SECTION II.

OF MOVING POWERS AND THEIR RECIPIENT MACHINES.

CHAPTER I.

OF THE MEASURE OF POWERS AND THEIR EFFECTS.

§ 42. *Dynamometer.*—In order to determine the mechanical effect produced by powers and machines, in terms of the *horse power* unit, three elements are necessary, viz: The *magnitude of the power or effort*, the *distance passed through by it*, and the *time during which the power has acted.*

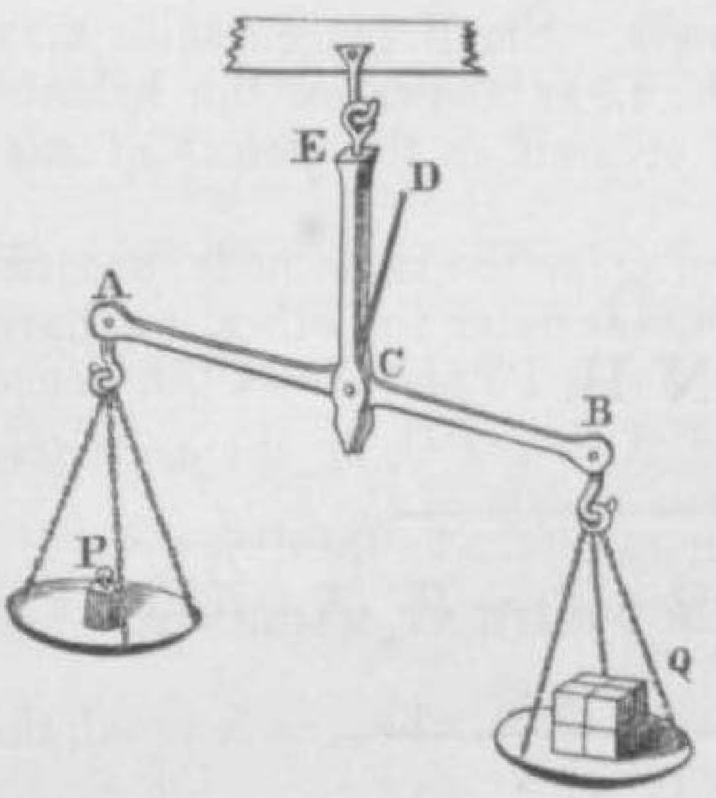
To enable us to represent the effect of forces, we must, therefore, have measures of the force applied, the distance, and the time. *Dynamometers* serve to measure the force applied, the *chain* is generally used for measuring space, and *clocks* or *watches* measure time. If P be the magnitude of the force indicated by the dynamometer, and s the distance throughout which it has acted during the time t, then, the *work* or *mechanical effect* produced in this time is $= Ps$, and the work per second $L = \frac{Ps}{t}$.

Of dynamometers, there are various forms. The common balance is a dynamometer, and is used to measure the force of gravity or *weight.* Modifications of spring balances, and the friction brake are the dynamometers applied to measure forces producing mechanical effect. The friction brake is applied to measuring the mechanical effect given off by revolving axles.

Balances are simple or compound levers, on which the force or weight to be measured is set in equilibrium with standard weights. Balances are either equal or unequal-armed levers, and the latter are variously combined, according to the purposes to which they are applied.

§ 43. *Common Balance.*—The common balance is a lever with equal arms, Fig. 92, on which the weight Q to be measured is equilibriated by an equal weight P. AB is the beam with its points of suspension, (Fr. *fléau*, Ger. *Waagebalken*,) CD the index or

Fig. 92.

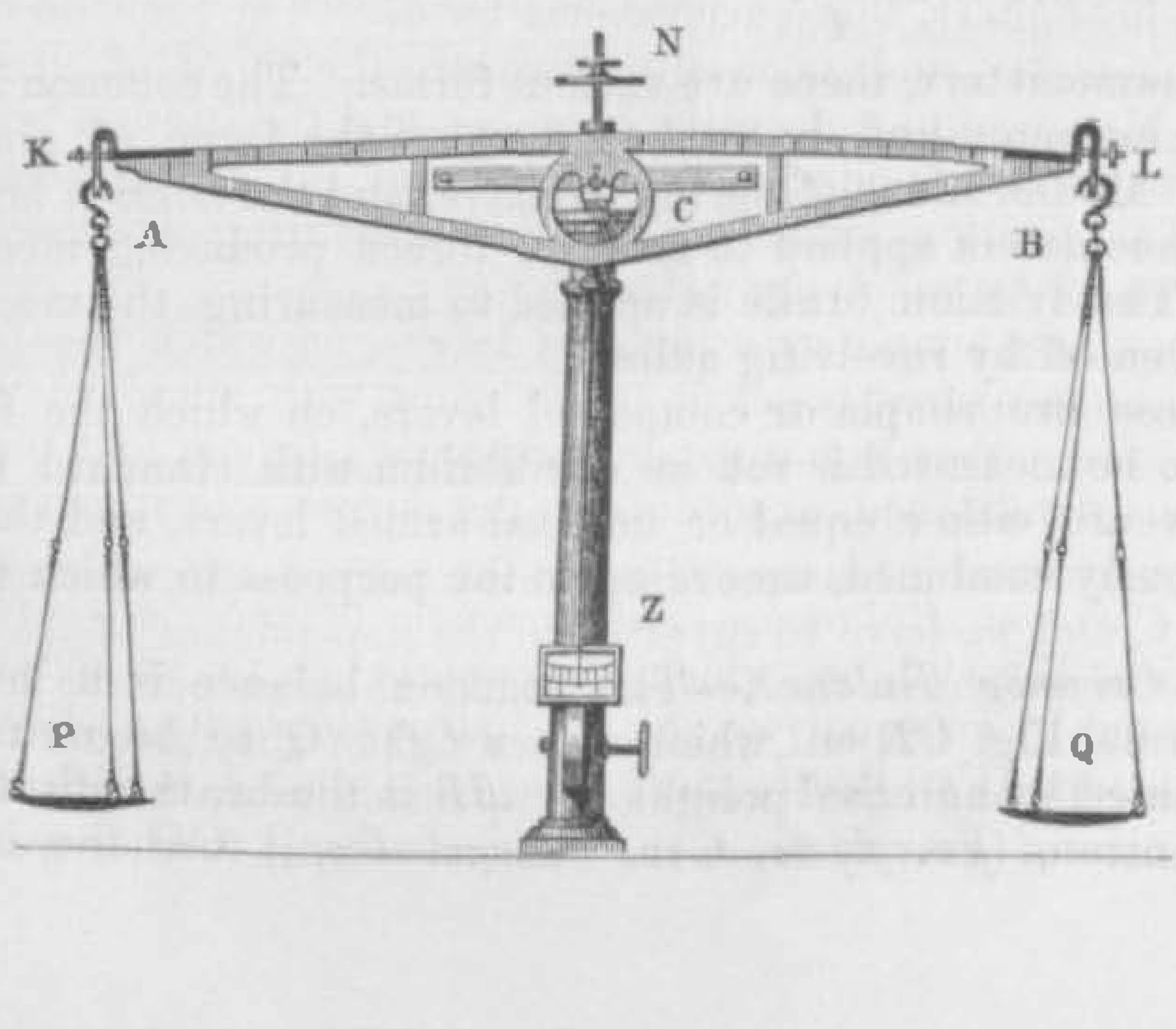


point, (Fr. *aiguille*, Ger. *Zunge*,) *CE* the support or fork, (Fr. *support*, Ger. *Scheere*,) *C* is the knife-edge or fulcrum, a three-sided prism of hard steel.

The requisites of a balance are: 1. That it shall take a horizontal position when the weights in the two scales are equal, and only then. 2. The balance must have *sensibility* and *stability*, that is, it must *play* with a very slight difference of weight in either of the scales, and must readily recover its horizontal position, when the weights are again made equal.

That a balance with equal weights in the two scales may be in adjustment, the arms must be perfectly equal. If a be the length of the one, and b be that of the other arm; P the weight in the one scale, and Q that in the other. Then when the beam is horizontal $Pa = Qb$. If, however, we transpose the weights P and Q, we have again $Pb = Qa$, if the beam retain its horizontal position. From the two equations we have $P^2\,ab = Q^2\,ab$, therefore, $P = Q$, and likewise $a = b$. When, therefore, on transposing the weights, the equilibrium is not disturbed, it is a test of the truth of the balance. A balance may also be tested in the following manner. If we put one after the other two weights P and P into equilibrium with a third Q in the opposite scale, the two weights P and P are equal to each other though not necessarily equal to Q. If, then, we lay the two equal weights in the opposite scales, removing Q, we should have in case of equilibrium $Pa = Pb$, and hence $a = b$. Thus the hori-

Fig. 93.



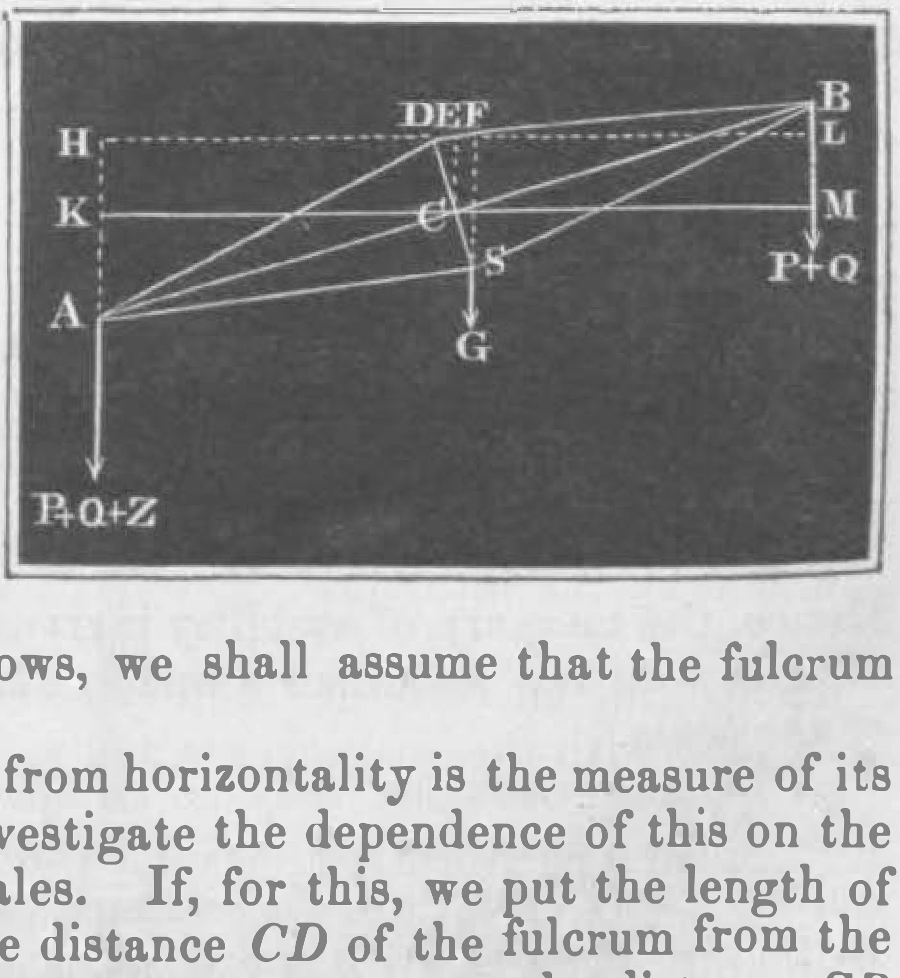
zontality of the balance when two equal weights are laid upon it, is a direct proof of its *truth* or *justness*. Small inaccuracies may be adjusted by means of the screws K, L, as shown on the balance (Fig.193), which serves to *press out* or *pull in* the *points of suspension.*

If a balance indicates weights P and Q for the same body, according as it has been weighed in the one scale or the other, we have for the true weight X of that body: $Xa = Pb$ and $Xb = Qa$, hence $X^2 \,.\, ab = PQ \,.\, ab$, or, $X^2 = PQ$, and $X = \sqrt{PQ}$, or the *geometric mean between the two values is the true weight of the body.*

We may also put $X = \sqrt{P(P + Q - P)} = P\sqrt{1 + \frac{Q-P}{P}}$, and approximately $= P\left(1 + \frac{Q-P}{2P}\right) = \frac{P+Q}{2}$, when, as is usual, the difference $Q - P$ is small; we may, therefore, take the *arithmetical mean of the two weighings as the true weight.*

§ 44. *Sensibility of Balances.*—That the balance may move as freely as possible, and particularly that it may not be retarded by friction at the fulcrum, this is formed into a three-sided prism or *knife-edge* of steel, and it rests on hardened steel plates, or on agate, or other stone. In order, further, that the direction of the resultant of the loaded or empty scale may pass through the point of suspension uninfluenced by friction, in order, in short, that the leverage of the scale may remain constant, it is necessary to hang the scales by knife-edges. In whatever manner such a balance is loaded, we may always assume that the weights act at the points of suspension, and that the points of application of the resultant of these two forces is in the line joining the points of suspension. As, according to Vol. I. § 122, a suspended body is only in equilibrium when its centre of gravity is under the point of suspension, it is evident that the fulcrum D of the balance, Fig. 94, should be above the centre of gravity S of the empty beam, and also not below the line AB drawn through the points of suspension. In what follows, we shall assume that the fulcrum D is above AB and above S.

Fig. 94.



The deviation of a balance from horizontality is the measure of its sensibility, and we have to investigate the dependence of this on the difference of weight in the scales. If, for this, we put the length of the arms CA and $CB = l$, the distance CD of the fulcrum from the line passing through the points of suspension $= a$, the distance SD of the centre of gravity from the fulcrum $= s$, if further we put the angle of deviation from the horizontal $= \phi$, the weight of the empty beam $= G$, the weight on the one side $= P$, and that on the other $= P + Z$, or the difference $= Z$, and, lastly, the weight of each scale, and its apurtenances $= Q$, we have the statical moment on the one side of the balance: $(P + Q + Z) \,.\, DH = (P + Q + Z)(CK - DE) = (P + Q + Z)(l \cos. \phi - a \sin. \phi)$ and on the other side: $(P + Q) \,.\, DL + G \,.\, DF = (P + Q)(CM + DE) + G \,.\, DF = (P + Q)(l \cos. \phi + a \sin. \phi) + G\, s \sin. \phi$; therefore, equilibrium: $(P + Q + Z)\, l \cos. \phi - a \sin. \phi) = (P + Q)(l \cos. \phi + a \sin. \phi) + G\, s \sin. \phi$, or, if we introduce *tang.* ϕ, and transform: $[[2(P + Q) + Z]\, a + G\, s]\ \text{tang.}\ \phi = Z\, l$, therefore,

$$\text{tang.}\ \phi = \frac{Z\, l}{[2(P + Q) + Z]\, a + G\, s}.$$

This expression informs us that the deviation, and, therefore, the *sensibility* of the balance, increases with the length of the beam, and decreases as the distances a and s increase. Again, a heavy balance is, *cæteris paribus*, less sensible than a light one, and the sensibility decreases continually, the greater the weights put upon the scales. In order to increase the sensibility of a balance, the line AB joining the points of suspension and the centre of gravity of the balance, must be brought nearer to each other.

If a and s were equal to 0, or if the points D and S were in the line AB, we should have $\text{tang.}\ \phi = \frac{Zl}{o} = \infty$, therefore $\phi = 90^\circ$; and therefore the slightest difference of weights would make the beam *kick* or deflect 90°. In this case for $Z = 0$, we should have: $\text{tang.}\ \phi = \frac{0}{0}$, *i. e.* the beam would be at rest in any position, if the weights were equal in each scale, and the balance would therefore be useless. If we make only $a = 0$, or put the fulcrum in the line AB, then $\text{tang.}\ \phi = \frac{Zl}{Gs}$, or the sensibility is independent of the amount weighed by the balance. By means of a counterweight N with a screw adjustment, Fig. 93, the sensibility may be regulated.

§ 45. *Stability and Motion of Balances.*—The stability, or statical moment, with which a balance with equal weights returns to the position of equilibrium, when it has inclined by an angle ϕ, is determined by the formula:

$$S = 2(P + Q) \,.\, DE + G \,.\, DF = [2(P + Q)\, a + G\, s] \sin. \phi.$$

Hence, the measure of stability increases with the weights P, Q, and G, and with the distances a and s, but is *independent of the length of the beam.*

A balance vibrating may be compared with a pendulum, and the time of its vibrations may be determined by the theory of the pendulum. $2(P + Q)\, a$ is the statical moment, and $2(P + Q) \,.\, AD^2 = 2(P + Q)(l^2 + a^2)$ is the moment of inertia of the loaded scales, and $G\, s$ is the statical moment of the empty beam. If we put the moment of inertia of the beam $= G y^2$, we have for the length of the pendulum which would be isochronous with the balance (Vol. I. § 250):

$$r = \frac{2\,(P+Q)\,(l^2+a^2)+Gy^2}{2\,(P+Q)\,a+Gs},$$

and hence the time of vibration of the balance:

$$t = \pi \sqrt{\frac{2\,(P+Q)\,(l^2+a^2)+Gy^2}{g\,[2\,(P+Q)]\,a+Gs}};$$

for which, when a is very small or 0, we may put:

$$t = \pi \sqrt{\frac{2\,(P+Q)\,l^2+Gy^2}{g\,G\,s}}.$$

It is evident from this that the time of a vibration increases as P, Q, l increase, and as a and s diminish. Therefore, with equal weights, a balance vibrates the more slowly, the more sensible it is, and therefore weighing by a sensible balance, is a slower process than with a less sensible one. On this account it is useful to furnish sensible balances with divided scales (as Z, Fig. 93). In order to judge of the indication of such a scale, let us put Z the additional weightl= 0 in the denominator of the formula:

$$tang.\ \phi = \frac{Z\,l}{[2\,(P+Q)+Z]\,a+Gs},$$

and write ϕ instead of $tang.\ \phi$, we then get:

$$\phi = \frac{Z\,l}{2\,(P+Q)\,a+Gs}.$$

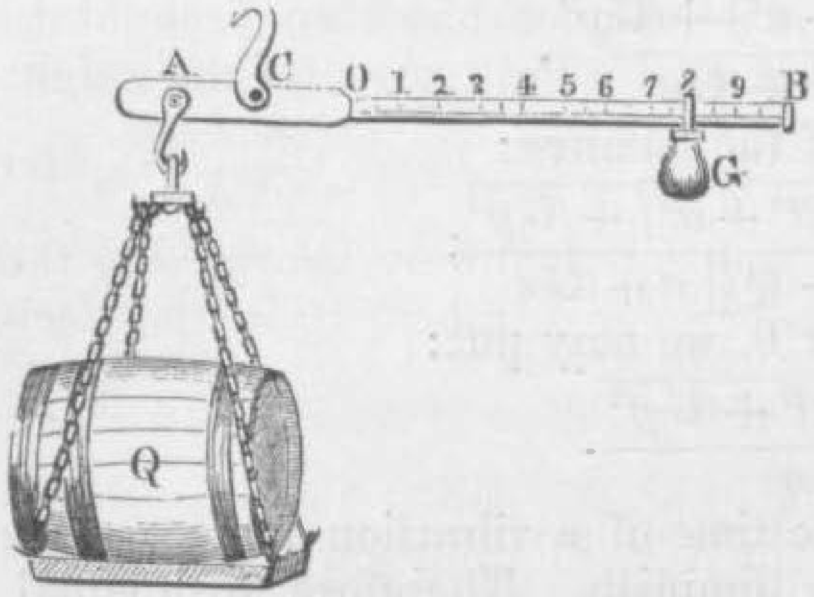
If we then put Z for Z_1, and ϕ for ϕ_1, we get:

$\phi_1 = \dfrac{Z_1\,l}{2\,(P+Q)\,a+Gs}$, hence $\phi : \phi_1 = Z : Z_1$; or, for small differences of weight, the angle of deviation is proportional to that difference. Hence, again $\phi : \phi_1 - \phi = Z : Z_1 - Z$; and therefore $Z = \dfrac{\phi}{\phi_1 - \phi}(Z_1 - Z)$. We can, therefore, find the difference of weights corresponding to a deviation ϕ, by trying by how much the deviation is increased, when the difference of weights is increased by a given small quantity, and then multiplying this increase $(Z_1 - Z)$ by the ratio of the first deviation to the greater deviation obtained.

Remark. Balances such as we have been considering, are used of all dimensions, and of all degrees of delicacy and perfection. Fig. 92 is the usual form of this balance used in trade, and Fig. 93 represents the balances used in assaying, analysis, and in physical researches. Such balances as Fig. 93, are adapted to weigh not more than 1 lb.; but they will turn with $\frac{1}{50}$ of a grain, or with $\frac{1}{350000}$ of a pound. The finest balances that have been made, render $\frac{1}{1{,}000{,}000}$ part of the weight appreciable, but such balances are only for extremely delicate work. Even large balances may be constructed with a very high degree of sensibility. For minute details on this subject, the student is referred to Lardner's and Kater's Mechanics, in "Lardner's Cyclopædia."

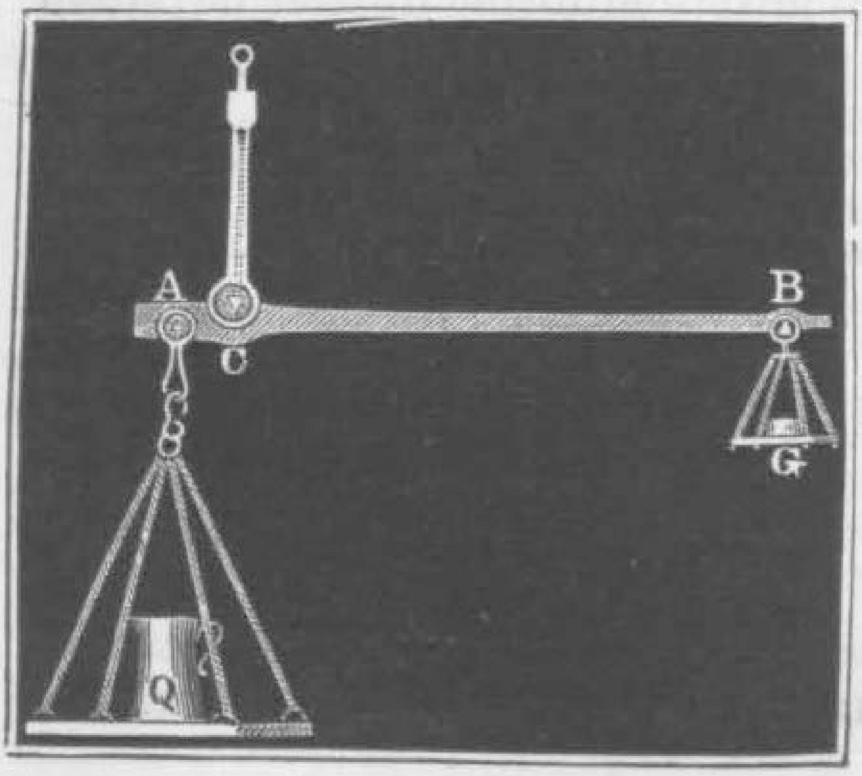
§ 46. *Unequal-armed Balances.*—The balance with unequal arms, termed *statira*, or Roman balance and *steelyard*, presents itself in

Fig. 95.



three different forms, viz: steelyard with movable weight, steelyard with proportional weights, and steelyard with fixed weight. The steelyard with running weight is a lever with unequal arms AB, Fig. 95, on the shorter arm of which CA, a scale is suspended, and on the longer divided arm of which there is a running weight, which can be brought into equilibrium with the body to be weighed Q. If l_0 be the leverage CO of the running weight G, when it balances the empty scale, we have for the statical moment with which the empty scale acts $X_0 = Gl_0$, but if l_n = the leverage CG, with which the running weight balances a certain weight Q, we have for its statical momentt $X_n = Gl_n$; and hence, by subtraction, the moment of the weight $Q, = X_n - X_0 = G(l_n - l_0) = G \,.\, \overline{OG}$. If again $a = CA$, the leverage of the weight, and if b be the distance OG of the running weight from the point O, at which it balances the empty scale, we have $Qa = Gb$, and therefore the weight $Q = \frac{G}{a} b$. Hence the weight Q of the body to be weighed is proportional to b the distance of the running weight from the point O. $2\,b$ corresponds to $2\,Q$, $3\,b$ to $3\,Q$, &c. And, therefore, the scale OB is to be divided into equal parts, starting from O. The unit of division is obtained by trying what weight Q_n must be put on the scale to balance G, placed at the end B. Then Q_n is the number of division, and therefore $\frac{OB}{Q_n}$ the scale or unit of division of OB. If, for example, the running weight is at B, when $Q = 100$ lbs., then OB must be divided into 100 equal parts, and, therefore, the unit of the scale $= \frac{OB}{100}$. If for another weight Q, the weight G has to be placed at a distance $b = 80$, to adjust the balance, then $Q = 80$ lbs.; and so on.

Fig. 96.



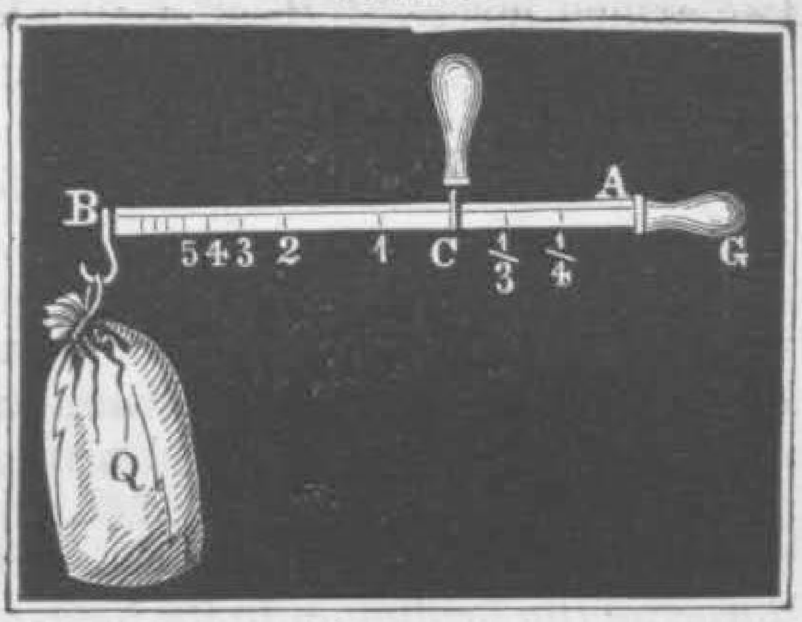
In the steelyard, Fig. 96, with proportional weights, the body to be weighed hangs on the shorter, and the standard weights are put on the longer arm. The ratio $\frac{CB}{CA}$

$=\frac{b}{a}$ of the arms is generally simple, as $\frac{10}{1}$, in which case the balance becomes a *decimal balance.* If the balance has been brought to adjustment or horizontality by a standard weight, then for the weight Q of the body in the scale, we have $Qa = Gb$; hence $Q = \frac{b}{a} G$, and therefore the weight of the merchandise is found by multiplying the small weight G by a constant number, for instance, 10 in the decimal balance, or if the latter be assumed $\frac{b}{a}$ times as heavy as it really is.

The steelyard with *fixed weight*, Fig. 97, called the Danish balance, has a movable fulcrum C, (or it is movable on its fulcrum,) which can be placed at any point in the length of the lever, so as to balance the weights Q hung on one end, by the constant weight G, fixed at the other.

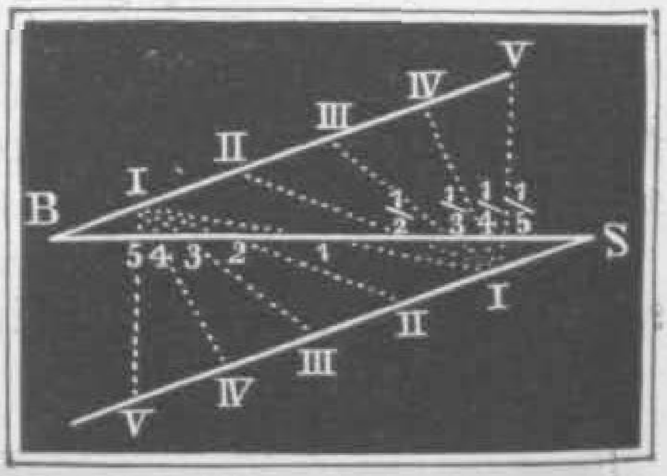
The divisions in this case are unequal, as will appear in the following remark:—

Fig. 97.



Remark. In order to divide the Danish steelyard, Fig. 98, draw through its centre of gravity S and its point for suspension B two parallel lines, and set off on these from S and B equal divisions, and draw from the first point of division of the one lines to the points of division I, II, III, IV of the other parallel straight line. These lines cut the axis BS of the beam in the points of division required. The point of intersection (1) of the line I, I, bisects SB, and, by placing the fulcrum there, the weight of the merchandise is equal to the total weight of the steelyard, if it be horizontal, or in a state of equilibrium. The point of intersection (2) in the line I, II, is as far again from S as from B; and, therefore, when this point is supported $Q = 2\,G$, when equilibrium is established similarly, the point of division (3) in the line I, III, is 3 times as far from S as from B; and hence for $Q = 3\,G$, the fulcrum must be moved to this point. It is also evident that by supporting the points of division $\frac{1}{2}$, $\frac{1}{3}$, &c., the weight $Q = \frac{1}{2}\,G, \frac{1}{3}\,G$, and so on, when the beam is in a state of equilibrium. We see from this that the points of division lie nearer together for heavy weights, and further apart for light weights, and that, therefore, the sensibility of this balance is very variable.

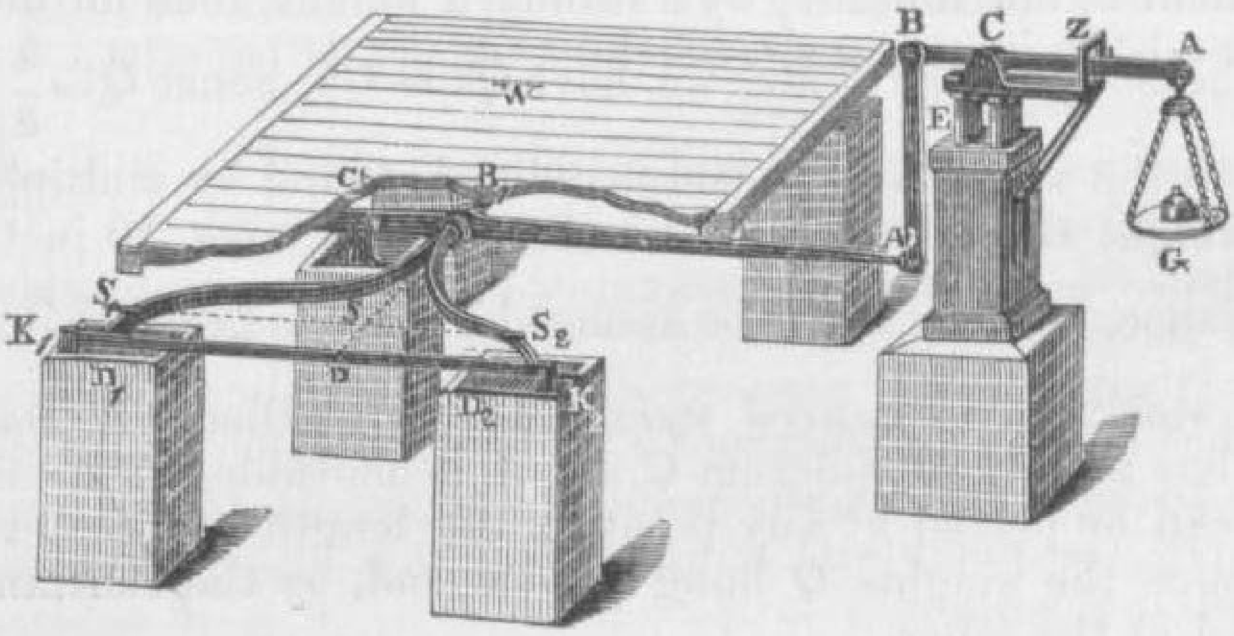
Fig. 98.



