ESSAYS on ASSET PRICING MODELS: THEORIES and EMPIRICAL TESTS

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My dissertation contains three chapters. Chapter one proposes a nonparametric method to evaluate the performance of a conditional factor model in explaining the cross section of stock returns. There are two tests: one is based on the individual pricing error of a conditional model and the other is based on the average pricing error. Empirical results show that for value-weighted portfolios, the conditional CAPM explains none of the asset-pricing anomalies, while the conditional Fama-French three-factor model is able to account for the size effect, and it also helps to explain the value effect and the momentum effect. From a statistical point of view, a conditional model always beats a conditional one because it is closer to the true data-generating process.

Chapter two proposes a general equilibrium model to study the implications of prospect theory for individual trading, security prices and trading volume. Its main finding is that different components of prospect theory make different predictions. The concavity/convexity of the value function drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive correlation between returns and volumes. On the other hand, loss aversion predicts exactly the opposite, namely a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volumes. In a calibrated economy, when

prospect theory preference parameters are set at the values estimated by the previous studies, our model can generate price momentum of up to 7% on an annual basis.

Chapter three studies the role of aggregate dividend volatility in asset prices. In the model, narrow-framing investors are loss averse over fluctuations in the value of their financial wealth. Persistent dividend volatility indicates persistent fluctuation in their financial wealth and makes stocks undesirable. It helps to explain the salient feature of the stock market including the high mean, excess volatility, and predictability of stock returns while maintaining a low and stable risk-free rate. Consistent with the data, stock returns have a low correlation with consumption growth, and Sharpe ratios are time-varying.

BIOGRAPHICAL SKETCH

Yan Li graduated from Peking University with a B.A. degree in economics in 1997. She continued her study in Peking University and obtained her master degree in economics in 2001. In August of 2003, she began her doctoral studies in economics at Cornell University.

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CHAPTER 1

A NEW TEST of TIME-VARYING FACTOR MODELS [1]

1.1 Introduction

Tests of time-varying factor models have caught a lot of attention in recent

literature. On the one hand, abundant empirical evidence have shown that

betas from a factor model are time-varying (e.g., Fama and French, 1997;

Lewellen and Nagel, 2006). On the other hand, tests of the unconditional

CAPM, one of the most important factor models, fails to explain the cross

section of stock returns (e.g., Fama and French, 1993). As a result, a lot of

research efforts have been devoted to exploring the performance of a

conditional factor model by allowing betas and expected returns to vary over

time. A long-standing approach to testing a time-varying factor model is to

allow factor loadings to depend on observable state variables (e.g., Shanken,

1990; Lettau and Ludvigson, 2001).[2]

Recently, Lewellen and Nagel (2006; henceforth, LN) don't use state variables

but divide data into non-overlapping small windows such as months, quarters,

half-years or years, and directly estimate the time series of alphas and betas

from short-window regressions.[3] If the conditional CAPM holds period-by-

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[1]This chapter is based on a joint paper with Liyan Yang.

[2]Numerous studies rely on state variables in the estimation of CAPM and other
asset pricing models. See, for example, Campbell (1987), Ferson, Kandel and
Stambaugh (1987), Ferson and Harvey (1991, 1993), Cochrane (1996), Wang
(2003), Petkova and Zhang (2005), and Santos and Veronesi (2006).

[3]It is arguable whether dividing data into small windows is a right way to
condition on information. In this paper, we don't attempt to participate in this
debate and follow Lewellen and Nagel (2006) by assuming that investors'

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period, then the average pricing errors from small window regressions should be equal to zero. Contrary to some other recent studies (e.g., Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001; Santos and Veronesi, 2006; Lustig and Van Nieuwerburgh, 2005), LN find that conditioning doesn't improve the performance of the simple and consumption CAPMs.

The time-series test proposed by LN possesses a special advantage over traditional cross-sectional tests which ignore important restrictions on cross-sectional slopes. However, it also has its own limitation. As argued in Boguth, Carlson, Fisher, and Simutin (2008; henceforth, BCFS), the procedures in LN can lead to potentially large biases in alphas, which arises when the division of windows is too fine. In other words, the test of LN is subject to a small sample bias. After correcting for this small sample bias with standard instruments, BCFS manage to obtain much smaller alphas for momentum portfolios, leading them to conclude that the conditional CAPM is superior to the unconditional CAPM in explaining momentum portfolios.

In essence, the small sample bias in LN will eventually vanish as the window size increases. However, when estimated using data from a large window size, betas will generally not be stable. This makes the test subject to the underconditoning bias, which occurs when empirical tests of a conditional model fail to account for the investor's time-varying information set (e.g., Hansen and Richard, 1987; Jagannathan and Wang, 1996). Therefore, the ideal test would rely on an optimal window size which takes into account both

information sets change gradually and thus betas are stable within certain time periods.

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the underconditioning bias and the small sample bias.

The first goal of this paper is to propose such a test.[4] Following LN, we don't rely on state variables, but assume that the investor's information set is relatively stable within certain time periods. Rather than dividing the windows arbitrarily, we use a nonparametric method to find the data-driven window size, such that within the window (i) investors' information sets don't greatly change; (ii) there are sufficient observations to achieve estimation efficiency. In other words, our estimation aims to minimize both the underconditioning and the small sample biases. We find that the optimal window size varies greatly across different portfolios. For instance, in the test of the conditional CAPM, the optimal window size varies from as short as 47 days to as long as 333 days for different value-weighted portfolios. To compare our results with those obtained by LN, we also estimate the model using their non-overlapping window approach. We find that the estimates from LN's method are very sensitive to the window size. When the window size changes from one month to three months, the monthly average pricing error can differ by as much as 1%! More importantly, different window estimates can also lead to different inferences. Therefore, arbitrarily fixing the window size as three months or six months for all portfolios may lead to unreliable estimates and inconsistent inferences.

The second goal of this paper is to consider a more general nonlinear relationship between asset returns and factor returns. Ang and Chen (2002),

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[4]In a contemporaneously proposed paper, Ang and Kristensen proposed a similar test to ours.

Ang, Chen and Xing (2006), and Hong, Tu and Zhou (2007) show that many securities covary differently when the market goes down from when it goes up, providing evidence of payoff nonlinearity[5]. Our empirical studies are based on a nonparametric methodology that avoids the misspecification between asset returns and factor returns, and hence is immune from potential nonlinearity biases. In testing the conditional CAPM, Wang (2003) also uses a nonparametric method to avoid nonlinearity biases, but his focus is on the nonlinear relationship between betas (risk premia) and state variables that represent conditioning information. We, in contrast, don't rely on state variables; we are concerned with the nonlinear relationship between asset returns and factor returns.

Our estimation method possesses further advantages. First, we use an overlapping window estimation, which allows a gradual change in betas rather than a drastic change through the non-overlapping window estimation as in Grundy and Martin (2001) and LN. Moreover, previous studies on time-varying betas by Campbell and Vuolteenaho (2004), Fama and French (2006), and LN, among others, assume that betas are constant within subsamples, thereby ignoring the variations in the betas within each window. Our method estimates the conditional alphas and betas at every point in time, and hence directly captures the variations that are overlooked by these studies. Another advantage is that our estimation is conducted in the spirit of generalized least

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[5]The nonlinear relationship considered in Ang and Chen (2002), Ang, Chen and Xing (2006), and Hong, Tu and Zhou (2007) depends on realized data, hence it is an *ex-post* relationship. Our nonlinear relationship is *ex-ante* because at time t we don't observe the realization at $t+1$.

squares, which puts more weight on recent data than on remote data, thus improving estimation efficiency.

Our estimation method applies to any time-varying factor models. In this paper, we focus on two models: the conditional CAPM and the conditional Fama-French three-factor model (1993; henceforth, the FF model). These two models have been widely used in empirical applications but whether they are able to explain the cross section of stock returns are in the spotlight of current research. After estimating these two models, we propose two tests to examine their performance in explaining asset-pricing anomalies. The null hypothesis is that if a conditional model holds at every point in time, then the pricing error should be zero at all time periods. Our first test focuses on individual pricing errors, i.e., we examine whether a conditional model holds at any given time. The unique advantage of this test is that it enables us to identify the exact time periods in which a conditional model holds or fails. Investigating the time periods when a model holds might sharpen our understanding of the conditions under which a model better applies, and examining the periods in which a model fails might help us identify the missing factors to further improve the model.

Our second test looks at the average pricing error. That is, if a conditional model holds, then the average pricing error should be zero. Under a general assumption of heteroskadastic innovations, we derive the asymptotic distribution of the average pricing error, which turns out to follow a normal distribution. For value-weighted portfolios, our results show that the conditional CAPM fails to explain any of the asset-pricing anomalies. For these portfolios,

the conditional FF model explains the size and value effect quite well, thus standing in sharp contrast to the results in Ferson and Siegel (2003) among others; it also helps to explain the momentum portfolios, but unlike Wang (2003), we strongly reject the model. For equally-weighted portfolios, it's rather difficult for either the conditional CAPM or the conditional FF model to explain return variations.

In addition to evaluating the conditional models from an economics point of view, i.e., whether they are able to explain asset-pricing anomalies, we also perform a statistical test to evaluate the goodness of fit for the conditional versus the unconditional models. We are interested in which model, the conditional or the unconditional, is closer to the true data-generating process. Our results show that the conditional models invariably outperform their unconditional counterparts for all portfolios, implying that the conditional models fit the data better.

The paper proceeds as follows. Section 1.2 introduces the methodology used to estimate and test the conditional models. Section 1.3 describes the data and presents the empirical results for the conditional CAPM, the conditional FF model, and the test on goodness of fit. Section 1.4 concludes the paper.

1.2 Methodology

In this section, we introduce a nonparametric approach to estimating and testing a conditional factor pricing model, for which the conditional CAPM and the conditional FF model are special cases. We first define the econometric specification of a conditional factor pricing model. We then discuss how to

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choose the optimal window for estimating it. Finally we propose two tests to

evaluate its performance in explaining the cross section of stock returns.

1.2.1 Econometric Framework

If a conditional factor model holds, then we have the following relationship

$$E_t(R_{i,t+1}) = \beta_{i,t} E_t(f_{t+1}), \qquad (1)$$

where $R_{i,t+1}$ is the excess return for portfolio i at time $t+1$, and f_{t+1} stands for

the factors at time $t+1$ in the corresponding factor model. For the CAPM, the

market excess return is the only factor, so $f_{t+1} = R_{m,t+1}$; for the FF model, there

are two additional factors SMB and HML other than the market factor, so

$f_{t+1} = \left(R_{m,t+1}, SMB_{t+1}, HML_{t+1} \right)'$. The notation $E_t(\cdot)$ indicates the conditional

expectation, given a common public information set I_t at time t.

In order to estimate (1), econometricians must know the investor's information

set I_t, but a significant practical obstacle is that I_t is unobservable. Standard

empirical methods use state variables observable to investors, such as the

dividend yield or term spread, to proxy I_t, and specify beta as a linear function

of lagged instruments (e.g., Shanken, 1990). This method therefore requires

that the state variables be the right ones in the investor's information set. It is

rather difficult, however, to identify which state variables are the right ones.

In recent literature, an alternative approach has been proposed for doing away

with state variables and estimating factor loadings directly from short-windows.

Grundy and Martin (2001) use monthly returns in the window from t to $t+5$ to

estimate the factor loading in month t. LN argue that (a)s long as betas are relatively stable within a month or quarter, simple CAPM regressions estimated over a short window---using no conditioning variables---provide direct estimates of assets' conditional alphas and betas. (P. 291)

Like Grundy and Martin (2001) and LN, we also dispense with state variables. Since the investor's information set is time-varying, we can let time t index her information set, and so the conditional alphas and betas change with time t. More specifically,

$$R_{i,t+1} = \alpha_{i,t} + \beta_{i,t} f_{t+1} + \varepsilon_{i,t+1}, \ t = 1, 2, ... T, \qquad (2)$$

where $E_t\left(\varepsilon_{i,t+1} \mid f_{t+1}\right) = 0$. $\alpha_{i,t}$ and $\beta_{i,t}$ are portfolio i's alpha and beta at time t, respectively.

Unlike Grundy and Martin (2001) and LN, we apply a data-driven method to obtain the optimal estimation window size. As will be seen from the empirical results later on, the estimates of a model from LN's method vary greatly as the window size enlarges or shrinks. The monthly average pricing error can differ by as much as 1.00% when the window size changes from one month to six months. Moreover, it is possible to arrive at totally different inferences based on different window estimates. Dividing data into arbitrary windows may therefore lead to inconsistent and unreliable conclusions.

Another important difference from BCFS is that we allow a more general relationship between asset payoffs and factor payoffs. BCFS assume an

asymmetric nonlinear relationship between asset returns and factor returns, i.e., betas are different for up and down markets. They demonstrate that with this special structure, the beta estimated in any window can covary with contemporaneous market returns, generating large biases in LN. In general, however, the true relationship between asset returns and factor returns is more complicated than merely asymmetric. Adjustments based on a particular structure, as assumed by BCFS, could potentially lead to large biases as well. Our estimation of (2) doesn't impose any special structure between $R_{i,t+1}$ and f_{t+1}, thereby avoiding the misspecification between asset returns and factor returns.

Another advantage of our method is that we directly capture the variation of betas over time. In practice, new information keeps arriving, and the investor keeps adjusting her portfolio according to the changing information sets. Betas therefore keep changing. Campbell and Vulteennaho (2004), Franzoni (2004), Adrian and Franzoni (2005), Fama and French (2006), and LN, among others, consider the variation of betas only across different non-overlapping windows, but ignore the variation of betas within each window. Our estimation, on the other hand, utilizes overlapping windows, permitting continuous information updating and thus capturing the gradual changes in betas.

1.2.2 Estimation of the Model

To estimate (2), we first find an optimal window size, to be discussed in the next subsection. With the optimal window in hand, at every time t, we use the data within this window to obtain the conditional alpha and beta corresponding to time t. Our goal is to choose parameters to minimize the following local

sum of squared residuals:

$$\min_{a_0, b_0, a_1, b_1} \sum_{s=t-[Th]}^{t+[Th]} [R_{i,s} - \alpha_{i,s} - \beta_{i,s} f_s]^2 k_{st}$$

$$\text{s.t.} \quad \alpha_{i,s} = \alpha(\tfrac{s}{T}) = a_0 + a_1 \cdot \tfrac{s-t}{T},$$

$$\beta_{i,s} = \beta(\tfrac{s}{T}) = b_0 + b_1 \cdot \tfrac{s-t}{T},$$

$$k_{st} = \tfrac{1}{h} k(\tfrac{s-t}{Th}).$$

$$(3)$$

Here, s is a particular data point within the window, so $R_{i,s}$ is the portfolio i's excess return at time s, and f_s is the factor return at time s. T is the total sample size, and h is the optimal window size. Thus, after fixing a time point t, we use observations from $t - [Th]$ to $t + [Th]$ to estimate $\alpha_{i,t}$ and $\beta_{i,t}$, where $[Th]$ denotes the integer part of Th. To simplify notation, we drop the portfolio index i and the time index t for $\alpha(\cdot)$ and $\beta(\cdot)$. Note that $\alpha(\cdot)$ and $\beta(\cdot)$ are functions of $\tfrac{t}{T}$ rather than t, because, as shown by Robinson (1989), it's necessary to let these functions depend on the sample size T in order to achieve asymptotic consistency.

In essence, we are approximating the unknown functions $\alpha(\cdot)$ and $\beta(\cdot)$ with a first-order Taylor expansion within the window, thus introducing four unknown coefficients a_0, b_0, a_1 and b_1. There are two main approaches to estimating $\alpha(\cdot)$ and $\beta(\cdot)$ in the nonparametric literature: local constant smoothing and local linear smoothing. If a_1 and b_1 are zero, then $\alpha(\cdot)$ and $\beta(\cdot)$ are constants within the estimation window, which corresponds to the local constant smoothing method; on the other hand, if a_1 and b_1 are not zero, then $\alpha(\cdot)$ and $\beta(\cdot)$ are different even within each estimation window, which corresponds to

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the local linear smoothing method. These two approaches yield qualitative similar results, so to save space, we report the estimation results of (2) based on the local constant smoothing method only. The time-t conditional alpha $\alpha_{i,t}$ and the conditional beta $\beta_{i,t}$ are the estimates for a_0 and b_0, respectively.[6]

$k(\cdot)$ is a weighting function satisfying certain statistical properties.[7] In our empirical work, we present results based on the following Epanechniov kernel

$$k(u) = \frac{3}{4}(1 - u^2)1(|u| \le 1),$$

which has been proven to achieve the highest estimation efficiency. This kernel function also gives higher weight to observations close to the point t at which the conditional alpha and beta are estimated and discounts the observations far away from t, which is consistent with the idea that recent data contain more relevant information than remote data. We also try two other popular kernels in the nonparametric literature, the uniform kernel and the Daniel kernel. Our main results are robust to the choice of these alternative kernels.

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[6]At a different time t, we use different data to estimate a_0 and b_0. Therefore, a_0 and b_0 are time-varying.

[7]The kernel function $k(\cdot)$ is a pre-specified symmetric probability density function such that (i) $\int_{-\infty}^{+\infty} k(u)du = 1$, (ii) $\int_{-\infty}^{+\infty} k(u)udu = 0$, (iii) $\int_{-\infty}^{+\infty} k(u)u^2 du < \infty$, and $\int_{-\infty}^{+\infty} k^2(u)du < \infty$.

An important issue in the nonparametric literature is the boundary problem. Simply put, there are no symmetric data for estimating the models in the boundary areas. For instance, if we want to estimate the model at time $t = 1$, we have data only after $t = 1$ but lack data before $t = 1$. Following the literature, we use a reflection method to obtain pseudodata $R_{i,t} = R_{i,-t}$, $f_t = f_{-t}$ for the left boundary when $-[Th] \leq t \leq -2$, and $R_{i,t} = R_{i,2T-t}$, $f_t = f_{2T-t}$, for the right boundary when $T + 1 \leq t \leq T + [Th]$.[8]

In our empirical implementation, for each portfolio i, we first estimate its optimal window size and then, at every time t, we solve the minimization problem (3) to obtain the conditional alpha $\alpha_{i,t}$ and the conditional beta $\beta_{i,t}$. Since the optimal window size serves to minimize the underconditioning and the small sample biases, let us now turn our discussion to how to find it.

1.2.3 The Choice of Window Size

When we dispense with state variables and assume the investor's information set to be relatively stable in adjacent periods, the optimal window size approximates the right amount of information to be used in the estimation. To reduce the underconditioning bias, we want the window size to be as small as possible. If h chosen is too large, the information set may have already changed within the window. As a result, if we estimate the model according to this large window, we are more likely to miss the variations in risk and are

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[8]For a robustnes check, we also estimate the model only for the interior points which have symmetric data.

therefore subject to the underconditioning bias.[9] On the other hand, to mitigate the small sample bias, we would like the window size to be as large as possible. If h chosen is too small, there will be too few observations within the window, so that the data are too noisy to yield reliable estimates. In this case, we run the risk of the small sample bias. Therefore, the optimal window size ought to minimize both the underconditioning *and the small sample biases*. This is exactly what the extensive nonparametric literature has been centering on.

We obtain the optimal window size from a standard nonparametric method called the cross-validation method. Define the leave-one-out estimators $\hat{\alpha}_{0_{-i,t}}$ and $\hat{\beta}_{0_{-i,t}}$ from the following regression

$$\min_{a_0,\,b_0,\,a_1,\,b_1} \sum_{s=t-[Th],\ s\neq t}^{t+[Th]} [R_{i,s} - \alpha_{i,s} - \beta_{i,s} f_s]^2 k_{st}$$

$$\text{s.t.}\quad \alpha_{i,s} = \alpha(\tfrac{s}{T}) = a_0 + a_1 \cdot \tfrac{s-t}{T},$$

$$\beta_{i,s} = \beta(\tfrac{s}{T}) = b_0 + b_1 \cdot \tfrac{s-t}{T},$$

$$k_{st} = \tfrac{1}{h} k(\tfrac{s-t}{Th}),$$

$$\tag{4}$$

with $\hat{\alpha}_{0_{-i,t}}$ and $\hat{\beta}_{0_{-i,t}}$ being the estimates for a_0 and b_0 for portfolio i at time t. The only difference between (4) and (3) is that when doing the minimization problem at time t, we exclude the data point at t in (4). The optimal window size h is then chosen to minimize

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[9]An extreme example is a model estimated using all observations, which corresponds to the largest window size. In this case, we simply estimate the unconditional model, totally ignoring the predictable variations in risk.

$$CV(h) = \sum_{t=1}^{T} \left(R_{i,t} - \hat{\alpha}_{0-i,t} - \hat{\beta}_{0-i,t} f_t \right)^2.$$

Intuitively, for any portfolio i, we first fix an arbitrary window size. At every time t, we use all data within this window except the data at time t to do the minimization in (4), obtaining the predicted value and prediction error corresponding to t. Intuitively, since the data in the vicinity of time t contain similar information to the data at time t, we can use them predict the time-t observation. We do this for all time periods ($t = 1, 2, ... T$), and sum up all the prediction errors denoted by $CV(h)$. The optimal window is chosen to minimize $CV(h)$.

For any given portfolio, the optimal window size obtained from the leave-one-out cross-validation method is the same for all time periods. It is possible that betas might change faster in some periods than in others, thus a time-varying window size might seem to be needed. We leave this for future research. Since existing studies relying on the simple window approach use a uniform window size, in order to better compare our results with the literature, we stick to the uniform window size in this paper.

1.2.4 Two Tests of a Conditional Factor Model
Our null hypothesis is that a conditional factor model holds at each point in time. If factors themselves are excess returns, as is the case with the conditional CAPM and the conditional FF model, then testing this hypothesis is equivalent to testing $\alpha(\frac{t}{T}) = 0$ for all t [10]. First, we test if the individual pricing

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[10]As has been mentioned, to achieve estimation consistency as proved by

errors are zero, i.e., $\alpha(\frac{t}{T}) = 0$ for any t. Second, we test if the average pricing error is equal to zero, i.e., $\frac{1}{T}\sum_{t=1}^{T}\alpha(\frac{t}{T}) = 0$.

1.2.4.1 Test on Individual Pricing Errors

Under usual technical assumptions, Cai (2007) shows that the individual alpha obtained from (2) follows an asymptotic normal distribution. Let $\tau = \frac{t}{T}$, under the null hypothesis that alphas are equal to zero at every point t, the interior alphas follow[11]

$$\sqrt{Th}\hat{\alpha}(\tau)\xrightarrow{d}N(0, \nu_0\Sigma(\tau)), \qquad (5)$$

where T is the sample size, and h is the optimal bandwidth or the window size. Since the effective data used to estimate $\alpha(\tau)$ is Th, $\hat{\alpha}(\tau)$ converges at the rate of $\sqrt{Th}$. The details for the variance of $\hat{\alpha}(\tau)$ are provided in Appendix 1.A.

The asymptotic behavior of the estimated boundary alphas is different from that of the interior ones. But the boundary alphas are not particularly interesting in our context,[12] and they also make up only a small proportion of

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Robinson (1989), the pricing errors are functions of t/T instead of t.

[11]The interior alphas are those corresponding to the time periods which doesn't suffer the boundary problem.

[12]One scenario in which the boundary alphas are particularly interesting is when the conditional alphas (also betas) are functions of, for example, the market return rather than time. In this case, the boundary alphas correspond to the pricing errors under extreme market conditions, such as market crashes or market frenzies.

the estimated time series of the conditional alphas. To save notation and space, we report the results only for interior alphas. Incorporating the boundary alphas won't change our results dramatically.

1.2.4.2 Test on Average Pricing Errors

If two models are both rejected at, for example, 80% of the time periods, the test on individual pricing errors alone cannot tell us which model is relatively better. Thus we need to turn to the second test, which focuses on the implication that if a conditional factor model holds, then the average pricing error should be equal to zero. This measure is also adopted by LN.

The average pricing error is

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}\left(\frac{t}{T}\right).$$

In Appendix 1.B, we derive the asymptotic distribution of $\hat{\alpha}$ when the random error process $\{\varepsilon_t\}_{t=1}^{T}$ is heteroskedastic. We find it follows a normal distribution:

$$\sqrt{T}\hat{\alpha} \xrightarrow{d} N(0,V), \qquad\qquad (6)$$

where V is the asymptotic variance and equals the $(1,1)th$ element of $\Omega_0^{-1} \sum_{j=-\infty}^{\infty} E\left(X_t X_{t+j}' \varepsilon_t \varepsilon_{t+j}\right)\Omega_0^{-1}$, with $X_t = (1, f_t)'$, $\Omega_0 = E\left(X_t X_t'\right)$, and j denoting the lag order. Even though we use a nonparametric estimation method, the asymptotic variance V resembles the standard Newey-West estimator. In implementation, we use the corresponding sample moments to estimate V.

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1.3. Empirical Results

1.3.1 Data

Our data are obtained from Professor Kenneth French's website.[13] From the 25 size-B/M portfolios, we form six size and B/M portfolios. S is the average return of the five portfolios in the lowest size quintile, B is the average return of the five portfolios in the highest size quintile, and S-B is the difference. G is the average return of the five portfolios in the low-B/M quintile, V is the average return of the five portfolios in the high-B/M quintile, and V-G is the difference. The three momentum portfolios are directly obtained from Professor Kenneth French's website, where we let W stand for the return of the winner portfolio, L for the return of the loser portfolio, and W-L for their difference.

To compare our findings with existing studies, we look at both the value-weighted and the equally-weighted portfolios, using daily data from 1963 to 2007.[14] The long time series of daily data not only provide rich information about the underlying information structure, but also help improve estimation efficiency. Moreover, the debate on the small sample bias of LN's procedure also focuses on daily data. For a robustness check, we also conduct tests using monthly data and obtain qualitatively similar results not reported here.

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[13]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[14]LN examine the performance of the conditional CAPM using value-weighted portfolios, while BCFS focus on momentum portfolios which are equally-weighted.

It is well known that nonsynchronous trading can have a great impact on short-horizon betas (Lo and MacKinlay, 1990). Since we use high-frequency daily data, we need to consider the microstructure issues such as the bid-ask spread. To address these issues, when estimating the unconditional models, we use Dimson (1979) regressions with the structure suggested by LN:

$$R_{i,t} = \alpha_i + \beta_{i,1} f_t + \beta_{i,2} f_{t-1} + \frac{\beta_{i,3}}{3} \sum_{p=2}^{4} f_{t-p} + \varepsilon_{i,t}, \quad (7)$$

where p denotes lag. The estimated pricing error for portfolio i is α_i, and the estimated beta is $\beta_{i,1} + \beta_{i,2} + \beta_{i,3}$.

Table 1.1 and Table 1.2 present the summary statistics for the value-weighted and equally-weighted portfolios, respectively. The daily estimates are multiplied by 21, the average number of trading days per month, so that all estimates are expressed as monthly percentages. With respect to the value-weighted portfolios, excess returns exhibit the usual cross section patterns. Overall, the small stocks outperform the big stocks (0.63% vs. 0.51%), the value stocks outperform the growth stocks (0.84% vs. 0.30%), and the winner stocks outperform the loser stocks (1.12% vs. -0.32%). Except for the size portfolios, the unconditional CAPM alphas are all significant, implying that the unconditional CAPM fails. In line with prior research (e.g., Fama and French, 1993), the unconditional FF model improves upon the unconditional CAPM because the alphas for the size and B/M portfolios are much smaller. However, the alphas for the B/M portfolios are still significant. For the momentum portfolios, the unconditional FF alphas are highly significant, and they are also

of the same magnitude of the unconditional CAPM alphas, indicating that the unconditional FF model doesn't help to explain the momentum effect.

Table 1.2 shows that the equally-weighted portfolios display some interesting patterns. First, the size effect is very pronounced. The equally-weighted S-B has an excess return of 1.05%, compared to only 0.13% for the value-weighted S-B. The unconditional CAPM alpha is also much higher: 1.04% for equally-weighted S-B vs. 0.07% for value-weighted S-B. The unconditional FF alpha for equally-weighted S-B is 1.00%, which is close to the unconditional CAPM alpha, indicating that the unconditional FF fails to explain the size effect in the equally-weighted portfolios. This is not surprising because the equally-weighted portfolios put more weight on small stocks, which, as shown in Fama and French (1996), the unconditional FF model doesn't explain quite well. Second, momentum portfolios have a very different pattern from the one usually observed in the monthly data. The loser portfolio actually earns a higher average return than the winner portfolio (2.08% vs. 1.71%), implying that the equally-weighted momentum strategy is not profitable at daily horizon. Neither the CAPM nor the FF model is able to account for the return variations in the momentum portfolios.

We now allow the factor loadings to vary over time, and investigate whether the conditional versions of the CAPM and the FF model are able to account for the return variations in these portfolios. To correct for the impact of nonsynchronous trading, we also append two lags in the estimation of (2):

$$R_{i,t} = \alpha\left(\frac{t}{T}\right) + \beta_1\left(\frac{t}{T}\right)f_t + \beta_2\left(\frac{t}{T}\right)f_{t-1} + \frac{\beta_3\left(\frac{t}{T}\right)}{3}\sum_{p=2}^{4}f_{t-p} + \varepsilon_{i,t}, \; t = 1, 2, \ldots T. \quad (8)$$

Thus, portfolio i's pricing error at time t is $\alpha\left(\frac{t}{T}\right)$, and its conditional beta at t is $\beta_1\left(\frac{t}{T}\right) + \beta_2\left(\frac{t}{T}\right) + \beta_3\left(\frac{t}{T}\right)$.

1.3.2 Testing the Conditional CAPM

In subsection 1.3.2.1, we report the data-driven window size for the conditional CAPM obtained from the cross-validation method described in subsection 1.2.3. In subsections 1.3.2.2 and 1.3.2.3, we estimate the conditional CAPM and evaluate its performance through the two tests proposed in subsection 1.2.4.

1.3.2.1 Data-Driven Window Size

If the investor optimally rebalances her portfolios according to changes in her information set, then the realized data structure should reflect changes in the underlying information structure and, as a result, the estimated window size serves as a proxy for the stability of the information structure. A larger window size implies that the relationship between asset returns and factor returns, or the underlying information structure, is generally more stable. Consequently, betas will change less frequently with a larger window than with a smaller window.

