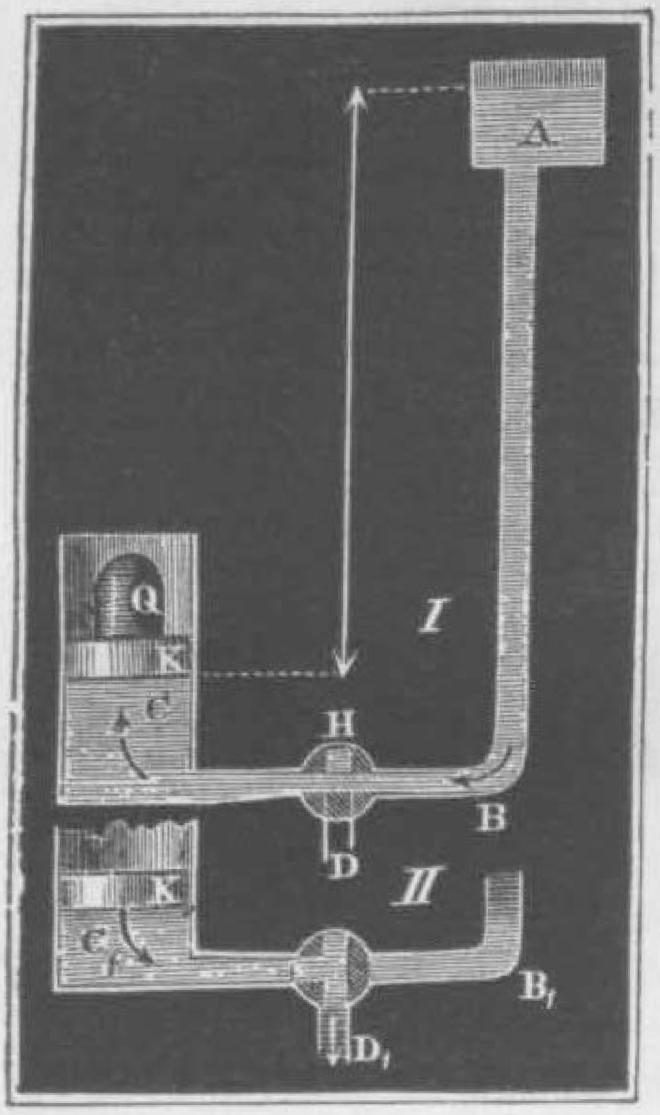
CHAPTER VI.

WATER-PRESSURE ENGINES.

§ 177. *Water-Pressure Engines.*—Water-pressure engines, as their name indicates, are set in motion by a column of water. Their motion is a reciprocating rectilinear motion, and not rotatory as in the turbine. The leading features of a water-pressure engine are delineated in Fig. 284. *A* is a reservoir at the upper end of the pipe. *AB* is the *pressure pipe*. *C* is the working cylinder, in which the water moves the loaded *piston K*. In the pipe *BC*, by which the pressure pipe communicates with the cylinder, the regulating valve or cock is placed. It is here represented as a three-way cock, serving alternately to open and close the communication between the working cylinder and the pressure pipe. When the way is open, the water presses on the piston, and raises it, with its load, through a certain height—the *length of stroke*—when the communication between the pressure pipe and the cylinder is shut, a way is opened for the discharge of the water from the cylinder by the pipe *D*, and the piston then descends by its own gravity.

Fig 284.



Water-pressure engines are either *single* or *double acting*. Fig. 284 shows the general arrangement of the single-acting engine, in which the piston is made to move in one direction by the pressure of the water, and to return by its own weight.

In the double-acting engine, the up stroke and down stroke, or both strokes of the piston, are made under the hydraulic pressure. Fig. 285 shows the general arrangement of a double-acting engine. The cock is in this case a four-way cock. In I. the pressure is on the upper side of the piston through *ABC*, and the discharge goes on through C_1B_1D. In II. the pressure is on the under side through AB_1C_1 of the piston, and the discharge through *CBD*.

Water-pressure engines are also made with two cylinders, each single-acting, but connected together, as in Fig. 285, so that while the one piston is ascending by the pressure of the water, the other is descending, the water being discharged therefrom. The relative position of the passages in the four-way cock are shown in Figs. 286 and 287.

Fig. 285.

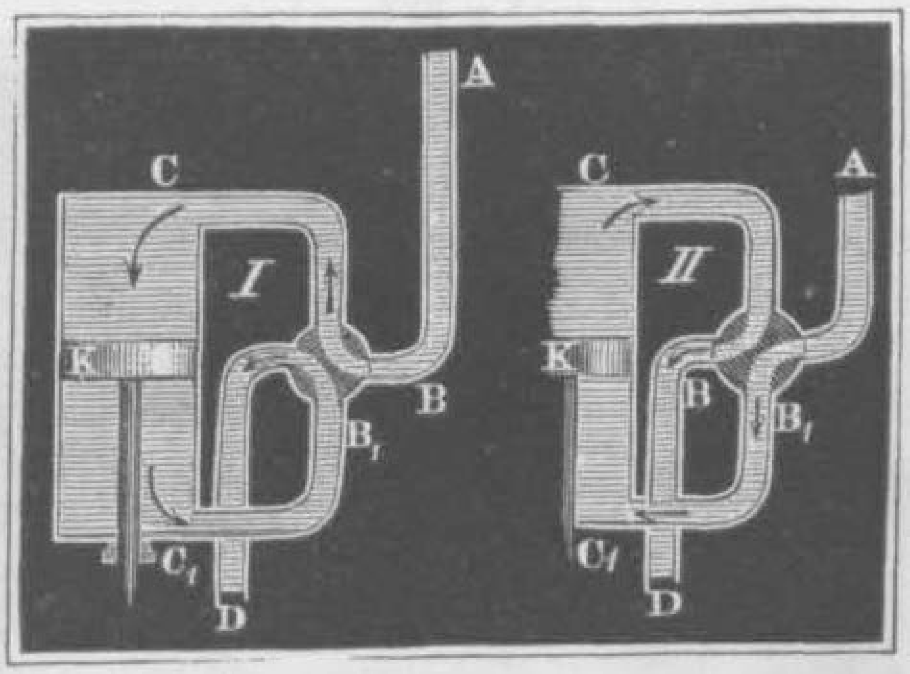


Fig. 286. I.

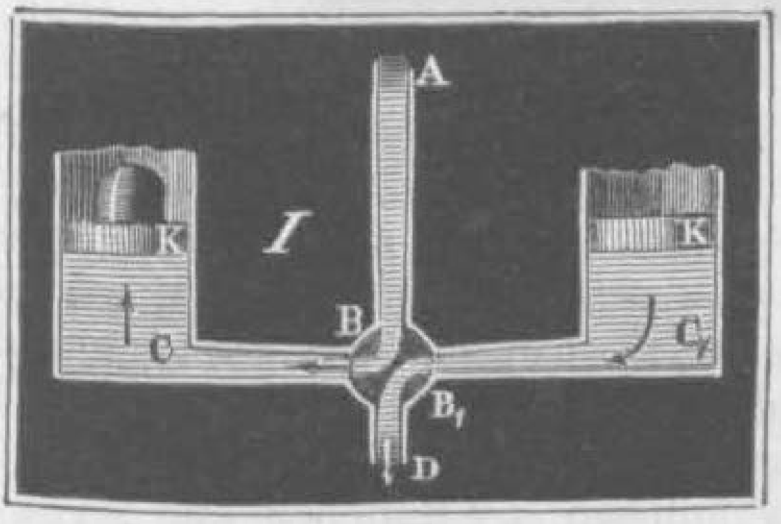
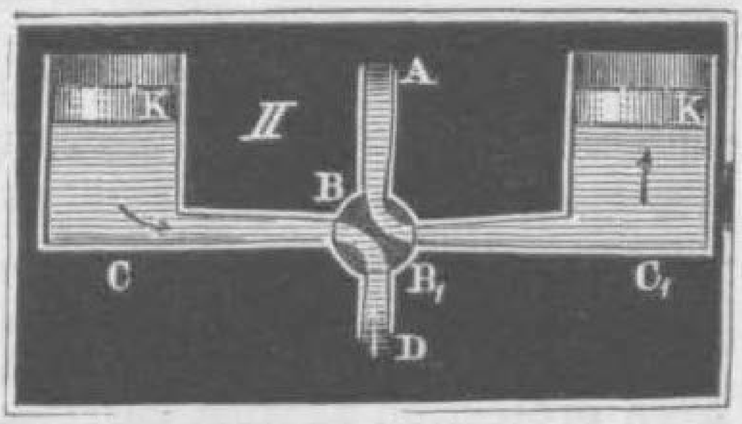


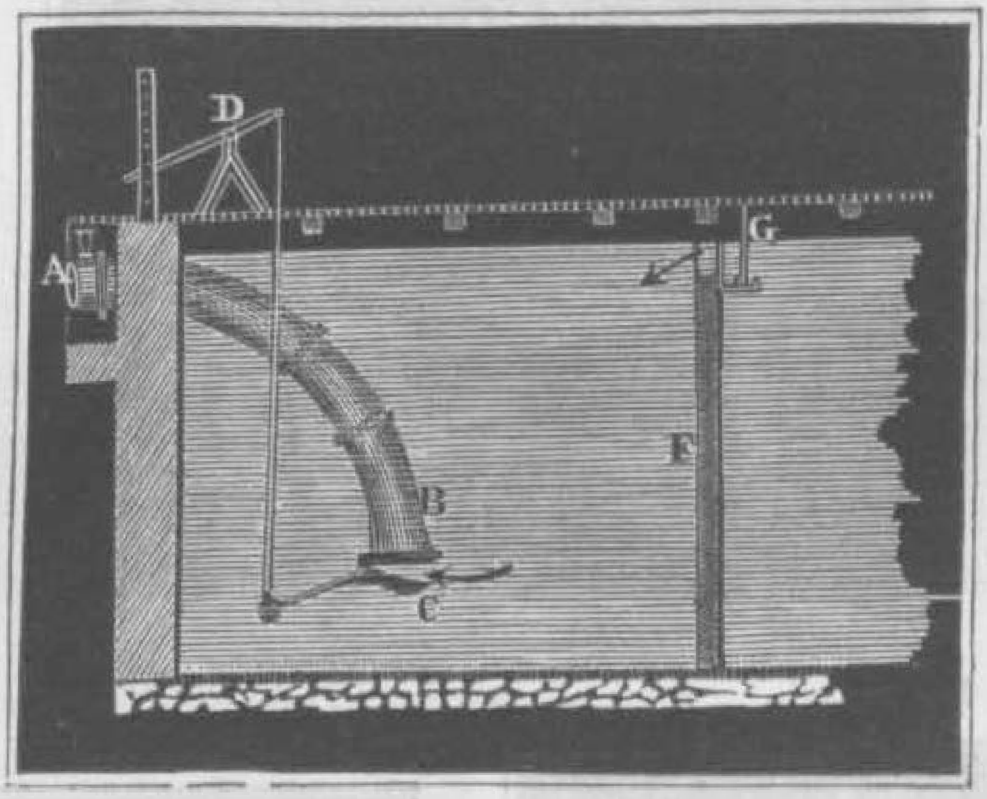
Fig. 287. II.



§ 178. *Pressure Pipes.*—The pressure pipes should take the water from a feeding cistern or settling reservoir, in which the water has time to deposit the foreign matters it may have carried so far along with it. In front of this a grating must be placed, to keep back leaves, ice, &c. &c.

The end of the pressure pipes should dip so as to be 1½ foot, at least, above the bottom of the feed cistern, and 3 to 4 feet under the surface of the water in it, so as to prevent the influx of heavier particles, and to render the indraught of air impossible. For this object the end of the pipe may be conveniently curved with the mouth downwards, as shown in Fig. 288. C being a valve for shutting off the water from the pipe B, when required. F is a division plate in the cistern. G is a grating to keep back floating bodies. The pressure pipes may be either of wood or iron, but are usually of the latter material, and made from $\frac{1}{3}$ to $\frac{1}{2}$ the internal diameter of the working cylinder. The pipes for great heads are made to increase in thickness from the top downwards proportionally to the pressure. The formula: $e = 0{,}0025\, n\, d_1 + 0{,}66$ inches may be used for calculating the strength required for any given pressure n in atmospheres = 33 feet of water; d_1 being the internal diameter of the pipes. The formula given in Vol. I. § 283, is applicable to ordinary water conduits, but is inapplicable to the present case, because the pressure of the water here varies frequently, and even acts with impact, when the valves are suddenly closed. The pipes

Fig. 288.



must be carefully *proved* by an hydraulic or Bramah press. The *porosity* of pipes, which at first proving is very sensible, gradually becomes insensible as oxidation goes on. In the case of the pipes for the pressure engine, at Huelgoat, described hereafter, boiled oil was used in proving the pipes, by which they become impregnated to a certain depth with the oil, and thus their porosity stopped, and even protection against corrosion insured.

The pressure pipes are usually jointed by flanches and screw-bolts; a ring of lead, or of *iron rust* being interposed, as shown in Figs. 289 and 290. A mixture of lime water, linseed oil, varnish,

Fig. 289.

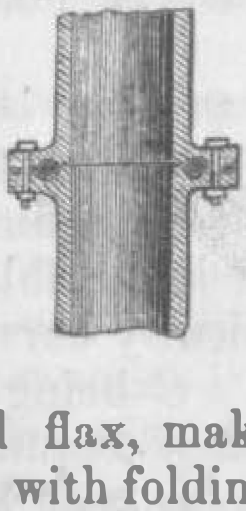
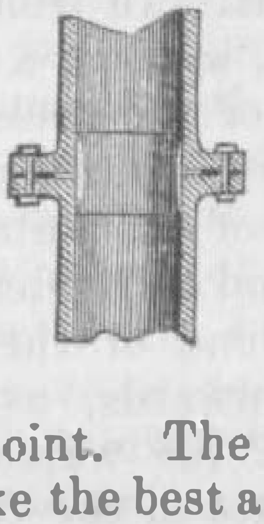


Fig. 290.



and chopped flax, makes a very good pipe-joint. The spigot and faucet joint, with folding wedges of wood, make the best and cheapest joint for cast iron pipes.

§ 179. *The Working Cylinder.*—The working cylinder is made of cast iron or of gun metal. The number of strokes is limited to from 3 to 6 per minute, so that there may be the least possible loss of effect; and, therefore, the capacity of the cylinder is made to depend rather on its length than its diameter. The stroke s is made from 3 to 6 times the diameter d of the cylinder. The mean velocity v, of the piston, is usually 1 foot per second, in order that the mean velocity v_1 of the water in the pressure pipes, and hence the hydraulic resistances may be as small as possible. It is not advisable in any case to have the latter velocity greater than 10 feet per second, and 6 feet is a better limit. If we assume $v = 1$, and $v_1 = 6$ feet, the quantity of water being: $\frac{\pi d^2 v}{4} = \frac{\pi d_1^2 v_1}{4}$, we get for the proportion of the diameter of the pressure pipe to that of the cylinder, $\frac{d_1}{d} = \sqrt{\frac{v}{v_1}} = \sqrt{\frac{1}{6}} = 0,408$, or about 0,4.

If Q be the quantity of water supplied, per second, then for a double-acting engine, or for a double-cylinder engine, $Q = \frac{\pi d^2}{4} \cdot v$, and hence we have the diameter of the working cylinder required $d = \sqrt{\frac{4\,Q}{\pi\, v}} = 1,13\sqrt{\frac{Q}{v}}$, that is, for $v = 1$, $d = 1,13\sqrt{Q}$ feet. For a single-cylinder, single-acting engine, $Q = \frac{1}{2} \cdot \frac{\pi d^2}{4}\, v \therefore d = 1,60\sqrt{\frac{Q}{v}}$, and if $v = 1$, $d = 1,60\sqrt{Q}$ feet. If the stroke of the piston $= 3\,d$ to $6\,d$, the time for one stroke of a single-acting engine is $t = \frac{s}{v}$, or, if $v = 1$, $t = s$ in seconds, and hence the number of single strokes per minute:

$$n_1 = \frac{60''}{t} = \frac{60\,.\,v}{s} \therefore \text{ when } v = 1,\ n_1 = \frac{60}{s},$$

and the number of double strokes:

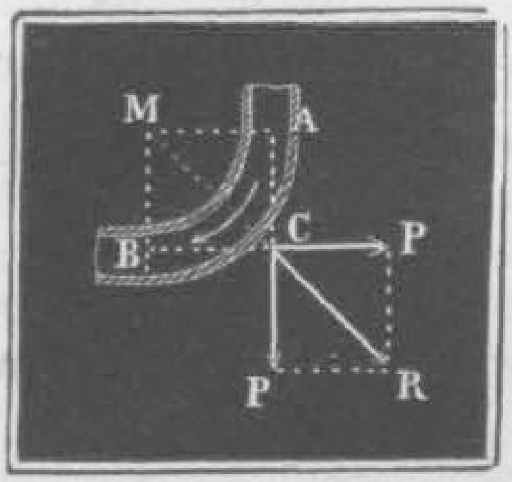
$$n = \frac{n_1}{2} = \frac{30\,v}{s}, \text{ or if } v = 1,\ n = \frac{30}{s}.$$

It is, however, better, in the case of a single acting, single cylinder, water-pressure engine, to begin the stroke somewhat more slowly, or to cause the descent of the piston to take place more rapidly than with the mean velocity, because the hydraulic resistances are greater for the working or up stroke, than for the return of the piston.

The working cylinder must be accurately bored. The thickness of the metal is made greater than the usual rules of calculation indicate as enough, to compensate for *wear*, and because of the shock at entrance of the water. The formula $e = 0,0025\, n\, d + 1$ will be found useful in guiding to the proper dimensions. The cylinder may be strengthened by mouldings or ribs cast round it.

The working cylinder is subject to a pressure in the direction opposite to that in which the piston moves, equal to the weight of a column of water $F\,h\,\gamma$, F being the area of the base, h the height, and γ the weight of a cubic unit; h being not unfrequently several hundred feet, this pressure of the water is very considerable, and, hence, the substructure on which the cylinder rests must be very strong. Water-pressure engines are erected in the shafts of mines for raising water, more frequently than in any other position, and cannot, therefore, be placed on the solid rock, or foundations laid thereon, but have to be supported on cross beams or arches of stone, or of iron.

Fig. 291.



Remark. Besides this pressure, the cylinder has to withstand a horizontal pressure in the direction of the water entering it, and proportional to its section. The effect of this is less observable, because the pressure acts at a point only a little above the base of the cylinder, and because the pressure pipe, which is firmly connected with the cylinder, is equally pressed in the opposite direction. In any bend or knee piece AB, Fig. 291, there is a resultant pressure $CR = R$, which may be put $= P\sqrt{2} = F_1 h \gamma \, . \sqrt{2}$, F_1 being the area of the pipe and h the pressure height.

§ 180. *The Working Piston.* — The main piston which moves under the pressure of the water, consists essentially of a cylindrical disc fitting smoothly into the cylinder. To make this piston perfectly tight, and at the same time not to cause thereby too great a resistance to motion, a *packing* (Fr. *garniture;* Ger. *Liderung*) of hemp, leather, or metal is applied, either on the piston, or in the cylinder, in which latter case the piston becomes what is termed a *plunger* or *ram.* The packing of the pistons of water-pressure engines is usually either leather or metallic rings. They are adjusted

Fig. 292.

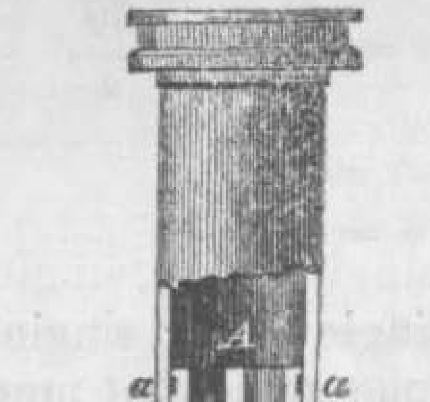
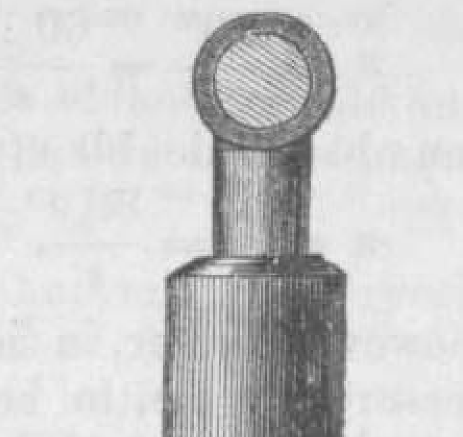


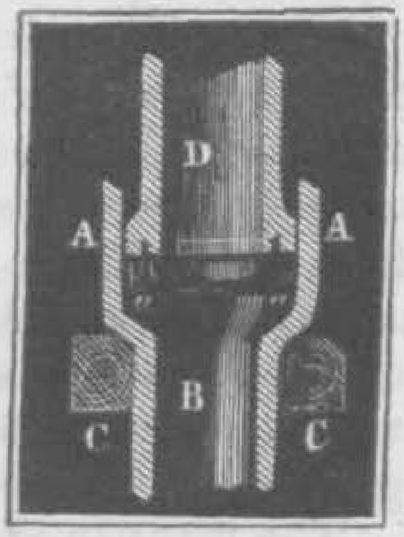
Fig. 293.



to a pressure proportional to the column of water, so that, on the one hand, no water may escape or pass, and on the other, that there may be no unnecessary friction. The best packing that can be employed, is that in which the water itself presses the leather or packing against the surface of the cylinder, or of the ram. The packing is made so that it can be gradually compressed as it wears, by means of a ring fitting upon it, and adjusted by screws. Fig. 292 is the piston of a water-pressure engine at Clausthal, in which the manner of laying in the packing is clearly represented. *A* is the piston, properly so called, and *BB* the piston rod, *a a* and *b b* are the packing rings, and *c c* two fine channels communicating with the back of the packing *b b*. Other methods of packing we shall describe hereafter.

For the plunger or ram, or *Bramah piston*, the packing may likewise be kept tight hydrostatically. *A*, Fig. 293, is the piston, *B* the cylinder, *C* the pressure pipe, *DD* the packing or stuffing box, screwed on to the piston, *a a* is the packing ring, and *b b* the five channels of communication. This manner of keeping the packing tight is more applicable to the case of a stuffing box, than to the ordinary piston.

Fig. 294.

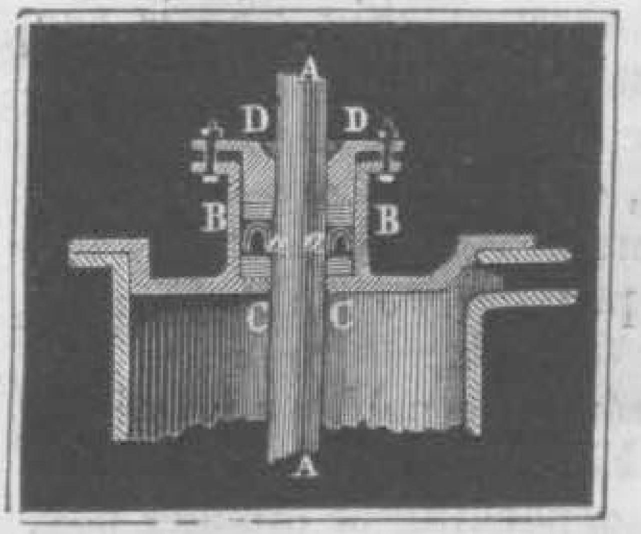


Remark. The compressed ring packing is also applied at the compensation joints, which must be introduced in the length of the pressure pipe. Fig. 294 shows such a pipe, *AA* being the enlarged end of one pipe *B*, accurately bored out, and resting on supports *CC*; *a a* are packing rings compressed by screws and nuts on to the thickened end of the upper pipe *D*.

§ 181. *The Piston Rod and Stuffing Box.*—The piston rod goes either upwards or downwards to the open end, or through the cover of the cylinder. In the first case, it requires very little special arrangement, and may be, in fact is, frequently made of wood. In the second case, it must go through a *stuffing box*, must, therefore, be turned, and can only be made of iron or gun metal. The dimensions of the piston rod is to be calculated according to the received theory of the strength of materials. If d be the diameter of the working cylinder, and p the pressure of the water, on each square inch of the piston, the force $P = \frac{\pi d^2}{4} \cdot p$; and if d_2 be the diameter of the piston rod, and K the modulus of strength of its material, then its strength $= P = \frac{\pi d_2^2}{4} K$, and by equating the two forces, we have: $d_2 = d \sqrt{\frac{p}{K}}$. K is to be taken from the table in Vol. I. § 186, and p is given by the formula $p = \frac{h \gamma}{144}$.

The stuffing box (Fr. *boîte à garniture;* Ger. *Stopfbüchse*) is a box placed on the cylinder cover, so lined with leather or hempen rings, that the piston rod, in passing through it, has freedom to move, but the passage is rendered water, or air, or steam tight, according to circumstances. For water-pressure engines, a leather packing is found to answer best. Fig. 295 shows the apparatus in question.

Fig 295.



AA is the piston rod, *BB* the stuffing box, *BaC* its *packing*, *DD* the cover with the screws for compressing the packing. A grease cup is sunk in the cover *D*, and kept filled with a grease composed of 6 parts hog's lard, 5 parts tallow, and 1 part palm oil, or with pure olive oil, or neat's-foot oil.

In the engine at Clausthal, oiling presses are applied, having a small piston, worked by a weight, and which forces the grease into the packing through a fine tube communicating with the channels of a brass ring, having a section of the I form, and round which the packing is lapped.

§ 182. *The Valves.*—The valves and their gear are, as it were, the very heart of the water-pressure engine, for it is by them the machine is made continuously self-acting. The valves cover and uncover apertures for the admission and discharge of the water from the cylinder, and these are worked so as to open and shut the apertures alternately, by means of *gear* connected with moving parts of the engine, so that the engine is thereby made *self-acting.* The valves are either *cocks*, or *sliding pistons.* The latter form is now generally adopted.

The manner of applying a cock as a valve has been already explained, so that we shall now only further describe the sliding piston valves. The arrangement of piston valves for a single acting, single cylinder engine is shown in Figs. 296 and 297. *E* is the pressure

Fig. 296.

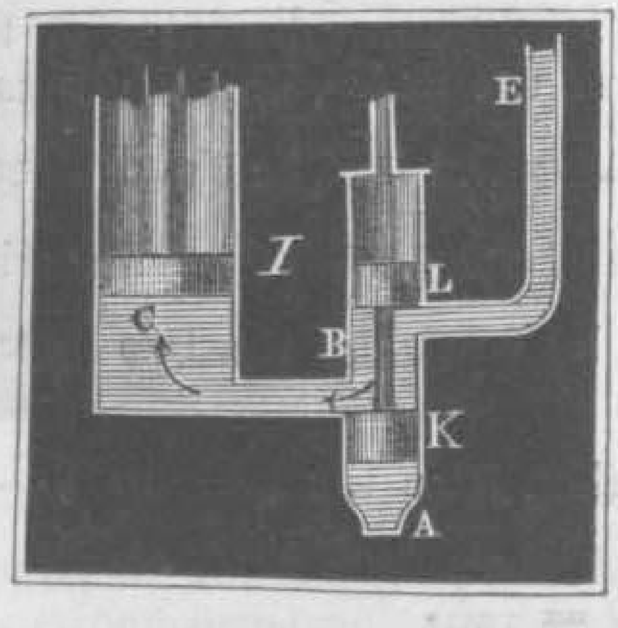
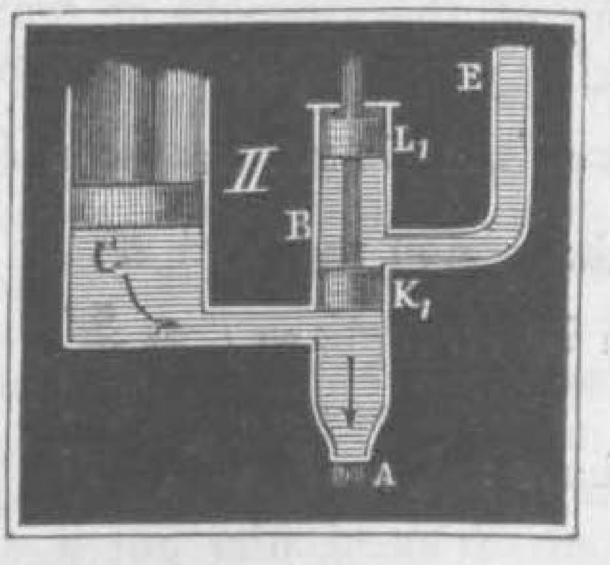


Fig. 297.



pipe, *C* the working cylinder, *B* the valve cylinder, *A* the discharge pipe, *K* the piston valve, and *L* its *counter pistdn*, which, by taking the equal and opposite pressure, renders the movement of the valves more easy. When, as in Fig. 296, K is lowered, the working cylinder and pressure pipes are in communication, and when, as in Fig. 297, K is raised to the position K_1, the communication between the pressure pipes and cylinder is shut, and the passage for discharge of water from the cylinder is open. In the *double-acting* engine, or in the double-cylinder engine, the slide pistons must be arranged as in Figs. 298 and 299. E is the pressure pipe, C the pipe going

Fig. 298

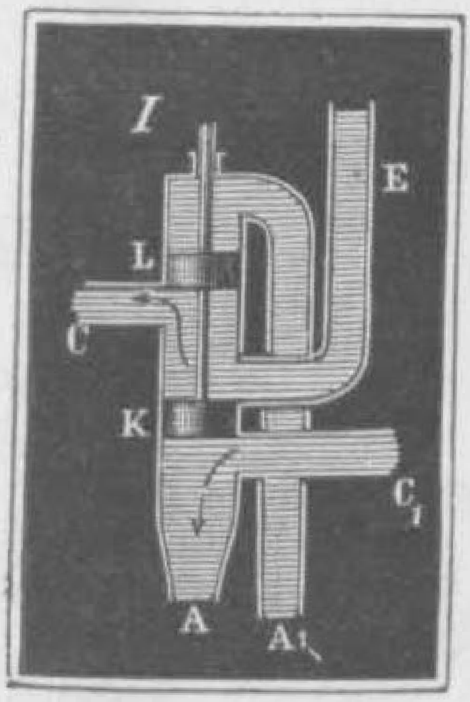
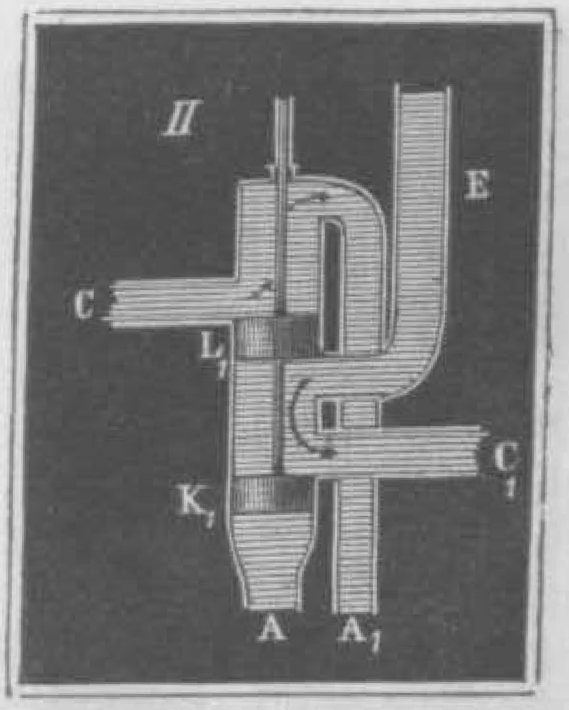


Fig. 299



to the top, and C_1 that going to the bottom of the working cylinder (or going respectively to the bottom of the two cylinders in the double-cylinder engine). A is the discharge pipe for the water supplied by the first, and A_1 that for the water supplied by the second. From Fig. 298, we see that, when the slide valve is up, the pressure pipe is in communication with C, and the discharge made through A, and when the slide pistons are lowered, as in Fig. 299, the communication is open to C_1, and the used water discharged from C by the pipe A_1.

