SOME AUGMENTED ROW-COLUMN DESIGNS

by

W. T. Federer, R. C. Nair and D. Raghavarao

BU-519-M*

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ABSTRACT

A number of specific augmented n-row by n-column designs for n=3,4,5,6,7 for v replicated check varieties and for v_1, unreplicated new varieties, are presented. Most of the designs are constructed such that any unreplicated variety plot is surrounded by two, three, or four replicated variety plots. The designs are compared using the average variance of a difference between unreplicated varieties for specified v_1 and n . Three generalizations of classes of designs are discussed. The specific application discussed pertains to screening new strains in a plant breeding program.

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1. INTRODUCTION

Eight sugarcane variety screening trials were designed as plan D-7-4 given below and were installed in Brazil in 1973. Dr. A. J. Mangelsdorf of the Experiment Station of the Hawaiian Sugar Planters' Association, was a consultant on these experiments, and it was he who raised questions about the statistical analysis for such designs. These questions led to a consideration of a group of row-column designs similar to the one he constructed. The purpose of this paper is to present a number of n-row by n-column designs for n^2 plots for n=2,3,4,5,6,7 which meet the requirement that $n^2/2$ for n even or $(n^2+1)/2$ for n odd plots are devoted to replicated check varieties and the remaining $n^2/2$ for n even or $(n^2-1)/2$ for n odd plots being devoted to unreplicated trials for testing new strains. The designs also need to meet the requirement that any unreplicated plot is surrounded by three or four check variety plots when n is odd; all unreplicated varieties except two are surrounded by three or four replicated variety plots when n is even, with the two exceptions being bordered by two replicated variety plots. Some additional row-column designs not meeting the above requirements are included for comparison.

The group of designs considered are presented in section two of the paper. Then, in section three, a randomization procedure is given which retains the

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requirements of the design. The statistical analysis is presented for each of the designs in the next section. A comparison of the designs with the augmented row-column designs of Federer and Raghavarao [1973] is made for n=7 and for the two cases of three and of four check varieties replicated seven times. In the final section some generalizations of specific designs in section two are discussed.

2. TWO-WAY AUGMENTED ROW-COLUMN DESIGNS FOR n=3,4,5,6,7

In the designs given below let capital letters A,B,C,··· represent the v replicated varieties and let the numbers 1,2,··· represent the v_1 unreplicated new varieties. Furthermore, let r_i denote the number of times the i^{th} variety is replicated in the n-row by n-column design; $r_1 = r_2 = \cdots = r_{v_1} = 1$ for each of the unreplicated varieties. Then, the following are some of the available designs with a number of them meeting the requirements stated in the introduction.

For n=3, v=2 two possible designs are:

D-3-1

A	1	B
2	A	3
B	4	A

$v = 2$; $r_A = 3$, $r_B = 2$
$v_1 = 4$ new varieties

D-3-2

A	B	1
2	A	B
B	3	A

$v = 2$; $r_A = r_B = 3$
$v_1 = 3$ new varieties

For n=4 the following designs may be used (new treatments numbered 1 to 8):

D-4-1

A	1	B	2
3	A	4	B
B	5	A	6
7	B	8	A

$v = 2;\ r_A = 4 = r_B$

$v_1 = 8$ new varieties

D-4-2

A	1	B	2
3	C	4	A
B	5	C	6
7	A	8	B

$v = 3;\ r_A = 3 = r_B,\ r_C = 2$

$v_1 = 8$ new varieties

For n=5, the following designs may be utilized (new treatments numbered 1 to 12):

D-5-1

A	1	B	2	A
3	B	4	A	5
B	6	A	7	B
8	A	9	B	10
A	11	B	12	A

$v = 2;\ r_A = 7,\ r_B = 6$

$v_1 = 12$ new varieties

D-5-2

A	1	B	2	C
3	A	4	B	5
C	6	A	7	B
8	C	9	A	10
B	11	C	12	A

$v = 3;\ r_A = 5,\ r_B = 4 = r_C$

$v_1 = 12$ new varieties

D-5-3

A	1	B	2	C
3	D	4	A	5
B	6	C	7	D
8	A	9	B	10
C	11	D	12	A

$v = 4;\ r_A = 4,\ r_B = r_C = r_D = 3$

$v_1 = 12$ new varieties

D-5-4

A	1	B	2	C
3	D	4	E	5
B	6	C	7	D
8	E	9	A	10
C	11	D	12	E

$v = 5;\ r_A = r_B = 2,\ r_C = r_D = r_E = 3$

$v_1 = 12$ new varieties

D-5-5

A	1	C	2	D
3	D	4	B	5
B	6	7	8	A
9	A	10	C	11
C	12	D	13	B

$v = 4;\ r_A = r_B = r_C = r_D = 3$

$v_1 = 13$ new varieties

(Note that the center plot is surrounded by checks [capital letters] <u>only</u> on the diagonals and not on adjacent sides.)

For n=6 and $v_1 = 18$ unreplicated new treatments, the following designs are available:

D-6-1

A	1	B	2	A	3
4	B	5	A	6	B
A	7	B	8	A	9
10	B	11	A	12	B
B	13	A	14	B	15
16	A	17	B	18	A

$v = 2;\ r_A = r_B = 9$

D-6-2

A	1	B	2	C	3
4	A	5	B	6	C
B	7	C	8	A	9
10	C	11	A	12	B
C	13	A	14	B	15
16	B	17	C	18	A

$v = 3;\ r_A = r_B = r_C = 6$

D-6-3

A	1	B	2	C	3
4	D	5	A	6	B
C	7	D	8	A	9
10	B	11	C	12	D
B	13	C	14	D	15
16	A	17	B	18	C

$v = 4;\ r_A = r_D = 4,\ r_C = r_B = 5$

D-6-4

A	1	B	2	C	3
4	D	5	E	6	A
B	7	C	8	D	9
10	E	11	A	12	B
C	13	D	14	E	15
16	A	17	B	18	C

$v = 5;\ r_A = r_B = r_C = 4,\ r_D = r_E = 3$

D-6-5

A	1	B	2	C	3
4	D	5	E	6	F
B	7	C	8	A	9
10	E	11	F	12	D
C	13	A	14	B	15
16	F	17	D	18	E

$v = 6;\ r_A = r_B = r_C = r_D = r_E = r_F = 3$

For n=7, the following designs may be utilized:

D-7-1

A	1	B	2	A	3	B
4	A	5	B	6	A	7
B	8	A	9	B	10	A
11	B	12	A	13	B	14
A	15	B	16	A	17	B
18	A	19	B	20	A	21
B	22	A	23	B	24	A

$v = 2;\ r_A = 13,\ r_B = 12$

$v_1 = 24$ new varieties

D-7-2

A	1	B	2	C	3	A
4	B	5	C	6	A	7
B	8	C	9	A	10	B
11	C	12	A	13	B	14
C	15	A	16	B	17	C
18	A	19	B	20	C	21
A	22	B	23	C	24	A

$v = 3;\ r_A = 9,\ r_B = 8 = r_C$

$v_1 = 24$ new varieties

D-7-3

A	1	B	2	C	3	D
4	A	5	B	6	C	7
D	8	A	9	B	10	C
11	D	12	A	13	B	14
C	15	D	16	A	17	B
18	C	19	D	20	A	21
B	22	C	23	D	24	A

$v = 4;\ r_A = 7,\ r_B = r_C = r_D = 6$

$v_1 = 24$ new varieties

D-7-4

A	1	B	2	C	3	D
4	E	5	A	6	B	7
C	8	D	9	E	10	A
11	B	12	C	13	D	14
E	15	A	16	B	17	C
18	D	19	E	20	A	21
B	22	C	23	D	24	E

$v = 5;\ r_A = r_B = r_C = r_D = r_E = 5$

$v_1 = 24$ new varieties

D-7-5

A	1	B	2	C	3	D
4	E	5	F	6	A	7
B	8	C	9	D	10	E
11	F	12	A	13	B	14
C	15	D	16	E	17	F
18	A	19	B	20	C	21
D	22	E	23	F	24	A

$v = 6;\ r_A = 5,\ r_B = r_C = r_D = r_E = r_F = 4$

$v_1 = 24$ new varieties

D-7-6

A	1	B	2	C	3	D
4	E	5	F	6	G	7
B	8	A	9	D	10	C
11	F	12	G	13	E	14
C	15	D	16	A	17	B
18	G	19	E	20	F	21
D	22	C	23	B	24	A

$v = 7;\ r_A = r_B = r_C = r_D = 4,\ r_E = r_F = r_G = 3$

$v_1 = 24$ new varieties

D-7-7

A	1	B	2	C	3	D
D	A	4	B	5	C	6
7	D	A	8	B	9	C
C	10	D	A	11	B	12
13	C	14	D	A	15	B
B	16	C	17	D	A	18
19	B	20	C	21	D	A

$v = 4;\ r_A = r_B = r_C = r_D = 7$

$v_1 = 21$ new varieties

D-7-8

A	1	2	B	3	C	D
D	A	4	5	B	6	C
C	D	A	7	8	B	9
10	C	D	A	11	12	B
B	13	C	D	A	14	15
16	B	17	C	D	A	18
19	20	B	21	C	D	A

$v = 4;\ r_A = r_B = r_C = r_D = 7$

$v_1 = 21$ new varieties

D-7-9

A	B	1	C	2	3	4
5	A	B	6	C	7	8
9	10	A	B	11	C	12
13	14	15	A	B	16	C
C	17	18	19	A	B	20
21	C	22	23	24	A	B
B	25	C	26	27	28	A

$v = 3;\ r_A = r_B = r_C = 7$

$v_1 = 28$ new varieties

D-7-10

A	1	B	2	C	3	4
5	A	6	B	7	C	8
9	10	A	11	B	12	C
C	13	14	A	15	B	16
17	C	18	19	A	20	B
B	21	C	22	23	A	24
25	B	26	C	27	28	A

$v = 3;\ r_A = r_B = r_C = 7$

$v_1 = 28$ new varieties

Designs for other n, v, and v_1 may be constructed following the above procedure. As is indicated in Table 1, the error degrees of freedom for these designs may be quite small or nonexistent. In general, the number of degrees of freedom available for estimating the error mean square is equal to $\left(\sum_{i=1}^{v} r_i = \text{number of replicated plots}\right) - v - 2(n-1)$. Only two designs considered, D-7-7 and D-7-8, have as many as 12 degrees of freedom, but these two designs do not meet the requirement (i) that $n^2/2$ for n even or $(n^2-1)/2$ for n odd of the plots are devoted to unreplicated plots. All designs meet this requirement except D-3-2, D-5-5, D-7-7, D-7-8, D-7-9, and D-7-10. In design D-5-5, $v_1 = (n^2+1)/2 = 13$, and this type of design can be constructed easily for any odd n for $(n-1) = v$ check varieties each replicated $(n+1)/2$ times. For these designs the error degrees of freedom for the error mean square is equal to $(n-1)(n-6)$. Hence, n should be at least 9 since n=7 produces only five degrees of freedom for error.

Designs meeting the requirement that (ii) every new variety be surrounded by three or four check variety plots are D-3-1, D-3-2, D-5-1, D-5-2, D-5-3, D-5-4, D-7-1, D-7-2, D-7-3, D-7-4, D-7-5, D-7-6, D-7-7, and D-7-8. Designs which meet the requirement that every new variety is surrounded by three or four check variety plots except two which are surrounded by two check variety plots are designs D-4-1, D-4-2, D-6-1, D-6-2, D-6-3, D-6-4, and D-6-5.

3. RANDOMIZATION PROCEDURE

A randomization procedure for the above designs is to proceed as follows:

(i) Randomly allot the even-numbered columns of a plan to the even-numbered columns in the experiment.

(ii) Randomly allot the odd-numbered columns to the odd-numbered columns in the experiment.

(iii) Randomly allot the even-numbered rows of the plan to the even-numbered rows in the experiment.

(iv) Randomly allot the odd-numbered rows to the odd-numbered rows in the experiment.

(v) Randomly allot the capital letters to the check varieties and then randomly allot the numbers to the new-unreplicated varieties.

This procedure retains the basic arrangement and requirements of the design. Special precaution must be used for designs like D-5-5 where the $(n+1)/2^{th}$ row and column cannot be allotted to and outside row and column, respectively.

4. SOLUTIONS FOR PARAMETERS OF DESIGNS AND ANOVA'S

For the row-column designs considered, suppose that the yield equation is

$$Y_{hij} = \mu + \rho_h + \gamma_i + \tau_j + \epsilon_{hij}$$

where μ is a general mean effect, ρ_h equals the effect for h^{th} row, $h=1,2,\cdots,n$, γ_i equals the effect for i^{th} column, $i=1,2,\cdots,n$, τ_j equals the effect of j^{th} treatment, $j=A,B,\cdots,v,\ 1,2,\cdots,v_1$, the ϵ_{hij} equals random error components which are identically and independently distributed with zero mean and variance σ^2_ϵ, and $E[Y_{hij}]=\mu+\rho_h+\gamma_i+\tau_j$. The normal equations are obtained by minimizing $\sum_{h=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{v} n_{hij}\epsilon^2_{hij}$ where n_{hij} equals one if j^{th} check variety is in the h^{th} row and the i^{th} column and zero otherwise. As shown by Federer and Raghavarao [1973], the yields for the unreplicated varieties may be ignored in obtaining solutions for the other effects.

A solution for a design (d) is given by $\hat{\tau}^{(d)} = A^{(d)}Q^{(d)}$, where $A^{(d)}$ is a generalized inverse of the matrix:

$$D = \text{diag}(r_1,\cdots,r_v) - \frac{1}{n} LL' - \frac{1}{n} MM' + \frac{1}{n^2}\begin{bmatrix} r_1 \\ \vdots \\ r_{v^*}\end{bmatrix}(r_1 \cdots r_{v^*}) \qquad (4.1)$$

and

$$Q = B - \frac{1}{n} LR - \frac{1}{n} MC + \frac{1}{n^2}\begin{bmatrix} r_1 \\ \vdots \\ r_{v^*}\end{bmatrix} Y_{\cdots} ,$$

where

$v^* = v + v_1$,

B = a vector of treatment totals,

R = a vector of row totals,

C = a vector of column totals,

$Y_{\cdots}$ = a total of all observations,

r_j = number of replicates of j^{th} treatment,

L = row-treatment incidence matrix, and

M = column-treatment incidence matrix.

