Assessing the Effects of Icing the Body for 20 Minutes

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Executive Summary

Icing is one of the most inexpensive and convenient treatments available to reduce inflammation in sore and injured muscles. A commonly purported icing regimen follows a "20 minutes on, 20 minutes off" cycle, so we investigated how much skeletal muscle cools during the 20-minute icing period. To model the temperature distribution, we used an axisymmetric geometry consisting of five layers: the ice, a plastic bag, skin, subcutaneous fat, and muscle. Our initial results showed cooling of the most superficial muscle tissue by approximately 15°C. We found that changes in properties such as density, specific heat, and conductivity did not affect temperature contours at the 20-minute time point; however, heating via perfusion, which was initially neglected, had a substantial effect on the final results. When blood flow was introduced into the model, the temperature of superficial muscle decreased only 3.5°C. We thus conclude that although icing is an effective means of cooling superficial layers of muscle, it is not particularly efficacious at increasing depths.

1 Introduction

Cooling therapy is used as both a preventative and responsive treatment for skeletal muscle injuries, reducing metabolic rate, perfusion, inflammation, and pain. Commercial gel packs, cooling gels, and ice are among several available treatments. In this project we chose to model ice as the cooling agent for two reasons. First, it is the most effective means of cooling tissue, since ice requires extra energy transfer from the tissue to account for the latent heat of fusion associated with melting [1]. Second, it is the most popular and inexpensive way to treat its target injuries.

The principal treatment goal in ice therapy is to reduce muscle temperature enough to achieve the aforementioned effects while avoiding temperatures that would damage either the muscle or the tissues superficial to it (subcutaneous fat, skin). Symptoms of hypothermia first appear when core body temperatures hit 36.1°C, but since the area being iced is small with respect to total body area, grand-scale hypothermia will not occur. Local hypothermia is a concern, however, so here we assess this risk by determining the maximum muscle temperature at a depth of 4 cm below the layer of fat, as well as at intervening points between the muscle, fat, skin, and ice.

1.1 Design Objectives

The physical problem, as translated into a mathematical model, will be further discussed in Sections 2.1 and 2.2 below. Briefly, an axisymmetric section of tissue (Figure 1a – 1c, p.2) was meshed using Gambit software and imported into FIDAP, a program which uses iterative methods to solve the equations governing the thermodynamic processes involved in this problem (see Appendices A and B for detailed descriptions of the problem definition, mathematical statement, and commands used in FIDAP). This process allowed us to verify the time required to effectively reduce muscle temperature and to assess the extent and depth to which this cooling occurred.

1.2 Assumptions

Although a seemingly straightforward process, cooling biological tissues is a quite complex phenomenon, and several assumptions need to be made to simplify the problem into a workable model. Since we are interested only in temperature variations as a function of depth, we neglect heat transfer processes occurring at the edges of the ice pack. We also assume constant thermal properties (specific heat, thermal conductivity, and density) with respect to time, temperature, and space. Importantly, in modeling the phase change from ice to liquid water, we needed to assume that ice melted over a small range of temperatures rather than exactly at 0°C (see Appendix A for a full description of this assumption). A basic