§ 183. *The Valve Cock.*—The cock is used for smaller engines, as shown in Fig. 300. HH is the cock, BB its cover, K is the squared end on to which a lever for turning it fits, D is a screw for raising or lowering the cock in its cover. The passages of the cock are made so as to suit the purposes to which it is applied, as we have explained above.

In Fig. 300, a means of counteracting the effects of greater pressure coming on one side of the cock is shown; b b_1 are two cuts on the cock, communicating with the passage a, by the openings c, c_1, so that a counter pressure is obtained, which, by proper adjustment of the parts, cut out at b and b_1, balance the diagonal pressure in the main passage.

In order to equalize the wear of the cocks on all sides, Mr. Brerdel, of Freiberg, introduced the method of turning them round continuously in the same direction, instead of turning them backwards and forwards through only 90°. We shall see the application of this valve in a description of a water-pressure engine erected by M. Brendrel, in the sequel.

§ 184. *The Slide-piston Valve.*—The pistons are generally made of slips of leather, placed one above the other, and closely packed

Fig. 300.

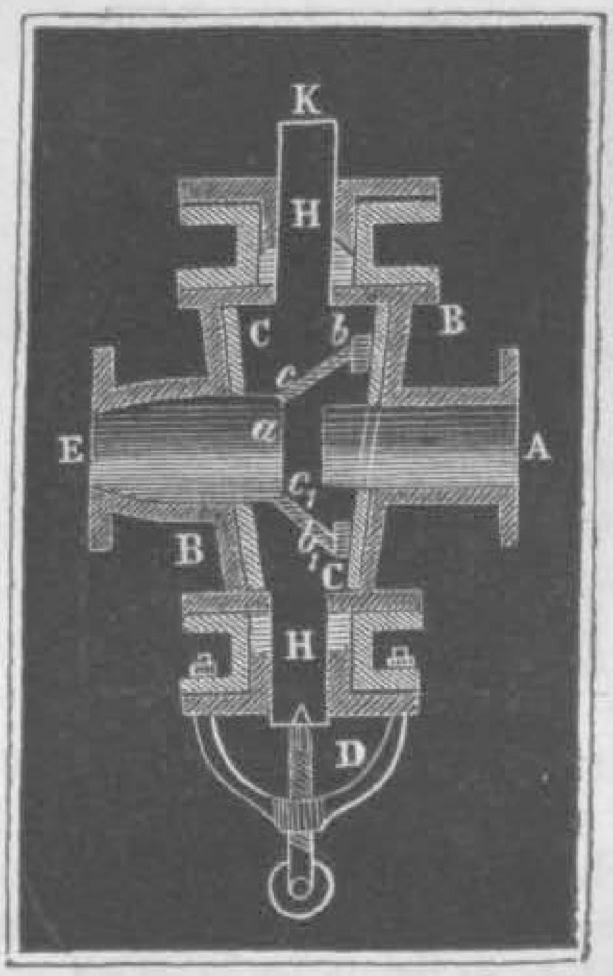
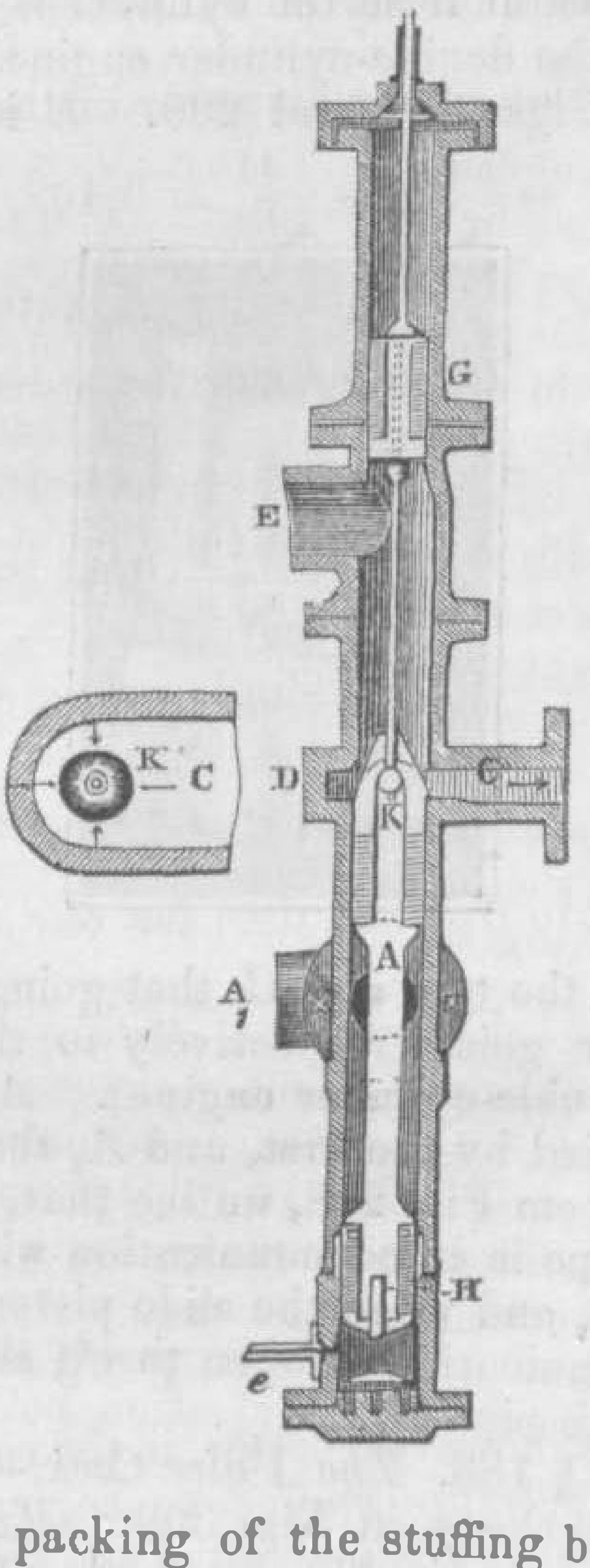


Fig. 301.



together, as we have mentioned for the packing of the stuffing box in § 181. The engine at Huelgoat, was originally made with cylindrical slide valves of gun metal. These lasted, without repair, for seven years; but in 1839, the valves having worn loose, a depth of 5 inches, consisting of 24 discs, or rings of leather pressed together and accurately turned down, was substituted. Reichenbach made the cylinder valves of *tin*, and the engines in Bavaria, in most recent times, have had the valves made by a combination of leather and tin rings.

At the end of the stroke of the working piston, the valve piston *AK* (Fig. 301) rises, gradually shutting off the water from the cylinder, but in gradually checking the flow of water in the course *EC*, the piston is pressed on one side, and this gives rise to a very rapid wear. To prevent this, the end of the pipe *CD* communicating with the working cylinder is carried quite round the valve cylinder, so that it incloses it, and the water then presses equally on every side of the piston, as it moves up and down. The packing suffers by this arrangement, as it has room to expand at this point, and has to be compressed as it passes into the cylinder above or below it. On this account the supply of water to the cylinder is carried through a series of openings, as shown in the horizontal section in Fig. 302. The objection to this arrangement is, that it increases the hydraulic resistances. The *form* of the valve piston *K* is of great importance. The communication between *C* and *E* must not be suddenly opened or shut, so that the column of water, in motion, may not be suddenly brought to rest; for this acts violently on the engine, on the same principle as is more fully developed in the so-called *hydraulic ram.* The gradual opening of the communication may be managed by giving the piston a particular form. We shall hereafter show how a slow motion of the valve piston is effected, and in the mean time point out, that, by giving a conical shape to the *head*, or that part of the piston which begins the closing of the *ports*, a ring-formed opening is made between *C* and *E*, which is gradually diminished as the piston ascends, until it is finally closed. Besides this arrangement, the top of the slide piston is perforated by slits that gradually diminish, but leave a narrow communication between *C* and *E*, even when the ring-formed opening above mentioned is quite closed, so that the passage is not perfectly closed until the slide-piston stroke is completed. This system of coning out the top, and perforating the upper part of the piston proper, is applied in the Clausthal water-pressure engine.

Fig. 302.

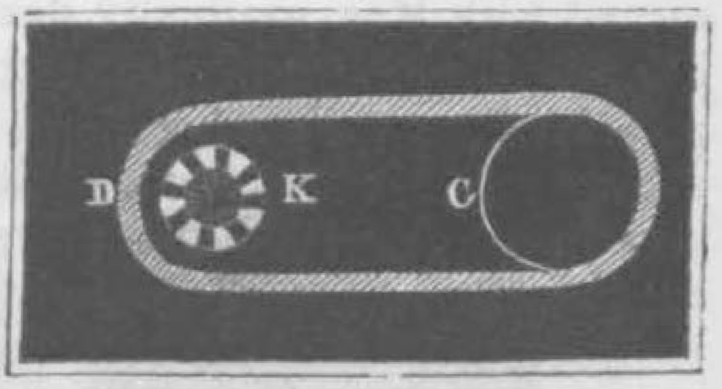
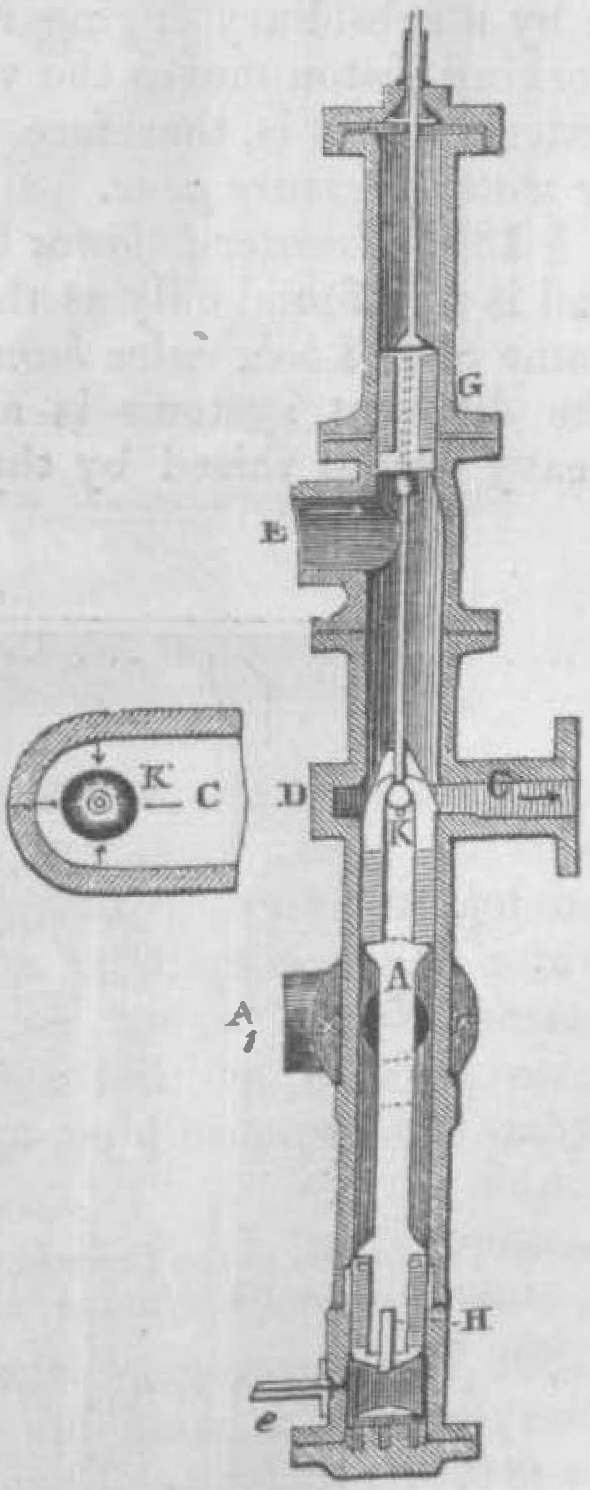


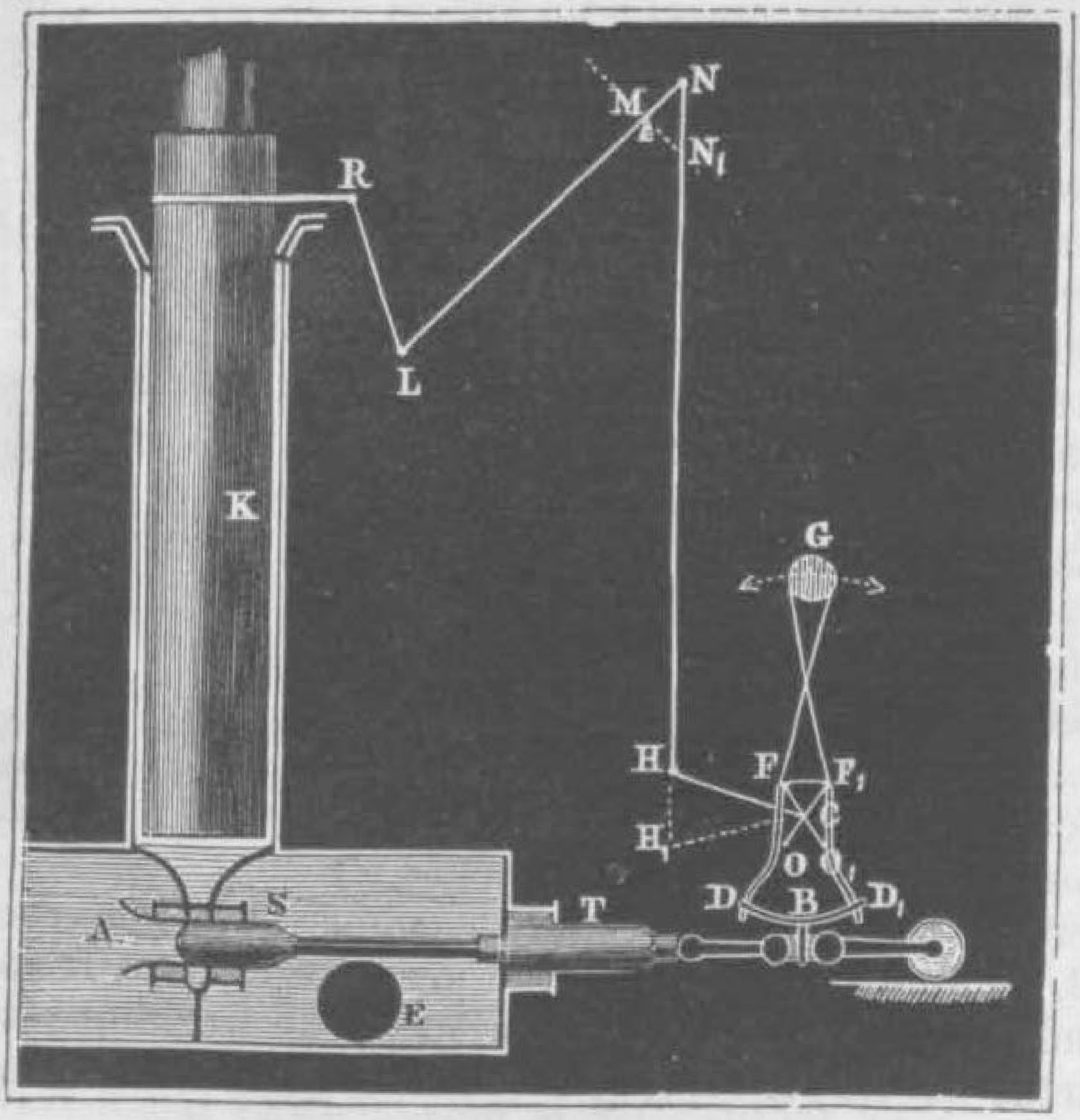
Fig. 303.



§ 185. *The Valve Gear.*—The gear for moving the valves of water-pressure engines is generally complicated, more so, for instance, than in the steam engine, because water is practically an incompressible fluid, exerting no pressure when cut off from the pressure column. When the piston *K*, Fig. 303, in ascending, cuts off the pressure column from the working cylinder *C*, then either the motion of the working piston ceases, or, in virtue of its *vis viva*, it moves away from the water in the cylinder, as this has no expansive capability. But this formation of a vacuum under the piston must be carefully avoided, and, therefore, the valve piston should begin to rise, while the main piston stroke is still unfinished, and thus the *vis viva* of all the parts connected with it is gradually destroyed by the gradual cutting off of the pressure column. But although the stroke of the piston is completed as the slide valve closes the communication, the motion of the slides must not stop here. The water in the working cylinder must now be discharged. The valve must rise somewhat higher, in order to open the orifice of discharge. Hence it is not *possible* to work the valve gear *directly* from the moving parts of the engine, for then the motion of both would cease simultaneously. Intermediate *gear* must be introduced, by which the motion of the valve piston is continued after the working piston has come to rest. This gear may be worked either by weights, raised by the piston in its ascent, and let fall at a particular part of the course, or by springs, bent during the motion of the piston, and disengaged at the end of the stroke, or by a subsidiary engine regulated by the main engine, but whose working piston moves the valves of the main engine. The gear of water engines is, therefore, either *counter-balance gear*, *spring gear*, or *water-pressure gear*.

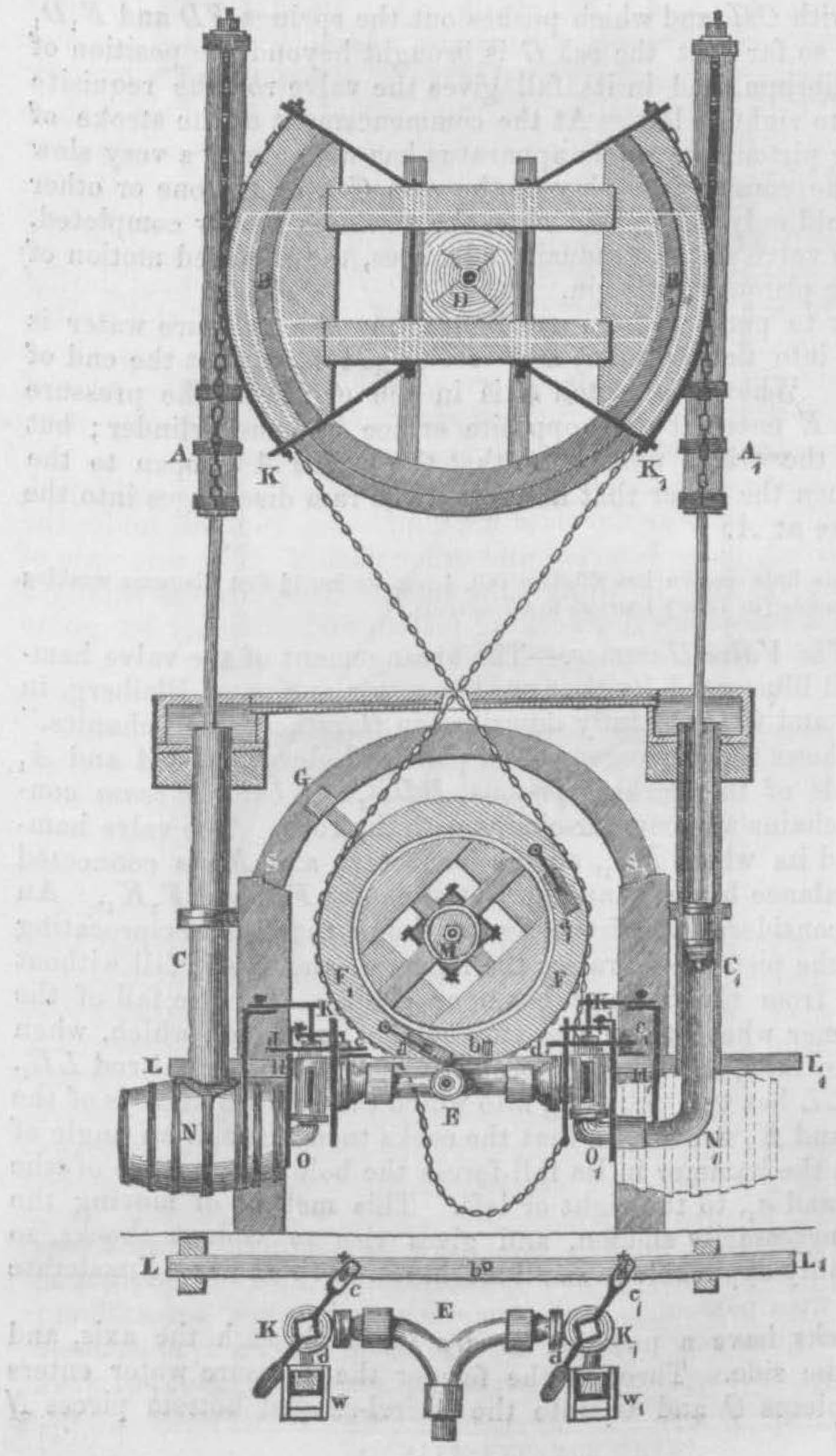
§ 186. *Counter-balance Gear.*—This gear was the first employed, and is now found only as the older water-pressure engines, under the name of *fall bob*, *valve hammer*, and other names. The principle of the different systems is always the same. They are essentially a heavy weight raised by the working piston, and suddenly let go to

Fig. 304.



work the cocks, or valves, by means of linked levers. We shall here describe only two of these arrangements. The small engine in the Pfingstwiese mine, near Ems, has gear connected with a pendulum or fall bob, moving two pistons S and T, lying horizontally under the working cylinder K, Fig. 304. The pendulum swings on an

Fig. 305.



axis C, and consists of a heavy bob G, and two fork-like springs FD and F_1D_1, carrying a cross head DBD_1, having a projecting piece B, in the centre, passing between two small rollers on the valve rod. The bob is raised so as to exceed the summit of its arc, by means of link work $CHNMLR$, connected with the *ram* of the engine at R. Motion is not communicated from the axis of the pendulum C, but by means of an arm CO, on a separate axis, and forming a single bent lever with CH, and which pushes out the springs FD and F_1D_1 alternately, so far that the *bob* G is brought beyond the position of stable equilibrium, and in its fall gives the valve rod the requisite extra push to right or left. At the commencement of the stroke of the working piston, the whole apparatus has necessarily a very slow motion. The coming into play of the arm CO, on the one or other spring, should only take place when the stroke is nearly completed, that, as the valve piston gradually advances, the retarded motion of the working piston may begin.

It is easy to perceive from our figure, how the pressure water is introduced into the cylinder, and discharged from it at the end of the stroke. When the piston S is in the orifice A, the pressure water from E enters by the opposite orifice into the cylinder; but if S be in the orifice next E, so that the orifice A is open to the cylinder, then the water that has raised the ram discharges into the waste-course at A.

Remark. This little engine has 60 feet fall, 4 feet stroke, 1½ foot diameter working cylinder, and made (in 1839) 1 stroke in 65 seconds.

§ 187. *The Valve Hammer.*—The arrangement of the valve hammer, is well illustrated by that on the water engine at Bleiberg, in Karinthia, and which is fully described in *Gerstner's* "Mechanics." Fig. 305 shows this arrangement in plan and elevation. A and A_1 are the rods of the working pistons, BDB_1 is a *balance beam* connected by chains and counter-chains with the rods. The valve hammer G, and its wheel FF_1, on the horizontal axis M, is connected with the balance beam by another set of chains FK and F_1K_1. An attentive consideration of the figure shows that the reciprocating motion of the piston rods raises the hammer, and lets it fall without hindrance from the balance beam or chains. On the fall of the valve-hammer wheel, there are two catches, a and a_1, which, when the hammer falls, catch upon a projection on the horizontal rod LL_1. This rod LL has two nobs c c_1, into which the handles or keys of the cocks, K and K_1 are set, so that the cocks turn through an angle of 90°, when the hammer in its fall forces the bolt b, by means of the catches a and a_1, to the right or left. This method of moving the cocks is necessarily sudden, and gives rise to violent shocks, so that it is only applicable to small machines, or those having moderate falls.

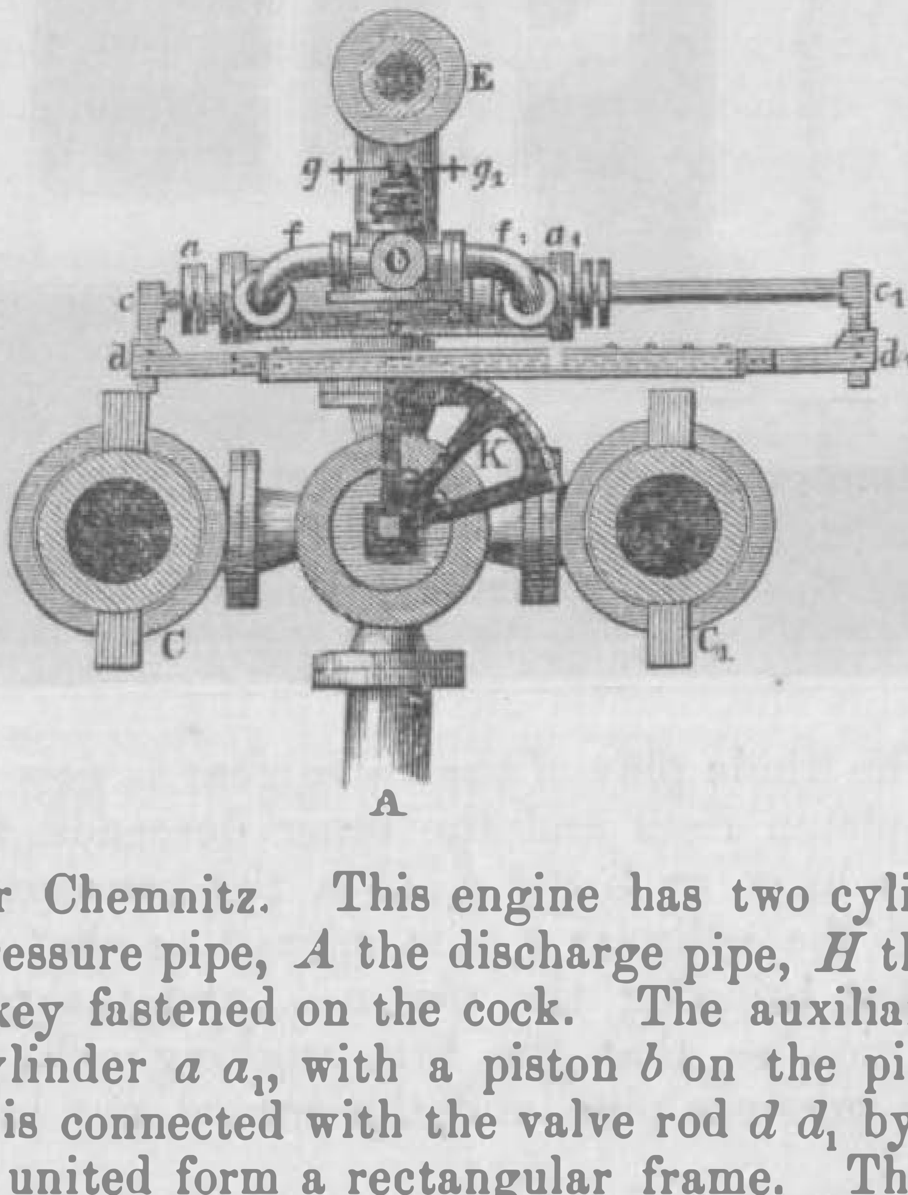
The cocks have a passage, or *are bored* through the axis, and through the side. Through the former the pressure water enters by knee pieces O and O_1 into the barrel-shaped bottom pieces N

and N_1 at the bottom of the working cylindersl; and through the side passage, the pressure water is brought to the cock from the cylinder. In order that only as much water may be used as is necessary to fill the space passed through by the working piston, the discharge is made to take place *under* water into special reservoirs W and W_1.

Remark. The engine now described has a fall of 260 feet, stroke 6½ feet, cylinder 7 inches diameter, 8 strokes per minute. It is in many respects an imperfect engine; but it is economically adapted to its position. We have not only to consider mechanical *perfection* in the construction of engines in general, but we have to weigh well the circumstances in which the engine is to work, the facilities for repair in the particular locality, and the relative supply and demand for the water power.

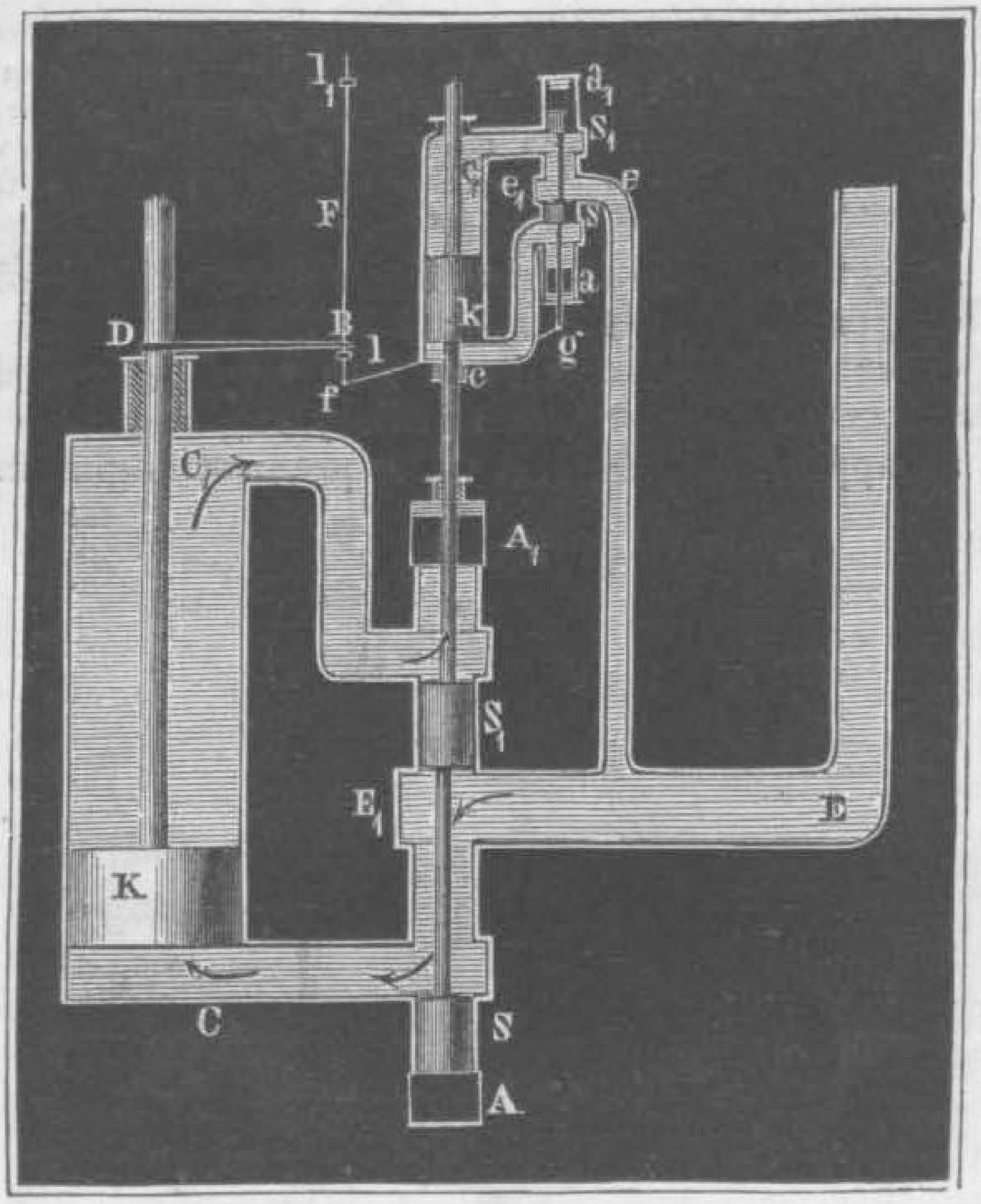
§ 188. *Auxiliary Water-Engine Valve Gear.*—No application of spring-valve gear has been made; but the method of using an auxiliary water engine is now come into very general use. The general arrangement of such an auxiliary engine gear is shown in Fig. 306, as applied to the great water-pressure engine in the *Leopold* shaft, near Chemnitz.

