LASER FREQUENCY STABILIZATION WITH POUND-DREVER-HALL TECHNIQUE

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A Thesis

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Master of Science

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by

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ABSTRACT

Laser frequency stabilization is important in laser science and various researches requiring laser beams with high frequency precision. However, the changes in physical parameters of the lasers themselves and in the external environments often cause instability in the output frequency of the laser.

This thesis studied the frequency locking system for a continuous-wave diode laser. Using the Pound-Drever-Hall technique, the system can significantly reduce the linewidth of an input laser with an un-stabilized linewidth. It uses a high-finesse Fabry-Perot cavity, which is mechanically and thermally isolated, as a frequency reference to measure the time-varying frequency of the input laser. An electronic feedback loop works to correct the frequency error and maintain constant optical power.

BIOGRAPHICAL SKETCH

Yin Feng earned his Bachelor of Science degree in Physics from Peking University, Beijing, China in 2012. After graduation, she came to Cornell University and joined the Master of Science program in School of Applied Physics.

Feng received Outstanding Student Award from School of Physics, Peking University regarding to his academic record and participation in campus activities. In addition to his academic studies, Feng joined the Student Union in Peking University. She coordinated and organized several activities as the vice-president of the Literature and Art Department.

While pursuing his undergraduate degree, Feng began to participate in research projects to practice what she learned from class. She studied micro-ring-resonator sensors for bio-molecular in Professor Yun-feng Xiao's group. During her graduate study, Feng joined Professor Alexander Gaete's group in a frequency comb project in Cornell University. She studied Pound-Drever-Hall technique used for frequency stabilization.

Yin's thesis, entitled 'Laser Frequency Stabilization With Pound-drever-hall Technique' was supervised by Dr. Alexander Gaete in School of Applied and Engineering Physics.

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I would like to dedicate this thesis to my family, my friends and

to my supervisor, Professor Alexander Gaeta.

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ACKNOWLEDGMENTS

I could not write this thesis without the help and support of the kind people around me, and I would like to mention them here.

Above all, I would like to thank my parents and my dear friends for their great patience and personal support. My words and expressions of thanks could not suffice This thesis would not have been possible without the help, support and patience of my supervisor, Professor Alexander Gaeta. I am extremely grateful for his good advice and support, which have been very helpful on both an academic and a personal level.

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Last but not the least, I thank my friends, both in Ithaca and elsewhere for their support and encouragement.

For any errors or inadequacies that may remain in this work, of course, the responsibility is entirely my own.

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1 Introduction

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. An ideal laser generates perfect spatial coherent, single frequency, highly focused spot of light. But in practice, noise is unavoidable. Noise of lasers is a short term for random fluctuations of various output parameters. This is a frequently encountered phenomenon which has a profound impact on many applications, particularly in the area of precision measurements. Among the noise in different parameters of lasers, the one of interest in this thesis is the frequency noise, which produces a broadened frequency spectrum and therefore limits the performances of some laser-employed optical processes such as frequency comb generation.

External cavity diode lasers (ECDLs) are a common example of narrow linewidth lasers. ECDLs use semiconductor diode lasers in an external cavity with dispersive feedback, often from a diffraction grating. The linewidth is greatly reduced with respect to the diode alone, and the laser can be tuned through the broad gain curve of the diode. ECDLs exhibit varying forms of frequency noise from environmental, fundamental, and artificial source. Environmental noise includes 50 or 60 Hz per-line induced noise, and acoustically coupled noise, in particular at frequencies corresponding to mechanical resonances. Fundamental noise is typically dominated by white phase noise at high frequencies and flicker frequency noise at low frequencies. The flicker frequency noise is often the dominant component of the linewidth, and so it is common practice to stabilize the laser to an external frequency reference, for example a Fabry-Perot etalon or sub-Doppler atomic resonance. Infrared lasers, particularly those with a wavelength of 1550 nm, lend themselves quite well to all kinds of optical fiber-based experiments because the vast majority of modern optical communications and sensing equipment is designed for this wavelength. A narrowlinewidth laser at 1550 nm could be used to calibrate precision fiber-based sensors, filters and multiplexers. It could be used in conjunction with these highly sensitive optical devices to make precision measurements of physical phenomena.

Pound-Drever-Hall (PDH) technique, invented in 1982, is the most popular laser-locking method currently in use. It was conceived by Ron Drever, based on similar microwave techniques used by R. V. Pound [1]. Employing a stabilized feedback system, it is able to lock the laser frequency with respect to an external Fabry-Perot cavity, which has a higher stability compared to the in-built cavity of the laser and thus can be used as an frequency reference. To date, the PDH technique is employed in various researches such as high signal-to-noise ratio acoustic sensor using phase-shifted gratings, molecucular spectroscopy experiments and locking lasers [2].

This project is part of frequency comb project in Alexander Gaeta's Quantum and Nonlinear Photonics Group. The purpose of the project is to provide a pre-stabilization of the tunable contiuous wave diode laser used for frequency comb generation at wavelength of 1550 nm. Future plans for the project includes further stabilizing the control signal in the feedback loop and precision measurements.

2 Background

2.1 Frequency noise

The term frequency noise refers to random fluctuation of the instantaneous frequency of an oscillating signal. For lasers, frequency noises can be classified into two types: slow frequency noise and fast frequency noise.

Slow frequency noise, which can be also called frequency drifting, is an unintended and generally arbitrary offset of an oscillator from its nominal frequency. It can be caused by thermal fluctuation of the system, poor power supply regulation and machanical pertubation. In this project, it is the main goal to elimilate frequency drfting of the laser [3].

Specifically, for a diode laser, high stability and narrow linewidth are often key requirements. Theoretically, the linewidth of grating stabilized diode lasers is given by Schawlow-Townes formula and therefore very narrow. In reality, however, a number of processes and disturbances have an effect on the laser frequency [4]. Laser current noise for example causes fluctuations of the refractive index within the laser diode itself, and changes the overall optical length of the laser resonator. Acoustic noise and vibrations have a direct influence on the mechanical length of an external resonator, while temperature and air pressure fluctuations cause frequency drifts by changing the refractive index of air.

