

CHAPTER XIII.

KINEMATIC SYNTHESIS.

"In magnis et voluisse sates."—PROPERTIUS.

§ 138.

General Nature of Kinematic Synthesis.

HAVING now examined at some length and through a great variety of cases the problems of kinematic analysis, we come to the consideration of the reversed operation—kinematic synthesis. While the former showed us the nature of the constrained motions obtained by the use of given combinations of elements, links, or chains, the province of the latter (which has already been mentioned in § 3) is the determination of the pairs, chains, or mechanisms necessary to produce a given constrained motion.

This problem is the highest of those which here come before us, and perhaps the most important of the whole series, for it has for its immediate object the creation of new machines. For this reason, and also because its solution presupposes an acquaintance with kinematic analysis, it fitly forms the conclusion of our investigations. The reader who has followed these so far, however, cannot failed to have noticed that various synthetic propositions have presented themselves in the course of our work, both in the general view of machinery to which the history of its development led us, and also in our special examination of single

elements and of several series of mechanisms and complete machines.

These propositions have more and more limited the road to the solution of the problem, so that we already know something of the general nature of the results to which we may expect kinematic synthesis to lead us. The methods, however, in which the problem may be treated differ very greatly, and we must in the first place endeavour to determine which form of application of the synthesis promises the best results in the treatment of its problems.

There are, I think, two principal methods by which the desired end can be attained;—these may be called direct and indirect synthesis respectively. Each of these again divides itself into two branches, according as its treatment is general or special. We shall attempt to determine, *a priori*, the usefulness of these different forms of synthesis.

§ 139.

Direct Kinematic Synthesis.

The general direct synthesis should give us immediately the mechanisms which are required in each machine to effect a given change of place or form in its work-piece, or to utilise in it a given natural force. It is evident at once, however, that we cannot hope to arrive at useful results by this method. For our analytical investigations have shown us that one and the same motion can be obtained in different and often very many different ways. The synthesis must therefore either give us a great number of different simultaneous solutions to the one problem, or must be able to furnish us with that one out of them all which is the best. The latter is, however, impossible,—for the practical merits or defects of each single result lie to a considerable extent beyond the sphere of kinematics (§ 3). Two steam-engines, for instance, in different circumstances may be equally good, equally useful, equally “practical,” although they may be kinematically very different. We cannot expect, therefore, to build up any system of general direct analysis which can be useful to us.

The function of special direct synthesis would be to furnish us

directly with a pair of elements suited for any required place- or form-change. This is generally possible, for if we know the required motion we can determine (Chap. II.) its axoids, and these themselves may be used (as was shown in Chap. III.) as the profiles of the elements. In the cases, however, when the centroids are infinite (§ 9) this cannot be done, and the problem requires a wider treatment, which presents the same difficulties as those of general direct synthesis. But it is not necessary to consider this further, for we saw long ago that the practical value of solutions of these problems by pairs of elements was far less than that of solutions based upon kinematic chains. This second method of synthesis, therefore, cannot furnish us with results of any practical value.

§ 140.

Indirect Kinematic Synthesis.

It is the province of the indirect synthetic method to give us beforehand the solutions of all those problems under which it is possible for the given problem to fall; to solve, that is, all the problems of machine-kinematics in advance. At first sight this problem may seem so extended, in fact so measureless, that any attempt to solve it may appear to be nothing more than a mere theoretic proposition. We must not forget, however, that some of the investigations we have already made point to the conclusion that the region covered by the problems of machine-kinematics is not unlimited. We may remember, for instance, the remarkable smallness of the number of lower pairs (§ 15), or the definite number of mechanisms which can be formed from a given chain (§ 3). So here also we shall find on closer examination that all these kinematic problems lie within a region which it is possible for us at least to survey. If the requirements of the case be not made too high, the difficulties attending the use of this form of synthesis, although great, are by no means insurmountable, especially within the limits ordinarily covered by machine-construction.

That the special indirect synthesis is really practicable is proved by the results of our analytical investigations. Its function is to show us what kinematic pairs actually exist. Now we know

(§ 56) that the number of elements is not very large, for we have been able to express them all by a very moderate number of symbols. It follows necessarily that the pairs built up from these elements can only vary within tolerably narrow limits. This is really the case; we see at once therefore that a field for the application of kinematic synthesis is becoming visible.

The object of general indirect synthesis is to do for the kinematic chain what the special synthesis does for the pair of elements. The great number of possible cases at once presents itself as a difficulty. On examination, however, it will be found that these fall together very much. The number of simple chains especially—that is, of chains in which no link contains more than two elements—is by no means as large as might be at first sight imagined. The determination of these possible simple kinematic chains alone, however, forms no inconsiderable part of the whole problem.

There is, of course, no limit to the number of compound chains which can be formed, so that in this direction the solution of the problem can never be complete, and these compound chains demand investigation on just the same terms as the simple ones. In actual machinery, however, the compounding of chains is not carried very far. In those cases which appear most complicated it is almost always possible to subdivide the whole according to the purpose of each of its groups of parts, and to treat it as a series of separate mechanisms, no one of which is in itself very complex. The method of descriptive analysis (§ 135) has given us very extensive and satisfactory illustration of this, and we shall further on find a more exact method of distinguishing between different cases of compound chains. There are certainly, however, many compound chains which cannot be subdivided in this way. Some of the more important of these we can already treat fully synthetically without extending their investigation to any excessive length; others will no doubt yield in time to synthetic methods.

We see, therefore, that we may apply the method of indirect synthesis to our subject with every prospect of obtaining by its means results which are really of practical value.

§ 141.

Diagram of the Synthetic Processes.

The importance of this part of the subject is so great that I have thought it worth while to add the accompanying diagram (Fig. 362) in the hope that it may make the connection between the different synthetic methods somewhat more clear to the reader.

Kinematic synthesis as a whole divides itself into direct and indirect, and each of these classes again subdivides into general and

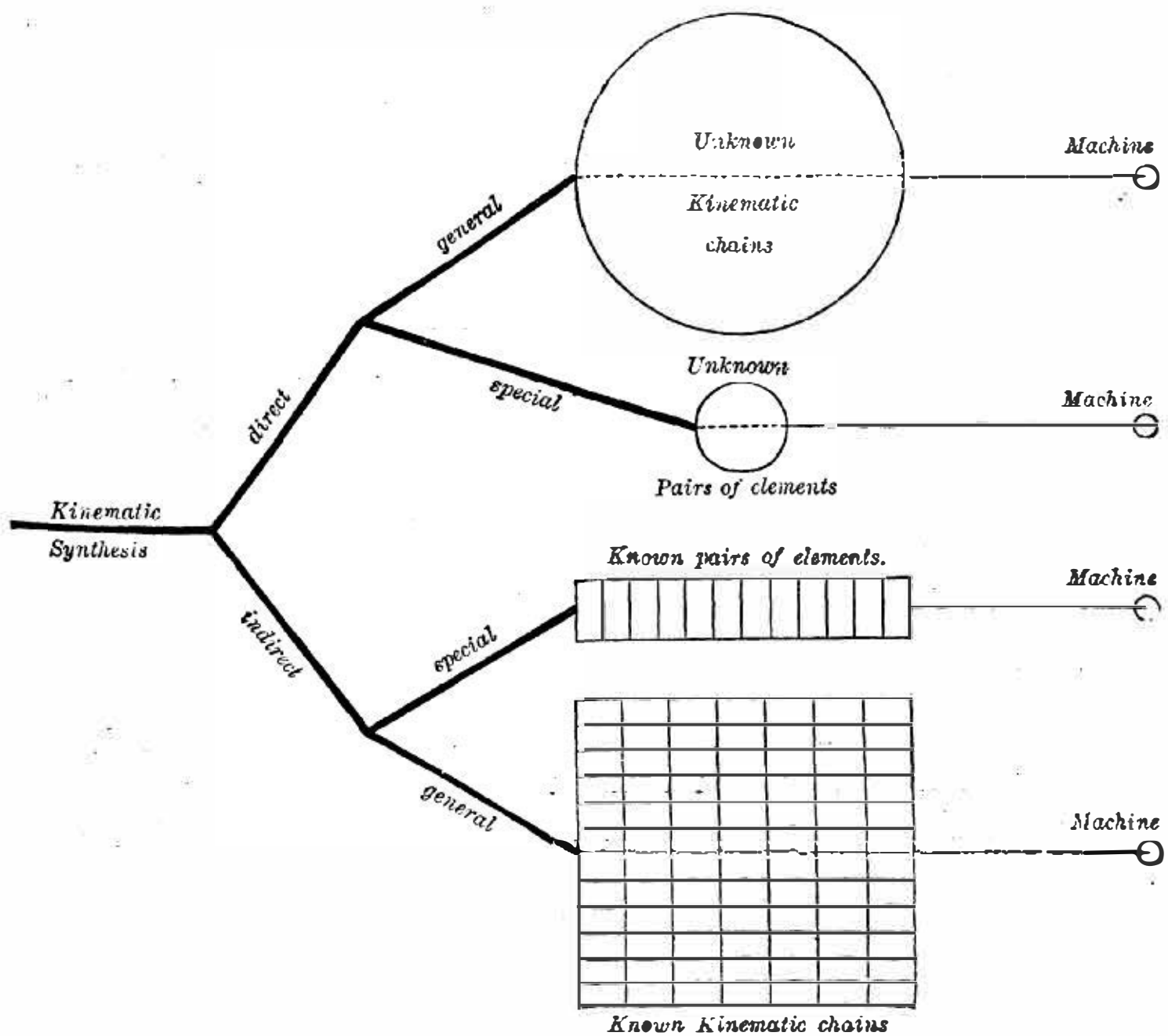


FIG. 362.

special. The direct synthesis should combine the kinematic elements at its command into the required pairs or chains according to the laws of pair- or chain-formation. In part it strikes upon insoluble difficulties, in part it furnishes results which have no practical value. The indirect synthesis first (as special

synthesis) forms and arranges all the possible pairs of elements, and then (as general synthesis) finds all the combinations of these pairs into chains. From this systematised arrangement of pairs and chains the special combinations best suited to each particular case can then be chosen by an inductive process. When the required chains have thus been found the remaining processes of forming them into mechanisms and machines present no difficulties.

We must now examine the results to which this indirect synthetical method leads us.

§ 142.

Synthesis of the Lower Pairs of Elements.

In § 55 we chose twelve class symbols for the kinematic elements, of which ten were for rigid elements :—

<i>S</i> Screw,	<i>H</i> Hyperboloid,
<i>R</i> Revolute,	<i>G</i> Sphere,
<i>P</i> Prism,	<i>A</i> Sector (portion of a revolute),
<i>C</i> Cylinder,	<i>Z</i> Tooth,
<i>K</i> Cone,	<i>V</i> Vessel,

and two for the flexional elements,

<i>T</i> Tension-organ.	<i>Q</i> Pressure-organ.
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We shall first look at the combination of the rigid elements into pairs. We may therefore omit the element *V*, which is always paired with the pressure-organ *Q*. *G*, also, stands only for a particular revolute, and *A* for a portion of the same element, so that these symbols may both be included under *R*. Seven elements therefore,

$$S, R, P, H, K, C, Z$$

remain for synthetic treatment. Three of the pairs which can be formed from these elements are already well-known to us; the three common lower pairs—

$$\begin{aligned} S^+S^- &\text{ or } (S) \text{ the screw- or twisting-pair,} \\ R^+R^- &\text{ or } (R) \text{ the revolute- or turning-pair,} \\ P^+P^- &\text{ or } (P) \text{ the prism- or sliding-pair.} \end{aligned}$$

Strictly speaking the word "lower" should be added when speaking of the first two pairs, but we have seen that in most cases it may be omitted without fear of misunderstanding. For (R) we commonly write (C) , calling the pair often a cylinder-pair; we can, however, always return to the more general symbol when necessary.

The two pairs (R) and (P) may, as we saw in § 3, be treated as special cases of the form (S) . If we place the tangent of the pitch angle as an exponent to the symbol S (as was done, for instance, in the case of the plane hyperboloid, p. 254), we have at once $(R) = (S^0)$ and $(P) = (S^\infty)$. We may also, in places where we require only to distinguish between two classes of elements, include all the lower pairs under the symbol (S) . We require, in other words, in such a general classification as is required for the synthesis, to consider only the one lower pair (S) .