§ 47. *Weigh-bridges.*—Compound balances consist of two, three, or more simple levers, and are chiefly used as *weighing-tables* or *weigh-bridges* for carts, wagons, animals, &c. Being used for weighing great weights, they are generally *proportional* balances. The *scale* of the ordinary steelyard is replaced here by a *floor*, which should be so supported, and connected with the levers, that the receiving and removing of the body to be weighed may be as conveniently done as possible, and that the indication of the balance may be independent of the position of the body on the floor.

Fig. 99 represents a very good kind of weigh-bridge by Schwilgue in Strasbourg (*balance à bascule*). This weigh-bridge consists of a

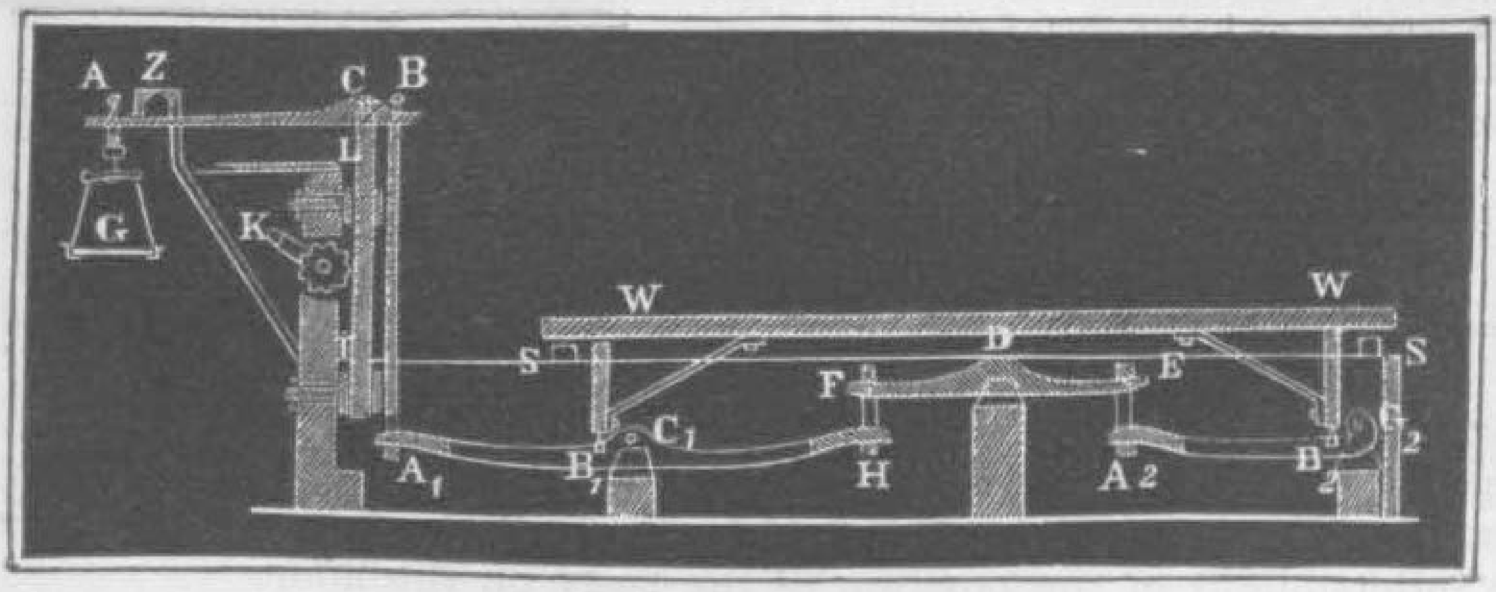
Fig. 99.



two-armed lever ACB, of a simple single-armed lever $A_1C_1B_1$, and of two fork-like single-armed levers B_1S_1, DS_2, &c. The fulcrum of these levers are C, C_1 and D_1D_2. The bridge or floor W is only partially shown, and only one of the fork-formed levers is visible. The bridge usually rests on four bolts K_1, K_2, &c., but during the weighing of any body, it is supported on the four knife-edges S_1, S_2, &c., attached to the fork-shaped levers. In order to do this, the support E of the balance AB is movable up and down by means of a pinion and rack (not visible in the drawing). The business of weighing consists in raising the support EC, when the wagon has been brought on to the floor, in adjusting the weight G in the scale, and finally in lowering the bridge on to its bearings K_1, K_2, &c. The usual proportions of the levers are: $\frac{CA}{CB} = 2$, $\frac{CA_1}{CB_1} = 5$, and the arms $\frac{DB_1}{BS} = 10$. If, therefore, the empty balance has been adjusted, the force at B or $A_1 = 2\ G$, the force at $B_1 = 5$ times the force in $A = 10\ G$, and lastly, the force in $S = 10$ times that in $B_1 = 100\ G$. And, therefore, when equilibrium is established, the weight on the floor is 100 times that laid on the scale at G, and this makes a centesimal scale.

Another form of weigh-bridge such as is constructed at Angers, is shown in Fig. 100. The bridge W of this balance, rests by means of

Fig. 100.

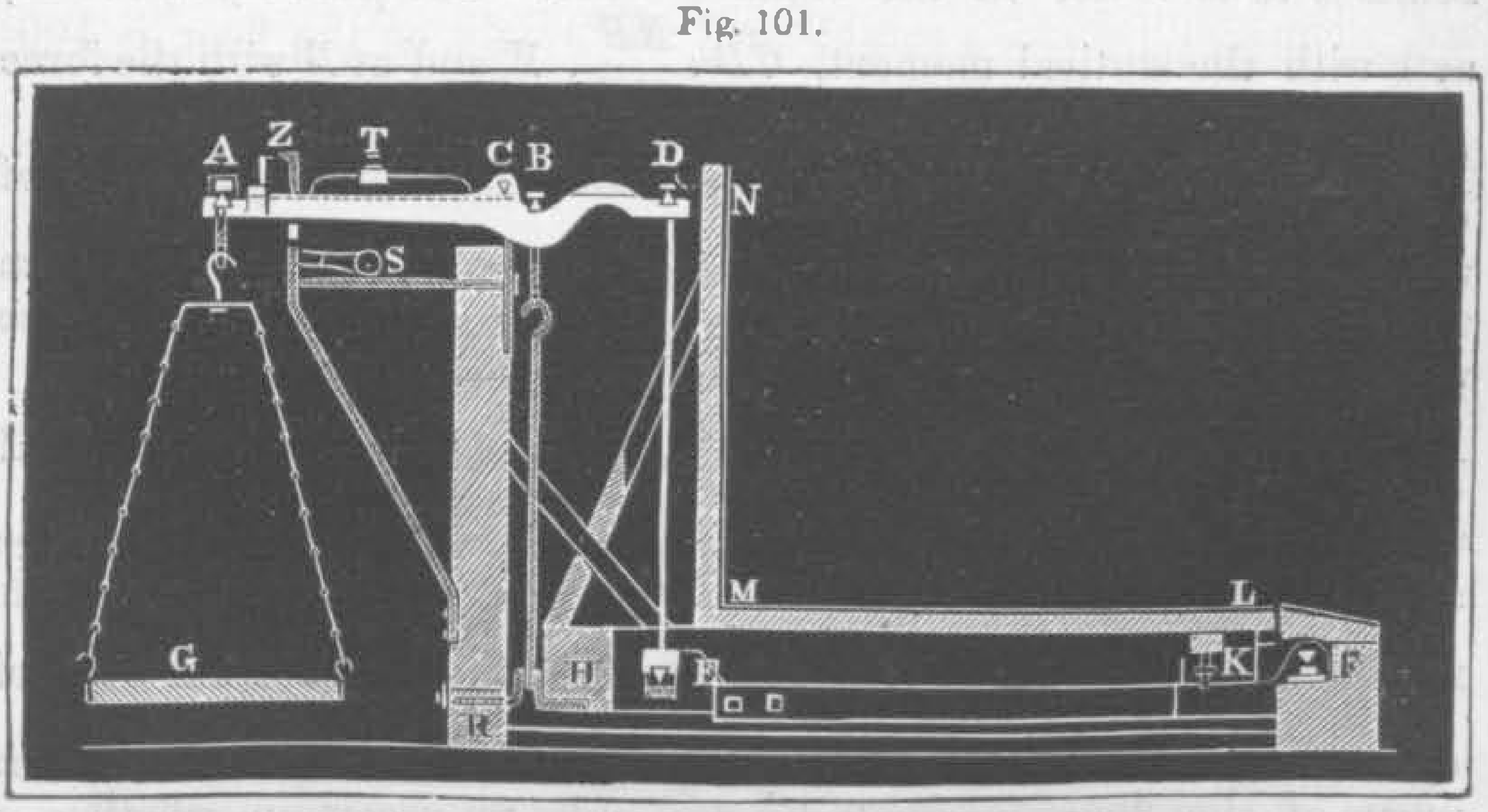


four supports at B_1, B_2, &c., on the fork-shaped single-armed levers $A_1B_1C_1$, $A_2B_2C_2$, of which the latter is connected with the prolongation C_1H of the former, by a lever DEF with equal arms. Until the bridge is to be used, it rests on beams S, S, but when the load is brought on, the support LL of the balance AB is raised (and with it the whole system of levers), by means of a pinion and rack-work, and then so much weight is laid in the scale at G, as is necessary to produce equilibrium.

In whatever manner the weight Q is set upon the bridge W, the sum of the forces at B_1, B_2, &c., is always equal to that weight. But the ratio $\frac{C_2A_2}{C_2B_2}$ is equal to the ratio $\frac{C_1A_1}{C_1B_1} = \frac{a_1}{b_1}$ of the length of the arms, and the length of the arm DE = length of arm DF, as also $C_1H = C_1A_1$. It is, therefore, the same in effect, whether a part of the weight Q is taken up on B_2, or directly on B_1; or the conditions of equilibrium of the lever $C_1B_1A_1$ are the same, whether the whole weight Q act directly on B_1, or only a part of it in B_1, and the rest in B_2, and only transferred by the levers $C_2B_2A_2$, EDF and C_1H to $C_1B_1A_1$. If, further, $\frac{a}{b}$ be the ratio of the arms $\frac{CA}{CB}$ of the upper balance, the force on the connecting rod $BA_1 = \frac{a}{b} \cdot G$, and hence the weight on the floor supposed previously adjusted, is: $Q = \frac{a_1}{b_1} \cdot \frac{a}{b} G$. Generally $\frac{a}{b} = \frac{a_1}{b_1} = \frac{10}{1}$, hence, $\frac{Q}{B} = \frac{100}{1}$, and the balance is centesimal.

§ 48. *Portable Weigh-bridge.* — In factories and warehouses, various forms and dimensions of weighing-tables, after the design of that of Quintenz, are used. This balance, which is represented in Fig. 101, consists of three levers ACD, EF, and HK. On the first lever hangs the scale-pan G, for the weights, and two rods DE and BH.

Fig. 101.

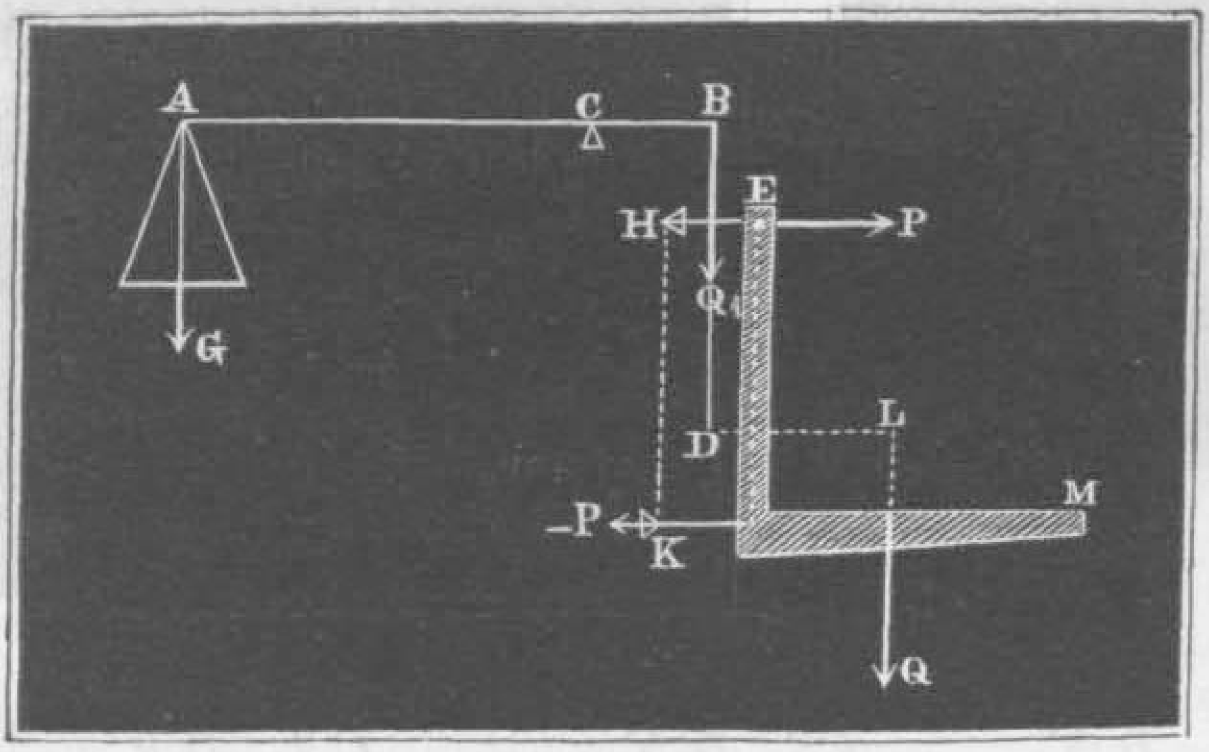


The rod DE carries the lever turning on the fixed point F, and the second rod BH carries the lever HK, the fulcrum of which rests upon the lever EF. In order to provide a safe position for the two latter levers, they are made fork-shaped, and the axes F and K on which they turn, are formed by the two knife-edges. On the lever HK, the trapezoidal floor ML is placed to receive the loads to be weighed, and it is provided with a back-board MN, which protects the more delicate parts of the balance from injury. Before and after the act of weighing, the lever formed by the border of the floor rests on three points, of which only one, R, is visible in our section; and the balance beam AD is supported by a lever-formed *catch* S, provided with a handle. When the merchandise is laid on the table, the catch is put down, and weights are laid on at G, till the balance AD is in adjustment. The catch is again raised, so that HK comes again to bear on the three points, and the merchandise can be removed without injury to the balance. The balance AD is known to be horizontal by an index Z, and the empty balance is adjusted by a small movable weight T, or by a special adjusting weight laid on the scale at G.

In this, as in other *weigh-bridges*, it is necessary that its indications be independent of the manner in which the goods are placed upon the table or floor. That this condition may be satisfied, it is necessary that the ratio $\frac{EF}{KF}$ of the arms of the lever EKF, be equal to the ratio $\frac{CD}{CB}$ of the arms of the balance beam AD. A part X of the weight Q on the floor is transferred by the connecting rod BH to the balance beam AH, and acts on this with the statical moment, $\overline{CB} \,.\, X$; another part Y, goes through K to the lever EF, and acts at E with the force $\frac{KF}{EF} \,.\, Y$. But this force goes by means of the rod DE to D to act on the balance beam. The part Y, therefore, acts with the statical moment, $\overline{CD} \,.\, \frac{KF}{EF} \,.\, Y$, and at B with the force $\frac{CD}{CB} \,.\, \frac{KF}{EF} \,.\, Y$ on the balance beam AD. That the equilibrium of the balance beam may not depend on either X or Y alone, but on the sum of them $Q = X + Y$, it is requisite that Y should act on the point B, exactly as if it were applied there directly, or that: $\frac{CD}{CB} \,.\, \frac{KF}{EF} \,.\, Y = Y$, i. e. $\frac{CD}{CB} \,.\, \frac{KF}{EF} = 1$, therefore, $\frac{CD}{CB} = \frac{EF}{KF}$. If we denote the arms CA and CB by a and b, we have here as in the simple balance $Ga = (X + Y)\, b = Q\, b$, and, therefore, the weight required $Q = \frac{a}{b}\, G$, for example, $= 10\, G$, if the length CB is $= \frac{1}{10}$ of CA. Such a balance is tested by laying a certain weight on different parts of the floor, particularly the ends, to ascertain whether it everywhere equilibrates the weight $G \frac{a}{b}$ times smaller than itself, placed on the scale.

Remark 1. Messrs. George, at Paris, manufacture weighing-tables of a peculiar construction described in the "Bulletin de la Société d'Encouragement, April, 1844." This balance has only one suspending rod BD, Fig. 102; but to provide against the floor FM

Fig. 102.



turning, there are two knife-edge axes on the back, which are united with two pair of knife-edges H and K, by four parallel rods EH and FK. According to the theory of couples, the tension Q_1 on the rod BD is equal to a weight Q laid on the floor; but besides this, the floor itself acts outwards with a force P in E, and with an opposite force $-P$ in F inwards. If d be the distance DL of the weight Q from the rod BD, and e the distance of the knife edges E and F, then $eP = dQ$, and, therefore, each horizontal force $P = \frac{d}{e} Q$. These forces do not influence the lever, and therefore the weight $Q = \frac{a}{b} G$, if, as hitherto, a and b denote the lever arms CA and CB, and G the weight in the scale.

Remark 2. Weigh bridges are treated of in detail in the "Allgemeinen Maschinen Encyclopädie, Bd. 2, Art. Brückenwaagen," under the art. "Weighing Machine, in the Encyclo. Britannica Edinensis."

§ 49. *Index Balances*, or *Bent Lever Balances*.—The bent lever balance is an unequal-armed lever ACB, Fig. 103, which shows the weight Q of a body hung on to it at B, by an index CA moving over a scale DE, the weight G of the index head being constant. To determine the theory of this balance, let us first take the simple case, in which the axis of the pointer passes through the point of suspension B of the scale, Fig. 104. When the empty balance is in equilibrium, i.e. its centre of gravity S_0 vertically under the centre of motion C, let the index stand in the position CD_0, and let the point of suspension be in B_0. If now we add a weight Q, then B_0 comes to B, and D_0 to D, and S_0 to S, and thus the weight Q acts with the leverage CK, and the weight G of the empty balance, with the leverage CH. Therefore, for the new state of equilibrium $Q \,.\, CK = G \,.\, CH$. If now D_0N falls perpendicularly on CD, we have CD_0N and SCH, two similar triangles, and, therefore, $\frac{CH}{CS} = \frac{D_0N}{CD_0}$, and as besides, the triangles D_0PN and CBK are similar, we have also $\frac{CK}{CB} = \frac{D_0N}{D_0P}$,

Fig. 103.

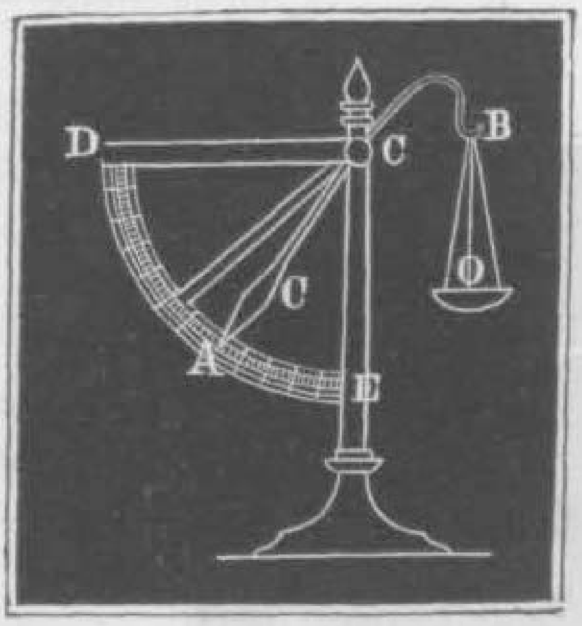
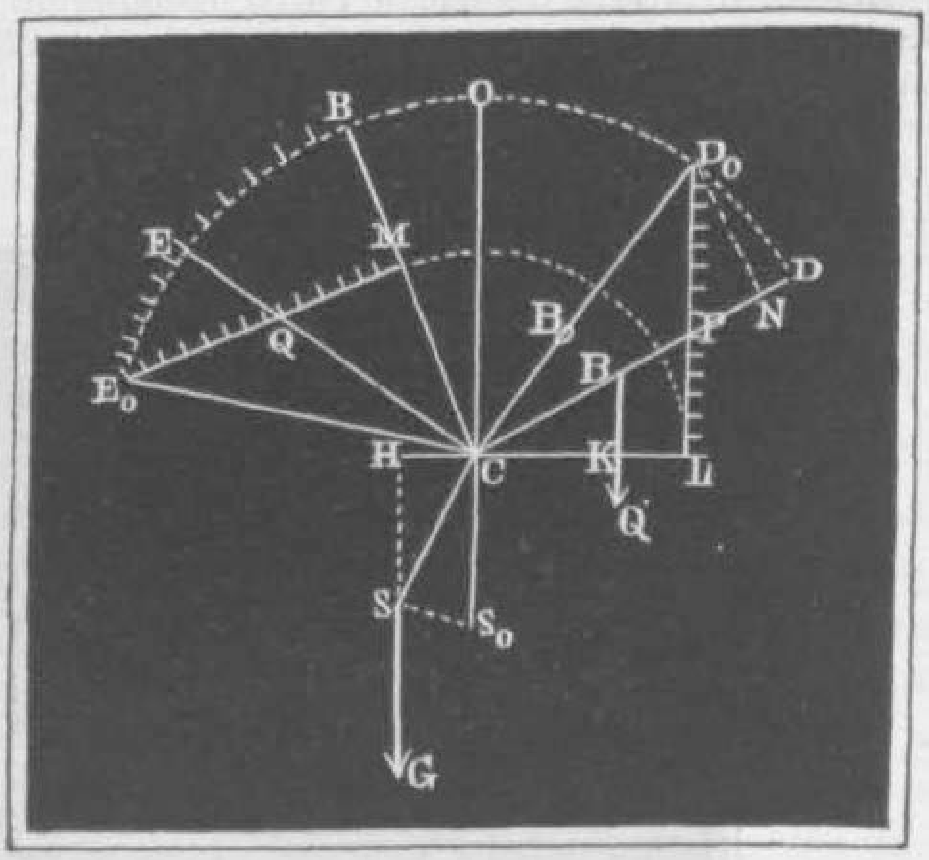


Fig. 104.



and hence:

$$Q \cdot \frac{CBB\,D_0N}{D_0P} = {}^{h}G \cdot \frac{CSB\,D_0N}{CD_0},\ \text{i. e}\ Q = \frac{CS}{CB} \cdot \frac{D_0P}{CD_0};\ \text{or, if:}$$

$CS = a$, $CB = b$, $CD_0 = CD = d$, and $D_0P = x$, $Q = \frac{a}{b} \cdot \frac{x}{d}$. Therefore the weight Q increases with the portion $D_0P = x$, cut off by the index from the vertical D_0L, and therefore D_0L may be divided as a scale of equal parts. If a point P on the scale has been found for a known weight put on the balance, other points of division are got by dividing D_0P into equal parts. If the index centre line of the CD_0 does not pass through the point of suspension, but has another direction CE_0, the scale of equal parts corresponding to it, is found, if we place the right-angled triangle CD_0L as CE_0M or CE_0, and lastly, in order to get the circular scale E_0R, we have to draw radii from the centre C through the points of division of the line E_0M to the periphery of the circle passed over by the point of the index.

Remark. There are other index balances described in Lardner's and Kater's Mechanics. Such balances are chiefly used for weighing letters and paper, thread, and such like manufactures, where samples have frequently to be weighed.

§ 50. *Spring-balances* or *Dynamometers.*—Spring-balances are made of hardened steel springs, upon which the weights or forces act, and are furnished with pointers, indicating on a scale the force applied in deflecting the spring. The springs must be perfectly tempered, or they must resume their original form on removal of the force applied. Thus spring-balances should never be strained beyond a certain point proportional to their strength; for if we surpass the limits of their elasticity in any application of them, they are afterwards useless as accurate measures of weight. The springs applied for such balances are of many different forms. Sometimes they are wound spirally on cylinders, and enclosed in a cylindrical case, so as to indicate the forces applied in the direction of the axes of the cylinder by the compression or extension of the spiral. In other balances the spring forms an open ring *ABDEC*, Fig. 105, and the index is attached by a hinge to the end *C*, and passed through an opening in the end *A*. If the ring *B* be held fast, and a force *P* applied at *E*, the ends *A* and *C* separate in the direction of the force applied, and the index *CZ* rises to a certain position on the scale fastened at *D* to the spring. If the scale has been previously divided by the application of standard weights, the magnitude of any force *P* applied, though previously unknown, is indicated by the pointer.

Fig. 105.

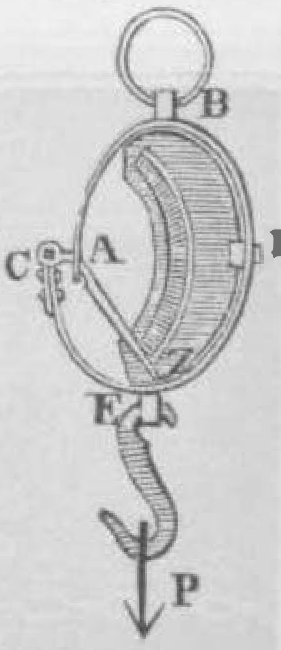
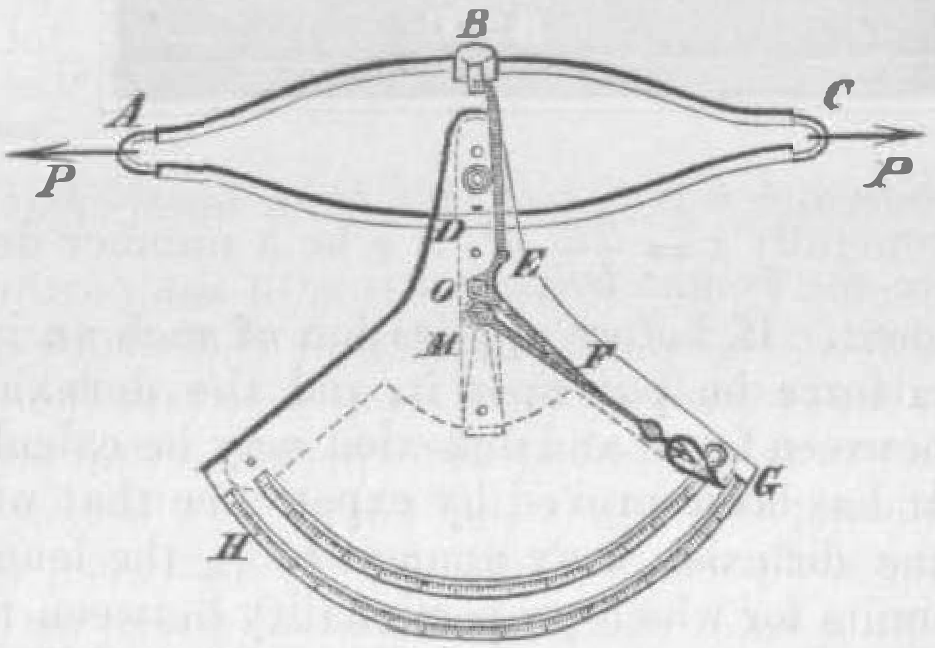


Fig. 106 is a representation of Regnier's dynamometer. *ABCD* is a steel spring forming a closed ring, which may either be drawn out by forces *P* and *P*, or pressed together by *OB* and *D*; *DEGH* is a sector connected with the spring, on which there are two scales; *MG* is a double index turning on a centre at *M*, and *EOF* is a bent lever, turning on *O*, and which is acted upon by a rod *BE*, when the parts *B* and *D* of the spring approach each other in consequence of the application of weights or other forces as above mentioned. That the index may remain where the force has put it, for more convenient reading, a friction leather is put on the under side.

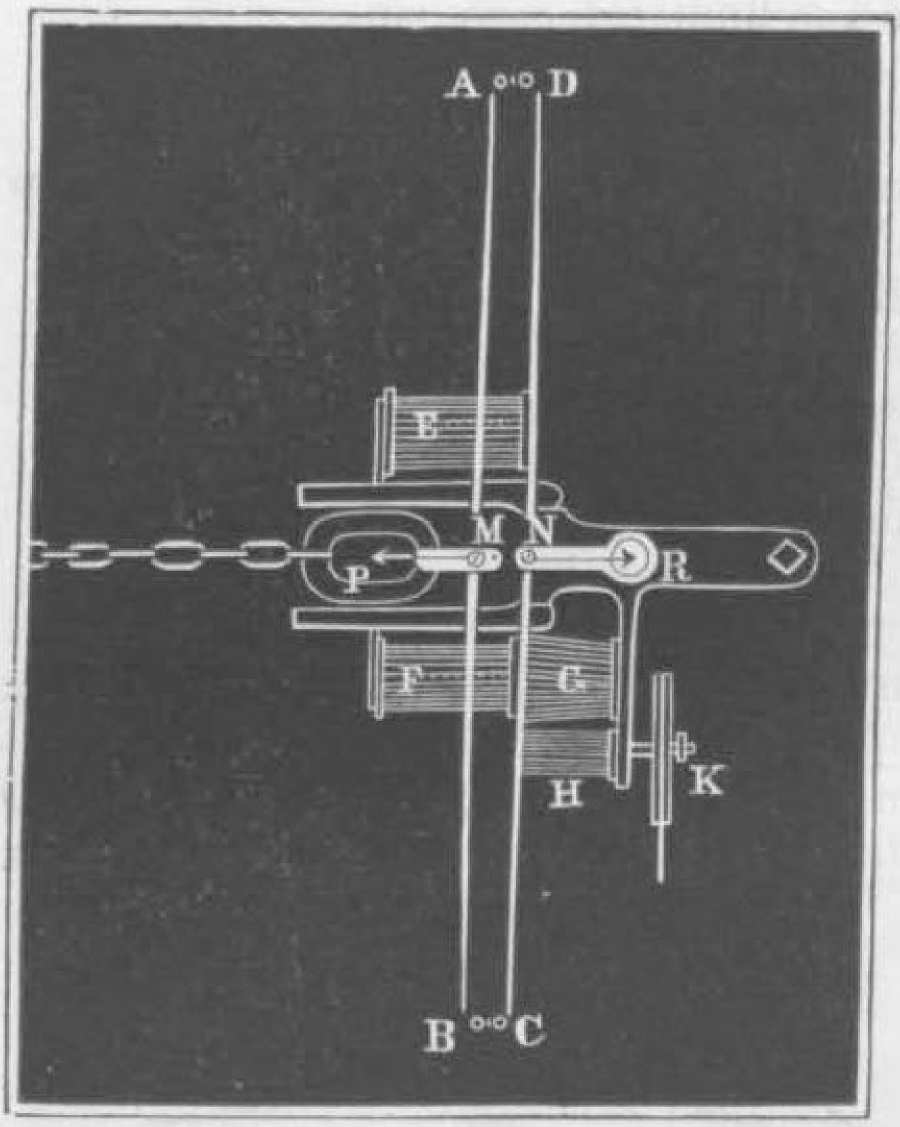
Fig. 106.