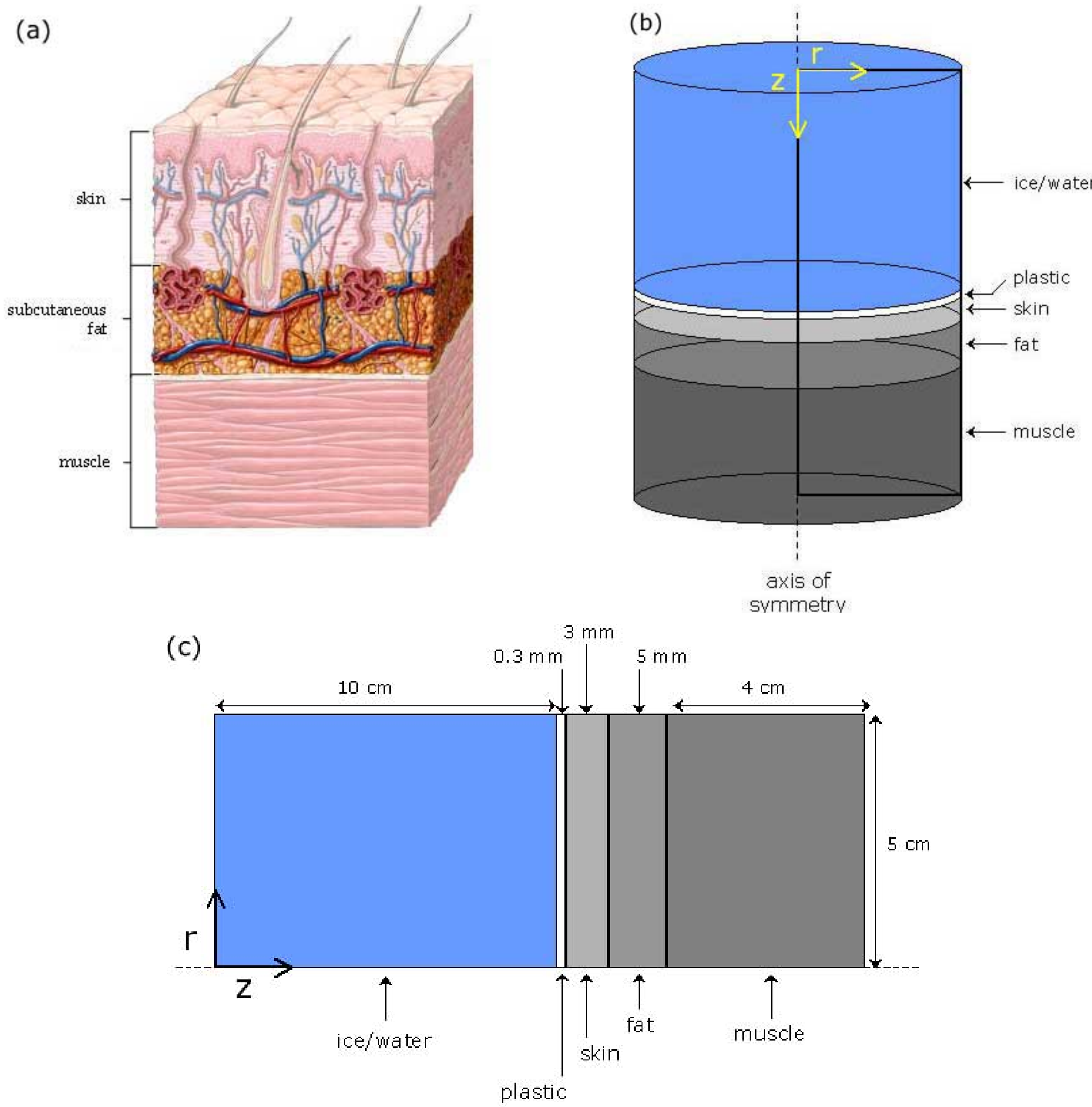

Figure 1. (a) Cartoon representation of the three tissue layers under consideration. (b) Cylindrical section of an ice pack on top of an arbitrary tissue element comprised of skin, subcutaneous fat, and skeletal muscle, such as a quadriceps (anterior thigh muscle) or soleus (calf muscle). Note: not to scale. (c) Axisymmetric model, rotated 90° counterclockwise, so that the axis of symmetry lies horizontally at the bottom of the figure.

understanding of the physical problem initially led us to neglect the heating effects of perfusion (blood flow) – since the process resulted in overall tissue cooling, we assumed that the heating due to blood flow was insignificant when compared to the heat flow out of the tissue and into the ice. This assumption was re-thought and discarded in the sensitivity analysis, as discussed in Section 2.2 below.

2 Results and Discussion

As discussed previously, the problem was simulated by creating a model with layers of ice/water, plastic, skin, fat, and muscle. The ice and plastic layers were set to an initial temperature of -10°C whereas the skin, fat, and muscle layers were set to 37°C. The simulated time was stopped at 25 minutes, and temperature profiles were assessed at 10 and 20 minutes.

The first set of simulations ran without a convective blood flow (perfusion) term, and produced the temperature distribution shown in Figure 2. In this initial run, even the deepest layers of muscle (z = 4 cm sub-adipose) were cooled, albeit slightly (Figure 3). More dramatic effects were seen in the superficial muscle layers (0 cm < z < 2 cm sub-adipose), as well as in the skin and fat layers, as can be seen in Figures 2 and 4.

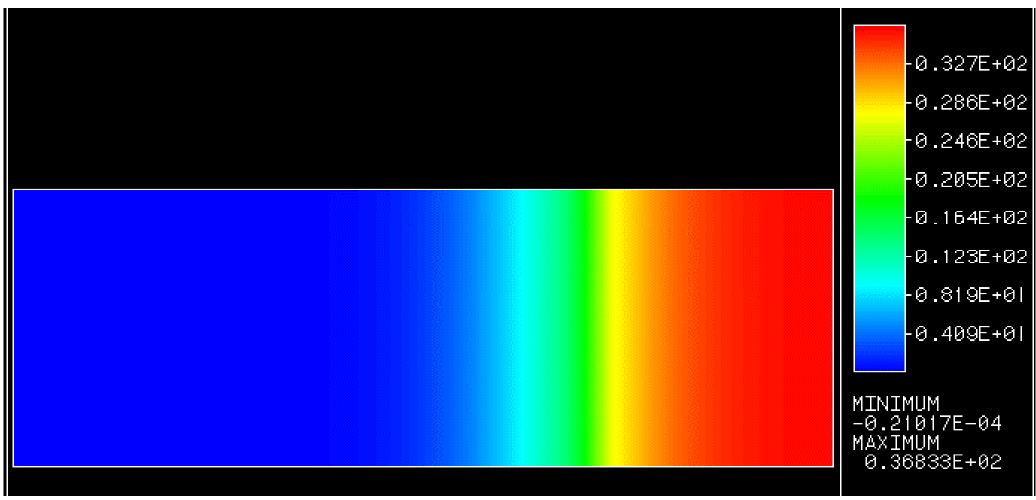

Figure 2. Temperature contour profile at 20 minutes. The maximum temperature, which occurred in the muscle at 4 cm below the fat layer, was 36.8ºC, demonstrating that even deep muscle tissue is cooled. In this initial solution there was no blood perfusion term.