§ 143.

The Simpler Higher Pairs.

The element C not only forms the closed pair (C) , but is used also in higher pairs, such, for example, as the cylindric friction-wheel, Fig. 363, which would be written C, C or more generally

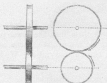


FIG. 363.



FIG. 364.

C, C . The class of pairs of which this new form is a special case is the pair of general hyperboloids, H, H .¹⁰ Non-circular cones, K, K , and non-circular cylinders, \tilde{C}, \tilde{C} , are also special cases of the same class. We had numerous examples of the last-named elements in the higher pairs of § 21 *et seq.*, of which Figure 364 represents a general case.

The pair C, C results from a further simplification of form in the same class, and it again takes several definite but general forms.

We may use for the symbol of these pairs, as we have done for former ones, a single letter. This presents no difficulty, as the

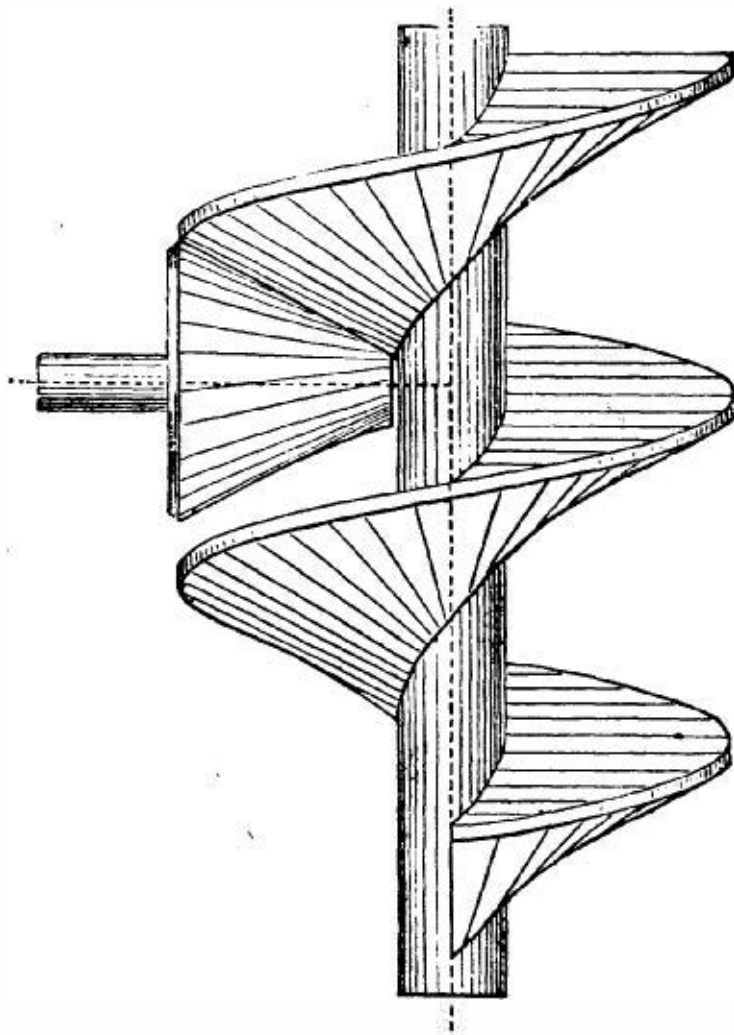


FIG. 365.

two elements always have the same name-symbol. We have only to provide means for distinguishing them from the lower pairs, (the name-symbol being sometimes the same), and this can easily be done by adding a comma to the name-symbol. We have therefore the class

\tilde{H}, \hat{H} or $(\tilde{H},)$ the general pair of hyperboloids,

and within this the following subdivisions :

\tilde{K}, \hat{K} or $(\tilde{K},)$ the pair of non-circular cones,

\tilde{C}, \hat{C} or $(\tilde{C},)$ the pair of non-circular cylinders,

as well as the three special cases of greatest simplicity :—

H, H or $(H,)$ the pair of hyperboloids of revolution,

K, K or $(K,)$ the pair of circular cones,

C, C or $(C,)$ the pair of circular cylinders.

Forms intermediate between the general and the special also occur, such as the pair H, S , which is represented in Fig. 365, and the pair HP, K , the plane-hyperboloid and cone, both of which pairs we have mentioned before (pp. 81 and 83). These forms would be indicated by the symbols (H_s) and (H_p) respectively, for the screw in Fig. 365 enters the pair because it is a ruled surface, i.e. H , and not because it is a screw S .

The pair C, C is to be distinguished from the closed cylinder-pair; this is easily done by using the symbol (C_s) for the higher pair. The pair C, P is the special case of (C_s) when one of the cylinders is of infinite radius. The sign (C_s) may be used for it, the comma indicating sufficiently that the pairing between C and P is higher.

§ 144

Synthesis of Toothed-wheel Pairs.

We come now to toothed-wheels. These might also be included under the class (H_s) , but the repetitions of the same profile occur

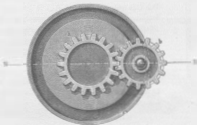


FIG. 365.

in them so characteristically that it appears well to treat them as a class by themselves, as it has hitherto been usual to do. It must not be forgotten here that chain-closure does not necessarily

accompany the use of toothed-wheels. We have already seen (§§ 43 to 50) that they may be employed with pair-closures alone, in such a way, for instance, as is represented in Fig. 366.

Taking first toothed-wheels for which the centroids are circular, we may include them all under the symbol H , H_1 , or ψH_1 , which stands for a pair of hyperboloidal toothed-wheels, and consider

$$\begin{array}{ll} K, K_1 \text{ or } (K_1) & \text{the pair of bevel wheels, and} \\ C, C_1 \text{ or } (C_1) & \text{do.} \quad \text{spur wheels} \end{array}$$

as special cases under this general class.

The teeth of these wheels are in general formed as ruled surfaces of the same character as the axoids for the motion which they transmit; they may, however, be made helical, and in that case H_1 becomes H_2 . The most general class formed in this way has the higher screw \tilde{S} for its tooth-form, it would therefore be written ψH_2 , H_2 , or $\psi(H_2)$. As special cases under this class we have

$$\begin{array}{l} (K_2) \text{ conical bevel-screw-wheels, and} \\ (C_2) \text{ cylindrical-screw-wheels.} \end{array}$$



FIG. 367

The pair of elements S, S or (S) , represented in Fig. 367, is included in the last subdivision. For this pair the symbol (C_2) is generally preferable. The symbol (S_1) is, however, valuable, in relation to the higher form $\psi(\tilde{S})$ as pointing out that the general closed screw-pair (S) may be looked at as a subdivision of the general class of reciprocally enveloping screws.

We have also to distinguish whether and which classes of toothed-wheels, those namely which have non-circular centroids. Of these we have the general cases

$$(\tilde{H}_1) \text{ and } (\tilde{H}_2),$$

which include as special cases (\tilde{K}_1) and (\tilde{K}_2) , (\tilde{C}_1) and (\tilde{C}_2) . The forms (\tilde{H}_1) , (\tilde{K}_1) , etc., may also be considered as subdivisions under (\tilde{H}_1) , $\psi(\tilde{K}_1)$, etc.

The intermediate forms H_2, K_2 —(hyperboloidal face-wheel with bevel-wheel (Fig. 36) and H_2, S (Fig. 365)—may be included under

these class signs. This holds good also for those pairs in which a spur-wheel becomes a rack P_* in which P may be treated as a special case of C .

§ 143.

Cam Pairs.

The pairing⁴ between the links a and b in the cam trains Fig. 368 and 369 may be considered a special case falling under the class (\tilde{H}) . In the cases shown we have a non-circular cylinder \tilde{C}



FIG. 368.



FIG. 369.

paired with a tooth \tilde{Z} ; the general case would be a non-circular hyperboloid \tilde{H} paired with a tooth of general form \tilde{Z} . The pair may be called the general cam pair and written

$$\tilde{H}, \tilde{Z} \text{ or } (\tilde{H}, \tilde{Z})$$



FIG. 370.



FIG. 371.

As a subdivision we have, besides (\tilde{H}, \tilde{Z})

$$(\tilde{K}_*) \text{ and } (K_*)$$

$$(\tilde{C}_*) \text{ and } (C_*)$$

Among cam pairs we have also to include (as we know from § 120) the click-pairings in such trains as are shown in Figs. 370 and 371. These may be written in the general case

$$(\tilde{H}_z;) \text{ and } (\tilde{H}_z:),$$

respectively (see § 119), and these classes subdivide themselves as in the case of toothed-wheels. The case of the toothed-rack occurs here also, as the limiting case of $C = P$.

§ 146.

Recapitulation of the Pairs of Rigid Elements.

We have seen in the foregoing sections that the pairs which are obtained from the rigid elements can be systematically arranged in divisions and subdivisions so that each special form may fall under the more general case to which it naturally belongs. Of the divisions obtained in this way we may call the highest and most general the order, and the next lower the class, while special subdivisions of the latter we have treated as groups. The following table gives a general view of the pairs of elements which we have considered, arranged in this way.⁵⁹

Pairs of Rigid Elements.

	Orders.	Classes.	Groups.
I.	(\tilde{S})	(S)	(S) , (R) , (P)
II.	(\tilde{H}_z) . . . (\tilde{H}_z) ,	(\tilde{K}_z) , (\tilde{C}_z) . . .	(H_z) , (K_z) , (C_z)
III.	(\tilde{H}_z) . . . (\tilde{H}_z) ,	(\tilde{K}_z) , (\tilde{C}_z) . . .	(H_z) , (K_z) , (C_z)
IV.	(\tilde{H}_z) . . . (\tilde{H}_z) ,	(\tilde{K}_z) , (\tilde{C}_z) . . .	(H_z) , (K_z) , (C_z)
V.	$(\tilde{H}_{z,z})$. . . $(\tilde{H}_{z,z})$,	$(\tilde{K}_{z,z})$, $(\tilde{C}_{z,z})$. . .	$(H_{z,z})$, $(K_{z,z})$, $(C_{z,z})$
VI.	$(\tilde{H}_{z;})$. . . $(\tilde{H}_{z;})$,	$(\tilde{K}_{z;})$, $(\tilde{C}_{z;})$. . .	$(H_{z;})$, $(K_{z;})$, $(C_{z;})$
VII.	$(\tilde{H}_{z:})$. . . $(\tilde{H}_{z:})$,	$(\tilde{K}_{z:})$, $(\tilde{C}_{z:})$. . .	$(H_{z:})$, $(K_{z:})$, $(C_{z:})$

We have here all the pairs already considered, in their numerous varieties, included in seven orders. Those special classes and groups which are obtained by the use of the limiting case $C = P$,

$H = H^*$ and $K = K^*$ may always be considered as falling into the divisions given above. Some cases which were, apparently, extremely complicated as, for example, the pairings occurring in Rose-engines, which are often constructed in the freest form, belong to the second order. Even the first order includes necessarily many quite free-forms. Although the number of primary divisions which we have employed is so small, we include among them, so as to make the classification as useful practically as possible, several which in strictness are only varieties of other orders, as, for instance, Nos. VI, and VII.

§ 167.

Pairs of Elements containing Tension-organs.

The characteristics of the tension-organs, band T_p , rope T_w , wire, T_w , chain T_c , or T_m , so far as their manner of pairing goes, may be sufficiently indicated by the use of the symbol T for the whole of them, and this accordingly will suffice us here. We have seen that the pairing of T with other elements can take place only

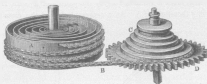


FIG. 375.

under a tensile force-closure. It can be laid upon, or wound round, rigid elements, but obviously only upon "positive" (see § 56) elements, so that its pairing with rigid elements is restricted to a certain class of forms.

Beginning with the screw, we find both higher and lower forms of the pairing of T with S frequently in use. The common chain drum of a crane forms an element of the pair S, T , which we may shortly write $(S_m)T$ and exactly the same formula represents the cylindric rope-drum, on which the rope is spirally coiled, the

cylinder itself here becoming a screw (§ 15). Higher screws \hat{S} , are not unfrequently paired with T , the fusee of a clock or watch, for example, Fig. 372, or the conic rope-drum which has lately been much used in winding engines, or the fusee in Robert's mule, Fig. 373. The conoid with a rope enveloping it (Fig. 374) belongs also to this class. The order to which these pairs belong is therefore (\hat{S}_n) , under which the class (S_n) is a lower form. At



Fig. 372.



