Ideological Platforms and Probabilistic Voting Equilibria

Jeremy Staum
School of Operations Research
and Industrial Engineering
Cornell University
Ithaca, NY 14853

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Abstract

This paper considers multicandidate competition from a new perspective. By Duverger's law, plurality rule tends to lead to the dominance of two main parties, such that a third party has almost no chance of entering government. Nonetheless, competition from the third party can influence the main parties' choices of platform. This influence can be either positive or detrimental from the point of view of the third party's candidate, who could then choose a platform in order to have an effect on the platform of the winning party. This paper describes an approach that such a decision-maker could take, and illustrates with numerical examples.

1 Introduction

What happens when some candidates or political parties choose their campaign platforms in pursuit of electoral success, while ideology motivates others' positions? Within the public choice paradigm, research on electoral competition often assumes that all platforms maximize an objective such as vote share or rank of vote share: the electoral competitors select platforms purely instrumentally, in the hope of winning office. The focus is then on studying the Nash equilibria of the game among these competitors: characterizing equilibria and finding conditions for their existence or uniqueness. What if some candidates or parties are not "playing the game" of office-seeking, but rather adopting platforms that reflect ideological convictions?

The present paper treats this question primarily in the context of plurality rule (first past the post) electoral competition. Duverger's law suggests that plurality rule tends to result in a two-party system, where third parties have almost no chance of entering into a governing coalition in the legislature or of winning a contest for an executive position. (See, for instance, Mueller 1989 §12D.) However, the activities of third parties may significantly influence the outcome of an election or the main parties' choices of platforms: a good example is Ross Perot's Reform Party in the 1992 U.S. presidential election. To model this situation, I study a game of electoral competition in which two main parties choose platforms in order to maximize vote rank (i.e. they seek to win the election, or failing that, at least to reach a statistical dead heat) in the presence of a third party that chooses its platform differently. The present analysis may also apply to parliamentary systems with proportional representation when there are two main parties whose actions are explicable in terms of vote maximization, for instance, Israel (Schofield et al. 1998b).

A more widely applicable approach to elections with proportional representation accounts for parties' incentives not only to win many votes but to position themselves well for post-election coalition bargaining (Schofield et al. 1998a, 1998b). The related work of Schofield and Parks (2000), which further takes into account policy motivation, is a bold attempt to tackle a problem of monumental scale. In this model, all parties (effectively considered as unitary actors) desire both to gain office (i.e. participate in a governing coalition) and that the policy of the governing coalition be as close as possible to their own ideal points.

Then we can understand party platforms as *Downsian*, *strategic*, or *sincere*. Downsian platforms are vote- or rank-maximizing and chosen by instrumental deliberation, while sincere platforms result from an aggregation of the sincerely expressed preferences of party members or elites. Strategic platforms are also instrumental, but directed toward both office-seeking and ideological goals. Since both these goals appear to motivate political actors in reality, this is one reason to study strategic platforms. Another reason is that both Downsian and sincere platforms can be suboptimal on their own terms. In a multiparty democracy with proportional representation, Downsian platforms need not maximize a party's chance of entering a governing coalition, because coalition formation requires compromise among coalition partners' platforms: thus a party might be better off with a platform that wins somewhat fewer votes but makes it a more attractive coalition partner. Similarly, sincere platforms need not be the best way of bringing about desired policy changes. The present paper explores this issue in detail.

The research just cited provides evidence that party platforms are often not Downsian—at least, they do not appear to be vote-maximizing within the models estimated. Sorting out the influence of office-seeking and ideological motives, and identifying the effect of partisan, electoral, and legislative institutions on the three-stage process of caucus or primary, general election, and coalition bargaining, appears to be a formidable problem of tremendous scope. The present work takes place in a simpler setting, but the same threefold categorization of platforms informs it. In the absence of coalition bargaining, Downsian behavior makes sense for parties without policy motivations. Therefore, in the present paper, the term *ideological*, when applied to platforms, means either strategic or sincere, since both of these types of platforms can be motivated only by attempts to influence or expressions of preferences about policy.

In a two-party plurality rule system, a third party has virtually no chance of entering government, and thus one might expect its platform to depend primarily on ideology, whether sincerely or strategically. There is a possibility for strategic third-party platforms because they can affect the existence and location of Nash equilibria in the game among Downsian parties. The apparent contradiction between this conclusion and some previously reported work is explained in an appendix. In Section 2, I discuss criteria for determining which strategic platforms are feasible and whether they are successful, and consider the problem of strategic optimization for an ideological third party. Section 3 examines the outcomes that can result from the introduction of ideological party platforms by computing equilibria in simple examples.

In Section 4, I conclude by summarizing this paper's main results. These are engineering recommendations for campaign strategists and reformers or designers of electoral systems, as well as suggestions about the orientation of future research in this area. This paper's model is sufficiently simplistic that its point is not to elaborate a testable hypothesis about the platforms of candidates for president of the U.S., for instance. Parties are not unitary actors, so it seems hard to exclude the influence of ideological motivation on main parties' platforms; likewise, even strongly ideologically motivated third parties have reasons to care about the magnitude of their vote share, such as desire to raise awareness of their message in the long run. The advantages of the simple setting of this paper include its directly comparability to earlier work (Adams 1999, 2000) and the transparency it allows in the demonstration of the way in which the entry of a third party changes the incentives of main parties. The goal is to raise awareness of the underappreciated subtleties of this situation, and provide a framework within which to address questions such as "Did Ralph Nader's candidacy hurt the left in the 2000 U.S. presidential election?" In particular, the goal is to begin to develop tools that will help decision-makers to answer such questions in the future, before taking action. In the end, this analysis will have served its purpose if it enables such engineering to be done.

2 Assessing Ideological Platforms

How are we to assess the effects of ideological third party platforms in a two party, plurality rule system? One expects the third party's candidate to lose because its platform is not designed to maximize vote rank, while the main parties' are. In models with party loyalty, the third party is also at a great disadvantage because the main parties have more partisans. Since the third party then has almost no chance of participating in government, in typical electoral situations, I model its choice of platform as being ideological.

