Ithaca Got Your Lips Chapped?
A Performance Analysis of Lip Balm

BEE 4530

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EXECUTIVE SUMMARY

The transfer of heat and moisture in the lip with and without lip balm present was modeled in COMSOL multiphysics in order to determine how effective lip balm is in moisture retention.The effects of weather conditions on moisture loss were also considered as the model was done for both normal conditions (indoors, 25°C, normal humidity) and winter conditions (windy, -5°C, low humidity). Heat and moisture in the lips is delivered through blood perfusion from capillaries in the dermis which is dependent on the temperature of the lips as the blood vessels constrict in cold weather and dilate in warm weather, thus changing the surface area from which moisture can diffuse. When lip balm is applied to the lips, it acts as a sealant, thereby preventing moisture loss through evaporation. This protection allows the lips to rehydrate through the accumulation of moisture at the lip balm-stratum corneum interface. \

For our model, we assumed the transfer of heat and moisture in the lip to be in one dimension but used a two dimensional geometry in COMSOL for better visualization. Our geometry included four layers: dermis, epidermis, stratum corneum, and lip balm. In modeling temperature, the epidermis stratum corneum, and lip balm had simple conduction governing equations while in the dermis, the governing equation also contained a generation term due to blood perfusion. For moisture, the epidermis and stratum corneum had simple diffusion governing equations while the dermis again had a governing equation containing a generation term due to blood perfusion. This generation term depended on the temperature of the lip. The lip balm layer had no governing equation and was inactive because it acted as a perfect sealant with a diffusivity of zero. Through experimentation we found it takes approximately two and a half hours for lip balm to fully degrade. In our model we assume that as long as lip balm is present no evaporation can occur and as soon as it is gone, evaporation can occur. As a result, for the first two and half hours the model is run, the lip balm layer acts as a sealant. After two and a half hours, the boundary condition at the lip surface becomes convective, as the lip surface now interacts with the environment.

For normal conditions, the model showed an overall temperature change of only 0.09°C in the dermis. In terms of moisture, the model showed that moisture was replenished in the lips with and without lip balm present, though when the lip balm was removed, moisture was replenished at a slower rate due to the evaporation at the lip surface. For winter conditions the model showed a more drastic change in lip temperature of 6°C. With lip balm present, moisture was steadily replenished but as soon as the lip balm was gone, moisture was quickly lost from the lips due to the dry and windy conditions. The results of our model confirm that lip balm is effective in helping the lips retain moisture and that weather conditions play an important role in the lips ability to retain moisture as well.

INTRODUCTION:

Is lip balm effective at maintaining the moisture of our lips? What is its stability in climates such as Ithaca? Does the weather affect the lip balm's main function of moisture retention? Does it actually have a purpose? We will complete a performance analysis of lip balm to determine how it maintains the moisture of our lips and how boundary conditions (wind, humidity, temperature) affect its ability to keep our lips moist and uncracked.

The composition of lips differs from that of regular skin. Lips have a thinner layer of skin cells, and are therefore more translucent, revealing the redness from the underlying blood vessels. Additionally, the skin on the lips does not contain hair follicles, sweat glands, or sebaceous glands that secrete sweat and oils, all of which are molecules that the skin usually uses for protection [13]. The lack of these features causes the lips to be more susceptible to drying out and becoming chapped in dry weather.

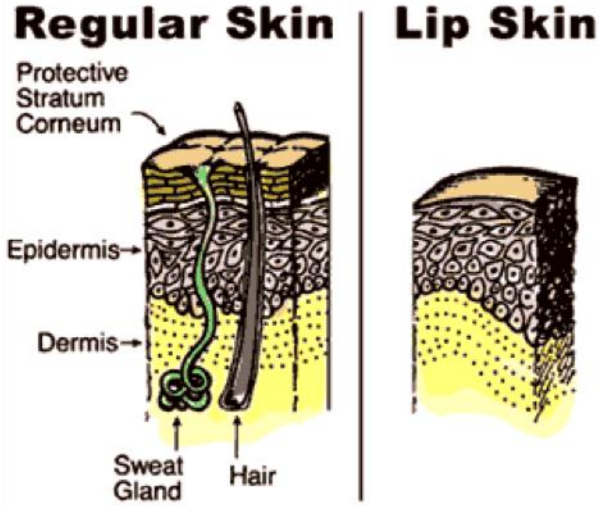

Figure 1: Schematic of lip balm and its layers in both regular skin, and lip skin. http://www.blistex.com/lip-care/what-are-lips

Our lips' moisture comes from the capillaries. Through the mechanism of mass transport, moisture diffuses from the capillaries to the tissue at a rate termed diffusive flux [14]. In addition to having a prominent role in the mass transfer of moisture, our capillaries are also a major source of heat for our skin and set up a temperature gradient within our tissue. This temperature gradient is complicated by the fact that the weather conditions also affect the temperature of our skin, causing the temperature gradient to be modifiable from either boundary. The diffusive flux of moisture is dependent on this temperature gradient since the diffusive flux is a function of the blood vessels' surface area, and this depends on temperature. During cold weather, our blood vessels will constrict to conserve heat [6]. The physiological effect of this constriction is a decrease in diameter of the blood vessel, and a reduction of blood flow near the skin surface [6]. The reduction in diameter reduces the surface area of the blood vessel, and it is this area that the diffusive flux depends on. The end result is less moisture transfer between the capillaries and tissue, and this often results in a physical discomfort known as chapped lips (Figure 2). Warm weather, in contrast, leads to vasodilatation of the capillaries, and causes the

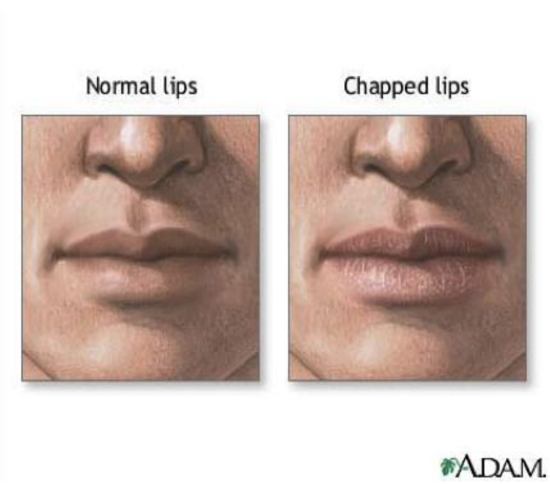

Figure 2: Schematic of Normal Lips versus Chapped Lips http://medicalimages.allrefer.com/large/chapped-lips.j

diameter of the blood vessels to increase. Using similar logic as in the vasoconstriction situation, the vasodilatation will cause the surface area of the blood vessel to enlarge, and increase the diffusive flux of moisture between the capillaries and tissue.

The application of lip balm creates a layer of immiscible oil on our lip surface [15]. Regular lip balms serve as sealants for our lips; the layer created by the lip balm is a protective coat between our lips and whatever boundary condition the weather creates at the air-lip balm interface. As a sealant, the lip balm prevents moisture loss, providing our lips the opportunity to regain their original moisture content through the diffusive flux between our capillaries and tissue. In the presence of lip balm, the moisture will accumulate at the protective stratum corneum-lip balm interface. Due to its role as a sealant, the degradation of the lip balm layer equates to an increase in moisture loss between the air and the lip. The complete removal of lip balm leaves zero protection between the lips and the external environment.

Figure 3: Visual of lip balm layer created by application of chapstick.
http://media.fa bulously40.com/images/chapstick.jpg

We will model the effect of lip balm on the moisture retention of our lips by completing a performance analysis of lip balm under a range of weather conditions. This model contains two governing equations –heat transfer with a bioheat term, and mass transfer of moisture from the capillaries to the dermis, and to the edge of our lips where the presence of lip balm determines its evaporation rate. The heat transfer equation will model the temperature gradient established by the conditions at the two boundaries, while the mass transfer equation will model the temperature-dependent diffusion of moisture from the capillaries of the dermis to the surface of the lip.

GOALS:

The performance analysis of lip balm has three main goals, which are outlined below.

1. Determine how temperature affects the moisture content of our lips

   One known physiological phenomenon is that temperature affects the diameter of the capillaries in the dermis. For example, we know that the capillaries will constrict in cold temperatures. The dependence of the diameter of the capillaries on temperature yields a varying diffusive flux of moisture, and the modeling of this behavior will allow us to quantify the moisture content. Our heat transfer equation will use a bioheat term which utilizes a blood perfusion expression that is a function of temperature based on literature data. This blood perfusion term is used to calculate the diffusive flux of moisture, which is also based on the percent of blood plasma, the average amount of moisture that diffuses out, and the densities of the tissue and water.

2. Model the evaporation of water from the lip surface.

   Through experimentation it was found that the average time that lip balm lasts on an individual's lips is approximately 2.5 hours. To model the sealant behavior of lip balm, we will implement a logic function that makes the diffusivity of water through lip balm 0 while the lip balm is on (t<9000sec). After 2.5 hours, the evaporation of moisture will begin to occur at the stratum corneum-air boundary. This will be modeled by a convective mass transfer boundary condition, based on the water vapor just above the lip surface and the relative humidity of the surrounding air.

3. Analyze how weather conditions affect our lips' moisture retention

   Two weather conditions will be studied in the performance analysis. Conditions that will be varied include temperature and humidity.
      a. Normal Conditions: room temperature, normal humidity
      b. Winter Weather Conditions: cold temperature, low humidity with wind conditions

The compilation of results will answer the question we've all asked – is lip balm really effective at moisture retention?

DESIGN:

The COMSOL design will have a temperature equation that models the effect of temperature on the blood flow rate in the dermis. This temperature parameter will include not only the temperature gradient maintained by the body, but also the temperature gradient established by the external boundary conditions (weather). The blood perfusion rate of the capillaries will change depending on the resulting temperature in the dermis.

The next part of the design will model the moisture content of the lips. In the dermis there is a moisture generation term that depends on the blood perfusion expression that is used in the bioheat term. The constant in front of the blood perfusion reflects the density of the tissue and water, the percent of blood plasma (essentially water) in the blood, and the percent that actually diffuses out into the dermis tissue. t=0 is defined as the time when lip balm is applied. When time is less than 2.5 hours, moisture will diffuse to the edge of the lips, and accumulate at the boundary, yielding a boundary condition of zero flux. After 2.5 hours, the original internal boundary between the lip balm and skin will become active and evaporation will be modeled through a convective mass transfer boundary.

To model the different weather conditions, the boundary conditions for the two sets of multiphysics (temperature and mass transfer) will account for indoor (natural convection), room temperature (25°C) and normal humidity (50% relative humidity) conditions and outdoor (forced convection) winter cold (-5°C), and dry (20% relative humidity) conditions.

SCHEMATIC:

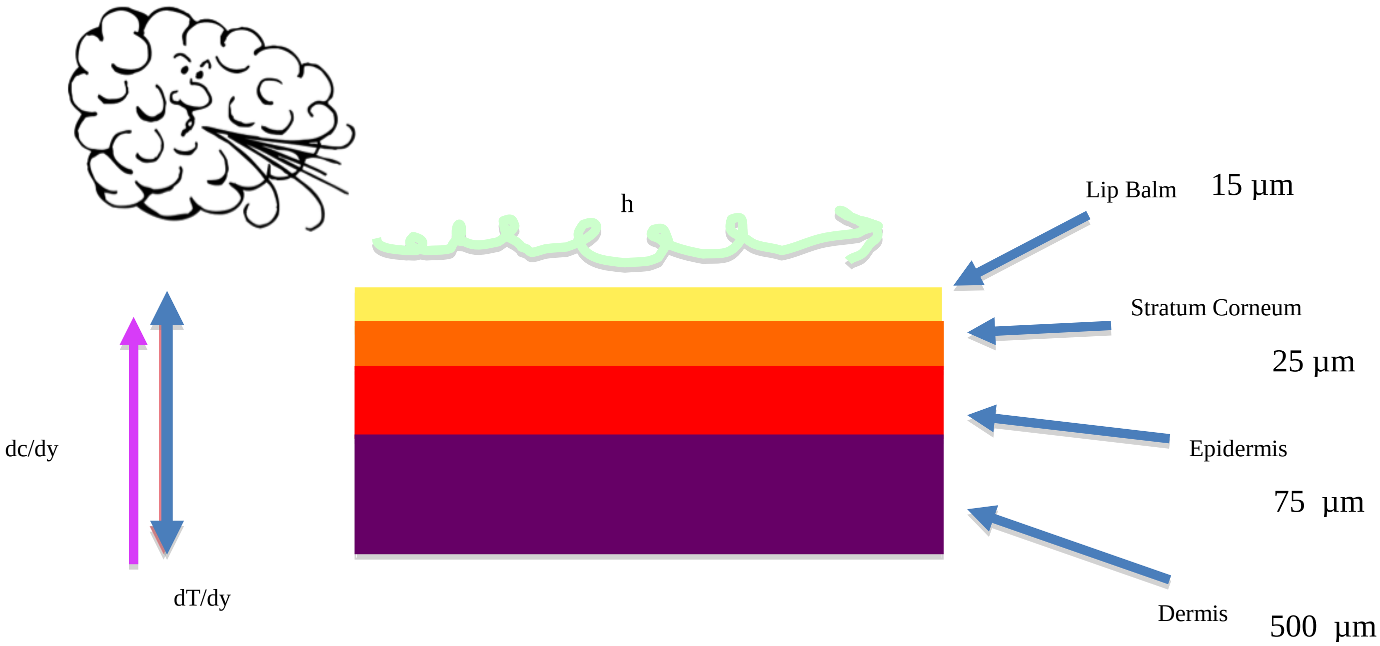

Figure 4: Schematic of physical situation in lip balm problem formulation.

The geometry of our problem formulation is composed of four sub-domains – lip balm, stratum corneum, epidermis, and dermis. The thicknesses of these subdomains are 15µm, 25µm, 75µm, and 500µm, respectively [5, 10]. Our geometry is technically 1D and will be solved using 1D equations. However, it will be implemented into COMSOL using a 2D geometry to better visualize the activity in the lips. This will be done by specifying the sides of our geometry as insulated boundaries. All four subdomains specified above are active for the heat transfer equation, since a temperature gradient forms through the lip balm and lip. For the mass transfer equation, though, only the three lip layers are active because no mass transfer occurs in the lip balm since it is a hydrophobic material and essentially acts an insulated boundary. Between t = 0 and t = 2.5 hours of our model, the lip balm is still present on the lip; thus it has a boundary condition of zero flux. After t=2.5 hours, the lip balm has degraded and disappears changing the boundary to convective conditions dependent on the relative humidity of the outside air and the water vapor concentration at the surface of the lip. Both of these values are based on temperature, rather than the moisture in the lip causing the convective boundary to simply become a constant flux. The total geometry contains two gradients – a moisture gradient and a temperature gradient. We are starting our modeling at time = 0 seconds with extremely chapped lips that contain no moisture, the temperature of the lip balm at the same temperature as the outside conditions, and the lip at body temperature, 37°C. The schematic for our project is shown in Figure 4.