The most perfect, and most easily applied dynamometer for mechanical purposes, is that described by Morin, in his Treatise, "Description des Appareils chronométriques à style, et des Appareils dynamométriques, Metz, 1838," and used by him in his various researches on friction and other important mechanical inquiries. Morin's dynamometer consists of two equal steel springs *AB* and *CD*, Fig. 107, of from 10 to 20 inches in length, and the force applied is measured by the separation which it produces between the two spring plates at *M*. In order to determine the force, for example, the force of traction of horses on a carriage, the spring plate *N* is connected by a bolt with the carriage, and the horses are attached to the chain

M in any convenient manner. There is a pointer on M which indicates on a scale attached to N, the separation of the plates produced by the force P applied. If the springs be plates of uniform breadth and thickness, and be $l =$ the length, $b =$ the breadth, and $h =$ the thickness; according to Vol. I. § 190, we have for the deflexion corresponding to a force P:

Fig. 107.



$$a = \tfrac{1}{48}\frac{Pl^3}{WE} = \tfrac{1}{4}\frac{Pl^3}{E\,b\,h^3},$$ or

the deflexion increases as the force applied, and, therefore, a scale of equal parts should answer in this dynamometer.

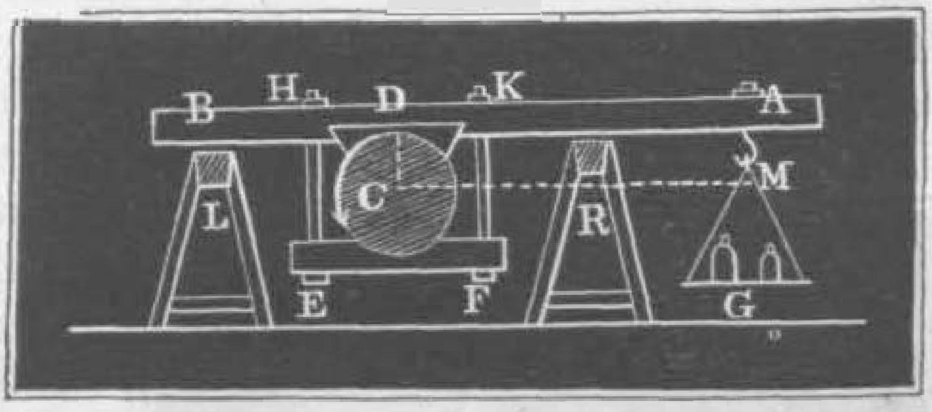
As the deflexion s of two springs is called into action, the amount is double of that of one of them, or it is $= \frac{1}{2}\frac{Pl^3}{E\,b\,h^3}$, and, therefore, generally $s = \frac{\phi\, l^3}{b\,h^3}P$, if ϕ be a number determined by direct experiment. If, before application of such an instrument, a known weight or force be put upon it, and the deflexion s ascertained, the ratio between force and deflexion may be calculated, and a scale prepared. It has been proved by experience that when the best steel is used, the deflexion may amount to $\frac{1}{10}$ the length, without surpassing the limits for which proportionality between force and deflexion subsists. The springs that have been employed by Morin and others are made into the form of beams of equal *resistance* throughout their length (Vol. I. § 204), and have, therefore, a parabolic form, or thicker in the centre than at the ends.

Remark. Forces do not generally act uniformly, but are continually changing, and therefore the usual object is to ascertain the *mean effort.* The usual index dynamometers only give the force as it has acted at some particular instant, or only the *maximum effort.* There is, therefore, extreme uncertainty in the indication of such dynamometers, modified as they have been by M'Neill and others, when applied to measuring the effort of horses applied to ploughing, canal traction, &c. Morin has completely provided against this defect, by attaching to his dynamometer, a self registering apparatus, first suggested byePoncelet, (see Morin's work above quoted,) by which, in one case, the force for each point of a distance passed over is registered in the form of a curved line, drawn on paper, and in the other case, the force as applied at each instant is summed up or integrated by a machine Both apparatuses give the product of the force into the distance described, and, therefore, the *mean effort* may be produced when the *mechanical effect* is divided by distance passed over—by a canal boat, for example.

In the dynamometer with pencil and continuous scroll of paper, the measure of the force is marked by a pencil passing through M, till its point touches a scroll of paper passing under it. This scroll is wound from the roller E, (Fig. 107,) to the roller F, which is set in motion by bands or wheel-work, by the wheels of the carriage itself. When no force acts on the springs, the pencil would mark a straight line on the paper, supposing it set in motion; but by application of a force P, the springs are deflected, and therefore a line more or less tortuous is drawn by the pencil at a variable distance from the above alluded to zero line, but on the whole parallel to it. The area of the space between the two lines is the measure of the mechanical effect developed by the force; for the basis of it is a line proportional to the distance passed over, and the height is itself proportional to the force that has acted to bend the spring.

§ 51. *Friction Brake.*—The dynamometrical brake (Fr. *frein dynamométrique de M. Prony*), is used to measure the power applied to, and mechanical effect produced by a revolving shaft, or other revolving part of a machine. In its simplest form, this instrument consists of a beam AB, Fig. 108, with a balance scale AG; and of two wooden segments D and EF, which can be tightened on the revolving axis C, by means of screw-bolts EH and FK. To measure, by means of this arrangement, the power of the axis C for a given number of revolutions, weights are laid in the scale, and the screw-bolts drawn up until the shaft makes the given number of revolutions, and the beam maintains a *horizontal* position, without support or check from the blocks L or R. In these circumstances the whole mechanical effect expended is consumed in overcoming the friction between the shaft and the wooden segments, and this mechanical effect is *equal* to the work or useful effect of the revolving shaft. As, again, the beam hangs freely, it is only the friction F acting in the direction of the revolution that counterbalances the weight at G, and this friction may be deduced from the weights. If we put the lever CM of the weight G referred to the axis of the shaft $= a$, the statical moment of the weight, and therefore also the *moment of the friction*, or the friction itself acting with the lever equal to unity $= G a$, then, if ϵ represent the angular velocity of the shaft, the mechanical effect produced $L = Pv = Ga \cdot \epsilon = \epsilon\, a\, G$ per second.

Fig. 108.

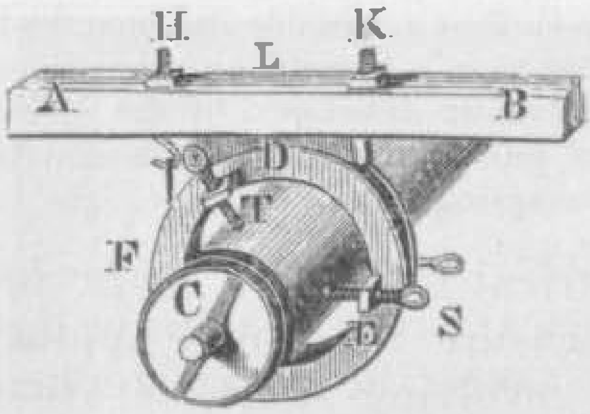


If, again, $u =$ the number of revolutions of the shaft per minute, then $\epsilon = \dfrac{2\pi u}{60} = \dfrac{\pi u}{30}$, and, therefore, the work required $L = \dfrac{\pi u a}{30} G$.

The weight G must of course include, not only the weight in the scales, but the weight of the apparatus reduced to the point of suspension. To do this, the apparatus is placed upon a knife-edge at D, and a cord from A attached to a balance would give the weight required.

The friction brake as represented in Fig. 109, with a cast iron friction ring DEF is a convenient form of this instrument. This ring is fastened by three pairs of screws on any sized shaft that will pass through the ring. For the wooden segment an iron band is substituted, embracing half the circumference of the iron ring. The band ends in two bolts passing through the beam AB, and may be tightened at will by means of screw nuts at H and K.

Fig. 109.



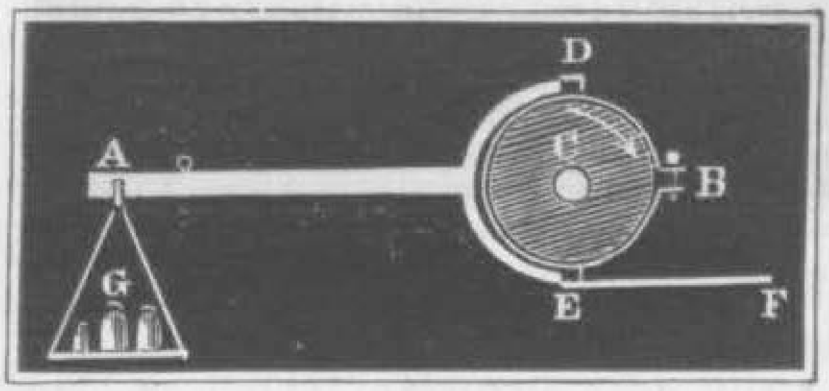
To hinder the firing of the wood, or excessive heating of the iron, water is continually supplied through a small hole L. This apparatus is known in Germany as "Egen's Friction Brake."

Example. To determine the mechanical effect produced by a water wheel, a friction brake was placed on the shaft, and when the water let on had been perfectly regulated for six revolutions per minute, the weight G including the reduced weight of the instrument was 530 lbs., the leverage of this weight was $a = 10{,}5$ feet. From these quantities we deduce the effect given off by the water wheel to have been:

$L = \frac{\pi \cdot 6 \cdot 10{,}5}{30} \cdot 530 = 3497$ ft. lbs. $= 6{,}3$ horse power.

§ 52. In more recent cases, various forms of friction brake have been adopted, some of them very complicated. The simplest we know of is that of Armstrong, shown in Fig. 110. This consists of an iron ring, which is tightened round the shaft by a screw at B, and of a lever ADE with a scale for weight G on one side, and a fork-shaped piece at the other, which fits into snuggs projecting from the ring. There is a prolongation of one prong of the fork, by which the weight of the instrument itself can be counterbalanced, and which is otherwise convenient in the application of the instrument.

Fig. 110.

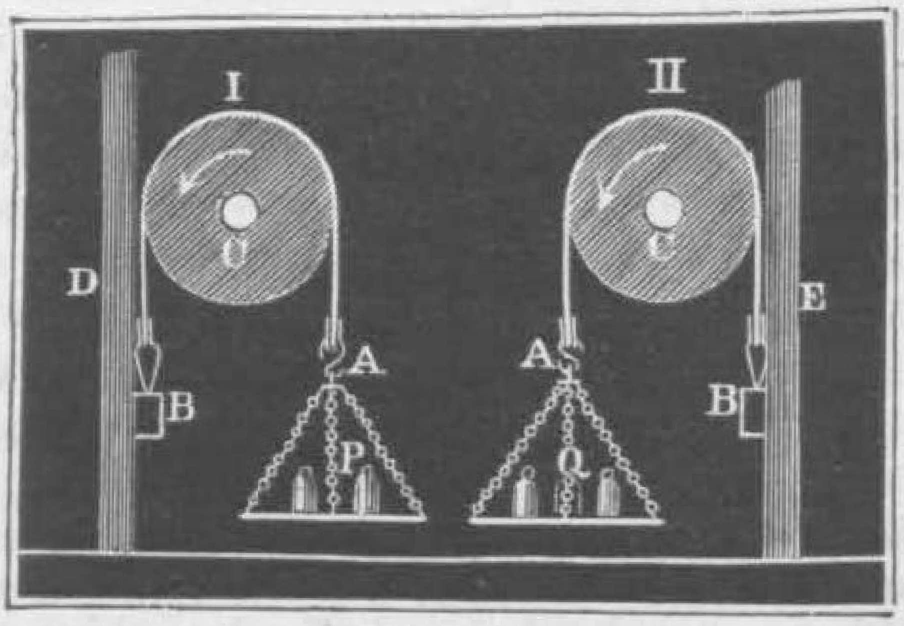


Navier proposed a mode of determining the effect given off at the circumference of a shaft by laying an iron band round the shaft, attaching the one end of this to a spring balance, the other end being weighted, so that the friction on the wheel causes a resistance, in overcoming which only the required number of revolutions take place. The difference of this weight Q and that indicated by the spring balance P, is of course $=$ the friction F between the shaft and the band. If then p be the circumference of the shaft, and n the number of revolutions per minute, the effect produced

$$= L = F \frac{up}{60} = \frac{up}{60} (Q - P).$$

When a spring balance cannot be obtained, a simple band, as shown in Fig. 111, is sufficient for the purpose, if the experiment be made twice, and the end B be fastened to an upright or other fixture, first on the one side and then on the other of the shaft. In this way one experiment gives us $Q = P + F$, and in the other $Q = P$, because in the one case, the friction F acting in the direction of the revolution of the shaft, counteracts the weight hanging in the scale on the end A, and in the other it acts with this friction. For this arrangement, used by the author in many experiments, the mode of calculation already explained applies precisely. As the power has only a small leverage in this arrangement, it is only suitable for cases in which the effort exerted is small. The strain on the band may be multiplied by means of an unequal-armed lever attached at A, instead of the direct application of the weight in the scale. The author has successfully applied a leverage of 10 to 1 in this way. In order to avoid the objectionable increase of friction of the axle or gudgeons induced by this apparatus, the band may be made to pass round the shaft, carrying the one end upwards and the other down.

Fig. 111.



Remark. Egen treats of the different forms of dynamometers in his work, entitled "Untersuchungen über den Effect einiger Wasserwerke, &c.," and Hülsse, in article Bremsdynamometer in the "Allgemeinen Maschinenencyclopädie." James White, of Manchester, invented the friction brake in 1808. See Hachette "Traité elémentaire des Machines, p. 460." Prony's original paper is in the "Annales des Mines, 1826. There are remarks on its use in the writings of Poncelet and Morin, worthy attention. W. G. Armstrong's paper is in the "Mechanic's Magazine," vol. xxxii. p. 531. Weisbach's papers on the friction band, in thee "Polytechnishes Central Blatt," 1844.

CHAPTER II.

OF ANIMAL POWER, AND ITS RECIPIENT MACHINES.

§ 53. *The Power of Animals.*—The working power of animals is, of course, not only different for individuals of different species, but for animals of the same species. The work done by animals of the same species depends on their race, age, temper, and management, as well as on the food they get, and their keeping generally, and also on the nature of the work to which they are applied, or the *manner* of putting them to their work, &c. We cannot discuss these different points here, but for each kind of animal employed by man, wet shall assume as fair an average specimen as possible,—that the animal is judiciously applied to work it has been used to perform, and that its food is suitable. But the working capabilities of animals depend also on the effort they exert, and on their speed, and on the time during which they continue to work. There is a certain *mean effort*, *speed*, and length of *shift* for which the *work done* is a maximum. The greater the effort an animal has to exert, the less the velocity with which it can move, and *vice versâ*. And there is a maximum effort when the speed is reduced to nothing, and when, therefore, the work done is nothing. It is evident, therefore, that animal powers should work with only a certain velocity, exerting a certain mean effort, in order to get the maximum effect; and it also appears that a certain mean length of shift, or of *day's work*, is necessary to the same end. Small deviations from the circumstances corresponding to the maximum effect produced, are proved by long experience to be of little consequence. It is also a matter of fact that animals produce a greater effect, when they work with variable efforts and velocities, than when these are constant for the day. Also pauses in the work, for breathing times, makes the accomplishment of the same amount of work less *fatiguing*, or the more the work actually done in a unit of time differs from the mean amount of work, the less is the fatigue.

The main point to be attended to in respect to animal powers is the "*day's work*." If this be compared with the daily cost of maintaining the animals, and interest on capital invested, we have a measure of the value of different animal powers.

§ 54. The manner and means of employing the power of men and animals is very different. Animal powers produce their effects either with or without the intervention of machines. For the different means of employing labor, the degree of fatigue induced is not proportional to the work done. Many operations fatigue more than others; or what amounts to the same thing, the mechanical effect produced is much smaller in some modes of applying labor than in others. Again, all labor cannot be measured by the same standard as is involved in our definition of mechanical effect. The work done in the transport of burdens on a horizontal road cannot be referred to the same standard as the raising of a weight is referred to. According to the notions we have acquired hitherto, the mechanical effect produced in the transport of burdens on a horizontal road is nothing, because there is no space described *in the direction* of the force (Vol. I. § 80) exerted, that is, at right angles to the road; whilst in drawing or lifting up a weight, the work done, or mechanical effect produced, is determined by the product of the weight into the distance through which it has been raised. It is true, that walking or carrying fatigues as much as lifting does, *i. e.* the "day's work" is consumed by the one as by the other kind of labor; and, therefore, a certain day's work is attributable to the one as there is to the other, although they are essentially different in their nature. According to experience, a man can walk, unburdened, for ten hours a day at $4\frac{3}{4}$ feet per second (something under $3\frac{1}{2}$ miles per hour). If we assume his weight at 140 lbs. we get as the day's labor $140 \,.\, 4{,}75 \,.\, 10 \,.\, 60 \,.\, 60 = 23{,}940000$ ft. lbs.

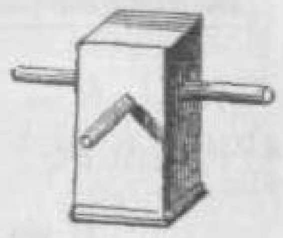
If a man carry a load of $85\frac{1}{2}$ lbs. on his shoulders, he can walk for 7 hours daily with a speed of 2,4 feet per second, and, therefore, produces daily the quantity of work = 85,5 . 2,4 . 7 . 60 . 60 = 5′,171000 ft. lbs., neglecting his *own* weight.

A horse will carry 256 lbs. for 10 hours daily, walking $3\frac{1}{2}$ feet per second, so that its day's work amounts to 256 . 3,5 . 10 . 60 . 60 = 32′256000 ft. lbs., or more than 6 times as much as a man doing the same kind of work. If the horse carries only 171 lbs. on his back, he will trot at 7 feet per second for 7 hours daily, and the work done in this case is only 171 . 7 . 7 . 60 . 60 = 30′164400 ft. lbs. daily.

The amounts of work done in raising burdens is much smaller, for in this case mechanical effect, according to our definition, is produced, or *the space is described in the direction of the effort exerted.*

If a man, unburdened, ascend a flight of steps, then, for a day's work of 8 hours, the velocity measured in the vertical direction is 0,48 feet per second; therefore, the amount of work done daily = 140 . 0,48 . 8 . 60 . 60 = 1′935000 ft. lbs. It thus appears that a man can go over $12\frac{1}{2}$ times the space horizontally that he can vertically.

Fig. 112.



In constructing a reservoir dam, the author observed that 4 practised men, raised a dolly, Fig. 112, weighing 120 lbs., 4 feet high 34 times per minute, and after a *spell* of 260 seconds, rested 260 seconds; so that, on the whole, there were only 5 hours work in the day. From this it appears that the day's work of a man $= \frac{120}{4} . 4 . 34 . 5 . 60 = 1'224000$ feet lbs.

Remark 1. In the "Ingenieur," there is detailed information on the work done by animal power. In the sequel, the effect produced by animals by aid of machines is given for each machine respectively.

Remark 2. The effect produced by men and animals is far from being accurately ascertained. The effect produced by men working under disadvantageous circumstances, or by unpracticed laborers, is not one-half of that produced by well-trained hands. Coulomb, in his "Théorie des Machines simples," first entered on investigations of the effect of animal powers. Desaguiliers ("Cours de Physique expérimentale,") and Schulze ("Abhandlungen der Berliner Akademie,") had previously occupied themselves with the subject. Many experiments have been made and recorded in more recent times. See Hachette, "Traité élémentaire, &c.," Morin, "Aide Mémoire," Mr. Field in the "Transactions of the Institution of Civil Engineers, London," Sim's "Practical Tunneling," and Gerstner's "Mechanik," Band 1.

§ 55. *Formulas.*—Effort and velocity have a very close dependence in the application of animal power; but the law of their dependence is by no means known, and is still less deducible *à priori*. The following empirical formulas, given by Euler and Bouguer, are only to be considered as approximations. If K_1 be the maximum effort which an animal can exert without velocity, and c_1 the greatest velocity it can give itself when unimpeded by the necessity of extraneous effort, we have for any other velocity and effort:

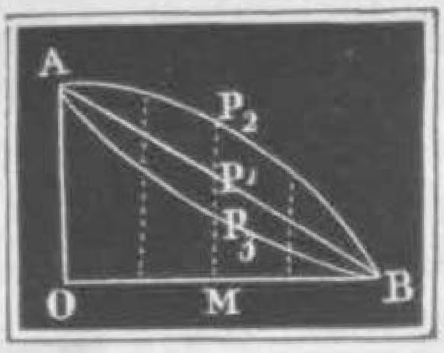
$$\text{according to Bouguer : } P = \left(1 - \frac{v}{c_1}\right) K_1,$$

according to Euler: $P = \left(1 - \frac{v^2}{c_1^2}\right) K_1$,

" Euler: $P = \left(1 - \frac{v}{c_1}\right)^2 K_1$.

The first of these is the most simple, and that which, according to Gerstner, corresponds best with observation. According to Schulze's observations, on the other hand, the last formula appears to be most consistent with experiment. If we draw v as abscissa, and P as ordinates to a curve, the first formula corresponds to a straight line AB, Fig. 113, the second with a concave parabolic curve AP_2B, and the third with a convex parabolic curve AP_3B, and the ordinates MP_1 of the straight line always lie between the ordinates MP_2 and MP_3 of the two parabolic curves. The abscissa OM, for example, $= v = \frac{1}{2} c_1$ corresponds to the ordinates $MP_1 = \frac{1}{2} K = \frac{1}{2} OA$, also $MP_2 = \frac{3}{4} K = \frac{3}{4} OA$, and $MP_3 = \frac{1}{4} K = \frac{1}{4} OA$. The formula of Bouguer, therefore, gives values of the *effort* which lie between the values given by the two formulas of Euler; and we may, therefore, make use of Bouguer's formula until some special reason for adopting Euler's formula be adduced. If we introduce into Bouguer's formula, instead of the maximum values K_1 and c_1, the halves of these, or their mean values $K = \frac{1}{2} K_1$, and $c = \frac{1}{2} c_1$, we get a formula first applied by Gerstner:

Fig. 113.



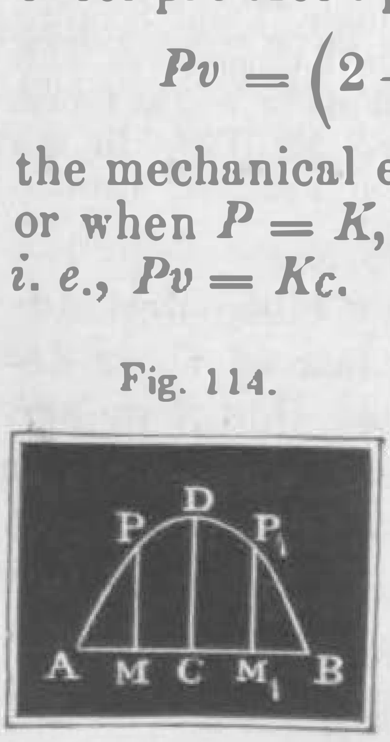
$$P = \left(1 - \frac{v}{2c}\right) 2\,K, \text{ or } P = \left(2 - \frac{v}{c}\right) K,$$

and inversely, $v = \left(2 - \frac{P}{K}\right) c$. Although this formula can be but little depended upon as accurate for extreme values of v and P, yet it may be presumed, that for values not very different from the mean, they are sufficiently near for practical uses. The mechanical effect produced per second would follow from this:

$$Pv = \left(2 - \frac{v}{c}\right) v\,K. \quad \text{As} \left(2 - \frac{v}{c}\right) v\,K = (2\,c - v)\, v \frac{K}{c},$$

the mechanical effect is a maximum, as in Vol. I. § 386, when $v = c$, or when $P = K$, or when the velocity and effort are mean values, *i. e.*, $Pv = Kc$. If we try to get a greater or less velocity, or a greater or less effort, we get an effect $L = Pv$ less than Kc. If we set off the velocities as abscisses, and the amounts of mechanical effect produced as ordinates, we get as the projected curve a parabola ADB, Fig. 114; and it is evident that not only for abscissa $AM < AC$, but also for $AM_1 > BC$, the ordinates MP, M_1P_1, are less than for the abscissa $AC = c$. For $v = \frac{c}{2}$, as also for $v = \frac{3}{2}\,c$, it follows from the above that $L = \frac{3}{4}\,Kc = \frac{3}{4}\,CD$.

Fig. 114.



According to Gerstner, the following table represents the *draught* of animals applied properly to draw by *traces*.

Animals.	Weight.	Mean effort K in lbs.	Mean speed c in feet per second.	Mean period of day's work. Hours.	Effect produced p. sec. in ft. lbs.	Daily effect feet lbs.
Man .	150	30	2,5	8	75	2'160000
Horse .	600	120	4	8	480	13'824000
Ox . .	600	120	2,5	8	300	8'640000
Ass . .	360	72	2,5	8	180	5'184000
Mule .	500	100	3,5	8	350	10'080000

Example 1. According to the above table, a man working with an effort of 30 lbs., and mean velocity of $2\frac{1}{2}$ feet per second, produces in a day an amount of mechanical effect represented by 2'160000 feet lbs. If he be urged to work at 3 feet per second, the effort will be reduced to $P = \left(2 - \frac{3}{2,5}\right) 30 = 24$ lbs., and his daily effect would only be 24 . 3 . 8 . 60 . 60 = 2'073600 feet lbs.

Example 2. If a horse be obliged to draw with an effort of 150 lbs., it can only be done with a velocity $v = \left(2 - \frac{150}{120}\right) 4 = 3$ feet per second, and thus his effective work is reduced to only 3 . 150 = 450 feet lbs. per second.

Remark. Fourier, in the *Annales des Ponts et Chaussées*, 1836, gives a complicated formula for the effect produced by horses. See also Crelle's *Journal der Baukunst*, Band xii. 1838.

§ 56. *Work done by aid of Machines.*—If we follow Gerstner's notion, that the period or time of each *shift*, or day's work, has the same influence on the amount of work done as the velocity, we must then put for the effort:

$$P = \left(2 - \frac{v}{c}\right)\left(2 - \frac{z}{t}\right)K,$$

and from this we get the daily effect produced:

$$L = \left(2 - \frac{v}{c}\right)\left(2 - \frac{z}{t}\right)K v z.$$

There can be no doubt that the effect produced is a maximum, that is $= K c t$, when the animal is made to work, not only with a mean velocity and effort, but also when the time of work is kept within the mean for this. It is to be kept in mind, however, that this formula only applies when the values of v, z, and P do not differ widely from c, t, and K.

M. Maschek, of Prague, recommends the expression:

$$P = \left(3 - \frac{v}{c} - \frac{z}{t}\right)K,$$

which is certainly more convenient for calculation.*

Eight to ten hours per day is a good average day's work, and, therefore, the factor $\left(2 - \frac{z}{t}\right)$ may generally be neglected, or the day's effect may be written $L = \left(2 - \frac{v}{c}\right) K v z$. If, however, an

* Neue Theorie der menschlichen und thierischen Kräfte, &c., von F. J. Maschek, Prag.

animal be applied to a machine, its effort P would be divided into an effort P_1 for doing the work, and an effort P_2 for overcoming prejudicial resistances, or $P = P_1 + P_2$, both resistances being reduced to the point of application of the effort. It is also usual, as we shall learn in the sequel, to find the prejudicial resistances P_2 composed of a constant part R, independent of the strain on the machine, and a part $\delta \,.\, P_1$, proportional, or nearly so, to the useful effect produced or work done, where δ is co-efficient derived from experiment, thus $P_2 = R + \delta \,.\, P_1$; and, therefore,

$$P = (1 + \delta)\, P_1 + R;\ \text{and again}\ \left(2 - \frac{v}{c}\right) K = (1 + \delta)\, P_1 + R.$$

The total effect produced per second is, therefore,

$$Pv = \left(2 - \frac{v}{c}\right) Kv = (1 + \delta)\, P_1 v + Rv.$$

and, therefore, the useful effect produced:

$$P_1 v = \frac{(2K - R)\, v - \frac{Kv^2}{c}}{1 + \delta} = \left[\left(2 - \frac{R}{K}\right) c - v\right] v \,.\, \frac{K}{(1 + \delta)\, c}.$$

That this effect may be the greatest possible (see previous paragraph), we must have $v = \frac{1}{2}\left(2 - \frac{R}{K}\right) c = \left(1 - \frac{R}{2K}\right) c$, or the velocity less than the mean velocity; and so much the less, the greater the constant part of the prejudicial resistance is. The effort corresponding would be, according to this:

$$P = \left(1 + \frac{R}{2K}\right) K = K + \frac{R}{2},$$

or greater than the mean effort. The useful resistance, on the other hand, is $P_1 = \dfrac{K - \frac{R}{2}}{1 + \delta}$. The total effect produced is:

$Pv = \left[1 - \left(\dfrac{R}{2K}\right)^2\right] K\, c$, and the useful effect produced is:

$P_1 v = \left(1 - \dfrac{R}{2K}\right)^2 \dfrac{Kc}{1 + \delta}$, and the efficiency of the machine:

$$\eta = \frac{\left(1 - \frac{R}{2K}\right)^2}{1 + \delta}.$$

Example. If in a machine turned by two horses, the *constant* prejudical resistance reduced to the point of application of the horses' effort $= 60$ lbs., the velocity at which the horses should work when $K = 2 \,.\, 120 = 240$ lbs., and $c = 4$ feet, is reduced to $v = \left(1 - \frac{60}{480}\right) c = \frac{7}{8} \,.\, 4 = 3{,}5$ feet. Further, the effort of the horses $= 240 + \frac{60}{2} = 270$ lbs., and, therefore, that of one horse $= 135$ lbs. If, now, the constant part of prejudicial resistance be 15 per cent. of the useful resistance, then $\delta = 0{,}15$, and, therefore, the resistance to be put on the machine $P_1 = \frac{240 - 30}{1{,}15} = 182{,}5$ lbs., and the efficiency of the machine would be $\eta = (\frac{7}{8})^2 \div 1{,}15 = 0{,}67$.

Remark. Gerstner reduces the calculation of the effect of animal power to motion on an inclined plane. If G be the weight of the animal, P the effort exerted, and a the angle of inclination of the inclined plane, upon which the moving power ascends with its load, then the effort is $P + G \sin. a$ (see *Theory of Inclined Plane*, Vol. I. § 134), and hence $\left(2 - \frac{v}{c}\right) K = P + G \sin. a$. Hence, we have the load with which an animal can ascend an inclined plane, and, conversely, the inclination corresponding to a given load, viz.: $\sin. a = \frac{\left(2 - \frac{v}{c}\right) K - P}{G}$, thus when $P = 0$, and $v = c$, or when the animal has no resistance to overcome, and goes with the mean velocity $\sin. a = \frac{K}{G}$. But, the weight of an animal is almost always five times as great as the mean effort it can exert, therefore $\sin. a = \frac{1}{5}$ and $a = 11\frac{1}{2}°$ is the angle of inclination of a plane which an animal can ascend with the mean amount of exertion and fatigue. This corresponds to a rise of one foot in five feet, or nearly so.

