## CHAPTER IV.

# INCOMPLETE PAIRS OF ELEMENTS.

### § 39.

### Closure of Pairs of Elements by Sensible Forces.

In our examination both of the lower and higher pairs of elements, we have hitherto assumed that the reciprocal restraint of the two elements forming the pair was complete; *i.e.*, that each of the two bodies by the resistant qualities of its material and the form given to it, both enveloped and constrained the other. We have made this assumption also, expressly or tacitly, in investigating the forms to be given to bodies in order that any sensible forces tending to alter their required relative motion might be balanced by latent forces. Under certain circumstances, however, the strictness of this condition may be somewhat relaxed,—when, namely, precautions are taken to prevent the possibility of sensible forces having certain directions ever affecting the pair. If this can be done, it is obvious that it is no longer absolutely necessary to make the pair entirely self-closed, bodily envelopment being no longer essential for restraint in those directions.

In order thus to prevent the disturbing action of sensible forces acting in any given direction upon an element *a* which is to be constrained, we allow another sensible force to act upon it continuously, and make the direction of this force opposite to, and its magnitude not less than, those of the former. If, for example, the greatest anticipated disturbing force be $= P$, then in order to neutralise it we must cause an opposite force of the magnitude $P$ to act upon the supposed element $a$. This force restrains $a$ in the place of a portion of the enveloping partner-element $b$; if for the sake of security we give to it the magnitude $P + Q$, then in the worst case $a$ is held at the opposite point of restraint by the force $P - P + Q = Q$, this pressure being balanced by latent forces in the element $b$. The general conditions of equilibrium are thus fulfilled. Any such force as the supposed one $P + Q$ closes, as it were, in the direction $-(P + Q)$, the pair of elements left unclosed or incomplete in that direction; we shall therefore call it a closing force. Such pairs of elements as require closing forces are evidently incomplete in themselves; their usefulness depends upon the applica-

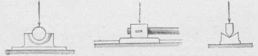

FIG. 120. FIG. 121. FIG. 122.

tion of the closing force, or upon what we may call in one word force-closure.

Force-closed pairs occur frequently in machinery. The shafts and bearings of most water-wheels are illustrations of them, where the great weight of the wheel is almost always sufficient in itself to prevent any vertical motion of the axle without the employment of a plummer-block cover (Fig. 120). The crosshead sometimes used for large horizontal engines gives us another illustration (Fig. 121); the heavy pistons and rods here prevent any rising of the crosshead, which is guided only beneath and at the sides. The knife-edge of a balance (Fig. 122) is also kept in continuous contact with its bearings by the weight of the beam and scales. The railway turn-table is another example, the whole table being here held by its own weight and that of the load upon it on a roller path completely open above; a similar thing often occurs in wharf-cranes. Railway-wheels, lastly, are common and well-known illustrations of force-closed pairs; they are kept in continual contact with their partner-elements the rails, by vertical downward closing forces.

Inhall these andhsimilar cases force-closure present slitself obviously and naturally, and often greatly slimplifies thehconstruction. This, however, is only onehof theways lin which itlocclrs; we must proceedto consider others.

§ 40.

## Force-closure in the Rolling of Axoids.

Whilein the cases just mentioned the object of thelclosing force is essentially to prevent the separation of the incompletely formed elements, it may under certain circumstances have a much wider purpose. This occurs when the action of a force derived fromlit, fr iction, is used to complete the reciprocal restraintlof the elements. An illustration of this is furnished by the friction wheels (Fig. 123) already mentioned in § 37. Here thefo rce-cl osure has notlmerely tohholdhthe two cyl inders inhcontact, but alsohto press them so strongly together as to prevent sliding under a given tangential force,—in other words, to cause the cylindric surfaces, which here are the axoids themselves, to roll upon one another.

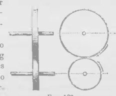

Fig. 123.