GOVERNING EQUATIONS:

The governing heat transfer equation for the conservation of thermal energy in 1D along the y-axis is:

$$\underbrace{\frac{\rho C_p \partial T}{\partial t}}_{storage} + \underbrace{u\rho C_p \frac{\partial T}{\partial y}}_{flow\ or\ convection} = \underbrace{k\frac{\partial^2 T}{\partial y^2}}_{conduction} + \underbrace{Q}_{generation}$$

Since there is no convection or bulk flow in our schematic, the convection term is reduced to zero.

$$u\rho C_p \frac{\partial T}{\partial y} = 0$$

$$\underbrace{\frac{\rho C_p \partial T}{\partial t}}_{storage} = \underbrace{k\frac{\partial^2 T}{\partial y^2}}_{conduction} + \underbrace{Q}_{generation}$$

Unlike the other subdomains, the heat transfer equation for the dermis contains a bioheat term known as convection due to blood flow since it contains capillaries. This replaces the Q term for the dermis region only.

$$\underbrace{\frac{\rho C_p \partial T}{\partial t}}_{storage} = \underbrace{k\frac{\partial^2 T}{\partial y^2}}_{conduction} + \underbrace{\rho_{blood} c_{blood} \dot{V}^{v}_{blood} (T_a - T)}_{convection\ due\ to\ blood\ flow} \qquad \text{(Dermis)}$$

Note thatblood flow only occurs in the dermis. As can be seen in the term, the convection due to blood flow depends on the temperature in the dermis, T, and the temperature of the arterial blood, T_a. The blood perfusion rate, $\dot{V}^{v}_{blood}$, will be linearly related to the temperature of the blood, and will also determine the diffusion of moisture into the lip tissue. The blood perfusion expression was derived from literature values (See Appendix B) and is denoted as ‘V_Blood’ in our COMSOL implementation. It is only valid for temperatures above 308K.

$$V_{blood} = 1.67 \times 10^{-7} T - 5.1103\ x 10^{-5} \frac{m^3_{blood}}{kg_{tissue} \cdot s}$$

There is no blood perfusion in the lip balm, epidermis, or protective stratum corneum. We will also assume no significant heat generation occurs in these subdomains since no major energy-consuming processes occur there. This causes the Q term to drop out in the remaining domains, making the governing equation:

$$\frac{\rho C_p \partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} \qquad \text{(Lip Balm, Stratum Corneum, Epidermis)}$$

The mass transfer governing equation begins with the mass species conservation equation.

$$\underbrace{\frac{\partial c_A}{\partial t}}_{storage} + \underbrace{\left(\upsilon_x \frac{\partial c_A}{\partial x} + \upsilon_y \frac{\partial c_A}{\partial y} + \upsilon_z \frac{\partial c_A}{\partial z}\right)}_{flow or\, convection} = \underbrace{D_{AB}\left(\frac{\partial^2 c_A}{\partial x^2} + \frac{\partial^2 c_A}{\partial y^2} + \frac{\partial^2 c_A}{\partial z^2}\right)}_{diffusion} + \underbrace{R_A}_{generation}$$

As previously discussed, the schematic of our simplified geometry is 1D. As a result, there are no x or z terms in the mass transfer governing equation.

$$\frac{\partial c_{H2O}}{\partial t} + \left(\upsilon y \frac{\partial c_{H2O}}{\partial y}\right) = D_{H2O}\left(\frac{\partial^2 c_{H2O}}{\partial y^2}\right) + R_A$$

Since there is no convection in our schematic, removing $\left(\upsilon y \frac{\partial c_{H2O}}{\partial y}\right)$ from our equation

$$\frac{\partial c_{H20}}{\partial t} = D_{H20}\left(\frac{\partial^2 c_{H20}}{\partial y^2}\right) + R_A$$

The moisture concentration will be transient in our physical situation. Also, there will be a generation term in our equation that will reflect the moisture diffusing from the blood capillaries. Since this diffusive flux depends on the blood perfusion rate, which depends on temperature, our moisture flux into the dermis is dependent on temperature through this relationship. The coefficient was calculated by accounting for the percent of water in blood, the percent of the water that diffuses into tissue, and the density of tissue and water (See Appendix B). There will also be a value that caps the maximum concentration to 70% of the tissue can be water, since the tissue cannot hold more moisture than this. The governing equation below will only be used in the dermis.

$$\frac{\partial c_{H20}}{\partial t} = D_{H20}\left(\frac{\partial^2 c_{H20}}{\partial y^2}\right) + 27{,}500 V_{blood}\left(700\, \frac{kg_{H2O}}{m^3_{tissue}} - c_{H2O}\right) \qquad \text{(Dermis)}$$

The epidermis and stratum corneum subdomains will not have this moisture generation term because there are no capillaries in these layers from which water can diffuse.

$$\frac{\partial c_{H20}}{\partial t} = D_{H20}\left(\frac{\partial^2 c_{H20}}{\partial y^2}\right) \qquad \text{(Stratum Corneum, Epidermis)}$$

There is no governing equation in the lip balm domain because no moisture diffuses through it, thus it is not active.

BOUNDARYCONDITIONS:

Temperature Equation

Defines flux at both sides of the lip boundary to make our 1D problem 2D:

$$\left.\frac{\partial T}{\partial x}\right|_{x=0\ mm} = \left.\frac{\partial T}{\partial x}\right|_{x=1mm} = 0$$

Defines conduction and convection at lip surface, y=0.615 mm, which will be dependent on outside weather conditions:

$$-k\left.\frac{\partial T}{\partial y}\right|_{y=0.615\ mm} = h_{air,T}\left(T_{y=0.615} - T_{\infty}\right)$$

Defines conduction at internal boundaries, showing continuity:

$$-k_{lip\ balm}\frac{\partial T}{\partial y} = -k_{\ statum\ corneum}\frac{\partial T}{\partial y}$$

$$-k_{\ statum\ corneum}\frac{\partial T}{\partial y} = -k_{epidermis}\frac{\partial T}{\partial y}$$

$$-k_{epidermis}\frac{\partial T}{\partial y} = -k_{dermis}\frac{\partial T}{\partial y}$$

Defines constant temperature boundary at the edge of the dermis as body temperature:

$$T|_{y=0mm} = 310.15K$$

Mass Transfer Equation

Defines flux at both sides of the lip boundary:

$$\left.\frac{\partial c_{H2O}}{\partial x}\right|_{x=0\ mm} = \left.\frac{\partial c_{H2O}}{\partial x}\right|_{x=1\ mm} = 0$$

The lip surface-lip balm boundary has a flux of zero to model the lip balm as a sealant for the first 2.5 hours that our simulation is run:

$$-D\left.\frac{\partial c_{H2O}}{\partial y}\right|_{y=0.600mm} = 0$$

After 2.5 hours, the following defines convective boundary at lip surface, which will be dependent on outside weather conditions:

$$-D\frac{\partial c_{H2O}}{\partial y}\bigg|_{y=0.600mm} = h_{m,T}(C_s - C_\infty)$$

The variable C_s reflects the water vapor concentration at 100% relative humidity at the lip boundary and depends on the outside temperature. The assumption is that the liquid water at the surface of the lip is at equilibrium with the air adjacent to it, resulting in saturated air with a relative humidity of 100% right at the lip-air boundary. C_∞ is the concentration of water vapor derived from the relative humidity of normal or winter weather conditions.

Defines internal boundaries, showing continuity:

$$-D_{statum\ corneum}\frac{\partial c_{H2O}}{\partial y} = -D_{epidermis}\frac{\partial c_{H2O}}{\partial y}$$

$$-D_{epidermis}\frac{\partial c_{H2O}}{\partial y} = -D_{dermis}\frac{\partial c_{H2O}}{\partial y}$$

Defines the bottom boundary as having zero flux:

$$-D\frac{\partial c_{H2O}}{\partial y}\bigg|_{y=0\ mm} = 0$$

INITIAL CONDITIONS:

Temperature in lip layers = 310.15K

Temperature of lip balm layer = Outside temperature (-5°C, 25°C)

Moisture content at time zero in entire lip (dry & cracked): C_{H2O} (t = 0) = 0

MESH CONVERGENCE:

COMSOL uses the finite element method in order to come up with the heat and mass transfer solutions over the geometry we have created. The map of the elements COMSOL uses in this method is called the mesh. The solution COMSOL gives us depends on our design of the mesh and the size of the elements. As the size of the elements decreases more elements can fit into the geometry and the final solution will be more accurate as more elements are included. Unfortunately, as the number of elements increases more computing power is needed and it will take COMSOL longer to compute the final solution. As a result, the ideal situation is to have the least number of elements with as accurate a solution as if more elements had been used. The process of finding this optimal number of elements is called a mesh convergence. In this process, a mesh is created, the solution is run, and a value in one of the areas (at a point) is recorded. The mesh is then refined to have more elements, the solution is run again, and the value in the same area (and therefore same point) ais recorded. These steps are repeated until there is no significant difference in the values recorded for two consecutive meshes. When this happens, the first of the two meshes is the optimal mesh.

For our mesh convergence we used a map mesh to make rectangular elements. For each mesh, we found the temperature in the dermis and concentration in the stratum corneum since the most amount of change occurred in these areas. The temperature and concentration values we found, along with their corresponding number of mesh elements used can be seen in Table 1. We found that temperature did not vary with mesh size, as seen in Figure 5, so any mesh size can be used. The concentration curve in Figure 6 converges at 9800 elements, representing our optimal mesh. The actual mesh we used with 9800 elements can be seen in Figure 7. It is finer in the topmost layers because that is where the concentration changes the most in our solution.

At time = 14,400 seconds
Temperature @ (0.5mm, 0.4mm)
Concentration @ (0.5mm, 0.58mm)

Mesh: 530 Elements
Concentration value:96.845097mol/m^3
Temperature value: 304.320779 K

Mesh: 800 Elements
Concentration value:96.768947mol/m^3
Temperature value: 304.320779 K

Mesh: 1410 Elements
Concentration value:96.764281mol/m^3
Temperature value: 304.320779 K

Mesh: 2200 Elements
Concentration value:96.754952 mol/m^3
Temperature value: 304.320779 K

Mesh: 3400 Elements
Concentration value:96.763638mol/m^3
Temperature value: 304.320779 K

Mesh: 5250 Elements
Concentration value:96.746149mol/m^3
Temperature value: 304.320779 K

Mesh: 7300 Elements
Concentration value:96.742118mol/m^3
Temperature value: 304.320779 K

Mesh: 9800 Elements
Concentration value:96.558545mol/m^3
Temperature value: 304.320779 K

Mesh: 12750 Elements
Concentration value:96.563273mol/m^3
Temperature value: 304.320779 K

Mesh: 15200 Elements
Concentration value:96.551726mol/m^3
Temperature value: 304.320779 K

Table 1. Mesh convergence values found in the dermis and stratum corneum for temperature and concentration, respectively, with corresponding number of elements used.

Number of Elements	Temperature at (0.5,0.4) [K]	Concentration at (0.5,0.58) [mol/m^3]
530	304.320779	96.845097
800	304.320779	96.768947
1410	304.320779	96.764281
2200	304.320779	96.754952
3400	304.320779	96.763638
5250	304.320779	96.746149
7300	304.320779	96.742118
9800	304.320779	96.558545
12750	304.320779	96.563273
15200	304.320779	96.551726

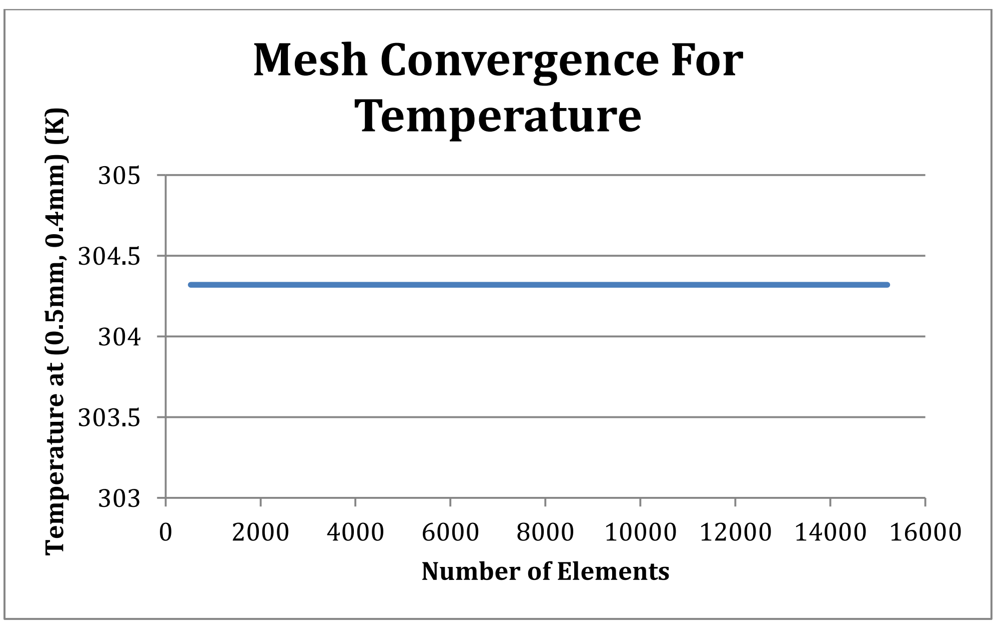

Figure 5. Mesh convergence for temperature using values in the dermis at (0.5mm, 0.4mm)

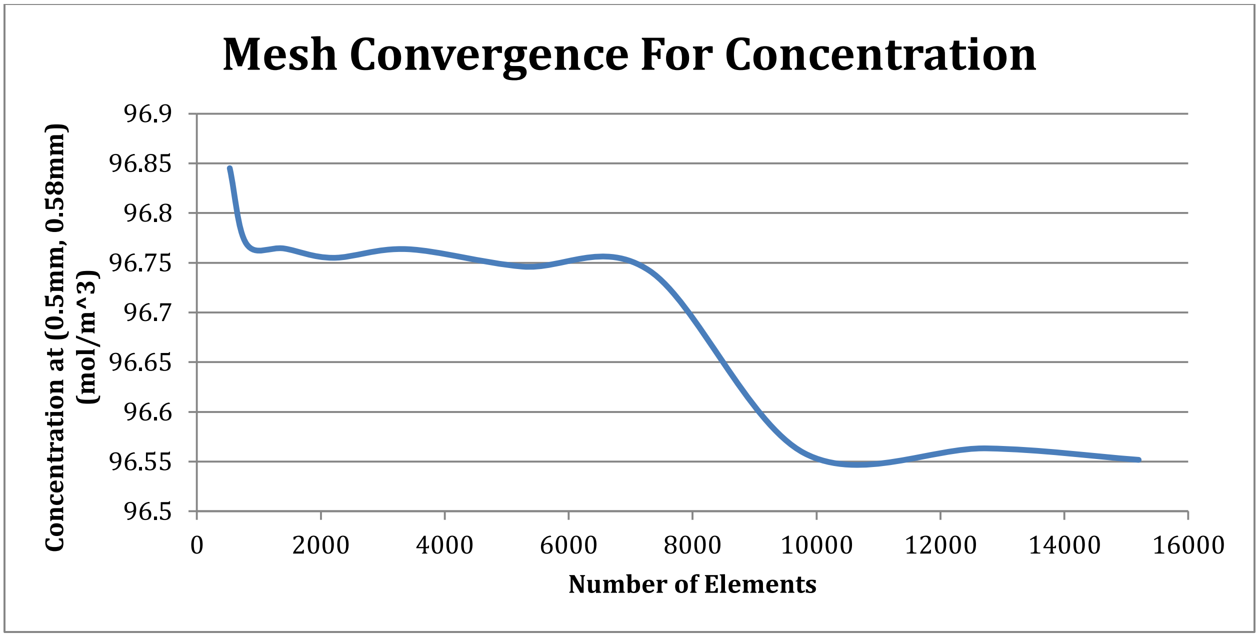

Figure 6. Mesh convergence for concentration using values in the stratum corneum at (0.5mm, 0.58mm)

