

**Large Sample Power of Some Nonparametric Test Statistics
for the Location and Mean Slippage Problems**

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ABSTRACT

Many nonparametric test statistics for the location problem or mean slippage problem are based on extreme order statistics. Some new test statistics related to those of Rosenbaum (1954) and Mosteller (1948) are considered in this paper. Analytic formulae for the large sample power of these test statistics are derived for Lehmann alternatives and for pure-shift alternatives for various common population distributions. Under Lehmann alternatives, the large sample power is independent of the population distributions and the limiting power is less than one, which implies that these test statistics are asymptotically poor. The large sample power under pure-shift alternatives depends greatly on the population distributions. For example, it tends asymptotically to one for uniform or normal distributions but converges to the significance level for lognormal or Cauchy distributions.

KEY WORDS AND PHRASES: Lehmann alternatives; Order statistics; Negative binomial distribution; Asymptotic distribution; Tail behavior.

1. INTRODUCTION

Many nonparametric test statistics for the location problem or the mean slippage problem are based on extreme order statistics. For example, Rosenbaum (1954) for the location problem and Mosteller (1948) for the mean slippage problem, considered the statistic equal to the number of observations from one population greater than the largest observation from the other populations. Related work on the mean slippage problem has been done by Bofinger (1965), Conover (1965; 1968), Joshi and Sathe (1978; 1981), Neave (1966; 1972); 1973; 1975) and Neave and Granger (1968). Related work on the location problem has been done by Tukey (1959) and Rosenbaum (1965). In this paper we consider generalizations of the statistics of Mosteller and Rosenbaum.

All of the referenced nonparametric procedures are simple to implement and distribution-free under the null hypothesis of no differences in location or no slippage. However, the alternative hypotheses are usually not well specified so that the implications of rejecting the null hypothesis are not clear. Tests based on ranks will not necessarily distinguish in any obvious way the slippage of mean, tail or dispersion of the population distribution (Barnett and Lewis 1984). In all of the above studies simulation was used to evaluate the power of the various test statistics. However, this approach cannot quantify large sample power. In this paper we show that the large sample power of the nonparametric tests considered are highly sensitive to the form of the alternative distribution. For distributions which are quite similar in practice, the large sample power can converge to one, to the significance level or to a constant in between.

To investigate the power behaviors of these nonparametric slippage tests, the problem primarily considered in this paper consists of k populations among which the k 'th one is suspected to have slipped to the right. Section 2 proposes a new test procedure for which the distribution of the test statistics under the null hypothesis is derived and the critical region is determined. Then, two kinds of alternative hypotheses are considered to investigate their power behaviors, especially in cases of large sample sizes. In Section 3, we consider Lehmann alternatives. The distribution and power of the test statistics are derived and the effects on the large sample power behaviors due to different test statistics and alternatives are investigated. The limiting power against pure-shift alternatives is discussed in Section 4. Several common population distributions are included to depict the dependence of power behavior on the tails of the population distributions. In Section 5 we relate our problem to the slippage problem and finally, some discussions and conclusions are presented in Section 6.

2. TEST STATISTICS FOR THE LOCATION PROBLEM

The setting considered here will be random samples of size n_i from each of k populations ($i=1,2,\dots,k$). For simplicity we will use $N = \sum_{i=1}^{k-1} n_i$ to denote the total sample size for the first $k-1$ populations and $n=n_k$ to denote the sample size for the population suspected to be to the right of the others. The first $k-1$ populations are identically distributed with cumulative distribution function (c.d.f.) $F(x)$ and the c.d.f. for the k 'th population will be denoted $F_k(x)$. The null hypothesis of no slippage is then given by

$$H_0: F_k(x) = F(x) \quad (2.1)$$

and the alternative hypothesis will be

$$H_A: F_k(x) < F(x) \quad \text{for all } x. \quad (2.2)$$

Generalizing Mosteller's (Mosteller 1948) and Rosenbaum's (Rosenbaum 1954) test procedures, we consider test statistics T_r , which are defined by counting the number of observations in the k 'th population which are greater than the r 'th largest observation in the remaining $k-1$ populations. In other words, let $X_{i,j}$ denote the j 'th observation from the i 'th population and Y_r be the r 'th maximum in the observations $X_{i,j}$ ($j=1,2,\dots,n_i$; $i=1,2,\dots,k-1$); then

$$T_r = \sum_{j=1}^n S_{r,j} \quad (2.3)$$

where $S_{r,j}$ is an indicator function defined as

$$S_{r,j} = \begin{cases} 1 & \text{if } X_{k,j} > Y_r \\ 0 & \text{if } X_{k,j} \leq Y_r \end{cases} \quad (2.4)$$

For example, T_1 is the number of observations in the k 'th population larger than the largest observation from the first $k-1$ populations.