At one extreme, there is the sincere platform, the outcome of democratic intra-party processes in which party members vote sincerely on the basis of their convictions. The members' motivation for participating in this process might be pleasure in expressing their beliefs, associating with like-minded individuals, and identifying themselves with a cause of which they approve. Perfectly sincere and non-strategic behavior takes no account of the effect that the third party's activity has on the outcome of the election. The logic of collective action (Olson 1965) suggests that party members will be motivated by their selective benefits, described above, not group benefits, namely the effect their party has on electoral outcomes.

As the other extreme, consider the situation of a decision-maker who has the ability to change the third party's platform. For instance, candidates such as Ross Perot and Ralph Nader have overshadowed (or even created) their parties and thus appeared to have wide latitude in choosing a platform or at least determining public perception of it. Suppose such a candidate is willing to act strategically and insincerely to maximize policy utility from the winning platform. What platform should such an ideologically motivated decision maker adopt?

Finding the answer depends on the ability to determine how the third party's platform influences implemented policy. This calculation relies on the approximate truth of the following assumptions:

1. All candidates share knowledge of how voters will respond to any combination of platforms.[1]

2. The Downsian candidates' platforms constitute a Nash equilibrium, if one exists.

3. Implemented policy reflects the platform of the winning candidate.

The notation to be used is that there are three candidates, of which the kth adopts platform y_k, which is the kth component of the ordered K-tuple $\mathbf{y}$.[2] Voter i's probability of voting for

[1] Theoretically, it would possible to consider the calculations of a candidate with accurate knowledge of voter responses and knowledge of the other candidates' erroneous beliefs, but this would seem to be of little interest.

[2] In a spatial model, $\mathbf{y}$ is a matrix and its kth row y_k is a point in "issue space."

candidate k given the platforms $\mathbf{y}$ is $p_{ik}(\mathbf{y})$. Candidate k's expected vote is then

$$P_k(\mathbf{y}, s) = \sum_{i=1}^{I} p_{ik}(\mathbf{y}, s), \tag{1}$$

where I is the number of voters in the electorate.

Then the third party candidate can compute $y_1^*(y_3)$ and $y_2^*(y_3)$, the subgame perfect equilibrium platforms of the Downsian candidates given that she adopts platform y_3. Her prediction is that the platforms will be

$$\mathbf{y}^* = (y_1^*(y_3), y_2^*(y_3), y_3).$$

Then the third party candidate's objective is

$$\max_{y_3} u(z^*(y_3)) \quad \text{s.t.} \quad z^*(y_3) = \left\{ y_k^*(y_3) | P_k(\mathbf{y}^*) = \max_j P_j(\mathbf{y}^*) \right\} \tag{2}$$

where u is her utility function. To make this objective well-defined, this discussion focuses on the simple scenario in which there exists a unique Nash equilibrium and any tie in expected vote is between candidates with the same platform.[3]

This provides a criterion by which to judge whether an ideological platform is successful or not. Let $\tilde{y} = (\tilde{y}_1, \tilde{y}_2)$ be the Nash equilibrium in the game between the two Downsian candidates in the absence of a third party, and let

$$\tilde{z} = \left\{ \tilde{y}_k | P_k(\tilde{\mathbf{y}}) = \max_j P_j(\tilde{\mathbf{y}}) \right\} \tag{3}$$

be the winning platform. (Again, assume this is well-defined.) Then say that an ideological platform y *succeeds* when $u(z^*(y)) > u(\tilde{z})$ and *backfires* when $u(z^*(y)) < u(\tilde{z})$, i.e. when it respectively increases or decreases the candidate's policy utility from the winning platform.

Describing success or backfiring with respect to an entire party, for instance when a party caucus or primary selects a sincere platform, is not so simple. One might just change the definition to use the sum of members' utilities, but this would rely on interpersonal comparison of cardinal utility, which is highly suspect.[4] Still, it is clear that a platform succeeds or backfires when it respectively increases or decreases the policy utility of all a party's members.

There also have to be constraints on insincere strategic behavior. For instance, would it be sensible to attempt to advocate a right-wing platform in an attempt to secure an outcome desirable

[3] In this scenario, which is quite common in the models under study here, there is near-certainty about the winning platform. Generalizing this framework for third party strategic optimization to cases where the electoral outcome is not certain given the platforms appears not to be trivial. It would be necessary to put a probability distribution over multiple Nash equilibria, or over all platforms when there is no equilibrium. With probabilities of outcomes specified, each choice of third party platform yields a random variable for electoral outcome, and it is not necessarily easy to model the third party candidate's preferences over these random variables. It is easy to use a utility function to describe an agent's preferences over sure outcomes, but harder to use expected utility maximization to describe her preferences over lotteries—indeed, research on the psychology of decision-making suggests that it is impossible to do so in certain settings.

[4] In this context it is worth stressing that probabilistic voting theory does *not* have to rely on interpersonal comparison of cardinal utility, despite appearances. Models such as that in equation (7) can have a purely behavioral interpretation: they link candidate platforms $\mathbf{y}$ to observable behavior, the voting decision. The relative magnitude of the evaluations $e_i(y)$ of voter i and $e_j(y)$ of voter j tells us not about the relative number of utils they derive from policy y, but about their relative likelihood of voting for a candidate proposing y, which is in principle observable.

from a left-wing perspective? It is difficult to imagine such a conspiracy successfully fooling the public or perpetuating itself.

For one thing, there is the danger that the electorate will perceive insincere strategic behavior as insincere, resulting in a great diminution of votes. The behavioral model of voter response to candidate platforms we are studying is predicated on the assumption that voters would find a candidate's advocacy of any platform equally credible. The implausibility of this assumption has prompted research on platform inertia, where parties change their platforms only gradually over time. (See, for instance, Miller and Stadler 1998.) The explicit introduction of the time dimension to the model complicates matters, as opposed to implicit reliance (in the present paper as elsewhere) on the assumption that parties are capable of adjusting their platforms to reach Nash equilibrium between elections or over the course of a campaign.

