# UNDERSTANDING FIRMS' STRATEGIC BEHAVIORS AND THEIR IMPLICATIONS 

A Dissertation<br>Presented to the Faculty of the Graduate School of Cornell University in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

by
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# UNDERSTANDING FIRMS' STRATEGIC BEHAVIORS AND THEIR IMPLICATIONS 

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Firms are key participants in the market. Compared to individual consumers, firms, particularly those with significant market power, are often seen by economists as more resourceful to overcome information frictions, more likely to retain past information and less susceptible to bounded rationality. As a result, firms are capable of acting as sophisticated strategic players. Under the assumption of being driven by a clearly defined profit motive, firms may behave strategically when interacting with other market participants, including consumers, other firms and governments. Understanding firms' strategic behaviors is a critical step in assessing market efficiency, analyzing the wellbeing of consumers, and designing or evaluating government policies.

This dissertation consists of three essays that analyze firms' strategic behaviors in different settings. Chapter 1 studies firms' strategic interactions in government organized spectrum auctions. In these ascending-bid auctions, firms as bidders are able to communicate their private information with one another using jump bidding as signals. The signals are credible since bidders with lower private information incur a higher ex ante cost for choosing a jump bid with any given size. This prevents the bidders with lower private information from mimicking those with higher private information. In equilibrium, the signaling model predicts lower expected revenue to the seller, in this case the government, than in the "open exit" model in which jump bidding is not allowed. Using data from a spectrum auction held by the Federal Communications Commission in the United States, the mean valuation estimated using the signaling model is higher compared to that of
the open exit model. This implies that if bidders are indeed using jump bids as signals, ignoring it leads to estimates of the mean values that are biased downwards. This result is consistent with the prediction of the theoretical model that bidders pay lower prices with jump bidding than in an open exit auction. I estimate that if jump bidding was prohibited, the government could have had $8 \%$ higher revenues from the auction.

In Chapter 2, my coauthors and I evaluate a government policy that subsidizes the agricultural equipment rental markets in India. We observe that private rental firms favor farmers located in dense areas and demanding higher machine-hours because equipment needs to be moved in space. Using our own census of 40,000 farmers, we document that costly delays and price dispersion in rentals are ubiquitous, and that small-scale farmers are rationed out by private rental firms. This rationing could be detrimental to aggregate productivity if small farmers have the highest marginal return to capital. A government subsidized first-come-first-served dispatch system grants small-scale farmers timely access to equipment at the expense of travel time. In a calibrated model of frictional rental services, optimal queueing and service dispatch we show that, while the constrained efficient allocation prioritizes large-holder farmers, small-scale farmers in dense areas are valuable because they help maximize capacity utilization. Through counterfactuals, we show that when the induced increase in subsidized equipment supply is high enough, service finding rates for small-farmers increase relative to large-holders farmers even when providers prioritize large-scale.

In Chapter 3, I offer an alternative explanation to the existing theories on why firms make the strategic decision to carry out planned obsolescence. Planned obsolescence refers to the practice of firms choosing durability levels for their products below the cost-efficient ones. Motivated by the Phoebus cartel, whose reason for engaging in planned obsolescence cannot be explained by existing theories, I introduce a new theory that centers on an important concept from behavioral economics: present-biased preferences. I construct
a theoretical model which demonstrates that when consumers are present biased, that is, when they exhibit time-inconsistent preference in favor of immediate gratification, a monopolist chooses a profit-maximizing level of durability below that chosen by a perfectly competitive market with the same production technology.

## BIOGRAPHICAL SKETCH

Haimeng Zhang grew up in China and Singapore. She holds a Bachelor of Arts degree in Economics and Management from the University of Oxford and a Master of Arts degree in Economics from New York University.

Before attending graduate school, Haimeng worked for 5 years in investment banking at UBS Investment Bank and Moelis \& Company based in London, covering the telecoms, media, and technology sectors. She also interned as a research assistant to the chief economist at the Antitrust Bureau of the New York State Attorney General's Office.

I dedicate this dissertation to my loving parents, Aihua Li and Yaxin Zhang, who have wholeheartedly supported every life decision I made.

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## CHAPTER 1

## JUMP BIDDING AS A SIGNALING GAME

This paper studies jump bidding in an ascending-bid auction with affiliated values using a multi-round signaling model. Bidders communicate their private information with one another via the sizes of jump bids. These signals are credible since bidders with lower private information incur a higher ex ante cost for choosing a jump bid with any given size. This prevents the bidders with lower private information from mimicking those with higher private information. In equilibrium, the signaling model predicts that the size of a jump bid placed each round is bounded above by a strategy that is equivalent to one in a first-price sealed-bid auction. The expected revenue to the seller is reduced due to bidders' abilities to send signals through jump bidding. Using data from a spectrum auction held by the Federal Communications Commission in the United States, the mean valuation estimated using the signaling model is higher compared to that of the "open exit" model. This implies that if bidders are indeed using jump bids as signals, ignoring it leads to estimates of the mean values that are biased downwards. This result is consistent with the prediction of the theoretical model that bidders pay lower prices with jump bidding than in an open exit auction. I estimate that if jump bidding was prohibited, the government could have had $8 \%$ higher revenues from the auction.

### 1.1 Introduction

Jump bidding in an ascending-bid auction refers to the action of placing a bid in excess of what is required by the auctioneer for a particular round. It is a common phenomenon in art auctions, spectrum auctions and online auctions. It is also observed in auction-like settings such as takeover bids.

In the auction literature, an ascending-bid auction is often modeled as an "open exit" auction (also called a button auction, or a clock model), in which the price in each round
increases by a small fixed amount set by the auctioneer. Bidders choose at which prices to drop out of the auction irrevocably. The open exit model leaves no scope to study the behavior of jump bidding, as bidders do not submit bids. In alternative models of the ascending-bid auction where bidders actively submit bids, jump bidding is weakly dominated by the strategy of bidding the minimum required price each round in the absence of two elements: transaction costs and signaling using bids between bidders. The two main theoretical strands of literature which explain jump bidding are concerned with these two elements.

This paper focuses on the informational role of jump bidding in an ascending-bid auction. There is substantial anecdotal evidence that bidders engage in jump bidding to intimidate their competitors so that they drop out of the auction sooner. In the earlier spectrum auctions organized by the Federal Communications Commission ("FCC"), jump bidding was pervasive. For example, in the very first FCC auction (the nationwide narrowband PCS auction) in 1994, $49 \%$ of all new high bids were jump bids that exceeded the high bid by more than two bid increments (Cramton, 1997). While the transaction cost theory could be applicable in this case, many economists believed that there were other factors at play given the large sizes of some of the jump bids and the fact that some were placed towards the end of the auctions. They suspected that bidders were using jump bids as signals of high valuation or "toughness" McAfee and McMillan, 1996; Cramton, 1997).

In 1997, the FCC introduced click box bidding which limited the size of a jump bid. One of the motivations was arguably to reduce the potential anticompetitive effects of jump bidding. Before this change in auction rules, 15 spectrum auctions had been conducted by the FCC, bringing in a total revenue of $\$ 23$ billion. Given the high stakes, if bidders were indeed using jump bids as signals and were successful in reducing the prices paid due to early withdrawal of competitors, the revenue effect could be significant.

Any attempt to quantify this effect will require a theoretical framework that describes a signaling equilibrium in an ascending-bid auction.

Avery's 1998 paper provides an excellent starting point. The paper solves for equilibria of ascending-bid auctions with two symmetric bidders and affiliated values when jump bidding strategies may be employed as signals. In this model, each bidder receives a private observation, which is a noisy signal of her true valuation of the auctioned object. The private observations are strictly affiliated, meaning a large realized private observation for one bidder makes the other bidder more likely to have large realized private observations as well. The paper shows that if bidders are allowed to have a one-off opportunity to simultaneously choose from a discrete set of jump bids as their opening bids and continue according to prespecified asymmetric equilibrium in favor of the higher bidder, then there exists a unique symmetric signaling equilibrium where the size of the opening bid is weakly monotone in a bidder's private observation. In other words, the choices of jump bids are partially separating. The bidder with the higher private observation chooses a weakly larger jump bid. Each possibility of jump bidding provides a Pareto improvement for the bidders from the symmetric equilibrium of a second-price sealed-bid auction.