Fig. 306.



This engine has two cylinders, C and C_1; E is the pressure pipe, A the discharge pipe, H the main cock, K a quadrant key fastened on the cock. The auxiliary engine has a horizontal cylinder $a\ a_1$, with a piston b on the piston rod $c\ c_1$. The piston rod is connected with the valve rod $d\ d_1$ by cross pieces, so that the two united form a rectangular frame. The valve rod is connected with the quadrant by two chains, so that the reciprocating motion of the piston b communicates a rotary motion of 90° to the cock. The auxiliary engine is worked by means of the cock $h\ h_1$ lying horizontally, with two *bdres*, or passages, as in the case of the main cock H. The little pipe e communicating with the pressure pipe E, takes the pressure water to the cock $h\ h_1$, from which it passes through the pipes $f\ f_1$ to one side or the other of the piston b, so that it is moved backwards and forwards, the water used in each alternate stroke being discharged by the other passage in the cock, and thence by a pipe from h. The small cock $h\ h_1$ is turned by the double-handled key $g\ g_1$ connected by slender chains to a double-armed lever parallel to it, and which is on the same axis as the balance beam to which the piston rods of the two cylinders are

Fig. 307.

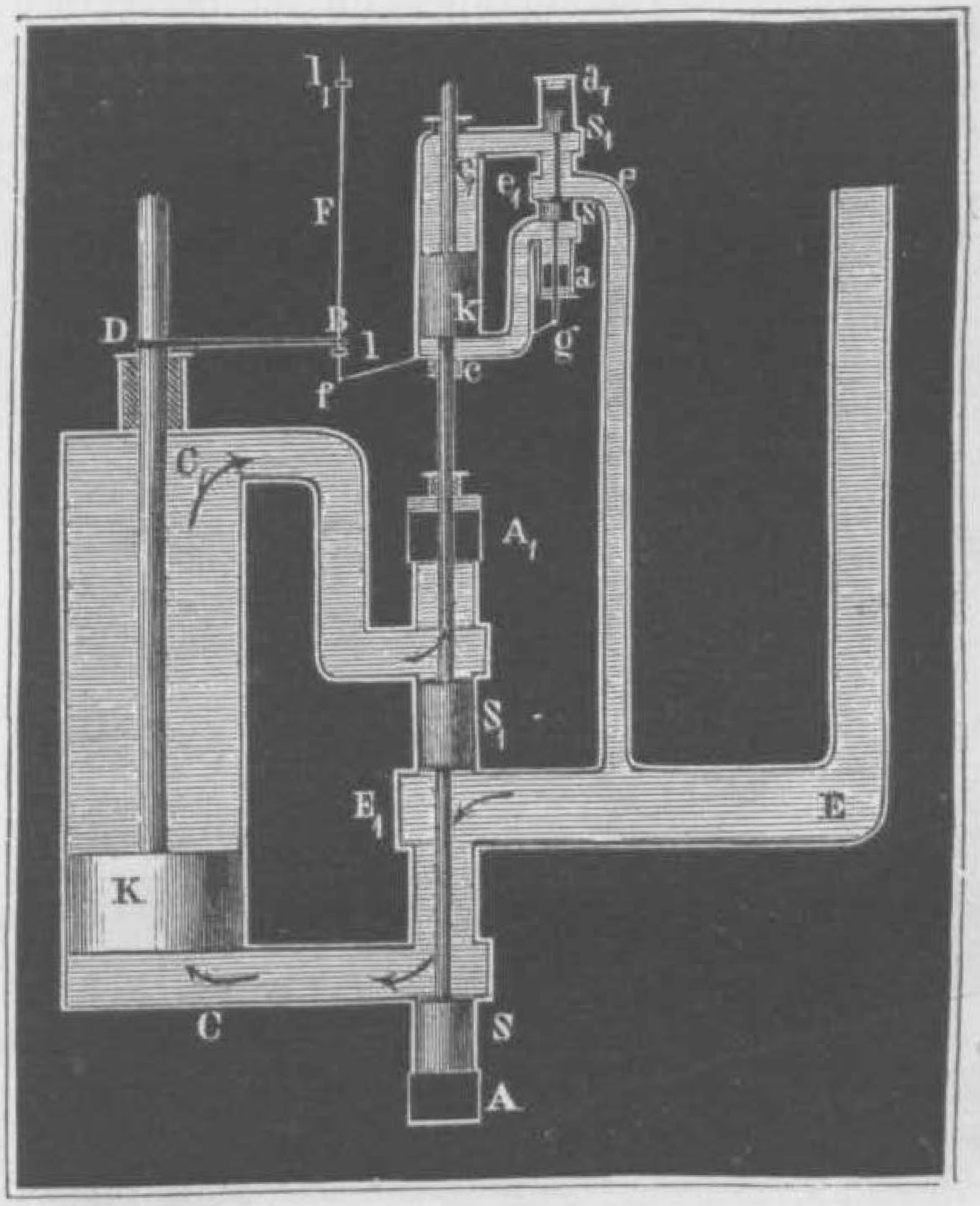


attached. The whole play of the valve gear is now evident. While the working piston rises and the other descends, the cock $h\ h_1$ is turned by the lever or key $g\ g_1$, thus the communication between the water and the cylinder $a\ a_1$ is opened or shut, and thus power is obtained for bringing the piston b, and the cock H into the opposite position, so that the first working cylinder is now shut off from the pressure pipe, and the second put in communication with it.

Remark. The engine in the Leopold shaft has 710 feet fall (Austrian measure), 8 feet stroke, 11 inch diameter of cylinder; each piston makes 3 strokes per minute.

§ 189. The working of the valves (Fig. 308) by means of an auxiliary engine, is well illustrated by that of the double-acting, water-pressure engine at Ebensee, in Salzburg; the auxiliary engine being, in this case, an exact model of the working engine. CC_1 is the cylinder of the principal engine, and cc_1 that of the auxiliary. K is the piston of the one, and k that of the other cylinder. S and S_1 are the valve pistons of the working, and s and s_1 those of the auxiliary engine. EE_1 (Fig. 308) is the main pressure pipe, and $e\ e_1$ the pipe communicating with the auxiliary engine. Lastly, A and A_1 are the orifices of discharge of the main, and $a\ a_1$ those of the auxiliary engine. Thus the one engine is an exact counterpart

Fig. 308.



of the other, the dimensions being, however, very different in the two. The valve gear of the auxiliary engine consists in the cantilever BD attached to the main piston rod at D—of the valve piston rod $g\ s$, connected by the link $f\ g$ to the rod $l\ l_1$, on which there are two studs placed, so that the lever DB catches upon them a little before the end of the up and down strokes, respectively, of the main piston, and thus the valve piston is moved. It is easy to trace how this motion admits the pressure water alternately above and below the piston K, so as to raise or depress the valve pistons $k\ S_1\ S$, giving the required alternation of admission of the pressure water above and below the main piston K.

Remark. The engine at Ebensee has a fall of only 36 feet, a stroke of 17 inches, and a cylinder of 9½ inches diameter. It makes 6 strokes per minute, and moves two double acting pumps.

§ 190. *The Valve Cylinder.*—In the larger engines of recent date, the valve pistons of the main cylinder are inclosed in the same pipe, or cylinder, as the piston of the auxiliary engine; and in some engines the counter-pressure valve, or piston balancing the pressure on the valve, is the working piston of the auxiliary engine, and thus great simplicity of construction is attained.

Fig. 309 shows a simple arrangement adopted in two engines in the Freiberg mining district. *S* is the main piston valve, and *G* the counter-pressure piston, *C* an intermediate pipe communicating with the main cylinder, *E* the entrance for the pressure water, and *A* the orifice of discharge for the water used, *e* is the communication with the valve of the auxiliary engine, which in this case is a cock. The piston *G* is larger than *S*, and, therefore, the valve

Fig. 309.

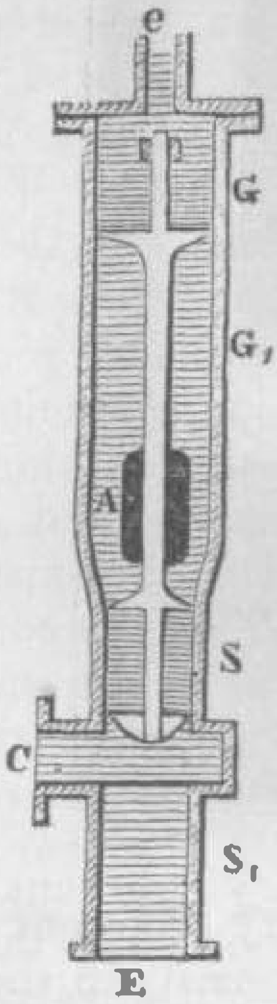
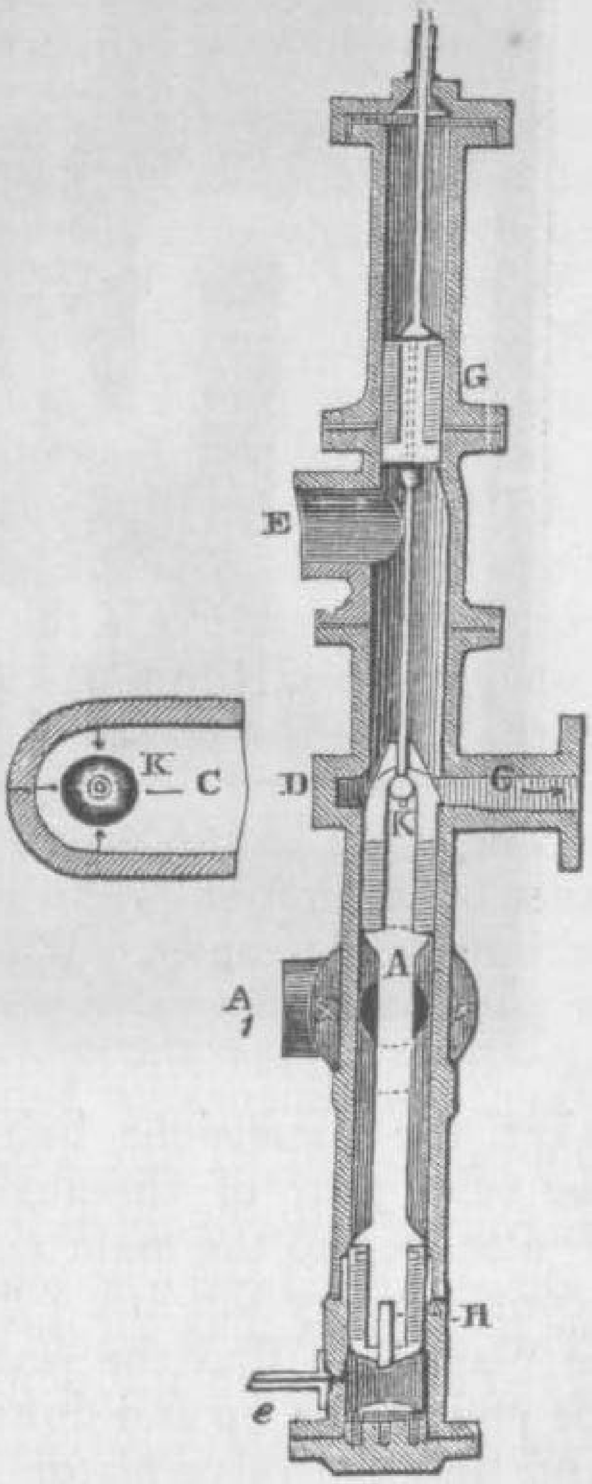


Fig. 310.



apparatus *S G* descends, when the pressure is admitted from above at *e*, and ascends when the pressure water is cut off at *e*, and the pressure acts underneath. For each stroke there is a consumption of a certain quantity of water for the valves, which is lost for useful effect. This amounts to the contents of the space passed through in the up or down stroke. In the construction, now under consideration, this is not so little as in some others, for the piston *G* must have, at least, one and a half times the area of the piston *S*, the sectional area of which is the same, or even greater than that of the pressure pipe.

The system of valves shown in Fig. 310, is that of the Clausthal engine, and here the waste of water is less than in the last mentioned system. For there are three pistons; namely: the main valve piston AK, the counter piston G, and the auxiliary piston H; the latter being somewhat less in area than the former. The water is brought into the valve cylinder by the pipe e, and the reverse motion of the piston is effected by a small cock through which the water enters before coming into e, and through which, also, when the revolution is completed, it is let off. The cock is moved by link work, by means of a *tap* on the main piston rod.

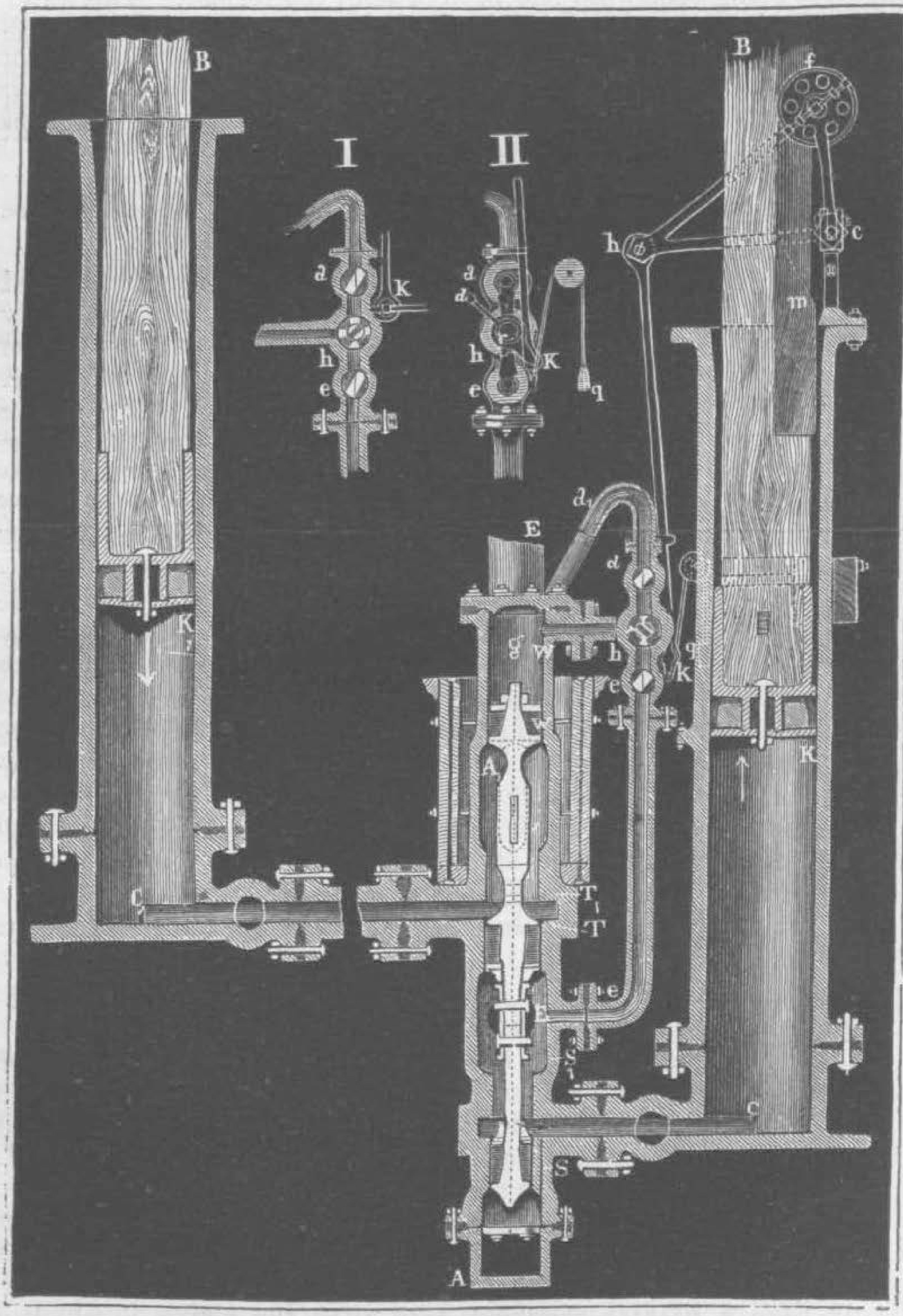
Remark. The engines at Clausthal have 612 feet fall, diameter of cylinder 16½ inches, stroke 6 feet, and make 4 strokes per minute.

§ 191. *Saxon Water-Pressure Engine.*—The arrangement and motions of a double cylinder water-pressure engine may be clearly understood by a study of a sectional view of the engine, erected in the Alte Mordgrube, near Freyberg, in Saxony, delineated in Fig. 311. CK and C_1K_1 are the two working cylinders, K and K_1 being the working pistons, S and T are the two valve pistons, W is the auxiliary piston, and S_1 T_1 and W_1 are the points in the valve cylinder ATW_1 at which the pistons are for the return stroke of the working pistons. E is the entrance of the pressure pipe E_1E into the valve cylinder, CS is the intermediate pipe communicating with the one, and C_1T the pipe communicating with the other working cylinder. A is the orifice of discharge of the one, and A_1 that of the other (this latter orifice is nearly covered by the piston rod in the drawing). The two piston rods BK and B_1K_1 are connected by a balance beam (not shown in the figure), so that as the one piston ascends the other descends. It is hence easy to perceive, that, for the lower position of the valve piston, here represented, the pressure water takes the course ES_1C, driving the piston K upwards, and that the piston K_1 is pushed downwards, the used water taking the course $C_1T_1A_1$ to the discharge orifice A_1.

The auxiliary valve consists of a four-way cock h (already described) shown at I. in the second position, and external elevation at II. This cock gives passage between the pipe e e_1 and the pressure pipe, and between g h and the valve cylinder.

It is evident that in the one position of h, the pressure water takes the course Ee_1 e h g W, and presses down the auxiliary piston W, whilst for the second position of h, the pressure water is shut off from W, and hence the ascent of the valve piston system STW, the return of the valve water through g h, and its discharge at a a_1, can take place. That the valve piston system may rise when the water is shut off from W, and may descend when it is let on, it is necessary that the piston T, pressed upwards by the pressure water, should have a greater sectional area than the piston S, which is pressed downwards by the pressure water; and also, the auxiliary piston must have sufficient area that the water pressure, or W and S together, may exceed the opposite pressure on T.

Fig. 311.



The valve gear of this machine is composed of a ratchet wheel r, a catch $r\,k$, a rod $k\,h$, and a bent lever $h\,c\,f$ with its friction wheel f, and the two wedge-formed pieces $m\,m_1$ set on each piston rod. The catch $r\,k$ is connected with the axis of the cock, and is held by a small balance weight q in its place on the ratchet wheel. When the piston K has reached nearly the end of its stroke, the wedge m (or m_1) passes under the friction wheel, and turns the lever $f\,c\,h$ to a certain extent, so that the rod $h\,k$ is drawn up, and the wheel and cock h are turned through a quadrant. As the working piston makes its return stroke, the lever falls back again,tand the catch slides back over the next tooth of the ratchet, and is ready at about the end of this return stroke to push round the ratchet, &c.

Remark. The water-pressure engine in the Alte Mordgrube, has a fall of 356 feet, a stroke of 8 feet, 18 inches diameter of cylinder, and makes 4 double strokes per minute.

§ 192. *Huelgoat Water-pressure Engine.*—One of the largest and most perfect water-pressure engines hitherto erected, is that at Huelgoat, in Brittany. It is a single-cylinder, single-acting engine. Fig. 312 represents the essential parts of this engine, and its valve gear. CC_1 is the working cylinder,.KK_1 the working piston, and BB_1 the piston rod working through a stuffing box at B. In the Saxon engine, the piston is packed by a single sheet of leather; but in this engine, the rim of the piston is packed, and there is also a sheet of leather, held in its place by a ring. The valve cylinder ASG is united to the working cylinder by the pipe DD_1, into which the pressure pipe opens at E, and the discharge pipe at A. To the valve piston S, a counter-balance piston T of greater diameter is connected by the rod ST. This system will, therefore, be forced upwards by the pressure water, if a third force be not brought into play. This third force is, however, produced by bringing the pressure water above T, through the pipe $e_1\,e\,f$, and in order to use only a small quantity of water for working this valve system, a hollow cylinder GH is placed on T, passing through a stuffing box at H, and, therefore, exposing only an annular area to the pressure of the water.

The alternate admission and exclusion of the pressure water of the hollow space $g\,g$, is effected by an auxiliary valve system, resembling the main valve system in every respect; consisting like it of a valve pistonts, a counter-balance piston t, which is a solid piston passing through a stuffing box at h. For the position $s\,t\,h$, shown in our figure, the pressure water has free circulation through $e\,f$ to g; but if $s\,t\,h$ be raised, so as to bring s above f, this passage is stopped, and the valve water, in the hollow space $g\,g$, escapes through $a\,a_1$, when ST goes up. Lastly, to derive the motion of the auxiliary valve-piston system from the engine itself, there is let into the working piston KK_1 an upright rod with a feather edge attached to the side. This feather has a series of holes drilled in it, into which catches can be put as X_1, X_2, at the required distance apart. The link $b\,h$ is connected to two levers, centred at c and o, and connected

Fig. 312.

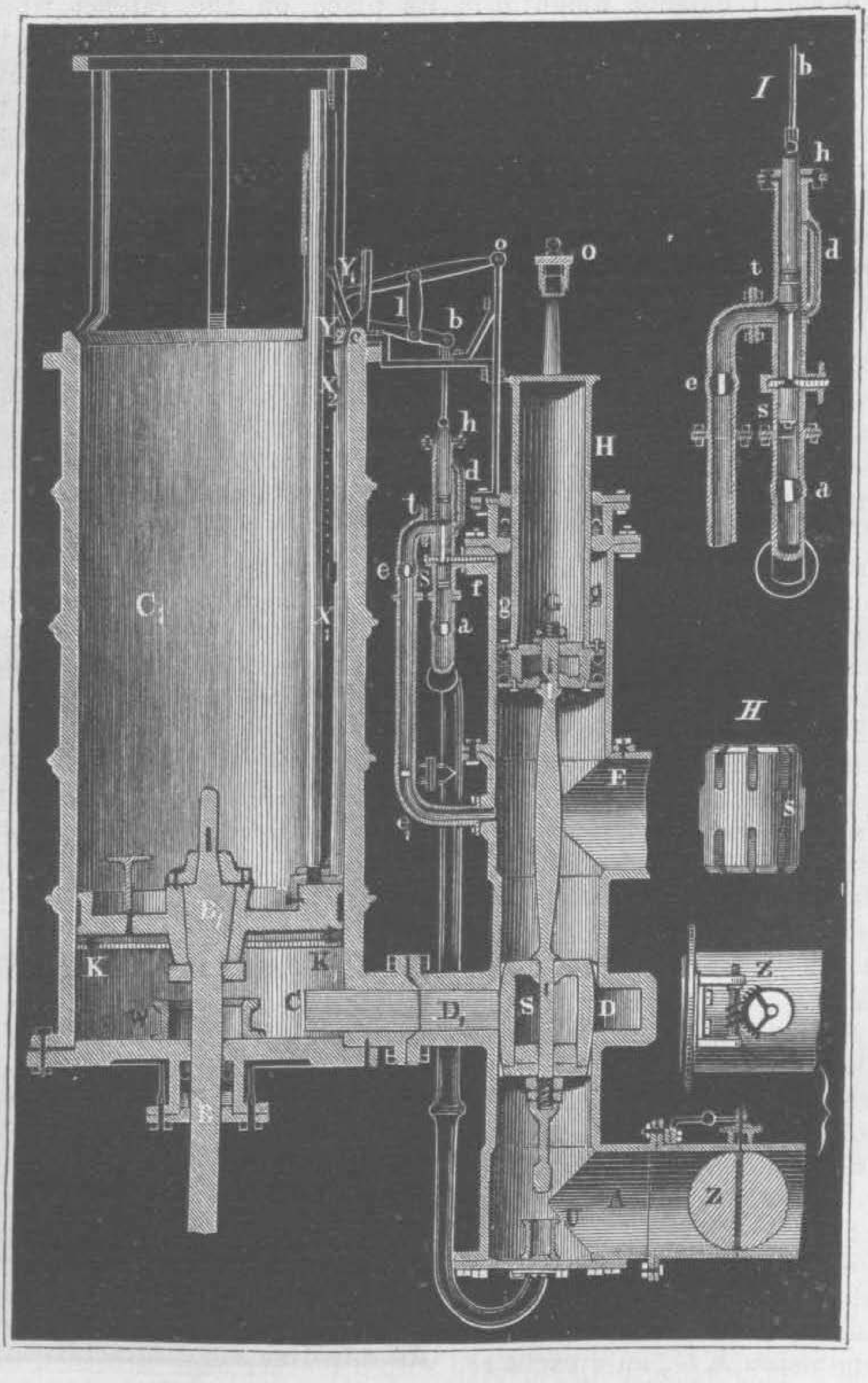
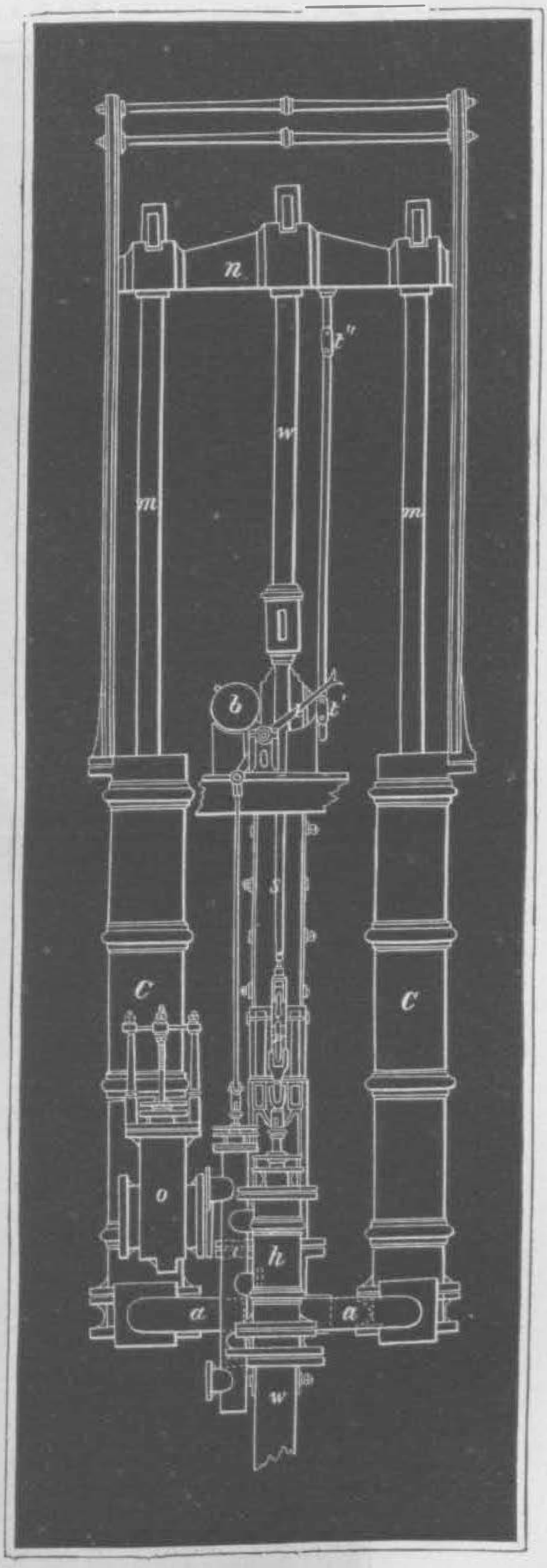


Fig. 313.

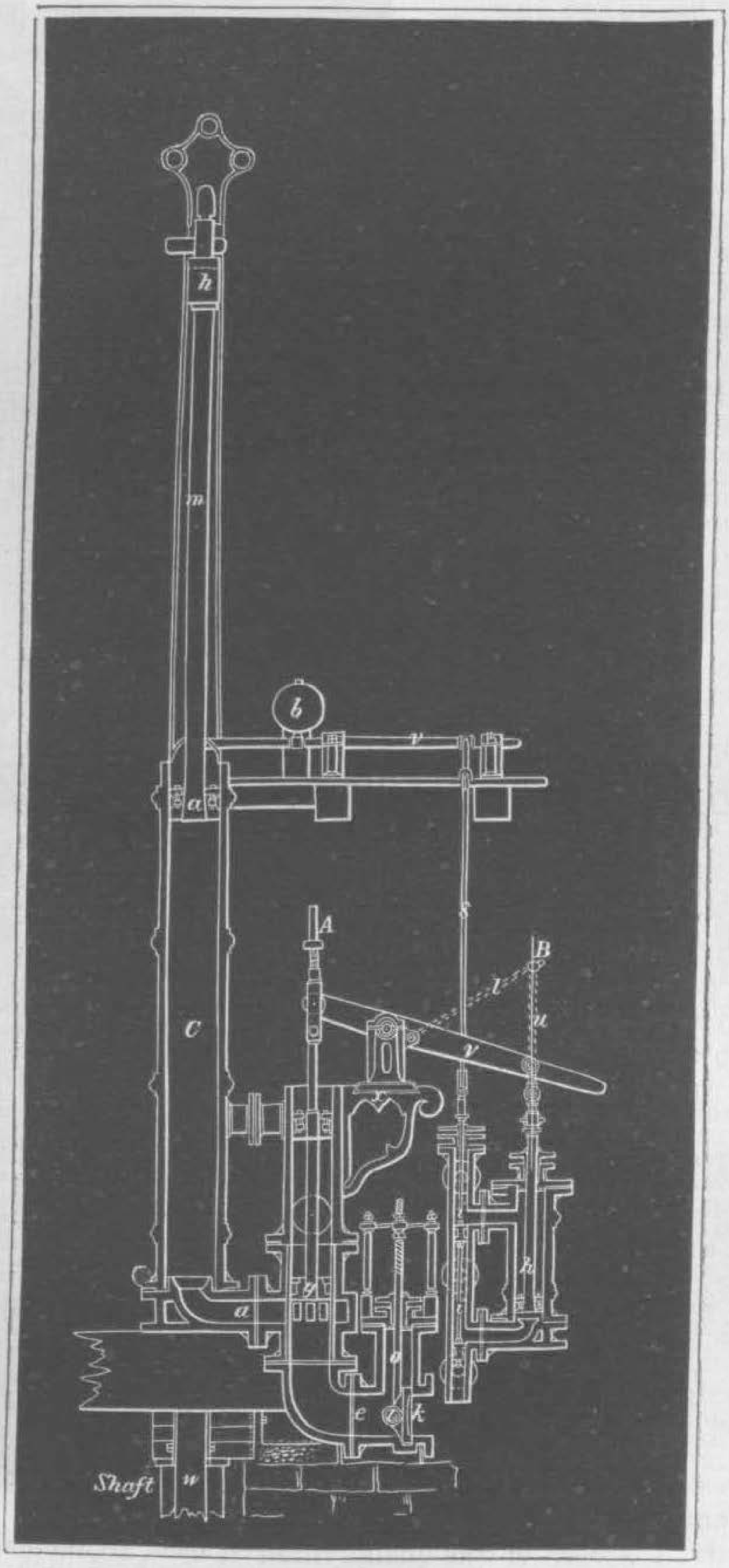


by the link l. The end of the one lever has an arc, on which there are two projections or catches Y_1, Y_2. As the up stroke of the piston comes to an end, the catch X_1 strikes on Y_1, and thus $s\ t\ h$ is moved to its upper position, and at nearly the end of the down stroke of the piston, the catch X_2 strikes Y_2, and the valve system $s\ t\ h$ descends to a lower position. It is now easy to perceive how the alternate positions of ST, necessary for the reciprocating motion of the piston, KK_1, are produced.

