STRATEGIC PLANNING FOR SHELTER LOCATIONS AND TRANSPORTATION

UNDER HURRICANE CONDITIONS

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Responding to hurricanes is an exceedingly complex task whose effectiveness can significantly influence the final impact of a hurricane. Despite a lot of progress, recent events and unchecked population growth in hurricane-prone regions make it clear that having appropriate shelter options and shelter evacuation plans is very important. This research proposes a scenario-based shelter location model that identifies a set of shelter locations to maintain over time. These locations are chosen such that they are robust across a range of major hurricane events. This model considers the influence of changing the selection of shelter locations on drivers' route choice behavior and the resulting traffic congestion. The problem is formulated as a two-stage stochastic bilevel programming model where the evacuees' route choice follows dynamic user equilibrium (DUE). Aiming for large-scale realistic applications, a heuristic approach is developed to efficiently solve the formulation. A case study in the state of North Carolina is presented to illustrate the applicability and efficacy of the proposed model formulation and solution approach.

BIOGRAPHICAL SKETCH

Chunying (Anna) Li grew up in the city of Tianjin, the largest coastal city in Northern China and 120 km away from the capital Beijing. In the same place, she obtained her Bachelor and Master of Engineering degrees from Tianjin University with a major in port, coastal and offshore engineering. She also earned a Master of Philosophy degree from the University of Hong Kong under the supervision of Professor Joseph H.W. Lee. In August 2006, she started her journey of doctoral studies at Cornell University within the research group of Professor Linda Nozick. After the long and extremely rewarding five years, in August 2011, she will receive the degree of Doctor of Philosophy in Civil Engineering from Cornell University.

She enjoys both the nice summers and the frigid snowy winters in Ithaca. Her work balance philosophy is "work hard, sleep well and play hard".

To My Parents, for their faith in me

To Evan Schwartz, for his love and support

To Evan's Family, for their care and support

To Mocha and Abby, for their fun companionship

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LIST OF ABBREVIATIONS

ARC American Red Cross

BPR Bureau of Public Roads

CTM Cell Transmission Model

CV Coefficient of Variation

DSO Dynamic System Optimal

DTA Dynamic Traffic Assignment

DUE Dynamic User Equilibrium

KKT Karush-Kuhn-Tucker

LA State of Louisiana

NC State of North Carolina

OD Origin-Destination pair

SUE (Static) Stochastic User Equilibrium

UE (Static) User Equilibrium

VI Variational Inequality

VPH Vehicles Per Hour

CHAPTER 1

BILEVEL OPTIMIZATION FOR INTEGRATED SHELTER LOCATION ANALYSIS AND TRANSPORTATION PLANNING FOR HURRICANE EVENTS[1]

Abstract

Responding to hurricanes is an exceedingly complex task whose effectiveness can significantly influence the final impact of a hurricane. Despite a lot of progress, recent events and unchecked population growth in hurricane-prone regions make it clear that many challenges remain. Hurricane Katrina has shown that having appropriate shelter options and an appropriate shelter evacuation plan is very important for hurricane evacuations. This paper proposes a scenario-based shelter location model for optimizing a set of shelter locations among potential alternatives that are robust across a range of hurricane events. This model considers the influence of changing the selection of shelter locations on driver route choice behavior and the resulting traffic congestion. The state of North Carolina is used as a case study to show the applicability of the model.

Keywords: Hurricane evacuation; Bilevel programming; Shelter location; Traffic assignment; Stochastic programming; Traffic congestion; Lagrangian heuristic

1.1. Introduction

The task of moving tens or even hundreds of thousands of people from a wide geographic area in only a few days or hours under uncertain, dangerous conditions and

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getting them to safe locations is a complicated process, and as hurricane Katrina made abundantly clear, the stakes are high. Hurricane response requires coordination of many organizations, including a range of local, state, federal, and non-governmental organizations. Stakeholders from these organizations must make a series of interrelated decisions. In long-term planning efforts, they must identify facilities as possible shelters and prepare them for that role, establish evacuation routes, and plan for the decisions that will be made as each hurricane event unfolds. In the short period immediately before a hurricane makes landfall, they must decide which shelters should be open and how best to issue evacuation orders, which includes deciding who should evacuate, what type of evacuation it should be (e.g., voluntary or mandatory), and possibly where they should evacuate to.

This paper develops a scenario-based location model for determining a set of shelter locations that are robust for a wide range of hurricane events. Standard practice today is to focus on a small number of conservative representations of hurricane events to make the majority of these decisions with little understanding of the impact of these choices on the range of events that are possible. By representing a range of hurricane events explicitly and their associated probabilities of occurrence, it is possible to capture hurricane-specific features, such as which areas are no-longer suitable for shelters with specific characteristics and the spatial distribution of people in "harm's way". In this study, the set of hurricane events and their annual occurrence probabilities are chosen such that they match the regional hazard described by the exceedence curve for peak wind speed in each census tract (Legg et al., 2010).

A key element of the model developed in this paper is the explicit inclusion of driver route selection behavior to support predictions of congestion across the network under each hurricane scenario and to understand how that congestion is influenced by the selection of shelter locations. There are several different traffic assignment models that could be employed to describe the behavior of drivers and to estimate the resulting traffic pattern. For static analyses in congested networks, user equilibrium (UE) and stochastic user equilibrium (SUE) assignment have been studied extensively in the literature. When drivers are well informed about delays, UE can be a reasonable assumption. Under evacuation conditions, driver knowledge of delays is likely to be significantly less than perfect therefore, we assume evacuees have imperfect information about traffic conditions and select the best routes according to their perceptions of the travel times; hence SUE is adopted to describe driver behavior.

The contributions of this paper are three-fold: (i) this is the first model that uses a hazard- consistent scenario representation in optimizing the selection of shelters for natural disasters; (ii) this model considers individual driver's route choice behavior as captured by SUE when optimizing the shelter locations under uncertainty; and (iii) we apply the model to a large-scale realistic case study. This case study illustrates the importance of understanding the relationship between shelter location and traffic congestion when making shelter location decisions.

The remainder of this paper is organized as follows. The next section describes the literature of direct relevance to this research. The third section gives the model formulation. The fourth section presents the solution procedure. The fifth section describes a case study for the state of North Carolina and discusses the computational

results. The last section concludes this paper and recommends opportunities for future research.

1.2. Literature Review

There is a vast amount of literature focused on developing simulation and optimization models to support evacuation decisions (Hobeika and Jamei, 1985; Han, 1990; Rathi and Solanki, 1993; Dunn and Newton, 1992; Yamada, 1996; Cova and Johnson, 2003). These evacuation models assume that shelter locations are specified as input data and prescribe evacuation routing strategies to achieve a certain goal. Very few evacuation models to date have considered shelter locations as variables. Perhaps the most interesting of these is that developed in Sherali et al. (1991). The authors assumed that a central authority had the power to control the evacuation flow (a system optimal approach) and considered an evacuation planning problem by jointly optimizing traffic flow distribution and shelter location. Their model focused on the selection of shelter locations for a given hurricane event and hence the selected shelters are not necessarily robust across the full range of scenarios that are possible. Also, it is not realistic for emergency planners to precisely direct flow volumes to links when drivers are generally free to select their own travel routes to their destination (Cova and Johnson, 2003). To overcome these limitations, the study in this paper extends Sherali et al.'s research in the following ways: (1) instead of a single hurricane event, the decision of shelter selection is based on a suite of hurricane events and their associated probabilities of occurrence consistent with the hazard; (2) while Sherali et al. (1991) assumed a system optimal approach, we focus on the situation where drivers select routes based on their own behavioral preferences; (3) we account for not only the total travel time in the objective function, but also a penalty for failing to

meet the shelter demand (e.g. as a result of budget and/or staffing limits); (4) we provide computational results for large-scale problems.

Stochastic programming represents an approach for the analysis of problems characterized by uncertainty. Over the last 20 years there has been substantial progress developing and solving two-stage stochastic facility location models (Louveaux, 1986; Daskin et al., 1997; Santoso et al., 2005; Snyder and Daskin, 2006). In these models, the facility planner is assumed to have full control over the flow on the network. This simplification ignores the interdependencies which can occur between the facility planner and the network users in some applications. When the behavior of network users is considered in facility location problems arising in transportation systems, the studies reported in the literature have been mainly limited to deterministic problems (Hansen et al., 2004; Sun et al., 2008; Taniguchi et al., 1999). There are a few recent models that formulated network design problems under uncertainty where UE or SUE are assumed for route choice behavior, including Ukkusuri et al. (2007), Yin et al. (2009), Fan and Liu (2010). However, none of this address large-scale problem instances. For example, Ukkusuri et al. (2007) applied their methods to a network described in Nguyen and Dupius (1984) which consists of 19 links and 13 nodes with 4 origin-destination (OD) pairs including 19 binary variables. The same network was used in Yin et al. (2009) for problems with up to 50 scenarios. Fan and Liu (2010) used up to 64 scenarios for the Sioux Fall city road network (24 nodes and 76 links) including 12 binary variables. In contrast, we focus on a large scale application of about 100,000 OD pairs in a network with over 15,000 directed links for 33 hurricane scenarios and almost 6,500 binary variables.

In this paper, we develop a two-stage stochastic bilevel programming model for shelter facility location in the context of evacuations associated with hurricane hazards. Bilevel programming or Stackelberg leader-follower games exist where the leader (upper level) represents the central authority and the follower (lower level) is the set of network users. In the proposed formulation, the central authority identifies the shelters to be made available for use and among these the shelters, which should be opened under each hurricane scenario. The traffic pattern is then developed as a result of the decisions made by the network users in each scenario. The proposed bilevel stochastic model is applied to the state of North Carolina for a range of hurricane scenarios.

1.3.   Model Formulation

The structure of the formulated bilevel stochastic programming problem is illustrated in Figure 1.1. The upper level problem is to determine where to locate shelters before observing a hurricane scenario and, after observing a hurricane scenario, which shelters to open and how to assign the evacuees to these shelters. This creates an OD trip matrix for the allocation of evacuees seeking shelters in each scenario. For each scenario, a SUE is used in the lower-level to describe the evacuees' route choice behavior. By taking into account the evacuees' route choice behavior, the upper level model makes decision at each stage to minimize the system's cost. During an evacuation, a portion of the evacuees is directed to shelter locations and the others travel to hotels or the homes of friends and relatives. This model takes into account the travel patterns induced by both the evacuees directed to shelters and evacuees destined for elsewhere under each hurricane event.

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┌─────────────────────────────────────────────────────┐
│      Upper-level: Shelter location-allocation model   │
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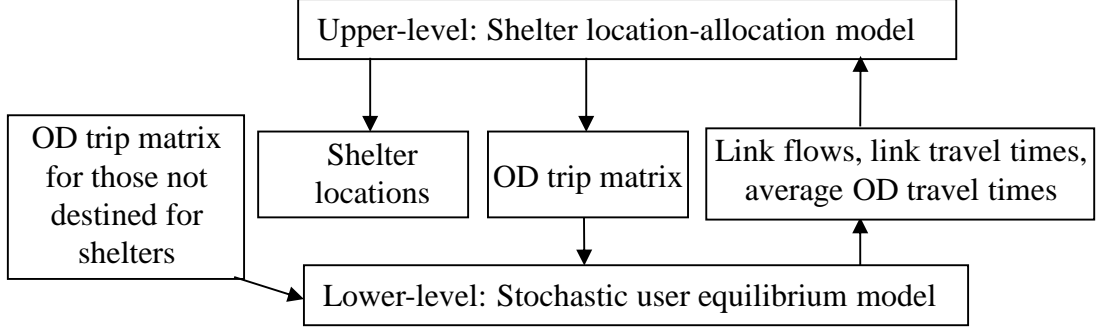
Figure 1.1. Structure of the bilevel model

We consider a transportation network represented by $\mathbf{G} = (\mathbf{N}, \mathbf{A})$, where $\mathbf{N}$ is the set of nodes and $\mathbf{A}$ the set of directed arcs. The arcs are highway arcs and the nodes in the network are intersections, sites to be potentially used as public shelters, or locations at which trips originate and/or terminate. The trip origins are census tracts where people live and therefore are assumed to be the departure points for evacuees. For evacuees not seeking public shelters, a trip table indicating their destinations must be provided or we assume their goal is to simply leave the evacuation zone as quickly as possible. Let $\mathbf{N_d} \subset \mathbf{N}$ be the set of sites where shelters may be located and $\mathbf{N_o} \subset \mathbf{N}$ be the set of demand origin nodes. A super sink node, denoted by e, is introduced to represent the exit for the evacuees destined for places other than shelters. For cases in which there are multiple exits, we add dummy links in the network which connect these exits to the super sink. The capacities of these dummy links are set to be very large and the travel times are set to be zero. Notice that this implies that the last location the evacuees pass as they exit the evacuation zone is not important. Since each hurricane scenario has a different impact zone, each scenario has a different set of exits outside the impact zone.

1.3.1. Upper-Level Problem

The upper-level is a two-stage stochastic programming problem. The first stage is to determine where to locate shelters before observing a hurricane scenario. Since funds for locating public shelters are usually limited, we assume that there is a limit on the maximum number of shelters that can be located. The second stage is to determine which shelters to open in a particular hurricane event and how to assign evacuees to these shelters. We assume that the public is well informed of available shelter locations during the hurricane warning and evacuation process.

We first describe the first-stage decision variables and constraints. We define a binary decision variable X_j to be one if a public shelter is located at node j and zero otherwise. Let P be the maximum number of shelters that can be located. Then the total number of shelters cannot exceed this maximum.

$$\sum_{j \in \mathbf{N_d}} X_j \leq P \tag{1.1}$$

Next we define the second-stage decision variables and constraints. Let $\mathbf{L}$ denote the set of hurricane scenarios. Let p^l be the probability that hurricane scenario l occurs. We define a binary decision variable W_j^l to be one if a shelter at location j is open and used to shelter people under hurricane scenario l, and zero otherwise. Clearly a shelter at location j cannot be used unless a shelter has been located there. Further, the intensity and path of a storm may preclude the use of the shelter even if one has been located there. Let η_j^l be a parameter that is one if a shelter located at node j is safe to use for hurricane scenario l. This constraint can be stated in Equation (1.2).

$$W_j^l \leq \eta_j^l X_j \qquad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{1.2}$$

A key limit on the number of shelters that can be opened in a hurricane event is the number of people available to staff the shelters. To ensure that each shelter used under each hurricane scenario has sufficient staff to operate the shelter, let S be the total shelter personnel available for the region and s_j be the staffing requirement for the shelter at location j. For each scenario l, the total staffing requirement for the region cannot exceed the total shelter personnel that are available. Equation (1.3) assumes that the number of staff available in an event is independent of the event and that they can work at any shelter. If this is not the case, this limit can be relaxed as scenario-dependent.

$$\sum_{j\in\mathbf{N_d}} s_j W_j^l \le S \qquad \forall l \in \mathbf{L} \tag{1.3}$$

For each hurricane scenario it is important to ensure that the demand destined for a shelter j does not exceed its capacity, c_j, over the evacuation period. We define a nonnegative continuous variable Y_{ij}^l to indicate the number of evacuees from origin i that use a shelter located at node j under scenario l. As described in Equation (1.4), the total number of people accommodated in shelter j under hurricane scenario l cannot exceed its capacity under this scenario.

$$\sum_{i\in\mathbf{N_o}} Y_{ij}^l \le c_j W_j^l \qquad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{1.4}$$

Note that the assignment variables ($\mathbf{Y}$) and shelter opening and closing variables ($\mathbf{W}$) are dependent on scenario l while the location variables ($\mathbf{X}$) are not. This reflects the two-stage nature of the problem.

Let h_i^l be the number of evacuees from origin node i that use public shelters under hurricane scenario l. The total number of people allocated to these shelters from origin node i cannot exceed this number under this scenario.

$$\sum_{j \in \mathbf{N_d}} Y_{ij}^l \leq h_i^l \qquad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \qquad (1.5)$$

In addition, for evacuees from origin node i to use the shelter at node j under scenario l, they must be able to reach that location and that location must not be at risk. We define the parameter δ_{ij}^l to reflect the accessibility from node i to j under scenario l. δ_{ij}^l is one if it is possible for people from node i to access shelter location j under scenario l and zero otherwise. Then the Equation (1.6) below must hold.

$$Y_{ij}^l \leq \delta_{ij}^l h_i^l \qquad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d}, l \in \mathbf{L} \qquad (1.6)$$

Unfortunately it may not be possible to accommodate all of those evacuees wishing to use shelters. We define a nonnegative decision variable Z_i^l to indicate the number of evacuees at origin node i that cannot be accommodated in shelters under scenario l. Since people wishing to use a shelter at each origin node are either assigned to a shelter or not, then

$$\sum_{j \in \mathbf{N_d}} Y_{ij}^l + Z_i^l = h_i^l \qquad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \qquad (1.7)$$

Recall that a portion of evacuees travels to the homes of friends/relatives or hotels/motels. Let U_i^l be the number of evacuees from origin node i going to places other than shelters under scenario l. Let τ_{ij}^l be the average travel time from origin node i to destination node j for hurricane scenario l. The objective of the upper-level is to minimize

$$\alpha \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} Z_i^l + \gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \tau_{ij}^l Y_{ij}^l + \gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \tau_{ie}^l U_i^l \qquad (1.8)$$

subject to constraints (1.1) - (1.7) and the non-negativity and binary restrictions given

earlier in this section. Parameters α and γ in the objective function reflect the relative

importance of each objective term in the model. The first term is the weighted expected

unmet shelter demand. The second and third terms compute the weighted expected total

evacuation travel time spent by evacuees seeking public shelters and evacuees going to

other locations, respectively. The average travel time τ for each OD pair and each

scenario in the objective function (1.8) is obtained from the SUE in the lower-level

problem.

1.3.2. Lower-Level Problem

Given the decisions made in the upper level problem ($\mathbf{Y}$ and $\mathbf{W}$) for each hurricane

scenario, the lower-level problem is to assign the trips to the network routes according to

SUE and to find the flows and travel times on the highway network. We define

nonnegative continuous variables v_a^l and t_a^l to indicate respectively the volume and the

travel time on link a under scenario l. We assume there are no link interactions and the

travel time on a given link under a given hurricane scenario depends only on the traffic

flow through that link.

$$t_a^l = g_a^l \left(v_a^l \right) \qquad\qquad \forall a \in \mathbf{A}, l \in \mathbf{L} \qquad (1.9)$$

Let $\mathcal{K}^{\mathbf{ij}}$ be the set of paths that connect origin node i to destination node j. We

define a nonnegative decision variable c_k^{ijl} to indicate the measured travel time on path k

between origin node i and destination node j under hurricane scenario l. The parameter

$\zeta_{a,k}^{ij}$ is one if link a is on route k from origin node i to destination node j, and zero otherwise. The measured path travel time can be obtained from the following incidence relationship.

$$c_k^{ijl} = \sum_{a \in \mathbf{A}} t_a^l \zeta_{a,k}^{ijl} \qquad \forall k \in \mathcal{K}^{\mathbf{ij}}, i \in \mathbf{N_o}, j \in \mathbf{N_d} \bigcup \{e\}, l \in \mathbf{L} \qquad (1.10)$$

Let C_k^{ijl} be a random variable to indicate the perceived travel time on path k between demand origin node i and destination node j under hurricane scenario l. Let ξ_k^{ijl} be the random variable with zero mean and a known distribution to indicate the error of the perceived travel time for path k of OD pair i-j under scenario l. Equation (1.11) shows the relation between measured and perceived travel time.

$$C_k^{ijl} = c_k^{ijl} + \xi_k^{ijl} \qquad \forall k \in \mathcal{K}^{\mathbf{ij}}, i \in \mathbf{N_o}, j \in \mathbf{N_d} \bigcup \{e\}, l \in \mathbf{L} \qquad (1.11)$$

We assume that the probability that path k is chosen is the probability that its travel time is perceived to be the smallest of all the alternative routes. Then

$$\mathbb{P}_k^{ijl} = Pr\left(C_k^{ijl} \le C_m^{ijl}, \forall m \ne k\right) \quad \forall k \in \mathcal{K}^{\mathbf{ij}}, i \in \mathbf{N_o}, j \in \mathbf{N_d} \bigcup \{e\}, l \in \mathbf{L} \quad (1.12)$$

We define a nonnegative decision variable f_k^{ijl} to indicate the traffic flow on path k from origin node i to destination node j under hurricane scenario l. The link flows and the route flows must satisfy the following link-route incidence relationship.

$$v_a^l = \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d} \bigcup \{e\}} \sum_{k \in \mathcal{K}^{\mathbf{ij}}} f_k^{ijl} \zeta_{a,k}^{ijl} \qquad \forall a \in \mathbf{A}, l \in \mathbf{L} \qquad (1.13)$$

Let q_{ij}^l be the vehicle flow rate from origin node i to destination j under hurricane scenario l. Since OD trip matrices $\mathbf{Y}$ and $\mathbf{U}$ are based on total number of people evacuating, we need to convert these quantities to rates for the traffic assignment calculation. In this paper, we assume a constant rate of evacuation. If an S-curve is more

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appropriate to model departures, a peak analysis can be conducted. Let T be the duration

of the evacuation in hours and μ be the vehicle occupancy factor. If the destination is a

shelter at node j, we have

$$q_{ij}^{l} = \frac{Y_{ij}^{l}}{\mu T} \quad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d}, l \in \mathbf{L} \tag{1.14}$$

If the destination is the super sink node e, then the following relationship must hold.

$$q_{ie}^{l} = \frac{U_{i}^{l}}{\mu T} \quad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \tag{1.15}$$

The traffic flow assigned to each path is based on the probability this path is chosen,

therefore

$$f_{k}^{ijl} = q_{ij}^{l} \mathbb{P}_{k}^{ijl} \quad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d} \cup \{e\}, k \in \mathcal{K}^{ij}, l \in \mathbf{L} \tag{1.16}$$

Sheffi and Powell (1982) show that the traffic assignment problem under the SUE is to

minimize

$$-\sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d} \cup \{e\}} q_{ij}^{l} E\left[\min_{k \in \mathcal{K}^{ij}} \left\{ C_{k}^{ijl} \right\} \right] + \sum_{a \in \mathbf{A}} v_{a}^{l} t_{a}^{l} - \sum_{a \in \mathbf{A}} \int_{0}^{v_{a}^{l}} g_{a}^{l}(\omega) d\omega \tag{1.17}$$

subject to constraints (1.9) - (1.16).

Note that for the given OD demand trip matrices $\mathbf{Y}$ and $\mathbf{U}$, we solve minimization

problem (1.9) - (1.17) for each scenario to obtain the measured SUE route travel times.

The average travel time for each OD pair and each scenario can be calculated as follows:

$$\tau_{ij}^{l} = \sum_{k \in \mathcal{K}^{ij}} c_{k}^{ijl} \mathbb{P}_{k}^{ijl} \quad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d} \cup \{e\}, l \in \mathbf{L} \tag{1.18}$$

Let $v_a^{ijl} = \sum_{k \in \mathcal{K}^{ij}} f_k^{ijl} \delta_{a,k}^{ijl}$, the flow on link a for scenario l under a stochastic loading

between origin i and destination j. From Equations (1.10), (1.16) and (1.18), we have

the following equivalent representation for average OD travel times.

$$\tau_{ij}^l = \frac{\sum_{a \in \mathbf{A}} v_a^{ijl} t_a^l}{q_{ij}^l} \qquad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d} \bigcup \{e\}, l \in \mathbf{L} \qquad (1.19)$$

Note that through Equation (1.14) the upper-level variable $\mathbf{Y}$ affects the lower-level

problem. Also the lower-level choice $(\mathbf{v}, \mathbf{t})$ affects the OD travel time variable τ through

Equation (1.19) and therefore the upper-level objective given in (1.8).