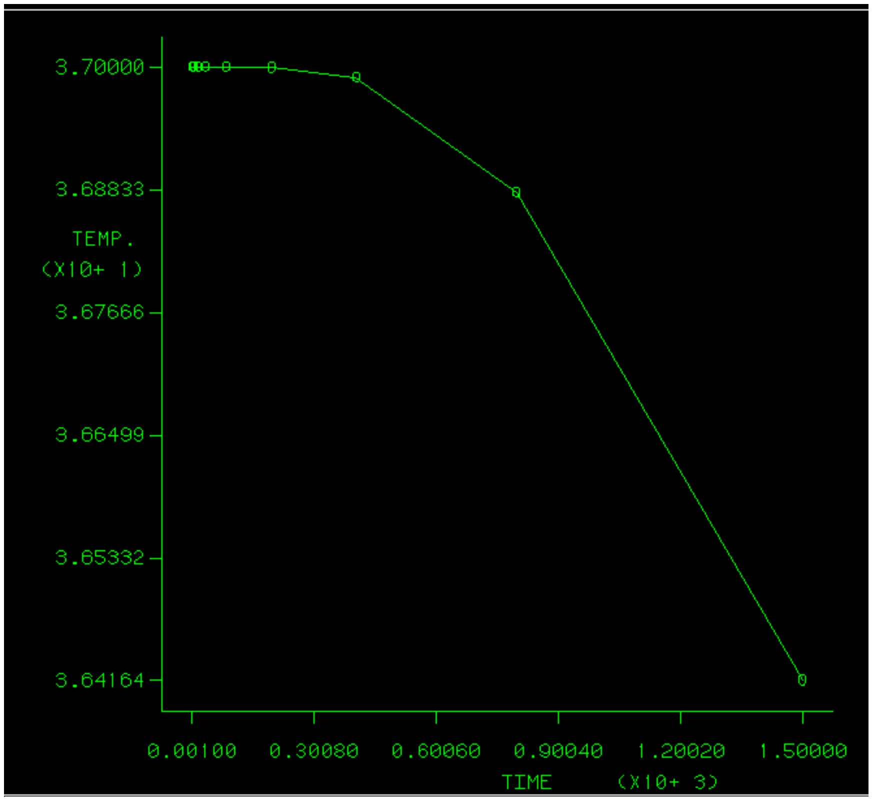

Figure 3. Temperature history plot at the "muscle end" boundary (z = 4 cm subadipose). The muscle is only cooled slightly at this depth.

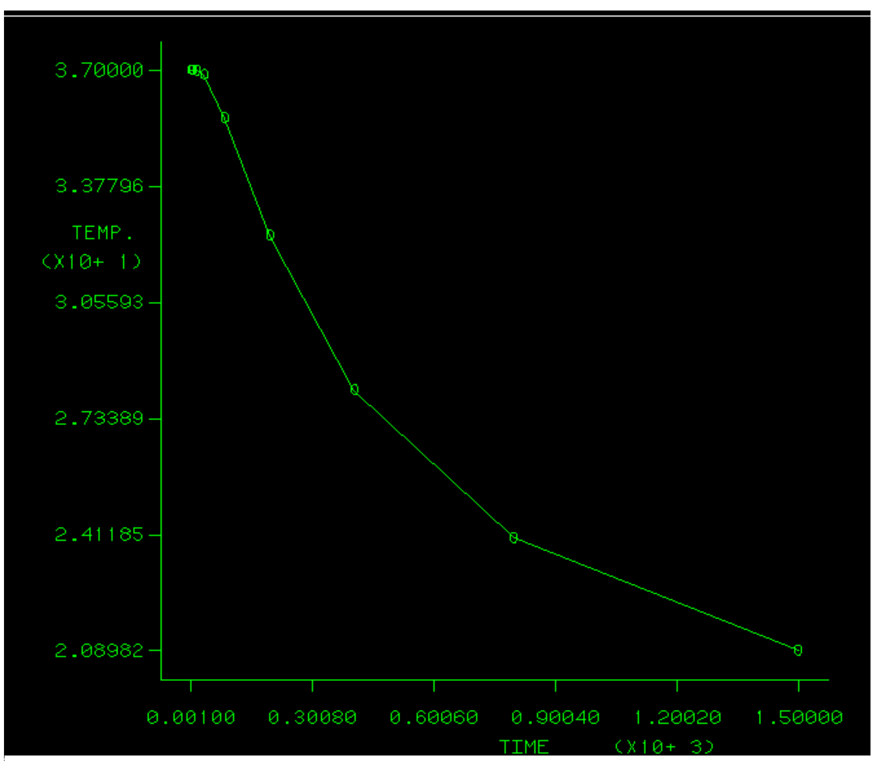

Figure 4. Temperature history plot at the fat-muscle boundary. Since this is the beginning of the most superficial muscle layers, there is significant cooling (approximately 22ºC at 20 minutes).

2.1 Sensitivity Analysis

Our sensitivity analysis aimed to determine how responsive the model was to the parameters used. The first step was to vary each set of parameters (specific heat, density, and thermal conductivity) by ±10% and run simulations to assess the changes in temperatures. For each of these variations, the temperature distributions did not change noticeably and were indistinguishable from that shown in Figure 2. To quantify any changes that were visually imperceptible, the minimum and maximum temperatures were recorded at both 10 minutes and 20 minutes for each trial (Table 1). Here, we also failed to find significant variations as a result of altered physical properties. The same trends (identical temperature distribution and minimal change in minimum and maximum temperatures) held true when we compared a refined mesh solution to our original mesh solution.

Table 1. Results of sensitivity analysis. Properties were varied by +/- 10% of their original values, and maximum and minimum temperatures compared for each independent test. None of the properties had an appreciable impact on final results.

	600 sec		**1200 sec**	
Normal	-9.69	36.9	-8.78	36.4
+10% Specific Heat	-9.77	37.0	-8.82	36.7
-10% Specific Heat	-9.69	36.9	-8.80	36.2
+10% Density	-9.77	37.0	-8.99	36.6
-10% Density	-9.60	36.9	-8.56	36.2
+10% Conductivity	-9.61	36.9	-8.59	36.2
-10% Conductivity	-9.78	37.0	-9.00	36.6
Refined Mesh	-9.76	37.0	-8.80	36.5

We can gather from these results that holding these properties held constant for plastic, skin, fat, and muscle was a valid assumption within this model. Even had we accounted for the changes in thermophysical properties that do occur with temperature (in the relatively small temperature range under consideration here), these changes would have been so small as to have a negligible impact on the final results.

Sensitivity analyses that were not performed but that could have significant impacts on the final solution include varying the thicknesses of the adipose layer, which has been shown to greatly affect the time it takes for muscle to cool [1], varying the skin thickness, and changing the initial temperature of the ice.

The second step in our sensitivity analysis focused on an assumption made early in the model: that the heat introduced to the muscle via blood flow was negligible. We reconstructed our governing equation (see Appendix A) to include the effects of perfusion, held all other factors constant, and found that the addition of this convective blood flow term had a marked effect on the final solution (Figure 5).