§ 57. *The Lever.*—Animal powers are applied to work by means of the lever or the wheel and axle. The latter are either horizontal or vertical. We shall first speak of the lever as a machine for receiving (and transmitting) animal power. The general theory of this machine is known from Vol. I. §§ 126, 127, and 170. The lever is either single as ACB, Fig. 115, or double, as $ACBA_1$, Fig. 116; the one has only one arm for the application of the power CA, whilst the other has two arms CA and CA_1. The lever produces an oscillating circular motion, and is, therefore, chiefly applied, when a reciprocating *up* and *down* motion is desired, as in

Fig. 115.

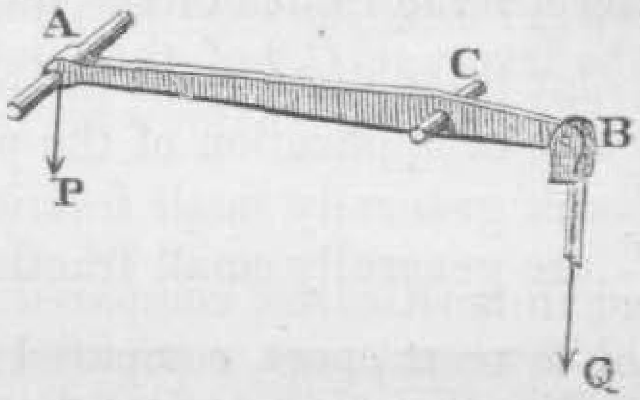
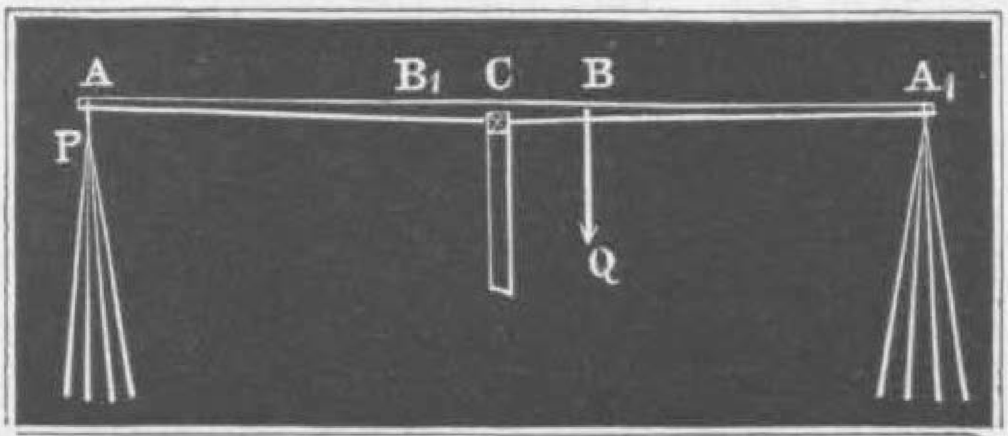


Fig. 116.



pumping. Handles, suited to the number of hands to be applied, are affixed to the lever. As the strength can be better exerted in pulling downwards than in lifting upwards, it is usual to make the *down-stroke* the *working-stroke*, and counter-balances are attached so as to aid the workmen in the *up-stroke*, or the double lever is used, on which the workers alternately pull downwards. When the down-stroke is the effective stroke, ropes, hanging from the end of the lever, are frequently substituted for handles. Levers are sometimes moved by the tread of the feet.

That there may not be too great a change of direction during a stroke, the lever's motion is confined to an arc of not more than

$60°$, and, in order to facilitate the exertions of the power, the space passed through at each stroke is kept proportional to the length of arm of the workers, or at from $2\frac{1}{2}$ to $3\frac{1}{2}$ feet. Again, the handles should not come within from 3 to $3\frac{1}{2}$ feet from the floor. According to experience, men work 8 hours per day, exerting an effort of $k = 10{,}7$ lbs. on the end of a lever, with a velocity $c = 3{,}5$ feet. Therefore, the mechanical effect produced by a man applied to a lever, as in pumping, is per second: $L = 10{,}7 \,.\, 3{,}5 = 37{,}45$ ft. lbs., and, therefore, the *day's work*

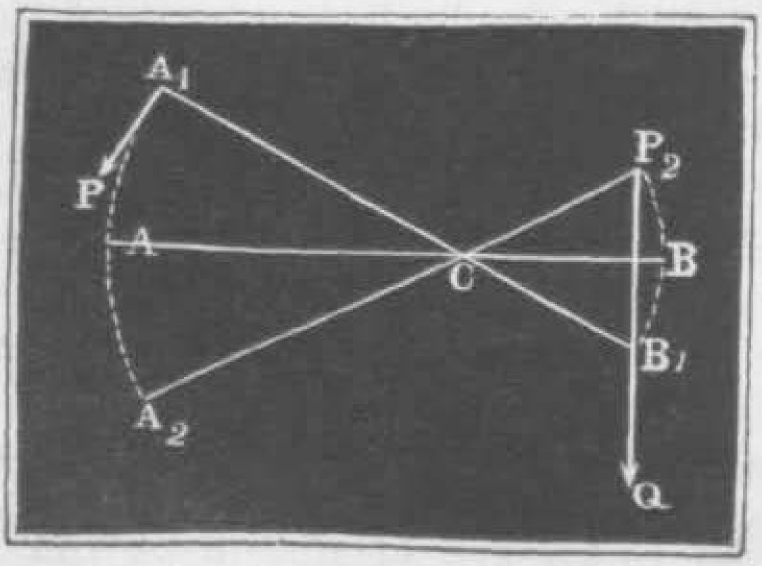
$$= Kct = 37{,}45 \,.\, 8 \,.\, 3600 = 1'078560 \text{ ft. lbs.}$$

In putting up a lever, it is necessary to take care that the workmen shall be applied so as to exert the ascertained mean effort with the mean velocity; or rather, that the effective effort shall exceed the mean effort by only one-half of the constant prejudicial resistance.

The lever itself is subject to only one prejudicial resistance, viz.: the friction of the fulcrum. If R be the pressure on the fulcrum arising from the weight of the lever and from the effort and resistance, r the radius of the fulcrum, f the co-efficient of friction, and a the leverage CA of the power, then the axle friction reduced to the point of application of the power $F = \frac{fr}{a} R$; as, however, f, and also $\frac{r}{a}$ are generally small fractions, F is so small that it may be neglected in most cases, compared with the other resistances.

If we suppose a useful resistance Q and a prejudicial resistance $\delta\, Q + W$ acting at the point B, and if we put the leverage CB of these resistances $= b$, the moment of the effort becomes:
$Pa = [(1 + \delta)\, Q + W]\, b$, and, therefore, the effort itself:
$P = \frac{b}{a} [(1 + \delta)\, Q + W]$. But, that the power of men may be most advantageously applied: $P = K + \frac{b}{a} \,.\, \frac{W}{2}$ and, therefore, $\frac{a}{b} K = (1 + \delta)\, Q + \frac{W}{2}$, and, therefore, the ratio of the lever arms $\frac{a}{b} = \frac{(1 + \delta)\, Q + \frac{1}{2} W}{K}$ is to be employed.

Fig. 117.



Remark. The arms of the lever are variable to a certain extent during the stroke, and, therefore, it may be well to determine the amount of this variation.

If the arm CB, Fig. 117, be horizontal at the *half-stroke*, and if the angle B_1CB_2 passed through in a stroke $= \beta°$, the height through which the resistance is overcome $s = \overline{B_1B_2} = 2\, b \sin. \frac{\beta}{2}$, and, therefore, the mechanical effect produced or expended in one stroke $= 2\, b \sin. \frac{\beta}{2} \,.\, Q$. If, however, the resistance were constant during the stroke at a leverage $CB = b$, the space passed over at each stroke would be $=$ arc $B_1BB_2 = \beta b$, and, therefore, the resistance would be $= \frac{2\,b\,sin.\frac{\beta}{2}}{\beta b}\,Q = \frac{2\,sin.\,\frac{1}{2}\,\beta}{\beta}\,.\,Q$, and the statical moment $= \frac{2\,sin.\,\frac{1}{2}\,\beta}{\beta}\,Q\,b$. Conversely, we may assume that the resistance Q acts during a stroke on the mean length of arm $\frac{2\,b\,sin.\,\frac{1}{2}\,\beta}{\beta}$. For $\beta^\circ = 60^\circ$ this lever $= \frac{b}{arc.\,60^\circ}$ $= \frac{b}{1{,}0472} = 0{,}955\,b$, or not quite 5 per cent less than b, and for smaller arcs of oscillation, the difference is still much less.

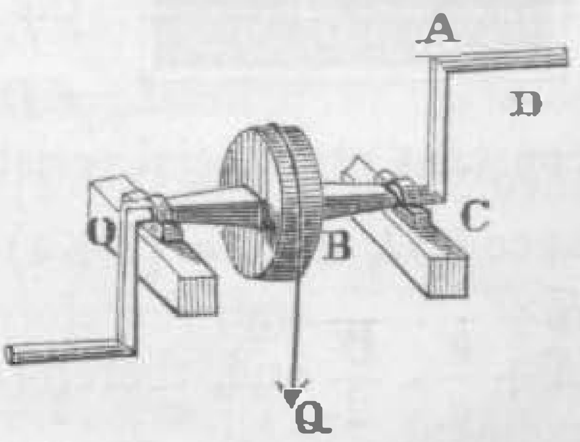
Example. What proportion of arms should be chosen for a lever, that for a useful resistance of 160 lbs. and a prejudicial resistance $Q_1 = 0.15\,Q + 55 = 0{,}15 e\,160 + 55 = 79$ lbs., four men may work to the best advantage? $K = 4\,.\,10{,}7 = 42{,}8$ lbs. therefore $\frac{a}{b} = \frac{1{,}15\,.\,160 + \frac{1}{2}\,55}{42{,}8} = \frac{2115}{428} = 4{,}9$. If the resistance passes through 1 foot for each stroke, the power must at the same time pass through 4,9 feet, and if we take the angle of oscillation $\beta = 50^\circ$, we get for the suitable length of lever $b = \frac{s}{2\,sin.\,\frac{\beta}{2}}$ $= \frac{0{,}5}{sin.\,25^\circ} = 1{,}183$ feet, and the length of arm $a = 4{,}9\,.\,b = 4{,}9\,.\,1{,}183 = 5{,}80$ ft. The *effort* necessary is $P = \frac{160 + 79}{4{,}9} = 48{,}78$ lbs., therefore, the effort of each man $=$ 12,195 lbs., and the efficiency

$$\eta = \frac{\left(1 - \frac{55}{2\,.\,4{,}9\,.\,42{,}8}\right)^2}{1{,}15} = \frac{(1 - 0{,}1311)^2}{1{,}15} = 0{,}657.$$

We see, therefore, that four men capable of a day's work each $=$ 1'075860 ft. lbs., or 4'303440 ft. lbs. in all, would only produce 0,657 . 4'303440 $=$ 2'800000 feet lbs. *useful effect* with this machine.

§ 58. *Windlass.*—The best means of applying the power of men, is the windlass (Fr. *treuil*, *tour*; Ger. *Haspel*). This machine consists of a horizontal axle, at the circumference of which the resistance acts, and of a crank, handle, or *winch*, Fig. 118, or series of handles on a wheel, Fig. 120, or of fixed or movable levers (hand-spikes), Fig. 119. With the winch, the laborers have a continuous hold throughout the revolution, whilst with the wheel or hand-spike, the action is hand over hand, or otherwise at short intervals. The winch is the form used for general purposes. The wheel is applied principally in working the tiller on board-ship, and the movable levers are chiefly used for weighing anchor by means of the capstan.

Fig. 118.



That a laborer may produce the best effect by means of the crank-handled windlass, the length of the lever must not be more than from 16 to 18 inches, corresponding to the length of arm of the laborer, and the axis of the barrel must not be more than 36 to 39 inches above the floor on which the laborer stands, for men of average height. The handle of the windlass is adapted for one, two, or more men, according to circumstances. As a man can work with less fatigue while pushing and pressing, than while lifting and pulling, the effort required at each point of a revolution of the handle is not

Fig. 119.

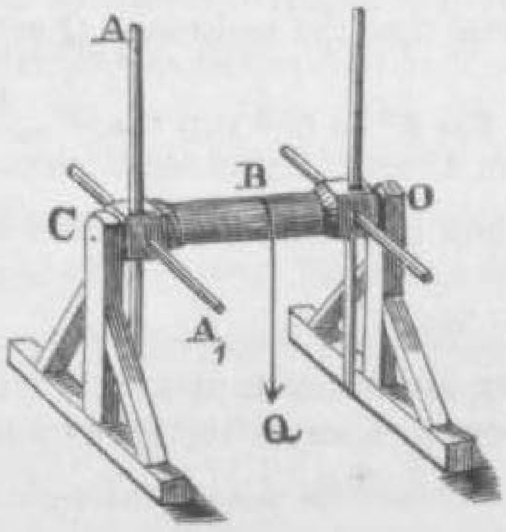
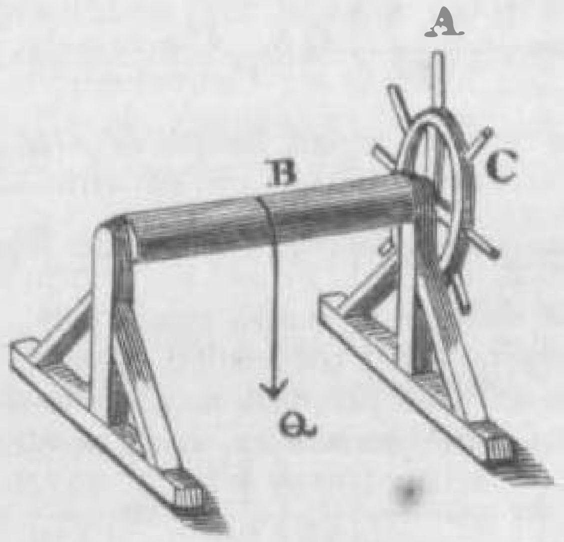


Fig. 120.



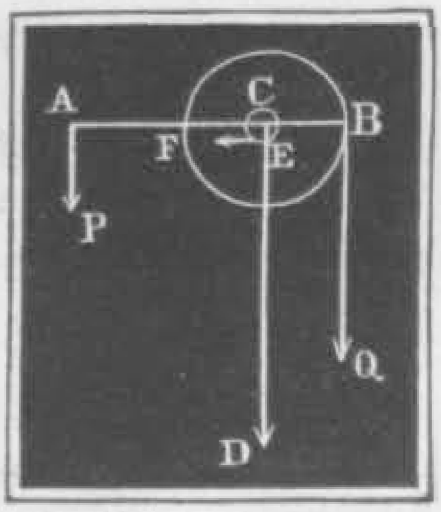
equal, and, therefore, it is well in double-handled windlasses to set the handles 180° apart, and, when more handles are applied, to distribute them equally.

The day's work of a man working a windlass has been found to be 1'175040 feet lbs. with a mean effort $K = 17$ lbs., and mean velocity $c = 2{,}4$ feet, and length of day 8 hours. The calculations for the windlass are the same as for the wheel and axle.

If the resistance Q, Fig. 121, act with the lever $CB = b$, and the power P on the lever $CA = a$, then $Pa = Qb$; and, therefore, the power corresponding to a given resistance is $P = \frac{b}{a} Q$. If, again, D be the pressure on the *journals* or *gudgeons* and r the radius of the gudgeons CE, then $Pa = Qb + f\,Dr$, and hence $P = \frac{b}{a} Q + \frac{r}{a} \cdot f D$. If the resistance Q, together with the friction $\frac{r}{b}$ $f D$, consist of the useful resistance Q_1, the constant prejudicial resistance W, and the variable prejudicial resistance $\delta\, Q$, or, $Q = (1 + \delta)\, Q_1 + W$, then $P = \frac{b}{a} \left[(1 + \delta)\, Q_1 + W\right] = K + \frac{b}{a} \cdot \frac{W}{2}$ and, therefore, the proportion of the winch and barrel radius should be: $\frac{a}{b} = \frac{(1 + \delta)\, Q_1 + \frac{1}{2} W}{K}$. But as the winch has a prescribed height of 16 to 18 inches, the leverage of the resistance, or radius of the barrel is to be determined by this, viz:

$b = \frac{K\,a}{(1 + \delta)\, Q_1 + \frac{1}{2} W}$, in order that the laborer may work to the greatest advantage.

Fig. 121.



Example. On a two-handled windlass the resistance is 200 lbs., viz.: 150 lbs. of useful resistance, and 30 lbs. constant, and 20 lbs. variable prejudicial resistance. The leverage of the resistance is 4 inches, that of the power 18 inches, the radius of the journal $\frac{1}{2}$ inch, the co-efficient of friction $f = 0,1$, and the weight of the barrel, &c., 80 lbs.; required the useful effect of such a machine. The whole power required, if the pressure on the journals be taken $200 + 80 = 280$ lbs., is $P = \frac{4}{18} \cdot 200 + 0,1 \frac{1}{2 \cdot 18} \cdot 280 = 44,44 + 0,5 = 44,94$ lbs., and, therefore, the effort of each laborer must be 22,47 lbs., and, according to Gerstner's formula, the velocity of the power, or of the handle of the windlass:

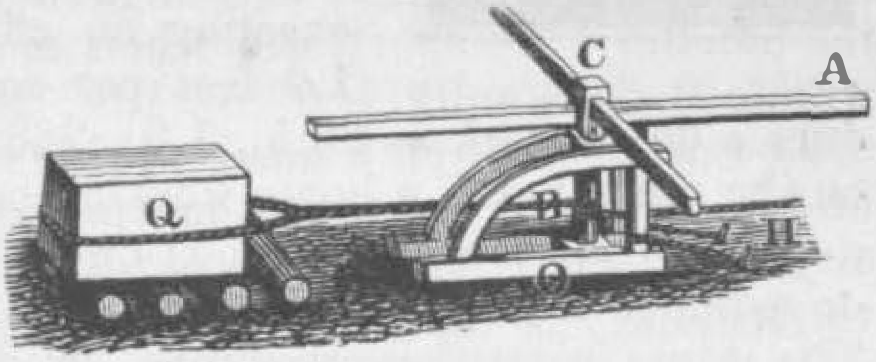
$v = \left(2 - \frac{P}{K}\right) c = \left(2 - \frac{22,47}{17}\right) \cdot 2,4 = 1,628$ feet, and that of the resistance:

$w = \frac{a}{b} v = \frac{2}{9} \cdot 1,628 = 0,362$ feet, and the useful effect per second:

$Q_1 w = 0,362 \cdot 150 = 54,3$ feet lbs., and daily $= 1,563840$ lbs., and the efficiency of such an application of the power of two laborers, the day's work of each of whom is assumed to be $1,175040$ e. $\eta = \frac{1,563840}{2,350080} = 0,665$.

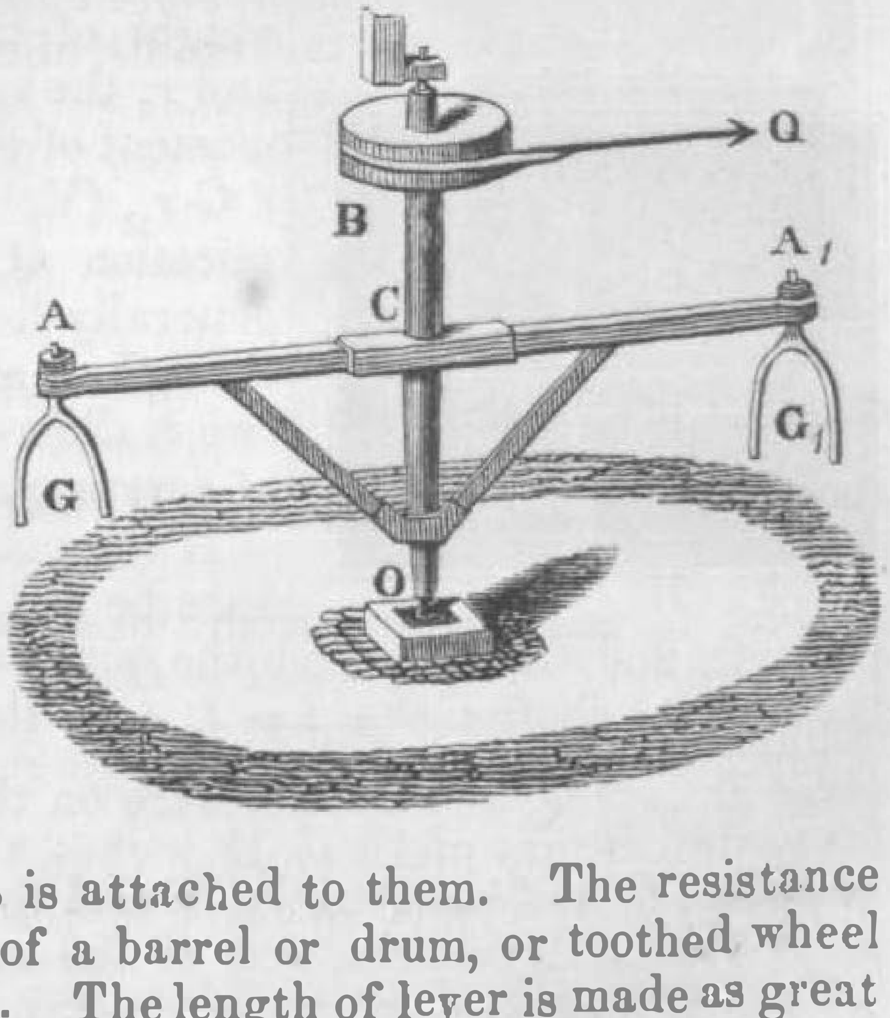
§ 59. *Vertical Capstan.*—When the axis or barrel of the windlass is vertical, it is termed a capstan, Fig. 122. It is chiefly used on land for moving great weights a short distance, or for removing great weights, as blocks of stone from a quarry, or for the erection of obelisks, &c. Its use on board-ship is well known.

Fig. 122.



The *horse-capstan*, in its different applications as the prime mover of mill work, or as a *whim-gin*, as it is termed, by miners, is a modification of the windlass easily comprehended. The cattle employed in working the vertical windlass or *gin*, go round in a given path, pushing or pulling at the arm of the machine. Fig. 123 shows the usual construction of the whim-gin (Fr. *baritel à chevaux manège*; Ger. *Pferdegöpel*, *Handgöpel*). BO is the axis, having a *pivot* at O resting in a *footstep*, ACA_1 is the double arm or lever, with fork-shaped *shafts* G, G_1. These cross the backs of the horses, and the harness is attached to them. The resistance Q acts at the circumference of a barrel or drum, or toothed wheel B, either directly or indirectly. The length of lever is made as great as conveniently can be done, that the animals may have the largest possible circle to move in. The radius should not be less than from 20 to 30 feet. The *line of traction* must be as nearly horizontal as possible, and, therefore, the height of the lever should be fixed, according to the height of the animals working on it. By the ar-

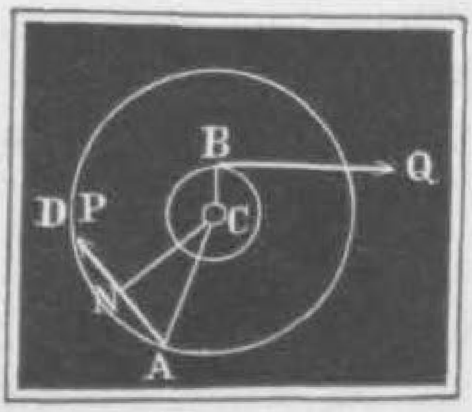
Fig. 123.



rangement shown in Fig. 123, the horses or other cattle work very nearly at right angles to the beam or lever; but if the horses be attached by traces to a cross bar and hook, the direction of traction makes a certain angle with the beam, becoming, in fact, a *chord* of the circular path.

From the length of beam $CA = a$, Fig. 124, and the length of traces $AD = d$, the length of levers of the horses is:

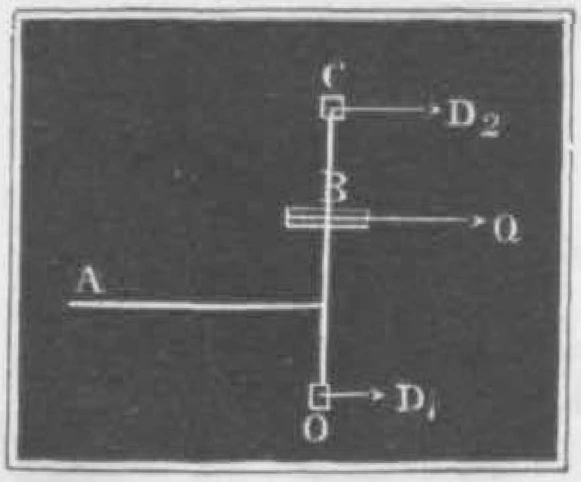
Fig. 124.



$$CN = a_1 = \sqrt{a^2 - \frac{d^2}{4}},$$

or, approximately, $= a - \frac{d^2}{8a}$. It is a result of experiment, that a man can work eight hours daily on the beam of capstan or gin, exerting an effort of $25\frac{1}{2}$ lbs. at the rate of 1,9 feet per second, and can, therefore, produce a day's work = 25.5 . 1,9 . 28800 = 1'395360 feet lbs.; that, on the other hand, a horse working on a gin for 8 hours daily, with a speed of 2,9 feet per second (a walk) can exert an effort of 95 lbs., or produce a day's work = 95 . 2,9 . 28800 = 7'934400 feet lbs. The *power* is to the resistance, on the capstan or gin, as for any wheel and axle, or $P = Q\frac{b}{a}$, when b and a are the arms or leverages of the resistance Q, and power P respectively. The frictions at the footstep and at the periphery of the pivots at top and bottom have to be considered; for these require an increase of the power. If G be the weight of the gin or capstan complete, and r_1 the radius of the pivot, the statical moment of the friction on the footstep $= \frac{2}{3} f\, G\, r_1$, (Vol. I.1§ 171.) The point of application of the resistance B, Fig. 125, generally lies nearer the one pivot, than the other, and thus the pressure on the two is different, and their dimensions are, of course, proportional to the strain.

Fig. 125.



If the point of application of the resistance be at the distance $BO = l_1$, from the pivot O, and $CB = l_2$ from the pivot C, and if the whole length of the upright shaft $CO = l = l_1 + l_2$, then the pressure on the lower pivot $D_1 = \frac{l_2}{l} Q$, and the pressure on the upper pivot $D_2 = \frac{l_1}{l} Q$, as is manifest if we first consider C and then O, as the fulcrums of the lever CBO. Thus, the sum of the statical moments of the lateral friction on the pivots $= f\, D_1 r_1 + f\, D_2 r_2 = \frac{r_1 l_2 + r_2 l_1}{l} \cdot f\, Q$, and the equation of equilibrium for the gin, is

$$Pa = Qb + \tfrac{2}{3} f\, Gr_1 + f\, Q \cdot \frac{r_1 l_2 + r_2 l_1}{l}.$$

Remark 1. The application of the *whim gin*, for drawing from mines, is treated of in the third section.

Remark 2. French authors assert that a horse, going at a trot, can work daily $4\frac{1}{2}$ hours, exerting an effort of 30 kilog. = 66 lbs. at a speed of 2 metres = 6,6 feet, and, therefore, can produce a day's work of 7'055000 feet lbs. If we apply Gerstner's formula, and put $K = 120$ lbs., $c = 4$ feet, $v = 6{,}6$ feet, $t = 8$ hours, and $z = 4\frac{1}{2}$ hours, we get the power $P = \left(2 - \frac{6{,}6}{4}\right)\left(2 - \frac{4{,}5}{8}\right) . 120 = 60$ lbs., and, therefore, the day's work = 60 . 6,6e 4,5e 3600 = 6'415200 feet lbs., or pretty nearly the result alluded to. If, however, we take the velocity 2,9 feet of a walk as the basis, we get by Gerstner's formula a much greater effort, viz: $\left(2 - \frac{2{,}9}{4}\right) . 120 = 153$ lbs., and, therefore, the day's work (8 hours)e = 12'778560 lbs.

§ 60. *Tread-wheels*, or *Tread-mills.*—The weight of men and cattle is sometimes used as the moving power of machines, the effort being exerted by *climbing* on the periphery of the wheel. The wheels consist of two *crowns*, connected with an axle by arms, and with each other by a *flooring*. The laborer treads either at the internal or on the external circumference, cross pieces, as steps, being provided for his steadier support at intervals of $1\frac{1}{2}$ feet. Figs.

Fig. 126.

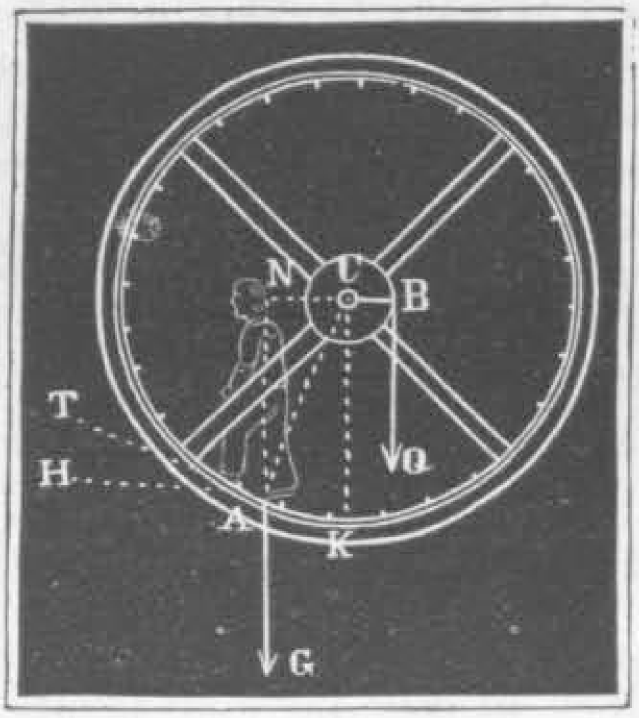
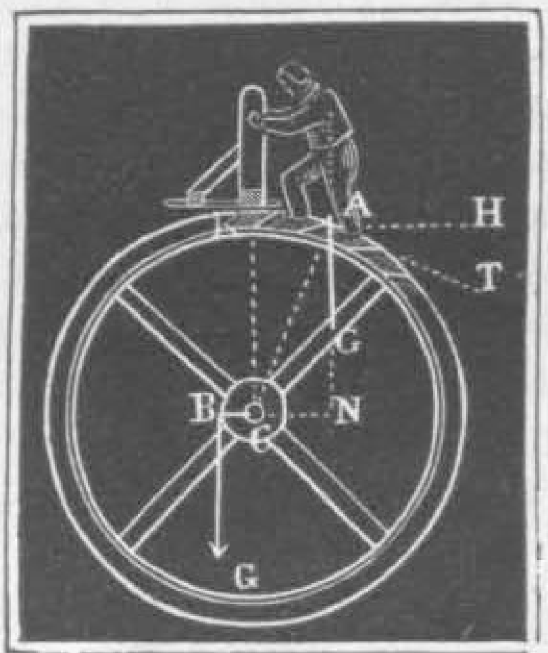
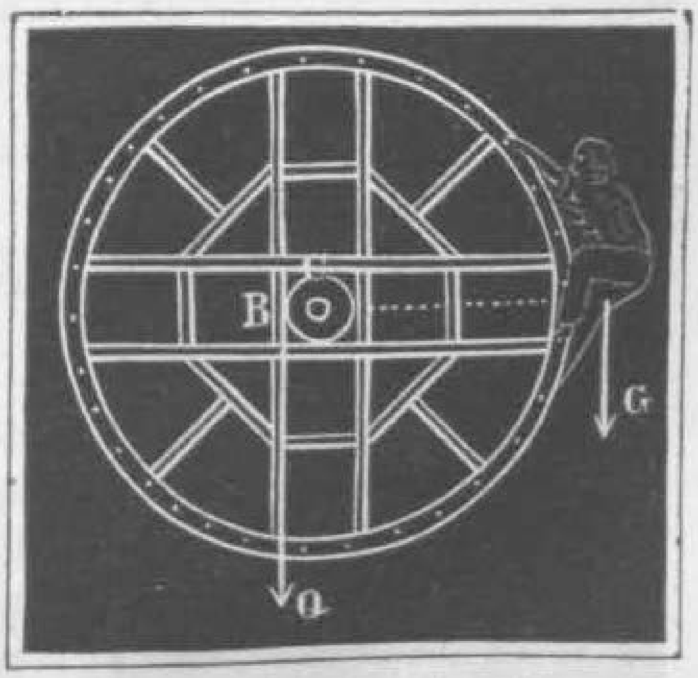


Fig. 127.