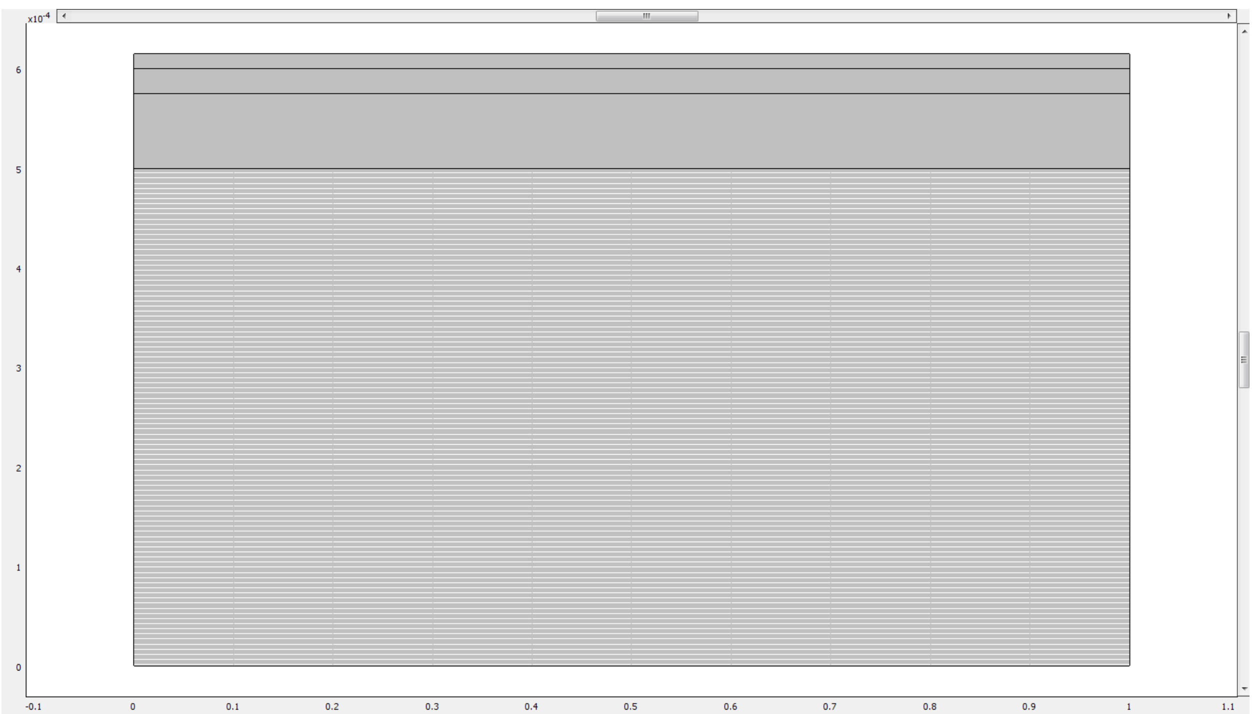

Figure 7. Mesh used in COMSOL for the lip balm, stratum corneum, epidermis, and dermis

COMPLETE SOLUTION:

Normal Weather Conditons (T=25°C and 50%RH)

Temperature Solution:

Shown below (Figure 8) is the temperature surface plot at initial conditions. As can be seen, the statrum corneum, epidermis, and dermis are all at body temperature, 310.15K. The lip balm domain is at a uniform temperature of 298.15K.

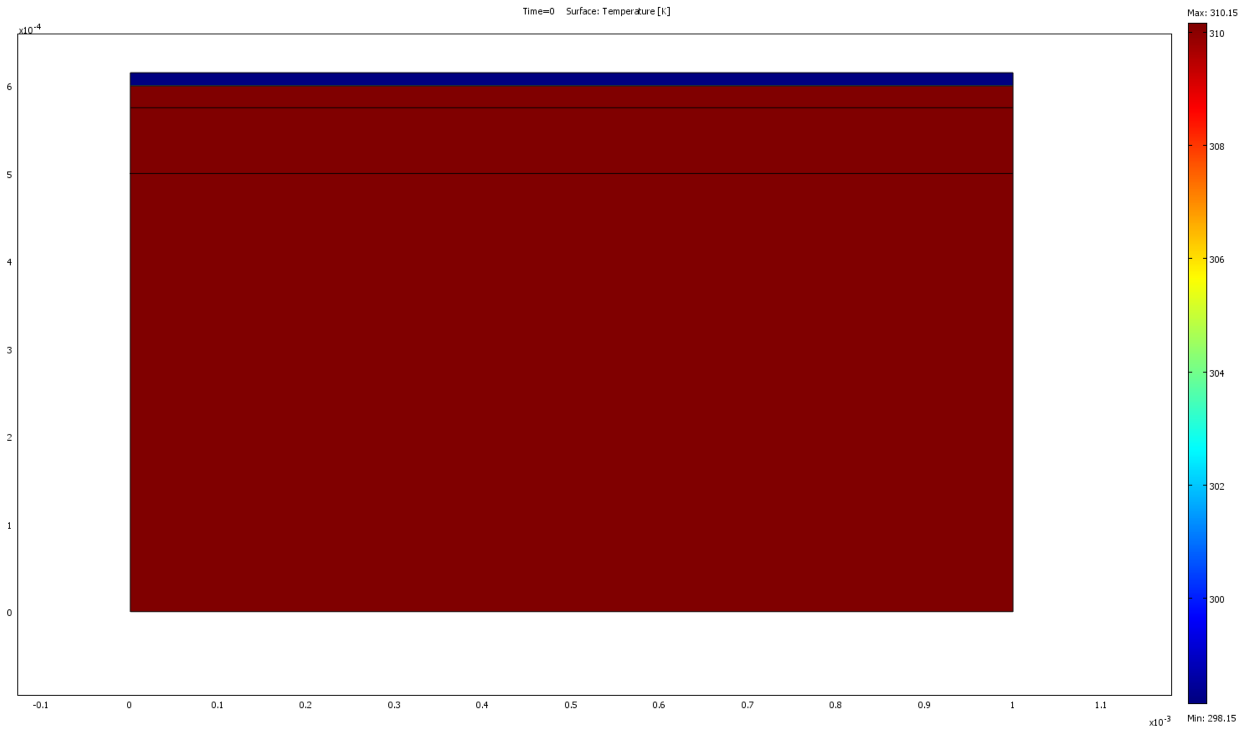

Figure 8: Schematic showing our initial conditions (t=0 sec) for temperature in which our lips are at body temperature, but the lip balm is at the weather condition's temperature, in this case T=25°C.

After running the temperature simulation for t=14,400 seconds, or 4 hours, another surface plot was generated (Figure 9). A run time of four hours was selected since the mass transfer equation is dependent on temperature, and we are interested in modeling the mass transfer equation to see the moisture content of the lips with (t>2.5 hours) and without lip balm (t>2.5 hours). The temperature of the lip balm increased to 310.09 from its 298K initial condition, and stratum corneum appears to be at the same temperature as the lip balm. A slight temperature gradient has been established in the epidermis and dermis layers, from the outside room temperature. Recall that dermis contains the capillaries, and their blood perfusion (modeled as the bioheat term in the governing equation) depends on the temperature within the dermis.

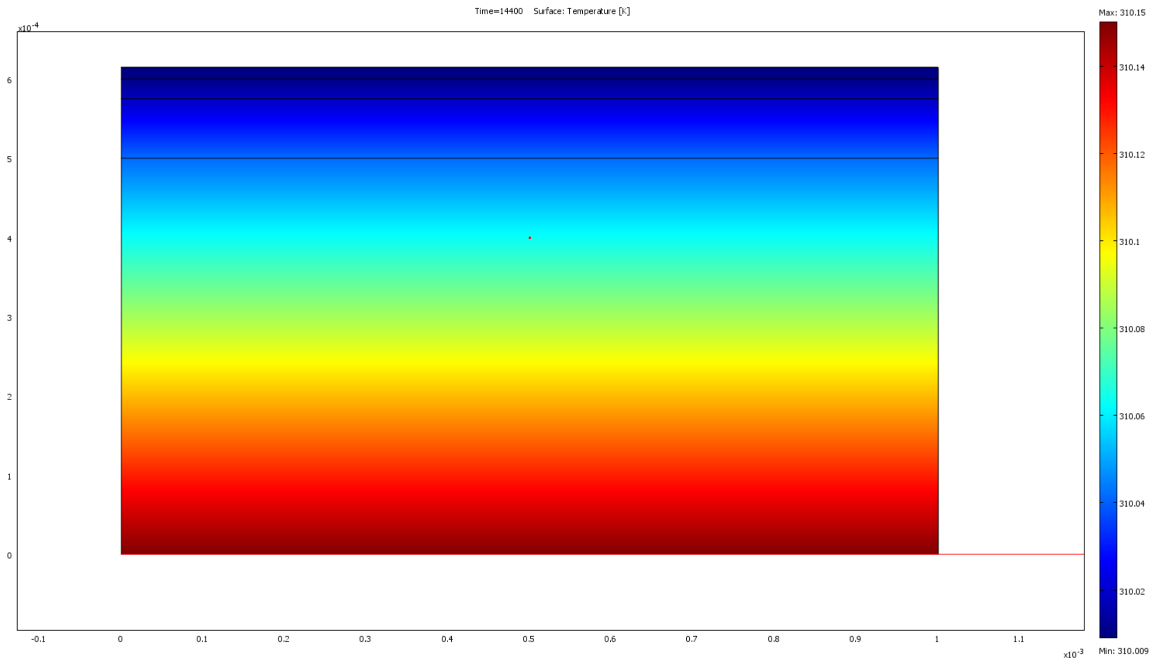

Figure 9: Surface plot of temperature at t = 14,400 sec (4 hours). The dermis provides heat through blood perfusion, while the outside temperature slightly cools the edge creating a gradient of about 0.09°K

To further evaluate the temperature change that occurs within the dermis, a temperature profile (Figure 10) for the first 10 minutes of simulation was generated at a point in the dermis (x=0.5mm and y=0.4mm). A temperature change of 0.09K occurred in the first 2 minutes, and steady state was achieved after these 2 minutes.

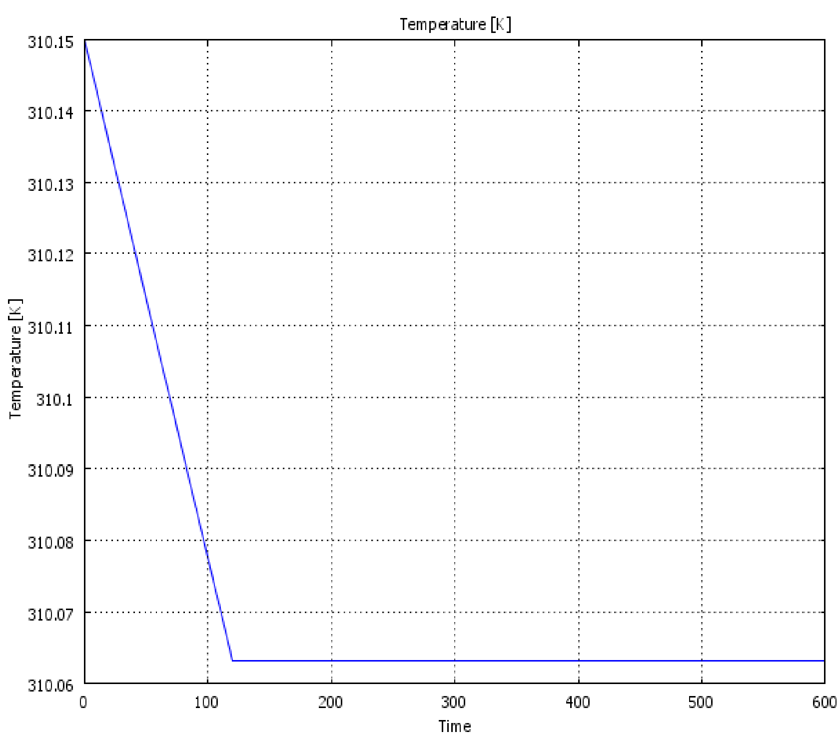

Figure 10: Plot of the temperature profile for the first 10 minutes at point x=0.5mm and y=.4mm (in dermis). The temperature only changed slightly due to the room temperature conditions outside. For most of our modeling time, the temperature stayed relatively constant.

Mass Transfer Solution

The mass transfer governing equation of the dermis utilizes the termperature solution of the dermis, but is run at the same time as the temperature equation. A mass surface plot was generated for the initial conditions of c=0 mol/m^3 in all subdomains at t=0 (Figure 11).

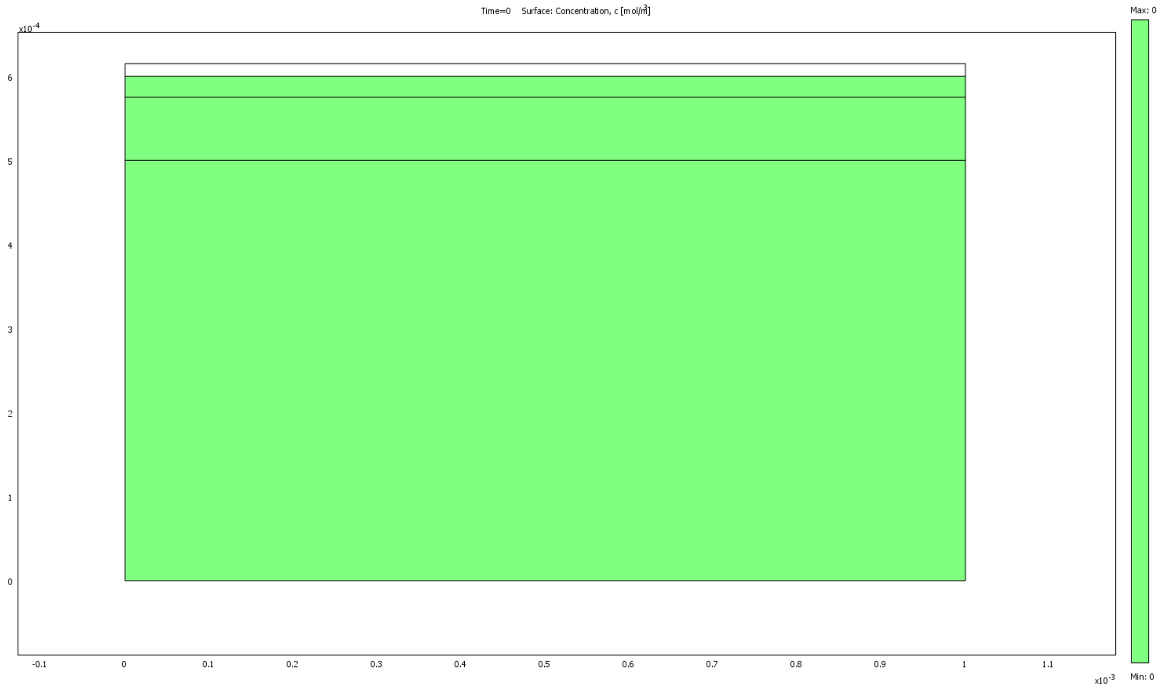

Figure 11: Schematic showing our initial conditions (t=0 sec) for mass transfer in which our lips are extremely dry with no moisture anywhere.