§ 193. The following are the details of the construction of Mr. Darlington's water-pressure engine. The first engine erected in England with cylinder or piston valves, was that put up in the Alport mines, Derbyshire, in the year 1842. This was a single cylinder engine. Its success was complete, and others were erected on the same plan. But in 1845, a *combined cylinder engine* was designed, and erected by the same engineer, which is found practically to have several advantages for such large supplies of water as that consumed by the pumping engine, of which are subjoined accurate reductions of the working drawings.

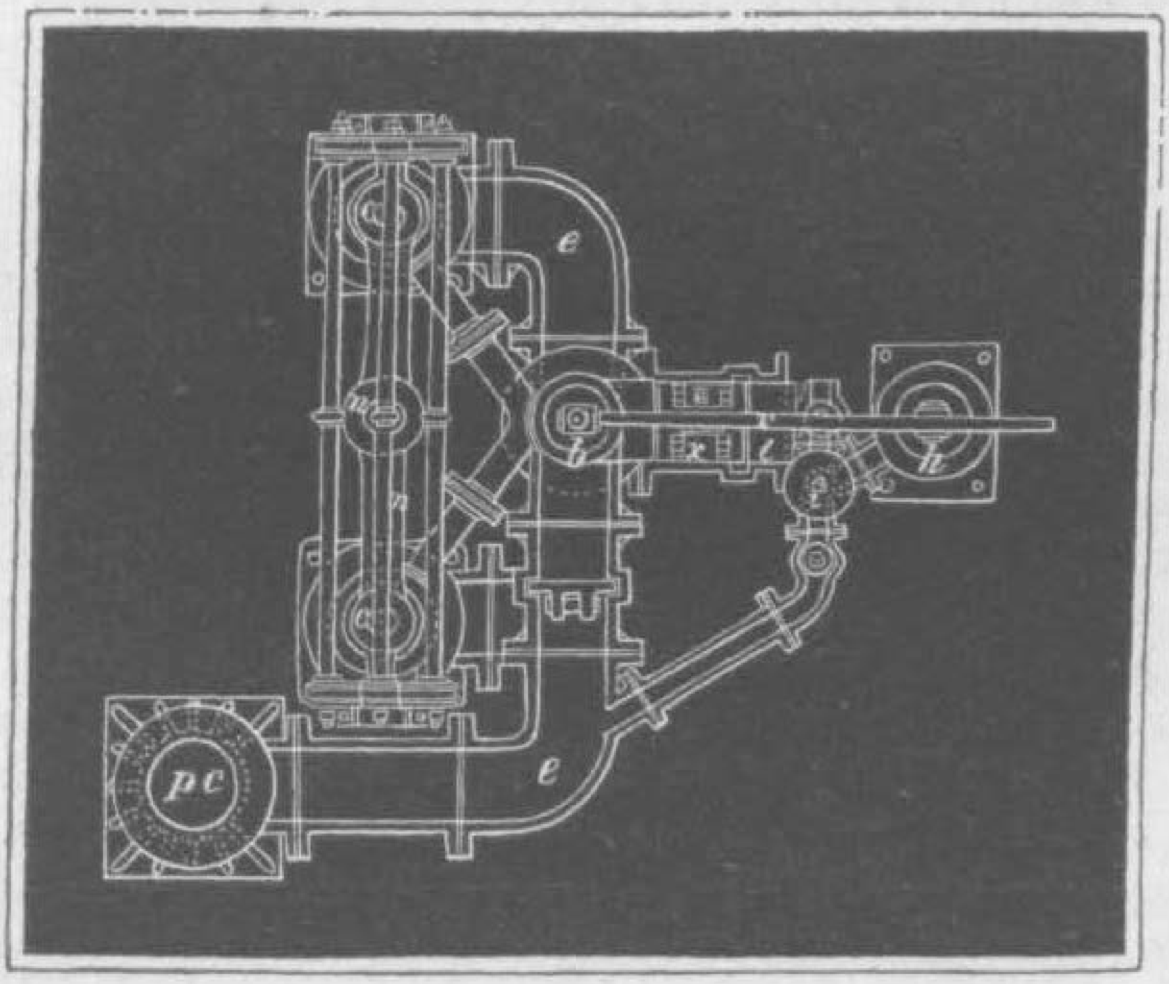
Fig. 313 is a front elevation of the combined cylinder engine. Fig. 314 is a sectional view, and Fig. 315 is a general plan. PC, is the bottom of the pressure column, 130 feet high, and 24 inches internal diameter, CC are the combined cylinders, each 24 inches diameter, open at top, with hemp-packed pistons a (Fig. 314), and piston rods

Fig. 314.



m, combined by a cross head *n*, working between guides in a strong frame. The admission throttle valve is a sluice valve, shown at *o*, Fig. 313, and between the letters *b* and *e* and Fig. 315. The main

Fig. 315.



or working valve, is a piston *g*, 18 inches in diameter, Fig. 314, with its counter or *equilibrium piston* above. The orifice for the admission of the pressure water is between the two pistons. The intermediate pipe *a* is a flat pipe, into which numerous apertures lead from the valve cylinder (seen immediately under *g*, Fig. 314). The valve piston is in the position for discharging the water from the cylinders through the pipe *e*, Fig. 314, by the sluice valve *k*.

The valve gear is worked by an auxiliary engine *h*, by means of the lever *v*. The auxiliary engine valves, are piston valves in the valve cylinder *i*, Figs. 314 and 315, communicating with the pressure pipes by a small pipe, provided with cocks, as shown in Fig. 315. The motion of the auxiliary engine valves is effected by a pair of tappets t', t'', set on a vertical rod attached to the cross head *n*. These tappets move the fall bob *b*, by means of the canti-lever *t*, Fig. 313, the other end of the lever being linked to the rod *s*, Fig. 314, which again is linked to the auxiliary piston valve rod.

The play of the machine is now manifest. It is in every respect analogous to the Harz and Huelgoat engines, described above. The average speed of the engine is 140 feet per minute, or 7 double strokes per minute. This requires a velocity of something less than $2\frac{1}{2}$ feet per second of the water in the pressure pipesi; and as all the valve apertures are large, the hydraulic resistances must be very small. The engine is direct acting, drawing water from a depth of 135 feet, by means of the spear *w*, *w*, Figs. 313 and 314. The "box," or *bucket* of the pump, is 28 inches in diameter, so that the discharge is 266 gallons per stroke, or, when working full speed, 1862 gallons per minute. The mechanical effect due to the fall and quantity of water consumed is nearly 140 horse power. The mechanical effect involved in the discharge of the last-named quantity of water is nearly 74 horse power, so that, supposing the efficiency of the engine and pumps to be on a par with each other, the efficiency of the two being (§ 203), $\eta_1 = 71,15$, the efficiency of the engine alone $\eta = \frac{1+\eta_1}{2} = \frac{1+,71}{2} = ,85$, or, in the language of Cornish engineers, 85 per cent. is the duty of the engine.

The cost of maintenance, grease, &c., of the engine, is only £40 per annum. In every particular, it redounds to the credit of Mr. Darlington's skill as an hydraulic engineer.

Balance.—For regulating the motion of water-pressure engines, several auxiliary arrangements are necessary, which we shall explain hereafter. The ascent and descent of the working piston is regulated by an arrangement called a *balance*, or *counter-balance*, which aids the motion of the piston in the one direction, and retards it in the other, so that the working of the machine goes on with a nearly uniform velocity. In the double-cylinder engine, the balance is effected by a simple beam, connecting the two cylinders. In the double-acting, single-cylinder engine, a fly wheel is necessary, and in the single-acting, single-cylinder engine, a counterbalance weight, either of a solid body, or of a column of water, an *hydraulic balance*, is employed. On the subject of "Regulators of Motion," we, for the present, make only a few general remarks. The *mechanical balance* consists of a beam with a weight at one end, and having the other end attached to the piston of the engine, so that the weight assists during the working stroke of the piston to counterbalance the piston and rods; and during the down stroke, or discharge of the used water, prevents the too rapid return of the piston and rods; the adjustment being such as to allow of the discharge stroke being made in about half the time that the working stroke occupies. The hydraulic balance consists of a second column of pipes, which ascends from the discharge orifice to such a height, that the water it contains counterbalances the extra weight of the piston and rods. The machine at Huelgoat, and also those at Clausthal, have hydraulic balances.

There is evidently neither loss nor gain of effect by the use of a counter-balance, save by the *prejudicial* resistances they give rise to in their motion. The balance beam has the advantage that its *balance weight* can be varied as required; and the hydraulic balance has the advantage of *simplicity*, when other circumstances do not interfere with its application.

§ 194. *Throttle Valves.*—The cocks or throttle valves of water-pressure engines are important organs, their function being to regulate the supply of water to, and its discharge from, the engine—that is *the speed* of the engine. These valves must have a prejudicial effect on the efficiency of the engine, and yet, they are a necessary evil. In order to regulate the ascent and descent of the working piston and of the valve pistons, there are necessary, four cocks or valves—one in the pressure pipe, and one in the discharge pipe (as Z, Fig. 312); also a cock in the pipe leading the valve water above the auxiliary piston, and another similar in the pipe which discharges the water used in the valves, as e and a in Fig. 312.

To get the highest efficiency from a water-pressure engine, its work should be such as to render any contraction of the pressure pipes, by a throttle valve, unnecessary for its uniform motion. If, however, the useful effect of the engine is greater than is required by the work to be done, the excess must be taken away by checking the supply by means of the throttle valve, or by shortening the stroke of the engine.

If it be an object to save water, the latter means is the best when possible, because the efficiency of the machine is not thereby interfered with.

A change in the length of stroke of the piston is easily effected by altering the position of the catches on the rod X_1, X_2, Fig. 312. The nearer X_1 and X_2 are brought together, the earlier the reversing of the stroke ensues; and, therefore, the shorter is the stroke of the working piston.

§ 195. *Mechanical Effect of Water-pressure Engines.*—In computing the effect of water-pressure engines, we shall make use of the following symbols:—

F = the area of the working piston.
F_1 = the area of the pressure pipes.
d = the diameter of the working piston.
d_1 = the diameter of the pressure pipe.
d_2 = the diameter of the discharge pipe.
h = the fall from surface of reservoir to surface of water in discharge channel.
h_1 = the vertical distance from the surface of reservoir to the surface of piston at half stroke.
h_2 = the distance from surface of discharged water to the piston at half stroke.
s = the stroke of the piston.
l_1 = length of pressure pipe.
l_2 = length of discharge pipe.
v = mean velocity of piston.
v_1 = mean velocity of water in pressure pipe.
v_2 = mean velocity of water in discharge pipe.

We shall assume the engine to be single acting, making:
n = the number of strokes per minute.
Q = the quantity of water used per second.

The mean pressure of the water on the piston surface F is $P_1 = F h_1 \gamma$, and, therefore, the mechanical effect produced *per stroke*, prejudicial resistances neglected, is $P_1 s = F s h_1 \gamma$, and per minute $n P s = n F s h_1 \gamma$, and, therefore, the mean effect per second, is:

$$L_1 = \frac{n}{60} P s = \frac{n}{60} F s h_1 \gamma, \text{ or as } \frac{n F s}{60} = Q,\ L_1 = Q h_1 \gamma,$$

In the return of the piston, the mean effective resistance is: $P_2 = F h_2 \gamma$, and, therefore, the mechanical effect consumed is: $P_2 s = F h_2 s \gamma$, and hence the loss of effect per second is: $L_2 = Q h_2 \gamma$, and, therefore, the effect available

$$L = L_1 - L_2 = Q (h_1 - h_2) \gamma = Q h \gamma,$$

as in many other hydraulic recipient machines.

This formula is evidently not changed should the working piston not fill up the cylinder, *i. e.*, supposing a plunger is used, round which there is a free space, or supposing the piston does not descend to touch the bottom of the cylinder. Nor would the circumstance of the discharge taking place *below* the mean position of the piston —that is, of h_2 being negative, or $h = h_1 + h_2$ alter the formula. F is the area of a section of the piston at right angles to its axis, or $F = \frac{\pi d^2}{4}$, and, therefore, the form of the piston can have no effect.

§ 196. *Friction of the Piston.*—Of the prejudicial resistances, the friction of the piston is a principal one. As there are no accurate experiments on this subject, we must content ourselves by estimating it from the pressure of the water, and a co-efficient of friction ascertained in the nearest possible analogous circumstances. If the packing be on the *hydrostatic* plan, the force with which each element e of the packing is pressed against the cylinder during the up stroke is $= e h_1 \gamma$, and during the down stroke it is $= f e h_2 \gamma$, and hence the friction $= f e h_1 \gamma$, and $f e h_2 \gamma$, respectively. The total friction will be the sum of the frictions of all the elements, or of the area of the whole packing. If the breadth of the packing be b, then $\pi d b$ is the area, and then the piston friction is $R_1 = f \pi d b h_1 \gamma$ for up stroke, and $R_2 = f \pi d b h_2 \gamma$ for down stroke.

It is convenient to express the various prejudicial resistances in terms of a column of water of the area of the piston, and whose height h_3 or h_4 is the head *lost* (in the present case) by the friction of the piston. Let us, therefore, put:

$R_1 = F h_3 \gamma$, and $R_2 = F h_4 \gamma$, or $F h_3 = f \pi d b h_1$, and $F h_4 = f \pi d b h_2$, or putting $\frac{\pi d^2}{4}$ for F, we have $\frac{d h_3}{4} = f b h_1$, and $\frac{d h_4}{4} = f b h_2$; and hence the loss of fall, corresponding to the friction of the piston $h_3 = 4 f \frac{b}{d} h_1$, and $h_4 = 4 f \frac{b}{d} h_2$.

If we deduct these heights, we get for the mean power during the up stroke:

$$P_1 = F (h_1 - h_3) \gamma = \left(1 - 4 f \frac{b}{d}\right) F h_1 \gamma,$$

and during the down stroke:

$$P_2 = F (h_2 + h_4) \gamma = \left(1 + 4 f \frac{b}{d}\right) F h_2 \gamma,$$

and hence the resultant mean effect:

$$L = \frac{n}{60}(P_1 - P_2)\,s = \frac{n}{60}\left((h_1 - h_2) - 4f\frac{b}{d}(h_1 + h_2)\right)F s \gamma$$
$$= \left(h - 4f\frac{b}{d}(h_1 + h_2)\right) Q \gamma.$$

If the rising pipe height $h_2 = 0$, or be very small, then we have more simply

$$L = \left(1 - 4f\frac{b}{d}\right) Q h \gamma.$$

We see from this that the loss of effect from friction of piston is so much the greater, the greater $\frac{h_1}{h}$ and $\frac{h_2}{h}$ are, that is, the greater the head, and the greater the counter-balance head.

To reduce this friction, the packing should not have unnecessary width. In existing machines $\frac{b}{d} = 0{,}1$ to $0{,}2$. The co-efficient of friction is to be taken as determined by Morin, $f = 0{,}25$. This being assumed, we see that $4f\frac{b}{d} = 0{,}1$ to $0{,}2$, or that the friction of the piston absorbs from 10 to 20 per cent. of the whole available power.

§ 197. *Hydraulic Prejudicial Resistances.*—Another source of loss of effect in water-pressure engines, is the friction of the water in the pressure and discharge pipes. According to the theory given in Vol. I. § 329, the pressure height or head corresponding to this loss, ζ being the co-efficient of friction, is

$h = \zeta \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$. This applied to the pressure pipe, becomes

$h_5 = \zeta \cdot \frac{l_1}{d_1} \cdot \frac{v_1^2}{2g}$, and applied to the discharge pipe it is

$h_6 = \zeta \cdot \frac{l_2}{d_2} \cdot \frac{v_2^2}{2g}$. But the quantity of water, is

$$\frac{\pi d_1^2}{4} \cdot v_1 = \frac{\pi d_2^2}{4} \cdot v^2 = \frac{\pi d^2}{4} v, \text{ therefore,}$$

$d_1^2 v_1 = d_2^2 v_2 = d^2 v$, or $v_1 = \left(\frac{d}{d_1}\right)^2 v$, and $v_2 = \left(\frac{d}{d_2}\right)^2 v$, and, hence, we may put

$$h_5 = \zeta \cdot \frac{l_1 d^4}{d_1^5} \cdot \frac{v^2}{2g}, \text{ and } h_6 = \zeta \cdot \frac{l_2 d^4}{d_2^5} \cdot \frac{v^2}{2g},$$

and for velocities (v_1 and v_2) of from 5 to 10 feet,

$$\zeta = 0{,}021 \text{ to } 0{,}020.$$

In order to reduce these resistances, the pipes must be of as great diameter as possible, and the number of strokes as few as possible.

The motion of water in the pipes of a water-pressure engine is different from that in ordinary conduit pipes, inasmuch as in the former the velocity continually varies, whilst in the latter it is sensibly uniform.

Hence the *inertia of the water* plays a more conspicuous part in the one than in the other. In order to put a mass, M, into motion with a velocity v, there is required to be expended an amount of mechanical effect represented by $\frac{M\,v^2}{2}$; and hence to communicate to the column of water in the pressure pipes a velocity v_1, the weight being $F_1\,l_1\,\gamma$, there is required an amount of mechanical effect $= F_1 l_1 \gamma \, . \, \frac{v_1^2}{2g}$. If the water column be cut off from the working cylinder only at the end of the stroke of the piston, this amount of effect would not be lost, for this column would restore, or give back the mechanical effect, during the gradual cessation of the piston's motion; but the cutting off of the water pressure from the working piston takes place, although near the end of the stroke, yet gradually and while the piston is in motion, so that the working piston and column of water come to rest at the same instant; and hence the valve piston causes a gradual absorption of all the *vis viva* of the water column during the first half of its ascent, inasmuch as it brings a gradually increasing resistance in the way, by gradually decreasing the passage, and hence we may assume that the mechanical effect due to inertia, $F_1\,l_1\,\gamma \, . \, \frac{v_1^2}{2g}$ is *lost* at each stroke.

If we introduce $v_1 = \frac{d^2}{d_1^2}\,v$, and $F_1 = \frac{\pi\,d_1^2}{4}$, then we have for the above amount of mechanical effect $\frac{\pi\,d^2}{4} \, . \, \frac{d^2\,l_1}{d_1^2}\,\gamma \, . \, \frac{v^2}{2g}$, and, hence, the mean *effort* during the whole stroke s,

$$= \frac{\pi\,d^2}{4} \, . \, \frac{d^2\,l_1}{d_1^2\,s}\,\gamma \, . \, \frac{v^2}{2g},$$

and the corresponding loss of fall or pressure head:

$$h_7 = \frac{d^2\,l_1}{d_1^2\,s} \, . \, \frac{v^2}{2g}.$$

A loss, that would be expressed in a similar manner, takes place on the return stroke, during which the water is forced out of the cylinder with a velocity v_2, and the *vis viva* communicated to it at the commencement of the stroke is of course lost to the efficiency of the engine. The pressure head lost would be:

$$h_8 = \frac{d^2\,l_2}{d_2^2\,s} \, . \, \frac{v^2}{2g}.$$

To keep these two losses by inertia as small as possible, it is requisite to have the pressure and discharge pipes of greatest diameter, and least length possible, to have a small velocity of the piston, and a long stroke.

Remark. To mitigate or to get rid of the prejudicial effect of shock, which the sudden cutting off of the water gives rise to, an air vessel has been introduced in many engines, at the lower end of the pressure pipes, and near the valves. This is a cylinder filled with compressed air, analogous to the air vessels on fire engines. The air in this case absorbs the excess of *vis viva* in the water, being compressed by it; and the air expanding again, restores this *vis viva* at the commencement of the next stroke, the water being forced from the air vessel into the working cylinder, nearly as if under the original hydrostatic pressure. In the application of this arrangement to machines having very great falls, the air in the vessel has been found to mix with the water, so that it is gradually removed from it entirely. To prevent this, either a piston must be fitted into the air cylinder, or air must be continually supplied to it by a small air pump to make up the absorption of it by the water.

§ 198. *Changes in direction and in sectional areas* of the various pipes of a water-pressure engine are further causes of diminished efficiency. Although these losses may be calculated by the formulas given in the third and fourth parts of the sixth section of the first volume, it appears necessary that we should here bring together the formulas to be applied.

In the pressure and discharge pipes, there are bent knee pieces, the motion of the water through which involves a loss of head, which may be expressed, according to Vol. I. § 334, by the formula $h = \zeta_1 \frac{\beta}{\pi} \cdot \frac{v^2}{2g}$. Here β is the arc of curvature, generally $= \frac{\pi}{2}$, ζ_1 is a co-efficient depending on the ratio between the radius r of the sectional area of the pipe, and the radius of curvature of the axis of the pipe, and which may be calculated by the formula

$\zeta_1 = 0,131 + 1,847 \left(\frac{r}{a}\right)^{\frac{7}{2}}$, or may be taken from the tables given at the place cited. For a bend in the pressure pipe, the head due to the resistance is

$$h_9 = \zeta_1 \cdot \frac{\beta_1}{\pi} \cdot \frac{v_1^2}{2g} = \zeta_1 \frac{\beta_1}{\pi} \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and for a bend in the discharge pipe:

$$h_{10} = \zeta_1 \cdot \frac{\beta_2}{\pi} \cdot \frac{v_2^2}{2g} = \zeta_1 \frac{\beta_2}{\pi} \cdot \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

At the entrance of the water into the valve cylinder, as well as at its discharge from it, the water is suddenly turned aside at a right angle, exactly as in an elbow, or rectangular knee piece. There is, therefore, a loss of head in this case, which, according to Vol. I. § 333, may be put: $h = 0,984 \frac{v^2}{2g}$, or almost equal to $\frac{v^2}{2g}$. For uniformity's sake, we shall put this loss of head for the pressure pipe:

$$h_{11} = \zeta_1 \cdot \frac{v_1^2 e}{2g} = \zeta_2 \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and for the discharge pipe:

$$h_{12} = \zeta_1 \cdot \frac{v_2^2}{2g} = \zeta_2 \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

Sudden *changes in sectional area*, as, for example, at the entrance and discharge of the water into and from the working cylinder, give rise, in like manner, to a loss of pressure head. According to Vol. I. § 337, such a loss is determined by the formula, $h = \left(\frac{F}{F_1} - 1\right)^2 \frac{v^2}{2g}$. For the entrance of the water into the working cylinder, this formula applies directly, if F and F_1 be the areas of the cylinder and pressure pipes respectively. For the discharge on the other hand $\frac{F}{F_1} = \frac{1}{a}$, in which a is the co-efficient of contraction. If $a = 0,6$, then $\left(\frac{1}{a} - 1\right)^2 = \frac{4}{9}$; and hence the head due to the resistance to the entrance of the water into the cylinder:

$$h_{13} = \left(\frac{F}{F_1} - 1\right)^2 \cdot \frac{v^2}{2g},$$

and for the discharge:

$$h_{14} = \left(\frac{1}{a} - 1\right)^2 \cdot \frac{v_1^2}{2g} = \frac{4}{9} \cdot \left(\frac{F}{F_1}\right)^2 \cdot \frac{v^2}{2g}.$$

For simplicity's sake, however, we shall put

$$h_{13} = \zeta_3 \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g}, \text{ and } h_{14} = \zeta_4 \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g},$$

so that, when $F = \frac{\pi d^2}{4}$ is introduced, and $F_1 = \frac{\pi d_1^2}{4}$, then

$$\zeta_3 = \left(\frac{F}{F_1} - 1\right)^2 \left(\frac{d_1}{d}\right)^4 = \left[1 - \left(\frac{d_1}{d}\right)^2\right]^2, \text{ and}$$

$$\zeta_4 = \frac{4}{9} \left(\frac{F}{F_1}\right)^2 \left(\frac{d_2}{d}\right)^4 = \frac{4}{9} \left(\frac{d_2}{d_1}\right)^4.$$

To avoid loss of effect by sudden variations of velocity generally, the intermediate pipes, and parts of the valve cylinder through which the water passes, should have the same area as the pressure and discharge pipes, or, at all events, the intermediate passages should gradually widen out to the area of the main pipes.

There are further special losses of effect occasioned by the cocks and throttle valves. These are to be calculated by the formula $h = \zeta_5 \cdot \frac{v^2}{2g}$ and the co-efficients ζ_5 depend on the position or angle of the cocks, &c., and are to be taken from the tables, Vol. I. § 340. Hence for the ascent of the working piston:

$$h_{15} = \zeta_5 \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v_1^2}{2g}, \text{ and for the descent, } h_{16} = \zeta_5 \cdot \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

By setting the regulating cock or valve, the co-efficient of resistance may be varied to any amount from 0 to ∞, or any excess of power may be absorbed, and the velocity of the piston checked at pleasure.

§ 199. *Formula for the Useful Effect.*—If in the mean time we leave the valves out of consideration, we can now construct a formula representing the useful effect of a water-pressure engine. The mean effort during an ascent of the piston is

$$P = [h_1 - (h_3 + h_5 + h_7 + h_9 + h_{11} + h_{13} + h_{15})] F \gamma,$$

and the resistance in the descent:

$$P_1 = (h_2 + h_4 + h_6 + h_8 + h_{10} + h_{12} + h_{14} + h_{16}) F \gamma;$$

and hence the effect for a double stroke:

$$(P - P_1) s = [h_1 - (h_2 + h_3 + h_4 + \ldots + h_{16})] F s \gamma,$$

and the mechanical effect produced per second:

$$L = [h_1 - (h_2 + h_3 + h_4 + \ldots + h_{16})] \cdot \frac{n}{60} \cdot F s \gamma.$$

If, again, we put:

$$\zeta \frac{l_1 d^4}{d_1^5} + \frac{d^2 l_1}{d_1^2 s} + \zeta_1 \frac{\beta_1}{\pi} \left(\frac{d}{d_1}\right)^4 + \zeta_2 \left(\frac{d}{d_1}\right)^4 + \zeta_3 \left(\frac{d}{d_1}\right)^4 + \zeta_5 \left(\frac{d}{d_1}\right)^4, \text{ or}$$

$$\left(\zeta \frac{l_1}{d_1} + \frac{d_1^2 l_1}{d^2 s} + \zeta_1 \cdot \frac{\beta_1}{\pi} + \zeta_2 + \zeta_3 + \zeta_5\right) \left(\frac{d}{d_1}\right)^4, = \varkappa_1 \left(\frac{d}{d_1}\right)^4, \text{ and}$$

$$\zeta \frac{l_2 d^4}{d_2^5} + \frac{d^2 l_2}{d_2^2 s} + \zeta_1 \cdot \frac{\beta_2}{\pi} \left(\frac{d}{d_2}\right)^4 + \zeta_2 \left(\frac{d}{d_2}\right)^4 + \zeta_4 \cdot \left(\frac{d}{d_2}\right)^4 + \zeta_5 \left(\frac{d}{d_2}\right)^4, \text{ or}$$

$$\left(\zeta \frac{l_2}{d_2} + \frac{d_2^2 l_2}{d^2 s} + \zeta_1 \frac{\beta_2}{\pi} + \zeta_2 + \zeta_4 + \zeta_5\right) \left(\frac{d}{d_2}\right)^4 = \varkappa_2 \left(\frac{d}{d_2}\right)^4,$$

then we may express the useful effect very simply and comprehensively, by

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\varkappa_1 \left(\frac{d}{d_1}\right)^4 + \varkappa_2 \left(\frac{d}{d_2}\right)^4\right] \cdot \frac{v^2}{2g}\right)\right] \cdot \frac{n}{60} F s \gamma.$$

On account of the greater length of the pressure pipes, $\varkappa_1$ is considerably more than $\varkappa_2$; and, therefore, the time for the up stroke t_1 is usually allowed to be longer than that for the down stroke t_2.

If we make the ratio $\frac{t_1}{t_2} = \nu = \frac{2}{1}$, then

$$t_1 = \frac{\nu}{\nu + 1} \cdot \frac{60''}{n}, \text{ and } t_2 = \frac{1}{\nu + 1} \cdot \frac{60''}{n};$$

and if we retain v as the value of the mean velocity of a double stroke $\frac{2 s}{t_1 + t_2} = \frac{2 n s}{60''}$, then the mean velocity during the up stroke

$$= \frac{s}{t_1} = \frac{\nu + 1}{\nu} \cdot \frac{n s}{60} = \frac{\nu + 1}{\nu} \cdot \frac{v}{2},$$

and that during the down stroke

$$= \frac{s}{t_2} = (\nu + 1) \cdot \frac{n s}{60''} = (\nu + 1) \cdot \frac{v}{2},$$

and, hence, the useful effect may be expressed more generally:

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\varkappa_1 \left(\frac{\nu + 1}{2 \nu}\right)^2 \left(\frac{d}{d_1}\right)^4 \right.\right.\right.$$
$$\left.\left.\left. + \varkappa_2 \left(\frac{\nu + 1}{2}\right)^2 \left(\frac{d}{d_2}\right)^4\right] \frac{v^2}{2g}\right)\right] \cdot \frac{n}{60} F s \gamma,$$

or, introducing $\frac{n}{60} \cdot F s = Q$,

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\varkappa_1 \left(\frac{1}{\nu}\right)^2 \left(\frac{d}{d_1}\right)^4 \right.\right.\right.$$
$$\left.\left.\left. + \varkappa_2 \left(\frac{d}{d_2}\right)^4\right] \left(\frac{\nu + 1}{2}\right)^2 \cdot \frac{v^2}{2g}\right)\right] Q \gamma,$$

or, introducing $v = \frac{Q}{F} = \frac{4 Q}{\pi d_2}$,

$$L = \left(h - \left[4 f \frac{b}{d}(h_1 + h_2) + \left(\frac{x_1}{\nu^2 d_1^4} + \frac{x_1}{d_2^4}\right)\frac{\nu + 1}{2}\right)^2 \cdot \frac{1}{2g} \cdot \left(\frac{4Q}{\pi}\right)^2\right]\right) Q \gamma.$$

In the double-acting, water-pressure engine, the mechanical effect produced is of course doubled.

