# The off-specular peak and polarisation effects of an undulating underwater surface* 

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## Introduction

The effect on the reflectance observed above a water surface when an undulating bottom surface layer is apparent is estimated. Given a relatively calm water surface, the observed polarisation at observation angles along the plane of incident light is shifted away from the specular direction.
A model surface akin to an egg carton used in previous work [1] is again employed. It is represented by $z=$ $a^{2} \sin \left(2 \pi \frac{x}{l}\right) \cos \left(2 \pi \frac{y}{l}\right)+a$, where $(x, y, z)$ are Cartesian coordinates, $a$ the amplitude and $l$ the length of the basic sinusoid function. A roughness parameter is defined to be, simply, $\sigma=\frac{a}{T}$ and is associated with some surface scale parameter $s$. The incidence direction is defined as the radial distance $\theta$ from the global normal with an azimuth $\phi$ away from a chosen reference direction $\left(0^{\circ}\right)$; all other directions are defined similarly.
To simplify the analysis, assume an infinitely distant, collimated, randomly polarised, and monochromatic light source against a black sky. Furthermore, all considerations are based on geometric optics.
Formulation. The radiance distribution is expressed by

$$
\begin{aligned}
L_{r}\left(\lambda, \theta_{r}, \phi_{r}\right)= & \int_{0}^{2 \pi} \int_{0}^{\frac{\pi}{2}} \frac{1}{\pi} \rho(\lambda) L_{i}\left(\lambda, \theta_{i}, \theta_{r}, \tau_{i}, \tau_{r}, \Omega_{s}\right) \\
& \times \exp \left[-k_{w}(\lambda) D\left(\hat{\xi}_{1}, \hat{\xi}_{r}\right)\right] \\
& \times \sin \hat{\xi}_{1} \cos \hat{\xi}_{1} G\left(\hat{\xi}_{i}, \hat{\xi}_{1}, \hat{\xi}_{r}\right) \mathrm{d} \theta_{i} \mathrm{~d} \phi_{i}
\end{aligned}
$$

A direction vector $\xi_{V}=\left(\theta_{V}, \phi_{V}, r_{V}\right)$ describes the radial and azimuthal direction, as well as the magnitude of radiance, where $V=i, 1$ or $r$ represent above-water incidence, in-water reflected, and first-order in-water incidence directions, respectively. The material reflectance is given by $\rho, \tau$ the air-water interface transmission factor, $\Omega_{s}$ the solid angle subtended by the source, $k_{w}$ the water attenuation factor, $D$ the attenuation due to distance travelled in the water, and $G$ a geometrical radiance transfer factor from an infinitesimally small area on the surface. The effects of shadowing and self-shading are equivalent to applying a geometrical attenuation factor to specular reflectance, which is similar to an analysis of morphological effects using triangular waves by [2].

## Off-Specular Peaks

Neglecting any scattering in the water column, the light entering the water will be refracted once, reflected off the bottom, and refracted another time. The result is a distribution in the plane of incidence whose peak will tend toward the specular direction, see Figs. 1, 2 and 3. Although the bottom is Lambertian, the combined effects of its geometry does not render its behaviour diffuse [3]. This and the attenuation by the wa-


Figure 1. Normalised first-order reflectance (contours) for an undulating Lambertian bottom and a flat water surface with return directions along the $x$ axis and roughness on the $y$-axis, sun at $\theta=18^{\circ}$. The dark dotted line indicates specular direction, the light dashed line the location of reflectance peak.
ter, however, cause the peak to shift away from the specular direction. Similar results have been shown for oil films on ocean surfaces using Monte Carlo methods by [4] and [5]. Minnaert [6] was the first to propose this peak shift toward the horizon (see also [7]).
For a flat water surface, when the sun is high in the sky, the reflectance peak will tend to be lower (Fig. 1) but will be higher when the sun is very low (Fig. 2). At some intermediate roughness (Fig. 3), for a sun in mid-sky, the reflectance peak will coincide with the specular direction.


Figure 2. As in Fig. 1, but for sun at $\theta=84^{\circ}$.

igure 3. As in Fig. 1, but for sun at $\theta=42^{\circ}$.
Following the statistical distribution of slopes on the water surface as determined by [8] and applied by [9], for a relatively calm water surface and the sun at $\theta=42^{\circ}$ (wind also coming from this direction), we see in Fig. 4 brightness peaking near the specular direction. Depending on the relative brightness of the bottom, its effect will contribute to a shift in the peak. If the incident light is randomly polarised, there will be virtually no polarisation in the incidence direction (Fig. 5). The degree of polarisation will increase the farther away from the incidence direction the observation angle. We assume that most of the polarisation effects come from the water surface and that perceived internal reflections contribute little ([10], [11]). Reflections from the bottom, however, will be depolarised and is likely to contribute to the depolarisation ratio peak around the specular direction, see Fig. 5 .

## Future Endeavours

Geometrical effects by a rough bottom play a significant role in the reflectance distribution and polarisation as observed above-water. Work on the full BRDF expression is still in progress, incorporating the directional distribution of polarisation effects that includes effects from in-water reflections.

Furthermore, the determination of a BRDF with either a standard clear sky or overcast sky is part of the current work. Preliminary results show that the BRDF can become diffuse so that polarisation is virtually nil at nadir and increases away from it.


Figure 4. Bi-directional reflectance distribution (thick contours) of a relatively calm ocean water surface with sun at $\left(\theta=42^{\circ}, \phi=90^{\circ}\right)$, marked by the asterisk, against a black sky. The specular direction is indicated by a circle; dotted lines represent 15degree angular increments from nadir. Degree of polarisation are the contours with lighter dashed lines.


Figure 5. As in Fig. 4, but for the depolarisation ratio.

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