2.2 Feedback System

In a dynamic system, physical variables are always required modification or regulation. One effective way of doing that is using the method of feedback. Feedback occurs when outputs of a system are fed back as some or all of the inputs of the system which affects the values of the system outputs. In such case, the system forms a circuit or a loop and when a change occurs in its output, such change is detected by the system itself and the output will be modified according to this change. When changes occurs in the output of a feedback system, it can perform two types of reaction towards such changes:

to amplify them, or to reduce them [5]. The feedback systems that amplify changes are called positive feedback system, in which the effects of a small disturbance on a system include an increase in the magnitude of the perturbation. Positive feedback tends to cause system instability. In contrast, the negative feedback systems which reduce the change of its output, generally promote stability. The output of a negative feedback system tends to be settling to an equilibrium value, resists environmental or inner-system perturbation within a certain range. In this thesis, a negative feedback system is built in order to perform stabilization of a frequency signal.

3 Pound-Drever-Hall Theory

The essential part of the Pound-Drever-Hall (PDH) laser frequency stabilization technique is a negative frequency feedback loop. An external Fabry-Parot cavity, whichhas a more stable resonant frequency than the frequency of the incoming laser, serves as a frequency reference [6]. If the frequency of a laser can be dynamically adjusted to match the resonant mode of the external cavity, the laser will then have the same stability as the cavity. The way to perform such adjustment according to the frequency jittering of the laser, is to generate an error signal which is proportional to the laser frequency deviation, and send the error signal back to the optical frequency shifter of the laser which corrects the laser output to eliminate the frequency error.

3.1 The Fabry-Perot Cavity

The external Fabry-Perot (FP) cavity plays an important part in the PDH system. By mounting two parallel high mirrors with high reflective index to a stable spacer opposite to each other, a high finesse cavity with a resonant frequency in accordance with the distance between the two mirrors is created. When light is normally incident on the cavity such that the distance between the two mirrors is a multiple of the wavelength of the light, it will bounce back and forth between the two mirrors and therefore generates a constructive interference field which continues to build up inside the cavity to an

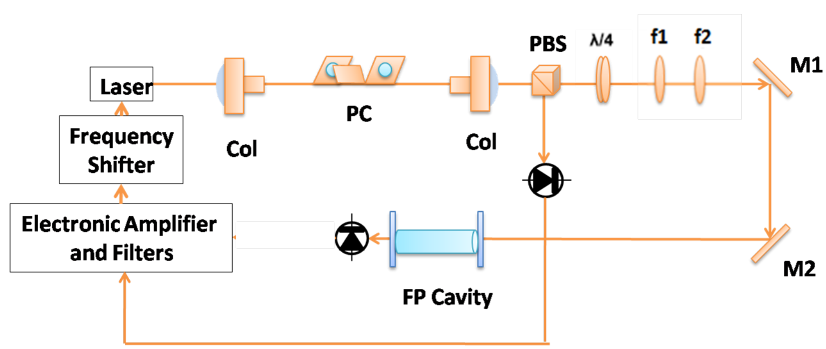

Figure 1: Pound-Drever-Hall System Block Diagram

equilibrium value. At equilibrium condition, the power exiting the cavity equals to the power of the incident light. The light exiting the cavity can go from both mirrors, but because the light exiting from the front mirror is 180 degree out of phase with the light directly reflected from the front mirror without entering the cavity, it appears to be no power coming out from the front mirror and all of the power exits from the rear mirror of the cavity [7]. The cavity seems to be transparent. On the other hand, if the cavity could not form an equilibrium condition, which means the incident light is not at resonant wavelength, all the light will be reflected back from the front mirror of the cavity. In this case, the cavity acts as a mirror with high reflection index. Figure 2 is the reflection and transmission spectrum of the cavity.

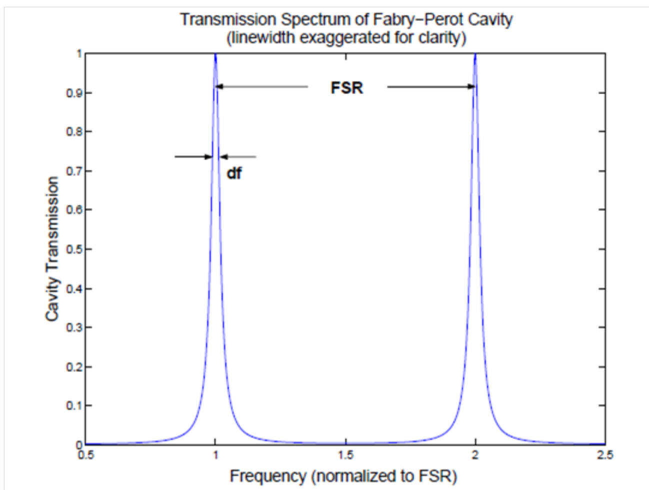

Figure 2: Transmission Spectrum of Fabry Perot Cavity

Assume the reflectivity of the front and rear mirror of the FP cavity are $R_1$and R_2, and transmittance to be $T_1$and T_2, respectfully. In order to calculate the reflection coefficient and transmition coeffeciton of the FP cavity, the parameters are defined as following.

$$r_1 = \sqrt{R_1} \tag{1}$$

$$r_2 = \sqrt{R_2} \tag{2}$$

$$t_1 = \sqrt{T_1} \tag{3}$$

$$t_2 = \sqrt{T_2} \tag{4}$$

Then the relation of incident, reflected and transmited electric field can be calculated.

$$\frac{E_{transmitted}}{E_{incident}} = \frac{t_1 t_2 e^{-i\omega_e \frac{L}{c}}}{1 - r_1 r_2 e^{-i2\omega_e \frac{L}{c}}} \tag{5}$$

$$\frac{E_{treflected}}{E_{incident}} = \frac{r_{1-} r_2 (r_1^2 + t_1^2) e^{-i2\omega_e \frac{L}{c}}}{1 - r_1 r_2 e^{-i2\omega_e \frac{L}{c}}} \tag{6}$$

$$\omega_e = \omega - \omega_0 \tag{7}$$

In the equations, L is the length between two mirrors, c is the speed of light in a free cavity, ω is the frequency of the incident light and ω_0 is the resonant frequency of the cavity. Because all the normally incident light with frequency , where n is an integer number, can be transmitted through the FP cavity, there are multiple peaks in the transmission spectrum. The spacing between theses peaks is called Free Spectral Range FSR, and it is a function of L and c.