The matrix $A^{(d)}$ for each design is given in the appendix for the unreplicated varieties. These were obtained, using the constraint $\sum_{j=1}^{v^*} \hat{\tau}_j = 0$. Variances of estimable parametric functions may be computed, using this matrix; for example, $V(\hat{\tau}_j - \hat{\tau}_{j'}) = \left[A_{jj} + A_{j'j'} - 2A_{jj'}\right]\sigma_\epsilon^2$ where the element of row j and column j' in $A^{(d)}$ is denoted as $A_{jj'}$. The procedure is illustrated for design D-3-2, below:

The yields are:

				Mean
	Y_{11A}	Y_{12B}	Y_{131}	$\bar{y}_{1\cdot\cdot}$
	Y_{212}	Y_{22A}	Y_{23B}	$\bar{y}_{2\cdot\cdot}$
	Y_{31B}	Y_{323}	Y_{33A}	$\bar{y}_{3\cdot\cdot}$
Mean	$\bar{y}_{\cdot 1\cdot}$	$\bar{y}_{\cdot 2\cdot}$	$\bar{y}_{\cdot 3\cdot}$	$\bar{y}_{\cdots}$

$$L' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}; \quad M' = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}; \; r_A=3=r_B,\; r_1=r_2=r_3 = 1 \; .$$

Then,

$$D = \frac{1}{9}\begin{bmatrix} 18 & -9 & -3 & -3 & -3 \\ -9 & 18 & -3 & -3 & -3 \\ -3 & -3 & 4 & 1 & 1 \\ -3 & -3 & 1 & 4 & 1 \\ -3 & -3 & 1 & 1 & 4 \end{bmatrix}$$

and

$$\underset{\sim}{Q} = \begin{bmatrix} Y_{..A} - \bar{y}_{1..} - \bar{y}_{2..} - \bar{y}_{3..} - \bar{y}_{.1.} - \bar{y}_{.2.} - \bar{y}_{.3.} + 3\bar{y}_{...} \\ Y_{..B} - \bar{y}_{1..} - \bar{y}_{2..} - \bar{y}_{3..} - \bar{y}_{.1.} - \bar{y}_{.2.} - \bar{y}_{.3.} + 3\bar{y}_{...} \\ Y_{..1} - \bar{y}_{1..} - \bar{y}_{.3.} + \bar{y}_{...} \\ Y_{..2} - \bar{y}_{2..} - \bar{y}_{.1.} + \bar{y}_{...} \\ Y_{..3} - \bar{y}_{3..} - \bar{y}_{.2.} + \bar{y}_{...} \end{bmatrix} = \begin{bmatrix} Q_{..A} \\ Q_{..B} \\ Q_{..1} \\ Q_{..2} \\ Q_{..3} \end{bmatrix}$$

where $Y_{..j}$ is the j^{th} treatment total. A solution for the variety effects is given by

$$\underset{\sim}{\hat{\tau}} = \begin{bmatrix} \hat{\tau}_A \\ \hat{\tau}_B \\ \hat{\tau}_1 \\ \hat{\tau}_2 \\ \hat{\tau}_3 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 7 & 2 & 0 & 0 & 0 \\ 2 & 7 & 0 & 0 & 0 \\ 0 & 0 & 33 & -12 & -12 \\ 0 & 0 & -12 & 33 & -12 \\ 0 & 0 & -12 & -12 & 33 \end{bmatrix}\begin{bmatrix} Q_{..A} \\ Q_{..B} \\ Q_{..1} \\ Q_{..2} \\ Q_{..3} \end{bmatrix} .$$

The various variances of differences between variety effects may be computed from the above matrix as:

$$V\left(\hat{\tau}_A - \hat{\tau}_B\right) = (7+7-2-2)\sigma^2/15 = 2\sigma^2/3 \ ;$$

$$V\left(\hat{\tau}_A - \hat{\tau}_j\right) = V\left(\hat{\tau}_B - \hat{\tau}_j\right) = (7+33-0-0)\sigma^2/15 = 8\sigma^2/3, \ j=1,2,3 \ ;$$

$$V\left(\hat{\tau}_j - \hat{\tau}_{j'}\right) = (33+33-[-12]-[-12])\sigma^2/15 = 6\sigma^2, \ j=1,2,3 \ .$$

It should be noted that any of the new varieties may be left unharvested and a zero inserted for the yields of those new varieties omitted from the harvest. Since the solution for the row and column effects depends solely upon the replicated check variety yields, the solutions for the differences between check varieties, between check and new varieties retained, and between new varieties retained will not be affected. If zero yields are inserted, formula (4.1) still holds. Likewise, the variance-covariance matrix remains the same, and we can simply drop the rows and columns of the matrix $A^{(d)}$ which pertain to the omitted new varieties; the variances of differences will remain unaffected. For example, suppose that unreplicated varieties 1 and 2 in design D-3-2 were not harvested. Then, the variance-covariance matrix for the remaining varieties would be:

$$V\begin{bmatrix}\hat{\tau}_A\\ \hat{\tau}_B\\ \hat{\tau}_3\end{bmatrix} = \begin{bmatrix}7 & 2 & 0\\ 2 & 7 & 0\\ 0 & 0 & 33\end{bmatrix}\sigma^2/15 \quad .$$

In variety screening experiments, the dropping of poor performers in an experiment is commonplace. Thus, the fact that the computation of the check variety, the row and the column effects is the same whether or not the unreplicated varieties are included (see Federer and Raghavarao [1973]) is of importance in experiments of this type.

Design 3-1 is disconnected but is an interesting example for statistical analyses of a disconnected design. The analysis of variance for this design using only the five check variety yields is:

Source of Variation	D.F.	Sum of Squares
Correction for mean (ignoring all else)	1	$\left[Y_{11A}+Y_{13B}+Y_{22A}+Y_{31B}+Y_{33A}=Y_{\cdots}\right]^2 / 5$
Rows (ignoring varieties and columns)	2	$\frac{(Y_{11A}+Y_{13B})^2}{2}+\frac{Y^2_{22A}}{1}+\frac{(Y_{31B}+Y_{33A})^2}{2} - Y^2_{\cdots}/5$
Columns (eliminating rows and mean) = columns 1 versus 3	1	$\left[Y_{11A}+Y_{31B}-Y_{13B}-Y_{33A}\right]^2 / 4$
Variety A versus B (eliminating all other effects)	1	$\left[Y_{11A}+Y_{33A}-Y_{31B}-Y_{13B}\right]^2 / 4$

In terms of orthogonal contrasts, the above may be written as:

Contrast	Y_{11A}	Y_{13B}	Y_{22A}	Y_{31B}	Y_{33A}
Correction for mean	+	+	+	+	+
Row 1 versus row 3	+	+	0	-	-
Rows 1 + 3 versus row 2	+	+	-4	+	+
Column 1 versus column 3	+	-	0	+	-
Variety A versus variety B	+	-	0	-	+

It should be noted that designs D-3-1, D-4-2, D-6-5, and D-7-6 are not over-all connected. The generalized inverse was obtained with the particular computer program used. The computer output gives the rank of the input matrix, or equivalently, the number of estimable contrasts but not the estimable contrasts themselves. For each of the designs, the generalized inverse part of the matrix for the new treatments is given in the appendix. Because of the symmetry of the matrices, only the top (or bottom) part of the matrix above (or below) the diagonal elements is given. Thus, it was possible to put two matrices on each page. The average variance of a difference between two unreplicated varieties may be computed as:

$$2\left\{(v_1-1)(\text{sum of diagonal elements}) - \text{sum of off-diagonal elements}\right\} \Big/ v_1(v_1-1) \quad . \tag{4.2}$$

5. COMPARISON OF DESIGNS FOR SPECIFIED n

As stated by Federer and Raghavarao [1973] there are four ways in which these designs may be compared; that is, through

(i) A comparison of variances of estimable contrasts of replicated or check varieties.

