APPLICATIONS OF SPECTRAL ANALYSIS TO HYDROLOGY AND CHEMICAL TRANSPORT

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APPLICATIONS OF SPECTRAL ANALYSIS TO HYDROLOGY AND CHEMICAL TRANSPORT

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This dissertation investigates and describes several ways of using spectral analysis as a frequency domain approach for mathematical hydrological modeling. Hydrologists have used spectral analysis for modeling in the past in various ways. However, one of the novelties of this research is that it is a simplification of previous techniques, which involved acquiring information about several parameters. Many of these parameters are time consuming to collect as data or are only estimated using intricate mathematical equations. In the first chapter the relationship between stream discharge and wetland water elevations in a watershed located in North Madison, CT was successfully modeled using exclusively independent measurements of discharge and wetland water elevations. This relationship was previously modeled using six parameters, three of which had to be estimated for their instantaneous behavior. Using the methods developed in this study conserved time and effort and produced the same results. The second chapter describes how to model discharge with measures of water table elevations in a runoff source area using the same techniques described in the first chapter. Using water table elevations across a lower part of the hillslope and in a near stream area of Townbrook watershed in the Catskills of New York, spectral analysis was used to determine the rate of water transport at these various locations and to successfully model stream discharge. The third chapter describes a method of using spectral analysis to determine chemical transport throughout a catchment area. Three watersheds were analyzed to describe a relationship between wet deposition and stream water concentrations of chloride (Cl) and nitrate (NO₃). Spectral analysis was also used to define a distribution of travel times associated with the transport of input concentrations of Cl, NO₃, ammonium (NH₄), total Phosphorus (TP), total dissolved phosphorus (TDP), total particulate phosphorus (TPP), soluble reactive phosphorus (SRP) suspended solids (SS), total kjeldahl nitrogen (TKN), and total organic carbon (TOC). All of these studies combined indicate that spectral analysis is a tool than can be of further use in many aspects of hydrology and in studies of water quality and chemical transport.

BIOGRAPHICAL SKETCH

Joie Taylor was born on March 24th 1977 to Karen V. Taylor in Tuscaloosa, Alabama. After living her first year in the care of her mother and grandparents, she and her mother were moved to Pontiac, Michigan where she was in the care of her mother, great aunt, Etta Lawrence and her oldest children. After 4 years Joie and her mother moved back to Tuscaloosa where she was in the primary care of her grandparents, great grandparents, aunts, uncles, and cousins. Joie discovered a talent for mathematics during her school years in Tuscaloosa and decided to pursue a career where she could use the talent. Her interest in environmental science was sparked during her high school education at Sidney Lanier in Montgomery, Alabama. Joie went on to earn a Bachelor of Science in Civil Engineering at Clark Atlanta University, a Master of Science in Agricultural and Biological Engineering and a Doctorate of Philosophy in Agricultural and Biological Engineering. This dissertation is dedicated to me, Joie Chiana Taylor for all of my will, strength, self-faith, and perseverance.

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PREFACE

This study was done to investigate the effectiveness of spectral analysis to the applied sciences of hydrology. Spectral analysis is a statistical method that analyzes times series of data and then converts this series into the frequency domain. In the time domain hydrologists can see clearly when a hydrological event occurs. The frequency domain allows hydrologists to view how many times an event occurs over the course of a time period. This can be useful in several ways. It can allow hydrologists to determine if the frequency of events in rainfall are correlated with stream flow or reservoir storage, which can give implications to transport process throughout a catchment area. In this same manner it can be used to investigate pollution by determining the processes associated with chemical and pollutant transport, subsequently providing evidence about the behavior of pollution sources that can be used for water quality purposes. The applications of spectral analysis to hydrology have included determining the characteristic scales of precipitation, runoff, and groundwater levels, studying the time and frequency response of chemical tracers and to modeling groundwater quality.

In this extensive study, spectral analysis was used in four ways. First we describe the relationship between wetland water elevations and discharge for a catchment area in North Madison Connecticut. We were able to successfully reproduce the results of a more involved model, which required measures of the elevation of water in the recharge area, surface area measurements of the wetland, estimations of the total contribution from the hillslope surrounding the recharge area, and approximations of the instantaneous behavior of evapotranspiration and precipitation. Using spectral analysis we simplified this approach and reproduced the same results. Secondly we used the same methods developed from our first study to model stream discharge from

measures of water table heights along a hillslope and saturated area in a watershed located in the Catskills of New York. We selected three wells that exhibited the same behavior as the stream flow in time and found that this is a pre-requisite for the methods to work. We were successful in estimating stream discharge and cumulative stream discharge for the spring and summer months preceding the change in behavior seen in this catchment area after a long dry period.

Thirdly we were able to apply the above methods to modeling stream water chemistry in three locations of the Catskills Watershed in NY. The focus of this study was using spectral analysis to determine phosphorous and nutrient transport with the goal of developing models that could be used to prevent eutrophication, a natural water pollution process. With the technique we developed where we define stream water chemistry as a function of rainwater chemistry we were able to approximate concentrations of Cl as a conservative tracer and NO₃. In this study we also investigated a fourth method of using spectral analysis where we proposed that stream water chemistry was the result of a convolution of rainwater chemistry and a distribution of travel times throughout a catchment. This relationship can be described in the frequency domain more simply, the results of which were used to find retardation factors for each chemical measured in the catchment.

This research describes new methods for how spectral analysis can be used in hydrology and chemical transport. This is particularly important when analyzing large quantities of data and searching for relationships that are not as obvious in a long time series. The utility of the analysis is defined in each technique developed in the study.

CHAPTER 1

SPECTRAL ANALYSIS TO DETERMINE A RELATIONSHIP BETWEEN WETLAND WATER ELEVATIONS AND DISCHARGE

Abstract

The model developed here allows users to determine the transport behavior of groundwater from a wetland to a stream with independent measurements of discharge and wetland elevations using spectral analysis. This frequency domain approach to transport modeling is a simplification of a previous model by Stagnitti et al. 1992 in the same area, which required measures of several parameters and the instantaneous behavior of evapotranspiration, precipitation and hillslope contributions. Spectral analysis is a promising technique used to reduce the error in data collection processes and to simplify techniques that determine the relationship between groundwater and catchment response. Here this mathematical model is proposed for a watershed located in North Madison, Connecticut. The watershed encompasses a wetland with an area of 3600 m^2 fed by a perched water table along a hillslope. We convert times series data into the frequency domain by using the Fourier Transform and describe the behavior of discharge from the wetland as a function of wetland water elevations. Our results show that during wet seasons the watershed behaves as a linear reservoir, transporting water from the wetland at faster rates than in dry periods. With spectral analysis we were able to reproduce the results of Stagnitti et al. 1992, which proves that frequency domain analysis is a reliable method of describing hydrological relationships.

Introduction

Hydrological modeling is an essential part of understanding catchment response for best watershed management practices that are being introduced into the watershed. A reliable mathematical model is based on the relationship between the key components of the hydrologic cycle, (e.g. precipitation, interception, infiltration, discharge, evapotranspiration, and runoff, soil physical parameters, etc.). Many models have been developed to study hydrological processes. In the past mathematical hydrological models have fallen short because they have been based on assumptions that are valid for only a few specific cases. For example, many models initially used infiltration excess overland or Horton's flow (Horton, 1933) to model transport in forested areas only to find out later that this process was mainly applicable to watersheds with thin vegetation or disturbed land use located in semiarid and arid regions (Dunne, 1983). Since then Dunne found that saturation excess overland flow and rapid subsurface storm flow are other extremes that may govern streamflow Preferential flow paths in the soil layer during both saturated and discharge. unsaturated conditions have been found to be responsible for the rapid subsurface storm flow response in forested watersheds (Freer et al. 2004, Weiler and McDonnell 2004, Mecke et al. 2000, and Moore et. al. 1986).

Generally, a distinction is made between lumped and distributed models; lumped models can predict stream discharge at the outlet of a watershed relatively well with a few lumped parameters (Donelly and Moore, 1999) where distributed models require spatial input data on topography, soils, and plant cover (Vivoni et al. 2005). In addition to rules for calculating surface runoff, each of these models maintains a water balance that calculates evaporation as a function of potential evaporation and moisture

status of the soil. For lumped models the water balance usually holds for the entire watershed contributing to the stream flow where distributed models calculate moisture content for partitions of the watershed.

Stochastic analysis has been used in the past in watershed studies (Molenat et al. 2000, Jukic and Denic-Jukic 2004, Zhang and Schilling 2004, MirallesWilhelm and Gelhar 1996, Duffy and Gelhar 1985, Kirchner et al. 2001, Kirchner et al. 2004) to simplify parameters for a lumped parameter modeling. Stochastic modeling involves converting time series of data into the frequency domain by using the results of a Fourier Transform in spectral analysis. We choose spectral analysis as our stochastic method because it has been proven to be effective in describing transport processes from a dampened signal of precipitation inputs to discharge (Kirchner et al. 2004 and 2001). This technique also reduced the number of parameters needed for the water balance described below in equation 1.1.

In this study we compare the results of a model developed by Stagnitti et al. 1992 to the results of a stochastic model using data from the same site. The model developed by Stagnitti et al. (1992) was composed of two sub models: a storage model describing the dynamic variation in water table elevation in the wetland and a hillslope model used to predict runoff and seepage throughflow from surrounding hillsides. Stagnitti's model was based on soil physical principles and is capable of predicting hillslope and watershed discharge, evapotranspiration demands, soil moisture status, surface and sub-surface flow rates. In this paper we use spectral analysis as a simplified approach to determining the relationship between discharge from a perched water table to wetland elevations in this region.

Motivation and Purpose

A mathematical model as a water balance for water storage can be represented as

$$\frac{d[A_{s}h(t)]}{dt} = A_{s}[P(t) - E_{a}(t)] + R(t) - Q(h) \qquad t \ge t_{a}$$
(1.1)

where h(t) is the elevation of water in the recharge area as measured as a height above some datum level [L], A_s is the surface area of the wetland [L²], t is time, t_a is an arbitrary starting time, P(t) is the precipitation rate per unit area [L/t], $E_a(t)$ is the evapotranspiration rate per unit area [L/t], R(t) is the total contribution from the hillslope surrounding the recharge area [L³/t], and Q(h) is the rate of outflow to the wetland area from the perched water table [L³/t]. In our case h(t) is the elevation of the wetland area as discussed later in the text , and Q is only a function of h. Using equation 1.1 for a model requires that the instantaneous behavior of P(t), Ea(t), and h(t) are known. Using this water balance also requires that and wetland water elevations are measured at exactly the same time, which is the stringent data collection requirement used previously be Stagnitti et al. (1989).

Using spectral analysis to interpret Q(t) and h(t) into the frequency domain we eliminate the necessity of finding the instantaneous behavior of the function P(t), $E_a(t)$, and R(t). We also eliminate the necessity of measuring discharge and wetland water elevation simultaneously and can use independent measurements of both to see their relationship. Spectral analysis is a frequency domain approach that uses the results of the Fourier transform to convert a time series of data to the frequency domain. The results are examined graphically using a plot known as the power spectrum. To be more explicit, "the spectrum of a time series shows the contributions of oscillations with various frequencies to the variance of time," as defined by Panofsky and Brier (1958).

The Fourier Transform decomposes a time series of data into sine and cosine functions of different frequencies. The series of h(t) expressed in the frequency domain by the Fourier transform is

$$H(f) = \int_{-\infty}^{\infty} e^{-ift} h(t) dt$$
 (1.2)

where H(f) is the Fourier Transform of $h(\tilde{t})$, t is time, and (f) is frequency (cycle/time) (Gelhar 1993). Spectral analysis takes the square of the amplitude of the wave to yield what is called the power spectra (S (f)). Therefore the power spectra of h(t) is expressed as

$$S_h(f) = |H(f)|^2$$
 (1.3)

and the power spectra of Q (S_Q) is expressed in the same manner. For the purposes of this study the program Applied Statistical Time Series Analysis (ASTSA), a statistical analysis package developed by Robert Shumway (Shumway and Stoffer 2000), will be used to perform the Fourier Transform and find their spectral powers at different frequencies.

It should be noted that a time series where the power spectra, has a power law dependence on frequency, i.e.,

$$S(f) \propto f^{-\beta}$$
 (1.4)

where S is the power spectrum, the power β =5-2D and D is the fractal dimension (Turcotte 1992). Equation (1.4) allows us to describe the behavior of the data series

in the frequency domain. For example, S(f) is constant at $\beta=0$ and the series is described as white noise, meaning that the process in the frequency domain is completely constant. If we integrate white noise over time we get brown noise, $\beta=2$, and can be generated by repeatedly summing independently random numbers (Shroeder, 1991). At $\beta=1$, the noise has been labeled 1/f noise, flicker noise, or pink noise, and indicates that the spectral power has a strong correlation to time (Takayasu, 1990), though scaling over any frequency will show the same pattern. This may seem counter-intuitive but it has been confirmed that the form of the spectrum does not change regardless of how long the observation period might be (Takayasu, 1990).

Site Description and Data Collection

The study area is a 2.3 ha wetland located in North Madison, Connecticut. The catchment is surrounded by low-lying hills and forms part of a deciduous forest. The forest soil is composed of a well-developed highly absorbent humus layer, which lies on a shallow layer of fine sandy loam to a depth of 0.3-1 m. Beneath the sandy loam is a layer of impervious granite, which extends to the boundaries of the catchment area. Located directly in the center of the watershed and about 47m above sea level is a 0.4 ha wetland that feeds an underground stream, which exits the catchment at the dotted lines on illustration 1.1. The muddy leafy bed of the wetland prohibits transmission losses.



Illustration 1.1: Sketch of the 2.3 ha catchment: illustrating the 0.4 ha wetland and one meter contours. The grid points are located at ten-meter interval. The dotted lines indicate the underground flow path connecting the swamp to the weir. (Stagnitti et. al. 1989).

Hourly wetland elevations and discharge were measured during 30 weeks in 1973, 22 weeks in 1974, and 19 weeks in 1975 by Stagnitti et al. 1989. Recording commenced in June and July of each year and was discontinued in December when the water and soil were frozen. A float device connected to a revolving, seven-day recording chart, monitored wetland water elevations continuously. A V shaped weir, which recorded the water level of the wetland near the outlet, measured catchment discharge during the periods of study. Water from the wetland flowed through the soil layer into a small collecting channel before being discharged at the weir. These measurements are shown in the hydrograph in figure 1.1, which shows the time series data of discharge values and wetland water elevations for 1973 and 1975.



Figure 1.1: Wetland water elevations and discharge at Cowetta, CT 1973 -1975.

Methods

First we look at a hydrograph of our components of interest, wetland elevation and discharge. Wetland elevations range between 0.38 and 0.635 m where the discharge ranges between 0 and $6.53 \text{m}^3/\text{hr}$. It is obvious that there is a relationship between the two in time. When the wetland elevation is the highest, we see our highest values for flow and the elevations are lowest when there is no flow. We use spectral analysis to determine if the overall long term trends of each series are equivalent. The spectral analysis in figure 1.2 shows that it is true that trends in the discharge are the same as trends in the wetland elevation. This is determined by the fact that they share the same β value (equation 1.4) of 0.9.



Figure 1.2: Spectral powers of discharge and wetland water elevations, 1973 and 1975. This graph shows the results of the spectral analysis of figure 1.1. Here the β value for each are 0.9 (approximately 1/f noise).

We can develop a model to describe and determine the discharge rates that correspond to elevations in the wetland using spectral analysis by assuming that discharge from equation 1.1 is a function of wetland elevations h(t). This relationship should be true both in the time and frequency domain. We perform a spectral analysis (equations 1.2 and 1.3) of various ranges of both data sets to determine the discharge from the aquifer that corresponds to particular wetland elevations.

Given that Q(t) is a function of h(t) it is should also true that Q(f) is a function of h(f). For a given range of spectral powers we can express Q(t) in terms of h(t) with the relationship

$$Q(t) = \Omega^* h(t) \tag{1.5}$$

where Ω is a constant over a range of Q and h(t) in m²/hr. With spectral analysis we can the determine the rate of flux (Ω) for a range of values from the spectral powers of discharge wetland elevations where

$$\Omega = \frac{\sqrt{\frac{1}{n} \sum_{1}^{n} S_{\varrho}(f)_{1...n}}}{\sqrt{\frac{1}{n} \sum_{1}^{n} S_{h}(f)_{1...n}}}$$
(1.6).

We validate our Ω value by checking that when weighted against an interval of h(t) it produces the given range of Q. To be more explicit, for a range $Q_i < Q < Q_e$ it must be true that

so that
$$Q_{c} = Q_{\Omega} - Q_{e} \approx 0$$
 (1.7)
 $\Omega_{ave}^{*} \Delta h(t) + Q_{i} = Q_{\Omega} \text{ and } Q_{\Omega} = Q_{e}$

where $\Delta h(t) = h(t)_{initial} - h(t)_{end}$, Q_{Ω} is the flux obtained from the results of equation 1.6, Q_i is the initial flux at $h(t)_{initial}$, Q_e is the final flux at $h(t)_{end}$, and Q_c is the corroboration of Q_{Ω} and Q_e. If the conditions of equation 1.6 are not met then the Ω value is invalid and experiments for a correct Ω_{ave} must continue.