126 and 127 represent the more usual construction of tread-mills. Fig. 128 is a construction of wheel analogous to an endless ladder, but is not much used. On it, the laborer is placed at the level of the axis, so that his weight acts entirely, and with the radius $CA = a$ surpassing that of the wheel itself. In tread-mills the laborer is placed at an acute angle $ACK = \alpha$ from the summit, or the bottom of the wheel, and, therefore, the leverage of his weight G, is less than the radius of the wheel $CA = a$, viz: $CN = a_1 = CA \; sin. \; CAN = a \; sin. \; \alpha$. But then the fatigue of the laborer on the endless ladder is greater than on the tread-mill. In

Fig. 128.



the former case, it is the effort necessary to mount a vertical ladder; in the other, it is that for going up an inclination given by the tangent AT, making the angle $TAH = CAN = \alpha$. The effort P in the case of the ladder is, therefore, G, while in the tread-mill it is G $sin.\ \alpha$. If the resistance Q act with the leverage $CB = b$, then for the ladder-wheel $Ga = Qb$, while for the tread-mill $Ga\ sin.\ \alpha = Qb$, by substituting the power or effort, as in the wheel and axle; $Pa = Qb$. Mathematically considered, therefore, the tread-mill gives no advantage over the windlass or capstan; but the laborer can produce a much greater day's work by the one than by the other, and, therefore, they are often advantageously employed. The application of four-footed animals on these wheels is inconvenient, and not advantageous in any point of view.

It has been deduced from experiment that a man can work near the centre of the wheel, *i. e.*, near the level of the axis for 8 hours daily, exerting an effort of 128 lbs., and going at 0,48 feet per second, while he can work for the same time, exerting an effort of $25\frac{3}{8}$ lbs., and going at $2\frac{1}{4}$ feet per second, when his position is 24° from the vertical. In the one case, the day's work amounts to 1'769000 feet lbs., and in the other 1'663000 feet lbs. Horses and other cattle produce less effect on such machines than by means of a gin. A part of the advantage arising from the use of tread-wheels is lost in the increased friction of their axles beyond that of windlasses or capstan. If $n\ G$ be the weight of the laborers, G_1 the weight of the machine, and if the resistance Q act vertically downwards, the pressure on the journals $D = n\ G + G_1 + Q$, and if r be the radius of the journal, the moment of friction $= f\ (n\ G + G_1 + Q)$ r, and the ratio of power to resistance is: $n\ G\ a\ sin.\ \alpha = Qb + f$ $(n\ G + G_1 + Q)\ r$.

If the resistance be given, the angle of ascent may be determined, viz.:

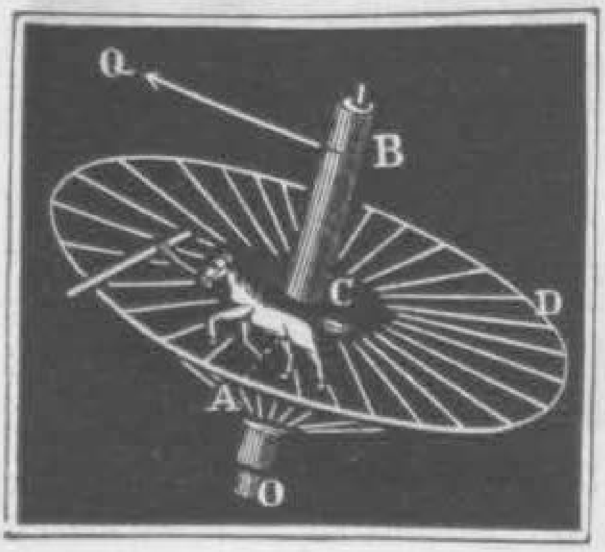
$$sin\,\alpha = \frac{Q\,b + f\,(n\ G + G_1 + Q)\,r}{n\,G\,a},$$

or the number of laborers

$$n = \frac{Q\,b + f\,(G_1 + Q)\,r}{G\,(a\ sin.\ \alpha - fr)}.$$

Men work to the greatest advantage when their effort: $n\,P = n\,G$ $sin.\ \alpha = n\ K + \frac{b}{a} \cdot \frac{W}{2}$, or when $sin.\ \alpha =$ $\left(K + \frac{b}{a} \cdot \frac{W}{2n}\right) \div G.$

Fig. 129.



§ 61. *Movable Inclined Planes.*—For farming purposes, in breweries, &c., the arrangement sketched in Fig. 129, is sometimes applied. The horse or ox works on such an inclined plane for short spells. The machine has this advantage, that the animal may be left without a driver. The action of the animals is in every respect the same as in tread-mills, when they work near the horizontal radius. The machine consists of a shaft BO, the axis of which is inclined 20° to 25° from the vertical, and of a plane, from 20 to 25 feetlin radius, set at right angles to the shaft, and, therefore, having an inclination of 20° to 25° to the horizon. If the animal moving the machine work at a distance $CA = a$ from the axis of the shaft, and if the angle of inclination of the plane, or the inclination upon which the animal may be supposed to be moving $= \alpha$, then the power $P = G \sin. \alpha$, and, therefore, the moment of rotation $= Pa = Ga \sin. \alpha$.

If the resistance be applied with a leverage b, its moment is $Q\,b$; and if G_1 be the weight of the machine when in work, and r be the radius of the pivot, the moment of friction on the footstep $= \frac{2}{3} f\,(G + G_1) \cos. \alpha \,.\, r$, and the moment of friction on the periphery of the pivots $= f\,(G + G_1 - Q) \sin. \alpha \,.\, r$; because the weight $G + G_1$ resolves itself into the components $(G + G_1) \cos. \alpha$, in the direction of the axis, and $(G + G_1) \sin. \alpha$, in the direction of the inclination of the plane, whilst Q acts in the opposite direction to this latter. Whence follows

$$G\,a \sin. \alpha = Q\,(b - f\,r \sin. \alpha) + f(G + G_1)\,(\tfrac{2}{3} \cos. \alpha + \sin. \alpha) \,.\, r.$$

Example. How many men are required to be put upon a tread-mill of 20 feet diameter, in order to raise a weight of 900 lbs., acting with a leverage of 0,8 feet? If we estimate the weight of the wheel, and its load at 5000 lbs., and taking the radius of the pivot at $2\frac{1}{2}$ inches, and the co-efficient of friction at 0,075, then the statical moment of the resistance $= 0{,}8 \,.\, 900 + 0{,}075 \,.\, \frac{5}{24} \,.\, 5000 = 720 + 78 = 798$ feet lbs., and, therefore, the power at the circumference of the wheele $= \frac{798}{10} = 79{,}8$ lbs. A laborer placed 24° back from the summit of the wheel, exerts an effort of $25\frac{3}{5}$ lbs., and, therefore, the number of men required is $\frac{79{,}8}{25\frac{3}{5}} = 3$. These men could produce $3 \,.\, 1663000 = 4989000$ feet lbs. per day of 8 hours, and, therefore, they could raise the weight Q daily through $\frac{4989000}{900} = 5890$ feet high; or, supposing the load had to be raised only 200 feet high, the three men could raise $\frac{5980}{200} = 30$ times 900 lbs. to the height of 200 feet.

CHAPTER III.

ON COLLECTING AND LEADING WATER THAT IS TO SERVE AS POWER.

§ 62. *Water-conduits.*—Water that is to serve as power (Fr. *l'eau motrice;* Ger. *Aufschlagewasser*), to be applied to machines, is collected from streams and rivers, or from springs. In most cases the machines have to be erected at some distance from the point at which the water can be collected, and must be led to the machine in what is termed the *lead* or *lete*, or *water-conduit*, or *water-course.*

The lead may either be an open *channel* or canal (Fr. *canales,* *rigoles*), or it may be a close pipe (Fr. *tuyaux de conduite;* Ger. *Röhrenleitungen*). Pipes are best adapted for smaller quantities of water. They have this great advantage, that they may be led in any way within the hydraulic range of variation of level, whilst canals as letes, must have a continuous fall. Valleys and hills may often be passed by pipes without trouble or expense, while the open channel requires the cutting of *drifts* or *tunnels*, and the erection of *aqueducts*.

§ 63. *Dams.*—The *vis viva* of running waters, of brooks and rivers having velocities of from 1 to 6 feet per second, is seldom sufficient to allow of their direct application as power to drive machines. To increase the *vis viva*, or to bring the weight of the water into action, it is necessary to *dam* it up to create a *head* or *fall* (Fr. *chute;* Ger. *Gefälle*). Water is dammed up by *weirs*, *dams*, or *bars* (Fr. *barrages ;* Ger. *Wehre*).

Weirs are either *overfall weirs*, or they are *sluice weirs*. Whilst in the former the water flows freely over the *saddle-beam cill*, or highest edge of the weir, in the latter movable sluice-boards dam the water above the summit of a weir, which may be either natural or artificial. The overfall weir is usually laid down with the view of constraining a portion at least of the water of a river or stream to enter a side canal above it, or a lete by which it is conducted to the machine by which the power of the water is to be applied; and the sluice weir, is used when the object is to get an increased *vis viva* to the water, which is then directly applied to a machine immediately below the weir.

In large rivers, dams are frequently built to occupy only a part of the width of the stream. These dams are termed *incomplete* wiers, in contradistinction to *complete* weirs, which are laid from side to side of the stream. The piers of bridges are examples of incomplete weirs (Fr. *barrage discontinus;* Ger. *Lichte Wehre*), contracting the passage for the stream to a certain extent.

Overfall weirs, too, may either be complete or imperfect. The summit of the complete overfall rises above the surface of the water in the part of the stream below it, whilst the top of the incomplete weir lies below that level, so that a part of the water flowing over undergoes a resistance from the water *below-weir*.

§ 64. *Swell*, or *Back-water.*—Any of the constructions we have above alluded to, *dam back* the water, produce a *swell* above the weir, an elevation of the water's surface, and, therefore, a decrease of velocity. The height and amplitude or extent backwards to which this rise of the water surface extends, is a matter important to be determined with reference to the dimensions of the weir.

A knowledge of this relation between the weir, and its effects on the river above it, is not only necessary because by damming up the water too high, we should involve the district above in *floods* to which they had not been previously subjected, but we may interfere with other establishments, robbing them of a part of their fall, by throwing *back-water* upon them. The level of the summit of weirs is often fixed by law or prescription according to a standard peg, or mark; any alteration of which is an offence liable to penalties (see, in reference to the English law on this subject, "Fonblanque on Equity"). The peg generally has a *scale* attached to it, by which the supply of water may be read off at a glance.

The water flowing over an overfall, or through an incomplete weir, acquires a waving eddying motion, the action of which is very severe on the bed of the river immediately below the weir, so that particular arrangements have to be made in the erection of weirs to withstand this action.

The quantity of water contained in or flowing through streams or rivers, is different at different times, so that we have the expressions *full*, *average*, and *dry*, applied to the *state* of rivers, corresponding in Britain, to winter, autumn, and summer, though not very definitely fixed as to the particular period of the seasons. It is evidently necessary to have accurate information as to the mean supply of water yielded by a brook or stream, proposed to be applied as water power. The state of the stream in autumn and spring may be taken as the *mean state*, but for any important undertaking of this nature a series of hydrometrical observations should be instituted, that the question of the supply of water may be accurately determined. Any one of the methods discussed in Vol. I. § 376, &c., may be adopted for this purpose.

§ 65. *Construction of Weirs.*—For obtaining water power, the *overfall* weir is the most important means. Weirs are built either square across the stream, or inclined to the axis of it. They are often built in two parts inclined to each other, the angle, which is laid up-stream, being rounded or not; they are formed as polygons also, and as segments of a circle, the convexity being always turned *to the* stream. Weirs are built of wood, or of stone, or of both combined. They have frequently to be founded on piles, from the difficulty of getting a sound foundation. The cross section of wooden, or other dams, is more or less of the form of a five-sided figure *ABCDE*, Fig 130, in which *AB* is termed the *breast*, *BC* the *front slope*, *CD the apron*, *DE* the *back*, and *EA* the sole, and *C* the saddle or cill. The cross section of stone weirs is generally composed of curved lines, as regards the apron and back, the object being to get the rush of water smoothly away from the foot of the apron, so as to prevent corrosion in time of floods.

Fig. 130.

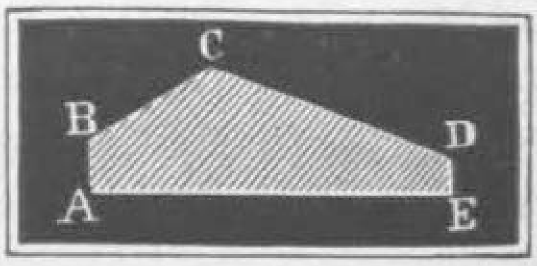
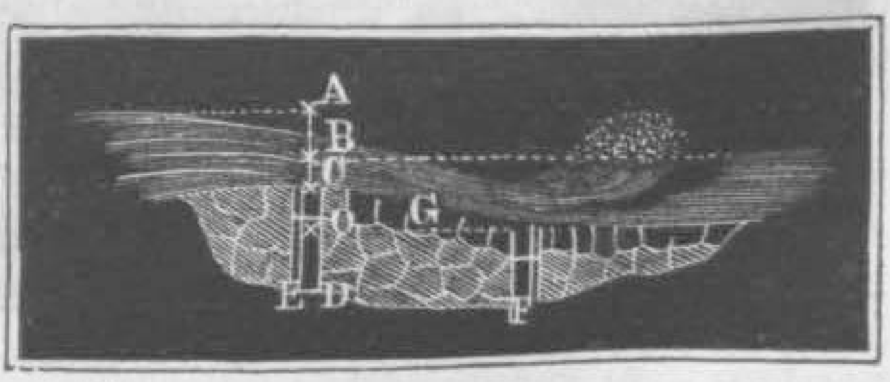
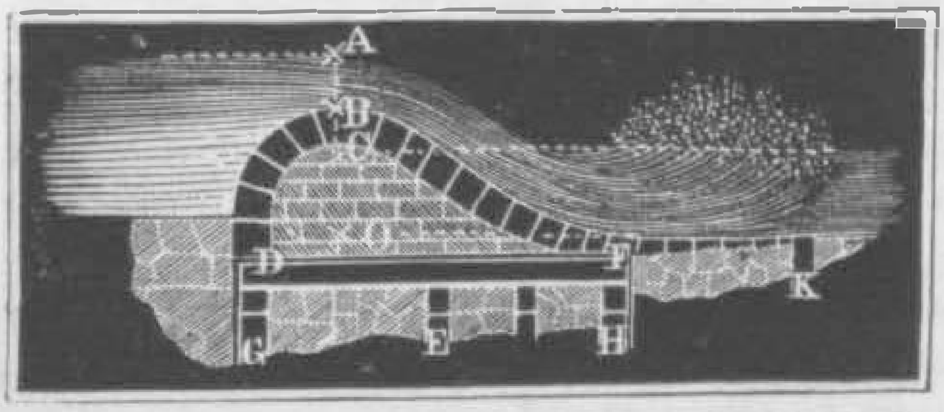


Fig. 131.



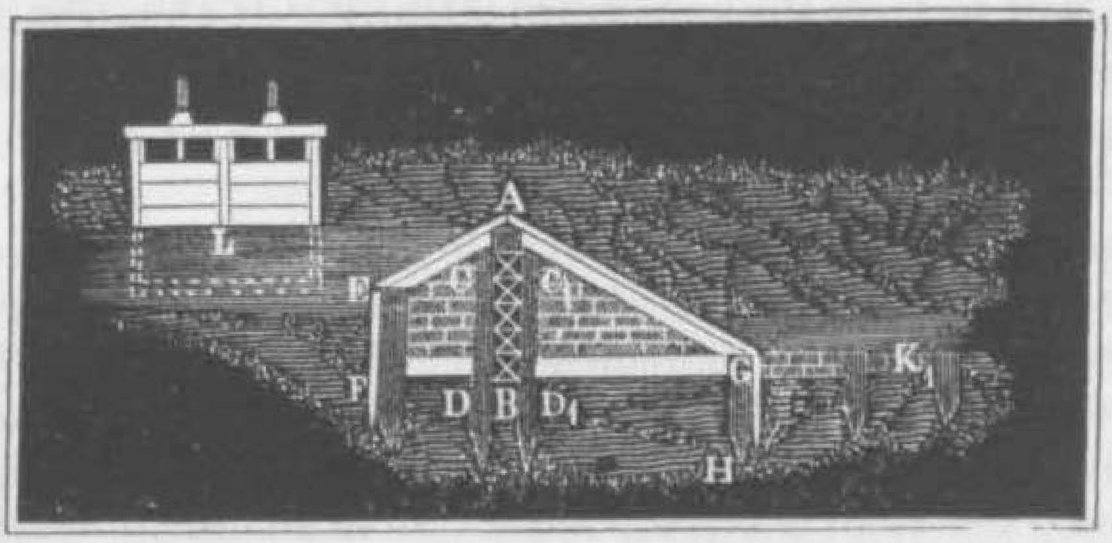
An overfall weir, such as is represented in Fig. 131, consists of a row of piles *D*, going across the stream, and a walling-piece, or *saddle-beam* C on the top—of walling *E* in front of the piles—a second row of piles *F* further down-stream and parallel to the first—of a casing of hard laid pavement *G*, between the two, and which is continued onwards with the same curvature, forming an apron (which should be continued so that it turns slightly *upwards*). The weir in Fig. 132, shows the manner of founding on piles, the intervals between the piles being cleared out as far as possible, and rammed with concrete, and upon this the superstructure is raised.

Fig. 132.



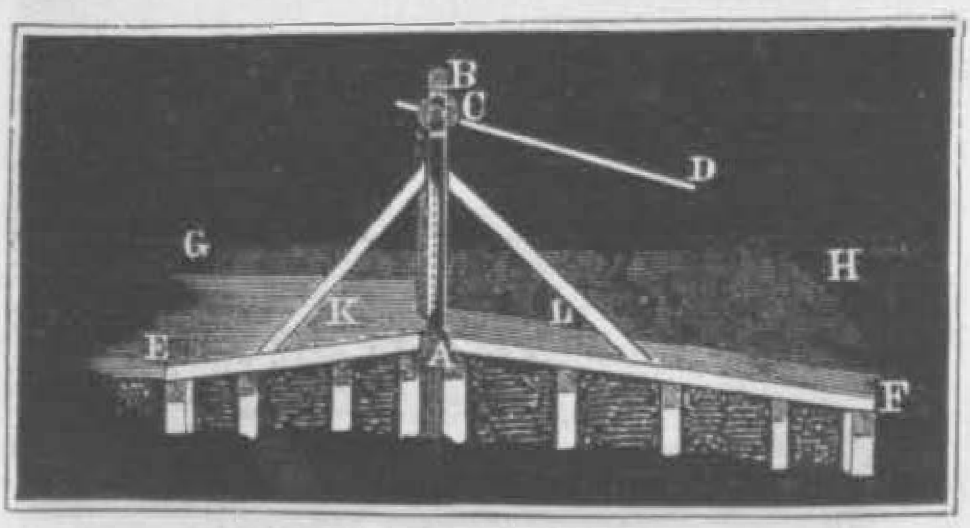
The construction of wooden wiers is sketched in Fig. 133. *AB*

Fig. 133.



is a wall of beams, lying tight, one on the other, on the top of which comes the saddle-beam *A*. These beams are confined by a double row of piles *CD* and C_1D_1, and the piles *EF* and *GH*, driven as breast and back of the dam, form resting points for the planking of the dam. The interior of the dam is filled with stone, clay, concrete, or such material. The apron *K* of the dam is continued onwards as substantially as possible, in the manner shown in the sketch. This latter is a point of great importance. At *L* the sluice of the lete is visible. A submerged weir is shown in Fig. 134. *A* is the saddle-beam, *AB* are the guide-columns, in grooves, in which the sluice works. The arrangements for raising or *opening* and lowering, or *shutting* the sluice are various. A capstan-like arrangement is shown in the figure, the

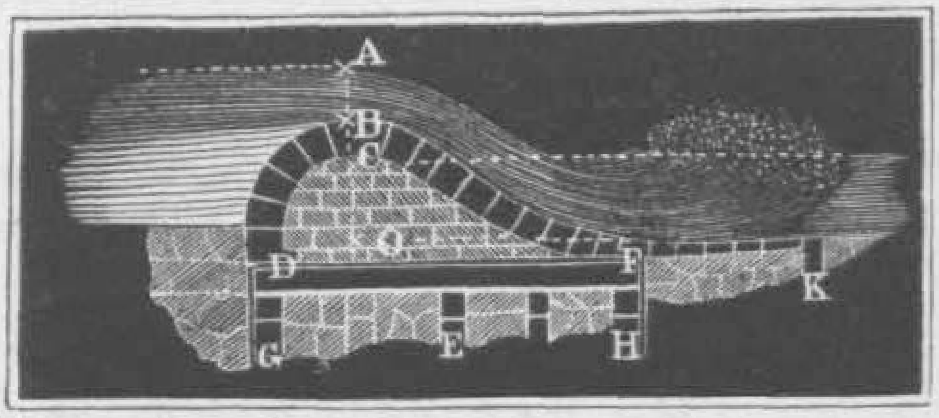
Fig. 134.



sluice-board hanging by chains. The piles in such a construction must be cleared for some depth, and the interstices well rammed with *puddle* or concrete, to prevent leakage.

§ 66. *Height of Swell.*—By aid of the hydraulic formulas we have investigated (Vol. I.), the height and amplitude of the *back-water* for any given dam may be easily determined. If, in the case of a dam represented in Fig. 135, h be the head AB, and if b, be the breadth, and k, the height due to the velocity c of the water as it flows up to the weir, or:

Fig. 135



$k = \frac{c^2}{2g}$, then the quantity of water discharged by the weir is (Vol. I. § 321) $Q = \frac{2}{3}\ \mu\ b\ \sqrt{2g}\,[h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}]$. If, on the other band, the quantity discharged be known, the *head* corresponding to it upon the saddle-beam: $h = \left(\frac{\frac{3}{2} Q}{\mu b \sqrt{2g}} + k^{\frac{3}{2}}\right)^{\frac{2}{3}} - k$. In order, therefore, to give the height $BO = x$ of a weir to produce a given *head*, or rise of the water surface at the weir $= h_1$, we put $AC + CO = AB + BO$, or, if the original depth of the water *down-stream* CO be put $= a$, then $h_1 + a = h + x$, and hence $x = a + h_1 - h$.

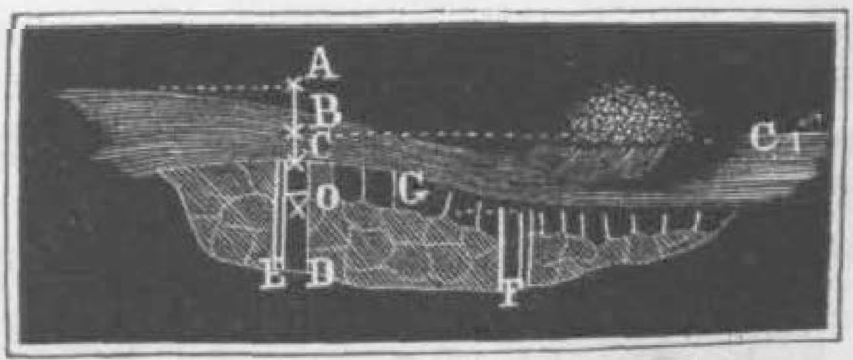
When the back-water or head raised is considerable, say $x =$ at least 2 feet; the velocity of the water, as it comes to the weir k, may be neglected, and, therefore, we may put:

$$x = a + h_1 - \left(\frac{\frac{3}{2} Q}{\mu\ b \sqrt{2g}}\right)^{\frac{2}{3}},$$

and according to experiments of the author, the co-efficient μ may be taken $= 0{,}80$ for this case.

In the case of the submerged weir, Fig. 136, the calculation is somewhat more complicated, because in this case two different discharges are combined. The height $AC = h$ of the water above the saddle-beam is greater in this case than the height $AB = h_1$, to which the water is raised by the dam, and, therefore, only the water above the level B flows away freely, whilst the water under B flows away under the head or pressure $AB = h_1$. The discharge through

Fig. 136.



$$AB = Q_1 = \tfrac{2}{3}\ \mu e b \sqrt{2g}\ [h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}],$$

and that through $BC = h - h_1$, ise

$$Q_2 = \mu b (h - h_1) \sqrt{2g} (h_1 + k)^{\frac{1}{2}},$$

and consequently the whole quantity, or,

$$Q_1 + Q_2 = Q = \mu b \sqrt{2g} \left[\tfrac{2}{3} \left[(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right] + (h - h_1)(h_1 + k)^{\frac{1}{2}} \right].$$

From the quantity of water Q, and the height h_1 to which the water is raised, we have the height of water above the saddle:

$$h = h_1 + \frac{Q}{\mu b \sqrt{2g(h_1 + k)}} - \tfrac{2}{3} \cdot \frac{(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}}{(h_1 + k)^{\frac{1}{2}}},$$

from which the height of weir $CO = x = a + h_1 - h$ may be deduced. It is evident that $h > h_1$ or the weir, is a submerged or imperfect weir, when

$$Q > \tfrac{2}{3} \mu b \sqrt{2g} \left[(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right].$$

Example. A stream of 30 feet width, and 3 feet in depth, discharges 310 cubic feet of water per second. It is required to raise it $4\frac{1}{2}$ feet by means of a weir. What height of weir is necessary? As the height of the water to be raised is considerable in this case, we may confidently use the simpler formula:

$x = a + h_1 - \left(\frac{3\,Q}{2\,\mu\, b \sqrt{2g}} \right)^{\frac{2}{3}}$. In this formula $a = 3$, $h_1 = 4{,}5$, $Q = 310$, $b = 30$, $\mu = 0{,}80$, and $\sqrt{2g} = 8{,}02$ for the case in question. Hence:

$x = 3 + 4{,}5 - \left(\frac{3\,.\,10}{2\,.\,0{,}8\,.\,30\,.\,8{,}02} \right)^{\frac{2}{3}} = 5{,}7$ feet; and, therefore, the overfall is a *perfect* weir, as was presumed. If it were required to raise the water up only 2 feet, x would be 3,2 feet, or the weir would still be *perfect.* If $1\frac{1}{2}$ feet only were required, the dam would not require to rise above the level of the water down-stream, or the natural level of the water in the stream; and would be a submerged weir. Applying the complete formulas to this case, and putting

$$k = \frac{c^2}{2g} = 0{,}0155 \left(\frac{Q}{(h + h_1) b} \right)^2 = 0{,}0155 \left(\frac{310}{4{,}5\,.\,30} \right)^2$$

$= 0{,}0155 \,.\, 5{,}27 = 0{,}084$ feet, and taking μ again $= 0{,}80$ we get:

$$h - h_1 = \frac{310}{0{,}8\,.\,30\,.\,8{,}02 \sqrt{1{,}584}} - \tfrac{2}{3} \cdot \frac{(1{,}584)^{\frac{3}{2}} - (0{,}084)^{\frac{3}{2}}}{1{,}584^{\frac{1}{2}}}$$

$$= 1{,}28 - 1{,}06 + 0{,}01 = 0{,}23 \text{ feet.}$$

The saddle overfall must, therefore, be about $\frac{1}{4}$ foot, or 3 inches under the surface of the water on the lower side of the weir; and, therefore, the height of the weir itself $x = a + h - h_1 = 3{,}25$ feet.

§ 67. The height and amplitude of the *back-water* in the case of sluice weirs may be determined according to the theory of the discharge by sluices. Three cases may occur. Either the water flows away unimpeded, or it flows under a counter pressure of water, or it flows partly unimpeded, partly under water. In the case of a free discharge, as in Fig. 134, the velocity of discharge depends upon h above, measured from the centre of the opening to the water's surface. If, then, a be the height of opening, and b the breadth, then $Q = \mu a b \sqrt{2gh}$, and, therefore, inversely

$h = \frac{1}{2g} \left(\frac{Q}{\mu ab} \right)^2$, or, taking into consideration the velocity k with which the water comes up to the sluice,

$h = \frac{1}{2g} \left(\frac{Q}{\mu ab} \right)^2 - k$. For the height of opening, we have the formula:

$a = \frac{Q}{\mu\, b \sqrt{2gh}}$, or, if $h_1 =$ the height to which the water is raised by the dam above the sill, be given:

$a = \frac{Q}{\mu\, b \sqrt{2g\left(h_1 - \frac{a}{2}\right)}}$. According to the author's experiments μ is here μ = .60.

If the under-water lie back to the sluice, as in Fig. 137, then the difference of level $AB = h$, is the *head* to be introduced as pressure in the above formula. In this case, therefore, the opening corresponding to a given head h is : $a = \frac{Q}{\mu\, b \sqrt{2gh}}$.

When the level of the under-water is within the range of the

Fig. 137.

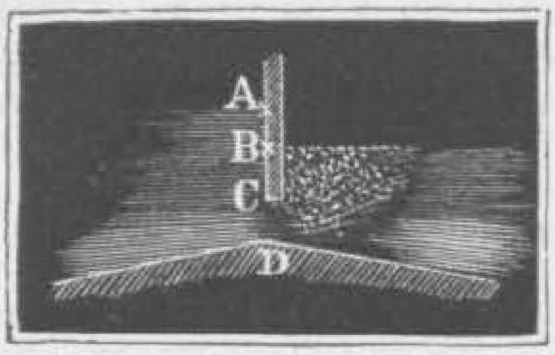
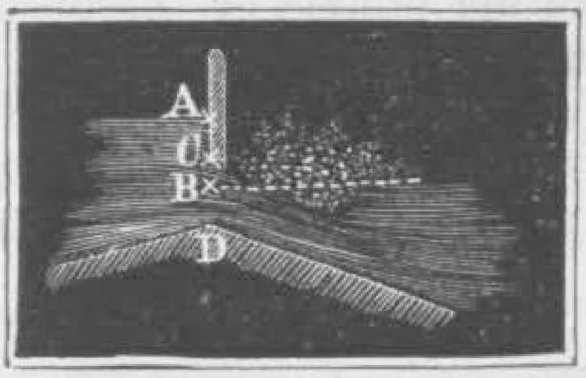


Fig. 138.



sluice's opening, as shown in Fig. 138, one part flows away unimpeded, whilst the other flows under water. If the height of the water is raised, or the difference of level AB, Fig. 138 $= h$ the height BC of the part of orifice of discharge above the surface of the water $= a_1$, and BD the height of the part under this surface $= a_2$, then the quantity of water for the former part:

$Q_1 = \mu\, a_1\, b \sqrt{2g\left(h - \frac{a_1}{2}\right)}$, and for the other:

$Q_2 = \mu\, a_2\, b \sqrt{2g\, h}$, therefore, the whole quantity:

$$Q = Q_1 + Q_2 = \mu\, b \sqrt{2g}\left(a_1 \sqrt{h - \frac{a_1}{2}} + a_2 \sqrt{h}\right).$$

From the quantity of water discharged Q, the height to which the water is raised h, and the depth a_2 of the sill or saddle of the weir under the under-water surface, we deduce the distance of the sluice-board from this surface:

$$a_1 = \left(\frac{Q}{\mu\, b \sqrt{2g}} - a_2 \sqrt{h}\right) : \sqrt{h - \frac{a_1}{2}}.$$

Example 1. How high must the boards of the sluice weir, Fig. 134, be raised, which has to let off 250 cubic feet of water per second, the breadth b being $=$ 24 feet, and the height h_1, to which the water is dammed above the sill $=$ 5 feet? In the case of unimpeded discharge:

$$a = \frac{250}{0{,}6 \,.\, 24 \,.\, 8{,}02 \sqrt{5 - \frac{a}{2}}} = \frac{2{,}16}{\sqrt{5 - \frac{a}{2}}},$$

approximately: $a = 1$, hence: $\sqrt{5 - \frac{a}{2}} = \sqrt{4,5} = 2,12$, therefore, the height of opening required: $a = \frac{2,16}{2,12} = 1,02$ feet $= 12,24$ inches.