The distribution of the test statistics can be determined from the order statistics Y_r . Since all the N observations from the first $k-1$ populations are identically and independently distributed with c.d.f., $F(x)$, the distribution of Y_r is given by Mood, Graybill, and Boes (1974),

$$F_{Y_r}(y) = \sum_{h=0}^{r-1} \binom{N}{h} [F(y)]^{N-h} [1-F(y)]^h, \quad (2.5)$$

and the density function is

$$f_{Y_r}(y) = r \binom{N}{r} [F(y)]^{N-r} [1-F(y)]^{r-1} f(y). \quad (2.6)$$

It is noted by inspecting (2.3) that the distribution of T_r conditional on $Y_r=y$ is binomial with parameters n and $p=1-F_k(y)$. Hence, the mass function of T_r can be expressed by

$$\begin{aligned} P(T_r=t) &= \int_{-\infty}^{\infty} P(T_r=t | Y_r=y) f_{Y_r}(y) dy \\ &= \int_{-\infty}^{\infty} \binom{n}{t} F_k(y)^{n-t} [1-F_k(y)]^t \cdot r \binom{N}{r} [F(y)]^{N-r} [1-F(y)]^{r-1} f(y) dy, \end{aligned} \quad (2.7)$$

for $t=0,1,2,\dots,n$.

Under the null hypothesis (2.1), the mass function of T_r can be found explicitly from (2.7), i.e.,

$$P(T_r=t) = \frac{\binom{t+r-1}{t} \cdot \binom{n+N-r-t}{n-t}}{\binom{n+N}{n}}, \quad t=0,1,2,\dots,n. \quad (2.8)$$

For the large sample case where $N \rightarrow \infty$ and $n/N \rightarrow \lambda$, (2.8) can be simplified to

$$P(T_r=t) = \binom{t+r-1}{t} \left(\frac{\lambda}{1+\lambda}\right)^t \left(\frac{1}{1+\lambda}\right)^r \quad t=0,1,2,\dots$$

This is a negative binomial distribution with parameters r and $p=\lambda/(1+\lambda)$.

This simplifies to

$$P(T_r=t) = \binom{t+r-1}{t} \left(\frac{1}{k}\right)^t \left(1 - \frac{1}{k}\right)^r \quad (2.9)$$

for equal sample sizes.

Intuitively, the statistical inference is to reject H_0 if the test statistic T_r is too large. Since T_r is a discrete random variable, a randomized test must be employed to exactly achieve most prescribed significance levels α . Therefore, to achieve an α -level test we use the following procedure. Choose the integer Z_c and probability p_c such that

$$\alpha = P(T_r > Z_c) + p_c P(T_r = Z_c).$$

We then reject H_0 if $T_r > Z_c$ or, with probability p_c , if $T_r = Z_c$.

3. POWER AGAINST LEHMANN ALTERNATIVES

Lehmann alternatives are mathematically natural nonparametric alternatives since, for them, the distribution of ranks is independent of the population distribution (Lehman 1953). We therefore derive the power of the test statistics T_r against these alternatives, which are defined in terms of a parameter m by

$$H_A: F_k(x) = [F(x)]^{1/m} \quad 0 < m < 1. \quad (3.1)$$

Obviously, (3.1) satisfies the relation (2.2). Also, the mean of the k 'th population moves to the right as m gets smaller. However, the dependence of the variance on m is complicated. For example, as m gets smaller, the variance gets smaller for uniform or normal distributions; gets larger for lognormal or Cauchy distributions; and stays approximately the same for exponential distributions. These arguments will be made clear in Section 4.

Equation (2.7) can be applied to derive the distribution of T_r . By a change of variable $F_k(y) = g$, i.e., $F(y) = g^m$ and $f(y)dy = mg^{m-1}dg$, as well as a binomial expansion for the term $[1-F(y)]^{r-1} = (1-g^m)^{r-1}$, (2.7) can be integrated to be

$$P(T_r=t) = N \cdot m \cdot \binom{n}{t} \binom{N-1}{r-1} \sum_{h=0}^{r-1} (-1)^h \binom{r-1}{h} \cdot B(t+1, n-t+mN-mr+mh+m), \quad (3.2)$$

where $B(\cdot, \cdot)$ denotes the Beta function.