Here I propose a different restriction on platform mobility: a constraint that an ideological candidate k's utility from her own platform should be no less than her utility from any other candidate's platform. Given the other candidates' platforms $\mathbf{y}_{-k}$, the vector containing all platforms except y_k, her platform must be in the set

$$\mathcal{Y}_k(\mathbf{y}_{-k}) = \left\{ y | u_k(y) \geq \max_{j \neq k} u_k(y_j) \right\}. \tag{4}$$

In the present situation, we consider one ideological candidate who can predict the Downsian candidates' equilibrium platforms $y_j^*(y_3)$ as a function of her platform. Her set of credible platforms is

$$\mathcal{Y}_3 = \{ y | u_3(y) \geq \max\{u_3(y_1^*(y)), u_3(y_2^*(y))\} . \tag{5}$$

This constraint does not apply to Downsian candidates, who are assumed to lack policy motivation.

It would also seem to be difficult to run a party organization of members who prefer another party's platform to their own. Presumably party activists would then tend to drop out, unwilling to be associated with a position of which they do not approve, and new recruits would tend to be those who do prefer this party's platform. However, a general equilibrium analysis of party membership is well beyond the scope of this paper. Because of this and the difficulties inherent in aggregating members' preferences, in the examples of the following section, I focus on strategic platforms of ideological candidates and sincere platforms of ideologically homogenous parties.

3 Examples of Outcomes

3.1 Setup

The following examples illustrate how ideological platforms can either succeed or backfire. As explained in the previous section, success or backfiring depends on the difference between the outcomes of Game 2, of competition between two Downsian candidates alone, and of Game 2+1, in which two Downsian candidates select platforms after an ideological third candidate has already announced her platform. In both games, the Downsian objective is maximization of the margin of victory. The kth candidate's margin of victory is

$$V_k(\mathbf{y}) = P_k(\mathbf{y}) - \max_{j \neq k} P_j(\mathbf{y}). \tag{6}$$

The justification for this choice is presented in Example 2. I present several examples, all using the conditional logit model with squared Euclidean distance loss.

The conditional logit model corresponds to choosing the function $F = \exp$ in the Luce model equation

$$p_{ik}(\mathbf{y}, s) = \frac{F(se_i(y_k))}{\sum_{j=1}^{3} F(se_i(y_j))} \tag{7}$$

for voter i's probability of voting for candidate k. The function e_i describes the ith voter's evaluation of platforms. A common example in spatial modeling is squared Euclidean distance loss, $e_i(y) = -\|x_i - y\|^2$, where x_i is the voter's ideal point, but it is not necessary to use a spatial framework, nor to assume that the evaluation equals the economic utility the voter would have if the platform were enacted. The function F specifies the influence of evaluation on voting probabilities. The issue salience parameter s controls the strength of this influence: for instance, at $s = 0$ platforms have no effect on voting decisions. For comments about the realism of this widely-used model, see Adams (1999).

The conditional logit model it is also a special case of the class of random utility models, used for example by Adams (2000), Lin et. al. (1999), and Schofield et. al. (1998a,b). Here voter i's utility from voting for candidate k with platform y_k is

$$U_{ik} = bu_i(y_k) + cv_{ik} + \epsilon_{ik} \tag{8}$$

where $u_i(y_k)$ is i's policy utility,[5] v_{ik} is a constant nonpolicy utility, and ϵ_{ik} is a random term. The nonnegative parameters b and c govern the salience of policy and nonpolicy factors respectively. Now i votes for the candidate k providing the highest total utility U_{ik}. If the random utility terms ϵ_{ik} are i.i.d. type I extreme value, then the probability that i votes for k is

$$p_{ik}(\mathbf{y}) = \frac{\exp(bu_i(y_k) + cv_{ik})}{\sum_{j=1}^{3} \exp(bu_i(y_j) + cv_{ij})} \tag{9}$$

(Adams 2000).

Because sufficient interesting outcomes occur within a narrow class of simple examples, there is no need to venture outside this class of examples for purposes of this paper. The advantage of using the conditional logit model is not that it is realistic, but that it is possible to say that any behavior seen in the examples presented here is possible *a fortiori* in the class of Luce models, random utility models, and all generalizations thereof.

The examples are based on the simplified model of the 1992 U.S. presidential election in Adams (2000), which uses equations (8) and (9). The utility from policy is Euclidean squared distance loss, $u_i(y) = -\|x_i - y\|^2$, where x_i is the voter's ideal point. The nonpolicy utility has the interpretation of party loyalty: voters tend to vote for the candidate of the party with which they identify themselves. The model for this phenomenon is that v_{ik} takes the value $+1$ when both voter i and candidate k are Democrats or both are Republicans, -1 when one is a Democrat and the other a Republican, and 0 when either is an independent (i.e. has no main party affiliation). All voters with the same party affiliation have the same ideal point. Table 1 gives the ideal point and fraction of the electorate for each type of voter. The issue space in which voters and candidates locate themselves is one-dimensional, with lower values representing policies that are further "left" in the ideological sense of contemporary American politics. The base case for examples has policy salience parameter $b = 5$ and nonpolicy salience parameter $c = 1$.

[5] This is entirely analogous to the "platform evaluation" e_i of equation (7). It is related to voter i's enthusiasm for voting for a platform, not necessarily to his economic utility if that platform's policies were to be implemented.

Party	Weight	Ideal Point
D	50%	$x_D = 0.25$
R	40%	$x_R = 0.75$
I	10%	$x_I = 0.5$

Table 1: Voters in the 1992 U.S. Presidential Election

To analyze these examples, it will be necessary to compute Nash equilibria. Because of the difficulty of analytical computations, many authors rely on numerical methods. Lin et. al. (1999) tested twenty-seven platform vectors (in which three candidates could be located at any of three voters' ideal points) as potential equilibria using grid search. A drawback of this approach is that it has no chance of identifying equilibria other than those specifically pre-selected for testing. Adams (2000) and Schofield et. al. (1998b) use a technique they call "simulation," because it simulates what would happen if candidates took turns changing their platforms to the best response to everyone else's platform. Schofield et. al. (1998b) refer to this as "iteration of one-step optimization by all parties." See Adams (2000) for an account of the shortcomings of this method, which is time-consuming and unreliable.