We begin our analysis by transforming defined ranges of $\Delta h(t)$ into the frequency domain. For this catchment area we analyze ranges where $\Delta h(t)=0.02m$ and 0.04m. We then start at zero for the discharge values and increase our range until we find results where $Q_c \approx 0$. We often find a Q_{Ω} that is higher or lower than anticipated values for Q_e . We use these results to identify a final solution. When the Q_{Ω} is too high $(Q_e < Q_x)$ we lower the Q_e or when Q_Ω is too low we raise Q_e until we find our solution. We set our initial Q_e for the first range of h(t) at Q=0. Subsequently we find the final Q_e that produces zero error in this region and begin the next zone at this Q_e .

Because the Fourier Transform has specific requirements as a stochastic analysis that the raw data do not meet, we satisfy the numerical conditions using the following guidelines for the spectral analysis:

Guidelines:

- Analyze ranges of discharge and wetland elevation for as many numbers of points within as series as possible. Spectral Analysis gives more accurate results for longer time series (Kirchner et al., 2004).
- Adjust series to get a number of points as close to a power of 2 as possible to find the closest solution to meet the requirements of the fast Fourier transform that input data series be of some power of 2.
 - When the data series falls short 2ⁿ, ASTSA pad the series with zeros, which results in slightly higher powers at lower frequencies. We instead pad our series with the average. Padding with other values (i.e. the arithmetic mean) is also acceptable as long as there is consistency with each transform.
 - When numbers are only slightly above a power of two, instead of padding to the next higher power of two, which would greatly decrease the value of powers at lower frequencies, we truncate these numbers to the lower power of two.

- When a series is evenly in between a power of two either padding or truncating can be used however, it is important to note the difference between padding and truncating for that particular range of data. In all of our cases we find that padding produced powers that are lower than truncating as discussed later in the results. Therefore a Q_{Ω} solution resulting from padding your results will predict solutions lower than truncation and vice versa.
- Remain consistent within a trial run; do not mix padding and truncation to derive an expected solution.
- We do not accept solutions of Q_{Ω} where the number of points in the $\Delta Q(t)$ falls below 2⁸ (10.66 days). In these cases we increase $\Delta h(t)$ so that $\Delta Q(t)$ can be increased to obtain more points. This usually occurs at higher values of h(t)and Q(t) where there a are fewer number of events. In this case it is necessary to reduce the values of $h(t)_{initial}$.
- For any given region there may be more than one possible solution. The final solution must be a near zero difference solution between Q_{Ω} values that are too high or too low. We select solutions with points closest to the power of 2 where there is less padding or truncation.
 - Note: In many regions values tend to oscillate between positive and negative values as it reaches closer to the solution. In such cases we choose the solution with more points of data in the series, though choosing either solution would not greatly affect the results because of the scatter of the data.

Results

As an example of how spectral analysis transforms times series into the frequency domain, we show in figure 1.2 the spectral analysis of the time series data from figure 1. We plot the spectral powers against wavelength rather than frequency to show how that the powers increase proportionally to wavelength across the entire range. We find that the wetland and discharge both have a slope where β =0.9, which indicates that their nature of behavior is also similar in the frequency domain. The discharge and the wetland heights follow similar patterns and are almost exactly the same scaling, implying that there is no dampening between the discharge and the wetland water elevations as shown by their similar β values. This implies that during the wet periods, wetland water elevations respond very quickly to Q inputs. There are 5022 numbers of hours (points) measured for our entire range of time series data. Because this number is much closer to 2¹² that 2¹³, we analyze this series truncate to 4096 points.

As an example of what padding does over truncation, we show an example where the number of hours available for a range of Q(t) is at the midpoint between 2^9 and 2^{10} . This occurs where $2.05m^3/hr < Q < 6.531 m^3/hr$ which is shown in figure 1.3. We also show the results of the spectral analysis for wetland water elevations above 0.5 m. In figure 1.3, the first Y axis shoes the spectral powers found for discharge. The second Y axis shows the spectral powers of wetland water elevations. For ease in interpretations this graph is plotted in term of wavelength instead of frequency. In this case, when set proportional to swamp elevations above 0.5 m our model predicts a maximum Q value of $6.33 m^3/hr$ for our series padded with the average, and 7.8 m3/hr for our truncated series. The measured Qmax= $6.531m^3/hr$.



Figure 1.3: Example of truncating versus padding. This figure shows the results of a spectral analysis of elevation where h(t)>0.05m and where $Q>2.05m^3/hr$. The number of points for $\Delta Q(t)$ here is 769 which is the midpoint of 2⁹ and 2¹⁰ where either padding or truncating can be used. We show the difference between truncating and padding where truncating has produced spectral powers slightly greater than padding the series with the average. The β values for h(t), Q_{pad} , and $Q_{truncated}$ are 1.82, 1.83, and 1.87 respectively.

Truncating this series predicts slightly higher values than padding the series with the average. This is a range of Q that should correspond to higher values in the wetland heights. If we use our methods from equations 1.5-1.7, and analyze where 0.05 < h(t) < 0.635m, we find that the results of our padding produces a $Q_{\Omega}=6.33m^3/hr$ where the truncated series results in $Q_{\Omega}=7.8 m^3/hr$. Table 1 shows the results of our techniques developed from equations 1.5, 1.6, and 1.7. The interval of $\Delta h(t)$ is

indicated in bold in Column 1. Rows following bolded cells in column 1 indicate discharge values that satisfy the conditions of the model developed by Stagnitti et al. 1992. Column 2 lists the number of points (hours) for which the range of values occur and to what number these points were padded or truncated. Column 3 gives our Ω from equation 1.6. And column 4 shows Q_{Ω} where column 5 shows the Q_c from equation 1.7.

For the $\Delta h(t) = 0.02m$ we find an exact solution for 0.38 < h(t) < 0.48m where Q(t)=0. At the next interval we show one example of our adjustments between high and low values in the range 0.4 < h < 0.42m. At this interval we see that between $0 < Q < 0.046m^3/hr$ our Q_{Ω} is lightly too low (Qc= -0.003) so we then try 0 < Q < 0.49 and find that are results are too high. Through searching between those two results we find our solution at $0 < Q < 0.048 m^3/hr$. For each range of h(t) following we show the solutions obtained using the same method. We continue with 0.02m increments until we find that we have too few points for analysis for 0.44 < h < 0.46m. So we follow the third point of our guidelines and take a larger interval of 30mm from 0.44 < h < 0.47mand find a solution for $0.14 < Q < 0.4m^3/hr$. We encounter the same situation for 0.47 < h < 0.5m so we expand our analysis beyond this region to 0.55m and find a solution where $0.4 < Q < 3.35 m^3/hr$. Beyond this region we that our $Q_{\Omega} = 6.41 m^3/hr$, which produces Qc=-0.144 which is the nearest zero values in this region.

Table 1.1: Results for spectral analysis of wetland water elevations and discharge: Ranges of h(t) data are indicated in bold in Column 1. Rows following bolded cells in column 1 indicate discharge ranges. Column 2 lists the number of points available and to what power of two the numbers were padded or truncated for each range of Q(t) and h(t). Column 3 lists the Ω from equation 1.6. Column 4 shows \mathbf{Q}_{Ω} the results of experiments using equation 1.7 where column 5 lists Q_c.

∆h(t)=0.02m	Points	Ω (m²/hr)	Q _Ω (m³/hr)	Q _c (m³/hr)
0.38≤h(t)≤0.4m	1079 to 1024	<u></u>	()	
Q=0.000	801 to 1024	0.00	0.000	0.000
0.4≤h(t)≤0.42m	404 to 512			
0≤Q(t)≤0.048	871 to 1024	2.42	0.048	0.000
0.42≤h(t)≤0. 44m	850 to 1024			
0.048≤Q(t)≤0.153	977 to 1024	5.11	0.150	-0.003
0.048≤Q(t)≤0.154	985 to 1024	5.28	0.154	0.000
0.048≤Q(t)≤0.156	990 to 1024	5.62	0.160	0.004
0.44≤h(t)≤0.47m	1599 to 2048			
0.154≤Q(t)≤0.4	897 to 1024	8.15	0.398	-0.002
0.47≤h(t)≤0.55m	892 to 1024			
0.4≤Q(t)≤3.35	1000 to 1024	36.86	3.348	-0.002
0.55≤h(t)≤0.635m	436 to 512			
3.35≤Q(t)≤6.531	486 to 512	35.72	6.386	-0.144
∆h(t)=0.04m				
0.38≤h(t)≤0.42m	1079 to 2048			
0≤Q(t)≤0.075	1921 to 2048	1.88	0.075	0.000
0.42≤h(t)≤0.46m	1835 to 2048			
0.075≤Q(t)≤0.65	1932 to 2048	14.51	0.655	0.005
0.46≤h(t)≤0.635m	1726 to 2048			
0.65≤Q(t)≤6.531	1179 to 1024	36.77	7.122	0.591
0.46≤h(t)≤0.5m	926 to 1024			
0.65≤Q(t)≤1.5	352 to 512	21.29	1.502	0.002
0.5≤h(t)≤0.63 <mark>5</mark> m	436 to 512			
1.5≤Q(t)≤6.531	827 to 1024	37.90	6.567	0.04

For the $\Delta h(t)=0.04$ m we also find an exact solution for the dry region (0.38<h<0.42m) where 0<Q<0.075m³/hr. For the low flow region, 0.42m<h<0.46m, we find a solution where 0.075<Q<0.65m³/hr. If we take our high flow zone from this region as 0.46<h(t)<0.635m we see that our number of points requires that we truncate, which predicts a Q_{max} value of 7.125 m³/hr. However if we choose to pad and stay with our increments of 0.04m, for the medium flow region we find a solution where 0.65<Q<1.5 m³/hr. And for the high flow zone where 1.5<Q(t)<6.53 m³/hr we find that our resulting Q_{max}=6.567 m³/hr gives a Q_c=0.04 which is the nearest zero values in this region.

We plot the result shown in Table 1.1 in figure 1.4 where we can see that for 0.35 < h(t) < 0.5m our results for $\Delta h(t)=0.02m$ predicts much lower than $\Delta h(t)=0.04m$. However beyond that region we find the same results if we use the values obtained from padding the range of $1.5 < Q < 6.531 \text{ m}^3/\text{hr}$ and the relatively few points for 0.05 < h(t) < 0.635m.

To produce these two curves there was quite a bit of padding for this range of Q data so we compare figure 1.4 with the results of truncating points of Q(t) where 0.046 < h(t) < 0.635m shown in figure 1.5. We find that using this value clearly defines an upper and lower boundary around the points calculated from the hydrograph and weir water level.



Figure 1.4: Discharge Q(t) as a function of the wetland water elevation h(t): the resulting relationship using our guidelines and equations1.5-1.7 with upper and lower envelopes.



Figure 1.5: Upper and lower envelops of data points with truncation at 0.04m. This figure shows that the Stagnitti model falls in between the upper and lower bounds found using the technique described in this paper.

We take the average of the two bounds shown in figure 1.5 as our final model and find that the model fits the data as four straight lines that can be distinguished by flow zones described by equations 1.8-1.11 as:

Zone 1: high flow (h>0.46m);
$$Q(h)=35.57m^2/hr^*h -16.01m^3/hr$$
 (1.8)
Zone 2: medium flow (0.42mQ(h)=20.32 m^2/hr^*h -8.97 m^3/hr (1.9)
Zone 3: low flow (0.38mQ(h)=3.84 \text{ m}^2/\text{hr *h-1.55 m}^3/\text{hr} (1.10)
Zone 4: no flow (h<0.42m)
$$Q(h)=0 \text{ m}^2/\text{hr}+0 \text{ m}^3/\text{hr}$$
 (1.11)

These four zones are similar to what Stagnitti et al. 1989 found from their model which is shown explicitly in figure 1.6 below.



Figure 1.6: Average of the upper and lower envelope and flow zones: This figure shows the average of the two curves from figure 1.5. Lines 1 through 4 represent high flow, medium flow, low flow and no flow zones respectively.

The implications of these results show that stochastic analysis is a reliable method for interpreting the relationship of hydrological components. In the case the major discharge regulating mechanism operating in the catchment is the wetland and accurate measurement of the wetland's water elevation will produce discharge values that are reliable.

Discussion and Conclusions

The technique we used here can be described as a set of mathematical experiments to determine wetland water elevations determine particular flow rates using spectral analysis. Our experiments using spectral analysis, resulted in producing flow zones similar to those found by Stagnitti et. al. 1989. Stagnitti et al. 1992 found that hillslope contributions to recharge or storage areas are important during large storm events or in periods of high flow. Here we find that when during wet seasons the watershed behaves as a linear reservoir transferring water from the wetland at higher rates than in dry periods. We find, just as Stagnitti et al. 1992 that the rate of attenuation for water transfer on a hillslope depends on soil saturation. In the dry seasons when soil is ready to absorb any amount of precipitation, the resulting rise in wetland water elevation is minimal. During the wet seasons, because of the increased rate of transfer, wetland water elevations respond quickly to very little amounts of input precipitation.

We were able to reproduce the same results as obtained by Stagnitti et al. 1992 where they used a simple first order partial differential equation to describe the dynamics of a hillslope discharge. Although the system we model here is fed by a perched water table the technique developed with the model is applicable to any groundwater system. The results of this research indicate that by studying the fractal the behavior of flow and storage areas, the nature of groundwater transport in real time can be determined. To produce the curve in figure 1.7 Stagnitti et al. 1989 measured or calculated all of the terms in equation 1.1, while estimating the instantaneous behavior of p(t), Ea(t), and h(t). The stochastic model developed here allows the user to determine the behavior of an aquifer with independent measurements of discharge and wetland water height, simplifying data collection and eliminating the necessity of knowing the instantaneous behavior of functions p(t), Ea(t), and h(t). Though our method involves a bit of artistry in meeting the requirements of the Fourier Transform, it is clear that the stochastic model developed here produces a reliable model. However, it should be noted that there is room for the technique of padding and truncating to be refined for a more sound and definitive model.

The results of our analysis also indicate that the behavior of groundwater transport is fractal in nature and therefore scale independent. This allows the user to validate the model be determining past values of discharge or wetland water heights and possibly project the model for future values.

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CHAPTER 2

HYDROLOGICAL MODELING WITH A SPECTRAL ANALYSIS OF WATER TABLE ELEVATIONS AND WATERSHED DISCHARGE

Abstract

In a watershed of the Catskills, NY spectral analysis is used to identify the relationship between stream discharge and the height of a perched water table. The water table elevations from two wells on a hillside and one well in a near-stream saturated area are used for this analysis. With spectral analysis the time series data of well water heights and stream discharge were converted into the frequency domain by using the Fourier Transform. By dividing the times series into a number of intervals a relationship between well water table height and discharge was established. This relationship was found to be different for wet spring and early summer months than for the fall after a prolonged late dry summer. Estimations of discharge from water table elevations measured on the hillslope were reasonable. However, predictions of discharge from the determined relationship produced the most reliable results from the measurements of the well located in the near-stream saturated area. With these three probes we were able to reproduce the measured cumulative discharge. The results of this paper show that stochastic analysis can be used to estimate discharge with measures water table elevations through the theory that over time the frequency of water table elevations are equivalent to the frequency of discharge.

Background

The study of water transport to streams or reservoirs involves a thorough understanding of the basic hydrological components including evapotranspiration, precipitation, streamflow, groundwater recharge, etc. of a catchment area. Recently many techniques have been used to refine models that have previously made various assumptions about contributions to streamwater. These studies have included detailed observation of interflow and groundwater transport (Bogena et. al. 2005, Valstar et. al., 2004, Weiler and McDonnell, 2004) and have accounted for the assumption that overland flow is the predominant source to streamwater. Soulis et al. 2000 improved upon the model Canadian Land Surface Scheme (CLASS) by approximating horizontal gradients of the internal topography and determined that interflow was generated from the near surface layer Frankenberger et al. (1999) were able to develop a more thorough model that uses land use practices to predict the spatial distribution of soil moisture as well as determining the portion of contribution from saturation excess overland flow. However, most of these models have assumed that aquifers are evenly distributed across the landscape. Models that assume this relationship of runoff to groundwater do not account for the affect of location of groundwater to runoff and calculates uphill and downhill sources of water as contributing equal amounts of runoff into streams. This assumption neglects evidence that water tables located nearer to streams create saturated areas, which generate overland flow and are a greater contributor to runoff than locations upslope (Seibert et al. 2003, Moore and Thompson 1996).

It may be possible to study the elevation of water tables over different points throughout a landscape to determine the portion of contribution from areas upslope versus downslope. Studies of water table elevation have been used to investigate hydrological transport by determining their relationship to storm runoff (Savabi et al. 2004), sloping aquifers and hydraulic conductivity (Stangitti et al. 2004). Water Table elevations have been used to refine models (Freer, et. al. 2004) and they have also been studied to model groundwater relationship to streamflow to investigate the assumption that streamflow can be modeled as a series of steady state groundwater flows (Seibert et al. 2003).