1.4. Solution Procedure

Recall that the average OD travel times τ is a function of OD trip matrices $\mathbf{Y}$ and $\mathbf{U}$

where $\mathbf{U}$ is input data. For OD pair i-j and scenario l, we express the OD time as

$\tau_{ij}^l(\mathbf{Y})$ given the matrix $\mathbf{Y}$ from upper-level problem. The objective of the bilevel

minimization problem is equivalent to minimizing the following:

$$\alpha \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} Z_i^l + \gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \tau_{ij}^l(\mathbf{Y}) Y_{ij}^l + \gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \tau_{ie}^l(\mathbf{Y}) U_i^l \qquad (1.20)$$

This is a non-convex, mixed integer, and two-stage stochastic program. Computationally,

this type of problem is known to be notoriously difficult and time-consuming to solve

(Bard, 1991). Hence it is necessary to explore heuristic methods to efficiently solve this

problem.

The heuristic adopted in this study is based on the following observation: if the

network flow $\mathbf{v}$ and travel times $\mathbf{t}$ are given, the average OD travel times τ can be

calculated by Equation (1.19); given the OD travel times $\boldsymbol{\tau}$, the upper-level decision variables $\mathbf{X}$, $\mathbf{W}$, $\mathbf{Y}$ and $\mathbf{Z}$ can be determined by solving the minimization problem (1.1) - (1.8) and let M be the upper-level objective value; once the OD trip tables are known, the network flow and travel times can be updated by solving the SUE. The structure of this heuristic is known to be capable of handling large-size problems and has been used to solve deterministic bilevel problems (Gartner et al., 1980; Yang et al., 1992). In each iteration of the bilevel algorithm, we record the best solution so far and let $\mathbf{BX}, \mathbf{BW}$, $\mathbf{BY}$ and $\mathbf{BZ}$ be the best upper level solution and BM be the best objective value. The solution procedure is summarized in the following steps:

Step 0: Compute the average travel time $\boldsymbol{\tau}^{(0)}$ based on free flow conditions; initialize the best objective value $BM^{(0)} = +\infty$; set $\rho = 1$;

Step 1: For given $\boldsymbol{\tau}^{(\rho-1)}$, solve the upper-level minimization problem. This gives first-stage variable $\mathbf{X}^{(\rho)}$ and the second-stage variables $\mathbf{W}^{(\rho)}$, $\mathbf{Y}^{(\rho)}$, and $\mathbf{Z}^{(\rho)}$.

Step 2: Given $\mathbf{Y}^{(\mathbf{p})}$ from step 1, solve the SUE problem for each scenario to obtain the link flows $\mathbf{v}^{(\rho)}$ and travel times $\mathbf{t}^{(\rho)}$. Compute $\boldsymbol{\tau}^{(\rho)}$ based on Equation (1.19).

Step 3: Compute the value of the upper-level objective $M^{(\rho)}$.

Step 4: If $M^{(\rho)} < BM^{(\rho-1)}$, set $BM^{(\rho)} = M^{(\rho)}$ and $\mathbf{BX}^{(\rho)} = \mathbf{X}^{(\rho)}$, $\mathbf{BW}^{(\rho)} = \mathbf{W}^{(\rho)}$, $\mathbf{BY}^{(\rho)} = \mathbf{Y}^{(\rho)}$, $\mathbf{BZ}^{(\rho)} = \mathbf{Z}^{(\rho)}$.

Step 5: If the objective value $BM^{(\rho)}$ has not improved for the last specified number of iterations, stop and report solution $BM^{(\rho)}, \mathbf{BX}^{(\rho)}$, $\mathbf{BW}^{(\rho)}$, $\mathbf{BY}^{(\rho)}$, $\mathbf{BZ}^{(\rho)}$. Otherwise, let $\rho = \rho + 1$ and go to Step 1.

Notice that sequence of the best objective value $BM^{(\rho)}$ is non-increasing and is bounded below ($BM^{(\rho)} \geq 0$ for all ρ). This implies the solution sequence $BM^{(\rho)}$ converges; however it does not guarantee a convergence to the true optima. In this study, we let the algorithm run at least a pre-specified number of iterations to prevent the algorithm from premature convergence in the heuristic scheme.

In Step 1, the upper-level problem is solved as a mixed integer program given the network travel times. We apply a Lagrangian relaxation heuristic which is known to be capable of solving a mixed integer problem of a large size. The idea is to relax certain "hard" constraints to the objective function by penalizing a violation of those constraints in the objective function. The penalty takes the form of multipliers that are associated with the corresponding constraints (see Geoffrion, 1974 for the details of the method). In this study, we relax constraints (1.2) and (1.4) because these two constraints bind the decision variables under all scenarios together. Relaxing them allow us to decompose the problem by scenarios.

In Step 2, we adopt a logit-based stochastic assignment algorithm because of its closed-form analytical expression and computational advantage. To overcome the convergence problem with Dial's logit model (Dial, 1971) when carried out as part of SUE, we apply the logit model proposed by Leurent (1997) with the method of successive averages. Leurent's logit model recommended a fixed set of efficient paths based on free-flow link travel times, which does not depend on the iteration number. It should be noted that the lower-level variable $\mathbf{v}$ is uniquely determined by the upper-level decision variables.

1.5. Case Study

To test the applicability of the above model and solution procedure, we have conducted a case study for the state of North Carolina. North Carolina is a hurricane-prone state which consists of 1,555 census tracts covering 139,391 km^2 (53,819 sq. mi.). From an analysis of historical hurricane data from year 1887 to 1998, Huang et al. (2001) estimated that the state had an annual hurricane occurrence rate of 0.277. Within the state, the hazard is most severe on the coast, decreasing as it moves inland. In 2000, North Carolina had 8.05 million people, up from 4.56 million in 1960 (U.S. Census Bureau, 2002). Importantly, much of this population growth occurred from 1960 to 1990 during a period of very little hurricane activity (Barnes, 2001). After seven remarkable hurricanes affected North Carolina from fall 1953 to fall 1955, no significant hurricane damage occurred in the state in the 1960s and relatively little in the 1970s and 1980s (Barnes, 2001). As a result of the coincidence of a quiet hurricane period and unprecedented population growth, hurricane risk may not have been adequately considered in construction during that time. During the 1990s, several significant hurricanes occurred in North Carolina, including Hurricanes Bertha, Fran, Bonnie, and Floyd.

Legg et al. (2010) developed an optimization model to select a subset of hurricanes and their annual occurrence probabilities so that the regional hazard estimated from the reduced set matched the regional hazard as closely as possible. The authors used the curves of annual probability of exceedence vs. wind speed given in HAZUS-MH (FEMA, 2006) by census tract to quantify the regional hazard in North Carolina. The set of 100 hurricanes identified by the authors are the basis for constructing the scenarios in the

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formulation developed in this paper. Of the 100 events, only 33 events require mass evacuation due to excessive wind; therefore, this case study is based on these 33 scenarios. For each hurricane scenario, HAZUS-MH is used to estimate the total number of people evacuating by census tract and the population seeking public shelters. Since the majority of evacuation comes from the coast, the 529 census tracts of the eastern third of the state are chosen as the origins.

A list of 187 existing and potential shelters from the American Red Cross are used in this study. These shelter locations include existing public building that have already been used as shelters and building that have never been used as shelters but by proper retrofitting can be used as shelters. The Red Cross estimates that the capacities of these shelters range from 700 to 4000 people based on the 1.86 m^2 (20 sq. ft.) per person standard. In a hurricane event, shelters located in at-risk areas are assumed to be not viable for use. Further, because storms generally approach from the coast, and given the uncertainty in their paths, we assume people do not evacuate towards the coastline regardless of storm track. For example, a shelter located at beachfronts or barrier islands is considered not safe and should not be open.

The highway network data are based on the primary roads (Interstate highways, US routes and NC routes) provided by North Carolina Department of Transportation. Each road segment on the network is assumed to be bi-directional and the free flow travel time for each direction is the same. The free flow travel speed on all links is assumed to be 88.5 kilometers per hour (55 miles per hour) and the capacity of each lane 1500 vehicles per hour. The network consists of 5055 nodes and 15,382 directed links.

The model is implemented in MATLAB R2009 on an Intel Xeon 2.26 GHz PC running Windows XP. Seven processors are used in this study to speed up the computation. The computation time for this case study is about two hours per bilevel iteration. Note that we could have decreased the computation time significantly if more processors were available. For example, the computation of the travel time for each OD pair is independent of the computation for another OD pair; therefore those computations could also benefit from additional processors. As another example, at each iteration of the Lagrangian heuristic, the computation time could have been reduced if each of the 33 scenarios had been assigned to a different processor. Note that the computational complexity for the lower-level is linear in the number of scenarios and the SUE computation is approximately linear in the number of OD pairs. However, the upper level problem has a staffing constraint, which yields a 0-1 knapsack problem and therefore the upper level problem is NP hard. In this case study, we solved the knapsack 3,510 times within the algorithm.

To apply the developed model to this case study, the following parameters and assumptions have been chosen: (i) There are two people occupying each vehicle; (ii) Five shelter staff are needed per hundred people of shelter capacity (based on conversations with American Red Cross personnel) and there are 3,000 staff available for each event; (iii) A maximum of 50 shelters can be selected. This, along with the staffing constraint, limits which shelters may be used under each scenario; (iv) The standard BPR (Bureau of Public Roads) volume-to-delay function holds: $t_a = t_a^0 \left(1 + \alpha \left(v_a / Q_a \right)^{\beta} \right)$, where t_a^0 is the free-flow travel time on link a, Q_a represents the practical capacity of link a with $\alpha = 0.15$ and $\beta = 4.0$. Note that this α takes on a different definition from the α in model

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objective (1.8); and (v) Only a portion of evacuees on the network is directed to public shelters, and for those evacuees seeking destinations other than public shelters, we assume they simply try to leave the impacted area as quickly as possible since there is little information available as to where these people actually might go. At the end of this section, we hypothesize a reasonable set of major city destinations and OD trips for these evacuees to broaden the insights from this case study.

Figure 1.2 illustrates the 50 shelters selected in this analysis. Only these 50 may be used to shelter people under each of the scenarios. It is useful to notice that most of the shelter capacity is to the west of I-95 because many of the storms which hit North Carolina primarily impact the eastern third of the state. This figure also gives the shelter location recommendations obtained from both the first iteration and the optimized iteration (i.e. the iteration where the optimized solution is found). In the initial iteration, the shelter locations are determined based on free-flow network travel times. While in the optimized iteration, congestion-related travel times are considered while making shelter location decisions. Both solutions give approximately the same shelter capacity but they recommend slightly different shelter locations. In the following, one of the hurricane scenarios will be used to show how different shelter locations can influence the evacuation traffic.

Among the 33 hurricane scenarios in this study, scenario 2 is primarily a coastal storm with about 410,000 people evacuating. The storm track first comes ashore near Wilmington and travels north along the coast with the peak wind gusts in the Wilmington area exceeding 175 mph. The estimated annual occurrence probability for this storm and those with a similar spatial distribution for wind speeds is about 0.04%. The location of
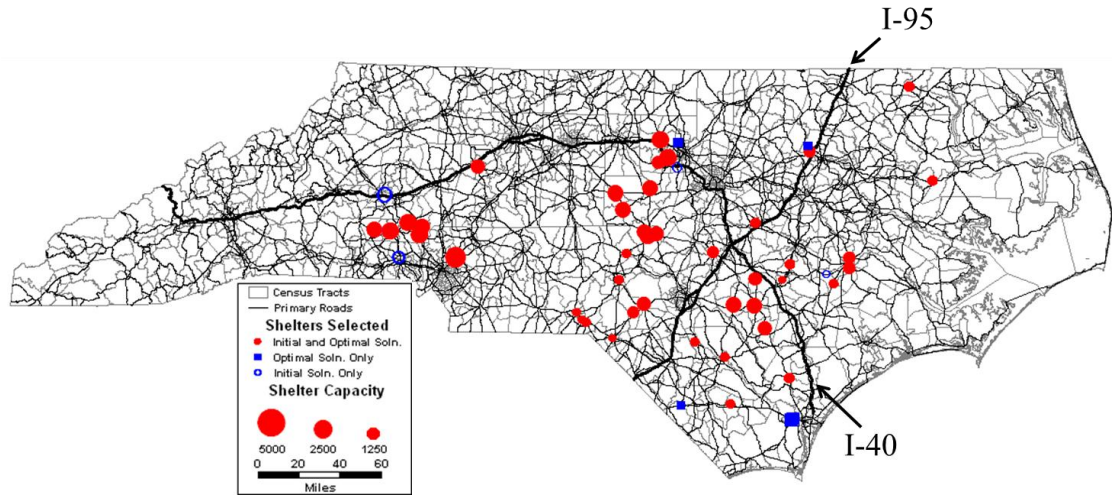
Figure 1.2. Recommended shelter locations
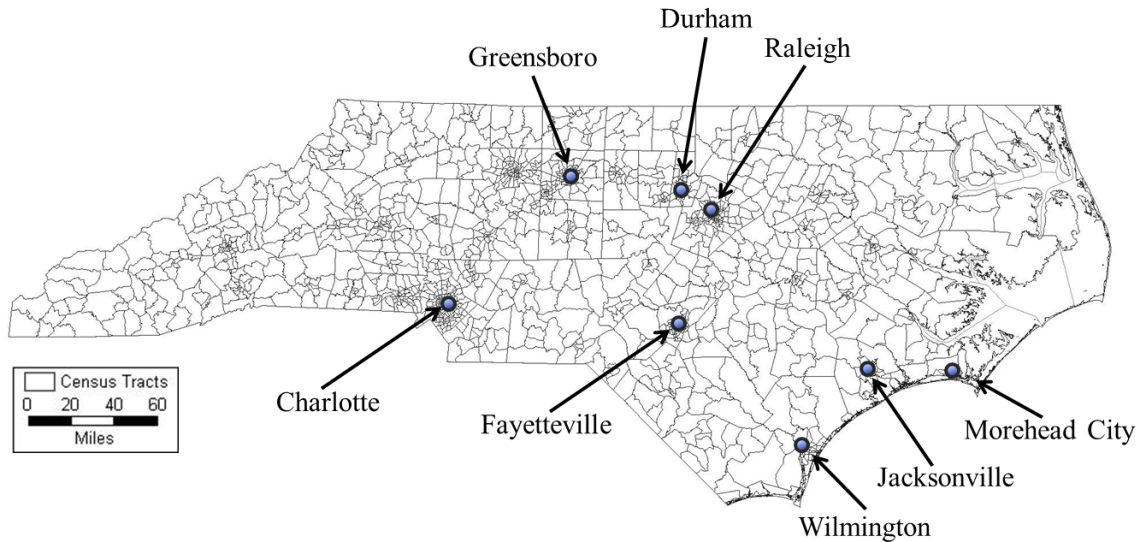
Figure 1.3. Major inland and coastal cities in North Carolina

the city of Wilmington and the locations of the high-population areas and some other coastal cities are shown in Figure 1.3. Figure 1.4 illustrates the traffic flow pattern and the shelters opened under this scenario. Most of the people evacuating originate from the southeastern part of the state, and therefore, northbound I-40 (towards Raleigh/Durham) and US route 74 west toward Charlotte and South Carolina are heavily travelled. In the central coastal area, US route 70 from Morehead City, through New Bern, Kingston and

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Goldsboro and state route 24 from Jacksonville to I-40 are also heavily used. Under this scenario, the assumption that evacuees destined for places other than shelters try to exit the evacuation area as quickly as possible results in significant traffic destined for Raleigh/Durham, the intersections of I-95 and I-40 and the area just south of Fayetteville. It is useful to notice that there are several shelters in the evacuation area that cannot be used, and there are several shelters in the western part of the state that are not needed.

Figure 1.5 shows just the traffic flow generated by evacuees destined for public shelters under hurricane scenario 2. It shows there is relatively little of this traffic on I-40 and many of the already congested roads shown in Figure 1.4. About 10% of the evacuating population in North Carolina uses a public shelter. The results show that the model attempts to place these shelters such that evacuees destined for shelters can take the routes that are lightly used by those evacuees destined for places other than shelters. Figure 1.6 depicts, in the initial solution (solution from the first iteration of the model), the traffic flow destined for public shelters and the shelters opened under scenario 2. Comparing the model results of the initial solution (Figure 1.6) and the optimized solution (Figure 1.5) reveals that by taking into account network congestion, evacuees destined for shelters can save about 14.2% in travel time (Column 6 and Scenario 2 in Table 1.1). Much of this benefit stems from two sources: the use of state route 24 is significantly lessened and some evacuees are shifted to the shelters in the Durham area.
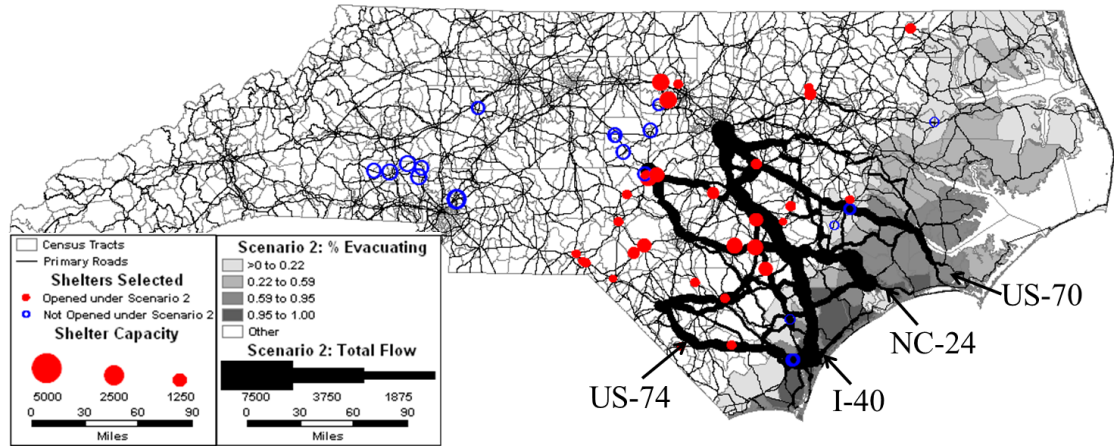
Figure 1.4. Total vehicular flow (flow to shelters and flow to other places) in Scenario 2 based on optimized solution
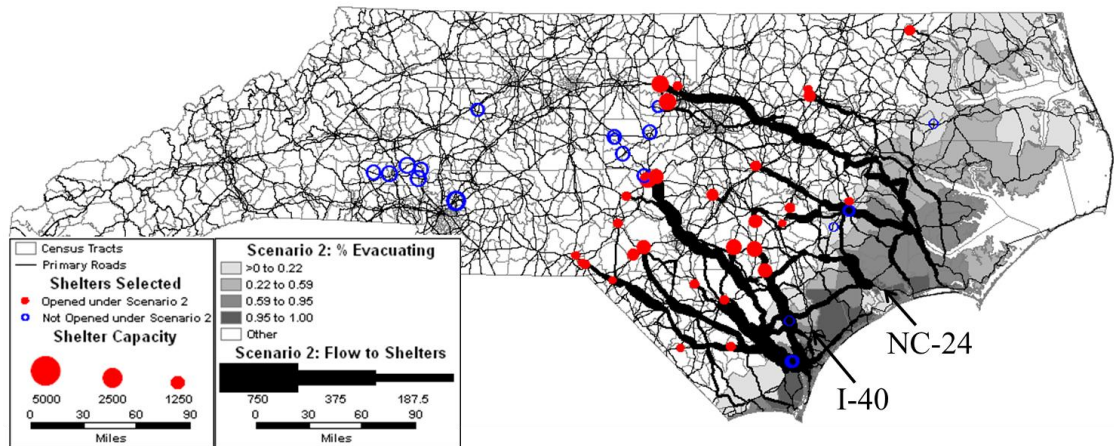
Figure 1.5. Vehicular flow to shelters in Scenario 2 based on optimized solution
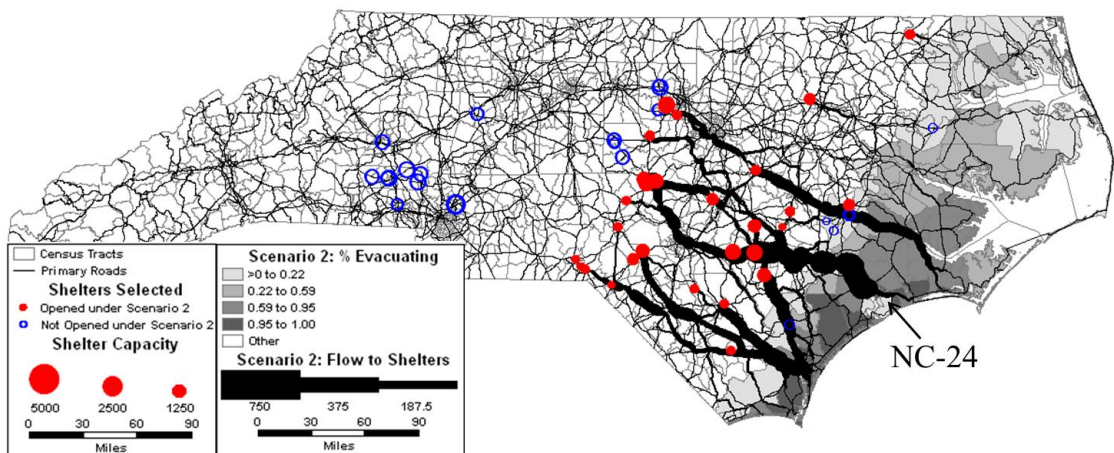
Figure 1.6. Vehicular flow to shelters in Scenario 2 based on initial solution

As mentioned earlier, we also experimented with fixed destinations for those evacuating to places other than public shelters. We assume that the population from the Outer Banks would evacuate into Virginia and that the rest of evacuees traveling to places other than shelters would be distributed between Raleigh, Durham, Greensboro, Fayetteville and Charlotte based on their relative populations and if these destinations are outside the evacuation area. Under this assumption, we compare in Table 1.1 the model results of the average travel times when the model selects shelter locations based on free-flow condition to the average travel times when the model incorporates the traffic congestion in selecting shelter locations. The travel time reduction across all trips and all scenarios is about 7.2% on average; however, there are several individual scenarios for which the travel time to shelters has been reduced by 20% or more, such as scenarios 1 to 4, and 10. Additionally, for some scenarios, there are travel time savings realized by those going to places other than shelters. For instance, Scenario 1 and Scenario 4 have benefits that exceed 10%. In both Scenario 1 and Scenario 4, in the initial solution where traffic congestion is ignored, we see significantly more evacuees housed near Charlotte than in the optimized solution where congestion is considered. This creates significant congestion which can be alleviated by shifting evacuees to the shelters near Durham and to the west of Raleigh.