Fortunately, we now possess better methods, such as the elegant and insightful gradient-following algorithm of Merrill and Adams (2001). The results of this paper come from a more general algorithm for computing Nash equilibria of multiplayer infinite games, the relaxation algorithm described by Krawczyk and Uryasev (2000). Despite these advances, the absence of wholly satisfactory numerical methods is a severe handicap for computational probabilistic voting: little has been said about the important cases of games with zero or multiple Nash equilibria. Inability to demonstrate absence of equilibrium remains particularly troublesome, and I therefore refrain from presenting examples in which there is equilibrium in Game 2+1 given the third candidate's platform, but there is not in competition between two or three Downsian candidates (*stabilization*), or vice versa (*destabilization*).[6]

3.2 Examples

First, let us examine the special case $c = 0$, where there is zero utility from nonpolicy factors, in particular, no party loyalty. This corresponds to the setting of Adams (1999), discussed in the appendix.

Example 1 (Backfiring without party loyalty) *The parameters are $b = 5$ and $c = 0$. First consider Game 2, where just two Downsian candidatees compete. The only Nash equilibrium found is given in Table 2. Next consider Game 2+1 with the third candidate adopting platform $y_3 = 0.25$. The only Nash equilibrium found is given in Table 3.*

Table 2 shows the classic equilibrium in which both candidates adopt the central platform $M = 0.475$. This is the mean of all voters' ideal points. Because of the use of Euclidean distance, it is also the "minimum-sum point," which minimizes the sum of distances from all voters' ideal points. It is also the "most popular platform," which maximizes the sum of all voters' utilities.

[6] I have some examples that I conjecture are of these types, but as they rely on unrealistic symmetry, they are of little interest.

Under assumptions of concavity or sufficiently low policy salience, standard theorems assert that there is a unique Nash equilibrium where both candidates adopt this central platform M (e.g. Adams 1999, Coughlin 1992, Lin et. al. 1999). In this case, all groups of voters split their votes equally among the two candidates, the winner is determined by chance, but the winning platform must be M.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate 1	0.475	50%	50%	50%	50%
Candidate 2	0.475	50%	50%	50%	50%

Table 2: Equilibrium for Game 2 in Example 1

In Table 3, Downsian candidates still adopt the same platform, but it has moved to the right, at $y_1 = y_2 = 0.516$. This happens because the presence of the third candidate on the left, at 0.25, makes voters on the left most likely to vote for her, and thus less attractive to the Downsian candidates. A move to the right away from $M = 0.475$ wins more votes on the right than it loses on the left. Voters on the left have become less responsive to the Downsian candidatees' moves, because of their propensity to vote for the third candidate. Note that policy salience $b = 5$ is low in the sense that Democratic voters have probability only 41.6% of voting for a candidate located at their ideal point versus two right-of-center candidates; in this example, random factors such as evaluation of candidates' characters must be very important. Again, it is a matter of chance whether candidate 1 or 2 wins the election, but the winning platform is certainly 0.516.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate 1	0.516	35.1%	29.2%	42.1%	36.6%
Candidate 2	0.516	35.1%	29.2%	42.1%	36.6%
Candidate 3	0.250	29.8%	41.6%	15.9%	26.8%

Table 3: Equilibrium for Game 2+1 in Example 1

The platform 0.516 provides less utility to Democrats than $M = 0.475$ did. Therefore the sincere platform $y_3 = 0.250$ backfires. A Democrat disgruntled with the centrist policy M would be ill-advised to form a splinter party of voters with ideal point $x_D = 0.250$ and campaign on a sincere platform, because it would produce an outcome even worse than M, from their point of view.

The next example justifies the use of margin of victory maximization throughout the rest of the paper.

Example 2 (Maximizing vote share vs. margin of victory) *The parameters are $b = 5$ and $c = 1$. First consider Game 3, where three Downsian candidates compete. In the two cases where they all maximize vote share, and where they all maximize margin of victory, the only Nash equilibrium found is given in Tables 4 and 5 respectively. Next consider Game 2+1 with the third candidate adopting platform $y_3 = 0.26$. In the two cases where the two Downsian candidates maximize vote share, and where they both maximize margin of victory, the only Nash equilibrium found is given in Tables 6 and 7 respectively.*

Up to rounding, the results in Table 4 for the stylized 1992 U.S. presidential election with vote share maximization agree with those reported by Adams (1999). If the candidates were attempting to maximize the margin of victory, the equilibrium outcome would be different, as in Table 5. The Democratic candidate still wins, but with a slightly lower vote share and margin of victory. The main parties have somewhat more centrist platforms, reflecting their motivation to take votes away from each other. The independent candidate, instead of being the centrist option, runs on nearly the same platform as the winning Democrat, because of the motivation to take votes away from her.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.358	41.4%	72.3%	5.2%	31.7%
Candidate R	0.600	35.8%	5.6%	74.1%	33.4%
Candidate I	0.471	22.8%	22.1%	20.7%	34.9%

Table 4: Equilibrium for Game 3 with Vote Share Objective in Example 2

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.397	40.1%	68.3%	6.6%	33.0%
Candidate R	0.558	36.9%	6.4%	75.7%	34.2%
Candidate I	0.394	23.0%	25.3%	17.7%	32.8%

Table 5: Equilibrium for Game 3 with Margin of Victory Objective in Example 2

It is debatable what the objective is or should be for office-seeking parties in multiparty competition (such as Game 3) when it seems that some candidates are doomed to lose no matter what platforms they adopt, as long as their opponents do not behave foolishly. In Game 2, the objectives are the same, because maximizing one's own vote implies minimizing one's opponent's vote, and thus maximizing the margin of victory. In Game 2+1, the objectives are different, and maximizing margin of victory makes more sense. In Table 6, the objective is maximizing vote share, and the Democrat loses while earning 39.76% of the vote. In Table 7, the objective is maximizing margin of victory, and the Democrat wins while earning 39.57% of the vote. Assuming the margin of victory objective ensures that candidates will attempt to win when possible, which the vote share objective does not.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.3616	39.76%	67.6%	6.2%	34.5%
Candidate R	0.5672	39.82%	5.9%	82.9%	37.1%
Candidate I	0.2600	20.42%	26.5%	10.9%	28.4%

Table 6: Equilibrium for Game 2+1 with Vote Share Objective in Example 2

The next example explores the effect of the third party's platform on the outcome of the election.