While Avery's paper provides a useful framework to empirically study the use of jump bidding as a signaling device, it has a number of stylized features that are incompatible with most empirical settings. It is a model limited to an auction with 2 bidders. More important, it only allows bidders a one-off opportunity to send signals through jump bidding at the beginning of the auction. Bidders' abilities to send signals in multiple rounds will change the predictions of the model.

This paper builds upon Avery's model and extends it to more than two players. By focusing on the equilibrium in which bidders are willing to drop out of the auctions as soon as they find out their private observations are not the highest, and extending the
jump bid space from a bounded discrete set to a continuous set without an upper bound, the 2-player 2-stage game is transformed into a first-price sealed-bid auction.

One challenge of building a multi-round signaling model is the existence of multiple equilibria. Instead of focusing on equilibrium selection employing various refinement criteria, I instead construct a model that predicts characteristics of auction outcomes that are common to all equilibria. In particular, the model predicts the upper bounds for each bid placed depending on whether it is a jump bid. A jump bid is bounded above by a strategy that is equivalent to one in a first-price sealed-bid auction. On the other hand, the upper bound for a non-jump bid is the equilibrium strategy in an open exit model following Milgrom and Weber (1982). These model predictions suggest that the signaling model works like a hybrid model of a first-price sealed-bid auction and an open exit ascending-bid auction. To structurally estimate such a model involves estimating each of these two auction formats.

The data used in the structural estimation are from the FCC broadband PCS auction (C-block), or "Auction 5" that took place between December 1995 and May 1996. In this auction, the U.S. was divided into 493 regional markets and one license was offered for each market. The reason I chose this auction is two fold. First, Auction 5 brought in over $\$ 10$ billion of revenue, the highest among all 15 FCC spectrum auctions before jump bids were restricted by the introduction of click box bidding. Second, only small businesses (defined as those with annual revenues less than $\$ 40$ million) were eligible to participate. Compared to other auctions where the sizes of the participating firms are much more heterogeneous, Auction 5 is more suited to be described with the theoretical model of this paper which involves symmetric bidders.

The empirical context of a spectrum auction has an implication on the choice of information structure of the model (that is, a common value model vs. a private value
model). One would expect the value of a spectrum license to consist of both a common value component to reflect market attributes that are valued by all bidders, such as market size measured by population, and a private value component to allow values to differ across bidders. This suggests that a common value model, which is nested into the affiliated value framework of the theoretical model, is the most appropriate,.

Nonparametric identification of a common value model has been proven infeasible in both a first-price sealed-bid auction (Laffont and Vuong, 1996) and an ascending-bid auction (Athey and Haile, 2007). Given the challenges with nonparametric identification, I adopt a parametric approach by choosing a multiplicative specification and making distributional assumptions following Hong and Shum (2003).

The theoretical model places bounds on the bids observed. While a partial identification approach may be the most appealing, the implementation of such an approach is difficult. Haile and Tamer. (2003) point out that the lack of sufficient structure of an ascending-bid auction makes a mapping between the bids observed and the underlying demand structure challenging. They instead construct bounds on observed bids and partially identify the distribution functions using an independent private value model. However, it is infeasible to replicate this approach in a common value model due to the interdependence in distributions of bidder information and valuation. In this case, stronger assumptions become necessary. I follow the approach of Donald and Paarsch (1996), Paarsch (1997) and Hong and Shum (2003) where the last bid placed by each bidder (except the winner of the auction) is equal to the upper bound predicted by the theoretical model. This approach allows for point identification.

I estimate the distributional parameters of the signaling model using a simulated nonlinear least-squares approach following Laffont, Ossard and Vuong (1995) and Hong and Shum (2003). The estimation involves the computation of the equilibrium strategies
for an open exit auction and a first-price sealed-bid auction respectively. The former has a closed form solution under the parametric assumptions. However, the equilibrium strategy of a first-price sealed-bid auction does not have a closed form solution. I compute it with a combination of simulation and numerical approximations.

In order to understand the impact of jump bidding on bidder valuation, I repeat the estimation using an open exit model. The mean valuation obtained is lower than the one estimated using the signaling model. This suggests that if bidders are indeed using jump bids as signals, ignoring it leads to an underestimation of the mean value. A counterfactual analysis using the estimation results from the signaling model suggests that if the FCC forbade the action of jump bidding and designed the auction following the open exit format, the total revenue would have been $8 \%$ higher. The result is consistent with the prediction of the theoretical model: by sharing information and coordinating among themselves using jump bids as signals, bidders are able to lower the prices they pay to the seller.

Understanding the revenue effect of jump bidding is imperative in designing an ascending-bid auction that maximizes the seller's revenue. This has direct welfare consequences if the seller is the government, as in the case of a spectrum auction. Theoretical models predict the direction of this revenue effect. It is up to the empirical research to quantify it. There is a growing body of empirical literature that studies the relation between auction revenue and jump bidding using reduced form methodologies, which I will discuss in more details in the next section. In short, these papers find a revenue effect in the opposite direction as predicted by the theoretical models potentially due to the endogenous effects of missing variables. The adoption of a structural approach in this case is particularly useful in shedding light on quantitative effect of jump bidding.

The remainder of the paper is organized as follows: Section 2 reviews the literature
relevant to this paper; Section 3 introduces the theoretical model; Section 3 discusses the empirical strategy and identification; Section 4 describes the estimation methodology and presents the results; Section 5 discusses the counterfactual analysis; Section 6 concludes.

### 1.2 Literature Review

As briefly discussed earlier, there are mainly two strands of the theoretical literature that explain jump bidding. The first deals with transaction costs. If bidders incur a transaction cost every time a bid is placed, placing fewer but larger bids could reduce this cost. The transaction costs could be either pecuniary costs associated with revising and submitting a bid (Fishman, 1988; Daniel and Hirshleifer, 1998), or time costs due to bidders' impatience ( $\overline{\text { Isaac, Salmon and Zillante, } 2007 ; ~ K w a s n i c a ~ a n d ~ K a t o k, ~ 2009) . ~}$ This paper treats all costs of participating in an auction as sunk and assumes away any significant marginal transaction costs associated with placing an additional bid.

The second strand of literature is concerned with the informational role played by jump bids when bidders interact strategically. Jump bids are used as signals of private information either to deter the entry of new bidders (Easley and Tenorio, 2004) or to induce early withdrawal of existing bidders (Avery, 1998; Hörner and Sahuguet, 2007). This paper contributes to the theoretical literature by extending Avery's well-known signaling model to a form that is more compatible with auction settings in real life. It abstracts away from the effect of entry deterrence of an opening jump bid and treats entry as exogenous. Instead of information sharing, jump bids could also be used as tools to conceal information when some bidders have an information advantage over others (Ettinger and Michelucci, 2015, 2016).

The econometric literature on structural estimation of auctions with independent pri-
vate values is vast. Hickman, Hubbard and Sağlam (2012) presents a comprehensive survey of structural econometric methods in auctions. In addition, Perrigne and Vuong (1999) survey on structural econometrics of first-price auctions and Athey and Haile (2007) discuss the nonparametric estimation methods of first-price auctions and ascending-bid auctions. However, when it comes to auctions with affiliated values, the literature becomes sparse due to considerable difficulties with identification. Laffont and Vuong (1996) show that a common value model in a first-price sealed-bid auction is unidentified nonparametrically. In fact, any affiliated value model is observationally equivalent to some affiliated private value model. Similarly in ascending-bid auctions, Athey and Haile (2002) prove that a common value model is generally not identified nonparametrically.

This paper is the first to analyze the revenue effect of jump bidding in ascendingbid auctions using a structural approach. The empirical research on jump bidding in spectrum auctions is limited and adopts a descriptive approach. McAfee and McMillan (1996) and Cramton (1997) analyze the first 3 and 6 FCC spectrum auctions respectively. Both papers document prevalent jump bidding behavior in these auctions but argue that it had little effect in deterring competition since most of the jump bids were eventually overtaken.

Based on the signaling model of this paper, whether an auction ends with a jump bid depends on the realization of the private observations of the top 2 bidders. If they are sufficiently dispersed, the auction ends with a jump bid with probability 1. Otherwise the auction ends with a jump bid with a positive probability. While it is true that if an auction does not end with a jump bid its revenue is the same as the one in an open exit model, the revenue reduction for those that do end with a jump bid is significant as demonstrated by the counterfactual analysis.

