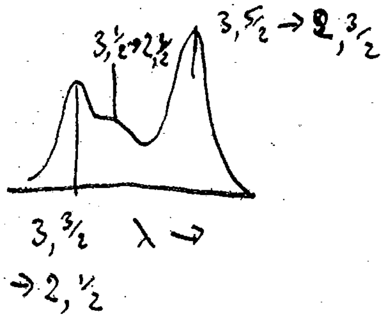
Lamb

Notes on Shelter Island Conference

$0.365 \text{ cm}^{-1} = 11{,}000$ Mc



Kemble & Present

to explain 0.0[illegible] cm^{-1} raising ~~shift~~ of S level

cutting Coulomb potential off flat, or replace by infinite repulsion: Needs range 10^{-12} cm of repulsion

Houston } Deuterium spectrum
Williams }

interpreted by Pasternack: Raise S level by .030 cm^{-1}

Richardson, Drinkwater & Williams:
Could be H impurity in D

Positron theory Uehling 1936

In region $5 \cdot 10^{-11}$ cm, potential changed; effectively increased attraction (because decrease by polarization is less at high wave number ~~frequency~~ than for slowly varying fields), so S state lower than $P_{1/2}$; also magnitude of shift is much less than required.

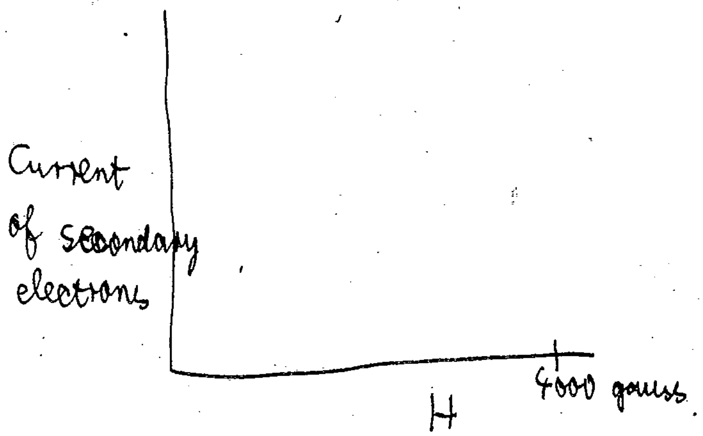
Lifetime of $S_{1/2}$ level about 2 × life of P levels

Apparatus: $H^2 \rightarrow H \rightarrow H^* \rightarrow$ Interaction space with RF → Detector

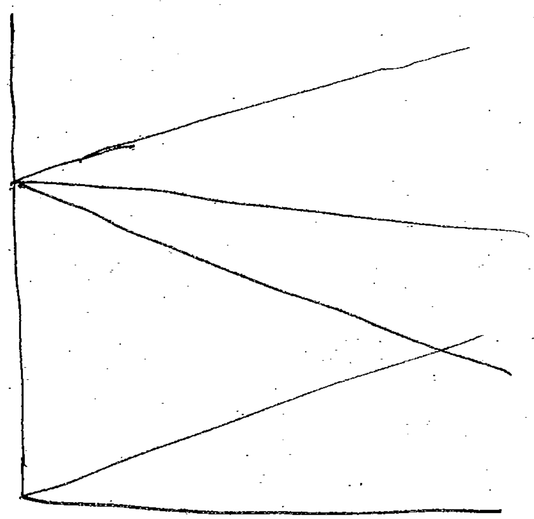
Detection: Secondary electrons can be ejected from W by metastable H atoms. About $\frac{1}{10}$% efficiency

Excitation by a cross current of electrons

Magnetic field (few 100 gauss) to split S and P levels



Frequency set, observe current as fn. of H
λ between 2.6 and 3.6 cm



Expected Zeeman frequencies

Result: S level shifted up by about 0.037 cm^{-1} (1100 Mc)