With stochastic analysis we can determine the portion of area contribution to runoff in a various locations on a landscape by using spectral analysis to convert long-term time series data of water table elevations at several wells. Spectral analysis is a long-term frequency domain time series analysis that has been successfully used to analyze longterm time series of data, quantify travel time distributions, and measure the watershed scale retardation factor for reactive solutes (Feng et. al., 2003, Kirchner et al., 2000a; Kirchner et al., 2001). We use spectral analysis because it has been proved to produce the least biased results in many hydrological studies when compared to other times series analyses such as the re-scaled range method, autocorrelation, and relative dispersion (Schepers et al 1992, Pelletier and Turcotte 1997). Spectral analysis has also been used in transport modeling in general (Kirchner et al 2001) and specifically with water tables to analyze fluctuations in determining hydraulic diffusivity of aquifers, (Shih, 1999) and to establish the effects of fluctuations to nearby ocean tides (Marechal et al. 2002). This method of analysis can also be used to analyze water movement in a landscape by determining the relationship between water table fluctuation and the discharge at the outlet of a particular watershed.

In this study we use spectral analysis to compare the behavior of water table elevations

for wells located upslope to aquifers located in a near stream wet area using measures of water table elevations from two wells at different location on a hillside and two wells in a downslope extremely saturated area near the stream. We make predictions of discharge using the relationship that discharge is a function of water table elevation.

Site Description and Data Collection

We use the site of the New York State Department of Environmental Protection (DEP) located in the upper region of the Cannonsville watershed in the Catskills Mountain region in New York. This primarily grassy study site is 2.44 ha in area and is part of a 2 km² watershed located in the southwest corner of the Townbrook watershed. More than 200 points along the study area were surveyed in addition to an existing 10m digital elevation model (DEM) to derive 1-m interval contours for identifying topographic details. Soil Survey Geographic Database (SSURGO) soil maps were used to determine soil types and properties. Two soil types dominate the study site. Both soils have shallow topsoil overlying a dense subsoil. The topsoil on the northern (down slope) half of the study site it consisted of 30 cm of a gravely silt loam. The southern (up slope) half of the study site it consisted of approximately 55 cm of a silt loam. The area stretches 120m along the stream and extends 180m upslope with slopes ranging between 0° and 8° and elevations between 585 and 600m above sea level, which is shown in illustration 2.1.



Illustration 2.1: Map of Townbrook, NY: Hillslope steepness is indicated by contour lines shown in white. Probes used for this paper are labeled at their locations.

Rainfall amounts were measured at 15-minute intervals using a tipping bucket gage placed approximately 0.5 km north of the study. Water tables were monitored with a water level logger at 44 sampling locations on 5-minute intervals and averaged over one hour for a study period between March 2004 and August 2004. The water levels in the upper 30-50 cm of the soil were recorded using WT-HR 500 capacitance probes manufactured by TruTrack, Inc, New Zealand, manufactured by TruTrack, were WT-HR 500 capacitance measured and acted as piezometers. The piezometers were located on two grid systems. The first system held 20 loggers on a 10x10m space near the stream. The second system held the remaining 24 loggers on a large spacing 30mx 40m upslope of the stream. For this study we chose water table elevation measurements from two loggers along the hillslope (Probes 15 and 51 in illustration 2.1) and one in the near stream area (probe 107) that were placed to the depth of the bedrock. These loggers recorded stream stage, which was then converted to flow using rating curves developed for the stream at both locations. Each rating curve was based on seven current-meter discharge measurements. We describe how we selected these 3 probes out of the 44 in the following section. All probe measurements in this paper were collected downloaded by Steve Lyon (Lyon et. al. 2006).

Method and Materials

Water table heights, rainfall, and stream discharge data retrieved from the probes over nine months was compiled into hourly time steps to be analyzed for their spectral powers. In figure 2.1 we show the hydrograph of precipitation in the area, which is highest in the spring and winter (between 20 and 150 mm/day) and lowest in the summer months (0-20 mm/day).



Figure 2.1: Hydrograph of rainfall and stream discharge

The discharge values follow the same seasonal pattern and range at the lowest $0.002m^3/s$, with just a few events above 1m3/s to a maximum of $4.2 m^3/s$. We use the hydrograph of each well to select the probes that show the least error in measurement and are to the depth of the semi-permeable layer. From the 44 probes we have selected 3, two probes on the hillslope and tone probe in the near stream area whose fluctuation patterns follow the pattern of the discharge so that discharge Q is a function of water table elevations h(t). For the frequency analysis to determine a relations it is necessary that each measured constituent has the came frequency of events. This was true of the three probes chosen. The discharge is also plotted against well water elevations at these probes to observe their relationship. This is shown in hydrographs of figures 2.2-2.4.



Figure 2.2: Hydrograph of water table elevations at probe 15 and stream discharge.



Figure 2.3: Hydrograph of water table elevations at probe 51 and stream discharge.



Figure 2.4: Hydrograph of water table elevations at probe 107 and stream discharge.

All of the probes measure the lowest elevations in summer and highest in the spring months. Probe 15 located furthest away from the stream measures water elevations between 96 to 335mm. A little further down hill probe 51 shows that water elevations range between 0mm in the summer months and 350 mm in rainy seasons. In the saturated area water elevations tend to be much higher. Probe 107 measures elevations as high as 440mm. After the summer dry season each probe shows a different behavior than the streamflow. Because of this we will analyze these graphs both including and excluding the period following the summer dry season.

We now analyze these hydrographs in the frequency domain to give an indication of the time independence of both components. Spectral Analysis coverts time series into the frequency domain using the Fourier Transform. The analysis decomposes the series into components of sin and cos and takes the square of the amplitude of these waves to yield the spectral power (Bracewell , 2000; and Scargle, 1982). A time series where the power spectra, has a power law dependence on frequency is defined as a fractal time series where the power β =5-2D and D is the fractal dimension (Turcotte 1992).

For the Fourier Transform we use the statistical package ASTSA which was developed by Robert Shumway at the University of Pittsburg (Shumay and Stoffer 2000). This program computes the periodogram, the squared magnitude of the Discrete Fourier Transform (DFT)

$$P(f_k) = |X(f_k)|^2$$
(2.1)

where $X(f_k)$ is the DFT of the sample series, $f_k = k/n$, k=0,...,n-1 defines the set of frequencies over which the DFT is computed (Shumway and Stoffer 2000). The evaluation continues using the fast Fourier Transform (FFT) assuming that the length of the data series (n) is in some power of 2. When the series is not a power of two ASTSA pads the series by adding zeros to extend the length to the next power of two (Shumway and Stoffer 2000). This padding technique is not acceptable for our purposes because it may greatly distort the results of our predictions at higher flow rates. To correct for stipulation with the Fourier Transform we use the guidelines described in the previous chapter.

Given that Q is a function of h(t), the power spectrum (equation 2.1) of Q can be written in terms of h using the following

$$|Q(f_{1...n})|^2 = \Omega^* |H(f_{1...n})|^2$$
 or $|Q(f_{1...n})|^2 = \Omega^* |H(f_{1...n})|^2$ (2.2)

where Ω is a term that associates stream discharge to water table elevations with units of m²/s, Q(f) and H(f) are the Fourier Transforms of Q(t) and h(t). We find our solutions for equation 2.2 by transforming a set range of h(t). There are several way to set the ranges of $\Delta h(t)$ which depend on data availability. For probe 15 we analyze changes of 0.05m of h(t), where we take $\Delta h=0.08m$ for probe 51. However for probe 107 we take ranges of h(t) that gives us a number of points closest to the power of two. We do this because for Probe 107 fixed interval of $\Delta h(t)$ gives a number of points too far from a power of two. We use the Fourier transform for a range of h(t) to find the range of Q(t) that produces an Ω that satisfies

$$\Omega * \Delta h(t) + Q_i = Q_\Omega$$
 where $Q_\Omega = Q_e$ and $Q_c = Q_\Omega - Q_e \approx 0$ (2.3)
and

$$\Omega = \frac{\sqrt{\frac{1}{n} \sum_{1}^{n} |Q(f_{1...n})|^{2}}}{\sqrt{\frac{1}{n} \sum_{1}^{n} |H(f_{1...n})|^{2}}}$$
(2.4),

where $Q_e = \Delta Q + Q_i$, $\Delta h(t) = h_e - h_i$, Q_i is the initial flux at $h(t)_i$, Q_e is the final flux at $h_e(t)$, and Q_e is the corroboration of Q_e . Using this relationship we often find Q_Ω that is higher or lower than anticipated values for Q_e . We use these results to identify a final solution. When $Q_\Omega > Q_e$ we lower our range of Q(t). When $Q_\Omega < Q_e$ we raise Q_e until we find a zero value. We set our initial Q_i at the lowest values of Q(t) (0.002 m³/s). Subsequently we find the final Q_e that produces zero error and begin the next range at this Q.

Results

When we convert our time series from each probe into the frequency domain we find that they follow similar patterns. We choose probe 51 to show an example of the spectral analysis of the entire series of discharge and water table elevations. Figure 2.5 shows that water table elevations measured by the probes scale the same as the discharge with β =2. However the spectral powers in water table elevations are much higher than in the stream because of greater amplitudes. We find these results also for Probes 15 and 107.



Figure 2.5: Spectral analysis of discharge and water table elevations at Probe 51. Using our guidelines to estimate the relationship to discharge at each probe for particular ranges of events we find our results in Table 2.1.

Table 2.1 Results for spectral analysis of water table elevations and discharge: Probe numbers are in larger bolded print. The ranges of water table elevations per trail run are listed in Column 1. Rows following the bolded italicized cells in column 1 indicate the range if discharge values that our predictions produced to correspond with the listed range. Column 2 lists the number of points available for each range and determines to what number we padded or truncated values for the Fourier criterion of 2^n . This is the number of hours within the times series that these events occur. Column 3 lists the results of the transport rate associated with the well elevations related to discharge (related flux from equation 4). Column 4 and 5 show the results of experiments using equation 2.3.

PROBE 51	# of points to			
	2 ⁿ	Ω (m²/s)	Q_{Ω} (m³/s)	Q _c (m³/s)
0 <h(t)<0.1m< td=""><td>845 to 1024</td><td></td><td></td><td></td></h(t)<0.1m<>	845 to 1024			
Q(t)=0.002	54 to 64	0.000	0.002	0.000
0.10 <h(t)<0.18 m<="" td=""><td>404 to 512</td><td>,</td><td></td><td></td></h(t)<0.18>	404 to 512	,		
0.002 <q<0.03< td=""><td>976 to 1024</td><td>0.332</td><td>0.029</td><td>-0.002</td></q<0.03<>	976 to 1024	0.332	0.029	-0.002
0.002 <q<0.035< td=""><td>1110 to 1024</td><td>0.407</td><td>0.035</td><td>0.000</td></q<0.035<>	1110 to 1024	0.407	0.035	0.000
0.002 <q<0.045< td=""><td>1315 to 2048</td><td>0.565</td><td>0.047</td><td>0.002</td></q<0.045<>	1315 to 2048	0.565	0.047	0.002
0.18 <h<0.26m< td=""><td>1081 to 1024</td><td></td><td></td><td></td></h<0.26m<>	1081 to 1024			
0.035 <q<0.172< td=""><td>2035 to 2048</td><td>1.700</td><td>0.171</td><td>-0.001</td></q<0.172<>	2035 to 2048	1.700	0.171	-0.001
0.26 <h<0.315m< td=""><td>1017 to 1024</td><td></td><td></td><td></td></h<0.315m<>	1017 to 1024			
0.172 <q<0.295< td=""><td>486 to 512</td><td>2.189</td><td>0.292</td><td>-0.003</td></q<0.295<>	486 to 512	2.189	0.292	-0.003
0.315 <h<0.35m< td=""><td>850 to 1024</td><td></td><td></td><td></td></h<0.35m<>	850 to 1024			
0.295 <q<0.368< td=""><td>238 to 256</td><td>2.085</td><td>0.368</td><td>0.000</td></q<0.368<>	238 to 256	2.085	0.368	0.000
0.1 <h<16m< td=""><td>252 to 256</td><td>;</td><td></td><td></td></h<16m<>	252 to 256	;		
0.002 <q<0.01< td=""><td>600 to 526</td><td>0.134</td><td>0.010</td><td>0.000</td></q<0.01<>	600 to 526	0.134	0.010	0.000
0.16 <h<0.2m< td=""><td>380 to 512</td><td></td><td></td><td></td></h<0.2m<>	380 to 512			
0.01 <q<0.043< td=""><td>1259 to 2048</td><td>0.845</td><td>0.044</td><td>0.001</td></q<0.043<>	1259 to 2048	0.845	0.044	0.001
0.2 <h<0.30m< td=""><td>300 to 256</td><td></td><td></td><td></td></h<0.30m<>	300 to 256			
0.043 <q<0.2< td=""><td>2043 to 2048</td><td>1.598</td><td>0.203</td><td>0.003</td></q<0.2<>	2043 to 2048	1.598	0.203	0.003
0.16 <h<0.24m< td=""><td>979 to 1024</td><td></td><td></td><td></td></h<0.24m<>	979 to 1024			
0.01 <q<0.1< td=""><td>2437 to 2048</td><td>1.150</td><td>0.102</td><td>0.002</td></q<0.1<>	2437 to 2048	1.150	0.102	0.002
0.24 <h<0.33m< td=""><td>1800 to 2048</td><td>•</td><td></td><td></td></h<0.33m<>	1800 to 2048	•		

Table 2.1 (continued)

0.1 <q<0.3< td=""><td>1180 to 1024</td><td>2.450</td><td>0.321</td><td>0.001</td></q<0.3<>	1180 to 1024	2.450	0.321	0.001
0.18 <h<0.22m< td=""><td>547 to 526</td><td></td><td></td><td></td></h<0.22m<>	547 to 526			
0.035 <q<0.063< td=""><td>618 to 512</td><td>0.890</td><td>0.071</td><td>0.001</td></q<0.063<>	618 to 512	0.890	0.071	0.001
0.180 <h<0.28m< td=""><td>1339 to 1024</td><td></td><td></td><td></td></h<0.28m<>	1339 to 1024			
0.035 <q<0.18< td=""><td>2090 to 2048</td><td>1.456</td><td>0.181</td><td>0.001</td></q<0.18<>	2090 to 2048	1.456	0.181	0.001
PROBE 15				
0.134 <h(t)<0.184m< td=""><td>648 to 512</td><td></td><td></td><td></td></h(t)<0.184m<>	648 to 512			
0.002 <q<0.005< td=""><td>256</td><td>0.067</td><td>0.005</td><td>0.000</td></q<0.005<>	256	0.067	0.005	0.000
0.184 <h(t)<0.234m< td=""><td>428 to 512</td><td></td><td></td><td></td></h(t)<0.234m<>	428 to 512			
0.005 <q<0.01< td=""><td>345 to 512</td><td>0.103</td><td>0.010</td><td>0.000</td></q<0.01<>	345 to 512	0.103	0.010	0.000
0.234 <h(t)<0.284< td=""><td>950 to 1024</td><td></td><td></td><td></td></h(t)<0.284<>	950 to 1024			
0.01 <q<0.085< td=""><td>1501 to 2048</td><td>1.456</td><td>0.083</td><td>-0.002</td></q<0.085<>	1501 to 2048	1.456	0.083	-0.002
0.284 <h(t)<0.334m< td=""><td>2135 to 2048</td><td></td><td></td><td></td></h(t)<0.334m<>	2135 to 2048			
0.085 <q<.17< td=""><td>1021 to 1024</td><td>1.696</td><td>0.170</td><td>0.000</td></q<.17<>	1021 to 1024	1.696	0.170	0.000
0.234 <h(t)<0.264m< td=""><td>348 to 512</td><td></td><td></td><td></td></h(t)<0.264m<>	348 to 512			
0.01 <q<0.057< td=""><td>994 to 1024</td><td>1.612</td><td>0.058</td><td>0.001</td></q<0.057<>	994 to 1024	1.612	0.058	0.001
0.284 <h(t)<0.304m< td=""><td>395 to 512</td><td></td><td></td><td></td></h(t)<0.304m<>	395 to 512			
0.085 <q<.133< td=""><td>765 to 1024</td><td>2.367</td><td>0.132</td><td>-0.001</td></q<.133<>	765 to 1024	2.367	0.132	-0.001
PROBE 107				
0.1 <h<0.158m< td=""><td>512</td><td></td><td></td><td></td></h<0.158m<>	512			
Q=0.002	100 to 128	0.000	0.002	0.000
0.158 <h<0.235m< td=""><td>510 to 512</td><td></td><td></td><td></td></h<0.235m<>	510 to 512			
0.002 <q<0.085< td=""><td>2105 to 2048</td><td>1.072</td><td>0.085</td><td>0.000</td></q<0.085<>	2105 to 2048	1.072	0.085	0.000
0.235 <h<0.291m< td=""><td>511 to 512</td><td></td><td></td><td></td></h<0.291m<>	511 to 512			
0.085 <q<0.188< td=""><td>1109 to 1024</td><td>1.841</td><td>0.188</td><td>0.000</td></q<0.188<>	1109 to 1024	1.841	0.188	0.000
0.291 <h<0.346m< td=""><td>505 to 512</td><td></td><td></td><td></td></h<0.346m<>	505 to 512			
0.188 <q<0.29< td=""><td>402 to 512</td><td>1.820</td><td>0.288</td><td>-0.002</td></q<0.29<>	402 to 512	1.820	0.288	-0.002
0.291 <h<0.321m< td=""><td>244 to 256</td><td></td><td></td><td></td></h<0.321m<>	244 to 256			
0.188 <q<0.22< td=""><td>108 to 128</td><td>1.109</td><td>0.222</td><td>0.002</td></q<0.22<>	108 to 128	1.109	0.222	0.002
0.346 <h<0.442m< td=""><td>915 to 1024</td><td></td><td></td><td></td></h<0.442m<>	915 to 1024			
0.29 <q<0.4< td=""><td>204 to 256</td><td>0.844</td><td>0.371</td><td>-0.029</td></q<0.4<>	204 to 256	0.844	0.371	-0.029
.158 <h<.228m< td=""><td>455 to 526</td><td></td><td></td><td></td></h<.228m<>	455 to 526			
0.002 <q<0.071< td=""><td>1850 to 2048</td><td>0.982</td><td>0.071</td><td>0.000</td></q<0.071<>	1850 to 2048	0.982	0.071	0.000
.228 <h<.278m< td=""><td>461 to 526</td><td></td><td></td><td></td></h<.278m<>	461 to 526			
0.071 <q<0.16< td=""><td>1124 to 1024</td><td>1.720</td><td>0.157</td><td>-0.003</td></q<0.16<>	1124 to 1024	1.720	0.157	-0.003
0.321 <h<0.355m< td=""><td>516 to 512</td><td></td><td></td><td></td></h<0.355m<>	516 to 512			
0.22 <q<0.3< td=""><td>381 to 512</td><td>2.413</td><td>0.302</td><td>0.002</td></q<0.3<>	381 to 512	2.413	0.302	0.002