Table 1.1. Average travel time to shelters for each hurricane scenario

Scenario	Evacuation Demand (persons)	Shelter Demand (persons)	Average Travel Time to Shelters (hour/person)					
			Assumption 1			Assumption 2		
			Initial Iteration	Optimized Iteration	% Reduction	Initial Iteration	Optimized Iteration	% Reduction
1	566,530	62,550	4.109	3.411	20.5%	10.189	3.164	222.0%
2	411,860	44,260	2.844	2.491	14.2%	3.275	2.46	33.1%
3	323,110	35,537	2.693	2.571	4.7%	3.329	2.696	23.5%
4	325,360	34,154	2.177	2.060	5.6%	4.894	2.294	113.3%
5	298,420	33,189	2.148	1.793	19.8%	2.147	1.802	19.1%
6	236,920	26,036	2.146	1.894	13.3%	2.219	1.855	19.6%
7	228,150	25,894	2.223	2.016	10.3%	2.198	1.96	12.1%
8	190,440	20,518	1.786	1.704	4.8%	1.764	1.654	6.7%
9	171,730	17,576	1.141	1.128	1.2%	1.147	1.143	0.3%
10	155,220	17,160	2.160	2.096	3.1%	2.432	1.97	23.5%
11	148,220	14,652	0.762	0.760	0.2%	0.741	0.719	3.1%
12	122,570	12,954	1.488	1.495	-0.5%	1.775	1.494	18.8%
13	111,840	10,204	0.993	0.994	-0.1%	1.062	1.051	1.0%
14	87,790	9,632	1.660	1.547	7.3%	1.666	1.57	6.1%
15	87,314	9,466	1.194	1.195	-0.1%	1.265	1.234	2.5%
16	101,940	9,093	0.896	0.901	-0.5%	0.936	0.959	-2.4%
17	91,369	8,935	0.972	0.973	-0.1%	0.969	0.86	12.7%
18	70,817	7,801	1.024	1.014	1.0%	1.01	1.002	0.8%
19	66,467	7,115	1.076	0.701	53.4%	1.081	0.852	26.9%
20	66,732	7,037	1.274	1.274	0.0%	1.3	1.285	1.2%
21	44,137	4,677	1.236	1.229	0.5%	1.235	1.228	0.6%
22	35,247	3,927	1.214	1.195	1.5%	1.214	1.196	1.5%
23	33,266	3,612	1.014	1.012	0.2%	1.014	1.012	0.2%
24	36,178	3,498	1.910	1.910	0.0%	1.906	1.906	0.0%
25	31,147	3,305	0.987	0.861	14.7%	0.988	0.861	14.8%
26	25,242	2,898	1.615	1.364	18.4%	1.615	1.351	19.5%
27	24,894	2,605	1.358	1.358	0.0%	1.358	1.358	0.0%
28	21,395	2,417	0.805	0.805	0.0%	0.805	0.805	0.0%
29	20,258	2,192	1.205	1.219	-1.1%	1.205	1.195	0.8%
30	20,180	2,111	1.482	1.483	-0.1%	1.482	1.482	0.0%
31	18,036	1,987	1.047	1.047	0.0%	1.047	1.047	0.0%
32	10,055	1,182	0.990	0.990	0.0%	0.99	0.99	0.0%
33	8,797	1,018	1.325	1.325	0.0%	1.325	1.325	0.0%

Note: Assumption 1 – the evacuees not using shelters try to leave the evacuation area as quickly as possible; Assumption 2 - the OD trip matrices for the evacuees not using shelters are given.

1.6. Conclusions and Future Research

This paper has proposed a stochastic bilevel optimization model for shelter location analysis under uncertainty. The model explicitly considers the transportation impacts of location decisions on all the evacuees and exploits the interdependence between the shelter locations and evacuees' route choice. The route choice behavior is formulated as a SUE in each hurricane scenario. To illustrate the applicability of the model to large scale problems, we have focused on North Carolina and a collection of realistic hurricane events.

This paper makes several important contributions. This is the first paper that uses a scenario representation for the hazard when optimizing the location of emergency shelters. It is also the first paper that effectively combines bilevel optimization and stochastic programming to address large-scale shelter location and transportation planning issues. Finally, the case study illustrates that significant benefits can be gained by jointly optimizing transportation and sheltering strategies. In the case study, when we assume those not headed to a shelter simply wish to leave the evacuation zone as quickly as possible the reduction in travel times to the shelter can be as high as 20% and average about 9.5% across all scenarios. When we estimate destinations for those not traveling to a shelter, the reductions in travel times are significantly greater.

There are opportunities for future research in several related areas. First, this model assumes the capacities of each highway facility are known. This implies that if contraflow is to be used, the plan is known. Based on the results in this paper, it is also important to optimize those decisions while optimizing the shelter location decisions. Since contraflow can be confusing for the public, and many different kinds of events are

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possible, an extension to the existing model structure would allow for the identification of the best "compromised" contraflow plan across all events. Second, this model focuses on the evacuating population using private vehicles. However, this is not possible for those who have no means of private transportation. For areas that have large low mobility populations such as New Orleans, it is important to extend the transportation model to consider this portion of the population and appropriate vehicles given their needs. Finally, this model has focused on a static transportation analysis so it cannot address questions of how to stage an evacuation over time. There is the opportunity to use the ideas from dynamic traffic assignment to develop a model with a deeper representation of how the trips occur over time. This is particularly important in modeling the evacuation of institutionalized populations. For example, people in hospitals often require significantly more time to get prepared to evacuate.

Acknowledgements

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CHAPTER 2

A COMPUTATIONALLY EFFICIENT ALGORITHM FOR DYNAMIC

TRAFFIC ASSIGNMENT

Abstract

This paper proposes a computationally effective dynamic traffic assignment algorithm for the analysis of traffic conditions in large-scale road networks over several days. The time-dependent origin-destination trips are assumed to be known and travelers' route choice behavior is described through a dynamic version of user equilibrium. A case study for evacuation of the New Orleans metropolitan area prior to the landfall of Hurricane Katrina is presented. The model results are compared to the traffic counts collected during the evacuation and also a simulation model developed in TRANSIMS. The study shows the traffic pattern produced by the proposed model is a realistic representation of the real world traffic.

2.1. Introduction

Dynamic traffic assignment (DTA) problems have attracted significant attention in the last decade in the areas of transportation systems, telecommunications and computer science as well as emergency planning. A dynamic traffic system can provide real-time traffic information to guide or assist pretrip and en route travel decisions. A real-time

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system utilizes data from traffic surveillance systems and incident detection systems to provide information for traveler information systems and real-time route guidance systems under the intelligent transportation system framework. As technology advances, it becomes more and more important to have a dynamic traffic model capable of addressing large-scale networks to utilize the wealth of real time data which is increasing available.

The DTA models for emergency evacuations have evolved since the Three-Mile Island nuclear incident in 1979. Hurricane evacuations typically cover wide geographic areas and require evacuations of tens to hundreds of thousands of people over a couple of days. They can be costly and result in loss of life. For example, Hurricane Rita in 2005 led to the evacuation of about 2.7 million people, making it one of the largest evacuations in US history. Seven deaths were directly linked to hurricane conditions but at least 55 additional fatalities were linked to either accidents that occurred during the evacuation or the conditions evacuees experienced during the evacuation (Knabb et al., 2006). The massive movement of people often leads to highly congested traffic conditions over at least some portion of the traffic network for part of the duration of evacuation. Therefore, understanding how traffic conditions evolve over the course of an evacuation is important so that mitigation and appropriate protective actions can be developed. A DTA model can keep track of the vehicles both temporally and spatially and provides dynamic information on the traffic conditions for each road in the network over time.

For a realistic application, it is important for a DTA model to handle a large number of origin-destination (OD) pairs on a large-scale highway network. However, the research efforts to date have focused primarily on developing DTA models for small (on the order

of part of a city) and idealized networks with a relatively small number of OD pairs for peak-period analysis (on the order of 2 to 4 hours). The goal of this paper is to develop a DTA model that is capable of dealing with large networks and a large number of origins and destinations with minimal computation time while producing reasonable results.

The contributions of this research are threefold. First, a computationally efficient dynamic traffic assignment algorithm is developed for large-scale regional networks with planning horizons on the order of several days. The computational time of this algorithm is sufficiently short to allow the algorithm to be integrated with optimization models for contraflow designs or for investment planning outside of the realm of evacuation. Second, the model is applied to a large-scale traffic network in southeastern Louisiana with nearly 4,000 OD pairs, 600 five-minute time intervals as well as about 11,000 links and 6,000 nodes for a two-day evacuation. Third, the proposed model is validated using the traffic count data collected during the Hurricane Katrina evacuation. This paper is among the few studies to validate a DTA model with real traffic data.

The remainder of this paper is organized as follows. Section 2 gives an overview of past research in DTA, including their limitations. The new DTA algorithm is presented in Section 3. Section 4 describes the case study for the evacuation of New Orleans metropolitan area during Hurricane Katrina evacuation. Section 5 concludes this paper and points to future research opportunities.

2.2. Literature Review

DTA problems have been studied either via a simulation-based or an analytical approach. The analytical approach is generally classified into mathematical

programming, optimal control and variational inequality approaches. Depending on the behavioral assumption of route choice, these approaches may be used for dynamic system optimal assignment (DSO) where the total travel cost in the network is minimized or dynamic user equilibrium assignment (DUE), in which any individual traveler chooses a route that dynamically minimizes his or her travel time along the route to the destination. A comprehensive review of the DTA literature can be found in Peeta and Ziliaskopoulos (2001). In the remainder of this section, each of the approaches and their development for evacuation are briefly reviewed.

Simulation-based models approximate route flows and route travel times through simulation runs. Microscopic simulation describes the traffic at the level of individual vehicles and represents the interaction between vehicles and the details of road infrastructure. Examples of microscopic models are CORSIM and VISSIM. Such models offer the capability to produce detailed data on a large scale, but require high computational efforts for a medium to large sized network. A number of simulation models follow a mesoscopic simulation paradigm where the traffic entities are described at a high level of detail (microscopic), but their behavior and interaction are described at a lower level of detail (macroscopic). Mesoscopic traffic models exit in various forms: (i) individual vehicles are not accounted for and vehicles are divided into "packets", as in CONTRAM (Taylor, 2003); (ii) flow dynamics are determined by simplified dynamics, as in DYNASMART (Jayakrishman et al., 1994), DynusT (Chiu, 2010) and DynaMIT (Ben-Akiva et al., 1998); (iii) TRANSIMS (Smith et al., 1995) uses a cellular automata where the road is discretized into a finite number of cells. Compared to microscopic models, mesoscopic models are less demanding of data but the computational

requirement is still very high for large-scale applications. For this reason, many studies concentrated on small urban networks over relatively short time spans (Jha et al., 2004; Mahut et al., 2004; Sbayti and Mahmassani, 2006). A few studies focused on the evaluation of alternative evacuation strategies (Chiu et al., 2008; Lim and Wolshon, 2005; Theodoulou and Wolshon, 2004; Williams et al., 2007). Motivated by the Three-Mile Island nuclear reactor incident in 1979, Sheffi et al. (1982) developed NETVAC for simulating nuclear power plant evacuations. It focused on the evacuation of a population from one specific area to another and hence is of limited use for events covering wide geographic areas such as hurricanes. The evacuation models that followed NETVAC include DYNEV (KLD Associates, 1984), MASSVAC (Hobeika and Jamei, 1985; Hobeika and Kim, 1998) and OREMS (Franzese and Sorensen, 2004; Rathi and Solanki, 1993). These evacuation models are macroscopic and generally used to estimate evacuation clearance time; therefore they provide limited information on traffic dynamics.

Merchant and Nemhauser (1978a, b) represent one of the earliest attempts to model the DTA problem as a mathematical program. The authors formulated a DSO route choice with a single destination as a discrete-time nonlinear nonconvex mathematical program and presented a decomposition algorithm for a piecewise linear version of the model. Janson (1991) proposed a formulation for DUE and a heuristic algorithm, for which two key assumptions were made. First, route choice decisions were made at the time of trip departure based on projected link impedances over future time intervals. Second, a trip could not use the same link for more than one time interval and hence the model performance largely depends on the selection of the time interval length. This

model was later incorporated by Southworth et al. (1992) into a combined destination and route choice evacuation model, named DYMOD, where departure time was assumed to follow a logistic traffic loading curve. Janson and Robles (1995) proposed a quasi-continuous time DTA model with scheduled departure times, which allowed trips to be split among successive time intervals to maintain continuity in time. Carey (2000) utilized a time-expanded network to develop a linear programming model for DSO. Daganzo (1994, 1995) introduced a cell transmission model (CTM) for traffic flows based on hydrodynamic theory. Subsequently, a group of studies emerged on the further development of CTM-based traffic models for DSO assignment (Chiu et al., 2007; Ziliaskopoulos, 2000; Yazici and Ozbay, 2010) and DUE assignment (Waller and Ziliaskopoulos, 2006). These CTM-based models do not rely on link performance functions and are capable of capturing shockwaves and queue formation. However, they rely on a dense space and time discretization, requiring considerable computing resources. Yperman (2007) and Yperman et al. (2007) proposed a link transmission model for dynamic network loading that was consistent with Newell's simplified theory of kinetic waves (Newell, 1993). The model assumed vehicles were uniformly spread over each link at each time instant and required the time step to be smaller than the smallest link travel time so that vehicles could not traverse a link within one time interval.

Friesz et al. (1989) analyzed some of the fundamental properties of the continuous-time optimal control formulations for single-destination DTA problems. This work was later extended by Wie et al. (1990) to multiple-destination networks and Wie (1991) to include elastic travel demand with departure time choice. A similar optimal control

model to Friesz et al. (1989) was given in Ran et al. (1993) by using a different definition of DUE and defining the outflow as another set of control variables in addition to inflow. No efficient algorithms for general networks are available for these types of formulations. The proposed algorithm by Boyce et al. (1995) involved diagonalization, which is computational challenging for networks of realistic sizes over a reasonable number of time periods.

Friesz et al. (1993) showed the existence of a variational inequality (VI) formulation for simultaneous departure time and route choice problem in continuous time. Ran and Boyce (1996b) introduced a VI formulation for a DUE problem where the optimality condition was defined for the actual travel times experienced by travelers instead of the instantaneous travel times at a given instant. This model was extended by Ran et al. (1996) for combined departure time and route choice. The necessity and sufficiency proofs of the VI model were given, but no solution algorithm or computational experience was reported. Chen and Hsueh (1998) proposed a VI formulation and showed that travel time on a link can be represented as a function of link inflow. This approach requires three levels of iterative loops and appears to be prohibitively expensive for implementing on medium- to large-size networks. Lo and Szeto (2002) developed a cell-based VI formulation and modeled the traffic dynamics as a unique mapping of route travel costs given route flows. This formulation encapsulated the Daganzo's CTM as the underlying dynamic traffic model to capture realistic traffic dynamics. Han (2003) investigated solution algorithms to a VI formulation for logit-based dynamic stochastic user equilibrium over multiple origin-destination pairs. The author explored three ways of

solving that problem for which a globally optimal solution is rather difficult to obtain for even a modest-size network (24 nodes and 76 links).

Most existing DTA studies concentrate on small to medium sized networks over relative short time spans. Mahut et al. (2004) applied a DTA to a part of the road network of the city of Calgary in Alberta, Canada (734 links, 314 nodes, 77 zones). The network used by Sbayti and Mahmassani (2006) in testing DYNASMART was extracted from the Fort Worth, Texas, network and consisted of 168 nodes, 441 links and 13 zones. A small idealized network (3 origins, 1 destination, 8 nodes and 9 links) with 10 time intervals was used in Chiu et al. (2007). Lo and Szeto (2002) implemented their cell-based VI model on Nguyen and Dupius (1984)'s network (13 nodes, 19 links and 4 OD pairs) with 10 time intervals.

In this paper, a DUE traffic assignment algorithm is developed for a modified version of the formulation given in Janson (1991) that can be applied to large metropolitan sized networks with many origins and destinations for time periods on the order of a couple of days or more. The model is tested on the southeastern Louisiana road network to reproduce the evacuation traffic patterns for the Hurricane Katrina evacuation. The problem scale contains nearly 4000 OD pairs, 600 five-minute time intervals as well as about 11,000 links and 6,000 nodes for a two-day evacuation. The proposed DUE model is calibrated and validated against the observed traffic counts collected during Hurricane Katrina evacuation. Statistical techniques are employed to evaluate the model performance and compared to the observed traffic counts. The model outputs are also compared to an accepted commercially available system, TRANSIMS, to measure the model performance.

2.3. Model Formulation and DTA Procedure

In this section, the formulation developed by Janson (1991) is presented. Then a new solution procedure is described to identify an approximate DUE solution to a slightly modified version of that formulation.

Input data

N	set of all nodes
Z	set of all zones (i.e., origins and destinations)
A	set of all links (directed links)
$\mathbf{A}_n$	set of all links incident from node n
P	set of all paths between all OD pairs
$\mathbf{P}_{rs}$	set of all paths from origin r to destination s
$\mathbf{K}_p$	set of all links on path p
$\mathbf{K}_{pn}$	set of all links on path p prior to node n
Δt	length of the time interval
D	set of all time intervals in the entire analysis horizon
q_{rs}^d	number of trips that leave origin r in time interval d for destination s

Decision variables

h_p^d	number of trips assigned to path p that departed in time interval d
$x_k(t)$	number of trips between all OD pairs assigned to link k in interval t
$\alpha_{pk}^d(t)$	0-1 variable, 1 if the trips departing in time interval d assigned to path p use link k in time interval t, and 0 otherwise.

b_{pn}^d travel time of path p from its origin to node n for trips departing in time interval d

$f_k^t(x_k(t))$ link travel time on link k in time interval t

It is assumed that the time-dependent OD trips are known and the travelers' route choice behavior can be described by DUE. The assignment problem is to determine the dynamic link flows $\mathbf{x} = \{x_k(t)\}$ when all OD trips by departure time $\mathbf{q} = \{q_{rs}^d\}$ are assigned to the network. The formulation developed in Janson (1991) is given below in Equation (2.1) through Equation (2.9).

$$\text{Minimize} \quad \sum_{k \in \mathbf{A}} \sum_{t \in \mathbf{D}} \int_0^{x_k(t)} f_k^t(\omega) d\omega \tag{2.1}$$

Subject to:

$$q_{rs}^d = \sum_{p \in \mathbf{P}_{rs}} h_p^d \quad \text{for all } r \in \mathbf{Z}, s \in \mathbf{Z}, d \in \mathbf{D} \tag{2.2}$$

$$h_p^d \geq 0 \quad \text{for all } p \in \mathbf{P}, d \in \mathbf{D} \tag{2.3}$$

$$x_k(t) = \sum_{p \in \mathbf{P}} \sum_{d \in \mathbf{D}} h_p^d \alpha_{pk}^d(t) \quad \text{for all } k \in \mathbf{A}, t \in \mathbf{D} \tag{2.4}$$

$$\alpha_{pk}^d(t) = \{0,1\} \quad \text{for all } p \in \mathbf{P}, k \in \mathbf{K}_p, d \in \mathbf{D}, t \in \mathbf{D} \tag{2.5}$$

$$\sum_{t \in \mathbf{D}} \alpha_{pk}^d(t) = 1 \quad \text{for all } p \in \mathbf{P}, k \in \mathbf{K}_p, d \in \mathbf{D} \tag{2.6}$$

$$b_{pn}^d = \sum_{t \in \mathbf{D}} \sum_{k \in \mathbf{K}_{pn}} f_k^t(x_k(t)) \alpha_{pk}^d(t) \quad \text{for all } p \in \mathbf{P}, n \in \mathbf{N}, d \in \mathbf{D} \tag{2.7}$$

$$\left[b_{pn}^d - (t-d+1)\Delta t \right] \alpha_{pk}^d(t) \leq 0 \quad \text{for all } p \in \mathbf{P}, n \in \mathbf{N}, k \in \mathbf{A}_n, d \in \mathbf{D}, t \in \mathbf{D} \tag{2.8}$$

$$\left[b_{pn}^d - (t-d)\Delta t \right] \alpha_{pk}^d(t) \geq 0 \quad \text{for all } p \in \mathbf{P}, n \in \mathbf{N}, k \in \mathbf{A}_n, d \in \mathbf{D}, t \in \mathbf{D} \tag{2.9}$$

The objective function (2.1) is the sum of the integrals of the link travel time functions over all the links in the network for the entire planning horizon. The objective function is a temporal generalization of the Beckmann's transformation for static user equilibrium by adding time superscripts. Equation (2.2) conserves the flow over all the paths for each OD pair and departure time interval. Equation (2.3) guarantees all path flows to be non-negative. Equation (2.4) defines the total number of trips on link k in time interval t as the sum of trips that use link k in time interval t for all the travel paths and departure times. The indicator $\alpha_{pk}^{d}(t)$ is a binary variable and take value of one if the trips departing in d assigned to path p (which includes like k) use link k in interval t, and zero otherwise, as defined in Equation (2.5). Equation (2.6) states that the trips that depart in d assigned to path p use each link on the path in exactly one time interval. Equation (2.7) gives the travel time to node n for the trips departing in d assigned to path p by summing the link travel times along the links prior to node n used by these trips. Equations (2.8) and (2.9) identify the time interval index that corresponds to the path travel time b_{pn}^{d}. The optimal solution of the formulation given by Equations (2.1)-(2.9) characterizes the traffic flow assigned to each link in each time interval. It is noteworthy that the indicator variable $\alpha_{pk}^{d}(t)$ that appears in Equations from (2.4) to (2.7) is a 0-1 integer variable. Hence, the formulated problem is nonlinear and nonconvex.

It is important to notice that this formulation through Equation (2.6) prohibits a trip from using the same link for more than one time interval. This requires the link travel times to be short (even during congested periods) relative to the length of the time interval. This constraint is relaxed in the proposed solution algorithm to allow a trip to

travel on a link through as many intervals as needed to fully traverse the link. Equation (2.7) requires that valid departure times coincide with the beginning of time intervals. In the proposed algorithm, the paths for trips in transit are updated at each time interval. The travel time incurred by a trip in a given time interval is computed as the weighted sum of the travel times of the links used by this trip, where the weight for each link is the proportion of the link traversed by this trip in the given time interval.

Two key assumptions are made in the proposed solution algorithm: (i) trips from the same origin to the same destination departing in the same time interval travel together as a group or platoon. The travelers in each platoon experience the same driving conditions and share the same travel route through the network; and (ii) the path first-in-first-out (FIFO) condition which prohibits trips that depart later from arriving at a destination earlier is not considered. Link FIFO is naturally satisfied based on (i).

Since all trips from each origin to each destination that depart at the same time are assigned to a platoon, it is the platoon that is routed through the network in this DTA procedure. The procedure sequentially moves through time assigning trips that depart in each time interval to their shortest paths and updating the path for each platoon that is en route. The procedure terminates when all platoons have reached their destinations. To support these computations, the movements of all the platoons on the network are tracked between successive time intervals. Two types of spatial attributes are used to identify the location of each platoon in each time interval. The first attribute identifies the link the platoon is currently on. The second attribute gives the distance from the current location on the link to the tail node of the link relative to the total distance of this link.