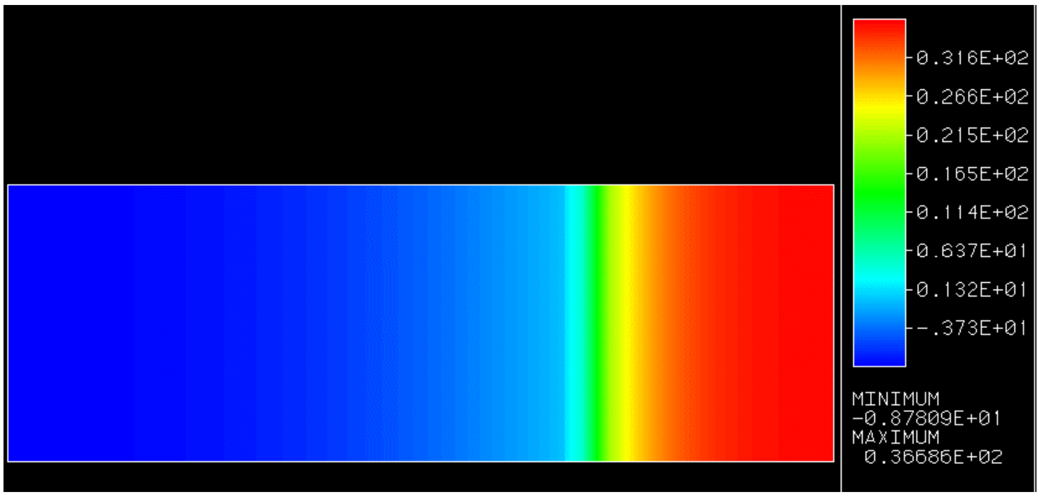

Figure 5. Temperature contour at 20 minutes. In this solution, a term accounting for the heating effects of blood flow was included in the problem formulation. Note the difference between these results and those shown in Figure 2.

Though the maximum temperatures did not change significantly, the minimum temperature (which corresponds to the maximum temperature of the ice) increased by over 6ºC. The differences in temperature distribution, however, are clearly observable. In this latter case, the temperature distribution in the intervening layers between the ice and the muscle is much tighter – a greater temperature change occurs over a smaller distance, most of it occurring in the skin, fat, and near the fat-muscle interface, as shown in the following history plots at 1 cm (Figure 6) and 2 cm from this boundary (Figure 7). At these depths, temperatures still reached 29.5ºC and 34.5ºC, respectively – temperatures that are substantially lower than the normal 37ºC body temperature, and would still lend therapeutic benefit to the patient.

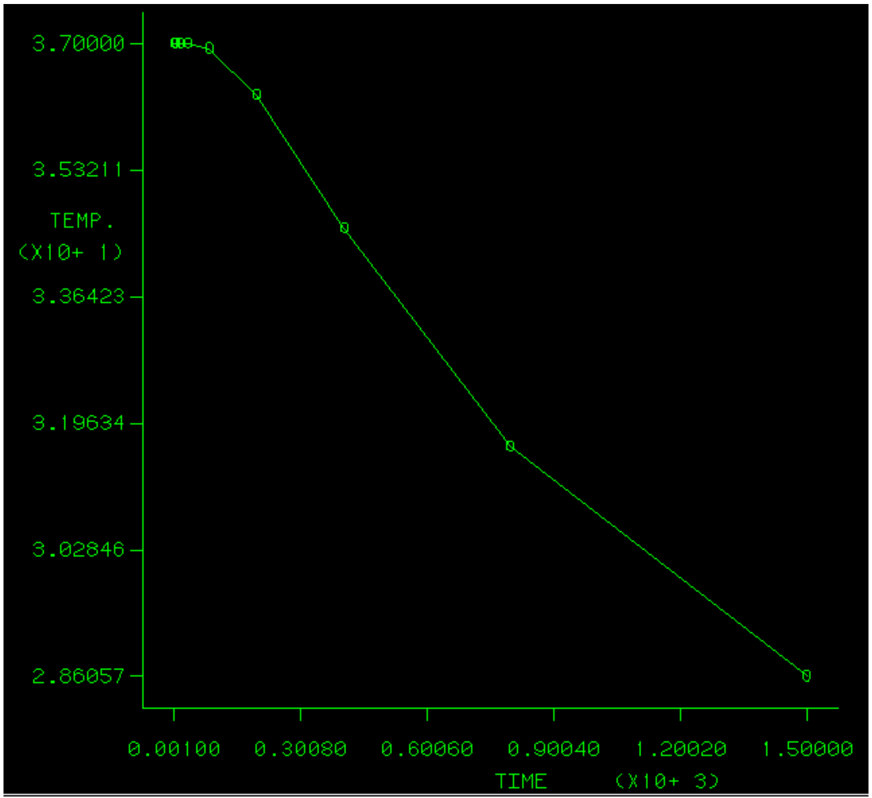

Figure 6. Time history plot in the muscle at 1 cm from fat layer. This solution accounts for the heat contribution from blood flow through muscle layers.