By a more stri ct examination into this case, we can see that the closing f orce presses the small roughnesses of thehsurfaces hogetherhso as to make the cylinders work like spur-wheels, such force-component s as tend to separate the wheels being resisted by thelclosing force. It is this consequehcelof the pressing together of two bodies which is considered in mechanics under the name friction.[22]

This employment of force-closure occurs very frequently. Its application in the case of the driving-wheels of locomotivesis important in the highest degree. The whohe development of our railways has been directly dependent on it. Every one is familiar with the fact that at the first the notion of "adhesion" between the wheels and the rails was thought so illusory that it could scarcely obtain a trial,—restraint being obtained by the use of suitably profiled pairs of elements. Blenkinsop's toothed-rail, and those of the Liverpool and Manchester Railway, Brunton's revolving legs, and other even less practical constructions all illustrate this.

The constrained rolling of axoids under force-closure differs essentially from the mere closure of an incomplete pair of elements. The two things may, however, occur together as well as singly. In the driving-wheels of locomotives they are united ; in the wheels of the carriages there is nothing more than closure of elements by pressure.

In the latter case it would be possible to bring about the element-closure by the addition of a second pair of elements, an additional rail, for instance, which could be so embraced by a suitably formed piece connected with the carriage as to render any rising of the latter from the main rails impossible; but this would not in any way make the wheels more like the driving-wheels. Such an arrangement has indeed been employed on the Rigi railway. With it the motion of the carriage on the line may be more accurately described as that of an ordinary closed pair than as occurring with force-closure.

We see that force-closure finds important and numerous applications. It always retains, however, a certain incompleteness. If the closing forces be not sufficiently large, or if unforeseen disturbances occur, the constraint may be destroyed or temporarily broken. Notwithstanding this, force-closure—as the examples given show—is of most essential service in machinery. It leads us besides to an entirely different kind of pairs of elements, which are in machinery of even greater importance than those just considered. We shall examine these more closely in the following paragraphs.

## § 41

### Flectional Kinematic Elements.

We have hitherto supposed the capacity for resistance, which we recognised as an essential for those bodies from which a machine could be constructed, to be attained by giving to the elements complete rigidity, molecular immovability. We assumed that the material and dimensions of the element had been suitably chosen for this purpose, this being the function of machine design. The admissibility of force-closure, however, shows us that bodies may serve for the formation of elements which cannot be considered as sensibly rigid. If, namely, we choose such bodies as, while not resistant in all directions, can maintain approximate molecular immovability under the action of sensible force in at least one direction, and employ these bodies under such force-closure as

FIG. 124.

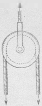

FIG. 125.

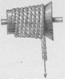

FIG. 126.

corresponds to their special capability of resistance, they will act exactly as if they were completely resistant.

As bodies possessing these peculiarities we have ordinary cords or ropes, woven or leathern bands and belts, bands or ropes of metal or wire, every kind of chain, all those organs in short which, offering no sensible resistance except to tension, can yet be made sufficiently rigid under the action of tensile forces of any magnitude. We may include them all under the name of tension-organs.

On account of their want of rigidity in other directions, the tension-organs can be very readily united into pairs of elements with rigid bodies of various forms. Thus we may have a rounded bar (Fig. 124) over which they slide, force-closed in both directions,—a pulley (Fig. 125), the tension-organ moving upwards on one side of it, downwards on the other;—a drum (Fig. 126) upon which it can be coiled, and so on. We find these pairs of elements used and applied in the most various ways, in pulleys and cranes, in belt-trains, in rope-trains, in submerged rope towing, etc. The elements always reciprocally envelope each other,—but although the envelopment passes sometimes into enclosure, they have not the latter as an essential characteristic; they must be classed therefore as higher pairs of elements.

Directly opposed to tension-organs there stand others which possess molecular immovability only in reference to compressive forces, and which may therefore be called pressure-organs. To this class belong all fluids, liquid and gaseous,—water, oil, steam,


Fig. 127.

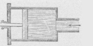
Fig. 128.

air, etc. The force-closure applied to them must be such as continually presses their molecules together. In order, however, that they may not alter their form by extension on either side, all the surfaces of the fluid body besides those normal to the direction of motion must be pressed together with the same force. This is done by the help of latent forces,—that is, by enclosing the fluid in vessels of suitable form and resistance. This occurs, for example, in steam- or water-pipes (Fig. 127) and in the cylinders of pumps, or of steam- or blowing-engines (Fig. 128), and so on. It hardly needs to be pointed out how extremely important a part pressure-organs of this kind play in the construction of machines.