Table 2.1 (continued)

0.321 <h<0.401m< th=""><th>1068 to 1024</th><th></th><th></th><th></th></h<0.401m<>	1068 to 1024			
0.22 <q<0.35< td=""><td>508 to 512</td><td>1.650</td><td>0.352</td><td>0.002</td></q<0.35<>	508 to 512	1.650	0.352	0.002
0.321 <h<0.431m< th=""><th>1170 to 1024</th><th></th><th></th><th></th></h<0.431m<>	1170 to 1024			
0.22 <q<0.337m< td=""><td>456 to 512</td><td>0.980</td><td>0.335</td><td>-0.002</td></q<0.337m<>	456 to 512	0.980	0.335	-0.002

In table 2.1 we show one example of how our selected values should measure between a value lower and higher than the Q_f in the range of data for Probe 51. This example is in rows 4-7 for elevations between 0.1m and 0.18m. The Ω found for 0.002<Q<0.03 was too low where the Ω for 0.002<Q<0.045 was too high. The following rowns only show solutions of the corroborated Q_c values at or near zero. For Probes 51 we begin our analysis for $\Delta h(t)=0.08m$ until we reach the 0.26m. From here, because our number of available point were to far from a power of two, we reduced this range of h(t). We found more points to refine our model by searching for different increments of h(t). We do the same for Probe 15 where our initial $\Delta h(t)=0.05m$. From table 2.1 we find discharge can be desdefined as a function of water table elevations through various flow zones. We show this explicitly for each probe in figures 2.6-2.7. There were 3 flow zones found to be associated with Probe 51, and two flow zones associated with water table elevations measured by Probes 15 and 107.



Figure 2.6: Discharge as a function of water table elevation at probe 51: Results for Probe 51 from experiments shown in Table 2.1. The numbers 1, 2, and 3 indicate the three flow zones. Zone 1 is a low flow zone where zone 2 is a medium flow zone, and zone 3 is a high flow zone.



Figure 2.7: Discharge as a function of water table elevation at probe 15. Results for Probe 15 from experiments shown in Table 2.1. The numbers 1 and 2 indicate the two flow zones. Zone 1 is a low flow zone where zone 2 is a high flow zone.



Figure 2.8: Discharge as a function of water table elevation at probe 107. Results for Probe 107 from experiments shown in Table 2.1. The numbers 1 and 2 indicate the two flow zones. Zone 1 is a low flow zone where zone 2 is a high flow zone.

We find that Probe 51 (figure 2.6) can be described as four linear zones described by the following where Q is in m^3/s :

(1)Low flow; h(t) < 0.15m where Q(t) = 0*h(t)+0.002(2)Medium Flow; 0.15 < h(t) < 0.34m where $Q(t) = 6.1*h(t)^2 - 1.29*h(t)+0.056$. (3)High Flow, 0.34 < h(t) where Q(t) = 3.4*h(t)-0.822.

For Probe 15 (figure 2.7) we also find two linear zones described by $Q(t) = \Omega^* h(t) + b$:

(1)Low flow; $0.134 \le h(t) \le 0.234 \text{m}$ a linear equation $\Omega = 0.08 \text{m}^2/\text{s}$ and $b = -0.0091 \text{m}^3/\text{s}$ (2)High Flow; $h(t) \ge 0.234 \text{m}$ a linear equation $\Omega = 1.746 \text{m}^2/\text{s}$ and $b = -0.402 \text{m}^3/\text{s}$.

Closest to the stream in the more saturated area Probe 107 (figure 2.8) shows similar behavior to probe 51. For this probe we find the following equations define two flow zones where Q is in m^3/s :

(1)low flow ; $0 \le h(t) \le 0.19$ m a linear equation Q(t)=0 *h(t)+0.002m3/s (2)High flow; $h(t) \ge 0.19$ m a linear equation Q(t)=1.6 *h(t)-0.3m³/s.

When we use the equations above to predict stream flow we find that our predictions simulated similar trends however, because the probes cannot measure water levels above the ground surface, we were unable to reproduce the peaks in the outflow. We show the results of each probe in figure 2.9-2.11.



Figure 2.9: Discharge predictions using water table elevations from Probe 51: Results using zones described in figure 2.6.



Figure 2.10: Discharge predictions using water table elevations from Probe 15:



Results using zones described in figure 2.7.

Figure 2.11: Discharge predictions using water table elevations from Probe 107: Results using zones described in figure 2.8.

It is obvious that for time periods beyond July 29th our predictions fail to produce reliable results. This is because water table elevations fluctuation patterns after a long dry period do not coincide with discharge fluctuation patterns. This may be due to the fact that during this period, the predominant source of runoff is governed by Hortonian flow. However, these patterns coincide for seasons preceding this relatively short dry season. Therefore it must be true that the cumulative estimated and calculated values agree. In figures 2.12-2.15 we do find that our calculations successfully estimate the cumulative discharge.



Figure 2.12: Predicted cumulative discharge using water table elevations probe 51.



Figure 2.13: Predicted cumulative discharge using water table elevations at probe 15.



Figure 2.14: Predicted cumulative discharge using water table elevations at probe 107.

Using the equations derived to describe flow zones at each probe in figures 2.6-2.8, we were also able to estimate the height of the water tables in time at each probe from measurements of discharge. We show as an example of this the results for Probe 51 in figure 2.15.



Figure 2.15: Estimation of water table heights at probe 51 using measures of discharge.

We see in figures 2.12-2.15 that after the dry season our predictions estimate an inverse behavior of discharge to the actual. When we investigate the measurements of all 44 probes in the study area, we find that after the dry period that water table behaved differently and is not correlated to the streamflow until the winter. Because of this we repeat our analysis for each probe to explore the effect this behavior could have on our prediction.

We find a summary of our newly found points for March-July 14th 2005 in Table 2. The number of point available in each reason decreased as a result of excluding post summer values. Fore Probes 51 and 15 this resulted in decreased predicted values, however probe 107 remained relatively the same.

Table 2.2: Spectral analysis of water table elevations and discharge excluding post
dry season months. Probe numbers are in larger bolded print. The ranges of water
table elevations per trail run are listed in Column 1. Rows following the bolded
italicized cells in column 1 indicate the range if discharge values that our predictions
produced to correspond with the listed range. Column 2 lists the number of points
available for each range to the Fourier criterion of 2^n . Column 3 lists related flux from
equation 4. Column 4 and 5 show the results of experiments using equation 2.3.

PROBE 51	# of points to 2 ⁿ	Ω (m²/s)	${ m Q}_{\Omega}$ (m³/s)	Q _c (m³/s)
0 <h(t)<0.085 m<="" td=""><td>637 to 512</td><td></td><td></td><td></td></h(t)<0.085>	637 to 512			
0.002 <q<0.005< td=""><td>218 to 256</td><td>0.049</td><td>0.006</td><td>0.001</td></q<0.005<>	218 to 256	0.049	0.006	0.001
0.002 <q<0.006< td=""><td>251 to 256</td><td>0.051</td><td>0.006</td><td>0.000</td></q<0.006<>	251 to 256	0.051	0.006	0.000
0.002 <q<0.007< td=""><td>280 to 512</td><td>0.052</td><td>0.006</td><td>-0.001</td></q<0.007<>	280 to 512	0.052	0.006	-0.001
0.085 <h<0.175m< td=""><td>128</td><td></td><td></td><td></td></h<0.175m<>	128			
0.006 <q<0.013< td=""><td>234 to 256</td><td>0.083</td><td>0.013</td><td>0.000</td></q<0.013<>	234 to 256	0.083	0.013	0.000
0.175 <h<0.208m< td=""><td>256</td><td></td><td></td><td></td></h<0.208m<>	256			
0.013 <q<0.31< td=""><td>288 to 256</td><td>0.499</td><td>0.029</td><td>-0.002</td></q<0.31<>	288 to 256	0.499	0.029	-0.002
0.208 <h<0.265m< td=""><td>522 to 512</td><td></td><td></td><td></td></h<0.265m<>	522 to 512			
0.031 <q<0.07< td=""><td>513 to 512</td><td>0.662</td><td>0.069</td><td>-0.001</td></q<0.07<>	513 to 512	0.662	0.069	-0.001
0.265 <h<0.301m< td=""><td>511</td><td></td><td></td><td></td></h<0.301m<>	511			
0.07 <q<0.175< td=""><td>1120 to 1024</td><td>2.920</td><td>0.175</td><td>0.000</td></q<0.175<>	1120 to 1024	2.920	0.175	0.000
0.301 <h<0.324m< td=""><td>514 to 512</td><td></td><td></td><td></td></h<0.324m<>	514 to 512			
0.175 <q<0.216< td=""><td>128</td><td>1.612</td><td>0.212</td><td>0.000</td></q<0.216<>	128	1.612	0.212	0.000
0.324 <h<0.339m< td=""><td>258 to 256</td><td></td><td></td><td></td></h<0.339m<>	258 to 256			
0.216 <q<0.33< td=""><td>266 to 256</td><td>6.936</td><td>0.327</td><td>-0.003</td></q<0.33<>	266 to 256	6.936	0.327	-0.003
0.339 <h<0.346m< td=""><td>128</td><td></td><td></td><td></td></h<0.346m<>	128			
0.33 <q<0.447< td=""><td>132 to 128</td><td>16.789</td><td>0.448</td><td>0.001</td></q<0.447<>	132 to 128	16.789	0.448	0.001
0.301 <h<0.313m< td=""><td>257 to 256</td><td></td><td></td><td></td></h<0.313m<>	257 to 256			
0.175 <q<0.2< td=""><td>100 to 128</td><td>2.123</td><td>0.200</td><td>0.000</td></q<0.2<>	100 to 128	2.123	0.200	0.000
0.175 <h<0.186m< td=""><td>147 to 128</td><td></td><td></td><td></td></h<0.186m<>	147 to 128			
0.013 <q<0.027< td=""><td>203 to 256</td><td>1.298</td><td>0.027</td><td>0.000</td></q<0.027<>	203 to 256	1.298	0.027	0.000
0.208 <h<0.230m< td=""><td>254 to 256</td><td></td><td></td><td></td></h<0.230m<>	254 to 256			
0.031 <q<0.057< td=""><td>478 to 526</td><td>1.212</td><td>0.058</td><td>0.001</td></q<0.057<>	478 to 526	1.212	0.058	0.001
0.265 <h<0.287m< td=""><td>259 to 256</td><td></td><td></td><td></td></h<0.287m<>	259 to 256			
0.07 <q<0.14< td=""><td>900 to 1024</td><td>3.100</td><td>0.138</td><td>-0.002</td></q<0.14<>	900 to 1024	3.100	0.138	-0.002
PROBE 15				
0.134 <h(t)<0.184m< td=""><td>548 to 526</td><td></td><td></td><td></td></h(t)<0.184m<>	548 to 526			
0.002 <q<0.006< td=""><td>251 to 256</td><td>0.081</td><td>0.006</td><td>0.000</td></q<0.006<>	251 to 256	0.081	0.006	0.000
0.184 <h(t)<0.234m< td=""><td>107 to 128</td><td></td><td></td><td></td></h(t)<0.234m<>	107 to 128			
0.006 <q<0.010< td=""><td>181 to 256</td><td>0.103</td><td>0.011</td><td>0.001</td></q<0.010<>	181 to 256	0.103	0.011	0.001

Table 2.2 (continued)

_				-
0.234 <h(t)<0.274< td=""><td>128</td><td></td><td></td><td></td></h(t)<0.274<>	128			
0.01 <q<0.043< td=""><td>500 to 512</td><td>0.826</td><td>0.043</td><td>0.000</td></q<0.043<>	500 to 512	0.826	0.043	0.000
0.274 <h(t)<0.284< td=""><td>462 to 512</td><td></td><td></td><td></td></h(t)<0.284<>	462 to 512			
0.043 <q<.107< td=""><td>1022 to 1024</td><td>6.355</td><td>0.107</td><td>0.000</td></q<.107<>	1022 to 1024	6.355	0.107	0.000
0.284 <h(t)<0.308m< td=""><td>519 to 512</td><td></td><td></td><td></td></h(t)<0.308m<>	519 to 512			
0.107 <q<0.13< td=""><td>210 to 256</td><td>0.959</td><td>0.130</td><td>0.000</td></q<0.13<>	210 to 256	0.959	0.130	0.000
0.308 <h(t)<0.314m< td=""><td>555 to 512</td><td></td><td></td><td></td></h(t)<0.314m<>	555 to 512			
0.13 <q<0.156< td=""><td>146 to 128</td><td>4.353</td><td>0.156</td><td>0.000</td></q<0.156<>	146 to 128	4.353	0.156	0.000
0.314 <h(t)<0.322m< td=""><td>536 to 512</td><td></td><td></td><td></td></h(t)<0.322m<>	536 to 512			
0.156 <q<0.1718< td=""><td>132 to 128</td><td>1.949</td><td>0.172</td><td>0.000</td></q<0.1718<>	132 to 128	1.949	0.172	0.000
0.322 <h(t)<0.332m< td=""><td>284 to 256</td><td></td><td></td><td></td></h(t)<0.332m<>	284 to 256			
0.171 <q<0.19< td=""><td>84 to 64</td><td>1.987</td><td>0.190</td><td>0.000</td></q<0.19<>	84 to 64	1.987	0.190	0.000
0.184 <h(t)<0.244m< td=""><td>122 to 128</td><td></td><td></td><td></td></h(t)<0.244m<>	122 to 128			
0.006 <q<0.030< td=""><td>260 to 256</td><td>0.394</td><td>0.030</td><td>0.000</td></q<0.030<>	260 to 256	0.394	0.030	0.000
0.274 <h(t)<0.280m< td=""><td>244 to 256</td><td></td><td></td><td></td></h(t)<0.280m<>	244 to 256			
0.043 <q<.076< td=""><td>413 to 512</td><td>6.148</td><td>0.077</td><td>0.001</td></q<.076<>	413 to 512	6.148	0.077	0.001
PROBE 107				
0m <h<0.140m< td=""><td>931 to 1024</td><td></td><td></td><td></td></h<0.140m<>	931 to 1024			
0.002 <q<0.005< td=""><td>218 to 256</td><td>0.021</td><td>0.005</td><td>0.000</td></q<0.005<>	218 to 256	0.021	0.005	0.000
0.140 <h<0.200m< td=""><td>163 to 128</td><td></td><td></td><td></td></h<0.200m<>	163 to 128			
0.005 <q<0.018< td=""><td>334 to 512</td><td>0.233</td><td>0.019</td><td>0.001</td></q<0.018<>	334 to 512	0.233	0.019	0.001
0.200 <h<0.250m< td=""><td>300 to 256</td><td></td><td></td><td></td></h<0.250m<>	300 to 256			
0.018 <q<0.05< td=""><td>498 to 512</td><td>0.586</td><td>0.047</td><td>-0.003</td></q<0.05<>	498 to 512	0.586	0.047	-0.003
0.25 <h<0.30m< td=""><td>338 to 512</td><td></td><td></td><td></td></h<0.30m<>	338 to 512			
0.05 <q<0.065< td=""><td>197 to 256</td><td>0.326</td><td>0.066</td><td>0.001</td></q<0.065<>	197 to 256	0.326	0.066	0.001
0.3 <h<0.325m< td=""><td>215 to 256</td><td></td><td></td><td></td></h<0.325m<>	215 to 256			
0.065 <q<0.095< td=""><td>495 to 512</td><td>1.194</td><td>0.095</td><td>0.000</td></q<0.095<>	495 to 512	1.194	0.095	0.000
0.325 <h<0.350m< td=""><td>248 to 256</td><td></td><td></td><td></td></h<0.350m<>	248 to 256			
0.095 <q<0.149< td=""><td>514 to 512</td><td>2.216</td><td>0.150</td><td>0.001</td></q<0.149<>	514 to 512	2.216	0.150	0.001
.35 <h<.375m< td=""><td>510 to 512</td><td></td><td></td><td></td></h<.375m<>	510 to 512			
0.149 <q<0.275< td=""><td>450 to 512</td><td>5.135</td><td>0.277</td><td>0.002</td></q<0.275<>	450 to 512	5.135	0.277	0.002
.375 <h<.41m< td=""><td>612 to 256</td><td></td><td></td><td></td></h<.41m<>	612 to 256			
0.275 <q<0.52< td=""><td>270 to 256</td><td>7.127</td><td>0.524</td><td>0.004</td></q<0.52<>	270 to 256	7.127	0.524	0.004
0.300 <h<0.335m< td=""><td>297 to 256</td><td></td><td></td><td></td></h<0.335m<>	297 to 256			
0.065 <q<0.13< td=""><td>910 to 1024</td><td>1.790</td><td>0.128</td><td>-0.002</td></q<0.13<>	910 to 1024	1.790	0.128	-0.002
0.325 <h<0.360m< td=""><td>536 to 512</td><td></td><td></td><td></td></h<0.360m<>	536 to 512			
0.095 <q<0.25< td=""><td>1001 to 1024</td><td>4.501</td><td>0.253</td><td>0.003</td></q<0.25<>	1001 to 1024	4.501	0.253	0.003
0.300 <h<0.390m< td=""><td>1091 to 1024</td><td></td><td></td><td></td></h<0.390m<>	1091 to 1024			
0.065 <q<0.44< td=""><td>1790 to 2048</td><td>4.151</td><td>0.439</td><td>-0.001</td></q<0.44<>	1790 to 2048	4.151	0.439	-0.001

We show the new graphs of our results in figures 2.16-2.18.