Example 2. What amount must the sluice, Fig. 137, be drawn up, in order that it may discharge 120 cubic feet of water per second, under a head of 1,5 feet, the width of opening being 30 feet? This is a case of discharge under water, therefore,

$a = \frac{120}{0,6 \cdot 30 \cdot 8,02 \sqrt{1,5}} = 0,678$ feet $= 8,14$ inches.

Example 3. It is required to determine the quantity of water which flows through a sluice opening (Fig. 138) of breadth $b = 18$ feet, height $CD = a_1 + a_2 = 1.2$ feet, when the head $AB = 2$ feet $= h$, and the height of water above the sill, $a_2 = 0,5$ feet. In this case $\mu\, b \sqrt{2g} = 0,6 \cdot 18 \cdot 8,02 = 86,6$. Further $a_2 \sqrt{h} = 0,5 \sqrt{2} = 0,707$, and $a_1 \sqrt{h - \frac{a_1}{2}} = 0,7 \sqrt{1,65} = 0,899$, therefore, the quantity of water required $Q = 86,6\,(0,707 + 0,899) = 86,6 \cdot 1,606 = 139,07$ cubic feet.

§ 68. *Discontinuous Weirs.*—The height of the back-water in the case of incomplete or discontinuous weirs, such as piers of bridges, jetties, &c., may be calculated in very much the same way as that for overfalls. For the jetty BE, Fig. 139, there results a damming back of the waters, because the stream is contracted from the width AC to AB. If, therefore, the *lead* be closed, which it is well to

Fig. 139.

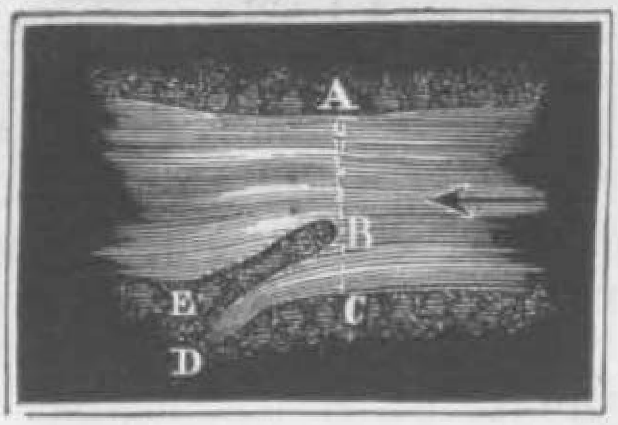
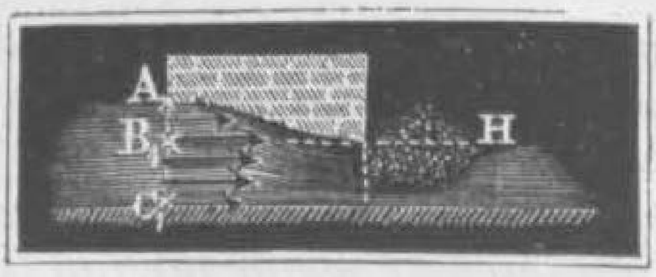


Fig. 140.



assume, the whole of the water of the stream Q must pass through the contracted passage AB. If we put the width $AB = b$, the height of dammed water $= AB_1 = h$, Fig. 140, and the depth B_1C_1 of the under-water $= a$, then the quantity flowing freely above the under-water is $Q_1 = \frac{2}{3}\,\mu\, b \sqrt{2\, g h^3}$, and the quantity flowing away as under-water $= Q_2 = \mu\, b\, a \sqrt{2\, g\, h}$. Therefore, the whole quantity going away: $Q = \mu\, b \sqrt{2\, g\, h}\,(\frac{2}{3}\, h + a)$. Hence, inversely, the breadth of weir corresponding to given height h of dammed water, is

$b = \frac{Q}{\mu\,(\frac{2}{3}\, h + a)\sqrt{2\, g\, h}}$. If the height of back-water h, be small, or the velocity of the water great, the velocity of the water as it comes up to the jetty, must be taken into consideration. If k be again taken to represent the height due to the velocity of the water as it comes to the weir, we have:

$Q_1 = \frac{2}{3}\,\mu\, b \sqrt{2g}\,[(h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}]$, and $Q_2 = \mu\, b\, a \sqrt{2g\,(h + k)}$,
and, therefore

$$Q = \mu\, b \sqrt{2g} \left[\tfrac{2}{3} \left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right] + a\,(h+k)^{\frac{1}{2}} \right],$$

and inversely:

$$b = \frac{Q}{\mu \sqrt{2g} \left[\tfrac{2}{3} \left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right] + a\,(h+)^{\frac{1}{2}} \right]}.$$

Whilst in the unimpeded motion of water in river channels, the velocity is greatest at the surface, and decreases gradually as we go downwards in the vertical depth, the case is different when the water is dammed up by any obstruction in the stream. Then the velocity *increases* from the surface of the upper-water down to that of the under-water, and diminishes very little from thence downwards to the bottom. There is, therefore, a change of velocity as represented by the arrows in Fig. 140. This must necessarily be the case, because the water *above* the under-water surface flows away under a pressure or head increasing from O to h, and the water *under* it, flows away under the constant pressure h, whilst for unimpeded motion, the pressure or head at all depths $= 0$. This formula is likewise applicable in the case of bridge piers, if b be put to represent the sum of the openings between the piers. In order to prevent as much as possible injurious effects from the eddying motion of the water behind and in front of the piers, the *starlings* are added, presenting a rounded or angled *prow* to the water. If the starling of the piers be round, or form a very obtuse angle, then μ is to be taken $= 90$, if the angle be acute $\mu = .95$, and if the acute angle be formed by the meeting of two elliptical or circular arcs, as in Fig. 141, μ becomes even .97, or very nearly 1.

Fig. 141.

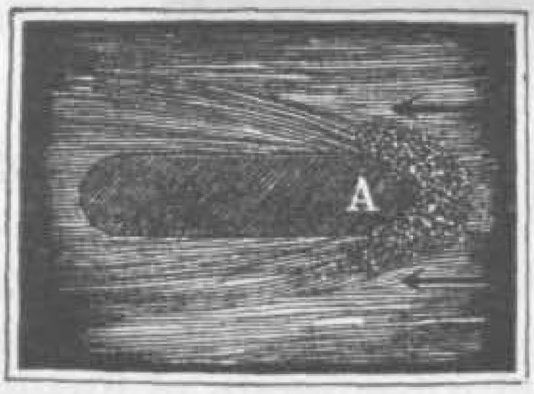
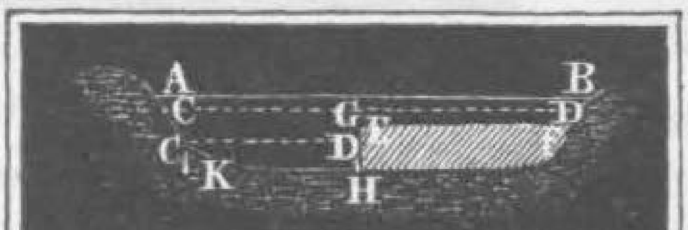


Fig. 142.



Remark. If a jetty, or other building contracting a stream, does not reach above the surface, the whole quantity of water Q may be considered as composed of 3 parts. If the top of the construction be beneath the under-water surface CD, Fig. 142, then the quantity of water flowing away through the section $ABDC$, is:

$$Q_1 = \tfrac{2}{3}\, \mu\, b \sqrt{2g} \left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right].$$

h being the height of the back-water, and b the breadth AB.

Secondly, the remaining part above the top of the building, and under the constant head h_1 or $Q_2 = \mu\, b_1\, (a - a_1) \sqrt{2g\,(h+k)}$, where $a = GH$ the depth of under-water $b_1 =$ the breadth EF of the building, and $a_1 =$ its height EH.

Lastly, the part flowing away at the end of the building under the constant head h, is $Q_3 = \mu\, b_2\, a \sqrt{2g\,(h+k)}$, b_2 being the free width CD. Thus:

$Q = \frac{2}{3} \mu\, b \sqrt{2g} \left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}} \right] + \mu \left[b\, a - b_1\, a_1 \right] \sqrt{2g\,(h+k)}$, and, therefore, we can calculate the length and height of building necessary to produce a given amount of dam. If, on the other hand, $C_1 D_1$ be the under-water surface, or if the construction reach above the surface,

$$Q = \tfrac{2}{3}\mu\, b_1 \sqrt{2g}\left[(a + h - a_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right]$$
$$+ \tfrac{2}{3}\mu\, b_2 \sqrt{2g}\left[(h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] + \mu\, a\, b_2 \sqrt{2g\,(h + k)}.$$

Example. What width BC must be given to the dam BE, Fig. 139, in order that the river, which is 550 feet wide, and 8 feet deep, and delivers 14000 cubic feet of water per second, may be dammed up 0,75 feet?

$$k = 0{,}0155\left(\frac{14000}{550\,.\,8}\right)^2 = 0{,}0155 e\ 3{,}18^2 = 0{,}156,$$

and if $\mu = 0{,}9$, then the width of the *contracted stream:*

$$b = \frac{14000}{09\,.\,8{,}02\,[\tfrac{2}{3}\,(0{,}906^{\frac{3}{2}} - 156^{\frac{3}{2}}) + 8\,.\,0{,}906^{\frac{1}{2}}]},$$

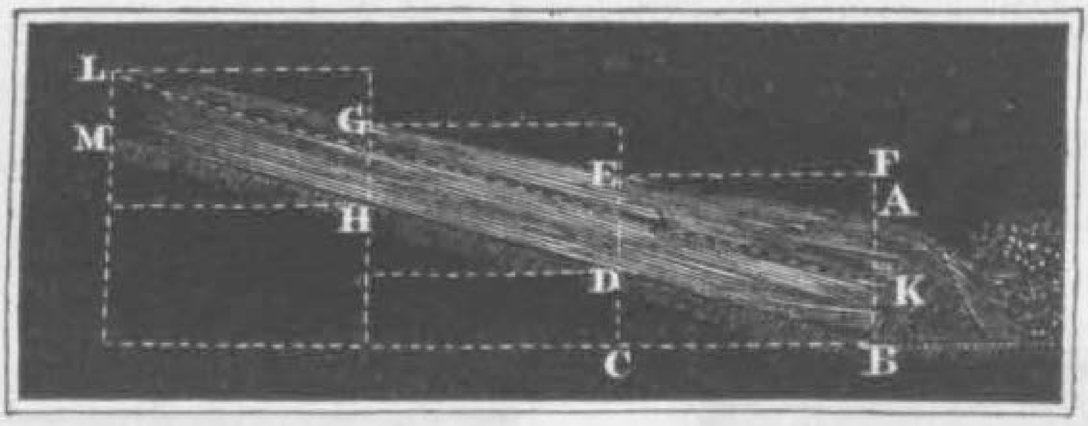
$$= \frac{14000}{7{,}218\,(0{,}522 + 7{,}608.)} = \frac{14000}{7{,}218 e\ 8{,}13} = 238{,}5 \text{ feet},$$

and, therefore, the length or projection of the dam $= 550 - 238{,}5 = 311{,}5$ feet.

§ 69. *Amplitude of the Back-water.*—We have now to resolve the other important question. According to what law does the height of the dammed water diminish in stretching back, up stream? Without having resort to any peculiar theory, this problem can be solved by the theory of the variable motion of water in river channels, explained Vol. I. § 369, § 370.

Let us suppose the length of river on which back-water from the dam ABK, Fig. 143, is perceptible, divided into separate lengths,

Fig. 143.



and let us submit each length separately to calculation. If a_0 be the depth of water AB at the weir, a_1 the depth DE at the upper end of such a length; $ABDE$, F_0 the section of the flowing water at the weir, F_1 the section at DE, Q the quantity of water, p the mean circumference of the section for this length, and a the angle of inclination of the river's bed, then, from (Vol. I. § 370) the length of the first division, (a_0 and a_1, and F_0 and F_1 being substituted for each other) is:

$$l = \frac{a_0 - a_1 - \left(\dfrac{1}{F_1^2} - \dfrac{1}{F_0^2}\right)\dfrac{Q^2}{2g}}{\sin.\, a - \zeta\,.\,\dfrac{p}{F_0 + F_1}\left(\dfrac{1}{F_0^2} + \dfrac{1}{F_1^2}\right)\dfrac{Q^2}{2g}}.$$

If a_2 be the depth of water GH at the upper end of a second length $DEGH$, F_2 its section, and p_1 the mean perimeter of the water section of this part, then its length

$$DH = l_1 = \frac{a_1 - a_2 - \left(\dfrac{1}{F_2^2} - \dfrac{1}{F_1^2}\right)\dfrac{Q^2}{2g}}{\sin.\, a = \zeta\,\dfrac{p_1}{F_1 + F_2}\left(\dfrac{1}{F_1^2} + \dfrac{1}{F_2^2}\right)\dfrac{Q^2}{2g}}.$$

Continuing in this manner, namely, assuming arbitrary decreases of depth $a_0 - a_1$, $a_1 - a_2$, $a_2 - a_3$, &c., and calculating from this the sections F_1, F_2, F_3, &c., and the mean perimeters, we get by the formula, the distances l, l_1, l_2, corresponding, or the distances l, $l + l_1$, $l + l_1 + l_2$, &c., from the weir.

To find the depth y corresponding to a given distance x, we may either apply the method of interpolation to the values l, $l + l_1$, $l + l_1 + l_2$, &c., just found, or we may make use of this other formula, likewise given, Vol. I. § 370, viz.:

$$a_0 - a_1 = \frac{\left(\sin. \alpha - \zeta \cdot \frac{p_0}{a_0 b_0} \cdot \frac{v_0^2}{2g}\right)}{1 - \frac{2}{a_0} \cdot \frac{v_0^2}{2g}} l.$$

If we put in this instead of b_0, the breadth, and instead of p_0, the perimeter, and for v_0 the velocity at the weir, this formula gives the decrease $(a_0 - a_1)$ of the height of back-water on the first short length l, and for a next following short length l_1 this decrease is:

$$a_1 - a_2 = \frac{\left(\sin. \alpha - \zeta \cdot \frac{p_1}{a_1 b_1} \cdot \frac{v_1^2}{2g}\right)}{1 - \frac{2}{a_1} \cdot \frac{v_1^2}{2g}} l_1, \text{ \&c.,}$$

and, lastly, for a given distance $l + l_1 + l_2 + \ldots$ the depth: $a_0 - (a_0 - a_1) - (a_1 - a_2) - \ldots$ may be calculated.

Example 1. A weir is to be built in a river 80 feet wide, 4 feet deep, and discharging 1400 cubic feet per second, in order to dam up the water 3 feet high. Required the relative amount of damming, at distances back from the weir. Without the dam, the velocity of the water $c = \frac{1400}{8 \cdot 04} = 4{,}375$ feet, and, therefore, according to the table, Vol. I. p. 447, the co-efficient of resistance $\zeta = 0{,}00747$, and the inclination of the channel $\sin. \alpha = 0{,}00747 \cdot \frac{p}{F} \cdot \frac{c^2}{2g}$. If, therefore, $p = 84$, $F = 80 \cdot 4 = 320$, $c = 4{,}375$, and $\frac{1}{2g} = 0{,}0155$, then the inclination:

$$\sin. \alpha = 0{,}00747 \frac{84}{320} \cdot 0{,}0155\ (4{,}375)^2 = .0{,}0005818, \text{ or, } .00058,$$

near enough. The depth of water close to the weir is $4 + 3 = 7$ feet, and we shall now determine the distances at which the depths $6\frac{1}{2}$, 6, $5\frac{1}{2}$, and 5 feet occur. If we first introduce into the formula:

$$l = \frac{a_0 - a_1 - \left(\frac{1}{F_1^2} - \frac{1}{F_0^2}\right)\frac{Q^2}{2g}}{\sin. \alpha - \zeta. \frac{p}{F_0 + F_1}\left(\frac{1}{F_0^2} + \frac{1}{F_1^2}\right)\frac{Q^2}{2g}},\ a_0 - a_1 = 0{,}5,\ F_0 = 80 \cdot 7 = 560,$$

$F_1 = 80 \cdot 6{,}5 = 520$, $Q = 1400$, $\sin. \alpha = .00058$, $p = 86$, and then for the mean velocity $= \frac{2\,Q}{F + F_1} = \frac{2800}{1080} = 2{,}59$ feet, $\zeta = .0075$, the value of

$$l = \frac{0{,}5 - (0{,}0000036982 - 0{,}0000031888) \cdot 30434}{0{,}00058 - 0{,}0075 \frac{86}{1080} (0{,}0000036982 + 0{,}0000031888) \cdot 30434}$$

$$= \frac{0{,}5 - 0{,}0155}{0{,}00058 - 0{,}000128} = \frac{0{,}4845}{0{,}000452} = 1071 \text{ feet.}$$

To find the distance back at which a depression of 1 foot in the water's surface occurs, we must again put $a_0 - a_1 = 0{,}5$, but $F_0 = 520$, $F_1 = 80 \cdot 6 = 480$, $p = 85{,}5$ and the mean velocity $\frac{2800}{1000} = 2,80$ gives $\zeta = 0,00749$. Hence, by means of the same formula as above, we get for the distance in which the surface lowers, so that the depth becomes 6 feet instead of 6,5,

$$l = \frac{0.5 - 0,0000006421 e . 30434}{.00058 - .00749 \frac{85.5}{1000} . 0,0000080385 . 30434} = \frac{0.4845}{.000424} = 1142 \text{ feet.}$$

The water at a distance $1071 + 1142 = 2213$ feet, is, therefore, only 6 feet deep, or the height of the back-water is 2 feet. If, again, we put $a_0 - a_1 = 0,5$, and $F_0 = 480$, $F_1 = 440$, $p = 85,1$, and $\zeta = 0,00749$, then $l = 1205$ feet, and for a further depression of 0,5 feet, $l = 1413$ feet, so that at 2213 feet + 1203 feet + 1413 feet = 4829 feet back from the weir there is still a rise of 1 foot, occasioned by it. For the $4\frac{1}{2}$ feet deep length $l = 1922$ feet, for $4\frac{1}{4}$ feet, $l = 1584$ feet, and for 4,1 feet, $l = 1850$ feet, so that there is still a difference of $\frac{1}{10}$th of a foot at a distance 4829 + 1922 + 1584 + 1850 + 10185 feet back from the weir, and diminishes upwards; but for 4 feet, or *complete cessation of back-water*, $l = \infty$ by our formula.

Example 2. Required the height of the back-water at the distance 2,500 feet back from the weir of the last example. According to the calculations above, there is a rise of 2 feet at 2122 feet above the weir, and the question, therefore, is, how does the rise diminish in the distance $2500 - 2122 = 378$ feet? The distance back from the 6 feet depth at which a further reduction of 0,5 feet takes place, has been found above to be 1205 feet. Therefore, for each foot a depression of $\frac{0,5}{1205}$ feet, so that for 377 feet, we should have $\frac{0,5 \,.\, 378}{1205} = 0,157$ feet, and, therefore, the rise of the back-water at 2,500 feet back from the weir is $2 - 0,157 = 1,843$ feet, and, therefore, the depth of water $= 5,843$ feet. If we calculate according to the second formula:

$$a_0 - a_0 = \frac{\left(\sin. a - \zeta \,.\, \frac{p_0}{a_0 b_0} \,.\, \frac{v_0^2}{2g}\right)}{1 - \frac{2}{a_0} \,.\, \frac{v_0^2}{2g}} \, l,$$ and if we put into this $l = 800$, $p_0 = 86$, $a_0 = 7$, $a_0 b_0 = 560$, $v_0 - \frac{1400}{560} = 2,5$, and $\zeta = 0075$, we get the depression corresponding $= 0,399$ feet, and if we again put $l = 800$, $p_0 = 85{,}8$, $a_0 = 7 - 0,399 = 6,601$, $a_0 b_0 = 528$, $v_0 = \frac{1400}{528} = 2,652$, and $\zeta = .0075$, the depression is found to be 0,383 feet. Continuing in this manner, but setting l this time $= 900$, $p_0 = 85,5$, $a_0 = 6,601 - 0,383 = 6,218$, $a_0 b_0 = 497,44$, $v_0 = \frac{1400}{497,44} = 2,88$, and $\zeta = 00749$, we get the depression $a_0 - a_1 = 0,403$ feet, so that for $800 + 800 + 900 = 2,500$ feet back from weir, the depth of water is $6,218 - 0,403 = 5,818$ feet, and the height of the back-water here is 1,815 feet. The first method gave 1,843, so that the difference in the results of the two methods is not quite $\frac{1}{3}$ of an inch.

§ 70. *Back-water Swell.*—If we consider somewhat closely the equation of the *curve* of the back-water, that is, of its longitudinal section, viz:

$$a_0 - a_1 = \left(\frac{\sin. a - \zeta \,.\, \frac{p}{F} \,.\, \frac{v^2}{2g}}{1 - \frac{2}{a} \,.\, \frac{v^2}{2g}}\right) l,$$

we discover several interesting circumstances in reference to the back-water. In the fraction:

$$\frac{\sin. a - \zeta \,.\, \frac{p}{F} \,.\, \frac{v^2}{2g}}{1 - \frac{2}{a} \,.\, \frac{v^2}{2g}}$$

the numerator and denominator become more nearly equal to 0, the greater the velocity v, and according as the one or the other first becomes 0, we havee

$$l = \frac{(a_0 - a_1)\left(1 - \frac{2}{a} \cdot \frac{v^2}{2g}\right)}{0} = \infty \text{ or } l = \frac{(a_0 - a_1) \cdot 0}{sin.\, \alpha - \zeta \cdot \frac{p}{F} \cdot \frac{v^2}{2g}} = 0.$$

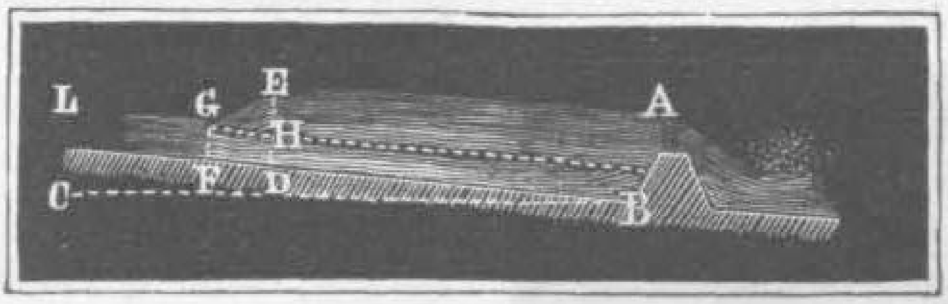
We perceive from this that when the numerator becomese= 0, the division l, or the limit of the back-water becomes infinitely distant, and in the case of the denominator becoming = 0, the length $l = 0$, or there is no back-water. Now the numerator becomese= 0, when $\zeta \cdot \frac{p}{F} \cdot \frac{v^2}{2g}$e= $sin.\, \alpha$, or, when the velocity of the dammed water differs in an infinitely small degree from the velocity $v = \sqrt{\frac{2g\, F\, sin.\, \alpha}{\zeta\, p}}$ of the uniformly flowing water of the stream, and the denominator becomese= 0, when:

$$\frac{2}{a} \cdot \frac{v^2}{2g} = 1, \text{ or, } \frac{v^2}{2g} = \frac{a}{2},$$

that is, when the height due to the velocitye= half the depth of the stream.

When the height due to the velocity of the water before the introduction of a weir, is less than half the depth of the undammed water, the back-water takes the form shown in Fig. 143, *and if the height due to the velocity be greater than half the depth, the back-water has the form* Fig. 144, there being a rise or swelling at the point EG.

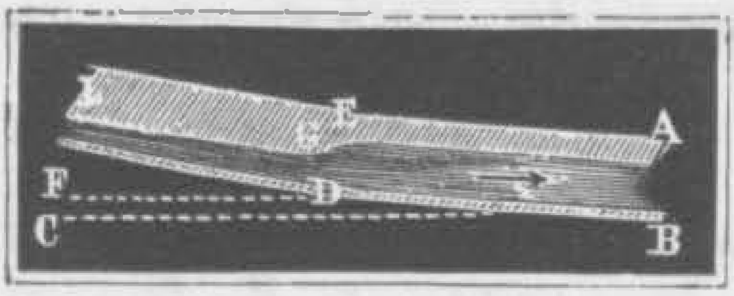
Fig. 144.



If in the equation $sin.\, \alpha = \zeta \frac{p}{F} \cdot \frac{v^2}{2g}$, we put $\frac{v^2}{2g} = \frac{a}{2}\, F = ab$, and p (though it be only approximately) $= b$, we have: $sin.\, \alpha = \frac{1}{2}\, \zeta$. Thus the circumstances represented in Fig. 144, are likely to occur when the fall or inclination of the stream α, is greater than $\frac{1}{2}$ the co-efficient of resistance $\zeta = .0075$, that is when $\alpha > .00375$, or $\alpha > \frac{3}{800}$, or 1 in 266. As rivers and water-courses have generally a less inclination than this, the sudden depression EG, Fig. 144, is eldom observable in them.

Remark 1. This sudden depression of the back-water was first observed by Bidone, in a 12 inch wide trough, in which α was $= 0{,}033$. The sameappearance is manifested when the inclination of the channel changes, as shown in the Fig. 145. If the degree of inclination of the upper part be greater than $\frac{1}{2}\, \zeta$, and inclination of the lower part

Fig. 145.



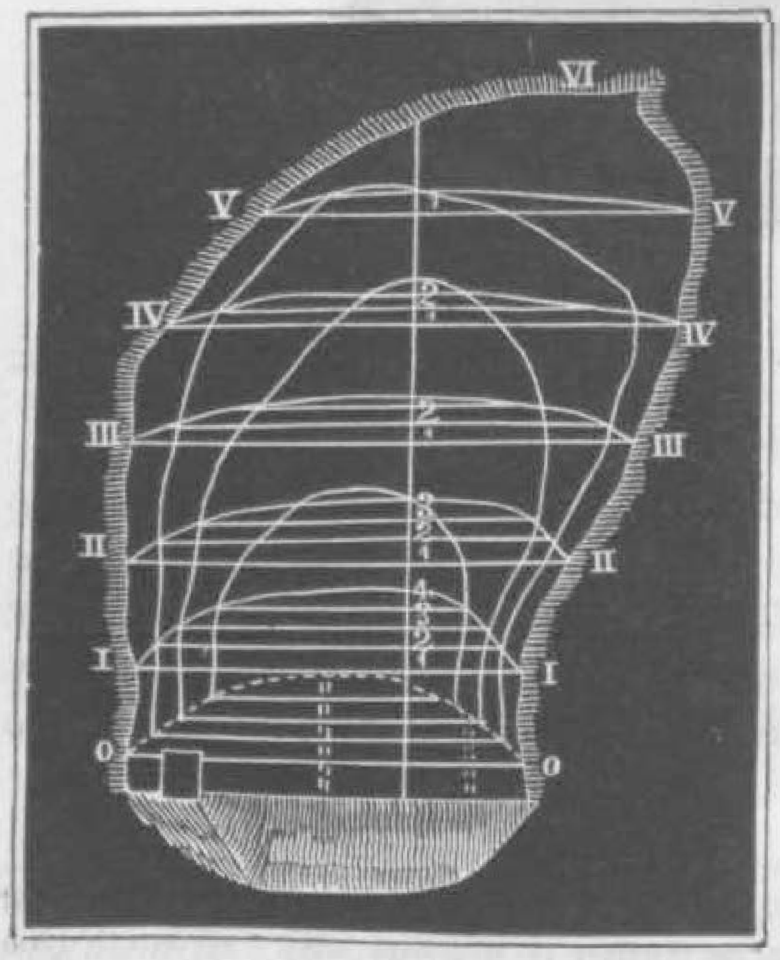
less, there is formed at the point of change a *swell*, or sudden rise where the less depth corresponding to the greater inclination, passes into the greater depth corresponding to the less inclination.

Remark 2. Saint-Guilhem has given an empirical equation for the curve of the back-water, but the author has given one more simple and accurate in the "*Allgemeinen Maschinen Encyclopädie*," article "Bewegung des Wassers."

§ 71. *Reservoirs.*—In districts where the supply of water is small, but where powerful machines are nevertheless required, as in mining districts generally, the construction of reservoirs (Fr. *étangs* ; Ger. *Teichen*), or large artificial ponds, that fill during seasons of rain, and supply the demands of drier seasons, is a matter of practical importance. The site to be chosen for a reservoir is regulated by a variety of circumstances. The main question is that of the relative level of the machines to which the water is to be applied. This being satisfied, they are most advantageously placed in a deep *dean*, or part of the valley where they can collect, not only the rain-water, but the streamlets and springs of as large a surrounding district as possible. In such a situation a single dyke or dam going square across the valley is sufficient to *enclose* the reservoir. The shorter the dyke, and the less the superficial area of a reservoir for a given cubical contents, the better. The steeper the banks, therefore, the more economically a reservoir is formed. The lower the level of the reservoir compared to the surrounding district, the greater supply of water may be led into it, or will flow to it naturally.

In selecting the site for a reservoir, great attention must be paid to the nature of the bottom, that is, its impermeability must be thoroughly ascertained; also its fitness for bearing the weight of the dyke or dam. Artificial puddling is, of course, a resource available in many cases; but for very extensive reservoirs, it is a precarious and expensive remedy for want of natural impermeability. Fissures in rocks, deposits of sand and gravel, morasses or bogs are to be avoided by all means.

Fig. 146.



Remark. On this subject, see Smeaton's "Reports," Sganzin, "*Cours de Construction*," and Hagen "*Wasserbaukunst*."

The value of a reservoir depends chiefly on its superficial and cubical contents. For ascertaining these, an accurate survey is necessary. The points I, II, III, &c., of Fig. 146, are laid down from a survey with the chain or theodolite, and *cross sections* are then taken by leveling (and sounding, when there exists a natural reservoir), on equi-distant parallel lines 0—0, I—I, &c.