Though (3.2) is valid for any finite sample size, the expression becomes much simpler in the limiting case. It is noted that the sequence of limit and integration in (2.7) can be interchanged according to the dominated convergence theorem (Folland 1984). By defining a new variable $v = -n \cdot \log[F_k(y)]$, the limit of (2.7) as $N \rightarrow \infty$ and $(n/N) \rightarrow \lambda$ can be expressed in terms of $A \equiv 1+m/\lambda$ by

$$\begin{aligned}
 P(T_r=t) &= \frac{1}{t!} \cdot \frac{1}{(r-1)!} \cdot (A-1)^r \int_0^{\infty} v^{t+r-1} e^{-Av} dv \\
 &= \binom{t+r-1}{t} \left(\frac{1}{A}\right)^t \left(1 - \frac{1}{A}\right)^r, \quad t=0,1,2,\dots \quad (3.3)
 \end{aligned}$$

This is again a negative binomial distribution, but this time with parameters r and $p=1/A$.

It is noted by inspecting (3.3) and (2.9) that both are identical except that the group number k in (2.9) is replaced by A in (3.3). Hence, A can be called an equivalent group number which is equal to $1+m(k-1)$ for equal-sample-size cases or $1+m/\lambda$ for unequal-sample-size cases.

By employing (3.2) or (3.3), the power of T_r against the Lehmann alternative can be calculated. Shown in Figure 1 is the large sample power of test statistics T_r with $r=1,5,10,20$ or 70 versus the parameter m when the number of populations is $k=10$ and the significance level is $\alpha=0.1$. Obviously, the power of the test statistic T_1 , is very poor even though the population size tends to infinity. It seems that a better way to improve the power is to increase the order of the test, r , instead of the sample size n .

The power of T_r for a fixed r gets poorer for a larger number of populations, k . Hence, in Figure 2, the large sample power is plotted versus k for constant ratio of $r/k = .1, .2, .5, 1, 2, 3$ or 4 while choosing $m=.5$ and $\alpha=.1$. It is interesting to note that the large sample power is approximately the same for constant r/k , while the population number may change from 2 to 50 or even larger. This means that, at least in large sample size cases, given a population number k , we can choose a proportionate value of r to achieve the wanted power. For example, in testing against the Lehmann alternative with $m=.5$ given $k=10$, we can choose $r \geq 40$ to have a probability of type II error about .4 while keeping the significance level $\alpha=.1$.

4. POWER AGAINST PURE-SHIFT ALTERNATIVES

The most common alternative used in the mean slippage problems is the pure-shift alternative, which is defined as

$$H_A: F_k(x) = F(x-\theta), \quad \theta > 0. \quad (4.1)$$

The distribution of the k 'th population here is uniformly shifted to the right of the first $k-1$ populations by a distance θ . However, the distribution of ranks as well as the distribution of T_r is now dependent not only on θ but also on the c.d.f. $F(x)$. It is thus very difficult to investigate the power under a pure-shift alternative analytically for finite sample-size cases. Therefore, only the limiting distribution of T_r , which is simpler but still illustrative, is emphasized here.

To find the limiting distribution of T_r , it is necessary to use the asymptotic distribution of order statistics. Let $Y_{n-j+1:n}$ be the j 'th maximum in a random sample of size n drawn from the population. For most of the common population distributions, an asymptotic distribution for the extreme order statistic $Y_{n:n}$ exists (Galambos 1978), i.e., there exist a_n and b_n such that

$$\lim_{n \rightarrow \infty} P\left(\frac{Y_{n:n} - a_n}{b_n} \leq y\right) = G(y).$$

Here, a_n and b_n can be treated roughly as a sequence of "means" and "standard deviations" of $Y_{n:n}$. The c.d.f. $G(y)$ is the asymptotic distribution of the normalized extreme order statistic $Z_1 \equiv (Y_{n:n} - a_n)/b_n$. Table 1 lists a_n , b_n , and $G(y)$ for some typical distributions (uniform, normal, exponential, lognormal, and Cauchy).

If the asymptotic distribution $G(y)$ for the normalized extreme order statistic Z_1 exists, the asymptotic distribution $G_j(y)$ for the normalized j 'th maximum $Z_j = (Y_{n-j+1:n} - a_n)/b_n$ also exists and is related to $G(y)$ by (Galambos 1978)

$$\begin{aligned} G_j(y) &= \lim_{n \rightarrow \infty} P\left(\frac{Y_{n-j+1:n} - a_n}{b_n} \leq y\right) \\ &= G(y) \sum_{h=0}^{j-1} \frac{1}{h!} \left[\log \frac{1}{G(y)} \right]^h. \end{aligned} \quad (4.3)$$

It is interesting to note that the distribution $F_k(x)$ under the Lehmann alternative (3.1) with $m=(1/n)$ is identical to the distribution of the extreme order statistic $Y_{n:n}$. This implies that to discuss the change of distribution as the Lehmann shift m tends to zero is no more than to discuss the behavior of extreme order statistics as n tends to infinity. Hence, the normalization constants a_n and b_n discussed here can give an indication of the mean and standard deviation of $F_k(x)$ under the Lehmann alternative as discussed in Section 3.

