not.

The model is presented in section 2. Four results using this model are developed in section 3. Conclusions appear in section 4.

2. Modeling workers' choices of ex ante search strategies and the ex post rate of employment, unemployment, and underemployment with on-the-job search

2.1. Overview of the model

The economy I am modeling is atemporal and consists of a large number of homogeneous, risk-neutral individuals and a small number of economic sectors.[1] Following the familiar terminology of development economics, these three sectors are termed modern, traditional, and agriculture, though they might equally well be called high-wage, free-entry, and low-wage, respectively. Primary attention is given to the supply side of the labor market. The production side of the model is specified only to the extent that it is necessary for the formulation of the labor market model. The various sectors differ from one another in two major respects: wages and job search opportunities.

The highest-paying sector is named 'the urban modern sector' and is denoted by M. The wage in this sector, W_M, is set by some combination of market and/or institutional forces, above market-clearing levels and higher than the wage elsewhere in the economy. All workers, being risk-neutral income-maximizers, aspire to jobs in the modern sector. They may elect to search for these jobs by being openly unemployed and searching full time (assuming for simplicity that there is no unemployment compensation) or by accepting low-wage employment elsewhere in the economy at a positive wage and searching part-time. Two such low-wage sectors are assumed to exist. One of them, termed 'rural agriculture', is assumed to be located some distance from the modern sector. Because of this distance, and because workers in the agricultural sector are occupied with their jobs for some number of hours each day, agricultural workers would probably face reduced job search prospects compared to their search chances if searching full-time in the urban economy. However,

[1] Heterogeneity of labor adds additional complexity that is not warranted at this point. In the case of worker heterogeneity, labor market outcomes have been shown in Fields (1974, 1975) to be very sensitive to the way in which one models the behavior of firms and different groups of workers and the interrelationships among the markets for different labor categories.

the wage in agriculture, W_A, exceeds the wage if unemployed (zero).[2] The other economic sector, termed the 'urban traditional sector', is assumed to be located in the same place as the modern sector. The wage paid there is W_T.

2.2. Search strategies

To search for modern sector jobs, three search strategies are possible.

2.2.1. Search Strategy I: The strategy of open unemployment

This is the search strategy specified in the models of Harris and Todaro (1970), Harberger (1971), Mincer (1976), Grämlich (1976), and Stiglitz (1982) and one of the search strategies allowed for by McDonald and Solow (1985). Those who adopt Search Strategy I begin unemployed and search full time. Each such worker faces the same probability n of obtaining a job in the modern sector; Π is endogenous and is specified further below. Those who are successful become employed at wage W_M, while those who are unsuccessful end up unemployed and earn zero.

2.2.2. Search Strategy II: The strategy of remaining in agriculture

This is the other option offered in the models of Harris and Todaro (1970), Harberger (1971), and Stiglitz (1982).[3] In their models, those who elect to remain in agriculture give up any chance of obtaining a job in the modern sector. They then settle for the agricultural wage W_A.

This formulation is unduly restrictive. In practice, it is quite common for those working in low-paying sectors of the economy to have a non-zero chance of finding jobs in the modern sectors of their economies. For instance, they may search for jobs at night and on weekends, hear of jobs through friends and relatives already at work, or secure a position through an employment agency or labor exchange.

To allow for the possibility of on-the-job search by workers in agriculture, I formulate a model in which each person not in the modern sector has a positive but reduced chance of finding a job in the modern sector. But unlike those job search models such as Burdett and Mortensen (1978) in which searchers have discretion over the amount of search they will engage in, and hence the margin of interest is the intensive margin (how intensively to search on the job), the formulation here treats the amount of on-the-job search as parametric, so the action takes place on the extensive margin (i.e., how many workers engage in on-the-job search rather than search while unemployed.[4]

Denote the relative efficiency of on-the-job search while in the agricultural sector as compared with search while unemployed by θ, which is best thought of as a relative job search parameter. For instance, suppose θ were equal to 1/2. This would mean that any given job searcher has only half as good a chance of obtaining a modern sector job if he or she is working in agriculture compared to the chance he or she would have were (s)he searching full time.

2.2.3. Search Strategy III: Searching while employed in the urban traditional sector

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[2] In cases where unemployment insurance exists, the wages Wj, $j = M$, A, T may be thought of as the amount by which wage exceeds unemployment benefits.