Fig. 373.

the limit $S = R$ we obtain the group (R_n) of which Fig. 375 shows two examples.

The next order is furnished us by the pairing of \hat{T} with \hat{H} , the pair being therefore (\hat{T}_n) . As a representative of the class (\hat{U}_n)



Fig. 377.

we have the pair formed by a rope-drum and a flat belt laid spirally round it.

With \hat{H} , we can pair the chain T . It gives us the order (\hat{H}_n) , of which there are many applications.

The combinations of the element T with \tilde{H}_i and \tilde{H}_j may be included in the order (\tilde{S}_{ij}) ; it does not require, therefore, special

at h

Click-trains containing tension-organs exist, and their number has increased of late years. They are both single acting (free-clicks) as in pulley tackle, and double-acting (fast-clicks). Fowler's well-known clip-drum (Fig. 376), which does excellent service in agricultural and in towing operations, is a case of the latter. We have thus the two orders (\tilde{H}_{ij}) and (H_{ij}) not merely theoretically possible, but actually in practical use. If in a click-train



Fig. 376



Fig. 377

of this kind the element T be used in the form of a chain, T , (as e.g. in Bernier's pulley) the element H_i takes the place of \tilde{H}_i . We thus may have both the orders (\tilde{H}_{ij}) and (H_{ij}) .

We find therefore that it is possible to employ a tension-organ in every one of the seven orders into which we divided the pairs of rigid elements. One other pairing may also be carried out, that, namely, of two tension-organs. We have already mentioned one such case in speaking of the spinning process (§ 131). In the wrenching-spring, Fig. 377, we have another example of it, so that the pair T, T was one which came very early into use. The symbol for this order of pairs is (T) .

§ 148.

Pairs of Elements containing Pressure-Organs.

We have seen (§ 56) that the pressure-organ Q takes several special forms: the liquid Q_λ , the gaseous Q_γ and the globular or grained Q_g or Q . Although these forms have very important points of difference, yet so far as their kinematic pairing is concerned they may all be denoted by the single symbol Q .

The characteristic of the element Q that it has no resistance except to compression, allows it to be paired with rigid elements in the most various ways. It can be used as one of the elements in all the first five orders of pairs mentioned in § 146, as a substitute for a rigid element. The turbine, the screw-propeller, the water-wheel, the chamber-wheel train, the pug-mill and so on, give us numerous and various examples of such pairs. We have therefore the orders

$$(\tilde{S}_{,q}) \quad , \quad (\tilde{H}_{,q}) \quad , \quad (\tilde{H}_{z,q}).$$

The order $(\tilde{H}_{s,q})$ and its lower form $(\tilde{H}_{s,q})$ may also be distinguished, but it is more convenient to include both in the order $(\tilde{S}_{,q})$.

We have already found, further, that pressure-organs are paired both in free and in fast click-trains,—in those namely, in which the clicks are valves. If we consider the valve in such pairs as a tooth, Z , (which the analogy of the rigid click-trains allows us readily to do), we can indicate the two orders of pairs thus obtained by the symbols

$$(Q_{,z}) \quad \text{and} \quad (Q_{,z}).$$

In none of the cases mentioned is the pairing possible without another pairing taking place at the same time, that namely of the pressure-organ with its vessel or chamber, V^- , Fig. 378. Besides this we have also the pairing with the piston V^+ (Fig. 379), so that we may write this order of pairs in general as

$$(V_{,q}).$$

We have noticed before (§ 41) that this kind of pairing can be and has been also extended to the tension-organs, as in the link-

chain-guided in a pipe Fig. 380 and the spring brake Fig. 381. The same principle is also utilised in certain machines for the manufacture of wire-work. It is unnecessary, however, to treat these pairs as an order (V_{20}), for in all cases the sectional element of the pair may be considered as a pressure-organ. They therefore are all included in the order (V_{10}).



FIG. 379.

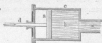


FIG. 378.

Another and very remarkable pairing of Q is that with T . This can be very easily carried out if the latter have the form T_1 , or T_{10} , if it be made, that is, as some form of chain. The Paternoster pump, whether with buckets or piston discs, the grain carriers of mills, the ladders of dredging machines, etc. all furnish illustrations



FIG. 380.



FIG. 381.

of this pairing. It would be possible to indicate by a special sign the peculiar construction of the chain, calling a tension-organ provided with scoops T_1 , for instance. This particularity is not necessary, however, for if a symbol shows a pairing between T_1 and Q it may be held to indicate that the extension organ is arranged in such a way that the pairing is possible. In some cases also no special alteration of the form of the tension-organ has to be made for this purpose, as in Veré's "rope-pump," where a mere rope T_1 lifts water by adhesion.* All these pairs together form there-

* The rope-pump, or water-rope machine, has been frequently ascribed to the elder Brunel. It is certainly older than his, however. See Langsdorf, *Maschinenkunde*, III., p. 226; Hildebrand, *Verfahren der Wasserversorgung*, p. 234.—E

fore an order for which we may use the symbol (T, q) and one of which very frequent use is made in machine practice.

We have, lastly, pairings between two pressure organs, analogous to the pairing of T with T which we have already examined. These occur somewhat frequently in a form which we may express by the symbol (Q, λ) , and of which we have illustrations in the air vessels of pumps and various hydraulic machines, in spiral pumps, hydraulic blowers and so on. We may indicate this order of pairs generally by the symbol (Q) .

§ 149.

Recapitulation of the Pairs containing Flectional Elements.

In summarising the pairs of elements considered in the last two sections it will be sufficient to tabulate the symbols for the orders alone, those for the classes and groups can be formed from these as in the case of the pairs of rigid elements. We have to add to the seven orders already tabulated:—

(a) The six following orders containing tension-organs:—

VIII. $(\bar{S}_{,t})$	XI. $(\bar{H}_{,t})$
IX. $(\bar{H}_{,t})$	XII. $(\bar{H}_{,t})$
X. $(\bar{H}_{,t})$	XIII. $(\bar{T}_{,t})$

(b) The eight following orders containing pressure-organs:—

XIV. $(\bar{S}_{,q})$	XVIII. $(Q_{,z})$
XV. $(\bar{H}_{,q})$	XIX. $(V_{,q})$
XVI. $(\bar{H}_{,q})$	XX. $(T_{,q})$
XVII. $(Q_{,z})$	XXI. $(Q_{,z})$

If it be required to indicate that the pressure-organ is a fluid, the symbol λ or γ (as the case may be) can be used instead of the q .

Examples of the majority of these twenty-one orders of pairs already exist in machinery, others of them have not yet found practical application. Our object here does not allow us to treat them further in detail. Our investigation has, however, gone far

enough to show us the notable fact that the number of possible pairs of elements is limited, and that the whole can be determined collectively by a synthetic treatment. We may now assume this to have been done, and proceed to the synthetic determination of kinematic chains.

§ 150.

The Simple Chains.

We cannot adopt so direct and definite a treatment in the case of kinematic chains as was possible with pairs of elements. In treating the latter we could build directly upon the definite and limited series of axoidal forms discussed in Chapter II; here, however, we can make but accidental use of these, for the relative motions of the links of very different chains may be identical, and have, therefore, similar axoids. We might indeed treat chains by working through every possible combination of two, three, four, &c. pairs, and take systematically all the relative positions for the pairs in each combination. But the extreme unwieldiness of such a method, and the certainty that very many of the combinations so found would prove useless, unpractical, or altogether impracticable makes it very desirable that some other treatment should be adopted, even at the expense of external uniformity in our methods.

We shall adopt in general an inductive method, as is so frequently done in mathematical investigations, and choose for each series of problems the treatment which seems best to suit the special conditions of the case. It must be remembered, at the same time, that our object here is not to complete the synthesis of the chain, but merely to note its general direction. On these grounds we shall not begin with the general case of compound chains, but with the simple ones, the essential characteristics of which we have already examined somewhat closely. The conclusions arrived at in § 128, where we found that the constrained chain took its place in the series of possible combinations of links between the unconstrained and the fixed chain, will be of great service to us here. If, that is to say, we find a chain to be fixed, we can convert it into a constrained closed chain by inductive addition of

links to it, or can obtain the same result from an unconstrained chain by the removal of links from it. We shall follow in general the same order in treating the simple chains that we have adopted in classifying the pairs of elements, without adhering to it rigidly.

§ 151.

The Screw Chain.

If we combine three coaxial screw pairs into a simple chain we obtain such an arrangement as is shown in Fig. 382, for which we see at once that we may use (S_3) as a contracted symbol. This chain forms an order by itself. The three mechanisms which can be obtained from it are essentially the same.

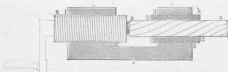


FIG. 382.

Besides the class which the order (S_3) itself represents we obtain special classes from the limiting cases in which one or other of the screws becomes $S^* = P$, or $S^* = R$ or C .

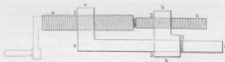


FIG. 383.

In the chain shown in Fig. 383 the pairs 1 and 2 remain (S) as before, while 3 has been made $(S^*) = (P)$; its formula runs (S_1^*P) . From this chain two different mechanisms can be formed, the trains $(S_1^*P)^*$ and $(S_1^*P)^*$ being similar. They are the well-

known "differential screws," whose invention has been ascribed both to Prony and to White. Our previous investigations have put the reader in a position from which he will recognise in Hunter's press,* the differential screw vice† and even, only such alterations of (S_2P) as are due to the reversal of pairs or to external differences in constructive form. The train (S_2P) does not appear to have been hitherto applied.

If we make the pairs 2 and 3 — (S) , and pair 1 — $(S') = (C)$, we obtain the chain shown in Fig 384, which gives the two mechan-

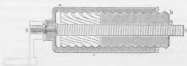


FIG. 384.

isms $(S_2U) = (S_2L)$ and $(S_2L)P$. The latter appears new, the first has been more than once applied, as for instance, very happily by Skinner‡ in his steering-gear, which, however, is a compound train.

If the pair 1 be made — $(S') = C$, the pair 3 — $(S'') = (P)$, the pair 2 alone remaining (S) , we get the chain $(S'PC)$, Fig. 385, which



FIG. 385.

has already several times come under our notice. Of the three mechanisms which it gives us $(S'PC)$ especially has found, as we know, numerous applications. (Cf. §§ 43 and 107.)

The synthesis has therefore given us where three classes of chains furnishing seven mechanisms, and three of the latter appear new. Let us now apply our method to the cases in which one of the rigid elements is replaced by a flexible one.

* *Woolley, Mechanical Principles, &c.*, vol. 1.

† *High's & Webster, Mechanics*, iii., p. 222.

‡ *The Engineer*, 1863, vol. xvi., p. 182.

The use of a tension-organ in this way does not give us any useful results. It is otherwise, however, with the pressure-organs. If, in the first place, we replace the link b in the chain $(S'P'C)$ by a fluid we can form several practical mechanisms from it. The complete formula would in this case run:

$$C^+ \dots | \dots S, Q \dots Q, P \dots | \dots C^-$$

which would be contracted, the link a being supposed fixed, into $(S'P'C)^1$. If we now take a as driving-link we get the machine $(S'P'C)^2$. This formula represents the screw-pump, the Archimedian waterlifting screw, the screw-ventilator, the main-train of Schlickeysen's clay press, &c.

If with the same mechanism the pressure organ b be made the driver, which would give us $(S'P'C)^3$ —we have the simple screw turbine.*

If we place the chain on b and make a the driver we obtain the mechanism $(S'P'C)^4$ which is the leading train of the screw-steamer. a is the propeller, c the vessel and b the water.

The train of Fig. 384 is also applied in a well-known machine. If we replace b once more by a pressure organ (here specially by a liquid) we obtain a chain of which the complete formula is:—

$$C^+ \dots | \dots S, Q_A \dots Q_A, S \dots | \dots C^-$$



FIG. 386.

