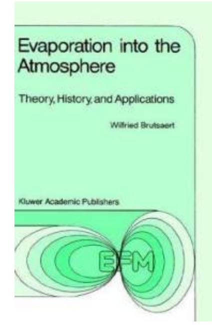
The mean velocity profile in a sheared and thermally stratified turbulent atmospheric boundary layer

Gabriel Katul[1,2], Amilcare Porporato[1,2], and Alexandra Konings[1,3]

[1]*Nicholas School of the Environment* and
[2]*Civil and Environmental Engineering* Department, Duke University

[3]Department of Civil and Environmental Engineering, Parsons Laboratory, Massachusetts Institute of Technology

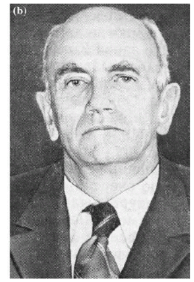
Hydrology, Evaporation, and Monin-Obukhov Similarity Theory



- Given the numerous contributions of professor Brutsaert to *hydrology* and *atmospheric* sciences, it is only befitting to honor these contributions by selecting a topic that intersects both.

- More important, the topic of evaporation and similarity theory in the Atmospheric Boundary Layer (ABL) is of great interest to professor Brutsaert[1].

[1] Evaporation OR Similarity theory OR Monin-Obukhov appeared in some 40% of professor Brutsaert's scientific manuscripts.



Monin and *Obukhov* Similarity Theory – The Foundation of Micro-meteorology

A.S. Monin and A.M. Obukhov developed their similarity theory (Monin and Obukhov, 1954) on the basis of the following findings:

- Fundamental experimental work at the Geophysical Main Observatory in Leningrad, directed by several scientists including Budyko.
- Logarithmic wind profile (Prandtl, 1925),
- Zero-plane displacement (Paeschke, 1937)
- Obukhov length (Obukhov, 1946).

Main Result

ON PHYSICALLY SIMILAR SYSTEMS; ILLUSTRATIONS OF THE USE OF DIMENSIONAL EQUATIONS.

By E. Buckingham.

Phys. Rev. 4, 345–376 (1914)

- Proposed using dimensional analysis by *Monin* and *Obukhov*[1], a '*stability correction function*' accounts for distortions to the logarithmic mean velocity profile (MVP) due to surface heating (or cooling).
- The universal shape is confirmed by many field experiments (e.g. the Kansas experiment[2]) and Large Eddy Simulations[3].
- Theories that predict this universal shape are currently lacking.

[1]A. Monin and A. Obukhov, Akad. Nauk. SSSR. Geoz. Inst. Trudy 151, 163 (1954). [2]J. A. Businger, J. C. Wyngaard, Y. Izumi, and E. F. Bradley, J. Atmos. Sci. 28, 181 (1971). [3]Khanna and J. G. Brasseur, J. Fluid. Mech. 345, 251 (1997).

The Stability Correction Function

Von Karman Constant

Height from the ground surface

$$\varsigma = z / L$$

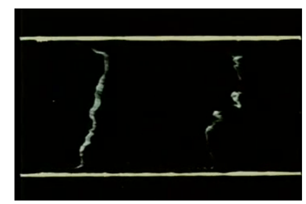
Stability Parameter

MVP

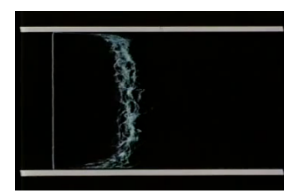
$$\frac{du}{dz}\,\frac{k_v z}{u_*} = \phi_m(\varsigma)$$