$$FSR = \frac{c}{2L} \tag{8}$$

An mirror with a large reflectivity R will cause the average photon to bounce inside the FP cavity many times before exiting it. A term finesse $\mathcal{F}$ is used to describe such bouncing. The higher finesses is, the more bouncing times, and therefore a photon can survive longer in the cavity. In this case, the FP cavity is more sensitive to small perturbations in the incident laser frequency. In the spectrum prospective, a high finesse cavity have very narrow transmission spectra and are said to have narrow Gaussian linewidth $\Delta\nu$.

$$\Delta\nu = \frac{FSR}{\mathcal{F}} \tag{9}$$

Normally, the same type of mirrors are unsed at the front and rear of the cavity. Therefore we have $R_1 = R_2 = R$ and $T_1 = T_2 = T$. From all the parameters above, it is able to generate an equation for the electric field reflection coefficient $F(\omega_e)$ as a function of the difference in the frequencies of the incident laser and the cavity resonance ω_e and FSR.

$$F(\omega_e) = \frac{E_{treflected}}{E_{incident}} = \frac{R[\exp(i\frac{\omega_e}{FSR}) - 1]}{1 - R^2 \exp(i\frac{\omega_e}{FSR})} \tag{10}$$

It is apparent from the Figure 3 that the reflected optical intensity is symmetric about the cavity resonance frequency, which means $F(\omega_e)$ is an even function of ω_e. If the feedback loop is designed such that the error signal is proportional to the reflected intensity, the system will never be able to determine which side of resonance the laser is on. However, it is evident that the phase is antisymmetric function of frequency shift [3], thus something proportional to this would make and ideal signal. A stable, effective feedback loop must rely on the phase of the reflected light to determine the required adjustments to the input laser frequency.

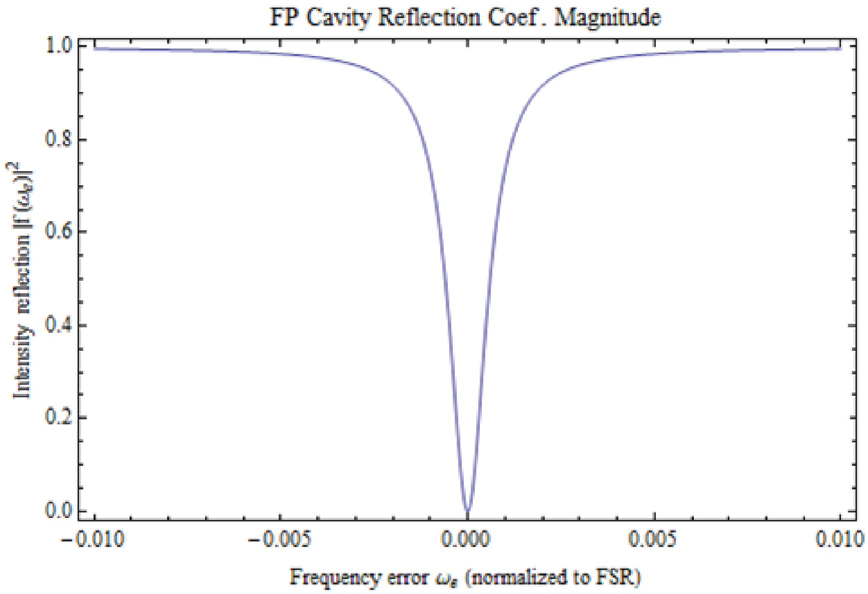

Figure 3: Magnitude of Fabry Perot Cavity Reflection Coeffcient

3.2 The Pound-Drever-Hall Technique

In order to generate an error signal antisymmetric across the cavity resonance frequency, the phase modulation is introduced into the laesr beam. An Electro-Optic Modulator (EOM), can be used for obtain the phase information of the reflected light. An EOM could be considerd similar to a variable waveplate. It is crystal which can rapidly its index of refraction in response to electrical stimuli. This induces a small phase change in light passing through it, which can be rapidly varied. The electric field of the laser beam passing through an EOM modulated at frequency $\Omega/2\pi$ will be the following.

$$\begin{aligned} E_{inc} &= E_0 e^{i(\omega t + \beta \sin \Omega t)} \\ &\approx E_0 \left[J_0(\beta) + 2iJ_1(\beta) \sin \Omega t \right] e^{i\omega t} \\ &= E_0 \left[J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\Omega+\omega)t} - J_1(\beta) e^{i(\omega-\Omega)t} \right] \end{aligned} \quad (11)$$

In the equation, J's are Bessel functions. It can be tell from the expansion that after phase modulation, there are three different frequency components in the beam. The three frequencies are ω, $\omega+\Omega$, and $\omega-\Omega$. The two J_1 components are called 'sidebands', being offset to either side of the primary frequency component by the modulation frequency Ω, which is a single tone in the RF frequency range. This high frequency modulation is required to push the sidebands far enough from the center frequency that they are totally reflected by the FP cavity when the center frequency is on resonance. β is called modulation depth, which is set such that the majority of the power is in the carrier and first order sidebands.

When this phase-modulated beam reaches the cavity and reflects, each frequency component will simply be transformed by $F(\omega)$, which gives the electric field of the reflected beam.

$$E_{ref} = E_0 \bullet \left[F(\omega_e)J_0(\beta)e^{i\omega t} + F(\Omega+\omega_e)J_1(\beta)e^{i(\Omega+\omega)t} - F(\omega_e-\Omega)J_1(\beta)e^{i(\omega-\Omega)t}\right] \quad (12)$$

However, the reflected electric field can not be measured directly. When light reflects back from the cavity, it is collected by a photodiode, which measures the intensity of the field. The photodiode acts as a mixer, obtains the signal of $I_{ref} = |E_{ref}|^2$.

$$\begin{aligned} I_{ref} = I_c|F(\omega_e)|^2 + I_s\left(|F(\omega_e+\Omega)|^2 + |F(\omega_e-\Omega)|^2\right) \\ + 2\sqrt{I_c+I_s}\Re\left[F(\omega_e)F^*(\omega_e+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right]\cos\Omega t \\ + 2\sqrt{I_c+I_s}\Im\left[F(\omega_e)F^*(\omega_e+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right]\sin\Omega t + \mathcal{O}(2\Omega) \quad (13) \end{aligned}$$