If $b_0, b_1, b_2 \ldots b_n$, be the widths 0—0, I—I, II—II, &c., and if the distance between the parallels be a, the area of the dam is:

$$G = \left[b_0 + b_n + 4\,(b_1 + b_3 + \ldots + b_{n-1}) + 2\,(b_2 + b_4 + \ldots + b_{n-2}) \right] \cdot \frac{a}{3},$$

and if, in like manner, F_0, F_1, F_2, &c., be the area of the cross sections corresponding to the widths b_0, b_1, b_2, &c., respectively, the volume of the dame

$$V = \left[F_0 + F_n + 4\,(F_1 + F_3 + \ldots + F_{n-1}) + 2\,(F_2 + F_4 + \ldots + F_{n-2}) \right] \cdot \frac{a}{3}.$$

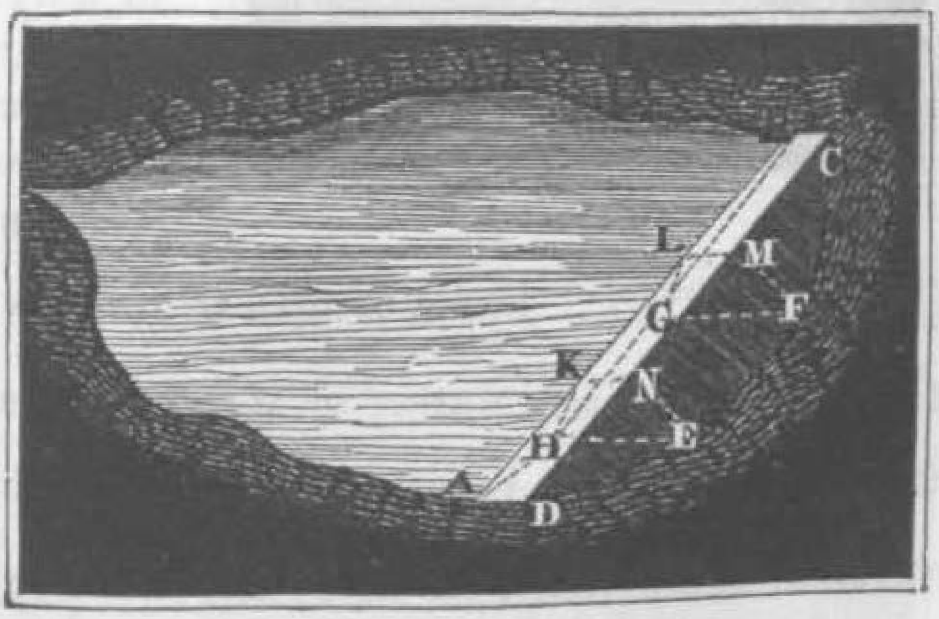
By dividing the cross sections by parallel lines, drawn at equal depths, we get the means of laying down contour lines of equal depth, and so ascertain the contents of the dam for each depth.

Remark. The author's work "*Der Ingenieur*," contains detailed instructions for measuring reservoirs and dykes.

§ 72. *Dykes.*—The dykes or dams of reservoirs are generally of earth-work, seldom of stone. The face inside, or next the reservoir is covered with clay puddle, and with a carefully laid course of gravel. They are carried up of a uniform slope, or with *offsets* or *terraces.* They are carefully rammed at every foot of additional height laid upon them. Especial care must be taken with the foundation, which must be carried down to an impermeable stratum with which the superstructure must be connected, so that the bed of junction may be perfectly water-tight. When a water-tight substance cannot be found, a system of piles must be used to insure this most important point of the reservoir's efficiency. The depth of the foundations depends on the nature of the ground, as above explained, and 5, 10, and 20 feet deep foundations have been executed.

The dyke, in its main features, is shown in Fig. 147, having a trapezoidal section *EK* or *FL*. *AC*, is sometimes termed the *crown* of the dam; it must be well paved, and generally has a parapet wall to prevent the wash of water during high winds from damaging the crown, or washing over to injure the back of the dam *NE* or *ME*. The piece *KME* of the dyke is termed the *middle* or centre piece, and the pieces *ANH* and *BMC* are termed the *wings* of the dam. As to the dimensions of dams, the breast is generally made to slope at the rate of 1 to 3, and the back at the rate of 1 to 2. The width on

Fig. 147.



the top is very various. For high dykes it varies from 10 to 20 feet. A common rule is, to make the width at top equal to the height, but this only applies to dams of small height. The dyke should be carried from 3 to 6 feet higher than the highest water intended to be in the reservoir.

Fig. 148.

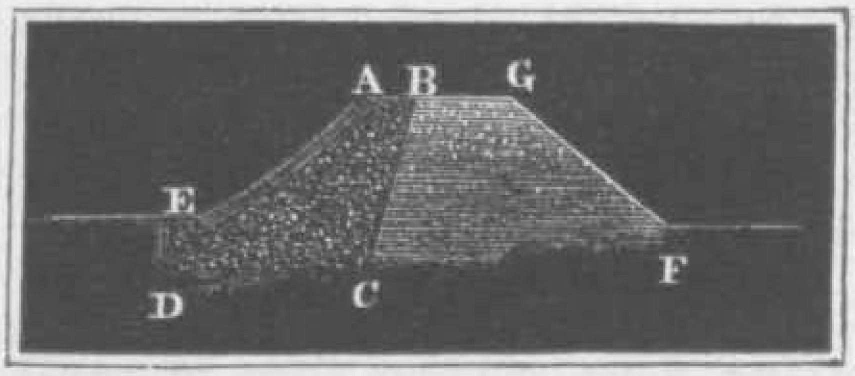


Fig. 148 represents a cross section of a dyke for a reservoir. $ABCE$ is the breastwork of clay carried down to water-tight substratum, $BGFC$ is the backing of earth-work, AE is the paved face, the paving being 4 feet thick at bottom, and 2 feet at top.

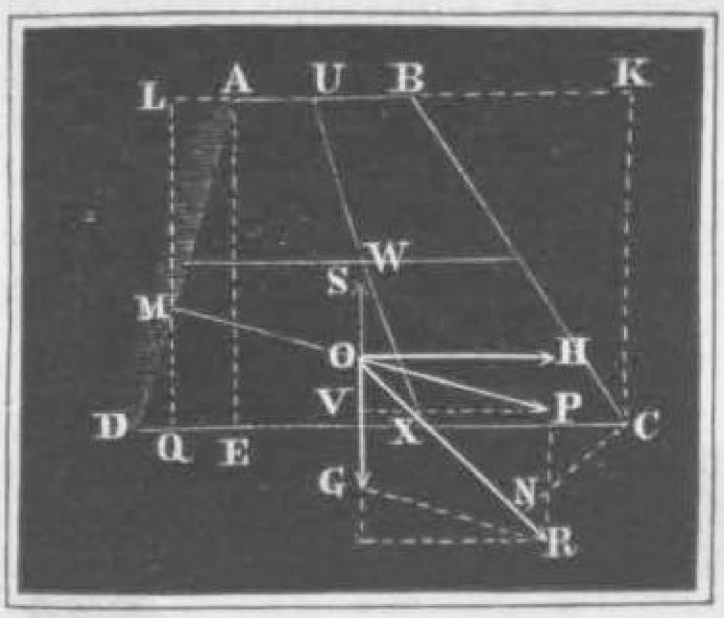
Remark. If l be the length along the top. and l_1 the length along the bottom, b the breadth on top, and b_1 the breadth at bottom, and if h be the height of a dyke such as Fig. 148, the cubic contents of the dam are:

$$V = [l b_1 + l_1 b + 2\,(l b + l_1 b_1)]\,\frac{h}{6}.$$

In applying this formula, it must be borne in mind, that the well-rammed clay does not occupy quite one-half of that of the earth-work that has not been rammed.

§ 73. *Stability of Dykes.*—Dykes are exposed to the pressure, and sometimes, though rarely, to the shock or impetus of water. They must, therefore, be of proportions that will resist either being overturned or shoved forward by the action of the water. The conditions under which they resist being shoved forward have been examined, Vol. I. § 280; and we shall now consider the question of stability in reference to dislocation by rotation. The water acts on the internal slope or breast AD of a dyke, Fig. 149, with a normal pressure $OP = P$, the point of application of which is M is at the distance $LM = \frac{2}{3}$ the depth $CK = \frac{2}{3}\,h$ from the surface of the water

Fig. 149.



(Vol. I. § 278). For a length of dam $= 1$, $P = AD \cdot \gamma \cdot \frac{h}{2}$, γ being the density of the water, or weight of cubic unit. The horizontal component of this pressure is: $H = h \cdot 1 \cdot \gamma \cdot \frac{h}{2} = \frac{1}{2}\,h^2\,\gamma$, and the vertical component, (if m be the relative batter, or mh the absolute batter DE of the breast,) $V = mh \cdot 1 \cdot \gamma \cdot \frac{h}{2} = \frac{1}{2}\,mh^2\,\gamma$. The weight of the piece of the dyke of length $= 1$, acting at the centre of gravity

S of the trapezoidal section $ABCD$, is $G = \left(b + \frac{m+n}{2} h\right) h \gamma_1$, in which $b =$ the breadth AB, and n relative, or $n h$ the absolute batter or slope of the back of the dyke. From P and G, or from H, V, and G, there arises a resultant force $OR = R$, the statical moment of which $CN \,.\, R$, referred to the corner C, represents the stability of the dam. If we suppose P, and also H and V, acting in M, the statical moment of $P =$ statical moment of H minus statical moment of $V = \frac{1}{2} h^2 \gamma \,.\, \overline{MQ} - \frac{1}{2} m h^2 \gamma \,.\, CQ = \frac{1}{2} h^2 \gamma\, (\overline{MQ} - m\,.\, CQ) = \frac{1}{2} h^2 \gamma\, [\frac{1}{3} h - m\,(nh + b + \frac{2}{3} m h)]$;

hence we have the statical moment of G working in a contrary direction:

$$= \tfrac{1}{2} n h^2 \gamma_1 \,.\, \tfrac{2}{3} n h + b h \gamma_1 \left(n h + \frac{b}{2}\right) + m h^2 \gamma_1\, (n h + b + \tfrac{1}{3} m h)$$

$$= h \gamma_1\, (\tfrac{1}{3} n^2 h^2 + n b h + \tfrac{1}{2} b^2 + \tfrac{1}{2} m n h^2 + \tfrac{1}{2} m b h + \tfrac{1}{6} m^2 h^2)$$

$$= h \gamma_1 \left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right];$$ and, hence, we have the stability of the dykes:

$$S = h \left(\left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right] \gamma_1 - [\tfrac{1}{3} h - m\,(n h + b + \tfrac{2}{3} m h)] \frac{h}{2} \gamma\right).$$

In order now to find the point X, in which the line of resistance UWX cuts the base CD of the dyke, we must determine the distance CX of this point from the edge C, and for this we put: $\frac{CX}{CN} = \frac{OR}{HR} = \frac{R}{V + G}$; and from this

$$CX = a = \frac{CN \,.\, R}{V + G} = \frac{S}{G + V} = \left(\left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right] \gamma_1 + \left[\left(\frac{2 m^2 - 1}{3} + m n\right) h + m b\right] \frac{h}{2} \gamma\right) : \left(\left[\left(\frac{m + n}{2}\right) h + b\right] \gamma_1 + \tfrac{1}{2} m h \gamma\right);$$ or,

$$a = \frac{[(m^2 + 2 n^2 + 3 m n) h^2 + (2 n + m) \,.\, 3 b h + 3 b^2]\, \gamma_1 + [(2 m^2 - 1 + 3 m n) h + 3 m b]\, h \gamma}{3\, ([(m + n) h + 2 b]\, \gamma_1 + m h \gamma)}.$$

By aid of this formula, other points W in the line of resistance may be found, if for h different heights of dyke be introduced, or we may ascertain the stability of any part of the dam bounded by a horizontal plane.

For a dyke with vertical sides, $m = n = o$, hence $a = \frac{3 b^2 \gamma_1 - h^2 \gamma}{6 b \gamma_1} = \frac{1}{2} b - \frac{h^2 \gamma}{6 b \gamma_1}$ (Vol. II. § 10). If the inclination of the breast and back be 1 to 1, or 45°, $m = n = 1$, therefore,

$$a = \frac{3\,(2 h^2 + 3 b h + b^2)\, \gamma_1 + (4 h + 3 b)\, h \gamma}{3\,[2\,(b + h)\, \gamma_1 + h \gamma]};$$

and if $b = h$, then $a = \frac{18 \gamma_1 + 7 \gamma}{4 \gamma_1 + \gamma} \,.\, \frac{h}{3}$, and if $\gamma_1 = 2\, \gamma$, then

$a = \frac{43}{27}\, h = \frac{43}{27}\, b$, or, as in this case the breadth at the base $b_1 = 3\, b$, or $b = \frac{1}{3}\, b_1$, $a = \frac{43}{81}\, b_1$. According to Vauban's practice, there is ample security when $a = \frac{5}{9} \cdot \frac{b_1}{2} = \frac{5}{18}\, b_1$ (Vol. II. § 11), so that for the last case there is an excess of stability. All things considered, it is well in dykes, for great reservoirs, to make a at leastl$= 0{,}4\, b_1$, or the line of resistance should cut the base at $\frac{4}{10}$ths of the width of the base from the heel of the dyke.

Example. Required the line of resistance of a dyke, the batter of inclination of the breast of which $m = 1$, that of the back $n = \frac{1}{2}$, the breadth on the summit, or crown being $b = 10$ feet. Assuming that the mass of the dyke has a specific gravity $= 2$. We have:

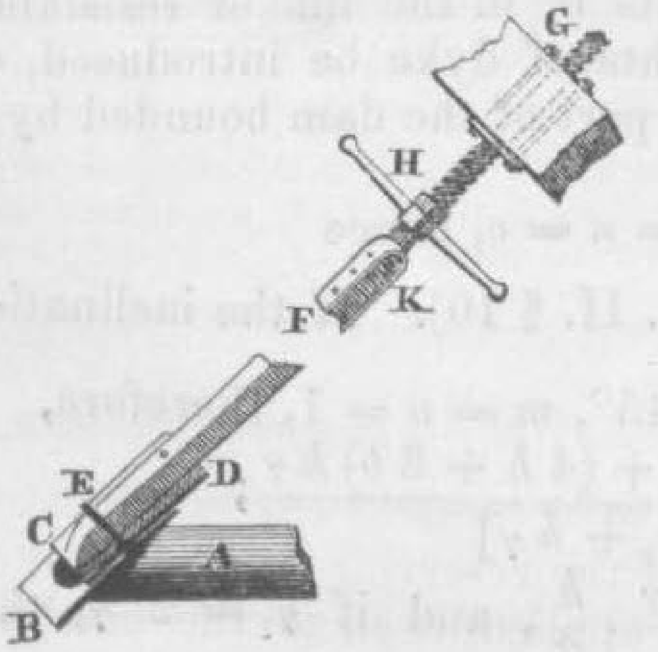
$$a = \frac{2\,(3\,h^2 + 60\,h + 300) + (\frac{5}{2}\,h + 30)\,h}{3\,(3\,h + 40 + h)} = \frac{1200 + 300\,h + 17\,h^2}{24\,(10 + h)};$$

hence for $h = 0$, $a = 5$ feet; for $h = 5$ feet, $a = \frac{3125}{360} = 8{,}68$ feet; for $h = 10$ feet, $a = \frac{5900}{480} = 12{,}29$ feet, for $h = 15$ feet, $a = \frac{9525}{600} = 15{,}87$ feet, for $h = 20$ feet, $a = \frac{14000}{720} = 19{,}44$ feet, &c. If the height of dyke be very great, we may put: $a = \frac{17\,h}{24}$, and $b = \frac{3}{2}\,h$, hence $\frac{a}{b} = \frac{17}{36}$. As $\frac{17}{36}$ is more than 0,4, such a dam would be safe for an infinite height.

Remark. According to the formula $b = \frac{3\,h - a}{2}$ in the example Vol. I. § 280, if we put $a = mh$, then $2\, b = (3 - m)\, h$, hence $h = \frac{2\, b}{3 - m}$, and, therefore, in our last example, in which $m = 1$, $h = b = 10$ feet.

§ 74. *Offlet Sluices of Dykes.*—Offlet sluices and discharge-pipes, or culverts, must be provided in the reservoir dyke. The offlet sluice or regulator, serves for the discharge of any excess of water that would accumulate in times of extraordinary wet. The discharge-pipe or culvert, is for supplying the *lead* or water-course as circumstances require. There may be one or more of each of these accessories in a dyke. For instance, in some dykes an offlet is arranged at the very lowest level, so that the dam may be completely emptied when occasion requires, and above this, a second offlet is laid, by which the water-course is supplied with water to be led to the machine that is to receive it as *power*.

Fig. 150.



The offlet-pipes may be either of wood or iron, or of stone, or may be built culverts. Fig. 150, in the margin, gives a general idea of the arrangement of the drawing-sluice or discharge-sluice of a dyke. *A* is the end of the pipe or culvert, on the face of which is a flat piece of wood or iron *B*, *CD* is a cast iron or wooden *sluice-board*, fitting into guides, DE is the *sluice-rod*, reaching to the surface or top of the dyke, E is a cross piece by which, in the absence of grooves or guides on the plate on the end of the pipe, the sluice is kept pressed upon its bed, G is a strong beam having a female screw, through which the screw GH passes, and the handle or key, H, being turned, the screw elevates or depresses the sluice-rod, as may be desired, for opening and shutting the sluice.

The discharge-pipe must have a sectional area, such that the discharge when the water, or rather its *head*, is lowest, may be sufficient for the supply of the power required for the machine. If Q be the quantity of water to be discharged per second, h the given least *head*, l the length, and d the diameter of the discharge-pipe, ζ the co-efficient of resistance at entrance, and ζ_1 the co-efficient for internal friction, then, according to Vol. I. § 332,

$$d = \sqrt[5]{\frac{(1+\zeta)\,d+\zeta_1\,l}{2\,g\,h}\cdot\left(\frac{4\,Q}{\pi}\right)^2},$$

or, more simply:

$$d = 0,4787\sqrt[5]{[(1+\zeta)\,d+\zeta_1\,l]\,\frac{Q^2}{h}}.$$

If, therefore, we take ζ from the table in Vol. I. § 325, and ζ_1 from the table in Vol. I. § 331, we can determine by approximation the required width of pipe. As the head is higher, a greater part of the aperture must be closed, so that, according to Vol. I. § 338, there must be introduced a greater co-efficient of resistance for the entrance. If the entrance aperture be very small, the water does not fill the pipe, and, therefore, the calculation is simply referable to the area of the opening $F = \frac{Q}{\mu\sqrt{2\,g\,h}}$, where μ is to be taken from Vol. I. § 325. With table of areas of segments, the calculations are very simple. The prolongation of the discharge-pipe through the dyke must be of very substantial cement-built masonry, and in large dykes should be from 5 to 6 feet high.

Example 1. A discharge-pipe of 100 feet long is required to let off 10 cubic feet per second, when the head is reduced to 1 foot, what must be the diameter? Supposing the inclination of the sluice to be 40° (equal that of the breast of the dyke) then $\zeta = 0,87$; and the co-efficient ζ_1 corresponding to a velocity of 5 feet $= 0,022$, we have $d = 4787\sqrt[5]{(1,870\,d + 2,2)\,e.\,100}$, and $d = 1,7$ satisfies this equation very nearly. Thus, a discharge pipe of $1,7\,.\,12 = 20,4$ inches would fulfil the required conditions.

Example 2. In what position must this sluice-board be placed, in order to discharge only 10 cubic feet of water per second, when the head is 16 feet? If we assume that the pipe does not fill in this case, then

$$F = \frac{Q}{\mu\sqrt{2\,g\,h}} = \frac{10}{0,731\,e.\,802\sqrt{16}} = \frac{5}{11,6} = .431 \text{ square feet.}$$

This segment of radius $\frac{1,7}{2}$ reduced to radius $1 = 0,431\,.\,\frac{4}{2,89} = 0,598$, and from a table of areas of segments, we find the height of such a segment to be 5 inches.

§ 75. *Water-courses.*—The water of the reservoir is conductedtor led to the point at which it is to be applied, *i. e.*, to the machine through which it is to expend its mechanical effect, by *canals*, *water-*

courses, and *mill-leads*. These channels are generally dug out of the natural soil, raised upon embankments and aqueducts over the deeper valleys, and cut as drifts or tunnels through the greater elevations that occur in their course. The bed of the canals are formed of sand or gravel, on a bottom of clay, or are hand-laid stones, or concrete formed with cement, and not unfrequently it consists of a wooden, an iron, or a stone *trough*. The sides of this canal form right lines, or its section is a gently curved trapezium, or it is rectangular when it becomes a *trough*. The section of water-courses is from 1½ to 3 times as wide as its depth. The slopes of the sides of the course are generally very slight, or none at all in the case of masonry set in cement. An inclination of 1 in 2 is given to dry stone sides, an inclination of 1 in 1 in the case of compact earth or clay, and of 2 to 1 in the case of sand or loose earth. Fig. 151 gives an idea of the construction of a water-course in loose ground,

Fig. 151.

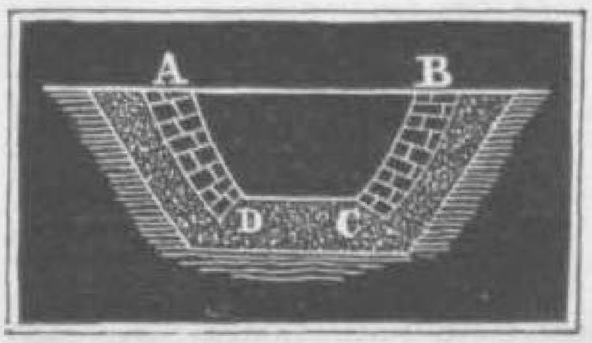
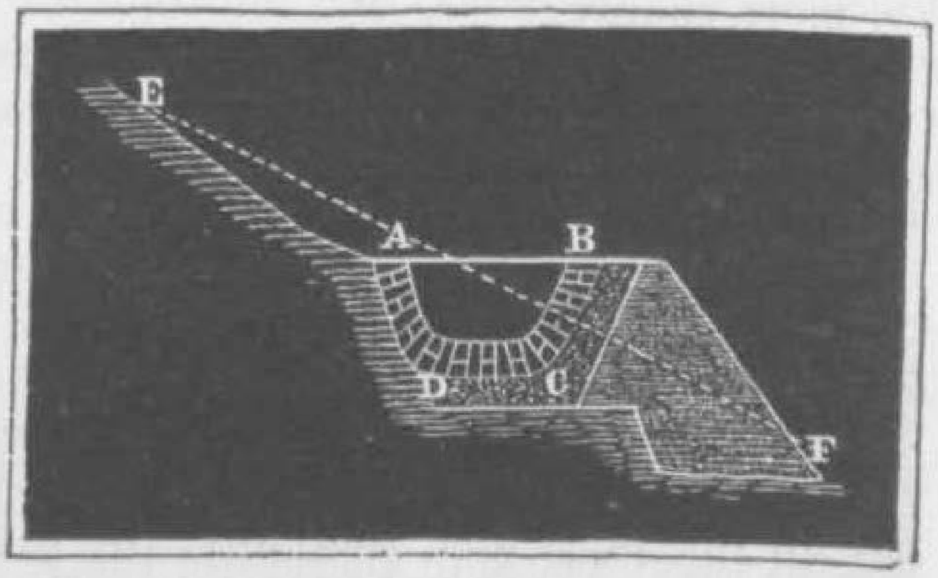


Fig. 152.



not water tight. Fig. 152 represents the manner of forming such a course on the side of a hill, where the earth taken from the cut is made the supporting bank on the under side. Fig. 153 shows the

Fig. 153.

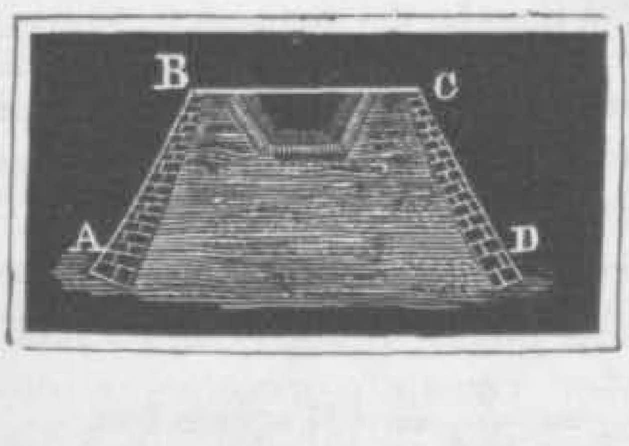


Fig. 154.

manner in which it is sometimes necessary to construct the embankments for aqueducts.

Fig. 154 is a section of a walled drift or tunnel, through ground not considered impermeable to water, and incapable of standing unsupported. The manner of putting troughs together is indicated in the sketches in Fig. 155 for wood, and Fig. 156 for iron, where the flanges, bolted together, are further made water-tight by what is

Fig. 155.

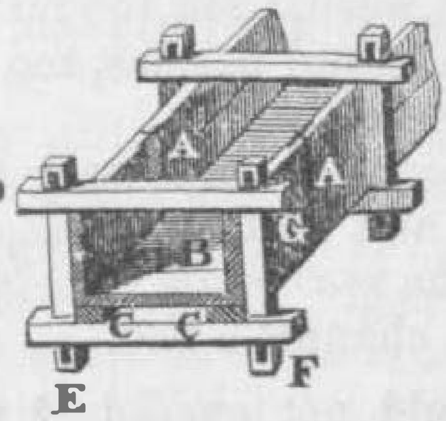
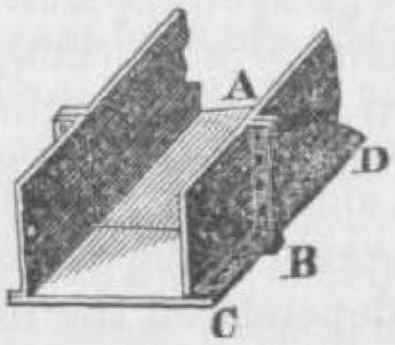


Fig. 156.



termed a *rust-joint* (a cement composed of sal ammoniac and iron filings or turnings).

The junction of a water-course with a river *AA*, Fig. 157, should be gradually widened and rounded off, and the head *D* substantially finished, so that it may not be injured by freshes, or objects carried against it in time of floods. Flood-gates or sluices have to be arranged along the course, if this be of any considerable extent. These sluices should be made self-acting, that no damage may be done to

Fig. 157.

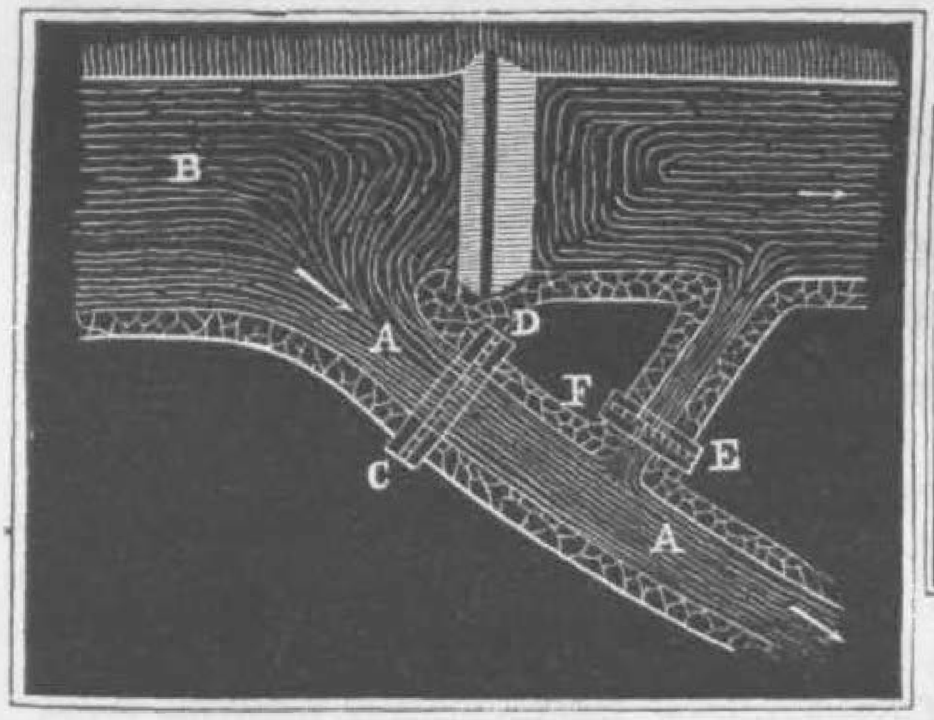
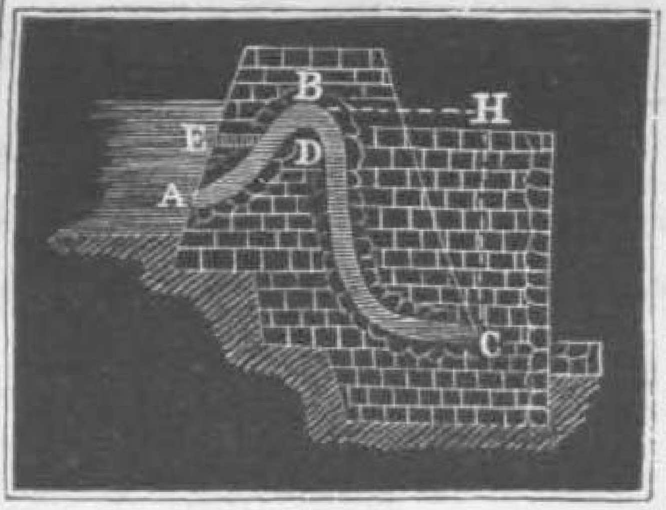


Fig. 158.



the banks by even a momentary overflow (the self-acting sluices on *Shaw's water-works*, in Scotland, is the most notable case of this self-acting arrangement on record). They act generally by a float being raised as the water in the channel rises, which float opens a valve or sluice to discharge the surplus water in convenient localities. Sometimes a case fills as the water rises, overcomes a counterbalance, and in its descent opens a valve or sluice, by which the surplus water is discharged. The syphon, properly adapted, forms a simple contrivance, and is shown in Fig. 158, where *ABC* is the syphon with an air-pipe *DE*. When the water in the water-course rises to the height of the summit of the syphon, which is the highest point for safety, the syphon fills with water, and the water is drawn off and discharged at *C*, the head being *CH*, the depth of *C* under the water surface. When the water has sunk to the level of *DE*, the air rushes in and stops the action of the syphon. If the water does not fill the section BD of the pipe, the discharge is made under the conditions of a weir.

§ 76. The velocity of the water in a water-course should be neither too slow, for then the course chokes with weeds; nor too fast, for then the bed of the channel may be disturbed; and besides, too much fall must not be lost in the inclination of the course.

A velocity of 7 to 8 inches per second is necessary to prevent deposit of slime and growth of weeds, and 1¼ feet per second is necessary to prevent deposit of sand. The *maximum velocity* of water in canals depends on the nature of the channel's bed.

On a	slimy bed the	velocity should	not exceed	¼	foot.
"	clay	"	"	½	"
"	sandy	"	"	1	"
"	gravelly	"	"	2	"
"	shingle	"	"	4	"
"	conglomerate	"	"	5	"
"	hard stone	"	"	10	"

This applies to the mean velocity.

From the assumed mean velocity c, and the quantity of water to be led through the course Q, we have the section F, and hence the perimeter p of the water section. If we put this in the formula $\delta = \frac{h}{l} = \zeta \cdot \frac{p}{F} \cdot \frac{c^2}{2g}$ (Vol. I. § 367), we get the required inclination δ of the canal, and hence the fall required for the lead, whose length $= l$ is $h = \delta\, l$.