Using (4.3), it is easy to derive the asymptotic distribution of test statistics T_r under pure-shift alternatives. Obviously, the event $T_r \geq t$ means equivalently that the r 'th maximum in the first $k-1$ populations, denoted $Y_{N-r+1:N}^{(I)}$, is smaller than the t 'th maximum in the k 'th population, denoted $Y_{n-t+1:n}^{(II)}$, i.e.,

$$P(T_r \geq t) = P\left(Y_{n-t+1:n}^{(II)} > Y_{N-r+1:N}^{(I)}\right). \quad (4.4)$$

Suppose that the normalization constants a_n , b_n and asymptotic distribution $G(y)$ for the distribution function $F(x)$ exist. Since the distribution of the k 'th population, $F_k(x)$, is identical to $F(x)$ except for a shift by θ , it is advantageous to define the following normalized order statistics

$$Z_r^{(I)} = \frac{Y_{N-r+1:N}^{(I)} - a_N}{b_N}, \quad Z_t^{(II)} = \frac{(Y_{n-t+1:n}^{(II)} - \theta) - a_n}{b_n}. \quad (4.5)$$

Then, $Z_r^{(I)}$ and $Z_t^{(II)}$, which are independent of each other, have asymptotic distributions $G_r(y)$ and $G_t(y)$, respectively.

Substituting (4.5) into (4.4), the asymptotic distribution of T_r satisfies

$$P(T_r \geq t) = \lim_{n \rightarrow \infty} P\left(Z_t^{(II)} - Z_r^{(I)} B_{k-1} > A_{k-1} - \theta/b_n\right) \quad (4.6)$$

where

$$A_{k-1} = \lim_{n \rightarrow \infty} \frac{a_N - a_n}{b_n}, \quad B_{k-1} = \lim_{n \rightarrow \infty} \frac{b_N}{b_n}. \quad (4.7)$$

By (4.6), the mass function and thus the power of T_r can be expressed in terms of the constants A_{k-1} , B_{k-1} and b_n . Among these, the constant b_n is crucial since it governs the large sample power. Typically, the power tends to one no matter how small θ is if b_n tends to zero, while it tends to the significance level α no matter how large θ is if b_n tends to infinity.

For distributions with finite upper range such as the uniform distributions, it can be shown that b_n tends to zero as n tends to infinity. Hence, the large sample power tends to one, i.e., the test statistics T_r are asymptotically ideal. For distributions with infinite upper range, the situation becomes much more complicated since the constant b_n may tend to zero, a nonzero constant, or infinity, as depicted in Table 1. It is interesting to notice that the tail behavior of the c.d.f. $F(x)$ reflects the limiting behavior of the constant b_n . In Figure 3 the density functions in \log scale of the normal, exponential, lognormal and Cauchy distributions are plotted. They can be categorized into light-, medium- and heavy-tailed distributions according as the term $\lim_{n \rightarrow \infty} \left[\frac{-\log_{10} f(x)}{x} \right]$ is infinite, a non-zero constant or zero, respectively.

For light-tail distributions such as normal distributions, the "standard deviations" b_n of extreme order statistics tends to zero. Therefore, T_r are ideal test statistics asymptotically, i.e., the large sample power converges to one. However, the rate of convergence is much slower for normal distributions than for uniform distributions since the constant $b_n = 1/\sqrt{2 \log n}$ for normal(0,1) distributions tends to zero more slowly as compared with the constant $b_n = 1/n$ for uniform (0,1) distributions.

For medium-tail distributions such as exponential distributions, b_n tends to a nonzero constant. Thus, the limiting power is some value strictly between the significance level and one. However, for heavy-tail distributions such as lognormal or Cauchy distributions, the "standard deviation" b_n of extreme order statistics tends to infinity as n tends to infinity. The power of T_r decreases as n increases and finally becomes α in the limiting case as $N \rightarrow \infty$ and $(n/N) \rightarrow \lambda$. In other words, the test statistics T_r are worse for larger sample sizes and finally become useless as n tends to infinity.

5. RELATION TO SLIPPAGE TESTS

We next indicate how the results described in this article apply to nonparametric slippage tests. The easiest case to analyze is Mosteller's test (Mosteller 1948). The null hypothesis for the slippage test is again given by (2.1) but the alternative is simply that one of the populations (unspecified) has slipped to the right of the others. Mosteller selects as the population which is suspected to have slipped to the right as the one with the largest observation. The test statistic is then the number of observations in the suspected population larger than all the observations in the remaining populations. We will denote this test statistic T_1^* .