In the equation , $I_c$and I_s are the power of the carrier and sideband components, repectively. The three terms beat with each other to produce signal with frequencies of 0, Ω, and 2Ω. The beat signal with frequency Ω is generated by the mixing between the carrier frequency and each of the sidebands. This is the useful portion of the intensity signal, and bandpass filtering around Ω generates frequency error signal $I_e(t)$.

$$I_e(t) = 2J_0(\beta)J_1(\beta)\{\Re\left[F(\omega_e)F^*(\omega_e+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right]\cos\Omega t$$
$$+ 2\Im\left[F(\omega_e)F^*(\omega+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right]\sin\Omega t\} + \mathcal{O}(2\Omega) \quad (14)$$

Because the modulation frequency Ω is chosen large enough, when $F(\omega_e) \approx 0$, $F(\omega_e \pm \Omega)$ is relatively large. In other words, Ω is large enough so that the sidebands of the incident light are completely reflected. As the phase of the reflection coefficient approaches ± 180 degree when the frequency of the incident light is far from resonance, $F(\omega_e \pm \Omega) \approx -1$. Then we can have the following approximation.

$$\Re\left[F(\omega_e)F^*(\omega_e+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right] \approx \Re\left[-F(\omega_e) + F^*(\omega_e)\right]$$
$$\approx \Re\left[-2\Im\{F(\omega_e)\}\right] = 0 \quad (15)$$

Under this condition, the function of error signal can be simplified as following.

$$I_e(t) = -2J_0(\beta)J_1(\beta)\Im\left[F(\omega_e)F^*(\omega+\Omega) - F^*(\omega_e)F(\omega_e-\Omega)\right]\sin\Omega t \quad (16)$$

Ploting the error signal amplitude $I_e(t)$ on frequency error ω_e, it is clear that $I_e(\omega_e)$ has odd symmetry, and there is a large frequency over which the sign corresponds to which side of cavity resonance the laser is on. Additionally, the error signal has a high slope near resonance, which is the designed operate region for the feedback system. Therefore it can be used as an control signal that stablilizes the laser at the resonant frequency.

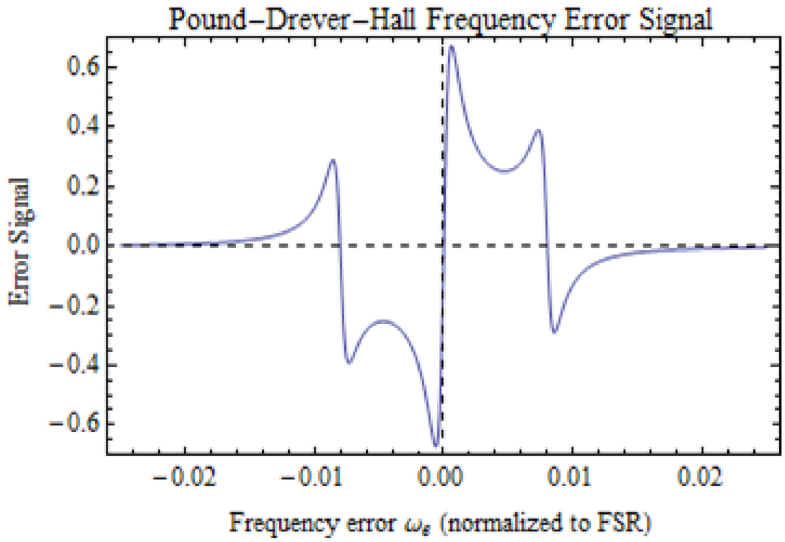

Figure 4: Pound-Drever-Hall Normalized Error Signal

3.3 System Sensitivity

As the system operates at the steep linear-like region around cavity resonance, the slope of that region determine the ability to correct the frequency error of the laser for the system. Normally, mirrors used in the FP cavity have extremely high reflectivity R. By approximating $R \approx 1$, the formular for cavity finesse can simplified [8].

$$\mathcal{F} \approx \frac{\pi}{1 - R^2} \tag{17}$$

Compare the resonant frequency, the frequency error generated due to environmental fluctuation is considerably small and locates amoung the linear-like region which has a very small frequency range. Therefore it ts safe to Taylor expand the reflection coefficient $F(\omega_e)$ about $\omega_e = 0$, and keep only two terms of the series under the condition $\omega_e/FSR \ll 1$.

$$F(\omega_e) = \frac{R[(1 + i\frac{\omega_e}{FSR}) - 1]}{1 - R^2(1 + i\frac{\omega_e}{FSR})} \tag{18}$$

For a small deviation $\delta\omega$ on ω_e, the equation can be further approximated as following.

$$F(\delta\omega) = i\frac{\mathcal{F}}{\pi FSR}\delta\omega \tag{19}$$

Plugging this expression into the error signal function and asumming $F(\omega_e \pm \Omega) \approx -1$, an approximation for the error signal when the laser is locked can be obtained.

$$I_e(\delta\omega) \approx \frac{4J_0(\beta)J_1(\beta)\mathcal{F}}{\pi FSR}\delta\omega \tag{20}$$

The PDH system sensitivity can then be calculated using the equations above.

$$Sensitivity = \frac{\delta I_e}{\delta f} = \frac{\delta I_e}{\delta\omega/2\pi} = \frac{8J(\beta)J_1(\beta)\mathcal{F}}{FSR} \tag{21}$$

It can be calculated that at $\beta = 1.08$, the systerm reaches the highest sensitivity.