(ii) A comparison of variances of estimable contrasts among the unreplicated or new varieties.

(iii) A comparison of variances of estimable contrasts of check varieties versus new varieties.

(iv) A comparison of variances of estimable contrasts among all varieties.

These authors chose (ii) in describing a class of optimal row-column designs. In screening experiments one might wish to use (iii) in comparing new varieties with check varieties. However, here we shall use (ii) even though one may use any of the four comparisons.

The average variance of a difference between estimated new variety effects was computed using formula (4.2) for each of the designs. The results are presented in Table 1 where each design, its type if balanced or orthogonal, values for v_1, values for degrees of freedom for estimating the error variance and replication numbers for the various check varieties are also given.

In studying the results of Table 1, one should note that if there were no row and column effects and if a completely randomized design had been used the variance of a difference between two new variety effects would be $2\sigma^2$. For the designs discussed herein, only one, D-7-8, has an average variance as small as $3.20\sigma^2$ with the next best one being D-7-10 where the average variance is $3.59\sigma^2$. As n increases and for designs of type X:00 or T:00, the average error variance

Table 1. Average coefficient of the variance of a difference between two new varieties

Type*	Design	No. of new varieties	Degrees of freedom	(Average variance)/σ^2	Replication numbers
X:00	D-4-1	8	0	4.57	$r_A = r_B = 4$
	D-4-2	8	-1	4.27	$r_A = r_B = 3, r_C = 2$
	D-5-1	12	3	4.06	$r_A = 7, r_B = 6$
	D-5-2	12	2	4.44	$r_A = 5, r_B = r_C = 4$
	D-5-3	12	1	5.15	$r_A = 4, r_B = r_C = r_D = 3$
	D-5-4	12	0	7.57	$r_A = r_B = 2, r_C = r_D = r_E = 3$
	D-5-5	13	0	5.35	$r_A = r_B = r_C = r_D = 3$
X:TT	D-6-1	18	6	4.34	$r_A = r_B = 9$
X:00	D-6-2	18	5	4.00	$r_A = r_B = r_C = 6$
	D-6-3	18	4	4.21	$r_A = r_B = 4, r_C = r_D = 5$
	D-6-4	18	3	5.16	$r_A = r_B = r_C = 4, r_D = r_E = 3$
	D-6-5	18	4**	4.24	$r_A = r_B = r_C = r_D = r_E = r_F = 3$
X:TT	D-7-1	24	11	3.81	$r_A = 13, r_B = 12$
	D-7-2	24	10	3.83	$r_A = 9, r_B = r_C = 8$
	D-7-3	24	9	3.86	$r_A = 7, r_B = r_C = r_D = 6$
	D-7-4	24	8	4.07	$r_A = r_B = r_C = r_D = r_E = 5$
	D-7-5	24	7	4.61	$r_A = 5, r_B = r_C = r_D = r_E = r_F = 4$
	D-7-6	24	8**	4.01	$r_A = r_B = r_C = r_D = 4, r_E = r_F = r_G = 3$
X:00	D-7-7	21	12	3.63	$r_A = r_B = r_C = r_D = 7$
T:00	D-7-8	21	12	3.20	$r_A = r_B = r_C = r_D = 7$
T:00	D-7-9	28	6	3.78	$r_A = r_B = r_C = 7$
X:00	D-7-10	28	6	3.59	$r_A = r_B = r_C = 7$

* O denotes orthogonality, T denotes balance, and X denotes unknown balance where the relationships are in the order rows-columns-varieties.

** One check variety, one row, and one column contrast are completely confounded with each other.

approaches $2\sigma^2$. Also, one should note the relatively small number of degrees of freedom available for error for n=7. Only four designs, D-7-1, D-7-2, D-7-7, and D-7-8, have ten or more degrees of freedom available for error. This may not be serious, if there are several sets of experiments and the errors for the experiments can be pooled. Designs with n=8 and 9 should have sufficient degrees of freedom for error estimation.

A rather surprising result from the examples is that the average variance of a difference for design D-7-10 is smaller than design D-7-9, whereas it is smaller for D-7-8 as compared to D-7-7. One would expect that if the rows and columns were in a balanced arrangement with each other that the average variance would be smaller. However, this is not the case and it would appear that the average variance also depends upon how many unreplicated plots are surrounded by replicated ones. Thus, the optimal row-column designs of Federer and Raghavarao [1973] are variance optimal for estimating row and column effects but are not variance optimal for estimating the average variance between new treatments. The Mangelsdorf criterion of surrounding unreplicated plots with replicated plots appears to have variance-optimality properties. However, note that the average variance of a difference within rows and within columns is smaller for D-7-8 and D-7-9 than for D-7-7 and D-7-10, respectively.

For constant v_1 and n, several comparisons are possible from Table 1. For example, for v_1=18 and n=6, D-6-2 has the smallest average error variance, has next to the largest error degrees of freedom, and is equi-replicated for three check varieties. For v_1=24, D-7-1 has the smallest average error, has the largest number of error degrees of freedom, and has the fewest check varieties. In general, as v tends to increase so does the average error, but there are exceptions caused by orthogonality and perhaps other items. D-5-4 has the highest average variance, 7.57, owing to the nonorthogonality of the check varieties with rows and columns and to the small replication number of the check varieties.

6. SOME POSSIBLE GENERALIZATIONS

It should be noted that analyses for specific designs are given in this paper. There are, however, some general classes of augmented row-column designs that could be considered. Three such classes are described below. The straightforward analyses and solutions are not given herein. If a need arises, they can quickly and easily be obtained.

For the first class of augmented row-column designs, consider the following: for n even, $v=n/2$ checks with $r=n$ replicates, and for $v_1=n^2/2$ new treatments, it would be possible to obtain an analysis for any n given that the $v=n/2$ checks were designed in an $n/2 \times n/2$ latin square in the odd rows and odd columns and in an $n/2 \times n/2$ latin square in the even rows and even columns. Designs D-4-1 and D-6-2 are members of this class of designs.

For the second class of augmented row-column designs, we consider the following: for n even, $v=n$, $r=n/2$, and for $v_1=n^2/2$ new varieties it would be possible to construct a row-column design for which the treatments formed a group-divisible design with rows and a group-divisible design with columns. An example follows:

A	1	B	2
3	C	4	D
C	5	A	6
7	D	8	B

This design is not connected; therefore n must be greater than four.

A third class of augmented row-column designs could be obtained for the following situation: for n odd, $v=n=2$ check varieties with $r=(v+5)/2=v-k$ replicates on $v-k$ of the entries and with $r=(v+5)/2-1$ replicates on the remaining k checks for $k=(v-5)/2=(n-7)/2$, and with $v_1=(n^2-1)/2$ new varieties. Designs D-5-2 and D-7-4 are of this type (the Mangelsdorf type). The n-2 check varieties enter cyclically in the $n \times n$ square. For n=5 it should be noted that k is a negative one. This means that one check variety is replicated five times and two varieties are replicated four times each. The above formula then holds for odd $n \geq 7$.

Still other classes of augmented row-column designs could be constructed. For example, instead of squares rectangular designs could be used. If rows 5 and 6 are deleted from D-6-2, then a 4-row by 6-column design is formed such that instead of latin squares in odd rows and columns and in the even rows and columns, a Youden design is formed. These problems are left as future research topics.