The mass transfer simulation ran for t=14,400 seconds, or 4 hours, and the concentration surface plot below depicts the final moisture content of the lips (Figure 12). Note that the lip balm subdomain was inactive in this simulation since it has a diffusivity of zero. At 14,400 seconds, the dermis and epidermis had an approximate moisture content of 227.914 mol/m^3. This confirms that the dermis is actively replenishing the moisture of the lips, and is successful during normal weather conditions. The stratum corneum, on the other hand, has a significant concentration gradient, with the interface between the stratum corneum and lip balm having a moisture concentration of 0.0768 mol/m^3. This dryness can be explained by the fact that the lip balm layer degraded after 2.5 hours (9000 seconds) in the simulation. Since the lips no longer had the protection from the lip balm, the natural convection of the air (which, again, modeled the indoor weather conditions) was able to remove the moisture from the lips.

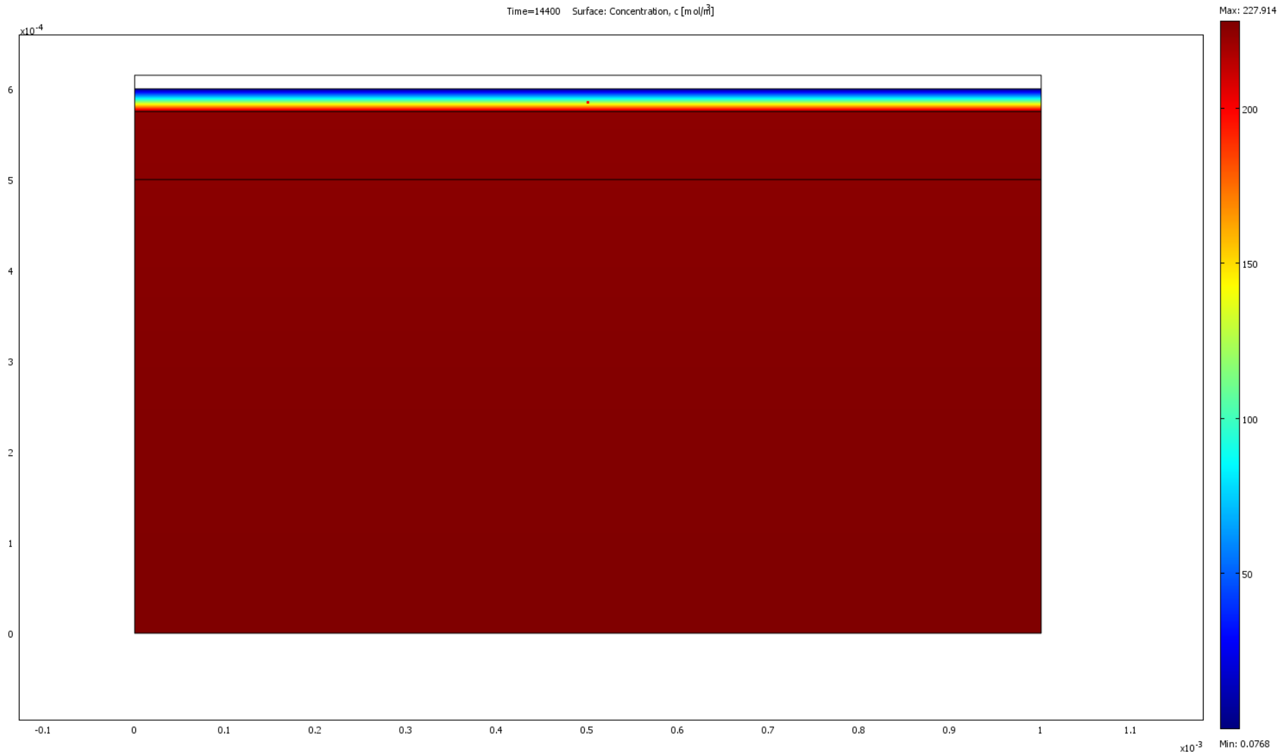

Figure 12: Surface plot of surface concentration at t = 14,400 sec (4 hours). Most of the lip has recovered and regain maximum moisture in the dermis.Since the lip balm degraded about 1.5 hours ago (@ t=9000 sec), the stratum corneum has become dry due to the naturalconvection in the air.

Since such a significant concentration gradient was established in the stratum corneum after 4 hours of simulation, it was deemed to have the highest sensitivity to the mass transfer that occurs in our lips (similar to how the dermis was deemed most sensitive in the temperature solution). To further evaluate how moisture is lost in the stratum corneum, a concentration profile was generated for a point within the stratum corneum (x=0.5mm and y = 0.585mm). The time domain studied was four hours. As can be seen in Figure 13, there is a constant increase in moisture in the stratum corneum until t=9000 seconds (2.5 hours). At t=9000 seconds (the point at which the lip balm is gone) there is a sharp change in slope. The rate of moisture accumulation

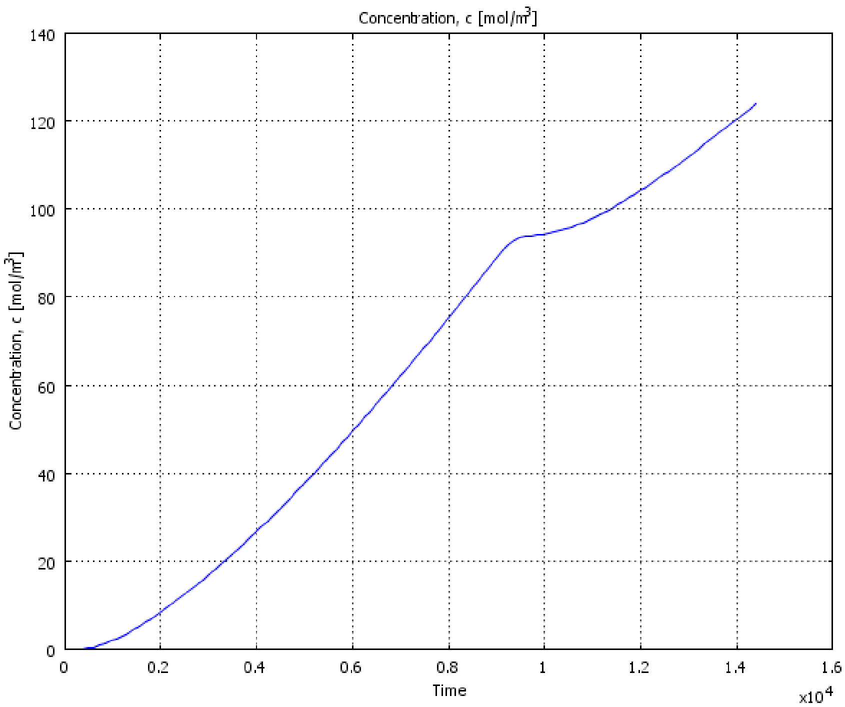

Figure 13: Plot of the concentration profile for 4 hours at point x=0.5mm and y=.585mm (in the stratum corneum). Our graph shows the increase in moisture diffusing from the dermis with the lip balm acting as a sealant till t= 9000 sec. After 9000 sec, the graph starts to drop indicating that the boundary condition changed to convective conditions and moisture is being lost due to evaporation.

occurs at a slower rate due to the presence of natural convection and evaporation at the boundary.

<u>Winter Weather Conditions (T= -5°C and 25%RH)</u>

Temperature Solution

The temperature surface plot for the winter weather conditions at t = 0 is similar to that of the normal weather conditions, except the lip balm domain is at -5°C instead of 25°C (Figure 14). The stratum corneum, epidermis, and dermis are all at the body temperature of 310.15K.

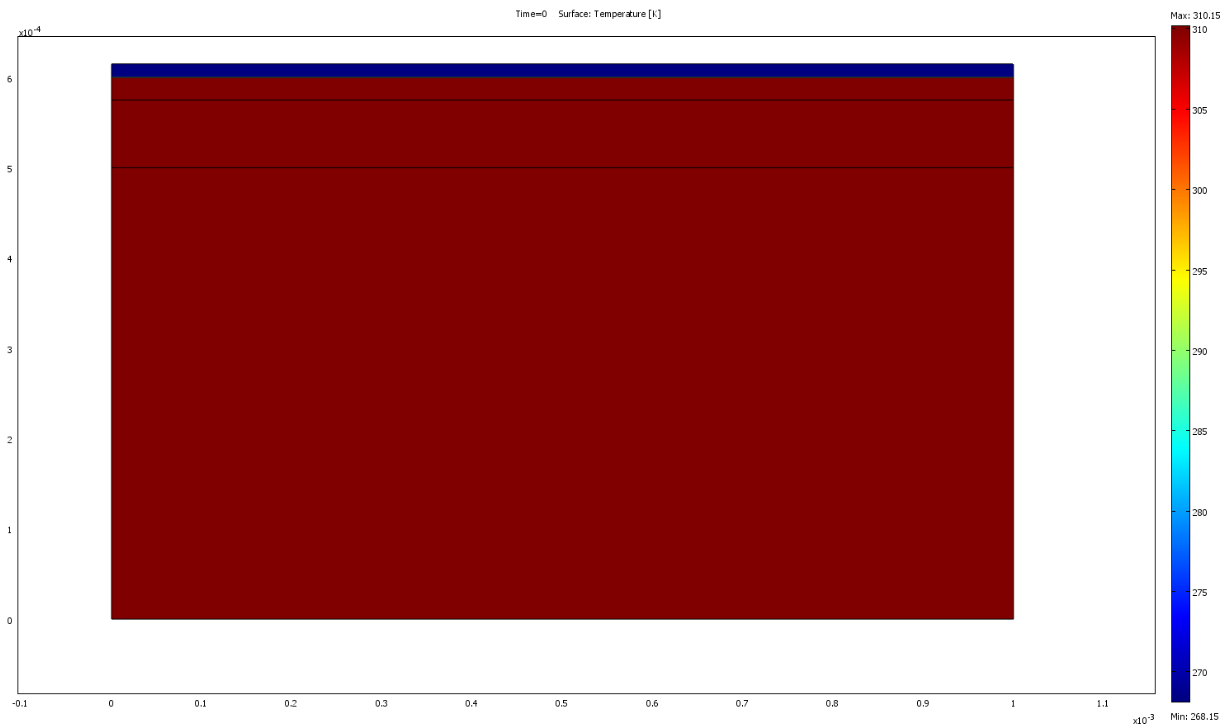

Figure 14: Schematic showing our initial conditions (t=0 sec) for temperature in which our lips are at body temperature, but the lip balm is at the weather condition's temperature, in this case T=-5°C.

After running the temperature simulation for t=14,400 seconds, or 4 hours, a surface plot was generated (Figure 15). The temperature of the lip balm increased to 300.701K from its 298K initial condition, and stratum corneum appears to be at the same temperature as the lip balm. The temperature gradient in the epidermis is more established in the winter weather conditions than the normal weather conditions, and the temperature gradient in the dermis is more severe. Both of these are caused by the colder boundary temperature at the lip balm surface, as the dermis provides heat through the capillaries, and the effect of the weather condition is more dramatic since it is -5°C instead of 25°C.

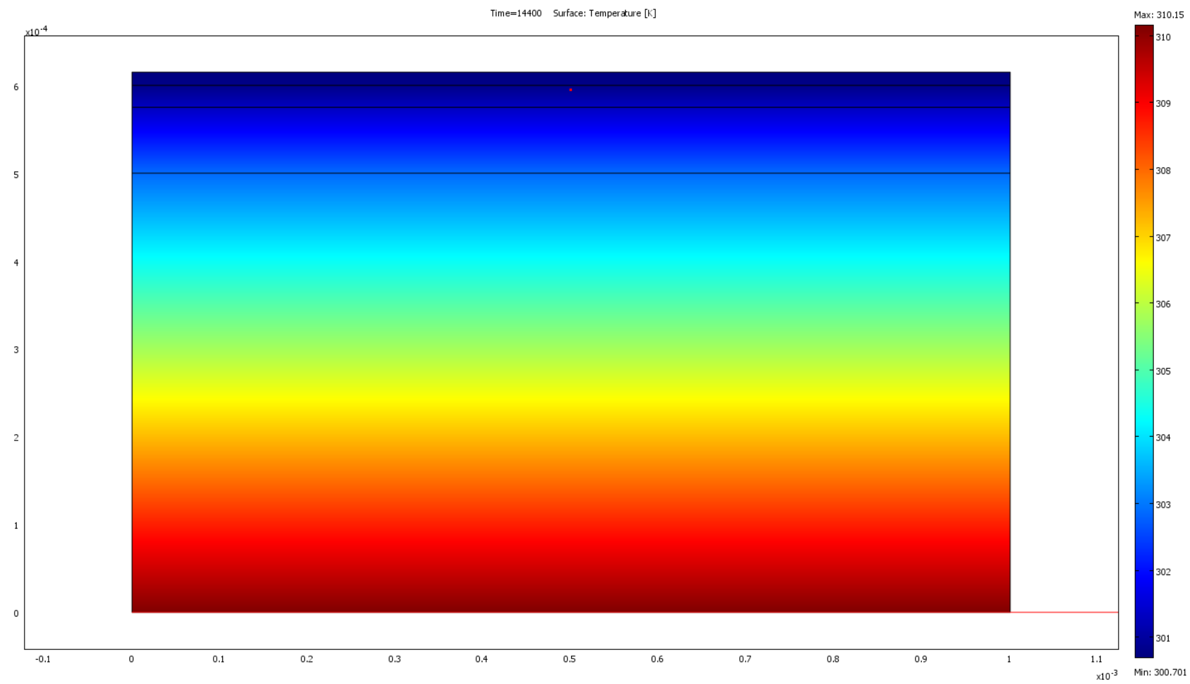

Figure 15: Surface plot of temperature at t = 14,400 sec (4 hours). The dermis provides heat through blood perfusion, while the outside winter temperature quickly cools the outer edge of our lips turning them blue (literally from the picture!), creating a gradient of about 10°C.