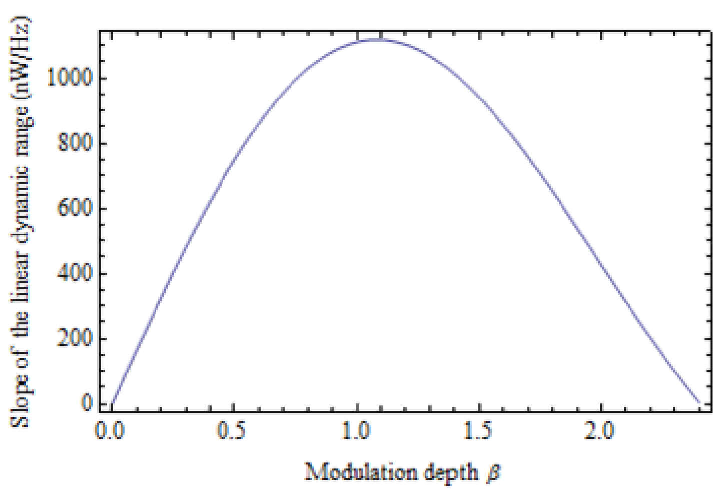

Figure 5: Modulation Depth Optimization Curve

4 System Design

4.1 External Cavity Diode Laser

An external cavity diode laser (ECDL) is a diode laser based on a laser diode chip which typically has one end anti-reflection coated, and the laser resonator is completed with a collimating lens and an external mirror. Another type of external-cavity laser uses a resonator based on an optical fiber rather than on free-space optics. Narrowband optical feedback can then come from a fiber Bragg grating. In ECDLs, wavelength tuning is possible by including some adjustable optical filter as tuning element. Tunable external-cavity diode lasers usually use a diffraction grating as the wavelength-selective element in the external resonator. They are also called grating-stabilized diode lasers.

4.2 Optical components

4.2.1 Fabry-Perot Cavity Design

The external FP cavity used for frequency reference contains three parts: a front mirror, a rear mirror, and an piezo actuator with inner hole that connects the two mirrors.

The mirrors used in the cavity are 10CV00SR.70F High Finesse SuperMirrors. It is a 25.4 mm diameter, 6.35 mm thickness concave SuperMirror, optimized for 1457-1659 nm. They have one concave and one flat surface. The concave surface has radius of curvature of 1 m, reflectivity $R \geq 99.97\%$, and polished to < 1 Å rms microroughness over central 50% of diameter. The flat surface has reflectivity $R_{avg} < 0.5\%$ and $R_{max} < 1.5\%$.

In order to accurately adjust the resonant frequency of the FP cavity, the two mirrors are mounted only to the piezo actuator. A piezo actuator converts an electrical signal into a precisely controlled physical displacement. The precise movement control afforded by piezoelectric actuators is used to finely adjust the distance between the two mirrors and therefore control the resonant frequency of the FP cavity.

The piezo used in this system is P-025.40H PICA Thru Ring Actuators has length of $L_p = 66$ mm, and displacement 80 μm. Adding the length of the mechanical parts, the total length of the cavity is $L = 76$ mm.

With these parameters, the basic properties of the FP cavity can be calculated as below.

$$FSR = \frac{c}{2L} = 1.972\ GHz \tag{22}$$

$$\mathcal{F} \approx \frac{\pi\sqrt{R}}{1-R} = 10470.4 \tag{23}$$

$$\Delta\nu = \frac{FSR}{\mathcal{F}} = 188.37\ kHz \tag{24}$$

The field inside the FP cavity can be modeled as a superposition of Hermite-Gaussian modes propagating back and forth between the two mirrors. These Hermite-Gaussian modes are orthogonal solutions to Maxwell's equations under the assumption that the beam propagates along the z axis The following equation describes the axial field distribution the fundamental Gaussian mode in the free space etalon. It is acceptable to consider only the fundamental transverse mode because a system of modematching lenses eliminates all other modes in the cavity.

$$E(r,t) = E_0 \exp\left[i(\omega t - kz) - \frac{x^2+y^2}{W^2(z)} - ik\frac{x^2+y^2}{2R(z)} - i\phi(z)\right]$$

In the equation, $W(z)$ is the beamwidth, $R(z)$ is the radius of curvature of the phase front, and $\phi(z)$ is the total phase. All of these terms are functions of the Rayleigh range z_0, which is a function of the wavenumber k.

$$\begin{aligned} W(z) &= W_0\sqrt{1+z^2/z_0^2} \\ R(z) &= z + z_0^2/z \\ \phi(z) &= \arctan(z/z_0) \\ z_0 &= kW_0^2/2 \end{aligned} \tag{25}$$

Plugging in all the values of the parameters, the waist of the beam inside the FP cavity can be obtained.

$$W_0 = 0.29\ mm \tag{26}$$

4.2.2 Mode Matching System

In order to excite the fundamental mode inside the FP cavity, a mode matching system is needed. There are multiple ways to shape the incident beam into the mode-matching beam waist before it reaches to the front mirrors of the cavity. But when the distance between the laser and the FP cavity is set to a certain value, using two focal lenses for beam shaping is the simplest way that provides the system enough degree of freedom for ajustment.

Before entering into the free space, the laser beam passes through a piece of fiber and comes out from a collimator. Because the light exits a single-mode fiber with a highly-polished FC connector, the beam exiting the collimator can be assumed to be almost entirely composed of the fundamental Gaussian mode. Any higher order Hermite-Gaussian modes in the Fabry-Perot etalon can only be excited by coupling from the fundamental mode due to beam misalignment.

The method of ABCD matrices is employed to obtain the values of the properties of the lenses for mode matching, which includes the focal length, and the distances between different optical components. A lens, mirror, or other optical element generally changes the parameters of a Gaussian beam. One approach to calculating the change is through the ABCD matrix representation of an element or a series of elements. An input q-parameter q_1 is transformed to an output qparameter q_2 according to the ABCD law.

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \tag{27}$$

And the q-parameter in the FP cavity$q_c(z)$, which is a known value, can be calculated by the following equation.

$$\frac{1}{q_c(z)} = \frac{1}{R(z)} - i\frac{2}{kW^2(z)} \tag{28}$$

The ABCD matrices for some simple optical elements are given in the table below [9].