First, we will argue that the probability of a correct decision for the slippage test is less than the test proposed in Section 2 (which "knows" which population is largest if any of them are). It is clear that $T_1^* \geq T_1$, since if the k th population (assumed without loss of generality to be the one with the largest population mean) has the largest observation, then $T_1^* = T_1$, otherwise $T_1 = 0$. Thus the critical values for T_1^* must be larger than the ones derived for T_1 . Since a correct decision as to the slipped population is only made in the case when the k th population gives the largest observation and since, for that case, $T_1^* = T_1$, the power of the test in Section 2 is larger than the probability of a correct decision using Mosteller's test. Thus, for heavy-tailed distributions like the lognormal, the probability of a correct decision will decrease to the significance level as the sample size increases.

Second, we argue that if the power of the test converges to one, then the probability of a correct decision in Mosteller's test also converges to one. If the power of the test based on T_1 converges to one then the largest observation comes from the k th population with probability approaching

one. Thus $T_1^* = T_1$ with probability approaching one. Investigation of equation (4.6) shows that the critical value, t , has no bearing on the limiting result if b_n converges to zero (the light-tailed case). Thus, T_1^* and T_1 will behave similarly in the limit and both have limiting probability one.

The above discussions show that the nonparametric slippage tests will exhibit the same qualitative behavior as the location test. That is, they will have limiting probability of correct decision equal to one in some cases and equal to the significance level in other cases. This will be true even with distributions which, in practice, can look very similar, like the normal and lognormal distributions.

6. CONCLUSIONS AND DISCUSSIONS

By generalizing Mosteller's and Rosenbaum's methods, nonparametric test statistics, T_r , based on the r 'th maximum, are proposed in this paper. We use T_r as a typical example of a statistic used for nonparametric tests. The distribution of T_r has been derived analytically and it tends to a negative binomial distribution for large sample sizes. Two kinds of alternative hypotheses, Lehmann alternatives and pure-shift alternatives are included to investigate the power of T_r . Characterized in this paper is the analytical analysis of large sample power which is simple and illustrative.

Under Lehmann alternatives, the large sample power of T_r is independent of the population distributions but depends greatly on the order of the test statistic, r . Though the test T_r with $r=1$ is very quick and simple in operation, its power is rather small and can be improved by a choice of larger r . In the large sample-size cases, the power of T_r improves monotonically as r increases. However, in practical cases with finite sample sizes, intuitively, there exists an optimal choice of r to achieve the maximal power. The optimal r , which may be a function of sample size n , Lehmann shift m , and population number k , deserves future investigation, perhaps by simulation.

Under pure shift alternatives, the large sample power of T_r depends greatly on the tail behavior of the population distribution. For distributions which have finite upper range (no tails) such as uniform distributions, the large sample power tends to one no matter how small the shift is. For light-tail distributions such as normal distributions, the large sample power also tends to one but at a much slower rate. However, for

medium-tail distributions such as exponential distributions, the increase in sample size does not contribute much in improving the power. The worst case is with heavy-tail distributions such as lognormal and Cauchy distributions for which increasing the sample size only decreases the power and ultimately makes the test based on T_r useless.

Many nonparametric mean-slippage tests, such as those proposed by Mosteller (1948), Conover (1965; 1968), Joshi and Sathe (1978), and others, are based on the comparison of maximum or minimum among the populations. It is reasonable to infer that they will exhibit properties similar to T_1^* . From the limiting behaviors of T_r , it is concluded that one should examine their data a little bit before jumping into nonparametric methods. Though the nonparametric test statistics are distribution-free under the null hypothesis, their power depends tremendously on the alternative and the population distributions. To sum up, without any idea of the population distributions, these nonparametric methods can be as unreliable as, or sometimes less reliable than, parametric methods.

Also, it should be noted that, in the limiting case, the proposed test T_r with any finite r discriminates the pure-shift populations based only on the tail portion of the distributions. This may be the reason why the test is asymptotically satisfactory for no- or light-tailed distributions, but not for medium- or heavy-tailed distributions. This argument also suggests that T_r with different ratios of order r to sample size n may emphasize different portions of the distributions for testing pure-shift alternatives. It thus will be interesting to evaluate the effect of different r 's on the finite sample-size power of T_r for different kinds of population distributions. The investigation is in progress and will be presented in the near future.

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Table 1. Normalizing constants a_n , b_n and asymptotic distribution $G(y)$ for several different distributions. Also listed is the limiting power of T_r under pure-shift alternatives.