Figure 2.16: Discharge as a function of water table elevations excluding measures after a long dry season at probe 51. There are two zones where Ω is a constant, a low flow zone at region 1 and a high flow zone at region 2.



Figure 2.17: Discharge as a function of water table elevations excluding measures after a long dry season at probe 15. There are two seasons where Ω is a constant, low flow periods for region 1 and high flow period at region 2



Figure 2.18: Discharge as a function of water table elevations excluding measures after a long dry season at probe 107. Zones for where Ω is a constant are for a dry flow zone at 1, a medium flow zone at 2, and a low flow zone at 3.
We find new zones for Probe 51 shown in figure 2.16 described as two zones where Q is in m^3/s :

(1): dry-mid flow for h(t)<0.339 as an exponential where $Q(h(t))=0.0008e^{17.6*h(t)}$.

(2): high flow for h(t) > 0.339 as a line where Q(h(t)) = 16.71 * h(t) - 5.34.

We also find another curve to describe probe 15 shown in figure 2.17 with the two linear zones describes as:

(1): dry zone where 0.134m< h(t)<0.234m Q(h(t))=0.08m²/s*h(t)-0.0087m³/s,
(2):Wet season where h(t)>0.234m Q(h(t))=1.96m²/s*h(t)-0.458m³/s.

And for Probe 107 we find a new description for three zones where Q is in m^3/s :

(1): Dry zone where $h(t) \le 0.14 \text{m Q}(h(t)) = 0 * h(t) + 0.002$

(2): Mid-flow where 0.14 m < h(t) < 0.335 m and Q(h(t)) = 0.4673 * h(t) - 0.0656,

(3): High flow season where h(t)>0.335m and Q(h(t))=5.90*h(t)-1.91.

Figures 2.19-2.24 show that the both the prediction of the stream flow and the cumulative discharge improved when we exclude post summer seasons.



Figure 2.19: Discharge predictions using water table elevations at probe 51 excluding measures after a long dry period.



Figure 2.20: Predicted cumulative discharge using water table elevations at probe 51 excluding measures after a long dry period.



Figure 2.21: Discharge predictions using water table elevations at probe 15 excluding measures after a long dry period.



Figure 2.22: Predicted cumulative discharge using water table elevations at probe 15 excluding measures after a long dry period.



Figure 2.23: Discharge predictions using water table elevations at probe 107 excluding measures after a long dry period.



Figure 2.24: Predicted cumulative discharge using water table elevations at probe 107 excluding measures after a long dry period.

Discussion and Conclusion

Our results show that stochastic analysis is a useful tool for estimating the relationship between water table elevations and stream discharge. We were able to determine transport equations for three locations on our site. This estimation is based on the relationship of the frequencies of events in stream flow to the frequency of events in water table elevations. This is an alteration of the theory developed by Kirchner et al. 2004 where they defined the relationship of rainfall input frequencies as a damped signal in stream discharge. In Kirchner's theory stream discharge frequencies were described as a convolution of rainfall signals and a distribution of travel times throughout the catchment using spectral analysis. Without the convolution integral we show that spectral analysis can be used to describe discharge as a function of water table elevations.

The results in figures 2.9-2.11 indicate that it is first necessary to see that stream flow patterns in time are the same as those of the water table elevations to produce a reliable model. Though the results in these figures are adequate we can see that after the dry period we cannot successfully predict the discharge because there is a significant difference in the behavior of the water table elevations to the behavior of the stream flow. However when we neglect this time period, we find in figures 2.19-2.24 that our predictions for discharge and cumulative discharge are more accurate.

Since the predictions for the discharge is based on probe measurements that could not record water table elevations above the surface of the ground, it was difficult to reproduce peaks in the discharge. However the integration over time of our estimated and measured values agree. With our estimations we were also reproduced reasonable results for the height of the water table at each location from discharge measurement.

Discharge estimations from Probe 51 was significantly more difficult than the other two probes. Even after excluding the dry period there is an obvious delay in response in the stream. This difference may be due to the fact that Probe 51 is located at the bottom of the hill, between the hill and the saturated area where transport processes are at an interphase. Though probe 15 successfully estimated discharge patterns, it greatly under predicted the peaks in discharge. Probe 107 shows the most accurate estimations for discharge perhaps due to the fact that it is located in the near stream saturated area.

If we compare the hydrograph of our water table elevations (figure 2.2-2.4) to the results in figure 2.9-2.24, we find that the measured elevations of the well indicate that events have been dampened (or filtered) by the soil catchment area. This supports theories developed by Kirchner et al. where he proposed that a catchment behaves as a fractal filter and find that spectral analysis is a useful to describe groundwater transport processes.

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CHAPTER 3

SPECTRAL ANALYSIS AS A TOOL TO INVESTIGATE WATER AND NUTRIENT TRANSPORT IN THREE WATERSHEDS IN THE CATSKILLS, NY

Abstract

Though spectral analysis has been used to study hydrology and soils, it has only recently been paired with chemical transport and never to such a highly reactive pollutant as phosphorus (P). This paper describes how spectral analysis can be used as a tool to study nutrient and dissolved contaminant transport in three watersheds areas in the Catskill Mountains of New York. These sites include a forested watershed, a forested previously farmed area, and a current operating farm. Runoff is generated by saturated excess from variable source areas that expand during rainstorms. The comparison of these watersheds allows us to determine the long-term effect on stream quality of dairy farms in which manure spreading is an integral part of the farming practice. Using spectral analysis in three techniques we define a relationship between wet deposition and stream water chemistry, estimate chemical travel times, and determine retardation factors for reactive chemicals.

The first technique predicts stream water chemistry by defining it as a function of wet deposition. The results of this technique show that although chloride (Cl) can be described as a piecewise linear function of wet deposition of Cl. The same type of description was also true for nitrate (NO₃). The results of this technique also indicated that a source outher than rainfall is responsible Cl stream concentrations making it

unlikely that the saturated areas are responsible for generating the runoff. Moreover, we find that during high stream flow NO₃ and Cl concentrations in the stream are much higher than low flow periods implying that flushing of the expanding saturated runoff areas is largely responsible for the observed behavior.

The second technique defines stream water chemistry as the results of wet deposition that has been dampened throughout a catchment area over a distribution of travel times. Historically catchment areas have been modeled as well mixed reservoirs transporting chemicals throughout the catchment over an exponential distribution of travel times. From the results of the frequency domain analysis it was found that transport processes in are best represented as the result of a gamma distribution of travel times that have a much longer travel time than an exponential distribution. However, time domain results imply that transport processes are best described as an exponential distribution. Frequency domain results also show that suspended solids (SS) and total particulate phosphorus (TPP) follow the same behavior as rainfall, which implies that SS and TPP in the stream are transported via surface runoff. Since high rainfall events are related to high flow events this implies that preferential flow pathways are largely responsible for transport processes during high rainfall events with little effect from soil matrix, resulting in faster responses (shorter transfer rates) to high rainfall events. During low flow periods we find that the water flows through the soil matrix and results in longer travel times. Indicating that the travel time distribution is a function of velocity differences and not related to path of travel.

Finally, in the third technique retardation factors for reactive chemicals were estimated as a ratio of conservative and reactive chemicals in stream water and wet deposition. It was found that NO_3 has a retardation of 2.8 for each location. Travel times for all of our chemicals on the previously farmed forested area are much longer than the currently operating farms, with the exception of total dissolved phosphorus, soluble reactive phosphorus, and total phosphorus, which have retardation factors that are slightly higher than the currently operated farm. These results imply that farming practices may have a significant effect on phosphorus transport.

Introduction

Understanding non-point source pollution is essential to protecting water quality. Phosphorus is particularly of concern since it is the dominant source of eutrophication and has associated negative impacts on aquatic habitats and drinking water supplies (Sharpley et al, 2002). It has been increasingly recognized that many watershed processes, leading to non-point source pollution, operate on timescales of decades or longer. Many studies on non-point source pollution have been based on models that use only a few years of experimental data for validation (Lindenschmidt et al. 2004). Applying these models over extended periods ignores possible long-term trends in pollution, resulting in miss-targeted point source pollution. Current nutrient transport models consider only a narrow range of transport processes, specifically, those that link terrestrial P to waterways over time frames of a storm to a year (Kleinman et al. 2003, Scott et al. 2001, Kim et al., 2004). By studying time periods of a year or less, these models fail to consider discovering retention times that may be longer than a year. Even though research has shown good evidence of long-term P retention in the landscape (Cahill et al., 1974, Baker and Richards 2002, Reinhardt et al. 2005).

Spectral analysis is a technique that may be used to analyze and determine long-term retention times (Kirchner et. al. 2000). In the recent past, spectral analysis has been

successfully used in hydrology for water resource management (Fleming et a. 2002), to determine characteristic scales of precipitation, runoff, and groundwater levels (Skoien et. al. 2003), to study the time and frequency response of chemical tracers (Duffy and Alhassan 1988), and to model water quality in groundwater (Duffy and Gelhar 1985). To determine the retention time of water and chemical tracers in catchments Kirchner et al. (2000) and Feng et al. (2004) proposed a relationship between the travel time distributions of water and chemical tracers in the landscape and subsequently describe their landscape transport processes using spectral analysis. This is notable because the pollution of stream water due to contaminants, such as dissolved phosphorus is directly linked to the travel time of water through a catchment (Langmuir, 1997; Schnoor, 1996). The travel time distributions of a component, for example water, denote the various pathways that water can take before it reaches an outlet such as a lake or stream. Mathematically it is the probability distribution of the amounts of water reaching an outlet after it has traveled a given time through the catchment (Kirchner et al. 2001). These distributions are directly linked to flow paths that dictate transport (McDonnell et al., 1991), and when calculated can provide distinctive information for the prediction of the transport and of soluble contaminants (Kirchner et. al. 2001).

It is necessary to analyze distributions over a long time span to gain a more in depth view of the transport processes that dominate catchment response. Historical records of rainstorms and chemical concentrations can be used to understand the internal processes of an ecosystem (Church, 1997). Large sets of historical monitoring data have been used widely in hydrological models to validate and predict the flow paths of water, nutrients, and contaminants (Ferrier et al., 1995; Hooper et al., 1998; Kirchner, 1992; Kirchner et al., 1992. This paper employs three techniques for using spectral

analysis to analyze data from three watersheds in the Catskills Mountains, NY: Biscuit Brook, the Crow Road (R farm), and the Shaw farm. The first technique estimates stream water chemistry by defining a relationship between stream water chemistry as a function of wet deposition. The second technique assumes that the relationship between inputs (wet deposition or rainfall) and outputs (stream water chemistry or runoff) are associated by a distribution travel time (Kirchner et. al. 2000). The third technique determines a retardation factors for reactive tracers in the catchment area.

Site and Data Description

Biscuit Brook is located in Ulster County of the Catskills Mountains with an area of 960-ha at an elevation of 634 meters above sea level and drains into the West Branch of the Neversink Watershed (Soren 1963, Mehta et al. 2004,) as indicated in the map of illustration 3.1. Elevations in the watershed range from 627 to 1129 m. This watershed is entirely forested and is characterized by steep slopes and shallow permeable soils over an impermeable layer of bedrock (Mehta et al. 2004). Daily measures of precipitation and streamflow were provided by USGS from January 1991 to November 2004. We show this data provided in the hydrograph of figure 3.1 for 5 years of the sampling period as an example. We also use weekly wet atmospheric deposition concentrations for Cl and NO₃ collected by the National Atmospheric Deposition Program for the years of 1991-2004 (NADP 2005) collected on site at Biscuit Brook. Streamflow concentrations of Cl and NO₃ were provided for the years between January 1991-November 2005 found at the USGS website for Biscuit Brook which are shown in figures 3.2 and 3.3 (USGS, 2006).



Illustration 3.1: Map of study sites in the Catskills watershed: Cannonsville and Biscuit Brook. The R and Shaw farms are located just under 60m from Biscuit Brook.



Figure 3.1: Hydrograph of runoff and rainfall at Biscuit Brook (1993-2000).



Figure 3.2 : Stream and wet deposition concentrations of chloride at Biscuit Brook.



Figure 3.3: Stream and wet deposition concentrations of nitrate (NO₃) at all sites.

Peaks in Cl concentrations in the stream correspond to peaks in the rainfall. Average concentrations in the stream exceed average concentrations in rainfall although the maximum values in the stream do not go above maximum values in rainfall. The higher average concentrations in the stream imply that there is another significant source of Cl.

Just under 60 km away is the R farm of the Cannonsville Reservoir (illustration 3.1). The R farm catchment is a 164-ha rural watershed that hosts a third-generation dairy farm with approximately 80 milking cows and 35 replacement heifers. Since 1993, the study watershed has been the subject of a long term monitoring study (Bishop *et al.*, 2003). A detailed description of the study watershed, and a description of the raster maps for land use, soil type, and manure spreading zones, is given in Bishop *et al.*

(2003), Gérard-Marchant *et al.* (2005a), and Hively (2004). The R farm is a 164-Ha dairy farm located in Delaware County, New York (Bishop et al. 2004). The farm is comprised of 53% forest, 13% unimproved pasture, 25% improved pasture/hay, 7% tilled crop rotation, and 2% impermeable surface (Hively et al. 2004).

The Shaw farm is a forested watershed that was previously farmed until 1983. It is located 6.4 km east of the R farm and covers 86 ha. This farm is comprised of 78% deciduous forest, 22% shrub and grasses with less than 1% of impermeable area. The abandoned watershed contains several seasonal residences and one septic system, but has no recent history of manure application and no significant anthropogenic P inputs. On both the R and Shaw farms measures of daily rainfall, stream flow, and stream chemistry data was provided for the years between 1993-2000. There were no stream data collected between July 1995 and September 1996. Values for this year were interpolated since we used the Discrete Fourier Transform (DFT) for the spectral analysis, which requires evenly spaced data. We use the simplest form of interpolation, the linear interpolation,

$$y_1(x) = y(x_0) + \frac{y(x_1) - y(x_0)}{x_1 - x_0} (x - x_0)$$
(3.1)

where x_0 represents the initial position in time, x_1 the final position in time, x the point of time in question, and y denotes the concentration (mg/L) or magnitude (mm) values at these intervals. We show the results of the interpolation for rainfall and runoff in the hydrograph in figure 3.5 for the same years as we have shown for Biscuit Brook. This hydrograph reveals that both the Shaw farm and R farm experience, as expected for catchments in this area, high flows during winter and spring months and low flows during summer and fall.



Figure 3.4: Hydrograph of runoff and rainfall at the Shaw and R farms. This graphs shows the results of the interpolation for July 1995 through September 1996.

To provide thorough information on nutrient transport, we observe the behavior of all the available chemical concentrations in the stream for the Shaw and R farms which includes: nitrate (NO₃), ammonia nitrogen (NH₃), total organic carbon (TOC), total kjeldahl nitrate (TKN), total phosphorous (TP), total dissolved phosphorus (TDP), and soluble reactive phosphorus (SRP), and suspended solids (SS). There was no Cl information provided for the R and Shaw farms, however we can compare the concentrations of NO₃ in the stream to atmospheric wet deposition of NO₃ at Biscuit Brook. Figure 3.3 shows the NO₃ stream concentrations at our two other sites the Shaw and R farm. Figure 3.5 shows that on the Shaw farm, total phosphorus (TP) concentrations in the stream range between 0-2 mg/L where TP stream concentrations on the R farm are twice as high.