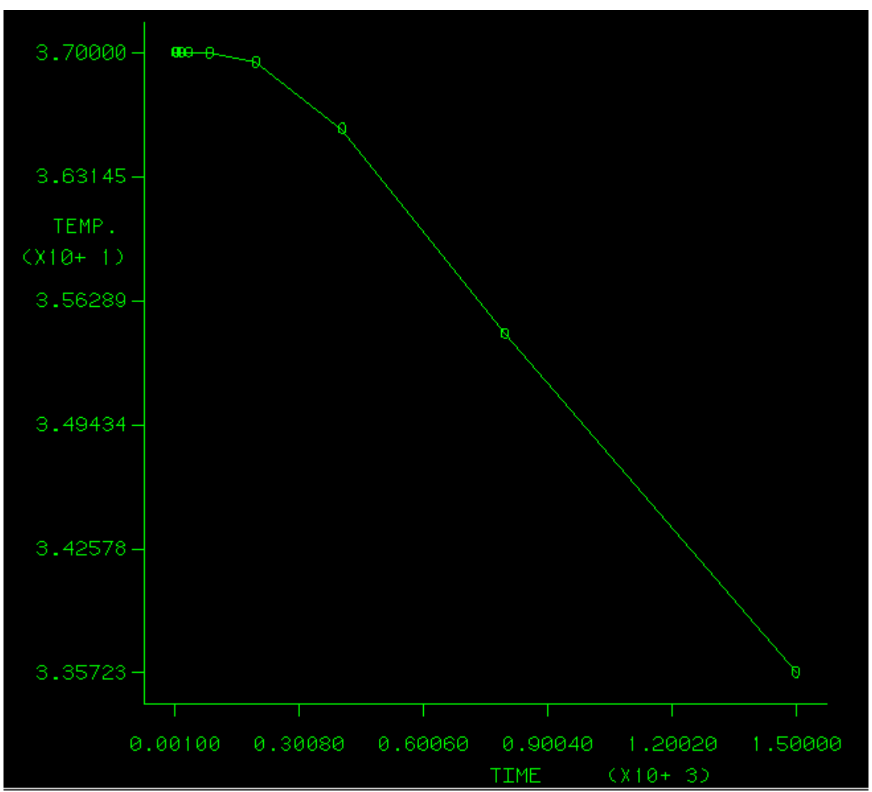

Figure 7. Time history plot at 2 cm below fat boundary layer. The temperature at this depth (for t = 20 minutes) is approximately 5°C warmer than at 1 cm sub-adipose (Figure 6).

When the resulting data was compared to that found in a 2003 study by Merrick et al [1], it was found that our results were within ±2°C of the published results. It is evident that this heat source cannot be ignored, and produces more realistic results than our initial model, which neglected perfusion.

2.2 Realistic Constraints

There are no ethical, social, or political considerations in our simulation. Economically speaking, the treatment is quite inexpensive, which is one reason why it is preferred over other forms of muscle therapy. Environmentally and medically, the process is safe. The ice will melt to water, which is environmentally sound. The plastic bag may be recycled, thereby reducing the amount of waste produced. For health and safety constraints, we have proven that the temperatures of the tissues, including the skin which is in closest contact with the ice, will not get down to damaging temperatures. Therefore, we can confidently state that the general rule of 20 minutes of icing is both safe and effective.

3 Conclusions

It is clear that after 20 minutes of icing the temperature of the entire modeled area decreases, even if only slightly at maximum muscle depths. There are, however, drastic temperature differences in the skin, fat, and superficial muscle layers. Our model, while not sensitive to small (±10%) changes in the thermophysical properties used or on the mesh density, was found to be highly dependent on the effects of perfusion; hence, the most realistic model tested included a convective blood flow term. From this, we can conclude that the temperature within the muscle decreases enough to get a therapeutic response, but not low enough to pose a threat due to extremely cold conditions.

APPENDIX A

A.1 Mathematical Statement and Governing Equations

We start with the general heat transfer equation:

$$\rho c_p\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+\frac{v_\theta}{r}\frac{\partial T}{\partial \theta}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right)+Q$$

Since there is no fluid flow, the convective terms are eliminated, and since an axisymmetric geometry is used (as discussed in Section 2 above), terms with respect to the r- and θ-axes are also dropped. The heat generation term is neglected in the original problem statement, as it is considered insignificant compared to the heat lost to the ice pack. The simplified governing equation is as follows:

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}$$

We now take into consideration that each layer has different thermal properties and can be described by inserting the appropriate constants (ρ, c_p, and k), as described below.

This equation was further modified during our sensitivity analysis to determine whether perfusion is indeed negligible, as we originally assumed. To account for blood flow, we reintroduced the heat generation term:

$$Q=\rho_b c_{p,b}\,\omega(T_b-T)$$

where ρ_b and $c_{p,b}$ are the density and specific heat capacity of blood, as tabulated below, and ω is the flow rate, in (m^3 blood)/(m^3 tissue·second). T_b is the temperature of arterial blood, while T is the temperature of the tissue at time t. With this term included, the governing equation used in this part of the sensitivity analysis becomes:

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2}+\frac{\rho_b c_{p,b}\,\omega(T_b-T)}{\rho c_p}$$

A.2 Properties

Early in the problem definition we assumed that the thermophysical properties for the plastic and for the tissues were constant (Table A.1).

Table A.1. Thermophysical data for the plastic and for the tissues of interest [2-5].

Material	Temperature (^oC)	k (W/m·K)	ρ (kg/m^3)	c_p (kJ/kkg·K)
plastic (LDPE)	20	0.33	920	1.90
plastic (HDPE)	20	0.46	950	1.90
skin	37	0.63	1030	3.79
fat	37	0.20	938	2.43
muscle	37	0.43	1044	3.55
blood	37	n/a	1035	2.30

This simplification, however, cannot be extended to the ice. Although we hold the specific heats of ice and water constant, we must account for the phase change that occurs as the ice melts. To do so, we first assume that instead of melting at exactly 0^{o}C, the transition from ice to water occurs over a narrow range of temperatures (-1^{o}C to +1^{o}C). From here we are able to construct an apparent specific heat curve, as follows:

There is a discontinuity between the specific heats of ice and water at 0^{o}C, which is accounted for by the latent heat of fusion, λ, which for water is 334.944 kJ/kg.

$$\int_{T_1}^{0} c_{p,right}\,dT + \int_{0}^{T_2} c_{p,left}\,dT = \lambda \qquad (1)$$

We can also construct relationships between temperature and specific heat for both ice and water at their respective limits.