Table 1.1: Summary statistics for value-weighted size, B/M, and momentum portfolios, 1963-2007

The table reports the average excess returns, the unconditional CAPM alphas and the unconditional FF alphas for value-weighted size, B/M, and momentum portfolios using daily data. The unconditional CAPM alphas are obtained from the regression in (7) by letting $f = R_m$, and the unconditional FF alphas are obtained from the regression in (7) by letting $f = (R_m\ SMB\ HML)'$. Average returns and alphas are expressed in percentage monthly. Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: Excess returns									
Ave.	**0.63**	**0.51**	0.13	0.30	**0.84**	**0.54**	**-0.32**	**1.12**	**1.44**
Std. err	0.16	0.18	0.13	0.21	0.16	0.10	0.25	0.25	0.21
Panel B: Unconditional CAPM alphas									
Est.	0.16	0.09	0.07	**-0.28**	**0.42**	**0.69**	**-0.94**	**0.56**	**1.50**
Std. err	0.09	0.05	0.12	0.07	0.07	0.09	0.14	0.12	0.21
Panel C: Unconditional FF alphas									
Est.	-0.05	-0.05	0.003	**-0.11**	-0.04	**0.08**	**-0.92**	**0.60**	**1.52**
Std. err.	0.04	0.03	0.04	0.03	0.03	0.04	0.15	0.11	0.21

Table 1.2: Summary statistics for equally-weighted size, B/M, and momentum portfolios, 1963-2007

The table reports the average excess returns, the unconditional CAPM alphas and the unconditional FF alphas for equally-weighted size, B/M, and momentum portfolios using daily data. The unconditional CAPM alphas are obtained from the regression in (7) by letting $f = R_m$, and the unconditional FF alphas are obtained from the regression in (7) by letting $f = \left(R_m \ SMB \ HML \right)^{'}$. Average returns and alphas are expressed in percentage monthly. Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: Excess returns									
Ave.	**1.65**	**0.60**	**1.05**	**0.58**	**1.17**	**0.60**	**2.08**	**1.71**	**-0.37**
Std. err	0.14	0.17	0.13	0.21	0.15	0.11	0.19	0.21	0.15
Panel B: Unconditional CAPM alphas									
Est.	**1.21**	**0.16**	**1.04**	-0.01	**0.75**	**0.76**	**1.53**	**1.16**	**-0.37**
Std. err	0.09	0.05	0.10	0.08	0.07	0.09	0.13	0.11	0.14
Panel C: Unconditional FF alphas									
Est.	**0.99**	-0.01	**1.00**	**0.14**	**0.31**	**0.17**	**1.37**	**1.07**	**-0.31**
Std. err.	0.06	0.04	0.06	0.05	0.04	0.05	0.11	0.08	0.14

Table 1.3 presents the estimated window size for the conditional CAPM and the conditional FF model. For now, we focus only on the conditional CAPM, and discuss some notable patterns of the estimated window size. First, the window size is much larger for the big portfolio than for other portfolios, 333 if value-weighted and 285 if equally-weighted, indicating a less frequent change in betas of large stocks. This is consistent with Shanken (1990), in which the T-bill rate serves as the state variable, and betas of large stocks are far less sensitive to changes in the T-bill rate than betas of small stocks.

Second, the window size is always larger for the value-weighted portfolios than for the equally-weighted portfolios. For example, we use 87 observations to estimate the value-weighted loser portfolio, while we use only 33 observations for the equally-weighted one. This is mostly likely due to the fact that the value-weighted portfolios put more weight on large stocks, whose underlying information structure turns out to be less volatile. We caution that we are not attempting to map one-to-one the window size to the underlying information structure. But we do argue that the data-driven window size reveals important information about the unknown information structure.

Our results in Table 1.3 show that the estimated window size ranges from as short as 31 days to as long as 333 days, varying widely from portfolio to portfolio. This suggests that fixing a window size as one month or three months for all portfolios may incur estimation biases, which generally become larger if the underlying relationship between asset returns and factor returns changes in a more complicated way.

Using the optimal window size, we estimate (8) with $f = R_m$ to get the time series of the conditional CAPM alphas for every portfolio. In order to evaluate whether the conditional CAPM explains the return variations, we apply the two tests on the pricing errors proposed in subsection 1.2.4.

1.3.2.2 Individual Pricing Errors

Panel A of Figures 1.1 and 1.2 plot the conditional alphas for the value-weighted and equally-weighted S-B, V-G, and W-L, respectively. The conditional alphas of all portfolios fluctuate greatly over time, but W-L displays the largest variation, with the daily alpha ranging from a minimum of -0.88% to a maximum of 1.38% if value-weighted, and from -2.11% to 0.97% if equally-weighted.

Unlike the existing studies, ours obtains the conditional alpha at every point in time, which enables us to investigate whether the conditional CAPM holds at any given time. Based on the distribution of the individual pricing error in (5), we can calculate the standard error sd_t, for α_t, the conditional alpha at time t. Define the difference between α_t and $1.96sd_t$, 1.96 times the corresponding time t standard error, as follows:

$$Diff_t = 1.96sd_t - |\alpha_t|. \qquad (9)$$

The sign of $Diff_t$ indicates whether we should accept or reject the conditional CAPM at the 5% significant level. If $Diff_t$ is positive, then we don't have the evidence for rejecting the conditional CAPM at time t; if $Diff_t$ is negative, then we find evidence that indicates the failure of the model at time t. The series of

$Diff_t$ for S-B, V-G, and W-L are plotted in Panel B of Figures 1.1 and 1.2. These graphs show that for all portfolios, $Diff_t$ tend to be negative most of the time, implying that the conditional CAPM may hold for only a small fraction of the time periods.

To examine the persistence of the model's explanatory power, we plot the autocorrelation function of the $Diff$ measure for the value-weighted S-B, V-G and W-L in Panel A of Figure 1.3 and the corresponding equally-weighted ones in Panel A of Figure 1.4. These figures show that the autocorrelation of $Diff$ generally declines to zero in an AR(1) fashion, because the information structure is more stable within adjacent time periods. Today's information structure, for example, is most like yesterday's, so that if we reject (accept) the model today, it's most likely that we rejected (accepted) the model yesterday. As we move further away from today, the similarity in information structure typically declines, and it becomes less likely for us to reject (accept) the model, given that we reject (accept) it today. After certain periods, the information structure may have totally changed, sharing no commonality with today's information structure, which explains why the autocorrelation usually drops to zero after certain lags.

Table 1.3: Optimal data-driven window size

This table reports the estimated window size using the cross-validation method described in subsection window size. The window size is measured in terms of days. For example, a window size of 60 days means that when estimating the model at day t, we use the 60-day data from $t - 30$ to $t + 30$.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Value-weighted Portfolios*									
conditional CAPM	60	333	62	131	91	56	87	133	47
conditional FF	356	349	322	596	378	502	205	160	165
Panel B: *Equally-weighted Portfolios*									
conditional CAPM	44	285	44	100	58	56	33	51	31
conditional FF	278	349	331	331	209	298	145	116	122

To get a quantitative idea of the overall performance of the conditional CAPM, for each portfolio, we calculate the fraction of the time when the conditional CAPM holds, the results of which are shown in Table 1.4.[15] Panel A shows that, for the value-weighted portfolios, the conditional CAPM performs best for L, holding 22.31% out of all time periods; it performs worst for S, holding 15.89% out of all time periods. In other words, out of the 44 years of data we are considering, the conditional CAPM roughly holds 9.8 years for the loser portfolio and 7.0 years for the small portfolio. Interestingly, examining the performance of the conditional CAPM period-by-period reveals that it fails most often for size portfolios, rather than, as generally assumed, momentum portfolios. In fact, among all the portfolios, the conditional CAPM seems to perform best for momentum portfolios, yielding 22.31% for L, 22.21% for W, and 17.25% for W-L.

Panel B of Table 1.4 shows that, for the equally-weighted portfolios, the conditional CAPM works best for G, holding 20.25% out of all time periods, and worst for S, holding only 12.76% out of all time periods. Therefore, the small portfolios, both value-weighted and equally-weighted, represent the greatest challenge to the conditional CAPM. Overall, the conditional CAPM holds for fewer periods for the equally-weighted portfolios than for the value-weighted ones, which is especially true for momentum portfolios. For example, it holds 14.24% of the time for the equally-weighted L, much less than 22.31% for the value-weighted L.

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[15]A statistical test needs to be constructed to rigorously evaluate the time periods in which a model holds. Here we propose this preliminary intuitive measure.

Panel A · Panel B

Conditional Alphas for S-B · Diff for S-B

Conditional Alphas for V-G · Diff for V-G

Conditional Alphas for W-L · Diff for W-L
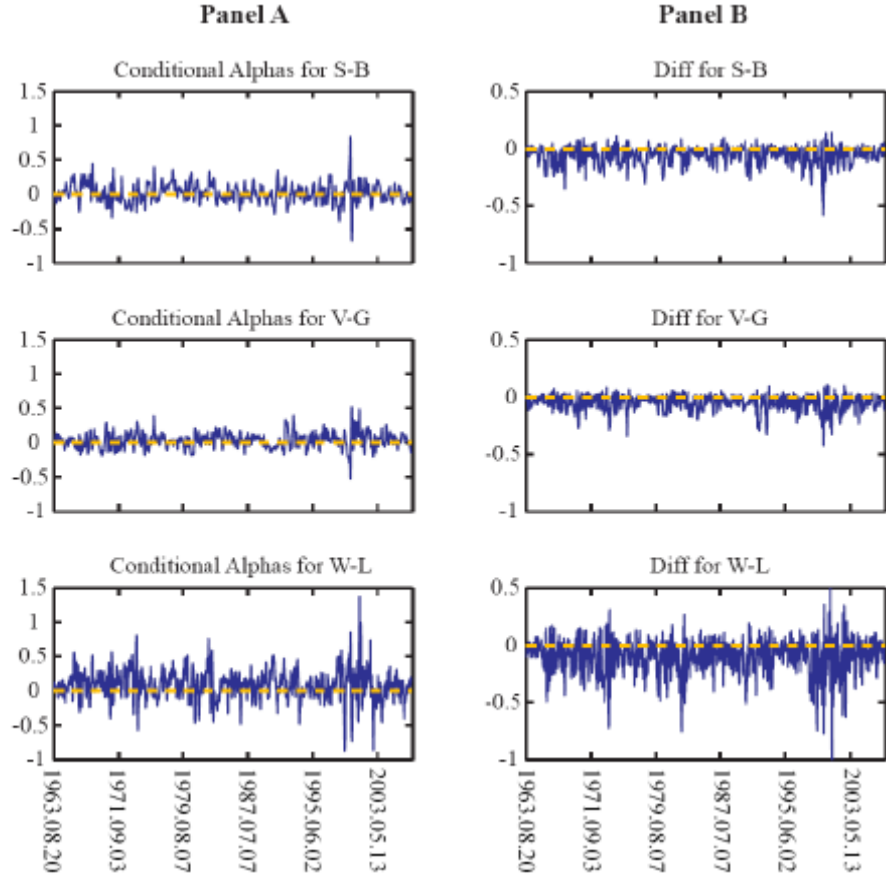
Figure 1.1: Conditional CAPM alphas and the $Diff$ measure for the value-weighted S-B, V-G, and W-L.

Panel A plots the series for the conditional alphas, which are obtained from the nonparametric estimation of (8) with $f = R_m$. The conditional alphas are reported as daily percentages. Panel B plots the series of $Diff$ which are calculated from (9). Positive values of $Diff$ correspond to the periods in which the conditional CAPM is accepted while negative values indicate the failure of the model.
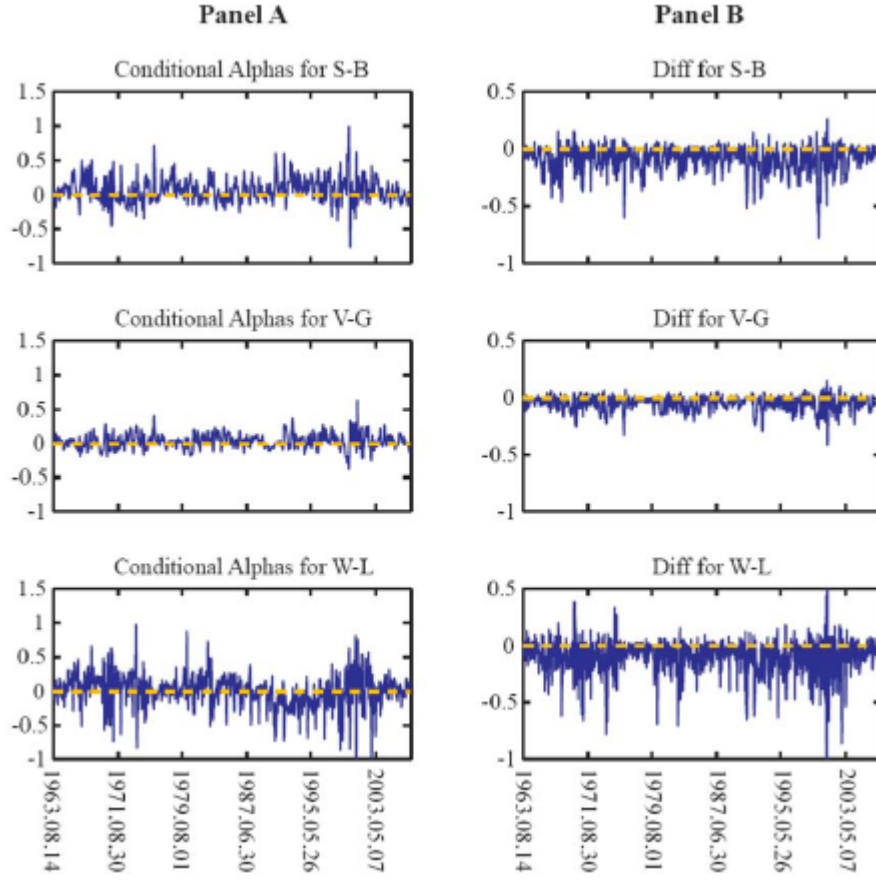
Figure 1.2: Conditional CAPM alphas and the $Diff$ measure for the equally-weighted S-B, V-G, and W-L.

Panel A plots the series for the conditional alphas, which are obtained from the nonparametric estimation of (8) with $f = R_m$. The conditional alphas are reported as daily percentages. Panel B plots the series of $Diff$ which are calculated from (9). Positive values of $Diff$ correspond to the periods in which the conditional CAPM is accepted while negative values indicate the failure of the model.

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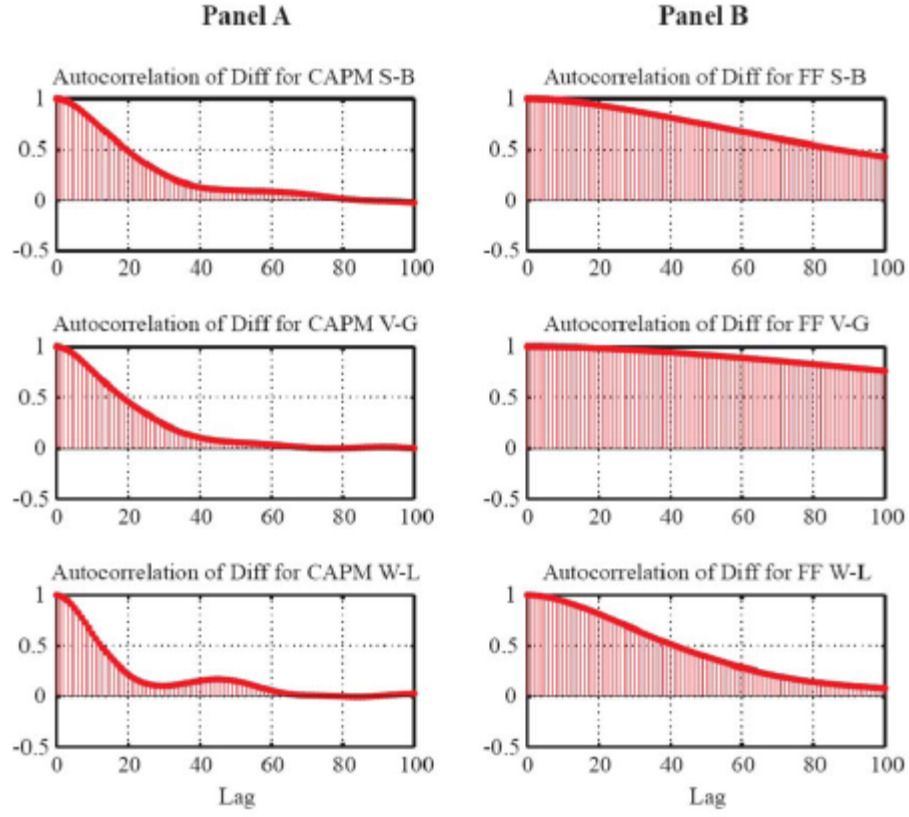
Figure 1.3: Autocorrelation function of $Diff$ for value-weighted portfolios.

The series of $Diff$ are calculated from (9). Panel A plots the autocorrelation of $Diff$ for the conditional CAPM, and Panel B plots the autocorrelation of $Diff$ for the conditional FF model.
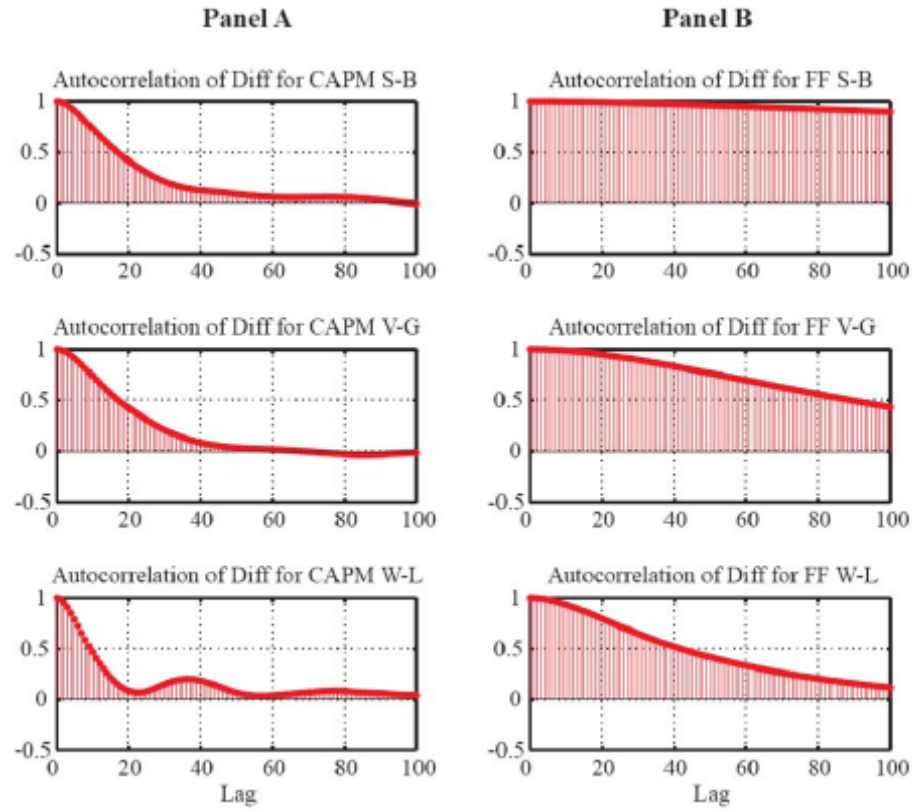
Figure 1.4: Autocorrelation function of *Diff* for equally-weighted portfolios.

The series of *Diff* are calculated from (9). Panel A plots the autocorrelation of *Diff* for the conditional CAPM, and Panel B plots the autocorrelation of *Diff* for the conditional FF model.

The tests on individual pricing errors thus show the inadequacy of the conditional CAPM to explain the dynamics of stock returns. For every portfolio, the conditional CAPM holds less than 1/4 of the time. More importantly, to claim success, a model has to be able to price all portfolios simultaneously. If we consider the time when the conditional CAPM holds for all three portfolios of the value-weighted S-B, V-G, and W-L, it will be even less than 4%!

Compared to existing methods in the literature, these tests on individual pricing errors possess a unique advantage, i.e., they enable us to identify the exact time periods in which the conditional CAPM holds or fails. For instance, referring to Figures 1.1 and 1.2, we observe that the most extreme values of alphas for S-B, V-G, and W-L all appeared around March 2001, when the technology bubble burst, thus representing the greatest failure of the conditional CAPM. We can also identify the periods when the conditional CAPM holds, and by investigating these periods' important variables, such as the market conditions and the economic situations, we will be able to discover the conditions under which the market risk factor will determine investors' portfolio choice. This has important theoretical and empirical implications but hasn't yet been pursued in the literature. We leave this for future research.

Table 1.4: Test of individual pricing errors

This table reports the proportion of time in which the conditional CAPM and the conditional FF model are accepted. At each time t, using the distribution in equation (5), we calculate the test statistic for the alpha at time t, and compare it with 1.65, the 5% critical value for the standard normal distribution. If the test statistic is less than 1.65, we accept the conditional model to hold at t. Summing up all the periods in which the model holds and dividing by the total number of periods gives the proportion, which is reported as the percentage.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Value-weighted Portfolios (%)*									
conditional CAPM	15.89	16.01	16.50	20.39	17.91	16.33	22.31	22.21	17.25
conditional FF	20.30	23.00	18.64	19.23	17.59	26.58	20.98	24.44	19.75
Panel B: *Equally-weighted Portfolios (%)*									
conditional CAPM	12.76	17.74	12.84	20.25	15.02	17.08	14.24	14.46	15.42
conditional FF	9.74	20.44	9.06	18.91	13.78	19.75	14.06	12.48	17.79

1.3.2.3 Average Pricing Errors

Now let us examine how the conditional CAPM explains the nine portfolios based on tests of the average pricing errors. We first conduct our discussions for the value-weighted portfolios and then for the equally-weighted. Since BCFS challenge the results of LN through the momentum portfolios, we first look at the results of the momentum portfolios, and then analyze the results of the size and B/M portfolios.

Value-weighted Portfolios

The results from our nonparametric method are presented in Panel A of Table 1.5. They show that for the momentum portfolios, the average conditional alphas are -0.88% (z-stat -6.77),[16] 0.54% (z-stat 5.40), and 1.72% (z-stat 11.47) for L, W and W-L, respectively. The estimates for L and W are slightly smaller than the unconditional alphas of -0.94% and 0.56%, but the estimate for W-L is larger than its unconditional alpha of 1.50%. Therefore, the conditional CAPM performs even worse than the unconditional CAPM in explaining W-L. Moreover, all these estimates are highly significant, providing strong evidence that the conditional CAPM fails to explain the momentum portfolios.

BCFS point out that the method in LN suffers potentially serious small sample biases. But how large are these biases? Are they as large as BCFS have

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[16]We use "z-stat" to stand for the statistics calculated based on the normal distribution of (6). That is, $z = \frac{\text{estimate of } \alpha}{\text{standard error of } \alpha}$. If $z > 1.65$, we reject the model at 5% significant level.

claimed? To answer these questions, we also estimate the model using the method in LN by choosing the non-overlapping window as $N = 1, 3, 6$ months. Following Fama and Macbeth (1973), we obtain the standard error of the estimates from the time series variation of the conditional alphas.

The results from LN's method are presented in Panel B of Table 1.5, which show that the average conditional alphas for W-L are 2.49% (t-stat 7.78) when $N = 1$, 1.96% (t-stat 7.54) when $N = 3$, and 1.58% (t-stat 6.32) when $N = 6$. Therefore, the LN method provides estimates that are very sensitive to the window size, where the difference in average pricing errors is as large as 0.91% (2.49%-1.58%) for W-L. This sensitivity to window size highlights the importance of using the data-driven window to estimate the model.

An important feature of the momentum portfolios is that they are typically rebalanced every month, and the entering and exiting stocks may not have similar betas.[17] Another shortcoming of the non-overlapping window estimation is that it fails to account for changing composition in the momentum portfolios, because by fixing $N = 6$, for instance, it assumes that betas are constant over periods of as long as six months. Our method can account for the high turnover in the momentum portfolios because we estimate the conditional alphas and betas continuously at each point in time.

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[17]Grundy and Martin (2001) show that due to selection, betas of newly added winner and loser stocks vary with the market return in the formation period.

Table 1.5: Test of average conditional CAPM alphas for the value-weighted portfolios, 1963-2007

The table reports the average conditional alphas for value-weighted size, B/M and momentum portfolios (% monthly). Panel A reports the nonparametric estimates from equation (8) with $f = R_m$ using daily data. The standard error is obtained from equation (6). Panel B presents estimates using the non-overlapping window estimation as in LN, with window size $N = 1,3$ and 6. The standard error is calculated from the time series variation of the conditional alphas, in the spirit of Fama and Macbeth (1973). Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Nonparametric Conditional Alphas*									
Est.	**0.42**	0.05	**0.39**	**-0.19**	**0.45**	**0.64**	**-0.88**	**0.54**	**1.72**
Std. err	0.07	0.04	0.09	0.07	0.06	0.07	0.13	0.10	0.15
Panel B: *Non-overlapping Window Estimated Conditional Alphas*									
$N = 1$									
Est.	**0.83**	-0.05	**0.88**	0.15	**0.63**	**0.48**	**-1.45**	**1.04**	**2.49**
Std. err	0.17	0.07	0.21	0.12	0.11	0.15	0.23	0.17	0.32
$N = 3$									
Est.	**0.35**	-0.02	0.37	-0.05	**0.45**	**0.49**	**-1.18**	**0.79**	**1.96**
Std. err	0.17	0.05	0.19	0.11	0.11	0.14	0.19	0.14	0.26
$N = 6$									
Est	0.15	0.01	0.14	-0.16	**0.33**	**0.49**	**-0.97**	**0.6**	**1.58**
Std. err	0.17	0.06	0.2	0.12	0.11	0.16	0.18	0.15	0.25

Despite the shortcomings of LN's method, we agree with them on the ability of the conditional CAPM to explain the momentum portfolios. Our estimates of the conditional alphas are of similar magnitude to the unconditional alphas, indicating that the conditional CAPM is not superior to the unconditional CAPM. This is very different from the conclusion of BCFS, who estimate the conditional alphas to be 20-40% smaller than the unconditional estimates. The reason BCFS achieve these results is most likely their special payoff structure, i.e., betas tend to be smaller when the market goes up and larger when the market goes down. Imposing a specific payoff structure in the estimation, however, may lead to serious misspecification bias. In our estimation, we don't assume any specific payoff structure, thus avoiding the misspecification bias, and we obtain very different results from BCFS.

The results for the size and B/M portfolios are also shown in Table 1.5, with the results from our nonparametric method in Panel A and those from LN's method in Panel B. Panel A shows that for the size portfolios, the S and S-B's average conditional alphas are 0.42% (z-stat 6.00) and 0.39% (z-stat 4.33), which are economically large and statistically significant. For the B/M portfolios, V's average conditional alpha is 0.45% (z-stat 7.50), G's average conditional alpha is -0.19% (z-stat -2.71), and V-G's average conditional alpha is 0.64% (z-stat 9.14). We therefore reject the conditional CAPM for all B/M portfolios.

Panel B of Table 1.5 shows that the estimates of the size and B/M portfolios using LN's method display wide variations. More importantly, they provide inconsistent results. For instance, S-B's average alphas are 0.88% (z-stat 4.19) for $N=1$, showing strong evidence for rejecting the conditional CAPM, but

when $N = 6$, the average alpha drops sharply to 0.14% (z-stat 0.70), providing no evidence for rejecting the model.

To sum up, our method provides consistent and efficient estimates, which enable us to accurately evaluate the performance of the conditional CAPM. We find that after taking into account the underconditioning and small sample biases, the conditional CAPM fails miserably to explain either the momentum effect, the value effect, or the size effect.

Equally-weighted Portfolios

Panel A of Table 1.6 presents the results of the equally-weighted portfolios using our nonparametric estimation method. For the momentum portfolios, W-L's average conditional alpha is 0.005% (z-stat 0.06), which is neither economically nor statistically significant. The average pricing error test therefore provides no evidence for the failure of the conditional CAPM.[18] However, the conditional alphas for L and W are 1.78% (z-stat 19.78) and 1.57% (z-stat 19.63), which are very large and significant, indicating the failure of the conditional CAPM.

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[18]A test that relies on average absolute pricing error or average squared pricing errors may reject the conditional CAPM in the equally-weighted W-L. But in this paper we choose the most conservative test, i.e., the test on average pricing errors.

Table 1.6: Test of average conditional CAPM alphas for equally-weighted portfolios, 1963-2007

The table reports the average conditional alphas for equally-weighted size, B/M and momentum portfolios (% monthly). Panel A reports the nonparametric estimates from equation (8) with $f = R_m$ using daily data. The standard error is obtained from equation (6). Panel B presents estimates using the non-overlapping window estimation as in LN, with window size $N = 1, 3$ and 6. The standard error is calculated from the time series variation of the conditional alphas, in the spirit of Fama and Macbeth (1973). Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Nonparametric Conditional Alphas*									
Est.	**1.57**	**0.15**	**1.43**	**0.16**	**0.86**	**0.68**	**1.78**	**1.57**	0.00
Std. err	0.07	0.04	0.09	0.07	0.06	0.07	0.09	0.08	0.09
Panel B: *Fixed Window Conditional Alphas*									
$N = 1$									
Est.	**1.82**	0.09	**1.72**	**0.4**	**0.93**	**0.54**	**1.68**	**2.22**	**0.53**
Std. err	0.17	0.07	0.2	0.13	0.11	0.14	0.25	0.2	0.24
$N = 3$									
Est.	**1.36**	**0.12**	**1.24**	0.23	**0.78**	**0.55**	**1.45**	**1.56**	0.11
Std. err	0.2	0.06	0.21	0.13	0.12	0.14	0.3	0.18	0.25
$N = 6$									
Est	**1.16**	**0.14**	**1.02**	0.13	**0.68**	**1.02**	**1.36**	**1.23**	-0.13
Std. err	0.21	0.06	0.23	0.13	0.11	0.23	0.33	0.19	0.28

For the B/M portfolios, V's average conditional alpha is 0.86% (z-stat 14.33), G's average conditional alpha is 0.16% (z-stat 2.29), and V-G's average conditional alpha is 0.68% (z-stat 9.71). We therefore reject the conditional CAPM for all B/M portfolios. Moreover, our rejection of the equally-weighted B/M portfolios is stronger than our rejection of the value-weighted ones.

For the size portfolios, our nonparametric estimation provides strong evidence that the conditional CAPM is unable to explain S, B, or S-B. The conditional alphas are 1.57% (z-stat 22.43), 0.15% (z-stat 3.75), and 1.43% (z-stat 15.89), respectively, which are all large and significant. Our rejection of the conditional CAPM is again much stronger than that for the value-weighted size portfolios.

Panel B of Table 1.6 presents the results using LN's non-overlapping window method. The conditional alphas vary greatly with the window size. V-G's average conditional alpha, for example, increases from 0.54% (t-stat 3.86) when $N = 1$ to 1.02% (t-stat 4.43) when $N = 6$. Based on different window estimates, we also draw inconsistent inferences for B, G, and W-L.

Overall, these tests on average pricing errors provide strong evidence that the conditional CAPM fails to account for the size and value effect. The failure is more pronounced for the equally-weighted portfolios than for the value-weighted ones. This is consistent with the results obtained from the tests on individual pricing errors.

1.3.3 Testing the Conditional Fama-French Three-Factor Model

The FF model has become the workhorse in empirical asset pricing. Fama and French (1993, 1996) have provided ample evidence that the unconditional version of the model captures much of the return variation in portfolios sorted by size and B/M. For these portfolios, by allowing the betas and risk premia to

vary, we would expect the conditional version of the model to perform even better. Surprisingly, empirical tests of the conditional FF model by He, Kan, Ng and Zhang (1996), Ferson and Harvey (1999), and Ferson and Siegel (2003) strongly reject the conditional model, providing strong evidence that the conditional FF model fails to explain the dynamics of asset returns. Wang (2003), in contrast, uses a nonparametric estimation method and finds that the conditional FF model performs well in explaining the size and B/M portfolios. Despite the different conclusions reached by these studies, they all rely heavily on state variables. Ferson and Harvey (1999), for example, specify the intercept in the conditional FF model to be linear in lagged state variables, and test the collective significance of the coefficients. Wang (2003) assumes a more flexible form, relating the stochastic discount factor to the state variables.

The momentum effect documented by Jegadeesh and Titman (1993) has become one of the most serious challenges to existing asset pricing models. Many studies have shown that the unconditional FF model is unable to explain this return anomaly, but very few studies have explored the conditional version of this model to explain the momentum effect. Wang (2003) evaluates the nonparametric version of the conditional FF model for momentum portfolios and finds that the model can't be rejected. Since the results of Wang (2003) heavily depend on state variables, it is unknown how the conditional FF model will perform when we dispense with state variables.