Jump bidding aside, there are two other papers that adopt a structural approach
in estimating value distributions in an FCC spectrum auction. Hong and Shum (2003) develop an econometric model of ascending-bid auctions with affiliated values and bidder asymmetries. Fox and Bajari (2013) estimate the deterministic component of bidder valuations using a pairwise stability condition which results in a matching game.

There is a growing body of empirical literature on jump bidding in the Swedish and Norwegian housing markets, where houses are sold through broker-assisted ascendingbid auctions (Hungria-Gunnelin, 2018; Sommervoll, 2020; Khazal et al., 2020; Sønstebø, Olaussen and Oust., 2021). Using a reduced form approach, these papers investigate the motivation behind jump bidding and its effect on entry and withdrawal of bidders and the selling prices. The key findings are: 1) bidders' primary motive of jump bidding is to intimidate their competitors as indicated by the survey data; 2) jump bidding is effective in deterring entry and inducing early withdrawal of competitors; 3) auctions containing jump bids achieve a premium. While the first two findings are consistent with the signaling model of this paper, the third one appears to contradict its prediction. There are two potential explanations. First, the OLS regressions may not have controlled for all important attributes of the houses. It is even harder to control for the heterogeneity in individual tastes. Second, the participants in the housing markets are individuals, who are likely more susceptible to bounded rationality than the bidders in the spectrum auctions which are firms.

### 1.3 Theoretical Model

### 1.3.1 Timing

The timing of the auction game is as follows:

1. Before the auction, the auctioneer sets a reservations price $\rho^{1}=0$.
2. In round $r=1$, all bidders submit bids $b_{i}^{1} \geq \rho^{1}$ simultaneously.
3. At the end of round 1 , all bidding information, including the value and identity of the bidder associated with each bid, is made public to all bidders. Each bidder (except the one(s) with the highest bid) publicly announces whether to drop out. The auction ends if only 1 bidder remains, who then becomes the winner and pays her bid.
4. If more than 1 bidder remains, the auctioneer sets the minimum required price for round $r=2$ using the function $\rho^{2}=P_{\min }\left(p^{1}\right)$, where $p^{1}$ is the highest bid in round 1. $P_{\min }(a) \geq a$, is strictly increasing, and is known to all bidders.
5. Repeat steps 2-4 until the auction ends.

### 1.3.2 Assumptions

There are $n \geq 2$ risk neutral bidders taking part in an ascending-bid auction of a single object. Each bidder $i$ values the object at $U_{i}$, but does not observe $U_{i}$ directly. Instead, each receives a scalar private observation $\tilde{X}_{i}$ about the object. $\tilde{X}_{i}$ 's are identically distributed over the support $(0, \bar{X})$ and are strictly affiliated. Intuitively, strict affiliation means that large realized values for some of the variables make the other variables more likely to be large than small ${ }^{\mathbb{T}}$. Formally, let $z$ and $z^{\prime}$ be points in $\mathbb{R}^{n}$. Let $z \vee z^{\prime}$ denote the element-wise maximum of $z$ and $z^{\prime}$, and let $z \wedge z^{\prime}$ denote the element-wise minimum. Strict affiliation requires that for all $z$ and $z^{\prime}$,

$$
\begin{equation*}
f\left(z \vee z^{\prime}\right) f(z \wedge z) \geq f(z) f\left(z^{\prime}\right) \tag{1.1}
\end{equation*}
$$

[^0]Bidder valuations are affiliated in the sense that bidder $i$ 's expected value of the object $V_{i}$ is a function of private observations of all bidders $V_{i}=v\left(x_{i},\left\{x_{j}\right\}_{j \neq i}\right)=E\left[U_{i} \mid\right.$ $x_{i},\left\{x_{j}\right\}_{j \neq i}$, where $v$ is continuous and increasing in each argument. By assumption, the value function $v(\cdot)$ is the same for all bidders. This value function encompasses the common value case (which consists of both a private value component and a common value component) and two special cases: the private value case and the pure common value case. For the private value case, $V_{i}=v\left(x_{i},\left\{x_{j}\right\}_{j \neq i}\right)=v\left(x_{i}\right)$, where bidder $i$ 's expected value only depends on her own private observation. For the pure common value case, $V_{i}=v\left(x_{i}, x_{k},\left\{x_{j}\right\}_{j \neq i, j \neq k}\right)=v\left(x_{k}, x_{i},\left\{x_{j}\right\}_{j \neq i, j \neq k}\right), \forall k \neq i$, that is, private observations of all bidders (including bidder $i$ ) enter into bidder $i$ 's value function symmetrically. In contrast, when both a private value component and a common value component are present (i.e. the common value model), a bidder's own private observation enters her value function differently from those of other bidders. Assume one places weakly more weight on her own private observation than her rival bidders', i.e. $V_{i}=v\left(x_{i}, x_{j},\left\{x_{k}\right\}_{k \neq i, k \neq j}\right) \geq$ $V_{j}=v\left(x_{j}, x_{i},\left\{x_{k}\right\}_{k \neq i, k \neq j}\right)$ iff $x_{i} \geq x_{j}$.

A jump bid is one that is "substantially" higher than the minimum required price set by the auctioneer for that round. Formally, a jump bid in round $r$ is defined as $b \geq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$, where $\bar{\kappa}\left(\rho^{r}\right) \gg 0$, and is known to all bidders.

### 1.3.3 A two-player, two-stage game by Avery

Placing a jump bid is not cheap talk because the bid may win the auction. Jump bids are costly to bidders since by placing a jump bid, bidders forego the possibility of winning at a lower price. However, if the cost of placing a jump bid (of the same size) differs across different "types" of bidders, it has the potential of being used as a signaling tool. In a symmetric equilibrium where the strategy is strictly increasing in the private observation
$x$, a bidder prefers to lose if she can be convinced that there exists another bidder with a private observation higher than hers because winning requires her to pay above her expected value. If a jump bid of a given size is more costly to bidders with lower $x^{\prime} s$ than those with higher $x^{\prime} s$, then it can potentially be used as a signaling tool of the underlying value.

Before presenting the general model, I will first use a simple two-player, two-stage game taken from Avery's 1998 paper to illustrate the intuition behind using jump bids as signals for underlying values when bidder values are affiliated ${ }^{2}$ I will demonstrate that affiliation in private observations makes it more costly ex ante for the player with a lower $x$ to place a jump bid than the one with a higher $x$.

The assumptions of this game follow the previous section. Additionally, assume $P_{\min }\left(p^{r}\right)=p^{r}+\epsilon$, where $\epsilon$ is a small positive number, i.e. the auctioneer raises the minimum required price by a small amount from round to round. This is a standard assumption for an ascending-bid auction.

The game (referred to as the "metagame" hereafter) proceeds in two stages. In the first stage, the bidders simultaneously choose between two choices of bids: an ordinary bid 0 and a jump bid $K>0$. The second stage resembles a standard ascending-bid auction, with the reservation price equal to the maximum bid from the first stage. In this stage, jump bids are not seen as signals by all bidders.

Let $S^{*}(x)$ denote a symmetric strategy played by both players at the second stage of the game, which represents the drop-out price in the ascending-bid auction. $S^{*}(x)$ is assumed to be increasing in $x$. Note that this strategy does not predict how a bidder behaves from round to round, but simply at which price the bidder will drop out. One example of such a symmetric strategy is $S^{*}(x)=v(x, x)$, derived by Milgrom and Weber

[^1](1982). There is an obvious subgame perfect equilibrium where both players bid 0 in the first stage, and follow $S^{*}(x)$ in the second stage. However, there exists another class of subgame perfect equilibria which deliver higher ex ante payoff to the bidders.

Let $S_{a}(x)$ denote a bidding strategy such that $S_{a}(x)<S^{*}(x)$ and $S^{*}(x)-S_{a}(x)$ is increasing. It can be shown that there exists a threshold $x^{*} \in(0, \bar{X})$ such that the following symmetric strategy is a subgame perfect equilibrium of the two-stage metagame: in the first stage, bid K if $x>x^{*}$, bid 0 otherwise; in the second stage, play $S_{a}(x)$ if outbid by the opponent in the first stage, play $S^{*}(x)$ otherwise. In this equilibrium, if the actions in the first stage are symmetric/asymmetric, the strategies played in the second stage are also symmetric/asymmetric. Table 1.1 summarizes the bidding functions of the second stage as a function of the first stage bidding. In addition, the threshold $x^{*}$ is unique for a fixed two-tuple $\left\{S_{a}, K\right\}$.