[3] In the minimum wage models of Mincer (1976) and Grämlich (1976), the analogue is 'remain in non-covered employment'.

[4] Fields (1975) and McDonald and Solow (1985) do likewise.

This is defined analogously to the situation facing those searching while in agriculture. On-the-job search while working in the urban traditional sector is possible but it is less efficient than full-time search. Analogously, therefore, denote the relative job search parameter while working in the urban traditional sector by φ. On-the-job search prospects are assumed to be better for workers in the urban traditional sector than for workers in rural agriculture, because the urban traditional sector workers are in closer proximity to the modern sector jobs than are rural agricultural workers. Therefore, the model is restricted so that $\theta < \varphi$.[5]

2.3. Formulation of the model

Workers are assumed to choose among alternative search strategies on the basis of the expected wage associated with each.[6] These expected values are denoted by V_j. The respective values are:

$$V_{\mathrm{I}} = W_{\mathrm{M}}\pi, \tag{1}$$

$$V_{\mathrm{II}} = W_{\mathrm{M}}\theta\pi + W_{\mathrm{A}}(1-\theta\pi), \quad \text{and} \tag{2}$$

$$V_{\mathrm{III}} = W_{\mathrm{M}}\varphi\pi + W_{\mathrm{T}}(1-\varphi\pi). \tag{3}$$

The expression for V_1 requires no explanation. The expressions for V_{II} and V_{III}, each consist of two terms. Take the expression for V_{II}. The first term, $W_M \theta\pi$, is the wage in the modern sector multiplied by the probability of obtaining a job at that wage given that the individual has elected Search Strategy II. That probability is the probability π associated with full-time search multiplied by the relative search parameter θ applicable to an individual who accepts a reduced search opportunity while working in agriculture. The product $\theta\pi$ is then the probability of successfully obtaining a modern sector job while working in agriculture. In the event that this search is unsuccessful, which occurs with probability $(1— \theta\pi)$, those who adopt this search strategy end up remaining in agriculture and earning W_A. V_{II} then consists of the wages in the two sectors weighted by the respective probabilities of receiving them under Search Strategy II. The expression for V_{III} in eq. (3) is derived analogously.

Let us turn our attention now to the probability of employment, π, defined as the ratio of modern sector jobs E_M to job-seeker equivalents (defined below) J_M:

$$\pi = E_{\mathrm{M}}/J_{\mathrm{M}}. \tag{4}$$

Of course, the amount of employment in the modern sector, E_M, is a function of the wage in the modern sector and

[5] From this restriction, it follows that the urban traditional sector wage W_T will be less than the agricultural wage W_K - which helps explain the miserable slums found in the cities of most developing countries.

[6] A concave utility function could easily be introduced but is omitted in order to highlight the result on intersectoral wage differentials derived in section 3. It would also be possible to extend the model to a multi-period context and allow for decisions to be made on the basis of expected present values. In this case, the expected wage in each period would depend on the individual's search strategy, which in turn depends on the transition probabilities between various labor market states and others - all of which are endogenous. While it is indeed possible to do such an extension [see Fields (1975) and McDonald and Solow (1985)], it is not at all clear that it would add anything in the present context. It is best to leave multi-period complexities for another time.

perhaps of other things as well; it may be written as

$$E_M = h(W_M) + \xi. \tag{5}$$

where $h(W_M)$ is the usual labor demand function and ξ is a shifter reflecting the possibility of resources being made available to modern sector employers (say, by the government or an external aid agency) for purposes of job creation.

J_M is the number of 'job-seeker equivalents' and has the following interpretation. A worker searching from the agricultural sector has only θ as great a chance for getting a job as does an unemployed worker who is searching full time; the relative chance for a worker searching from the urban traditional sector is φ. Denoting the number of workers electing each search strategy by L_i, i = I, II, III, each of the L_I, workers has a full chance of obtaining any given job, each of the L_{II} workers has *6* of a chance, and each of the L_{III} workers has φ of a chance. If each job-seeker is weighted according to the relative chance of obtaining a job, then the number of jobseeker equivalents is

$$J_M = L_I + \theta L_{II} + \varphi L_{III}. \tag{6}$$

Note that it is the number initially choosing the three search strategies i = I, II, III and *not* the number ending up in the three employment sectors j=M, A, T that enters into the expression for job-seeker equivalents. By distinguishing between the ex ante allocation of searchers in (6) and the ex post allocations between employment in the various sectors and unemployment in (7)—(10), the formulation presented here improves upon that of Fields (1975) and McDonald and Solow (1985), who did not draw this distinction. On the assumption that modern sector jobs are filled in exact proportion to the number of job-seeker equivalents, it is easily verified that the formulation given by (6) indeed yields just the right amount of modern sector employment.[7]