$$\frac{c_{p,left} - c_{p,T_1}}{T - T_1} = \frac{c_{p,\max} - c_{p,T_1}}{T_{\max} - T_1} \quad \Rightarrow \quad c_{p,left} = \left(T - T_1\right)\left(\frac{c_{p,\max} - c_{p,T_1}}{T_{\max} - T_1}\right) + c_{p,T_1} \qquad (2)$$

$$\frac{c_{p,right} - c_{p,T_2}}{T - T_2} = \frac{c_{p,max} - c_{p,T_2}}{T_{max} - T_2} \quad \Rightarrow \quad c_{p,right} = \left(T - T_2\right)\left(\frac{c_{p,max} - c_{p,T_2}}{T_{max} - T_2}\right) + c_{p,T_2} \qquad (3)$$

We can now solve equation (1) by substituting these expressions into the respective integrals.

$$\int_{T_1}^{0}\left[\left(T-T_1\right)\left(\frac{c_{p,\max}-c_{p,T_1}}{T_{\max}-T_1}\right)+c_{p,T_1}\right]dT + \int_{0}^{T_2}\left[\left(T-T_2\right)\left(\frac{c_{p,\max}-c_{p,T_2}}{T_{\max}-T_2}\right)+c_{p,T_2}\right]dT = \lambda \tag{4}$$

$$\left[\left(\tfrac{1}{2}T^2-T_1T\right)\left(\frac{c_{p,\max}-c_{p,T_1}}{T_{\max}-T_1}\right)+c_{p,T_1}T\right]_{T_1}^{0} + \left[\left(\tfrac{1}{2}T^2-T_2T\right)\left(\frac{c_{p,\max}-c_{p,T_2}}{T_{\max}-T_2}\right)+c_{p,T_2}T\right]_{0}^{T_2} = \lambda \tag{5}$$

As stated above, we let $T_1 = -1^oC$, $T_2 = +1^oC$, $c_{p,T1} = c_{p,ice} = 2.092$ kJ/kg·K, and $c_{p,T2} = c_{p,water} = 4.184$ kJ/kg·K, and then solve for $c_{p,max}$.

$$\left[\left(-\tfrac{1}{2}+1\right)\left(\frac{c_{p,max}-2.092}{1}\right)+2.092\right]+\left[\left(\tfrac{1}{2}+1\right)\left(\frac{c_{p.max}-4.184}{-1}\right)+4.184\right] = 334.9\frac{kJ}{kg} \tag{6}$$

$$c_{p,\max} = 331.8\frac{kJ}{kg\cdot K} \tag{7}$$

We arrive at the equations describing the apparent specific heat curve (Figure A.1) from -1^oC to $+1^oC$ by substituting this value in equations (2) and (3).

$$c_{p,left}(T) = (T+1)\left(\frac{331.8-2.092}{0-(-1)}\right)+2.092 = 329.7T+331.8 \tag{8}$$

$$c_{p,right}(T) = (T-1)\left(\frac{331.8-4.184}{0-1}\right)+4.184 = 331.8T-327.6 \tag{9}$$

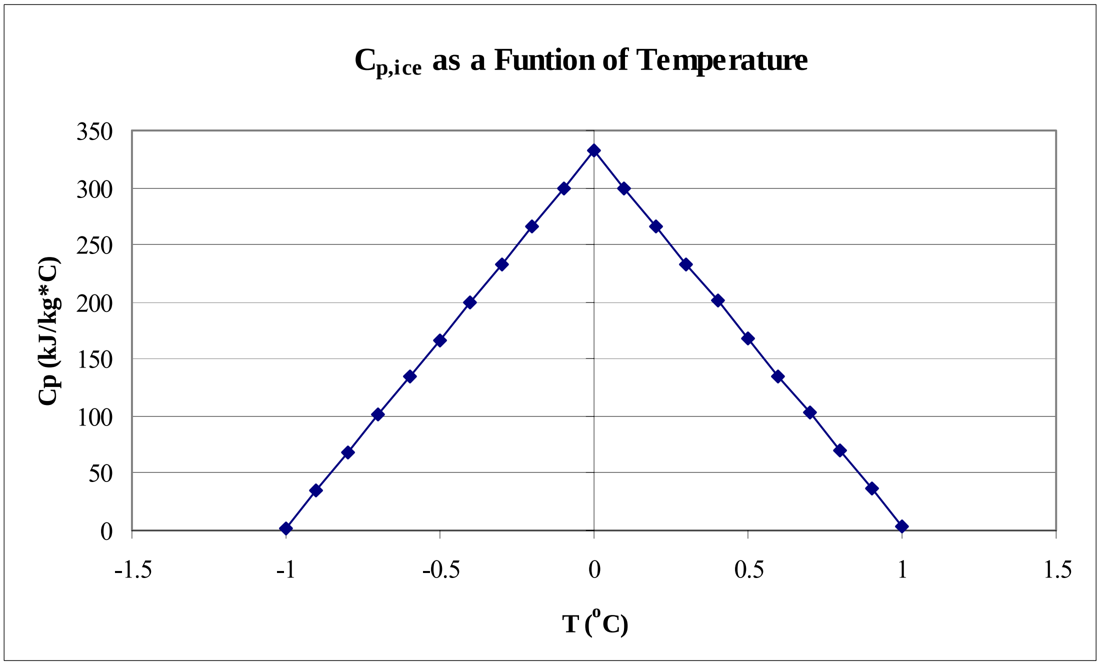

Figure A.1. The apparent specific heat of ice as a function of temperature. To construct this relationship, we first assume that the melting process occurs over a short range of temperatures (-1°C to +1°C) instead of occurring only at 0°C. Outside of this narrow range, the specific heats of ice and water were assumed constant.

So, at temperatures below -1°C, the specific heat of ice is held constant at 2.09 kJ/kg·K; between -1°C and +1°C, the apparent specific heat is described by the above equations; and, above +1°C, the specific heat of water is a constant 4.18 kJ/kg·K (Table A.2).

Table A.2. Thermophysical data for ice and water, including apparent specific heat as a function of temperature for the transitional ice-to-water phase change.