Present width of lines probably increased by RF interaction

Oppy Positron theory (wave length dependence)
" " (high field)
Nuclear interaction
Electrodynamic term shift e^2

$$\sum_{p,r} \frac{S_{ir} S_{ri}}{(E_p + E_r - E_i) E_p}$$ E_p = energy

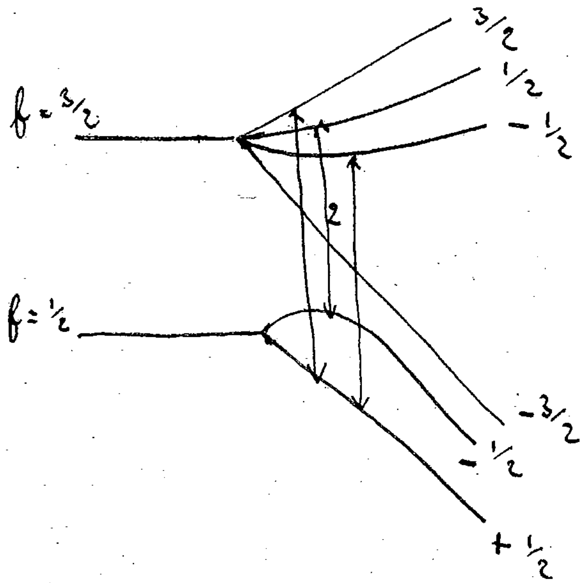
First term $\frac{S_{ir} S_{ri}}{E_p^2} = S_{ii}^2$ same for all levels

Second term $\frac{S_{ir} S_{ri} (E_r - E_i)}{E_p^3}$ ~~including~~ if Dirac theory is used also supposed to be same (Breit)

Third term $\frac{(E_r - E_i)^2}{E_p^4}$ converges and might explain Lamb's result.

Rabi   Nelson & Nape

Deuterium ground state



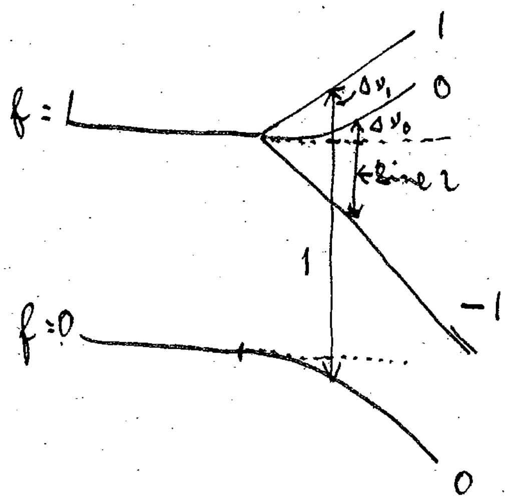
Line 2

$$\nu = \nu_D \left(1 + \frac{4}{9} x^2\right)$$

$$x = \frac{g \mu_0 H}{\nu_D}$$

$$\nu_D = 327.38_5 \pm 0.005 \text{ Mc}$$

Hydrogen



$$\nu_1 = \nu_H + \Delta\nu_1 + \Delta\nu_0$$

$$\nu_2 = \Delta\nu_1 + \Delta\nu_0$$

$$\nu_1 - \nu_2 = \nu_H$$

$$\nu_H = 1421.3 \pm 0.2 \text{ Mc}$$

$$h\nu_{theor.} = \frac{8\pi}{3} \frac{2I+1}{I} \mu_N \mu_0 \psi^2(0)$$

$$\nu = \frac{4}{3} \frac{2I+1}{I} \frac{g_N}{1836.6} \left(\frac{m_r}{m_0}\right)^3 \alpha^2 R_\infty$$

g_N = nuclear moment in ~~nuclear~~ magnetons

Calculated $\nu_H = 1416.9 \pm 0.54$ observed

$\nu_D = 326.53 \pm 0.16$

$\frac{\nu_H}{\nu_D} = 4.3393 \pm 0.0014$ 4.3416 ± 0.0007

$\frac{23}{42000} = 0.0005$

$\frac{4.4}{1400} \sim 0.3\%$

$\frac{0.85}{325} \sim 0.26\%$

$\frac{3/2}{3600} = 0.083\%$

Breit

$$\left\{ p_0 + \alpha_e \cdot p_e + \alpha_p \cdot P_p + \beta_e mc + \beta_p Mc - \frac{e^2}{2}\left[-\frac{\underline{\alpha}_e \cdot \underline{\alpha}_p}{r} + \frac{\underline{\alpha}_e \cdot \underline{r} \; \underline{\alpha}_p \cdot \underline{r}}{r^3} \right] \right\} \psi = 0$$