Optical Element	ABCD Matrix
Propagation through a medium having index-of-refraction n and length d	$\begin{bmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{bmatrix}$
Refraction at a spherical boundary of radius R, entering a medium of index n_2 from a medium of index n_1. R is positive if the center of curvature lies in the positive direction of ray propagation.	$\begin{bmatrix} 1 & 0 \\ -\frac{n_2-n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$
Transmission through a thin lens of focal length f	$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$
Reflection from a spherical mirror having radius$R$. R is positive if the center of curvature lies in the positive direction of incident ray propagation.	$\begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$

Table 1: ABCD Matrices of Optical Elements

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With the ABCD matrices above, the overall ABCD matrices M can be calculated as below.

$$\mathrm{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & L_0 - d_1 - d_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \tag{29}$$

In the above equation, L_0 is the distance between the FP cavity and the output collimator, d_1 is the

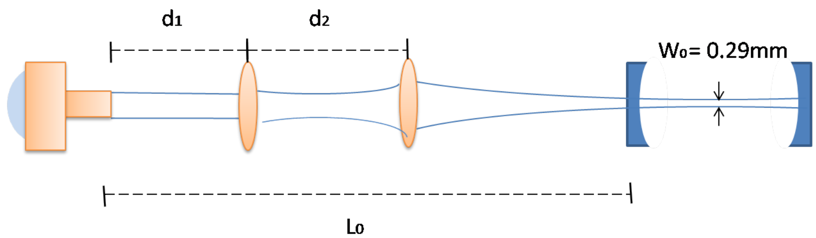

Figure 6: Optical Mode-Matching System

distance between the first lens and the collimator, d_2 is the distance between the two lenses, and f_1, f_2 are the focal length of the first and second lens, respectively.

The detail calculation is done through the software Mathematica, which is included in the Appendix I.

After calculation, the following set of parameters are chosen.

$$\begin{aligned} f_1 &= 0.25\ m \\ f_2 &= 0.10\ m \\ d_1 &= 0.205\ m \\ d_2 &= 0.372\ m \end{aligned} \tag{30}$$

4.2.3 Polarization Control

To achieve the proper low-noise operation of the PDH system, the polarization of the incident light must be controlled at two critical locations: the input to the EOM and the input to the Fabry-Perot Cavity. The $MgO : LiNbO_3$ crystal inside the EOM requires that the incident light be vertically polarized. Polarization of the input beam is performed by a Glan-Taylor (GT) polarizing cube, which filters out the unwanted horizontal component. A simple fiber polarization controller is used to precondition the beam coming from the laser in order to minimize the amount of power lost at the Glan-Taylor cube.

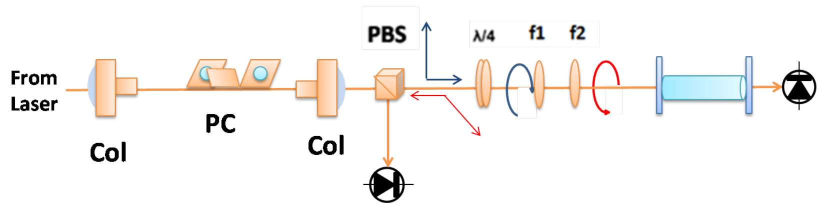

Figure 7: Optical Polarization Control System

The rationale behind the use of circular polarization in the etalon is more complicated. Even the highest quality Fabry-Perot mirrors exhibit some small level of birefringence due to stresses induced during fabrication of the mirror substrates and mounting to the etalon spacer. The result is that the light experiences a slightly different phase shift upon reflection (on the order of 0.1 μrad), depending on its state of polarization. In a high-finesse etalon, this minuscule effect multiplies to the point where there is a noticeable difference between the resonant frequencies of two orthogonal polarizations. Additionally, linearly polarized light can cause the mirror to develop a photorefractive "memory", which can lead to time-dependent birefringence noise. All of these effects can be avoided by using circularly polarized light, which rapidly averages the two birefringent states, and effectively erases the photorefractive memory as the polarization rotates.

A quarter-waveplate positioned in front of the etalon performs the conversion from vertical linear polarization to right-handed circular polarization. Upon reflection at the planar mirror, the polarization flips, and the reflected light is left-handed. As it passes back through the quarter-waveplate, the backward-traveling beam is converted to horizontal linear polarization. Half of this light is directed toward the photodiode module by a non-polarizing beamsplitter. The other half continues propagating toward the input, where it is completely rejected by the Glan-Taylor polarizer. In this way, the polarization control system also acts as an isolator, eliminating any reflected power that might destabilize the laser.

4.3 Electronic components

When the reflected beam is detected by a photon detector, the light signal is transformed into electric signal. As mentioned above, a photodiode acts as a mixer and generates signal contains three components, which are DC signal, signals with frequency of Ω, and 2 Ω. Among them, only the Ω frequency signal is used to feedback into the laser for eliminating the frequency fluctuation. Therefore electronic components are needed for signal amplification and filtering.

In the experiment, an InGaAs Transimpedence Amplified Photodetectors is used for collecting the reflected signal in free space. It consists of a photodiode and amplifier in a compact, low-profile package, are sensitive to light in the NIR region from 700 nm to 2600 nm. The slim profile housing enables use in light paths with space constraints. All connections and controls are located perpendicular to the light path, providing increased accessibility. Amplification is provided by low noise transimpedance or voltage amplifiers that are capable of driving 50 Ω loads.

The DC signal and the 2 Ω signal is filtered out by a high-pass RF filter and a low-pass RF filter.

5 Exprimental Results and Discussion

5.1 Mode Coupling

In order to excite the fundamental mode in the Febry Perot cavity, all the optical components such as collimators, polarizing cube and lenses are required be precisely adjusted to a specific place and orientation to make sure the incident beam is vertical to the front mirror of the cavity, and is well-shaped into the cavity mode. Each components have four degrees of freedom: x, y, z and axis orientation, which makes the alignment a challenging work. Firstly the output collimator is ajusted such that the beam coming out of it is parallel to the floating table. Next the FP cavity is placed at a proper distance from the collimator, making sure that the lasre beam is incident normally on the front mirror and the two mirrors are parallel to each other. Then all the other optical components are placed in between the output collimator and the FP cavity such that the path of the laser beam in free space is not changed. Finally, the position of the two lenses are finely adjusted to shape the beam into the mode that can be coupled into the cavity.

By analysing the reflected light and the transmitted light of the FP cavity, it can be tell whether the cavity fundamental mode is excited.