Tail	Example	a_n , b_n , and $G(y)$	$\lim_{n \rightarrow \infty} b_n$	Power
No	$U(0,1)$	$a_n = 1, \quad b_n = 1/n$ $G(y) = \begin{cases} e^y & y < 0 \\ 1 & y \geq 0 \end{cases}$	0	1
Light	$N(0,1)$	$a_n = \sqrt{2 \log n} - \frac{\log(4\pi \cdot \log n)}{2 \cdot \sqrt{2 \log n}}$ $b_n = 1/\sqrt{2 \log n}$ $G(y) = \exp\{-e^{-y}\} \quad y \in (-\infty, \infty)$	0	1
Medium	$\text{Exp}(1)$	$a_n = \log n, \quad b_n = 1$ $G(y) = \exp\{-e^{-y}\} \quad y \in (-\infty, \infty)$	1	$(\alpha, 1)^*$
Heavy	Log-normal (0,1)	$a_n = \exp\left[\sqrt{2 \log n} - \frac{\log(4\pi \cdot \log n)}{2 \cdot \sqrt{2 \log n}}\right]$ $b_n = a_n / \sqrt{2 \log n}$ $G(y) = \exp\{-e^{-y}\} \quad y \in (-\infty, \infty)$	∞	α
	Cauchy (0,1)	$a_n = 0, \quad b_n = \tan(\pi/2 - \pi/n)$ $G(y) = \begin{cases} e^{-1/y} & y > 0 \\ 0 & y \leq 0 \end{cases}$		

NOTE: $(\alpha, 1)^*$ means the power is some value strictly between α and 1.

Here, α is the significance level.