Figure 3.5: Total phosphorus concentrations in manure and stream water.

Manure is applied regularly to the R farm, resulting in higher concentration values in the stream than on the Shaw farm. On the Shaw farm, stream concentrations of TP are associated only with what has been left over in the soil profile over time from farming 10 years prior. R farm manure data were provided in terms of load per day over an area of 2000m² where each load of manure represents 4.3 kg of P per load. Typically, during spring and summer, more than 1 load and up to 35 loads per day may be applied to the farm. To express TP in mg/L, we find the load distributed per unit and divide this value by the rainfall. We show the average concentration of our other chemical constituents in the stream in Table 3.1.

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Table 3.1: Average measurements for rainfall, runoff, and chemical concentrations at all 3 locations. Column 1 lists the chemical constituent per site, column 2 lists the average concentrations for each constituent in (mg/L).

Component	Average
Rain Biscuit Brook (mm/day)	3.62
Runoff Biscuit Brook (mm/day)	2.69
Rain R and Shaw (mm/day)	2.68
Runoff Shaw (mm/day)	1.41
Runoff R farm (mm/day)	1.5
*CI Wet Deposition (mg/L)	0.15
*CI Biscuit Brook (mg/L)	0.54
*NO ₃ Wet Deposition (mg/L)	0.51
*NO ₃ Biscuit Brook (mg/L)	1.13
NO₃ Shaw (mg/L)	0.17
NO₃ R farm (mg/L)	1.38
NH₃ Shaw (mg/L)	0.01
NH₃ R farm (mg/L)	0.09
TKN Shaw (mg/L)	0.22
TKN R farm (mg/L)	0.48
TOC Shaw (mg/L)	3.43
TOC R farm (mg/L)	5.39
SS Shaw (mg/L)	9.58
SS R farm (mg/L)	24.51
TP Shaw stream (mg/L)	0.03
TP R farm stream (mg/L)	0.18
TP R farm manure (mg/L)	121.79
TDP Shaw stream (mg/L)	0.01
TDP R farm stream (mg/L)	0.10
TPP Shaw stream (mg/L)	0.01
TPP R farm stream (mg/L)	0.09
SRP Shaw stream (mg/L)	1.83E-03
SRP R farm Stream (mg/L)	0.07

* weekly values

Techniques

Spectral Analysis:

Spectral analysis transforms time domain signals (here rainfall, streamflow, and concentration) into wavelengths (Bracewell,2000; Scargle 1982) and then finds the spectral power by using the Fourier transform to convert time signals into the frequency domain. A time series characterizes data in observed terms, where the frequency domain represents data in terms of contributions occurring at different characteristic frequencies by representing each time scale as a pair of sine and cosine functions (Wilks, 1995). The spectral power S(f), is defined as

$$S(f) = \frac{1}{T} |X(f,T)|^2$$
 (3.2)

and T is time, f is frequency, and X (f, T) is the amplitude in the frequency domain resulting from the Fourier Transform (Turcotte 1992). A time series where the power spectra, has a power law dependence on frequency, i.e.,

$$S(f) \propto f^{-\beta} \tag{3.3}$$

is defined as a fractal time series where β =5-2D and D is the fractal dimension (Turcotte 1992). The exponent β , ranges from 0 to 4, and characterizes the behavior of the power spectrum often known as noises (Shroeder 1926). In this paper, Applications of Statistical Time Series Analysis (ASTSA), a statistical analysis package developed by Robert Shumway (Shumway and Stoffer 2000), will be used to perform the spectral analysis. This program computes the periodogram, which is the squared magnitude of the Fourier Transform, using the fast Fourier Transform (FFT)

assuming that the length of the data series (n) is in some power of 2. When the series is not a power of two ASTSA pads the series by adding zeros to extend the length to the next power of two (Shumway and Stoffer 2000). For the purposes of this paper we have padded series with the average when the series is in between powers of two. However when the series is less than 10% above a power of two we truncate remaining values above the power. We use two techniques to investigate the utility of spectral analysis in chemical transport that are described in the following.

Technique 1

The relationship between the spectral power of inputs and outputs at each wavelength reflect the behavior of catchment transport processes (Kirchner et. al. 2001). The behavior of these transport processes can be determined by assuming that stream concentrations are a function of rainfall concentrations both in the time and frequency domain, meaning that the values of stream concentrations can be written in terms of the rainfall concentrations using the following

$$C_{\rm S}(t) = \Omega^* C_{\rm R}(t) \tag{3.4},$$

where Ω is a term which associates stream concentrations (C_s (t)) and wet deposition or rain concentrations (C_R(t)). This relationship should hold true in both the frequency and time domain so that from the frequency domain Ω is found from the spectral powers of the series using the equation

$$\Omega = \frac{\sqrt{\frac{1}{n} \sum_{1}^{n} S_{C_{s}}(f)_{1...n}}}{\sqrt{\frac{1}{n} \sum_{1}^{n} S_{C_{R}}(f)_{1...n}}}$$
(3.5),

We validate this Ω value by checking that when weighted against an interval of C_R it produces the measured values of C_S . To be more explicit, for a range $C_{s_i} < C_S < C_{s_e}$ it must be true that

$$\Omega * \Delta C_R(t) + C_{s_i} = C_{s_\Omega} \quad \text{and} \quad C_{s_\Omega} = C_{s_e}$$

$$C_c = C_{s_\Omega} - C_{s_e} \approx 0 \quad (3.6).$$

so that

where
$$\Delta C_R(t) = C_R(t)_{initial} - C_R(t)_{end}$$
, $C_{s_{\Omega}}$ is the calculated stream value from the results
of the spectral analysis, C_{s_i} is the initial stream value at $C_R(t)_{initial}$, C_{s_e} is the final
value at $C_R(t)_{end}$, and C_c is the corroboration of $C_{s_{\Omega}}$ and C_{s_e} . We use this technique in
three experiments:

Experiment (1): We analyze intervals of wet deposition data with spectral analysis and then find the interval of stream data that satisfies the conditions of equation 3.6. Because results for C_c tend to oscillate the solution of $C_c\approx 0$ must fall between $C_c<0$ and $C_c>0$.

Experiment (2): The above is repeated for rain and steam chemistry to check if rainfall is the driving force using measures of rainfall and stream concentrations.

Experiment (3): This is also carried out for stream concentration of total phosphorus substitution manure as our input source of TP instead of rain.

Technique 2

We use the assumptions proposed by Kirchner et. al. 2000 to purport that stream concentrations are the results of a convolution of rainfall concentrations as they move through a catchment area over a distribution of travel times. This relationship is mathematically described (Duffy and Gelhar, 1985, 1986) as

$$C_s(t) = \int_0^\infty h(\tau) C_R(t-\tau) d\tau$$
(3.7)

where C_S are stream concentrations at any time t, $h(\tau)$ is a distribution of travel times, and C_R are the rainfall concentrations throughout the past, where τ is the lag time between rainfall and runoff. The Fourier transform of (3.7) produces

$$C_{s}(f) = H(f) * C_{R}(f)$$
 or $|C_{s}(f)|^{2} = |H(f)|^{2} |C_{R}(f)|^{2}$
or $S_{C_{s}}(f) = S_{h}(f) * S_{C_{r}}$ (3.8)

where f is the frequency (cycles/time); $C_s(f)$, H(f) and C_R (f) are the Fourier transforms of $C_s(t)$, h(τ), and C_R (1- τ); and the square of the absolutes represent their power spectra (S(f)) (Gelhar, 1993). If the travel time distribution of runoff is long compared to the travel time distribution of rainfall this implies that input fluctuations have be averaged together and dampened (Kirchner et al., 2001) which result in long retention times. Conversely, travel time distribution of rainfall that are longer than travel time distribution of runoff indicate that rainfall and chemistry will be transmitted relatively fast without significant dampening (Kirchner et al., 2001).

As we find travel time distributions, we would also like to describe the nature of the behavior. In the past, catchments have been predicted as well mixed reservoirs (Stagnitti et al., 1992). However, Kirchner et al. 2001 propose that travel time distribution which have power law slopes near 1 are inconsistent with commonly used conceptual predicts that imply exponential travel time distributions, which have a power law slope of 2 for wavelengths of $\lambda << 2\pi\tau_0$. Spectral scaling where $\beta=1$ which we have seen in this paper is however consistent with a gamma distribution of transfer rates

$$h(\tau) = \frac{\tau^{\alpha - 1}}{\varphi^{\alpha} \Gamma(\alpha)} e^{\frac{-\tau}{\beta}}$$
(3.9)

where α is a shape parameter and $\varphi = \tau_0/\alpha$ is a scale parameter. At $\alpha=0.5$, $\Gamma=1.773$ (source web gamma function finder) and at $\alpha=1$, $\Gamma=1$. The Fourier Transform of the gamma distribution gives

$$|H(f)|^{2} = \frac{\alpha^{2\alpha}}{\left[\alpha^{2} + 4\pi^{2}f^{2}\tau_{o}^{2}\right]^{\alpha}}$$
(3.10)

where τ_0 is the average transfer rate. When α =0.5, the power spectrum of (3.10) is described as a gamma distribution which has a power law slope of 1 for wavelengths<<2 $\pi\tau_0$. When α =1 equation 3.10 behaves as an exponential which has a power law slope of 2 for wavelengths<<2 $\pi\tau_0$.

Technique 3

A retardation factor for reactive chemicals can be defined as a ratio of the concentrations of our non reactive tracer in the stream and rainfall to our reactive tracer where

$$\frac{(R_d)_{reactive}}{(R_d)_{non-reactive}} \approx \frac{|H(f)|^2_{non-reactive}}{|H(f)|^2_{reactive}} = \frac{|C_s(f)|^2_{non-reactive}}{|C_s(f)|^2_{reactive}} \frac{|C_R(f)|^2_{reactive}}{|C_R(f)|^2_{non-reactive}}$$

for $f > 1/R_d \tau_0$ (3.11)

For passive tracers R_d =1 where as for reactive tracers R_d >1 (Feng et al., 2004; Bouwer, 1991; Freeze and Cherry, 1979; Vermeulen and Heister, 1952). This retardation gives an indication of the residence time of chemicals relative to a conservative tracer. Higher numbers indicate longer residence times where lower numbers indicate shorter residence times.

Results

Spectral Analysis: From time to frequency domain

The daily rainfall and flow per unit area for each catchment were first converted into the frequency domain. For ease in interpretation the power spectra are shown as function of wavelength (1/f) rather than frequency. Figure 3.6 shows that the spectral powers of the rain for Biscuit Brook scale as $\beta=0$ (white noise), where the signal for the stream flow scale with $\beta=1$.



Figure 3.6: Spectral analysis of Biscuit Brook hydrograph: runoff and rainfall.

We find similar results for the Shaw and R farms, which are shown in figure 3.7.



Figure 3.7: Spectral analysis of hydrograph for R and Shaw farms: Spectral powers for runoff and rainfall on the Shaw farm are slightly larger than for the R farm. The rainfall spectrum scales as white noise (β =0), where the flow exhibits a fractal power law scaling of 1/f noise (β =1).

In figure 3.6, slightly larger spectral powers for the rainfall at Biscuit Brook are the results of more frequent, higher intensity storms at Biscuit Brook than on the R and Shaw farm which, can be reviewed in figures 3.1 and 3.3. From equation 3.3 we understand these results to indicate that the catchment has dampened signals of rainfall in the resulting stream signal, converting constant noise signals from rainfall (S(f)= f^0), to a lower power where S(f)= f^1 at each location. Figure 3.8 shows similar results for the spectral analysis of chloride in Biscuit Brook.



Figure 3.8: Results of the spectral analysis for Cl at Biscuit Brook.

The atmospheric deposition scales as $\beta=0$ where the stream flow scales $\beta=1$ again implying that there is dampening. That the spectral powers in the stream are larger than the powers in the atmosphere support the implication from hydrograph figure 3.2 that there is another source of chloride other than atmospheric deposition. NO₃ concentrations at each location behave similar to Cl as shown in Figure 3.9.



Figure 3.9: Spectral analysis of NO_3 wet atmospheric deposition and stream concentrations at all locations.

The results for both Cl and NO₃ show the same conditions as were found in Plymnon Whales by Kircher et. al. (2004), which implies that the catchments filters nonconservative and conservative tracers in a similar manner. For the reactive chemicals TP in the manure on the R farm scales as $\beta=0$, white noise the same as the rainfall and scales as $\beta=1$ in the stream showing the same conditions as the water, Cl, and NO₃ where the input signal is dampened in the stream signal. TP on the Shaw farm also scales closest to white noise with $\beta=0.3$. TPP and SS on both farms scale closest to white noise scaling in the stream, implying that they are transported to the stream via surface runoff. A list of the scale factors for all of the chemicals measured in the catchment can be found in Table 3.2.

Table 3.2. β values from spectral analysis: Column 1: constituent of interest (chemicals are stream concentrations unless indicated), Column 2: β values resulting from equation 3.3. Dashed lines indicate where data was not measured.

	Biscuit Brook	Shaw farm	R Farm
Rainfall	0.0	0.0	0.0
Runoff	1.0	1.0	1.0
Cl wet deposition	0.0		
Cl	1.0		
NO _{3 wet deposition}	0.0		
NO ₃	1.0	1.0	1.0
NH ₃		0.7	0.8
TKN		0.7	0.6
TOC		0.7	0.7
TP		0.3	0.6
TDP		0.7	1.0
SRP		1.0	1.0
TPP		0.3	0.3
SS		0.0	0.0

Table 3.2 shows that the power spectra of SRP on the Shaw farm scales as 1/f, where TDP, NH₃, TKN, and TOC scale with β =0.7. SS and TPP on both farms scale closer to white noise implying that they are transported to the stream via surface runoff at both locations. The R farm shows similar scaling (β ≈0.7) for NH₃, TKN, and TOC which indicates that these chemicals follow the same fractal scaling independent of farming practices. However P constituents scale differently which implies that farming practices are have an effect on P transport processes. TDP and SRP scale as 1/f noise, implying dampening, where TP scales with β =0.6.

Technique 1, Experiment 1

We now apply our techniques to check the proportionality of rainfall signals to our stream signal. The results of equations 3.5 and 3.6 for Cl and NO₃ are shown in Table 3.3. NO₃ wet deposition from Biscuit Brook was used to predict NO₃ on the Shaw and R farms. In the table we include the intervals of data for $C_c < 0$, $C_c \approx 0$ and for $C_c > 0$.

Table 3.3: Results of technique 1, experiment 1 for stream chemistry as a function of wet depositions. Column 1 lists the range of data for wet deposition and stream concentrations (C_S). Column 2 lists the number of points in the range and to what power of 2 the ranges was padded or truncated. Column 3 lists constants calculated for the range (Ω , equation 3.5), columns 4 lists the calculated stream value ($C_{s_{\Omega}}$ (mg/L)) and column 5 lists the corroboration (C_c(mg/L)). Each catchment is sectioned by bolded larger text.