The wage determination process must be specified. As already stated, the wage in the modern sector W_M is determined exogenously above market-clearing levels. This may occur for a number of possible reasons including minimum wages, trade unions, or firms deliberately setting the wage above market-clearing levels for efficiency wage reasons. But whatever those reasons may be, they do not carry over to agriculture and the urban traditional sector. In those sectors, wages are market-clearing, determined as functions of their respective labor forces. Certainly, we wish these functions to be non-increasing. They may either be constant or decreasing. The wage being invariant with respect to the size of the agricultural sector's labor force would be relevant, for example, in a land abundant economy in which anyone who wishes to enter the agricultural sector can take up selfemployment and earn a wage W_A. A decreasing wage as a function of the size of the labor force would be consistent with the wage equaling a diminishing marginal product of labor or even with a situation

[7] If E_M jobs are to be filled, the number filled by those adopting Search Strategy I is πL_I, the number filled by those adopting Search Strategy II is $\pi\theta L_{II}$, and the number filled by those adopting Search Strategy III is $\pi\varphi L_{III}$. Summing these and substituting $\pi = E_M/J_M$, one obtains the total amount of employment as

$$(E_M/J_M)L_I + (E_M/J_M)L_{II}\theta + (E_M/J_M)L_{III}\varphi - (E_M/J_M)(L_I + \theta L_{II} + \varphi L_{III}) - (E_M/J_M)(J_M) - E_M,$$

satisfying the adding-up condition.

of zero marginal product of labor and income-sharing among all those working in that sector. Some special cases will be dealt with below. But at this point, it is useful to stick to the general wage functions:

$$W_A = f(L_A), \qquad f' \leqq 0, \tag{7}$$

and

$$W_T = g(L_T), \qquad g' \leqq 0. \tag{8}$$

A bit of explanation may be in order regarding the sizes of the labor forces in the different sectors, from which W_A and W_T are derived. L_A and L_T in eqs. (7) and (8) are the numbers of workers who ultimately end up in the agricultural and traditional sectors, respectively. These numbers are *not* the same as the numbers electing Search Strategies II and III. Take, for instance, those who elect on-the-job search from the agricultural sector. Of those persons (L_{II} in number), the fraction who successfully obtain modern sector employment is $\theta\pi$: only (1 — $\theta\pi$)% are left in agriculture. Hence,

$$L_A = L_{II}(1 - \theta\pi). \tag{9}$$

By identical reasoning,

$$L_T = L_{III}(1 - \varphi\pi). \tag{10}$$

The adding-up conditions for the labor force are, ex ante,

$$L = L_I + L_{II} + L_{III}, \tag{11}$$

and ex post

$$L = L_M + L_A + L_T. \tag{12}$$

Finally, we must have the rule specifying the allocation of the labor force between search strategies and sectors. The rule is expected wage maximization. If the solution is interior, meaning that all three search strategies have the same expected value, for some allocation of the labor force among search strategies, then

$$V_I = V_{II} = V_{III}. \tag{13}$$

For the solution to in fact lie in the interior, the parameters of the model are restricted to take on certain values. In the three-sector model below, it is assumed that they do.

2.4. A note on non-interior solutions and two-sector models

Some of the results derived below do not require three sectors, though they are indeed valid in the three-sector case. In the derivation of these results, a two-sector model is used. The two-sector model may be thought of as a non-interior solution to the three-sector model.

A non-interior solution might arise for either of two reasons. One is that in a particular country, one of the three sectors might simply not exist. Examples might be the absence of agriculture in a city-state or the absence of a free-entry traditional sector in a relatively advanced economy. The other reason for a non-interior solution is that one search strategy may be dominated by others. For example, if opportunities for on-the-job search from agriculture are sufficiently high and income opportunities in the urban traditional sector sufficiently low, it may pay *nobody* to enter the urban traditional sector.