Table 1.1: Bidding functions as a function of opening bids

| Player 1's Bid | Player 2's Bid |  |
| :--- | :---: | :---: |
|  | K |  |
| K | $\left(S^{*}(x), S^{*}(x)\right)$ | $\left(S^{*}(x), S_{a}(x)\right)$ |
| 0 | $\left(S_{a}(x), S^{*}(x)\right)$ | $\left(S^{*}(x), S^{*}(x)\right)$ |

To prove this strategy is a subgame perfect equilibrium, I will start from the second stage of the metagame. Suppose it is common knowledge that $x_{1}>x_{2}$, then the strategy pair $\left(S^{*}\left(x_{1}\right), S_{a}\left(x_{2}\right)\right)$ form an equilibrium for the second stage. The reason is given $x_{1}>x_{2}$, bidder 1 prefers to win since $V_{1}=v\left(x_{1}, x_{2}\right)>v\left(x_{2}, x_{2}\right)=S^{*}\left(x_{2}\right)>S_{a}\left(x_{2}\right)$. Similarly, bidder 2 prefers to lose since $V_{2}=v\left(x_{2}, x_{1}\right)<v\left(x_{1}, x_{1}\right)=S^{*}\left(x_{1}\right)$. Neither bidder has an incentive to deviate, the strategy pair is therefore an equilibrium of the second stage.

Given the equilibrium of the second stage, it remains to show that there exists a (partially) separating signaling equilibrium in the first stage with threshold $x^{*}$. Assume bidder 2 plays the following strategy: in the first stage, bid K if $x_{2}>z, z$ is fixed and is public knowledge; bid 0 otherwise. In the second stage, play $S_{a}\left(x_{2}\right)$ if outbid by bidder 1


Figure 1.1: Symmetric equilibrium with jump bids
in the first stage; play $S^{*}\left(x_{2}\right)$ otherwise. Conditional on $x_{2}<z$, bidder 1 is able to induce bidder 2 to shift from a bidding strategy $S^{*}\left(x_{2}\right)$ to $S_{a}\left(x_{2}\right)$ in the second stage by placing a jump bid K. As shown in Figure 1.1, the yellow path $P^{K}\left(x_{2}\right)=\max \left(K, S_{a}\left(x_{2}\right)\right)$ represents the price path faced by bidder 1 triggered by an unmatched jump bid. If $x_{2}<x^{\prime}$, bidder 1 is better off not placing a jump bid and face $S^{*}\left(x_{2}\right)$ since $S^{*}\left(x_{2}\right)$ lies below $P^{K}\left(x_{2}\right)$. If $x^{\prime}<x_{2}<z$, bidder $i$ prefers to place a jump bid and faces the lower curve $P^{K}\left(x_{2}\right)$. Since $\tilde{X}_{1}$ and $\tilde{X}_{2}$ are strictly affiliated, then intuitively the larger the private observation $x_{1}$, the more likely that $x_{2}$ lies between $x^{\prime}$ and $z$, the more likely that bidder 1 chooses to place a jump bid.

Suppose that bidder 1 receives the private signal $x$ and define

$$
\begin{equation*}
\phi(x, z)=E\left[S^{*}\left(x_{2}\right)-P^{K}\left(x_{2}\right) \mid x_{1}=x, x_{2} \leq z\right] . \tag{1.2}
\end{equation*}
$$

$\phi(x, z)$ measures the expected reduction in price that bidder 1 has to pay conditional on placing a jump bid unmatched by bidder 2. $\phi(x, z)>0$ implies that jump bidding by bidder 1 would be profitable conditional on bidder 1 winning. Since $S^{*}(x)-S_{a}(x)$
is increasing, $S^{*}(x)-P^{K}(x)$ is also increasing and continuous. As a result, $\phi(x, z)$ is continuous and strictly increasing in both arguments by the property of strict affiliation between $x_{1}$ and $x_{2}$. In addition, as depicted in Figure 1.1, $\phi(x, z)$ is negative for $x$ near zero and positive for large $x$. By the intermediate value theorem, there is a unique value $x^{*}$ such that $\phi\left(x^{*}, x^{*}\right)=0$. Update bidder 2's strategy by replacing the threshold $z$ with $x^{*}$. To complete the proof, it remains to show that player 1 does not want to deviate from a symmetric strategy that also uses $x^{*}$ as the jump bid threshold in the first stage. There are 4 scenarios to consider: 1) $x_{1}>x^{*} \geq x_{2}$; 2) $x_{1}>x^{*}, x_{2}>x^{*}$; 3) $x_{1} \leq x^{*}, x_{2} \leq x^{*}$; 4) $x_{1} \leq x^{*}<x_{2}$. Scenario 1 is straight forward to show. Since $x_{1}>x_{2}$, bidder 1 wins the auction regardless of her choice in the first stage. However, if she chooses to jump bid, the expected reduction in price is positive, i.e. $\phi\left(x_{1}, x^{*}\right)>0$. Bidder 1 therefore does not deviate from the decision to jump bid. For the remaining 3 scenarios, Avery (1998) provides a detailed proof, which I will not repeat here.

The equilibrium in the first stage is partially separating as all bidders with $x^{\prime} s$ below threshold $x^{*}$ ("low-value" types) choose one action and those with $x^{\prime} s$ above ("high-value" types) choose the other. Low-value types are not able to mimic high-value types because they face a higher ex ante cost attributed to the property of strict affiliation of private observations. If the outcome in the first stage is asymmetric, the bidder that chooses not to jump bid learns that her private observation is lower than her opponent's. As a result, she prefers to lose the auction than to win it. She is therefore indifferent between strategies $S^{*}$ and $S_{a}{ }^{3}$ When $S_{a} \leq K$, she chooses to drop out right away at the end of the first stage. The willingness of lower types to drop out at prices lower than the equilibrium prices without signaling results in lower expected revenue to the auctioneer and higher expected profit to the bidders.

[^2]With independent private observations, there remains a unique signaling equilibrium with jump bids for each two-tuple $\left\{S_{a}, K\right\}$. However, this equilibrium is pooling, instead of partially separating. Both bidders choose the same action in the first stage, always leading to a symmetric outcome in the second stage. The expected revenue to the auctioneer is therefore equivalent to that under an ascending-bid auction without signaling. This result is in line with the what revenue equivalence theorem predicts $\int^{4}$

### 1.3.4 A symmetric equilibrium with one-round signaling

The simple 2-player 2-stage game has demonstrated that the existence of a partially separating signaling equilibrium is directly attributed to the property of strict affiliation. The same intuition applies when the auction is extended to allow for endogenous jump bids. Proposition 1.1 by Avery describes the conditions for the existence and the characteristics of a unique symmetric signaling equilibrium of a 2-player 2-stage game with endogenous jump bids in the first stage.

Proposition 1.1. (Avery) $5^{5}$ Suppose that the two bidders choose their opening bids endogenously and simultaneously from a fixed set of $n$ possible bids and that they continue according to a prespecified asymmetric equilibrium in favor of the higher bidder. Then there is a unique symmetric signaling equilibrium, with strategies identical to those strategies for an n-stage descending signal game with the same set of possible jump bids.

I will broadly outline the idea behind this proposition. For a detailed proof, see Avery (1998). First consider a fixed set of jump bids $\left\{K_{1}, K_{2}, \ldots, K_{n}\right\}$ where $K_{1}>K_{2}>\ldots>$ $K_{n}>0$. Unlike in the two-stage game, bidders may now signal in more than one round.