Placing the chain on c and making the fluid link b the driver we obtain a mechanism $(S'P'C)^5$ which is that of the Jonval or Henschel turbine, Fig. 386. To improve the working of the machine the screws 2 and 3 are higher screws, so that strictly the formula should contain S'_2 instead of S_A .

Once more we have a series of machines placed together which differ immensely in their objects and in their constructive form, but which are formed upon one and the same kinematic chain. We shall have to mention other screw chains in § 154.

* Such for example as the turbines at the mill of St. Maur described by Letour.

§ 152.

Cylinder-Chains.

We have already (Chap. VIII.) studied the chains (C_4'') and (C_4^L), and found that they divided themselves into twelve classes containing fifty-four mechanisms. Our investigation resembled so very much a synthetic treatment of these chains that it is not necessary here to repeat the investigation. Let us look what other simple chains containing none but cylinder pairs can be formed.

If we attempt to form a chain from three cylinder pairs we see at once that its closure is fixed (Fig. 387), we need not, therefore, examine it further.

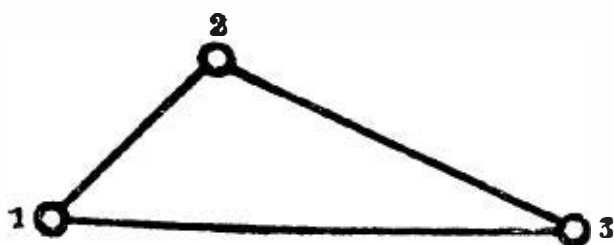


FIG. 387.

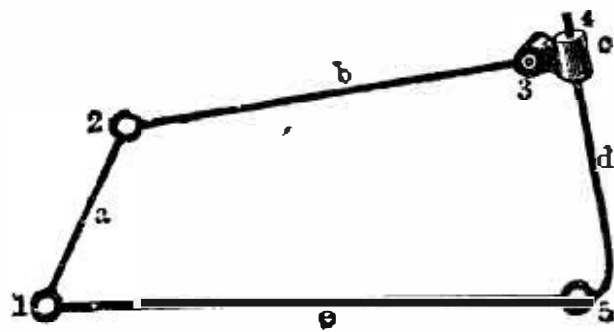


FIG. 388.

Five parallel or conic cylinder pairs give us a simple chain which is unconstrained, so that this combination also is of no importance to us. If we put a normal or normally crossed pair in place of one of the parallel pairs we obtain a chain which is constrained, and which contains five cylinder pairs, but here no motion can take place in the normal pair. It might thus be altogether omitted without affecting the motion of the chain, which is therefore really one of four links only. If the pair instead of being normal be oblique or obliquely crossed, as in Fig. 388, the chain becomes fixed, no motion whatever can take place in it. We have not, however, reached the limits of the cylinder chain, the two last mentioned may be considered to be only special cases of one containing a larger number of links. Working upwards to this from Fig. 388 we obtain first, by the addition of another cross-joint, and by destroying the parallelism of 1 and 6, the six-linked chain of Fig. 389, which again, like Fig. 388 is immoveable. If, however, we divide the link joining the cross-blocks into two parts, as in Fig. 390 for example, where a cylinder pair whose axis passes

through 3 and 5 has been inserted in it, the chain becomes moveable. The links a and f move in different planes, but the linkage bcd allows them to be constrainedly connected. The chain now consists of seven turning-pairs, and may be written generally (C_7^+) . Crossed-crank-trains formed from this chain are used in machinery; as e.g. in planing machines for giving motion to the fork which moves the belt from one pulley to another. The chain is wound with win



FIG. 389.

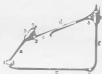


FIG. 390.

special cases, and out of many of these mechanisms can be formed. It deserves a complete and systematic examination; a glance at the subdivisions of the much simpler chain (C_4^+) gives some idea of the number of special cases to which such an examination would lead us.

Under certain conditions the six-linked chain, which we may call (C_6^+) can also be made moveable and constrainedly closed. Applications of this

occur in practice, but not in an easily recognisable form. Indeed it is especially noticeable that in these applications the principle of chain reduction (§ 76) is almost always employed. In order to fully understand them it becomes necessary to add or

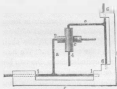
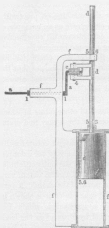


FIG. 391.

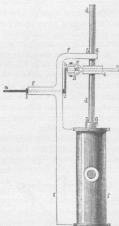
suppose added the white links

The mechanism represented by Fig. 391 gives us an illustration of this. It serves here and there for working shunting signals

which require to be turned through 90° ,—the signal is connected with the link e . Of the links in this reduced train e and f are normal crossed-links, a and c have parallel cylinders; the links b and d are omitted. The two open cylinders 2 and 4 of the link e have therefore higher screw motions relatively to the full cylinders of a and c upon which they work. In order to complete the chain it would be necessary to add the two links b and d , (which are each in the form $C^+ \dots P^+$, and to make the elements in e a pair of torcentrisms.



1000



1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679, 2680, 26

Robertson's steam-engine * is another mechanism of the same kind, its leading train is represented by Fig: 392. The link a has the form $C^+...J...C^+t$; the link b , $C^-...J...P^+$, is omitted, but c is still left. On account of the omission of b the latter has no longer

² *Artisan*, vol. LXIII, 1871, p. 2; *Revue Industrielle*, June, 1874, p. 129; *Dingler's Journal*, 1874, vol. 212, p. 183.

The "ball and socket" joint which we have here is simply the result of the omission of one of the two cylinder pairs, the axes of which intersect at right angles in one point. It is therefore marked 345. The block $f = P^+ \dots 1 \dots P^+$ serves as the piston-rod, the driving-link, of the steam-engine. The contracted formula of the mechanism is therefore $(C^+ C^+ P^+ P^+)^4 - c - d$. In order to make

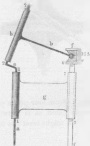


FIG. 345.



FIG. 346.

a comparison with the general form (C^+) more easy, I have represented the chain in Fig. 396 so that the two prism pairs, which we may consider as R^+ or C^+ , are replaced by the two cylinder pairs 6 and 7. The formula for the mechanism in this form, placed on g , runs $(C^+ C^+ C^+ C^+)^4$.

The foregoing examples are sufficient to show the importance of the chains (C^+) and (C^+) and to serve as an introduction to their complete synthesis. The former appears to have the largest number of links that can be used in any constrained simple chain formed from the lower pairs of elements.

Prism Chains.

We have already more than once examined (§§ 64 and 108) the three-linked prism chain, or two-wedge-chain, (P^+) . Fig. 397 represents

it in a form with which we are now familiar, Fig. 398 shows it in another form, in which all three prism pairs are closed. If instead of three we attempt to combine two prism pairs into a chain we

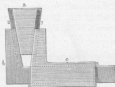


Fig. 397.

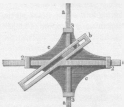


Fig. 398.

obtain either a single pair or a fixed chain. From four prism pairs, however, we can very easily form a simple chain, as is shown in Fig. 399, of which the formula is (P_4^4) . We might treat the chain

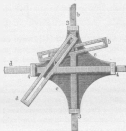


Fig. 399.

(P_4^4) as derived from this one by making the angle between the pairs 3 and 4 infinitely small. The chain (P_4^4) itself, however, might be considered as obtained from (C_4^4) by making all four links infinitely long. But closer examination shows that the chain (P_4^4) is not constrainedly closed. If we suppose, for example,

that the links b and c be fixed together,—the pair 3, that is, made immovable,—we see at once that the chain still remains moveable, and indeed has become simply that of Fig. 397.

§ 154.

The Crossed and Skew Screw Chains.

In our development of the cylinder chain, (§ 152), we found seven to be the limiting number of links in a simple chain. We were not there dealing, however, with the most general case, for the cylinder pair is not the highest form of the lower or closed pairs. This position is occupied, as we know, by the screw-pair (S). We shall therefore obtain the most general form of chain containing



Fig. 400.

only lower pairs if we place (S) instead of (C) in the most extended cylinder chain. The highest chain formed from closed pairs will therefore be the chain (S^4). The complete synthetic examination of this chain, of which those already treated in §§ 151 to 153 are really special cases, is part of the work still before synthetic kinematics. Further on we shall have to return to the question, it would suffice here merely to indicate the existence of the chain were it not necessary to look at some of the forms in which it is practically applied in machinery.

Referring again to the chain (C^2) which was shown in Fig. 394, it will be noticed that we can make the cylinder 2 oblique or crossed instead of normal, as for instance in Fig. 400, where an exceedingly complex motion can be obtained. Leaving this

unexamined, however, we may go a step further, and make the cylinders 1 and 2 coaxial. We obtain in this way the chain shown in Fig. 401. It is no longer, however, constrainedly closed, for the link a can be turned about its axis, (the coinciding axes of 1 and 2), without any motion occurring in the other links, while in these



FIG. 401.

remaining links the chain is fixed, they move together like a single link, or rather the element, relatively to the

These conditions can be entirely altered, however. If either of

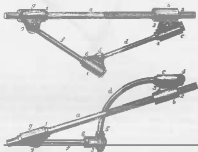


FIG. 402.

the pairs 1 or 2, let us say 2, be changed from $(C)-(S)$ to (S) , and at the same time the chain be so arranged that the axes of 6 and 4 are not in the same plane, we obtain such a chain as is shown in

Fig. (402). In order to make the relative positions of the axes 2, 4 and 6 more distinct we have here given a plan as well as an elevation of the chain, which is now constrainedly closed, and which we may call a crossed screw-chain. Its formula is (beginning with the pair 1); $(C^+ S^+ C_2^+ C^+ C^+ C^+)$. A more general



FIG. 402.

case of the same chain could be formed from the chain shown in Fig. 400. It must be noticed as a condition of the moveability of the chain that the axes of the pairs 4 and 2 must always intersect each other, and also those of the pairs 6 and 1.

If in the chain Fig. 401 we replace 2 by a screw pair but leave the axes 4 and 6 still co-plane, there is no longer any motion in the pairs 4 and 6, and they may be altogether omitted. The chain therefore takes the form shown in Fig. 403.

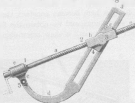


FIG. 403.

Here there are five links only instead of seven; f and e have been united and d and c have also become a single link. The formula for this chain is $(C^+ S^+ C_2^+ C^+)$. This five-linked chain has very many practical applications. By placing it on d , for example, we get a screw-reversing gear which has been used for locomotives, by placing it on e we have a train which has been used as steering gear, knuckle lever presses, &c.

If the link c in such a chain be made infinitely long we obtain the chain shown in Fig. 404, which also finds a number of applications. Its formula is $(C^1C^1P^1C^1)$.

If now the length of the link c be also made infinite, or in other

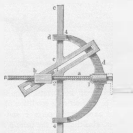


FIG. 404.

words the axis 5 removed to an infinite distance, the chain takes the form shown in Fig. 405. The varying angle between the links b and c in Fig. 404 has here become constant, the cylinder pairing at 3 is therefore superfluous, and to obtain the four-linked chain $(C^1S^1P^1C^1)$ which we may call a skew screw-chain. Placed upon



FIG. 405.

d it has received a very neat application by Nasmyth in his dividing machine.* Fig. 406 shows the arrangement adopted by him. The frame d is here the bed of the dividing machine and c the slide. The angle between the pair 3 and 4 is made variable, so that the motions of the slide for each revolution of the screw can be altered with great nicety within very wide limits.

We see that chains derived from (S_7^+) take in themselves many forms which are of practical value; they can and do also receive very numerous useful applications in compound chains.

§ 155.

Substitution of Higher Pairs for Pairs of Revolutes.

We have seen that we can regard the chains (C_7) , (C_6) , (C_5) and (C_4) with all their special cases as derived from the chain, (S_7) , as the highest form of the chains formed from closed pairs. This, however, we are not obliged to do. For the element C may be considered as a special case not only of S but of other higher forms, namely the general cylinders and cones \tilde{C} and \tilde{K} , with which (as we saw in § 21, etc.), we can form higher pairs of elements. Considering, then, the circular cylinder C as a particular case of the general cylinder \tilde{C} , we can substitute the pair (\tilde{C}) , formed from the latter, for the pair (C) where it occurs, and thus obtain entirely new motions in the cylinder chains. In this way an immense series of chains can be formed, and an almost inexhaustible series of constrained motions obtained.