Another class, differing but little from the one just described, has been formed of late years from some of the tension-organs, by enclosing them in suitably shaped vessels, and having thus rendered sideway motion impossible, using them as pressure-organs. The flat-link chain of the Neustadt cranes (Fig. 129) is enclosed in a tube, so that it can be pressed forward; the thin brakeband of steel has been made capable of resisting pressure at its free ends by bedding it in a hollow cylinder (Fig 130), its particles then acting like the stones of an arch. Wire-rope has also been used in a somewhat similar way.

The pressure-organs form closed pairs of elements like the lower pairs; on account, however, of their molecular moveability, they must be classed with the higher pairs.

If we compare the two classes of pairs of elements to which the consideration of force-closure has led us, we see that they are closely related. They both have the peculiarity that they can be used in one way only; with a closing force, that is, of one particular kind or direction, the tension-organ only with tension, the pressure-organ only with thrust. If the rope in Fig. 126 be pushed

FIG. 129.

FIG. 130.

upwards, it does not set the drum in motion, nor can the piston (Fig. 128) be moved by withdrawing the fluid from behind it. The pair are closed on one side only; they are mono-kinetic, a peculiarity which we shall find further on to belong to other pairs also. They owe this to their molecular yieldingness, or want of fixedness in all except a single or a small number of directions. This quality is what is known in pressure-organs as fluidity, in tension-organs as flexibility or pliability. As a common designation for both, we may use the word flectional, and therefore call both tension- and pressure-organs, when employed as kinematic elements, flectional elements.

The two sets of organs stand opposed to each other as positive and negative, the relation directly indicated also by the nature of their closing forces. The pipe filled with water, Fig. 127, stands opposite to the rope in Fig. 124; the cylinder having a piston moved forward by a fluid pressure on one side, Fig. 128, corresponds to the drum in Fig. 126. The application of a column of water for transmitting pressure, which has lately been made in mining operations, furnishes again the opposite of a rope used in tension. Thus the tension- and pressure-organs are contra-positive. They must therefore be equally reckoned among kinematic elements. Willis' exclusion of all mechanisms of which fluid organs form a part, to which we alluded in the Introduction, was therefore incorrect. If belts, pulleys and so on, are to be considered as forming portions of "pure mechanism," it is logically impossible to omit water- and wind-mills, or steam-engines. We have only to think of the importance of the latter to be astonished that one of the most valuable and most extensively applied of machines, one possessing also the greatest delicacy and accuracy in its motion, should ever have been considered unkinematic,—incapable of scientific kinematic treatment,—"impure." We shall on the other hand be able to see further with what scientific force and with what important consequences kinematic science can be brought to bear upon these machines. Although Willis' view of the matter is not acknowledged as a principle, its incorrectness has not been specially pointed out; practically it has had the result that this class of machinery has scarcely ever been treated kinematically by English writers, and by others only seldom, and even then generally not with the requisite thoroughness.

## § 42.

## Springs.

While in the tension- and pressure-organs we have had elements in which the application of force could occur only in a certain very simple manner, there is in machinery another class of elements which can be arranged so as to be used with any possible application of force. These elements are springs. They are familiar in many forms and for many purposes; always, however, under the condition which we found to be necessary in the case of the flectional elements,—with the limitation, that is, that in each special case a single force-application only can be used. The various constructions of springs may be classified according to the nature of this force, so that we can distinguish them as springs for tension, thrust, bending or torsion. Bending- and torsion-springs are most often of metal, but also sometimes of wood; for the thrust- and (less often) the tension-springs india-rubber and other organic materials are much used.

The ease with which the most various forms can be given to springs permits them often to have a form which, considered as a whole, is adapted for working under a force-application quite different from that for which the cross-section of the material from

Fig. 131

which they are made would seem to adapt them. The helical spring (Fig. 131) for instance is closed, when used as a whole, by a force in the direction of its axis,—in other words, is employed as a tension-organ; as regards its cross section it is suited for working with torsion.* If the same spring be so formed that its coils are not in contact in its normal position it can be used as a whole as a pressure-organ; for this purpose, however, it must be enclosed in a suitable case to prevent lateral deflection. A spring which is itself adapted for tension can be, and is used as a whole as a torsion or wrenching spring in the well-known form shown in Fig. 132

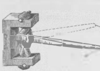

Fig. 132

Springs, both simple and compound, working under a bending force are familiar in their many applications to railway work, as are several kinds of torsion-springs, and among the pressure-springs one of a peculiar kind (*Strebe-fieder*)—the steel ring used between the tread and tyre of the wheel in the proposed "spring wheels" of Mr. Adams.†

Springs are very well suited for supplying the force-closure of

* See *Der Constructeur*, 3rd Ed. p. 59.