For each time interval, this method involves three steps: (i) dynamically calculate shortest paths, (ii) track each platoon's location, and (iii) update link travel times. The platoons that depart in the current time interval are routed using the current shortest path. The path for each platoon that is en route is revised as needed given evolving conditions. By tracking each platoon's location in the network in time, the number of trips entering each link (inflow) and exiting each link (outflow) are identified for each time interval. Once a platoon reaches its destination, the total travel time experienced by that platoon is given by the sum of the travel times the platoon spends on the links from the time it departs to the time it arrives at its destination.

For notational convenience, the trips that leave origin r in time interval d for destination s are referred to as "rsd trips". The decision variables used in the proposed algorithm are listed below.

$u_k^{rsd}(t)$ number of rsd trips entering link k in interval t

$v_k^{rsd}(t)$ number of rsd trips exiting link k in interval t

$u_k(t)$ number of trips between all OD pairs entering link k in interval t (inflow)

$v_k(t)$ number of trips between all OD pairs exiting link k in interval t (outflow)

$x_k(t)$ number of trips between all OD pairs assigned to link k in interval t (average flow)

$c_k(t)$ average travel time on link k in time interval t

c_k^0 free-flow travel time on link k

The step-by step procedure of the proposed solution algorithm is described as follows.

Step 0: Initialization.

Set the time interval $t = 0$; initialize link travel time $c_k(t)$ to c_k^0 for all $k \in \mathbf{A}$.

Step 1: Increment the current time interval t to $t = t+1$.

Step 2: For each destination node $s \in \mathbf{Z}$, do:

> **Step 2.1: Dynamic shortest-path calculation.** Based on the link travel time
>
> $c_k(t-1)$ for all $k \in \mathbf{A}$, compute the minimal travel time from each node in the
>
> network to destination s. For computational efficiency, the shortest path
>
> computation terminates when the shortest path to destination s is found for all
>
> nodes that are either origins for new traffic departing in the current interval t or
>
> the tail nodes of the links with non-empty traffic. Go to step 2.2.
>
> **Step 2.2: All-Or-Nothing assignment**. For each origin $r \in \mathbf{Z}$, assign the trips
>
> that depart in the current time interval t to their shortest paths and update the
>
> paths for the trips that are en route based on the shortest path computations
>
> performed in Step 2.1. This information is used to update the location of each
>
> platoon at end of the current time interval (identified by the link spatial attributes
>
> described earlier). The inflow and outflow for each link $k \in \mathbf{A}$ are calculated
>
> through tracking as the platoons pass the initial node and/or tail node of this link.
>
> Store the inflow and outflow for each link k in the current time interval t to
>
> $u_k^{rsd}(t)$ and $v_k^{rsd}(t)$, respectively. If the locations of all the platoons for
>
> destination s are updated, go to Step 3. Otherwise, go to Step 2.1 and select the
>
> next destination node.

Step 3: Aggregate the total inflows, outflows and average flows on all links.

$$u_k(t) = \sum_{r \in \mathbf{Z}} \sum_{s \in \mathbf{Z}} \sum_{d \in \mathbf{D}} u_k^{rsd}(t) \quad \text{for all } k \in \mathbf{A} \tag{2.10}$$

$$v_k(t) = \sum_{r \in \mathbf{Z}} \sum_{s \in \mathbf{Z}} \sum_{d \in \mathbf{D}} v_k^{rsd}(t) \quad \text{for all } k \in \mathbf{A} \tag{2.11}$$

Step 4: Calculate average flows on all links.

The average flow on a link is considered to be the average of the inflow and outflow for

this link, i.e. $x_k(t) = 0.5\left[u_k(t) + v_k(t)\right]$ for all $k \in \mathbf{A}$.

Step 5: Update link travel times.

After all the trips are assigned for the current interval t, update the link travel time $c_k(t)$

based on the average link flow $x_k(t)$ for all $k \in \mathbf{A}$. If all the time intervals in the analysis

horizon have been processed, stop; otherwise, go to Step 1 for the next time interval.

This procedure steps through time until all trips reach their destinations or all the time

intervals in the planning horizon are processed. The solution procedure is conceptually

straightforward, involving dynamic shortest-path calculations and traffic assignments at

successive time steps. The idea of the procedure is to trace trips across the network in

both spatial and temporal domains. The procedure described above involves two main

functions: finding the shortest path from the current platoon locations to their

destinations, as in step 2.1, and tracking each platoon and advancing the platoon to their

new locations based on its current shortest path, as in step 2.2.

The execution time of the algorithm is driven by the number of platoons (governed by

the number of origins, destinations and departure time intervals) and the number of links

and nodes in the network. Each time step requires only one shortest path tree for every

destination. The complexity of the heuristic is proportional to the number of destinations

but grows quadratically with the number of time intervals, and is bounded by $|Z|^2|D|^2$. Based on the model assumptions mentioned earlier, more accurate results are expected if a smaller time interval is used. However, as the time discretization gets denser, the computational burden increases.

2.4. Case Study: Katrina Evacuation in the New Orleans Metropolitan Area

Hurricane Katrina formed over the Bahamas on August 23, 2005 and crossed southern Florida as a moderate Category 1 hurricane. It then strengthened rapidly in the Gulf of Mexico and made landfall in southeastern Louisiana at 6:10 AM on August 29, 2005 as a Category 3 hurricane with sustained winds of 125 miles per hour (205 kilometers per hour). Since much of New Orleans city is below sea level, the city relies on a system of levees for flood protection from the Mississippi River, Lake Pontchartrain and Lake Borgne.

2.4.1. Highway Network and Contraflow Operations

The road network in this study includes Interstate highways, US routes, state routes, and some local streets in southeastern Louisiana (Figure 2.1). The geographic extent of the network is Baton Rouge in the west, Hammond in the north, Slidell in the east, and Paradis in the south. These four cities in the four directions are either evacuation destinations or key pass-through points for the majority of evacuees. Each network link is

associated with its length, number of lanes, posted speed, and functional classes. The network consists of 11,061 directed links and 5,594 nodes.

During the Katrina evacuation, a contraflow plan was implemented to support the evacuation of New Orleans. The goal of the contraflow plan was to quickly move people out of the low-lying area toward the west and the north. This plan operated from about 4:00 PM on August 27th to 5:00 PM on August 28th. As illustrated in Figure 2.1, the lanes on portions of I-10, I-59 and I-55 leading into New Orleans were reversed during contraflow operations. Evacuees from the New Orleans region traveling to the west entered the I-10 contraflow lanes from Williams Boulevard, Veterans Boulevard, Clearview Parkway, and I-10 westbound via a crossover. At I-10/I-55 Interchange, this contraflow traffic was routed back into the normal westbound lanes of I-10 toward Baton Rouge. The evacuees toward the east were diverted onto I-59 contraflow lanes at I-10/I-59 interchange. The traffic on I-55 northbound was routed onto I-55 contraflow lanes at I-12/I-55 interchange. To consider the contraflow in the dynamic traffic model, the input data are required to specify the links being reversed and the ramps being restricted or reversed during contraflow operations.

During the two-day evacuation (from August 27th to August 28th), hourly traffic volumes were recorded at six traffic count stations situated on the major outbound evacuation routes from New Orleans (Figure 2.1). These stations consist of five Louisiana Department of Transportation and Development (LA DOTD) stations (Stations 54, 27, 42, 15 and 88) and one Mississippi Department of Transportation (MDOT) station on I-59 northbound. Station 54 is on I-10 in LaPlace immediately after the I-10 contraflow termination. Station 27 is on US-61 in LaPlace parallel to I-10. MDOT

counter is located just over border on I-59. Station 42 and Station 15 are on I-55 about 20 miles from the Mississippi border. Station 42 on the I-55 contraflow lanes measures the contraflow traffic during contraflow operations. Station 88 is located on US-90 the southwest bound route out of the metro area.
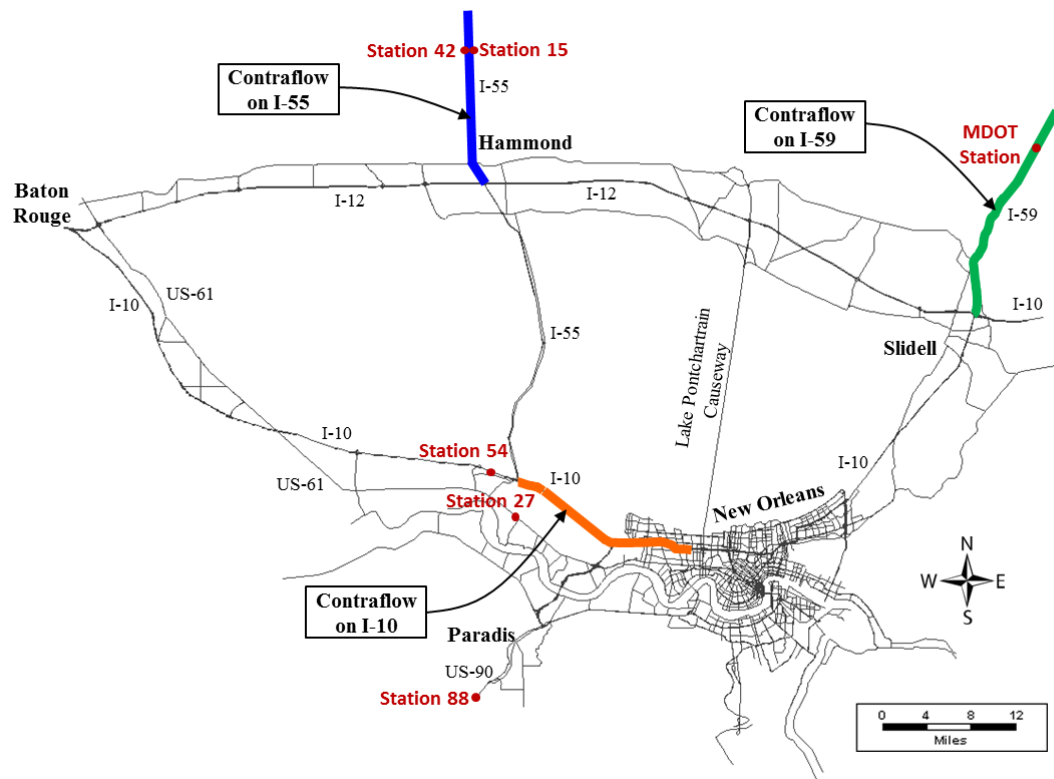
Figure 2.1. Contraflow sections and traffic count stations on southeastern Louisiana road network

2.4.2. Time-Dependent Traffic Demand

The proposed DTA algorithm requires a time-dependent OD trip table, which is estimated as follows. The origins are the 935 census block groups in the five parishes in the New Orleans metro area—St. Charles, Jefferson, Orleans, St. Bernard and Plaquemines. According to 2000 census data, there are about 415,000 households and 1.08 million people residing in the five parishes. A total of 326,128 vehicles are observed at the six traffic count stations over the 48 hours and this traffic is assumed to stem from

the 935 census block groups. The fraction of the trips that originate from each census block group is assumed to be proportional to its population.

Evacuees from the New Orleans metro area can leave the area via four outbound directions: westbound I-10 toward Baton Rouge, eastbound I-59 to Hattiesburg in Mississippi, northbound I-55 to McComb and Jackson in Mississippi, and southbound US-90 to Lafayette and Houston. Therefore, the four exit points (West, East, North and South) on the perimeter of the evacuation network are used as evacuation destinations. The traffic counts at the traffic count stations suggest that 38%, 15%, 27%, 20% of the total evacuating vehicles are headed west, east, north and south, respectively. Since there is no data to indicate the fraction of evacuees at each origin destined for each of these destinations, the same fraction for each destination is applied to all the origins.

Finally, it is assumed that the departure time curve for each origin is the same as the aggregate departure curve for the region (Figure 2.2). The cumulative departure time curve is obtained by aggregating the hourly traffic counts from all six traffic stations and then calculating the cumulative percentage of evacuating vehicles entering the road network at each hour over the 48-hour evacuation period. The result is a characteristic double S-shaped profile for the two-day evacuation. About 36% of the total evacuating vehicles left the New Orleans metro area on Saturday and the remaining 74% evacuated on Sunday. The departure curve indicates that the significant drop-off in evacuations occurs during the evening hours, followed by increases during the mornings and afternoons for each 24-hour period. This finding is consistent with the behavioral studies in the literature (e.g., Lindell and Prater, 2007). It is important to note that the traffic count data show the hourly traffic volumes that are observed at each station rather than at

each departure point. To account for the travel time to go from a departure point to a traffic station, the departure times are offset by one hour. For example, if 6000 vehicles from New Orleans are observed to pass the six count stations from 6:00 PM to 7:00 PM, these trips are assumed to depart at 5:00 PM to 6:00 PM. In reality, the departure process is likely more complex but there is no basis in data to further refine this approximation.
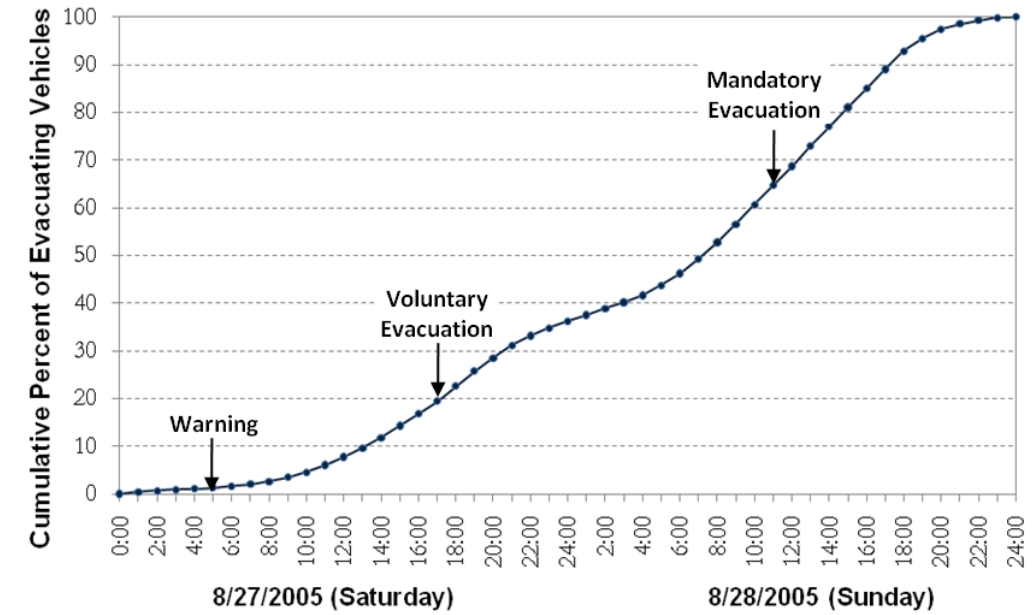
Figure 2.2. Cumulative Katrina evacuation in New Orleans metropolitan area

The size of the time-dependent OD trip table depends on the numbers of origins, destinations, and time intervals for the entire analysis horizon. In this study, an interval of 5 minutes is implemented for the 48-hour evacuation window for a total of 576 departure time intervals. The size of the time-dependent OD table is $935 \times 4 \times 576 \approx 2.2 \times 10^6$.

2.4.3. Volume-Delay Function

This study does not consider queue formation or spillbacks to upstream links and adopts the link performance function developed by US Bureau of Public Roads (BPR) to

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determine the congestion delay over each link at each time interval. For a given link k and time interval t, the link travel time is:

$$c_k(t) = c_k^0 \left[1 + \alpha \left(x_k(t) / Q_k \right)^\beta \right] \qquad (2.12)$$

where Q_k is the practical capacity of link k adjusted to the time interval Δt, c_k^0 is the free-flow travel time on link k. The practical road capacity assumes 1500 vehicles per hour per lane (or 125 vehicles per five-minute time interval and per lane). The values of parameters α and β depend on the link functional class: $\alpha = 0.3$ and $\beta = 4.0$ for freeways and expressways; $\alpha = 0.2$ and $\beta = 4.0$ for arterials and local streets. The BPR function has been known to be able to yield good responses for highways and has been used in various dynamic traffic models (e.g., Smith et al., 1995; Janson, 1991, 1995). It is useful to realize that although a BPR-type formula is adopted in this study, the proposed solution procedure can be applied with any other proper travel delay functions.

2.4.4. Computational Times

At each time interval, the shortest path calculation and traffic assignment for one destination are independent of that for the other destinations. This makes parallel computation attractive. The model is implemented using Java on an 8 core Intel Xeon 2.66 GHz PC running Windows 7 64-bit with 32 GB memory. The implementation for the southeastern Louisiana network (11,061 links and 5,594 nodes) for the two-day Katrina evacuation requires 8.7, 13.2, 28 and 226 seconds for time interval durations of 15, 10, 5 and 1 minutes, respectively. This is approximately three orders of magnitude faster than using TRANSIMS for the same application (Wolshon, 2009). Chiu et al.

(2008) reported that it took DynusT about 20 hours to complete the simulation for the Houston-Galveston area over a 24-hour evacuation.

2.4.5. Model Validation

Figure 2.3 shows the traffic volumes produced by the proposed DTA model, the traffic volumes from a TRANSIMS model developed using the same assumptions, and the hourly traffic counts for each of the six stations over the 48-hour window. Figure 2.3a gives the temporal profiles for traffic volumes at Station 54. It is observed that the traffic volumes given by the proposed model are similar to both the observed data and TRANSIMS results. The traffic volumes resulted from TRANSIMS are generally consistent with the observed data with overestimations during 3:00 PM - 11:00 PM for both days. At Station 27 (Figure 2.3b), between 9:00 PM on the first day and 4:00 AM the next day (i.e., the evening hours between the two peak periods), the observed volume is about 500 vehicles per hour (vph) while the proposed model indicates negligible amount of traffic. This might stem from the omission of the evacuation traffic from St. John Parish or these vehicles might be non-evacuation related traffic (e.g., grocery shopping).

Among the six sets of comparisons (Figures 2.3a-f), the most significant discrepancy occurs at Station 15 (Figure 2.3d) which is located on the normal northbound lanes of I-55. It can be seen that both our model and TRANSIMS predict too little traffic on I-55 northbound during contraflow hours (from 4:00 PM on the first day to 5:00 PM the second day). Before the contraflow operations started, all the traffic from New Orleans using I-55 to travel north continues on I-55 northbound lanes. During contraflow

operations, this traffic is diverted onto I-55 contraflow lanes (Station 42, in Figure 2.3e) at the interchange of I-12 and I-55, resulting in a significant drop of the traffic volumes originated from New Orleans traveling on I-55 northbound near Station 15 (Figure 2.3d).
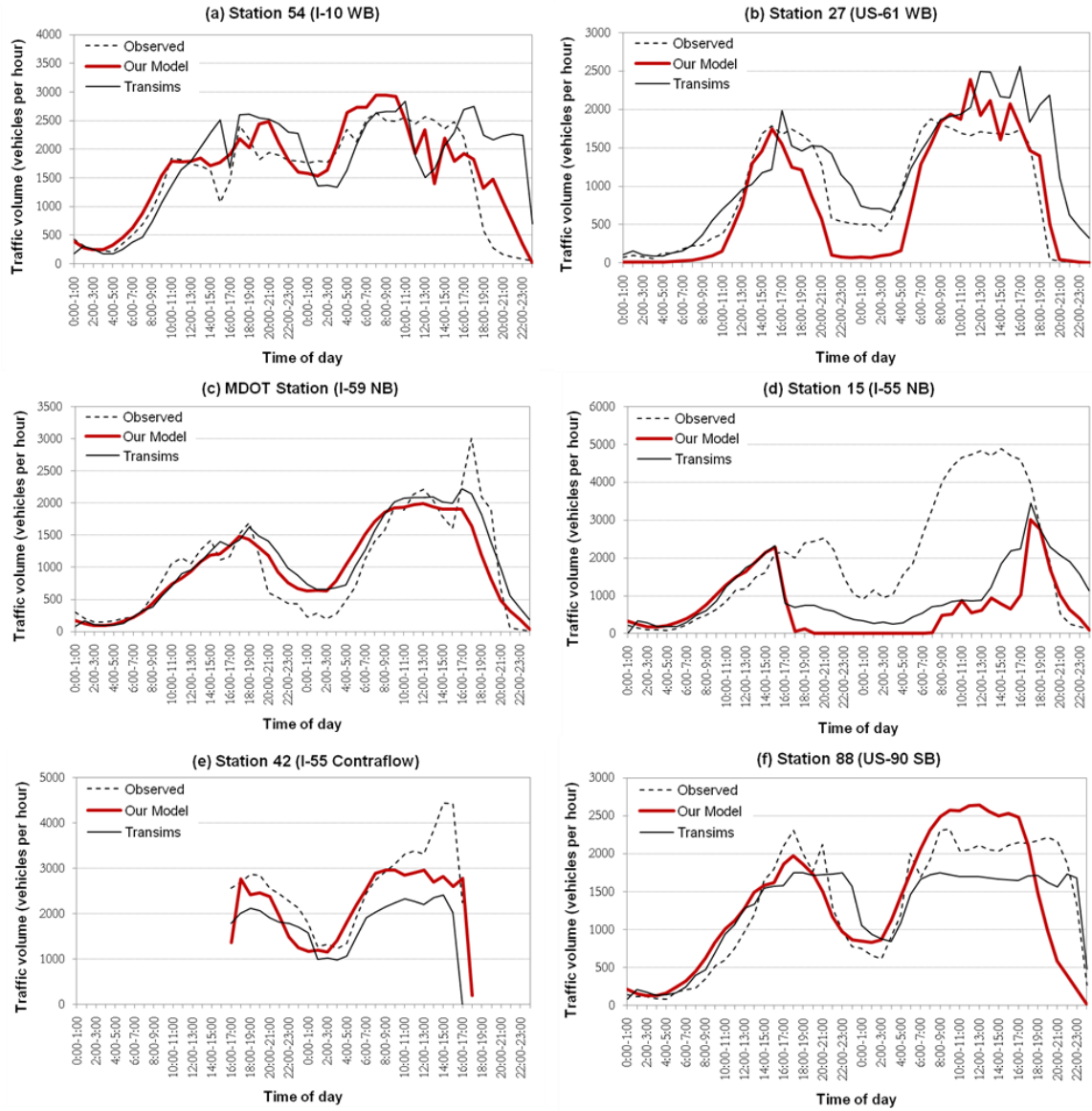
Figure 2.3. Comparison of temporal traffic patterns: (a) Station 54 on I-10 westbound, (b) Station 27 on US-61 westbound, (c) MDOT station on I-59 northbound, (d) Station 15 on I-55 northbound, (e) Station 42 on I-55 Contraflow lanes, and (f) Station 88 on US-90 southbound

However, the observed data indicate a traffic volume of about 2000 vph during the first-day peak hour and about 5000 vph during the second-day peak hour. This discrepancy is likely due to the evacuating vehicles outside the New Orleans metro area such as the neighborhoods north of Lake Pontchartrain, which are reflected in the observed traffic counts but not included in our model and TRANSIMS.

Table 2.1 shows the correlation (Pearson's correlation) for two pairs of data – our DTA model and the observed traffic counts, and TRANSIMS and the observed traffic counts at the six count stations. Except at Station 15, the correlation between our model and the observed data ranges from 0.733 to 0.91. For the MDOT station, Station 15 and Station 88, TRANSIMS performs slightly better than the proposed DTA model. However, the proposed model outperforms TRANSIMS at Station 54 and Station 27. None of these models predicts the traffic pattern perfectly; however, given the limitations in the data, both models perform reasonably well.

