A Remark on Construction of BIB Designs with v = 2k

by

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ABSTRACT

A unified method of construction of BIB designs with $v = 2k$ is described. The method follows from well-known properties of BIB designs and Hadamard matrices. However the existing literature fails to point out this method of construction. It is also noticed that this method may yield a variety of nonisomorphic BIB designs with the same parameters which may prove especially interesting to some researchers or experimenters.

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There are listings available in the literature of BIB designs with $r \leq 20$. Unfortunately, the authors do not make an effort to improve essentially on the emmunerations of their predecessors. The following observation may serve in support of the above statement. The listings include usually a source and method of construction of a BIB design with the parameters in question. This information is generally repeated and no information is incorporated which is available since the preceding listing was published. Here is one example of such a situation. For BIB designs with $v = 2k$, D. A. Preece (1967) enumerated nonisomorphic designs for $8 \leq v \leq 16$. The authors of subsequent books do not include these results in their listings of the BIB designs.

For unexplained reasons, the BIB of the type $v = 2k$ with $v = 14$, $b = 26$, $k = 7$, $r = 13$ does not appear in the available listings in spite of the fact that Preece gives twelve nonisomorphic designs with these parameters.

The purpose of this note is to point out further that if one intends to list just one source and method of construction for each set of the parameters of a BIB design then for $v = 2k$, $b = 2(2k-1)$ it seems reasonable to use a unified method rather than resort to some special construction available in the literature.

Presently such a method will be described which, although known to people working in construction of designs, is not mentioned as a method of construction. Consider a Hadamard matrix of size $4k$. Normalize this matrix and then pick any column of this normalized matrix and delete all rows which have in this column the element appearing in the normalizing row and column. Now delete the normalizing row and column and the chosen column. The remaining $2k \times 2(2k-1)$ matrix is an incidence matrix of a BIB design with arbitrary identification of the two distinct elements with 0 and 1. The parameters of the design were clearly $v = 2k$, $b = 2(2k-1)$, $r = 2k-1$. The deleted rows, excluding the normalizing column and the chosen column, form an incidence matrix of a BIB design with $v = 2k-1$, $b = 2(2k-1)$, $r = 2(k-1)$. This method of construction follows clearly from statements made often in the literature which read as follows: A normalized Hadamard matrix gives, after deleting the normalizing row and column, a symmetric BIB with $v = b = 4t-1$ and $r = 2t$. Usually the authors fail to mention that the derived design and its complement yield the designs described above.

In conclusion, it may be worthwhile to point out that by using different columns or no isomorphic Hadamard matrices of the same size it may be possible to obtain a variety of nonisomorphic BIB designs and some of them may have an interesting structure which could prove of value in construction of designs of particular interest to a researcher.

Remark: It may be worthwhile investigating which of the designs constructed by Preece could be augmented to a Hadamard matrix. This could yield new construction of some Hadamard matrices not available in the literature.

References

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