The temperature change within the dermis was again further evaluated using a temperature profile (Figure 16). This profile was for the first 10 minutes of simulation at a point in the dermis (x=0.5mm and y=0.4mm). A more drastic temperature change of 9.5K occurred in the first 2 minutes, and steady state was achieved after 2 minutes.

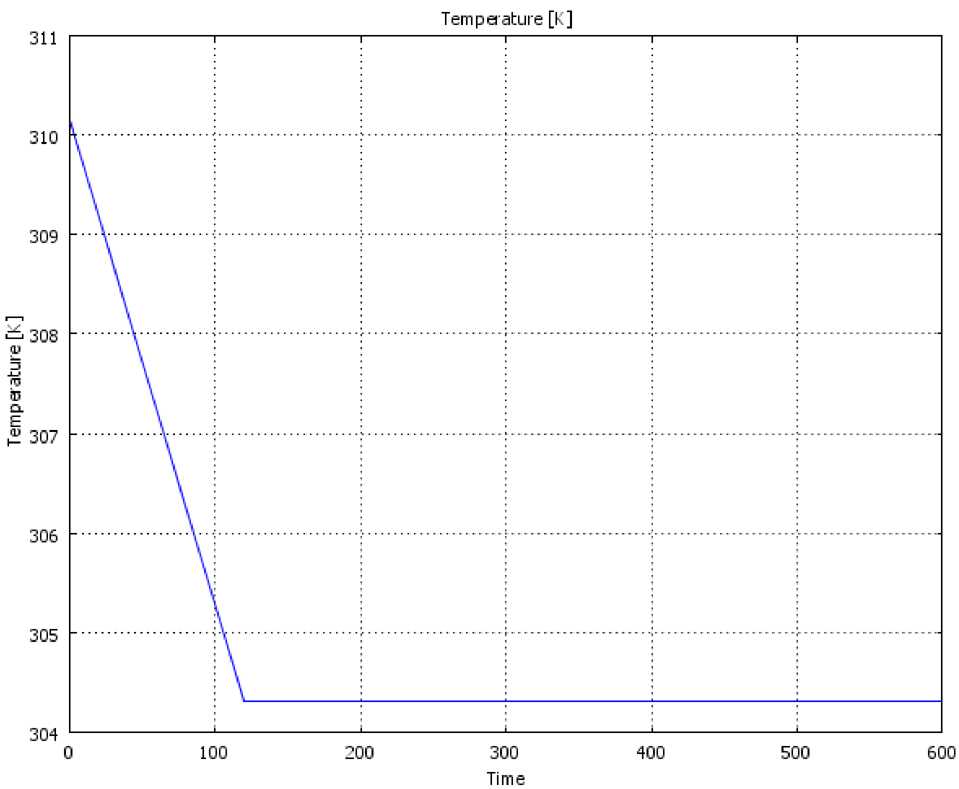

Figure 16: Plot of the temperature profile for the first 10 minutes at point x=0.5mm and y=.4mm (in dermis). The temperature changed rapidly (within the first few minutes) due to the frigid , -5°C, conditions outside. For the remainder of our modeling time, the temperature stayed relatively constant.

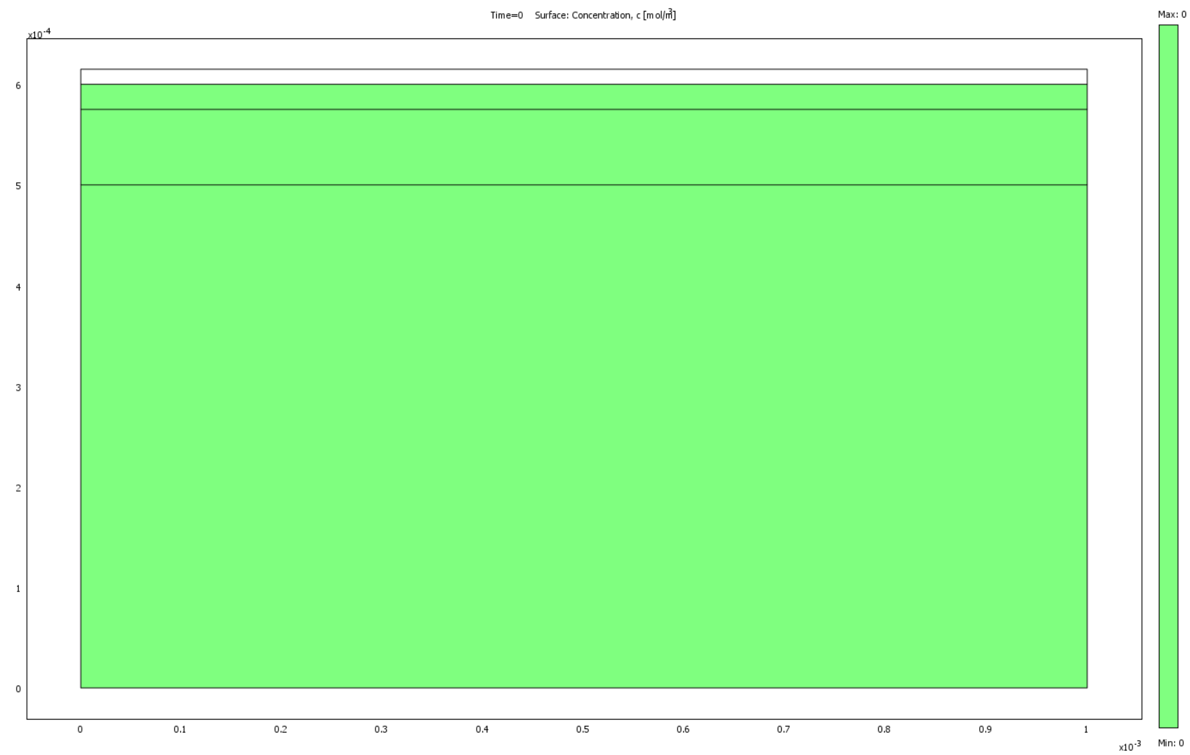

Figure 17: Schematic showing our initial conditions (t=0 sec) for mass transfer in which our lips are extremely dry with no moisture anywhere.

The mass transfer simulation was again run for t=14,400 seconds, or 4 hours, and the concentration surface plot below depicts the final moisture content of the lips (Figure 18). Again, the lip balm subdomain was inactive in this simulation since it has a diffusivity of zero. At 14,400 seconds, the dermis and epidermis had an approximate moisture content of 129.281kg/m^3. In comparison to the normal weather conditions, this is a 98.63 mol/m^3 decrease in moisture, showing that the cold weather conditions do significantly affect the moisture content of the lip, both in terms of temperature and convection. Although the dermis is actively replenishing the moisture of the lips, it is doing it at reduced rate and the moisture is convected away faster since it's forced rather than natural. The stratum corneum again has a significant concentration gradient, with the interface between the stratum corneum and lip balm having a moisture concentration of 1.754E-4 kg/m^3. The stratum corneum is therefore dryer in the winter weather conditions than the normal weather conditions. This is again explained by the forced convection. Ultimately, in colder and windier conditions, less moisture will be present in the lips and more moisture will be convected away, at a given time.

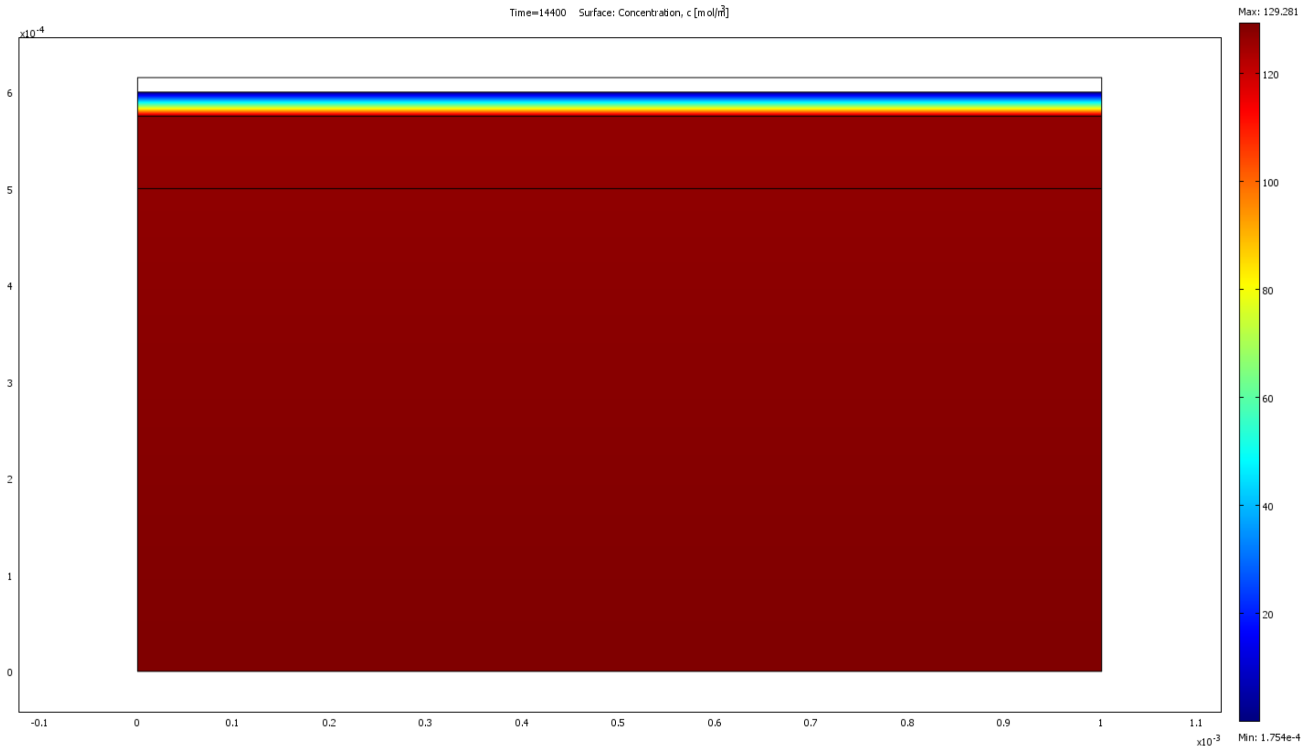

Figure 18: Surface plot of surface concentration at t = 14,400 sec (4 hours). Most of the lip has recovered and regain maximum moisture in the dermis, but since the lip balm degraded about 1.5 hours ago (@ t=9000 sec), the stratum corneum has become dry due to the forced convection from the winter wind,and moisture has been lost from the epidermis.

A concentration profile was again generated for a point within the stratum corneum (x=0.5mm and y = 0.585mm) to determine the behavior of the moisture over the 14,000 seconds. As can be seen in Figure 19, there is a constant increase in moisture in the stratum corneum until t=9000 seconds (2.5 hours). Note that in comparison to the normal weather conditions, it takes longer for the moisture to start accumulating, and it does so at a reduced rate. At t=9000 seconds (the point at which the lip balm is gone) there is a sharp decrease in moisture, and it continues to decrease rapidly as the forced convection and evaporation removes the moisture from the lips. The model is unfortunately only realistic up to 11,000 seconds. After this point, unrealistic values are shown that moisture begins to accumulate rather than decrease. This is due to the Boolean expression used in the model, where a rule was defined such that if the difference between the vapor concentration at the lip surface and the external vapor concentration is extremely small (≈0), there is no evaporation. This lack of evaporation leads to an accumulation of moisture.

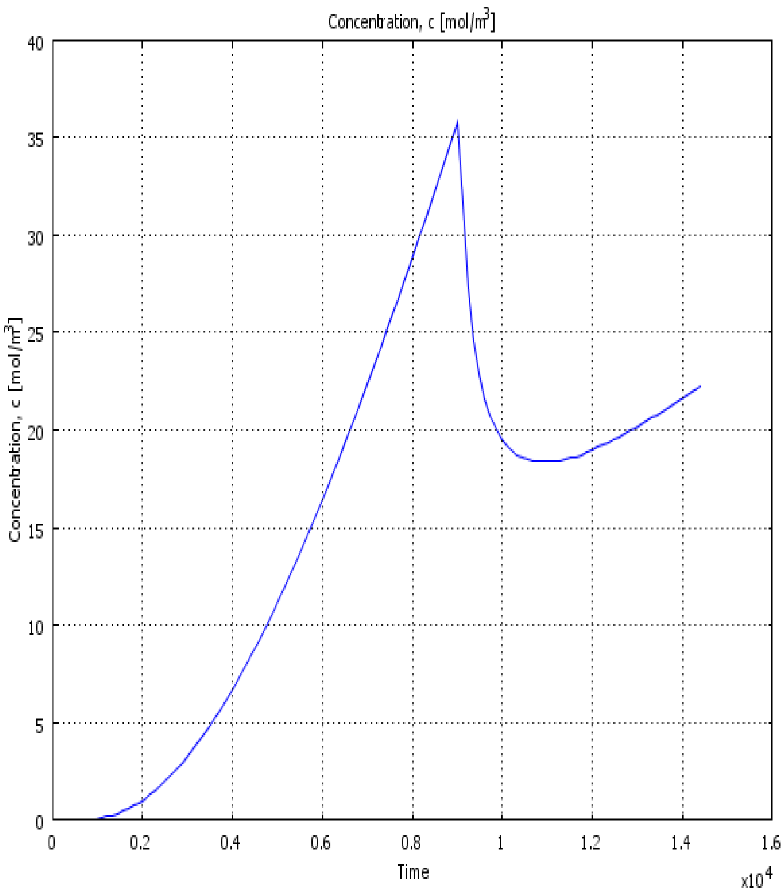

Figure 19: Plot of the concentration profile for 4 hours at point x=0.5mm and y=.595mm (in the stratum corneum). Our graph shows the increase in moisture diffusing from the dermis with the lip balm acting as a sealant till t= 9000 sec. After 9000 sec, the graph starts to drop indicating that the boundary condition changed to convective conditions and moisture is being lost due to evaporation.

ACCURACY CHECK:

Due to the lack of public experimentation in the lip balm field, an accuracy check of our solutions is limited by the readily available literature. There are no literature values for the amount of time lip balm lasts on lips. As a result, we experimentally determined how long lip balm is present and effective on our lips by applying it and timing how long it look to disappear. This value is highly dependent on the circumstances, such as eating, drinking and weather conditions. For the purposes of this project, we chose to simulate typical behavior. On average, we found that lip balm lasts approximately 2.5 hours on the lips. We used this value in COMSOL as the point at which the zero flux condition disappears, and the convective mass transfer can take place.

We also performed sensitivity analyses for the heat and mass transfer convective terms, the maximum concentration of moisture in the tissue, and diffusive flux by varying the percent of moisture that diffuses out from the capillaries.