The substitution of (\tilde{C}) for (C) in the general chain (C_7) is not, however, possible in every case. It cannot, for instance, be carried out in the chain (C_4') or (generally) in those cases where oblique cylinder-pairs are applied, for here the universal condition of the closed pairs, the coincidence of the axes of the two elements, becomes essential, and this is not fulfilled by the elements of the higher pair. In these cases, however, we can still use the higher pairing if we substitute (\tilde{K}) for (\tilde{C}) ,—the higher cone for the higher cylinder. The instantaneous axis of the pair then always passes through the same point.

It must not be supposed that this use of (\tilde{C}) or (\tilde{K}) instead of (C) is mere speculation. We find many instances of it in practice, especially in chains of the class (C_4') . As an example very often met with I may give Hornblower's curve-triangle train (Fig. 407), beside which is placed (in Fig. 408) the slider crank-train from which it is derived. The latter is a reduced turning double slider-crank

* *Civil Ingénieur*, 1863, p. 215, 1864, p. 21.

(§ 72 and 76), this formula runs $(C_2^2 P_1^1)^2 = b$. In Hornblower's train the curve-triangle \hat{C} , (which we have already examined in



FIG. 408.



FIG. 409.

§ 26) takes the place of the pin 2. The chain being reduced by the link b , the curve-triangle appears without its rectangular partner

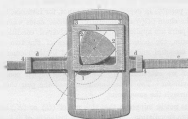


FIG. 410.

element. The formula of the chain runs therefore $(C^2 \hat{C} P_1^1)^2 = b$. If we bring both trains by augmentation into their complete condition we obtain the mechanisms shown in Figs. 409 and 410. The

centroids of a and c becomes somewhat complicated, we cannot here examine them.* This curve-triangle train has also sometimes been

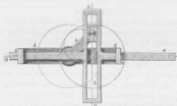


FIG. 410.

employed in practice unreduced; I know of one case, at least, in which the pair 2 has been used complete. Fig. 411 shows this



FIG. 411.



FIG. 412.

mechanism in our schematic form and with the addition of our symbols; this formulatist $(C^r C^r C^r P^1)_2^4$. It is used for driving the

* In the kinematic collection of modelist Berlin I have shown these for various mechanisms of this class.—R.

slide-valve of a 100 HP. Woolf steam-engine.* I place beside it in Fig. 412 the analogous mechanism $(C_3''P\perp)_s^d$, which we already know, in order to make the comparison between them more easy. In both trains the pair 2 is expanded.

I may just note here in passing that the whole series of forms obtained by pin-expansion from the chain (C_4'') , &c., (see § 71), can be used directly in the higher chains which we have been considering. This has scarcely been noticed as yet by machinists, and many forms possessing considerable constructive advantages have consequently not been utilised. There are many cases indeed, as the foregoing example shows, in which these cam-trains may be employed as easily and advantageously as the common eccentric-train of a steam-engine.

Chains of the class (C_7) containing more than one higher pair have not, to my knowledge, ever been practically applied. It is probable enough that really useful applications may be found for some of the numerous cases which we see here to be possible. It must suffice here to have noticed the general case.

§ 156.

Simple Wheel-chains.

Among the simple chains which consist of wheels with their shafts and bearings (cf. § 43) the friction-wheel chains naturally come first. The circular wheels with the frame which pair-closes them give us the chain (C_2^+H) , with the special forms (C_2^+K) and $(C_2''C')$. Hyperboloidal wheels seldom occur in this way, but are occasionally employed. Still higher forms, indeed, have occasionally found application, as *e.g.* the spiral friction wheels of Dick's cotton press. Friction-wheels generally occur in compound chains, I merely refer to them here because of their importance in some industries, in particular in rolling-mills, where the rolls themselves are really friction-wheels.

We need not non-examine the series of special forms which are taken by the simple toothed-wheel chain $(C_2\bar{H})$, for we have already (§ 144) investigated the various forms which the pairing \bar{H} .

* By Ad. Hirn in the Logelbach Works.

cam take. We must mention here, however, the use of a pressure-organ in the chain. The chamber-wheel trains of Chap. XI. belong to the compound chains; we have, however, simple chains in which a fluid—that is a pressure-organ—takes the place of one of the wheels, and the pairing of the fluid with its chamber takes the place of one of the cylinder pairs. Among these are the common water-wheel, the lift-wheel, and the paddle-wheel (cf. §§ 61 and 63), and also some turbines and centrifugal pumps.

§ 107.

Cam Chains.

We noticed the cam trains very briefly in § 120, and recognized their desirability to their separate treatment. Fig. 413 represents one of these trains. Its formula is $(C_2^+C_{2n})$;—we have already examined (p. 537) the nature of the pairing between the cam and the click or tooth. The special forms which this chain can take are very numerous. The cam chain, however, is not there represented in anything like its highest form. The latter, so far as is conditioned by the form of the cam and the tooth, would be the chain $(C_2^+ \hat{C}_{2n})$, which is formed from two pairs $(\hat{R}) = (C)$ of the first order, and the highest forms of the pairs of the fourth order (§ 146). The most general form of all will be obtained if we substitute (as in § 135) the higher pairs (\hat{C}) or (\hat{K}) for (C) , a method which also leads to the highest forms of the simple spur-wheel chains.

Under this most general form of the cam chain there come those important special cases which we have called click-trains. We obtain these by using in the chainst point of order VI. or VII. (§ 146). We thus obtain the chainst:

$$(C_2^+ \hat{C}_{2n}) \text{ and } (C_2^+ \hat{K}_{2n})$$

with their numberless simpler forms. As we have studied several typical cases of these in Chapter XI, we might now leave them

without further examination. There is one other click-train, however, the one represented by Figs. 414 and 415, which deserves a little notice here. In both these forms the train can also be used as a ratchet-train. The train of Fig. 414 was called by Redtenbacher the "one-toothed-wheel;" it is somewhat widely known by the names of Maltese cross or Geneva ratchet, the one being taken from the form of the wheel b , the other from the employment of this click-train in Geneva musical boxes. Fig. 415 shows that the essential condition of the train is not that the wheel a should have one tooth only; in the majority of cases, however, the wheel b is not a ratchet-shaped, on which ground it has been proposed *



FIG. 414.



FIG. 415.

to call the wheels Star-wheels. It is evident that we have here a special form of the train ($C_1\hat{C}_n$) or if it be preferred, of ($C_1\hat{H}_n$), and it is desirable to indicate by a special formula its relation to the other wheel-chains. The special characteristic of the wheels is their segmental arrangement. We may therefore use here the symbol A_1 and will indicate chains of the kind before us by the formula (C_1A_1), or more generally ($C_1A_1^*$).

If the radius of the wheel b in the click-train (C_1A_1) be made infinite, b becomes a rod or bar, carrying upon it the curved recesses and the hollows for receiving the teeth. Its formula then becomes ($C_1P^1A_1^*$). The bolt-train in the Bramah lock, Fig. 416 furnishes us with an interesting example of this. The piece $ABCD$ belongs to the bolt b ; the opening 2 in it is the hollow for the tooth 2 of a , AB and CD are adjacent curved recesses in the piece

* Polytech. Anzeiger, 1855, 2, 211; Maschinen.

b corresponding to the circular recesses which were made in w in Fig. 415. There is no longer any segment w or w' made to fit these curved recesses (as in Figs. 414 and 415), the pin 2 sufficiently answers the purpose of such a segment. Similar arrangements are to be found in other locks, although not often in such a



Fig. 416

distinct form. In the Branch lock the piece a is further connected with a fast click train, the ingenious nature of which is well known.

In cam and slider-cam trains, and also in their special forms—click-trains—pressure-organs are sometimes used. This always occurs, however, in compound chains.

Fig. 417

Pulley Chains.

The mono-kinetic properties of the tension-organ greatly increase the difficulty of arranging them in simple chains along with rigid elements (cf. § 41 *et seq.*). Such simple chains do exist, however, in belt-chains or rope-chains, as Fig. 417, and also in chain-gearing, where T takes the form T_c . The form taken by these chains, apart from the special form of the tension-organ, is (C, R_{∞}) . Of such higher forms as $(\tilde{C}, \tilde{H}_{\infty})$ or $(\tilde{C}, \tilde{H}_{\infty})$ but few applications exist. One special and limiting form of the chain, however, demands special mention. If we imagine a belt-train with crossed belts (Fig. 418) to have its two pulleys a and c brought into contact, and then the pulley c made infinite in radius, the pair

4 necessarily becomes a prism-pair, and the organ T is fixed at both ends to the prism into which it has been changed. Fig. 419 shows

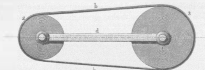


Fig. 419.

this chain, in which it is specially noteworthy that on account of the fixing of the organ T we have a chain of three links only instead of four. Its complete



Fig. 418.

$$E, T \pm \dots P \pm P' \dots \pm \dots G$$

In contracted form this would be $(C^* E, W^{(1)})$. The chain is shown here and there, placed both upon c and upon d .

A common pulley-tackle is an unconstrained closed chain, or at least is constrained only by force-closure; here therefore we need not consider it.

If it be made complete by pair-closure, it becomes (as we saw also in § 43, with the simplified roller arrangement) a compound chain. The uses thus obtained are very interesting



Fig. 420.

in themselves, but do not come into this part of our subject (cf. p. 568). This is true also of a number of other important applications of tension-organs.

§ 159

Chains with Pressure-organs.

In the foregoing treatment of the simple chains we have repeatedly had to consider the replacement of a rigid element by a pressure-organ, and in doing this have also examined the chains of which the pressure-organ became a part. The mono-kinetic properties of the pressure-organ have in all these cases shown themselves to be most important, and in the majority of cases force-closure has been necessary. If we wish to avoid this we come at once into compound chains. Complete simple constrained closed chains containing pressure-organs do not appear to be possible. Such a simple chain, for example, as that shown in Fig. 420, which would be written



Fig. 420.

$$\overbrace{P^+}^a \mid \overbrace{V_+ + Q_+}^b \mid \overbrace{Q_-}^c \mid \overbrace{V_-}^d \mid P^-$$

or in a contracted form $(PV_+ + V_-)$, is essentially force-closed. If the force-closure did not exist, for instance, the water would at once

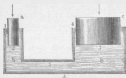


Fig. 421.

leave the chamber. The arrangement has none the less its own value, it is simply that of the common squirt.

If we arrange a second piston in the delivery pipe we obtain the arrangement of Fig. 421, already known to us. The chain has now four links instead of three; it is, however, a compound chain for

two of its links are ternary; the water is paired with the three elements 2, 3 and 5, and the frame d with the three elements 1, 4 and 5. Its formula is

$$\begin{array}{c} d \qquad \qquad \qquad a \qquad \qquad \qquad b \\ \hline V- \dots\dots \{ \dots (P) \dots\dots (V_{12}^+) \dots \} \dots\dots Q_2 \\ \qquad \qquad \qquad \underbrace{\dots\dots (P) \dots\dots (V_{12}^+) \dots\dots}_{c} \end{array}$$

By itself, however, the chain is still not constrainedly closed,—force-closure is always a condition of its working.

One means of making such chains, as well as others containing frictional elements, pair-closed, consists in doubling them.

We have found already that this method has been applied in the case of the belt-trains (cf. § 44), and have also seen the nature of the doubling of the chain now before us. Fig. 422 represents such a doubled chain. Its motions are now completely pair-closed; both the frame and time, although it contains only five links, it has become a compound chain even more completely than before. If we imagine the chain of Fig. 420, also, to be made pair-closed by any means,—the addition for instance of a suitable lever-train, which could make the motions of the piston dependent upon those of the other,—we

see at once that we are again brought into the region of compound chains. It may just be noticed that this method of doubling chains can be applied also to ordinary pulley-tackle in such a way as to transform it into a constrained closed chain.*

* I have illustrated this by a model in the kinematic collection of the König. Gewerh. Akademie in Berlin.—*F.*



Fig. 422.

§ 160.

Compound Chains.