This formula shows, very clearly, that the useful effect of a water-pressure engine is greater, the greater d, d_1 and d_2 are, or the wider the cylinder and pipes. It is also demonstrable, by aid of the higher calculus, that for a given number of strokes the useful effect is a maximum, or the prejudicial resistances are a minimum, when $\frac{x_1}{\nu^3 d_1^4} = \frac{x^2}{d_2^4}$, that is, when $\nu = \sqrt[3]{\frac{x_1 d_2^4}{x_2 d_1^4}}$. If, for example, $d_2 = d_1$, and $x_1 = 8 x_2$, then $\nu = \sqrt[3]{8} = 2$, or the time for the up stroke would be double that for the down stroke. By applying a balance beam, attached to the working piston rod, this ratio ν, between the time for the up and down stroke, may be adjusted by the counter-balance weight applied. Any regulation by means of the throttle valve, or cocks, on the pressure or discharge pipes, can only be effected at the cost of useful effect, as by these a loss of power measured by ζ_5 is occasioned, and which increases in proportion as the passages are contracted.

If the mechanical effect required be less than the best effect of the engine, the excess must be destroyed or checked by the throttle valves.

Example. It is required to make the calculations necessary for establishing a single-acting, single cylinder, water-pressure engine for a fall $h = 350$ feet, and a quantity of water $Q = 1$ cubic foot per second.

Suppose v the mean velocity of the up and down stroke $= 1$ foot, then for its area, we have $F = \frac{2Q}{v} = \frac{2 \cdot 1}{1} = 2$ square feet; and if we arrange that the water shall move through the pressure and discharge pipes with a velocity $v_1 = v_2 = 5$ feet, then for the section of these pipes, we have $F_1 = \frac{2Q}{v_1} = \frac{2}{5} = 0{,}4$ square feet. Hence the diameter of the working piston, $d = \sqrt{\frac{4F}{\pi}} = \sqrt{\frac{8}{\pi}} = 1{,}5958$ feet; and that of the pressure and discharge pipes, $d_1 = d_2 = \sqrt{\frac{4F_1}{\pi}} = \sqrt{\frac{1{,}6}{\pi}} = 0{,}71364$ feet. For simplicity and certainty, we shall assume $d = 20$ inches, and $d_1 = d_2 = 7$ inches.

If, for counter-balancing the rods, &c., we carry up the discharge pipe 50 feet above the mean height of the piston, or make $h_2 = 50$ feet, then $h_1 = h + h_2 = 400$ feet. We shall assume further, that the total length of pressure pipe $l_1 = 450$ feet, and that of the discharge pipe $l_2 = 66$ feet. For a diameter of 20 inches,

$$F = \frac{\pi d^2}{4} = \frac{\pi}{4} \cdot \frac{25}{9} = 2{,}182 \text{ square feet} \ldots v = \frac{2Q}{F} = \frac{2}{2{,}182} = 0{,}9166 \text{ feet.}$$

Suppose we have 4 strokes per minute, then the length of stroke

$$s = \frac{60\,v}{2\,n} = \frac{60 \cdot 0{,}9166}{8} = 6{,}8745 \text{ feet.}$$

If, again, we suppose the width of the packing of the piston $b = \frac{1}{8}d = 2\frac{1}{2}$ inches, we get as the pressure height absorbed by the friction of the piston:

$$4 f \frac{b}{d}(h_1 + h_2) = 4 \cdot 0{,}25 \cdot \tfrac{1}{8}(400 + 50) = \frac{450}{8} = 56{,}25 \text{ feet,}$$

or there remains, after deducting the piston friction, the head $350 - 56{,}25 = 293{,}75$ feet. To calculate the hydraulic resistances, we must, in the first place, determine $\varkappa_1$ and $\varkappa_2$. That for the pressure pipe,

$$\varkappa_1 = \zeta \frac{l_1}{d_1} + \frac{d_1^2 \, l_1}{d^2 \, s} + \zeta_1 \frac{\beta_1}{\pi} + \zeta_2 + \zeta_3 + \zeta_5,$$

and that for the discharge pipe:

$$\varkappa_2 = \zeta \frac{l_2}{d_2} + \frac{d_2^2 \, l_2}{d^2 \, s} + \zeta_1 \frac{\beta_2}{\pi} + \zeta_2 + \zeta_4 + \zeta_5,$$

and in these expressions:

$$\zeta = 0{,}021, \ \frac{l_1}{d_1} = \frac{450}{\frac{3}{4}} = 600, \text{ and } \frac{l_2}{d_2} = \frac{66}{\frac{3}{4}} = 88;$$

therefore, $\zeta \frac{l_1}{d_1} = 0{,}021 \ . \ 600 = 12{,}6$, and $\zeta \frac{l_2}{d_2} = 0{,}021 \ . \ 88 = 1{,}85$. Again,

$$\frac{d_1^2 \, l_1}{d^2 \, s} = \left(\frac{9}{20}\right)^2 e \, \frac{450}{6{,}87} = 13{,}26, \text{ and } \frac{d_2^2 \, l_2}{d^2 \, s} = \left(\frac{9}{20}\right)^2 \frac{66}{6{,}87} = 1{,}94.$$

If we further assume, that the bends in the pipes have radii of curvature $a = 4r$, or if $\frac{r}{a} = \frac{1}{4}$, we have as the co-efficient of resistance in bends:

$\zeta_1 = 0{,}131 + 1{,}847 \left(\frac{1}{4}\right)^{\frac{7}{2}} = 0{,}145$, and if the aggregate angle of deflexion by curves in the pressure and discharge pipes $= 270^\circ$, or if:

$$\frac{\beta_1}{\pi} = \frac{\beta_2}{\pi} = \frac{270^\circ}{180^\circ} = \frac{3}{2} \text{ then, } \zeta_1 \frac{\beta_1}{\pi} = \zeta_1 \frac{\beta_2}{\pi} = 0{,}145 e \, \frac{3}{2} = 0{,}22.$$

If, further, the water, before and after its work is done in the cylinder, makes two rectangular deviations in its progress through the valve cylinder, we have, in the formulas for $\varkappa_1$ and $\varkappa_2$, $\zeta_2 = 2 \ . \ 1 = 2$; and if the valve cylinder is of the same diameter as the pressure and connecting pipes, the co-efficient of resistance for the up stroke

$\zeta_3 = \left[1 - \left(\frac{d_1}{d}\right)^2\right]^2 = (1 - 0{,}2025)^2 = 0{,}64$, whilst for the down stroke $\zeta_4 = \frac{4}{9} = 0{,}44$.

If the throttle and other passage valves be fully open, then $\zeta_5 = 0$, and, therefore, we have $\varkappa_1 = 12{,}60 + 13{,}26 + 0{,}22 + 2{,}00 + 0{,}64 = 28{,}72$, and

$$\varkappa_2 = 1{,}85 + 1{,}94 + 0{,}22 + 2{,}00 + 0{,}44 = 6{,}45$$

Lastly, we have the best ratio of the times for the up stroke and down stroke:

$$\nu = \sqrt[3]{\frac{\varkappa_1}{\varkappa_2}} = \sqrt{\frac{28{,}72}{6{,}45}} = 1{,}646, \text{ or nearly 5 to 3.}$$

By introducing these values, we get the height of column remaining:

$$h - \left[4 f \frac{b}{d} (h_1 + h_2) + \left(\frac{\varkappa_1}{\nu^2 d_1^4} + \frac{\varkappa_2}{d_2^4}\right) \left(\frac{\nu+1}{2}\right)^2 \cdot \frac{1}{2g} \left(\frac{4 Q}{\pi}\right)^2\right]$$

$$= h - \left[4 f \frac{b}{d} (h_1 + h_2) + \left(\frac{\varkappa_1}{\nu_2} + \varkappa_2\right) \left(\frac{\nu+1}{2}\right)^2 \cdot \frac{1}{2g} \cdot \left(\frac{4 Q}{\pi d_1^2}\right)^2\right]$$

$$= 293{,}75 - \left(\frac{28{,}72 \ . \ 9}{25} + 6{,}45\right) \left(\frac{4}{3}\right)^2 . 0{,}0155 . \left(\frac{4 \ . \ 16 e}{9 \ . \ \pi}\right)^2$$

$$= 293{,}75 - 16{,}79 . \ 0{,}0155 \ . e \frac{16 \ . \ 4096 e}{729 \ . \ \pi^2} = 293{,}75 - 2{,}37 = 291{,}38 \text{ feet.}$$

From this we get the efficiency of this engine, neglecting the mechanical effect required for working the valves, $\eta = \frac{291{,}38}{350} = 0{,}832$, and the useful effect:

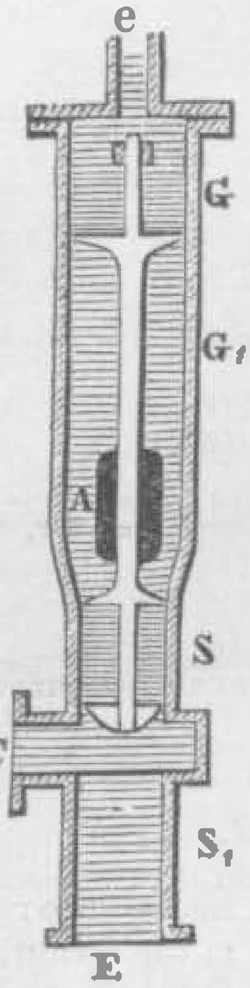
$L = 291{,}38 \ . \ 1 \ . \ 62{,}5 = 18211$ feet lbs., or 3,1 horse power, nearly.

§ 200. *Adjustment of the Valves.*—The arrangement and proper adjustment of the valves is a most important part of the water-pressure engine. As in all the engines we have described piston valves are used, we shall, in what follows, confine ourselves to the consideration of these arrangements.

We shall first consider the system having *two* pistons, as used in some of the Saxon engines, and represented in Fig. 316.

If we assume that the valve piston S is pressed upwards with a mean pressure h_1, and downwards with a pressure h_2; and if the height of the counter piston G above $S = e$, and, therefore, the height of the hydrostatic column under $G = h_2 - e$, and that above G according as the water is let on or shut off, $h_1 - e$, or $h_2 - e$. If further, d_1 = the diameter of S, and d_2 = that of G, and we shall assume that the packing of the two pistons consists of leather discs pressed together, and that they are about the same height or thickness. If, now, this piston valve system be *up*, as shown in Fig. 316, the letting on of the pressure water above G would occasion a descent of the valves, and, therefore, the difference of the water pressure on S and G, in combination with the weight R of the system, must be sufficient to overcome the friction of the piston S and G. The pressure downwards on

Fig. 316.



$$G = \frac{\pi d_2^2}{4}(h_1 - e)\gamma,$$

and the counter pressure under

$$G = \frac{\pi d_2^2}{4}(h_2 - e)\gamma.$$

The downward pressure on $S = \frac{\pi d_1^2}{4} h_2 \gamma$, and the counter pressure under $S = \frac{\pi d_1^2}{4} h_1 \gamma$, and, hence, the power to push the system down:

$$P = \frac{\pi d_2^2}{4}(h_1 - e - h_2 + e)\gamma + \frac{\pi d_1^2}{4}(h_2 - h_1)\gamma + R$$

$$= \frac{\pi}{4}(d_2^2 - d_1^2)(h_1 - h_2)\gamma + R,$$

or the fall $h_1 - h_2$ being represented by h:

$$P = \frac{\pi}{4}(d_2^2 - d_1^2)\, h\gamma + R.$$

The friction of the pistons, even though they be not on the hydrostatic principle, is proportional to the circumference of the piston, and to the difference of pressure on the two sides of the piston, and may be represented by $F = \phi \pi d h \gamma$. Hence, in the case in question $P = \phi\pi(d_1(h_1 - h_2) + d_2[h_1 - e - (h_2 - e)])\gamma = \phi\pi(d_1 + d_2)h\gamma$. Hence we have the following formula:

$$\frac{\pi}{4}(d_2^2 - d_1^2)\, h\gamma + R = \phi\pi(d_1 + d_2)\, h\gamma,$$

or, simplified:

$$1.)\ d_2^2 - d_1^2 + \frac{4R}{\pi h \gamma} = 4\phi(d_1 + d_2).$$

If, on the other hand, the valve has to rise from its lowest position after the water has been cut off, then the excess, or the difference of the water pressure on S alone, must overcome the weight of the valve system, and the friction of the pistons; because then the pressures on both sides of G cease, we must, therefore, have

$$\frac{\pi}{4} d_1^2 (h_1 - h_2)\gamma = R + \phi\pi(d_1 + d_2) h\gamma,$$

or, more simply:

$$2.)\ d_1^2 - \frac{4R}{\pi h\gamma} = 4\phi(d_1 + d_2).$$

These formulas will serve for calculating the diameters d_1 and d_2 of the two pistons. Neglecting R, which, in considerable falls, is almost always of small amount:

$$d_2^2 - d_1^2 = 4\phi(d_1 + d_2), \text{ and } d_1^2 = 4\phi(d_1 + d_2), \text{ therefore,}$$
$$d_2^2 - d_1^2 = d_1^2, \text{ or } d_2^2 = 2d_1^2,$$

and, hence, the diameter of the counter piston:

$$d_2 = d_1\sqrt{2} = 1{,}414\, d_1,$$

or about $\frac{7}{5}$ of the diameter of the piston valve, which is determined by the first equation:

$$d_2^2 - d_1^2 = 4\phi(d_1 + d_2), \text{ or } d_1 - d = 4\phi,$$

if we substitute in it: $d_1\sqrt{2}$ for d_2.

We then have:

$$d_1 = \frac{4\phi}{\sqrt{2} - 1} = (\sqrt{2} + 1)\,.\,4\phi = 2{,}414\,.\,4\phi, \text{ and } d_2 = 3{,}414\,.\,4\phi.$$

Taking the weight of the pistons into account, we have, with sufficient accuracy,

$$d_2 = \sqrt{2d_1^2 - \frac{8R}{\pi h\gamma}} = d_1\sqrt{2} - \frac{4R}{\pi h\gamma d_1\sqrt{2}}$$
$$= d_1\sqrt{2} - \frac{(\sqrt{2} - 1)R}{\phi\pi h\gamma\sqrt{2}},$$

and from this, we have by the equation 1:

$$d_2 - d_1 = 4\phi - \frac{4R}{\pi h\gamma(d_1 + d_2)}, \textit{ i. e.}$$

$$(\sqrt{2} - 1)d_1 = 4\phi + \frac{(\sqrt{2} - 1)}{\phi\pi h\gamma\sqrt{2}}R - \frac{(\sqrt{2} - 1)R}{\phi\pi h\gamma(1 + \sqrt{2})}, \textit{ i. e.}$$

$$d_1 = (\sqrt{2} + 1)4\phi + \frac{(2 - \sqrt{2})R}{2\phi\pi h\gamma}, \text{ and}$$

$$d_2 = (\sqrt{2} + 2)4\phi + \frac{(3\sqrt{2} - 4)R}{2\phi\pi h\gamma}.$$

For the sake of certainty in the working, both diameters are made somewhat greater, and the excess of power is absorbed by setting the regulating cocks, already mentioned, so as to exactly adjust the area of passage. Judging from the best existing engines, we may take $4\phi = 0{,}1$, or $\phi = \frac{1}{40}$. In order that, in the passage of the pressure water through the valve cylinder, there may be the least possible hydraulic resistance, it is usually made of equal area, at that part, with the area of the pressure and intermediate pipes; and supposing the formulas give a diameter d_1, which is less than that of the pressure pipes, we may consider that there exists an excess of power, which must be adjusted by the regulating cocks.

Example. It is required to determine the proportions of a two-piston valve system for a water-pressure engine of 400 feet fall. Suppose the weight of the pistons and rod, &c. = 150 lbs. Leaving this weight out of the calculation, the diameter $d_1 = 2{,}414 \,.\, 4\,\phi = 2{,}414 \,.\, 0{,}1 = 0{,}2414$ feet $= 2{,}897$ inches, and $d_2 = 3{,}414 \,.\, 0{,}1 = 0{,}3414 = 4{,}097$ inches. Taking the weight of pistons, &c., into account $d_1 = 0{,}2414 + \frac{0{,}586 \,.\, 150}{0{,}05 \,.\, 400 \,.\, 62{,}5\,\pi} = 0{,}2414 + \frac{0{,}586}{8{,}33\,.\,\pi} = 0{,}2414 + 0{,}0223 = 0{,}2637$ ft. $= 3{,}164$ inches, and $d_2 = 0{,}3414 + \frac{0{,}243 \,.\, 150}{0{,}05 \,.\, 400 \,.\, 62{,}5\,\pi} = 0{,}3414 + 0{,}0092 = 0{,}3516$ feet $= 4{,}219$ inches. It will be sufficient in this case, if we take $d_1 = 3\frac{1}{2}$, and $d_2 = 5$ inches. For so small a counter-balance to piston valve, only a small supply of water is necessary; but the resistance in the passage through the valve cylinder would be great. If, on this account, we put $d_1 = 6$ inches, then we should have to make d_2 at least $= d_1\sqrt{2} = 8{,}484$ inches, that is from $8\frac{3}{4}$ to 9 inches, the excess of power being absorbed by adjusting the cocks.

§ 201. In the three-piston valve system, the mode of calculation is very similar to that gone through above. The advantage of this system is, that we may make one of the pistons, the valve piston proper, for example, of the same diameter as the pressure pipes. The calculations for the valves in the engine represented in Fig. 311, may be made as follows: Putting d_1 = the diameter of the lower piston, or first valve piston, and d_2 that of the second, and d_3 that of the upper or counter piston; then, for the descent, we have

$$1.)\; d_1^2 - d_2^2 + d_3^2 + \frac{4\,R}{\pi\,h\,\gamma} = 4\,\phi\,(d_1 + d_2 + d_3),$$

and for the ascent:

$$2.)\; d_2^2 - d_1^2 - \frac{4\,R}{\pi\,h\,\gamma} = 4\,\phi\,(d_1 + d_2 + d_3).$$

From d_1 we can, by means of these formulas, determine d_2 and d_3, making d_2, however, somewhat greater than the calculation gives for insuring certainty of action. If we put the value thus found into the formula

$$2\,(d_1^2 - d_2^2) + d_3^2 + \frac{8\,R}{\pi\,h\,\gamma} = 0,$$

we get as the diameter of the third piston:

$$d_3 = \sqrt{2\,(d_2^2 - d_1^2)] - \frac{8\,R}{\pi\,h\,\gamma}},$$

which, for the reasons already given, should be made something more than the absolute result of calculation.

For the valve system of the engine in Fig. 312, we have the following formulas. Let h_1 = the mean height of the pressure column, and h_2 = the mean height of counter-balance column; d_1 the diameter of the valve piston, d_2 that of the counter piston, and d_3 that of the projection forming a third piston. The power in the descent, is then

$$\frac{\pi}{4}\,[d_1^2\,(h_1 - h_2) + (d_2^2 - d_3^2)\,h_1 - d_2^2\,h_1]\,\gamma + R,$$

and that in the ascent:

$$\frac{\pi}{4}\left[d_2^2 h_1 - (d_2^2 - d_3^2) h_2 - d_1^2 (h_1 - h_2)\right] \gamma - R;$$

therefore:

$$1.)\ d_1^2 - \frac{h_1}{h} d_3^2 + \frac{4R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3), \text{ and}$$

$$2.)\ d_2^2 - d_1^2 + \frac{h_2}{h} d_3^2 - \frac{4R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3).$$

If d_1 be given, we can then calculate d_2 and d_3, but we must keep d_2 somewhat above, and d_3 somewhat below the result of the formula. The formulas

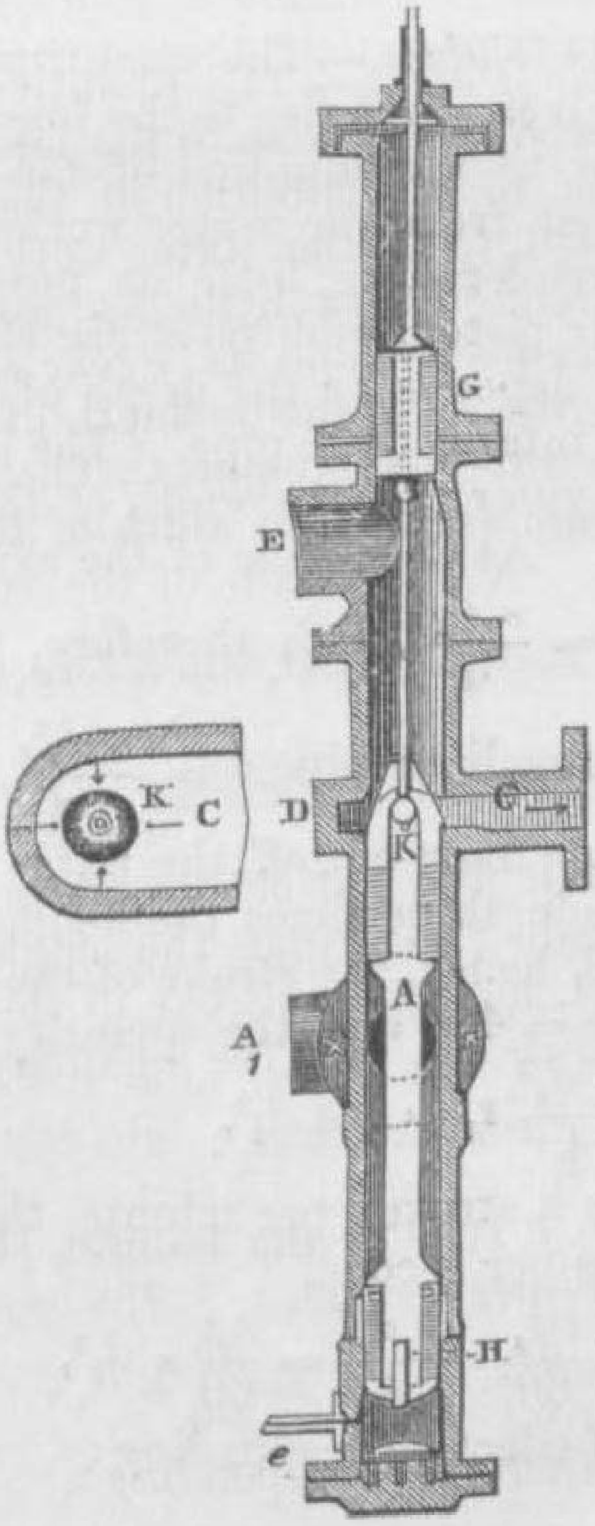
$$1.\ d_2^2 - d_3^2 = 8 \phi (d_1 + d_2 + d_3), \text{ and}$$

$$2.\ d_2^2 + \left(\frac{h_1 + h_2}{h}\right) d_3^2 = 2 d_1^2 + \frac{8R}{\pi h \gamma},$$

are of rather simpler application.

For the valve system shown in Fig. 317, already mentioned as that of the Clausthal engines, we have, when d_1 = the diameter of valve piston, d_2 the diameter of upper or counter piston, and d_3 that of the lower or auxiliary piston, the power for descent:

Fig. 317.



$$\frac{\pi}{4}\left[d_1^2 (h_1 - h_2) - d_2^2 h_1)\right] \gamma + R,$$

The power of ascent:

$$\frac{\pi}{4}\left[d_3^2 (h_1 - h_2) - d_1^2 (h_1 - h_2) + d_2^2 h_1\right] \gamma - R; \text{ therefore,}$$

$$1.)\ d_1^2 - \frac{h_1}{h} d_2^2 + \frac{4 R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3), \text{ and}$$

$$2.)\ d_3^2 - d_1^2 + \frac{h_1}{h} d_2^2 - \frac{4 R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3).$$

Example. Supposing, as in the last-mentioned engine, $h_1 = 688$ feet, and $h_2 = 76$ feet, $R = 170$ lbs., and $d_1 = \frac{1}{2}$ foot, we get the diameters of the other pistons as follows: $d_3^2 = 8 \phi (d_1 + d_2 + d_3)$, and also $= 2 d_1^2 - \frac{2 h_1}{h} d_2^2 + \frac{8 R}{\pi h \gamma}$, or, in numbers: $d_3^2 = 0{,}2 (0{,}5 + d_2 + d_3)$, and $= 0{,}5 - 2{,}248 d_2^2 + 0{,}0107$. If, now, we assume $d_2 = 0{,}3$ feet, we have by one formula $d_3^2 = 0{,}5107 - 0{,}2023 = 0{,}3084$, that is $d_3 = 0{,}555$; and by the second formula, $d_3^2 = 0{,}2 \,.\, 1{,}355 = 0{,}2710$, *i. e.* $d_3 = 0{,}5205$. But if we put $d_2 = 0.33$, then $d_3^2 = 0{,}5107 - 0{,}2448 = 0{,}2659$, or $d_3 = 0{,}516$, and, again, $d_3^2 = 0{,}2 \,.\, 1{,}346 = 0{,}2692$, or $d_3 = 0{,}519$. Hence $d_2 = 0{,}33 \,.\, 12 = 3{,}96$, or about 4 inches, and $d_3 = 0{,}52 \,.\, 12 = 6{,}24$, or $6\frac{1}{4}$ inches. Jordan, the engineer, who erected these machines, has made $d_2 = 4$ inches, 1,6 lines, and $d_3 = 5$ inches, $9\frac{3}{4}$ lines, from which we deduce that 4ϕ is somewhat less than 0,1 in this case.

Remark. To calculate more accurately, the diameter of the valve rod would have to be taken into account.

§ 202. *Water for the Valves.*—The quantity of water required, for the motion of the valves, gives rise to the loss of a certain amount of mechanical effect, or to a diminution of the engine's efficiency, because it is abstracted from the water working the engine. It should, therefore, be rendered as little as possible, that is d_3, the diameter of the counter piston, and its *stroke* should be as small as possible. The stroke depends on the depth of the valve piston, or on the diameter of the intermediate pipe. The intermediate pipe is, therefore, made rectangular: of the width of the working cylinder, and low in proportion. As it is made of the same area as the pressure pipes, we have $ad = \frac{\pi d_1^2}{4}$, and, therefore, the height, or least dimension of the intermediate pipe $a = \frac{\pi d_1^2}{4 d}$.

That the valve piston may cut off the water exactly at the end of the stroke, it is made three times the height of the pipe, or its depth is $a_1 = 3 a$; and, hence, the stroke of the valve piston proper $s_1 = a_1 + a = 3 a + a = 4 a$, and the quantity of water expended for each stroke is $= \frac{\pi d_3^2}{4} s_1 = \pi a d_3^2$.

If the engine makes n strokes per minute, the quantity of water expended by the valves per second.

$$Q_1 = \frac{n s_1}{60} \cdot \frac{\pi d_3^2}{4} = \frac{na}{60} \pi d_3^2,$$

and, hence, the loss of effect corresponding:

$$L_1 = \frac{n s_1}{60} \cdot \frac{\pi d_3^2}{4} \cdot h \gamma,$$

or the loss is the less the longer the stroke of the engine.

As to the valve gear, the power required to work it is so small that it may be left out of consideration. The study of the arrangement of the mechanism arises under another section of our work.

Example. If in the water-pressure engine, the subject of the example calculated, there be applied a valve piston of 9 inches diameter, and, therefore, a counter piston $= 9\sqrt{2} = 13$ inches diameter. If, further, the intermediate pipe have a height $a = \frac{\pi d_1^2}{4 d} = \frac{9^2 \pi}{4 \cdot 20} = \frac{81 e \pi}{80} = 3{,}18e$ inches, then the valve piston must have a height $a_1 = 3 a = 9{,}54$ inches, and its stroke $s_1 = a_1 + a = 12{,}72$ inches $= 1{,}06$ feet; and therefore the quantity of water expended each stroke $= \frac{\pi}{4}\left(\frac{13}{12}\right)^2 \cdot 1{,}06 = 0{,}977$ cubic feet; and hence the loss of effect per second:

$L_1 = \frac{n}{60} \cdot 0{,}977 e\, h \gamma = \frac{4}{60} \cdot 0{,}977 \cdot 350 \cdot 62{,}5 = 1424{,}9$ feet lbs., or nearly 3 *horse power.*

It would certainly be better in this case to make the piston valves less in diameter, and have a lower intermediate pipe; for although this would increase the hydraulic resistances, still it would not involve so great a loss as the waste of water we have calculated implies.

§ 203. *Experimental Results.*—There are not many good experiments on the effect of water-pressure engines. These engines are usually employed as pumping engines in mines, and the experiments that have been made involve the whole machinery, as well as the engines themselves, in the results as to the efficiency. But it is very easy to get an approximate determination of this efficiency, if we assume that the efficiency of water-pressure engines and pumps are in certain proportions to each other. This assumption we may make with perfect propriety, as the engine and machine are very analogous in their construction and movements. We shall not give any advantage to the water-pressure engine, nor be far from the truth, if we suppose the loss of effect of the whole apparatus to be one-half due to the water-pressure engine. The calculation then becomes very simple.

The effect at disposition is $\frac{n}{60}(F s + F_1 s_1) h \gamma$, in which F_1 is the section, and s_1 the stroke of the auxiliary piston. The effect produced, however, is $\frac{n s}{60} F_2 h_2 \gamma$; if $F_2 =$ the section of the pump piston, and h_2 the height, the water is raised by the pump. The loss of effect is, therefore,

$$= \frac{n}{60}(F s + F_1 s_1) h \gamma - \frac{n s}{60} F_2 h_2 \gamma,$$

the half of which is:l

$$= \frac{n \gamma}{120}[(F s + F_1 s_1) h - F_2 s h_2],$$

and hence the efficiency of the water-pressure engine:

$$\eta = 1 - \tfrac{1}{2}\frac{(F s + F_1 s_1) h - F_2 s h_2}{(F s + F_1 s_1) h} = \tfrac{1}{2} + \frac{F_2 s h_2}{2(F s + F_1 s_1) h} = \tfrac{1}{2}(1 + \eta_1),$$

if η_1 be the efficiency of the combined engine and pumps. In this mode of calculation, it is assumed that there are no losses of water, and when the machinery is in good order, this loss is so small that it may be neglected. Jordan found for the Clausthal engines, that the loss of water in the water-pressure engines is only $\frac{1}{4}$ per cent., and in the pumps $2\frac{1}{4}$ per cent. The experiments are made by opening the regulating apparatus in pressure and discharge pipes, and then raising the height of the *pump* column, or increasing the work to be done till the required number of strokes is performed *uniformly*.

By experiments on this principle, Jordan found that one of the Clausthal engines gave, when making 4 strokes per minute, $\eta_1 = 0,6568$, and making 3 strokes $\eta_1 = 0,7055$, and, therefore, in the first case, $\eta = \frac{1,6568}{2} = 0,8284$, and in the second: $\eta = \frac{1,7055}{2} = 0,8527$, and hence as a mean $\eta = 0,84$. When the greatest effect of a water-pressure engine cannot be determined by the method of heightening the pump column till a uniform motion is established, it may be done perhaps by diminishing the water-pressure column. This, however, can only be done when the excess of power of the engine is small, that is, when the part of the water-column to be taken off is small. The water may be kept at a certain level in the pressure pipes, ascertained by a float, and in this way the efficiency for a *certain head* be determined. The engine in Alte Mordgrube, near Freyberg, was experimented on in this way, and it was found that for 3 strokes per minute $\eta_1 = 0,684$, and hence the efficiency of the water-pressure engine alone is to be estimated as $\eta = \frac{1,684}{2} = 0,842$.