REFERENCE

Federer, W.T. and D. Raghavarao [1973]. On augmented designs. Number BU-480-M in the Mimeo Series of the Biometrics Unit, Cornell University (to appear in Biometrics).

D-4-1

Average variation of a treatment difference = 4.5714

2875	875	- 875	-375	1375	375	-875	-375
	1875	- 375	125	375	375	-375	125
1889		2875	875	- 875	-375	1375	375
1111	1889		1875	- 375	125	375	375
-333	1333	5000		2875	875	-875	-375
-111	1111	2667	1889		1875	-375	125
1111	-111	-1667	-889	889		2875	875
333	667	000	333	667	1000		1875
-111	2111	5667	2889	-1889	-333	7889	
111	1889	3333	2111	-1111	-667	5111	3889

Average variation of a treatment difference = 4.274

D-4-2

Not all functions are estimable

D-5-1

Average variation of a treatment difference = 4.06484

2773	773	-690	-787	-360	13467	347	-690	-787	-360	1440	440
	1773	-190	-287	140	347	347	-190	-287	140	440	440
2593		2507	1113	840	-303	-253	1007	613	340	-693	-193
593	1759		2427	780	-807	-307	613	927	280	-787	-287
-778	056	3167		1840	-420	080	340	280	340	-360	140
-778	056	1833	3167		2627	627	-753	-807	-420	1347	347
-444	389	1500	1500	2500		1627	-253	-307	080	347	347
1259	259	-778	-778	-444	2593		2507	1113	840	-693	-193
259	426	056	056	389	593	1759		1147	780	-787	-287
-778	-111	1333	1000	667	-778	-111	2667		1840	-360	140
-778	-111	1000	1333	667	-778	-111	1333	2667		2773	773
-444	222	667	667	667	-444	222	1000	1000	2000		1773
1259	259	-778	-778	-444	1259	259	-778	-778	-444	2593	
259	426	056	056	389	259	426	-111	-111	222	593	1759

Average variation of a treatment difference = 4.44466

D-5-2

D-5-3

Average variation of a treatment difference = 5.14990

2775	775	- 686	- 784	-353	1343	343	- 686	- 784	- 353	1441	441
	2150	439	216	522	218	593	814	591	897	191	566
3513		3591	1961	1507	- 983	142	2716	2086	1632	-1103	022
753	2313		3098	1294	- 980	020	1961	2098	1294	-1118	-118
-1113	847	4713		2257	- 556	309	1382	1169	1132	- 603	272
- 607	473	2407	3353		2679	554	-1108	-1105	- 691	1427	302
- 400	400	1400	1200	2000		1804	392	270	559	177	427
2687	047	-2087	- 993	-600	5113		5215	3461	2882	-1353	147
927	607	- 127	087	200	1473	2153		4473	2544	-1368	007
-1873	1407	4673	2487	1200	-3727	- 447	8953		3382	- 853	397
-1367	1033	3367	2433	1000	-2633	- 233	5767	5833		2941	691
-1160	960	2360	1280	800	-2240	- 120	4480	3400	3920		1816
2560	-360	-2760	-1480	-800	3840	920	-4680	-3400	-2720	5520	
800	200	- 800	- 400	000	1200	600	-1400	-1000	- 600	1600	2000

Average variation of a treatment difference = 7.57164

D-5-4

D-5-5

Average variation of a treatment difference = 5.35192

3275	775	-1275	-1450	-875	1850	-225	350	-975	-1150	-575	1675	175					
	1875	025	- 250	225	450	075	550	325	050	525	375	475					
2392		3675	2050	1675	-1050	625	250	2375	1750	1375	-1075	225					
1132	2569		3500	1450	-1100	1150	100	1650	2100	1050	-1250	-060					
725	798	1726		2475	- 650	425	450	1175	950	975	- 675	425					
-618	-596	- 285	2236		3500	650	1100	-850	- 900	-450	1650	250					
-691	-701	- 356	882	2226		2675	950	925	1450	725	- 225	075					
-285	-264	049	569	549	1569		2300	450	300	650	350	550					
1059	799	392	- 618	-691	- 285	2392		4675	2950	2475	- 975	325					
799	1236	465	- 597	-701	- 264	1132	2569		4300	2150	-1150	050					
392	465	392	- 284	-358	049	726	799	1726		3075	- 575	525					
-618	-597	- 285	903	549	236	-618	-597	-285	2236		2875	575					
-691	-701	- 358	549	882	215	-691	-701	-358	882	2226		1875					
-285	-264	049	236	215	236	-285	-264	049	569	549	1569						
986	694	319	- 639	-681	- 306	986	694	319	- 639	-681	- 306	2278					
726	1132	392	- 618	-691	- 285	726	1132	392	- 618	-691	- 285	986	2392				
319	361	319	- 306	-347	028	319	361	319	- 306	-347	028	611	653	1611			
-545	-493	- 212	924	538	257	-545	-493	-212	924	538	257	-597	-545	-264	2309		
-618	-597	- 285	569	882	236	-618	-597	-285	569	882	236	-639	-618	-306	361	2236	
-212	-160	122	257	205	257	-212	-160	122	257	205	257	-264	-212	069	642	590	1642

Average variation of a treatment difference = 4.34056

D-6-1

D-6-2

Average variation of a treatment difference = 4.0000

2333	1000	667	-556	-556	-222	1000	667	333	-556	-556	-222	1000	667	333	-556	-556	-222
	2333	667	-556	-556	-222	667	1000	333	-556	-556	-222	667	1000	333	-556	-556	-222
2749		1667	-222	-222	111	333	333	333	-222	-222	111	333	333	333	-222	-222	111
1525	3033		2333	1000	667	-556	-556	-222	1000	667	333	-556	-556	-222	1000	667	333
983	1117	1950		2333	667	-556	-556	-222	667	1000	333	-556	-556	-222	667	1000	333
-323	-208	000	2615		1667	-222	-222	111	333	333	333	-222	-222	111	333	333	333
-531	-458	-167	1156	2448		2333	1000	667	-556	-556	-222	1000	667	333	-556	-556	-222
-108	-033	217	792	708	1700		2333	667	-556	-556	-222	667	1000	333	-556	-556	-222
1296	1033	533	-479	-646	-200	2575		1667	-222	-222	111	333	333	333	-222	-222	111
1073	1542	667	-365	-573	-125	1312	2781		2333	1000	667	-556	-556	-222	1000	667	333
531	625	500	-156	-281	125	812	906	1781		2333	667	-556	-556	-222	667	1000	333
-421	-283	-033	1271	854	450	-575	-438	-188	2658		1667	-222	-222	111	333	333	333
-629	-533	-200	812	1146	367	-742	-646	-312	1242	2575		2333	1000	667	-556	-556	-222
-206	-108	183	448	406	358	-296	-198	094	838	796	1748		2333	667	-556	-556	-222
1206	942	483	-448	-573	-192	1129	865	406	-504	-629	-248	2415		1667	-222	-222	111
983	1450	617	-333	-500	-117	867	1333	500	-367	-533	-150	1150	2617		2333	1000	667
442	533	450	-125	-208	133	367	458	375	-117	-200	142	692	783	1700		2333	667
-213	-033	133	1396	896	533	-408	-229	-062	1408	908	546	-379	-200	-033	2908		1667
-421	-283	-033	938	1188	450	-575	-438	-188	992	1242	504	-504	-367	-117	1408	2658	
002	142	350	573	448	442	-129	010	219	588	463	456	456	017	225	1046	921	1915