Our synthetic investigation has now brought us, in a number of different directions, to the limits of the simple chains and to the ground covered by the compound ones. We see at once that the latter include so many important practical cases that it would be impossible to leave their synthetic treatment untouched. But there is no end to the possible combinations which can be made by joining one kinematic chain to another, and it is therefore very necessary to inquire if every problem arising in this way must necessarily fall within the region of kinematic synthesis, or if some distinction which may simplify our work does not exist between different classes of problems.

A distinction of this kind is, fortunately, furnished by the way in which the compounding has been carried out. A compounding may be a mere placing in sequence of known motions or chains, giving us therefore nothing new, or it may be so arranged as to give us some result in itself quite different from before. It is evident that these two methods of compounding may be treated in quite different ways. Let us first examine a few examples of them.

To take first a very simple case; we obviously obtain nothing kinematically new by placing one belt-train or one wheel-train behind another. The relative velocities of rotation of the different parts may be altered, the nature of these rotations is, however, exactly the same as in the simple chain, and the advantage of the compounding in such cases is connected simply with the repeated use of one and the same form of train.

Fig. 423 is a sketch of the leading train of a beam-engine. The chain here used, which consists of the seven links a, b, c, d, b_1, a_1 and d_1 , is clearly compound. It consists of a lever-crank $a, b, c, d = (C''_4)^d$ and a crossed swinging slider-crank $(C''_3P^+)^c$. The latter is shown separately, in a form already known to us, in Fig. 424. The compounding has been carried out by combining the fixed links d and c_1 of the two chains into one frame, and the links c and b_1 into a ternary link, the "beam" of the engine. The angle of swing of the lever c and the coupler b_1 becomes therefore equal, and the stroke of the slide d_1 is made dependent upon the length of the crank a .

Each train, however, might have precisely the motion which it now has were it entirely separated from the other.

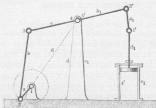


FIG. 115.



FIG. 116.

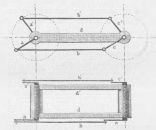


FIG. 117.

The case is quite different with the compound train shown in Fig. 118, which we have already studied. The mechanism is a pair

of parallel crank-trains,—it consists therefore of two chains of the form $(C_2''' \parallel C_2'')$ having their d links equal and common, and their links a and c combined into ternary links. We know that this chain 2 $(C_2''' \parallel C_2'')$ has the property that both its parallel cranks can pass their dead points, which are also change-points (cf. §§ 46 and 66) without stoppage or change of motion. This property, however, is characteristic of the combination, neither chain by itself

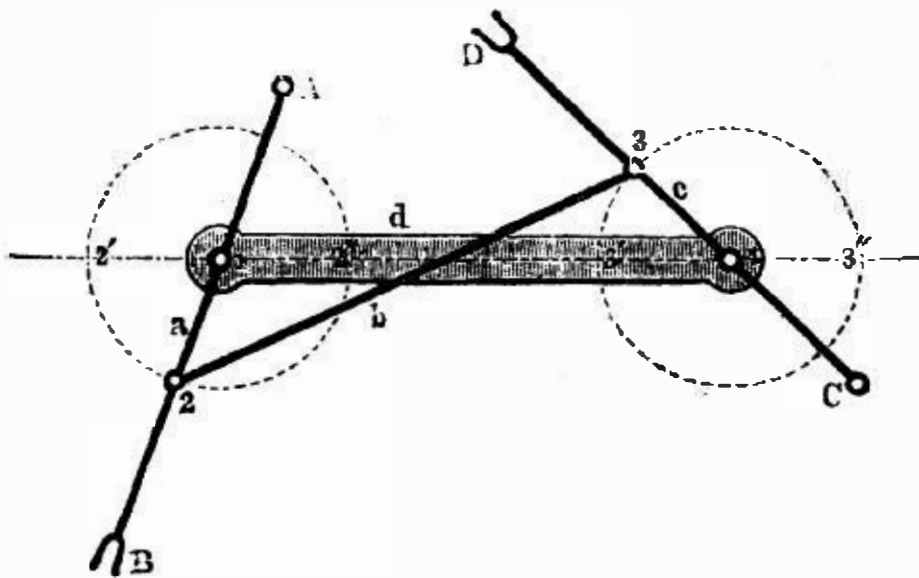


FIG. 426.

possesses it,—the compounding has therefore given us in this case something new.

The anti-parallel cranks, Fig. 428, give us another illustration of the same thing. The object of our examination of this train in §§ 47 and 67 did not lead us to notice that here also, although the number of links is not increased, we have a compound chain. It consists of our well-known four links a, b, c, d and a second chain having for its links, $A1 B, C4 D$, and d . The latter may be written in full

$$C^+ \dots \overset{a}{\parallel} \dots (Z) \dots \overset{c}{\parallel} \dots (C') \dots \overset{d}{\parallel} \dots C^-.$$

Its frame d is identical with the link d of the chain (C_4'') ; the two links $C' \dots \parallel \dots Z$ coincide with the links a and c ,—both are therefore made ternary links. By themselves neither of the chains could move continuously, (C_4'') would be stopped at the dead points, $(C_2''Z'')$ is unclosed in every other position.

These illustrations will suffice to show the difference between the two classes of compound chains. We shall call the class

last considered combined chains,* and the former class mixed chains. The consideration of the combined chains forms an essential part of kinematic synthesis, while that of the mixed chains is not in every case necessary.

§ 161.

Examples of Combined Chains.

The compound chains having a larger number of links than the simple ones, the mechanisms formed from them have a proportionately greater number of applications than those from the former. Their investigation, therefore, to be in any degree complete, would far exceed the space here at our command. Our object here, too, is rather to point out the existence and nature of problems than to attempt any complete treatment of them. I must therefore limit myself to a few examples.

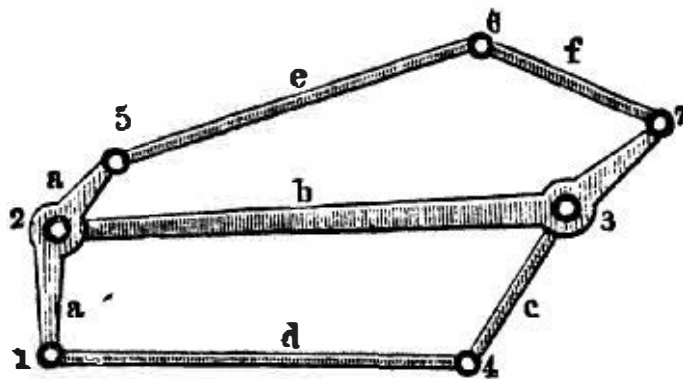


FIG. 427.

An immense number of compounds can be formed from chains of the class (C_4'') and its modifications. Among these compounds some of the mixed chains also give us something new if they be placed upon certain links. The chain shown in Fig. 427 is a combined chain. It consists of two chains of the form (C_4'') , the first is $a b c d$, the second $a e f b$.

When it is remembered that the lengths of the different links can all be changed, and also that they can be increased to infinity, it will be recognised what an enormous number of special cases arise out of the general one shown in the figure. If, for example, we

* Prof. Reuleaux uses the expressions *ächt* and *unächt zusammengesetzt*,—real and apparent compound,—for what I have called combined and mixed chains. I think I am justified in using the latter much shorter terms, especially as a very closely analogous use of them is familiar in chemical terminology.

make the links e and f infinite, and further make the axes of the pairs in each of the ternary links (1, 2, 5 and 2, 3, 7,) conplane, we obtain the combination shown in Fig. 428. If we make the original chain (C_4''), a, b, c, d , a parallelogram (as here shown) we obtain a combined chain which has some remarkable properties, although they have not yet been utilised. The line $7\cdot4'$ parallel to a is always $=2\cdot1$, and the length $1\cdot4'$ is constant, the lines $5\cdot6$ and $1\cdot4$ therefore always intersect in the same point O . If we place the chain on d we obtain a mechanism in which the bar e will

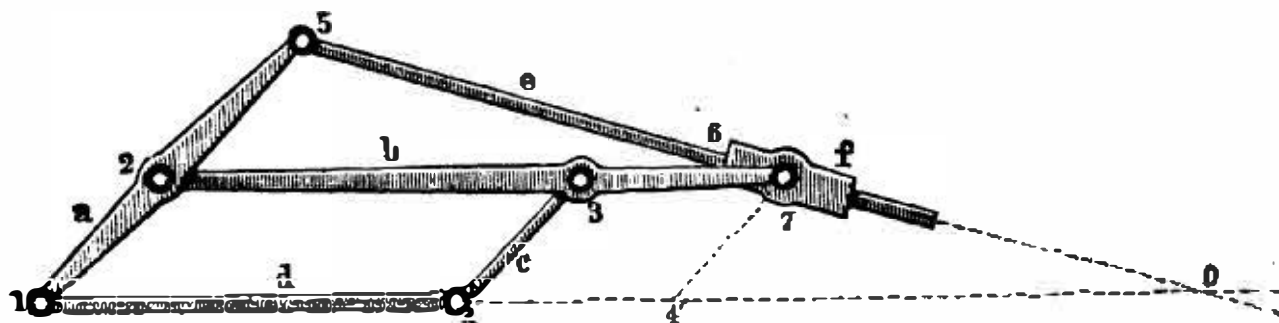


FIG. 428.

move so that its axis passes always through a fixed point beyond the mechanism, and which therefore may be itself inaccessible.

If we make e finite and therefore f and $3\cdot7$ infinitely long, we obtain the chain shown in Fig. 429, which is essentially different from the last.

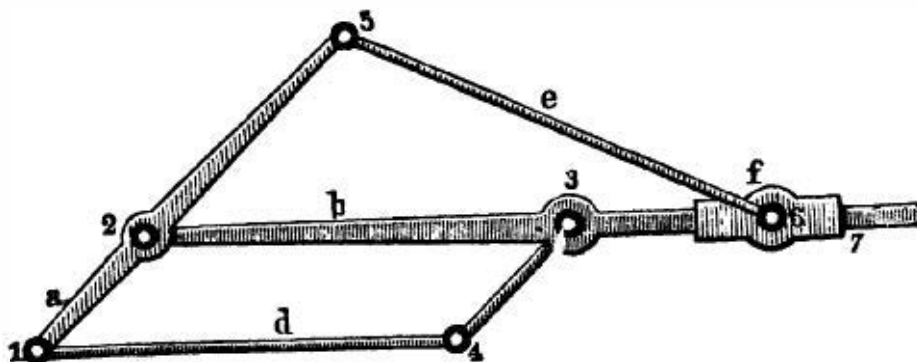


FIG. 429.

The combination of cylinder-pairs already described in § 60, which is again represented in Fig. 430, is a combined chain. The closure of the links $4\cdot7$ and $3\cdot6$ of the (C_4''') chain makes the otherwise incompletely closed five-linked chain 1. 2. 3. 4. 5 constrained. This chain finds several useful applications in "parallel motions," trains in which one or more points move in (accurately or approximately) straight paths. One of these, for instance, given by Tchebyscheff,* and another by Harvey,† are formed on this

* *Dingler's Journal*, 1862, vol. 163, p. 403.

† *Practical Mechanic's Journal*, 1850, vol ii., p. 174.