Teller Neutron - Electron

Interaction potential over range $\frac{e^2}{mc^2}$

0 ± 5 kv

First expts. Kr, later Xe

Corrections: Van der Waals Forces

Interaction of electric field of atom with neutron magnetic moment

Rabi Change of σ in liquid Pb about ≤ 0.3 b

out of 11 b total nuclear σ.

$$\sigma_e = \left(\frac{0.3}{2 \cdot 82 \cdot \sqrt{11}} \right)^2 \leq 3 \times 10^{-7} \text{ barns}$$

Acc. to Schwinger, this means ~ 10 kv interaction over $\frac{e^2}{mc^2}$

Mesons ~~Capture~~. (Rossi)

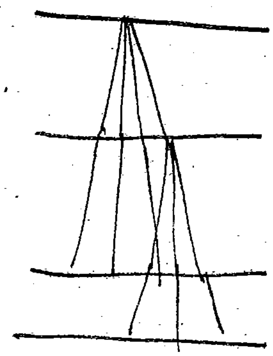
Capture! Oppy suggests $U_{meson} \varphi_{neutrino} \gamma^{\mu} \varphi_{electron} \cdot j^{\mu}_{nucleon}$

Production of mesons

Wataghin production of meson showers as fn. of height

Jansky barometer effect

Meson showers



apparently double showers frequent

∴ Showers not produced by mesons but probably nuclear particles.

a) Multiplicity cannot be proportional to energy because then altitude dependence should be the same at all latitudes

b) Multiplicity cannot be independent of energy because then altitude dependence should be stronger (?)

Kramers

$$\mathcal{H} = \frac{(p - \frac{e}{c}\underline{A})^2}{2m} + e\phi + \frac{1}{8\pi}\int(E^2 + H^2)\,d\tau$$

In external field

$$m\ddot{\underline{R}} = e\underline{E} + \frac{e}{c}\dot{\underline{R}} \times \underline{H}$$

Elimination of longitudinal field:

$$\underline{E} = \underline{E}_\perp - \nabla\phi$$

$$\nabla^2\phi = -4\pi\rho \qquad \mathrm{div}\,\underline{A} = 0$$

1. Replace m by m_0 (mechanical mass)
2. " $\tilde{\underline{A}}$ "at electron" by $\int\rho\,\underline{A}\,d\tau$
3. Try to get structure - independent part of theory
4. Don't integrate field equations yet (retarded or advanced potentials)

$$\underline{A} = \underline{A}' + \underline{A}_0 \qquad \underline{A}_0 = \text{proper field}, \ \underline{A}' = \text{external field}$$

$$\underline{A}_0(P) = T_r\ \frac{\dot{\underline{R}}}{c}\int\frac{\rho(Q)\,d\tau_Q}{r_{PQ}} \qquad T_r:\ \text{transverse part}$$

(so that $\nabla^2\underline{A}_0 = -\frac{4\pi}{c}(\rho\,\dot{\underline{R}})_{\text{transverse}}$
instead of $\Box\,\underline{A}_0 = \text{same}$)

At large distance, $A_0 = \frac{e}{2cr}\left(\dot{\underline{R}} + \frac{\underline{r}\,\underline{r}\cdot\dot{\underline{R}}}{r^2}\right)$