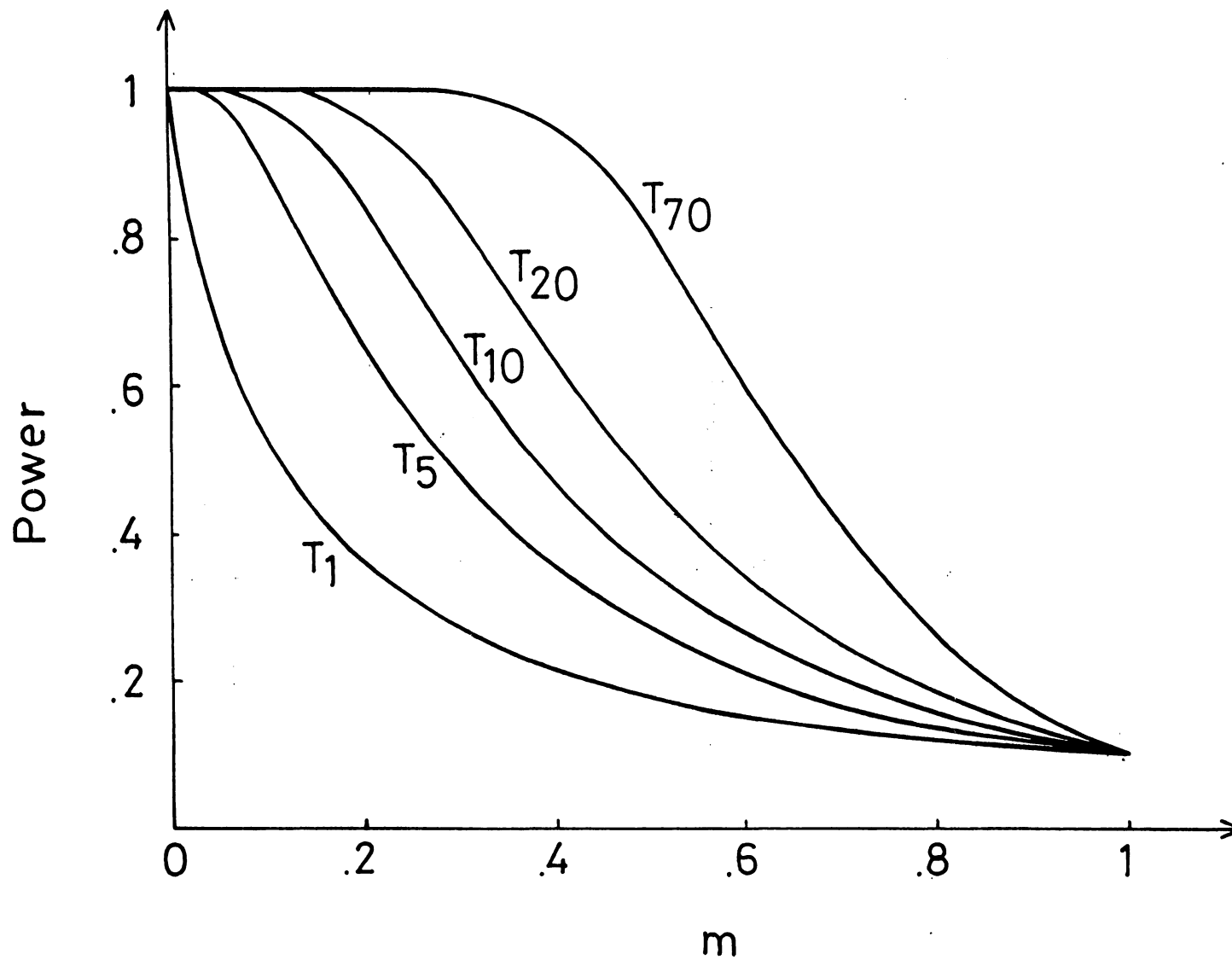


Fig. 1 Power versus m while keeping $k=10$, $\alpha=0.1$ and $r=1, 5, 10, 20$ or 70 .

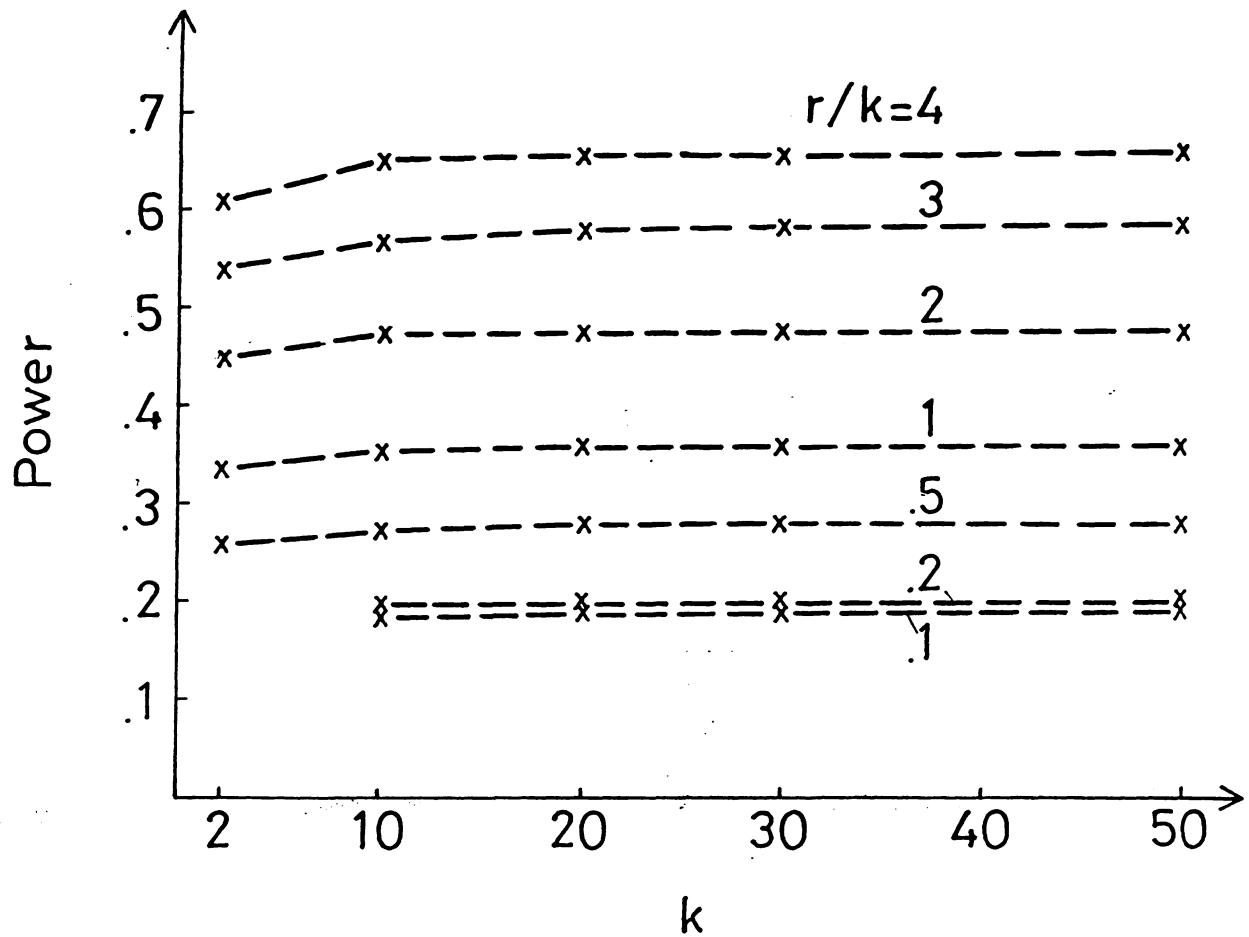


Fig. 2 Power versus k with $m=0.5$ and $\alpha=00.1$ for $r/k=0.1, 0.2, 0.5, 1, 2, 3$ or 4 .