The most of the results reported in reference to the effect of water-pressure engines are too uncertain to be worthy of much confidence, having been deduced from experiments in which essential circumstances were not noted. If we take ζ as the co-efficient of resistance corresponding to a certain position of the regulating valves or cocks, as given in the table, Vol. I. § 340, the fall y, lost by this contraction, may be estimated by the formula:

$$y = \zeta \cdot \frac{v_1^2}{2g} = \zeta \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and we can, therefore, estimate the efficiency by the formula:

$$\eta = \frac{1}{2}\left(1 + \frac{F_2 s h_2}{Fs\left[h - \zeta\left(\frac{d}{d_1}\right)^4 \frac{v^2}{2g}\right] + F_1 s_1 h}\right).$$

Example. A pressure engine consumes 10 cubic feet of water per second, besides 0,4 cubic feet for the valves. The fall = 300 feet, the mean velocity of the water in the pressure pipes = 6 feet per second, and the circular throttle valves in the main pipe stands at 60°. Suppose, that by this engine, there is raised at each stroke 3,5 cubic feet 420 feet high, at what is the efficiency of the engine to be estimated? According to Vol. I. § 340, for the position of the valve 60°,

$$\zeta = 118, \therefore \zeta \cdot \frac{v_1^2}{2g} = 118 \cdot 0{,}0155 \cdot 6^2 = 62{,}2 \text{ feet; and hence,}$$

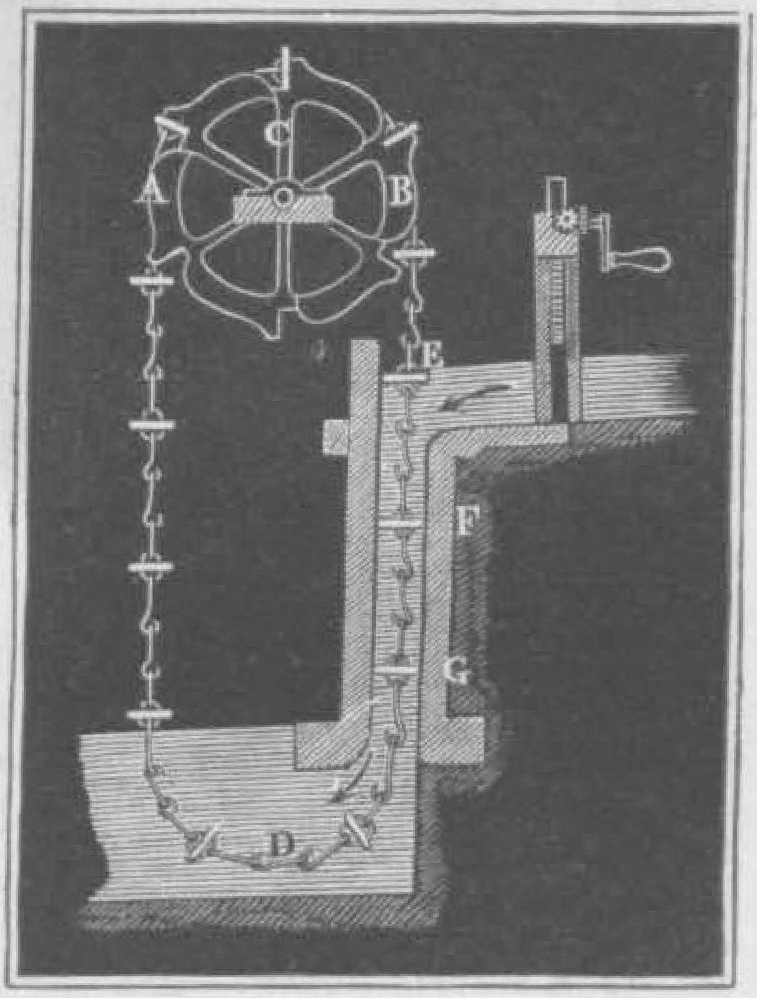
$$\eta = \tfrac{1}{2}\left(1 + \frac{3{,}5 \cdot 420}{10(300 - 62{,}2) + 0{,}4 \cdot 300}\right) = \tfrac{1}{2}\left(1 + \frac{3{,}5 \cdot 42}{237{,}8 + 12}\right) = \tfrac{1}{2} \cdot 1{,}588 = 0{,}794.$$

§ 204. *Chain Wheels.*—There are other water-power machines, neither wheels nor pressure engines, but which are to be met with from time to time. We may mention the following :—

The chain of buckets (Fr. *roue à piston ;* Ger. *Kolbenrad*) has recently been revived as a machine recipient of water power by Lamolières (*see* "Technologiste," Sept. 1845).

The principal parts of this machine are *ACB*, Fig. 318, over which passes a chain *ADB*, forming the axis connecting a series of pistons (called buckets or saucers), *E*, *F*, *G*, &c., and a pipe *EG*, through which the chain passes in such manner, that the pistons nearly fill the section of the pipe. The water flowing in at *E*, descends in the pipe *EG*, carrying the buckets along with it, thus setting the whole chain in motion, and turning the sprocket wheel *ACB* round with it. Lamolière's piston wheel, consists of two chains having from 10 to 15 buckets with leather packing. The buckets have an elliptical form, the major axis being 8 times the minor axis. The sprocket wheel consists of two discs with six cuts to receive the buckets.

Fig. 318.



For a fall of two metres (6′ — 8″), the surface of buckets being 0,25 square feet, the quantity of water 31 litres (6,82 gallons) per second, the number of revolutions 36 to 39, it is said that an efficiency $\eta = 0{,}71$ to 0,72 was obtained.

Remark. This machine is the chain pump used in the English navy, converted into a recipient of power. For a description of the chain pumps, *see* Nicholson's "Operative Mech.," p. 268.

The chain of buckets (Fr. *noria*, *chapelet*, *pater-noster;* Ger. *Eimerkette*) is a similar apparatus. The chain in this machine has a series of buckets attached, Fig. 319, of such form that no pipe is required. The water enters at *A*, fills the buckets successively, and sets the whole series in motion, so that the sprocket wheel *C* is made to revolve. This wheel should give a very high efficiency, seeing that the whole fall may be made use of; but from the great number of parts of which it is composed, their liability to wear, and other sources of loss of effect, it is practically a very inefficient machine.

Fig. 319.

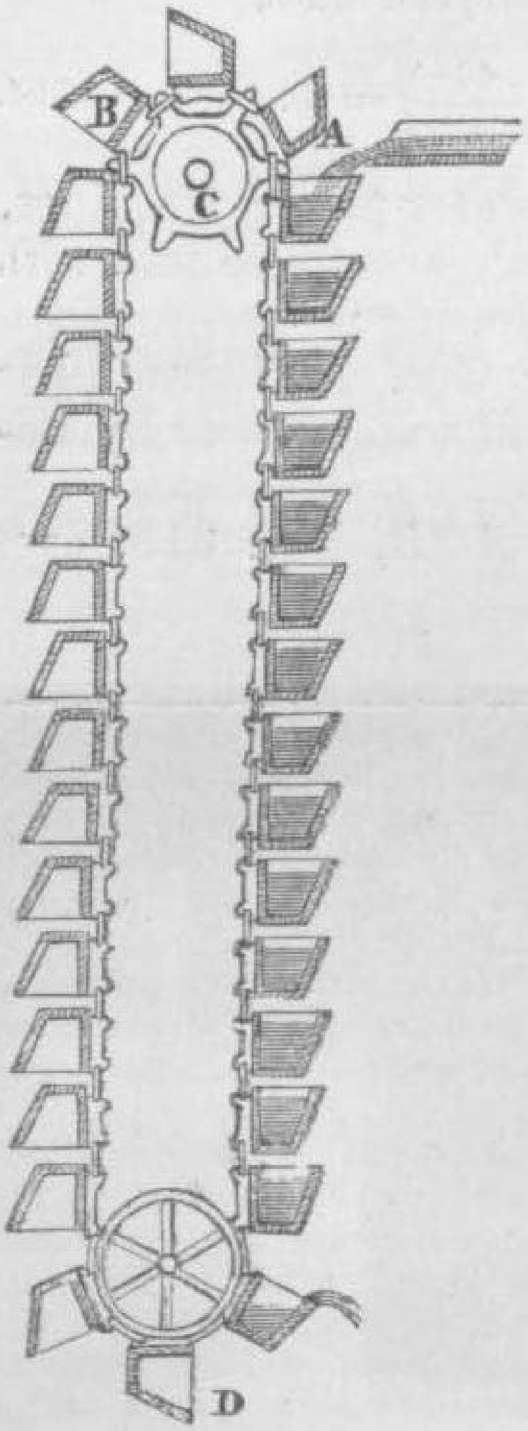
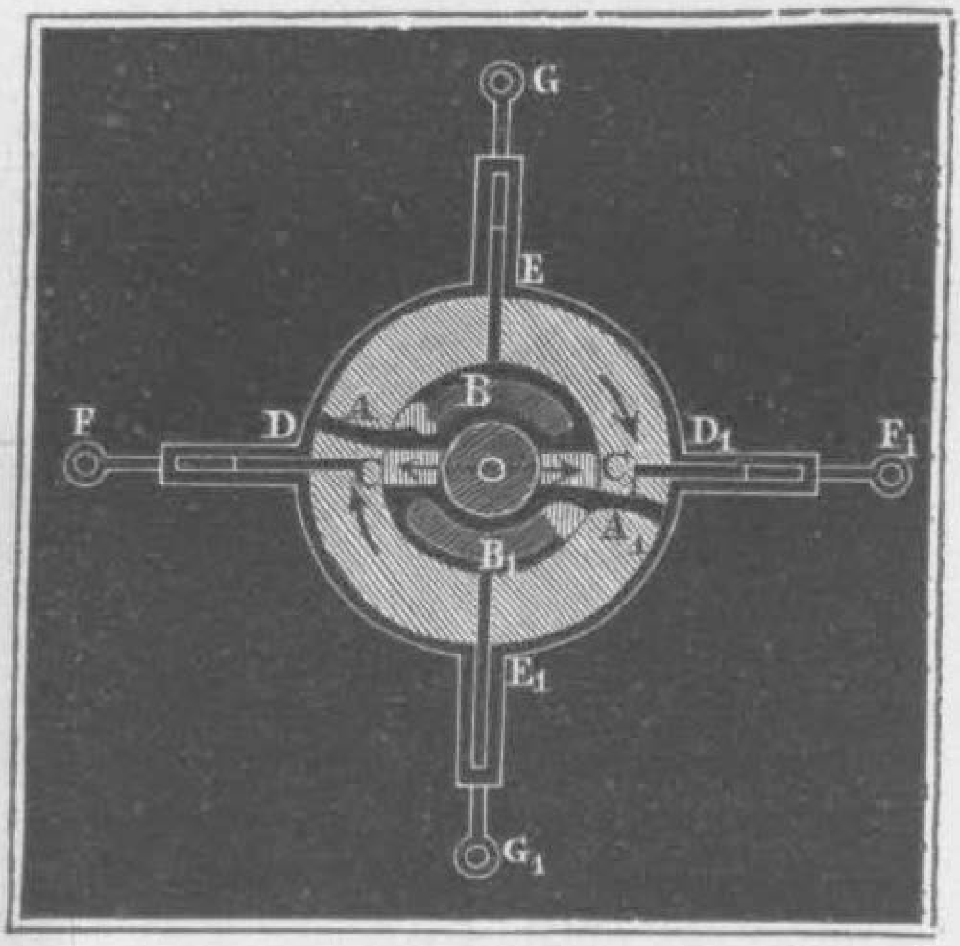


Fig. 320.



Remark. We may here mention, that the so called rotary pump, rotary steam engines, &c., may be adapted to receive water power. Fig. 320, represents a water pressure wheel, of which there is a detailed description and theory given in the "Polytechn. Centralblatt, 1840." It is Pecqueur's rotary steam engine adapted to water power. BOB_1 is a strong, accurately turned axis, A and A_1 being two wings connected with it, and which serve as pistons. These pistons are enclosed in a cover DED_1E_1, in which there are four slides moved by the engine itself, and performing the functions of valves. The axis is bored three times in the direction of its length, and each of the hollow spaces has a lateral communication within the cover. The pressure water flows through the inner bore O, enters through the side openings C and C_1, into the, in other respects, isolated space between the axis and the cover; presses against the pistons A and A_1, and in that way sets the axis in rotation. That the rotation may not be interrupted by the slides, they must always recede before the piston comes up to them; and on the other hand, that no water pressure may act on the opposite side of the piston, the slides must fall back instantly on the piston passing them, so that the spaces ABE and $A_1B_1E_1$, are shut off, and communicate only with the passages B and B_1, through which the water is discharged when it has done its work.

Mr. Armstrong, of Newcastle, constructed a water-pressure wheel of about 5 H. P., in 1841, a description of which will be found in the "Mechanic's Magazine," vol. xxxii.

Literature.—We shall conclude by some account of the literature and statistics of water-pressure engines. Belidor, in the "Architecture Hydraulique," describes a water-pressure engine with a horizontal working cylinder; and mentions, also, that, in 1731, MM. Denisard and De la Duaille had constructed a water-pressure engine. But this machine had only 9 feet fall, and raised about $\frac{1}{25}$ part of the weight of the power water to a height of 32 feet. It appears pretty certain, however, that the water-pressure engine was employed for raising water from mines, first by Winterschmidt, and soon afterwards by Höll. The details of this historical fact are to be found in Busse's "Betrachtung der Winterschmidt und Höll'schen Wassersäulenmaschine, &c., Freiberg, 1804." A drawing and description of Winterschmidt's engine is given in Calvör's "Historisch chronolog.

Nachricht, &c., des Maschinenwesens, &c., auf dem Oberharze, Braunschweig, 1763." Höll's engine is described in Delius's "Introduction to Mining," originally published at Vienna, 1773, and in the description of the engines erected at Schemnitz, by Poda, published at Prague, 1771.

Smeaton mentions the water-pressure engine in 1765, as an old invention, improved by Mr. Westgarth, of Coalcleugh, in the county of Northumberland, at which time several had been erected, in different mines, on Mr. Westgarth's plan. *See* Smeaton's "Report," vol. ii. p. 96.

Trevethick, the celebrated Cornish engineer, also invented or reproduced the water-pressure engine; and erected one, still at work in the Druid copper mine, near Truro, about the year 1793. *See* Nicholson's "Operative Mech."

The water-pressure engine is now in use in nearly every mining district in the world. The Bavarian engineer, Reichenbach, greatly improved and has made a most extensive application of this power for raising the brine to the boiling establishments in the Salzbourg district. These engines have never been accurately described, but notices of them will be found in Langsdorf's "Maschinenkunde," in Hachette's "Traité élémentaire des Machines," and in Flachat's "Traité élémentaire de Mécanique." The engines erected by Brendel, in Saxony, are described in Gerstner's "Mechanik," where also the engines in Karinthia and at Bleiberg are described in detail. The water-pressure engines in the Schemnitz district are described by Schitko, in his "Beiträgen zur Bergbaukunde." Jordan has given a very detailed account of the engines at Clausthal, in Karsten's "Archiv für Mineralogie," &c., b. x., published as a separate work by Reimer, of Berlin. Junker has described his engines, at Huelgoat, in the "Annales des Mines," vol. viii. 1835, and the description is published as a separate work, by Bachelier.

No description of the engines erected by Mr. Deans, of Hexham, has been published. They are, however, simple and efficient. The engine erected by him at Wanlockhead, in Scotland, in 1830 or 31, having the fall-bob for working the valves, is one of the largest, and considered very efficient.

But the water-pressure engine erected in 1842, at the Alport Mines, near Bakewell, in Derbyshire, and several others on nearly the same model, are perhaps the most perfect of this description of engine hitherto made. These engines have been constructed from the designs of Mr. Darlington, engineer of the Alport Mines, under Mr. Taylor, by the Butterly Iron Company. There is a beautiful model of the first erected at Alport, in the Museum of Economic Geology, but no description of it has yet been published. Its arrangement—the construction of its parts—the valves, and their gear—are each of them admirable and peculiar to this engine, though, in its general features, it resembles the engines of Brundel, and Junker, and Jordan, which have been described.—Tr.

CHAPTER VII.

ON WINDMILLS.

§ 205. *Windmills.*—The atmospheric currents caused by a local expansion of the air by the sun's heat, are a source of mechanical effect, as is the expansive force of air heated artificially.

The machines, recipients of this wind power, are windmills (Fr. *roues à vent;* Ger. *Windräder*). They serve to convert a portion of the *vis viva* of the mass of air in motion into useful effect. As the direction of the wind is more or less horizontal, windmills or sail wheels usually have the axis nearly horizontal, that is, they are themselves nearly vertical.

Horizontal windmills, having concave buckets or sails, have been erected. The force of the wind against a hollow surface is greater than against a plane or a convex surface, and hence such a wheel revolves under very light winds, but not advantageously for the production of mechanical effect.

Remark. For some account of Beatson's horizontal windmills, *see* Nicholson's "Practical Mechanic," and Gregore's "Mechanic," vol. ii.

§ 206. The advantage of *sail* wheels over any construction of *bucket* wheel is, that for the same weight, or in the same conditions generally, they produce a greater effect than these latter. We shall, therefore, in what follows, confine ourselves to the consideration of sail wheels, of which the general arrangement is as follows: First, there is the axle of wood, or better, of iron. This shaft, or axle, is inclined at an angle of from 5 to 15 degrees to the horizon, in order that the wheels may hang free from the structure on which they are placed, and also because the wind is supposed to blow at an inclination amounting to that number of degrees. This axle has a *head*, a *neck*, a *spur wheel*, and a *pivot*. At the head are the *arms*—the neck is the journal, or principal point of support on which it revolves. The spur wheel transmits the motion to the work to be done, and the pivot, at the low end of the axis, takes up a certain amount of the weight and counter-pressure of the machine. The loss of effect arising from the friction of the axle on the points of support is considerable, on account of the great weight and strain upon them, as also on account of the velocity with which it generally revolves, and hence every means must be taken to reduce it. On this account, iron shafts and bearings are to be preferred to wooden ones, as they may be made of much less diameter. The diameter of a wooden neck being 1½ to 2 feet; that of an iron one substituted, need not be more than 6 to 9 inches. The friction of wooden axles is also in itself greater than that of iron.

§ 207. *Windmill Sails.*—A windmill sail consists of the arm or *whip*, of the *cross bars*, and of the *clothing*. The whips are radial arms of any required length, up to 40 or even 50 feet, usually about 30 feet. The number of arms is generally four, less frequently 5 or 6. For 30 feet in length, these arms are made 1 foot thick by 9 inches broad at the shaft, and 6 inches by 4½ inches at the outer end. The mode of setting them in, or fastening them to the shaft, is various. When the axle is of wood, the arms are put through two holes, morticed at right angles to each other, thus getting 4 arms. The arms are sometimes made fast by screws to the shaft head, like the arms of a water wheel, and we refer to our description of water wheels for hints applicable to this subject. The bars are wooden cross arms, passing through the whip, which is morticed through at intervals of from 15 to 18 inches for the purpose, at right angles to the leading side of the whip. According as the sail is to have a rectangular or a trapezoidal form, the bars are all of the same length, or they increase in length from the shaft outwards. The first bar is placed at ⅙ of the length of the whip from the shaft, and its length is = to this ⅙ to ⅛ of the length of the whip. The outermost bar is made from $\frac{1}{3}$ to $\frac{2}{5}$ of the length of the whip. The whips are not generally made the *centre* line of the sails, but they divide them so that the part next the wind equals from $\frac{1}{4}$ to $\frac{1}{3}$ of the entire width of the sail. Therefore, the bars project much less from the one side than from the other. The narrower side is usually covered by the so-called *windboard*, and, on the wide side, the winddoor or a sail-cloth clothing is used.

The sails are made plane, or surfaces of double curvature, *i. e.*, warped, or concave. The slightly hollow surfaces of double curvature give the greatest effect, as we shall learn in the sequel. For plane sails, the bars have all the same inclination of from 12 to 18 degrees to the plane of rotation. In the double-curvature sails, the first bars are set at 24°, and the outer bars at 6° from the plane of rotation, and the inclinations of the intermediate bars form a transition between these two angles. To give the sails concavity, the whips must be curved, as also the bars. Although, according to the theory of the wind's impulse, this form gives an increased effect, the difficulty of execution renders it nearly inapplicable. The ends of the bars are connected or strung together by *uplongs*, and sometimes there are 3 of these uplongs on the driving and 2 on the leading side of the sail, to strengthen the lattice on which the sailcloth lies, on a series of frames of not more than 2 square feet each.

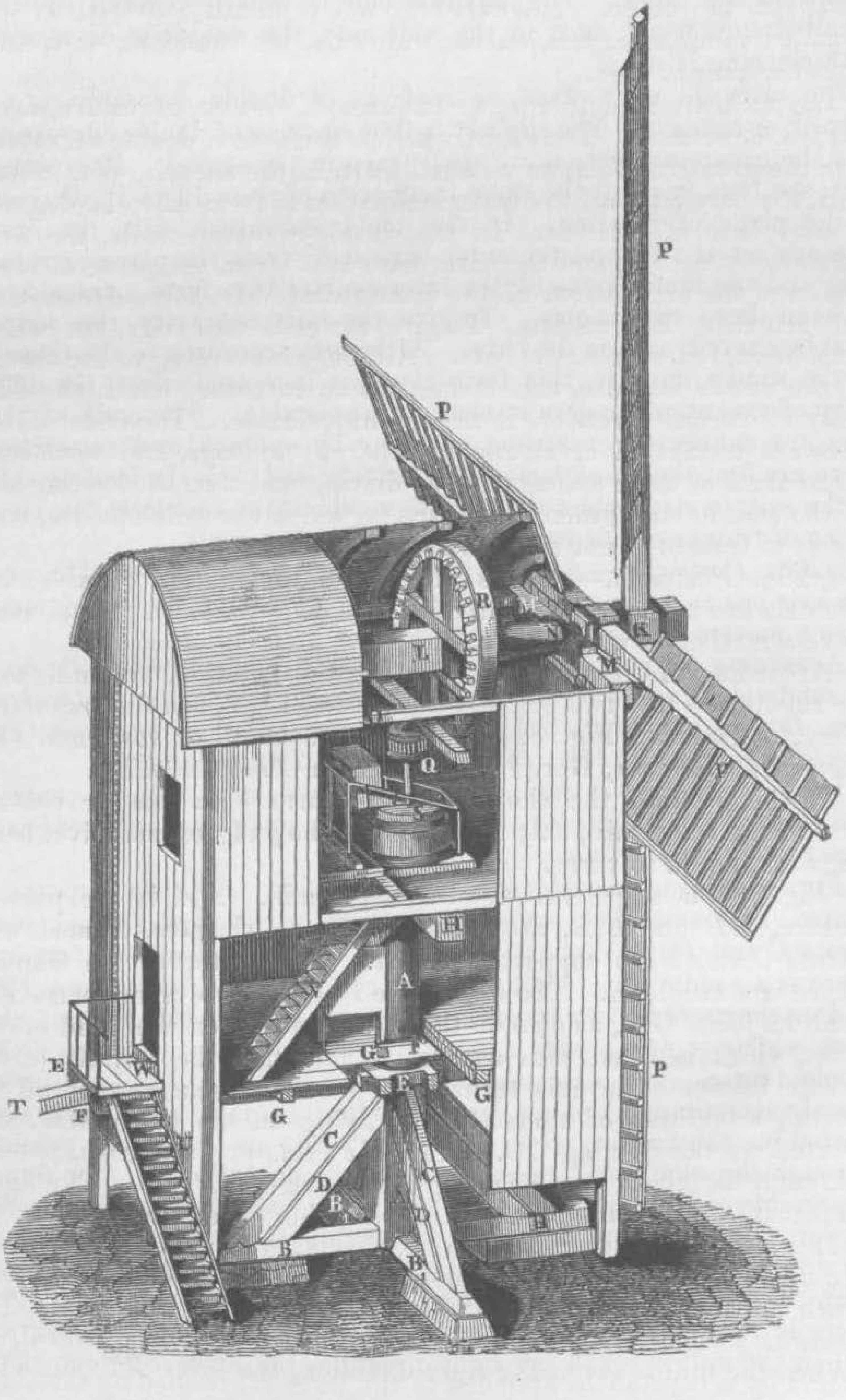
§ 208. *Postmills.*—As the direction of the wind is variable, and the axis has to be in that direction, the support of the wheel must have a motion on a vertical axis.

According to the manner of effecting this rotation, windmills may be subdivided into two classes: the postmill (Fr. *moulin ordinaire;* Ger. *Bockmühle*), Fig. 321, and the smockmill, or towermill (Fr. *moulin Hollandais;* Ger. *Holländische*, or *Thurmmühle*).

In the postmill, the whole structure turns on a foot, or centre; and in the smockmill, only the cap, with the gudgeon and pivot bearings resting on it, turns.

Fig. 321 is a general view of a postmill. *AA* is the post or centre, *BB* and B_1B_1 are cross bearers or sleepers, framed with struts *C* and *D*, to support the post. On the top of the framing there is a saddle *E*. The mill house rests on two cross beams *FF*, and on joists *GG*, as also on the cross beam *H* on the head of the post, which is fitted with a pivot to facilitate the turning of the whole fabric. The axis turns in a plumber block *N*, generally of metal, sometimes of stone (basalt), lying on the beam *MM*, supported on the framing *OO*. *KP*, *KP*, &c., are the arms, passing through the shaft and carrying 4 plane sails *PP*, &c. The figure represents a grindingmill, and, hence, the wheel transmitting the power, *R*, works into a pinion *Q*, driving the upper millstone *S*. In order to turn the whole house, a long lever, strongly connected with the beams *EF*, projects 20 to 30 feet from the back of it. This lever is loaded, to counterbalance the weight of the sail wheel, &c. When the mill is set in the right direction, the lever *FT* (cut off in the figure), is anchored and held fast, and generally there is a small movable capstan for getting power to turn the mill house, &c.

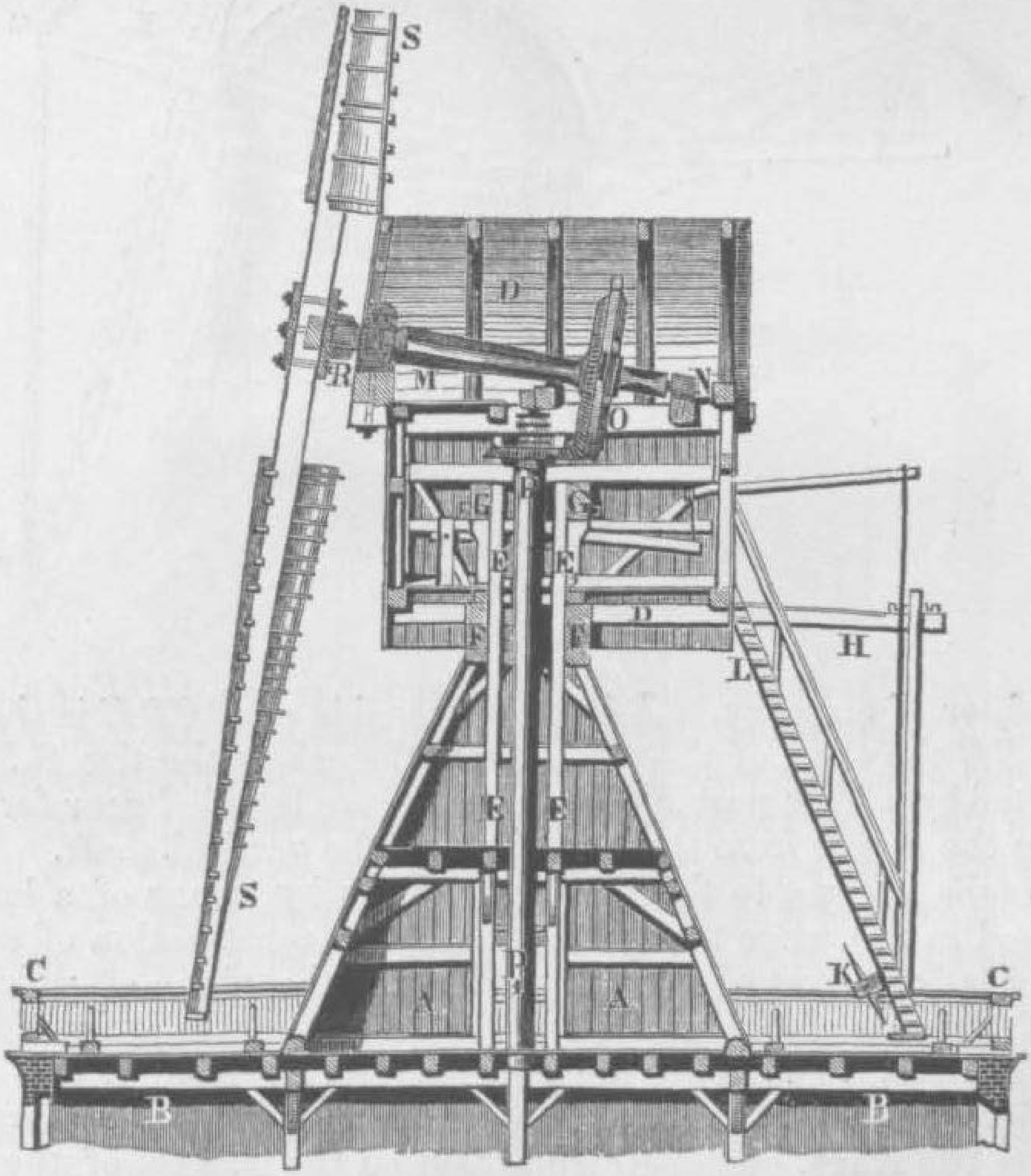
Fig. 321.



§ 209. *Smockmills.*—Smockmills are made in two different ways. Either the movable cap encloses the windshaft alone, or a greater part of the mill house, from the windshaft downwards, turns on a vertical axis. The motion of the sail wheel is transmitted by a pair of wheels to the *king post*, that is, a strong vertical axis going through the whole height of the mill house. In order that the wheels may be in gear in every position of the windshaft, it is necessary that the axis of the one shaft should intersect that of the other.