In this subsection, we will re-examine the performance of the conditional FF model in the size, B/M portfolios and momentum portfolios without using state variables. Similar to the conditional CAPM, we first use the cross-validation

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method to find the optimal estimation window for each portfolio, and then obtain the conditional alphas and betas from (8) by letting $f = (R_m$ $SMB \ HML)'$. To examine how the model explains the various portfolios, we conduct the tests on individual pricing errors and average pricing errors.

1.3.3.1 Data-Driven Window Size

The estimated window size for the conditional FF model is provided in Table 1.3. A couple of features deserve highlighting. Because the conditional FF model has more regressors, the optimal windows are now much larger than those for the conditional CAPM, ranging from a minimum of 116 days for the equally-weighted W to a maximum of 596 days for the value-weighted G. As in the conditional CAPM, the window size for the equally-weighted portfolios are smaller than those for the value-weighted ones, implying a less stable information structure for the equally-weighted portfolios.

We notice that the difference between the window size for S and B is not as dramatic as it is for the conditional CAPM. For the conditional CAPM, B has a much larger window size than S: 333 vs. 60 if value-weighted, and 285 vs. 44 if equally-weighted. For the conditional FF model, however, the estimation windows are comparable for S and B: 356 vs. 349 if value-weighted, and 278 vs. 349 if equally-weighted. This is most likely because the conditional FF model has included the SMB factor, which attenuates the difference between S and B.

Another notable pattern is that momentum portfolios have the smallest windows among all portfolios. This is to be expected, because momentum

portfolios typically involve high turnovers and, as a result, their information structure should be volatile and unstable. This pattern, however, is not obvious for the conditional CAPM, probably because in the one-factor model, the difference between S and B plays a larger role than the high turnover. The conditional FF model, on the other hand, introduces the additional factor SMB, which may help diminish this difference, so that the window size can reveal more about the information structure pattern unique to momentum portfolios.

1.3.3.2 Individual Pricing Errors

Panel A of Figures 1.5 and 1.6 plot the conditional alphas for S-B, V-G, and W-L associated with the conditional FF model. The individual pricing errors still fluctuate over time, but the magnitude is much smaller than in the conditional CAPM, especially for S-B and V-G. Our intuition is that the conditional FF model produces much smaller pricing errors than the conditional CAPM. Comparing Panel A of Figure 1.5 to Panel A of Figure 1.1, and Panel A of Figure 1.6 to Panel A of Figure 1.2, we find a striking difference between the conditional CAPM and the conditional FF model. That is, the explanatory power of the conditional FF model seems to be more persistent over time than that of the conditional CAPM. To confirm our intuition, we plot the autocorrelation function for the $Diff$ measure, defined in equation (9), with the value-weighted S-B, V-G and W-L in Panel B of Figure 1.3 and the corresponding equally-weighted ones in Panel B of Figure 1.4. These figures show that the autocorrelation of $Diff$ for the conditional FF model remains
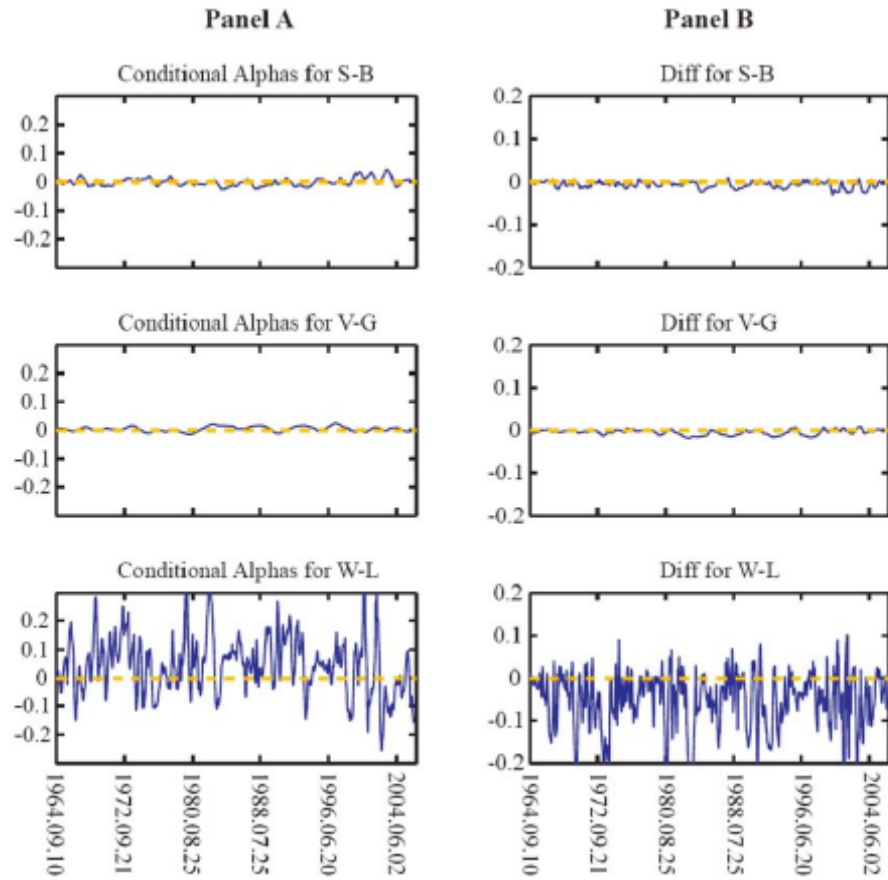
Figure 1.5: Conditional FF alphas and the *Diff* measure for the value-weighted S-B, V-G, and W-L.

Panel A plots the series for the conditional alphas, which are obtained from the nonparametric estimation of (8) with $f = (R_m \ SMB \ HML)'$. The conditional alphas are reported as daily percentages. Panel B plots the series of *Diff* which are calculated from (9). Positive values of *Diff* correspond to the periods in which the conditional FF model is accepted while negative values indicate the failure of the model.
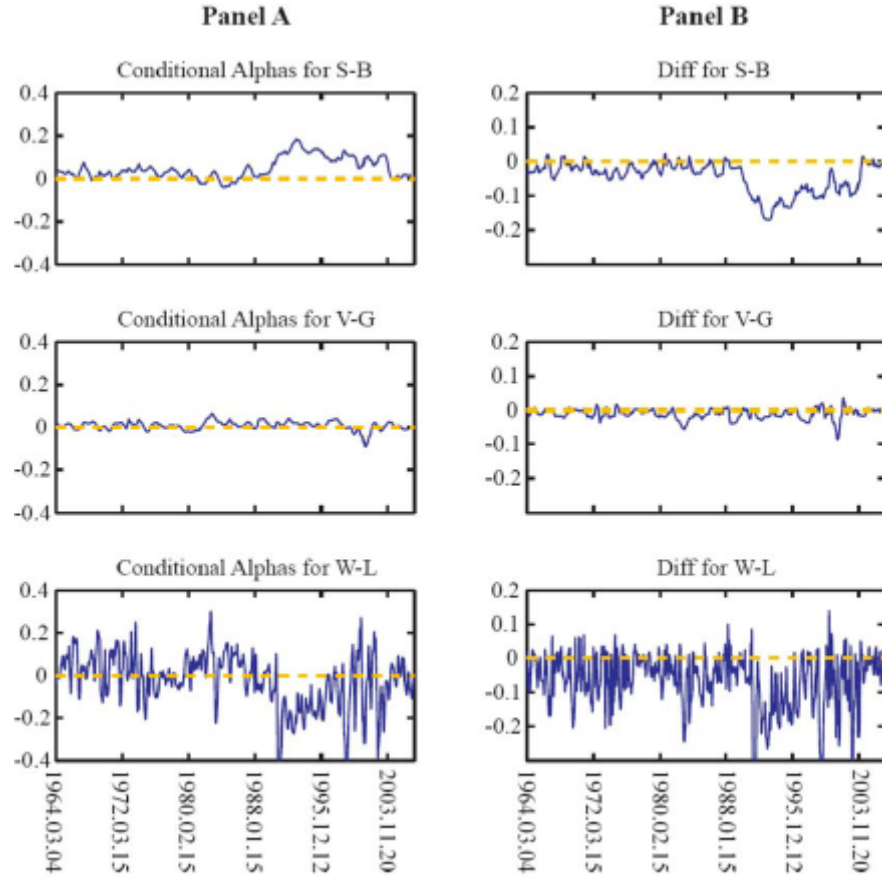
Figure 1.6: Conditional FF alphas and the *Diff* measure for the equally-weighted S-B, V-G, and W-L.

Panel A plots the series for the conditional alphas, which are obtained from the nonparametric estimation of (8) with $f = \left(R_m \; SMB \; HML\right)^{'}$. The conditional alphas are reported as daily percentages. Panel B plots the series of *Diff* which are calculated from (9). Positive values of *Diff* correspond to the periods in which the conditional FF model is accepted while negative values indicate the failure of the model.

very high for a long time before it gradually declines. For S-B and V-G, the autocorrelation of $Diff$ stays close to 1 for as long as 40 lags. For the conditional CAPM, however, the autocorrelation of $Diff$ declines much faster, generally declining almost zero within 60 lags. The persistence of the explanatory power for different portfolios is closely related to the optimal window size used to estimate these portfolios. In fact, it generally holds that the larger the window size, the longer it takes for the autocorrelation to approach zero. Consistent with the fact that the conditional FF model has a larger window size than does the conditional CAPM, it takes much longer for the autocorrelation to decline over time in the conditional FF model than in the conditional CAPM. Focusing on the conditional FF model, its explanatory power is least persistent for the momentum portfolios, which have the smallest windows and most volatile information structures.

As in the conditional CAPM, we also obtain the fraction of the time that the conditional FF model holds, which is shown in Table 1.4. Panel A shows that for the value-weighted portfolios, on average, the conditional FF model holds for more periods than the conditional CAPM. For instance, for the value-weighted V-G, the conditional FF model holds 26.58% of the time, which is much higher than the 16.33% in which the conditional CAPM holds. This implies that, as with the unconditional FF model, adding two additional risk factors helps explain the dynamics of most portfolios.

Panel B of Table 1.4 indicates that for the equally-weighted portfolios, the performance of the conditional FF model is somewhat mixed. For some portfolios, such as W-L, the proportion in which the model holds increases, but

for other portfolios such as S-B, the explanatory power decreases. Examining the conditional FF model, we find that the time periods drop sharply if portfolios are switched from value-weighted to equally-weighted; in the case of W, the decrease is as large as 12%. We therefore come to the same conclusion as in the conditional CAPM that it is much more difficult for the conditional FF model to explain the equally-weighted portfolios than the value-weighted ones.

Since the conditional FF model tends to hold for consecutive periods, we can identify the block of time periods in which it holds, and investigate under which conditions the model will perform best. This can shed light on when to use the conditional FF model to evaluate the profitability of certain trading strategies, the performance of mutual fund managers, etc. Alternatively, we can identify the periods in which the conditional FF model will fail. By examining the corresponding market and macroeconomic conditions, we can discover the missing factors, which is essential to improving the model. For example, one prominent pattern for the equally-weighted S-B is that the conditional FF model consistently failed in recent years, somewhere from the middle of the 1980s to 2003. Since the equally-weighted S-B puts more weight on small stocks, which are generally illiquid, investigating the behavior of small (illiquid) stocks during this period might help explain why the model fails. Moreover, as argued by Amihud and Mendelson (1986), Hasbrouck and Seppi (2001), Pastor and Stambaugh (2003), and Acharya and Pedersen (2007), among others, liquidity should be priced. Therefore, it would be interesting to find out whether incorporating the additional liquidity factor would help improve the performance of the conditional FF model in the equally-weighted S-B. In fact,

because neither the conditional CAPM nor the conditional FF model is able to explain the small portfolios, it would be interesting to find out whether the inclusion of the liquidity factor would help improve both models to explain these portfolios.

1.3.3.2 Average Pricing Errors

Table 1.7 shows the results for the value-weighted size, B/M, and momentum portfolios, where Panel A presents the results based on our nonparametric estimation using the optimal window size, and Panel B presents the results based on the non-overlapping window estimation.

Our results for the size and B/M portfolios in Panel A of Table 1.7 are similar to those obtained in Wang (2003), but stand in stark contrast to those in He, Kan, Ng and Zhang (1996), Ferson and Harvey (1999), and Ferson and Siegel (2003). Simply put, we find that the conditional FF model performs quite well for the size and B/M portfolios. For the size portfolios, S-B's average conditional alpha is -0.01% (z-stat -0.33), which is both economically and statistically insignificant. The average alphas for S and B are -0.07% (z-stat -2.33), and -0.05% (z-stat -2.50), respectively, which are statistically significant but economically small. V's average conditional alpha is -0.01% (z-stat -0.33), which is indistinguishable from zero. G's and V-G's average conditional alphas are -0.14% (z-stat -4.67), and 0.11% (z-stat 3.67), which are comparable to their unconditional FF model alphas of -0.11% and 0.08%, even though they are significant. All this indicates that the conditional FF model explains the

Table 1.7: Test of average conditional FF alphas for value-weighted portfolios, 1963-2007

The table reports the average conditional alphas for value-weighted size, B/M and momentum portfolios (% monthly). Panel A reports the nonparametric estimates from equation (8) with $f = (R_m \ SMB \ HML)'$ using daily data. The standard error is obtained from equation (6). Panel B presents estimates using the non-overlapping window estimation as in LN, with window size $N = 1, 3$ and 6. The standard error is from the time series variation of the conditional alphas, in the spirit of Fama and Macbeth (1973). Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Nonparametric Conditional Alphas*									
Est.	**-0.07**	**-0.05**	-0.01	**-0.14**	-0.01	**0.11**	**-0.49**	**0.44**	**0.91**
Std. err	0.03	0.02	0.03	0.03	0.03	0.03	0.12	0.08	0.16
Panel B: *Non-Overlapping Window Conditional Alphas*									
$N = 1$									
Est.	**0.30**	-0.14	**0.44**	0.08	-0.06	-0.14	**-1.60**	**0.91**	**2.50**
Std. err	0.14	0.09	0.15	0.13	0.14	0.13	0.40	0.33	0.57
$N = 3$									
Est.	-0.07	**-0.06**	-0.01	**-0.10**	-0.00	0.09	**-1.05**	**0.68**	**1.73**
Std. err	0.06	0.03	0.05	0.05	0.05	0.05	0.17	0.12	0.24
$N = 6$									
Est	-0.08	-0.04	-0.04	-0.09	0.03	**0.12**	**-0.63**	**0.46**	**1.09**
Std. err	0.05	0.03	0.04	0.05	0.04	0.05	0.34	0.11	0.23

return variation of the size and B/M almost as well as the unconditional FF model.

Interestingly, the conditional FF model helps explain the momentum effect. Panel A of Table 1.7 shows that the average conditional alphas for L, W and W-L are -0.49% (z-stat -4.08), 0.44% (z-stat 5.50), and 0.91% (z-stat 5.69), respectively. Even though these pricing errors are still large and significant, they are much smaller than those from the unconditional FF model, which are -0.92% (t-stat -6.57), 0.60% (t-stat 5.45), and 1.52% (t-stat 7.24), respectively. This implies that the conditional FF model explains the momentum portfolios much better than the unconditional version, indicating that incorporating time-varying betas and risk premia does help to explain the momentum anomaly. Moreover, the conditional FF model also produces much smaller average pricing errors than those from the conditional CAPM. For the conditional CAPM, the average pricing errors for L, W, and W-L are -0.88% (z-stat -6.77), 0.54% (z-stat 5.40), and 1.72% (z-stat 11.47), which are 25% to 80% larger than those from the conditional FF model. Even though we find that the conditional FF model helps to explain the momentum portfolios, we still strongly reject the model, which is different from Wang (2003).

Panel B of Table 1.7 shows the results from the non-overlapping window estimation. We observe that, like the conditional CAPM estimation, the results display great variations as the window size changes, especially for the momentum portfolios. The W-L's average conditional alpha, for example, decreases from 2.50% to 1.09% as the window increases from one month to three. Different window estimations moreover yield inconsistent results. The S-

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B's average alpha, for example, is 0.44% (t-stat 2.93) if $N = 1$, indicating the failure of the conditional FF model, whereas it becomes -0.01% (t-stat -0.20) if $N = 3$, providing no evidence to reject the model.

The results for the equally-weighted portfolios are presented in Table 1.8. Panel A shows that the conditional FF model is no longer able to explain the S-B, and the average pricing errors for S-B and V-G are also much larger than those for the value-weighted portfolios. The conditional FF model generally produces much smaller average pricing errors than does the conditional CAPM (except for W-L and G). For instance, V-G's average pricing error is 0.18% (z-stat 4.50) using the conditional FF, and 0.68% (z-stat 9.71) using the conditional CAPM. However, the conditional FF model performs no better than the unconditional FF, with W-L's average pricing error being -0.40% using the conditional FF model and -0.31% using the unconditional FF model.

In summary, with respect to the value-weighted portfolios, the average pricing errors from the conditional FF model are economically small for the size and B/M portfolios, providing evidence that the conditional model helps to explain the dynamics of the returns of these portfolios. For momentum portfolios, even though the conditional FF model still fails to capture their return variation, the average pricing errors are much smaller than those from the unconditional model, and they are also smaller than those from the conditional CAPM. As for the equally-weighted portfolios, their return variations are very difficult to capture by the conditional FF model.

Table 1.8: Test of average conditional FF alphas for equally-weighted portfolios, 1963-2007

The table reports the average conditional alphas for equally-weighted size, B/M and momentum portfolios (% monthly). Panel A reports the nonparametric estimates from equation (8) with $f = (R_m \quad SMB \quad HML)'$ using daily data. The standard error is obtained from equation (6). Panel B presents estimates using the non-overlapping window estimation as in LN, with window size $N = 1, 3$ and 6. The standard error is from the time series variation of the conditional alphas, in the spirit of Fama and Macbeth (1973). Bold values denote estimates greater than two standard errors from zero.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Nonparametric Conditional Alphas*									
Est.	**1.04**	0.04	**1.00**	**0.20**	**0.38**	**0.18**	**1.46**	**1.01**	**-0.40**
Std. err	0.05	0.03	0.06	0.04	0.03	0.04	0.10	0.05	0.11
Panel B: *Non-Overlapping Window Conditional Alphas*									
N = 1									
Est.	**1.18**	-0.07	**1.25**	0.22	0.20	-0.02	**0.85**	**1.57**	**0.72**
Std. err	0.18	0.15	0.25	0.16	0.14	0.16	0.31	0.21	0.36
N = 3									
Est.	**0.97**	0.06	**0.90**	**0.14**	**0.36**	**0.22**	**1.09**	**1.13**	0.05
Std. err	0.11	0.04	0.12	0.07	0.05	0.06	0.24	0.11	0.23
N = 6									
Est	**0.95**	**0.11**	**0.84**	**0.20**	**0.41**	**0.21**	**1.27**	**0.93**	**-0.34**
Std. err	0.12	0.04	0.13	0.07	0.05	0.06	0.27	0.10	0.26

1.3.4 Conditional Models or Unconditional Models?

So far, our focus has been on the test of the pricing errors, or the conditional alphas. We find that the conditional CAPM is not superior to the unconditional CAPM in explaining asset-pricing anomalies, and that even though the conditional FF model is superior to the unconditional FF model, it still fails to account for the momentum effect. In this subsection, we evaluate the models from the statistical point of view by asking the following question: Which model fits the actual data better? Or, which is closer to the true data generating process, the conditional or the unconditional model?

To answer this question, we need to directly compare the fit of an unconditional model to that of a conditional one. This is important because, even though a conditional model performs as badly as an unconditional one from the economic point of view, it may still fit the data better and thus represent a better model from the statistical point of view. In cases in which we have to rely on one model, it is better to choose the one which is closer to the true data generating process.

In this subsection, we directly compare the goodness of fit of the conditional and unconditional models by using a nonparametric test statistic proposed by Chen (2008). The idea is to compare the sum squared residuals (SSR) estimated from an unconditional model with those estimated from a time-varying conditional model. The test statistic is constructed in the spirit of F-test and follows a convenient standard normal distribution:

$$S = \frac{\sqrt{h}\left(SSR_0 - SSR_1\right) - A}{\sqrt{B}} \rightarrow^d N(0,1) \quad (10)$$

where SSR_0 is the SSR from an unconditional model of (7), and SSR_1 is the SSR from a conditional model of (8). h is the optimal window size, and A, B are centering and scaling factors. The details of A and B are provided in Appendix 1.C.

The null hypothesis is that there is no difference between a conditional model and an unconditional one in describing the true data generating process. This is a one-tail test. If the unconditional model is closer to the true data generating process than the conditional one, then we expect SSR_0 to be smaller than SSR_1. In this case, S will be small and the null hypothesis won't be rejected. On the other hand, if the conditional model is closer to the truth, then SSR_0 will be larger than SSR_1, generating a large S so that the null hypothesis is rejected.

We consider two pairs of models: the unconditional CAPM versus the conditional CAPM, and the unconditional FF model versus conditional FF model. For each portfolio, we calculate the test statistics from (10) associated with each model. The results are provided in Table 1.9. We see that for both pairs of models, the value of S far exceeds the 5% critical value of 1.65 for every portfolio, whether value-weighted or equally-weighted. This provides strong evidence that the conditional models are closer to the true data-generating process than the unconditional ones.

1.4. Conclusion

We propose a nonparametric method to estimate and test a time-varying factor model. Our method does away with state variables, and it also takes into account the nonlinear relationship between asset returns and factor returns. Rather than dividing windows in an arbitrary way as in existing studies, we obtain the optimal data-driven window size, which serves to minimize both the underconditioning and the small sample biases.

We then propose two tests to evaluate the performance of the conditional models. Our first test focuses on individual pricing errors and reveals whether or not a conditional model holds at a particular time period. In addition to the test on individual pricing errors, we derive the asymptotic distribution of average pricing errors under a very general distributional assumption, and test whether the average pricing errors are equal to zero.

Based on different estimation methods, there have been some controversies as to whether either the conditional CAPM or the conditional FF model can explain the well-known asset-pricing anomalies. In this paper, we use the nonparametric method to estimate the time series of the alphas and betas associated with these two conditional models and evaluate their performance in explaining cross section of stock returns.

Table 1.9: Test on Goodness of Fit

This table reports the test statistics calculated based on (10). The test statistic for the CAPM is calculated by comparing the sum squared residuals (SSR) estimated from equation (7) to the SSR estimated from equation (8), by letting $f = R_m$. The test statistic for FF is calculated by SSR estimated from equation (7) to the SSR estimated from equation (8), by letting $f = (R_m \quad SMB \quad HML)'$. If the test statistic is greater than 1.65, we reject the hypothesis that there is no difference between an unconditional and conditional model at 5% significant level.

	Size			B/M			Mom		
	S	B	S-B	G	V	V-G	L	W	W-L
Panel A: *Value-weighted Portfolios*									
CAPM	122.8	200.7	109.8	68.7	141.2	116.8	94.4	135.7	130.4
FF	227.8	271.4	149.4	132.4	199.9	106.1	197.3	216.8	276.0
Panel B: *Equally-weighted Portfolios*									
CAPM	134.9	148.9	128.5	81.7	129.3	123.7	124.3	139.7	162.6
FF	235.1	221.9	158.0	183.4	214.7	106.2	165.3	273.5	267.0

Our results show that the conditional CAPM holds at less than 1/4 of all the time periods for every portfolio. The conditional FF model holds at more time periods, and its explanatory power tends to be more persistent. The uniqueness of this test is that we are able to identify the exact time periods in which a conditional model holds, suggesting an interesting research direction for discovering the conditions in which a conditional model is better applied.

For the value-weighted portfolios, we provide strong evidence that the conditional CAPM fails miserably to explain the size effect, the value effect, and the momentum effect. The conditional FF model explains the size and value effect quite well, and it also helps to explain the momentum effect. However, it's rather difficult for either conditional model to explain the return variations in the equally-weighted portfolios, so the small portfolios remain a serious challenge to existing factor pricing models.

We also examine the goodness of fit for conditional versus unconditional models. Our results show that the conditional models are closer to the true data-generating process and are thus better models.

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CHAPTER 2

PROSPECT THEORY, THE DISPOSITION EFFECT AND ASSET PRICES[19]

2.1 Introduction

One of the mostly studied individual trading behaviors is the disposition effect:

investors have a greater tendency to sell assets that have risen in value since

purchase than those that have fallen.[20] This effect has been observed in many

markets, both for retail investors and for professional investors.[21] It is puzzling

because none of the most obvious rational explanations, such as portfolio

rebalancing or information story, can entirely account for the disposition effect

(Odean, 1998). As a result, an alternative view based on prospect theory has

gained favor.

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[19]This chapter is based on a joint paper with Liyan Yang.

[20]Shefrin and Statman (1985) coined the term the disposition effect. This effect is puzzling because the purchase price of a stock should not matter much for an investor's decision to sell it. In addition, tax laws encourage investors to sell losers rather than winners to reduce taxes. In a careful further study, Odean (1998) finds that the most obvious explanations, namely those based on information, taxes, rebalancing, or transaction costs, fail to capture important features of the data.

[21]Odean (1998), Grinblatt and Keloharju (2001), and Feng and Seasholes (2005) find disposition effects in the stock markets of the U.S., Finland, and China, respectively. Grinblatt and Keloharju (2001), Shapira and Veneezia (2001), Werners (2003), Garvey and Murphy (2004), Coval and Shumway (2005), Locke and Mann (2005), Frazzini (2006), and Scherbina and Jin (2006) document the disposition effect in the trading of professionals who trade on behalf of their firms; Genesove and Mayor (2001) and Heath *et al*. (1999) document disposition effects in the housing market and in the exercise of executive stock options, respectively. Webber and Camerer (1998) and Oehler *et al*. (2002) uncover disposition effects with experimental data. See Feng and Seasholes' (2005) Appendix A for more empirical studies on the disposition effect.

The literature has produced both *informal* arguments (e.g., Odean, 1998) and *formal* models (Kyle *et al.*, 2006; Hens and Vlcek, 2006; Barberis and Xiong, 2009), relying on prospect theory to explain the disposition effect. As a prominent theory of decision-making under risk, prospect theory was first proposed by Kahneman and Tversky (1979) and extended by Tversky and Kahneman (1992). A prospect theory investor evaluates gambles through gains and losses, not final wealth levels. The value function used by the investor to process gains and losses has a kink in the origin, indicating that investors are more sensitive to losses than to gains; this feature is referred to as *loss aversion* in the literature. Moreover, the value function is concave for gains and convex for losses, meaning that the investor is risk averse for gains and risk-loving for losses, which is known as *diminishing sensitivity.*[22]

Aside from using prospect theory to study the underlying cause of the disposition effect, recent empirical studies suggest that the disposition effect has pricing and volume implications: it can generate momentum in stock returns (Grinblatt and Han, 2005; Shumway and Wu, 2007), induce post-earnings announcement drift (Frazzini, 2006), and contribute to a positive correlation between returns and volumes (e.g., Statman *et al.*, 2006).

While existing studies have offered many insightful understandings on the link

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[22]For a review of prospect theory, see Barberis and Thaler's (2003) Section 3.2.1 or Barberis and Huang's (2008) Section 2. Another salient feature of prospect theory is probability weighting: the investor overweights small probabilities and underweights intermediate probabilities in computing the expectation. We don't incorporate this feature in our model due to the reasons discussed in Section 2.2.

from prospect theory to the disposition effect, and on the link from the disposition effect to return and volume patterns, they have almost always investigated these two links separately. On the one hand, the partial equilibrium models proposed by Kyle *et al.* (2006), Hens and Vlcek (2006) or Barberis and Xiong (2009) assume an exogenous stock return process, and are therefore silent about the pricing and volume implications of the disposition effect. On the other hand, Grinblatt and Han's (2005) theoretical model shows that the disposition effect can lead to price momentum, but it begins with a demand function featuring the disposition effect without exploring whether such a demand function can indeed be generated from prospect theory preferences. In particular, Barberis and Xiong (2009)'s partial equilibrium results suggest that when the expected stock return is high, the disposition effect leads to a reversed disposition effect, implying a *reversal* in stock returns and a *negative* correlation between returns and volumes. The literature thus lacks a theoretical foundation to support the intuition from prospect theory to the disposition effect and the intuition from the disposition effect to price momentum or volume patterns.

Without such a general equilibrium model, the following questions are thus left unanswered: Whether the intuitions emphasized in existing studies are coherent in a unified framework? Does prospect theory predict the disposition effect when stock returns are endogenous? Which component of prospect theory drives the momentum, and which drives the reversal? In a calibrated economy, how much can prospect theory explain the data? The challenges of proposing such a general equilibrium model come from: (i) an investor's decision involves solving an optimal stopping time problem with a non-smooth

and partially convex objective function, and (ii) the state vector in the general equilibrium model is high-dimensional, including the distribution of stock holdings and purchase prices (i.e., the reference points) for all investors in every possible state of nature.

In this paper, we develop an overlapping-generation (OLG) model to simplify an investor's optimal stopping time problem and to reduce the dimensions of the state vector, making it possible to simultaneously study the link between prospect theory and the disposition effect, as well as the impact of this effect on stock prices. In our model, over their lifetimes, investors can trade stocks and a risk-free asset in the financial market, and, at the end of their final periods, receive prospect theory utility based on their trading profits. The behavior of those investors who bought stocks in previous periods can potentially exhibit the disposition effect. Our model shows that different components of prospect theory make different predictions regarding trading behavior, return predictability and volume patterns.

Specifically, the *diminishing sensitivity* component, which posits that investors are risk averse (risk-loving) for gains (losses), or that the value function is concave (convex) in the gain (loss) domain, predicts the disposition effect in equilibrium, which in turn drives price momentum and a positive correlation between returns and volumes (See Subsection 2.4.2).[23] However, the *loss*

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[23]Throughout this paper, we follow the literature in using the terms diminishing sensitivity and concavity/convexity interchangeably to refer to the S-shaped value function of prospect theory.

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aversion component, which says that investors are more sensitive to losses than to gains, or that the value function has a kink at the origin, predicts exactly the opposite, namely, a reversed disposition effect in individual trading, reversal in the cross-section of stock returns and a negative correlation between returns and volumes (See Subsection 2.4.3). In a calibrated economy, when preference parameters are set at the values estimated by the previous studies, the concavity/convexity feature of prospect theory value function dominates, so that our model can generate an annual momentum of up to 7% (See Subsection 2.4.4).

The intuition for the implications of diminishing sensitivity is as follows. When a stock experiences good news and increases in value relative to the purchase price, these investors will be keen to sell it to lock in the paper gain, due to the concavity of the value function of prospect theory in the region of gains. Their selling increases volume. The selling pressure, moreover, depresses the stock price, generating subsequent higher returns. Similarly, when a stock experiences bad news and decreases in value relative to the purchase price, these investors are facing capital losses, and they are reluctant to sell, absent a premium, because of the convexity in the region of losses. In this case, the volume dries up, and the price is inflated, giving rise to subsequent lower returns. In this way, our model proves the internal consistency of the existing informal arguments which link prospect theory to the disposition effect (e.g., Odean, 1998) and which rely on the disposition effect to explain the momentum effect (e.g., Grinblatt and Han, 2005) and the positive relationship between price changes and volume (e.g., Odean, 1998; Statman *et al.*, 2006).