Table 2.1. Correlation between traffic volumes of our model, TRANSIMS model and observed traffic counts at six traffic count stations

	Station 54	Station 27	MDOT station	Station 15	Station 42	Station 88
r (Our model, Observed)	0.886	0.910	0.861	0.208	0.733	0.818
r (TRANSIMS, Observed)	0.592	0.791	0.910	0.360	0.739	0.911

2.4.6. Distribution of Errors

Figure 2.4 presents the distribution of errors between the traffic volumes produced by the proposed model and the observed hourly traffic counts. The error is defined as the modeled traffic volume minus the observed traffic volume, in vph. This histogram includes 217 error data points for the 48-hour evacuation at Stations 54, 27, MDOT, 42

(on I-55 contraflow lanes) and 88. The distribution suggests that the errors are approximately normally distributed with an expected value of zero. The majority of the traffic volume data have small and acceptable errors relative to the peak traffic volumes of 2000 to 3000 vph. Among the 217 traffic volume data points, 61% (132 out of 217) have errors within ±300 vph and 88% (190 out of 217) within ±600 vph.
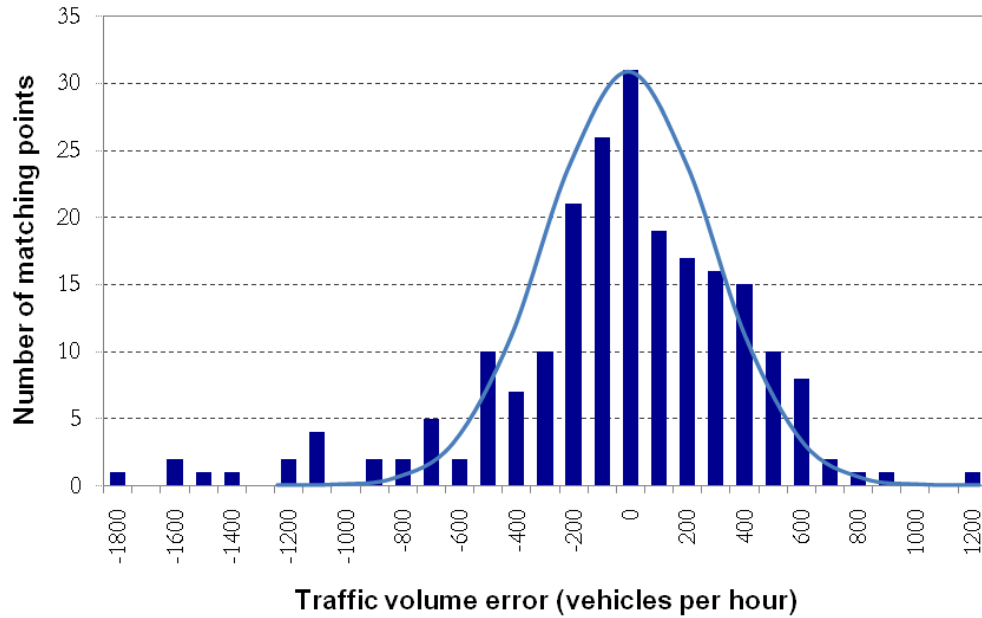
Figure 2.4. Histogram of errors between modeled and observed traffic volumes for all six stations except Station 15

2.4.7. User Equilibrium Analysis

Since the proposed solution procedure is heuristic, it is important to assess how well the resulting traffic patterns satisfy the DUE condition. According to Janson (1991) and Ran and Boyce (1996b), the DUE condition is stated as "*The dynamic traffic flows are in dynamic user equilibrium if for each OD pair at each time interval, the travel times experienced by drivers departing at the same time over used routes are equal and minimal.*" At equilibrium, the drivers have no incentives to switch from their current shortest travel time path.

In the DUE analysis, it is assumed that if vehicles with the same OD pair depart within a ten-minute slice of time, these vehicles should experience approximately the same travel time on the network. To prove this assumption is valid, an interval of $\Delta t = 1$ minute is used to perform this analysis. For all OD pairs and all one-minute departure time intervals, the proposed model outputs 11 million trip travel times. These trip travel times are grouped by ten-minute departure time widow, resulting in 1.1 million data sets (with 10 trip travel times in each set). If the DUE assumption holds, the trip travel times within each data set should be roughly the same. The coefficient of variation (CV) is used to measure the dispersion of the travel time distribution in each travel time data set. A small value of the CV for each data set would confirm the DUE assumption of the model holds.

Figure 2.5 presents the percentage of CV data points with a value less than or equal to the value specified on the horizontal axis. It shows that 80% of the CV data points have a standard deviation less than 1% of the corresponding mean and 88% have a standard deviation less than 3%. This result is consistent with the assumption of DUE. The plot in Figure 2.6 shows the CVs versus the corresponding means for trip travel times. It illustrates that the CV increases with the mean of the travel time. The implication is that longer trips are subject to relatively more variability.
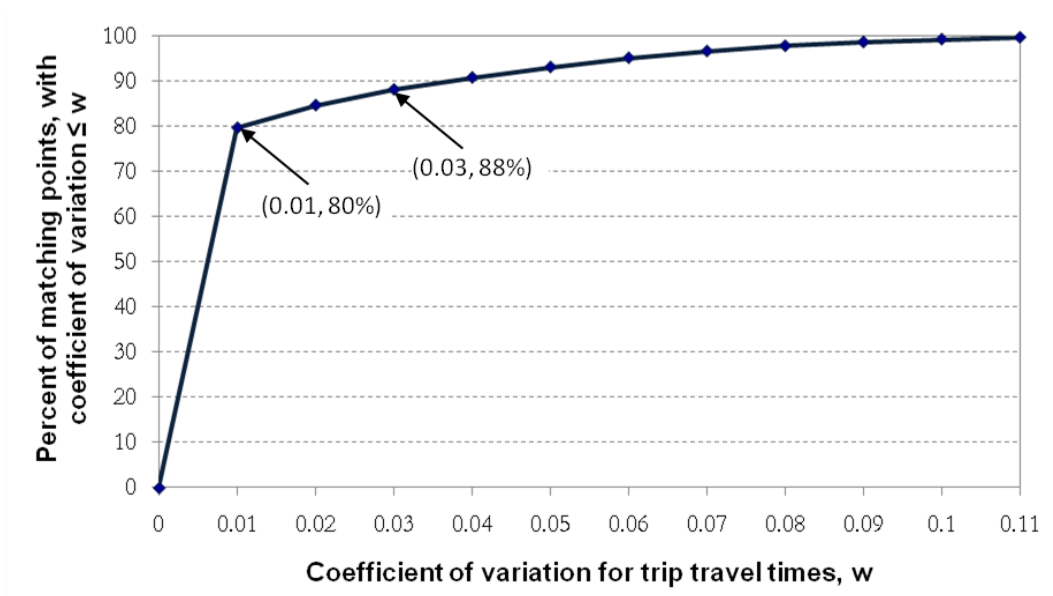
Figure 2.5. Frequency plot of coefficient of variations for the trip travel times
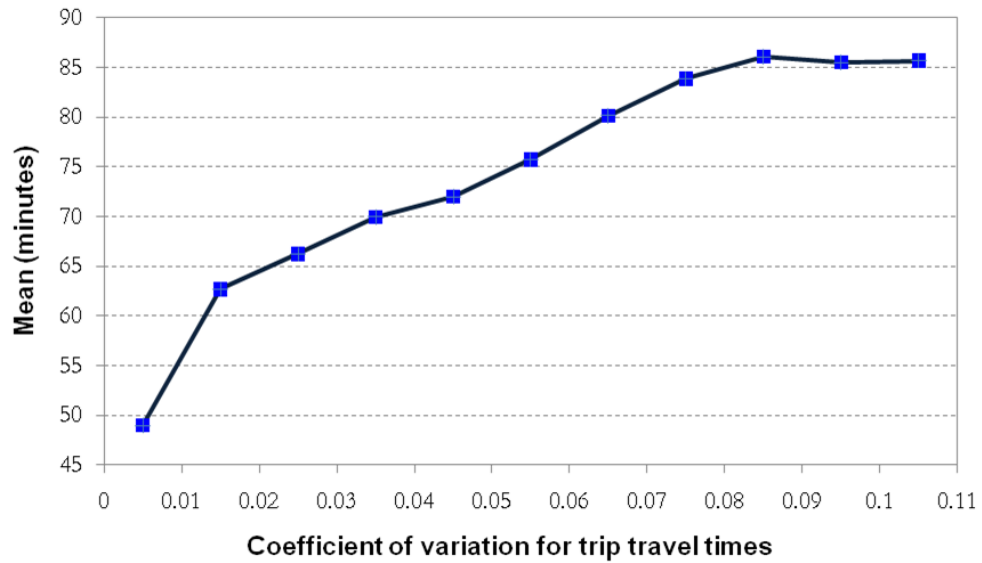
Figure 2.6. Coefficient of variations versus means for the trip travel times

2.5. Conclusions

This paper presents a dynamic traffic model in which drivers' route choice behavior is assumed to follow dynamic user equilibrium. The solution procedure developed in this paper does not rely on any iterative diagonalization or decomposition, hence allowing for

very fast computation. The model solution procedure developed in this paper is applied to the evacuation of the New Orleans metro area in preparation of landfall by Hurricane Katrina. Standard statistical techniques are utilized to evaluate the model performance and validate against the traffic volumes observed during the evacuation. As a further basis of comparison and to assess solution consistency, the model outputs are also compared to the results produced from a similar study of the New Orleans region recently conducted using TRANSIMS. The study presented in this paper shows that: (i) the results of the proposed DTA model are consistent with the results from TRANSIMS; (ii) the proposed model can reasonably predict the actual dynamic traffic patterns in a metropolitan-size network; (iii) analysis of the trip travel times for the same OD pair departing about the same time shows that the traffic patterns produced by the proposed model approximate the DUE conditions.

In addition to the theoretical contributions of the research, the results also have significant potential application in practice. The input requirements of the model as well as the relative levels of accuracy that it has been shown to achieve, make it an ideal tool for emergency evacuation planning and the sketch-plan level. From an operational context, the computational efficiency and overall speed of the modeling procedures developed in this paper allow the model to be applied shortly in advance of (or perhaps during) a major event to identify and/or anticipate where traffic problems are likely to occur under a range of varying hazard-response scenarios. Although the southeastern Louisiana regional network under threat of a major hurricane is used as the case study, the methods and computational procedures presented here can be applied to any location,

road network, and hazard scenario in which an evacuation maybe implemented as a protective action.

Further research opportunities exist in at least the following three areas. First, this paper does not differentiate the types of evacuating vehicles. For areas with large low-mobility populations, a multi-modal choice expansion of the model is important so that the transportation needs of all segments of the population are considered. Second, this model is applicable to peak-hour analysis for daily mixed traffic in the absence of evacuation conditions. For example, truck traffic can be represented explicitly in the traffic stream with passenger car equivalents. The representation of trucks will make the model valuable for the analysis of air quality implications. Finally, the model is designed for freeways rather than arterial roadways where intersection delay and queuing are major factors. Measures for arterial congestions may be explored to improve on the model's applicability for urban road networks with signalized or unsignalized intersections.

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CHAPTER 3

STOCHASTIC BILEVEL PROGRAMMING MODEL FOR SHELTER LOCATION AND TRANSPORTATION PLANNING UNDER HURRICANE CONDITIONS

Abstract

The effectiveness of the decision-making associated with hurricane response can significantly influence the ultimate consequences of the event. This paper proposes a scenario-based bilevel programming model for optimal shelter locations over a range of hurricane events. The model considers the influence of alternative shelter locations on drivers' route choice and the resulting traffic pattern. The drivers' route choice behavior is described by dynamic user equilibrium. Aiming for large-scale realistic applications, a heuristic approach is pursued in this study to efficiently solve the model formulation. A case study for the state of North Carolina is presented to illustrate the applicability and efficacy of the proposed model formulation and solution procedure.

3.1. Introduction

The task of moving tens or even hundreds of thousands of people from a wide geographic area in a few days or hours under uncertain and dangerous conditions and getting them to safe locations is a complicated process. As Hurricane Katrina made

abundantly clear, the stakes are high. Hurricane response requires coordination of many organizations, including a range of local, state, federal and non-governmental organizations, each of which must make a series of inter-related decisions. In the long-term, they must identify the facilities as possible shelters and prepare them for that role, establish evacuation routes, and plan for the decisions that will be made as each hurricane event unfolds. In the short period immediately before a hurricane makes landfall, they must decide which shelters should be opened and how best to issue evacuation orders, which includes deciding who should evacuate, what level of evacuation it should be (e.g., voluntary or mandatory), and possibly where they should evacuate to.

This paper proposes a scenario-based location model for identifying a set of shelter locations that are robust for a range of hurricane events. Standard practice today is to focus on a small number of conservative representations of hurricane events to make the majority of these decisions with little understanding of the impact of these choices on the range of possible events. Using a small number of conservative events can lead to decisions that are effective against those particular events, but are not robust against the range of events that could occur. By representing a range of hurricane events explicitly and their associated probabilities of occurrence, it is possible to achieve the long-term benefit of shelter planning effort and to capture the hurricane-specific features, such as the areas that are no longer suitable for shelters with particular characteristics and the spatial distribution of population in "harm's way".

A key element of the model developed in this paper is the explicit inclusion of drivers' route choice behavior to support predictions of congestion across the network in each hurricane scenario. This model addresses the two inter-related questions: (i) to what

extent can different shelter locations influence drivers' route choice by exploiting the inter-dependence between the shelter location plan and the dynamic traffic patterns? (ii) given a certain predictive model for drivers' route choice behavior, how should the facility planner choose a set of shelter locations, which together with the induced traffic pattern, is optimal with respect to a specific criterion? The shelter location and route choice decisions are made by two different decision makers, i.e. a facility planner who manages the shelter facilities and a set of network users who try to minimize their travel times on the network. This is a class of problem that can be represented as a bilevel programming problem or a Stackelberg leader-follower game in game theory, where the leader (upper-level decision maker) is the facility planner, and the follower (lower-level decision maker) is the set of network users. In this paper, the traffic dynamics is included by assuming the route choice behavior of network users follows dynamic user equilibrium (DUE), where any individual user chooses a route that dynamically minimizes his or her travel time along the route to the destination.

This paper makes the following contributions. First, this model is the first attempt, to the best of our knowledge, to incorporate user equilibrium-based traffic dynamics into the optimization of shelter locations under uncertainty. Second, the applicability of the model is demonstrated through a large-scale real world example for the state of North Carolina. The problem scale contains nearly 100,000 origin-destination (OD) pairs and 15,000 network links for 33 hurricane scenarios with about 6,500 binary location variables. The results illustrate the importance of understanding the relationship between shelter locations and traffic congestion.

The remainder of this paper is organized as follows. Section 2 gives an overview of the related literature. The model formulation and solution algorithm are presented in Section 3 and Section 4, respectively. Section 5 describes a case study for the state of North Carolina and discusses the computational results. The last Section concludes this paper and recommends future research opportunities.

3.2. Literature Review

There is a vast amount of literature on the development of simulation and optimization models to support evacuation decisions (e.g., Chiu et al., 2008; Cova and Johnson, 2003; Dunn and Newton, 1992; Franzese and Sorensen, 2004; Hobeika and Kim, 1998; Sbayti and Mahmassani, 2006; Yamada, 1996). Yazici and Ozbay (2007) studied the impact of roadway capacity changes on shelter capacity requirement and average travel times during evacuation. The shelter locations were assumed to be known and the impact of each shelter on the performance of evacuation was evaluated by eliminating one shelter at a time. Very few evacuation models to date have considered shelter locations as decision variables and sought the optimal locations that minimize total system cost. Sherali et al. (1991) assumed that a central authority had the power to control the evacuation flow (a system optimal approach) and considered an evacuation planning problem by jointly optimizing traffic flow distribution and the selection of shelter locations from a set of given candidate sites. However, it is not practical for emergency planners to precisely direct traffic flows to links when drivers are generally free to select their own travel routes to their destinations (Cova and Johnson, 2003). They also represent traffic not destined for shelters as "constant" background traffic and therefore do not allow this

traffic to be re-routed based on the shelters selected. Kongsomsaksakul et al. (2005) also considered the impact of shelter locations on evacuation traffic and developed a bilevel shelter location model where the upper level problem was a location problem which minimized the total network travel time and in the lower level problem, the evacuees simultaneously decide which shelter to use and the route to take under static user equilibrium. They do not consider traffic bound for other destinations in addition to shelters. A similar problem was formulated in Ng et al. (2010) where the shelter allocation was optimized in the upper level problem rather than a choice made by the evacuees in the lower level problem as in Kongsomsaksakul et al. (2005). They adopted the same representation as Sherali et al. (1991) for traffic not destined for shelters. All these shelter models focused on a given hazard scenario and hence the selected shelters may not be robust across the full range of hazard scenarios. They also all employed a static traffic assignment which assumed steady state for time-varying OD demand. Further, all of these papers either do not represent traffic not bound for a shelter or assume that traffic is constant. The study in this paper extends the previous studies on shelter locations in the following ways: (i) instead of a single evacuation scenario, the decision of shelter selection is based on a suite of hurricane events and their associated probabilities of occurrence; (ii) we focus on the situation where drivers are free to select the routes based on their own preferences and DUE is used to describe their behavior; (iii) we account for not only the total travel time in the objective function, but also a penalty for failing to meet the shelter demand due to limited budget or staffing; (iv) computational experience illustrating the efficacy of the modeling is provided for large-scale applications; and (v) we explicitly represent traffic not bound for shelters allowing

the optimization model to understand how all traffic will adapt to changes in shelter location decision.

The field of optimal investment that considers traffic dynamics is very new. When dynamic traffic assignment (DTA) is considered in the decision making to optimize investment strategies (e.g., network design, facility location modeling), the existing studies have been limited to the system-optimal approach where the problem is formulated as a single-level deterministic or stochastic program. Waller and Ziliaskopoulos (2001) introduced a stochastic linear programming formulation for a single-destination continuous network design problem based on Daganzo's cell transmission model (Daganzo, 1994, 1995) for traffic propagation. Continuous network design problems were also studied in Karoonsoontawong and Waller (2005) and Ukkusuri and Waller (2008). The continuous capacity enhancement variable (i.e., the amount of a link capacity to increase) may result in optimal solutions that are not easily implemented in practice. Tuydes and Ziliaskopoulos (2006) addressed a class of discrete network design problems that focused on contraflow strategies in evacuation planning. A Tabu Search algorithm was developed to determine which link and which lane of that link to reverse, which is easier to implement in practice than using continuous capacity variables. Kalafatas and Peeta (2009) developed a mixed-integer formulation for optimal contraflow strategies under simplifying assumptions (e.g., light traffic conditions, no backward propagation). All these studies considered DTA in an optimal investment problem as a single-level structure and assumed the planner had full control over the traffic flow on the network.

When drivers' route choice behavior is considered in the development of optimal investment strategies, most studies assume that static traffic patterns prevail on the transportation network (e.g., Fan and Liu, 2010; Hansen et al., 2004; Kongsomsaksakul et al., 2005; Ng et al., 2010, Patriksson and Rockafellar, 2002; Ukkusuri, et al., 2007; Wang, 2009; Yin et al., 2009). Static traffic assignment lacks realism for analyses spanning the peak hours and for evacuation applications. DTA, on the other hand, assigns traffic continuously or in very short time intervals and tracks the vehicles both temporally and spatially. This provides dynamic information on the traffic conditions on each link in the network. Additionally, DTA has the capability to assign traffic for varying road conditions, such as capacity changes due to incidents, road closures, or contraflow operation at certain times during the evacuation process. Compared to the static traffic assignment, the DTA provides a more accurate and realistic prediction of the traffic under evacuation conditions.

A DTA-based bilevel approach is more suitable for optimal investment problems where at the system level the planner decides the investment strategies to minimize the total system cost but has little control over the network users' route choice, while at the user level the network users follow the DUE and each individual chooses a route to minimize his or her travel time on the network. The solution method for DTA-based bilevel formulation is still ongoing research. Karoonsoontawong and Waller (2006) developed a linear bilevel programming model for a continuous network design problem where the drivers' route choice was based on a linear formulation of DUE as discussed in Ukkusuri (2002). Metaheuristics of genetic algorithm, simulated annealing and random search were explored for the solution. The model was tested on a modified Sioux Falls

networks (76 links, 24 nodes, 552 OD pairs) for 12 five-minute time intervals and the entire computation took at least 55 hours to complete. The results revealed the bilevel formulation was more desirable over the single-level DTA-based network design models (Karoonsoontawong and Waller, 2005). Meng et al. (2008) developed a bilevel optimization model for lane-based contraflow strategy design and a modified genetic algorithm that embeds with the traffic microsimulation software, PARAMICS, to solve the problem. A case study in the central business area in Singapore (2,046 links, 1,050 nodes, 36 zones) was conducted for the peak hour with 27 binary variables for lane reversal. However, none of these models considered the uncertainty in the OD demand. A decision maker must take into account the uncertainty about the environment as well as the influence of his or her decisions on the behavior of network users. This leads to a stochastic bilevel model. To the best of our knowledge, stochastic bilevel models that incorporate traffic dynamics for optimal shelter location problems are currently non-existent in the literature.

In this paper, we develop a DTA-based stochastic bilevel programming model for optimal shelter locations that is capable of handling large-scale networks with multiple OD pairs for a range of stochastic scenarios. In the model presented, the facility planner selects the shelters to be maintained over time and among these shelters, the shelters to be opened for a particular scenario. At the lower-level of the model, the dynamic route choice of network users is described by the DUE in each scenario. The proposed stochastic bilevel model is applied to a range of hurricane events that represent the hurricane hazard in the state of North Carolina.

The key ideas in Gartner et al. (1980) and Yang et al. (1992) are used to construct our solution procedure. Janson (1991) proposed a mathematical programming model for DUE and discussed the optimality conditions. An alternative approach to solve a DUE based bilevel formulation would be to convert the bilevel optimization problem into a single-level problem by applying Karush-Kuhn-Tucker (KKT) conditions to characterize the dynamic travel times under the DUE conditions. However, this would result in a nonconvex and mixed-integer program which prohibits this approach for large-scale applications. For examples of application of this general solution approach for small and illustrative problems, see Luo et al. (1996), Fletcher and Leyffer (2004), Fletcher et al. (2006), and Lawphongpanich and Hearn (2004). The largest problems addressed in these studies involved about 2,000 variables and 2,000 equations (Fletcher and Leyffer, 2004), at least a couple of orders of magnitude smaller than this research seeks to address. In addition, for several of their problem instances, they fail to identify a solution. Another potential solution strategy could be developed based on Yang (1995), who developed a heuristic to solve an OD trip estimation problem. The link travel times were approximated as a linear function of the OD trips and therefore the resultant iterations yielded quadratic convex programs. This strategy also presents difficulties when used to address our proposed problem. For example, it is not clear how to approximate the KKT conditions with a set of linear equations between the dynamic link travel times and the shelter locations and traffic volumes. Unlike the problem studied by Yang (1995), the problem this paper proposes contains integer variables and has a non-convex objective function.