† See Colburn's *Locomotive Engineering*, pp. 99, 100, etc.

incomplete pairs of elements and are much used for that purpose, as in the case of spring-packed pistons, spring pawls for ratchet-wheels, etc.; they play also a most important part in the storing of energy, to which we shall return further on. Springs of organic material, india-rubber, vegetable fibre, skin, etc. greatly resemble tension-organs; those of hard material seem more like the rigid elements. In their mode of action they rather resemble the pressure-organs, as being, within somewhat wide limits, elastic. They differ essentially from the rigid elements, however, notwithstanding the apparent resemblance, for in these the flexibility is supposed to be reduced to so small an amount that it may be neglected, while in the springs it is intentionally made very considerable.

§ 43.

## Closure of a Pair of Elements by a Kinematic Chain.

An incomplete pair of elements may also be closed by kinematic linkage. Two bodies $a$ and $b$ (Fig. 133) having for their axoids circular cylinders, and having their surfaces fluted,—in other words, two spurwheels furnished with accurately fitting teeth,—have by their profile forms the necessary restraint in the direction of the tangent $T\,T$; the teeth may also be so made that the wheels cannot be moved nearer together; there is required only restraint against divergent motion in the direction $N\,N$ of the normals.

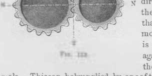


Fig. 133.

This can be supplied by one of the methods considered in a former article. Let us take the fifth method (§ 35), in which parallels to roulettes serve as profiles, then we have as a parallel to the (point-)path of the centre of one of the wheels a circular ring, and as a parallel to the other centre itself a circle, these giving us the profile shown in Fig. 134, an annular groove for the wheel $a$, and a cylindric pin moving in it for the wheel $b$,—and by these the necessary restraint both against convergent and divergent motion is obtained. If we suppose further that means are provided for preventing lateral motion, we have before us a closed pair.

This method of closure is not practicable in the cases commonly occurring; possibly it seems here somewhat far-fetched; we shall

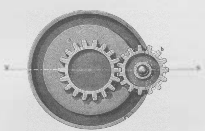


Fig. 134.

see presently, however, that it is in no way without precedent. In practice we use rather an easily arranged kinematic linkage between $a$ and $b$. If, for instance, we attach both to $a$ and to $b$ (coaxially with their axoids or pitch surfaces), solid cylinders, enclose these

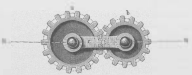


Fig. 135.

in open cylinders, and connect the latter by a rigid bar (Fig. 135), the restraint in the direction $NN$ of the normals is perfect (we may here, as also in the last case, allow the points of the teeth to be quite free, as they are no longer required for restraint). Instead of a closed kinematic pair, we have now a closed kinematic chain of three links. The two wheels $a$ and $b$, with their conaxial cylindric pins, form two of these links; the third is the bar $c$, with its two parallel cylindric holes, which form the bearings for the axles of the wheels.

The chain-closure by which the given incomplete pair has here been closed, or made into a constrained pair, is used constantly not only for cylindric wheels but also for bevel-wheels, hyperboloidal-wheels, screw-wheels, etc.

Very frequently it is employed merely in order to simplify a construction,—the element necessary for the pair-closure being actually present, although incompletely formed. The screw mechanism shown in two forms in Figs. 136 and 137, for example, consists in each case of the three following links: $a$ a screw with conaxial cylindric journals, $b$ a nut profiled externally as a prism

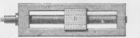

FIG. 136.

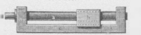

FIG. 137.