Average variation of a treatment difference = 4.20568

D-6-3

D-6-4

Average variation of a treatment difference = 5.15588

2407	980	703	-507	-450	-143	1130	703	427	-600	-543	-237	1103	677	400	- 543	- 487	-180
	2377	823	-180	-233	-037	593	990	437	-117	-170	027	457	853	300	- 003	- 057	140
2444		2068	547	458	545	282	402	647	667	578	665	035	155	400	912	823	910
1111	2444		3907	2300	1493	-780	-453	273	2900	2293	1487	-1253	- 927	-200	3293	2687	1880
778	778	1778		3492	1408	-592	-375	312	2183	2375	1292	-1008	- 792	-100	2542	2733	1650
-667	-667	-333	2444		2248	-228	-122	460	1267	1182	922	- 488	- 382	200	1515	1430	1170
-667	-667	-333	1111	2444		2652	1115	803	-983	-795	-432	1348	812	500	- 962	- 773	-410
-333	-333	0000	778	778	1778		2402	813	-500	-422	-168	702	988	400	- 422	- 343	-090
1111	778	444	-667	-667	-333	2444		2023	283	327	470	280	290	500	493	537	680
778	1111	444	-667	-667	-333	1111	2444		4717	3000	2083	-1567	-1083	-300	3833	3117	1970
444	444	444	-333	-333	0000	778	778	1778		4082	1888	-1322	- 948	-200	3082	3163	2200
-667	-667	-333	1111	778	444	-667	-667	-333	2444		2518	- 802	- 538	100	2055	1860	1490
-667	-667	-333	778	1111	444	-667	-667	-333	1111	2444		2868	1222	800	-1655	-1410	-890
-333	-333	0000	444	444	444	-333	-333	0000	778	778	1778		2398	700	-1115	- 980	-570
1111	778	444	-667	-667	-333	1111	778	444	-667	-667	-333	2444		1800	- 200	- 100	200
778	1111	444	-667	-667	-333	778	1111	444	-667	-667	-333	1111	2444		5748	3997	2970
444	444	444	-333	-333	0000	444	444	444	-333	-333	0000	778	778	1778		5043	2740
-667	-667	-333	1111	778	444	-667	-667	-333	1111	778	444	- 667	- 667	-333	2444		3260
-667	-667	-333	778	1111	444	-667	-667	-333	778	1111	444	- 667	- 667	-333	1111	2444	
-333	-333	0000	444	444	444	-333	-333	0000	444	444	444	- 333	- 333	0000	778	778	1778

Average variation of a treatment difference = 4.23498

D-6-5

<u>D-7-7</u> Average variation of a treatment difference = 3.63290

1742	478	448	-062	-039	-003	-003	129	099	448	063	098	-039	-051	062	478	214	129	-062	-074	-051
	1754	478	-074	-074	-039	099	478	202	099	-039	-005	-005	-039	098	202	478	098	-039	-074	-074
1714		1742	-051	-074	-062	129	214	478	062	-051	-039	098	062	448	099	129	-003	-003	-039	-062
429	1714		1742	478	448	-003	-062	-039	-003	129	098	098	448	062	-039	-051	062	129	478	214
429	429	1714		1754	478	-039	-074	-074	098	478	202	-005	098	-039	-005	-039	098	098	202	478
071	429	071	1714		1742	-062	-051	-074	129	214	478	-039	062	-051	098	062	448	-003	098	129
000	071	000	429	1714		1742	448	478	-062	-051	-074	478	129	214	-039	062	-051	448	098	062
000	071	000	429	429	1714		1742	478	-003	-062	-039	099	-003	129	098	448	062	062	-039	-051
000	000	071	071	429	071	1714		1754	-039	-074	-074	202	098	478	-005	098	-039	098	-005	-039
071	071	429	000	071	000	429	1714		1742	448	478	-074	-062	-051	478	129	214	-051	-039	062
000	000	071	000	071	000	429	429	1714		1742	478	-039	-003	-062	098	-003	129	062	098	448
000	000	071	000	000	071	000	071	000	1714		1754	-074	-039	-074	202	098	478	-039	-005	098
071	071	429	000	000	071	071	429	071	429	1714		1754	478	478	-074	-039	-074	478	202	098
000	000	071	071	071	429	000	071	000	000	429	1714		1742	448	-074	-062	-051	214	478	129
429	071	071	000	000	071	000	000	071	071	000	071	1714		1742	-039	-003	-062	129	098	-003
071	000	000	071	071	429	000	000	071	000	071	429	429	1714		1754	478	478	-074	-074	-039
071	000	000	000	000	071	071	071	429	000	000	071	429	429	1714		1742	448	-051	-074	-062
000	071	000	071	000	000	000	000	071	429	071	071	000	000	071	1714		1742	-062	-039	-003
071	429	071	429	071	071	000	000	071	071	000	000	000	000	071	429	1714		1742	478	448
000	071	000	071	000	000	071	071	429	071	000	000	071	071	429	429	429	1714		1754	478
071	000	000	000	071	000	071	000	000	429	071	071	071	000	000	429	071	071	1714		1742
429	071	071	000	071	000	071	000	000	071	000	000	429	071	071	071	000	000	429	1714	
071	000	000	071	429	071	429	071	071	071	000	000	071	000	000	071	000	000	429	429	1714

<u>D-7-8</u> Average variation of a treatment difference = 3.20000

[redacted]