3.3. Problem Formulation

The proposed stochastic bilevel approach for shelter location takes into account the influence of location decisions on the drivers' route choice behavior while capturing the stochastic nature of hurricane events and the induced evacuation demand. The model structure is illustrated in Figure 3.1. The upper-level problem is a two-stage stochastic location and allocation problem. The first stage is to identify the shelters to be maintained over time. After observing a hurricane scenario, the second stage is to select the shelters outside the affected zone to prepare to open and allocate evacuees to these shelters. This creates an OD trip matrix for the allocation of evacuees seeking shelters in each scenario. For each scenario, a DUE is employed in the lower-level to describe the reaction of evacuees to a shelter location alternative and the dynamic route choice of the individual evacuees. This gives the dynamic traffic flows and travel times on the road network. By taking into account the evacuees' route choice behavior, the upper level model makes the location and allocation decisions to minimize the total system cost. According to Mileti et al. (1992), during an evacuation, a portion of the evacuees go to the designated shelters (shelter evacuees) and many others evacuate to hotels or places of their friends and relatives (non-shelter evacuees). To evaluate the traffic congestion in each hurricane scenario, this model takes into account the traffic due to the non-shelter evacuees. For these non-shelter evacuees, a trip table indicating their destinations must be provided or it is assumed that their goal is to simply leave the evacuation zone as quickly as possible.
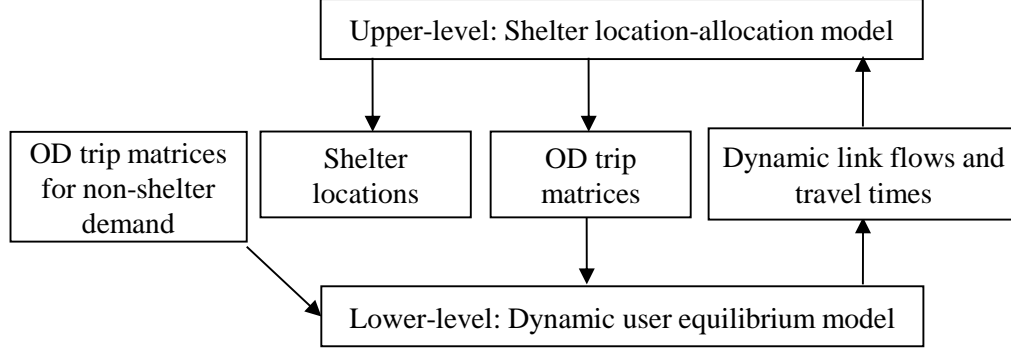
Figure 3.1. Structure of the bilevel model

We consider a transportation network represented by a connected graph $\mathbf{G} = (\mathbf{N}, \mathbf{A})$, where $\mathbf{N}$ is the set of nodes and $\mathbf{A}$ the set of directed links. The links are roadway links and the nodes are road intersections, sites to be potentially used as public shelters, or locations where trips originate and/or terminate. Let $\mathbf{N_o} \subseteq \mathbf{N}$ be the set of origins and $\mathbf{N_d} \subseteq \mathbf{N}$ be the set of sites where shelters may be located. A super sink node, denoted by e, is introduced to represent the exit for non-shelter evacuees. If there are multiple exits, dummy links are added in the network to connect these exits to the super sink and the capacities of these dummy links are set to be very large. Each hurricane scenario has a different impact area and therefore a different set of exits outside the evacuation zone.

The upper-level problem is a two-stage stochastic programming problem. Since funds for locating public shelters are usually limited, it is assumed that each shelter is associated with a budgetary cost. However, data on shelter retrofitting budget are currently difficult to obtain. The budget constraint is replaced with a constraint on the maximum number of shelters that can be located. This simplification can be relaxed once the shelter budget data are available. We define a binary decision variable X_j to be one

if a shelter is located at site j and zero otherwise. Let P be the maximum number of shelters that can be located. Then the total number of shelters cannot exceed this maximum.

$$\sum_{j \in \mathbf{N_d}} X_j \leq P \qquad (3.1)$$

Next we define the second-stage decision variables and constraints. Let $\mathbf{L}$ be the set of hurricane scenarios and p^l be the probability that hurricane scenario l occurs. We define a binary decision variable W_j^l to be one if a shelter at site j is opened and used to shelter people in hurricane scenario l, and zero otherwise. Clearly a shelter at site j cannot be used unless a shelter has been located there. Further, the intensity and path of a storm may preclude the use of the shelter even if it is located there. Let η_j^l be a parameter that is one if a shelter at site j is safe to use during hurricane scenario l. This constraint is stated in Equation (3.2).

$$W_j^l \leq \eta_j^l X_j \qquad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \qquad (3.2)$$

A key limit on the number of shelters that can be opened in a hurricane event is the number of trained personnel available to staff the shelters. To ensure that a shelter used in each hurricane scenario has sufficient staff to operate, the total staffing requirement cannot exceed the total shelter personnel that are available. Let S be the total shelter personnel available for the region and s_j be the staffing requirement for the shelter at site j. Equation (3.3) assumes that the number of staff available is independent of the event and that they can work at any shelter. If this is not the case, this limit can be relaxed to be scenario-dependent.

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$$\sum_{j \in \mathbf{N_d}} s_j W_j^l \leq S \qquad \forall l \in \mathbf{L} \tag{3.3}$$

For each hurricane scenario it is important to ensure that the demand at a shelter at site j does not exceed its capacity c_j over the evacuation horizon. We define a nonnegative decision variable $\overline{Y}_{ij}^l$ to indicate the number of evacuees from origin i that use a shelter at site j in scenario l. As described in Equation (3.4), the total number of people accommodated in the shelter at site j under hurricane scenario l cannot exceed its capacity.

$$\sum_{i \in \mathbf{N_o}} \overline{Y}_{ij}^l \leq c_j W_j^l \qquad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.4}$$

Note that the assignment variable $\overline{Y}_{ij}^l$ and shelter location variable W_j^l are dependent on scenario l while the first-stage location variable X_j is not. This reflects the two-stage nature of the problem.

Let h_i^l be the number of evacuees from origin i who seek to use shelters under hurricane scenario l. The total number of people from origin i allocated to shelters cannot exceed the total number of evacuees from origin i seeking to use shelters under this scenario.

$$\sum_{j \in \mathbf{N_d}} \overline{Y}_{ij}^l \leq h_i^l \qquad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \tag{3.5}$$

In addition, for evacuees from origin i to use the shelter at site j under scenario l, they must be able to reach that location and this location must not be at risk. We define the parameter δ_{ij}^l to reflect the accessibility from origin i to shelter location j in scenario l.

The value of δ_{ij}^l is one if it is possible for people from origin i to access the shelter at site j in scenario l and zero otherwise. Then Equation (3.6) below must hold.

$$\overline{Y}_{ij}^l \leq \delta_{ij}^l h_i^l \qquad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.6}$$

Unfortunately it may not be possible to accommodate all the evacuees wishing to use shelters. A nonnegative decision variable Z_i^l is defined to indicate the number of evacuees from origin i who seek shelters in scenario l but cannot be accommodated. Since evacuees from each origin seeking shelters in a scenario are either assigned to a shelter or not, then

$$\sum_{j \in \mathbf{N_d}} \overline{Y}_{ij}^l + Z_i^l = h_i^l \qquad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \tag{3.7}$$

Let $\overline{U}_i^l$ be the number of non-shelter evacuees from origin i in scenario l, which is input data. Let τ_{ij}^l be the average travel time from origin i to destination j in hurricane scenario l. The objective of the upper-level problem is to minimize the weighted sum of the expected unmet shelter demand and the expected total network travel time, that is

$$\sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} Z_i^l + \gamma \left[\sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \tau_{ij}^l \overline{Y}_{ij}^l + \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \tau_{ie}^l \overline{U}_i^l \right] \tag{3.8}$$

where parameter γ in the objective function reflects the relative importance of travel time in comparison to the number of people not accommodated in shelters. The first term is the expected unmet shelter demand. Recall that a portion of evacuees travel to places of their friends/relatives and hotels/motels. The second and third terms are the expected total travel time spent by shelter evacuees and non-shelter evacuees, respectively.

Since OD trip matrices $\overline{Y}_{ij}^{l}$ and $\overline{U}_{i}^{l}$ for all $i \in \mathbf{N_o}$, $j \in \mathbf{N_d}$ and $l \in \mathbf{L}$ in the upper-level problem are based on total number of people evacuating over the entire evacuation, it is necessary to convert these quantities to time-dependent OD matrices in terms of vehicle trips for the lower-level computation. Denote $\mathbf{K}$ as the set of discrete time intervals. Let Y_{ijk}^{l} be number of evacuees departing from origin i in time interval k to a shelter at site j in scenario l and U_{ik}^{l} the number of non-shelter evacuees departing from origin i in time interval k in scenario l. Time-dependent OD demand Y_{ijk}^{l} and U_{ik}^{l} are obtained by disaggregating $\overline{Y}_{ij}^{l}$ and $\overline{U}_{i}^{l}$ based on a given departure time curve. By using this curve, one can estimate the number of evacuees departing from their origins at each time interval. Let f_{ijk}^{l} be the fraction of evacuees entering the network in time interval k for OD pair i-j and scenario l. For all $i \in \mathbf{N_o}$, $j \in \mathbf{N_d} \cup \{e\}$ and $l \in \mathbf{L}$, the fractions over all departure times must sum to one, i.e. $\sum_{k \in \mathbf{K}} f_{ijk}^{l} = 1$. If the destination is a shelter site,

$$Y_{ijk}^{l} = f_{ijk}^{l} \overline{Y}_{ij}^{l} \quad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d}, k \in \mathbf{K}, l \in \mathbf{L} \tag{3.9}$$

If the destination is the super sink node e,

$$U_{ik}^{l} = f_{iek}^{l} \overline{U}_{i}^{l} \quad \forall i \in \mathbf{N_o}, k \in \mathbf{K}, l \in \mathbf{L} \tag{3.10}$$

In practice, the departure times are estimated by conducting post-hurricane surveys or studying evacuees' behaviors for the affected region. The cumulative percentage of evacuees leaving their origins versus time can be represented by an *S*-shaped curve (Hobeika and Kim, 1998), a Rayleigh distribution (Tweedie et al., 1986), or any other appropriate distribution to model the departure.

Let π_{ijk}^l be the trip travel time from origin i departing in time interval k to destination j (a shelter site or an exit) in scenario l. Then the average OD travel times in the upper-level objective (3.8) are defined as follows:

$$\tau_{ij}^l = \frac{\sum_{k\in\mathbf{K}} \pi_{ijk}^l \left(\mathbf{Y}^l, \mathbf{U}^l\right) Y_{ijk}^l}{\overline{Y}_{ij}^l} \qquad \forall i \in \mathbf{N_o}, j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.11}$$

$$\tau_{ie}^l = \frac{\sum_{k\in\mathbf{K}} \pi_{iek}^l \left(\mathbf{Y}^l, \mathbf{U}^l\right) U_{ik}^l}{\overline{U}_i^l} \qquad \forall i \in \mathbf{N_o}, l \in \mathbf{L} \tag{3.12}$$

where $\mathbf{Y}^l = \left\{Y_{ijk}^l \mid i \in \mathbf{N_o}, j \in \mathbf{N_d}, k \in \mathbf{K}\right\}$ and $\mathbf{U}^l = \left\{U_{ik}^l \mid i \in \mathbf{N_o}, k \in \mathbf{K}\right\}$. For each scenario l, the time-dependent trip travel times π_{ijk}^l and π_{iek}^l are computed by solving the DUE in the lower-level given the time-dependent OD demand $\mathbf{Y}^l$ and $\mathbf{U}^l$ from the upper-level.

It should be noted that through Equations (3.9) and (3.10), the upper-level variable $\overline{Y}_{ij}^l$ and input data $\overline{U}_i^l$ affect the lower-level problem. The trip travel times π_{ijk}^l and π_{iek}^l from the lower-level problem affect the upper-level objective given in Equation (3.8) through Equations (3.11) and (3.12). This reflects the bilevel nature of the formulation.

3.4. Heuristic Algorithms

From Equations (3.9)-(3.12), the average trip travel time τ_{ij}^l and τ_{ie}^l can be viewed as a function of the OD trip matrices $\overline{\mathbf{Y}}^l = \left\{\overline{Y}_{ij}^l \mid i \in \mathbf{N_o}, j \in \mathbf{N_d}\right\}$ and $\overline{\mathbf{U}}^l = \left\{\overline{U}_i^l \mid i \in \mathbf{N_o}\right\}$ for scenario l where $\overline{\mathbf{U}}^l$ is input data. The objective of the stochastic bilevel programming problem is to minimize the following:

$$F\left(\mathbf{X},\mathbf{W},\overline{\mathbf{Y}},\mathbf{Z}\right)=\sum_{l\in\mathbf{L}}p^{l}\sum_{i\in\mathbf{N_o}}Z_{i}^{l}+\gamma\left[\sum_{l\in\mathbf{L}}p^{l}\sum_{i\in\mathbf{N_o}}\sum_{j\in\mathbf{N_d}}\tau_{ij}^{l}\left(\overline{\mathbf{Y}}^{l},\overline{\mathbf{U}}^{l}\right)\overline{Y}_{ij}^{l}+\sum_{l\in\mathbf{L}}p^{l}\sum_{i\in\mathbf{N_o}}\tau_{ie}^{l}\left(\overline{\mathbf{Y}}^{l},\overline{\mathbf{U}}^{l}\right)\overline{U}_{i}^{l}\right]$$

$$(3.13)$$

where $\mathbf{X}=\left\{X_{j}\mid j\in\mathbf{N_d}\right\}$, $\mathbf{W}=\left\{W_{j}^{l}\mid j\in\mathbf{N_d},l\in\mathbf{L}\right\}$, $\overline{\mathbf{Y}}=\left\{\overline{Y}_{ij}^{l}\mid i\in\mathbf{N_o},j\in\mathbf{N_d},l\in\mathbf{L}\right\}$ and

$\mathbf{Z}=\left\{Z_{i}^{l}\mid i\in\mathbf{N_o},l\in\mathbf{L}\right\}$. The formulated problem is a non-convex, mixed integer, and two-

stage stochastic program. Computationally this type of problem has been known to be

difficult and time-consuming to solve (Bard, 1991). Hence it is necessary to explore

heuristic methods to efficiently solve this problem. The heuristic adopted in this study is

based on the following observation. If the time-dependent trip travel times

$\boldsymbol{\pi}=\left\{\pi_{ijk}^{l}\mid i\in\mathbf{N_o},j\in\mathbf{N_d}\cup\{e\},k\in\mathbf{K},l\in\mathbf{L}\right\}$ are given, the average OD travel times

$\boldsymbol{\tau}=\left\{\tau_{ij}^{l}\mid i\in\mathbf{N_o},j\in\mathbf{N_d}\cup\{e\},l\in\mathbf{L}\right\}$ can be calculated by Equations (3.11) and (3.12).

Once $\boldsymbol{\tau}$ is known, the upper-level decision variables $\mathbf{X}$, $\mathbf{W}$, $\overline{\mathbf{Y}}$ and $\mathbf{Z}$ can be

determined by solving the minimization problem (3.1)-(3.8). Given $\overline{\mathbf{Y}}$ from the upper-

level problem and the departure time distribution, the time-dependent OD travel times $\boldsymbol{\pi}$

can be updated by solving the DUE traffic assignment. The structure of this heuristic is

known to be capable of handling large-size problems and has been used to solve

deterministic bilevel problems (Gartner et al., 1980; Yang et al., 1992). In each iteration

of the bilevel algorithm, the current best solution is recorded and let $\mathbf{X}^{*},\mathbf{W}^{*},\mathbf{Y}^{*},\mathbf{Z}^{*}$

represent the best solution and F^{*} the best upper level objective value. Also let

$\mathbf{Y}=\left\{Y_{ijk}^{l}\mid i\in\mathbf{N_o},j\in\mathbf{N_d},k\in\mathbf{K},l\in\mathbf{L}\right\}$ and $\mathbf{U}=\left\{U_{ik}^{l}\mid i\in\mathbf{N_o},k\in\mathbf{K},l\in\mathbf{L}\right\}$. The general

solution scheme is summarized in the following steps:

Step 0 (Initialization): Compute $\boldsymbol{\tau}^{(0)}$ based on free flow conditions; set iteration number $\rho = 1$; set best objective value $F^* = +\infty$.

Step 1: Solve the upper-level problem in Equations (3.1)-(3.8) for given $\boldsymbol{\tau}^{(\rho-1)}$. This gives the values for the first-stage variable $\mathbf{X}^{(\rho)}$ and second-stage variables $\mathbf{W}^{(\rho)}$, $\overline{\mathbf{Y}}^{(\rho)}$ and $\mathbf{Z}^{(\rho)}$.

Step 2: Compute the time-dependent OD trips $\mathbf{Y}^{(\rho)}$ and $\mathbf{U}^{(\rho)}$ based on Equations (3.9)-(3.10).

Step 3: Given $\mathbf{Y}^{(\rho)}$ from Step 2, solve the lower-level DUE problem for each scenario. This step yields dynamic link flows and link travel times on the network, and also the time-dependent trip travel time $\boldsymbol{\pi}^{(\rho)}$.

Step 4: Compute $\boldsymbol{\tau}^{(\rho)}$ based on Equations (3.11)-(3.12) and the value of the upper-level objective $F^{(\rho)}$ in Equation (3.8).

Step 5 (Record the best solution): If $F^{(\rho)} < F^*$, set $F^* = F^{(\rho)}$ and $\mathbf{X}^* = \mathbf{X}^{(\rho)}$, $\mathbf{W}^* = \mathbf{W}^{(\rho)}$, $\mathbf{Y}^* = \mathbf{Y}^{(\rho)}$, $\mathbf{Z}^* = \mathbf{Z}^{(\rho)}$.

Step 6 (Stopping criteria): If the objective has not improved for the last pre-specified number of iterations, stop and output the best solution. Otherwise, set $\rho = \rho + 1$ and go to Step 1.

Since the best solution is recorded at each iteration, the best objective value is non-increasing and bounded below (always non-negative). This implies the solution converges; however it does not guarantee a convergence to the true optima. In this study, the algorithm is run for at least a pre-specified number of iterations to prevent the algorithm from premature convergence in the heuristic scheme. The main goal of this

paper is develop a DTA-based stochastic bilevel programming model that can be applied to large-size networks for a full range of stochastic scenarios. Therefore, it is important to explore computationally efficient algorithms in Step 1 and Step 3.

Step 3 involves a DUE calculation for each scenario. A highly efficient solution procedure to solve the DUE assignment is critical to the computational feasibility of this model and solution approach. The algorithm developed by Li et al. (2011) is used here to find the DUE solution in the lower-level computation. This solution procedure identifies an approximate equilibrium solution to a modified version of the DTA formulation developed in Janson (1991). The procedure is platoon based and steps through time assigning platoons that depart in each time interval to their shortest paths and updating the shortest path for each platoon that is en route. The algorithm terminates when all platoons reach their destinations. The procedure involves dynamic shortest-path calculations and traffic assignments at successive time steps which allow trips to be traced in both spatial and temporal domains. This solution procedure was validated against the Hurricane Katrina evacuation using a road network with more than 11,000 links and 5,500 nodes for nearly 4,000 OD pairs over a two-day period requiring a computation time on the order of seconds.

In Step 1, the upper-level problem is solved as a mixed integer program for fixed OD travel times. The upper level problem has a staffing constraint, which yields a binary knapsack problem and therefore the problem is NP hard. Lagrangian relaxation heuristics are known to be capable of solving large mixed integer problems with similar structure. The idea of the heuristic is to relax certain "hard" constraints to the objective function by penalizing a violation of these constraints in the objective function. The penalty takes the

form of multipliers that are associated with each of the relaxed constraints. The solution procedure for the Lagrangian relaxation in solving the upper level problem with the fixed τ is described as follows.

Lagrangian Relaxation Algorithm

We choose to relax constraints (3.2) and (3.4) which bind the decision variables under all scenarios together. Relaxing these two constraints allow us to decompose the problem by scenarios. Let ϕ_j^l be the multiplier associated with constraint (3.2) and λ_j^l be the multiplier associated with constraint (3.4). The corresponding Lagrangian dual problem is to choose multipliers $\phi = \left\{ \phi_j^l \,\middle|\, j \in \mathbf{N_d}, l \in \mathbf{L}, \phi_j^l \geq 0 \right\}$ and $\lambda = \left\{ \lambda_j^l \,\middle|\, j \in \mathbf{N_d}, l \in \mathbf{L}, \lambda_j^l \geq 0 \right\}$ to maximize the Lagrangian function $L(\phi, \lambda)$:

$$
L(\phi, \lambda) = \min \left\{ \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} Z_i^l + \gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \tau_{ij}^l \overline{Y}_{ij}^l + \sum_{l \in \mathbf{L}} \sum_{j \in \mathbf{N_d}} \phi_j^l \left(W_j^l - \eta_j^l X_j \right) + \sum_{l \in \mathbf{L}} \sum_{j \in \mathbf{N_d}} \lambda_j^l \left(\frac{\sum_{i \in \mathbf{N_o}} \overline{Y}_{ij}^l}{c_j} - W_j^l \right) \right\}
$$

(3.14)

Regrouping the terms in Equation (3.14) yields the following form:

$$
L(\phi, \lambda) = \min \left\{ \sum_{j \in \mathbf{N_d}} \left(-\sum_{l \in \mathbf{L}} \phi_j^l \eta_j^l \right) X_j + \sum_{l \in \mathbf{L}} \left[\sum_{j \in \mathbf{N_d}} \left(\phi_j^l - \lambda_j^l \right) W_j^l + \sum_{i \in \mathbf{N_o}} p^l Z_i^l + \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \left(\gamma p^l \tau_{ij}^l + \frac{\lambda_j^l}{c_j} \right) \overline{Y}_{ij}^l \right] \right\}
$$

(3.15)

subject to the model constraints (3.1)–(3.7) except (3.2) and (3.4).

The term $\gamma \sum_{l \in \mathbf{L}} p^l \sum_{i \in N_o} \tau_{ie}^l \overline{U}_i^l$ in the objective function (3.8) is omitted in the definition of the Lagrangian function because this term is constant when τ is fixed and thus does not have any impact on the dual solutions. It follows from the weak duality theory that the

minimum value of the relaxed problem for any fixed values of the multipliers provides a lower bound on the optimal value of the original problem. This allows us to assess how far the solution could possibly be from optimality. It can be checked that the Lagrangian function $L(\phi, \lambda)$ is concave. Then the Lagrangian dual problem, i.e., maximize $L(\phi, \lambda)$, is a concave maximization problem and thus any algorithms for convex optimization problems can be used to solve the Lagrangian dual problem. We adopt a line search heuristic based on the subgradient method to solve the Lagrangian dual problem. This method is capable of solving large scale problems because the calculation in each iteration is inexpensive. The algorithm requires choosing a search direction and a step size along the search direction in each iteration. The choice of the search direction is a combination of the previous and current subgradients. The choice of the step size requires the calculations of the current lower bound and the best upper bound found so far. The algorithm repeats the following three steps: (i) calculate $L(\phi, \lambda)$ for the fixed values of the multipliers ϕ and λ to obtain a lower bound of the original problem; (ii) construct a feasible solution to the original problem based on the solution to the lower bound; and (iii) update the values of the multipliers. The procedure is described as follows.