Example 3 (Effect of third party platform) *The parameters are $b = 5$ and $c = 1$. First consider Game 2, with only two Downsian candidates. The only Nash equilibrium found is given in Table 8. Next consider Game 2+1, with the two Downsian candidates maximizing margin of victory. The only Nash equilibrium found is illustrated in Figure 1, which graphs the equilibrium*

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.4085	39.57%	66.0%	7.5%	35.6%
Candidate R	0.5481	39.47%	6.5%	81.4%	36.6%
Candidate I	0.2600	20.96%	27.5%	11.1%	27.8%

Table 7: Equilibrium for Game 2+1 with Margin of Victory Objective in Example 2

Democratic (lower, dashed line) and Republican (upper, dashed-dotted line) platforms against the platform adopted by the third candidate.

Introducing party loyalty changes the equilibrium in the game between two Downsian candidates from that shown in Table 2, where there was no party loyalty. Then the equilibrium was for both candidates to adopt the platform $M = 0.475$, resulting in a dead heat. With party loyalty, the equilibrium shown in Table 8 is for both candidates to adopt the platform 0.478, and the Democratic candidate wins due to the greater number of Democratic voters. The explanation for the slight rightward shift of equilibrium platform is that voters are no longer equally important once there is party loyalty: it makes more sense for both candidates to court the "swing" or independent voters located at $x_I = 0.5$ whose responsiveness to platform changes is not overwhelmed by party loyalty. Thus the equilibrium shifts right from $M = 0.475$ towards $x_I = 0.5$. The change is small in this example because there are few independent voters and their ideal point is close to the previous equilibrium.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.478	53.8%	88.1%	11.9%	50.0%
Candidate R	0.478	46.2%	11.9%	88.1%	50.0%

Table 8: Equilibrium for Game 2 in Example 3

In Example 3, one of the Downsian candidatees wins Game 2+1, so in Figure 1 the winning platform (solid line) coincides with one of their platforms. One feature of this figure is that the dispersion of the Downsian candidates' equilibrium platforms is greatest when the third candidate adopts a centrist position. In this case the main candidatees have less to gain from courting the centrist independent voters, and more to gain from courting their own partisans, to reduce their defection to the third party candidate. On the other hand, as the third candidate adopts an increasingly extreme position in either direction, her vote share approaches zero, and the equilibrium platforms of the Downsian candidates approach 0.478, which is the equilibrium in the absence of the third party shown in Table 8. In this case the Democratic candidate wins due to the inherent advantage of more Democratic than Republican voters in this example. However, the Republican candidate wins when the third candidate adopts a platform in an interval whose left endpoint is between -0.038 and -0.037 and whose right endpoint is between 0.254 and 0.255.

This interval includes the sincere Democratic platform $x_D = 0.25$. (In other examples it does not.) The equilibrium for this choice of third party platform is in Table 9. This platform backfires, causing the Republican candidate to win with a platform of 0.547, which is worse than the 0.478 which wins in the absence of the third party. In Example 1, where there was no party loyalty, the sincere Democratic platform also backfired, resulting in winning platform 0.516. In this case, with party loyalty, the Democratic candidate can not effectively pursue the Republican candidate by moving right, which would gain few Republican votes and would lose substantial Democratic votes to the third candidate. The third candidate takes so many Democratic votes that the Democratic candidate loses by a tiny amount, causing a backfire worse than that which occurred without party loyalty.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.4095	39.60%	65.9%	7.6%	35.8%
Candidate R	0.5470	39.67%	6.5%	81.8%	36.9%
Candidate I	0.2500	20.74%	27.6%	10.6%	27.3%

Table 9: Equilibrium for Game 2+1 when $y_I = 0.25 = x_D$ in Example 3

Continuing to suppose an ideal point of $x_D = 0.25$ for the third candidate, consider her problem of strategic optimization. Maximum utility from the winning platform comes when she adopts a platform y_I between 0.50 and 0.55, resulting in a Democratic victory with a platform between 0.392 and 0.393. An example occupies Table 10. The objective function is so flat here that it hardly makes sense to attempt to identify a precise optimum, especially considering that, as discussed in Section 2, adopting a platform $y_I > y_D > x_D$ is hardly feasible: it would neither be credible to voters nor acceptable to party activists.

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.392	42.4%	73.5%	6.0%	32.3%
Candidate R	0.558	34.6%	6.9%	69.4%	33.7%
Candidate I	0.540	23.1%	19.7%	24.6%	34.0%

Table 10: Equilibrium for Game 2+1 when $y_I = 0.540$ in Example 3

The credibility constraint is tight when $y_I = 0.397$, resulting in nearly the same equilibrium platform for the Democratic candidate. Table 11 illustrates this equilibrium. This is the optimal strategic platform for a third candidate with ideal point $x_D = 0.25$, in terms of the maximization problem defined by objective (2) and constraint (5).

	Platform	Total Vote	Voter D	Voter R	Voter I
Candidate D	0.397	40.1%	68.4%	6.6%	32.9%
Candidate R	0.558	36.8%	6.4%	75.7%	34.1%
Candidate I	0.397	23.0%	25.2%	17.9%	32.9%

Table 11: Equilibrium for Game 2+1 when $y_I = 0.397$ in Example 3

The following examples show the effect of varying the salience parameters b and c.