For distinctness' sake I have added the names of the links in the formula, and also the numbers of the pairs. The formula makes the symmetrical construction of the chain very distinct. The lengths of the links can be altered within the widest limits ; so

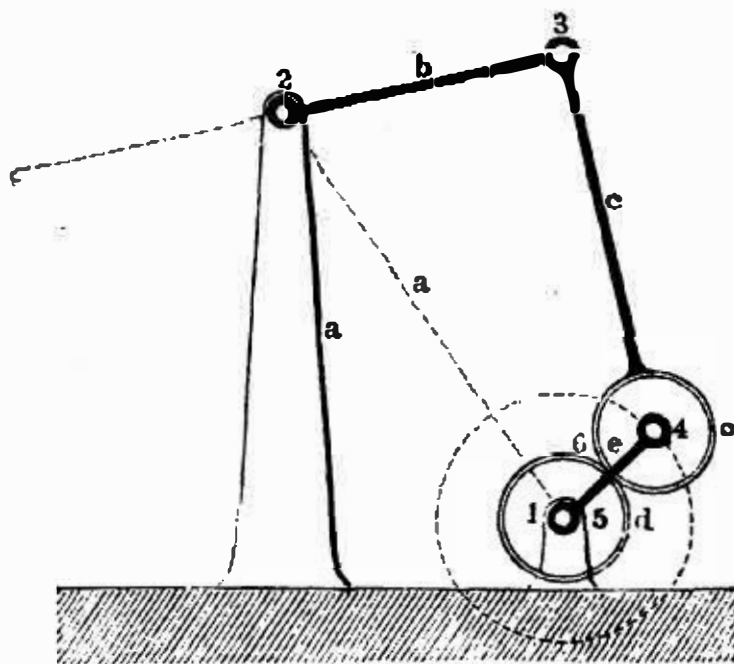


FIG. 432.

long as the closure be not made either fixed or unconstrained, any link may be fixed and any other made at the same time as the driving link, and in this way we can obtain from the chain most

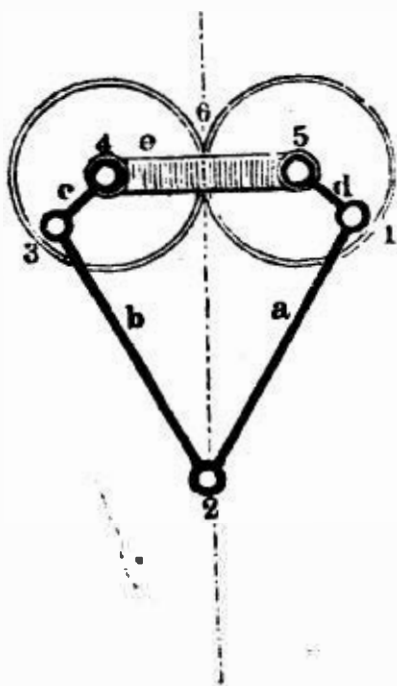


FIG. 433.

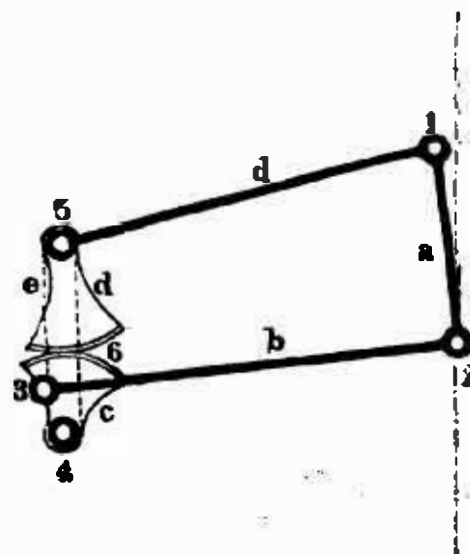


FIG. 434.

various mechanisms. We shall examine briefly a few important cases. For simplicity's sake I have omitted the teeth of the wheels and shown only their pitch circles in the figures.

(1.) Let the length 1·5 be made = 0,—the pairs 1 and 5 then

becoming coaxial. If the chain be then placed on a we obtain Watt's planet-train, Fig. 432, the motions in which we have already examined (§ 105). Following the name which Watt gave to the mechanism $(C''_5C_2)^*$ we may call the chain itself (C''_5C_2) the planet-wheel chain.*

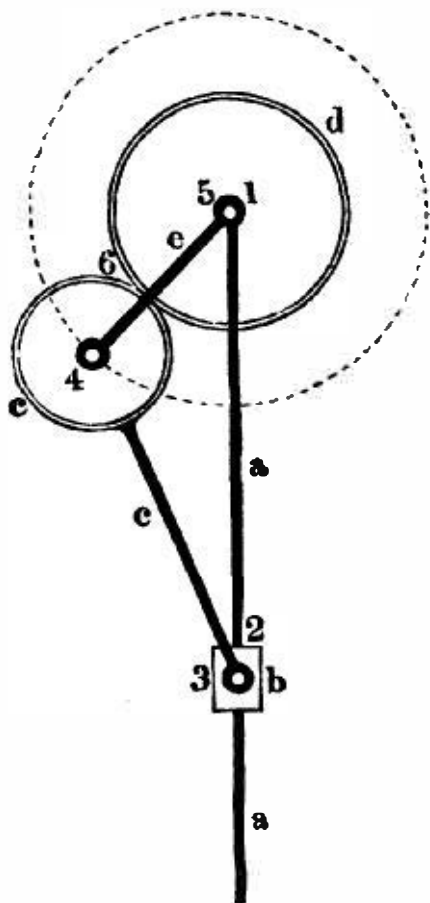


FIG. 435.

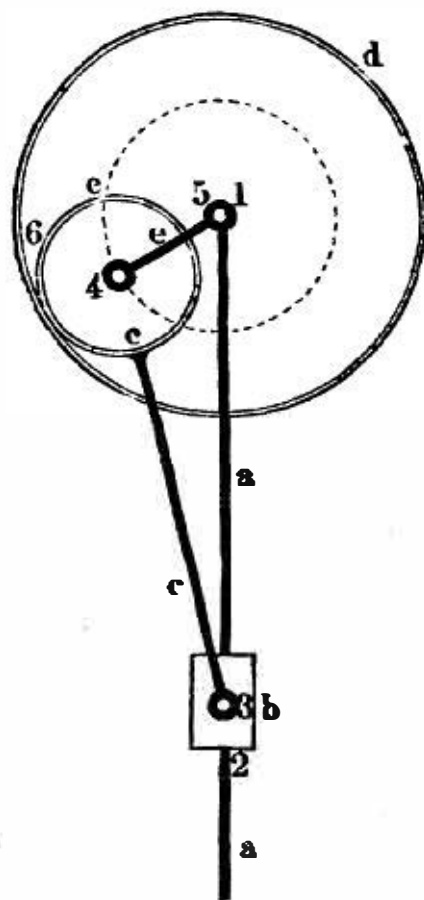


FIG. 436.

(2.) If we make $1 \cdot 5 = 3 \cdot 4$, $a = b$, and the two wheels also equal, so that the links c and d are equal and similarly placed, the whole chain becomes symmetrical about the line $6 \cdot 2$, Fig. 433, and placed on e it gives us Cartwright's parallel motion.

(3.) We can make the lengths $5 \cdot 1$, $1 \cdot 2$, $2 \cdot 3$ and $3 \cdot 4$ un-symmetrical, but by suitably proportioning them, and giving the wheels a particular diametral ratio, we obtain, by placing the chain on a , a mechanism in which 2 moves approximately in a straight line, Fig. 434. This arrangement is that proposed by Maudslay. The path of 2 is very nearly straight if the link d be not allowed to swing through too large an angle.

(4.) We obtain important special cases by making single links infinite. Let us do this first with b and a , using at the same time the simplification employed in Watt's planet-train, namely, making the length $1 \cdot 5 = 0$. We obtain in this way such a chain as is

* The planet-wheel train used by Galloway was more complex than the chain before us, and so does not come into consideration here.

shown in Fig. 435, of which the contracted formula, beginning with 1, is $(C^1 \cdot P^1 \cdot C''^3 C_z^1)$. If it be placed upon a it gives a planet-wheel train with a straight slider, a combination which has found numerous applications.

We obtain a special form of this by making d an annular wheel as in Fig. 436. In this form the chain, without recognition of its nature, has recently found several applications. Among others it has been used in an arrangement of steering-gear by Caird and Robertson.* They place the chain on a and use e as the driving-link, formula $(C^1 \cdot P^1 \cdot C''^3 C_z^1)^{\frac{1}{2}} \cdot e$. The diametral ratio of the wheels is made very nearly equal to unity, so that the rudder moving

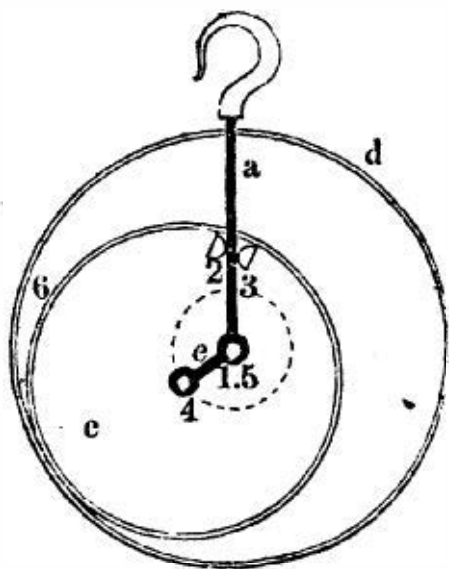


FIG. 437.

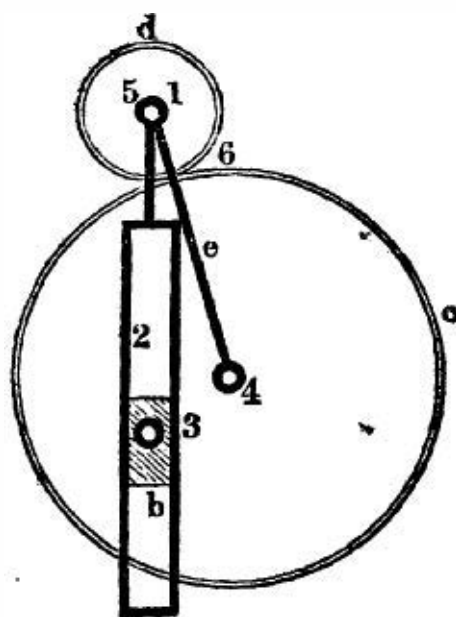


FIG. 438.

slowly is well under control. The rudder shaft is coaxial with d . With the wheel as the driving-link this mechanism is sometimes used in sewing-machines.

Eade's pulley-block,† schematically represented in Fig. 437, is another application of the same mechanism. It is again placed on a and driven by e . The link $b = C \dots \perp \dots P$ is omitted, and the higher pairing described in § 76, Figs. 269 and 270, is employed in its place. The formula of the train is therefore $(C^1 \cdot P^1 \cdot C''^3 C_z^1)^{\frac{1}{2}} \cdot b$.

The same mechanism, with the same reduction, has been used by Wilcox‡ and also by Taylor§ in counters or numbering machines.

* *Génie Industriel*, 1869, vol. xxxvii., p. 29. Caird and Robertson have applied the same mechanism also in capstans.

† *The Engineer*, 1867, p. 135.

‡ *Engineering*, January, 1869, p. 38.

§ *Engineering*, July, 1869, p. 1.

(5.) By making the length $3\cdot4$ less instead of greater than $4\cdot5$ we obtain in the chain a motion differing very greatly from any occurring in either of the former cases. Fig 438 shows this arrangement. While in Figs. 436 and 437 the whole motion of b relatively to a was equal to twice the distance $4\cdot5$, that is twice the length e , it is now equal to twice the distance $4\cdot3$. I have formerly suggested this mechanism as a leading train for punching, riveting or stamping-machines, and given it the name of toothed-eccentric* (cf. also p. 300).

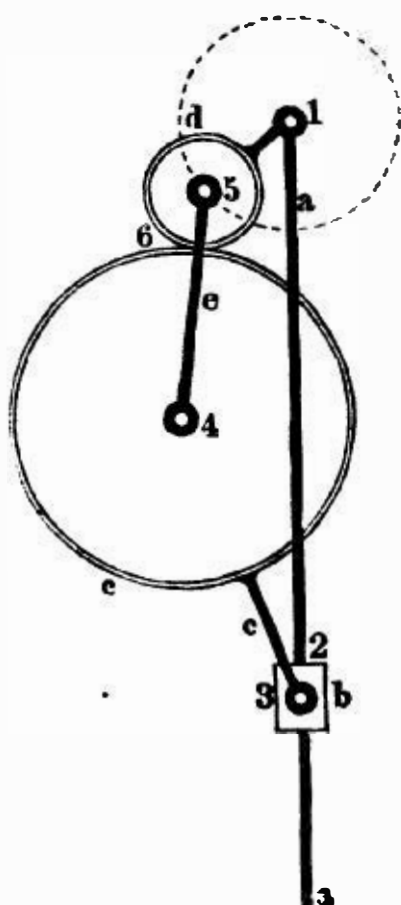


FIG. 439.