D-7-5 Average variation of a treatment difference = 4.61062

2310	903	580	-499	-484	-492	-202	1075	668	345	-573	-558	-566	-276	1067	660	072	-563	-548	-556	-265	1108	701	378
	2277	665	-313	-342	-414	-134	624	997	386	-272	-302	-373	-094	552	926	275	-217	-247	-318	-039	582	955	344
2292		1834	326	286	151	386	290	374	544	411	370	235	470	155	239	1048	580	539	404	640	140	224	394
958	2292		3021	1704	1473	1152	-567	-380	259	1874	1557	1327	1005	-797	-610	1339	2180	1862	1632	1311	-868	-682	-043
625	625	1625		2927	1461	1126	-511	-368	259	1512	1736	1270	935	-727	-584	1272	1807	2031	1564	1229	-812	-670	-042
-583	-583	-250	2278		2562	991	-505	-426	138	1219	1207	1308	737	-653	-575	957	1450	1437	1538	967	-724	-646	-082
-583	-583	-250	1028	2278		1962	-228	-160	360	886	860	726	696	-362	-295	1107	1073	1047	912	882	-392	-325	195
-583	-583	-250	1028	1028	2278		2382	930	596	-685	-629	-623	-346	1138	686	005	-686	-630	-624	-347	1165	713	379
-333	-333	000	778	778	778	1778		2259	636	-384	-372	-430	-164	622	952	208	-341	-329	-386	-120	638	967	344
1042	708	375	-583	-583	-583	-333	2292		1794	299	300	179	400	225	265	981	457	458	337	558	196	237	395
708	1042	375	-583	-583	-583	-333	958	2292		3509	2147	1853	1521	-979	-678	1542	2525	2163	1870	1537	-1061	-761	-077
375	375	375	-250	-250	-250	000	625	625	1625		3326	1797	1450	-909	-652	1476	2152	2332	1802	1456	-1005	-748	-076
-583	-583	-250	944	694	694	444	-583	-583	-250	2278		2834	1252	-835	-642	1160	1795	1738	1776	1194	-918	-724	-116
-583	-583	-250	694	944	694	444	-583	-583	-250	1028	2278		2211	-544	-362	1310	1418	1348	1149	1108	-585	-403	161
-583	-583	-250	694	694	994	444	-583	-583	-250	1028	1028	2278		2461	946	-060	-1043	-973	-899	-608	1252	737	339
-333	-333	000	444	444	444	444	-333	-333	000	778	778	778	1778		2212	143	-698	-672	-662	-382	726	991	305
1042	708	375	-583	-583	-583	-333	1042	708	375	-583	-583	-583	-333	2292		3999	2315	2249	1933	2083	-410	-207	566
708	1042	375	-583	-583	-583	-333	708	1042	375	-583	-583	-583	-333	958	2292		4323	2950	2592	2216	-1170	-825	-027
375	375	375	-250	-250	-250	000	375	375	375	-250	-250	-250	000	625	625	1625		4118	2525	2134	-1113	-812	-026
-583	-583	-250	944	694	694	444	-583	-583	-250	944	694	694	444	-583	-583	-250	2278		3499	1872	-1026	-789	-066
-583	-583	-250	694	944	694	444	-583	-583	-250	694	944	694	444	-583	-583	-250	1028	2278		278[illegible]	-694	-467	211
-583	-583	-250	694	694	944	444	-583	-583	-250	694	694	944	444	-583	-583	-250	1028	1028	2278		2584	1058	616
-333	-333	000	444	444	444	444	-333	-333	000	444	444	444	444	-333	-333	000	778	778	778	1778		2312	582
1042	708	375	-583	-583	-583	-333	1042	708	375	-583	-583	-583	-333	1042	708	375	-583	-583	-583	-333	2292		1632
708	1042	375	-583	-583	-583	-333	708	1042	375	-583	-583	-583	-333	708	1042	375	-583	-583	-583	-333	958	2292	
375	375	375	-250	-250	-250	000	375	375	375	-250	-250	-250	000	375	375	375	-250	-250	-250	000	625	625	1625

D-7-6 Average variation of a treatment difference = 4.01450

[redacted]

D-7-3 Average variation of a treatment difference = 3.85890

2158	804	513	-463	-463	-463	-213	908	554	263	-504	-504	-504	-254	908	554	263	-483	-483	-483	-233	908	554	263
	2158	513	-463	-463	-463	-213	554	908	263	-483	-483	-483	-233	554	908	263	-504	-504	-504	-254	554	908	263
2242		1575	-025	-025	-025	225	263	263	325	-088	-088	-088	163	263	263	325	-088	-088	-088	163	263	263	325
885	2311		2425	1175	1175	925	-463	-463	-025	988	738	738	488	-463	-463	-025	988	738	738	488	-463	-463	-025
435	409	3086		2425	1175	925	-463	-463	-025	738	988	738	488	-463	-463	-025	738	988	738	488	-463	-463	-025
-452	-481	866	2427		2425	925	-463	-463	-025	738	738	988	488	-463	-463	-025	738	738	988	488	-463	-463	-025
-391	-446	1012	1228	2562		1925	-213	-213	225	488	488	488	488	-213	-213	225	488	488	488	488	-213	-213	225
-426	-501	857	1148	1216	2402		2158	804	513	-504	-504	-504	-254	908	554	263	-483	-483	-483	-233	908	554	263
-187	-216	1111	877	943	863	1857		2158	513	-483	-483	-483	-233	554	908	263	-504	-504	-504	-254	554	908	263
946	615	034	-552	-510	-530	-272	2283		1575	-088	-088	-088	163	263	263	325	-088	-088	-088	163	263	263	325
589	1041	008	-582	-564	-604	-301	852	2304		2258	1008	1008	758	-504	-483	-088	904	654	654	404	-504	-483	-088
326	355	574	-112	-050	-110	168	544	572	1624		2258	1008	758	-504	-483	-088	654	904	654	404	-504	-483	-088
-346	-414	685	1121	968	888	591	-472	-541	-032	2599		2258	758	-504	-483	-088	654	654	904	404	-504	-483	-088
-285	-379	832	922	1301	956	657	-430	-523	030	1445	2825		1758	-254	-233	163	404	404	404	404	-254	-233	163
-320	-434	677	842	956	1141	577	-450	-563	-030	1365	1480	2665		2158	804	513	-483	-483	-483	-233	908	554	263
-081	-149	930	571	683	603	571	-192	-261	248	1069	1180	1100	2069		2158	513	-504	-504	-504	-254	554	908	263
931	600	039	-502	-474	-494	-252	904	572	264	-442	-415	-435	-192	2154		1575	-088	-088	-088	163	263	263	325
574	1026	013	-531	-529	-561	-281	572	1204	292	-511	-508	-548	-261	822	2274		2258	1008	1008	758	-483	-504	-088
311	340	579	-062	-014	-074	188	264	292	343	-002	045	-015	248	514	542	1594		2258	1008	758	-483	-504	-088
-432	-461	526	1027	848	768	517	-513	-541	-072	1081	902	822	571	-502	-531	-062	2347		2258	758	-483	-504	-088
-371	-426	672	828	1182	836	583	-470	-524	-010	928	1281	936	683	-474	-529	-014	1168	2521		1758	-233	-254	163
-406	-481	517	748	836	1022	503	-490	-564	-070	848	936	1121	603	-494	-569	-074	1088	1176	2362		2158	804	513
-167	-196	771	477	563	482	497	-232	-261	208	551	637	557	571	-252	-281	188	837	923	843	1857		2158	513
957	580	030	-532	-486	-491	-267	926	549	266	-426	-380	-385	-161	911	534	251	-512	-466	-471	-247	2202		1575
600	1006	004	-561	-541	-566	-296	595	1001	296	-494	-474	-499	229	580	986	280	-541	-521	-546	-276	825	2231	
337	320	570	-092	-026	-071	173	286	269	346	014	080	035	279	271	254	331	-072	-006	-051	193	542	525	1602

D-7-4 Average variation of a treatment difference = 4.07288

[redacted]