Example 4 (Effect of policy salience) *The parameters are $c = 1$ and $y_I = 0.25 = x_D$. Consider Game 2+1, with the two Downsian candidates maximizing margin of victory. The only Nash equilibrium found is illustrated in Figure 2, which graphs the equilibrium Democratic (lower, dashed line) and Republican (upper, dashed-dotted line) platforms against the policy salience b.*

The Democratic candidate wins when policy salience b is less than a critical value between 4.9 and 4.95, and loses when it is more. As policy salience approaches zero, the equilibrium platforms of the Democratic and Republican candidates approach limits near 0.409 and 0.549 respectively. (Of course, when policy salience is zero, any pair of platforms is a Nash equilibrium.) Thus we see that low policy salience allows the sincere third party platform $y_I = 0.25 = x_D$ to succeed, while high policy salience causes it to backfire. When policy salience is high, the third candidate at y_I takes so many votes from the Democratic candidate that she loses; when it is low, the effect of party loyalty is great enough that this does not happen. Equilibrium appears to break down above a level of policy salience of roughly $b \approx 26$, but this result is not very reliable.

Example 5 (Effect of party loyalty) *The parameters are $b = 5$ and $y_I = 0.25 = x_D$. Consider Game 2+1, with the two Downsian candidates maximizing margin of victory. The only Nash equilibrium found is illustrated in Figure 3, which graphs the equilibrium Democratic (lower, dashed line) and Republican (upper, dashed-dotted line) platforms against the nonpolicy salience c.*

With no party loyalty ($c = 0$) the equilibrium is for both candidates to adopt the platform 0.516, as described in Table 3. As c approaches infinity, the equilibrium platform for both candidates approaches $x_I = 0.5$. With infinite party loyalty, both parties can take their supporters wholly for granted and concentrate entirely on appealing to independents. The Republican candidate wins when nonpolicy salience c is less than a critical value between 1 and 1.1, and loses when it is more. Much as in the discussion of Example 4, low party loyalty means that the third candidate located at $y_I = 0.25$ takes many votes away from the Democratic candidate, who wins for high party loyalty because of a greater number of partisans.

4 Conclusion

Far from being irrelevant, a third party campaigning in a plurality rule contest can change the winner of an election and influence the platforms adopted by main candidates who try to win the election, even when there is virtually no chance of its candidate winning. Assuming a connection between the winner's platform and implemented policy, the third candidate's platform can either succeed, by altering the winning platform to one that she prefers, or backfire, when she regards the new winning platform as inferior to the old. Both these outcomes are possible, depending on the details of the voters' responses to candidates' platforms. In particular, sincere platforms, those reflecting a candidate's beliefs or preference aggregation among party members, can backfire.

This provides a strategic incentive for a third candidate to offer a platform that is neither sincere nor vote-maximizing. There are difficulties in analyzing strategic decision-making on the part of parties, which are not unitary actors, but one may take the standpoint of a candidate whose personal fame, charisma, or wealth grant substantial ability to influence public perception of a third party's platform: in recent American politics, consider Ross Perot and his Reform Party, Howard Stern and the New York State Libertarian Party, and Ralph Nader and the Green Party. This paper demonstrates the approach such a candidate or campaign strategist might take in choosing a strategically optimal platform.

To make this approach successful in practice will require further research. Most importantly, it depends on accurate predictions of how voters respond to candidates' platforms: the appropriate models seem to be those incorporating both policy and nonpolicy factors, such as that of Erikson and

Romero (1990) and later empirical work in the same tradition. Also desirable is a superior algorithm for computing Nash equilibria in games of electoral competition. This would be instrumental in promoting a computational probabilistic voting that can assess the impact of political institutions, such as high but surmountable barriers to entry for third candidates, and help political actors make better decisions, such as platform choice for ideological candidates.

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Appendix

This paper's conclusion that third party platforms can influence the equilibrium platforms of Downsian candidates is at odds with the main result of Adams (1999). There it was asserted that under a Luce model (7), if there exists a unique most popular platform M, then there exists a threshold η of issue salience such that

> For $0 < s < \eta$, M represents a dominant strategy for each vote- or rank-maximizing (candidate), regardless of the platforms proposed by rival (candidates).

because any candidate k's expected vote $P_k(\mathbf{y})$, as well as her margin $P_k(\mathbf{y}) - P_j(\mathbf{y})$ over any other candidate j, is increasing in her composite voter evaluation $E(y_k)$, regardless of the other components of the vector $\mathbf{y}$, that is, the other platforms. However, this conclusion is not quite correct.

The proof demonstrates that the expected vote function $P_k(\mathbf{y}, s)$ satisfies the properties

$$\forall \mathbf{y},\ \ P_k(\mathbf{y}, 0) = \frac{1}{J}$$

that is, with zero issue salience s, the expected vote is the same for each of the J candidates, and

$$E(y_k) > E(y'_k) \Rightarrow \left.\frac{\partial P_k(\mathbf{y}, s)}{\partial s}\right|_{s=0} > \left.\frac{\partial P_k(\mathbf{y}', s)}{\partial s}\right|_{s=0}$$

that is, the partial derivative of expected vote P_k with respect to issue salience at $s = 0$ is increasing in composite voter evaluation $E(y_k)$. However, this does not imply, as asserted, that expected vote is increasing in composite voter evaluation for all sufficiently small issue saliences $s \in (0, \eta)$:

$$\exists \eta > 0 \ni \forall s \in (0, \eta), \mathbf{y}, \mathbf{y}' \ni E(y_k) > E(y'_k),\ \ P_k(\mathbf{y}, s) > P_k(\mathbf{y}', s). \tag{10}$$

What is true is that for any two platform vectors $\mathbf{y}$ and $\mathbf{y}'$ there is some positive level $\eta = \eta(\mathbf{y}, \mathbf{y}')$ *which depends on the pair of platform vectors* such that for smaller issue salience s, candidate k's expected vote is greater when her composite voter evaluation is greater:

$$\forall \mathbf{y}, \mathbf{y}' \ni E(y_k) > E(y'_k),\ \ \exists \eta > 0 \ni \forall s \in (0, \eta),\ \ P_k(\mathbf{y}, s) > P_k(\mathbf{y}', s). \tag{11}$$

This is true by continuity of the partial derivative $\partial P_k(\mathbf{y}, s)/\partial s$, which holds by the assumption of continuous differentiability of F. Therefore in particular we can conclude that for any $\mathbf{y}$ whose kth component $y_k \neq M$, there exists a sufficiently low level of issue salience such that it is better for candidate k to choose the most popular platform M than y_k, given the other platforms (the other components of $\mathbf{y}$). However, there does not exist a positive level of issue salience that makes M better than all possible platforms y_k.