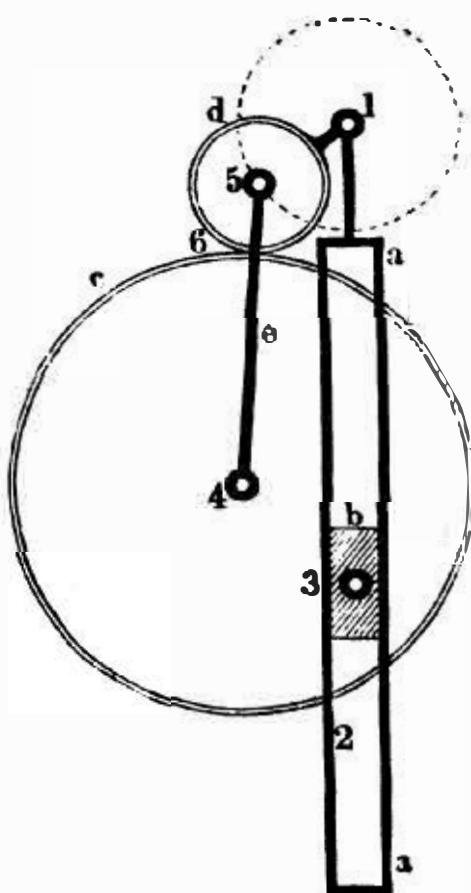


FIG. 440.

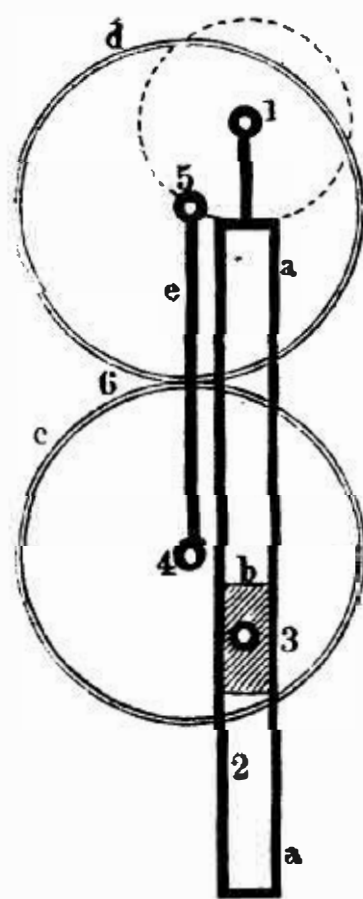


FIG. 441.

(6.) Leaving still the links a and b infinite, but giving to $1\cdot5$ some finite value, we obtain a chain represented in a general form in Fig 439. If we here make $3\cdot4 < e$, we have the chain represented in Fig. 440, which I formerly called the general case of the toothed-eccentric. Placing this chain on a we obtain a mechanism which may serve to give to a link (b) reciprocations of varying stroke. We obtain an interesting case if we make the toothed-wheels equal and the lengths $1\cdot5$ and $3\cdot4$ also equal, and at the same time place the latter symmetrically to a , as in Fig. 441. I have

* See *Civil Ingénieur*, 1858, p. 4; "Das Zahnexzentrik, ein neuer Bewegungsmechanismus." In this article I examined the whole series of these mechanisms. I had not then recognised their connection, above explained, with the planet-wheel trains. n—R.

called this mechanism the symmetric toothed-eccentric. The centroids of c and a and also those of d and b are Cardanic circles.

(7.) By making b and c infinite instead of b and a we obtain an altogether different mechanism, as is shown in Fig. 442. Its formula is $(C''C \perp P \perp C''C_2C_1)$. Placed on c and driven by e we obtain a somewhat complicated reciprocation of b . Among other applications of the train is one by Whitehill for the motion of the needle in a sewing-machine; he makes the two wheels equal.

If we make c an annular wheel we obtain the chain shown in Fig. 443. If here the diameter of d be made half that of c , and

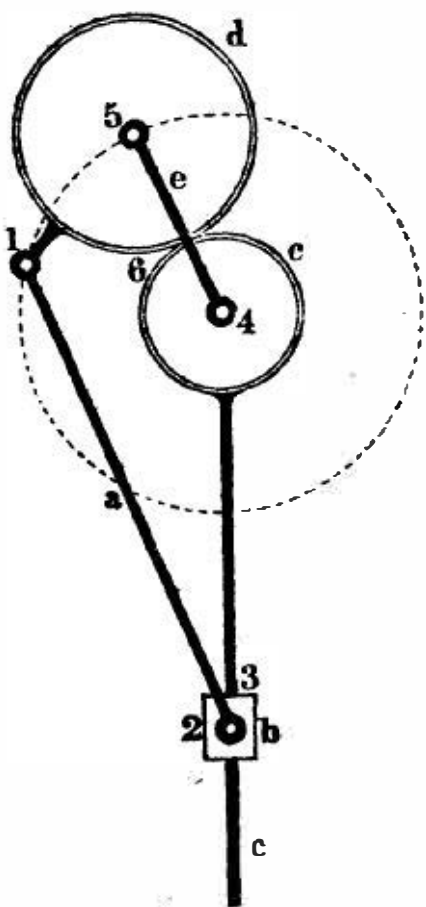


FIG. 442.

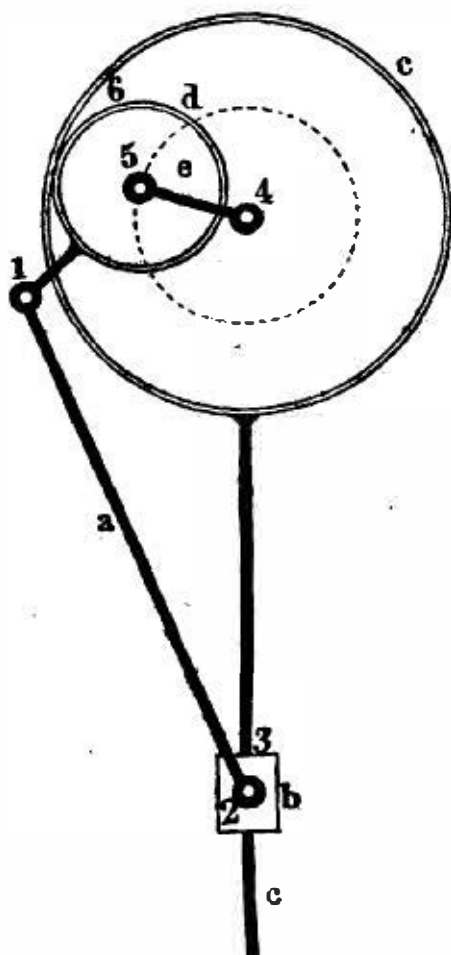


FIG. 443.

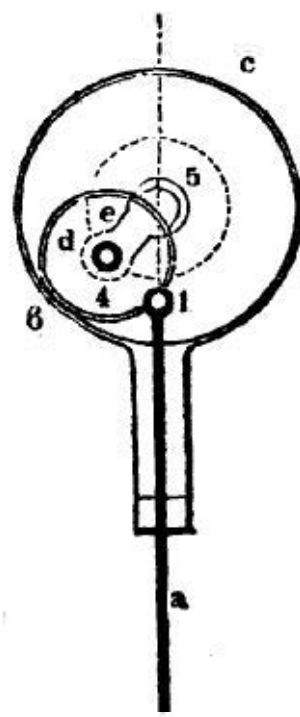


FIG. 444.

1.5 be made equal to 5.4, the chain takes the form shown in Fig. 444. The point 1, upon the circumference of a smaller Cardanic circle, moves along a diameter of c . Placing the chain on c , therefore, we obtain the well-known hypocycloidal "parallel-motion." This is a very old mechanism, called both after Lahire and after White, and is often used in printing-presses. There is no longer any motion in the turning pair 2, so that the link b may be altogether omitted. If the same chain be placed upon a instead of c we obtain again a parallel-motion, this time for the link c . So far as I know this mechanism is new.*

* There is a model of it in the Berlin kinematic collection.

I can here only mention further, that if the prism pairs in the chain ($C'_6 C_2$) be made crossed instead of normal to the axes of the cylinder pairs, a number of special cases occur: these must be here left altogether unexamined.

§ 162.

Closing Remarks.

The sketch of the synthesis of machines which we have now ended has given us several results differing greatly from those which have hitherto been deduced from a general and apparently scientific treatment of the subject. The most important discovery which we have made is undoubtedly that the region within which kinematic combinations are formed is much more narrowly limited than has usually been supposed. This is apart, I think, from the inexactness of the treatment with which so many former writers have been satisfied, for even the more accurate ideas as to combinations of elements with which we commenced our study of the problem did not in themselves indicate that the synthesis could be successfully used over so large a field as that in which we have found it available.

It is very noteworthy also, in regard especially both to practice and to instruction, that all the principal problems of machinery are connected with a comparatively very small number of kinematic chains. These are :—

the screw-chain,
the wheel-chain,
the crank-chain,
the cam-chain,
the ratchet-chain,
the pulley-chain,—

in all of which fictitious elements may take the place of rigid ones. The problems not covered by these chains are all more or less inferior in importance.

In § 92 I directed attention to the extraordinary unanimity with which the inventors of “rotary” engines and pumps have chosen crank-trains as the foundation for their chamber-gear. This now

explains itself. Among all the kinematic chains just mentioned as those most generally and easily applied, the crank-chain is that which contains the pairs of elements,—the cylinder and the prism-pair, most suitable for chambering and for the making of fluid-tight joints. Invention has thus, unconsciously, fallen generally upon this chain.

We have seen at the same time how extremely important it is that the synthetic treatment should be carried out to the fullest extent possible, for it is full of promise of new and valuable results. The question is, what form this treatment should take; for what we have here been able to accomplish in this direction has brought us only to the outer limits of the subject. It might appear at first sight that the best plan would be to make “Synthetic Kinematics” a special subject of study and instruction, treating it in separate books, and working completely through it, pair by pair and chain by chain. I do not think, however, that this is the best method. It appears to me far more advisable that under “Applied Kinematics” we should treat mechanisms, which might then be arranged according to their practical applications, both analytically and synthetically. Synthesis should be here simply one of the aids in the investigation, not its governing idea; it must be used with and beside other methods, the whole being combined for the most advantageous treatment of each particular branch of the subject.

Another remark, however, must be made here. After the satisfactory consciousness which our investigation has given us that we are not working in a field of which we can never see the boundaries, there may arise a doubt whether the material now placed at our command may not too soon be exhausted, whether our scientific treatment of it may not speedily work the mine altogether out. The doubt is made all the stronger by the stress which we have laid upon the simplifications of the matter to which we have been able to make our way. It is not one, however, about which we need to trouble ourselves.

We have carried the synthesis far enough to allow us to look round, forwards and backwards, and to compare the ground which has been explored with that which still lies untouched before us. And in the latter we can see an immense, indeed, an inexhaustible series of problems awaiting the earnest investigator. The short sketch which we have given of the planet-wheel chains gives some

indication of one of the thousand points in which the region of the compound chains awaits investigation. And here, after all, we considered only the abstract mechanism as formed from rigid elements. If we substitute for some of these flexional elements and use for all of them the materials actually employed in construction, each with its special natural characteristics, we find a multitude of new demands upon us which must be met before the abstract scheme is suitable for working under its altered conditions. Before these, that is before the never-ending demands of practical work, the doubter may well make himself once more happy in the knowledge of the essential simplicity of the means with which we have to work. We are encouraged by the conviction that the many things which have to be done can be done with but few means, and that the principles underlying them all lie clearly before us.

And now, finally, I have reached a matter upon which I touched long ago in the Introduction, and with which this whole chapter has been, without directly mentioning it, indirectly connected. This matter is the invention of mechanisms. What I meant in saying that the process of invention might become a scientific one, and might especially be performed synthetically, has now been made clear, and the truth of my assertion has, I believe, been proved. The kinematic synthesis, however, makes the finding of mechanisms easier only to those who have scientifically grasped their subject, while at the same time it places the goal which they attempt to reach ever higher and higher. It does not decrease, but rather raises, the intellectual work of the inventor, while it enables him to see more clearly, not only the object he wishes to attain, but also the means at his disposal for attaining it, and the best method of employing those means.