The inclination may, therefore, be very different according to circumstances. As, however, ζ as a mean is 0,007565, and c generally from 1 to 5 feet, and $\frac{p}{F}$ is something between $\frac{1}{5}$ and 2, the limits of the inclinations for the water-course would be

$$0{,}007565 \cdot \tfrac{1}{5} \cdot 1 \cdot {,}0155 = 0{,}000023, \text{ and}$$
$$0{,}007565 \cdot 2 \cdot 25 \cdot {,}0155 = 0{,}00578,$$

the courses leading from the machine have a greater fall, that the machine may be quite clear of back-water. The course leading from the machine is usually termed the *tail-race.*

Remark 1. The water-courses for the water wheels and general uses of the Freyberg mining districts, have inclinations varying from $\delta = 0{,}00025$ to $\delta = 0{,}0005$, or from 15 inches to 30 inches per mile, the tail-races generally .001 to .002. The Roman aqueduct, at Arcueil, near Paris, has an inclination $\delta = 0{,}000416$, or 2 feet per mile nearly. The New River, which supplies a great part of London, has an inclination $\delta = 0{,}00004735$. [The Croton aqueduct has $\delta = 0{,}000208$, or 1,1 foot; and the Boston aqueduct 0,0000473̄5, or 3 inches per mile, the same as New River.]—AM. ED.

Remark 2. All sudden changes of sectional area and of direction are to be avoided, because these not only occasion loss of fall, but entail other bad effects in the way of wear and tear and deposits. Bends or curves should have as great a radius as possible, or *the sectional area* should be increased there. If r be the mean width of the course, and R the radius of curvature, the fall lost by a curve may be calculated, according to Vol. I. § 334, by the formula:

$$h_r = \left[0{,}124 + 3{,}104 \left(\frac{r}{R}\right)^{\frac{7}{2}}\right] \frac{\beta^\circ}{180^\circ} \cdot \frac{c^2}{2g},$$

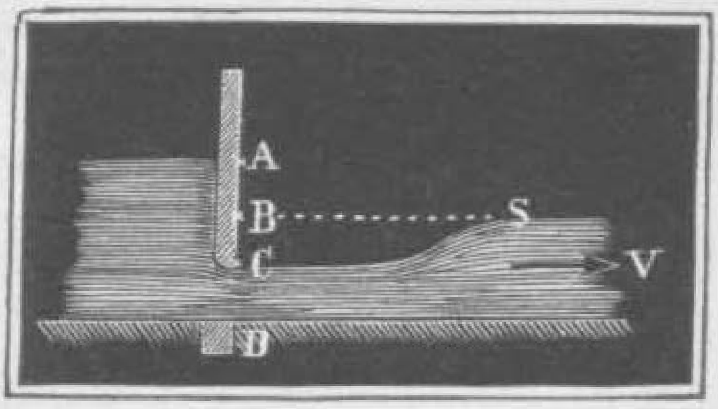
until we have further experimental data.

Remark 3. The deposit of slime, sand, and the growth of plants, diminishes the section of water-courses, and fall is thereby lost. The water-courses must, therefore, be carefully cleaned out from time to time.

§ 77. *Sluices.*—The entrance of water into a water-course is either free, or regulated by a sluice. If the water enter unimpeded from the weir-dam or reservoir, in which it may be considered to be *still*, the surface of the water sinks where the flow commences, and the depression is proportional to the initial velocity in the water-course, and therefore $= \frac{v^2}{2g}$, which height must be deducted from the total *fall* of the water-course. For moderate velocities of 3 to 4 feet per second, this depression amounts to only $1\frac{1}{2}$ to 3 inches.

If the entrance of water into the lead be regulated by a sluice, two distinct cases may present themselves. Either the water flows freely through the sluice, or it flows into and against the water of the lead. It will generally be found that the depth of the water in the lead, is greater than the height of the sluice-opening, and, therefore, there occurs a sudden rise S at a certain distance from the sluice AC, Fig. 159. The height $BC = x$ of this rise is a function of the velocity v of the water in the lead, and of the velocity v_1 of the water coming up to the sluice, such that

Fig. 159.



$x = \frac{v_1^2}{2g} - \frac{v^2}{2g}$, and if we deduct this height from that due to the velocity v_1, or $AC = h = \frac{v_1^2}{2g}$, then the head causing the initial velocity v_1 is:

$$AB = h_1 = h - x = \frac{v_1^2}{2g} - \left(\frac{v_1^2}{2g} - \frac{v^2}{2g}\right) \frac{v^2}{2g},$$

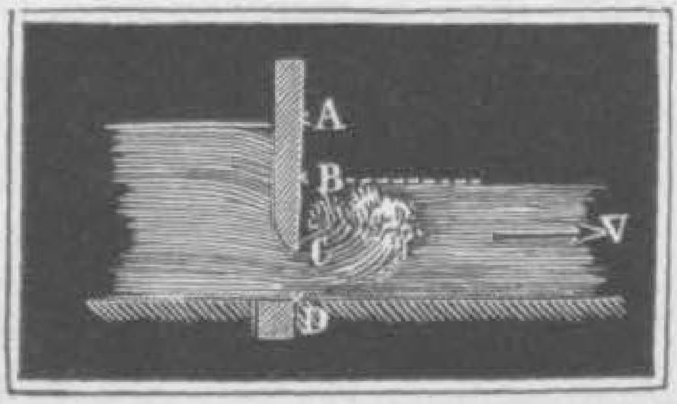
or exactly the same as if the water were discharging freely. As the sluice-opening is never perfectly smooth, there is, of course, a certain resistance increasing the head required by 10, or even more, per cent.

If we put $G =$ the area of the section of the water flowing in the lead, and $F =$ the area of the sluice-opening CD, then $Gv = Fv_1$, and, therefore, the rise

$$x = a - a_1 = \left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g},$$

and substituting for $\frac{v_1^2}{2g}$ the height due to the velocity or the head $AC = h$, $x = \left[1 - \left(\frac{F}{G}\right)^2\right] h$. If the difference $x = a - a_1$ of the depth of water a and a_1 be less than $\left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g}$, the rise occurs further *down* the lead; but, if it be greater, then the rise occurs nearer the sluice, till at last the discharge takes place under back-water, as shown in Fig. 160. In this case, the head $AB = h$ has not only to produce the velocity v in the water of the lead, but also to overcome the resistance arising from the sudden change of the velocity v_1 into the velocity v of the lead. If we put $F =$ the area of the opening, and $G =$ the area of the lead, the loss of head occasioned by this transition is:

Fig. 160.



$$= \frac{(v_1 - v)^2}{2g} = \left(\frac{G}{F} - 1\right)^2 \frac{v^2}{2g},$$

and hence the fall:

$$AB = h = \frac{v^2}{2g} + \left(\frac{G}{F} - 1\right)^2 \frac{v^2}{2g},$$

$$\text{hence} h = \left[1 + \left(\frac{G}{F} - 1\right)^2\right] \frac{v^2}{2g}.$$

It is obvious that the difference of level of the water before and behind the sluice, is so much the greater, the smaller the sluice-opening F in proportion to the section of the water in the lead G.

Example. A lead of 5 feet mean width, and 3 feet depth, supplies 45 cubic feet per second. It is fed through a sluice 4 feet wide, and 1 foot opening. Required how much higher the water will stand, before the sluice than behind it. $G = 5 \times 3 = 15$ square feet. $F = 4 \times 1 = 4$ square feet; $v = \frac{45}{15} = 3$ feet per second, and $v_1 = \frac{3 \cdot 15}{4} = \frac{45}{4}$ $= 11\frac{1}{4}$ feet.

Now as $\left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g} = [1 - (\frac{4}{15})^2]\ 2{,}02 = 1{,}88$ feet is less than $a - a_1 = 3 - 1$ $= 2$ feet, it is evident that there will not be a *free discharge*. The formula $h = \left[1 + \left(\frac{G}{F} - 1\right)^2\right] \frac{v^2}{2g}$ gives the difference of level required $h = (1 + 2{,}75^2)\ 0{,}139 = 8{,}56 \times 0{,}139 = 1{,}19$ feet, which must, however, be increased 10 per cent. at least, on account of the resistances at the opening.

§ 78. *Pipes, Conduit Pipes.*—Pipes are usually employed when smaller quantities of water are to be brought to supply machines, such as the water-pressure engine, and turbines of very high fall. They have the advantage of much greater *pliability* than open conduits, but their adoption instead of open canals, depends entirely on local circumstances in the question of relative advantage.

Pipes are made of wood, of pottery, of stone, of glass, iron, lead, &c. Wooden and iron pipes are those most usually employed in connection with water-power engines. Wooden pipes are usually formed from large trees, because straight pipes of 12 to 20 feet in length, and from $1\frac{1}{2}$ to 8 inches *bore*, or internal diameter, may be got from this timber. The bore is generally $\frac{1}{3}$ of the diameter of the tree. Wooden pipes are jointed or connected together as shown in Figs. 161 and 162.

Fig. 161 is a conical mortice with a *binding ring* and *packing* of hemp, or linen steeped in tar and oil. Fig. 162 is a connection by

Fig. 161.

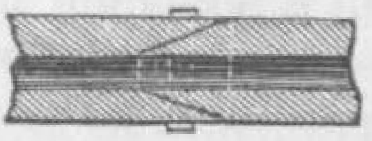
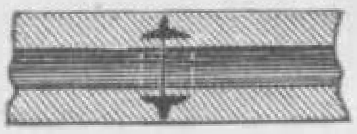


Fig. 162.



means of an iron double spigot going from 1 to 2 inches into the ends of the two pipes.

Iron pipes are the most durable and most universally employed of all pipes. They are cast of any diameter, and have been used as large as 5 feet bore. The length of each pipe rarely exceeds 12 feet, and is less as the diameter is greater. For 3 feet diameter, they are about 9 feet long each, in England. To prevent internal oxidation, they are sometimes boiled in oil, sometimes lined with wood, or with Roman cement. The thickness of metal must be proportional to the pressure they have to bear, and to their diameter, according to Vol. I. § 283. The jointing of iron pipes is effected either by flanges and bolts, as shown in Fig. 163, there being an annular packing between the flanges, or by the *spigot* and *faucet*, as shown in Fig. 164, (which is considered the best and cheapest mode,

Fig. 163.

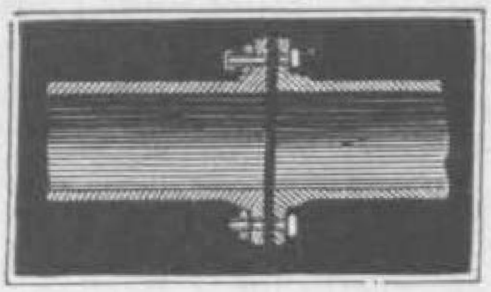
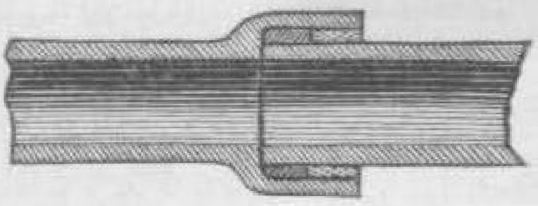


Fig. 164.



when the packing is properly done with small folding wedges of hard wood.) A collar, or ring, as shown in Fig. 165, is sometimes used. The packing is either leather, felt, lead, iron rust, or wood. The more effectually to prevent all leakage, there is sometimes a small internal ring put in (counter-sunk) to cover the joint. A flexible joint, as shown in Fig. 166, is sometimes necessary (as for crossing a river, where it is necessary to let the pipe rest on the original bed of the river). Where the pipes are exposed to changes of temperature, expansion joints, as shown in Fig. 167,

Fig. 165.

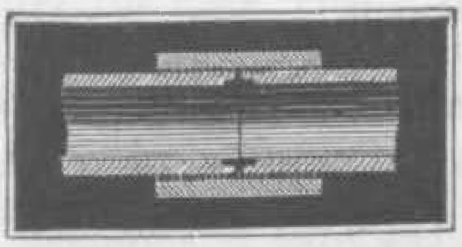


Fig. 166.

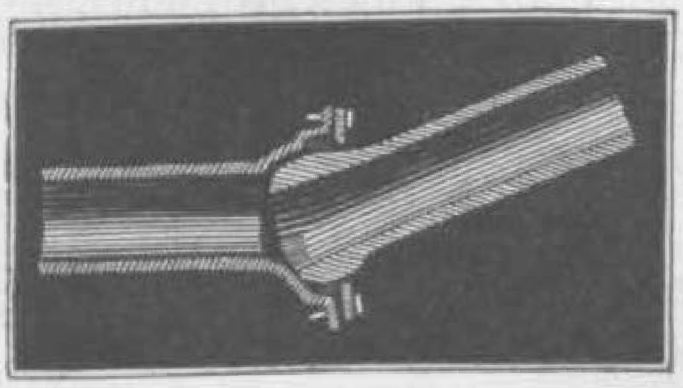
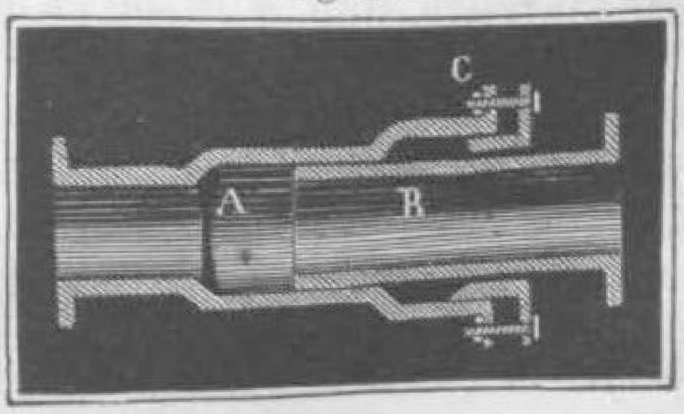


Fig. 167.



must be introduced, that the expansion and contraction of each considerable length may not injure the pipes or joints. The expansion of cast iron is .0000111 of its length, for each degree of centigrade; and, therefore, for a change of temperature of 50°, or from winter frost to summer heat, the expansion would be 0,000553. Therefore, for every 900 feet, there is an expansion and contraction of 6 inches. This is to be *compensated* by an arrangement, such as is shown in our last figure, where the pipe *B* is movable through the water-tight stuffing box *C*. There should be a compensation joint for every length of 300 feet exposed to a change of temperature.

§ 79. Pipes cannot of course be laid so as to maintain a straight line; but rise and fall, and turn from right to left in their course. It is a general maxim to avoid all *sudden* changes of direction in laying pipes. All bends should be effected by curved pipes, of as great radius as possible, or the bore of the pipe should even be increased at bends, to avoid loss of *vis viva* in the water. When a pipe bends over an elevation, as in Fig. 168, there is a disadvantage arises from the collection of air at *L*, as this contracts the section, and would gradually stop the flow of water. To prevent this accumulation of air, vertical pipes *AL*, called *ventilators* or *wind pipes*, are placed on the summit of the pipe, through which air, or other gases given off by the water, can be discharged from time to time, by means of a cock, to be turned by the inspector of the pipes. To make these ventilators self-acting, the arrangement shown in Fig. 169 has been adopted. In this ventilator the discharge valve *V* is connected with a float *S* of tinned iron, which is pressed upwards as long as it is surrounded by water, and thus keeps the valve shut, but falls or sinks downwards when the space about it becomes filled with air, and then the valve is opened to discharge the air. As air collects at the highest points of a conduit pipe, so the sand or slime collects at the lowest points. To remove any deposits of this nature, *waste-cocks* are placed at these points, by which the pipe is scoured, or separate receptacles for the deposits are attached to the pipes, and these are cleared from time to time, as may be found necessary. The deposit is favored by the greater section of these receptacles, and sometimes by the introduction of check or division plates, which still more retard the flow.

Fig. 168.

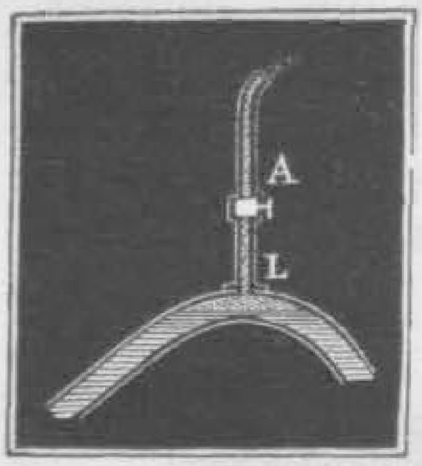


Fig. 169.

Cocks for *flushing* the pipes are introduced more or less frequently, according to the purity of the water, and the rate of flow through the pipes, and seldom at less intervals than 100 feet. For ascertaining the point in the pipe where any obstruction has occurred, *piezometers* (Vol. I. § 344) are very useful.

For regulating the discharge of water through pipes, cocks and slides, and valves are used. The effect of these has been shown in Vol. I. § 340, &c. In order to moderate the effects of the impulse or shock arising on the sudden closing of a cock, or other valve, it is useful to have a loaded safety valve, so placed that it will open outwards when the pressure exceeds a certain limit.

Remark. The most detailed treatise on the subject of conduit pipes, is Geniey's "*Essai sur les Moyens de conduire, d'élever, et de distribuer les eaux.*" Matthew's "*Hydraulia,*" and the "*Civil Engineer and Architect's Journal,*" contain much useful information on this subject. Hagen, "*Wasserbaukunst,*" Vol. I. has a chapter on water pipes.

§ 80. The general conditions of motion in conduit pipes have been already discussed. If h be the fall, and l the length, d the diameter of the pipe, ζ the co-efficient of resistance at entrance, ζ_1 the co-efficient for friction in the pipe, and ζ_2, &c., the co-efficients for resistances in passing bends, cocks, &c., and if v be the velocity of discharge, we have:

$$h = \left(1 + \zeta + \zeta_1 \frac{l}{d} + \zeta_2 + \ldots\right) \frac{v^2}{2g},$$

and if Q be the quantity of water:

$$h = \left(1 + \zeta + \zeta_1 \frac{l}{d} + \zeta_2 + \ldots\right) \left(\frac{4\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,d^4}.$$

We see from this, that for carrying a certain quantity of water Q, so much less fall is requisite, the greater the width of the lead. If there be two pipes instead of one, the two together having an area equal to the one, and supposing each to take half the whole quantity of water, the fall necessary is:

$$h_1 = \left(1 + \zeta + \zeta_1 \frac{l}{d\sqrt{\frac{1}{2}}} + \zeta_2 + \ldots\right) \left(\frac{2\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,(d\sqrt{\frac{1}{2}})^4}$$

$$= \left(1 + \zeta + \zeta_1 \cdot \frac{l\sqrt{2}}{d} + \zeta_2 + \ldots\right) \left(\frac{4\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,d^4};$$

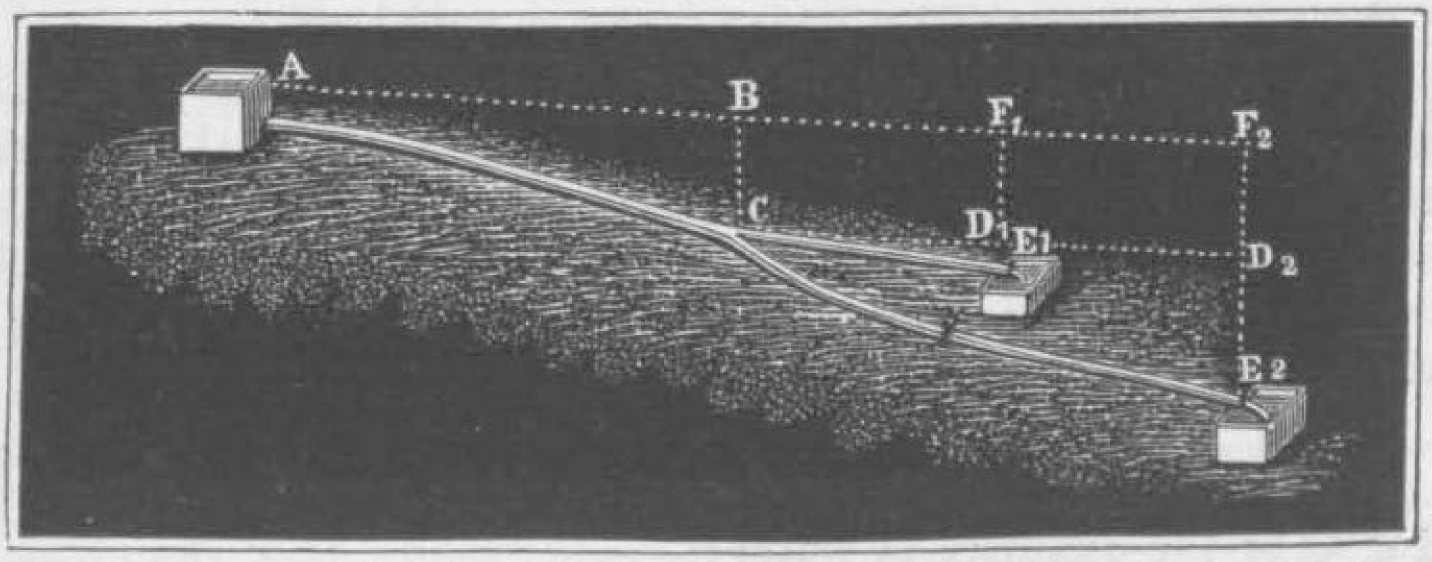
so that in this case the fall is greater, or the head required is greater, so that it is mechanically better to employ one large pipe, than two smaller of equal section when united.

Calculations for whole systems of pipes, where there are numerous subdivisions of branches, become exceedingly complicated. The case in which water is brought from different sources, and the pipes ultimately united, is of the same nature.

The general nature of such calculations is as follows. If the subdivision takes place in a reservoir which has a much greater sectional area than the *main* pipe, the water comes there again to rest, or the whole *vis viva* is destroyed, and has to be acquired again in the branch pipes. The same loss of *vis viva* occurs when several branches come together in a reservoir, from which one main pipe carries off their waters. In this case, the calculation reduces itself to a separate consideration of each branch and pipe, and requires no further elucidation. The collecting reservoir should be, if possible, placed at such levels as will ensure the same mean velocity in all the pipes, in order that the loss of head or of *vis viva* may be the least possible.

In the case of a simple subdivision or *fork*, it is mechanically advantageous to make such arrangements that the water may move in all the pipes with the same velocity. If, besides this, the branches be curved off properly, so that there is no sudden change of direction in the passage of the water from the main into the branches, it may be assumed that there is no loss of head or *vis viva*. In the case sketched in Fig. 170, let $h =$ the head BC, l the length, and

Fig. 170.



d the diameter of the main pipe, and let $h_1 =$ the head or fall D_1E_1, l_1 the length, and d_1 the diameter of the one branch, and $d_2 = D_2E_2$, l_2, and d_2, the fall, length, and diameter of the other branch, and also let c, c_1, c_2, be the velocities of the water in these three branches, and, lastly, let ζ be the co-efficient of resistance for entrance, and ζ_1 the co-efficient for friction of the water. Then, for the length of pipes ACE_1, we may put:

1. $F_1E_1 = BC + D_1E_1 = h + h_1 = \left(\zeta + \zeta_1 \frac{l}{d}\right)\frac{c^2}{2g} + \left(1 + \zeta_1 \frac{l_1}{d_1}\right)\frac{c_1^2}{2g}$ and

for the length of pipes ACE_2:

2. $F_2E_2 = BC + D_2E_2 = h + h_2 = \left(\zeta + \zeta_1 \frac{l}{d}\right)\frac{c^2}{2g} + \left(1 + \zeta_1 \frac{l_2}{d_2}\right)\frac{c_2^2}{2g}$.

But the quantity of water $Q = \frac{\pi d^2}{4} c$ of the main pipe, is equal to the sum of the quantities $Q_1 = \frac{\pi d_1^2}{4} c_1$, and $Q_2 = \frac{\pi d_2^2}{4} c_2$ of the two branches; and hence we may put:

$$3. \quad d^2 c = d_1^2 c_1 + d_2^2 c_2.$$

By aid of these three equations, three quantities may be determined. The more usual case is, that of the fall, the length and the quantity of water being given, the necessary diameter of the pipe is required. If, then, we assume a certain velocity c in the main, we get the width of this pipe by the formula:

$d = \sqrt{\frac{4\,Q}{\pi\,c}}$, and we have then only to solve the equations:

$$2\,g\,(h + h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2 = \left(1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{4}{\pi}\frac{Q_1}{d_1^2}\right)^2, \text{ and}$$

$$2\,g\,(h + h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2 = \left(1 + \zeta_1 \frac{l_2}{d_2}\right)\left(\frac{4}{\pi}\frac{Q_2}{d_2^2}\right)^2.$$

By transformation, we get similar equations for determining d_1 and d_2, as in Vol. I. § 332, viz.:

$$\zeta_1 \frac{l_1}{d_1^5} + \frac{1}{d_1^4} = \left[2\,g\,(h+h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2\right] \left(\frac{\pi}{4\,Q_1}\right)^2, \text{ and}$$

$$\zeta_1 \cdot \frac{l_2}{d_2^5} + \frac{1}{d_2^4} = \left[2\,g\,(h+h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2\right] \left(\frac{\pi}{4\,Q_2}\right)^2;$$

we can, therefore, as in Vol. I. § 332, put:

$$d_1 = \sqrt[5]{\frac{\zeta_1 l_1 + d_1}{2\,g\,(h+h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2} \cdot \left(\frac{4\,Q_1}{\pi}\right)^2}, \text{ and}$$

$$d_2 = \sqrt[5]{\frac{\zeta_1 l_1 + d_1}{2\,g\,(h+h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2} \cdot \left(\frac{4\,Q_2}{\pi}\right)^2},$$

and in order to obtain a first approximation to the values of d_1 and d_2, we may omit these from the part under the radical. If c_1 and c_2 come out to be very different from c, attention must be paid to the co-efficient ζ_1, being variable, and its value for each of the pipes introduced, and the determination of d_1 and d_2 repeated.

Example. A system of pipes, to consist of one main and two branches is intended to carry 15 cubic feet of water per minute by one branch, and 24 cubic feet by the other. The levels showed that in a length of 1000 feet of main, the fall was 4 feet, the first branch had a fall of 3 feet in 600 feet length, and the second 1 foot in 200 feet. What must be the diameters of the pipes respectively? If we suppose a velocity of $2\frac{1}{2}$ feet per second in the main, then its diameter

$d = \sqrt{\frac{4\,Q}{\pi\,c}} = \sqrt{\frac{4\,.\,39}{\frac{5}{2}\,.\,60\,\pi}} = \sqrt{\frac{26}{25\,\pi}} = 0{,}5754$ feet $= 6{,}9$ inches. If now (according to Vol. I. § 436), we put the co-efficient of resistance for entrance $\zeta = 0{,}505$, the co-efficient of friction (Vol. I. § 435) for velocity $c = 2{,}5$ feet, $\zeta_1 = 0{,}0253$, and as $2\,g = 64{,}4$, and $\left(\frac{4}{\pi}\right)^2 = 1{,}621$, we have for the diameter of the branches

$$d_1 = \sqrt[5]{\frac{0{,}0253\,.\,600 + d_1}{64{,}4\,.\,7 - (0{,}505 + 0{,}0253\,.\,1738)\,.\,\frac{25}{4}}\,.\,1{,}621\,.\,\left(\tfrac{15}{60}\right)^2}$$

$$= \sqrt[5]{\frac{15{,}18 + d_1}{450{,}8 - 277{,}98}\,.\,0{,}1013} = \sqrt[5]{\frac{15{,}18 + d_1}{1706}}, \text{ and}$$

$d_2 = \sqrt[5]{\frac{0{,}0253\,.\,200 + d_2}{322{,}0 - 277{,}98}\,.\,1{,}621\,.\,\left(\tfrac{24}{60}\right)^2} = \sqrt[5]{\frac{5{,}06 + d_2}{169{,}7}}$. If we first neglect d_1 and d_2 under the radical, we get the approximate values $d_1 = \sqrt[5]{\frac{15{,}18}{1706}} = 0{,}39$ feet, and $d_2 = \sqrt[5]{\frac{5{,}06}{169{,}7}} = 0{,}495$ feet. If we now introduce the value on the right-hand side of the equation, we get more accurately $d_1 = \sqrt[5]{\frac{15{,}57}{1706}} = 0{,}391$ feet and $d_2 = \sqrt[5]{\frac{5{,}555}{169{,}7}} = 0{,}505$ feet. The diameter $d_1 = 0{,}391$ corresponds to a velocity

$$c_1 = \tfrac{15}{60}\,.\,\frac{4}{\pi\,d_1^2} = \frac{1}{0{,}391^2\,.\,\pi} = 2{,}082 \text{ feet},$$

and the diameter $d_2 = 0{,}505$ corresponds to

$$c_2 = \tfrac{24}{60}\,.\,\frac{4}{\pi\,.\,0{,}505^2} = 1{,}997 \text{ feet},$$

and hence we should have more accurately for the first branch pipe $\zeta_1 = 00263$, and for the other $\zeta_2 = 0{,}0270$, and hence with the best accuracy which the formula admits

$$d_1 = \sqrt[5]{\frac{0{,}0263 \cdot 600 + 0{,}391}{1706}} = \sqrt[5]{\frac{16{,}171}{1706}} = 0{,}394 \text{ feet} = 4{,}7 \text{ inches, and}$$

$$d_2 = \sqrt[5]{\frac{0.0270 \cdot 200 + 0{,}505}{169{,}7}} = \sqrt[5]{\frac{5{,}905}{169{,}7}} = 0{,}511 \text{ feet} = 6{,}13 \text{ inches.}$$

CHAPTER IV.

OF VERTICAL WATER WHEELS.

§ 81. *Water Power.*—Water acts as a *moving power*, or *moves machines* either by its *weight*, or by its *vis viva*, and in the latter case it may act either by *pressure* or by *impact*. In the action of water by its weight, it is supported on some surface connected with the machine, that sinks under the weight; and in the action by its *vis viva* it comes against a surface yielding to it, in a horizontal direction generally, which is, in like manner, an integral part of the machine. If Q be the quantity of water (or $Q\gamma$ the weight of water) available as power, per second, and h, the *fall*, or the perpendicular height through which the water falls in giving out its mechanical effect, then the mechanical effect produced is: $L = Q\gamma \,.\, h = Q h \gamma$. If, again, c be the velocity with which the water comes upon any machine, the mechanical effect produced by its *vis viva*, is:

$$L = Q\gamma \frac{c^2}{2g} = \frac{c^2}{2g} Q\gamma.$$

That water may pass from rest to the velocity c, a fall, or height due to the velocity $h = \frac{c^2}{2g}$ is necessary, and, therefore, in the second instance we may also put $L = h\, Q\gamma$. So that *the mechanical effect inherent in water is the product of its weight into the height from which it falls*, as in the case of other bodies.

Water sometimes acts by its weight and *vis viva* simultaneously, by combining the effects of an acquired velocity c, with the fall h through which it sinks on the machine. In this case, the mechanical effect produced is again:

$$L = Q\gamma \,.\, h + Q\gamma \frac{c^2}{2g} = \left(h + \frac{c^2}{2g}\right) Q\gamma.$$

The mechanical effect Pv yielded by a machine is of course always less than the above available mechanical effect $Q h \gamma$; because many *losses* occur. In the first place, *all the water* cannot always be brought to work; secondly, a part of the fall is generally lost; thirdly, the water retains a certain amount of *vis viva* after having quitted the machine; and, fourthly, there are the passive