Friction velocity

Stability correction Function



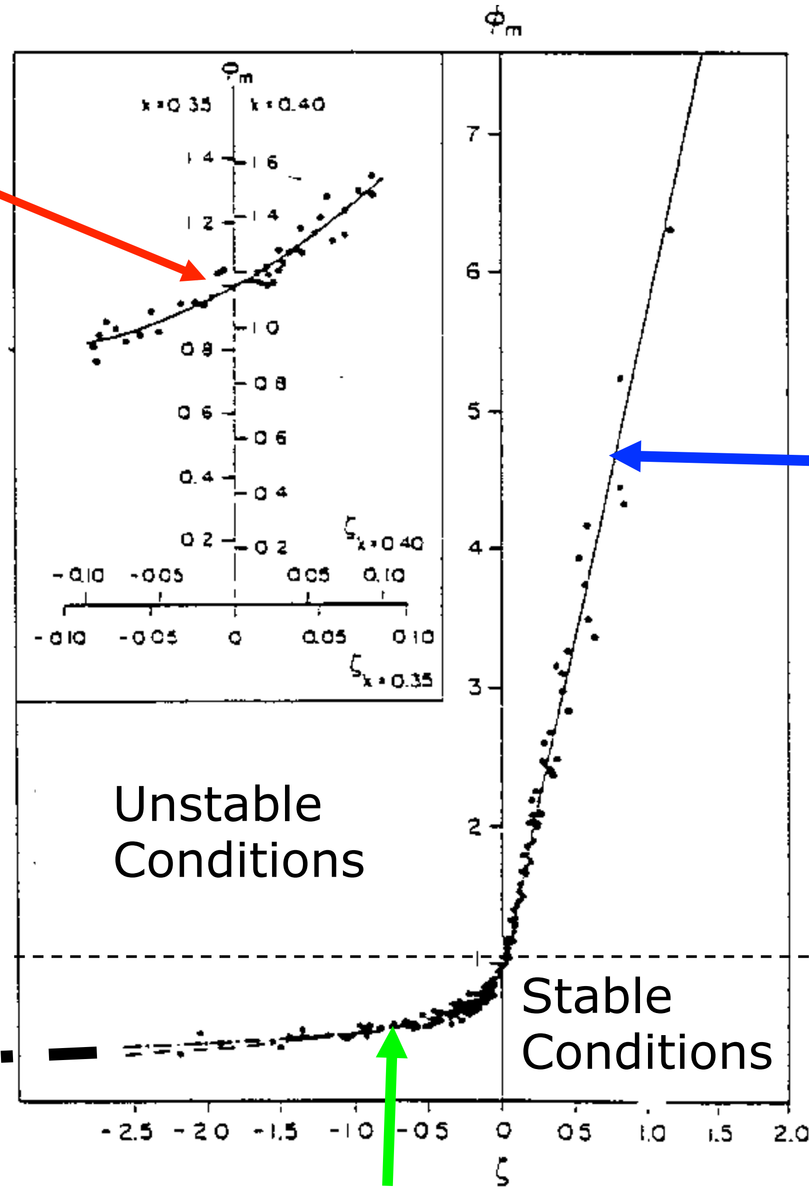
Individual Runs



MVP

KANSAS EXPERIMENT

Continuous and smooth



$$\phi_m(\varsigma) = (1 + 4.7\varsigma)$$

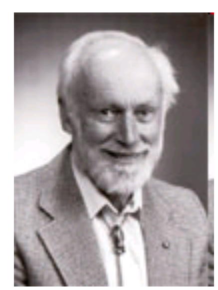
Log-profile

$$\phi_m(0) = 1$$

$$\sim (-\varsigma)^{-1/3}$$

Pure convective, u* independence[1]

$$\phi_m(\varsigma) = (1 - 16\varsigma)^{-1/4}$$



Businger-Dyer Stability Correction Function

[1]C. H. B. Priestley and W. C. Swinbank, P. Roy. Soc. Lond. A Mat. 189, 543 (1947).

Success for dimensional considerations – But no phenomenological theory

Optimism in collapsing field experiments via dimensional consideration - best reflected in Kaimal's statement[1] -

"*with proper non- dimensionalization, all flow statistics in the surface layer can be reduced to a set of universal curves*"

[1]J. C. Kaimal, Bound.-Lay. Meteorol. 4, 289 (1973).

Approach

- A theory based on the recent link[1] between the spectrum of turbulence and the MVP is expanded here to include:

(i) effects of thermal stratification on the turbulent kinetic energy dissipation rate and