What's the intuition for the implications of loss aversion? Loss aversion means that prospect theory value function has a kink at the origin, and investors are afraid of holding stocks if they are close to the kink. It is well understood in the literature that loss aversion can raise equity premiums in equilibrium (e.g., Benartzi and Thaler, 1995; Barberis *et al.*, 2001). So in equilibrium, good (bad) news will push investors far from (close to) the kink, making them more likely to hold (sell) stocks when facing gains (losses). This resulting reversed disposition effect, in turn, leads to a negative correlation between returns and volumes, as well as reversal in the cross-section of returns: when a stock experiences good (bad) news and increases (decreases) in value relative to the purchase price, investors, according to the reversed disposition effect, want to hold (sell) stocks, which reduces (raises) the trading volume and inflates (depresses) the stock price; from that higher (lower) base, subsequent stock returns will also be lower (higher).

To the best of our knowledge, this paper is the first to comprehensively study the implications of prospect theory for individual trading behavior, asset prices and trading volume in a dynamic setting. Previous research on the effect of prospect theory in the asset pricing literature has focused primarily on the loss aversion component and shown that it can increase the equity premium, i.e., the *mean* of stock returns in excess of the risk free rate (e.g., Barberis *et al.*, 2001).[24] Our model demonstrates that loss aversion also has implications for

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[24]Recently, in a one period (two dates) model, Barberis and Huang (2008) show that the *probability weighting* feature of prospect theory can cause a security's individual skewness to be priced in equilibrium.

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return *predictability* and the correlation between returns and volumes. In addition, our paper shows that the S-shaped value function of prospect theory helps explain the disposition effect, the momentum effect and the comovement between stock returns and turnovers. Over and above these results, in Subsection 2.4.2, we argue that diminishing sensitivity alone, in the absence of loss aversion, can raise equity premiums.

The rest of the paper is organized as follows. Section 2.2 describes the model, and Section 2.3 characterizes the equilibrium. Section 2.4 solves the price-dividend ratios and uses simulated data to analyze the implications of diminishing sensitivity and loss aversion for individual trading behavior, asset prices and trading volumes. In particular, Subsection 2.4.4 conducts a quantitative analysis to evaluate how well our model matches the historical data. Section 2.5 concludes the paper. The appendix discusses the robustness of our results to certain modeling assumptions.

2.2. The Model

Let us consider an OLG model with one consumption good. Time is discrete and indexed by t. In each period, there are 3 generations (age-1, age-2 and age-3), each with a unitary mass. We adopt an OLG setup simply to reduce the dimension of the state vector. In the context of the disposition effect, the reference points usually relate to the purchase prices, which enter the state of the economy via the disposition effect, making the state history dependent. In an OLG setup, investors live for a finite period of time, so their purchase prices involve only a finite number of periods, effectively reducing the dimension of the state vector. The OLG setup should therefore not be interpreted literally.

Generations should be understood as generations of transactions, not generations of people. Since the average holding periods of stocks are six months to one year, one generation corresponds to six months to one year. Why are there three generations in each period? First, in order to study the disposition effect, which concerns selling decisions, we need at least three generations. In the standard two generation models, old investors always sell stocks whether facing good news or bad, thereby automatically ruling out the disposition effect. On the other hand, one model with more than two generations allows some investors to decide when to liquidate stocks which they bought in previous periods. Second, if there were more than three generations, the state vector would be highly dimensional, making the model intractable. In Appendix 2.A.2, we intuitively argue that our results might still hold in a setup with more than three generations.

2.2.1 Financial Assets

There are two traded assets: a risk-free bond and a risky stock. The bond is in perfectly elastic supply at a constant gross interest rate $R_f > 1$. The stock pays a random dividend $D_t > 0$ in period t. The dividend growth rate $\theta_{t+1} = \frac{D_{t+1}}{D_t}$ is *i.i.d.* over time, and follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H & \text{with probability } \frac{1}{2} \\ \theta_L & \text{with probability } \frac{1}{2} \end{cases}, \text{ with } 0 < \theta_L < \theta_H. \qquad (1)$$

The stock is in limited supply (normalized as 1) and is traded in a competitive market at price P_t. Let R_{t+1} be the gross return on the stock between time t and $t+1$, i.e., $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$.

Investors can buy or short bonds at any level, but they can not short stocks, and if they buy stocks, they can hold exactly 1 unit in each period. We assume that people hold either zero or one unit of stock for several reasons. First, this specification is realistic in the sense that the lower (upper) bound of the holding position captures the shorting (borrowing) constraints in stock trading. Second, the assumption that people buy at most one unit of stock at one time captures the idea that they tend to form different mental accounts for the same stock bought at different prices. Third, a binary choice in stock holdings simplifies an investor's decisions, because otherwise it is very difficult to characterize the investor's demand function due to the convexity of the Kahneman and Tversky (1992) value function in the loss domain. Finally, a binary choice and an OLG setup combine to reduce the complicated optimal stopping problem of an age-2 investor owning a stock to a simple problem of choosing between an early liquidation and a late liquidation.

2.2.2 Beliefs

In order to study the impact of the disposition effect on trading volumes, we make two assumptions on investors' beliefs. First, investors hold heterogeneous beliefs about the dividend growth rate within one period. Due to this cross-sectional heterogeneity in beliefs, investors, in particular young investors, will make different investment decisions: more optimistic investors will purchase a stock, while more pessimistic investors will not. Second, an investor's one-period-ahead dividend forecast changes during his lifetime. The time-variation in an investor's belief will motivate the selling of a middle-aged investor who purchased the stock when he was young. With these two assumptions, we ensure that in each period, there is always a group of middle-

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aged investors who bought stocks last period and want to sell them this period. It is this group of investors that can potentially exhibit a disposition effect. Of course, these two assumptions are just a modelling device, and any other trading motives, such as liquidity shocks (e.g., Kaustia, 2008), can also serve the same purpose.

As a matter of fact, in the informal arguments that have been used to link prospect theory and the disposition effect, investors are often assumed to experience belief changes, i.e., time-variation in an investor's beliefs is often maintained as the following quotation from Odean (1998, p. 1777) illustrates.

(S)uppose an investor purchases a stock that *she believes to have an expected return high enough* to justify its risk. If the stock appreciates and the investor continues to use the purchase price as a reference point, the stock price will then be in a more concave, more risk-averse, part of the investor's value function. It may be that the stock's expected return continues to justify its risk. However, if the investor *somewhat lowers her expectation of the stock's return*, she will be likely to sell the stock. What if, instead of appreciating, the stock declines? Then its price is in the convex, risk-seeking, part of the value function. Here the investor will continue to hold the stock even if its expected return falls lower than would have been necessary for her to justify its original purchase. Thus *the investor's belief about expected return must fall further* to motivate the sale of a stock that has already declined than one that has appreciated. [Emphasis added as italics]

Formally, in period t, investor i believes that the dividend growth rate θ_{t+1}

follows a distribution given by

$$\theta_{t+1} = \begin{cases} \theta_H & \text{with probability } q_{i,t} \\ \theta_L & \text{with probability } 1 - q_{i,t} \end{cases}, \qquad (2)$$

where $q_{i,t}$ is a random variable with uniform distribution on $[0,1]$ and $q_{i,t}$ is *i.i.d.* across investors (index i) and over time (index t).[25] On average, investors have the correct beliefs, since the mean of $q_{i,t}$ is equal to $\frac{1}{2}$. Investors are forward looking, so that we can apply the standard dynamic programming techniques to solve their optimal decision problems.

2.2.3 Preference

An investor derives prospect theory utility from trading assets in the spirit of Kahneman and Tversky (1979, 1992).[26] When investor i is born, he is endowed with $W_{1,i}$ units of consumption good. He can trade when he is young and middle-aged, leaving his final wealth as $W_{3,i}$ and his capital gains/losses as $X_{3,i}$. Let E_t^i denote the investor's expectation operator at time t. His time t utility, U_t^i, is then given by

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[25]In reality, an investor's one-period-ahead dividend forecasts might be correlated. As a robustness check, we also try the following specification to capture this correlation: $q_{i,t+1} = \rho q_{i,t} + (1-\rho)\varepsilon_{i,t+1}$ with $\rho \in (0,1)$, where $q_{i,t}$ follows a beta distribution and $\varepsilon_{i,t+1}$ follows a uniform distribution. If $\rho = 0$, then we return to the specification in the main text in which his forecasts are independent over time; if $\rho = 1$, then an investor's forecasts about dividend growth rate are constant over time.

[26]We also considered a model, similar to Barberis *et al.* (2001), in which an investor derives two kinds of utilities --- the standard consumption utility and prospect theory utility --- and obtained similar results.

$$U_t^i = E_t^i [v(X_{3,i})], \qquad (3)$$

where

$$X_{3,i} = W_{3,i} - R_f^2 W_{1,i}, \qquad (4)$$

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\alpha & \text{if } x < 0 \end{cases}, \qquad (5)$$

with $0 < \alpha \leq 1$ and $\lambda \geq 1$.

Here, the function $v(\cdot)$ is the standard value function of prospect theory proposed by Tversky and Kahneman (1992). The argument of $v(\cdot)$ is the capital gain/loss, $X_{3,i}$, not the final period wealth, $W_{3,i}$. Function $v(\cdot)$ is concave for gains and convex for losses, meaning that investors are risk averse in the domain of gains and risk-seeking in the domain of losses; it has a kink at the origin, implying a greater sensitivity to losses than to gains of the same magnitude. Parameter α governs its concavity/convexity and parameter λ controls loss aversion. For simplicity, we don't explore prospect theory's probability weighting feature in the above preference specification and just apply the standard expectation operator E_t^i. The primary effect of probability weighting is to overweight small probabilities; it therefore has its biggest impact on securities with highly skewed returns. Since most stocks are not highly skewed, we do not expect probability weighting to be central to the link between prospect theory and the disposition effect. Indeed, Hens and Vlcek (2006) find that probability weighting only plays a minor role in determining

whether prospect theory predicts the disposition effect.

In equation (4), we follow the literature (e.g., Gomes, 2005; Barberis and Huang, 2008; Barberis and Xiong, 2009) and define the capital gain/loss as $X_{3,i} = W_{3,i} - R_f^2 W_{1,i}$.[27] That is, we take a reference point as an investor's final wealth which he could have earned by investing in bonds when he was young and middle-aged.[28] The gain/loss from a particular stock sale is calculated as the difference between the reference point and the investor's final wealth resulting from buying and selling this stock. For example, if investor i buys a stock at price P^B at age 1, sells it at price P^S and collects a dividend $D_{2,i}$ at age 2, and he then reinvests $P^S + D_{2,i}$ in bonds, getting back $R_f P^S + R_f D_{2,i}$ at age 3. If he had not bought the stock at age 1, but had invested P^B in bonds and held them till age 3, then he would have collected $R_f^2 P^B$ at age 3. Therefore, the gain/loss from this stock sale is $X_{3,i} = R_f P^S + R_f D_{2,i} - R_f^2 P^B$.

This definition reflects the idea that an investor usually starts considering the stock investment as a loss if he could have earned more from investing in the riskless bond.

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[27]Two implementations of propect theory have been proposed in the literature. The first implementation defines propect theory over *annual* gains/losses (Benartzi and Thaler, 1995; Barberis *et al.*, 2001; Barberis and Huang, 2008). Another implementation is to define prospect theoy over "*realized* gains/losses as in Barberis and Xiong (2008, 2009). The two implementations will be identical in our setup, because investors are allowed to hold only one unit of stock over their lifetimes.

[28]In Appendix 2.A.1, we further show that our results are robust to taking purchase prices as reference points.

2.2.4 Timeline

To summarize, in the model, the exogenous random variables are θ_t and $q_{i,t}$, and the exogenous parameters of the model are $\theta_H > 0$, $\theta_L > 0$, $R_f > 1$, $0 < \alpha \leq 1$ and $\lambda \geq 1$. The order of events in each period t is shown in Figure 2.1. At the beginning of period t, age-1 investors are born and receive consumption good endowments. The dividend growth rate θ_t is realized, and all investors observe θ_t. The idiosyncratic belief shock $q_{i,t}$ is realized, and investor i observes $q_{i,t}$. All investors trade in the stock and bond market; age-2 and age-3 investors carry stocks to the market; after trading, age-1 and age-2 investors hold stocks. At the end of period t, age-3 investors receive prospect theory utility and exit the economy.

Our OLG setup can be understood as a stylized way of describing how different types of investors existing in real markets interact with each other. Our model economy can be linked to reality as follows. The potential buyers, namely an age-1 investor and an age-2 investor without a stock, correspond respectively to a new participant and to a wait-and-seer who has been sitting in the market for some time. The potential sellers, namely an age-3 investor and an age-2 investor owning a stock, correspond respectively to a pure noise investor, one who has no discretion with regard to the timing of his trade, and
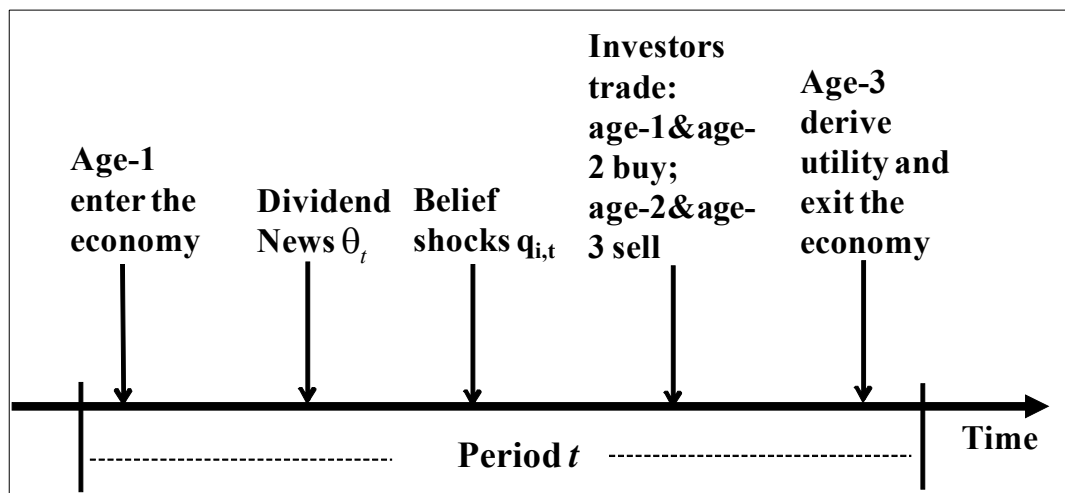
Figure 2.1 Timeline

Figure 2.1 plots the order of events in period t.

to a discretionary liquidity investor, one who can determine when to trade.[29]

2.2.5 Extension: A Multi-Stock Setting

So far, we have assumed just one risky asset, but our analysis has implications for the cross-section property of stock returns, so long as the investor engages in mental accounting or narrow framing (Thaler, 1980, 1985), thus deriving prospect theory utility separately from the trading profit on each distinct stock. This assumption is always present in the literature relating prospect theory to the disposition effect (e.g., Odean, 1998; Barberis and Xiong, 2009). Kumar and Lim (2008) also document that narrow framers indeed exhibit more of a disposition effect. Formally, we can consider an economy with N stocks, in which each stock has *i.i.d.* dividend processes with distribution given by equation (1), investors hold heterogeneous beliefs about the dividend growth rates and experience belief changes in their lifetimes, and these investors derive prospect theory utility from accumulative trading profits at the level of individual stocks. Then we can still use the conditions that characterize the equilibrium in the single stock setting --- more precisely, equations (6) through (23) --- to define an equilibrium, stock by stock, in this mutli-stock setting. In Section 2.4, we conduct such an analysis and calculate the average returns to the winners-minus-losers portfolio to examine whether price momentum exists in our model economies.

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[29]The importance of differentiating a pure noise investor from a discretionary liquidity investor has been emphasized in the microstructure literature, for example, Admati and Pfleiderer (1988).

2.3 Equilibrium

We now derive equilibrium asset prices. Let $f_t = \frac{P_t}{D_t}$ denote the price-dividend ratio in period t. To ease exposition, the investors of age 2 who *have* (*don't have*) a stock when they enter the market are referred to as *age-2-1 investors* (*age-2-0 investors*). Let z_t be the mass of age-2-1 investors in period t, i.e., z_t captures the distribution of stocks. Then in period t, the state of the economy is $S_t = (\theta_t, f_{t-1}, z_t)$. In equilibrium, the stock price-dividend ratios will be a function of the state vector, $f_t = f(S_t)$. The three variables θ_t, f_{t-1} and z_t affect stock prices because (i) θ_t and f_{t-1} affect age-2-1's investment decisions through the disposition effect, and (ii) z_t relates to aggregate effect on prices of age-2-1 investors as a whole. We construct the price-dividend function f by solving investors' optimal decisions backwards and using the market clearing condition.

2.3.1 Age-3 Investors' Decisions

A typical investor i of age 3 faces a state vector $(S_t, q_{i,t})$. His decision is simple: if he has a stock, he sells it and derives prospect theory utility from his trading profit; if he does not have a stock, he just waits until the end of the period and receives prospect theory utility. In sum, age-3 investors will sell $1 - z_t$ stocks as a whole.

2.3.2 Age-2 Investors' Decisions

A typical investor i of age 2 faces a state vector $(S_t, q_{i,t}, h_{i,t-1})$, where $h_{i,t-1} = 1$ if he belongs to age-2-1 and $h_{i,t-1} = 0$ if he belongs to age-2-0. An age-2-1 investor decides whether to sell the stock, and an age-2-0 investor decides whether to buy a stock.

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Let us first look at the age-2-1 investors. If an age-2-1 investor continues to hold the stock, what is his expected prospect theory utility? In the next period, he will sell the stock at price P_{t+1}, resulting in a gain/loss

$$P_{t+1} + D_{t+1} + R_f D_t - R_f^2 P_{t-1} = G_{1\to1}^{t+1} D_{t-1},$$
$$\text{with } G_{1\to1}^{t+1} = (f_{t+1} + 1)\theta_t \theta_{t+1} + R_f \theta_t - R_f^2 f_{t-1}. \tag{6}$$

As a result, his expected utility is

$$U_{1\to1}(S_t, q_{i,t}) = E_t^i[v(G_{1\to1}^{t+1})]D_{t-1}^\alpha, \tag{7}$$

where E_t^i is the subjective expectation operator conditional on investor i 's period t information set $\mathbf{F}_t^i = \{S_t, q_{i,t}\}$. Here, investor i takes expectation over the random variables θ_{t+1} and f_{t+1} according to his subjective belief [equation (2)] and the transition law of the state vector [equation (22)].

If he sells the stock, what is his expected prospect theory utility? Since he sells at price P_t, then his gain/loss is

$$R_f P_t + R_f D_t - R_f^2 P_{t-1} = G_{1\to0}^t D_{t-1},$$
$$\text{with } G_{1\to0}^t = (f_t + 1)R_f \theta_t - R_f^2 f_{t-1}. \tag{8}$$

Therefore, his expected utility is

$$U_{1\to0}(S_t) = v(G_{1\to0}^t)D_{t-1}^\alpha. \tag{9}$$

If $U_{1\to1}(S_t, q_{i,t}) \geq U_{1\to0}(S_t)$, then investor i will continue to hold the stock. That is,

those with sufficiently large belief shocks $q_{i,t}$ will not sell their stocks.

To sum up, the optimal decision of an age-2-1 investor is

$$h(S_t, q_{i,t}, 1) = 1_{U_{1\to1}(S_t, q_{i,t}) \geq U_{1\to0}(S_t)} = 1_{E_t^i[v(G_{1\to1}^{t+1})] \geq v(G_{1\to0}^t)}. \quad (10)$$

The corresponding *indirect* value function is[30]

$$V(S_t, q_{i,t}, 1) = \hat{V}(S_t, q_{i,t}, 1) D_{t-1}^\alpha,$$
$$\text{with } \hat{V}(S_t, q_{i,t}, 1) = h(S_t, q_{i,t}, 1) E_t^i[v(G_{1\to1}^{t+1})] + [1 - h(S_t, q_{i,t}, 1)] v(G_{1\to0}^t). \quad (11)$$

After trading, the fraction of those age-2-1 investors who continue to hold on to their stocks is

$$H_2(S_t, 1) = \int_0^{z_t} h(S_t, q_{i,t}, 1) di = z_t E[h(S_t, q_{i,t}, 1) | S_t], \quad (12)$$

where the second equality follows from the law of large numbers and the expectation is taken over the random variable $q_{i,t}$, which follows a uniform distribution over $[0,1]$.

Next, let us check the age-2-0 investors. If an age-2-0 investor decides to buy a stock, then he will have a gain/loss

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[30]Note that the *indirect* value function, $V(S_t, q_{i,t}, 1)$, is different from the value function of prospect theory $v(\cdot)$. Function $v(\cdot)$ corresponds to a standard Bernoulli utility function in the choice theory under uncertainty, but function $V(S_t, q_{i,t}, 1)$ is the indirect utility function which has taken into account the investor's optimal decisions.

$$P_{t+1} + D_{t+1} - R_f P_t = G_{0 \to 1}^{t+1} D_{t-1},$$
$$\text{with } G_{0 \to 1}^{t+1} = (f_{t+1} + 1)\theta_t \theta_{t+1} - R_f f_t \theta_t, \tag{13}$$

and have expected prospect theory utility

$$U_{0 \to 1}(S_t, q_{i,t}) = E_t^i \left[v\left(G_{0 \to 1}^{t+1} \right) \right] D_{t-1}^\alpha. \tag{14}$$

If he decides not to buy a stock, then his utility is 0. So an age-2-0 investor's optimal decision is

$$h(S_t, q_{i,t}, 0) = 1_{U_{0 \to 1}(S_t, q_{i,t}) \geq 0} = 1_{E_t^i \left[v\left(G_{0 \to 1}^{t+1} \right) \right] \geq 0}, \tag{15}$$

and the corresponding *indirect* value function is

$$V(S_t, q_{i,t}, 0) = \hat{V}(S_t, q_{i,t}, 0) D_{t-1}^\alpha, \text{with } \hat{V}(S_t, q_{i,t}, 0) = h(S_t, q_{i,t}, 0) E_t^i \left[v\left(G_{0 \to 1}^{t+1} \right) \right] \tag{16}$$

After trading, the aggregate stock holding of age-2-0 investors is

$$H_2(S_t, 0) = \int_0^{1-z_t} h(S_t, q_{i,t}, 0) di = (1 - z_t) E\left[h(S_t, q_{i,t}, 0) \mid S_t \right] \tag{17}$$

2.3.3 Age-1 Investors' Decisions

A typical investor i of age 1 faces a state vector $(S_t, q_{i,t})$. If he decides to buy a stock, then his expected prospect theory utility is

$$U_1(S_t, q_{i,t}) = E_t^i \left[V\left(S_{t+1}, q_{i,t+1}, 1 \right) \right] = \hat{U}_1(S_t, q_{i,t}) D_t^\alpha,$$
$$\text{with } \hat{U}_1(S_t, q_{i,t}) = E_t^i \left[\hat{V}\left(S_{t+1}, q_{i,t+1}, 1 \right) \right] \tag{18}$$

and if he decides not to buy a stock, then his expected utility is

$$U_0(S_t, q_{i,t}) = E_t^i[V(S_{t+1}, q_{i,t+1}, 0)] = \hat{U}_0(S_t, q_{i,t})D_t^\alpha,$$
$$\text{with } \hat{U}_0(S_t, q_{i,t}) = E_t^i[\hat{V}(S_{t+1}, q_{i,t+1}, 0)].$$

(19)

Therefore, his optimal decision is

$$h(S_t, q_{i,t}) = 1_{\hat{U}_1(S_t, q_{i,t}) \geq \hat{U}_0(S_t, q_{i,t})}.$$

(20)

So after trade, age 1 as a whole will hold

$$H_1(S_t) = \int_0^1 h(S_t, q_{i,t})di = E[h(S_t, q_{i,t}) \mid S_t].$$

(21)

2.3.4 Evolution of State Variables

The state vector S_t evolves according to the following equation

$$S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t)),$$

(22)

where functions $H_1(S_t)$ [given by (21)] and $f(S_t)$ are both endogenously determined. The random process $\{\theta_{t+1}\}_{t=1}^\infty$ is *i.i.d.* with distribution $\Pr(\theta_{t+1} = \theta_H) = \Pr(\theta_{t+1} = \theta_L) = \frac{1}{2}$ [i.e., equation (1)]. When investors make decisions, however, they believe that θ_{t+1} evolves according to $\Pr_t^i(\theta_{t+1} = \theta_H) = q_{i,t}$ [i.e., equation (2)]. Since S_t is in the investors' information set, they know the other two variables in S_{t+1}, i.e., f_t and z_{t+1}.

2.3.5 Market Clearing Condition

The market clearing condition is

$$H_1(S_t) + H_2(S_t, 0) + H_2(S_t, 1) = 1, \qquad (23)$$

which states that the stock holdings from age-1, age-2-0, and age-2-1 add up to the total stock supply 1. An equilibrium price-dividend function f is implicitly determined by equations (6) through (23).

We adopt the equilibrium concept of Radner (1972), known as equilibrium of plans, prices, and price expectations. An equilibrium is formally defined as follows.

Definition *An equilibrium consists of decision rules,* $h(S_t, q_{i,t}), h(S_t, q_{i,t}, 0)$ *and* $h(S_t, q_{i,t}, 1)$, *and a law of motion* $S_{t+1} = (\theta_{t+1}, f_t, z_{t+1}) = (\theta_{t+1}, f(S_t), H_1(S_t))$ *such that*

(1) the decision rules maximize investors' expected prospect theory utility conditional on their information;

(2) markets clear: $H_1(S_t) + H_2(S_t, 0) + H_2(S_t, 1) = 1$ *for almost every realization of* S_t; *and*

(3) the law of motion is generated by decision rules.

Note that the above definition of equilibrium has implicitly incorporated prices into the price-dividend ratio function in the law of motion.

2.3.6 Benchmark Case: Standard Risk Neutral Utility

Suppose $\alpha = \lambda = 1$. Concavity/convexity and loss aversion, two distinctive features of prospect theory, will vanish, reducing the preferences to a standard risk neutral utility representation. This works as a benchmark economy to illustrate that all our results are driven by prospect theory preferences. We don't use a standard, risk averse preference, such as power utility functions, as the benchmark, because risk aversion *per se* can *qualitatively* generate a disposition effect through portfolio rebalancing, although Odean (1998) argues that portfolio rebalancing cannot *quantitatively* account for the disposition effect.[31] Risk neutrality removes this contamination and therefore gives cleaner results.

When investors are risk neutral, i.e., when $\alpha = \lambda = 1$, both the price-dividend ratio and the mass of age 2-1 investors are constant:
$f_t \equiv f = \frac{E(\theta_{t+1})}{R_f - E(\theta_{t+1})} = \frac{\frac{1}{2}\theta_H + \frac{1}{2}\theta_L}{R_f - \left(\frac{1}{2}\theta_H + \frac{1}{2}\theta_L\right)}$ and $z_t = \frac{1}{2}$. This result can be obtained by examining equations (6) to (23).

In fact, the constant price-dividend ratio is consistent with the simple Gordon rule: $P_t = \frac{E(\theta_{t+1})D_t}{R_f - E(\theta_{t+1})}$. Intuitively, the potential buyers of stocks are those age-1 and age-2 investors who hold optimistic views about next period's dividend realization; the marginal buyer's subjective belief, coinciding with the true

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[31]If investors sell winners due to portfolio rebalancing, then they will partially reduce their position in a winning stock, rather than sell the entire position of the stock. Odean (1998) shows that the disposition effect still remains strong, even when the sample is restricted to transactions of investors' entire holdings of a stock, i.e., to those transactions not motivated by portfolio rebalancing. This suggests that portfolio rebalancing cannot entirely account for the disposition effect.

distribution of the dividend process, brings the stock price equal to the sum of the discounted expected dividends. In this special case, we have an *i.i.d.* return process,

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{f+1}{f}\theta_{t+1},$$

with mean equal to R_f. The age-2-1 investors no longer exhibit a disposition effect, because half of them, those who have received pessimistic belief shocks (i.e., $q_{i,t} < 1/2$), will always liquidate stocks no matter whether they face gains or losses.

For the general cases of $\alpha < 1$ or $\lambda > 1$, we have to numerically solve the price-dividend function $f(\cdot,\cdot,\cdot)$ and age-1 investors' stock demand function $H_1(\cdot,\cdot,\cdot)$. The basic methodology is as follows: starting from an initial conjecture of $f^{(0)}(\cdot,\cdot,\cdot)$ and $H_1^{(0)}(\cdot,\cdot,\cdot)$, solve $f^{(1)}(S_t)$ and $H_1^{(1)}(S_t)$ on a grid of S_t from equations (6)-(23), and continue this process until $f^{(n)}(\cdot,\cdot,\cdot) \to f(\cdot,\cdot,\cdot)$ and $H_1^{(n)}(\cdot,\cdot,\cdot) \to H_1(\cdot,\cdot,\cdot)$.

2.4 Numerical Results and Intuitions

In this section we solve equations (6) through (23) for the two endogenous functions in the law of motion: the price-dividend ratio function, $f(\cdot,\cdot,\cdot)$, and the aggregate demand function of age-1 investors, $H_1(\cdot,\cdot,\cdot)$. We then use simulations to show that the two components of prospect theory, diminishing sensitivity and loss aversion, make exactly opposite predictions regarding individual trading behavior, return predictability, and the correlation between

returns and volume. Specifically, Subsection 2.4.2 demonstrates that diminishing sensitivity drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive correlation between returns and volume. Subsection 2.4.3 shows, on the other hand, that loss aversion predicts a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. Subsection 2.4.4 conducts further quantitative analysis to examine how successful prospect theory is in explaining price momentum, and suggests testable empirical predictions.

2.4.1 Calibrating Technology Parameters

There are five exogenous parameters in our model: two preference parameters (λ and α) and three technology parameters (θ_H, θ_L and R_f). Since we are interested in the implications of preferences, we allow the preference parameters to vary over a certain range. But we calibrate the technology parameters as follows. We take one period to be one year, and thus set the net risk-free rate to $R_f - 1 = 3.86$ percent, a choice adopted by Barberis and Huang (2001). Since the disposition effect refers to the behavior of individual stocks, we choose dividend parameters to match the mean and standard deviation of the dividend growth rate of a typical individual stock. Barberis and Huang (2001) estimate the moments of individual stock dividend growth using the COMPUSTAT database, and based on their results, we set $\theta_H = 1.28$ and $\theta_L = 0.76$, such that the mean and volatility of the net growth rate of the dividend are 2.24% and 25.97%, respectively. Table 2.1 summarizes our choice of technology parameters.

2.4.2 Implications of Diminishing Sensitivity

We obtain the implications of diminishing sensitivity through comparative static analysis with respect to parameter λ, which governs the curvature of the value function. To ensure that our results are completely driven by the concavity/convexity component of prospect theory, in this subsection we also set parameter λ at 1 to remove the loss aversion feature of the preference. Table 2.2 presents the main results for a range of values of α: 0.2, 0.5, 0.88 and 1. In particular, when $\alpha = 1$, the investor is risk neutral, which provides a benchmark for highlighting the fact that our results stem from prospect theory preferences. The value of 0.88 is the number estimated by Tversky and Kehneman (1992). Our results demonstrate that, in a general equilibrium setting, diminishing sensitivity drives the disposition effect, the momentum effect and the comovement between returns and volume. We also find that diminishing sensitivity alone, in the absence of loss aversion, raises equity premiums.