FIG. 138.

parallel to the axis of the screw, $c$ a guiding prism for the nut, carrying bearings for the journals of the screw. In Fig. 136 each of the three pairs is completely closed, in Fig. 137 the closure of the pair $b$ $c$ is incomplete. Restraint against the upward motion of the nut $b$ is here given by the link $c$, which is of a suitable form for that purpose; a conveniently arranged chain-closure, that is, occurs, besides the incompletely applied pair-closure.

Chain-closure also occurs along with force-closure. The common ratchet work (Fig. 138) furnishes an example of this. The motion of the working end of the pawl is here force-closed in moving backwards over the teeth; the motion of its jointed end is chain-closed, taking place in circular arcs about the axis of the wheel. This mechanism is at the same time single-acting only, or monokinetic, a property already pointed out in connection with Fig. 126.

The hydraulic press gives us an instructive example of a mechanism combining chain- and force-closure. In Fig. 139 the pump valves are omitted for the sake of clearness. The vessel *d* carries the chain-closure between the pistons *a* and *b*, with which it is prismatically* paired. At the same time it encloses the fluid *c*, which is force-closed by the pressures upon the two pistons in the only direction of motion left possible to it. The hydraulic press forms the contra-positive of a machine apparently most unlike it, the pulley-tackle, the pressure-organ water in the one being replaced by the tension-organ rope in the other. If we substitute rigid rounded bars at the top and bottom of the tackle for the usual pulleys, as in Fig. 140, the logical correspondence

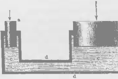

Fig. 139

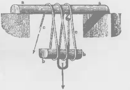

Fig. 140

between these two mechanisms becomes even more obvious.† The chain now, indeed, has three links only, but this is merely because the tension-organ does not need a confining vessel. The motion of the piece *b* as a whole, is still force-closed by the load. If we used a prism pair to compel *b* to move in a straight line in reference to *a*, the similarity would be still more complete. It is worth noticing how many illustrations occur in modern engineering of what we may call the interchangeability of tension- and pressure-organs, of which we have had here an illustration. The arrangement for ringing bells by air-pressure, now becoming extensively

* Each piston, although cylindric, forms a sliding pair with its stuffing-box, turning being prevented by some external means.

† An error is frequently made which stands greatly in the way of understanding this matter, that namely of supposing the action of the tackle to be absolutely connected with the rotating block, or pulley, which in reality has no other object than that of lessening destructive friction. In anatomy the trachlea surface over which a tendon slides is rightly called a *Rolle* or pulley.—R

used, is obviously the contra-positive of the old bell-rope; the (single-acting) "water-rods" used in mines, of the iron tension-rods,—and partly also the hydraulic crane of the rope or chain crane.

The water-wheel (Fig. 141) gives us a further illustration of a force-closed mechanism. The enclosure of the water in the channel is again two-fold. It is due in the first place to the action of latent forces in the channel walls, and then further to the force of gravity, which prevents the water moving upwards. In the bent portion of the channel the water is paired both with the floats,—virtually a toothed rim of the wheel—and also with the channel; the wheel itself, on the other hand, is again pair-closed with the channel through its shaft and bearings. The kinematic chain has three links,—the wheel $a$, with its shaft, the water $b$, and the channel $c$ with the shaft bearings. The pairing of the flectional element $b$ with $a$ is effected by the chain; the force-closure produces at the same time the envelopment (here enclosure) of the floats with the flectional element $b$, and the confinement of the latter within its channel, which we must look upon as a vessel only partially closed.

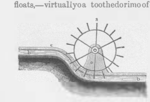


Fig. 141.

There are many cases in which the axoid rolling (§§ 37 and 40) takes place, or at least is rendered possible, by chain-closure instead of direct force-closure. The Fell railway is a good example of this. Here the driving-wheels, instead of being held on the rails, as usual, by the weight above them, are made horizontal and held against a central by an arrangement of springs forming a kinematic chain. In this and many similar cases the spring can be used with great advantage, for as an element at once flectional and elastic it admits of advantageous employment under varying forces. In this special arrangement, however, there is often a certain indeterminateness of motion; the possibility occurs of a sort of overbalancing of the force-closure.

§ 44.