	Temperature (°C)	k (W/m·K)	ρ (kg/m³)	c_p (kJ/kg·K)
ice	< -1	0.567	1000	2.09
ice (melting)	$-1 \leq T \leq 0$			329.7T + 331.8
water (melting)	$0 \leq T \leq +1$			-327.6T + 331.8
Water	$> +1$			4.18

A.3 Initial Conditions

The initial conditions specified temperatures only. We assumed that the ice was in thermal equilibrium with the plastic, and that all tissues were at a constant body temperature.

T_{ice}(t = 0 s, 0.0 m ≤ x < 0.10 m) = -10°C
$T_{plastic}$(t = 0 s, 0.10 m ≤ x < 0.1003 m) = -10°C

T_{skin} (t = 0 s, 0.1003 m ≤ x < 0.1033 m) = 37°C
T_{fat} (t = 0 s, 0.1033 m ≤ x < 0.1083 m) = 37°C
T_{muscle} (t = 0 s, 0.1083 m ≤ x 0.1483 m) = 37°C

A.4 Boundary Conditions

The axisymmetric geometry used in this model imposes the condition that there is no heat flux along the axis. This condition can be extended to the other boundaries as well (see Figure A.2).

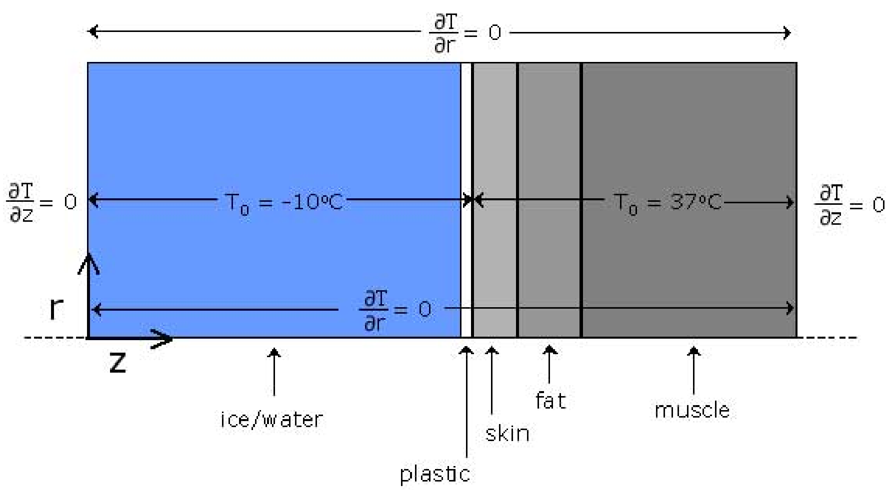

Figure A.2. Schematic showing boundary conditions. Heat flux was set to zero along each external boundary and along the axis of symmetry.

$$\left.\frac{dT}{dz}\right|_{z=0m} = 0$$ Heat flux into the ice is zero.

$$\left.\frac{dT}{dz}\right|_{z=0.1483m} = 0$$ Heat flux into the tissue is zero.

$$\left.\frac{dT}{dr}\right|_{r=0.05m} = 0$$ Heat flux along the "side" of the section is zero.

$$\left.\frac{dT}{dr}\right|_{r=0.0m} = 0$$ Heat flux along the axis of symmetry is zero.

See Appendix B for the FIDAP input file, which lists both the initial and boundary conditions in the format used in FIDAP.

APPENDIX B

B.1 Problem Statement Keywords

> PROB (AXI-, INCO, TRAN, LAMI, LINE, NEWT, NOMO, ENER, FIXE, NOST, NORE, SING)

Table B.1. Problem statement keywords used in FIDAP, with their respective explanations.

Descriptor	Value	Explanation
Geometry type	AXISYMMETRIC	System is symmetric about an axis, therefore only one calculation is needed
Flow regime	INCOMPRESSIBLE	All fluid is incompressible
Simulation type	TRANSIENT	The results depend on time
Flow type	LAMINAR	No fluid flow
Convective term	LINEAR	No convection
Fluid type	NEWTONIAN	No fluid flow
Momentum equation	NO MOMENTUM	No momentum in the system
Temperature dependence	ENERGY	Thermal conduction in the system
Surface type	FIXED	Surface is fixed
Structural solver	NO STRUCTURAL	No structural solver
Elasticity remeshing	NO REMESHING	Constant mesh inputted from Gambit
Number of phases	SINGLE PHASE	No phase changes in system

B.2 Solution Statement Keywords

> SOLU (S.S. = 10, ACCF = 0.000000000000E+00)

Table B.2. Solution Description

Descriptor	Value	Explanation
Solution method	STEADY STATE = 10	Steady state analysis with a maximum of 10 iterations per time step
Relaxation Factor	ACCF = 0	No relaxation

B.3 Time Integration Statement Keywords

> TIME (BACK, NSTE = 15000, TSTA = 0.000000000000E+00, TEND = 1500.0, DT = 1.0, VARI = 1.0, WIND, NOFI, INCM = 10.0)

Table B.3: Time Integration Description

Descriptor	Value	Explanation
Time Integration	BACKWARD	Backward Euler time integration method
Number of time steps	NSTEPS = 15000	Maximum number of time steps used
Starting time	TSTART = 0	Starting time is 0 seconds
Ending time	TEND = 1500	Ending time is 1500 seconds
Time increment	DT = 1.0	Time increment is 1 second
Time stepping algorithm	VARIABLE = 1.0	Time steps are varied according to a tolerance level of 1 for truncation error
Variable Window	WINDOW	Controls for error in variable time increment
Number of Fixed Steps	NOFIXED	No fixed time increment steps
Max Increase Factor	INCMAX = 10	Maximum increase between time steps is 10

B.4 Refined Mesh

The original (preliminary) mesh (schematic with coordinates shown in Figure B.1, mesh as shown in Gambit and FIDAP shown in Figure B.2) was created using both graded and uniform meshing in Gambit. While the plastic, skin, and fat layers were each meshed uniformly, the ice and muscle layers were not, and had increased node density near the ice-plastic and fat-muscle boundaries, respectively.