2.4.2.1 Disposition Effects

We use the following measure to test whether our model can generate a disposition effect,

$$DispEffect = \frac{E\left[\left.\frac{z_t - H_2(S_t, 1)}{z_t}\right| G_{1 \to 0}^t > 0\right]}{E\left[\left.\frac{z_t - H_2(S_t, 1)}{z_t}\right| G_{1 \to 0}^t < 0\right]}. \quad (24)$$

Table 2.1   Technology Parameter Values

We take one period to be one year. Dividend parameters (θ_H and θ_L) are calibrated to generate a dividend growth rate with the mean and standard deviation equal to 2.24% and 25.97%, respectively.

Parameter	Values
Risk-free rate	
R_f	1.0386
Dividend parameters	
θ_H	1.2821
θ_L	0.7628

Table 2.2 Implications of Diminishing Sensitivity

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$, and if $DispEffect > 1$, then a disposition effect exists. $MomEffect = E\left(R_{t+1} \mid \theta_t = \theta_H\right) - E\left(R_{t+1} \mid \theta_t = \theta_L\right)$. WML is the simulated average momentum portfolio return in the multi-stock setting. If $MomEffect > 0$ and $WML > 0$, then a momentum effect exists. $Q_t = 1 - H_2\left(S_t, 1\right)$ is the turnover, or aggregate selling, in period t. Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. The preference parameter $\lambda \geq 1$ determines loss aversion; in this table, we deliberately set λ as 1, so that the investor is not averse to loss.

	$\alpha = 0.2$	$\alpha = 0.5$	$\alpha = 0.88$	$\alpha = 1$
(i) Disposition Effect				
PGR	0.49	0.56	0.53	0.50
PLR	0.29	0.36	0.47	0.50
DispEffect	**1.73**	**1.56**	**1.12**	**1.00**
(ii) Momentum Effect				
$E(R_{t+1}\mid\theta_t = \theta_H)$	1.1295	1.0934	1.0516	1.0386
$E(R_{t+1}\mid\theta_t = \theta_L)$	1.0158	1.0439	1.0410	1.0386
MomEffect	**11.37**%	**4.95**%	**1.06**%	**0.00**%
WML	**10.91**%	**4.67**%	**1.06**%	**0.00**%
(iii) Turnover				
$Corr(R_t, Q_t)$	**0.52**	**0.83**	**0.92**	**0.00**
(iv) Equity Premium				
$E(R_t - R_f)$	**3.43**%	**3.01**%	**0.82**%	**0.00**%
$E[H_1(S_t)]$	0.43	0.46	0.49	0.50

If $DispEffect > 1$, then we conclude that investors exhibit the disposition effect in our model. The numerator of $DispEffect$ is the average fraction of age-2-1 investors who close their positions facing a capital gain. This term is the theoretical analog to Odean's (1998) proportion of gains realized (PGR), i.e., the number of gains that are realized as a fraction of the total number of gains that could have been realized. Similarly, the denominator of $DispEffect$ is the average fraction of age-2-1 investors who realize losses and corresponds to Odean's proportion of losses realized (PLR). Odean uses the difference between PGR and PLR to measure the disposition effect. In equation (24), we instead adopt a ratio of PGR to PLR to remove the effect of equity premiums on the magnitudes of PGR or PLR.[32]

To obtain the two conditional moments in equation (24), we simulate a long time series $\{\theta_t\}_{t=1}^{\infty}$ of 500,000 independent draws from the distribution described in equation (1). Then we use the solved functions $f(\cdot,\cdot,\cdot)$ and $H_1(\cdot,\cdot,\cdot)$ to calculate f_t and z_{t+1} and get the time series $\{S_t\}_{t=1}^{\infty}$. When we do this, we also compute $\{H_2(S_t,1)\}_{t=1}^{\infty}$ and $\{G_{1\to0}^t\}_{t=1}^{\infty}$ along the way, using equations (H21) and (G10). We compute sample moments from these simulated data to serve as approximations of population moments.

Table 2.2 reports the results for different values of α. The case of $\alpha = 1$ corresponds to a linear value function, when investors don't exhibit a disposition effect, so that $DispEffect = 1$. As we gradually decrease α from 1

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[32]Brown *et al.* (2006) also use the ratio of PGR to PLR to measure the disposition effect when examining Australian stock trading data.

to 0.2, the value function becomes more curved along the way, and the value of *DispEffect* increases monotonically from 1 to 1.73, giving rise to an even stronger disposition effect. The mechanism behind this result is exactly Odean's (1998) intuition: risk aversion (risk-seeking) for gains (losses) causes an age-2-1 investor more (less) likely to sell the stock.

Figure 2.2 graphs this intuition for the case of $\alpha = 0.5$. Here, from the simulated time series of state vectors, we randomly choose a realization of $\left(f_{t-1}, z_t \right) = \left(20.01, 0.50 \right)$, and then graph the possible gains/losses together with the associated prospect theory utilities faced by an age-2-1 investor in periods t and $t+1$.[33] The period t gains/losses as well as prospect theory utilities from liquidating the stock [i.e., $\left(G_{1 \to 0}^t, v \left(G_{1 \to 0}^t \right) \right)$] are marked with dots, while the period $t+1$ gains/losses and prospect theory utilities from keeping the stock [i.e., $\left(G_{1 \to 1}^{t+1}, v \left(G_{1 \to 1}^{t+1} \right) \right)$] are marked with circles.

Good dividend news ($\theta_t = \theta_H$) will bring an age-2-1 investor to the point of choosing a sure medium gain (6.5, Point H in the figure) versus a gamble which offers either a smaller gain (1.18, Point HL) or a larger gain (14.45, Point HH) with some probabilities. Whether an age-2-1 investor will continue to hold the stock depends on his one-period-ahead dividend forecast. In this example, those age-2-1 investors who believe, with probability higher than 0.54 (i.e., $\frac{v(6.5) - v(1.18)}{v(14.45) - v(1.18)}$), that the next period dividend growth rate (θ_{t+1}) will take a high value (θ_H) will continue to hold the risky stock.

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[33]The result is robust to the choice of $\left(f_{t-1}, z_t \right)$.
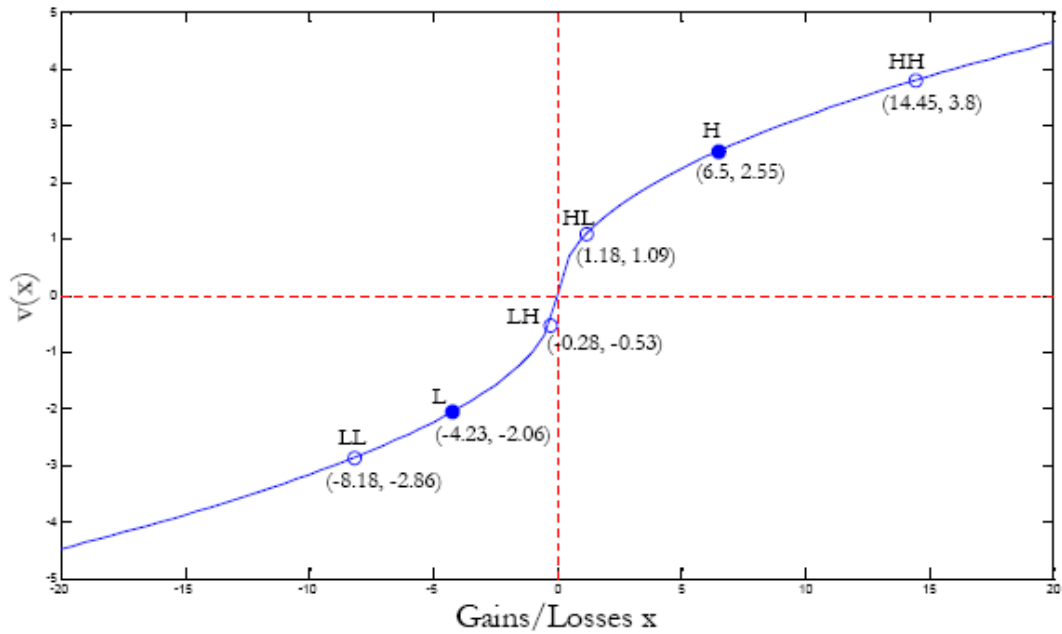
Figure 2.2 Diminishing Sensitivity Drives the Disposition Effect

Figure 2.2 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 20.01$ and $z_t = 0.50$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 1$ and $\alpha = 0.5$.

What will happen if the dividend news is negative ($\theta_t = \theta_L$) at period t? If an age-2-1 investor sells the stock, he experiences a sure loss (-4.23, Point L); if he continues to hold the stock, he faces the gamble of a smaller loss (-0.28, Point LH) or an even larger loss (-8.18, Point LL). In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability lower than 0.35 (i.e., $\frac{v(-4.23)-v(-8.18)}{v(-0.28)-v(-8.18)}$), will liquidate their stocks. Note that the cutoff probability in the low dividend realization case, 0.35, is lower than that in the high dividend realization, 0.54. This precisely supports the informal argument, which relies on prospect theory to explain the disposition effect: the investor's belief about expected return must fall further to motivate the sale of a stock that has already declined than one that has appreciated (Odean, 1998, p. 1777).

Table 2.2 suggests that PGR and PLR respond to a change in α differently: as α falls from 1 to 0.2, PGR first goes up from 0.50 to 0.56 and then goes down to 0.49, while PLR continuously decreases from 0.50 to 0.29. There are two forces at work here. As α becomes smaller, the value function is more concave for gains and more convex for losses, causing the investor to be more likely to sell winners and hold losers, and hence generating a higher PGR and a lower PLR. However, as α falls, the expected stock return rises and the stock becomes more attractive to the investor, which will be discussed shortly; this decreases the investor's propensity to sell the stock no matter whether he is facing gains or losses, and therefore leads to both a lower PGR and a lower PLR. In sum, as α decreases, both forces tend to lower PLR, while the first force tends to raise PGR and the second to lower PGR. As α falls slightly below 1, the first force dominates, and we observe a higher PGR, but once α falls sufficiently, the second force catches up and we obtain a

lower PGR.

2.4.2.2 Momentum

Following Barberis *et al.* (1998), who also rely on a model with one risky asset to explain the cross-section of stock returns, we measure momentum as

$$MomEffect = E\left(R_{t+1} \mid \theta_t = \theta_H\right) - E\left(R_{t+1} \mid \theta_t = \theta_L\right), \quad (25)$$

i.e., the difference in the expected return following a positive shock and following a negative shock. If $MomEffect > 0$, then we claim that there is momentum in the stock returns. The two moments in equation (25) are obtained using simulations. The results are also reported in Table 2.2. Since $MomEffect > 0$ for $\alpha < 1$, our model shows that the concavity/convexity feature of prospect theory preferences generates momentum in stock returns. Moreover, the momentum effect becomes stronger as we increase the curvature of the value function, i.e., decrease the value of α. For example, $MomEffect$ increases from 1.06% to 11.37% as α decreases from 0.88 to 0.2. The underlying reason for this momentum effect is simple. Following a positive shock ($\theta_t = \theta_H$), stock prices will rise, moving age-2-1 investors into their capital gain domain. Due to the concavity of the value function of prospect theory in the gain region, age-2-1 investors tend to close their stock positions, which depresses the stock price, generating higher subsequent returns. On the other hand, a negative shock ($\theta_t = \theta_L$) will decrease the stock price, driving age-2-1 investors into their capital loss domain. Convexity in the region of losses means that they are less likely to sell the stock absent a price premium; the stock price is therefore initially inflated, generating lower subsequent

returns.

We also conduct a cross-section analysis and replicate the momentum effect in the empirical literature (e.g., Jegadeesh and Titman, 1993; Liu and Zhang, 2008). As discussed in the end of Section 2.2, we can extend our model to an economy with N stocks. We simulate dividend data on $N = 2,000$ independent stocks over $T = 10,000$ time periods, and then compute the resulting equilibrium return sequence for each stock. We create the winners-minus-losers zero cost portfolios as follows. In each period, we sort stocks into two equal-sized groups based on their last period returns and record the equal-weighted return of each group over the next period; in particular, R_t^{winner} (R_t^{loser}) is the return on the portfolio containing stocks with better (worse) performance. Repeating this each period produces long time series of returns on the winner and loser portfolios, namely $\{R_t^{winner}\}_{t=1}^T$ and $\{R_t^{loser}\}_{t=1}^T$. Our second measure of momentum is the difference in the average returns on these two portfolios:

$$WML = \frac{1}{T} \Sigma_{t=1}^T \left(R_t^{winner} - R_t^{loser} \right) \quad (26)$$

Table 2.2 also reports the results for this alternative measure. We find that the two measures for momentum are almost identical, so that they behave in precisely the same way: both *MomEffect* and *WML* are greater than 0 for $\alpha < 1$, and both decrease with α.

2.4.2.3 Turnover

Empirical studies show that there is more trading in rising markets than in falling markets (Statman *et al.*, 2006; Griffin *et al.*, 2007). In our model, the age-2-1 investors have a much greater propensity to sell stocks facing good news ($\theta_t = \theta_H$) than facing bad news ($\theta_t = \theta_L$). This will contribute to a positive correlation between turnover and stock returns. Let $Q_t = 1 - H_2(S_t, 1)$ be the turnover or aggregate selling in period t. In Table 2.2, we report the simulated correlations between stock returns and turnovers, $Corr(R_t, Q_t)$. Indeed, we have $Corr(R_t, Q_t) > 0$ so long as $\alpha < 1$. This demonstrates that diminishing sensitivity drives a positive correlation between returns and volume.

As we gradually decrease α from 0.88 to 0.2, $Corr(R_t, Q_t)$ decreases from 0.92 to 0.52. The stock distributions (z_t) and price-dividend ratios (f_{t-1}) combine to contribute to this relationship, but they work in different ways when α varies. When α is close to 1, both z_t and f_{t-1} are almost constant at their values in the benchmark economy (i.e., $\alpha = 1$), so that the state of the economy is captured only by dividend growth rates (θ_t). Since the disposition effect causes returns and turnovers to vary with θ_t in the same direction, there is an almost perfect correlation between returns and volume. On the other hand, as α gets close to 0, both z_t and f_{t-1} will change over time and influence trading behavior. However, returns and volumes respond to the variation in z_t and f_{t-1} in opposite ways. For example, a larger z_t implies that more stocks are held by age-2-1 investors and fewer by age-3 investors; after trading, all age-3 investors will have to close their positions, even though this is not the case for age-2-1 investors; as a result, stock selling (i.e., trading volumes Q_t) will decrease with z_t, but at the same time, the decreasing selling

pressure causes stock returns R_t to rise with z_t. The variation in z_t and f_{t-1} will therefore attenuate the positive correlation between returns and volume generated by θ_t. As a result, for $0 < \alpha < 1$, a lower α implies a lower $Corr(R_t, Q_t)$.

2.4.2.4 Equity Premiums

Our model also demonstrates that the S-shaped value function of prospect theory can help explain the equity premium puzzle. Table 2.2 reports the simulated equity premiums, $E(R_t - R_f)$, as well as average stock purchases by young people, $E[H_1(S_t)]$. As α gets smaller, the curvature of the value function becomes larger, and equity premiums become higher. Note that the positive equity premium is not due to loss aversion, since we have set $\lambda = 1$ in this section. Notably, a low α is also associated with a low $E[H_1(S_t)]$, suggesting that equity premiums are driven by the behavior of young people. The young investor makes investment decisions by comparing the expected utility from buying the stock to that from not buying. These utility levels are determined by his belief $q_{i,t}$ (current belief about θ_{t+1}), and by how he evaluates his future reactions to $q_{i,t+1}$ (future belief about θ_{t+2}). Those who are extremely optimistic (pessimistic), i.e., those with extremely high (low) values of $q_{i,t}$, always buy (not buy) the stock. It is those who have intermediate values of $q_{i,t}$ that care more about their future reactions to $q_{i,t+1}$. It turns out that only high realizations of $q_{i,t+1}$ will matter, because there will be no extra benefit of holding a stock from middle-aged till old when $q_{i,t+1}$ is low. Only when $q_{i,t+1}$ is high will holding the stock from middle-aged till old bring an extra benefit: a young investor who buys a stock now will enjoy a further gain if he keeps the stock, and one who doesn't buy now will enjoy a new gain if he buys

the stock when middle-aged.

How does this extra gain associated with high $q_{i,t+1}$ relate to the current purchasing decision and the value function's curvature α? Not buying now means that when evaluating this gain, the young investor will stay in the origin of the value function, where the marginal utility is the highest; the more curved the value function, the higher is this marginal utility. In contrast, if he buys now, he will be pushed away from the origin because this gain has to be appended to an existing gain or loss, namely the one generated by holding the stock from young until middle-aged. In this case, the marginal utility is much smaller compared to that in the origin; the more curved the value function, the smaller is this marginal utility.

To summarize, the higher the curvature of the value function, the less a young investor will value the potential gain associated with high realizations of $q_{i,t+1}$, and the less they want to buy now, thereby depressing stock prices and raising equity premiums.

2.4.3 Implications of Loss Aversion

To obtain the implications of loss aversion, we conduct comparative static analysis with respect to the parameter λ. In Table 2.3, we present the results for a variety of values of λ: 1, 2.25, 3 and 4. In particular, $\lambda = 1$ is still our benchmark economy when the investor is risk neutral. The value of $\lambda = 2.25$ is the number estimated by Tversky and Kahneman (1992). To guarantee that our results are solely due to the loss aversion component, we always set parameter $\alpha = 1$ to remove the curvature feature of the prospect theory value

function. Table 2.3 demonstrates that loss aversion drives a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. In addition, Table 2.3 produces a well-known result in the asset pricing literature: loss aversion can raise equity premiums, such as Benartzi and Thaler (1995) and Barberis *et al*. (2001).

2.4.3.1 Reversed Disposition Effects

Again, when investors are risk neutral, i.e., when $\lambda = 1$, they don't exhibit a disposition effect, so that $DispEffect = 1$. When investors are loss averse, i.e., when $\lambda > 1$, we obtain a reversed disposition effect, since $DispEffect < 1$. Moreover, as we gradually increase λ from 1 to 4, investors become more loss averse, and the value of $DispEffect$ decreases monotonically from 1 to 0.83, giving rise to an even stronger reversed disposition effect.

What's the intuition behind this result? The mechanism works through a combination of two forces: one is the kink at the origin of the value function, which is a direct implication of loss aversion; the other is the positive equity premium, which is an indirect equilibrium implication of loss aversion preferences. Roughly speaking, when investors are close to (far from) the kink, they are reluctant (inclined) to take risk, and want to sell (keep) the stock; when the average stock returns are higher than the risk free rate, bad (good) dividend news will bring investors relatively close to (far from) the kink, so that they are more (less) likely to liquidate the stock.

Table 2.3   Implications of Loss Aversion

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$, and if $DispEffect < 1$, then a reversed disposition effect exists. $MomEffect = E\left(R_{t+1} \mid \theta_t = \theta_H\right) - E\left(R_{t+1} \mid \theta_t = \theta_L\right)$. WML is the simulated average momentum portfolio return in the multi-stock setting. If $MomEffect < 0$ and $WML < 0$, then there is reversal in the cross-section of stock returns. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t. Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Preference parameter α controls the curvature of the value function. In this table, we deliberately set α to be 1, so that the value function is piecewise linear.

	$\lambda = 1$	$\lambda = 2.25$	$\lambda = 3$	$\lambda = 4$
(i) Disposition Effect				
PGR	0.50	0.39	0.35	0.31
PLR	0.50	0.41	0.39	0.38
DispEffect	**1.00**	**0.97**	**0.91**	**0.83**
(ii) Momentum Effect				
$E(R_{t+1}\mid\theta_t = \theta_H)$	1.0386	1.1000	1.1255	1.1519
$E(R_{t+1}\mid\theta_t = \theta_L)$	1.0386	1.1006	1.1311	1.1632
MomEffect	**0.00%**	**−0.06%**	**−0.56%**	**−1.13%**
WML	**0.00%**	**−0.23%**	**−0.85%**	**−1.48%**
(iii) Turnover				
$Corr(R_t, Q_t)$	**0.00**	**−0.70**	**−0.91**	**−0.94**
(iv) Equity Premium				
$E(R_t - R_f)$	**0.00%**	**6.17%**	**8.97%**	**11.89%**

Figure 2.3 conducts an exercise to confirm this intuition for the case of $\lambda = 4$.
Now that we assumed $\alpha = 1$ to remove the curvature, the investor's value
function becomes piecewise linear with a kink at the origin due to loss
aversion. Similar to the exercise in Figure 2.2, we randomly choose a
realization of (f_{t-1}, z_t), which is $(7.75, 0.48)$ in this case, from the simulated time
series of state vectors. We then graph an age-2-1 investor's period t
gains/losses as well as prospect theory utilities from liquidating the stock [i.e.
$(G_{1\to 0}^t, v(G_{1\to 0}^t))$] with dots, and the period $t+1$ gains/losses and prospect theory
utilities from keeping the stock [i.e. $(G_{1\to 1}^{t+1}, v(G_{1\to 1}^{t+1}))$] with circles.

Good dividend news ($\theta_t = \theta_H$) will bring the investor to Point H. Bad dividend
news ($\theta_t = \theta_L$) will bring him to Point L, which is closer to the kink relative to
Point H. That is, the investor is more cautious in holding stocks at Point L than
at Point H. Specifically, at Point H, if the investor liquidates the stock, he will
lock in a medium gain of 3.29; if he keeps the stock, when he becomes old he
will arrive either at Point HH, enjoying a large gain of 7.35, or at Point HL,
enjoying a small gain of 1.46. Since both Point HH and Point HL are in the
gain domain, the investor's behavior at Point H can be described as risk
neutral. Of course, whether an age-2-1 investor will indeed continue to hold
the stock depends on his one-period-ahead dividend forecast. In this example,
those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability higher than
0.31 (i.e., $\frac{v(3.29) - v(1.46)}{v(7.35) - v(1.46)}$), will continue to hold the risky stock.
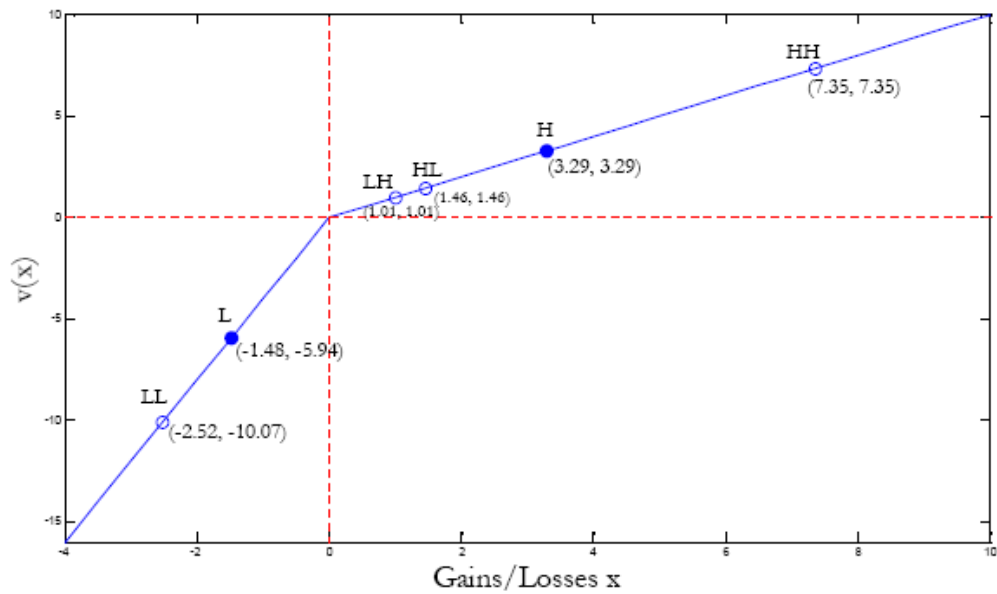
Figure 2.3 Loss Aversion Drives the Reversed Disposition Effect

Figure 2.3 graphs the possible capital gains/losses, as well as prospect theory utilities, faced by an age-2-1 investor. If this investor liquidates his stock, his capital gains/losses, together with his prospect theory utilities, are marked with dots; if he keeps the stock, then his possible future capital gains/losses and his prospect theory utilities are marked with circles. The two endogenous state variables are $f_{t-1} = 7.75$ and $z_t = 0.48$. The parameter values are $\theta_H = 1.2821$, $\theta_L = 0.7628$, $R_f = 1.0386$, $\lambda = 4$ and $\alpha = 1$.

At Point L, if the investor sells the stock, he will realize a loss of 1.48. If he keeps the stock, then he will arrive either at Point LH, enjoying a small gain of 1.01, or at Point LL, facing a large loss of 2.52. Because Point LH and Point LL straddle over the kink, the investor is reluctant to take a risk at Point L relative to Point H, at which point his behavior resembles risk neutrality. In this example, those age-2-1 investors who believe that $\theta_{t+1} = \theta_H$ with probability lower than 0.37 (i.e., $\frac{v(-1.48)-v(-2.52)}{v(1.01)-v(-2.52)}$), will liquidate their stocks.

In Table 2.3, we also observe that both PGR and PLR decrease with λ. This is because loss aversion raises equity premiums, making the investor less likely to sell stocks, whether facing good news or bad news. We also observe PGR decreases at a faster rate than PLR due to the reversed disposition effect.

2.4.3.2 Reversal

As discussed above, when $\lambda = 1$, the investor is risk neutral, and there is no momentum effect in the cross-section of stock returns, because both measures capturing momentum, *MomEffect* and *WML*, are equal to zero. But as long as $\lambda > 1$, i.e., as long as the investor is loss averse, we obtain reversal in the cross-section of stock returns, since both *MomEffect* and *WML* are negative. In particular, as we increase λ from 1 to 4, reversal gets stronger. This result demonstrates that the loss aversion feature of prospect theory has implications for return predictability.

The underlying reason for this result is similar to Grinblatt and Han's (2005). For example, facing good dividend news, age-2-1 investors are more likely to

hold stocks according to the reversed disposition effect. This generates extra buying pressure, which will inflate stock prices and lead to lower stock returns later. Similarly, facing bad dividend news, those investors are likely to sell stocks and depress prices, generating higher subsequent returns.

2.4.3.3 Turnover

Table 2.3 also shows that loss aversion can generate a negative correlation between returns and volumes: $Corr(R_t, Q_t) < 0$ as long as $\lambda > 1$. This result is also driven by trading by age-2-1 investors, who, due to the reversed disposition effect, have a much greater propensity to sell stocks in down markets ($\theta_t = \theta_L$) than in up markets ($\theta_t = \theta_H$), contributing to a negative correlation between turnover and stock returns.

As we gradually increase λ from 1 to 4, $Corr(R_t, Q_t)$ monotonically decreases from 0 to -0.94. This pattern is different from the relationship between $Corr(R_t, Q_t)$ and α in Table 2.2 and can be understood as follows. In Table 2.2, when we vary α while fixing λ, dividend news θ_t contributes to a positive $Corr(R_t, Q_t)$ via the disposition effect, while the other endogenous state variables, stock distributions (z_t) and price-dividend ratios (f_{t-1}), tend to generate a negative $Corr(R_t, Q_t)$. These two forces are counteracting. On the other hand, in Table 2.3, when we vary λ and fix α, dividend news θ_t also leads to a negative $Corr(R_t, Q_t)$ through the reversed disposition effect, which strengthens the impact of the two endogenous state variables on $Corr(R_t, Q_t)$.

2.4.3.4 Equity Premiums

Table 2.3 also reproduces the well-known result that loss aversion can raise equity premiums (e.g., Benartzi and Thaler,1995; Barberis *et al.*,2001). As we increase λ from 1 to 4, equity premiums rise from 0 to 12%. This result is intuitive: loss aversion means that investors are more sensitive to losses than to gains, and since stocks often perform poorly and investors often face losses, a large premium is required to convince them to hold stocks. The asset pricing literature studying loss aversion has focused primarily on its implications for the equity premium, that is, the average *level* of stock returns. Our model, on the other hand, shows that loss aversion can lead to reversal in the cross-section of stock returns, suggesting, in turn, that loss aversion may also be a useful ingredient for equilibrium models trying to understand return *predictability*.

2.4.4 Quantitative Analysis and Testable Predictions

In this Subsection, we conduct further quantitative analysis to examine how successful prospect theory is in explaining price momentum and derive testable empirical predictions which are either unique to our model or consistent with the existing empirical studies.

2.3.4.1 Quantitative Analysis: How Successful is Prospect Theory?

So far, we have shown that there are two counteracting forces in equilibrium --- diminishing sensitivity and loss aversion --- driving the disposition effect, the momentum effect and the correlation between returns and volumes. In order to understand how successful prospect theory is in explaining price momentum, we set preference parameters at certain empirical values and examine which

force will dominate, and to what extent.

What are the empirical values of preference parameters, λ and α? The existing evidence concerning parameter λ is relatively rich and remarkably consistent: both experimental data (e.g., Kahneman *et al.*, 1990; Tversky and Kahneman, 1991, 1992; Novemsky and Kahneman, 2005) and real data (e.g., Putler, 1992; Hardie *et al.*, 1993) suggest a number close to 2. This is true even for monkeys (Chen *et al.*, 2006). So in the following analysis, we fix λ at 2.25, the value estimated by Tversky and Kahneman (1992).

But there is not much evidence as to the value of α. As far as we know, only two studies have estimated this parameter, and the results differ markedly in the data sets used. Tversky and Kahneman (1992) estimate $\alpha = 0.88$ by offering subjects isolated gambles in experimental settings. Wu and Gonzalez (1996) use a different experimental data set and estimate $\alpha = 0.52$, but when they apply Camerer and Ho's (1994) data, they find $\alpha = 0.37$. Due to the small sample size in the experiments, none of those studies can estimate α with great precision. So our strategy is to report results for all these three possible values of α in Table 2.4.

Table 2.4 Quantitative Analysis

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E\left(R_{t+1} \mid \theta_t = \theta_H\right) - E\left(R_{t+1} \mid \theta_t = \theta_L\right)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2\left(S_t, 1\right)$ is the turnover, or aggregate selling, in period t. Technology parameter values are fixed at the values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter λ is set at 2.25, the value estimated by Tversky and Kahneman (1992).The empirical values of PGR/PLR and momentum are taken from Dhar and Zhu (2006) and Jegagdeesh and Titman (1993), respectively. The empirical values of $Corr\left(R_t, Q_t\right)$ and $E\left(R_t - R_f\right)$ are based on AMEX/NYSE data from 1926-2006.

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$	Empirical Value
(i) Disposition Effect				
PGR	0.40	0.41	0.41	0.38
PLR	0.18	0.23	0.37	0.17
DispEffect	**2.25**	**1.75**	**1.10**	**2.24**
(ii) Momentum Effect				
$E(R_{t+1} \mid \theta_t = \theta_H)$	1.1575	1.1431	1.1091	—
$E(R_{t+1} \mid \theta_t = \theta_L)$	1.0822	1.0927	1.1004	—
MomEffect	**7.54%**	**5.04%**	**0.87%**	—
WML	**7.20%**	**4.76%**	**0.76%**	**8.60%**
(iii) Turnover				
$Corr(R_t, Q_t)$	**0.84**	**0.88**	**0.91**	**0.28**
(iv) Equity Premium				
$E(R_t - R_f)$	**8.14%**	**7.94%**	**6.62%**	**7.84%**

Table 2.4 also presents the historical values for the disposition effect, the momentum effect and the correlation between returns and volumes. Unlike Odean (1998), who studies the disposition effect by aggregating across investors, Dhar and Zhu (2006) examine the disposition effect at the level of the individual. They report, in their Table 2.2, that the means of PGR and PLR for all individuals are 0.38 and 0.17, respectively. We adopt these numbers as the empirical values of PGR and PLR. Regarding the momentum effect, we use Jegagdeesh and Titman's (1993) estimate, that is, 8.60%, on an annual basis. Using AMEX/NYSE data from 1926-2006 from CRSP, we find that the correlation between returns and volumes, $Corr(R_t, Q_t)$, and the equity premium, $E(R_t - R_f)$, for a typical firm, are 0.28 and 7.84%, respectively.[34] Those historical values help us to evaluate how well our model matches the data. Even though, because we are not confident of the actual value of α among real investors, this evaluation should be interpreted with caution, our quantitative analysis makes a methodological contribution: a general equilibrium model, such as the one provided in the present paper, is the only way to link prospect theory preference to momentum, thereby explaining how much prospect theory preference can contribute to price momentum.