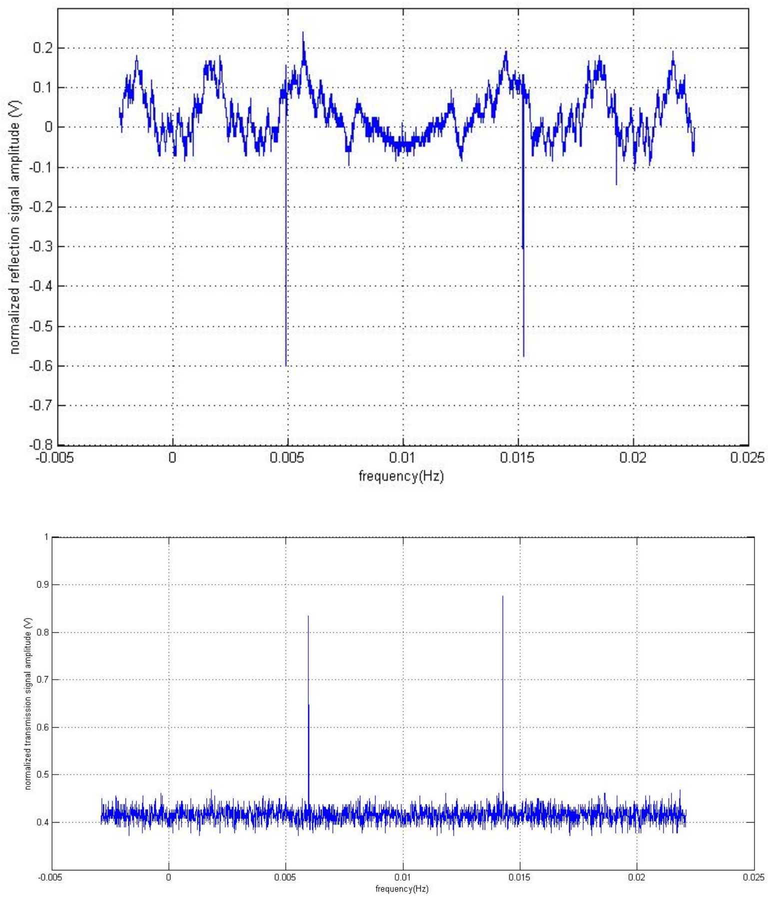

Figure 8: Reflected (top) and transmitted (bottom) signal from the febry perot cavity

In order to confirm that the mode excited in the cavity is actually the fundamental mode, a camera is set to capture the mode shape of the transmission light.

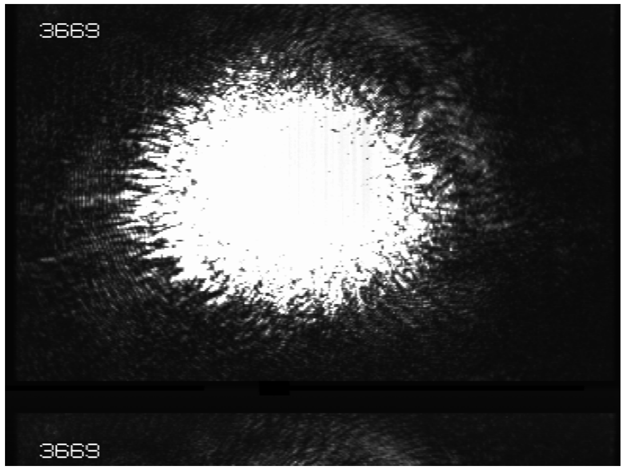

Figure 9: fundamental Mode in Fabry Perot Cavity

As it can be seen in the Figure 9, the spot of the transmission light is a perfect round shape, which assure that the mode being excited in the FP cavity is the fundamental mode.

5.2 Error Signal

During the experiment, the laser is being scanned by a function generator, which produces a triangular wave with frequency of 25 Hz, and peak to peak voltage of 100 mV.

The slope of the error signal can be directly read from the Figure 10.

$$Slope = 4.80 \times 10^9 \ V/s \tag{31}$$

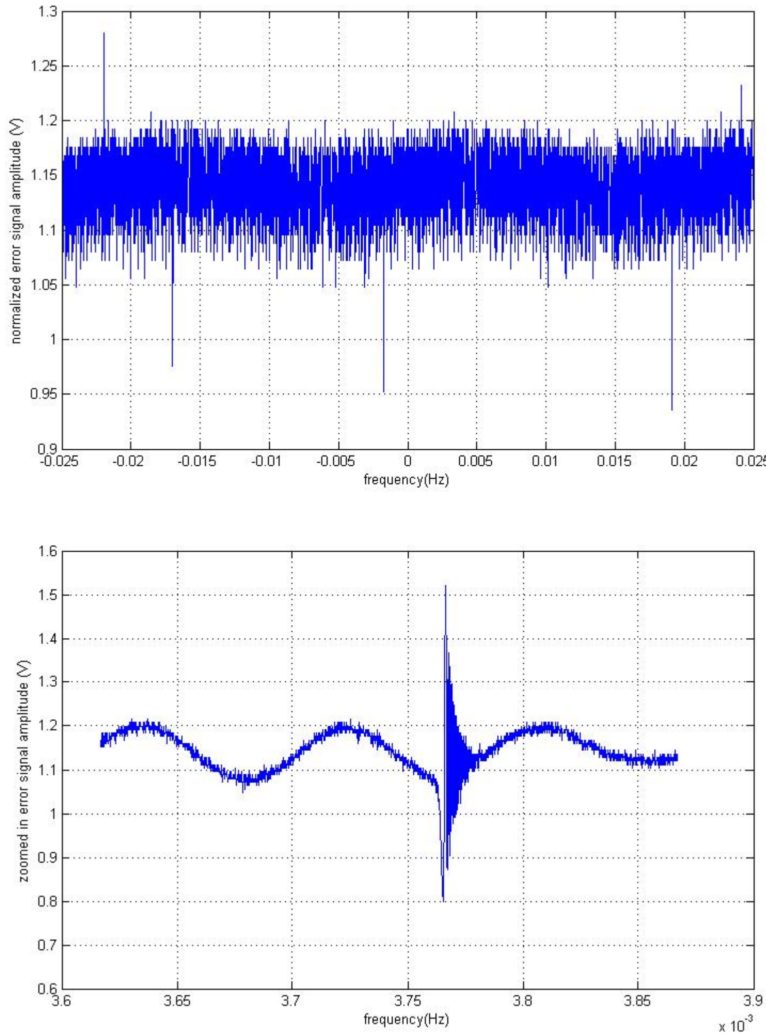

Figure 10: Error Signal of Pound-Drever-Hall System

5.3 Discussion

Alrhough it is properly generated, the error signal is high unstable. It fluctuates both in central frequency an amplitude. There is a low frequency modulation about 120 Hz in the signal, which is probably generated by the eletronic components. Also environmental changes such as mechanical pertubation will induce noise in intensity fluctuation of the error signal. The drifting of the cavity resonant frequency may be caused by the small changes in the cavity length. Because it is a very high finesse cavity, a small deviation from the origional cavity length will induce a large change in the resonant frequency. The relation between the resonant frequency deviation $\delta\omega_0$ and cavity length deviation δL is as below.