To sum up, any platform other than the most popular M is suboptimal once issue salience s is sufficiently small. However, for any positive level of issue salience, there may be platforms other than M that provide a greater expected vote than M does, given the other candidates' platforms. Thus M is not necessarily the unique Nash equilibrium of the game among vote-maximizing candidates when non-Downsian candidates are present.

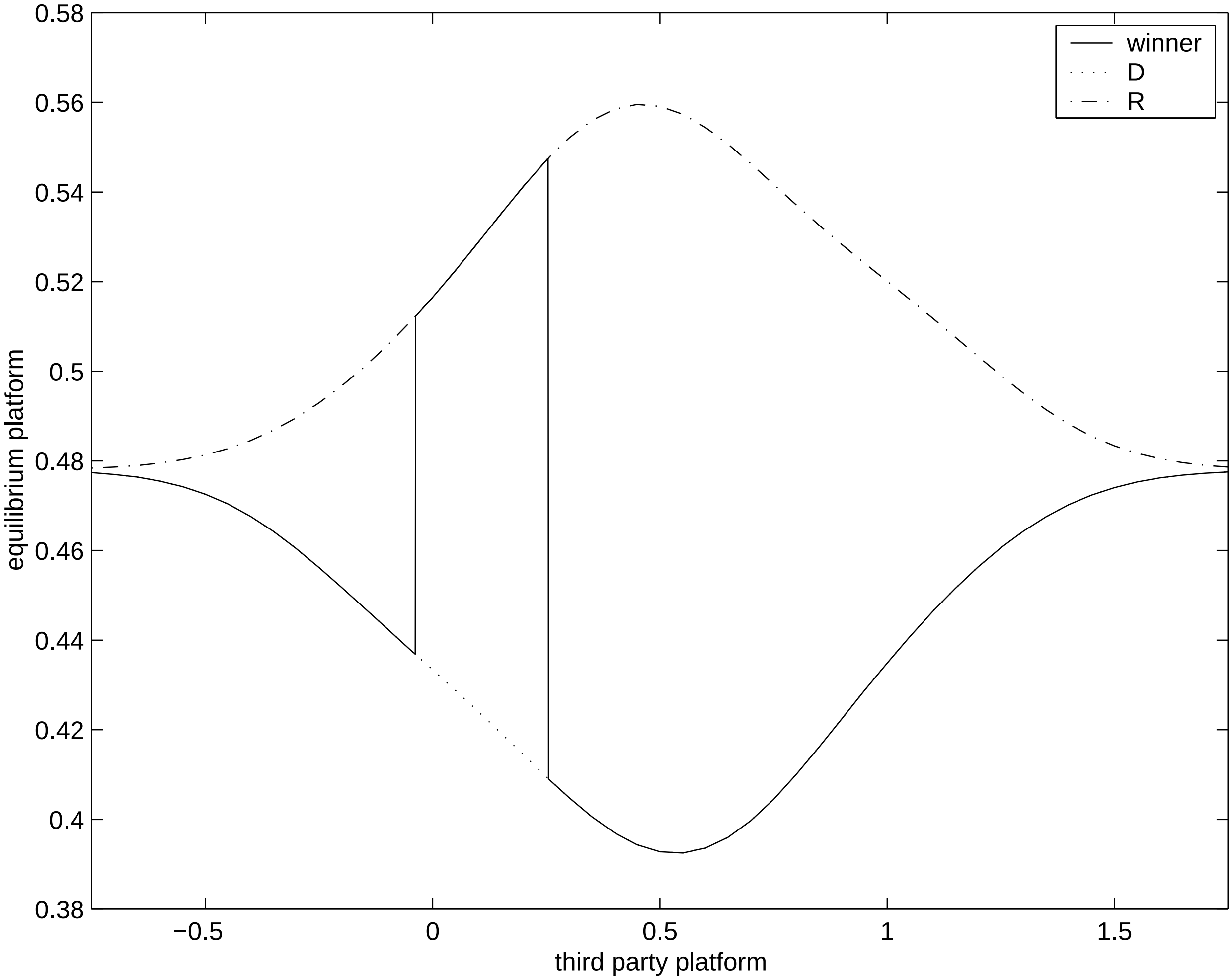

Figure 1: 2+1 Competition $b = 5$, $c = 1$

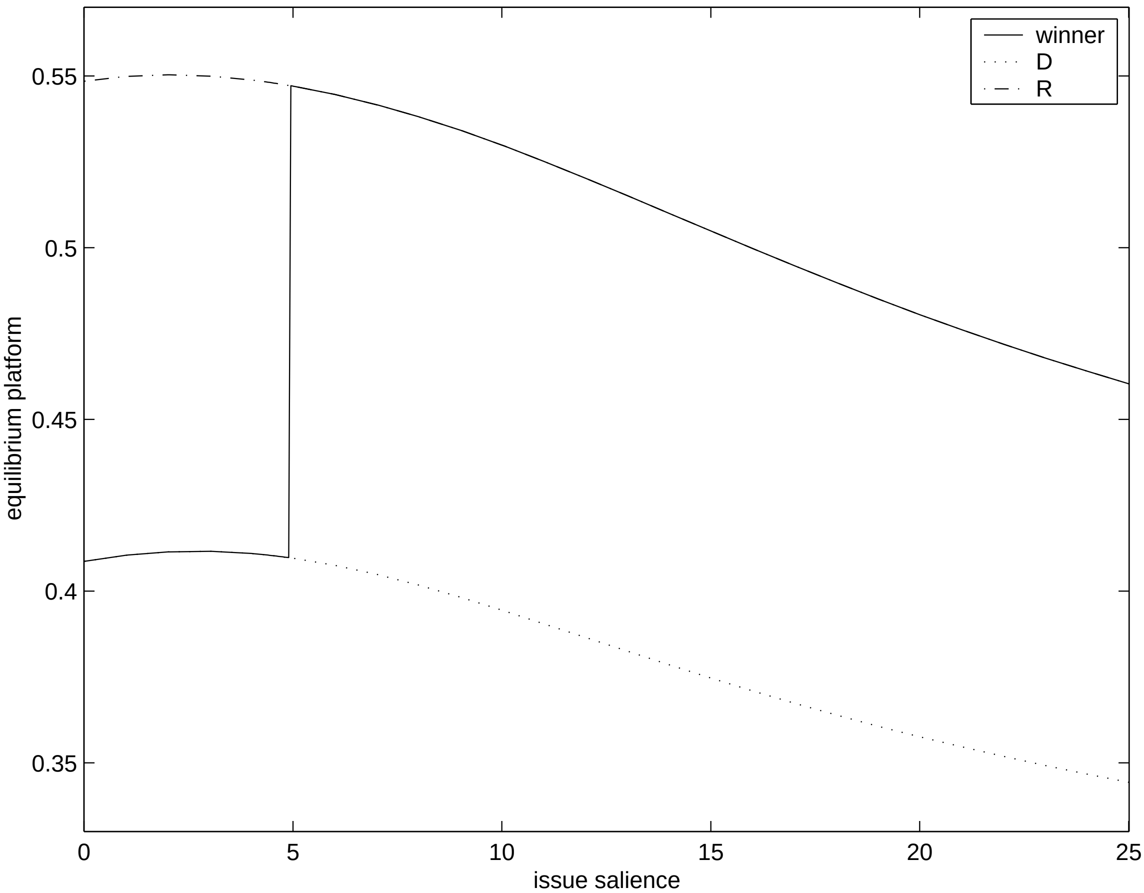

Figure 2: 2+1 Competition $y_I = 0.25$, $c = 1$

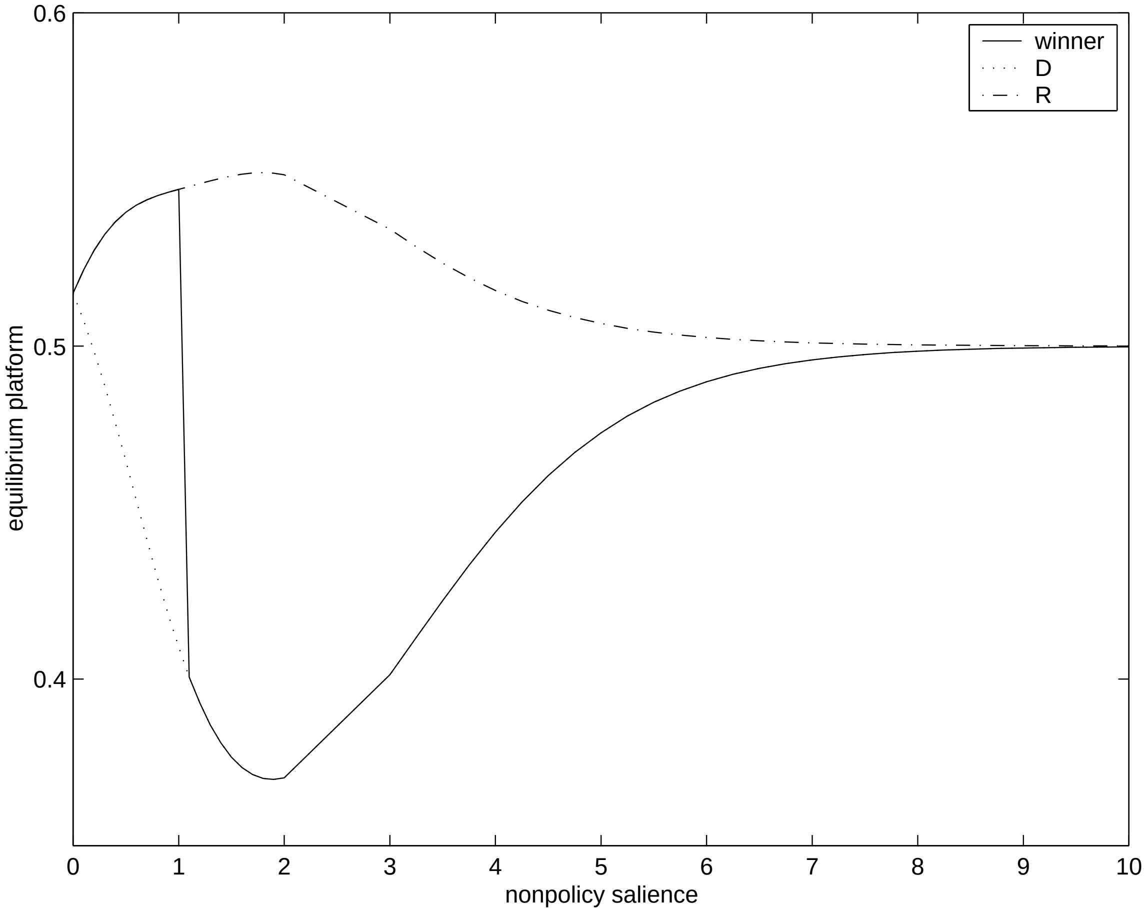

Figure 3: 2+1 Competition $y_I = 0.25$, $b = 5$