Fig. 322 represents the latter arrangement, which is, in fact, intermediate between the postmill and the smockmill. AA is a stationary

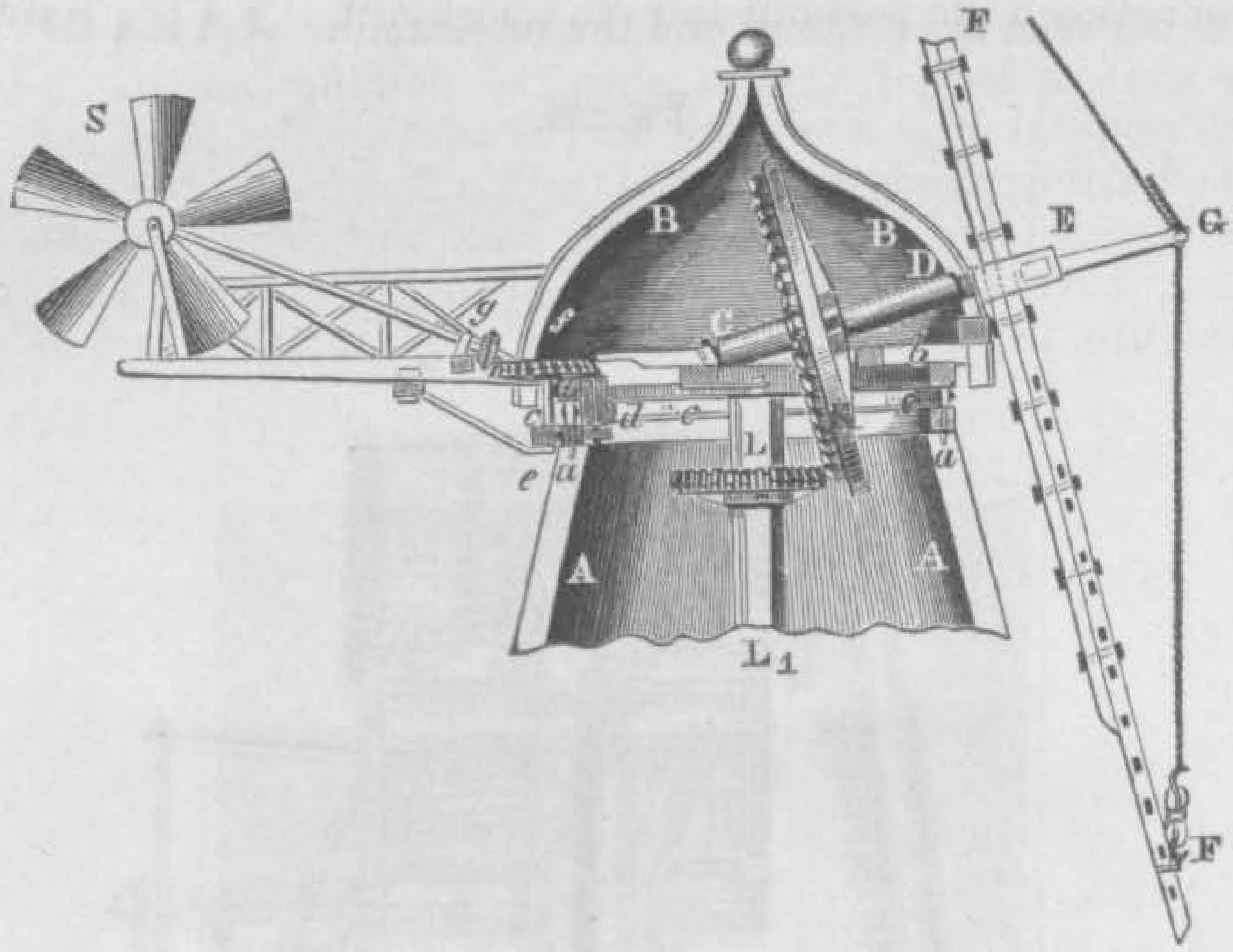
Fig. 322.



tower or pyramid, raised above which is the building containing the machinery, in driving which the power is consumed. DD is the movable top of the mill, supported by the wooden ring FF, and by the wooden ring GG, by means of the uprights EE and E_1E_1, and which only admits of rotation round these, which are, in fact, the substitute for the post in the postmill. The mill wheel is drawn round by a capstan R, attached to the lever H, framed by the stairs to the movable part of the structure. The windshaft is of cast iron, and rests at *M* and *N* in plumber blocks, lined with brasses. *O* and *P* are iron-toothed wheels, for transmitting the motion of the wind shaft to the upright, or *king post PP*. The windsails *R,S*, *RS*... are warped surfaces: the arms are fastened by screws into the cast iron socket piece *R*, attached by wedges to the head of the windshaft.

The upper part of a smockmill, properly so called, is shown in Fig. 323. *AA* is the upper part of the tower, or mill house, built

Fig. 323.



of wood, or of masonry. *BB* is the movable cap, *CDE* is the wind shaft, *EE* the arms of the sails, strengthened by the ties *GF'*, *GF*, supported by a king post *EG*. *K* and *L* are bevelled gear for transmitting the power from the windshaft to the vertical shaft.

The sails are set to the wind, sometimes by means of a lever, as described in reference to the last-mentioned construction of cap, but sometimes by means of a large wind vane, the plane of which is in that of the axis of the wind shaft, but more generally by means of a small windmill *S*. That the cap may revolve easily, it is placed on rollers *c*, *c*, *c*... connected together in a frame, and running between two rings, one of which is laid on the summit of the tower, the other is attached to the under side of the cap. To prevent the cap from being raised up and displaced, there is an internal ring *d*, which has likewise friction rollers running on the internal surface of *a a*. When this method of adjustment is used, the outer surface of *a a* is toothed, and a small pinion *e* working into it, is moved by the auxiliary windmill, by means of the bevelled gear *f* and *g*, Fig. 323, and thus the whole cap is made to revolve, until the auxiliary wheel, and therefore the axis of the windshaft is in the direction of the wind.

§ 210. *Regulation of the Power.*—As the wind varies in intensity as well as in direction, when the work to be done is a constant resistance, unless some means of regulating the power be applied. the motion of the machinery would not be uniform. One means of absorbing any excess of power, is a friction strap, applied to the outside of the wheel on the windshaft. Another means is, to vary the extent of sail, or the quantity of clothing exposed. When the sails are quite spread out, the maximum power depends on the intensity of the wind, and if this intensity be constant, the power may be varied by taking in more or less of the clothing of the sail. When the clothing is canvas, the regulation of power is easily managed by reefing more or less of it; and when the clothing consists of boarding, the removal of one or more boards answers the same end.

Self-adjusting windsails, that is, sails which extend their surface as the force of the wind decreases, and contract it as this force increases, have been successfully applied. The best windsails of this kind are those invented by Mr. Cubitt, in 1817, and of which

Fig. 324.

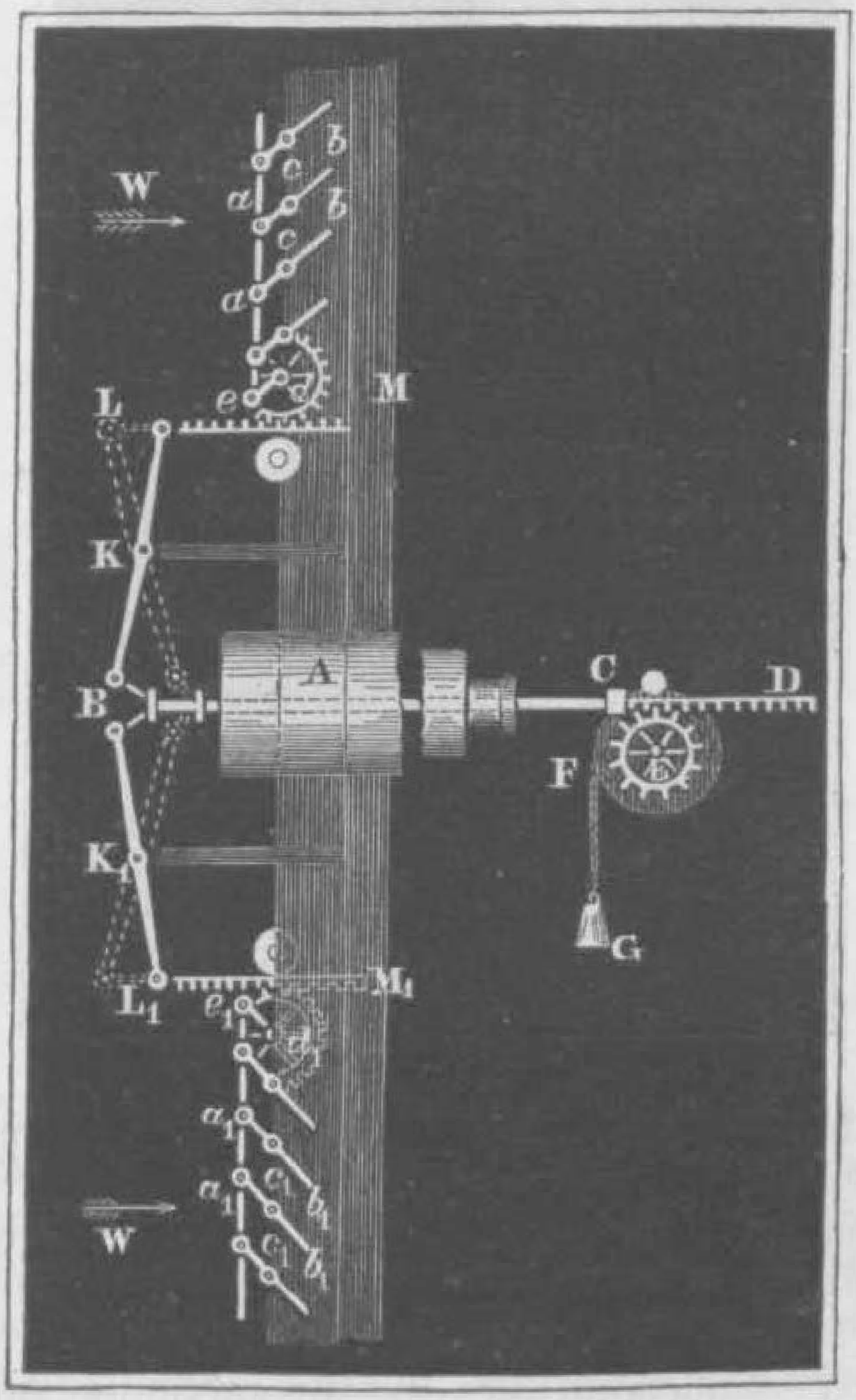
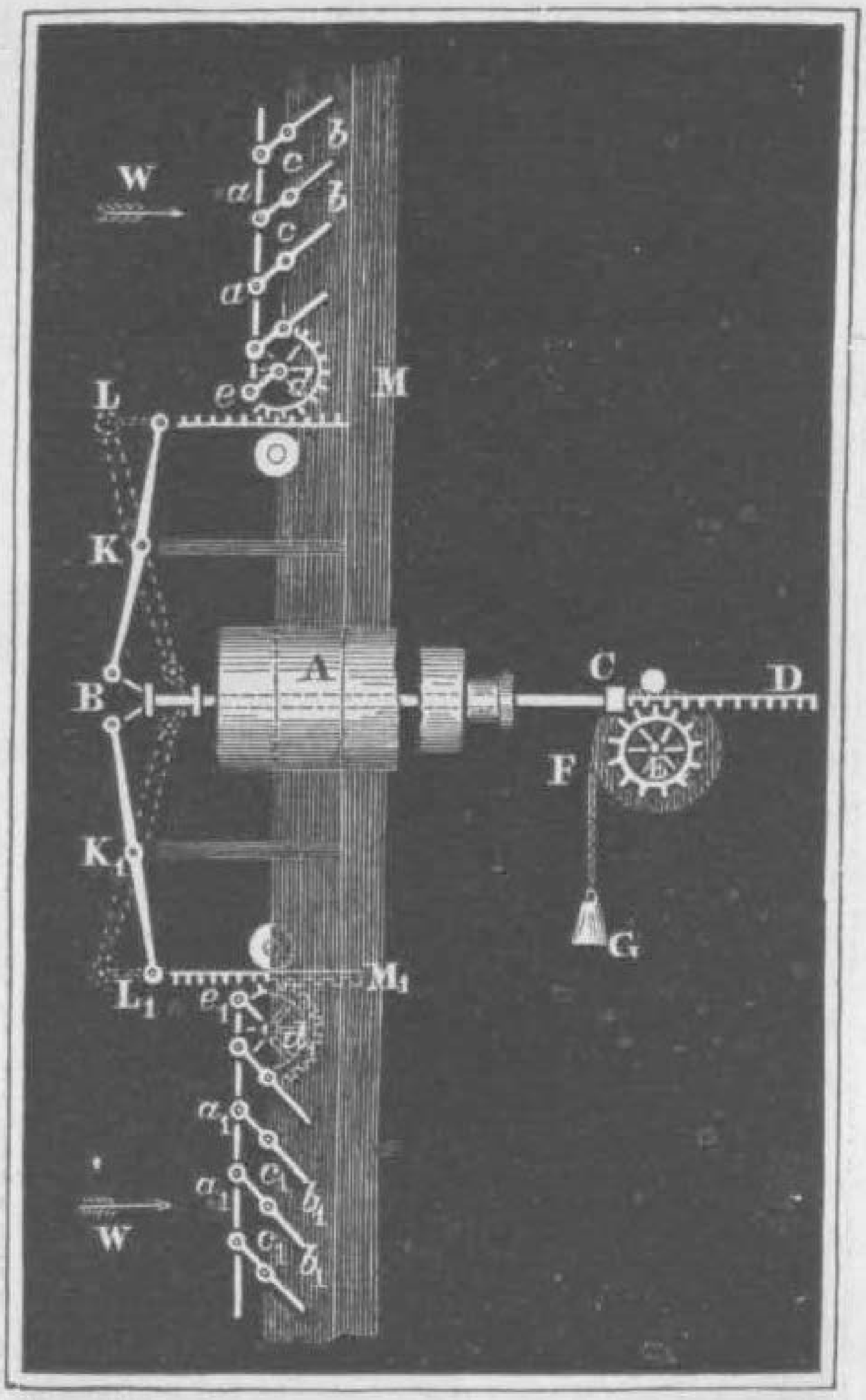


Fig. 324 represents the section of a part. *A* is a hollow windshaft. *BC* a rod passing through it, *CD* a ratchet fastened to *BC*, so that it does not turn with it, but serves to move it in the direction of the axis.

The ratchet works into a toothed wheel E, on the same axis as the pulley F, round which there passes a string with a weight G. The sail clothing consists of a series of boards, or sheet-iron doors bc, b_1c_1, &c., movable by the arms ac, a_1c_1, &c., round the axis c, c_1, &c. These arms are connected by the rods ae, a_1e_1, &c., and by the levers or cranks, de, d_1e_1, with toothed wheels d, d_1, so that, by the turning of the latter, the opening and closing, or, in general, the *adjustment* of the flaps or doors is possible.

There are besides, levers BL, BL_1, Fig. 325, revolving on centres

Fig. 325.



K and K_1, and attached at one end to the rod BC, and at the other to the ratchets LM and L_1M_1, working into the small wheels d and d_1. The drawing explains how the wind, coming in the direction W, works backwards on the counter-balance weight G, which is adjusted so that the surface exposed shall be that required to do the work regularly, always supposing that, for the maximum surface that can be exposed, there is wind sufficient.

Remark. Mr. Bywater invented a mode of furling and unfurling the clothing when it consists of sailcloth. There are two rollers moved by toothed wheels, and the action of these is to cover more or less of the sail frame, according to the force of the wind. This plan is described in detail in Barlow's "Treatise on the Manufactures and Machinery, &c. &c."

§ 211. *Direction of the Wind.*—The direction of the wind may be any of the 32 points of the compass, but the indications are generally noted as one of the 8 following: N., N.E., E., S.E., S., S.W., W., N.W., *i.e.*, north, north-east, east, south-east, south, south-west, west, north-west; or naming them according to the direction from which they blow. In the course of the year, the direction of the wind is more or less frequently from each of all these directions; some winds blowing more frequently than others. From Kämtz's "Meteorology," we extract the following table of the winds that blow during 1000 days, in different countries.

Country.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Germany . . .	84	98	119	87	97	185	198	131
England . . .	82	111	99	81	111	225	171	120
France . . .	126	140	84	76	117	192	155	110

We see from this that, in the three countries named, the south-west wind predominates; the passage of the wind from one direction to another is usually in the course from S., S.W., W., &c., and seldom in the opposite course of S., S.E., E., &c. That is, the latter course is generally only taken through a small angle, and then retraced.

The wind *vane*, or fane (Fr. *girouette*, *flouette*; Ger. *Wind-* or *Wetterfahne*), gives the direction of the wind. To give it facility of movement, the friction on its pivot or collar must be as small as possible, and hence the blade or plane of the vane has to be balanced by a counter-weight to bring the centre of gravity line to pass through the axis of rotation. (Whether the form resulting from this combination gave rise to the term *weathercock* (Fr. *coq à vent;* Ger. *Wetterhahn*), or whether "a king-fisher hanging by the bill, converting the breast to that point of the horizon from whence the wind doth blow, be the introducing of *weathercocks*," we cannot pretend to say.)

§ 212. *Intensity of the Wind.*—The miller is, however, dependent on the intensity of the wind, and not on its direction; for on the former the mechanical effect to be obtained from given wind-sails depends.

Accordingly, the velocity of the wind is

Scarcely sensible for	$1\frac{1}{2}$	feet per second.	
Very gentle wind for	3	"	"
Gentle breeze for	6	"	"
Brisk breeze for	18	"	"
Good breeze for windmills	22	"	"
Brisk gale for	30	"	"
High wind for	45	"	"
Very high wind for	60	"	"
Storm for	70 to 90	"	"
Hurricane	100 or more.		

A breeze of 10 feet per second is not in general sufficient to drive a loaded windsail, and if the velocity rises above 35 feet per second, the intensity becomes too much for the strength of the arms, unless the clothing be very close reefed, and stormy weather is dangerous even to "bare poles."

Windgauges, or *anemometers*, are used for ascertaining the velocity of the wind. Many anemometers have been proposed and adopted, but few of them are sufficiently convenient or trustworthy in their indications. The anemometers have great resemblance to the hydrometers described in Vol. I. § 376. The velocity of a current of air may be measured by noting the rate of progress of a body floating in it, as a feather, smoke, soap bubbles, small air balloons, &c. This means will not suffice in the case of high velocities, for the eddies, that invariably accompany wind, disturb the progress of such bodies.

Anemometers may be divided into three classes. Either the velocity of the wind is deduced from that of a wheel moved by it, or it is measured by the height of a column of fluid, counterbalancing the force of the wind, or the pressure on a given surface is determined. We shall give a succinct account of each of these methods.

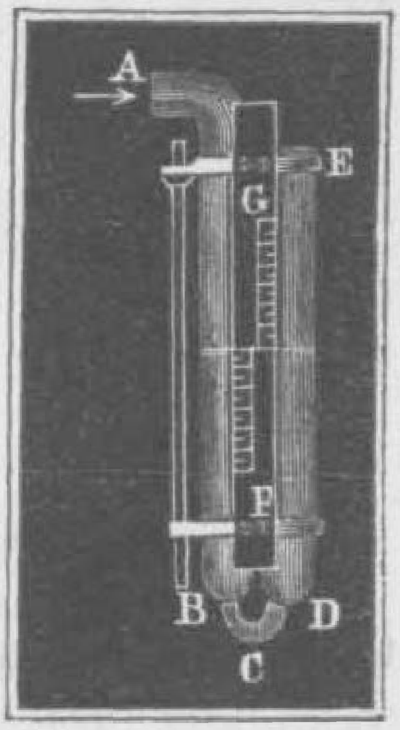
Remark. There is a very complete treatise on Anemometers, in the "Allgemeinen Maschinenencyclopädie, by Hülsse." In the transactions of the British Association for 1846, there is a report, by Mr. J. Phillips, on Anemometers, in which the essential points to be aimed at in these instruments, and the merits of those of Whewell, Osler, and Lind, respectively, are discussed. The chapters on Wind, in Kämtz's "Meteorology," and in Gehler's "Wörterbuch," are standards of reference on this subject.

§ 213. *Anemometers.*—Woltmann's wheel may be used for ascertaining the velocity of the wind as conveniently as it is for ascertaining the velocity of currents of water. When its axis of rotation is set in the direction of the wind, which is insured by means of a vane set on the same vertical axis with it, the number of revolutions made in a given time may be observed, and from this the velocity may be deduced, as explained, Vol. I. § 378, by the formula $v = v_0 + a\,u$, in which v_0 is the velocity of the wind, for which the wheel begins to stop, and a is the ratio $\frac{v - v_0}{u}$. If the impulse of the wind were of the same nature as that of water, and if they both increased exactly as the squares of the relative velocities, then $a = \frac{v - v_0}{u}$ would answer for windland water, but as this is only *nearly* true, we can only expect that the co-efficient a is nearly the same for wind and water. As to the initial velocity v_0, this is, in the case of wind, about $\sqrt{800} = 28{,}3$ times greater than for water, as the density of water is about 800 times greater than that of air; and thence a column of air 800 times as high as that of water, or the impact of a stream of $\sqrt{800} = 28{,}3$ times the velocity of the water. This high value of the constant v_0, makes it necessary to construct the anemometer sails with great lightness, and to have the axis of hard steel running on agate or other hard bearings; as, for instance, in Combe's anemometer. The constants v_0 and u are generally determined by moving the instrument through air at rest; but this method is objectionable, because the impact of a fluid in motion is not the same as the resistance of a fluid at rest (Vol. I. § 391).

It is better, on every account, to deduce the constants from experiments on currents of air, deducing the low velocities by direct observations on light, floating bodies. By placing the instrument in the main pipe of a blowing engine, the observations for calculating the constants might be made. The calculation of constants from a series of experiments for which v and u are known, should be done as shown in Vol. I. § 379.

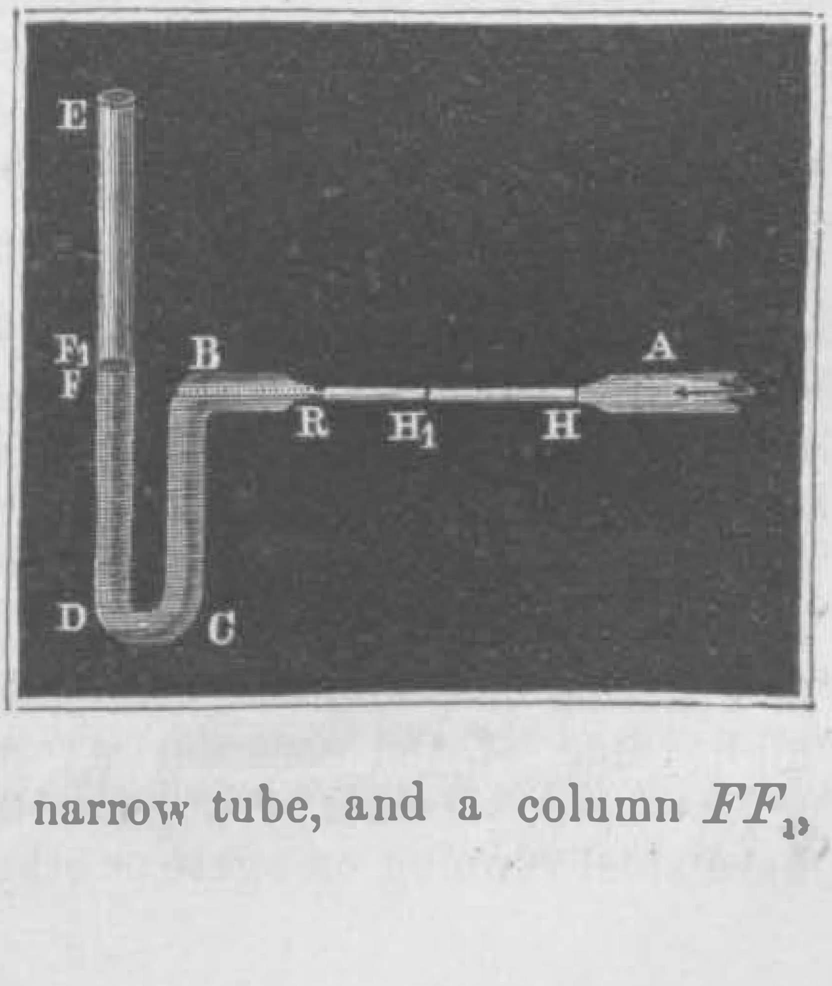
§ 214. Pitot's tube may also be very conveniently applied as an anemometer. This is Lind's anemometer, and its arrangement is shown in Fig. 326. AB and BE are two upright glass tubes $\frac{5}{12}$ of an inch in diameter, and filled with water, and BCD is a narrow, bent, connecting piece between the two, of only $\frac{1}{20}$ inch diameter. FG is a scale, by which to read off the height of the water. When the mouth A is turned to the wind, its force presses down the column in AB, and raises that in DE, and hence the difference of level between the two surfaces may be read on the scale h. From this, the velocity of the wind may be calculated by the formula $v = v_0 + a \sqrt{h}$; v_0 and a being co-efficients deduced from experiments for each instrument.

Fig. 326.



The use of this instrument is very limited, as pressures which move the water to a difference of level of $\frac{1}{20}$ of an inch can scarcely be noted accurately, but may be estimated to $\frac{1}{40}$ or $\frac{1}{50}$. This gives 5 to 7 miles per hour as the limit of wind velocity really measurable. To obviate these disadvantages, and render the instrument useful for moderate velocities, Robison and Wollaston introduced the following improvements.

Fig. 327.



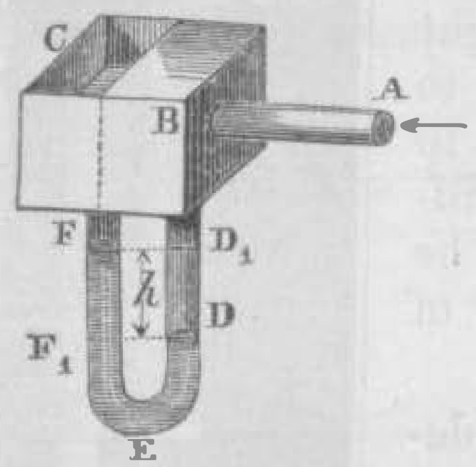
In Robison's anemometer there is a narrow, horizontal pipe HR, Fig. 327, between the mouthpiece A, and the upright pipe BC; and there is poured as much water into the instrument, before using it, as brings the surface F to the level of HR, and filling the small tube to H. When the mouth A is turned to the wind, the water is driven back in the narrow tube, and a column FF_1, counterbalancing the force of the wind, rises in the tube DE, but which is measurable by the length of tube HH_1, in which the water has been driven back. If d and d_1 be the diameters, and h and h_1 the height of the columns FF_1 and HH_1, then

$$\frac{\pi d^2}{4} h = \frac{\pi d_1^2}{4} h_1, \text{ and } \therefore h = \left(\frac{d}{d_1}\right)^2 h_1, \text{ or } h_1 = \left(\frac{d}{d_1}\right)^2 h \text{ or } h_1 \text{ is always}$$

greater in the ratio $\left(\frac{d}{d_1}\right)^2$ than h, and can, therefore, be read with much greater accuracy than h. If $\frac{d}{d_1} = 5$, then the indications in HH_1 are 25 times greater than in FF_1.

Again, the differential anemometer of Wollaston, shown in Fig. 328, may be used for ascertaining the velocity of the wind. This instrument consists of two vessels B and C, and of a bent pipe DEF, which unites the two vessels by their bottoms. The one vessel is shut at top, and has a side orifice A, which is turned to the wind. The instrument is filled with *water and oil.* The former fills the two legs to about $\frac{1}{2}$, and the oil fills them up, and occupies part of each of the vessels. The force of the wind raises the water in one leg higher than it stands in the other, and the amount of this force is measured by the difference of pressure between the water column FF_1, and the oil column DD_1. If h = the height of each of these columns, and ε = the specific gravity of the oil, we then have in the last formula $h\,(1 - \varepsilon)$, instead of h, and, therefore, $v = v_0 + \alpha\sqrt{(1 - \varepsilon)\,h}$. If, for example, the oil be linseed, $\varepsilon = 0{,}94$, $v = v_0 + \alpha\sqrt{(1 - 0{,}94)\,h} = v_0 + \alpha\sqrt{0{,}061.\,h} = v_0 + 0{,}245\,\alpha\sqrt{h}$, or h is $\frac{100}{6} = 16\frac{2}{3}$ times as high as in the case of the tubes being simply filled with water. By mixing, or combining the water with alcohol, the density of the water may be brought even nearer to that of the oil; and, therefore, $1 - \varepsilon$ becomes still less, or the difference of level to be read; and, therefore, the accuracy of the reading is increased.

Fig. 328.



§ 215. Anemometers, analogous to the *stream quadrant* (Vol. I. § 381), have also been proposed and applied on the same principle, there being substituted a thin plate for the spherical ball used in gauging water streams. But a very thin metallic sphere is certainly preferable to a thin plate, for then the force of the wind remains the same for all inclinations of the rod to which it is attached, whereas it changes with the angle of inclination of the thin plate; whilst, when a sphere is used, the formula $v = \psi\sqrt{tang.\,\beta}$ (in which β is the deviation of the rod from the vertical), is sufficient. The application of a thin plate involves a complicated expression for the calculation of the velocity.

Lastly, the velocity of the wind may be ascertained by the force with which it acts directly on a plane surface opposed to it at right angles, and for this the instruments used are more or less similar to the hydrometer, described in Vol. I. § 382. If the law of the impact of wind were accurately known, the velocity of the wind might be determined without further research by these means. But this is not the case, and the formulas given in Vol. I. § 390, and the co-efficient given in § 392, lead to only approximate results. Retaining these, however, for the present, or putting the impulse of the wind

$$P = \zeta \cdot \frac{v^2}{2g} F\gamma, = 186 \cdot \frac{v^2}{2g} F\gamma,$$

or, rendered in English measures, as

$$\frac{1}{2g} = 0{,}0155, P = 0{,}02883\, v^2 F\gamma,$$

and if the density of the air $\gamma = 0{,}07974$ lb. per cubic foot, then $P = 0{,}002299$ or $0{,}0023\, v^2 F$, and $\because$ for two square feet of surfacel:

$$P = 0{,}0023\, v^2, \therefore v = \sqrt{\frac{P}{0{,}0023}} = 20{,}85 \sqrt{P} \text{ feet.}$$

For velocities $v =$	10	15	20	25	30	35	40	45	50 feet.
The impulsive force of the wind on 1 square ft. =	0,2455	0,5524	0,982	1,534	2,209	3,0070	3,928	4,971	6,1375 lb.

Admitting the above premises, the force of the wind on any surface at right angles to its direction may be easily calculated.

§ 216. *Force of Wind.*—We shall now study more closely the effect of the impulse of wind on the sails of windmills. Let us, for this purpose, conceive the whole sail surface divided into an infinite number of normal planes on the axis of the sail or arm, and suppose CD, Fig. 329, to be such an elementary plane. Owing to the considerable extent, and particularly owing to the great length of a sail, we may assume that all the wind of the column pressing on the surface CD, coming in the direction AH, will be turned off by the impact in directions parallel to CD, and, therefore, we may make use of the formulas in Vol. I. § 388. If $c =$ the velocity of the sail, Q the quantity of wind striking on CD *per second*, $\gamma =$ the density of the wind, and $\alpha =$ the angle CAH, which the direction of the wind makes with CD, then, on the assumption

Fig. 329.

that the plane CD moves away in the direction of the wind, the normal impulse of the wind on CD, is $N = \frac{c - v}{g} sin. \alpha \cdot Q \gamma$.

Putting the section $CN = G$, then the quantity of wind Q coming into action, is not $G c$, but $G (c - v)$, as the sail, moving with the velocity v, leaves a space $G v$ behind it, which takes up a proportion of the quantity of wind $G c$ following it, equal to $G v$, without undergoing any change of direction. Hence, the normal impulse may be put

$$N = \frac{c - v}{g} sin. \alpha \cdot (c - v) G \gamma = \frac{(c - v)^2}{g} sin. \alpha\, G \gamma,$$

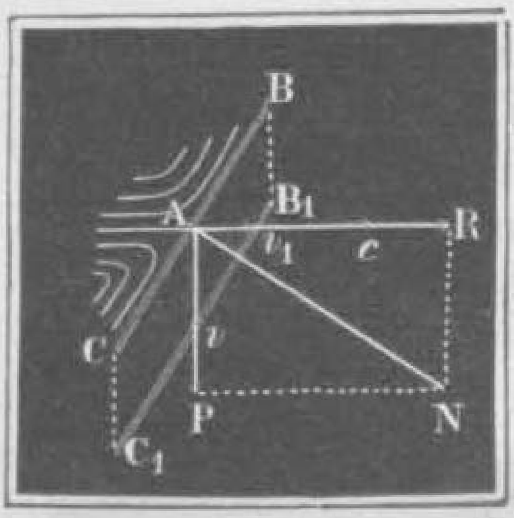
or, if F = the area of the element CD, and we substitute $F\, sin. \alpha$ for G, then $N = \frac{(c - v)^2}{g} sin. \alpha^2 F \gamma$.