$$\delta\omega_0 = \frac{\omega_0}{L}\delta L \tag{32}$$

Therefore a 2.6 nm change in the cavity length can introduce a change of 5 MHz in the cavity resonant frequency.

In the experiment, the two mirrors are mounted on the piezo by glue, with thickness can be relatively easy to change by the tempreture and radiation pressure. One way to build a more stable FP cavity is to highly polish the surfaces of the piezo actuator and the two mirrors such that they can be connected by the electrostatic force. By such direct connection the cavity length can be more precisely control by the piezo and therefore the stability of the resonant frequency can be improved.

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APPENDIX

Mathematica Code for Mode Matching Calculation

```
(* Solving for one lens *)
Mlens = (1  L - x) . (  1    0) . (1  x)
        (0    1  )   (-1 / f  1)   (0  1)
qi = 1 / (I λ / (d / 2) ^2 / Pi);
qf = (qi * Mlens[[1, 1]] + Mlens[[1, 2]]) / (qi * Mlens[[2, 1]] + Mlens[[2, 2]]);
Simplify[Refine[Re[qf], x ∈ Reals && L ∈ Reals && f ∈ Reals && d ∈ Reals]];
qfRe = Assuming[x ∈ Reals && L ∈ Reals && f ∈ Reals && d ∈ Reals, ComplexExpand[Re[qf]]];
qfIm = Assuming[x ∈ Reals && L ∈ Reals && f ∈ Reals && d ∈ Reals, ComplexExpand[Im[qf]]];

q0 = -0.0728027 - I 0.208462;
d = 0.002;
L = 0.76;
λ = 1550 * 10^(-9);
Solve[{Re[q0] == qfRe, Im[q0] == qfIm}, {x, f}]
```

$$\left\{\left\{1-\frac{0.76-x}{f},\ 0.76-x+\left(1-\frac{0.76-x}{f}\right)x\right\},\ \left\{-\frac{1}{f},\ 1-\frac{x}{f}\right\}\right\}$$

```
{{f → -1.0917082025685765`, x → 1.6432206056604646`},
 {f → 0.` - 2.8032243804449496`*^-16 i, x → 0.` + 2.026833970057931` i},
 {f → 0.` - 2.8032243804449496`*^-16 i, x → 0.` + 2.026833970057931` i},
 {f → 1.4016121902224748`*^-16 + 8.409673141334848`*^-16 i,
  x → 0.` - 2.0268339700579308` i},
 {f → 1.4016121902224748`*^-16 + 8.409673141334848`*^-16 i,
  x → 0.` - 2.0268339700579308` i}, {f → 0.666029684386937`, x → 0.2133333123612569`}}

(* d1=0.106 d2=0.3944 f1=0.3 f2=0.08 the better one * )
(* d1=0.1 d2=0.542 f=1.45 f2=0.08*)

(* Solving for two lenses *)
```

```
Clear[d1, d2, f1, f2];
Mlens2 = (1  L - d1 - d2) . (   1     0) . (1  d2) . (   1     0) . (1  d1);
         (0       1     )   (-1 / f2  1)   (0  1 )   (-1 / f1  1)   (0  1 )
(* collimator d1 f1 d2 f2 (L-d1-d2) InputMirror *)
qf2 = (qi * Mlens2[[1, 1]] + Mlens2[[1, 2]]) / (qi * Mlens2[[2, 1]] + Mlens2[[2, 2]]);

qfRe2 = Assuming[d1 ∈ Reals && d2 ∈ Reals && f1 ∈ Reals && f2 ∈ Reals && L ∈ Reals && d ∈ Reals,
   ComplexExpand[Re[qf2]]];
qfIm2 = Assuming[d1 ∈ Reals && d2 ∈ Reals && f1 ∈ Reals && f2 ∈ Reals && L ∈ Rxeals && d ∈ Reals,
   ComplexExpand[Im[qf2]]];

q0 = -0.0728027 - I 0.208462;
d = 0.002; (* diameter of the collimated light *)
(* d1 = 0.106; *)
f1 = 0.25;
f2 = 0.10;
L = 0.76;
λ = 1550 * 10^(-9);
(*NSolve[{Re[q0]==qfRe2,Im[q0]==qfIm2},{d2,d1}]*)
Solve[{Re[q0] == qfRe2, Im[q0] == qfIm2}, {d2, d1}]
```

```
{{d2 → -106.015 + 272.036 i, d1 → 0.25162 - 2.05459 i}, {d2 → 0., d1 → 0.25 - 2.02683 i},
 {d2 → 0., d1 → 0.25 - 2.02683 i}, {d2 → 0., d1 → 1.60047 × 10^14},
 {d2 → 0., d1 → 2.07151 × 10^14}, {d2 → 0. - 9.0251 i, d1 → 0.25 + 2.02683 i},
 {d2 → 0. - 9.0251 i, d1 → 0.25 + 2.02683 i}, {d2 → 0.382143, d1 → 0.204998},
 {d2 → 0.419112, d1 → 0.561812}, {d2 → 452.647 - 372.265 i, d1 → 0.25162 + 2.05459 i}}
```

```
(* f1 = 0.25; f2 = 0.10,d2→0.3726882835232522`,d1→0.2055868004039431`*)
(* f1 = 0.25; f2 = 0.10; d1 = 0.205; d2 = 0.3724*)
(* f1=0.125; f2=0.075; d2=0.2061;d1=0.2152 *)
(* f1 = 3;f2 = 0.1; d1 = 0.2618; d2 = 0.4118*)
```