Whether a non-interior solution arises because of non-existence of a sector or domination of one search strategy by others, the model reverts to a two- sector model. Some of the results below are proved for the case L_{III} and L_T identically zero and V_{III} irrelevant

2.5. A special case

If the solution to the model given in this section is interior (which must, of course, be verified), the equilibrium allocation is given by the condition that alternative search strategies have the same expected values; this is eq. (13). The solution would be found by substituting the remaining equations of the system into (13) and solving for the ex ante distribution of search strategies. From this, we may then derive the ex post allocation of the labor force between employment in various sectors and unemployment.

With the general wage-determination functions (7) and (8), it is not possible to obtain explicit expressions for the allocations of the labor force among ex ante search strategies and ex post labor market outcomes. This is because the wage functions $W_A = f(L_A)$ and $W_T = g(L_T)$ enter into the equilibrium conditions (13), and until we know those functions and can invert them, we cannot proceed. However, if specific functional forms are assumed, closed-form solutions can be obtained. This is the strategy followed when needed below.

The restricted functions are the following. Suppose, as in Fields (1975), Anand and Joshi (1979), and Stiglitz (1982), that the agricultural wage is invariant with respect to the size of that sector's labor force. This would be meaningful if either (a) the economy is land-abundant, so that any worker who wishes to work in agriculture can obtain a plot of land and earn the same self-employment income as others already in that sector, or (b) within the range of variation, the size of the agricultural sector labor force varies only negligibly in response to other changes in the economy. In either case, the agricultural sector wage function is restricted such that

$$W_A = \text{constant}. \tag{7'}$$

As for the urban traditional sector, suppose (a) whatever income is generated in the urban traditional sector is divided equally among the workers in that sector (e.g., each street vendor earns the same as any other), and (b) the total demand for traditional sector output is fixed in amount (e.g., because all of the demand comes from those employed in the modern sector, the number and wage of whom is constant). Under these conditions, the urban traditional sector wage function takes the form

$$W_T = Q_T / L_T, \tag{8'}$$

Q_T denoting the amount of income to be shared.

In what follows, eq. (7') is used in Propositions 1 and 2 and eq. (8') in Proposition 2. The general forms are used in Propositions 3 and 4.

3. Results

This section proves four propositions. Some are for two-sector models, the reason being that only two sectors are needed to prove them and the presence of a third sector obfuscates the issue. Others are for three-sector models, the third sector being included because it is needed.

3.1. On-the-job search from agriculture and the urban unemployment rate in the two-sector model

In Fields (1975), I introduced on-the-job search from agriculture into what was otherwise a two-sector Harris-Todaro model with a constant agricultural wage. I claimed (p. 171): '...there is a lower equilibrium [urban] unemployment rate in general than would be predicted by the Harris- Todaro model; and the greater the relative chance of rural workers finding urban jobs, the greater the discrepancy between the general result and the Harris-Todaro result'. In that paper (eq. 11), no ex ante/ex post distinction was made. Unemployment was calculated on the basis of what we are here calling L_I the number who choose Search Strategy I (search full time for a modern sector job, accept unemployment otherwise). While that is not wrong, it is also not the most interesting. A more appropriate unemployment rate is that using the ex post urban labor force, denoted here by L_M in the two-sector model. If we calculate urban unemployment (U) as a proportion of the urban ex post labor force (L_M), the claim that more efficient on-the-job search from agriculture lowers the urban unemployment rate must be verified. Indeed, it continues to hold.

Begin with the two-sector version of the model of section 2, i.e., only the modern and agricultural sectors but no traditional sector. Fix the agricultural wage. The model may be manipulated to yield

$$U = L + E_M\left(\frac{W_M}{W_A} - 1\right) + \frac{W_M L}{(W_M + W_A)\theta - W_M}, \qquad (14a)$$

and

$$L_M = L + E_M\frac{W_M}{W_A} + \frac{W_M L}{(W_M - W_A)\theta - W_M}. \qquad (14b)$$

By (14a), since an increase in θ increases the denominator, it follows that more efficient on-the-job search implies that fewer persons will be unemployed. But by (14b), a higher θ also implies that the ex post urban labor force, L_M, will be smaller. Thus, to determine what happens to the urban unemployment *rate, U/L_M,* we must do more work.