SENSITIVITY ANALYSIS:

The two parameters that were varied in the heat transfer component are the heat transfer coefficients due to air for both temperature conditions ($h_{air,T=25C}$ and $h_{air,T=-5C}$). At T=-5°C, the convection over the lip surface is forced (due to the presence of wind). As a result, $h_{air,T=-5C}$ was calculated using Reynolds number and Nusselt number. Rather than having forced convection, the T=25°C condition had natural convention. This meant that the Grashof number and the Rayleigh number were used instead of Reynolds number to find the correct Nusselt number. One variable that was varied in both equations is the length of the lip, termed L. The length of the lip is different depending on not only the person, but also the person's mood (ie, smiling, frowning, pouting, pursing) and facial covering (ie, mask, scarf). It's important to note that the solution to both the heat transfer component and mass transfer component was examined since the heat transfer variables also affect mass transfer.

Sensitivity analyses were performed at the point (.5 mm, .58 mm) in the stratum corneum at t=4 hours=14,400 seconds. The temporary boundary condition was -5°C with forced convection. We varied the lip length from 2 cm to 8 cm to mimic the variances in lip length among individuals and during different lip motions, such as smiling or pouting. Varying lip length changes Reynold's number, which in turn changes Nusselt number for heat transfer, and this changes the heat transfer coefficient, h. Changing Reynold's number also changes the mass transfer coefficient, h_m, through the equation $\frac{h_m L}{D} = 0.664 Re_L{}^{1/2} Sc^{\frac{1}{3}}$. That equation is also directly affected by varying lip length through L. Table 4 (See Appendix F) shows the changes to h and h_m that were found when lip length was varied and the corresponding temperatures and concentrations at the point mentioned above.

Varying the lip length lead to a five degree range in temperature values at the point (5 mm, 5.8 mm) in the stratum corneum and had a negligible effect on the water concentration at this position, since the change was only 6.6%.

For the natural convection condition at T=25°C, values of L=0.05, 0.06, 0.07, and 0.08 m were used (Table 5, Appendix F). Lengths less than those made the Rayleigh number out of range, and were ignored. The heat transfer coefficient, h, was calculated for each length through the Grashof, Rayleigh and Nusselt numbers as shown in Appendix F. Since the mass transfer coefficient, h_m, is dependent on h, it was also recalculated for the various lengths using the Grashof, Schmidt Rayleigh and Nusselt numbers.

The sensitivity analysis showed that the final solution is not sensitive to changes in the heat or mass transfer coefficient. This was determined by comparing the final values of concentration at the lower edge of the protective stratum corneum. There was a maximum of 3.8% change in concentration for the -5°C condition and a maximum of 0.0015% change in the 25 C condition. This indicates that these parameters have little effect on the final solution.

The diffusive flux term or the generation term used in the mass transfer equation was derived from unit conversions of the V_{blood} term. The unit conversions took into account the density of the tissue and water, the percent of water in the blood and the percent that diffused out of the capillaries into the tissue. The calculation is reproduced below:

$$\frac{kg_{H2O}}{m^3_{tissue} \cdot S} = V_{blood} x \rho_{tissue} x \ \% \ H2OinBloodx \ \% \ H2Odiffusesoutx \rho_{H2O}$$

$$\frac{kg_{H2O}}{m^3_{tissue} \cdot S} = \left(\frac{m^3_{blood}}{kg_{tissue} \cdot s}\right)\left(\frac{1000 \ kg_{tissue}}{1 \ m^3_{tissue}}\right)\left(\frac{0.55 \ m^3_{H2O}}{1 \ m^3_{blood}}\right)(0.05)\left(\frac{1000 \ kg_{H2O}}{1 \ m^3_{H2O}}\right)$$

These values were found from literature and are considered ‘true’ values in that there have been many experiments and general agreement on the densities values and the percent of blood that is plasma or water, except for the percent that diffuses out of the capillaries. This was estimated to be 5%, but it could vary. In this sensitivity analysis, we will vary the percent of water that leaves the capillaries and into the tissue from 1-6% and see how much this affects our solution. In Table 6 of Appendix F, the diffusive flux coefficient was calculated and COMSOL was re-implemented for the varying diffusive flux term. Since this term is only used in the mass transfer equation, only a change in H_2O concentration will be observed. The implementation and sensitivity analysis will use the winter weather boundary conditions because the concentration changes more due to the forced convection boundary thus will likely to be more affected by different diffusive flux terms. By observing the changes in concentration at point (0.5mm, 0.58mm) after 4 hours, we found that varying the amount of water that leaves from the capillaries from 1% to 6% causes the concentration to vary from approximately 20 kg/m^3 to 120 kg/m^3. This is a large variation and our solution is highly dependent on the diffusive flux term, which implies that this term requires accurate values. This number however is very difficult to find in literature since the amount that diffuses out is based on many different factors such as how hydrated the body is, what type of tissue the capillaries are in, metabolic activity of the person, etc. If it were possible, we would have tried to measure the diffusive flux of moisture into the lips through experiment, but this was impossible with our limited resources.

The last part of the sensitivity analysis was an evaluation of the maximum concentration of moisture in the tissue, and the solution's sensitivity to it. This evaluation was performed on the point (0.5mm, 0.58mm) at t=4 hours = 14,400 seconds. The temperature boundary condition was T = -5˚C, and it can be assumed that the concentration's dependency on C_{max} is the same, regardless of the temperature condition. This assumption allows us to examine just one of the temperature conditions, instead of the two.

To calculate the maximum concentration of water that the tissue can hold we will assume that x% of tissue is composed of water:

$$c_{max} = \left(\frac{1000\ kg_{tissue}}{1\ m^3_{tissue}}\right)\left(\frac{\frac{x}{100}kg_{H2O}}{1\ kg_{tissue}}\right) where\ c_{max}\ has\ the\ units\ of\ \frac{kg_{H2O}}{m^3_{tissue}}$$

As can be seen from Table 7 (See Appendix F), the behavior of C does not depend on C_{max}. The time period of our simulation is not long enough for C to reach C_{max}. The C in the R term in the dermis therefore serves as a Boolean expression to prevent C from ever exceeding C_{max}, no matter how unlikely such an occurrence is in our model.

CONCLUSION

At normal weather conditions of natural convection, 25°C and 50% relative humidity, our model showed that lip balm helps the lips restore moisture at a higher rate than than they would have without lip balm. This is realistic, because when lip balm is not present, moisture can evaporate from the lip surface. When the lip balm is not present, the capillaries are able to replenish the tissue with moisture, but at a lower rate than when the balm is present. Lip balm functions as a sealant and does not allow moisture to leave the lip. The concentration gradient occurs primarily in the protective stratum corneum once the lip balm is not present. It is the layer closest to the air, so moisture is evaporating from the surface while the capillaries are providing moisture to the tissue.

The lip's temperature varied over 0.09°C, so there was a small temperature gradient throughout the lip balm, stratum corneum, epidermis and dermis. The temperature changes occur within the first two minutes of our model while it is reaching equilibrium. The capillaries in the dermis are effective at maintaining constant heat in the layers. Similarly, in winter conditions the temperature gradient in the lip that varies over 6°C and establishes steady state quickly.

At winter weather conditions of windy, -5°C and 20% relative humidity, much more moisture is lost due to the constriction of the capillaries which results in reduced surface area and less diffusive flux of moisture, and the weather conditions which lead to more evaporation and convection of moisture than the normal conditions. The minimum moisture concentration was 0.22% of the minimum moisture content in normal conditions. This supports our initial idea that cold and windy weather makes lips drier than normal weather.

While the lip balm is present in the winter conditions, the tissue is gaining moisture from the capillaries and none is evaporating from the lip surface-lip balm boundary. However, the lip balm is removed after 2.5 hours and the lip loses moisture, as seen by the sharp drop in concentration of water in Figure 18. The concentration starts increasing again due to a Boolean expression in our COMSOL design to avoid negative concentration values. We would've expected our model to show a continuous decrease in moisture after the lip balm is removed. In order to correct this we should change our initial conditions to have some sort of moisture content such that when the evaporation starts to occur there is more moisture in the lip and even though moisture is lost there is no risk of getting negative values.

We can conclude from our model that lip balm is effective at maintaining moisture in the lip and allowing the lip to regain moisture. Weather conditions also have a significant effect on the moisture concentration in the lip, due to their effects on water perfusion from the capillaries and evaporation and convection from the lip surface.

APPENDIX

Appendix A: COMSOL Implementation of Complete Solution

Solution 1: Normal Conditions
COMSOL Model Report

1. Table of Contents

2. Model Properties

Property	Value
Model name	
Author	
Company	
Department	
Reference	
URL	
Saved date	Nov 16, 2010 12:04:28 AM
Creation date	Oct 17, 2010 10:02:23 PM
COMSOL version	COMSOL 3.5.0.603

File name: J:\School 2010-2011\BEE 4530\Project\Up to Date\BEE4530COMSOLT25.mph
Application modes and modules used in this model:

- Geom1 (2D)
  - Heat Transfer by Conduction
  - Diffusion

4. Geom1

Space dimensions: 2D
Independent variables: x, y, z

4.1. Scalar Expressions

Name	Expression	Unit	Description
cs	101325/(8.315*T*exp(13.122-(4894.768/T)))		

4.2. Expressions

4.2.1. Subdomain Expressions

Subdomain		1
V_blood	K	(-5.1103e-005+1.67e-007*T)*(T>308)+3.33e-007*(T<=308)

4.3. Mesh

4.3.1. Mesh Statistics

Number of degrees of freedom	79002
Number of mesh points	10791
Number of elements	9800
Triangular	0
Quadrilateral	9800
Number of boundary elements	2010
Number of vertex elements	10
Minimum element quality	0.004
Element area ratio	0.187

4.4. Application Mode: Heat Transfer by Conduction (ht)

Application mode type: Heat Transfer by Conduction
Application mode name: ht

4.4.1. Application Mode Properties

Property	**Value**
Default element type	Lagrange - Quadratic
Analysis type	Transient
Frame	Frame (ref)
Weak constraints	Off
Constraint type	Ideal

4.4.2. Variables

Dependent variables: T
Shape functions: shlag(2,'T')
Interior boundaries not active

4.4.3. Boundary Settings

Boundary		1, 3, 5, 7, 10-13	2	9
Type		Thermal insulation	Temperature	Heat flux
Heat transfer coefficient (h)	$W/(m^2 \cdot K)$	0	0	**5.497**
External temperature (Tinf)	K	273.15	273.15	**298.15**
Temperature (T0)	K	273.15	**310.15**	273.15

4.4.4. Subdomain Settings

Subdomain		1	2-3	4
Thermal conductivity (k)	$W/(m \cdot K)$	**0.3**	**0.21**	**.813**
Density (rho)	kg/m^3	**1000**	**1000**	**860**
Heat capacity at constant pressure	$J/(kg \cdot K)$	**2846.15**	**3181.82**	**1905**

(C)				
Heat source (Q)	W/m^3	**1100*3300*(310.15-T)*V_blood**	0	0
External temperature (Text)	K	**0**	**0**	**0**

Subdomain initial value		1	2-3	4
Temperature (T)	K	310.15	310.15	298.15

4.5. Application Mode: Diffusion (di)

Application mode type: Diffusion
Application mode name: di

4.5.1. Application Mode Properties

Property	**Value**
Default element type	Lagrange - Quadratic
Analysis type	Transient
Frame	Frame (ref)
Weak constraints	Off
Constraint type	Ideal

4.5.2. Variables

Dependent variables: c
Shape functions: shlag(2,'c')
Interior boundaries not active

4.5.3. Boundary Settings

Boundary		1-3, 5, 10-12	8
Type		Insulation/Symmetry	Flux
Inward flux (N)	mol/(m^2·s)	0	**-.0051*(t>9000)*((c/1000)*(c/1000<=0.01177)+(0.01177)*(c/1000>0.01177))**
Concentration (c0)	mol/m^3	0	**cs**

4.5.4. Subdomain Settings

Subdomain		1	2	3
Diffusion coefficient (D)	m^2/s	**2E-10**	**2E-10**	**5e-14**
Reaction rate (R)	mol/(m^3·s)	**27500*V_blood*(c<700)**	0	0

5. Solver Settings

Solve using a script: off

Analysis type	Transient
Auto select solver	On
Solver	Time dependent
Solution form	Automatic

Symmetric	auto
Adaptive mesh refinement	Off
Optimization/Sensitivity	Off
Plot while solving	Off

5.1. Direct (UMFPACK)

Solver type: Linear system solver

Parameter	Value
Pivot threshold	0.1
Memory allocation factor	0.7

5.2. Time Stepping

Parameter	Value
Times	range(0,120,14400)
Relative tolerance	0.01
Absolute tolerance	0.0010
Times to store in output	Specified times
Time steps taken by solver	Free
Maximum BDF order	5
Singular mass matrix	Maybe
Consistent initialization of DAE systems	Backward Euler
Error estimation strategy	Include algebraic
Allow complex numbers	Off

5.3. Advanced

Parameter	Value
Constraint handling method	Elimination
Null-space function	Automatic
Automatic assembly block size	On
Assembly block size	1000
Use Hermitian transpose of constraint matrix and in symmetry detection	Off
Use complex functions with real input	Off
Stop if error due to undefined operation	On
Store solution on file	Off
Type of scaling	Automatic
Manual scaling	
Row equilibration	On
Manual control of reassembly	Off

Load constant	On
Constraint constant	On
Mass constant	On
Damping (mass) constant	On
Jacobian constant	On
Constraint Jacobian constant	On

6. Postprocessing

7. Variables

7.1. Boundary

7.1.1. Boundary 1-6, 8, 10-12

Name	Description	Unit	Expression
nflux_ht	Normal heat flux	W/m^2	nx_ht * fluxx_ht+ny_ht * fluxy_ht
ndflux_c_di	Normal diffusive flux, c	mol/(m^2*s)	nx_di * dflux_c_x_di+ny_di * dflux_c_y_di

7.1.2. Boundary 7, 9, 13

Name	Description	Unit	Expression
nflux_ht	Normal heat flux	W/m^2	nx_ht * fluxx_ht+ny_ht * fluxy_ht
ndflux_c_di	Normal diffusive flux, c	mol/(m^2*s)	

7.2. Subdomain

7.2.1. Subdomain 1-3

Name	Description	Unit	Expression
fluxx_ht	Heat flux, x component	W/m^2	-kxx_ht * Tx-kxy_ht * Ty
fluxy_ht	Heat flux, y component	W/m^2	-kyx_ht * Tx-kyy_ht * Ty
gradT_ht	Temperature gradient	K/m	sqrt(Tx^2+Ty^2)
flux_ht	Heat flux	W/m^2	sqrt(fluxx_ht^2+fluxy_ht^2)
grad_c_x_di	Concentration gradient, c, x component	mol/m^4	cx
dflux_c_x_di	Diffusive flux, c, x component	mol/(m^2*s)	-Dxx_c_di * cx-Dxy_c_di * cy
grad_c_y_di	Concentration gradient, c, y component	mol/m^4	cy
dflux_c_y_di	Diffusive flux, c, y component	mol/(m^2*s)	-Dyx_c_di * cx-Dyy_c_di * cy
grad_c_di	Concentration gradient, c	mol/m^4	sqrt(grad_c_x_di^2+grad_c_y_di^2)
dflux_c_di	Diffusive flux, c	mol/(m^2*s)	sqrt(dflux_c_x_di^2+dflux_c_y_di^2)