Step 0: Initialization.

Set the initial values of the multipliers ϕ^0 and λ^0; set error tolerance ϵ; set best objective value $UB^* = +\infty$; set iteration number $n = 1$.

Step 1: Calculate lower bound.

Solve the relaxed problem $L(\phi^{n-1}, \lambda^{n-1})$ to find a lower bound. The Lagrangian function $L(\phi^{n-1}, \lambda^{n-1})$ can be decomposed by three independent subproblems.

Subproblem 1:

$$\text{Minimize} \sum_{j \in \mathbf{N_d}} \left(-\sum_{l \in \mathbf{L}} \phi_j^{l,n-1} \eta_j^l \right) X_j^n \tag{3.16}$$

$$\text{Subject to} \sum_{j \in \mathbf{N_d}} X_j^n \le P \tag{3.17}$$

$$X_j^n \in \{0,1\} \quad \forall j \in \mathbf{N_d} \tag{3.18}$$

This is a simple binary knapsack problem involving decision variable $\mathbf{X}^n$. It can be solved by inspection: let $M_j = -\sum_{l \in \mathbf{L}} \phi_j^{l,n-1} \eta_j^l$; sort the sequence $\{M_j\}$ in ascending order; if location j is in the first P locations, set $X_j^n = 1$ and otherwise, $X_j^n = 0$. The resulting $\mathbf{X}^n$ solves the subproblem.

Subproblem 2. for each scenario l, solve the following problem:

$$\text{Minimize} \sum_{j \in \mathbf{N_d}} \left(\phi_j^{l,n-1} - \lambda_j^{l,n-1} \right) W_j^{l,n} \tag{3.19}$$

$$\text{Subject to} \sum_{j \in \mathbf{N_d}} s_j W_j^{l,n} \le S \tag{3.20}$$

$$W_j^{l,n} \in \{0,1\} \quad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.21}$$

This is also a binary knapsack problem, which can be solved in two steps. Let $R_j = \phi_j^{l,n-1} - \lambda_j^{l,n-1}$. In the first step, if $R_j < 0$, set $W_j^{l,n} = 1$; otherwise $W_j^l = 0$. In the second step, based on the pre-calculated $\{W_j^l\}$ in the first step, if the constraint (3.20) is violated, $W_j^{l,n}$ is solved again over the subset $\{j | W_j^{l,n} = 1, j \in \mathbf{N_d}\}$. After the two steps, $\mathbf{W}^n$ is the solution.

Subproblem 3. _For each scenario_ l, _solve the following transportation problem_:

$$\text{Minimize} \sum_{i \in \mathbf{N_o}} p^l Z_i^{l,n} + \sum_{i \in \mathbf{N_o}} \sum_{j \in \mathbf{N_d}} \left(\gamma p^l \tau_{ij}^l + \frac{\lambda_j^{l,n-1}}{c_j} \right) \overline{Y}_{ij}^{l,n} \tag{3.22}$$

subject to constraints (3.5)–(3.7) where $\overline{\mathbf{Y}}^n$ and $\mathbf{Z}^n$ are nonnegative.

This is a classic transportation problem. It is solved as follows.

$$\forall i \in \mathbf{N_o}, \ R_i = \min_{j \in \mathbf{N_d}} \left(p^l \gamma \tau_{ij}^l + \frac{\lambda_j^{l,n-1}}{c_j} \right) \quad \text{subject to} \quad \delta_{ij}^l = 1 \quad \text{and let} \quad j_i \quad \text{be its solution.}$$

$\forall i \in \mathbf{N_o}$, if $R_i \leq p^l$, set $\overline{Y}_{ij_i}^{l,n} = h_i^l$, $Z_i^{l,n} = 0$, and $\overline{Y}_{ij}^{l,n} = 0$ for all $j \in \mathbf{N_d}$ and $j \neq j_i$;

otherwise, set $Z_i^{l,n} = h_i^l$, and $\overline{Y}_{ij}^{l,n} = 0$ for all $j \in \mathbf{N_d}$. Let $\overline{\mathbf{Y}}^n$ and $\mathbf{Z}^n$ be the solution.

Define LB^n as the sum of the above three minimization values. Then $LB^n = L\left(\boldsymbol{\phi}^{n-1}, \boldsymbol{\lambda}^{n-1}\right)$

Step 2: Calculate upper bound.

Based on the solution to the relaxed problem ($\mathbf{X}^n, \mathbf{W}^n, \overline{\mathbf{Y}}^n, \mathbf{Z}^n$), this step is to identify a feasible solution to the original problem for use as an upper bound solution.

    Step 2.1: Check constraint $W_j^{l,n} \leq \eta_j^l X_j^n$, $\forall j \in \mathbf{N_d}, l \in \mathbf{L}$. If a shelter at site j is not

located ($X_j^n = 0$) or this site is not usable ($\eta_j^l = 0$), this shelter should remain

closed in this scenario ($W_j^{l,n} = 0$).

    Step 2.2: If $\sum_{j \in \mathbf{N_d}} s_j W_j^{l,n} < S$, $\forall l \in \mathbf{L}$, check if more shelters can be opened while

this staff constraint remains satisfied. If so, among the located shelters ($X_j^n = 1$)

that can be used under scenario l but not yet opened ($\eta_j^l = 1$ and $W_j^{l,n} = 0$), find

the shelter that receives the largest shelter demand based on the solution from

Step 1 and open that shelter. This is repeated until no more shelters can be opened for the given staffing limit S.

Step 2.3: Check the allocation constraint $\sum_{i \in \mathbf{N_o}} \overline{Y}_{ij}^{l,n} \le c_j W_j^{l,n}, \ \forall j \in \mathbf{N_d}, l \in \mathbf{L}$. If the solution for $\overline{Y}_{ij}^{l,n}$ from Step 1 satisfies the constraint, they are kept for use in the upper bound solution. If this constraint is violated for a shelter at site j, reset $\overline{Y}_{ij}^{l,n} = 0$ and assign the demand from its closest origins to this shelter until its capacity is reached. Finally, if there is still uncovered demand for origin i in scenario l ($Z_i^{l,n} > 0$), assign the uncovered demand to its closest opened shelters that are used under capacity and can be accessed from origin i.

Let UB^n be the objective value based on the current upper bound solution. If $UB^n < UB^*$, set $UB^* = UB^n$.

Step 3: Update Lagrangian multipliers.

The process of updating the values of the Lagrange multipliers requires the calculation of a step size and a subgradient. The adaptive step size follows the rule suggested by Held et al. (1974). The step size depends upon the gap between the current lower bound (LB^n), the best upper bound (UB^*) and a user-defined parameter a^n with the sum of the square of the relaxed constraints as the scaling factor. The step size at the nth iteration is:

$$\Delta^n = \frac{a^n \left(UB^* - LB^n \right)}{\sum_{l \in \mathbf{L}} \left\{ \sum_{j \in \mathbf{N_d}} \left(W_j^{l,n} - \eta_j^l X_j^n \right)^2 + \sum_{j \in \mathbf{N_d}} \left(\frac{\sum_{i \in \mathbf{N_o}} \overline{Y}_{ij}^{l,n}}{c_j} - W_j^{l,n} \right)^2 \right\}} \tag{3.23}$$

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where X_j^n, $W_j^{l,n}$ and $\overline{Y}_{ij}^{l,n}$ use the lower bound solutions from Step 1. The parameter a^n is a scalar with $0 < a^n \leq 2$, which goes to zero at a linear rate as the number of iterations goes to infinity. The sequence a^n is determined by the rule that is expressed by $\Lambda(a_0, b_0, m, g)$: set $a = a_0$ for the first $b = b_0$ iterations; both a and b are then divided by m and yield new values of a for the next b iterations, at the end of which both a and b are again divided by m yielding new values of a and b; this process repeats until $b < g$, after which a is divided by m every g iterations.

The subgradient direction suggested by Camerini et al. (1975) and Crowder (1976) is used to make the subgradient optimization algorithm less sensitive to the heuristic choice of step size sequence. The modified direction is a linear combination of the preceding subgradient directions rather than just the current gradient.

$$r_j^{l,n} = \theta r_j^{l,n-1} + \left(W_j^{l,n} - \eta_j^l X_j^n \right) \quad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.24}$$

$$\kappa_j^{l,n} = \theta \kappa_j^{l,n-1} + \left(\frac{\sum_{i \in \mathbf{N_o}} \overline{Y}_{ij}^{l,n}}{c_j} - W_j^{l,n} \right) \quad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.25}$$

where θ is a weighting factor and let $r_j^{l,0} = \kappa_j^{l,0} = 0$. This scheme gives rise to a composite step direction for which the most contribution is made by the current subgradient with decreasing contributions from previous subgradients. The choice of value of θ should be less than 0.5. The value of $\theta = 0.3$ is used in all the examples presented in this paper.

Given the step size and subgradients, the updating equations for the multipliers are:

$$\phi_j^{l,n} = \max \left\{ 0, \phi_j^{l,n-1} + \Delta^n r_j^{l,n} \right\} \quad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.26}$$

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$$\lambda_j^{l,n} = \max\left\{0, \lambda_j^{l,n-1} + \Delta^n \kappa_j^{l,n}\right\} \quad \forall j \in \mathbf{N_d}, l \in \mathbf{L} \tag{3.27}$$

Step 4: Stopping condition.

If $\left(UB^* - LB^n\right)\big/LB^n < \epsilon$ or a predetermined number of iterations have been completed,

stop. Otherwise, set $n = n+1$ and go to Step 1.

3.5. Case Study in North Carolina

3.5.1. Introduction

To test the applicability of the proposed formulation and heuristic algorithms, the case

study is carried out for the state of North Carolina. North Carolina is among the most

hurricane-prone regions in the US. It consists of 1,555 census tracts covering 53,819

square miles (139,391 square kilometers). From an analysis of historical hurricane data

from year 1887 to 1998, Huang et al. (2001) estimated that on average a hurricane affects

the state about once every four years. Within the state, the hazard is most severe on the

coast. According to US Census 2000 (U.S. Census Bureau, 2002), the total population in

North Carolina was 8.05 million, up from 4.56 million in 1960. Importantly, much of this

population growth occurred from 1960 to 1990 during a period of very little hurricane

activity.

3.5.2. Input Data

Legg et al. (2010) developed an optimization model that selects a subset of hurricanes

and the hazard-consistent occurrence probabilities so that the regional hazard estimated

from the subset matches the wind loss maps produced by the software HAZUS-MH

(FEMA, 2006) at each census tract. The authors identified a set of 100 hurricanes and only 33 of those hurricanes require mass evacuation due to excessive wind. Hence, this case study is based on these 33 hurricane scenarios.

For each hurricane scenario, HAZUS-MH is used to estimate the total number of people evacuating at each census tract and the number of people seeking public shelters. Since the majority of evacuation comes from the coast, the 529 census tracts in the eastern third of the state including Raleigh and Fayetteville are chosen as the trip origins. According to the US Census 2000 (U.S. Census Bureau, 2002), there are about 1.1 million households and 3 million people residing in the 529 census tracts.

A list of 187 existing and potential shelters across North Carolina from the American Red Cross are used in this study. These shelter locations include existing public buildings that have already been used as shelters and buildings that have never been used as shelters but by proper retrofitting could be used. The Red Cross estimates that the capacities of these shelters range from 700 to 4,000 people based on the 20 square feet (1.86 square meters) per person standard (American Red Cross, 2002). Figure 3.2 indicates the location of these shelters and the capacity of each. In a hurricane event, shelters located in at-risk areas are assumed to be not viable for use. Further, because storms generally approach from the coast and given the uncertainty in their paths, we assume evacuees do not evacuate toward the coastline regardless of storm track. For example, a shelter located at beachfronts or barrier islands is considered not safe and should not be opened under any scenario.

The network data are based on the primary roads data (Interstate highways, US routes and state routes) provided by North Carolina Department of Transportation. The network

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consists of 5,055 nodes and 15,382 directed links (Figure 3.2). Each road segment on the

network is assumed to be bi-directional and the free flow travel times for both directions

of the same link are the same. The free flow travel speed on all links is assumed to be 55

miles per hour (88.5 kilometers per hour) and the practical capacity of each lane is 1200
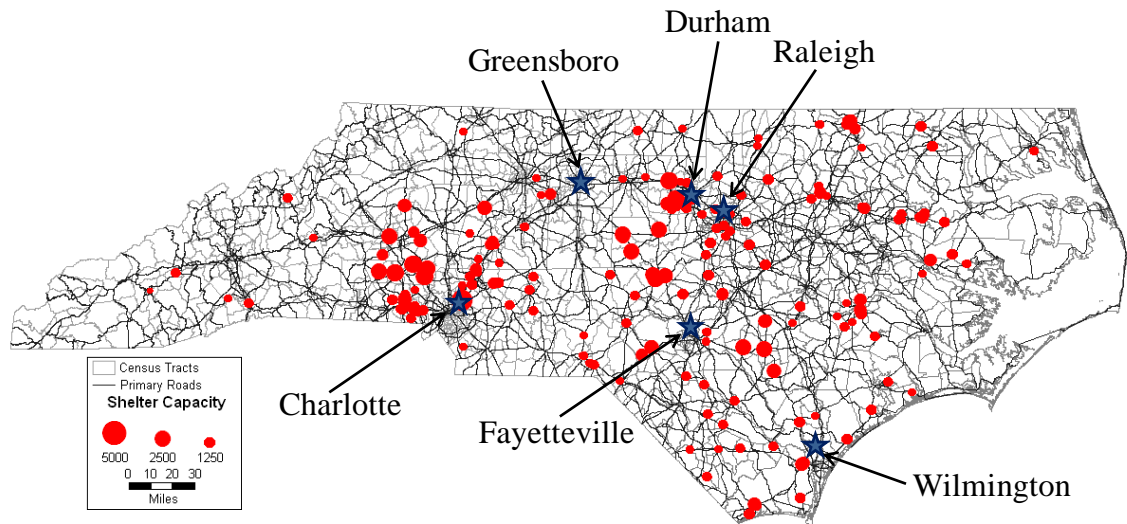
vehicles per hour.

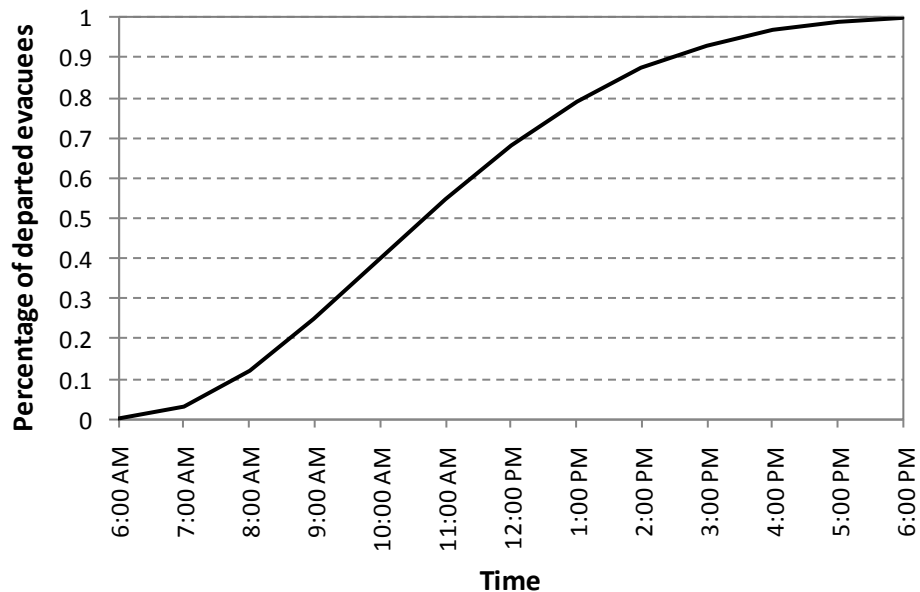Figure 3.2. Network, 187 shelter locations, and major cities in North Carolina

Figure 3.3. Rayleigh distribution for departure time

The departure time distribution is represented by a Rayleigh distribution (Tweedie et al., 1986) for a 12-hour evacuation from 6:00 AM to 6:00 PM with peak departure at 10:00 AM (Figure 3.3). The entire evacuation period is divided into 144 discrete time intervals and the size of each time interval is 5 minutes. Since there is no departure time data available at each census tract for each hurricane scenario, the same departure distribution is applied to all OD pairs and all hurricane scenarios.

The link performance function developed by US Bureau of Public Roads (BPR) is used in the DUE model at each time interval to determine the congestion delay over each link. For a given link a and time interval t, the link travel time is:

$$c_a(t) = c_a^0 \left[1 + \alpha \left(x_a(t)/Q_a \right)^\beta \right] \qquad (3.28)$$

where Q_a is the practical capacity of link a adjusted to the time interval. c_a^0 is the free-flow travel time on link a. Parameters $\alpha = 0.3$ and $\beta = 4.0$ (Note this α differs from the α in model objective). The BPR function is known to be able to yield good responses for highways and has been used in various dynamic traffic models (e.g., Smith et al., 1995; Janson, 1991, 1995). Although a BPR-type formula is adopted in this study for simplicity, the proposed formulation and solution procedure can be applied with any other proper travel delay function.

To apply the model to this case study, the following parameters are applied: (i) on average, there are two people per vehicle; (ii) five shelter staff are needed per hundred people accommodated in a shelter (based on conversations with American Red Cross personnel) and there are 3,000 staff available for each event ($S = 3000$); (iii) a maximum of 50 shelters can be selected ($P = 50$). This, along with the staffing constraint, limits the set of shelters that can be used in each scenario; and (iv) the value of parameter γ in the

objective function is set to 0.1. This implies that if more than 10 hours is needed for an evacuee to reach a shelter, it is assumed that there is no effective shelter available for that individual (hence that person is not accommodated in a shelter in the solution).

As mentioned, a portion of evacuees are directed to public shelters, and for those non-shelter evacuees, there is little information available as to where these people actually might go. We conduct the analysis under two alternative assumptions: (i) their goal is simply to leave the evacuation zone as quickly as possible (referred to as Assumption 1); and (ii) the non-shelter evacuees from the Outer Banks evacuate into Virginia and the rest are distributed among Raleigh, Durham, Greensboro, Fayetteville and Charlotte based on their relative populations and if these destinations are outside the evacuation zone (referred to as Assumption 2).

3.5.3. Computational Time

In the lower-level of the model, the dynamic traffic assignment for one scenario is independent from that for another scenario. This makes the use of parallel computing attractive in each iteration to speed up the overall computation time. The model is implemented in MATLAB R2010b on an Intel Xeon 2.66 GHz PC running Windows 7 64-bit. The implementation for the entire state of North Carolina over a 12-hour evacuation period required 60 hours to complete when eight parallel processors are used. Note that the computation time could have been reduced significantly if more processors were available and each of the 33 scenarios was assigned to a different processor.

3.5.4. Results

The heuristic approach proposed in Section 4 is implemented, starting with free-flow conditions. Table 3.1 shows the annual occurrence probability, evacuation demand and shelter demand for each hurricane scenario, and the percent reduction in average travel time to shelters from the initial solution (shelter locations are selected based on free-flow conditions) to the optimized solution (congestion-related travel times are considered) under both Assumption 1 (evacuees not using shelters try to leave the impacted area as quickly as possible) and Assumption 2 (OD trips are given for evacuees not using shelters). Scenario 1 causes the largest evacuation and demand for shelter use among the 33 scenarios. Because the objective is the minimization of the expected total travel time and unmet shelter demand over the 33 scenarios, the average travel time reduction for the Scenario 1 is the greatest under for assumptions. For Scenario 1 under Assumption 1, by taking into account network congestion in shelter selection, the evacuees destined for shelters can save 16.5% in travel time on average and 20.7% for the evacuees departing in the peak traffic hour (from 10:00 AM to 11:00 AM). We use the travel time of trips that depart during the peak traffic hour to create a measure of the reduction in congestion during the most intense periods of travel during the evacuation. Under Assumption 2, the model reduces the average travel time from 4.23 hours to 3.5 hours for Scenario 1, which is about 21.0% reduction. The model also suggests that the travel time reduction to shelters for evacuees departing during the peak traffic hour could be on the order of 27%.

It is useful to notice that there are deteriorations in the travel times to shelters in Scenarios 9, 13, 15 and 16 under Assumption 2. These four scenarios all make landfall to the south of North Carolina and cross into North Carolina along Interstate 95. In the initi-

Table 3.1. Percent reduction in average travel time to shelters from initial (free-flow) to optimized solution for each hurricane scenario

Scenario	Annual Occurrence Probability	Evacuation Demand (persons)	Shelter Demand (persons)	Percent Reduction in Average Travel Time to Shelters			
				Assumption 1		Assumption 2	
				Average	Peak Departure Hour	Average	Peak Departure Hour
1	0.0005	566,530	62,550	16.5%	20.7%	21.0%	27.0%
2	0.0004	411,860	44,260	5.0%	7.1%	2.8%	5.4%
3	0.001	323,110	35,537	2.7%	3.9%	1.6%	1.9%
4	0.0004	325,360	34,154	0.7%	2.2%	-0.5%	-0.5%
5	0.0006	298,420	33,189	2.7%	3.2%	0.8%	1.0%
6	0.0001	236,920	26,036	2.2%	4.8%	2.4%	4.7%
7	0.001	228,150	25,894	2.1%	1.7%	0.4%	1.0%
8	0.0007	190,440	20,518	0.9%	0.2%	0.3%	0.4%
9	0.0002	171,730	17,576	-0.9%	0.0%	-4.8%	-4.3%
10	0.0023	155,220	17,160	0.6%	1.0%	2.2%	4.2%
11	0.0001	148,220	14,652	0.3%	0.2%	2.4%	2.4%
12	0.0006	122,570	12,954	1.6%	3.7%	1.1%	1.6%
13	0.0002	111,840	10,204	0.3%	2.5%	-4.9%	-3.5%
14	0.0019	87,790	9,632	0.3%	0.1%	0.1%	0.1%
15	0.0004	87,314	9,466	2.0%	3.1%	-2.6%	-2.0%
16	0.0001	101,940	9,093	-1.4%	0.9%	-6.3%	-5.3%
17	0.0005	91,369	8,935	10.9%	10.8%	0.2%	0.3%
18	0.004	70,817	7,801	0.1%	-0.1%	0.0%	0.0%
19	0.0005	66,467	7,115	0.1%	0.5%	0.3%	0.5%
20	0.0014	66,732	7,037	1.6%	1.6%	-0.1%	0.0%
21	0.00246	44,137	4,677	0.0%	0.0%	0.0%	0.0%
22	0.00005	35,247	3,927	0.1%	0.1%	0.0%	0.0%
23	0.00155	33,266	3,612	0.0%	0.0%	0.0%	0.0%
24	0.0014	36,178	3,498	0.0%	-0.1%	0.0%	-0.1%
25	0.0003	31,147	3,305	0.0%	0.0%	0.0%	0.0%
26	0.00205	25,242	2,898	2.4%	2.5%	0.0%	0.0%
27	0.004	24,894	2,605	0.0%	0.0%	0.0%	0.0%
28	0.00205	21,395	2,417	0.0%	0.0%	0.0%	0.0%
29	0.0002	20,258	2,192	0.0%	0.0%	0.0%	0.0%
30	0.0002	20,180	2,111	0.0%	0.0%	0.0%	0.0%
31	0.0021	18,036	1,987	0.0%	0.0%	0.0%	0.0%
32	0.00205	10,055	1,182	0.0%	0.0%	0.0%	0.0%
33	0.0041	8,797	1,018	0.0%	0.0%	0.0%	0.0%

Note: Assumption 1 – non-shelter evacuees try to leave the evacuation area as quickly as possible; Assumption 2 - the OD trip matrices for the non-shelter evacuees are given.