[^3]In round 1 , bidders face a set of $\left\{0, K_{1}\right\}$. If at least one bidder chooses $K_{1}$, no more signaling is allowed and bidders proceed to play $S^{*}$ or $S_{a}$ as in the simple game. If both bidders choose 0 , then they proceed to round 2 and face the new choice set $\left\{0, K_{2}\right\}$. Bidders can now signal up to n rounds. The threshold $x_{r}^{*}$ in round $r$ is determined the same way as in the simple game and $x_{r}^{*}$ is decreasing in $r$. The subgame that starts from the last round of signaling is identical to the simple game. Conditional on the existence of a unique symmetric equilibrium for this subgame, adding one more signaling round to the beginning of this subgame retains the structure of the simple game and the proof that a unique symmetric equilibrium exists for the bigger subgame is identical to the proof of the simple game. Hence by iteratively adding a signaling round to the beginning of the expanded subgame, it can be shown that the multi-round signaling game with descending jump bids has a unique symmetric equilibrium.

Next, suppose that bidders again can only signal in the first round, but are now allowed to choose their opening bids from the descending set $\left\{K_{1}, K_{2}, \ldots, K_{n}, 0\right\}$. Consider the choice of the lowest jump bid $K_{n}$ in the simultaneous game of the first round. In equilibrium, the only bidders who will not make at least that bid are those with the lowest private observations. Suppose the threshold for such a bid is $z_{n}$. $z_{n}$ is identified by finding the level of private observation which makes the receiver indifferent between bidding 0 and $K_{n}$, conditional on the opponent's private observation being below $z_{n}$. This condition is identical to the condition for the threshold in the $n^{\text {th }}$ round of signaling in the multi-round signaling game. By inductive reasoning, the n thresholds in this singleround signaling game are equal to the thresholds in the multi-round signaling game with descending jump bids, thereby producing a unique equilibrium.

While Proposition 1.1 provides an excellent starting point for a structural model for estimation, there are two issues that need to be resolved. First, the equilibrium changes with the specification of $S_{a}$, the second-stage strategy played by the bidder with the lower
jump bid from the first stage. $S_{a}$ will need to be fixed to avoid multiple equilibria. The second is that the model needs to be generalized from a model with two bidders to a model with more than two.

Proposition 1.2. The following symmetric strategy is a subgame perfect equilibrium of the multi-player ascending bid auction: in the first round of the auction, place a bid following a symmetric first-price sealed bid strategy $S_{1 s t}(x)$; if outbid by any bidder in the first round, drop out immediately, otherwise move on to the second round; from the second round onward, follow an ascending-bid strategy with drop-out price equal to $S^{*}(x \mid \hat{\Omega})$, where $\hat{\Omega}$ denotes all information available up to that point of the auction.

Proof. This equilibrium can be written as $\left\{S_{1 s t}(x),\left(0, S^{*}(x \mid \hat{\Omega})\right\}\right.$. Proposition 1.1 holds for any discrete set. Consider an equally spaced discrete set defined by $\{\underline{K}, \bar{K}, \delta\}$, where $\underline{K}>0$ is the lower bound, $\bar{K}<\infty$ is the upper bound, and $\delta>0$ is the equal spacing. Fix $S_{a}(x)=0$. This implies that as long as the two bidders do not choose the same jump bid in the first round, the one with the lower jump bid drops out at the end of the first round. As $\underline{K} \rightarrow 0, \bar{K} \rightarrow \infty$, and $\delta \rightarrow 0$, the choice set approaches the set of non-negative (real) numbers. As the private observations are drawn from a continuous distribution, the probability that the two bidders draw the same private observations is zero. Hence the probability that they choose the same jump bid approaches 0 and the probability that the auction ends after the first round approaches 1 . In the limit, the bidders are free to choose any nonnegative bid in the first (and only) round of the auction. The one with the lower bid drops out and the other one wins the auction and pays her own bid. This describes a first-price sealed bid auction. Let $S_{1 s t}(x)$ denote the strictly increasing symmetric equilibrium strategy for a first-price sealed bid auction, where $S_{1 s t}\left(x_{i}\right)$ maximizes $E\left[\left(U_{i}-b_{i}\right) \mathbb{1}\left(S_{1 s t}\left(x_{j}\right) \leq b_{i}\right) \mid x_{i}\right]$. This equilibrium strategy can be easily extended to when there are more than 2 bidders by redefining $x_{j}=\max _{k \neq i} x_{k}$.

In the limit of $\underline{K} \rightarrow 0, \bar{K} \rightarrow \infty$, and $\delta \rightarrow 0$, the auction beyond the first round becomes degenerate. Nevertheless, a strategy is needed for a subgame perfect equilibrium. I will adopt the strictly increasing symmetric strategy $S^{*}(x \mid \hat{\Omega})$ for 2 or more bidders by Milgrom and Weber (1982). Note that the key difference between an ascending auction with 2 bidders and one with more than 2 bidders is that more information is revealed in the course of the auction in the latter case. When a bidder drops out of the auction, if the auction does not end (i.e. when there are two or more bidders left), the remaining bidders are able to infer the private observation received by the bidder who drops out from her drop-out price. The remaining bidders can therefore update their expectations since the value function is assumed to be affiliated. $\hat{\Omega}$ denotes the set of drop-out prices observed up to a particular point of the ascending-bid auction.

If either $\underline{K} \rightarrow 0$ or $\bar{K} \rightarrow \infty$ (or both) is relaxed, the first round can be interpreted as a first-price sealed bid auction with a price floor or a price ceiling (or both). In each case, there is a probability mass for the first-round bids at 0 or $\bar{K}$. The auction beyond the first round is no longer degenerate.

### 1.3.5 Multi-round signaling

This section explores an extension of the multi-player model with one round of signaling to one with multiple rounds of signaling.

Proposition 1.2 describes an equilibrium in which bidders send signals through jump bidding in the first round only. Beyond the first round, the implicit assumption is that any further jump bidding will not be interpreted as signals. If this assumption is relaxed, that is, jump bids in rounds beyond the first round are also seen as signals, then this provides further opportunities for bidders to increase the expected payoff.

Consider a simple case in which bidders can signal in the first 2 rounds. Again assume $\underline{K} \rightarrow 0, \bar{K} \rightarrow \infty$, and $\delta \rightarrow 0$. Consider the following strategy $\left\{S_{1 s t}(x),\left(0, S_{1 s t}(x)\right),\left(0, S^{*}(x \mid \hat{\Omega})\right)\right\}$. The difference between this strategy and the one in Proposition 1.2 is that in the second round, a bidder plays $S_{1 s t}(x)$ again if she is not outbid by any other bidders in the first round. If all bidders follow this strategy, the auction again ends after the first round. However, this strategy is no longer an equilibrium. Assume all bidders but $i$ play this strategy. Consider an alternative strategy for bidder $i$ where she places a reduced bid $\tilde{S}=S_{1 s t}\left(x_{i}\right)-\theta>0$ in the first round. Let $S_{1 s t}\left(x_{j}\right)$ be the highest bid in the first round among the rest of the bidders. If $S_{1 s t}\left(x_{j}\right)>S_{1 s t}\left(x_{i}\right)$ (that is, $x_{j}>x_{i}$ ), then bidder $i$ drops out right away and her payoff remains 0 . If $\tilde{S} \leq S_{1 s t}\left(x_{j}\right)<S_{1 s t}\left(x_{i}\right)$, bidder $i$ can move onto the second round with bidder $j$ and $\operatorname{bid} S_{1 s t}\left(x_{i}\right)$ to win the auction and receive the same payoff. In the last scenario when $\tilde{S}>S_{1 s t}\left(x_{j}\right)$, bidder $i$ wins the auction at a lower price. Since the probability of the last scenario is non zero, the alternative strategy delivers a strictly higher ex ante payoff.

The two-round signaling example demonstrates that there is an incentive to place a bid lower than $S_{1 s t}\left(x_{i}\right)$ in the earlier round due to the possibility of winning at a lower price. The same argument applies when bidders can signal in more than 2 rounds, and in an unlimited number of rounds.

There are additional benefits for placing a bid lower than $S_{1 s t}\left(x_{i}\right)$ in earlier rounds when bidders can signal in multiple rounds. Bidders are able to infer more information about the private observations other bidders receive from their decisions to drop out or to stay in the auction using the inverse of the equilibrium strategy. This information affects both the expected value of the object as well as the expected probability of winning. In particular, having more information will allow bidders to form an expected value closer to the true value of the object, thus reducing the winner's curse typical to a sealedbid auction. This ability to collect more information along the course of the auction
and update one's expectations accordingly is what differentiates an ascending-bid auction from a second-price sealed bid auction. In this case, it also marks the difference between a multi-round signaling game and a one-round signaling game. I use $\tilde{S}_{1 s t}\left(x_{i} \mid \Omega^{r}\right)$ to denote the strategy that maximizes $E\left[\left(U_{i}-b_{i}\right) \mathbb{1}\left(\tilde{S}_{1 s t}\left(x_{j} \mid \Omega^{r}\right) \leq b_{i}\right) \mid x_{i}, \Omega^{r}\right]$, where $\Omega^{r}$ is the set of information revealed up to round $r-1$.