At electron, $\tilde{A}_0 = \frac{c}{e}\,\mu\,\dot{\underline{R}}$

[Definition $\mu\dot{\underline{R}} = \frac{1}{4\pi c}\int\underline{E}\times\underline{H}\,d\tau$ is identical with this]

$$m_0\ddot{\underline{R}} = -\frac{e}{c}\frac{\partial\underline{A}}{\partial t} + \frac{e}{c}\dot{\underline{R}}\times\mathrm{curl}\,A - \nabla U$$

$$= -\frac{e}{c}\frac{\partial\underline{A}'}{\partial t} + \frac{e}{c}\dot{\underline{R}}\times\mathrm{curl}\,\underline{A}' - \nabla U - \frac{e}{c}\frac{\partial A_0}{\partial t} + \frac{e}{c}\dot{R}\times\mathrm{curl}\,\tilde{\underline{A}}_0$$

$$\frac{e}{c}\frac{\partial \underline{A}_0}{\partial t} = \mu \ddot{R} + \text{a term which does not act on the electron itself, because it is spherically symmetrical.}$$

$\underline{R}_0 \times \operatorname{curl} \underline{A}_0$ disappears by averaging over electron

$$m\ddot{R} = -\frac{e}{c}\frac{\partial \underline{A}'}{\partial t} + \frac{e}{c}\dot{\underline{R}} \times \underline{H}' - \nabla U \qquad \text{Eq. of motion}$$

Field equn.

$$\Box \underline{A}' + \underbrace{\cancel{\Delta \underline{A}_0} - \frac{1}{c^2}\frac{\partial^2 \underline{A}_0}{\partial t^2} = -\cancel{\frac{4\pi}{c}} T_+}_{\text{equal by definition}}\left(\dot{\underline{R}}\,\rho\right)$$

$\frac{\partial^2 A_0}{\partial t^2}$ contains $\dot{R}^3$, $\dot{R}\ddot{R}$ and $\dddot{R}$

Put $\underline{A}' = \frac{1}{c}\frac{\partial Z''}{\partial t}$ $\quad$ (Z'' = Hertz potential)

$$\Box Z'' = \frac{1}{c}\frac{\partial A_0}{\partial t} \qquad \operatorname{div} Z'' = 0$$

$$\operatorname{curl}\operatorname{curl} Z'' = -\frac{1}{c^2}\frac{\partial^2 \underline{Z}''}{\partial t^2} - \frac{1}{c}\frac{\partial \underline{A}_0}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}(\underline{A}' + \underline{A}_0) = -\frac{1}{c}\frac{\partial \underline{A}}{\partial t} = \underline{E} = \underline{E}' + E_0$$

Lagrangian

$$L = \frac{1}{2}m_0\dot{R}^2 + \frac{e}{c}\dot{\underline{R}}\cdot\underline{A} - U(R) + \frac{1}{8\pi}\int\left[(\operatorname{curl}\underline{A})^2 - \frac{1}{c^2}\left(\frac{\partial A}{\partial t}\right)^2\right]d\tau + \frac{1}{4\pi c}\int\rho\,\nabla\phi\, d\tau$$

Last term necessary to give $\operatorname{div}\underline{A} = 0$ __from__ the equn. of motion.

Want to put $\underline{A} = \underline{A}_0 + \underline{A}'$, $\underline{A}_0 = \ldots\dot{\underline{R}}$

Must introduce supernumerary variables

$$\underline{v} = \dot{\underline{R}} \text{ defined as } \frac{P - \frac{e}{c}\dot{A}}{m_0} = \frac{P - \frac{e}{c}A'}{m} \quad \left(\text{where } P = \frac{\partial L}{\partial R} = m_0\dot{R}\right.$$

New Lagrangian

$$M = L - V\cdot P - \underline{R}\cdot\dot{\underline{P}} = M(A, \dot{A}, R, P, \dot{P})$$

__Then__ can replace $\underline{A}$ by $\underline{A}' + \underline{A}_0$ because now $\underline{A}_0$ depends only on

$\underline{P}$ and $\underline{R}$, no longer on velocity, ~~so that~~

If this is done, $\dot{A}_0$ term introduces $\dot{R}$ into the Lagrangian

$$M = -\frac{1}{2m}\left(p^2 - \frac{e^2}{c^2}\tilde{A}'^2\right) - U(R) - \dot{P}R - \frac{1}{8\pi}\int(H'^2 - E'^2)$$

$$-\frac{e}{c}\underline{V}\cdot\dot{R}\frac{\partial Z'}{\partial R}$$

$$V = \frac{P - \frac{e}{c}A'}{m}$$

Z' defined by curl curl $\underline{Z}' = \underline{E}'$

This gives equns. of motion correctly.