-al solution, there were two shelters near Rockingham, North Carolina which were available for use, and very close to the origins of many evacuating under these scenarios. In the optimized solution, these shelters were discarded in favor of additional shelter capacity to the North.

There are some travel time savings for the evacuees not destined for shelters due to the relocation of the evacuees using shelters, but the benefits that accrue to this population are modest. For example, in Scenario 1, the reduction in peak hour travel time is 7.6% under Assumption 1 and 2.8% under Assumption 2. Similarly, reduction in the average travel time is on the order of 6.5% under Assumption 1 and 1.4% under Assumption 2.

In the interest of brevity, the remainder of this discussion focuses on insights gained under Assumption 2. Figure 3.4 illustrates the 50 shelters selected by the optimization model. Only these shelters may be used under any of the scenarios. It is useful to notice that the model tends to select larger shelters with an emphasis on those to the west of I-95 and south of I-40. The preference for shelters to the west of I-95 and south of I-40 stems from the fact that many of the storms that hit North Carolina primarily affect the eastern third of the state.
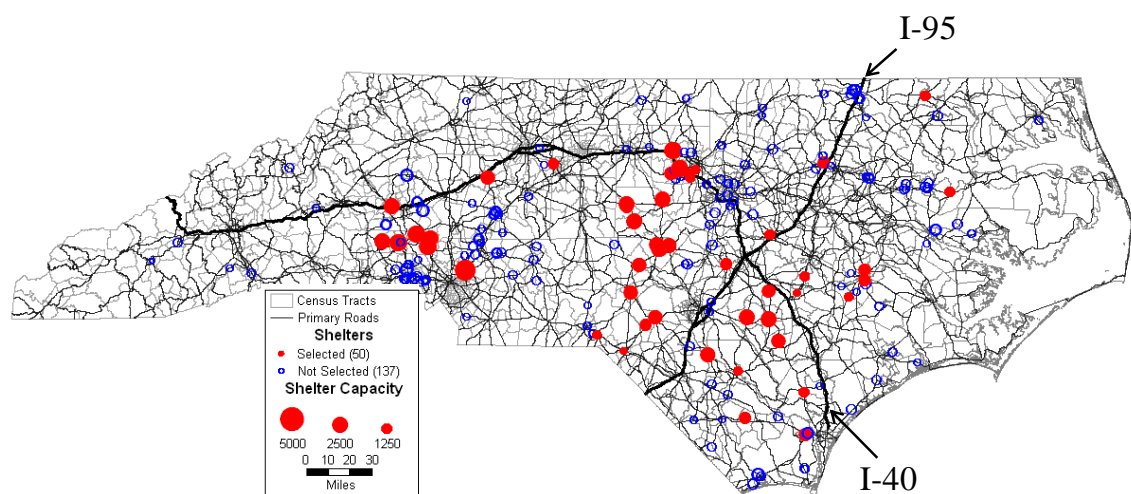

Figure 3.4. Recommended shelter locations

Among the 33 hurricane scenarios in this study the largest impacts stemming from the optimization under Assumption 2 occur in Scenario 1, hence for the remainder of this discussion we focus on this scenario. Scenario 1 makes landfall at Category 5 strength (Saffir–Simpson Hurricane Scale) near Cape Fear where the city of Wilmington is located. The storm travels along the I-40 corridor up passing Raleigh, resulting in 567,000 people evacuating. The track of this scenario is similar to Hurricane Fran in 1996 which caused significant damage in North Carolina. The estimated annual occurrence probability for this storm and those with a similar spatial distribution of wind speeds is about 0.05%.

According to Assumption 2, the non-shelter evacuees travel to the cities of Charlotte, Greensboro, and Durham with proportion of 59.9%, 21.5% and 18.6% in Scenario 1. Note that in this scenario, Raleigh and Fayetteville are affected and there is no evacuation from the Outer Banks. The large proportion of evacuees heading to Charlotte results in significant westbound traffic towards the Charlotte area. Since most evacuees in Scenario 1 originate from the southeastern part of the state, westbound US-74 from Wilmington to Charlotte is heavily travelled.

Figure 3.5a presents, in the initial solution, the shelters opened and the traffic pattern generated by shelter evacuees in Scenario 1. It can be observed that many evacuees take the already congested route US-74 to shelter locations near Charlotte area. Additionally, the portion of NC-73 near Charlotte is also heavily used. Figure 3.5b shows, in the optimized solution, the shelters opened and the traffic pattern generated by shelter evacuees under Scenario 1. The results show there is little of this traffic using US-74, which is heavily used by non-shelter evacuees. The implication is that the model attempts

to place these shelters such that evacuees using shelters can take the routes that are lightly used by the evacuees destined for others places. As shown in Table 3.1, evacuees can save an average of 21% in their travel time to shelters when traffic congestion is considered in the selection of shelters. Much of this benefit stems from two sources: the use of state route US-74 by evacuees destined for shelters is significantly lessened and many evacuees are shifted to the shelters located to the west of Raleigh via northbound I-40 and US-421.
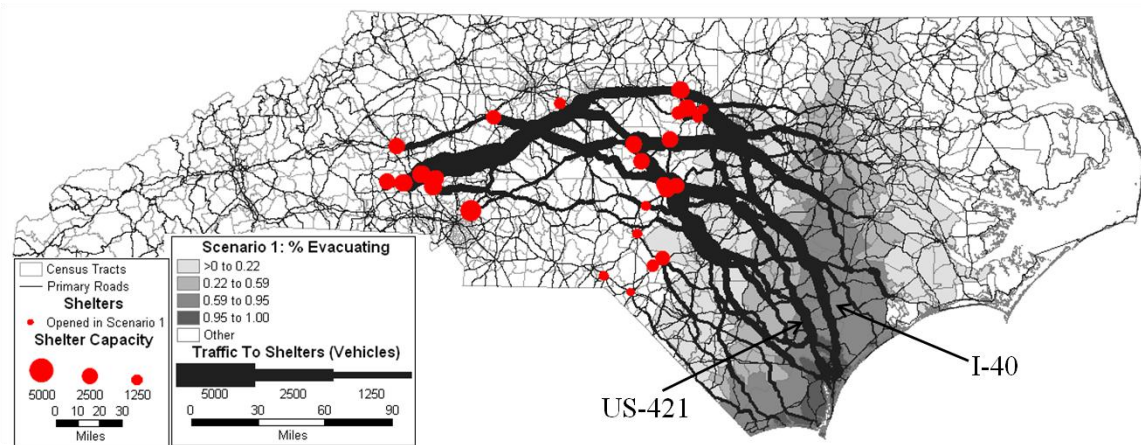
(a)
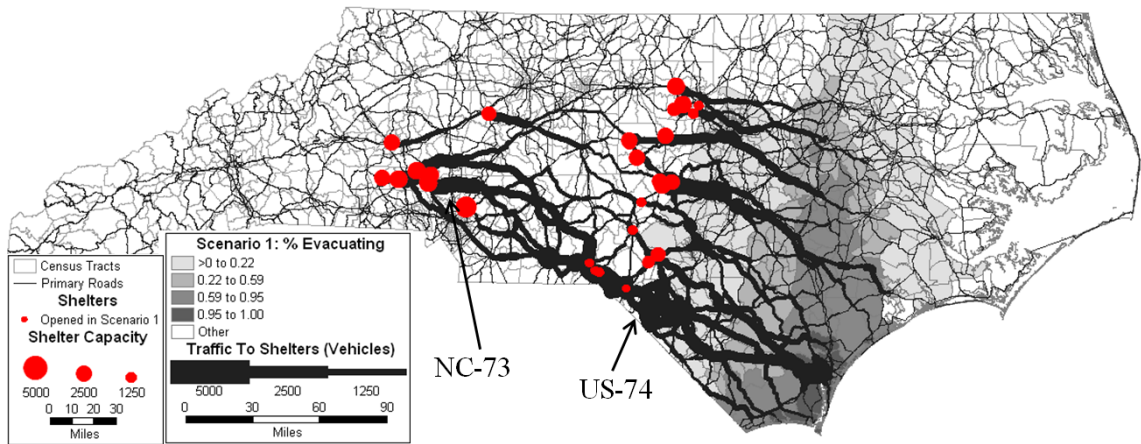
(b)

Figure 3.5. Traffic patterns to shelters in Scenario 1: (a) initial solution, (b) optimized solution

3.6. Conclusions

This paper proposes a DTA-based stochastic bilevel optimization model for the selection of shelter locations in the context of evacuations. This model explicitly takes into account the impact of location decisions on the evacuees' route choice and the traffic dynamics while capturing the stochastic nature of hurricane events. A DUE model is used to describe the evacuees' route choice behavior in each hurricane scenario. A heuristic solution method based on Lagrangian relaxation and scenario decomposition is developed to solve the proposed formulation. To illustrate the applicability of the model to large-scale problems, the case study is conducted for the state of North Carolina and a collection of possible hurricane events. The results illustrate the importance of jointly optimizing shelter options and transportation strategies. Although the state of North Carolina for hurricane events is used as the case study, the methods and computational procedures proposed in this paper can be applied to any region and type of hazard in which the operations of public shelters are needed to house evacuees.

The opportunities for future research exist in at least the following three areas. First, this model focuses on the evacuation population using private vehicles. This may not be possible for those who have no means of private transportation. As Hurricane Katrina demonstrated, it is important to develop a multi-modal choice expansion of the model so that the transportation needs of low-mobility and special-needs groups can be addressed. Second, this model assumes the capacity of each highway facility is known. For emergency evacuations, it is useful to extend the model to optimize the contraflow strategies while optimizing the choice of shelter locations. Third, the model developed in this paper assumes no evacuation from neighboring states into North Carolina occurs.

Further, we only assume that individuals from North Carolina's Outer Banks evacuate into Virginia. In reality, storms commonly move up the coast as do evacuees. By the time a storm hits North Carolina, it may have already hit South Carolina, resulting in people from South Carolina to evacuate into North Carolina. Therefore, it is important to take a multi-state perspective when considering sheltering and evacuation for major storms in the South Atlantic States. The model solution procedure developed in this paper is consistent with the computational demands posed by an application on this scale. An important opportunity for future research is the development of dataset to provide insight into how the different states might coordinate their evacuation and sheltering plans.

Acknowledgements

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APPENDIX

DTA CODE DOCUMENTATION (INPUT, OUTPUT, EXECUTION)

A.1.   Input Parameters and Files

The program requires five types of input data
1. The network data that consist of the locations of origins and destinations in the network, the highway network (links, nodes, link attributes).
2. Parameters that control the model
3. If contraflow is in place, the contraflow data that include initiating and terminating times, restricted links, the links to reverse during contraflow period, and contraflow crossovers.
4. Time-varying OD demand
5. Data used for generating useful output results

File Directories

dirIn   Input directory for all input files
dirOut   Output directory for all output files
All the parameters are set in the first cell of the MATLAB script file datmain.m

Network Data

nbLinks   Number of undirected network links (includes one-way or
                 bidirectional roads)
nbLinks2   Number of directed network links
nbNodes   Number of network nodes
OrgNodes.txt   Node ID for each origin in the network
DestNodes.txt Node ID for each destination in the network
HwyNet.txt   Network attributes of each directed network link. Table A.2 shows a
                 sample network file. It contains the following fields: TlinkID, Fnode,
                 Tnode, Length, Dir, Lanes, Speed0, Time0, Capacity, BPRa and
                 BPRb. The fields are described in Table A.1.

Table A.1. Definition of network attributes

Field	Definition
TlinkID	TransCAD link ID (note: a TransCAD geographic file contains both one-way and two-way links, while the network data file contains only directed or one-way links)
Fnode	Initial node
Tnode	Tail node
Length	Length (mile)
Dir	Direction of flow, = 1 for topology direction, = -1 for backward topology direction

Lanes	Number of lanes
Speed0	Free-flow travel speed (miles per hour)
Time0	Free-flow travel time (hour)
Capacity	Link capacity (vehicles per hour)
BPRa	BPR parameter, indicating the ratio of the travel time at capacity to free-flow travel time
BPRb	BPR parameter, which determines how abruptly the volume-delay curve increases from the free-flow travel time

Table A.2. Sample network file

TlinkID	Fnode	Tnode	Length	Dir	Lanes	Speed0	Time0	Capacity	BPRa	BPRb
1	5	6	0.6	1	2	47	0.0128	4000	0.4	4
1	6	5	0.6	-1	2	47	0.0128	4000	0.4	4
2	7	906	0.53	1	1	47	0.0113	2000	0.2	3
2	906	7	0.53	-1	1	47	0.0113	2000	0.2	3
3	3344	3709	0.34	1	3	60	0.0057	6000	0.4	4
3	3709	3344	0.34	-1	3	60	0.0057	6000	0.4	4
4	21	377	0.5	1	1	25	0.0200	2000	0.2	3
5	25	26	0.01	1	2	40	0.0003	4000	0.2	3
5	26	25	0.01	-1	2	40	0.0003	4000	0.4	4
6	3345	2413	1.16	1	2	40	0.0290	4000	0.4	4
6	2413	3345	1.16	-1	2	40	0.0290	4000	0.4	4
7	35	36	0.74	1	1	22	0.0336	2000	0.4	4
7	36	35	0.74	-1	1	22	0.0336	2000	0.4	4
8	45	46	0.44	1	2	35	0.0126	4000	0.4	4
8	46	45	0.44	-1	2	35	0.0126	4000	0.4	4
9	50	51	0.24	1	1	25	0.0096	2000	0.4	4
10	49	53	0.7	1	1	25	0.0280	2000	0.4	4
11	55	56	1.15	1	2	55	0.0209	4000	0.4	4
12	55	54	1.1	1	1	45	0.0244	2000	0.4	4
13	57	58	0.99	1	1	25	0.0396	2000	0.4	4
14	59	60	0.99	1	2	45	0.0220	4000	0.4	4
15	61	60	1.77	1	2	45	0.0393	4000	0.4	4
16	57	70	0.37	1	3	70	0.0053	6000	0.4	4
17	69	71	0.05	1	1	25	0.0020	2000	0.4	4

Model Parameters

nbhrs The duration of entire traffic analysis period in hours
Dtime The size of the simulation time interval in minutes. For 15-minute intervals, the full assignment period is discretized into small time intervals of 15 minutes each. Interval 1 begins at 0 minutes, interval 2 begins at 15 minutes, interval 3 begins at 30 minutes, etc.
offsethr The time lag relative to the evacuation area. For validation purpose, we consider the average time it takes an evacuee to travel from his/her departure point to the traffic sensor stations.

Contraflow Settings

opContraflow   Indication of whether contraflow is in place, =1 if contraflow is in place, and =0 otherwise.

If *opContraflow*=1, the following parameters and files are also needed:

tbhr, *tehr*	Initiating and terminating hour of the contraflow operation, respectively (i.e., contraflow starts at the beginning of tbhr and terminates at the beginning of tehr)
nbcontr	Number of contraflow routes
TCFlkres_n.txt	Column vector composed of the TlinkIDs for the links that have restricted access due to forced traffic movement for the nth contraflow route to reduce merging congestion. For example, at I-12 and I-59, the traffic from Mississippi was routed onto I-59 North back into Mississippi; at I-12 and US-190, the traffic on I-12 West between Slidell and Covington was diverted onto US-190 West; at I-12 and I-55, traffic traveling I-12 West was routed I-55 North and traffic from I-55 diverted to I-55 North Contraflow via a median crossover.
TCFlkresOn_n.txt	Column vector of the TlinkIDs for the normal entrance ramp(s) to the contraflow lanes of the nth contraflow route. These links are restricted during contraflow period. (TlinkID is a one-way road).
TCFcrss_n.txt'	Column vector of the TlinkIDs for the contraflow crossover(s) for the nth contraflow route (sorted in its flow direction during contraflow, if there are more than one link to represent the whole crossover). These links are open only during contraflow period. (TlinkID is a one-way road).
TCFonramp_n.txt	Column vector of the TlinkIDs for the exit ramp(s) that are reversed to be used as entrance ramps to the nth contraflow route. (TlinkID is a one-way road).
TCFlkcontr_n.txt	Column vector of the TlinkIDs for the links that made up the nth contraflow route, sorted in its normal flow direction. The contraflow segments in New Orleans during Katrina include: SB I-55 normal lanes, SB I-59 normal lanes, EB I-10 normal lanes. (TlinkID is a one-way road).
TCFload_n.txt	Each row contains the TlinkIDs (in contraflow progression direction) for an access point to the nth contraflow route. An access point can be a crossover (in *TCFcrss_n.txt*), contraflow entrance ramp(s) (in *TCFonramp_n.txt*), or a combination of both. (TlinkID is a one-way road).
TCFend_n.txt	Each row contains the TlinkIDs (in contraflow progression direction) for a termination point to the nth contraflow route. A termination point is usually a crossover (in *TCFcrss_n.txt*). (TlinkID is a one-way road).

Synthesis of the time-dependent OD demand

The two parameters *opODD* and *opDT* control how the temporal OD trips are generated.

opODD   Option for generation the time-varying OD demand, =1 if the time-varying OD demand is created from the population of each origin zone, given proportion to each destination and a departure timing curve, =2 if the trip table is based on a given OD vehicle trip table for entire analysis period and a departure timing curve.
The general assumptions for opODD=1 are: (1) The number of vehicles originated from each origin zone can be reasonably estimated by the population and average vehicle occupancy. (2) The evacuees from all origin zones choose the destinations in the same proportion. (3) The departure time distribution applies to all origin zones.

If *opODD*=1, the parameters and files below are needed:

pcocc	Average vehicle occupancy
Population.txt	Population of each census block group in New Orleans
Proportion.txt	Proportion of evacuees heading toward each destination of the given destinations, corresponding to the destination locations in *DestNodes.txt*

If *opODD*=2,

ODvehtrips.txt	OD vehicular demand between each origin (indicated by each row of the table) and each destination (in each column of the table) for the entire evacuation.

opDT   Option for departure time distribution, =1 for a uniform distribution, =2 for a nbhrs Rayleigh distribution (For more than one day, each day is represented by a 24-hour Rayleigh curve), =3 for an empirical distribution.

If *opDT*=2, the following additional parameters are required:

paramRayleigh	The parameter of a Rayleigh distribution, which indicates at the hour at which the mode of the distribution occurs. For example, if the analysis period is 24 hours from 0:00 to 24:00 and the peak occurs at 7:00-8:00am on each day, then *paramRayleigh=8*.

If *opDT*=3,

DepartTiming.txt	The cumulative percentage of evacuees that depart by each hour. Note that the data is the percentage value multiplied by 100. If the time interval is smaller than an hour, interpolation is performed the neighboring points. The coding in this module requires 60 divided by Dtime is an even number (refer to Tmpdist.m).

Data Used in Result Analysis

M	In the statistical analysis of the route travel times, the route travel times between each OD pair are grouped for every M time intervals. If there are 935 origins, 4 destinations, 2880 time

intervals, and M=10, there are $935 \times 4 \times 2880/10 = 1.1$ millions data sets for the route travel times, each with 10 data points.

binsize   The bin size in the analysis of the coefficient of variation for route travel times. Both *M* and *binsize* are used for generating *odtmStats.txt*.

opOutputStation Control whether to output flow data at specific traffic sensor stations, =1 if the observed traffic counts are available for comparing to the model results, =0 otherwise.

If *opOutputStation*=1,

  TRdStation.txt The links with initial nodes at the locations of traffic count stations. If TlinkID is a one-way road, the file is a column vector (i.e. TlinkID uniquely determines a directed link); Otherwise, another column is added to give its topology direction (1 or -1). This data is used for comparing the traffic patterns produced by the DTA model and observed data at specified traffic count stations. For Katrina study, the stations used for comparison purpose are Station 54, Station 27, MODT station, Station 15, Station 42, and Station 88.

A.2. Program Execution

The DTA program is performed by running the following script file in the current work directory:

dtamain.m

A.3. Results

The results include the following:

Interval_nct Number of time intervals required for all evacuees to clear out of the evacuation zones

invehTmp Number of trips entering each link during each time interval

outvehTmp Number of trips exiting each link during each interval

odtmTmp The OD travel time for the trips departing from each origin at each time interval towards each destination

A.4. Output Files

The output files are post-processed based on the model results listed in A.3.

ODDdem.txt   Hourly departure flow rate between each OD pair

ODDtime.txt   Route travel time between each OD pair for trips departing at

	each hour
odtm.txt	Average travel time between each OD pair departing at all time intervals
odtmStats.txt	The statistics for the coefficient of variation (CV) of the travel times between each OD pair for every M departure time intervals. It contains the fields of a vector that specifies the lower edge of each bin (F1), the number of values in each bin (F2), the mean of corresponding travel times (F3), the number of values below or equal to each element in the edge vector (F4), and the number of values above or equal to each element in the edge vector (F5). The bins are distributed between zero and the maximum of all CV values.
	An example is presented in Table A.3. In the first data entry of this example, there are 113,930 CV data points ranging from 0 to 0.01. The average travel time for all of the corresponding trips is 48 minutes. There are zero OD travel time values with a CV less than or equal to 0, and there are 142,120 OD travel time data sets with a CV greater than or equal to 0.
inflowTmp.txt	Number of vehicle trips entering each link during each hour
outflowTmp.txt	Number of vehicle trips exiting each link during each hour
linktimeTmp.txt	Travel time on each link during each hour
linkspeedTmp.txt	Travel speed on each link during each hour
inflowTCAD.txt	The traffic volume file for exporting to TransCAD. It consists of four fields, i.e. TlinkID, AB_flow, BA_flow, and TOT_flow, which are explained in Table A.4.
RdInFlow.txt (if *opOutputStation*=1)	Hourly traffic volumes at the location of traffic count stations, as specified in '*TRdStation.txt*'. The row represents the hour and the column represents the station. It is used to compare to the observed data at the station.

Table A.3. Example data of "odtmStats.txt*"*

F1	F2	F3	F4	F5
0	113,930	48	0	142,120
0.01	15,956	64	113,930	28,192
0.02	4,026	73	129,880	12,236
0.03	2,732	73	133,910	8,210
0.04	2,255	70	136,640	5,478
0.05	1,488	67	138,900	3,223
0.06	857	66	140,390	1,735
0.07	359	66	141,240	878
0.08	256	58	141,600	519
0.09	153	51	141,860	263
0.1	88	49	142,010	110
0.11	20	52	142,100	22
0.12	2	48	142,120	2

Table A.4. Description of the data in "inflowTCAD.txt"

Field	Definition
TlinkID	TransCAD link ID
AB_flow	Flow rate in topology direction
BA_flow	Flow rate in backward topology direction
TOT_flow	Sum of the flow rates from both directions