Table 2.4 demonstrates that, for all the three possible values of α, the diminishing sensitivity component of prospect theory dominates the loss aversion component. In particular, when $\alpha = 0.37$, our model matches the

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[34]More precisely, we take all stocks in the CRSP database for which at least 11 consecutive years of return and volume data are recorded, compute the correlation between real returns and volume as well as the mean returns in excess of the 30-day T-bill rate for each, and then calculate the medians.

historical data well, except for the dimension of the correlation between returns and volumes. To be specific, for $\alpha = 0.37$, our model predicts that $DispEffect = 2.25$, $WML = 7.20\%$ and $E(R_t - R_f) = 8.14\%$, while the historical counterparts for these variables are 2.24, 8.60% and 7.84%, respectively. The model predicts too high a correlation between returns and volumes, i.e., $Corr(R_t, Q_t) = 0.84$, but the empirical value is 0.28.

2.4.4.2 Testable Predictions

One testable prediction emerges from Table 2.4, which suggests that prospect theory simultaneously predicts momentum and a positive correlation between returns and volumes. So we expect the momentum effect to be stronger among those stocks whose returns are *positively* correlated with their own trading volumes.[35] This empirical prediction is unique to our mechanism and is easy to test. We can rely on this prediction to differentiate our story from other explanations of price momentum, such as the belief-based models proposed by Barberis *et al.* (1998), Daniel *et al.* (1998) or Hong and Stein (1999). Note that our prediction is different from that of Lee and Swaminathan (2000), who show that price momentum is more pronounced among those stocks with higher *levels* of trading volumes, while our predictions relates momentum to the *sensitivity* of returns to volumes.

Besides the above new prediction, our model also makes certain predictions which are consistent with the existing studies. For example, we do not expect

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[35]Note that we don't claim that momentum profits are monotonically increasing in $Corr(R_t, Q_t)$. Actually, Table 4 suggests that the opposite is true.

prospect theory utility to be equally important for all investors, expecting it to matter more for individual investors than for institutional investors. Indeed, some empirical studies find that mutual fund managers are less prone to the disposition effect than individual investors: the difference between PGR and PLR is 3% for managers, and 5% for retail investors (c.f. Shefrin, 2008). Since our results on momentum are completely driven by prospect theory, one prediction of our model is that a stronger momentum effect will exist among stocks with greater individual investor ownership. Hur *et al.* (2008) test precisely this prediction with a large sample of NYSE/AMEX/NASDAQ stocks between 1981 and 2005 and find strong evidence for this hypothesis. Further evidence comes from Hong *et al.* (2000) and Fama and French (2008), who find that the profitability of momentum strategies declines sharply with market capitalization; since small firms are traded more heavily by individuals, this finding is consistent with our prediction.

Our model can also relate momentum to the volatility of cash flow. Table 2.5 examines the effect of varying the volatility of the dividend growth rate. For a binary distribution given by equation (1), the dividend growth rate has a mean equal to $E(\theta_{t+1}) = \frac{\theta_H + \theta_L}{2}$, and a volatility equal to $\sigma(\theta_{t+1}) = \frac{\theta_H - \theta_L}{2}$. In Table 2.5, we maintain $E(\theta_{t+1}) - 1$ at 2.24% and change $\sigma(\theta_{t+1})$ from 21% to 26% to 31%.[36] The preference parameters are set at $\alpha = 0.52$ and $\lambda = 2.25$. Table 2.5 suggests that increasing $\sigma(\theta_{t+1})$ generates stronger momentum effects and

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[36]Barberis and Huang (2001) use COMPUSTAT data to estimate the dispersion in firm-level dividend growth volatilities to be 5 percent. So, we choose 5% as a step.

Table 2.5 Sensitivity Analysis w.r.t Dividend Growth Rate Volatility $\sigma(\theta_{t+1})$

PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E(R_{t+1} | \theta_t = \theta_H) - E(R_{t+1} | \theta_t = \theta_L)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2(S_t, 1)$ is the turnover, or aggregate selling, in period t. The risk-free rate is set at $R_f = 1.0386$. The preference parameters are $\alpha = 0.52$ and $\lambda = 2.25$.

	$\theta_L = 0.81586$ $\theta_H = 1.2289$	$\theta_L = 0.7628$ $\theta_H = 1.2821$	$\theta_L = 0.70865$ $\theta_H = 1.3362$
(i) Disposition Effect			
PGR	0.41	0.41	0.41
PLR	0.24	0.23	0.23
DispEffect	**1.71**	**1.75**	**1.79**
(ii) Momentum Effect			
MomEffect	**3.83%**	**5.04%**	**6.28%**
WML	**3.62%**	**4.76%**	**5.94%**
(iii) Turnover			
$Corr(R_t, Q_t)$	**0.88**	**0.88**	**0.88**
(iv) Equity Premium			
$E(R_t - R_f)$	**6.13%**	**7.94%**	**9.96%**

higher equity premiums. Since a higher $\sigma(\theta_{t+1})$ is also associated with a higher return volatility, the momentum effect is expected to be stronger among stocks both with higher dividend volatility and with higher return volatility. This observation is in fact consistent with Zhang's (2006) finding that momentum profits are higher among firms with higher cash flow volatility or return volatility.

2.5 Conclusion

In this paper, we propose a general equilibrium model to study the implications of prospect theory for individual trading, security prices and trading volume. We show that, in a general equilibrium setting, different components of prospect theory make very different predictions. The diminishing sensitivity component drives a disposition effect, which in turn leads to momentum in the cross-section of stock returns and a positive correlation between returns and volumes. On the other hand, the loss aversion component predicts exactly the opposite, namely a reversed disposition effect and reversal in the cross-section of stock returns, as well as a negative correlation between returns and volume. In a calibrated economy, when prospect theory preference parameters are set at the values estimated by the previous studies, our model can generate price momentum of up to 7% on an annual basis. One testable empirical prediction unique to our model is that the momentum strategy is most profitable, all else equal, among stocks whose returns are positively correlated with their trading volumes.

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CHAPTER 3

DIVIDEND VOLATILITY and ASSET PRICING[37]

3.1 Introduction

How does aggregate dividend volatility affect asset prices?[38] Until now the

literature has largely disregarded this question. To the best of our knowledge,

the only exception is Longstaff and Piazzesi (2004), who demonstrate that

volatile and procyclical dividends can raise equity premiums in a

representative agent model with power utility. However, their model explains

less than half as large as historical equity premiums, and they don't explore

whether dividend volatility can help explain other puzzling facts in the

aggregate stock market, such as return predictability and time-varying Sharpe

ratios. More importantly, their consumption-based model will inevitably predict

a high correlation between consumption and stock returns, contradicting our

observation. In this paper, we turn to a narrow-framing approach to

comprehensively study the pricing implications of dividend volatility, and find

that our model can explain key asset markets phenomena.

Narrow-framing means that, when people evaluate risk, they often appear to

pay attention to narrowly defined gains and losses. This behavior is uncovered

by experimental work on decision-making under risk (e.g., Kahneman and

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[37]This chapter is based on a joint paper with Liyan Yang.

[38]Throughout the paper, the term dividend volatility refers to the standard
deviation of the *growth rate* of (not the level of) the *aggregate* dividends paid
to all stocks. See equation (4) for a technical definition.

Lovallo, 1993; Kahneman, 2003). In the context of financial investment, narrow-framing states that investors tend to separate their financial wealth from their overall wealth, and are inclined to get utility directly from fluctuations in the value of their overall portfolio of stocks (Benartzi and Thaler, 1995; Barberis and Huang, 2001; Barberis, Huang, and Santos, 2001, henceforth BHS; Barberis and Huang, 2007).[39] Under this assumption, investors may perceive aggregate dividend volatility, which drives fluctuations in the value of their financial wealth, as a more appropriate metric to represent risk than consumption volatility, a commonly used measure in the literature. This immediately implies that dividend volatility has significant implications for asset prices.

In this paper, we first provide strong empirical evidence that (i) dividend volatility exhibits strong persistence, usually called volatility clustering, indicating the tendency of a big (small) change today to be followed by a big (small) change tomorrow, (ii) dividend volatility has declined dramatically in the postwar period.[40] The aggregate dividend time series we use is backed out from CRSP stock return data.[41] This imputed dividend series has accounted for stock repurchases as an increasingly significant component of dividends

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[39]In the literature, narrow-framing is sometimes applied to individual stocks that investors own (e.g., Barberis and Huang, 2001). For a deep discussion on narrow-framing, see Barberis and Huang (2007).

[40]Lettau, Ludvigson and Wachter (2008) also mention that the volatility of dividend growth has declined since 1990s. But their model assumes that this decline affects stock prices through consumption.

[41]This constructed dividend index is identical to Campbell (2000). The detailed data construction is given in the appendix.

since 1980. One may argue that the declining trend in dividend volatility is partly due to corporate managers' intention to smooth dividend. Whatever the reason is, however, an investor in our theoretical model takes the dividend process as exogenously given when making her investment decisions, which is a standard assumption in the asset pricing literature.

We further propose a theoretical model in which dividend volatility is persistent and investors exhibit loss aversion: they dislike fluctuations in their financial wealth; and the more persistent the dividend volatility, the more they dislike stocks. Loss aversion is a central feature of the prospect theory of Kahneman and Tversky (1979), which is based on a variety of experimental evidence and has been extensively used in behavioral finance literature (e.g., Benartzi and Thaler, 1995; BHS, 2001).

Our model is able to account for many of the stylized facts of asset prices, including the high mean and excess volatility of stock prices, predictability of stock returns, time-varying Sharpe ratios, a low and stable risk-free rate, and the low correlation between consumption and stock returns. Our model shows, moreover, the substantial decline in dividend volatility since the 1950s, signals a much more stable investment environment, which loss averse investors prefer; they therefore require a much lower return on holding stocks, resulting in lower equity premiums. This is consistent with Blanchard (1993), Fama and French (2002), and Buranavityawut, Freeman and Freeman (2006), who find that ex-ante equity premiums have declined in the past fifty years. Dividend volatility plays an essential role in explaining the intuitions of our model. As the state variable, it completely determines equilibrium price-

dividend ratios and helps explain the high mean and excess volatility of stock returns. In equilibrium, a rise (drop) in dividend volatility lowers (raises) asset prices, and hence price-dividend ratios fluctuate with the dividend volatility process, generating excess volatility in market returns. The high volatility of returns, in turn, means that stocks often perform poorly, causing loss averse investors considerable discomfort and leading to low stock prices or high risk premiums. Furthermore, dividend volatility tends to be higher in market troughs than in booms, which leads to the countercyclical expected excess returns observed in financial markets.

The persistence of dividend volatility leads to the persistence of the price-dividend ratio, producing predictability in stock returns, where the forecasting power increases with the forecast horizon. The conditional mean and conditional standard deviation of expected returns are driven differently by dividend volatility, hence the Sharpe ratio as a measure of the price of risk changes over time. Moreover, the model-generated stock returns correlate only weakly with consumption, because stock returns are ultimately driven by dividend news, which has a low correlation with consumption news.

Many studies have been devoted to explaining these puzzling facts in the literature. Our work is closely related to two prominent approaches,[42] but also differs in a variety of ways. The first approach, including Campbell and

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[42]Besides the two approaches mentioned here, another line of research relies on modifying the market and asset structure (e.g., Constantinides and Duffie, 1996; Heaton and Lucas, 1996).

Cochrane (1999) and BHS (2001), relies on stochastic changing risk aversion, whereas the second, including Bansal and Yaron (2004, henceforth BY), relies on the changing economic environment.

With respect to the first approach, we share with BHS (2001) the use of loss aversion to describe investors' preferences. However, we depart from them in two ways: we use loss aversion as the only psychological assumption, and our result isn't driven by the changing risk aversion of investors. BHS's result depends crucially on another psychological assumption, usually labelled the house money effect, which refers to the experimental finding that people are more (less) willing to bear risks when they have had prior gains (losses). The house effect together with loss aversion generates their model's results.

In terms of the mechanism, our model is similar to BY (2004) in that we all require a persistent component in the underlying processes. However, our model specification is less stringent than theirs. In BY's model, it's critical to model the growth rates of both consumption and dividends as containing a long-run predictable component, as well as containing persistent volatility to stand for fluctuating economic uncertainty. In conjunction with Epstein and Zin's (1989) preferences, they succeed in explaining the financial market phenomena. However, as BY have pointed out, since it's econometrically difficult to distinguish an i.i.d. process from a process containing a small persistent component, it's rather difficult to justify the forecastable persistent component in the consumption and dividend growth rates. In our model, we need the persistent component only in the volatility of the dividend growth rate, which is supported by strong econometric evidence; we don't rely on the persistent component in the growth rates *per se*, which lacks empirical

evidence. The consumption growth rate is still maintained to be a white noise process in our model.

The rest of the paper is organized as follows. Section 3.2 provides extensive econometric evidence to show that (i) dividend volatility is persistent over time and (ii) it changes with the business cycle and experiences significant declines in the postwar period. Section 3.3 presents the model and characterizes the equilibrium asset prices. Section 3.4 calibrates the model and solves the price-dividend ratios, then analyzes model simulation results. Section 3.5 concludes the paper.

3.2. Key Features of Historical Dividend Volatility

3.2.1 Dividend Volatility Clustering

In this subsection, we provide evidence that dividend volatility displays the property of clustering, which, as we will see more clearly later, plays an important role in explaining the high mean, excess volatility, as well as the predictability of stock returns. We perform a variety of standard econometrics tests: first identify whether volatility clustering in dividend in fact exists and, if so, run a unit-root test to check how strong this persistence is.

Volatility clustering, which characterizes the persistence in volatility, has been documented as a standard feature of many financial series. For instance, Bollerslev, Engle and Wooldridge (1988) show that conditional variance of market return fluctuates across time and is very persistent. For high-frequency return data, the ARCH literature finds a very high coefficient in the correlation of conditional standard deviations of returns. In our model, we consider

volatility clustering in the dividend growth rate and examine its impact on equilibrium asset prices. Even though our data are at a low-frequency, the estimated coefficient is very similar to those found in high-frequncy data. Before running the ARCH type tests, we first run two diagnostic tests to see if there is volatility clustering in the dividend growth rate, which is constructed from the value weighted NYSE/AMEX return data from CRSP. More specifically, we use two standard tests in the econometric literature, the Box-Pierce-Ljung test and ARCH test, to check whether there are strong correlations in the second moment of the dividend growth rate. Both tests have as the null hypothesis that there's no volatility clustering in the dividend growth rate, and under the null, both tests asymptotically follow a Chi square distribution. Panel A of Table 3.1 presents the test results. The statistics from both tests significantly reject the null hypothesis, indicating strong persistence in the volatility of the dividend growth rate.

The preliminary tests make us comfortable using the exponential GARCH (EGARCH) model to identify the persistent component in dividend volatility. We use EAGRCH for two reasons: first, it matches best with our theoretical dividend volatility specification in section 3.3; second, it can capture the asymmetric behaviors in volatility, i.e., larger (smaller) volatility is associated with negative (positive) news. Specifically, we consider the following regression:

Table 3.1 Dividend Volatility Estimates

Panel A reports the test statistics for Box-Pierce-Ljung test and ARCH test for lag=4, 8 and 12 on quarterly dividend growth rate from 1926.Q3 to 2006.Q3. Panel B models the dividend growth rate, $g_{D,t+1}$, as AR(1)-EAGRCH(1,1),

$$g_{D,t+1} = \beta_0 + \beta_1 g_{D,t} + \sigma_t Z_{t+1}, \ \log \sigma_t^2 = \kappa + \mathbf{G}_1 \log \sigma_{t-1}^2 + \mathbf{A}_1 \left[|Z_t| - E|Z_t| \right] + \mathbf{L}_1 Z_t, \text{ where}$$

σ_t^2 is conditional variance of $g_{D,t+1}$, and $Z_{t+1} \sim i.i.d. \mathbf{N}(0,1)$. Panel C reports an augmented Dicky-Fuller test on the log of the conditional volatility series estimated by an AR(1)-EAGRCH(1,1). Panel D models the dividend growth rate as a regime-switching process: $g_{D,t+1} = \mu_{s_t} + \sigma_{s_t} v_{t+1}$, $v_{t+1} \sim \mathbf{N}(0,1)$, where $\mu_{s_t} \in \{\mu_1, \mu_2\}$ and $\sigma_{s_t} \in \{\sigma_1, \sigma_2\}$ depend on the underlying state s_t, which follows a Markov chain characterized by transitional probabilities $\mathbf{p}_{11}$ and $\mathbf{p}_{22}$. In Panels B and D, the standard errors of the estimated parameters are reported in parentheses. ** and * mean that the estimates are significantly different from zero at 1% and 5% levels, respectively.

Panel A: Dividend Volatility Clustering Tests			
Lag	4	8	12
Box-Pierce-Ljung Test	32.57**	93.29**	101.37**
ARCH Test	24.96**	66.72**	73.58**

Panel B: AR(1)-EAGRCH(1,1) Estimation						
Parameters	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\kappa}$	$\hat{\mathbf{G}}_1$	$\hat{\mathbf{A}}_1$	$\hat{\mathbf{L}}_1$
Values	0.002*	0.451**	−0.224*	**0.968****	0.423**	0.037
	(0.001)	(0.0618)	(0.109)	(**0.014**)	(0.077)	(0.034)

Panel C: Augmented Dicky-Fuller Test			
	Critical Values		
Test Statistic	1%	5%	10%
−11.8	−20.3	−14.0	−11.2

Panel D: Regime-switching Estimation					
Parameters	$\hat{\mu}_1$	$\hat{\sigma}_1$	$\hat{\mu}_2$	$\hat{\sigma}_2$	$\hat{\mathbf{p}}_{11}$	$\hat{\mathbf{p}}_{22}$
Values	0.007**	0.018**	−0.013	0.078**	0.969**	0.797**
	(0.001)	(0.001)	(0.013)	(0.007)	(0.007)	(0.046)

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$$g_{D,t+1} = \beta_0 + \beta_1 g_{D,t} + \sigma_t Z_{t+1},$$
$$\log(\sigma_t^2) = \kappa + G_1 \log(\sigma_{t-1}^2) + A_1 \left[|Z_t| - E|Z_t| \right] + L_1 Z_t, \qquad (1)$$
$$Z_{t+1} \sim i.i.d. \, N(0,1),$$

where $g_{D,t+1}$ is the dividend growth rate, σ_t^2 is the conditional variance of $g_{D,t+1}$, and $\beta_0, \beta_1, \kappa, G_1, A_1$ and L_1 are coefficients.[43] Panel B of Table 3.1 reports the estimation result. In addition to this EGARCH specification, we also try the specifications in Bansal, Khatchatrian and Yaron (2005) and get similar results not reported here.

The coefficient for dividend volatility is $\hat{G}_1 = 0.968,$ indicating that persistent dividend volatility indeed exists, which is consistent with the standard findings in the ARCH literature. However, the coefficient that measures the persistence in the dividend growth rate *per se* is much smaller ($\hat{\beta}_1 = 0.451$). In the long-run risk literature (e.g., BY, 2004; Bansal, Kiku and Yaron, 2007), it is crucial to have the persistence in both the mean and the volatility of the dividend growth rate to explain the high equity premium, in other words, both $\hat{G}_1$ and $\hat{\beta}_1$ are assumed to be close to one. In contrast, our model requires persistence only in the volatility, but not in the mean of the dividend growth rate process. The current estimation result shows that the econometric evidence is weak for the persistence in the dividend growth rate, but that the persistence in dividend volatility is strong, providing strong econometric evidence for our model.

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[43]In what follows, we report results based on this AR(1)-EGARCH(1,1) specification. We also tried AR(2)-EGARCH(1,1) and other specifications, and the main results remain unchanged.

To further confirm that the persistence of dividend volatility is indeed very high, we resort to the augmented Dicky-Fuller unit root test by running the following regression:[44]

$$\log(\hat{\sigma}_t) = \alpha_0 + \alpha_1 \log(\hat{\sigma}_{t-1}) + \alpha_2 \Delta \log(\hat{\sigma}_{t-1}) + e_t$$

where $\hat{\sigma}_t$ is conditional dividend volatility obtained from the EGARCH estimation (equation [1]), and α_0, α_1 and α_2 are coefficients. Panel C of Table 3.1 reports the test statistics together with the critical values at 1%, 5%, and 10% levels. We can hardly reject the null hypothesis of $\alpha_1 = 1$ at the 10% critical level, which implies that dividend volatility is indeed very persistent.[45] For comparison, we also run the unit root test in the dividend growth rate, and the unreported result strongly rejects the unit root hypothesis at any critical level, which is not surprising given that $\hat{\beta}_1$ is only 0.451 in Panel B of Table 3.1.

Given the strong econometric evidence, we believe that dividend volatility clustering is an important feature of the actual dividend data. Our theoretical model incorporates this feature when we specify the dividend growth rate

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[44]An IGARCH (integrated GARCH) model will be able to nest the EGARCH estimation and the unit root estimation. However, we don't use IGARCH for two reasons: first, IGARCH is not stationary because it assumes a unit root in the volatility process; second, EGARCH fits more with our theoretical dividend volatility specification. We dispense with long memory GARCH models for similar reasons.

[45]The persistence of dividend volatility is going to generate important model results. Therefore, α has to be sufficiently high although it need not be close to 1.

process.[46]

3.2.2 Time-Varying Dividend Volatility

In this subsection, we examine the evolution of dividend volatility by asking

two questions. How does dividend volatility vary with the business cycle? Is

there any remarkable change in dividend volatility over the years? Since

dividend volatility is the state variable in our model, the answer to the first

question will enable us to analyze the procyclical stock prices through the

model. The answer to the second question can relate our measure of

macroeconomic risk to the measures in other papers, and provide empirical

support for our model to explain the dynamics of equity premiums.

To see how dividend volatility varies with the business cycle, Figure 3.1 plots

dividend volatility, the real GDP growth rate, and the recession periods

identified using NBER's business cycle chronology. In this figure, dividend

volatility is the conditional standard deviation estimated from the EGARCH

model (equation [1]); the real GDP growth rates are obtained from the website

of the Bureau of Economic Analysis and start from the second quarter of 1947;

and the shaded areas correspond to the economic recession periods

according to NBER's business cycle chronology.

We see that dividend volatility changes over time, with the highest values

appearing in the 1930's. Comparing dividend volatility with GDP growth rates,

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[46]That is, we require a high ϕ in equation (5).

we see roughly a negative relationship: high dividend volatility usually coincides with lower GDP growth rates. This pattern makes sense, because it's usually the case that more uncertainty is present when the economy is in a trough. Further comparing it with NBER identified recessions, we find that dividend volatility tends to be very high during most recessions. The evidence suggests that dividend volatility evolves in a counter-cyclical way, which can potentially generate procyclical stock prices as well as counter-cyclical equity premiums and Sharpe ratios. Although this direction is promising, this evidence is weak. We thus take a conservative view in next section, assuming that the dividend volatility process is uncorrelated with the consumption growth process.[47]

Observing the data through time, Figure 3.1 also shows that dividend volatility was relatively high before 1952 and became much smoother thereafter, except for a spike around 1989. Therefore, dividend volatility seems to have undergone a significant decline in the postwar years, which suggests that investors' perceived[48] financial risk, as an inseparable part of macroeconomic risk, has experienced a significant decline since the 1950s.

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[47]That is we assume $Cov(\eta_t, u_t) = 0$ in equation (6).

[48]Here, perceived is used to emphasize the notion that investors treat the dividend process as exogenous, although firms tend to smooth dividends in reality.
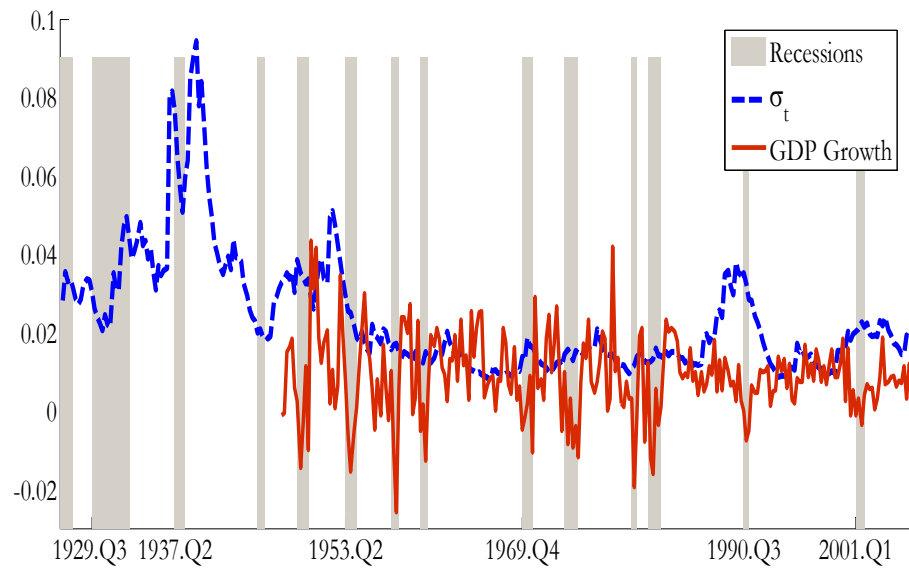
Figure 3.1 Dividend Volatility, GDP Growth and Recessions

Figure 3.1 plots quarterly consumption growth rates for period 1947.Q1-2006.Q3, and conditional dividend volatility for period 1926.Q3-2006.Q3. The dividend volatility σ_t is estimated from an AR(1)-EGARCH(1,1) regression. The shaded bars indicate the recessions according to NBER's website data.

To characterize the decline in dividend volatility more formally, we follow Hamilton (1989) to estimate a regime-switching model. The basic idea is to model the dividend growth rate as deriving from one of two regimes, a regime with a high dividend volatility or one with a low dividend volatility. The parameter values in each regime, together with the transitional probability can be obtained through maximum likelihood estimation. These parameter estimates can then be used to infer which regime the process was in at any historical date. Specifically, the dividend growth rate, $g_{D,t+1}$, is generated according to:

$$g_{D,t+1} = \mu_{s_t} + \sigma_{s_t} v_{t+1}, \ v_{t+1} \sim N(0,1),$$

where $\mu_{s_t} \in \{\mu_1, \mu_2\}$ is the mean, and $\sigma_{s_t} \in \{\sigma_1, \sigma_2\}$ is the volatility in state s_t. Thus, when $s_t = 1$, the observed dividend growth rate, $g_{D,t+1}$, is presumed to have been drawn from a $N(\mu_1, \sigma_1)$ distribution, whereas when $s_t = 2$, $g_{D,t+1}$, is drawn from another distribution $N(\mu_2, \sigma_2)$. The state evolves according to a Markov process, and we denote the transitional probability of the Markov chains

$$\Pr(s_t = 1 | s_{t-1} = 1) = p_{11},$$
$$\Pr(s_t = 2 | s_{t-1} = 2) = p_{22}.$$

The parameter values and their standard deviations are reported in Panel D of Table 3.1. The estimated two regimes are characterized as follows: the high-mean, low-volatility regime has an average growth rate of 0.65% per quarter, with a low standard deviation of 0.018; the low-mean, high-volatility regime

has an average growth rate of -1.3% per quarter, with a very high standard deviation 0.078. In addition, the high-mean, low-volatility regime seems more persistent, because its transitional probability is higher, $\hat{\mathbf{p}}_{11} = 0.969$.

Figure 3.2 plots the smoothed posterior probability of the dividend growth rate being in a low-mean, high-volatility state. The probability is very high in prewar data, but exhibits sharp declines after the 1950s. In much of the postwar period, the posterior probability of being in a high-mean, low-volatility regime is close to one.

The reported evidence clearly shows that dividend volatility has been declining since 1950s. This is broadly consistent with the findings in Kim, Morley and Nelson (2004), who document a similar pattern in stock returns. In Section 3.4.3.5, we incorporate this finding into our theoretical framework by doing comparative statics with respect to the exogenous parameters governing the dividend process, and find that the declining dividend volatility helps to explain the decreasing equity premiums after WWII.

3.3 The Model

3.3.1 Setup

Consider an economy populated with a continuum of identical, infinitely lived, narrow-framing and loss averse agents. Two assets are available to trade: a risk-free asset in zero net supply, paying a gross interest rate $R_{f,t}$, and one unit of risky asset, paying a gross return R_{t+1}, between time t and $t+1$.
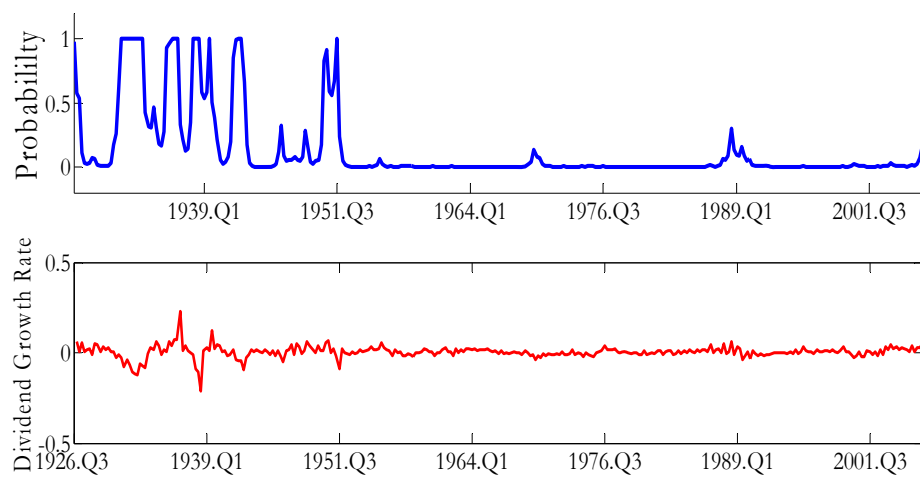
Figure 3.2 Smoothed Probability of a High Volatility Regime and Historical
Dividend Growth Rate

In Figure 3.2, the dividend growth rate is assumed to be generated from a
regime switching model. The estimation results in Panel D of Table 1 suggests
that one regime features a positive mean and a low volatility, while the other
one has a negative mean and a high volatility. The top panel plots the
posterior probability of dividend growth being in the high volatility regime given
the observed data process. The bottom panel plots the dividend growth rate
for period 1926.Q3-2006.Q3.