7.2.2. Subdomain 4

Name	Description	Unit	Expression

fluxx_ht	Heat flux, x component	W/m^2	-kxx_ht * Tx-kxy_ht * Ty
fluxy_ht	Heat flux, y component	W/m^2	-kyx_ht * Tx-kyy_ht * Ty
gradT_ht	Temperature gradient	K/m	sqrt(Tx^2+Ty^2)
flux_ht	Heat flux	W/m^2	sqrt(fluxx_ht^2+fluxy_ht^2)
grad_c_x_di	Concentration gradient, c, x component	mol/m^4	
dflux_c_x_di	Diffusive flux, c, x component	mol/(m^2*s)	
grad_c_y_di	Concentration gradient, c, y component	mol/m^4	
dflux_c_y_di	Diffusive flux, c, y component	mol/(m^2*s)	
grad_c_di	Concentration gradient, c	mol/m^4	
dflux_c_di	Diffusive flux, c	mol/(m^2*s)	

Solution 2: Winter Weather Conditions

COMSOL Model Report

1. Table of Contents

2. Model Properties

File name: J:\School 2010-2011\BEE 4530\Project\Up to Date\BEE4530COMSOLT-5.mph
Application modes and modules used in this model:

- Geom1 (2D)
  - Heat Transfer by Conduction
  - Diffusion

3. Geometry

Number of geometries: 1

3.1. Geom1

3.1.1. Point mode
3.1.2. Boundary mode
3.1.3. Subdomain mode

4. Geom1

Space dimensions: 2D

Independent variables: x, y, z

4.1. Scalar Expressions

Name	**Expression**	**Unit**	**Description**
cs	101325/(8.315*T*exp(13.122-(4894.768/T)))		

4.2. Expressions

4.2.1. Subdomain Expressions

Subdomain		1
V_blood	K	(-5.1103e-005+1.67e-007*T)*(T>308)+3.33e-007*(T<=308)

4.3. Mesh

4.3.1. Mesh Statistics

Number of degrees of freedom	79002
Number of mesh points	10791
Number of elements	9800
Triangular	0
Quadrilateral	9800
Number of boundary elements	2010
Number of vertex elements	10
Minimum element quality	0.0037
Element area ratio	0.1875

4.4. Application Mode: Heat Transfer by Conduction (ht)

Application mode type: Heat Transfer by Conduction
Application mode name: ht

4.4.1. Application Mode Properties

Property	**Value**
Default element type	Lagrange - Quadratic
Analysis type	Transient
Frame	Frame (ref)
Weak constraints	Off

Constraint type	Ideal

4.4.2. Variables

Dependent variables: T
Shape functions: shlag(2,'T')
Interior boundaries not active

4.4.3. Boundary Settings

Boundary		1, 3, 5, 7, 10-13	2	9
Type		Thermal insulation	Temperature	Heat flux
Heat transfer coefficient (h)	W/(m^2·K)	0	0	**134.31**
External temperature (Tinf)	K	273.15	273.15	**268.15**
Temperature (T0)	K	273.15	**310.15**	273.15

4.4.4. Subdomain Settings

Subdomain		1	2-3	4
Thermal conductivity (k)	W/(m·K)	**0.3**	**0.21**	**.813**
Density (rho)	kg/m^3	**1000**	**1000**	**860**
Heat capacity at constant pressure (C)	J/(kg·K)	**2846.15**	**3181.82**	**1905**
Heat source (Q)	W/m^3	**1100*3300*(310.15-T)*V_blood**	0	0
External temperature (Text)	K	**0**	**0**	**0**

Subdomain initial value		1	2-3	4
Temperature (T)	K	310.15	310.15	268.15

4.5. Application Mode: Diffusion (di)

Application mode type: Diffusion
Application mode name: di

4.5.1. Application Mode Properties

Property	**Value**
Default element type	Lagrange - Quadratic
Analysis type	Transient
Frame	Frame (ref)
Weak constraints	Off
Constraint type	Ideal

4.5.2. Variables

Dependent variables: c
Shape functions: shlag(2,'c')
Interior boundaries not active

4.5.3. Boundary Settings

Boundary		1-3, 5, 10-12	8
Type		Insulation/Symmetry	Flux
Inward	mol/(m^2	0	**-**

flux (N)	·s)		**1.26*(t>9000)*((c/1000)*(c/1000<=0.0045)+(0.0045)*(c/1000>0.0045))**
Concentration (c0)	mol/m^3	0	**cs**

4.5.4. Subdomain Settings

Subdomain		1	2	3
Diffusion coefficient (D)	m^2/s	**2E-10**	**2E-10**	**5e-14**
Reaction rate (R)	mol/(m^3·s)	**27500*V_blood*(c<700)**	0	0

5. Solver Settings

Solve using a script: off

Analysis type	Transient
Auto select solver	On
Solver	Time dependent
Solution form	Automatic
Symmetric	auto
Adaptive mesh refinement	Off
Optimization/Sensitivity	Off
Plot while solving	Off

5.1. Direct (UMFPACK)

Solver type: Linear system solver

Parameter	**Value**
Pivot threshold	0.1
Memory allocation factor	0.7

5.2. Time Stepping

Parameter	**Value**
Times	range(0,120,14400)
Relative tolerance	0.01
Absolute tolerance	0.0010
Times to store in output	Specified times
Time steps taken by solver	Free
Maximum BDF order	5
Singular mass matrix	Maybe
Consistent initialization of DAE systems	Backward Euler
Error estimation strategy	Include algebraic
Allow complex numbers	Off

5.3. Advanced

Parameter	**Value**
Constraint handling method	Elimination
Null-space function	Automatic
Automatic assembly block size	On
Assembly block size	1000
Use Hermitian transpose of constraint matrix and in symmetry detection	Off
Use complex functions with real input	Off
Stop if error due to undefined operation	On
Store solution on file	Off
Type of scaling	Automatic
Manual scaling	
Row equilibration	On
Manual control of reassembly	Off
Load constant	On
Constraint constant	On
Mass constant	On
Damping (mass) constant	On
Jacobian constant	On
Constraint Jacobian constant	On

6. Postprocessing

7. Variables

7.1. Boundary

7.1.1. Boundary 1-6, 8, 10-12

Name	**Description**	**Unit**	**Expression**
nflux_ht	Normal heat flux	W/m^2	nx_ht * fluxx_ht+ny_ht * fluxy_ht
ndflux_c_di	Normal diffusive flux, c	mol/(m^2*s)	nx_di * dflux_c_x_di+ny_di * dflux_c_y_di

7.1.2. Boundary 7, 9, 13

Name	**Description**	**Unit**	**Expression**
nflux_ht	Normal heat flux	W/m^2	nx_ht * fluxx_ht+ny_ht * fluxy_ht
ndflux_c_di	Normal diffusive flux, c	mol/(m^2*s)	

7.2. Subdomain

7.2.1. Subdomain 1-3

Name	**Description**	**Unit**	**Expression**
fluxx_ht	Heat flux, x component	W/m^2	-kxx_ht * Tx-kxy_ht * Ty

fluxy_ht	Heat flux, y component	W/m^2	-kyx_ht * Tx-kyy_ht * Ty
gradT_ht	Temperature gradient	K/m	sqrt(Tx^2+Ty^2)
flux_ht	Heat flux	W/m^2	sqrt(fluxx_ht^2+fluxy_ht^2)
grad_c_x_di	Concentration gradient, c, x component	mol/m^4	cx
dflux_c_x_di	Diffusive flux, c, x component	mol/(m^2*s)	-Dxx_c_di * cx-Dxy_c_di * cy
grad_c_y_di	Concentration gradient, c, y component	mol/m^4	cy
dflux_c_y_di	Diffusive flux, c, y component	mol/(m^2*s)	-Dyx_c_di * cx-Dyy_c_di * cy
grad_c_di	Concentration gradient, c	mol/m^4	sqrt(grad_c_x_di^2+grad_c_y_di^2)
dflux_c_di	Diffusive flux, c	mol/(m^2*s)	sqrt(dflux_c_x_di^2+dflux_c_y_di^2)

7.2.2. Subdomain 4

Name	**Description**	**Unit**	**Expression**
fluxx_ht	Heat flux, x component	W/m^2	-kxx_ht * Tx-kxy_ht * Ty
fluxy_ht	Heat flux, y component	W/m^2	-kyx_ht * Tx-kyy_ht * Ty
gradT_ht	Temperature gradient	K/m	sqrt(Tx^2+Ty^2)
flux_ht	Heat flux	W/m^2	sqrt(fluxx_ht^2+fluxy_ht^2)
grad_c_x_di	Concentration gradient, c, x component	mol/m^4	
dflux_c_x_di	Diffusive flux, c, x component	mol/(m^2*s)	
grad_c_y_di	Concentration gradient, c, y component	mol/m^4	
dflux_c_y_di	Diffusive flux, c, y component	mol/(m^2*s)	
grad_c_di	Concentration gradient, c	mol/m^4	
dflux_c_di	Diffusive flux, c	mol/(m^2*s)	

Appendix B: Source Term Calculations

Blood Perfusion Expression in the Capillaries (V_blood)

In order to model the change in blood perfusion as a function of temperature, we will use a linear equation that was created using two known values of blood perfusion at two different temperatures which were converted to proper units.

$$\mathbf{0.02}\,\frac{mL_{blood}}{g_{tissue}\cdot min}\left(\frac{10^{-6}m^3{}_{blood}}{1\,mL_{blood}}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{1000g}{1\,kg}\right)=\mathbf{3.33x10^{-7}}\,\frac{m^3_{blood}}{kg_{tissue}\cdot s}\qquad @\ T=308K$$

$$\mathbf{0.04}\,\frac{mL_{blood}}{g_{tissue}\cdot min}\left(\frac{10^{-6}m^3{}_{blood}}{1\,mL_{blood}}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{1000g}{1\,kg}\right)=\mathbf{6.67x10^{-7}}\,\frac{m^3_{blood}}{kg_{tissue}\cdot s}\qquad @\ T=310K$$

The form of the linear equation is V_{blood} = mT + b. The slope, m, was determined by

$$m=\frac{\Delta y}{\Delta x}=\frac{3.33x10^{-7}-6.67x10^{-7}}{35-37}=1.67\times10^{-7}$$

and the y-intercept was calculated by solving for b after plugging in T=308K and V_{blood} = $3.33x10^{-7}$:

$$3.33x10^{-7}\,\frac{m^3_{blood}}{kg_{tissue}\cdot s}=\ 1.67\times10^{-7}(308K)+b$$

$$b=\ -5.1103\ x10^{-5}\,\frac{m^3_{blood}}{kg_{tissue}\cdot s}$$

Thus the final equation for V_blood is

$$V_{blood}=1.67\times10^{-7}T-5.1103\ x10^{-5}\,\frac{m^3_{blood}}{kg_{tissue}\cdot s}$$

Since this equation is only valid from 308-310K, a Boolean term will also be multiplied to ensure that this equation is used for the valid temperature change, and any temperature below 308K will automatically have a V_{blood} of $3.33x10^{-7}\frac{m^3_{blood}}{kg_{tissue}\cdot s}$. The Boolean expression we will use to limit the blood perfusion term for temperatures higher than 308K is (T>308)+3.33e-007*(T<=308).

Diffusive Flux

The diffusive flux indicates how much moisture or water is diffusing from the capillaries into the tissue. This term is only used in the dermis where the capillaries are located. It has been derived from dimensional analysis of V_blood into appropriate units that can be used in the mass transfer equation which are $\frac{kg_{H2O}}{m^3_{tissue} \cdot S}$.

$$\frac{kg_{H2O}}{m^3_{tissue} \cdot S} = V_{blood} x \rho_{tissue} x \, \% \, H2OinBloodx \, \% \, H2Odiffusesoutx\rho_{H2O}$$

$$\frac{kg_{H2O}}{m^3_{tissue} \cdot S} = \left(\frac{m^3_{blood}}{kg_{tissue} \cdot s}\right)\left(\frac{1000\ kg_{tissue}}{1\ m^3_{tissue}}\right)\left(\frac{0.55\ m^3_{H2O}}{1\ m^3_{blood}}\right)(0.05)\left(\frac{1000\ kg_{H2O}}{1\ m^3_{H2O}}\right)$$

If we calculate the unit conversions our diffusive flux becomes:

$$27,500 \frac{kg_{tissue} \cdot kg_{H2O}}{m^3_{tissue} \cdot m^3_{blood}} (\mathbf{V_{Blood}})$$

Since we want diffusion from the capillaries to the tissue to only occur when the tissue can actually hold more moisture rather than when it is saturated, we will multiply our diffusive flux by the expression c < 700, where 700 kg_{H2O}/m^3_{tissue} is the maximum concentration of H2O allowed in the tissue. With this expression, no additional moisture will diffuse into the tissue once the tissue is saturated with water.

Maximum Concentration of H_2O in Tissue

To calculate the maximum concentration of water that the tissue can hold we will assume that 70% of tissue is composed of water:

$$c_{max} = \left(\frac{1000\ kg_{tissue}}{1\ m^3_{tissue}}\right)\left(\frac{0.7\ kg_{H2O}}{1\ kg_{tissue}}\right) = 700\ \frac{kg_{H2O}}{m^3_{tissue}}$$

Appendix C: Boundary Condition Term Calculations

1. Heat Transfer Coefficients

Calculation of $h_{T,Air}$, which is defined as the thermal boundary's convection of moisture (in a cylinder) from the air.