Besides this impulse on the face of CD, there is a counter action on the back; inasmuch as one part of wind, passing in the directions CE and DF, at the outside of the plane, takes an eddying motion to fill up the space behind, and consequently loses pressure corresponding to the relative velocity $(c - v)\, sin. \alpha$, and represented by $\frac{(c - v)^2}{2g} sin. \alpha^2 \cdot F \gamma$. If we combine the two effects, we get the normal impulse of the wind on the element F of the sail:

$$N = \frac{(c - v)^2}{g} sin. \alpha^2 F \gamma + \frac{(c - v)^2}{2g} sin. \alpha^2 F \gamma = 3 \cdot \frac{(c - v)^2}{2g} sin. \alpha^2 F \gamma.$$

§ 217. *Best Angle of Impulse.*—In the application of this formula to windmills, we have to bear in mind that the windsail BC, Fig. 330, does not move away in the direction AR of the wind, but in a direction AP at right angles to it, and hence, in the formula

Fig. 330.



$N = 3 \cdot \frac{(c - v)^2}{2g} sin. \alpha^2 \cdot F \gamma$, we have to substitute, for v, the velocity $A v_1 = v_1$, with which the windsail moves in reference to the direction of the wind. If v = the actual velocity of rotation $\overline{A v}$ (Fig. 330), then $A v_1 = v_1 = v\ cotg.\ \overline{A v_1 v} = v\ cotg. \alpha$; and, therefore, for the case in question:

$$N = 3 \cdot \frac{(c - v\, cotg. \alpha)^2}{2g} \cdot sin. \alpha^2 F \gamma, \text{ or} = 3 \frac{(c\, sin. \alpha - v\, cos. \alpha)^2}{2g} F \gamma.$$

This normal impulse is to be decomposed into two others, P and R, one acting in the direction of rotation, the other in the direction of the axis of the element of the sail; then

$$P = N\, cos. \alpha = 3 \frac{(c\, sin. \alpha - v\, cos. \alpha)^2}{2g} cos. \alpha \cdot F \gamma, \text{ and}$$

$$R = N\, sin. \alpha = 3 \frac{(c\, sin. \alpha - v\, cos. \alpha)^2}{2g} sin. \alpha \cdot F \gamma.$$

By multiplying by the velocity of rotation v, we get, from the formula for P, the mechanical effect of the windsail.

$$L = Pv = 3\frac{(c\ sin.\ a - v\ cos.\ a)^2}{2g}\ v\ cos.\ a\ .\ F\gamma.$$

The parallel or axial force R, gives no mechanical effect, but on the contrary increases the pressure on the pivot or footstep at the lower end of the windshaft, and so gives rise to a *loss* of effect.

The last formula indicates, and it is self-evident, that the effect increases with the velocity c, and with the area F; but it is not so evident from it, how the angle of impulse a, affects the mechanical effect produced. That L may not be $= 0$, $c\ sin.\ a$ must be $> v\ cos.\ a$; that is, $tang.\ a > \frac{v}{c}$, and $cos.\ a > 0$, and, therefore, $a < 90°$. There must, therefore, be a value of a between the limits $tang.\ a > \frac{v}{c}$, and $a < 90°$, corresponding to a maximum value of L. To find this value, let us instead of a put $a \pm x$, x being a very small angle. Then we have $sin.\ (a \pm x) = sin.\ a\ cos.\ x \pm cos.\ a\ sin.\ x$, or putting $cos.\ x = 1$, and $sin.\ x$ being put $= x$,

$sin.\ (a \pm x) = sin.\ a \pm x\ cos.\ a$, further:

$cos.\ (a \pm x) = cos.\ a\ cos.\ x \mp sin.\ a\ sin.\ x = cos.\ a \mp x\ sin.\ a$, and these values give us as the effect:

$$L = \frac{3\,c^2 v}{2g} F\gamma\left(sin.\ a - \frac{v}{c}\ cos.\ a\right)^2 cos.\ a,$$

$$L_1 = \frac{3\,c^2 v}{2g} F\gamma([sin.\ a \pm x\ cos.\ a - \frac{v}{c}(cos.\ a \mp x\,sin.\ a)]^2(cos.\ a \mp x\,sin.\ a))$$

$$= \frac{3\,c^2 v}{2g} F\gamma[sin.\ a - \frac{v}{c} cos.\ a \pm (cos.\ a + \frac{v}{c} sin.\ a)x]^2(cos.\ a \mp x\ sin.\ a)$$

$$= \frac{3\,c^2 v}{2g} F\gamma\ (sin.\ a - \frac{v}{c}\ cos.\ a)^2\ cos.\ a$$

$$\pm [2\,(sin.\ a - \frac{v}{c}\ cos.\ a)\ (cos.\ a + \frac{v}{c}\ sin.\ a)\ cos.\ a - (sin.\ a - \frac{v}{c}\ cos.\ a)^2$$

$$sin.\ a]\ x +),\ \&c.,\ \&c.$$

$$= L \pm \frac{3\,c^2 v}{2g} F\gamma\ ([2\ (sin.\ a - \frac{v}{c}\ cos.\ a)\ (cos.\ a + \frac{v}{c}\ sin.\ a)\ cos.\ a$$

$$- (sin.\ a - \frac{v}{c}\ cos.\ a)^2\ sin.\ a]\ x + \&c.)$$

In order that a may give the maximum value, L_1 must be less than L, a being increased or diminished by x, that is, x being positive or negative. But the last formula gives in the one case $L_1 > L$, and in the other $< L$, so long as the second member $\pm \frac{3\,c\,v}{2g} F\gamma\ [\ .\ .\ .\ ]\ x$ is a real quantity. Therefore, for obtaining the maximum value, it is necessary that this second member should be 0, or, that

$$2\left(\sin.\alpha - \frac{v}{c}\cos.\alpha\right)\left(\cos.\alpha + \frac{v}{c}\sin.\alpha\right)\cos.\alpha - \left(\sin.\alpha - \frac{v}{c}\cos.\alpha\right)^2 \sin.\alpha = 0,$$

or $2\left(\cos.\alpha + \frac{v}{c}\sin.\alpha\right)\cos.\alpha = \left(\sin.\alpha - \frac{v}{c}\cos.\alpha\right)\sin.\alpha,$

or $\sin.\alpha^2 - \frac{3v}{c}\sin.\alpha\cos.\alpha = 2\cos.\alpha^2.$

Dividing by $\cos.\alpha^2$, and putting $\frac{\sin.\alpha}{\cos.\alpha} = tang.\alpha$, we have

$$tang.\alpha^2 - \frac{3v}{c}tang.\alpha = 2,$$

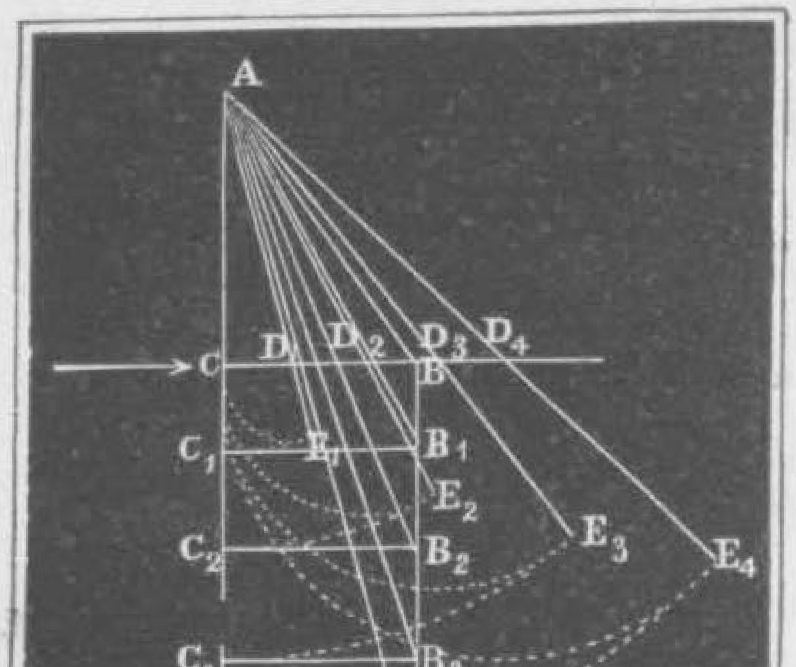
from which we deduce as the angle for the maximum effect:

$$tang.\alpha = \frac{3v}{2c} + \sqrt{\left(\frac{3v}{2c}\right)^2 + 2}.$$

As, in the windsails, the outer elements of the sail have a greater velocity than those nearer the axis of rotation, it follows that the outer part of the sail should be set at a greater angle of impulse than the inner, in order to insure a maximum effect. Hence the sails must not be plane surfaces, but "*surfaces gauches*," or surfaces of double curvature, warped so that the outer part deviates less from the plane of the axis of rotation, than the inner part.

Remark. The most advantageous angles of impulse of a sail, may also be ascertained by the following construction. Fig. 331, take $CB = 1$, set off $CA = \sqrt{2}$ at right angles to it, *i. e.*, the diagonal of a square on CB, and draw AB. The *tang.* $ABC = \sqrt{2}$, and, therefore, $\angle ABC = 54°. 44', 8''$, which is the angle of impulse close up to the axis of rotation. If in $y = \frac{3\omega x}{2c}$, we make c the velocity of the wind, and ω the angular velocity, and for x successively, the distance of the sail bars from the windshaft axis, and set off these values of y from C on CB as CD_1, CD_2, CD_3, &c. Further draw the hypothenuses AD_1, AD_2, AD_3, &c., and prolong them, so that $D_1E_1 = CD_1$, $D_2E_2 = CD_2$, $D_3E_3 = CD_3$, &c. Lastly, lay off AE_1, AE_2, AE_3, in the direction AC as AC_1, AC_2, AC_3, &c., raise at C_1, C_2, C_3, &c., the perpendiculars C_1B_1, C_2B_2, C_3B_3, &c., $= CB = 1$, and draw AB_1, AB_2, AB_3, &c., then AB_1C_1, AB_2C_2, AB_3C_3, &c., are the angles required; for

Fig. 331.



$$tang. AB_1C_1 = \frac{AC_1}{B_1B_1} = \frac{AE_1}{1} = D_1E_1 + AD_1 = y_1 + \sqrt{y_1^2 + 2},$$

$$tang. AB_2C_2 = \frac{AC_2}{A_2C_2} = \frac{AE_2}{1} = D_2E_2 + AD_2 = y_2 + \sqrt{y_2^2 + 2}, \text{ \&c.}$$

§ 218. *The Mechanical Effect.*—The formula for the best angle of the sails may be used conversely to determine the best velocity of rotation for a given angle α. For this

$$tang.\, \alpha^2 - \frac{3v}{c}\, tang.\, \alpha = 2,$$

and, therefore, very simply,

$$v = \left(\frac{tang.\, \alpha^2 - 2}{tang.\, \alpha}\right) \cdot \frac{c}{3} = (tang.\, \alpha - 2\, cotang.\, \alpha)\frac{c}{3}.$$

If we put this value in the formula for the mechanical effect, we have

$$L = \frac{3\, c^2}{2g} F\gamma \cdot \frac{tang.\, \alpha^2 - 2}{tang.\, \alpha} \cdot \frac{c}{3} \cdot \left(sin.\, \alpha - \frac{tang.\, \alpha^2 - 2}{3\, tang.\, \alpha}\, cos.\, \alpha\right)^2 cos.\, \alpha$$

$$= \tfrac{4}{9} \cdot \frac{c^3}{2g} F\gamma \cdot \frac{(tang.\, \alpha^2 - 2)\, cos.\, \alpha^2}{sin.\, \alpha^3} = \tfrac{4}{9} \cdot \frac{c^3}{2g} F\gamma \cdot \frac{(3\, sin.\, \alpha^2 - 2)}{sin.\, \alpha^3}.$$

The theoretical effect of a windsail, may hence be calculated for any given velocity of wind, and of rotation. From a given number of revolutions per minute, we have the angular velocity $\omega = \frac{\pi\, u}{30} = 0{,}1047 \,.\, u$. If the whole length of whip be divided into 7 equal parts, and if, as usual, the sail begins at the lowest point of division, so that its total length $= \frac{6}{7}\, l$, we can very easily, by means of the formula

$$tang.\, \alpha = \frac{3\, v}{2\, c} + \sqrt{\left(\frac{3\, v}{2\, c}\right)^2 + 2},$$

calculate the best angle of sail $\alpha_0, \alpha_1, \alpha_2$, &c., or for each of the points of division of the whip, by substituting successively

$$v_0 = \omega \cdot \frac{l}{7},\ v_1 = \omega \cdot \frac{2\, l}{7},\ v_2 = \omega \cdot \frac{3\, l}{7} \ldots \text{to } v_6 = \omega \cdot \frac{7\, l}{7} \text{ or } \omega\, l.$$

If, further, $b_0, b_1, b_2 \ldots b_6$ be the width of sail to be put on eachl of these points, we can calculate, by aid of Simpson's rule, from $\left(\frac{3\, sin.\, \alpha_0^2 - 2}{sin.\, \alpha_0^3}\right) b_0$, $\left(\frac{3\, sin.\, \alpha_1^2 - 2}{sin.\, \alpha_1^3}\right) b_1$, $\left(\frac{3\, sin.\, \alpha_2^2 - 2}{sin.\, \alpha_2^3}\right) b_2$, &c., a mean valuel k, and, hence, we arrive at the whole effect of the sail $L = \frac{4}{9}\, k\, \gamma \,.\, \frac{6}{7}\, l \,.\, \frac{c^3}{2g}$, or, more generally, l_1 being the length of *sail*, properly so called, $L = \frac{4}{9}\, \gamma\, k\, l_1 \frac{c^3}{2g}$.

If the sail were a plane surface, that is, if α were constant throughout its whole extent, then, by means of

$$v_0 = \frac{\omega\, l}{7},\ v_1 = \omega \cdot \frac{2l l}{7}, \text{ \&c.},$$

we should first calculate the corresponding values:

$$\left(sin.\, \alpha - \frac{v_0}{c}\, cos.\, \alpha\right)^2 \frac{v_0}{c}\, cos.\, \alpha \,.\, b_0,\ \left(sin.\, \alpha - \frac{v_1}{c}\, cos.\, \alpha\right)^2 \frac{v_1}{c}\, cos.\, \alpha \,.\, b_1, \text{ \&c.},$$

and then from these, by Simpson'sl rule, deduce the mean value k_1, and introduce this into the formula for the mechanical effect developed, $L = 3\, \gamma\, k_1 \,.\, l_1 \,.\, \frac{c^3}{2g}$.

If n be the number of sails, we have of course to multiply the last found value by this number, to get the whole mechanical effect developed by the windsail wheel, or $L = 3\, n\, \gamma\, k_1\, l_1 \frac{c^3}{2g}$.

Example 1. What angle of impulse is required for a windsail wheel, the velocity of the wind being 20 feet, the nnmber of sails 4, each being 24 feet in length, and 6 to 9 feet in width? Number of revolutions 16 per minute. What will be the theoretical effect of this windmill?

In the first place, the angular velocity $w = 0{,}1047 \,.\, 16 = 1{,}6755$ feet, and if the distance of the first sail bar be 4 feet from the axis of the shaft, or the total length of whips $= 24 + 4 = 28$ feet, then for the

Distancese	4	8	12	16	20	24	28 Feet.
The velocities:	6,702	13,404	20,106	26,808	33,510	40,212	46,914 ft.
The tangents of the angles of impulse:	2,004	2,740	3,575	4,469	5,397	6,347	7,311
The angles:	63°,29′	69°,57′	74°,22′	77°,23′	79°,30′	81°,3′	82°,13′
The values of $\frac{3\, \sin. a^2 - 2}{\sin. a^3}$:	0,5612	0,7810	0,8759	0,9220	0,9472	0,9622	0,9716
The width of sails:	6,0	6,5	7,0	7,5	8,0	8,5	9,0 feet
The product of the two last:	3,367	5,076	6,131	6,915	7,578	8,179	8,744

And from the last product the mean value:

$$k = \frac{3{,}367 + 8{,}744 + 4 \,.\, (5{,}076 + 6{,}915 + 8{,}179) + 2 \,.\, (6{,}131 + 7{,}578)}{18}$$

$$= \frac{12{,}111 + 80{,}680 + 27{,}418}{18} = \frac{120{,}209}{18} = 6{,}679,$$ and if we put:

$$\gamma = 0{,}07974 \text{ lbs. [illegible] } l = 24, \text{ and } \frac{c^3}{2g} = 0{,}0155 \times 20^3 = 124,$$

then the effect of this wheel:

$L = 4 \,.\,$ [illegible] $\,.\, 6{,}679 \,.\, 0{,}07974 \,.\, 24 \,.\, 124 = 11{,}874 \,.\, 1{,}91 \,.\, 124 = 2798$ feet lbs. $= 5$ horse power.

Example 2. What effect may be expected from a windmill wheel, having four plane sails, and the angle of impulse 75°, the other dimensions and proportions being the same as those of the wheel in the last example? In this case

The velocities of ratio $\frac{v}{c}$:	0,3351	0,6702	1,0053	1,3404	1,6755	2,0106	2,3457
The differences $\sin. a - \frac{v}{c} \cos. a$:	0,8792	0,7925	0,7057	0,6190	0,5323	0,4456	0,3588
The width b:	6,0	6,5	7,0	7,5	8,0	8,5	9,0 feet
The products $(\sin. a - \frac{v}{c} \cos. a)^2 \times \frac{v}{c} \cos. a\, b$:	0,4023	0,7081	0,9071	0,9967	0,9830	0,8783	0.7034

From the latter products we deduce, by Simpson's rule, the mean value

$k_1 = \frac{1}{18} [0{,}4023 + 0{,}7034 + 4\, (0{,}7081 + 0{,}9969 + 0{,}8783) + 2\, (0{,}9071 + 09830)]$

$= \frac{1}{18} (1{,}1057 + 10{,}3324 + 3{,}7802) = \frac{15{,}2183}{18} = 0{,}8455$, and from this we have the effect required $L = 4 \,.\, 3 \,.\, 0{,}8455 \,.\, 0{,}7974 \,.\, 24 \,.\, 124 = 2390 = 4{,}34$ horse power, instead of 5 horse power, found when the sails are *warped*.

§ 219. *Loss by Friction.*—A considerable part of the mechanical effect developed by the wind on the sails, is consumed by the friction of the windshaft at the neck, especially if the diameter of this be great, as is not unfrequently the case. We may assume that the whole weight of the sail wheel bears on the neck, and thus leave out of consideration the pressure on the lower or back bearing. Although we shall thus find an excess of friction, yet this is compensated by leaving out of consideration the friction arising on the back pivot from the force of the wind in the axial direction. As the back pivot is much less in diameter than the neck or front gudgeon, this simplification of the problem may be the more readily admitted. This being assumed, we have from the weight G of the whole wheel, $F = f\,G$ = the friction, and if r = the radius of the neck, and $\omega\, r$ the angular velocity, the mechanical effect consumed:

$$F\,\omega\, r = f\,G\,\omega\, r = 0{,}10471.\, u f\,G\, r = f\,G\,\frac{r}{l}\,v,$$

if v be the velocity at the periphery of the sail wheel.

This being allowed, the useful effect of a windmill with plane sails:

$$L = 3\, n\,\gamma\, k_1\, l_1\,.\,\frac{c^3}{2g} - f\,G\,\frac{r}{l}\,v,$$

and that of one with warped sails:

$$L = \tfrac{4}{9}\, n\,\gamma\, k\, l_1\,.\,\frac{c^3}{2g} - f\,G\,\frac{r}{l}\,v.$$

From the formula:

$$L = \frac{3\,(c\, sin.\, a - v\, cos.\, a)^2}{2g}\, v\, cos.\, a\,.\, F\gamma,$$

for the theoretical effect of an element of a sail, we may deduce the influence of the velocity of the sail on the mechanical effect, and we find that for $v\, cos.\, a = \frac{c\, sin.\, a}{3}$ (compare Vol. II. § 118), that is, for $v = \frac{c\, tang.\, a}{3}$, the effect is a maximum. If we introduce this value into the above formula, we get

$$L = 3\,.\,\tfrac{4}{27}\,.\,\frac{c^3\, sin.\, a^3}{2g}\, F\gamma,$$

and from this we deduce that the effect will be greatest when the angle $a = 90^\circ$, or $v = \infty$. These conditions cannot be fulfilled; because, even for moderately great velocities, the prejudicial resistances, and more particularly the friction at the neck, consume so much mechanical effect, that the useful effect remaining is very small. The velocity of rotation should be great to insure a good efficiency, but it must in each case be made a special subject of calculation, as to what number of revolutions will give the maximum effect. This can only be done by calculating the effect for a series of velocities of rotation, and from these choosing the greatest, or deducing it by interpolation.

Example. Supposing the windshaft, sails, &c., of the mill in the last example weighs 7500 lbs., that the radius of the neck or gudgeon $r = \frac{1}{3}$ foot, that the co efficient of friction $f = 0{,}1$, then the mechanical effect lost by friction at the neck $= 0{,}1\,.\,7500\,.\,\omega\, r = 419$ feet lbs. There remains, therefore, in the wheel with *warped* sails $2793 - 419 = 2590$ feet lbs., or about 86 per cent. of the theoretical effect. When the shaft is of wood, the neck is double the above diameter, and, hence, the loss of effect by friction is double, or the efficiency is only 0,70.

§ 220. *Experiments.*—Experiments or observations on windmills, of accuracy sufficient to test our theory, are not extant. There is no lack of general statements of the results of the effects of different windmills, but these are not of a nature to serve for judging of the *efficiency* of the machines referred to, inasmuch as the velocity of the wind has been either altogether undetermined, or ascertained by instruments not sufficiently trustworthy. The experiments of Coulomb and Smeaton are still the most complete, there being, in fact, none of recent date. Coulomb made his experiments on one of the many windmills in the neighborhood of Lille; and from the circumstance of the work done, being the pressing of oil by means of stampers, a kind of work, the mechanical effect consumed in which is easily calculated, deductions from these experiments may be very safely made. The four sails of this mill were *warped* in the Dutch style, with the angle of impulse from $63\frac{3}{4}°$ to $81\frac{1}{4}°$, and each of them contained about 20 square metres, or 215 square feet. The experiments were made when the velocity of the wind was from 7 to 30 feet per second, the velocity at the periphery being from 23 to 70 feet, and the results correspond, according to Coriolis (*see* "Calcul de l'effet des Machines), with those of the theory above given. It is, besides, easy to perceive that, for the better construction, when *warped* sails are used, the mean value of $\frac{3 \sin. a^2 - 2}{\sin. a^3}$, cannot vary very much from that which is deduced by calculation in the first example § 218, viz. = 0,880. If, now, we introduce this into the general formula, we obtain the following very simple expression for the effect of a windmill:

$$L = \tfrac{4}{9} \cdot 0{,}88 \cdot 0{,}0781 \cdot n\,F \frac{c^3}{2g} = 0{,}000473\, n\, F\, c^3 \text{ ft. lbs.}$$

The mean of Coulomb's observations, gives

$$L = 0{,}026\, n\, F\, c^3 \text{ kilogrammetres, or}$$

$$L = 0{,}000511\, n\, F\, c^3 \text{ ft. lbs.}$$

or a near approximation to the theoretical determination. We may with safety assume

$$L = 0{,}00048\, n\, F\, c^3 \text{ ft. lbs.}$$

This formula only gives satisfactory results, however, when the velocity at the extremity of the sails is about $2\frac{1}{2}$ times that of the wind, as indicated by theory to be the best velocity.

Example. Suppose a windmill of 4 horse power, when the velocity of the wind is 16 feet per second is required. What sail surface must it have? According to the last formula, $n\,F = \frac{4 \cdot 510}{0{,}00048 \cdot 16^3} = \frac{4249320}{4096} = 1030$ square feet; that is, for 5 sails each, 206 square feet. If l_1 the length $= 5$ times the mean breadth b, then $5\, b^2 = 206 \therefore b = \sqrt{41} = 6\frac{1}{3}$ feet, and the length $l_1 = 31\frac{2}{3}$ feet.

§ 221. *Smeaton's Maxims.*—The great English civil engineer, John Smeaton, instituted a very complete inquiry into the power of wind, and made a series of experiments, the results of which are given in the following table:—

TABLE,

Exhibiting the Results of Nineteen Sets of Experiments on Windmill Sails, of various Structures, Positions, and Extents of Surface.

The description of sails made use of.	Number.	Angle at the extremities.	Greatest angle.	Turns of the Sails unloaded.	Turns of the Sails at the Maximum.	Load at the Maximum.	Greatest Load.	Product.	Extent of Surface.	Ratio of greatest Velocity to the Velocity at a Maximum.	Ratio of greatest Load to the Load at a maximum.	Ratio of Surface to the Product.
		degrees	degrees			lb.	lb.		sq. in.			
Plane sails, at an angle of 55° . .	1	35	35	66	42	7.56	12.59	318	404	10.7	10:6	10:7.9
Plane sails, weathered according to the common practice . .	2	12	12	—	70	6.3	7.56	441	404	—	10:8.3	10:10.1
	3	15	15	105	69	6.72	8.12	464	404	10:6.6	19:8.3	10:10.15
	4	18	18	96	66	7.0	9.81	462	404	10:7	10:7.1	10:10.15
Weathered according to M'Laurin's Theory	5	9	26½	—	66	7.0	—	462	404	—	—	10:11.4
	6	12	29½	—	70½	7.35	—	518	404	—	—	10:12.8
	7	15	32½	—	63½	8.3	—	527	404	—	—	10:13.
Sails weathered in the Dutch manner, tried in various positions.	8	0	15	120	93	4.75	5.31	442	404	10:7.7	10:8.8	10:11.
	9	3	18	120	79	7.0	8.12	553	404	10:6.6	10:8.6	10:13.7
	10	5	20	—	78	7.5	8.12	585	404	—	10:9.2	10:14.5
	11	7½	29½	123	77	8.3	9.81	639	404	10:6.8	10:8.5	10:15.8
	12	10	25	108	73	8.69	10.37	634	404	10:6.8	10:8.4	10:15.7
	13	12	27	100	66	8.41	10.94	580	404	10:6.6	10:7.7	10:14.4
Sails weathered in the Dutch manner, but enlarged towards the extremities	14	7½	22½	123	75	10.65	12.59	799	505	10:6.1	10:8.5	10:15.8
	15	10	25	117	74	11.08	13.69	820	505	10:6.3	10:8.1	10:16.2
	16	12	27	114	66	12.09	14.23	799	505	10:5.8	10:8.4	10:15.8
	17	15	30	96	63	12.09	14.78	762	505	10:6.6	10:8.2	10:15.1
Eight sails, being sectors of ellipses, in their best positions .	18	12	22	105	64½	16.42	27.87	1059	854	10:6.1	10:5.9	10:12.4
	19	12	22	96	64½	18.06	—	1165	1146	10:5.9	—	10:10.1

The experimental wheel had whips 21 inches long, the sails being 18 inches long, and 5,6 inches broad. This wheel was not moved by the impulse of wind, but was moved round in air at rest, whence it was the resistance of the air, and not its impulse, which was observed—a circumstance taking considerably from the value of the experiments. The motion of the sails against the wind, was given by means of an upright shaft, from which projected an arm $5\frac{1}{2}$ feet long, at the end of which was a seat for the model mill wheel. This upright shaft was set in motion by the observer having a cord wound round it like the peg of a top. To measure the *resistances of the air*, supposed here to be identical with the impulse of wind of the same velocity, there was a scale with weights, attached by a fine cord to the shaft of the wind wheel, and this was wound up by the power communicated to the sails. The results of these experiments correspond well *qualitatively* with our theory. They show to demonstration that the warped sail gives the best effect, and that the angles of impulse deduced by theory are actually the best. In the example to § 218, we found the angles for 7 bars, starting from next the axle, to be: 63° 29′; 69° 57′; 74° 22′; 77° 23′; 79° 30′; 81° 3′, and 82° 13′, and Smeaton found the following 6 angles to be the best, or at least very good, 72°; 71°; 72°; 74°; $77\frac{1}{2}$°; 83°; or very little different from the theory.

Smeaton remarks, too, that a deviation of 2 degrees in the angle of impulse, has no sensible influence on the mechanical effect produced by the wheel.

Smeaton draws the following *maxims* from his experiments, made at velocities varying from $4\frac{1}{3}$ to $8\frac{3}{4}$ feet per second.

1. The velocity of the windmill sails, whether unloaded or loaded, so as to produce a maximum, is nearly as the velocity of the wind, their shape and motion being the same.

2. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.

3. The effects of the same sails at a maximum are nearly, but somewhat less than, as the cubes of the velocity of the wind.

4. The load of the same sails at the maximum is nearly as the squares, and their effects as the cubes of their number of turns in a given time.

5. When the sails are loaded so as to produce a maximum at a given velocity, and the velocity of the wind increases the load containing the same: first, the increase of effect, when the increase of the velocity of the wind is smaller, will be nearly as the squares of those velocities; secondly, when the velocity of the wind is double, the effects will be nearly as 10 to $27\frac{1}{2}$; but, thirdly, when the velocities compared are more than double of that where the given load produces a maximum, the effects increase nearly in a simple ratio of the velocity of the wind.

6. If sails are of a similar figure and position, the number of

turns in a given time will be reciprocally as the radius or length of the sail.

7. The load at a maximum that sails of a similar figure and position will overcome, at a given distance from the centre of motion, will be as the cube of the radius.

8. The effect of sails of similar figure and position are as the square of the radius.

9. The velocity of the extremity of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably quicker than the velocity of the wind.

According to these experiments, the effect of the wind on windmill sails is greater than theory indicates, or than Coulomb's experiments gave.

Literature. The most complete exposition of the theory of windmills is given in Weisbach's "Bergmaschinen Mechanik," vol. ii., and in Coriolis's "Traité du Calcul â l'effet des Machines." Smeaton's experiments are recorded in the "Philosophical Transactions," 1759 to 1776. They were collected into a separate volume, and published under the title "An experimental Enquiry concerning the natural powers of Water and Wind to turn Mills and other Machines depending on a circular motion." These papers were translated into French by Girard, in 1827. There are extracts from them in Barlow's "Treatise on the Manufactures," &c. In Nicholson's "Operative Mechanic," Brewster's, Ferguson's, &c., &c. Coulomb's experiments are given in his oft-quoted work "Théorie des Machines simples."

Mariotte wrote upon the impulse of wind, in his "Hydrostatics." He makes the impulse $P = 1,73 \frac{c^2}{2g} F \gamma$.

Borda, in the "Mémoires de l'Académie de Paris," 1763, has a paper; Rouse, Hutton, Woltmann, have all handled this subject. The two latter authors find P much smaller than Mariotte did, because they measured the *resistance*, not the *impulse* of the wind. The co-efficient $\zeta = \frac{4}{3}$, as found by Woltmann, is too small, or $P = \frac{4}{3} \frac{c^2}{2g} F \gamma$ is certainly too little, for he did not obtain the *constants* for his windsail wheel by direct experiment (*see* "Theorie und Gebrauch des Hydrometrischen Flügels," Hamburg, 1790). Hutton deduces from his experiments, that it is more accurate to consider the impulse and resistance of the air as increasing as $F^{0,1}$ (*see* "Philosophical and Mathematical Dictionary," vol. ii.). If we assume $\zeta = 1,86$ for a small surface of 1 square foot, then, for a sail of 200 square feet surface, we should have $\zeta = 200^{0,1} \cdot 1,86 = 1,7 \cdot 1,86 = 3,162$, which agrees well with the theoretical determination, and with what we have said above, where $\zeta = 3$ and $P = 3 \cdot \frac{c^2}{2g} F \gamma$. In Poncelet's "Introduction à la Mécanique industrielle," there is an admirable collection and discussion of the experiments on impulse and resistance of wind.