D-7-1 Average variation of a treatment difference = 3.81130

2147	785	481	-516	-516	-516	-266	897	535	231	-545	-545	-545	-295	897	535	231	-516	-516	-516	-266	897	535	231
	2109	452	-561	-561	-561	-311	535	859	202	-571	-571	-571	-321	535	859	202	-561	-561	-561	-311	535	859	201
2245		1481	-183	-183	-183	673	231	202	231	-212	-212	-212	039	231	202	231	-183	-183	-183	067	231	202	231
912	2245		2160	910	910	660	-516	-561	-183	782	532	532	282	-516	-561	-183	827	577	577	327	-516	-516	-183
579	579	1579		2160	910	660	-516	-561	-183	532	782	532	282	-516	-561	-183	577	827	577	327	-516	-516	-183
-477	-477	-144	2143		2160	660	-516	-561	-183	532	532	782	282	-516	-561	-183	577	577	827	327	-516	-516	-183
-490	-490	-157	894	2161		1660	-266	-311	067	282	282	282	282	-266	-311	067	327	327	327	327	-266	-311	067
-523	-523	-190	861	869	2094		2147	785	481	-545	-545	-545	-295	897	535	231	-516	-516	-516	-266	897	535	231
-227	-227	106	643	644	611	1644		2109	452	-571	-571	-571	-321	535	859	202	-561	-561	-561	-311	535	859	202
982	649	316	-476	-473	-515	-226	2235		1481	-212	-212	-212	039	231	202	231	-183	-183	-183	067	231	202	231
649	982	316	-476	-473	-515	-226	902	2235		2090	840	840	590	-545	-571	-212	782	532	532	282	-545	-571	-212
316	316	316	-143	-140	-182	107	569	569	1569		2090	840	590	-545	-571	-212	532	782	532	282	-545	-571	-212
-477	-477	-144	810	561	528	310	-476	-476	-143	2143		2090	590	-545	-571	-212	532	532	782	282	-545	-571	-212
-490	-490	-157	561	828	536	311	-473	-473	-140	894	2161		1590	-295	-321	039	282	282	282	282	-294	-321	039
-523	-523	-190	528	536	761	278	-515	-515	-182	861	869	2094		2147	785	481	-516	-516	-516	-266	897	535	231
-227	-227	106	310	311	278	310	-226	-226	107	643	644	611	1643		2109	452	-561	-561	-561	-311	535	859	202
949	616	282	-510	-515	-540	-260	944	610	277	-510	-515	-540	-260	2169		1481	-183	-183	-183	067	231	202	231
616	949	282	-510	-515	-540	-260	610	944	277	-510	-515	-540	-260	835	2269		2160	910	910	660	-516	-561	-183
282	282	282	-177	-182	-207	078	278	277	277	-176	-182	-207	074	502	502	1502		2160	910	660	-516	-561	-183
-477	-477	-144	810	561	528	310	-476	-476	-143	810	561	528	310	-510	-510	-176	2143		2160	660	-516	-561	-183
-490	-490	-157	561	828	536	310	-473	-473	-140	561	828	536	310	-515	-515	-182	894	2161		1660	-266	-311	067
-523	-523	-190	528	536	761	278	-515	-515	-182	528	536	761	278	-540	-540	-207	861	869	2094		2147	785	481
-227	-227	106	310	311	278	310	-226	-226	107	310	311	278	310	-260	-260	074	643	644	611	1644		2109	452
995	662	329	-477	-490	-523	-227	982	649	316	-477	-490	-523	-227	949	616	282	-477	-490	-523	-227	2245		1481
662	995	329	-477	-490	-523	-227	649	982	316	-477	-490	-523	-227	616	949	282	-477	-490	-523	-227	912	2245	
329	329	329	-144	-157	-190	106	316	316	316	-144	-157	-190	106	282	282	282	-144	-157	-190	106	579	579	1579

D-7-2 Average variation of a treatment difference = 3.82798

[redacted]

D-7-9 Average variation of a treatment difference = 3.77778

2000	571	571	571	-143	-143	000	000	-143	-143	000	000	000	000	571	143	000	571	000	143	000	571	000	143	-143	-143	000	000
	2000	571	571	-143	-143	000	000	000	000	571	143	-143	-143	000	000	-143	000	-143	000	000	143	000	571	000	000	571	143
1672		2000	571	000	000	571	143	-143	-143	000	000	000	000	143	571	-143	000	-143	000	-143	000	-143	000	000	000	143	571
307	1735		2000	000	000	143	571	000	000	143	571	-143	-143	000	000	000	143	000	571	-143	000	-143	000	-143	-143	000	000
404	352	2119		2000	571	571	571	571	000	000	143	571	000	000	143	-143	-143	000	000	571	000	143	000	-143	000	-143	000
404	530	544	2119		2000	571	571	000	-143	-143	000	000	-143	-143	000	000	000	571	143	143	000	571	000	000	571	000	143
014	-080	-237	-223	2119		2000	571	000	-143	-143	000	143	000	000	571	-143	-143	000	000	000	-143	000	-143	000	143	000	571
087	087	014	112	404	1672		2000	143	000	000	571	000	-143	-143	000	000	000	143	571	000	-143	000	-143	-143	000	-143	000
112	028	352	-031	530	307	1735		2000	571	571	571	571	143	000	000	000	-143	-143	000	571	000	000	143	000	-143	000	-143
112	206	-223	544	544	404	352	2119		2000	571	571	143	571	000	000	571	000	000	143	000	-143	-143	000	571	000	143	000
-059	-185	192	-237	544	112	206	-223	2119		2000	571	000	000	143	-143	000	-143	-143	000	143	000	000	571	143	000	571	000
014	066	348	192	-223	014	-080	-237	544	2119		2000	000	000	-143	-143	143	000	000	571	000	-143	-143	000	000	-143	000	-143
-059	307	014	-059	112	087	087	014	404	404	1672		2000	571	571	571	000	000	-143	-143	571	143	000	000	000	-143	-143	000
038	101	206	530	-031	112	028	352	352	530	307	1735		2000	571	571	571	143	000	000	000	000	-143	-143	571	000	000	143
038	004	-185	-080	530	038	101	206	352	-031	112	028	1735		2000	571	143	571	000	000	143	571	000	000	000	-143	-143	000
112	254	-028	348	-237	-059	-185	192	-223	544	112	206	352	2119		2000	000	000	-143	-143	000	000	-143	-143	-143	000	000	571
404	066	-310	-028	192	014	066	348	-237	-223	014	-080	530	544	2119		2000	571	571	571	-143	000	000	-143	571	143	000	000
136	112	404	112	-059	-059	307	014	014	112	087	087	307	404	404	1672		2000	571	571	000	571	143	000	000	000	-143	-143
112	004	254	066	-080	038	004	-185	206	530	038	101	028	352	-031	112	1735		2000	571	000	143	571	000	143	571	000	000
404	-185	-028	-310	348	112	254	-028	192	-237	-059	-185	206	-223	544	112	352	2119		2000	-143	000	000	-143	000	000	-143	-143
112	-080	348	-028	-028	404	066	-310	348	192	014	066	-080	-237	-223	014	530	544	2119		2000	571	571	571	-143	000	000	-143
136	038	112	404	112	136	112	404	014	-059	-059	307	087	014	112	087	307	404	404	1672		2000	571	571	-143	000	000	-143
-059	038	-059	014	404	136	038	112	404	112	136	112	307	014	-059	-059	087	014	112	087	1672		2000	571	000	571	143	000
307	101	-185	066	066	112	004	254	-185	-080	038	004	101	206	530	038	028	352	-031	112	307	1735		2000	000	143	571	000
014	206	192	348	-310	404	-185	-028	-028	348	112	254	-185	192	-237	-059	206	-223	544	112	404	352	2119		2000	571	571	571
-059	530	-237	192	-028	112	-080	348	-310	-028	404	066	066	348	192	014	-080	237	-223	014	404	530	544	2119		2000	571	571
087	112	112	014	014	-059	038	-059	112	404	136	038	112	404	112	014	307	014	-059	-059	087	087	014	112	1672		2000	571
087	028	206	-080	066	307	101	-185	254	066	112	004	004	-185	-080	038	101	206	530	038	112	028	352	-031	307	1735		2000
014	352	-223	-236	348	014	206	192	-028	-310	404	-185	254	-028	348	112	-185	192	-237	-059	112	206	-223	544	404	352	2119	
112	031	544	-223	192	-059	530	-237	348	-028	112	-080	066	-310	-028	404	066	348	192	014	014	-080	-237	-223	404	530	544	2119

D-7-10 Average variation of a treatment difference = 3.59350