After generating the refined mesh (Figure B.3), we ran FIDAP using the same input parameters as with the preliminary mesh. The refined mesh included twice as many nodes at actively changing areas. Temperature profiles from this run indicated that the finer mesh provides no additional accuracy, as the profiles were identical. While the time required for computation was not significantly longer, we found that the refined mesh was unnecessary.

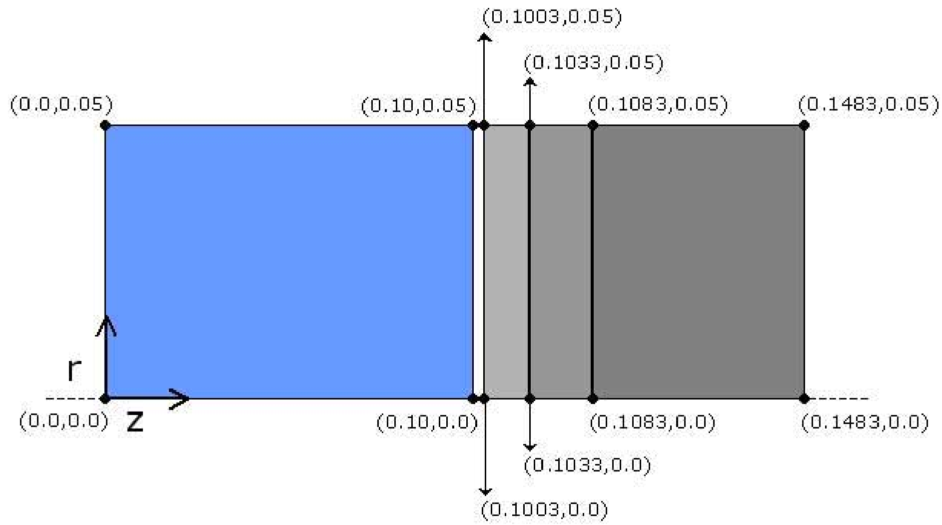

Figure B.1. Schematic shown with coordinates as entered in Gambit during mesh construction. Note: not to scale.

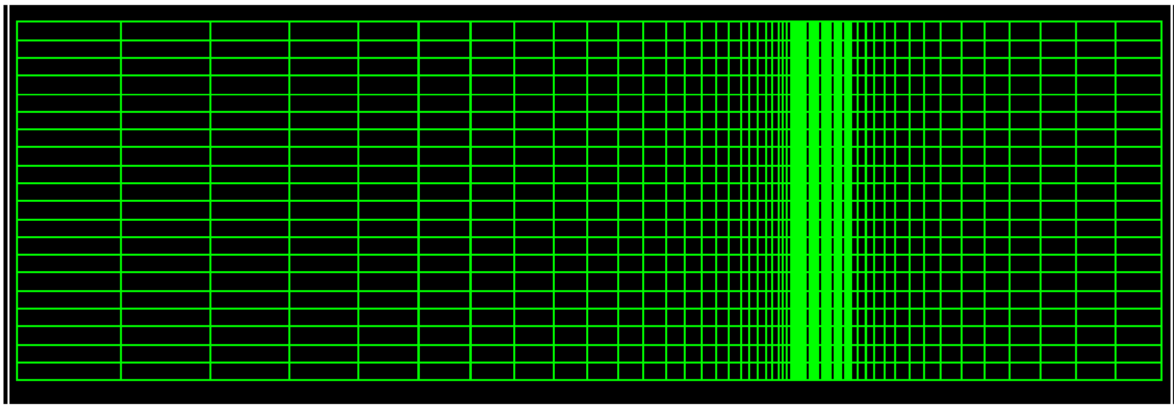

Figure B.2. Original mesh (1638 nodes) created in Gambit and used for all solutions, excepting the sensitivity analysis that compared the original to the refined mesh.

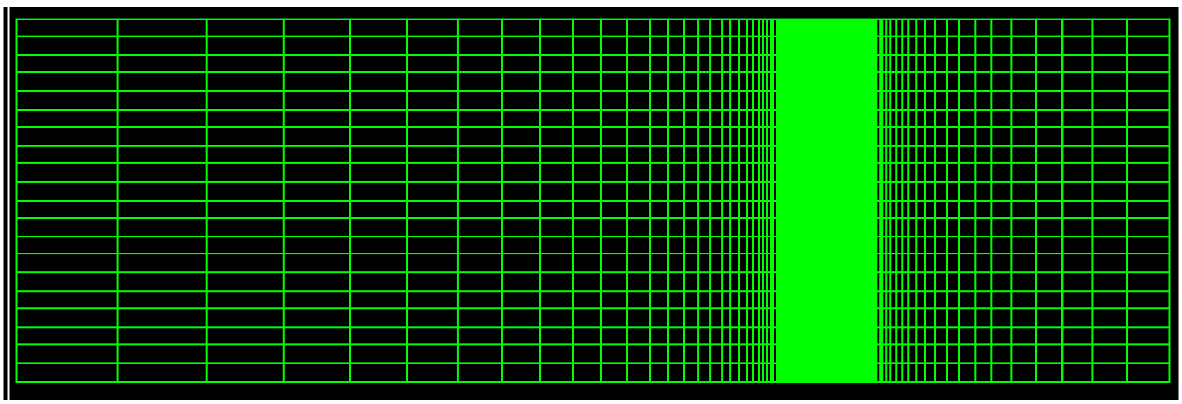

Figure B.3. **Refined mesh with twice as many nodes as the original mesh. Results found with the original mesh converged with the results obtained with this refined mesh, demonstrating the solution's independence from the mesh.**

Works Cited

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