	# of points			
Biscuit Brook CI (mg/L)	to 2 ⁿ	Ω	C _{sΩ}	Cc
0.01< C _R <0.055	128			
0.11 <c<sub>s<0.39</c<sub>	77 to 128	5.95	0.38	-0.01
0.11< C _S <0.4	80 to 128	6.72	0.41	0.01
0.11< C _S <0.43	129 to 128	11.09	0.61	0.18
0.055< C _R <0.08	140 to 128			
0.4< C _S <0.45	108 to 128	1.83	0.45	0.00
0.08< C _R <0.1	128			
0.45< C _S <0.52	185 to 128	3.44	0.52	0.00
0.1< C _R <0.14	128			
0.52< C _S <0.58	155 to 128	1.54	0.58	0.00
0.14< C _R <0.22	102 to 128			
0.58< C _S <0.79	200 to 256	2.75	0.80	0.01
0.22< C _R <1.46	106 to 128			
0.79< C _S <1.04	37 to 32	0.20	1.03	-0.01
Biscuit Brook NO ₃ (mg/L)				
0.12< C _R <0.71	128			
0< C _S <0.46	88 to 128	0.78	0.46	0.00

Table 3.3 (continued)

			1	
0.71< C _R <1.15	127 to 128			
0.46< C_s < 0.63	89 to 128	0.40	0.63	0.00
1.15< C _R <1.65	126 to 128			
0.63< C_s <1	166 to 128	0.76	1.01	0.01
1.65< C _R <2.25	128			
1< C _s <1.2	113 to 128	0.33	1.20	0.00
2.25< C _R <3.47	122 to 128			
1.2< C _s <1.5	106 to 128	0.25	1.51	0.01
3.47< C _R <6.65	75 to 64			
1.5< C_s < 2.32	132 to 128	0.26	2.32	0.00
	# of points	_	~	~
R Farm NO ₃ (mg/L)	to 2"	Ω	C _{sΩ}	Cc
0.12< C _R <0.69	65 to 64			
0.061< C _S <0.172	29 to 32	0.19	0.17	0.00
0.69< C _R <1.16	66 to 64			
0.172< C _S <0.288	64	0.25	0.29	0.00
1.16< C _R <1.52	65 to 64			
0.288< C _S <0.374	63 to 64	0.23	0.37	0.00
1.52< C _R <2.025	65 to 64			
0.374< C _S <0.397	16 to 32	0.04	0.40	0.00
2.025< C _R <2.69	64			
0.397< C _S <0.525	63 to 64	0.19	0.53	0.00
2.69< C _R <5.49	64			
0.525< C _S <0.7	70 to 64	0.06	0.70	0.00
Shaw NO₃ (mg/L)				
0.12< C _R <0.69	65 to 64			
0.01< C _S <0.03	44 to 64	0.04	0.03	0.00
0.69< C _R <1.16	66 to 64			
0.03< C _S <0.0522	52 to 64	0.05	0.05	0.00
1.16 <c<sub>R<1.52</c<sub>	65 to 64			
0.053< C _S <0.0761	64	0.06	0.08	0.00
1.52< C _R <2.025	65 to 64			
0.0761< C _s <.0934	32	0.03	0.10	0.00
2.025< C _R <2.69	64			
0.0934< C _s <0.114	32 pts	0.03	0.12	0.00
2.69< C _R <5.49	64			
0.114< C _S <0.297	131 to 128	0.06	0.30	0.00

From Table 3.3 we find that at Biscuit Brook Cl behaves as a function of wet deposition in 3 linear zones described by $C_s=a*C_R + b$ where b is the intercept of stream concentrations to rain concentrations (figure 3.6a). The 3 zones are distinguishable by the following:

(1) $C_R < 0.055 \text{ mg/L}$ (low concentration zone) a=6.44 and b=0.046 mg/L, (2) $C_R < 0.22 \text{ mg/L}$ (mid concentration zone) a=2.343 and b=0.27 mg/L, (3) $C_R > 0.22 \text{ mg/L}$ (high concentration zone) a=0.2016 and b=0.75 mg/L.



Figure 3.10: Stream Cl as a function of Cl atmospheric wet deposition. Zones 1, 2, and 3 indicate low, medium, and high concentration zones respectively.

The relationship shown in figure 3.10 predicts stream Cl concentrations that show much more scatter than the actual data as shown below in figure 3.11.



Figure 3.11: Results of prediction for stream Cl at Biscuit Brook using technique 1.

Though the calculations estimate more scattered in the data, we find that the summed concentration of Cl over time predicts accurately as shown in figure 3.12.



Figure 3.12: Predicted and actual summed stream Cl at Biscuit Brook.

Additionally we find in Table 3.3 that NO₃ stream concentrations at Biscuit Brook behaves as a function of wet deposition of NO₃ in two zones where $C_s=a_1 * C_R^2 + a_2*C_R+a_3$. The two zones in figure 3.13 are describes as:

(1) $C_R < 0.12 \text{ mg/L}$ (low concentration zone) $a_1=0 \text{ (mg/L)}^{-1}$, $a_2=0$, and $a_3=0 \text{ mg/L}^{-1}$, (2) $C_R > 0.12 \text{ mg/L}$ (low concentration zone) $a_1=-0.03 \text{ (mg/L)}^{-1}$, $a_2=0.58$, $a_3=0.1 \text{ mg/L}$.



Figure 3.13: Stream NO₃ as a function of wet deposition at Biscuit Brook. Zones 1 and 2 indicate low and high concentration zones.

Predicting stream NO₃ from this relationship produces increased scatter (figure 3.14).


Figure 3.14: Results of prediction for stream NO₃ at Biscuit Brook using technique 1.



However the predicted summed concentrations match with the actual (figure 3.15).

Figure 3.15: Results of predicted summed NO₃ at Biscuit Brook using technique 1.

From Table 3.3 we find that NO₃ on the Shaw farm as a function of wet deposition as shown in figure 3.8a fits as two linear zones $C_s=\Omega *C_R + b$ where b is the intercept of stream concentrations to rain concentrations defined as:

- (1) Low Concentration: $C_R < 5.7 \text{ mg/L} a=0.05 \text{ and } b=0.001$,
- (2) High Concentrations: $C_R > 5.7 \text{ mg/L} a=0.05 \text{ and } b=0.01$.



Figure 3.16: Shaw farm stream NO_3 as a function of wet deposition. Zones 1 and 2 indicate low and high concentration zones.

Better results were found for the prediction of stream NO_3 on the Shaw farm. Figure 3.17 shows less scatter and closer correlation than what was predicted for NO_3 at Biscuit Brook.



Figure 3.17: Results of prediction for NO₃ at the Shaw farm using technique 1.



We show the results of the summed NO₃ concentration predictions in figure 3.18.

Figure 3.18: Results of predicted summed NO₃ at Shaw farm using technique 1.

Additionally from Table 3.3 it was found that NO_3 stream concentrations on the R farm can be shown as a function of wet deposition in one zone

 $C_s = a1 * C_R^2 + a2 * C_R + a3$

where a_1 =-0.021 (mg/L)⁻¹, a_2 =0.238, and a_3 =0.031mg/L shown in figure 3.6d.



Figure 3.19: R farm stream NO_3 as a function of wet deposition .

From this relationship an estimation of stream NO_3 was calculated and shown in figure 3.20.



Figure 3.20: Results of predicted summed NO₃ at R farm using technique 1.





Figure 3.21: Predicted summed NO₃ at R farm using technique 1.

To check if the signals for Cl and NO₃ concentrations in the stream are indeed driven by the rainfall instead of another source (which may be indeed the case for Cl since our hydrographs have indicated that there is another significant source) we would like to repeat technique 1 but instead proposing that concentrations in the stream are a function of the rainfall, i.e. C_s (rain)= Ω *Rainfall. Table 3.4 shows the results of our technique for testing this assumption for weekly values of rainfall and stream concentrations.

We find that the relationship for stream Cl and rainfall can be described as a piecewise polynomial 1^{st} order polynomial with two zones where $C_s=a_1(mg*weeks/L*mm^2)$ *Rain² + $a_2(mg*weeks/L*mm)*Rain+ a_3 (mg/L)$. The two zones in figure 3.21 are describes as

(1) Rainfall<12.5mm/week (low rain):a1=-1.3*10⁻³, a2=3.2*10⁻², and a3 =0.24,
(2) R>12.5 mm (high rain): a1=-2*10⁻⁵, a2=6.6*10⁻², and a3 =0.37.

The relationship for stream NO₃ and rainfall at Biscuit Brook can be described as two linear zone where Cs=a (mg*weeks/L*mm) *Rain + b (mg/L). The two zones in figure 3.24 are describes as

(1) R<5.1 mm/week (low rain): a=9.8*10⁻², and b=0,
(2) R>5.1 mm (wet seasons): a=1.8*10⁻², and b=0.4.

Table 3.4: Results of technique 1, experiment 2 stream chemistry as a function of rain: Column 1 lists the range of rainfall and stream concentrations (C_s(mg/L)). Column 2 lists the number of points in the range and to what power of 2 the range was padded or truncated. Column 3 lists the results of Ω from equation 3.5, where columns 4 lists $C_{s_{\Omega}}$ and column 5 lists C_c. In the table we include the intervals of data for C_c<0, C_c~0 and for C_c>0. Each catchment is sectioned by bolded larger text.

Biscuit Brook Cl	# of points to 2 ⁿ	Ω	C _{sΩ} (mg/L)	C _c (mg/L)
0 <rain<5.1mm< th=""><th>126 to 128</th><th></th><th></th><th></th></rain<5.1mm<>	126 to 128			
0.24 <c<sub>S<0.37</c<sub>	61 to 64	0.025	0.367	-0.003
5.1 <rain<12.5mm< td=""><td>128</td><td></td><td></td><td></td></rain<12.5mm<>	128			
0.37< C _S <0.44	128	0.009	0.438	-0.002
12.5 <rain<22.5mm< td=""><td>127</td><td></td><td></td><td></td></rain<22.5mm<>	127			
0.44< C _S <0.52	221 to 256	0.008	0.521	0.001
22.5 <rain<35.5mm< td=""><td>127</td><td></td><td></td><td></td></rain<35.5mm<>	127			
0.52< C _S <0.58	155 to 128	0.005	0.579	-0.001
35.5 <rain<66.5mm< td=""><td>128</td><td></td><td></td><td></td></rain<66.5mm<>	128			
0.58< C _S <0.71	173 to 128	0.004	0.713	0.003
65 <rain<105mm< td=""><td>32 pts</td><td></td><td></td><td></td></rain<105mm<>	32 pts			
0.71< C _S <0.835	57 to 64	0.003	0.843	0.008
Biscuit Brook NO ₃				
0 <rain<5.1mm< td=""><td>126 to 128</td><td></td><td></td><td></td></rain<5.1mm<>	126 to 128			
0< C _S <0.5	108 to 128	0.099	0.505	0.005
5.1 <rain<12.5mm< td=""><td>128</td><td></td><td></td><td></td></rain<12.5mm<>	128			
0.5< C _S <0.65	75 to 64	0.020	0.649	-0.001
12.5 <rain<22.5mm< td=""><td>127</td><td></td><td></td><td></td></rain<22.5mm<>	127			
0.65< C _S <0.79	65 to 64	0.015	0.796	0.006
22.5 <rain<35.5mm< td=""><td>127</td><td></td><td></td><td></td></rain<35.5mm<>	127			
0.79< C _S <0.93	60 to 64	0.011	0.930	0.000
35.5 <rain<47.5mm< td=""><td>674</td><td></td><td></td><td></td></rain<47.5mm<>	674			
0.93< C _S <1.22	152 to 128	0.024	1.219	-0.001
47.5 <rain<106.5mm< td=""><td>98 to 105</td><td></td><td></td><td></td></rain<106.5mm<>	98 to 105			
0.93< C _S <1.16	120 to 128	0.008	1.163	0.003
35.5 <rain<66.5mm< td=""><td>128</td><td></td><td></td><td></td></rain<66.5mm<>	128			
1.22< C _S <2.3	215 to 256	0.018	2.299	-0.001

Table 3.4 (continued)

R farm NO₃	# of points to 2 ⁿ	Ω	C _{sΩ} (mg/L)	C _c (mg/L)
0 <rain<2.1mm< td=""><td>64 pts</td><td></td><td></td><td></td></rain<2.1mm<>	64 pts			
0.061< C _S <0.172	29 to 32	0.193	0.171	-0.001
2.1 <rain<6.86mm< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<6.86mm<>	65 to 64			
0.172< C _S <0.288	64	0.245	0.287	-0.001
6.86 <rain<13.47mm< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<13.47mm<>	65 to 64			
0.288< C _S <0.50	134 to 128	0.032	0.500	0.000
13.47 <rain<20.37mm< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<20.37mm<>	65 to 64			
0.5< C _S <0.58	32	0.012	0.584	0.004
20.37 <rain<32.5mm< td=""><td>64</td><td></td><td></td><td></td></rain<32.5mm<>	64			
0.58 <q<0.66< td=""><td>32 pts</td><td>0.006</td><td>0.660</td><td>0.000</td></q<0.66<>	32 pts	0.006	0.660	0.000
32.5 <rain<63.89mm< td=""><td>64</td><td></td><td></td><td></td></rain<63.89mm<>	64			
0.66< C _s <0.81	40 to 32	0.005	0.812	0.002
63.89 <rain<105mm< td=""><td>64</td><td></td><td></td><td></td></rain<105mm<>	64			
0.81< C _S <0.9	29 to 32	0.002	0.900	0.000
Shaw farm NO₃				
0 <rain<2.1mm< td=""><td>64 pts</td><td></td><td></td><td></td></rain<2.1mm<>	64 pts			
0.01< C _s <0.03	44 to 64	0.009	0.030	0.000
2.1 <rain<6.86< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<6.86<>	65 to 64			
0.03< C _S <0.053	57 to 64	0.005	0.053	0.000
6.86 <rain<13.47mm< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<13.47mm<>	65 to 64			
0.053< C _S <0.0637	32	0.002	0.064	0.000
13.47 <rain<20.37< td=""><td>65 to 64</td><td></td><td></td><td></td></rain<20.37<>	65 to 64			
0.761< C _S <.116	64	0.006	0.116	0.000
20.37 <rain<32.5< td=""><td>64</td><td></td><td></td><td></td></rain<32.5<>	64			
0.116< C _S <0.175	60 to 64	0.005	0.175	0.000
32.5 <rain<63.89< td=""><td>64</td><td></td><td></td><td></td></rain<63.89<>	64			
0.175< C _S <0.29	69 to 64	0.004	0.291	0.001
63.89 <rain<105< td=""><td>64</td><td></td><td></td><td></td></rain<105<>	64			
0.29< C _s <0.5	30 to 32	0.005	0.501	0.001



Figure 3.22: Figure of stream Cl as a function of Rainfall at Biscuit Brook. Zones 1 and 2 indicate low and high concentration zones.



This relationship predicts streamflow concentrations shown in figure 3.23.

Figure 3.23: Predicted stream Cl estimated as a function of rain.

The summed prediction for Cl calculated from the relationship of Cl as a function of rainfall (figure 3.24) does not produce the same results as figure 3.11 where Cl was modeled as a function of wet deposition. This strongly implies that Cl contributions to the stream are governed source other than rainfall.



Figure 3.24: Summed stream Cl predicted as a function rainfall.



Figure 3.25: Stream NO₃ as a function of Rainfall at Biscuit Brook.

This relationship predicts streamflow concentrations shown in figure 3.26.



Figure 3.26: Predicted stream NO₃ at Biscuit Brook estimated as a function of rain.

The results of the summed prediction (figure 3.26) imply that a source other than rainfall may be responsible for NO_3 stream concentrations at Biscuit Brook.



Figure 3.27: Summed stream NO₃ at Biscuit Brook estimated as a function of rain.

The relationship for NO₃ on the Shaw farm as a function of rainfall (figure 3.28) fits as two linear zones $C_s=\Omega *R + b$ where b is the intercept of stream concentrations to rain concentrations defined as:

(1) Rain <2.2mm/week (low intensity): Ω =9.2*10⁻² (mg*weeks/L*mm), b=0.01 mg/L (2) Rain>2.2mm/week (high intensity): Ω =4.5*10⁻² (mg*weeks/L*mm), b=0.02 mg/L.



Figure 3.28: Shaw farm stream NO₃ as a function of weekly rainfall.

This relationship predicts reasonable results for stream NO_3 as shown in figures 3.29 and 3.30.



Figure 3.29: Predicted stream NO₃ at the Shaw farm as a function of rainfall.



Figure 3.30: Summed stream NO₃ at the Shaw farm as a function of weekly rainfall.

The relationship for stream NO₃ and rainfall on the R farm (figure 3.31) can be described as a piecewise polynomial 1st order polynomial with two zones where $C_s=a_1(mg*weeks/L*mm^2) *Rain^2 + a_2(mg*weeks/L*mm)*Rain+ a_3(mg/L)$. The two zones in figure 3.13a are defined as:

(1) Rain<20.36mm/week (low intensity): a1=-8.1*10⁻⁴, a2=4.2*10⁻², and a3=0.07,
(2) Rain>20.36 m/week (high intensity): a1=-3.6*10⁻⁵, a2=8.1*10⁻³, and a3=0.42.



Figure 3.31: R farm Stream NO_3 as a function of rainfall. Zones 1 and 2 indicate low and high concentration zones.

This relationship predicts the following stream concentrations that indicate stream NO₃ on the R farm is a function of rainfall.



Figure 3.32: Prediction of stream NO₃ on the R farm estimated as a function of rain.



Figure 3.33: Summed stream NO₃ as a function of weekly rainfall at the R farm.

The results for NO_3 as a function of rainfall produce better correlations to the actual data on the R and Shaw farm than the predictions using NO_3 as a function of wet deposition. This may be due to the fact that NO_3 wet deposition from Biscuit Brook were used for the R and Shaw farm where the results for NO_3 as a function of rainfall for the R and Shaw farm used rainfall and stream data on location.

Technique 1, Experiment 3

We try this technique to predict TP using this technique, however since TP from manure in the hydrographs showed much fewer events than in the stream, we could not assume that stream concentrations were a function of manure TP. So when we attempted to model from technique 1 for TP we could not find solution that satisfied the conditions of equations 3.5 and 3.6 as shown in Table 3.5.

Table 3.5: Results of technique 1, experiment 3 for TP as a function of Manure spreading: Column 1: range of data for manure and stream concentrations (C_S). Column 2: # of points in the range and the power of 2 that the range was padded/ truncated. Column 3: constants for the range (Ω , equation 3.5), columns 4 lists the calculated stream value ($C_{s_{\Omega}}$) and column 5 lists the corroboration (C_c).