The only difference between U in (14a) and L_M in (14b) is the second term on the right-hand side. This is as it should be: the numerator is less than the denominator, so the urban unemployment rate is a positive fraction. To show how the urban unemployment rate changes with θ, note that in (14a) and (14b) the denominator of the last term is negative. When θ increases, the denominator is less negative, and therefore the third term is more negative as θ increases. When larger equal amounts are subtracted from a fraction less than one, the value of the fraction falls. This confirms my earlier claim, though now for an unemployment rate expressed as a fraction of the ex post rather than ex ante labor force.

Proposition 1. In the two-sector model with a constant agricultural wage, more efficient on-the-job search from agriculture lowers the urban unemployment rate in equilibrium.

3.2. On-the-job search from the urban traditional sector and the urban unemployment rate in a specialized three-sector model

In Fields (1975), in another extension of the Harris-Todaro model, I introduced an urban traditional sector, from which on-the-job search for high-paying modern sector jobs was possible. The relative job search parameter from that sector (φ in present notation) was assumed to be greater than the corresponding parameter from agriculture (θ) but less than the search parameter if unemployed in the urban area (normalized at 1). I claimed (p. 175) that in a specialized three-sector model with wage functions given by

$$W_A = \text{constant}, \tag{7'}$$

and

$$W_T = Q_T / L_T, \tag{8'}$$

and with θ = 0, the larger is φ, the larger is the equilibrium urban employment rate. Again, it must be verified that an earlier claim holds once due account is taken of the ex ante/ex post distinction. Once again, it does.

The model given by (l)-(6), (7'), (8'), and (9)—(13) and with $\theta \geqq 0$ may be solved to give the following results:

Ex ante allocations

$$L_I = -\frac{\theta L}{1-\theta} + \frac{E_M W_M}{W_A} + \frac{E_M \theta}{1-\theta} - \frac{Q_T(\varphi-\theta)}{W_A(1-\varphi)} - \frac{Q_T\theta(\varphi-\theta)}{W_M(1-\varphi)(1-\theta)}, \tag{15a}$$

$$L_{II} = \frac{L}{1-\theta} - \frac{E_M W_M}{W_A} - \frac{E_M \theta}{1-\theta} - \frac{Q_T}{W_A} - \frac{Q_T \theta}{W_M(1-\theta)}, \tag{15b}$$

$$L_{III} = \frac{Q_T\{W_M(1-\theta) + W_A \theta\}}{W_A W_M (1-\varphi)}. \tag{15c}$$

Ex post allocations

$$L_T = \frac{Q_T(1-\theta)}{W_A(1-\varphi)} - \frac{Q_T(\varphi-\theta)}{W_M(1-\varphi)}, \tag{15d}$$

$$L_A = \frac{L}{1-\theta} - \frac{E_M W_M}{W_A} - \frac{Q_T}{W_A} - \frac{\theta W_A L}{\{W_M(1-\theta)+W_A\theta\}(1-\theta)}, \quad (15e)$$

$$E_M \text{ is given}, \quad (15f)$$

$$U = L - L_T - L_A - E_M. \quad (15g)$$

Now let θ = 0, as in my earlier paper. From expressions (15a)—(15g), we may derive the equilibrium urban unemployment rate based on the ex post urban labor force $L_M + L_T = L—L_A$ as

$$\text{Urban Unem Rate} = \frac{L - L_T - L_A - E_M}{L - L_A} = 1 - \frac{L_T + E_M}{L - L_A}$$

$$= 1 - \{W_A[(Q_T/W_A(1-\varphi)) - (Q_T\varphi/W_M(1-\varphi) + E_M]$$

$$/[E_M W_M + Q_T]\}.$$

The only term that changes with φ is the bracketed term, and the urban unemployment rate varies inversely with it. To see how the bracketed term varies with φ, differentiate and simplify to obtain

$$\frac{\partial[\cdot]}{\partial\varphi} = \frac{Q_T}{(1-\varphi)^2}\left(\frac{1}{W_A} - \frac{1}{W_M}\right),$$

which is positive since $W_M > W_A$. Hence, the equilibrium urban unemployment rate *falls* with φ. Thus, we have proved

Proposition 2. In the specialized three-sector model given by (1)-(6), (7'), (8'), *and (9)-(13) and with θ = 0, more efficient on-the-job search from the urban traditional sector lowers the equilibrium rate of urban unemployment.*

The earlier result is confirmed even after the ex ante/ex post distinction is made and the ex post labor force is used instead as the base.