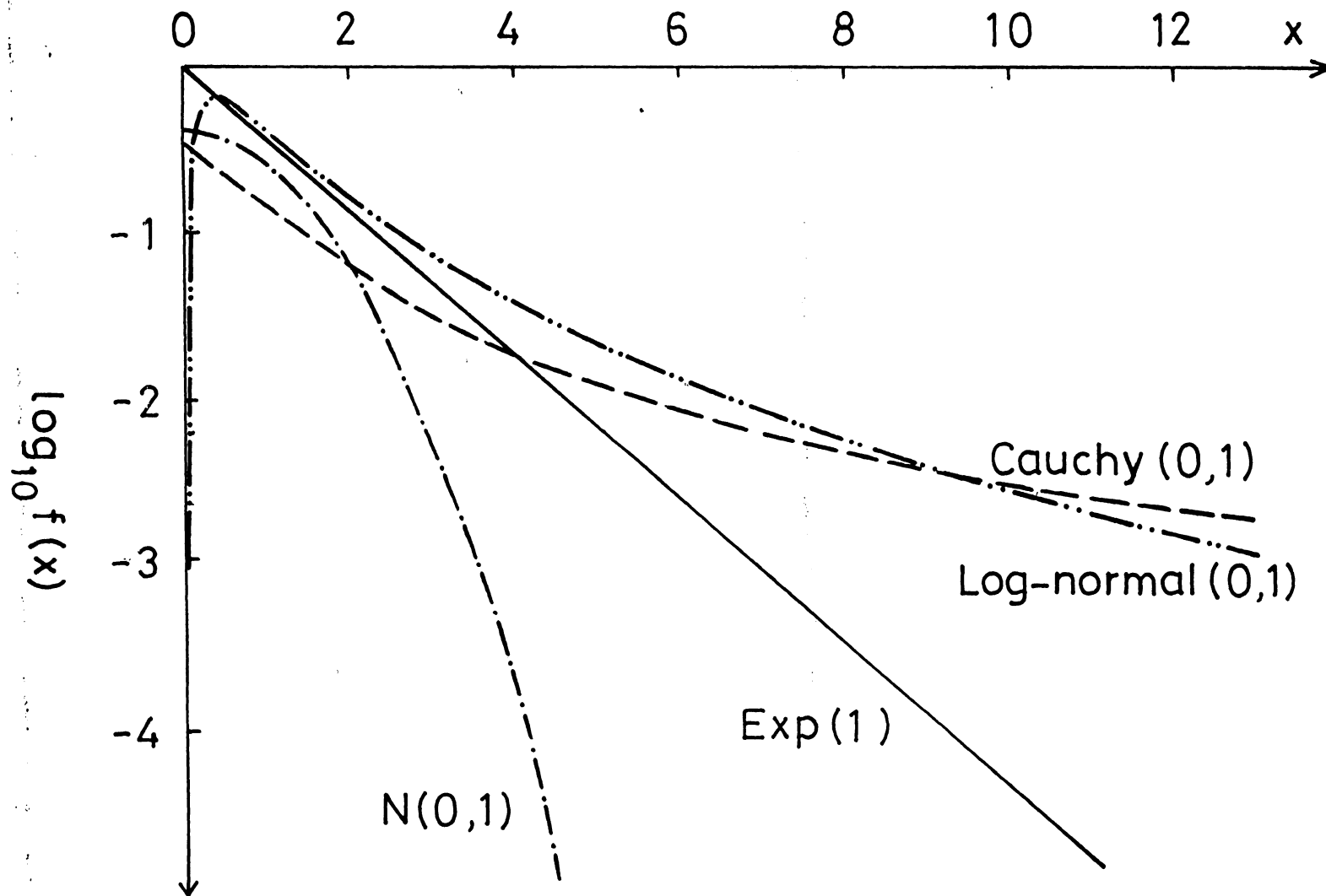


Fig. 3 Tail behaviors of several distributions.

List of captions

Table 1 Normalizing constants a_n , b_n and asymptotic distribution $G(y)$ for several different distributions. Also listed is the limiting power of T_r under the pure-shift alternatives.

Figure 1 Power versus m while keeping $k=10$, $\alpha=0.1$ and $r=1, 5, 10, 20$ or 70 .

Figure 2 Power versus k with $m=0.5$ and $\alpha=0.1$ for $r/k=0.1, 0.2, 0.5, 1, 2, 3$ or 4 .

Figure 3 Tail behaviors of several distributions.