			C _{so}	Cc
TP (mg/L)	# of points	Ω	(mg/L)	(mg/L)
0.015 <c<sub>manure<0.05</c<sub>	518 to 512			
C _S =0	2201 to 2048	0.000	0.020	-0.030
0 <c<sub>S<0.000001</c<sub>	2281 to 2048	469.280	0.067	0.017
0 <c<sub>S<0.0001</c<sub>	2346 to 2048	1.914	0.211	0.161
0.05< C _{manure} <0.075	520 to 512			
.0000001 <c<sub>s<0.70</c<sub>	256	4.14E-05	0.079	0.004
.0000001 <c<sub>s<0.8</c<sub>	263 to 256	3.28E-05	0.076	0.001
.0000001 <c<sub>S<0.9</c<sub>	266 to 256	2.68E-05	0.074	-0.001
0.075< C _{manure} <0.2215	1048 to 1024			
0.8< C _s <17.338	118 to 127	1.02E-05	0.241	0.019
0.8< C _S <21	128	7.38E-06	0.224	0.003
0.8< C _S <30	135 to 128	6.53E-06	0.266	0.044
0.2215< C _{manure} <5.05	660 to 512			
21< C _S <436	49 to 64	5.73E-06	2.607	-2.443
21< C _S <109	32	1.53E-05	1.600	-3.450
21< C _s <300	44 to 64	8.55E-06	2.617	-2.433

Using the values where C_c is closest to zero, we find that the TP predicts as a straight line as shown in figure 3.34.



Figure 3.34: Estimation of stream TP as a function of manure TP at the R farm.



However this does not yield a solution for the summed concentrations (figure 3.35).

Figure 3.35: Estimation of summed stream TP calculated as a function of manure.

Technique 2

We apply the results of our spectral analysis to equation 3.8 to determine distribution of travel times for water and chemicals. Figure 3.36 shows that on the Shaw farm the gamma distribution (β =1) scales more closely to the travel time distributions of rain water than the exponential distributions (β =2).



Figure 3.36: Power spectra of travel time distributions of water at the Shaw farm.

The exponential distribution is fitted to a travel time of 1 day where that gamma distribution is fit to a travel time of 8 days. The R farm also shows that gamma distributions fit more precisely to the power spectra of rain travel time distributions as shown in figure 3.37.



Figure 3.37: Power Spectra of travel time distributions of water at the R farm.

On the R farm the exponential distribution fits the power spectra with an average travel time of 8 days where the gamma distribution gives a longer average travel time of 14 days. We find the same results for Biscuit Brook. The gamma distribution, is fitted to the power spectra of rainwater distributions with an average travel time of 8 days (figure 3.38).



Figure 3.38: Power Spectra of travel time distributions of water at Biscuit Brook.

The gamma distribution also fit more closely than the exponential for power spectra of travel time distributions for Cl and NO₃ at Biscuit Brook, for NO₃ on the Shaw and R farm (figures 3.39-3.42).



Figure 3.39: Power Spectra of travel time distributions for Cl at Biscuit Brook.



Figure 3.40: Power Spectra of travel time distributions for NO₃ at Biscuit Brook.



Figure 3.41: Power Spectra of travel time distributions for NO₃ at the Shaw farm. These values were calculated using Biscuit Brook NO₃ wet deposition.



Figure 3.42: Power Spectra of travel time distributions for NO₃ at the R farm. These values were calculated using Biscuit Brook NO₃ wet deposition.

However TP on the R farm did not produce the same results. TP travel time distributions were calculated using power spectra of TP from manure and stream concentrations (figure 3.43). We find that that the power spectra of travel time distributions for TP on the R farm scale closest to β =0.6 instead of 1 for the exponential or 2 with the gamma. This distribution shows that over wavelengths less than a year, TP follows a white noise distribution, implying that it is transported via surface runoff. For wavelengths greater than 1 year, the distribution may be described as either an exponential or a gamma distribution.



Figure 3.43: Power Spectra of travel time distributions for TP at the R farm. These values were calculated using TP concentrations in manure.

We find that the power spectra of travel time distributions for SRP on the R farm scale closest to gamma distribution (figure 3.44).



Figure 3.44: Power Spectra of travel time distributions for SRP at the R farm. These values were calculated as 50% of TP concentrations in manure.

For the remaining stream chemistry atmospheric deposition data were not provided. However, assuming that the atmospheric deposition of these chemicals behave in the same manner as the Cl and NO₃ deposition, which were found to scale as white noise, equation 3.8 is simplified to

$$\left|Q_{s}(f)\right|^{2} = \left|H(f)\right|^{2} * \text{Constant}$$
(3.12)

where white noise scales as a constant. This indicates that stream concentrations are proportional to travel time distributions and follow the same patterns and spectral scaling. Using this equation on the Shaw farm a constant of 0.02 was estimated for TP. With this it was found that TP on the Shaw farm did not scale to either the gamma or the exponential because TP in the stream on the Shaw farm scaled to β =0.3 (table 3.6), closer to white noise (figure 3.45).



Figure 3.45: Power Spectra of travel time distributions for TP at the Shaw farm

The constants found for SRP on the Shaw farm was estimated to be 0.003. With this the gamma distribution scaled to the power spectra with an average travel time of 20.8 days.



Figure 3.46: Power Spectra of travel time distributions for SRP at the Shaw farm. The figure for other chemicals measured on the Shaw and R farms can be found in the Appendix. The results for all locations are summarized in Table 3.6 in the next

section. The power law slope of the gamma distribution fit closer to the power spectra of travel time distribution than the exponential for rain, Cl, NO₃, NH₃, TKN, TOC, TDP, and SRP on both farms. The chemicals that did not follow the gamma distribution are TP, SS, and TPP on the Shaw farm and R farms. SS on both farms and TPP on the Shaw farm scale the same as rainfall with power law slope of β =0 white noise. These figures can also be found in the appendix.

Technique 3

Using equation 3.11 we now attempt to find retardation factors for both reactive and non-reactive chemicals. With R_d=1 for Cl, results of the spectral analysis yields a retardation coefficient of 2.8 for NO₃ at Biscuit Brook. This retardation factor for NO₃ was used to estimate Cl travel time distributions at the R and Shaw farm to approximate all other retardation factor listed in Table 3.6. Rainfall at Biscuit Brook was found to scale as white noise where runoff scaled as 1/f noise as shown previously in figure 3.6. Using these values yielded a travel time distribution (equation 3.3) with an average of 1.34. The power spectra of these travel time distributions fit closest to a gamma distribution (β =1) with an average travel time of 8 days where the exponential distribution estimated travel times of 0.8 days. The power spectra of travel time distributions for Cl and NO₃ yielded an average of 2.09 and 0.74 respectively. Using Cl as the conservative tracer with a retardation factor of 1 produces a retardation factor of 2.8 for NO₃. These power spectra for NO₃ fit closets to the gamma distribution, which estimated that NO₃ takes 22 days to move through the catchment at Biscuit Brook. The exponential distribution fit to the Biscuit Brook catchment area with an average travel time of 2.24 days.

Table 3.6: Results of Techniques 2 and 3. Column 1 lists the chemical constituent of interest, column 2 lists the β values of the travel time distributions. Column 3 lists constants from equation 3.12. Column 4 lists average values of the travel time distributions found from equation 3.10. Column 5 lists the retardation factor from equation 3.11. Columns 6 and 7 list the average travel times of the exponential and gamma distribution.

Biscuit		Constant			Exponential Travel Time	Gamma Travel Time
Brook	β	(mg/L^2)	$\mathbf{H}(\mathbf{f})^2$	R _d	(days)	(days)
Rain water	1		1.34		0.8	8
Cl	1		2.09	1	0.8	8
NO ₃	1		0.74	2.8	2.24	22.7
Shaw farm						
Rain water	1		2.47		1	8
Cl	1		0.48	1.00	1	8
NO ₃	1	**	0.17	2.82	2.82	22.6
NH ₃	0.7	0.002	0.26	1.84	1.84	6.72
TKN	0.7	1	0.2	2.4	2.4	19.2
TOC	0.7	50	1	0.48	0.48	3.84
ТР	0.3	0.02	0.24			
TDP	0.7	0.002	0.17	2.8	2.84	22.4
SRP	1	0.0003	0.18	2.6	2.60	20.8
TPP	0.3	1	0.003			
SS	0.0	1000	1.38			
R Farm						
Rain water	1		2.52		8	14
C1	1	**	0.82	1	8	14
NO ₃	1	**	0.29	2.82	22.56	39.5
NH ₃	0.8	1	0.19	4.3	34.43	60.3
TKN	0.6	5.4	1.54	0.53	4.24	7.4
TOC	0.7	594	100	0.01	0.07	0.1
ТР	0.6	*	0.5	1.64	13.08	22.9

Table 3.6 (continued)

TDP	1	*	0.16	5.11	40.89	71.6
SRP	1	*	0.12	6.82	54.52	95.4
TPP	0.3	1	0.11			
SS	0.0	1000	7.9			

--not applicable. No retardation for rainwater, β values for chemicals are close to white noise.

*concentrations available from manure, TDP and SRP in manure are calculated as 50% of TP which was previously stated as 4.3 kg/load of manure.

**from NO₃ deposition at Biscuit Brook. Chemicals with β values less than one are rough approximations.

Rainwater on the Shaw farm shows similar scaling. The gamma distribution of rainwater scales closest to the power spectra of travel time distributions for rainfall estimated an average travel time of 8 days where the exponential scales to an average travel time of 1 day. Cl values of travel time distributions are approximated from the results of NO₃ that was estimated to have a retardation factor of 2.82 over the whole watershed and are assumed to have the same scaling as concentrations at Biscuit Brook. Retardation factors for TOC are very low. TDP and SRP retardation factors are approximately equal to NO₃.

Larger retardation factors for P constituents on the R farm also indicate that it takes much longer for P to move through the landscape of a farmed watershed than a forested watershed. There are unexpectedly large retardation factors for NH₃ on the Shaw farm and extremely small retardation factors for TOC and on both farms. The retardation for TKN on both farms is also shorter than NO₃. Our estimations for TDP and SRP on the Shaw farm imply a travel times of approximately 80 days. On the R farm we find a travel time for TP of 23 days, for TDP 72 days, for SRP 95 days, and for TPP 104 days.

Using the travel times that from Table 3.5 to solve equation 3.7, the stream concentrations of Cl and NO₃ were predicted using the wet deposition at Biscuit Brook for C_R. According to Niemi, 1977; and Rhode et. al. 1996) since flow rates vary over time, t and τ must be expressed in terms of summed flow, meaning that we will predict the average concentrations over time. We find that the convolution the gamma distribution and Cl rainfall concentrations at with an average travel time of 30 days under predicts the average concentrations of Cl in the stream in figure 3.45.



Figure 3.47: Prediction of stream Cl concentrations as a convolution of travel time distribution and wet deposition using technique 2.

In figure 3.47 the exponential distribution with an average travel time of 20 days is a slight improvement above the gamma.

Figure 3.48 shows better results for NO_3 predictions. Again we find that the exponential distribution fits better with a travel time of 2.24 days than the gamma distribution at 22.4 days of average travel time. We find similar results for the NO_3 on the R and Shaw farm (3.48-3.50) that show that exponential distributions are a more accurate prediction than a gamma distribution of travel times.



Figure 3.48: Estimation of summed stream NO₃ concentrations as a convolution of travel time distribution and wet deposition at Biscuit Brook.



Figure 3.49: Estimated summed stream NO₃ concentrations as a convolution of travel time distribution and wet deposition on the Shaw farm.



Figure 3.50: Estimated stream NO_3 concentrations as a convolution of travel time distribution and wet deposition on the R farm.

Discussion and Conclusions

The results of the spectral analysis indicate that rainfall at all locations and runoff at all locations scale as white noise and 1/f noise respectively. We also find the wet deposition of Cl to scale as white noise where stream concentrations scaled as 1/f noise. These are the same conditions found for Cl at Plymnon, Whales, the study site where Kirchner et. al. 2000 developed their theories on the fractal nature of catchment filtering. The results of this paper indicate that the same type of fractal filtering could also be pertinent to NO₃ at all locations, for SRP on the Shaw farm, and for TDP, TPP, and SRP on the R farm. SS and TPP on the Shaw and R farm were found to scale as white noise, implying that they are transported via surface runoff. TP on the Shaw farm also scales more closely to white noise, however farming practices may have altered the transport processes of TP on the R farm which scaled as β =0.6.

The results of our first technique indicate that it is possible to describe stream chemistry as a piecewise linear function of wet deposition using spectral analysis. However, It was found that although another source of Cl other than wet depositions contributed to stream concentrations, Cl in the stream can be represented as a function of wet deposition following a piecewise linear model. This is obvious in our results of figure 3.12 where the summed observed stream concentrations over time fit the predicted. The results also show that during high rainfall periods Cl concentrations in the stream increase at a much faster rate than during dry periods (figure 3.10). We also find this to be true for NO₃, although our prediction does not produce results that as close to our results for Cl. This could be due to the fact that stream water nitrification, which has been found to be a significant source of NO₃, could not be accounted. The combination of our results for Cl and NO₃ at all locations (figures

3.10-3.21) indicate that stream concentrations increase with high rainfall intensities, implying that preferential flow pathways are largely responsible for transport processes with little effect from soil matrix, resulting in faster responses (shorter transfer rates) to high rainfall events. During dry periods we find that the soil matrix where chemicals are retained at over long periods dominates chemical transport and results in slower transfer rates. This was also found to be true for NO₃ and rainfall in experiment 2 on the R and Shaw farm, meaning that in these locations NO₃ stream chemistry is driven by rainfall events. However, this was not true for Cl concentrations as all of our results supported that another source of Cl is responsible for stream concentrations at Biscuit Brook. Experiment 3 of this technique proved that TP stream concentrations could not be defined as a function of manure TP concentrations.

The results of our analysis from technique 2 shows the spectral powers of travel time distributions for rain water at all locations scale more closely to a gamma than exponential distributions (figures 3.36-3.40). Cl at Biscuit Brook and NO₃ at all sites also meet this conditions. However, we do not find this to be trued for TP on the R farm or Shaw farms. TP travel time distributions on the R farms showed white noise scaling for wavelengths less that 1 year, and exponential scaling for wavelengths greater than 1 year. Travel time distributions for TP on the Shaw farm scaled more closely to white noise, indicating that TP is transport via surface runoff 15 years after discontinuing farming practices. In table 3.6 we see that for TOC in particular travel times are shorter than water, meaning that there is another significant source of TOC to the stream at both locations. We also find that the power spectra of TPP on the Shaw farm and SS on both farms scaled as white noise, following the rainfall, which suggests that transport TPP and SS are governed by surface runoff. NH₃ on our

abandoned farm shows the longest travel times where we see travel times for TP on the Shaw farm between 2-3 months. We see slightly longer travel times of SRP and TDP on the R farm and also find that TP does not scale the same as the Shaw farm. These results imply that farming practices effect the retention time and transport process of phosphorus and its constituents. Beta values for solutes were similar on each farm though retardation factors were not. Conversely it was found that farming practices may have no effect on the transport and retention processes of NH₃, TKN, and TOC. On our forested watershed we were unable to calculate travel times for total phosphorus because its travel time distribution was not the same as NO₃ or stream runoff.

We find from our last results (figures 3.47-3.50) that describing chemical transport to a stream as an exponential distribution of travel times produces better results than the gamma distribution. Therefore representing catchment areas as well-mixed reservoirs and with relatively short travel times is a valid representation. However, neither the gamma or the exponential distribution reproduces the actual results. The gamma distribution will result in longer catchment retention times.

We found that in the frequency domain over the entire 15 years Cl, NO₃, NH₃, TKN, TOC, TDP, and SRP have power spectra that scaled more closely to the gamma distribution at all locations which yielded reasonable travel times (Table 3.6). However figures 3.47-3.50 from technique 3 show that in the time domain our prediction scales more closely to the exponential distribution of travel times with significantly shorter travel time. Though the exponential distribution results in summed concentration that fit more closely to the actual, figures 3.39-3.42 and 3.47-3.50 indicate that another form of describing these distributions may be necessary.

Figures 3.39-3.42 show that the gamma distribution closely approximates power spectra for certain wavelength where it predicts lower and higher values at others.

Though many of our results require further research before conclusions can be generalized, it is quite clear that spectral analysis can be used as a tool to investigate transport process and give details that were previously undetermined. The results of spectral analysis can be used to support information from hydrograph separations. This technique can also be combined with other modeling tools to make a stronger connection with the physical environment by determining how sources contribute to stream water quality.

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APPENDIX



Figure 1: Power spectra of travel time distributions for TDP at the Shaw farm.



Figure 2: Power spectra of travel time distributions for TDP at the Shaw farm.



Figure 3: Power spectra of travel time distributions for NH₃ at the Shaw farm.



Figure 4: Power spectra of travel time distributions for TKN at the Shaw farm.



Figure 5: Power spectra of travel time distributions for TOC at the Shaw farm.



Figure 6: Power spectra of travel time distributions for SS at the Shaw farm.



Figure 7: Power spectra of travel time distributions for TDP at the R farm. TDP is calculated as 50% of TP in manure.



Figure 8: Power spectra of travel time distributions for TPP at the R farm.



Figure 9: Power spectra of travel time distributions for NH₃ at the R farm.



Figure 10: Power spectra of travel time distributions for TKN at the R farm.



Figure 11: Power spectra of travel time distributions for TOC at the R farm.



Figure 12: Power spectra of travel time distributions for SS at the R farm.