The loss averse investor chooses consumption C_t and risky asset holdings S_t to maximize the utility function

$$E\left[\sum_{t=0}^{\infty}\left(\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \overline{C}_t^{-\gamma} \rho^{t+1} v(X_{t+1})\right)\right], \tag{2}$$

subject to the standard budget constraint, and

$$X_{t+1} = S_t\left(R_{t+1} - R_{f,t}\right),$$

$$v(X_{t+1}) = \begin{cases} X_{t+1}, & \text{if } X_{t+1} \geq 0, \\ \lambda X_{t+1}, & \text{if } X_{t+1} < 0. \end{cases}$$

The first term in the objective function is the standard utility over consumption, where $\rho \in (0,1)$ is the time discount factor; $\gamma > 0$ measures the curvature of the investor's utility over consumption;[49] and $\overline{C}_t$ is the aggregate per capita consumption at time t, which is exogenous to the investor. The exogenous scalor $\overline{C}_t$ is introduced to ensure that consumption utility and prospect utility are of the same order as aggregate wealth increases over time.[50]

The second term deserves more attention, as it captures the direct utility the investor derives from fluctuations in the value of her financial wealth. Depending on the return of the risky asset, her total portfolio excess return X_{t+1}

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[49]For $\gamma = 1$, we replace $C_t^{1-\gamma}/(1-\gamma)$ with $\log(C_t)$.

[50]Another tractable preference specification that incorporates narrow-framing but doesn't rely on a scaling to ensure stationarity can be found in Barberis and Huang (2007).

can be either positive or negative, a positive one indicating a financial gain, and a negative one a financial loss. The function $v(X_{t+1})$ describes how she feels about her investment performance. Since she is loss averse, the pain she receives from financial losses outweighs the happiness from financial gains. Therefore, $v(X_{t+1})$ takes different functional form with respect to the values of X_{t+1}: when X_{t+1} is positive showing that she makes money, $v(X_{t+1})$ is linear in X_{t+1} with slope one; in contrast, when X_{t+1} is negative meaning that she loses money, $v(X_{t+1})$ amplifies her utility loss by a magnitude of λ, with λ being greater than one. Figure 3.3 plots the function $v(X_{t+1})$.

The dynamics of the economy crucially depends on the value of b_0, which tells how much the second utility counts in her total utility. If $b_0 = 0$, loss aversion doesn't play a role in the overall utility, and the model is reduced to a traditional asset pricing setting studied by Hansen and Singleton (1983). In this case, higher dividend volatility leads to a higher dividend growth rate, resulting in a higher price-dividend ratio and a lower equity premium. However, as the value of b_0 increases, the investor suffers more utility loss from her financial loss and demands a higher risk premium in holding stocks. As will be clearer later, the balance of these two utility forces generates the pattern actually observed in financial markets.
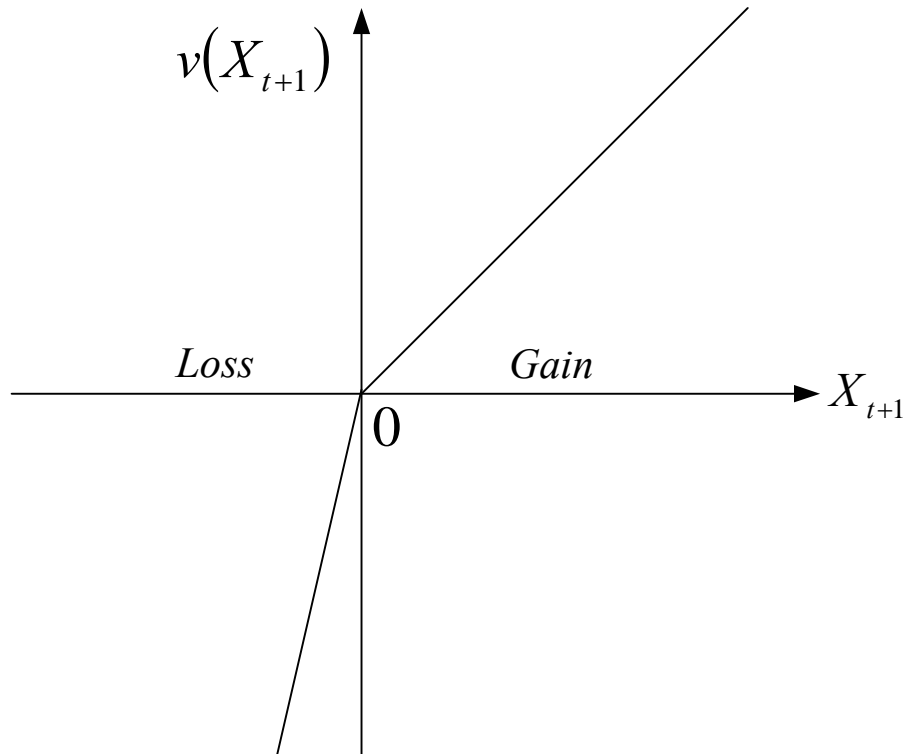
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Figure 3.3 Gain and Loss Function

Figure 3.3 plots the gain and loss function $v(X_{t+1}) = \begin{cases} X_{t+1}, & \textit{if } X_{t+1} > 0 \\ \lambda X_{t+1}, & \textit{if } X_{t+1} \leq 0 \end{cases}$ for

$\lambda > 1$.

Both consumption and dividend growth follow lognormal processes,

$$g_{C,t+1} = \log\left(\overline{C}_{t+1}/\overline{C}_t\right) = g_C + \sigma_C \eta_{t+1}, \tag{3}$$

$$g_{D,t+1} = \log\left(D_{t+1}/D_t\right) = g_D + \sigma_t \varepsilon_{t+1}, \tag{4}$$

$$\log(\sigma_{t+1}) - \log(\overline{\sigma}) = \phi\left[\log(\sigma_t) - \log(\overline{\sigma})\right] + \sigma_u u_{t+1} \tag{5}$$

Here $g_{C,t+1}$ is the growth rate of aggregate consumption $\overline{C}_t$. g_C and σ_C are the mean and standard deviation of consumption growth. D_t is dividend: its growth rate is denoted as $g_{D,t+1}$, with mean g_D and standard deviation σ_t. We draw special attention to equation (5), which characterizes the evolution of dividend volatility. To ensure the positiveness of σ_t, we model $\log(\sigma_{t+1})$ instead of σ_t as an $AR(1)$ process. In this sense, the dividend volatility equation (5) is very similar to an EGARCH specification (equation [1]). σ_u captures the magnitude of the innovation to the conditional volatility σ_t : a big σ_u will increase dividend volatility. A particular interesting parameter is the coefficient ϕ, which controls the strength of dependence on past volatilities. A larger ϕ implies that the impact of a shock to dividend volatility is very persistent. As has been shown in Section 3.2, this persistence parameter ϕ is very high in actual dividend data.

The innovations to the consumption growth η_t, the dividend growth ε_t, and the dividend volatility u_t are jointly normally distributed as

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \\ u_t \end{pmatrix} \sim \text{i.i.d. } \mathbf{N} \left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \omega & 0 \\ \omega & 1 & \theta \\ 0 & \theta & 1 \end{pmatrix} \right], \tag{6}$$

where ω is the correlation between consumption shocks and dividend shocks. Note that when allowing for persistence in dividend volatility, the unconditional correlation between $g_{C,t+1}$ and $g_{D,t+1}$ is $\omega e^{-0.5\sigma_u^2/(1-\phi^2)} < \omega$. As discussed in Section 3.2.2, we assume consumption growth shocks are independent of dividend volatility shocks, i.e., $Cov(\eta_t, u_t) = 0$, although data suggests a weak negative correlation between η_t and u_t, which has important implications for the time-variation pattern of the equity premiums. We allow for the interaction between shocks to the dividend growth rate ε_t and shocks to the dividend volatility u_t, and the interaction of these two shocks are denoted by θ. As will be shown later, θ also plays a role in generating certain model results.

3.3.2 Equilibrium Prices

This subsection derives the equilibrium asset prices. We first construct a one-factor Markov equilibrium, in which the risk-free rate is a constant and the state variable σ_t (dividend volatility) determines the distribution of future stock returns. Assume that the price-dividend ratio is a function of σ_t:

$$f_t \equiv P_t/D_t = f(\sigma_t).$$

We are going to verify that there is indeed an equilibrium satisfying this assumption.

Given the one-factor assumption, the stock returns R_{t+1} can be determined as

$$R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{1+P_{t+1}/D_{t+1}}{P_t/D_t}\frac{D_{t+1}}{D_t} = \frac{1+f(\sigma_{t+1})}{f(\sigma_t)}e^{g_D+\sigma_t\varepsilon_{t+1}}. \tag{7}$$

Intuitively, the change in stock returns can be attributed to either the news about dividend growth ε_{t+1}, or the financial market uncertainty σ_t, or changes in the price-dividend ratio f. Since the dividend process is exogenously given, the key to solving R_{t+1} is to solve the price-dividend ratio f.

In equilibrium, the Euler equations fully capture the dynamics of the economy[51]

$$1 = \rho R_f E_t \left[\left(\overline{C}_{t+1} / \overline{C}_t \right)^{-\gamma} \right], \tag{8}$$

$$1 = \rho E_t \left[R_{t+1} \left(\overline{C}_{t+1} / \overline{C}_t \right)^{-\gamma} \right] + b_0 \rho E_t \left[\hat{v}(R_{t+1}) \right], \tag{9}$$

where

$$\hat{v}(R_{t+1}) = \begin{cases} R_{t+1} - R_{f,t}, & \text{if } R_{t+1} \geq R_{f,t}, \\ \lambda \left(R_{t+1} - R_{f,t} \right), & \text{if } R_{t+1} < R_{f,t}. \end{cases}$$

Equation (8) and the i.i.d. assumption on the consumption growth together imply a constant risk free rate,

$$R_f = \rho^{-1} e^{\gamma g_C - \gamma^2 \sigma_C^2 / 2}. \tag{10}$$

After substituting in the respective consumption and dividend processes, equation (9) boils down to

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[51]The Euler equations are both necessary and sufficient to characterize the equilibrium. Refer to BHS (2001) for a proof.

$$1 = \rho E_t \left[\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} e^{-\gamma(g_C + \sigma_C \eta_{t+1})} \right] + b_0 \rho E_t \left[\hat{v} \left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} \right) \right], \forall \sigma_t. \quad (11)$$

In equilibrium, the function f must evolve according to equation (11), which also verifies the conjectured one-factor Markov equilibrium price function.

3.3.3 Methodology of Numerical Computation

We solve f numerically on a grid search of the state variable σ_t. We start out by guessing a solution to (11), $f^{(0)}$ say. According to (5), the distribution of σ_{t+1} is completely determined by σ_t and u_{t+1}. Then we get a new candidate solution $f^{(1)}$ by the following recursion

$$1 = \rho e^{g_D - \gamma g_C + \frac{1}{2}\gamma^2 \sigma_C^2 (1-\omega^2)} E_t \left[\frac{1 + f^{(i)}(\sigma_{t+1})}{f^{(i+1)}(\sigma_t)} e^{(\sigma_t - \gamma \omega \sigma_C)\varepsilon_{t+1}} \right]$$
$$+ b_0 \rho E_t \left[\hat{v} \left(\frac{1 + f^{(i)}(\sigma_{t+1})}{f^{(i+1)}(\sigma_t)} e^{g_D + \sigma_t \varepsilon_{t+1}} \right) \right], \forall \sigma_t.$$

We continue this process until $f^{(i)} \to f$.

3.4 Model Results

3.4.1 Calibrating Parameter Values

We calibrate the model at quarterly frequency, such that the model implied moments match those of the observed annual data. In reality, many companies issue their dividend policies and earning reports at quarterly frequency, hence it is reasonable for the investors to re-evaluate their investment performance at a quarterly basis. We also calibrate the model at monthly and annual frequency, in which cases investors re-evaluate their

performance more frequently or less frequently. We get qualitatively similar results, so we only report the results based on quarterly decision making throughout our analysis.

Table 3.2 summarizes our choice of parameter values. We choose similar values as BHS for the consumption growth parameters and the preference parameters. For g_C and σ_C, the mean and standard deviation of log consumption growth, we follow Cecchetti, Lam and Mark (1990) and set $g_C = 0.46\%$ and $\sigma_C = 1.90\%$, which corresponds to an annual growth rate of 1.84% with volatility of 3.79%. The curvature γ of utility over consumption and the time discount factor ρ are set as 1.0 and 0.995 respectively, bringing the net annual risk free rate close to 3.86 percent by equation (10) and the values of g_C and σ_C. The loss aversion parameter λ is equal to 2.25, since many independent experimental studies have estimated it as being around this level. Similar to BHS, the parameter b_0 does not have an empirical counterpart, and we present results for a range of values of b_0.

Using NYSE/AMEX data and Fama risk-free rate data from 1926.Q3 to 2006.Q4 from CRSP, we calibrate the unconditional mean of quarterly dividend growth rate as its empirical mean, $g_D = 0.39\%$. By matching the first moment of Equation (4), $E\left[\log\left(\left|g_{D,t+1} - g_D\right|\right)\right] = \log(\bar{\sigma}) + E\left[\log\left(\left|\varepsilon_{t+1}\right|\right)\right]$, we calibrate $\log(\bar{\sigma})$ as -3.91. The parameter ϕ, who governs the persistence of dividend volatility, takes the value 0.99, close to the estimated value from an EGARCH model in Section 3.2.

Table 3.2   Calibrated Parameters

This table reports the calibration values for the preference parameters and technology parameters in the theoretical model. γ is the curvature of utility over consumption, ρ is the time discount factor, λ is the loss aversion parameter, and b_0 controls the importance of the loss aversion relative to the consumption in the utility function. g_C and g_D are the means of the consumption and dividend growth rate, respectively. σ_C is the volatility of consumption growth. $\log(\bar{\sigma})$ is the mean of the log of conditional volatility of dividend growth. ϕ measures the persistence of dividend volatility, while σ_u controls the variation in dividend volatility. ω is the correlation between consumption news and dividend news, and θ is the correlation between dividend *level* news and *volatility* news. The calibration for the dividend parameters is based on the dividend sample 1926.Q3-2006.Q3 constructed from the value weighted NYSE/AMEX returns from CRSP.

Parameters	Calibration Values
Preference	
γ	1.0
ρ	0.995
b_0	range
λ	2.25
Technology	
g_C	0.46%
g_D	0.39%
σ_C	1.90%
$\log(\bar{\sigma})$	-3.91
ω	0.15
ϕ	0.99
θ	-0.67
σ_u	0.14

The parameter σ_u is very important since it measures the magnitude of dividend volatility. We calibrate this parameter as 0.14, such that the model implied annual dividend growth rate has a volatility equal to its empirical counterpart. Compared to BY (2004), the value of σ_u appears large. However, this is an artifact of our specification of the volatility process in equation (5), where the logarithm of dividend volatility rather than its square follows an AR(1) process. Indeed, given $\phi \approx 1$, taking a first order approximation of (5), we have $\sigma_{t+1}^2 \approx \sigma_t^2 + \sigma_w u_{t+1}$, where $\sigma_w = 2E(\sigma_t^2)\sigma_u = 8.2 \times 10^{-4}$, close to the value in BY (2004).

Two more model parameters remain to be calibrated: θ, which captures the interaction between innovations in dividend growth rate and dividend volatility; and ω, the correlation between consumption and dividend. By equations (4) and (5), we calibrate θ at -0.67. Following Campbell (2000), we set $\omega = 0.15$, which implies an unconditional correlation of 0.1 between consumption and dividend growth processes.

3.4.2 Price-dividend Ratio Function f

Figure 3.4 plots the price-dividend ratio f as a function of $\log(\sigma_t)$ for $b_0 = 0.7$, $b_0 = 2$ and $b_0 = 6$. We also try a variety of other values for b_0, for example, $b_0 = 0.1$, $b_0 = 20$, $b_0 = 200$, etc. The essential pattern, however, is fully depicted by Figure 3.4.
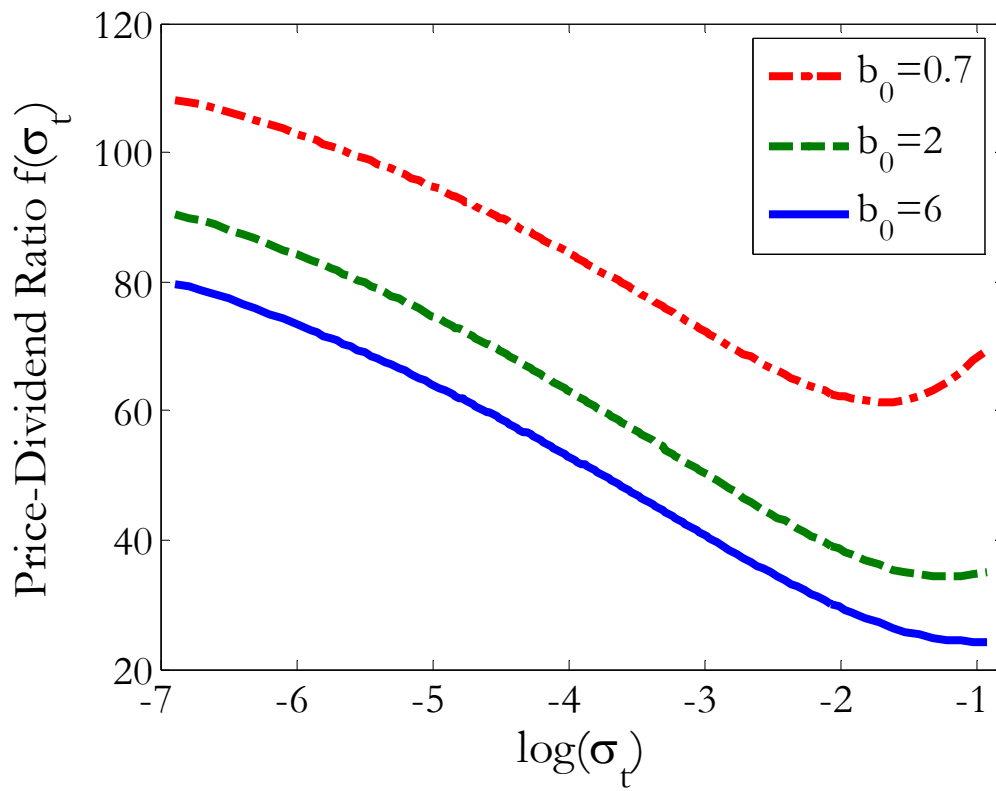
Figure 3.4 Price-Dividend Functions f

Figure 3.4 plots the equilibrium price-dividend ratios against the log of the conditional dividend volatility, $\log(\sigma_t)$, for $b_0 = 0.7$, 2 and 6.

Investors in our model care not only about consumption, the standard

expected log utility term in (2), but also about fluctuations in the value of their

investments, the additional prospect utility term in (2). These two forces jointly

determine the shape of the function f. Without loss aversion, a higher

dividend volatility implies a higher dividend growth rate in the future, and thus

higher expected cash flows from holding stocks. Since the stochastic discount

factor depends on the consumption process, which is weakly correlated with

dividend, it is relatively unchanged. Therefore, stocks are more attractive and

their prices are higher. The standard consumption utility contributes to a

positive relationship between σ_t and the price-dividend ratio $f(\sigma_t)$.

The presence of loss aversion, in contrast, contributes to a negative

relationship between dividend volatility σ_t and the price-dividend ratio $f(\sigma_t)$.

For a fixed b_0, the more volatile the dividend process, the more volatile the

returns, therefore, the more likely investors are to suffer financial losses. This

causes loss averse investors great pains, and makes stocks less desirable. As

a result, they require more compensations when faced with more volatile

dividend processes, causing lower stock prices or higher equity premiums.

The negative slope of f function is consistent with BY (2004) and Bansal,

Khatchatrian and Yaron (2005), who find that asset prices drop as economic

uncertainty rises, although their measure of economic uncertainty is

conditional consumption volatility rather than dividend volatility. It is rather

difficult to justify this negative relationship within the standard power utility

framework, where, as we have discussed before, a higher dividend volatility is

associated with higher expected dividend growth, hence price-dividend ratios

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always vary positively with dividend volatility. However, it can be easily understood with the introduction of loss aversion preferences.

The overall shape of f can now be summarized as follows. For low values of σ_t, the impact of loss aversion is dominant, hence the function f is downward sloping. As σ_t becomes larger, the impact of log utility catches up, and the function f eventually becomes upward sloping. That is, f is U-shaped, as shown in Figure 3.4. The smaller is b_0, the earlier f achieves its minimum. Moreover, larger values of b_0 say that investors care more about their wealth fluctuations, in which case the risk premiums for holding stocks are higher. Therefore, as b_0 increases, the function f will move downward.

How does f look like in the data? According to our calibration, $\log(\sigma_t)$ is normally distributed with mean -3.91 and standard deviation 0.99. Therefore, just reading from Figure 3.4, we will expect to see a negative relationship between price-dividend ratios and dividend volatility for most of the time. To see this more formally, we run an AR(1)-EGARCH(1,1) estimation on the quarterly dividend growth for 1926.Q3-2006.Q4 and plot the price-dividend ratios against the estimated conditional dividend volatilities $\hat{\sigma}_t$ in Figure 3.5. Indeed, more than 80 percent of the observed data display a negative relationship. In addition, we also notice an interesting positive relationship between price-dividend ratios and dividend volatility, which occurs for some extremely high realizations of $\hat{\sigma}_t$. For instance, when the logarithm of dividend volatility is larger than 0.05, price-dividend ratios actually rise with dividend volatility. Therefore, the data display a similar U-shaped pattern as predicted by our model.
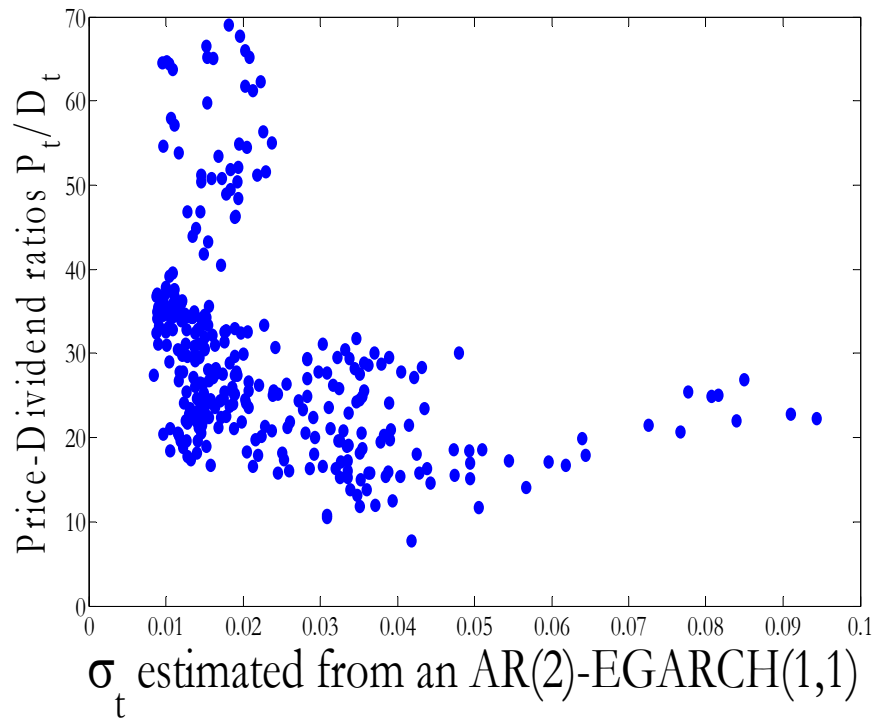
Figure 3.5 Historical Price-Dividend Ratios v.s.
Conditional Dividend Volatility

Figure 3.5 plots the historical price-dividend ratios against the conditional
dividend volatility estimated from an AR(2)-EGARCH(1,1) regression for
period 1926.Q3-2006.Q3.

3.4.3 Simulation Results

In this subsection, we generate artificial data under the parameter configuration in Table 3.2, and show that the model-simulated data replicate the interesting patterns found in actual data.

In order to facilitate a comparison with historical data, we simulate the model at a quarterly frequency and time-aggregate them to get annual data. We do 10,000 simulations each with 320 quarterly observations. We then calculate the interested statistics and report their sample moments. Given that the simulation number is large enough, the sample moments should serve as good approximations to population moments.

3.4.3.1 Stock Returns and Stock Volatility

Table 3.3 reports a variety of statistics calculated from model simulated data and the corresponding statistics from historical data. It is noteworthy that the model can match the mean and standard deviations of excess stock returns pretty well. When $b_0 = 6$, the model generates a sizable premium of 6.75% per annum, which is slightly higher than the empirical value 5.90%; the model also generates a standard deviation of 19.49%, which is almost equal to the corresponding value of 19.17% in the data.

We notice that as b_0 grows, both the mean and standard deviations from model simulated excess returns increase. This is because when b_0 increases, loss aversion becomes a more important feature of investors' preference, so investors become more and more fearful of risky assets, pushing down stock prices.

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We also report the mean and standard deviation of the simulated annual price-dividend ratios, $E(P_t^a / D_t^a)$ and $\sigma(P_t^a / D_t^a)$.[52] The empirical value $\sigma(P_t^a / D_t^a) = 12.43\%$ is relatively high to those found in other papers (BHS, 2001; BY, 2004; Campbell and Cochrane, 1999). This is due to the relatively high price-dividend ratios from 1996 to 2006, which includes the high-tech bubble period.

We are able to match stock returns volatility even though the volatility of price-dividend ratios is lower than their empirical counterparts, a common problem with one factor models. The reason to achieve excess volatility in stock returns is due to the positive relationship between price-dividend ratios and dividend innovations. To see this more clearly, consider the following approximate relationship (Campbell, Lo and MacKinlay, 1997):

$$r_{t+1} \approx A + \log[f(\sigma_{t+1})/ f(\sigma_t)] + \sigma_t \varepsilon_{t+1},$$

where A is a constant. The excess volatility of market return relative to that of the dividend growth (or the fundamental), $Var(r_{t+1}) - Var(\sigma_t \varepsilon_{t+1})$, comes from two sources: the volatility of price-dividend ratios, $Var\left(\log \frac{f_{t+1}}{f_t}\right)$, and the covariance between the price-dividend ratios and the news to the dividends, $Cov\left(\log \frac{f_{t+1}}{f_t}, \sigma_t \varepsilon_{t+1}\right)$. In actual data, since $\theta < 0$ in (6), good dividend news (positive ε_{t+1}) tends to be associated with negative dividend volatility shock (negative u_{t+1}), implying that next period price-dividend ratios will increase (see Figure 3.4). Therefore, the covariance term $Cov\left(\log \frac{f_{t+1}}{f_t}, \sigma_t \varepsilon_{t+1}\right)$ is positive.

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[52]The superscript a indicates *annualized* variables.

Table 3.3 Asset Prices and Annual Returns (1926-2006)

This table provides information regarding stock returns for the simulated data and historical data. The historical data correspond to the period 1926-2006. The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2. The expressions $E(r^a_{t+1} - r^a_{f,t})$ and $\sigma(r^a_{t+1} - r^a_{f,t})$ are, respectively, the mean and volatility of the annualized continuously compounded returns. $Corr(r^a_{t+1}, g^a_{C,t+1})$ is the correlation between the annual stock return and annual growth rate. $E(P^a_t / D^a_t)$ and $\sigma(P^a_t / D^a_t)$ are the mean and volatility of the annualized price-dividend ratios.

Variables	Empirical Value (1926-2006)	Model $b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
Annual Excess Stock Return				
$E\left(r^a_{t+1} - r^a_{f,t}\right)$	5.90	2.68	4.98	6.75
$\sigma\left(r^a_{t+1} - r^a_{f,t}\right)$	19.17	16.18	18.44	19.49
$E\left(r^a_{t+1} - r^a_{f,t}\right)/\sigma\left(r^a_{t+1} - r^a_{f,t}\right)$	0.31	0.16	0.27	0.35
$Corr(r^a_{t+1}, g^a_{C,t+1})$	0.1	0.1	0.1	0.1
Annual Price-Dividend Ratio				
$E(P^a_t/D^a_t)$	29.08	19.01	13.47	11.06
$\sigma(P^a_t/D^a_t)$	12.43	2.56	2.62	2.47

The model is also able to generate the low correlation between stock returns and consumption growth, $Corr\left(r_{t+1}^{a}, g_{C,t+1}^{a}\right) = 0.1$. This happens because the variation in stock returns is completely driven by the innovations in the dividend process, which is only weakly correlated with the consumption process.

3.4.3.2 Autocorrelations of Returns and Price-Dividend Ratios

Table 3.4 presents autocorrelations in returns and price-dividend ratios. Our model predicts negative autocorrelations in stock returns, as documented by Poterba and Summers (1988) and Fama and French (1988a). This negative correlation comes from the fact that returns and price-dividend ratios depend solely on a persistent AR(1) dividend volatility process. Moreover, our model closely matches the highly positively correlated price-dividend ratios in the data.

3.4.3.3 Return Predictability

To analyze the predictability pattern of returns, we run the following regression on both simulated and historical data:

$$r_{t+1}^{a} + r_{t+2}^{a} + ... + r_{t+j}^{a} = \alpha_{j} + \beta_{j}\left(D_{t}^{a} / P_{t}^{a}\right) + \varepsilon_{j,t},$$

where r_{t+j}^{a} refers to the annual cumulative log returns from year $t + j - 1$ to $t + j$. Table 3.5 presents the regression result for different values of b_{0}. This estimation result from model-simulated data resembles the classic pattern documented by Campbell and Shiller (1988) and Fama and French (1988b).

The coefficients are significant and negative, indicating that high prices tend to predict low expected returns. Moreover, the forecasting power increases with forecasting horizons, as reflected by the increasing coefficients and R^2's.

The pattern of return predictability generated by our model can be understood through the volatility test in Cochrane (1992). Starting from the accounting identity $1 = R_{t+1}^{-1} R_{t+1}$ with $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$, the log-linearization around the average price-dividend ratios, $\overline{P/D}$, implies that, in the absence of rational asset price bubbles,

$$Var(p_t - d_t) \approx \sum_{j=1}^{\infty} h^j Cov(p_t - d_t, g_{D,t+j}) - \sum_{j=1}^{\infty} h^j Cov(p_t - d_t, r_{t+j})$$

where lower case indicates log values and $h = \overline{P/D}/(1 + \overline{P/D})$. This suggests that the variation in the price-dividend ratio will forecast either the change in expected dividend growth rate, or the discount rate, or both.

In our model, even though dividend volatility is time varying, the dividend growth rate *per se* is still a white noise, meaning $Cov(p_t - d_t, g_{D,t+j}) = 0$. Given that the risk-free rate is maintained as a constant, the only thing remaining for the price-dividend ratio to predict is the excess return. A high price-dividend ratio is associated with a decline in dividend volatility, so the required expected return will be lower. Therefore, our model implies an extreme version of the volatility test results.

Table 3.4 Autocorrelations of Returns and Price-Dividend Ratios

This table reports the autocorrelations of annualized stock returns and price-dividend ratios for the simulated data and historical data. The historical data correspond to the period 1926-2006. The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2. The expressions $Corr\left(r_t^a, r_{t-j}^a\right)$ and $Corr\left(P_t^a / D_t^a, P_{t-j}^a / D_{t-j}^a\right)$ are, respectively, the autocorrelations of the annualized compound equity returns and P/D ratios.