## Complete Kinematic Closure of the Flectional Elements.

We have seen that the flectional elements may receive, by means of pair-closure and force-closure, important, practical and in the highest degree valuable kinematic applications. We found force-closure to be always necessary, but to a different extent in different cases. In the case of the cord in Fig. 124 we required the absolute prevention of every force not acting as a direct pull upon the cord itself; for the force-closure in the hydraulic press nothing more is necessary than that downward force should be caused to act upon the pistons. If we go one step further we can make even this unnecessary; we can, that is, admit the flectional elements unreserved into what we have called (§ 1) machinal systems.

The removal of the force-closure is effected by the use of suitably

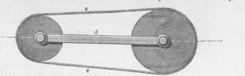


FIG. 142.

combined force-closed chains. The common arrangement of belt pulleys (Fig. 142) gives us a familiar example of this. Here we have two cylindric pulleys, $a$ and $c$, fitted with conaxial shafts, and connected by a bar or frame $d$ which carries the bearings for the latter; the belt belonging to $a$ is made in one piece with that of $c$,—the two forming the common endless band $b$. By this means we obtain a kinematic chain in which the flectional element no longer requires external force-closure. We might regard it as a combination of two chains, each of the form of Fig. 125, their closure acting continuously in opposite directions.

The chain-closure here is twofold. In the first place, as has been mentioned, it makes the force-closure of the flectional element $b$

unnecessary by substituting for it the action of the latent forces in the frame $d$. In the second place, it entirely constrains the axoid rolling, the securing of which by force-closure we discussed in § 40. The pulleys being cylindric, the axoids here are on the one hand the surfaces of the belts continually running off and on, and on the other the peripheral surfaces of the pulleys. These surfaces are brought so firmly into contact by the latent forces acting through $d$, that they cannot slide upon each other. The chain obtained has four links the two pulleys $a$ and $c$ with their shafts, the band $b$, and the frame $d$, with the shaft bearings. The motions in the chain occur exactly as if its elements were completely rigid. Every angular motion of the one pulley is accompanied by a corresponding motion of the other; the chain can also be inverted, that is, any one of its links may be fixed, assuming, of course, that no lateral disturbing force be allowed to act upon the belt itself.

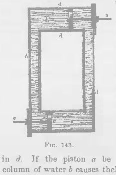


FIG. 143.

The kinematic chain shown in Fig. 143 may be considered the contra-positive of this mechanism. The two pistons $a$ and $c$, fitting tightly into a connecting vessel $d$, are made to form sliding pairs * with it, and are kinematically linked by means of the fluid columns $b$ and $e$ enclosed in $d$. If the piston $a$ be moved to the left the descending column of water $b$ causes the piston $c$ to move to the right; just as much water is moved up one column as moves down the other. If care be taken to fill the vessel $d$ perfectly, all air being removed from the water, the action of the mechanism is complete, and the chain is as perfectly closed as if it consisted of rigid elements only. Herr Anderssohn has shown this convincingly in his interesting experiments with tubes as long as 3000 metres, used as water-rods, as in Fig. 139.† Every small sliding of one piston is accompanied by the corresponding motion of the other. This mechanism, the tardily acknowledged contra-positive of the familiar

* See note p. 168.

† *Zeitschrift des V. deutscher Ingenieure.* Vol. xiii. (1869) p. 402

endless band, is now coming into more extended use as a double-acting water-rod.

Springs also, like tension- and pressure-organs, can be completely constrained and used in mechanisms by means of chain-closure. The common clock-spring, Fig. 144, illustrates this. The spring *a* is connected at one end with the cylinder *c*, and at the other with the barrel *b*, which is paired with *c*, and connected to the clock-work. It thus forms a link in a closed kinematic chain, and this is one of its special uses in machinery, where in some cases a constrained motion of special exactness becomes of great importance.

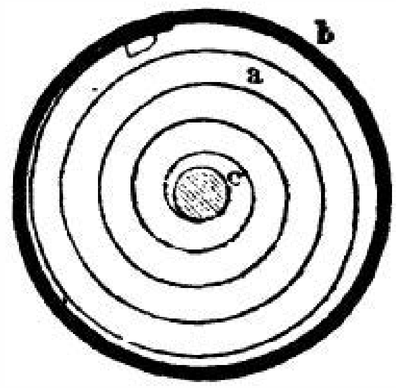

FIG. 144.