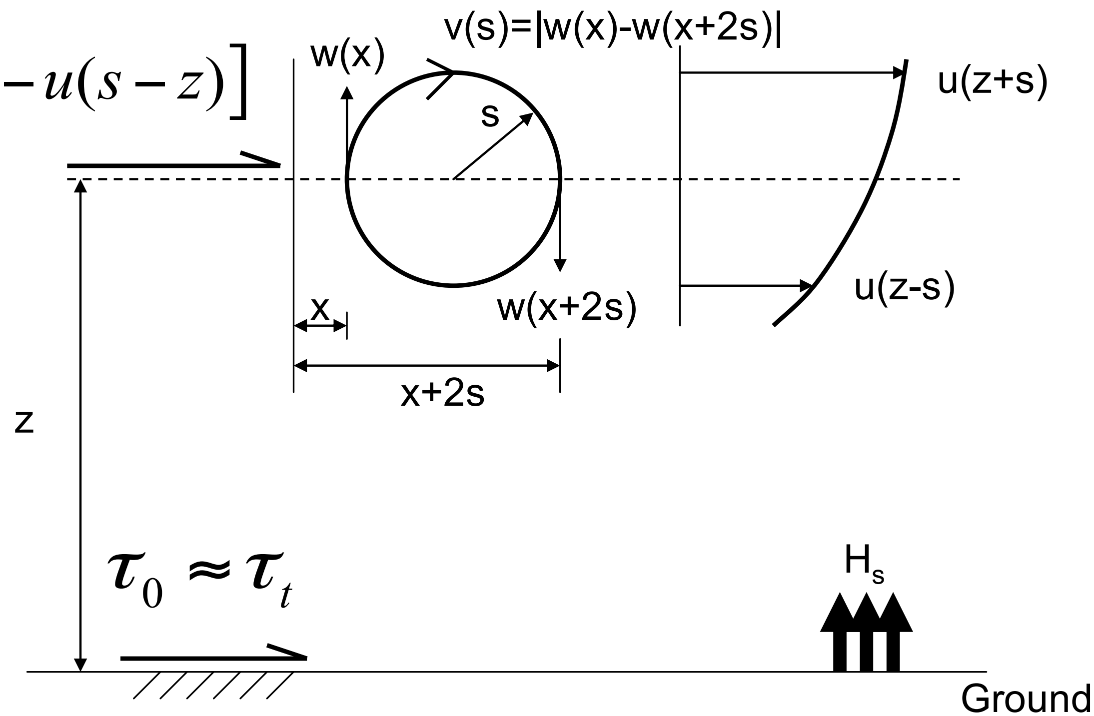
(ii) eddy-size anisotropy.

- The resulting theory provides a novel explanation for the power-law exponents and coefficients of the stability correction functions.

[1]G. Gioia, N. Guttenberg, N. Goldenfeld, and P. Chakraborty, Phys. Rev. Lett. 105, 184501 (2010)

Phenomenological Theory

$$\tau_t \sim v(s)\left[u(s+z)-u(s-z)\right]$$



Eddy structure most efficient in momentum transport is the one 'touching' the ground[1].

Essence of Attached Eddy Hypothesis of Townsend

Hence, **z=2s**.

[1]G. Gioia, N. Guttenberg, N. Goldenfeld, and P. Chakraborty, Phys. Rev. Lett. 105, 184501 (2010)

Phenomenological Theory

- Based on this theory, the turbulent stress is given as:

$$\overline{u'w'} \qquad\qquad\qquad\qquad\qquad\qquad\qquad w' \qquad u'$$

$$\frac{\tau_t}{\rho} = \frac{\tau_o}{\rho} = k_\tau v(s)\left[u(s+z) - u(s-z)\right] \approx k_\tau v(s)\left[\frac{du(z)}{dz} 2s\right]$$

Unknown

→ 2s = z (isotropic assumption).

→ v(s) is given by *Kolmogorov's 4/5 law* for locally homogeneous and isotropic turbulence[1].

$$v(s) = \left|k_\varepsilon \varepsilon s\right|^{1/3} ; k_\varepsilon = 4/5$$

[1]U. Frisch, Turbulence (Cambridge University Press, Cambridge, England, 1995).

Phenomenological Theory

- To determine *v(s)*, estimate the TKE dissipation rate from TKE budget:

$$\varepsilon = u_*^2 \frac{\partial u}{\partial z} + \frac{g}{T}\frac{H_s}{\rho C_p} + \cancel{\left(-\frac{1}{2}\frac{\partial \overline{w'e^2}}{\partial z} - \frac{1}{\rho}\frac{\partial \overline{w'p'}}{\partial z} \right)}$$

Insert into the turbulent stress equation results in the following expression:

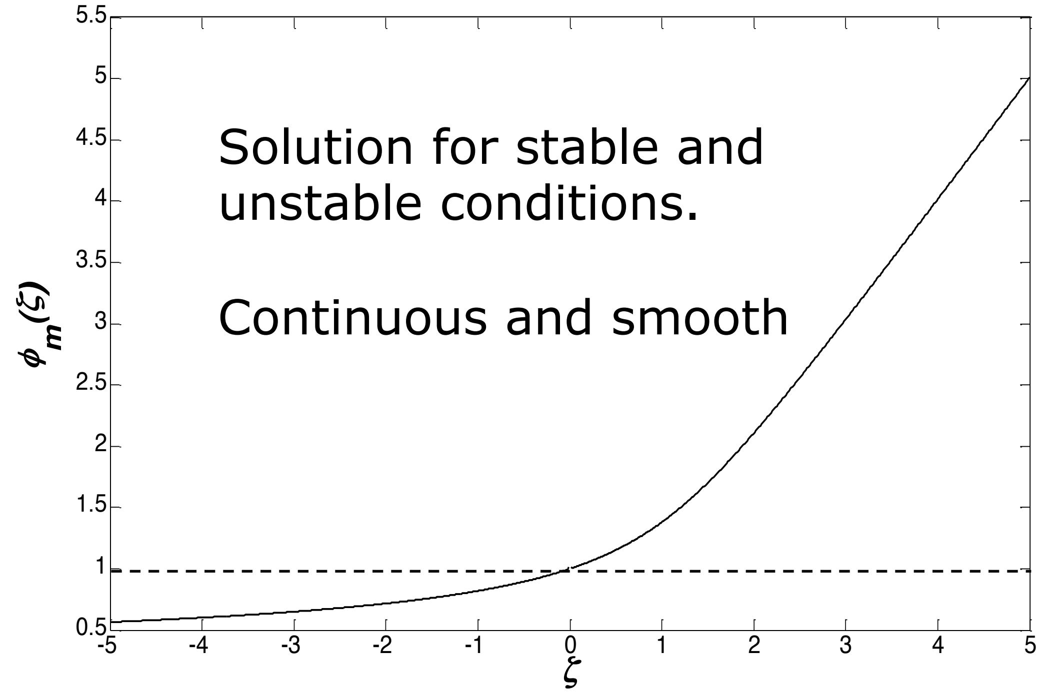
$$\frac{2k_\tau k_\varepsilon^{1/3}}{k_v^{4/3}}\left[\phi_m\right]\left[\frac{k_v z}{u_*^3}\left(u_*^2 \frac{\partial u}{\partial z} + \frac{g}{T}\frac{H_s}{\rho C_p} \right)\right]^{1/3} = 1$$

Phenomenological Theory

- Using the definition of the Obukhov length *L* results in:

$$\mathit{Imposing} \quad \phi_m(0) = 1$$

$$\left[\phi_m(\varsigma)\right]^4 \left[1 - \frac{\varsigma}{\phi_m(\varsigma)}\right] = 1.$$



Similar to the *OKEYPS* Equ.
(after Obukhov, Kazansky, Ellison, Yamamoto, Panofsky, and Sellers)

$$\left[\phi_m(\varsigma)\right]^4 - \gamma\varsigma\left[\phi_m(\varsigma)\right]^3 = 1$$

Discussion: Unstable Conditions

Phenomenological Theory: $$\left[\phi_m(\varsigma)\right]^4\left[1-\frac{\varsigma}{\phi_m(\varsigma)}\right]=1.$$

→ Recovery of the 1/4 exponent

For small stability parameter, $\left[\varsigma/\phi_m(\varsigma)\right]\approx\varsigma$

$$\left[\phi_m(\varsigma)\right]^4\left[1-\varsigma\right]\approx 1;\Rightarrow \phi_m(\varsigma)\approx\left(1-\varsigma\right)^{-1/4}.$$

RECALL $\boxed{\phi_m(\varsigma)=(1-16\varsigma)^{-1/4} \quad \text{Businger-Dyer}}$

→ Recovery of the 1/3 exponent

For very large stability parameter, $\left[-\varsigma/\phi_m(\varsigma)\right]>>1$

$$\left[\phi_m(\varsigma)\right]^4\left[-\varsigma/\phi_m(\varsigma)\right]\approx 1;\Rightarrow \phi_m(\varsigma)\approx\left(-\varsigma\right)^{-1/3}.$$