### 1.3.6 General predictions for equilibria with multi-round signal-

## ing

When multi-round signaling is allowed and the decision to stop placing a jump bid becomes endogenous, it is complicated to specify a pure-strategy equilibrium. The very nature of a multi-round auction means the cost for bidders to make a mistake is low, in particular in earlier rounds. This suggests that there are likely multiple equilibria. It is complicated to predict which equilibrium is more likely to be played in reality. Instead of trying to argue which equilibrium prevails, I will focus on predicting some common characteristics shared by all the equilibria. Note that there is no limit on how many rounds a bidder can signal through jump bids. The only restriction is that once a bidder stops placing a jump bid in a round, she will not be allowed to do so in all future rounds. I will call the earlier rounds where jump bids are placed the signaling stage and the later rounds where non-jump bids are placed the ascending-bid stage.

Proposition 1.3. For any symmetric (pure) strategy that constitutes a SPE of the multiplayer ascending bid auction with multi-rounds of signaling, the following holds:

1. if $b_{i}^{r}$, the bid in round $r$ by bidder $i$, is a jump bid, i.e. $b_{i}^{r} \geq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$, then $\tilde{S}_{1 s t}\left(x_{i} \mid \Omega^{r}\right) \geq b_{i}^{r} ;$
2. if $b_{i}^{r}$ is not a jump bid, i.e. $\rho^{r} \leq b_{i}^{r} \leq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$, then $\tilde{S}_{1 s t}\left(x_{i} \mid \Omega^{r}\right) \leq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$ and

$$
b_{i}^{r}<S^{*}\left(x_{i} \mid \hat{\Omega}\right) ;
$$

3. if a bidder $i$ drops out at the end of round $r$, then either there is a high enough signal, i.e. $b_{j}^{r} \geq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$ and $b_{j}^{r} \geq \tilde{S}_{1 s t}\left(x \mid \Omega^{r}\right)$, where $b_{j}^{r}$ is the highest jump bid of round $r$; or the expected value is lower than the minimum required price of the next round, i.e. $S^{*}\left(x \mid \Omega^{r+1}\right)<\rho^{r+1}$.

Proof. As discussed earlier, in round $r$, with additional information collected from the dropout decisions of other bidders, $\tilde{S}_{1 s t}\left(x_{i} \mid \Omega^{r}\right)$ gives the jump bid that makes bidder $i$ indifferent between placing a jump bid and placing a bid equal to the minimum required amount. As a result, $\tilde{S}_{1 s t}\left(x_{i} \mid \Omega^{r}\right)$ serves as the upper bound of bidder $i$ ' jump bid in round $r$ for all equilibria.

When bidder $i$ places a non-jump bid, this means she has endogenously chosen to end the signaling stage as the smallest jump bid now exceeds the upper bound. She then moves on to follow the strategy in an ascending-bid auction, which specifies a drop out price at $S^{*}\left(x_{i} \mid \hat{\Omega}\right)$.

A bidder decides to drop out at the end of round $r$ as soon as she receives a signal that puts the sender's private observation above her own. This means there must be at least one jump bid that is higher than the upper bound of bidder $i$ 's jump bid in round $r$. There is another reason for the bidder to drop out. This happens when $\rho^{r+1}$, the minimum required price for the next round exceeds the drop-out price given by $S^{*}\left(x \mid \Omega^{r+1}\right)$.

Proposition 1.3 places bounds on the observed bids based on whether a bidder places a jump bid and her dropout decision at the end of each round. These model predictions suggest that the signaling model works like a hybrid model of a first-price sealed-bid auction and an open exit ascending-bid auction. To structurally estimate such a model
involves estimating each of these two auction formats.

### 1.4 Empirical Strategy

### 1.4.1 Auction background and data

In 1997, the FCC introduced "click box bidding" in Auction 16. Participants placed bids by simply clicking on the license numbers and all bids were exactly one increment above the standing high bid. It was believed that the change in auction rule was to primarily address the issues of jump bidding and code bidding ${ }^{6}$ (Cramton and Schwartz, 2000; Bajari and Yeo, 2009). In later auctions, this rule was relaxed to allow bidders to place jump bids up to 9 bid increments. Nonetheless bidders' ability to signal using jump bids was restricted, making the later auctions less suitable for studying jump bidding.

Among the first 15 auctions where bidders faced no restriction on bid size, Auction 5 (broadband PCS auction (C-block)) was the largest in terms of total revenue raised (over $\$ 10$ billion). It was only open to small businesses with annual revenues less than $\$ 40$ million. Compared to other auctions, the bidders in Auction 5 are more homogeneous in size, therefore more appropriate to be described as symmetric, which is a key feature of the theoretical model. In this auction, the U.S. was divided into 493 markets ("Basic Trading Areas"). One license was offered for each market. 255 small businesses took part in the auction.

In the empirical exercise that follows, I treat the auction of each of the 493 licenses as an independent auction and assume each auction follows the rules detailed in the theoretical section. However, in real life, these auctions ran in parallel following a simultaneous

[^4]multiple round format. Each round, bidders simultaneously placed bids on all the licenses they were interested in. A key difference between the rules of the theoretical model and those of Auction 5 is that bidders did not need to announce whether they dropped out of the auction for a particular market at the end of each round. It is therefore unclear at which point the dropout took place. Following Donald and Paarsch (1996), Paarsch (1997), and Hong and Shum (2003), I assume this happened immediately after a bidder placed her last bid in a market and the other bidders made the same inference. While this assumption may not always hold, the eligibility rule of Auction 5 encouraged participants to actively bid in all markets they were interested in by making the maximum number of bids a bidder could place in future rounds dependent on the number of bids she placed in the current round. If a bidder adopted a "sit-and-wait" strategy in too many markets, she might eventually lose the eligibility to contest in some of the markets that she stood a chance to win. The eligibility rule could therefore be interpreted as a milder version of the irrevocable dropout assumption made in the theoretical model.

### 1.4.2 Define a jump bid

The theoretical model defines a jump bid in round $r$ as $b \geq \rho^{r}+\bar{\kappa}\left(\rho^{r}\right)$, where $\bar{\kappa}\left(\rho^{r}\right) \gg$ 0 , and is known to all bidders. In auctions where bidders can only place a bid that either equals the minimum required price, or exceeds it by whole multiples of a fixed increment set by the auctioneer (for example in FCC Auction 17 and some art auctions), the definition of a jump bid is unambiguous. However, in Auction 5, bidders faced no restrictions on the choice of bids, as long as they were not below the minimum required price. This begs the question: how should a jump bid be defined in this empirical context?

In the data, out of all the bids that exceed the minimum required price, over $20 \%$ exceed the minimum required price by $\$ 100$ or less, and over $40 \%$ exceed it by $\$ 1000$
or less (see Figure A.1 in Appendix A.1). In a signaling model, jump bids are intended as tools to disclose some information about one's private observation. However, bidders could aim to achieve other goals with small jumps. For example, some may simply want to avoid a tie with another bidder and become the highest bidder of that round. In Auction 5 , the highest bidder in a round does not need to raise her own bid in the following round. Further, since the FCC always sets the minimum required price in multiples of thousands, i.e. the last three digits are zeros, all code bids (bids with the market numbers attaching to the end) exceed the minimum required prices. Code bids send signals in a different way than jump bids.