3.3. Intersectoral wage differentials ex ante and ex post in the two- and three- sector models

This subsection shows that the pattern of wage differentials in the economy is affected fundamentally by the distinction between the ex ante distribution of workers among job search strategies on the one hand and, on the other, the ex post allocation of the labor force between work in the different sectors and unemployment. Begin with the familiar models of intersectoral labor force allocation with search while unemployed only, i.e., no on-the-job search. The Harris-Todaro model is probably the most famous of these. The equilibrium in their model is characterized by equality of expected wages for alternative search strategies. (Of course, in their model, one of the search strategies is to remain in agriculture which, in the absence of on-the-job search, is tantamount to not searching at all for a modern sector urban job.) Thus, in the Harris-Todaro model, the labor force allocates itself such that expected wages are equalized across alternatives ex ante.

For present purposes, it should be noted that in addition to ex ante wage equalization, in a Harris-Todaro world, another type of wage equalization occurs: the average wages in the urban and rural sectors are also equalized ex post. That is, in a Harris-Todaro economy, a researcher looking at the data would find that the average wage received in the urban economy (remembering to include zeroes for the unemployed) would exactly equal the average rural wage.

With on-the-job search, this is no longer so. Workers would still allocate themselves between search strategies so that the expected returns from alternative search strategies are equalized ex ante (if in fact parameter values are such that they can be equalized). But unlike the case analyzed by Harris and Todaro, in which on-the-job search is absent, when there *is* on-the-job search, average wages are *not* equalized across sectors ex post. This holds even under the rational expectations assumed here; that is, even though every worker knows that the average wages will not turn out to be equal in the urban and rural sectors, no worker has an incentive to alter his/her search behavior. This may be stated formally as

Proposition 3. Denote the average urban wage by W_U, the average rural wage by W_R, and the relative search parameter associated with the agricultural sector by θ. Then in both the general two-sector model and the general three-sector model:

(*a*) $\theta = 0 \Rightarrow W_U = W_R$,
(*b*) $\theta > 0 \Rightarrow W_U > W_R$.

Proof. See appendix.

It should be noted that the discrepancy between average urban and average rural wages ex post is *not* attributable to the existence of a third sector per se; the same discrepancy is shown to arise even in a two-sector model with on-the-job search. It should also be noted that the discrepancy between the average wages in urban and rural areas arises apart from other reasons such as higher cost-of-living in urban as opposed to rural areas, risk aversion in the part of workers, a greater concentration of better skilled workers in the urban economy, and multi-period decision-making - all of which might be considered in more general models.

In sum, in an economy with on-the-job search from the rural sector, a researcher looking at the data might observe a difference between the average urban and rural wages and conclude that expected wage equalization had not yet taken place. Proposition 3 implies that such a conclusion would be erroneous.

3.4. Effects of modem sector enlargement

By ‘modern sector enlargement’ in this context. I mean that the government or some other body creates additional modern sector jobs beyond those that would otherwise exist. This possibility is accommodated by the term ξ in eq. (5) above.

For policy purposes, the most significant implication of the Harris-Todaro model is that modern sector enlargement creates additional unemployment under fairly general conditions. This is because each additional high wage job attracts more than one job-seeker from the rural area, adding to unemployment. It is because of this that Harris and Todaro reached their famed conclusion that ‘the solution to urban unemployment is rural development’.

The question may then be asked: In a three-sector model, in which underemployment in the urban traditional sector is an additional option, are the adverse effects of modern sector enlargement mitigated? The answer hinges on whether the urban traditional sector absorbs any of the impact which would otherwise fall on unemployment. The following, possibly surprising, result emerges:

Proposition 4. Consider the effects of modern sector enlargement in the three- sector model given by eqs. (7)—(*13). Suppose that the wage rate in one of the other sectors is invariant with respect to the size of that sector's labor force. Then, in the* third *sector, the wage, the number of ex ante searchers, and ex post employment will all be* constant. *Specifically:*

a) *Suppose* $W_A=f(L_A)$, $f' = 0$ *and* $W_T=g(L_T)$, $g'<0$. *Then the effect of modern sector enlargement is to leave* W_T, L_T, *and* L_{III} unchanged.

b) *Suppose* $W_A=f(L_A)$, $f' < 0$ *and* $W_T=g(L_T)$, $g'= 0$. *Then the effect of modern sector enlargement is to leave* W_A, L_A, *and* L_{II} unchanged.