	Empirical Value	Model		
	(1926-2006)	$b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
$Corr\left(r_t^a, r_{t-j}^a\right)$				
$j = 1$	0.09	−0.01	−0.01	−0.01
$j = 2$	−0.17	−0.02	−0.02	−0.02
$j = 3$	−0.06	−0.02	−0.02	−0.02
$j = 4$	−0.12	−0.02	−0.02	−0.02
$j = 5$	−0.06	−0.02	−0.02	−0.02
$Corr\left(P_t^a/D_t^a, P_{t-j}^a/D_{t-j}^a\right)$				
$j = 1$	0.90	0.68	0.79	0.82
$j = 2$	0.81	0.61	0.71	0.74
$j = 3$	0.75	0.56	0.64	0.66
$j = 4$	0.68	0.50	0.58	0.60
$j = 5$	0.60	0.45	0.51	0.53

Table 3.5 Return Predictability Regressions (1926-2006)

This table provides evidence of predictability of future excess returns by price-dividend ratios. The entries correspond to regressing $r_{t+1}^a + r_{t+2}^a + ... + r_{t+j}^a = \alpha_j + \beta_j \left(D_t^a / P_t^a \right) + \varepsilon_{j,t}$, where r_{t+j}^a refers to the annual cumulative log returns from year $t + j - 1$ to $t + j$. The historical data correspond to the period 1926-2006. The entries for the model are based on 10,000 simulations each with 320 quarterly observations that are time-aggregated to an annual frequency. The parameter configuration in simulation follows that in table 3.2.

	Empirical Value	Model		
	(1926-2006)	$b_0 = 0.7$	$b_0 = 2$	$b_0 = 6$
β_1	2.55	2.42	2.08	1.83
β_2	5.99	4.90	4.08	3.58
β_3	8.28	7.23	5.95	5.20
β_4	11.26	9.44	7.71	6.69
$R^2(1)$	0.04	0.03	0.05	0.06
$R^2(2)$	0.09	0.05	0.09	0.11
$R^2(3)$	0.13	0.07	0.13	0.16
$R^2(4)$	0.18	0.09	0.16	0.20

It's worth noting that a central fact driving predictability of returns is the persistence of dividend volatility. As shown in Cochrane (2005), both the estimated coefficients and $R^{2\prime}s$ increase with the persistence of the price-dividend ratio, which depends on dividend volatility.

3.4.3.4 Time-varying Sharpe Ratios

Empirical evidence suggests that estimates of both conditional means and conditional standard deviations of returns change through time, but they do not move one for one. Hence Sharpe ratios are time-varying. Figure 3.6 presents the conditional means and conditional standard deviations as functions of the state variable $\log(\sigma_t)$. Overall, as σ_t increases, the dividend growth becomes more volatile; thus both the means and the standard deviations of expected returns increase.

Comparing the conditional means, $E_t(R_{t+1})$, and conditional standard deviations, $\sigma_t(R_{t+1})$, of expected returns, we see that they are different functions of dividend volatility. Most noticeably, for those values of $\log(\sigma_t)$ smaller than $\log(\overline{\sigma}) = -3.91$, the conditional standard deviation is almost a constant, whereas the conditional mean has more variations and increases with $\log(\sigma_t)$. Therefore, the Sharpe ratio of conditional mean to conditional standard deviation varies over time, with its variation due to the difference between $E_t(R_{t+1})$ and $\sigma_t(R_{t+1})$.
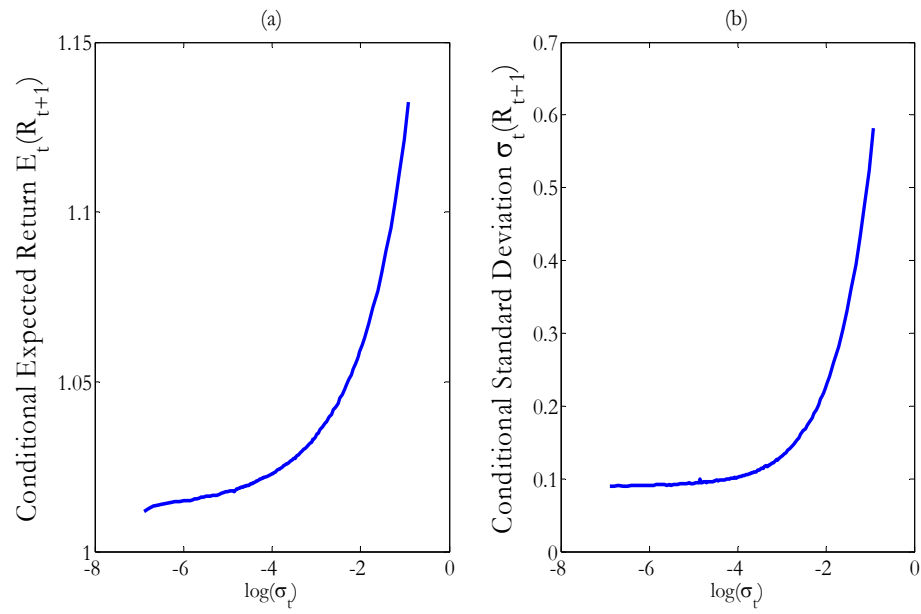
Figure 3.6 Conditional Moments of Stock Returns

Panel (a) and (b) plot the conditional expected stock return $E_t(R_{t+1})$ and conditional volatility of return $\sigma_t(R_{t+1})$ for the case b_0=6.

More formally, according to (4)-(7), the conditional mean and conditional variance of R_{t+1} are respectively,

$$E_t(R_{t+1}) = e^{g_D + \frac{1}{2}(1-\theta^2)\sigma_t^2} E_t\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{\sigma_t \theta u_{t+1}}\right),$$

$$Var_t(R_{t+1}) = e^{2g_D + 2(1-\theta^2)\sigma_t^2} E_t\left[\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)}\right)^2 e^{2\sigma_t \theta u_{t+1}}\right]$$

$$- e^{2g_D + (1-\theta^2)\sigma_t^2}\left[E_t\left(\frac{1 + f(\sigma_{t+1})}{f(\sigma_t)} e^{\sigma_t \theta u_{t+1}}\right)\right]^2.$$

To get a clearer picture of the distribution of Sharpe ratios, we numerically calculate the conditional Sharpe ratios from the above formula. Specifically, we make 160,000 random draws of ε_{t+1} and u_{t+1}, calculate the conditional mean and conditional standard deviation of expected returns by numerical integration, and then obtain the conditional Sharpe ratios as a function of $\log(\sigma_t)$ when $b_0 = 6$. Figure 3.7 presents the histogram of the simulated conditional Sharpe ratios, showing that the price of risk is changing over time. The unconditional mean and standard deviation of simulated Sharpe ratios are 0.14 and 0.05, matching their empirical values.
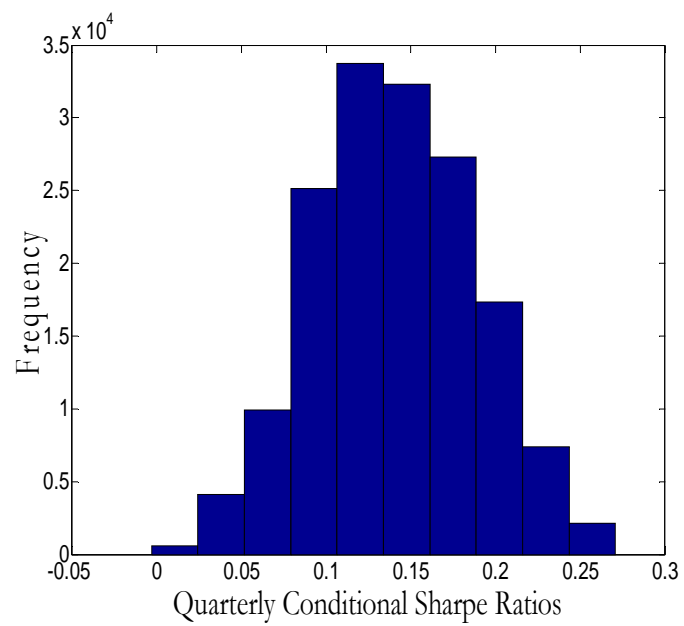
Figure 3.7 Distribution of the Conditional Sharpe Ratios

The distribution is based on a simulation for the case $b_0=6$.

3.4.3.5 Structural Break and Equity Premiums

Recent empirical evidence shows that the macroeconomic risk has declined over the past fifty years. It still remains an open question how this reduced risk affects ex-ante equity premiums, which are identified to have declined since WWII, by Blanchard (1993), Fama and French (2002), Freeman (2004), and Buranavityawut, Freeman and Freeman (2006). We use dividend volatility to stand for risk and study how this risk is priced in financial markets.

The econometric evidence in Section 3.2 suggests that dividend volatility has decreased dramatically since the 1950s. According to our model, lower dividend volatility means that stocks are less likely to perform poorly; thus loss averse investors are less worried about fluctuations in their financial wealth. As a result, they are more willing to hold risky stocks, pushing up stock prices and driving down expected returns. To test our model performance in the postwar period with declined dividend volatility, we re-calibrate the model according to the data for 1954-2006. The new parameter values are provided in Table 3.6. Comparing the new values with those calibrated from all data, we find that the mean dividend growth rate doesn't change a lot, however, the standard deviation of $\log(\sigma_t)$ decreased from 0.14 to 0.10, a decline of roughly 30%. Consistent with our intuition, the model-simulated data match the actual data very well in excess returns, in the standard deviation of excess returns, as well as in Sharpe ratios.

Table 3.6 Structural Break and Asset Prices

This table reports the mean and volatility of stock returns for the simulated data for two sets of dividend parameter configurations. Calibration I is based on the dividend sample 1926.Q3-2006.Q3; Calibration II is based on the dividend sample 1954.Q3-2006.Q3. The preference parameters and consumption parameters are the same as table 3.2 for both configurations. The expressions $E\left(r_{t+1}^a - r_{f,t}^a\right)$ and $\sigma\left(r_{t+1}^a - r_{f,t}^a\right)$ are, respectively, the mean and volatility of the annualized continuously compounded returns.

Dividend Parameter Configuration				
Parameters	σ_u	g_D	$\log(\bar{\sigma})$	θ
Calibration I	0.14	0.39%	-3.91	-0.67
Calibration II	0.10	0.35%	-4.16	-0.34

Annual Excess Returns			
Variables	$E\left(r_{t+1}^a - r_{f,t}^a\right)$	$\sigma\left(r_{t+1}^a - r_{f,t}^a\right)$	$\dfrac{E\left(r_{t+1}^a - r_{f,t}^a\right)}{\sigma\left(r_{t+1}^a - r_{f,t}^a\right)}$
Empirical Value			
1926–2006	5.90	19.17	0.31
1954–2006	4.87	15.35	0.32
Model ($b_0 = 6$)			
Calibration I	6.75	19.49	0.35
Calibration II	4.71	16.54	0.28

Our model suggests that the decline in equity premiums is a direct result of declining macroeconomic risk, which is characterized by dividend volatility. The existing literature has focused on other measures of macroeconomic risk. Pastor and Stambaugh (2001) and Kim, Morley and Nelson (2004, 2005) have examined structural changes in market volatility and argue that, if the market price of risk does not vary greatly, then falls in market volatility should be associated with a decline in the required rate of return for equity. Lettau, Ludvigson and Wachter (2008) use consumption volatility to measure economic risk, and argue that this reduced macroeconomic risk contributed to the recent run-up in price-dividend ratios. We prefer dividend volatility to other measures of macroeconomic risk because dividend volatility is an important feature of the underlying endowment process, which determines market volatility in equilibrium. More importantly, as in the data, stock returns are only weakly correlated with consumption, therefore, a model relying on consumption volatility will inevitably generate a high correlation between stock returns and consumption, contradicting our observation.

3.5 Conclusion

This paper proposes a model in which dividend volatility is used to represent fluctuating economic uncertainty, and investors are loss averse over fluctuations in their financial wealth. Experimental and psychological evidence supports the behavioral assumption of loss aversion. Our empirical analysis of the aggregate dividend (including all distributions) establishes that dividend volatility is highly persistent and has experienced a remarkable decline in the postwar period.

Our model-simulated data exhibit similar patterns to those observed in actual return data: stock returns have a high mean, high volatility and a low correlation with consumption; they are predicted by price-dividend ratios; the Sharpe ratios are time-varying.

To address the dynamic evolution of equity premiums, we also calibrate the models according to postwar data, in which dividend volatility is shown to be substantially lower than in prewar data. Based on the new calibrated parameter values, the model can generate much lower equity premiums (higher price-dividend ratios) thanks to a more stable economic environment.

In essence, this paper highlights the significant effect of dividend volatility on asset prices when investors are narrow-framing, i.e., they derive direct utility from their financial investments. In the face of an uncertain investment environment captured by dividend volatility, loss averse investors are fearful of holding risky assets; if this uncertainty is persistent, their fears are stronger. This mechanism can generate important asset price behaviors in financial markets.

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APPENDIX

Appendix 1.A: Distribution of individual pricing errors

Cai (2006) proves that for interior points, the distribution of individual pricing error $\alpha(\tau)$ is

$$\sqrt{Th}\alpha(\tau) \xrightarrow{d} N(0, v_0 \Sigma(\tau)).$$

The variance of $\alpha(\tau)$ is composed of two parts: v_0 and $\Sigma(\tau)$. $v_0 = \int K^2(u)du$, which represents some kernel adjustments. $\Sigma(\tau) = \Omega_0^{-1}\Sigma_0(\tau)\Omega_0^{-1}$, with $\Omega_0 = E(X_t X_t')$, where $X_t = (1, f_t)'$ and $\Sigma_0(\tau) = \sum_{t=-\infty}^{\infty} \text{cov}[\varepsilon_t X_t K_h(\tau_t - \tau), \varepsilon_{t+k} X_{t+k} K_h(\tau_{t+k} - \tau)]$, which captures the possible heteroskedasticity.

In implementation, we use the sample moment to estimate Ω_0. To estimate $\Sigma_0(\tau)$, we compute the residuals $\hat{\varepsilon}_t$, and then apply the method of moment to obtain a direct estimator as

$$\hat{\Sigma}_0(\tau) = \frac{h}{Tv_0}\left(\sum_{t=1}^{T}\hat{\varepsilon}_t X_t K_h(\tau_t - \tau)\right)\left(\sum_{t=1}^{T}\hat{\varepsilon}_t X_t K_h(\tau_t - \tau)\right)'.$$

See Cai and Chen (2005) for details about the asymptotic consistency of this estimator.

Appendix 1.B: Derivation of the asymptotic distribution of average pricing errors

The average of conditional alpha is

$$\hat{\alpha} = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}\left(\frac{t}{T}\right).$$

Under the general assumption that the random error process $\{\varepsilon_t\}_{t=1}^{T}$ is heteroskadastic, the asymptotic distribution of $\hat{\alpha}$ is

$$\sqrt{T}\hat{\alpha} \xrightarrow{d} N(0,V),$$

where V is the asymptotic variance and equals the (1,1) element of

$$\Omega_0^{-1}\sum_{j=-\infty}^{\infty}E\left(X_t X_{t+j}' \varepsilon_t \varepsilon_{t+j}\right)\Omega_0^{-1}.$$

Proof:

Cai (2007) shows that $\hat{\alpha}(\tau)$ can be approximated as the (1,1) element of $\Omega_0^{-1}T_{n,0}^{*}(\tau)$, where $T_{n,0}^{*}(\tau) = T^{-1}\sum_{t=1}^{T}X_t K_h(t/T - \tau)\varepsilon_t$. So $Var(\hat{\alpha})$ asymptotically approaches the (1,1) element of

$$\Omega_0^{-1}\frac{1}{T^2}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}Cov\left[T^{-1}\sum_{t=1}^{T}X_t K_h\left(\frac{t-\tau}{T}\right)\varepsilon_t, T^{-1}\sum_{s=1}^{T}X_s K_h\left(\frac{s-\tau'}{T}\right)\varepsilon_s\right]\Omega_0^{-1}$$

$$= \Omega_0^{-1}\left[\frac{1}{T^4}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}\sum_{j=1-T}^{T-1}\gamma(j)\sum_{t=1}^{T}K_h\left(\frac{t-\tau}{T}\right)K_h\left(\frac{t-j-\tau'}{T}\right)\right]\Omega_0^{-1}$$

$$= \Omega_0^{-1}\sum_{j=1-T}^{T-1}\gamma(j)\left[\frac{1}{T^4}\sum_{\tau=1}^{T}\sum_{\tau'=1}^{T}\sum_{t=1}^{T}K_h\left(\frac{t-\tau}{T}\right)K_h\left(\frac{t-j-\tau'}{T}\right)\right]\Omega_0^{-1}$$

$$= \frac{1}{T}\Omega_0^{-1}\sum_{j=-\infty}^{\infty}\gamma(j)\Omega_0^{-1}$$

where $\gamma(j) = E\left(X_t X'_{t+j} \varepsilon_t \varepsilon_{t+j}\right)$.

Note that the coefficient

$$\frac{1}{T^4} \sum_{\tau=1}^{T} \sum_{\tau'=1}^{T} \sum_{t=1}^{T} K_h\left(\frac{t-\tau}{T}\right) K_h\left(\frac{t-\tau'}{T}\right)$$

$$= \frac{1}{T^4} \sum_{t=1}^{T} \sum_{\tau=1}^{T} K_h\left(\frac{t-\tau}{T}\right) \sum_{\tau'=1}^{T} K_h\left(\frac{t-\tau'}{T}\right)$$

$$= \frac{1}{T^3} \sum_{t=1}^{T} \sum_{\tau=1}^{T} K_h\left(\frac{t-\tau}{T}\right) \left[\frac{1}{T} \sum_{\tau'=1}^{T} K_h\left(\frac{t-\tau'}{T}\right)\right]$$

$$\simeq \frac{1}{T^3} \sum_{t=1}^{T} \sum_{\tau=1}^{T} K_h\left(\frac{t-\tau}{T}\right) \text{ [by } \frac{1}{T} \sum_{\tau'=1}^{T} K_h\left(\frac{t-\tau'}{T}\right) \simeq \int k(u)du = 1]$$

$$= \frac{1}{T^2} \sum_{t=1}^{T} \frac{\sum_{\tau=1}^{T} K_h\left(\frac{t-\tau}{T}\right)}{T} \simeq \frac{1}{T^2} \sum_{t=1}^{T} 1 = \frac{1}{T}.$$

So $\sqrt{T}Var(\hat{\alpha})$ approaches the $(1,1)$ element of $\Omega_0^{-1} \sum_{j=-\infty}^{\infty} E\left(X_t X'_{t+j} \varepsilon_t \varepsilon_{t+j}\right) \Omega_0^{-1}$.

Appendix 1.C: Goodness of fit for conditional models versus unconditional models

Chen (2008) shows that A and B in (10) take different forms depending the specifications of the errors. We report the statistics based on i.i.d. errors, but the results are robust to more general error assumptions. Under i.i.d. errors,

$$\hat{S} = \frac{\sqrt{h}(SSR_0 - SSR_1) - \hat{A}}{\sqrt{\hat{B}}},$$

where the centering and scaling factors are

$$\hat{A} = h^{-1/2}\hat{\sigma}^2 \left\{ \begin{array}{c} 2dk(0) - \frac{1}{Th}\sum_{j=1-T}^{T-1}\left(1 - \frac{|j|}{T}\right)k^2\left(\frac{j}{Th}\right)\hat{C}(j) \\ +h\left[d - \frac{1}{Th}\sum_{j=-[Th]}^{[Th]}\left(1 - \frac{|j|}{T}\right)\hat{C}(j)k\left(\frac{j}{Th}\right)\int_{-1}^{1}k\left(\frac{j}{Th} + 2u\right)du\right] \end{array} \right\},$$

and

$$\hat{B} = 4\hat{\sigma}^4 \frac{1}{Th}\sum_{j=1}^{T-1}\left(1 - \frac{j}{T}\right)\hat{C}(j)\left[2k\left(\frac{j}{Th}\right) - \int_{-1}^{1}k(u)k\left(u + \frac{j}{Th}\right)du\right]^2,$$

where $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t} - \hat{\alpha}(t/T) - \hat{\beta}(t/T)f_t\right)^2$, and

$$\hat{C}(j) = \frac{1}{T - |j|}\sum_{t=|j|+1}^{T}X'_{t-|j|}\hat{M}^{-1}X_tX'_t\hat{M}^{-1}X_{t-|j|},$$

with $\hat{M} = T^{-1}\sum_{t=1}^{T}X_tX'_t$. Intuitively, $\hat{A}$ and $\hat{B}$ are approximately the mean and

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variance of $\sqrt{h}\left(SSR_0 - SSR_1\right)$. The third term of $\hat{A}$ involving the factor h arises due to the use of the pseudodata in the reflection method to correct the boundary issue, but it is proportional to h and will vanish to zero when $T \to \infty$. It is a finite sample correction. Note that the residual variance estimator $\hat{\sigma}^2$ here is based on the nonparametric residuals and is consistent for $\hat{\sigma}^2$ under both the null and alternative hypotheses. One could also use the OLS residuals to estimate $\hat{\sigma}^2$, and the asymptotic distribution of $\hat{C}$ remains unchanged. However, the OLS residuals will not give a consistent estimator for $\hat{\sigma}^2$ under the alternative hypothesis. Thus, the use of the nonparametric residuals is expected to deliver better power.

Appendix 2.A.1 Sensitivity Analysis

We have mentioned that generations in our model should be understood as generations of trades, so that one period corresponds to six months to one year. So far in our analysis, we have taken one period to be one year. Table 2.A1 analyzes the effect of changing this assumption, by assuming the decision interval of an investor to be six months. We recalibrate dividend parameters as $\theta_H = 1.19$ and $\theta_L = 0.83$, so that the time-aggregated annual growth rate of dividends has the same mean and volatility as the data. We also reset $R_f - 1$ to be 1.91 percent to maintain a net annual risk-free rate of 3.86 percent. The loss aversion parameter is still set at $\lambda = 2.25$, and the diminishing sensitivity parameter α can take three values: 0.37, 0.52 and 0.88. The variable $WML2$ is the simulated average cumulative annualized momentum portfolio returns:

$$WML2 = \frac{1}{T}\sum_{t=1}^{T}\left(R_t^{winner} R_{t+1}^{winner} - R_t^{loser} R_{t+1}^{loser}\right).$$

Comparing Table 2.A1 with Table 2.4, where one period is assumed to be one year, we find that changing the length of the decision interval affects the momentum effect and the equity premium. When the decision interval becomes shorter, a typical investor will experience more losses in one year, and since he is averse to losses, he will demand a higher premium. The higher equilibrium equity premium or, equivalently, the lower price-dividend ratio, means that the disposition effect, i.e., age-2-1 investors' different behavior facing good news versus bad news, will have a higher impact on the stock return predictability, thereby generating higher returns to the winners-minus-

losers portfolio.

As described in Section 2.2, we suppose that investor i uses $R_f^2 W_{1,i}$ as a reference level of wealth when calculating gains and losses. Odean (1998) and Genesove and Mayer (2001) assume that the investor uses the original purchase price as a reference point. That is, if an investor buys a stock at price P^B and sells at price P^S, he calculates gains/losses X as follows: if he holds the stock one period and receives a dividend D, then he perceives $X = P^S + D - P^B$; if he holds the stock two periods and collects dividends D and D', then he perceives $X = P^S + D + D' - P^B$. Table 2.A2 presents the results for this specification of gains/losses. We still take one period as one year, and the parameter values are fixed at $\theta_H = 1.28$, $\theta_L = 0.76$, $R_f = 1.0386$ and $\lambda = 2.25$. Comparing Table 2.A2 with Table 2.4, we find that this alternative definition of gains/losses has virtually no effect on our results except to deliver a lower equity premium. The reason for the low equity premium is that stock returns don't need to beat the risk-free rate to be counted as gain, which in turn makes the investor more willing to purchase a stock.

Table 2.A1   Results for a Decision Interval of Six Months

The decision interval of the investor is assumed to be six months. Dividend parameters are recalibrated as $\theta_H = 1.1913$ and $\theta_L = 0.8309$, so that the annualized dividend growth rate has a mean of 2.24% and a volatility of 25.97%. The risk-free rate is set as $R_f - 1 = 1.91\%$. Loss aversion parameter λ is set at 2.25. PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$. $WML2$ is the simulated average cumulative annualized momentum portfolio return. $E\left(R_t R_{t+1} - R_f^2\right)$ is the annualized equity premium. $Q_t = 1 - H_2\left(S_t, 1\right)$ is the turnover, or aggregate selling, in period t.

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$
(i) Disposition Effect			
PGR	0.40	0.41	0.40
PLR	0.18	0.24	0.37
DispEffect	**2.15**	**1.68**	**1.07**
(ii) Momentum Effect			
WML2	**10.33%**	**6.59%**	**0.78%**
(iii) Turnover			
$Corr\left(R_t R_{t+1}, \frac{Q_t + Q_{t+1}}{2}\right)$	**0.87**	**0.88**	**0.89**
(iv) Equity Premium			
$E\left(R_t R_{t+1} - R_f^2\right)$	**11.27%**	**11.22%**	**9.69%**

When we extend our model to a multi-stock setting and construct the winners-minus-losers portfolio, we have assumed that investors engage in narrow-framing. Is it plausible that people frame individual stocks narrowly? As argued by Barberis and Huang (2007), narrow framing is related to non-consumption utility such as regret: if one of the investor's stocks performs poorly, he may regret the specific decision to buy that stock. So, from a theoretical perspective, gains and losses on individual stocks can affect the investor's decisions. In addition, the extensive empirical evidence on the disposition effect documents that investors, including institutional investors, are reluctant to take losses on the level of individual stocks, suggesting that investors engage in narrow framing in the real market. Of course, a framework that allows the investor to derive utility directly from trading profits on individual stocks, but also, as in traditional models, to derive utility from consumption, namely a framework that allows for both narrow and traditional broad framing at the same time, might fit the data better. Although to construct such a formal model poses significant technical challenges and is beyond the scope of our current analysis, we believe our intuition will carry over, and our main results will survive in this more general setting, so long as the investor's preference can be partially captured by narrow framing and prospect theory, which are the two main drivers of our results.

Table 2.A2   Results for Using Purchase Prices as Reference Points

The investor uses the purchase price as the reference point when calculating capital gains or losses. PGR and PLR are the simulated proportion of gains realized and proportion of losses realized. We define $DispEffect = \frac{PGR}{PLR}$ and $MomEffect = E\left(R_{t+1} \mid \theta_t = \theta_H\right) - E\left(R_{t+1} \mid \theta_t = \theta_L\right)$. WML is the simulated average momentum portfolio return in the multi-stock setting. $Q_t = 1 - H_2\left(S_t, 1\right)$ is the turnover, or aggregate selling, in period t. Technology parameter values are fixed at their values in Table 2.1: $\theta_H = 1.2821$, $\theta_L = 0.7628$ and $R_f = 1.0386$. Loss aversion parameter λ is set at 2.25, the value estimated by Tversky and Kahneman (1992).

	$\alpha = 0.37$	$\alpha = 0.52$	$\alpha = 0.88$
(i) Disposition Effect			
PGR	0.40	0.41	0.49
PLR	0.18	0.24	0.43
DispEffect	**2.24**	**1.74**	**1.16**
(ii) Momentum Effect			
MomEffect	**7.39%**	**4.91%**	**0.86%**
WML	**7.07%**	**4.65%**	**0.68%**
(iii) Turnover			
$Corr(R_t, Q_t)$	**0.84**	**0.88**	**0.95**
(iv) Equity Premium			
$E(R_t - R_f)$	**3.86%**	**3.77%**	**2.76%**

Appendix 2.A.2 Heterogeneity, Aggregation and Price Impacts

In our model, all investors have prospect theory preferences, the preference parameters (α and λ) are the same across investors, and the disposition investors (age-2-1 investors) frame gains/losses in the same way. In reality, investors are likely to be heterogeneous in a number of ways. First, some of them might be better described by traditional, risk-averse expected utility preferences, for example, the standard power utility representation, and these investors might take advantage of prospect theory investors and kill their effects on prices. Second, even prospect theory investors may differ in many dimensions, and this heterogeneity might somehow cause their aggregate behaviors to wash out. So recognizing these heterogeneities raises the question of whether the results of our model still hold in this more realistic world.

A full analysis of this issue poses significant technical hurdles, but there is good reason to believe that a more general model might deliver similar results. On the one hand, as pointed out in the limits to arbitrage literature, there might be limits to the ability and willingness of traditional expected utility maximizers, or arbitrageurs, to offset the pricing effects of prospect theory investors, because by exploiting prospect theory investors, arbitrageurs face fundamental risk as well as noise trader risk, over and above the significant implementation costs they have to bear.[53] As a result, arbitrageurs will trade

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[53]See Barberis and Thaler (2003) Section 2.2, Barberis and Huang (2001) Section IV B, or Barberis and Huang (2008) Section III F for more detailed discussion of this point.

cautiously and only partially absorb the impact on prices of prospect theory investors, thereby allowing our results to persist.

On the other hand, even though prospect theory investors might be heterogeneous in many ways, their disposition related tradings are likely to be systematic and have implications for stock prices. For example, empirical evidence documents that both institutional investors and individual investors exhibit a disposition effect, although the former do so to a smaller extent. This suggests that prospect theory can indeed capture the preferences of both type of investors, albeit differently. Formally, we can model their preferences as prospect theory utility with different parameters (α and λ), or as a combination of consumption utility and prospect theory utility with different weights. This kind of heterogeneity should not wash out in the aggregate, so that prospect theory preferences should have pricing implications. Actually, Coval and Shumway (2005) have provided strong evidence that prospect theory investors indeed move prices.

Another heterogeneity of prospect theory investors relates to the framing of gains/losses. One may argue that different investors might buy into stocks at different prices, so that, in a given period, some investors face gains and others face losses, causing their disposition related tradings to cancel out in aggregate. However, this argument is flawed because it ignores the updating of reference points. When the investor has held a stock many periods, it is more reasonable for him to think of the reference point as some weighted average of the purchase price and other former prices. Once this updating process is taken into account, then in a rising (falling) market, most investors

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holding the stock will accumulate gains (losses), regardless of when they bought into the stock or at what price, making their disposition related tradings systematic. This idea can be formalized in a setup with more than three generations. It will, however, exponentially increase the dimension of state vector, making the problem intractable.

APPENDIX 3: Constructing Dividend Time Series

We follow Bansal, Khatchatrian and Yaron (2005) to impute the dividend time series from CRSP database. This appendix describes the details. The data covers quarterly sample from 1926.Q3 till 2006.Q3. In order to construct the quarterly dividend variable, the following series are used:

- P_{indx} : Monthly stock price index on NYSE/AMEX. The price index for month j is calculated as $P_{indx,j} = (VWRETX_j + 1) \cdot P_{indx,j-1}$, where $VWRETX$ is the value weighted return on NYSE/AMEX excluding dividends, taken from CRSP.

- D_{indx} : Monthly dividend index on NYSE/AMEX. The dividend for month j is calculated as $D_{indx,j} = \left(\frac{1+VWRETD_j}{1+VWRETX_j} - 1\right) \cdot P_{indx,j}$, where $VWRETD$ and $VWRETX$ are, correspondingly, the value weighted return on NYSE/AMEX including and excluding dividends, taken from CRSP.

- $\overline{D}_{indx}$: Quarterly dividend index on NYSE/AMEX. The dividend for a quarter is the sum of the monthly dividend indices for the 3 months comprising the quarter. Then a four period backward moving average is taken to remove seasonality. That is, $\overline{D}_{indx,t} = \frac{1}{4}\sum_{j=0}^{3}\left[D_{indx,3(t-j)-2} + D_{indx,3(t-j)-1} + D_{indx,3(t-j)}\right]$, where t indexes quarters.

- $Inflation$: Quarterly inflation index. The inflation index for a quarter is the inflation index in the last month of the quarter, taken from CRSP.

The resulting quarterly dividend series and dividend growth rate series are calculated as follows:

$$D_t = \frac{\overline{D}_{indx,t}}{Inflation_t}, g_{D,t} = \log\left(\frac{D_t}{D_{t-1}}\right).$$