It is entirely possible that bidders do not agree on how much a bid must exceed the minimum required price to be interpreted as a jump bid in the way described by the theoretical model. However, if the majority of them agree on a common definition, is there a way to identify it? Suppose a bidder believes a jump bid exceeds the minimum required price by at least $\bar{\kappa}$, then bidding $\rho^{r}+\bar{\kappa}-\epsilon$ is strictly dominated by bidding $\rho^{r}+\bar{\kappa}$ in most cases since the value of signaling exceeds $\epsilon$ when $\epsilon$ is small. ${ }^{7}$ This suggests that we may observe "bunching", defined as a spike in the probability distribution over sizes of jumps, at the true $\bar{\kappa}$. Figure A.2 is a histogram of the empirical distribution of jumps. Each bar correspond to a bin with width of 50 . We observe distinct spikes for the following bins with lower bounds at 0,100 (this bin contains jump sizes from 100 to 149), and whole thousands. Considering that numbers that are multiples of 100 and 1000 may contain additional informational value or associate with lower transaction cost, I remove bids with jump sizes equal to $100,500,1000$ and 2000 . The spikes with lower bounds at 100 and 1000 persist after the adjustment (see Figure A.4). The spike at 1000 become more prominent in the later rounds of the auction (see Figure A.5).

[^5]Based on the empirical evidence, I define $\bar{\kappa}$ at 1000. Compared to 100, 1000 has the added benefit of excluding all potential code bids (since there are 493 markets, a code bid exceeds the minimum required price by at most \$493). Further, a relatively stringent criterion for a jump bid will reduce the probability of over stating the revenue effect of jump bidding.

### 1.4.3 A parametric approach to estimation

As discussed earlier, the affiliated value framework of the theoretical model nests the private value model, the pure common value model and the common value model, which includes a common value component and a private value component. In the context of a spectrum auction, one would expect the presence of both a common value component to reflect market attributes that are valued by all bidders, such as market size measured by population, and a private value component to allow values to differ across bidders. This suggests that a common value model is the most appropriate in this empirical context.

The difficulties with nonparametric identification of a common value model are well recognized in the econometric literature. Laffont and Vuong (1996) show that a common value model in a first-price sealed-bid auction is unidentified nonparametrically. In fact, any affiliated value model is observationally equivalent to some affiliated private value model. Similarly in ascending-bid auctions, Athey and Haile (2002) prove that a common value model is generally not identified nonparametrically. Given the challenges with nonparametric identification of a common value model, I adopt a parametric approach following Hong and Shum (2003).

I assume that $U_{i}$, the value of the object to bidder $i$, takes a multiplicative form $U_{i}=A_{i} V$, where $A_{i}$ is a bidder-specific private value for $i$, and $V$ is a common value component unknown to all bidders. $V$ and $A_{i}$ 's are assumed to be independently log
normally distributed. Let $v \equiv \ln V, a_{i} \equiv \ln A_{i}, u_{i} \equiv \ln U_{i}$ :

$$
\begin{aligned}
v & =m+\epsilon_{v} \sim N\left(m, r_{0}^{2}\right), \\
a_{i} & =\bar{a}+\epsilon_{a, i} \sim N\left(\bar{a}, t^{2}\right), \\
u_{i} & =m+\bar{a}+\epsilon_{v}+\epsilon_{a, i} \sim N\left(m+\bar{a}, r_{0}^{2}+t^{2}\right)
\end{aligned}
$$

Each bidder receives private observation $X_{i}=U_{i} \cdot \exp \left(s \xi_{i}\right)$, where $\xi_{i}$ is an unobserved error term with a standard normal distribution, and $s$ is an unobserved parameter. Let $x_{i} \equiv \ln X_{i}$, then conditional on $u_{i}$,

$$
x_{i}=u_{i}+s_{i} \xi_{i} \sim N\left(u_{i}, s^{2}\right) .
$$

As a result, $\left(u_{i}, x_{i}, i=1, \ldots, N\right)$ are jointly normally distributed. The joint distribution is fully characterized by parameters $\left\{m, r_{0}, \bar{a}, t, s\right\}$. These parameters are common knowledge among the bidders and are what I will estimate using the signaling model.

### 1.4.4 Identification

As pointed out by Haile and Tamer. (2003), the lack of sufficient structure of an ascendingbid auction makes a mapping between the bids observed and the underlying demand structure challenging. Point identification typically relies on observing at which price a bidder drops out. Haile and Tamer. (2003) instead construct bounds on observed bids and partially identify the distribution functions using an independent private value model.

As the theoretical model of this paper also places bounds on the bids observed, a partial identification approach could naturally follow. However, it is difficult to implement the approach by Haile and Tamer. (2003) in a common value model due to the interdependence
in distributions of bidder information and valuation. In this case, stronger assumptions become necessary. I follow the approach by Donald and Paarsch (1996), Paarsch (1997) and Hong and Shum (2003) and assume that the last bid placed by each bidder (but the winner of the auction) is equal to the dropout prices predicted by the theoretical model. This assumption allows for point identification. As discussed in Section 1.4.1, whether this assumption is valid depends on whether the bidder indeed drops out immediately after placing the last bid. If instead the bidder stays in the auction for a few additional rounds without placing any bid, the last observed bid is below the real dropout price. While this scenario could happen in Auction 5 because a formal announcement of dropout was not required, an eligibility rule was put in place to discourage such a "sit and wait" strategy by reducing the maximum number of bids a bidder could place in each future round for bidders who did not place enough bids in a few consecutive rounds.

### 1.5 Structural Estimation

### 1.5.1 Simulated nonlinear least-squares (SNLS)

I use a simulated non-linear least squares estimator following the methodology of Laffont, Ossard and Vuong (1995) and Hong and Shum (2003). This estimator minimizes the nonlinear least-squares objective function:

$$
\begin{equation*}
Q_{T}(\theta)=\frac{1}{T} \sum_{t=1}^{T} \sum_{i=2}^{N_{t}}\left(p_{i}^{t}-m_{i}^{t}(\theta)\right)^{2} \tag{1.3}
\end{equation*}
$$

$p_{i}^{t}$ is the observed $\log$ dropout price for bidder $i$ in auction $t$. For tractability, I number the bidders in an auction in the reverse of the dropout order. For example, in auction
t , the winner is labelled bidder 1 , and the bidder who is the first to drop out is labelled bidder $N_{t}$, where $N_{t}$ is the total number of bidders in this auction. Notice that the bidder number in Equation 1.3 starts from 2. This is because the dropout price for the winner is not observed since she never dropped out. $m_{i}^{t}(\theta)$ is the model-predicted counterpart of $p_{i}^{t}$, which is the mean of a multivariate truncated distribution. Due to the difficulties of computing $m_{i}^{t}(\theta)$ analytically, it is natural to replace it with a consistent simulation estimator $\tilde{m}_{i}^{t}(\theta)$.

$$
\begin{equation*}
\tilde{m}_{i}^{t}(\theta)=\frac{1}{S} \sum_{s=1}^{S}\left(b_{i, s}^{t}\left(\vec{x}_{s} ; \theta\right)\right) \tag{1.4}
\end{equation*}
$$

where $b_{i, s}^{t}\left(\vec{x}_{s} ; \theta\right)$ is the model predicted dropout price for bidder $i$ in auction $t$ for a particular draw of $\vec{x}_{s}$ from the joint distribution.

The updated objective function is:

$$
\begin{equation*}
\bar{Q}_{S, T}(\theta)=\frac{1}{T} \sum_{t=1}^{T} \sum_{i=2}^{N_{t}}\left(p_{i}^{t}-\tilde{m}_{i}^{t}(\theta)\right)^{2} . \tag{1.5}
\end{equation*}
$$

Laffont, Ossard and Vuong (1995) show that while $\tilde{m}_{i}^{t}(\theta)$ is a consistent estimator of $m_{i}^{t}(\theta), \bar{Q}_{S, T}(\theta)$ is not a consistent estimator of $Q_{T}(\theta)$ for any fixed number of simulations $S$ as T goes to infinity. It can be shown that the size of the bias for each auction t is:

$$
\begin{equation*}
\Delta_{S}(\theta)=\frac{1}{S(S-1)} \sum_{s=1}^{S}\left(b_{i, s}^{t}\left(\vec{x}_{s} ; \theta\right)-\tilde{m}_{i}^{t}(\theta)\right)^{2} \tag{1.6}
\end{equation*}
$$

As a result, to obtain a consistent simulation estimator for $Q_{T}(\theta)$, simply subtract

Equation 1.6 from Equation 1.5 .

$$
\begin{equation*}
\tilde{Q}_{S, T}(\theta)=\frac{1}{T} \sum_{t=1}^{T} \sum_{i=2}^{N_{t}}\left\{\left(p_{i}^{t}-\tilde{m}_{i}^{t}(\theta)\right)^{2}-\frac{1}{S(S-1)} \sum_{s=1}^{S}\left(b_{i}^{t}\left(\vec{x}_{s} ; \theta\right)-\tilde{m}_{i}^{t}(\theta)\right)^{2}\right\} . \tag{1.7}
\end{equation*}
$$

As discussed in Section 1.4.4. $b_{i, s}^{t}\left(\vec{x}_{s} ; \theta\right)$ in the signaling model is predicted by the equilibrium strategy of a first-price sealed-bid auction if the observed bid is a jump bid; and is predicted by the equilibrium strategy of an open exit auction if the observed bid is not a jump bid. In the rest of this section, I will discuss how each of the two is computed in auction $t$ with simulated draw $\vec{x}_{s}$. For simplicity, I will suppress the superscript $t$.