The proofs are straightforward. Take (a) first. From (13), we have $W_M\pi = W_M\theta\pi + W_A(1 — \theta\pi)$. W_M and θ have been assumed constant throughout. With W_A constant by assumption in this special case, the only other variable, π, cannot change. Now consider $W_M\theta\pi + W_A(1 — \theta\pi) = W_M\varphi\pi + W_T(1 —\varphi\pi)$. With W_M, W_A, θ, φ, and π all constant, a unique value of W_T is determined. Given $W_T = g(L_T)$, $g'< 0$, a unique value of L_T is determined. And from $L_T = L_{III}(1 — (1 —\varphi\pi)$, a unique value of L_{III} is determined. This confirms that when the wage in the agricultural sector is invariant with respect to the size of that sector's labor force, then nothing in the *traditional* sector changes in response to modern sector enlargement. The proof of (b) goes through in precisely analogous fashion, starting with $W_M\pi = W_M\varphi\pi + W_T(1 — <\varphi\pi)$. This means that *all* of the adjustment will be felt in the sector with the constant wage or in unemployment, but not in the sector with the variable wage. This results from the need to preserve a three-way equality among expected wages when one of the wage functions, $W_A= f(L_A)$, $f' < 0$ or $W_T= g(L_T)$, $g'< 0$, is monotonically decreasing.[8]

This shows that the adverse effects of modern sector enlargement on unemployment are *not* mitigated by the existence of a low-wage, free-entry urban traditional sector. If modern sector enlargement draws additional unemployed into the urban economy when work in the urban traditional sector is not a possibility, it will continue to do so when that possibility exists.

4. Conclusion

I began in section 1 by claiming that that the desirable features of a multisector labor market model include wage dualism, open unemployment, underemployment, on-the-job search and expected wage equalization and noting that previous models had not taken adequate account of the distinction between the ex ante allocation of the labor force among search strategies and the ex post allocation of the labor force among labor market outcomes. A three-sector model containing the desired features was developed in section 2. Section 3 derived four results showing how on-the-job search

[8] If *all three* wages were invariant with respect to the sizes of their respective labor forces, the three expected values would be equal only by chance. One search strategy would be dominated by the others and would drop out, leaving us with a two-sector model.

matters using this model:

Proposition 1. In the two sector model with a constant agricultural wage, more efficient on-the-job search from agriculture lowers the urban unemployment rate in equilibrium.

Proposition 2. In the specialized three-sector model given by (1)-(6), (7'), (8'), and (9)-(13) and with $\theta = 0$, more efficient on-the-job search from the urban traditional sector lowers the equilibrium rate of urban unemployment.

Proposition 3. Denote the average urban wage by W_U, the average rural wage by W_R, and the relative search parameter associated with the agricultural sector by θ. Then in both the general two-sector model and the general three-sector model:

a) $\theta = 0 \Rightarrow W_U = W_R$,

b) $\theta > 0 \Rightarrow W_U > W_R$.

Proposition 4. Consider the effects of modern sector enlargement in the three- sector model given by eqs. (1)-(13). Suppose that the wage rate in one of the other sectors is invariant with respect to the size of that sector's labor force. Then, in the third sector, the wage, the number of ex ante searchers, and ex post employment will all be constant. Specifically:

a) *Suppose* $W_A = f(L_A)$, $f' = 0$ and $W_T = g(L_T)$, $g' < 0$. *Then the effect of modern sector enlargement is to leave* W_T, L_T, and L_{III} *unchanged.*

b) Suppose $W_A = f(L_A)$, $f' < 0$ *and* $W_T = g(L_T)$, $g' = 0$. *Then the effect of modern sector enlargement is to leave* W_A, L_A, *and* L_{II} *unchanged.*

Development policy-makers and analysts are concerned with such ex post outcomes as unemployment, poverty, and inequality. However, they cannot affect these directly. They must work instead on factors which influence workers' ex ante choices and determine the ex post effects of these choices. Among those factors which have been considered here are the relative efficiency of on-the-job search compared with search while unemployed and modern sector job creation.