[Original $H^2 = H'^2 + 2\underline{H}'\cdot\underline{H}_0 + H_0^2$

still here disappears $\frac{1}{2}\mu v^2$

$E^2 = E'^2 + 2\underline{E}'\cdot\underline{E}_0 + E_0^2$

gives last term structure dependent but of higher order]

Bad thing: cannot introduce canonical variables

But $\mathcal{H} = \Sigma p\dot{q} - M$ is constant of motion

$$\mathcal{H} = \frac{1}{2m}\left(p^2 - \frac{e^2}{c^2}A'^2\right) + U(R) + \frac{1}{8\pi}\int(H'^2 + E'^2)d\tau + \frac{e}{c}V\,\underline{\dot{R}}\cdot\frac{\partial\tilde{Z}'}{\partial R}$$

because H does not contain time derivatives, and E' is a time derivative, ∴ change of sign of E'^2 term and

$$\text{Old } \mathcal{H} = \frac{1}{2m_0}\left(P - \frac{e}{c}A\right)^2 + U(R) + \frac{1}{8\pi}\int(H^2 + E^2)d\tau$$

Try to get canonical equns. by assuming $\frac{\partial M}{\partial\dot{R}}$ small

$$\frac{d}{dt}\left(\frac{\partial M}{\partial\dot{R}}\right) = \frac{\partial M}{\partial R}$$

If ~~this is~~ terms of $\dot{R}$ are left out, ~~added to~~ this adds to equn. of motion of electron the term

$$-\frac{e}{c^2}\frac{d}{dt}\frac{\partial}{\partial R}(\underline{Z}\cdot\underline{v})$$

which vanishes for dipole radiation

This neglect is now done. Then

canonical conjugate of P is not $-R$ but $-R' = -\left(R + \frac{e}{mc^2}Z'\right)$

" " " $\underline{A}'$ is $-\frac{1}{4\pi c}\underline{E}' + \frac{e^2}{mc^2}\underline{Z}'\delta(\underline{R} - \underline{r})$

By comparison:

Dirac

$$\frac{1}{2m}p^2 - \frac{e}{mc}\,\underline{p}\cdot\underline{A} + \frac{e^2}{2mc^2}A^2 + U(R) + \frac{1}{8\pi}\int (H^2 + E_\perp^2)\,d\tau$$

Emission and absorption in Dirac from $\underline{p}\cdot\underline{A}$

<u>Now</u> from U term which should be written in terms of $\underline{R}'$

$$U(R) = U\left(R' - \frac{e}{mc^2}Z'\right) = U(R') - \frac{e}{mc^2}Z\cdot DU + \frac{1}{2}\frac{e^2}{m^2c^4}(Z\cdot D)(Z\cdot DU)$$

$Z\cdot DU$ term replaces old $\underline{p}\cdot\underline{A}$

$DU = m\cdot$ acceleration $= k = \dot{p}$

$Z = \int A\,dt$

New term $\underline{Z}\cdot\underline{k}$ is $\frac{\nu_0}{\nu}$ times old term $A\cdot p$ (ν_0 atomic frequency, ν light ")

$\therefore$ absorption & emission unchanged

Dispersion theory complicated cancellation. E.g. Compton scattering:

Dirac $+\frac{e^2}{mc^2}A^2$, Kramers $-\frac{e^2}{mc^2}A^2$ but this is compensated by the fact that E' is not conjugate to A'. If E'' is the conjugate, then

$$E' = E'' + \frac{4\pi e^2}{mc}Z'\delta(R' - r)$$

$$\frac{1}{8\pi}\int E'^2 = \frac{1}{8\pi}E''^2 + \frac{e^2}{mc}\,\tilde{E}''\cdot\tilde{Z}'$$

This doubly overcompensates ~~d~~ A^2 term

If we went on to e^4 term, this would be infinite, i.e. ~~prop. to~~ $\delta^2(R' - r)$