The Factor 16 in: $\phi_m(\varsigma) = (1-16\varsigma)^{-1/4}$

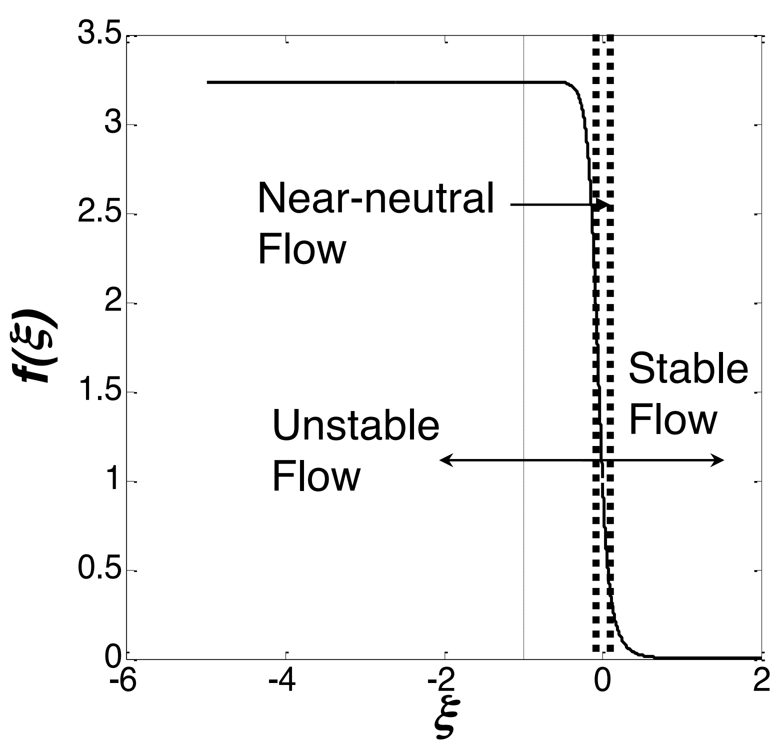
- Recall isotropic condition: $z/s = 2$
- Non-isotropic condition: $z/s = f(\varsigma)$

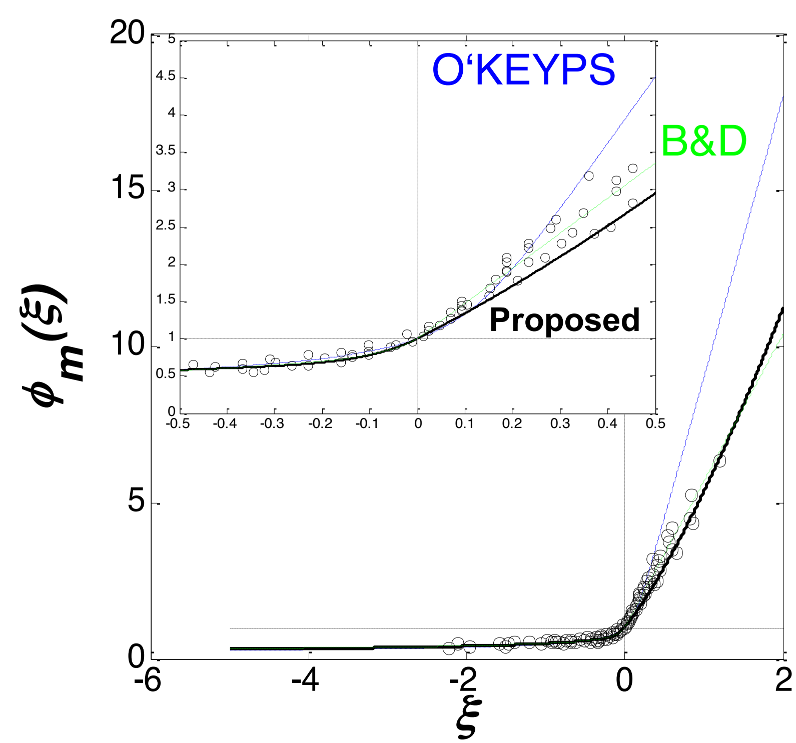
$$v(2s') = \left[k_\varepsilon \varepsilon \underbrace{z\, f(\varsigma)}_{adjusted} \right]^{1/3}$$

Infer f(.) from ‘spectral peaks’ of vertical velocity – again Kansas Data

Comparison with Businger-Dyer and O' KEYPS equation

- Assumed shape of anisotropy function – from vertical velocity spectra





Conclusions:

- First phenomenological approach linking Kolmogorov's theory for locally homogeneous and isotropic turbulence with the MVP to explain all the power-law exponents of the stability correction function for momentum[1].

- The factor '9 to 16' in Businger-Dyer, while not predicted, was shown to be primarily linked to anisotropy in 'eddy sizes' due to thermal stratification[1].

[1]Katul, G.G., A. Konings, and A. Porporato, 2011, *Physical Review Letters*, 107, 268502

Broader Implication: A general phenomenological theory for turbulence

- Similar arguments have been used to derive:

- Manning's equation[1]

- Darcy-Weisbach Friction-Factor with Reynolds number and roughness height (for 2D and 3D turbulence[2,3])

- [1]Gioia, G., and F.A. Bombardelli, *Physical Review Letters* **88**, 014501, 2002; [2]Tuan et al. *Nature Physics* **6**, 438-441, 2010 [3]Gioia, G., and P. Chakraborty, *Physical Review Letters* **96**, 044502, 2006

Reference and Acknowledgements

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