The call has gone out for development economists to develop more realistic models of labor markets. The present paper is a step in that direction. Other applications and extensions will be considered in future work.

Appendix: Proof of Proposition 3

Proposition 3. Denote the average urban wage by W_U and the average rural wage by W_R. Let θ be the relative search parameter associated with the agricultural sector. Then in both the general two-sector model and the general three-sector model:

a) $\theta = 0 \Rightarrow W_U = W_R$,

b) $\theta > 0 \Rightarrow W_U > W_R$.

Proof in the three-sector case

Proof of (a)

By definition, the average urban wage equals

$$W_U = \frac{W_M E_M + W_T L_T}{L_M + L_T}, \tag{A.1}$$

and the average rural wage equals

$$W_R = W_A. \tag{A.2}$$

Note that

$$L_M + L_T = L - L_A \tag{A.3}$$

and

$$E_M = \pi J_M. \tag{A.4}$$

When $\theta = 0$, the model simplifies to

$$W_M \pi = W_A = W_M \varphi\pi + W_T(1 - \varphi\pi),$$

$$L_A = L_{II},$$

$$L_T = L_{III}(1 - \varphi\pi),$$

$$J_M = L_I + \varphi L_{III}.$$

Substitute these into (A.1) as follows:

$$W_U = \frac{W_M \pi J_M + W_T L_T}{L - L_A}$$

$$= \frac{W_M \pi (L_I + \varphi L_{III}) + W_T(1 - \varphi\pi) L_{III}}{L - L_{II}}$$

$$= \frac{[W_M \varphi\pi + W_T(1 - \varphi\pi)] L_{III} + W_M \pi L_I}{L_I + L_{III}}$$

$$= \frac{W_A L_{III} + W_A L_I}{L_I + L_{III}} = W_A.$$

Given $W_U = W_A$ when $\theta = 0$ and $W_A = W_R$, it follows that $W_U = W_R$ when $\theta = 0$. This completes the proof of (a).

Proof of (b)

When $\theta > 0$, (A.1)–(A.4) hold as before. Now, however,

$$W_M \pi = W_M \theta\pi + W_A(1-\theta\pi) = W_M \varphi\pi + W_T(1-\varphi\pi) = V. \tag{A.5}$$

Substitute into (A.1) as follows:

$$W_U = \frac{W_M \pi J_M + W_T L_T}{L - L_A}$$

$$= \frac{W_M \pi(L_I + \theta L_{II} + \varphi L_{III}) + W_T L_{III}(1-\varphi\pi)}{L_I + L_{II} + L_{III} - L_{II}(1-\theta\pi)}$$

$$= \frac{V L_{III} + V L_I + V\theta L_{II}}{L_I + L_{III} + \theta\pi L_{II}}$$

$$= V\left\{\frac{[L-(1-\theta)L_{II}]}{[L-(1-\theta\pi)L_{II}]}\right\}. \tag{A.6}$$

Now,

$$\theta > 0,\ \pi < 1 \Rightarrow \theta\pi < \theta \Rightarrow -\theta\pi > -\theta \Rightarrow (1-\theta\pi) > (1-\theta) \Rightarrow$$

$$-(1-\theta\pi) < -(1-\theta) \Rightarrow L-(1-\theta\pi)L_{II} < L-(1-\theta)L_{II}.$$

Hence,

$$\{\cdot\} > 1 \quad \text{if} \quad \theta > 0. \tag{A.7}$$

Eqs. (A.6) and (A.7) establish

Lemma 1. $\theta > 0 \Rightarrow W_U > V.$

Given π, $\theta > 0$ and (A.5), we obtain

Lemma 2. $\theta > 0 \Rightarrow V > W_A.$

From Lemmas 1 and 2 and $W_A = W_R$, we have

$$\theta > 0 \Rightarrow W_U > W_R.$$

This completes the proof of (b).

Having shown (a) and (b), the theorem is now proved for the three-sector case. Observe that these results are completely general: they are valid for *any* wage eqs. (7) and (8).

Proof in the two-sector case

In the preceding proof, set L_T and L_{III} and identically equal to zero, drop $V_{III} = W_M\varphi\pi + W_T(1 - \varphi\pi)$ and proceed as before.

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