This is structure dependent

Weisskopf

$$(H_0 + U - E)\psi_0 + e^2 H' \frac{1}{H_0 + U - E} H' \psi_0 = 0$$

Perturbation theory gives $\quad H' \frac{1}{H_0 - E} U \frac{1}{H_0 - E} H' \psi_0$

$\psi = \psi_0 + e\psi_1$

$H\psi_0 + e^2 H' \psi_1 = E\psi_0$

$H\psi_1 + H'\psi_0 = E\psi_1$

$\mathcal{H} = H + H'$, $H = T + V$

Von Neumann

Space rotational symmetry, quantization desirable

This not possible with classical variables, but known to be possible for angular momenta

Lorentz group, infinitesimal rotations Y_{ij}, ~~these~~ coordinates x_k $(x_1, x_2, x_3, x_4 = x, y, z, t)$, called Y_{0k}

$[Y_{ij}, Y_{0k}] = 0 \quad k \neq i, j$

$, Y_{0i} \quad Y_{0j}$

$Y_{0j} \quad Y_{0i}$

New relation $[Y_{0i}, Y_{0j}] = Y_{ij}$

1+4 dimensional Lorentz group

Displacement

$x_1 \to x_1 + a_1$, $x_2\ x_3\ x_4$ invariant. This is impossible

Let ~~by~~ $Y_{0i} \to Y_{0i} + a_i$, then $[Y_{0i}, Y_{0j}]$ invariant $= Y_{ij}$

$\therefore [Y_{ij}, Y_{0i}]$ invariant, $\therefore Y_{0j}$ invariant in contradiction to assumption

$[\cancel{H}.x_i, x_j]$ has same symmetry as angular momenta, but is fixed, while Y_{ij} depends on origin about which ang. mom. are taken

Possibilities

1. Relative coordinates. 2 particles OK, more particles questionable
2. Absolute coord: Lost invariance for inhomogeneous Lorentz group.

Oppy

Reversibility of Meson Production

Nordheim $\sigma(\gamma - M) \simeq \sigma(M - \gamma)$

Schwinger & Oppy

1) Scattering: $\sigma \sim a^2$
2) Production: ~~Large~~ deflection of protons. Shaking off of mesons on change of direction

$N \sim g^2 \int k\, dk$, $E_0 \sim g^2 \int^K \varepsilon\, k\, dk$

$N \sim E_0^{2/3}$ $K \sim 1/a$

Couplings: dipole $\sim 1/r^3$, number of mesons increases with energy

Pseudoscalar: Pseudovector coupling $V_k\,(\underline{\sigma}_1 \cdot \underline{\sigma}_2 - \gamma_1^5 \gamma_2^5)$

$$d\sigma = \sigma_0\, g^{2N} \prod \frac{d\vec{k}_n}{\varepsilon_n}$$ increasing multiplicity

Pseudoscalar coupling: $\dfrac{dk_n}{\varepsilon_n} \dfrac{k_{\perp n}^2}{E^2_{proton}(\varepsilon_n - \underline{v}_p \cdot \underline{k}_n)^2}$ finite multiplicity
(~~Pseudovector~~)

$$\sigma_N = \left(\frac{4g^2}{\pi}\right)^N \frac{N^2}{N!^3} (\varepsilon_1 \varepsilon_2)^{N-1} d\varepsilon_1\, d\varepsilon_2$$ (ε_1 = energy loss of 1st proton)

$$N = 4\left(\frac{E_0}{10^9 \text{ev}}\right)^{1/3}$$ for $g^2 = 1/2$

Total cross section assumed unaffected by meson emission.

Why is Schwinger - Oppy wrong?

$\sigma = a^2$ only for emission of single mesons; multiple meson production not small in strong coupling theory; this will actually occur if enough energy is available: ∴ multiple production in scattering

Present theories require ~~theories~~ couplings

$(A_- + C_+)^n$ A_- = annihilation of negative particle
C_+ = creation " positive "

$A_-^n + C_+^n$ would be necessary to avoid multiple processes produced by mesons; but this is probably not relativistically invariant (according to Oppy)

Excited nucleon states (hypothesis of Weisskopf)