At T = -5°C, there is forced (due to wind) convection over a horizontal plate of distance L. The Nusselt number for this physical situation is defined below.

$$Nu_L = 0.664 {Re_L}^{\frac{1}{2}} Pr^{\frac{1}{3}} \; for\ laminar\ (Re_L < 2\ x\ 10^5)$$
$$Nu_L = 0.0360 {Re_L}^{\frac{4}{5}} Pr^{\frac{1}{3}} \; for\ laminar\ (Re_L > 3\ x\ 10^6)$$

We must first calculate the $Re_L$to determine which Nu relationship to use.

$$Re_L = \frac{u_{average} L}{\upsilon}$$

- $\mu_{average}$is defined as the average wind speed, which is $5.6 \frac{m}{s}$.
$$\therefore\ \mu_{average} = 5.6 \frac{m}{s}$$
- L is defined as the length of the lip, which is 0.05 meters. $\therefore\ L = 0.05\ meters$
- $\upsilon$is defined as the kinematic viscosity of the air at T = -5°C, which is $12.925\ Pa \cdot s$.
$$\therefore\ \upsilon = 1.2425x10 - 6\ m2/s$$

Plugging in, we get:

$$Re_L = \frac{\left(5.6 \frac{m}{s}\right)(0.05m)}{1.2425x10 - 6\ m2/s}$$
$$Re_L = 223570 < 2\ x\ 10^5$$
$$\therefore Nu_L = 0.664 {Re_L}^{\frac{1}{2}} Pr^{\frac{1}{3}}$$

Referring to Table 1, we know that Pr at T = -5°C is 0.716. Plugging and solving for Nu_D, we get:

$$Nu_L = 0.664(223570)^{\frac{1}{2}}(0.716)^{\frac{1}{3}}$$
$$Nu_L = 280.87$$

And we know that $Nu_L = \frac{hL}{k_{fluid}}$. Rearranging the equation for h gives:

$$h = \frac{k_{fluid} Nu_L}{L}$$

k_{fluid} at T = -5°C is 0.02391 $\frac{W}{mK}$. Plugging in, we get:

$$h_{air,T=-5°C} = \frac{(0.02391\,\frac{W}{mK})(0.087431)}{0.05m} = 134.31\,\frac{W}{m^2K}$$

At T = 25°C, there is natural convection over a horizontal plate of distance L. The Grashof number for this physical situation is defined below.

$$Gr = \frac{\beta g L^3 \Delta T}{v^2}$$

- β is the thermodynamic property of the fluid, and is defined as 3.373 x 10^{-3} at T = 25°C.
  ∴ β = 3.373 x 10^{-3}
- g is the acceleration of gravity, which is defined as $9.81\,\frac{m}{s^2}$. ∴ $g = 9.81\,\frac{m}{s^2}$
- ρ is the density of air at T = 25°C, which is defined as $1.1855\ \frac{kg}{m^3}$ ∴ $\rho = 1.1855\ \frac{kg}{m^3}$
- L is the length of the lip, which is 0.05 meters. ∴ $L = 0.05\,meters$
- ΔT is the change of temperature between the air and skin. The temperature of the skin is 37°C, and the temperature of the air is 25°C. ∴ $\Delta T = 12°C$
- v is the kinematic viscosity of the air at T = 25° , which is $15.575 \text{x} 10^{-6}\ m2/s$.

Plugging everything in and solving, we get:

$$Gr = \frac{(3.373 \text{ x } 10^{-3})\left(9.81\,\frac{m}{s^2}\right)(0.05\,m)^3(12°C)}{\left(15.575\text{x}10^{-6}\,\frac{m^2}{s}\right)^2} = 204607.0044$$

We know that Pr=0.7125 at T = 25°C, and that the Rayleigh number (Ra) is defined as $Ra = Gr\ x\ Pr$.

$$\therefore Ra = (204607.0044)(0.7125\,) = 145782.4906$$

Using Ra and Nu, we can calculate h.

$$Nu_L = 0.54 {Ra_L}^{\frac{1}{4}}\, for\ 10^5\ < Ra_L < 2\ x\ 10^7$$

$$Nu_L = 0.14\ {Ra_L}^{\frac{1}{3}}\, for\ 2\ x\ 10^7 < Ra_L < 3\ x\ 10^{10}$$

Since $10^5\ < 145782.4906 < 2\ x\ 10^7$, we can define $Nu_L = 0.54 {Ra_L}^{\frac{1}{4}}$

Plugging in Ra_L, we get $Nu_L = 10.55$ and we know that $Nu_L = \frac{hL}{k_{fluid}}$. Rearranging the equation for h gives:

$$h = \frac{k_{fluid} Nu_L}{L}$$

We know that k_{fluid} is 0.02605$\frac{W}{mK}$ and L= 0.05m. Plugging in and solving, we get:

$$h_{air,T=25℃} = \frac{k_{fluid} Nu_L}{L} = 5.497\frac{W}{m^2}$$

2. Mass Transfer Coefficients

Forced Convection

At -5° C, an average wind speed is assumed (5.6m/s), therefore forced convection is taking place. The calculations are shown below.

$$Sh_L = 0.664(Re)^{\frac{1}{2}}(Sc)^{\frac{1}{3}}$$

$$Schmidt\ Number = \frac{v}{D_{ab}} = \frac{Momentum\ Diffusivity}{Mass\ Diffusivity}$$

Using the kinematic viscosity of the air is 1.2425x10^{-6} and $D_{H20,\ air}$=2.527*$10^{-5}\frac{m}{s}$,

$$Sc = \frac{(1.2425\text{x}10-6)}{2.527*10-5} = .492$$

$$\frac{hL}{D} = 0.664 {Re_L}^{1/2} Sc^{\frac{1}{3}}$$

Rearranging and substituting values yields

$$h_{m,T=-5} = \frac{.664(225352)^{\frac{1}{2}}(0.492)^{\frac{1}{3}}\text{x}2.527*10-5\frac{m}{s}}{0.05m} = 0.126\frac{m}{s}$$

Natural Convection

At 25° C, using $D_{H20,\ air}$=2.527*$10^{-5}\frac{m}{s}$.

Grashof's value will first be calculated.

$$Gr = \frac{\beta g L^3 \Delta T}{v^2}$$

- $\beta$is the thermodynamic property of the fluid, and is defined as 3.373 x 10^{-3} at T = 25°C.
  ∴ β = 3.373 x 10^{-3}
- g is the acceleration of gravity, which is defined as $9.81\frac{m}{s^2}$. ∴ $g = 9.81\frac{m}{s^2}$
- L is the length of the lip, which is 0.05 meters. ∴ $L = 0.05\ meters$
- $\Delta T$is the change of temperature between the air and skin. The temperature of the skin is 37°C, and the temperature of the air is 25°C. ∴ $\Delta T = 12°C$
- $v$is the kinematic viscosity of the air at T = 25° , which is $15.575\text{x}10^{-6}\ m2/s$.

$$Gr = \frac{(3.373 \text{ x } 10^{-3})\left(9.81\frac{m}{s^2}\right)(0.05\ m)^3(12°C)}{\left(15.575\text{x}10^{-6}\frac{m^2}{s}\right)^2} = 204607.0044$$

$$Sc = \frac{v}{D_{ab}} = \frac{15.575x10^{-6}}{(2.527e-5)} = 0.616$$

$$Ra = GrxSc = (2.1733x10^{-8})x(438550.58) = 126037.91$$

For a horizontal surface with 10^5<Ra<2x10^7,

$$Nu = 0.54Ra^{1/4}$$

$$\frac{h_m L}{D} = 0.54Ra^{1/4}$$

Therefore,

$$h_{m,T=25} = \frac{0.54(126037.91)^{\frac{1}{4}}(2.527e-5\ m/s)}{0.05m} = 0.0051\frac{m}{s}$$

3. Relative Humidity to Vapor Concentration Calculations

The concentration of vapor at the surface of the lip is assumed to be at equilibrium with the liquid in the lip, resulting in a relative humidity 100%. To find the concentration of moisture, the corresponding concentration of moisture in air to 100% relative humidity was found from Figure 20. To convert this concentration to the appropriate units it must be multiplied by the air density at the appropriate temperature. The two concentrations at the lip surface for the two different temperatures are calculated below.

$$Amount\ of\ Water\ in\ Air\ x\ Air\ Density\ @-5°C = 0.004\ \frac{kg\ H_20}{kg\ air}\ x\ 1.31\ \frac{kg\ air}{m^3}$$

$$= 0.00524\ \frac{kg\ H_20}{m^3\ air}$$

$$Amount\ of\ Water\ in\ Air\ x\ Air\ Density\ @\ 25℃ = 0.02\ \frac{kg\ H_20}{kg\ air} x\ 1.177\ \frac{kg\ air}{m^3} = 0.02354\ \frac{kg\ H_20}{m^3\ air}$$

The same figure was used to find the concentration of vapor in the surrounding air at room temperature, denoted at c_∞ at 50% relative humidity.

$$Amount\ of\ Water\ in\ Air\ x\ Air\ Density\ @\ 25℃ = 0.01\ \frac{kg\ H_20}{kg\ air} x\ 1.177\ \frac{kg\ air}{m^3} = 0.01177\ \frac{kg\ H_20}{m^3\ air}$$

The concentration at c_∞ for winter conditions, which is assumed to be at a relative humidity of 20%, changes with temperature and whose information was not available from the figure was found from a relatively humidity to concentration calculator [2].

$c_\infty = 0.000714\ \frac{kg\ h20}{m3\ air}$ @ T = -5°C % 20% RH

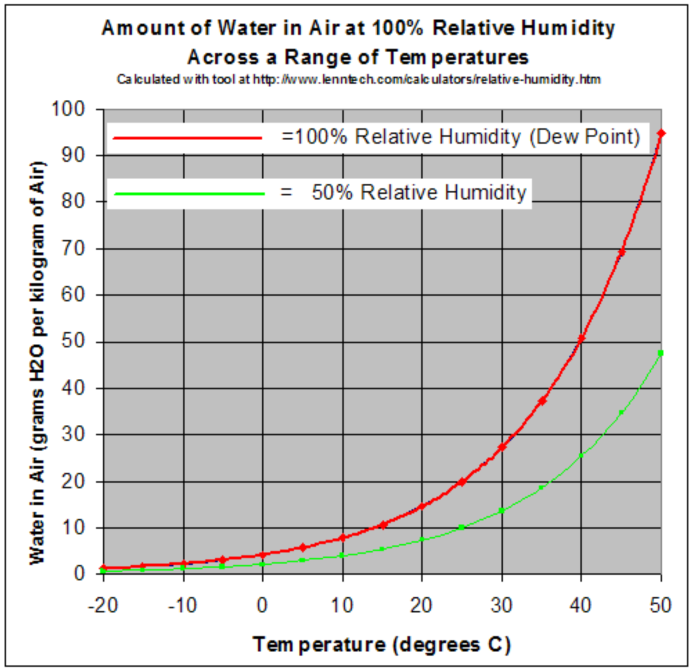

Figure 20: Graph used to calculate concentration of water in air (kg H_20/m^3 air) at 100% relative humidity for -5 and 25°C

At the lip-air boundary, the following boundary condition is imposed. Note that it is dependent on the outside temperature, and independent of the concentration in the lip.

$$-D \frac{\partial c_{H2O}}{\partial y}\bigg|_{y=0.600mm} = h_{m,T}(C_s - C_\infty)$$

When we implement this boundary condition, our solution does not converge because the outside air dries out the lip so much that we begin to see negative concentrations, which are not possible. Thus we have introduced a Boolean to catch this. If the concentration in the lip becomes lower than the difference of the absolute humidity (concentration of water vapor), then the flux becomes very low (1/1000th of the concentration at the boundary) until enough moisture accumulates from the capillaries to switch the boundary condition back.

-1.26*(t>9000)*((c/1000)*(c/1000<=0.0045)+(0.0045)*(c/1000>0.0045))

Additionally, this boundary condition does not become active until the lip balm has worn off after 2.5 hours, thus an additional statement activates this boundary condition once t is greater than 9000 seconds.

Appendix D: Parameters Properties Found from Literature:

Table 2: Properties and Parameters[1, 4, 5, 10]				
	Property	*Value*	*Unit*	*Source*
Layer Thickness				
	x_{dermis}	0.5	Millimeter	[10]-281
	$x_{epidermis}$	75	Microns	[10]-281
	$x_{lip\ balm}$	15	Microns	Experimental
	$X_{strateumcorneum}$	25	Microns	
Thermal Conductivity				
	Epidermis	0.21	$Wm^{-1}K^{-1}$	[10]-282
	Dermis	0.293-0.322	$Wm^{-1}K^{-1}$	[10]-428
	Lip Balm (Petroleum Jelly)	0.813	$Wm^{-1}K^{-1}$	[1]
Density				
	Blood	1100	kgm^{-3}	[10]-282
	Epidermis	1000	kgm^{-3}	[10]-282
	Dermis	1000	kgm^{-3}	[10]-282
	Lip Balm (Petroleum Jelly)	860	kgm^{-3}	[7]
Specific Heat				
	Blood	3300	$Jkg^{-1}K^{-1}$	[10]-282
	Epidermis	3181.82	$Jkg^{-1}K^{-1}$	[10]-282
	Dermis	2846.15	$Jkg^{-1}K^{-1}$	[10]-282
	Lip Balm (Petroleum Jelly)	1905	$Jkg^{-1}K^{-1}$	[1]
Temperatures				
	Capillary Blood Temperature	37	C	[10]-282
	Ambient Air Temperature	25	C	[10]-282
	Winter Conditions	-5	C	
Blood Flow in Skin				
	Forearm, in the cold	0.02	ml/min/g	[10]-419
	Forearm, thermoneutral	0.04-0.05	ml/min/g	[10]-419
	forearm, hyperthermic	>=0.20	ml/min/g	[10]-419
Diffusivity of Water in Skin Layers				
	Stratum Corneum	5E-14	m^2/s	[4]
	Epidermis	2E-10	m^2/s	[4]
	Dermis	2E-10	m^2/s	[4]
Environmental Boundary Condition Values				
	Room Temperature	25	C	--
	Normal Relative Humidity	50	%	[2]
	Cold Temperature	-5	C	--
	Low Relative Humidity	20	%	[2]

Appendix E: Properties Used in Convective Coefficient Calculations

Table 3: Properties Used in Convective Coefficient Calculations[11]		
Temperature [C]	-5	25
Density [kg/m3]	1.3171	1.1855
Kinematic Viscosity [m2/s]	12.925	15.575
Thermal conductivity [W/mK]	0.0239	0.02605
D, water through air [m2/s]	2.11E-05	2.53E-05
Beta, thermal expansion coefficient of air [K^{-1}]		3.37E-03
Prandtl Number of air	0.716	0.7125

Appendix F: Sensitivity Analysis Charts

Table 4: Sensitivity Analysis of Lip Length for Winter Conditions				
Lip Length (m)	**h (W/m^2K)**	**h_m (m/s)**	**Temperature (K)**	**Concentration (kg/m^3)**
0.02	213.21	0.199	297.598041	92.415772
0.03	174.09	0.162	299.271449	93.846511
0.04	150.77	0.141	300.370525	95.05926
0.05	134.85	0.126	301.170112	96.116554
0.06	123.1	0.115	301.788135	97.077444
0.07	113.97	0.106	302.28574	97.961896
0.08	106.61	0.099	302.698495	98.785397

Table 5: Sensitivity Analysis of Lip Length for Normal Conditions				
Length [m]	**h (W/m^2K)**	**h_m (m/s)**	**Temperature [K]**	**Concentration [kg/m3]**
0.05	5.5	0.00514	310.016445	170.617072
0.06	5.25	0.00491	310.022447	170.719751
0.07	5.05	0.00472	310.027254	170.801973
0.08	4.89	0.00457	310.031102	170.867802

Table 6: Sensitivity Analysis of Diffusive Flux in Winter Conditions		
% of H2O diffusing out	Diffusive Flux Coefficient	[H2O] @ (0.5mm, 0.58mm) (kg/m^3)
0.01	5500	19.35064
0.02	11000	38.701733
0.03	16500	57.946901
0.04	22000	77.400816
0.05	27500	96.558545
0.06	33000	116.092418

Table 7. C_{max} values calculated using a range of water in tissue percentages		
Percentage of Water in Tissue	**C_{max}(kg/m^3)**	**C (kgl/m^3)**
55	550	96.746149
60	600	96.746149
65	650	96.746149
70	700	96.746149
75	750	96.746149
80	800	96.746149
85	850	96.746149

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