### 1.5.2 Computing the open exit strategy

The equilibrium strategy of an open exit auction with symmetric affiliated value follows Milgrom and Weber (1982). In equilibrium, the price $\beta_{i}$ at which bidder $i$ drops out of the auction is defined as follows:

$$
\begin{equation*}
\beta_{i}\left(X_{i}\right)=E\left[U_{i} \mid X_{i} ; X_{j}=X_{i}, j=1,2, \ldots, i-1 ; X_{k}, k=i+1, \ldots, N\right] . \tag{1.8}
\end{equation*}
$$

Since bidder valuations are affiliated, the expected valuation of each bidder depends on the private observations of all bidders. For each bidder $i$, the private observations can be partitioned into 3 groups. The first group includes bidder $i$ 's own private observation $X_{i}$. The second group consists of $X_{j}$ 's in Equation 1.8, the private observations of the bidders that remain in the auction. Since these private observations are unobserved by bidder $i$, she needs to form expectations on them. Given the assumption of symmetric bidders, bidder $i$ simply assumes $X_{j}=X_{i}$. The last group includes $X_{k}$ 's, the privates observations of the bidders that dropped out before bidder $i$. While bidder $i$ does not
observe these private observations at the beginning of the auction, she can infer them by inverting the bidding strategy after observing at which prices these bidders drop out.

Given the parametric assumptions per Section 1.4.3, the log dropout price has a closedform solution that can be easily computed with each simulated draw $\vec{x}_{s} \equiv\left(x_{s}^{1}, \ldots, x_{s}^{N}\right)^{\prime}$. Let $\hat{x}_{i}=(\underbrace{x_{s}^{i}, \ldots, x_{s}^{i}}_{\text {first i elements }}, \underbrace{x_{s}^{i+1}, \ldots, x_{s}^{N}}_{\text {last N-i elements }})^{\prime}$.

$$
\begin{align*}
b_{i}\left(\vec{x}_{s}\right) & =\ln \beta_{i}\left(X_{i}\right) \\
& =E\left[u_{i} \mid \hat{x}_{i}\right]+\frac{1}{2} \operatorname{Var}\left[u_{i} \mid \hat{x}_{i}\right] . \tag{1.9}
\end{align*}
$$

Denote the marginal mean-vector and variance-covariance matrix of $\left(u_{i}, x_{1}, \ldots, x_{N}\right)$ by $\mu_{i} \equiv\left(u_{i}, \mu^{*}\right)^{\prime}$ and $\Sigma_{i} \equiv\left(\begin{array}{cc}\sigma_{i}^{2} & \sigma_{i}^{* \prime} \\ \sigma_{i}^{*} & \Sigma^{*}\end{array}\right)$. Then using the formulas for conditional mean and variance of a multivariate normal distribution:

$$
\begin{gather*}
E\left[u_{i} \mid \hat{x}_{i}\right]=\left(u_{i}-\mu^{* \prime} \Sigma^{*-1} \sigma_{i}^{*}\right)+\hat{x}_{i}^{\prime} \Sigma^{*-1} \sigma_{i}^{*}  \tag{1.10}\\
\operatorname{Var}\left[u_{i} \mid \hat{x}_{i}\right]=\sigma_{i}^{2}-\sigma_{i}^{* \prime} \Sigma^{*-1} \sigma_{i}^{*} . \tag{1.11}
\end{gather*}
$$

### 1.5.3 Computing the first-price strategy

The equilibrium strategy of a first-price sealed-bid auction with symmetric affiliated value according to Milgrom and Weber (1982) is:

$$
\begin{equation*}
b_{i}\left(X_{i}\right)=\ln \int_{0}^{X_{i}} v(\alpha, \alpha) \frac{f_{Y_{1}}(\alpha \mid \alpha)}{F_{Y_{1}}(\alpha \mid \alpha)} L\left(\alpha \mid X_{i}\right) d \alpha \tag{1.12}
\end{equation*}
$$

where $Y_{1}=\max \left\{X_{j}\right\}, j \neq i, L\left(\alpha \mid X_{i}\right)=\exp \left(-\int_{\alpha}^{X_{i}} \frac{f_{Y_{1}}(s \mid s)}{F_{Y_{1}}(s \mid s)} d s\right)$.

This strategy does not have a closed-form analytical solution. To compute it, I employ a combination of simulation and numerical approximation. First, I divide the integral into n bins and rewrite the it into a summation:

$$
\begin{equation*}
b_{i}\left(X_{i}\right) \approx \ln \sum_{s=1}^{n} \underbrace{v\left(\alpha_{s}, \alpha_{s}\right) \frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)} L\left(\alpha_{s} \mid X_{i}\right) \omega_{s}}_{\rho_{s, i}}, \tag{1.13}
\end{equation*}
$$

where $\alpha_{s}$ is the mid-point of bin $s$ and $\omega_{s}$ is the width of bin $s$. I use $\rho_{s, i}$ to denote the value of bin $s$ conditional on $X_{i}$. Next, I estimate components of $\rho_{s, i}\left(v\left(\alpha_{s}, \alpha_{s}\right), \frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}\right.$ and $\left.L\left(\alpha_{s} \mid X_{i}\right)\right)$ separately for each bin $s$. Last, I sum up the values of all n bins and take the natural $\log$ to obtain an estimate for the equilibrium strategy of a first-price sealed-bid auction.

The number of bins I select for approximation is 10 . I conduct tests for robustness using various bin numbers up to 50 . The value estimated is not very sensitive to the choice of bin numbers. Further, as the number of bins increases, the estimation for $\frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}$ tends to yield undefined values for bins with lower values since $F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)$ is more likely to be zero. The reason for this will become clear once I discuss how $F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)$ is estimated.

For a given number of bins, an intuitive way of defining the bins is to draw a large number of observations from the empirical distribution ${ }^{8}$, and divide the distance between the minimum and maximum observations into equal intervals. However, given the normal distribution, the majority of the observations drawn will fall into the middle bins, rendering estimations for $\frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}$ and $L\left(\alpha_{s} \mid X_{i}\right)$ for the bins at the two ends inaccurate due to a lack of observations. As a result, I define the bins using percentiles of the observations drawn. In particular, when the number of bins is 10 , the bins are defined by

[^6]the deciles (i.e. the 10th, 20th, $\ldots$ and 90 th percentiles). This makes sure the probability of a randomly drawn observation falling into any of the bins is the same. I number these bins from 1 to 10 starting from the lowest decile.
$v\left(X_{i}=\alpha_{s}, Y_{1}=\alpha_{s}\right)$ measures the expected value of bidder $i$ when both bidder $i$ and the bidder with the highest private observation among all other bidders receive a private observation equal to $\alpha_{s}$. The analytical solution to $v(\cdot)$ is a multi-dimensional integral on the joint distribution of $X$ 's, which is difficult to compute. I therefore approximate $v\left(\vec{x}_{s}\right)$ by fitting a function on the largest two elements of 500 simulated draws. $v\left(\vec{x}_{s}\right)$ is evaluated using Equation 1.10. By assumption, $v\left(\vec{x}_{s}\right)$ is monotone in each element of vector $\vec{x}_{s}$ and the elements are strictly affiliated, $v\left(\vec{x}_{s}\right)$ is therefore likely to be monotone in the largest two elements, too. I experiment with a number of specifications up to a cubic term of each element. Higher level functions do not significantly improve the fitting than a linear specification, which is what I adopt.

The cumulative distribution function ("cdf") $F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)$ can be re-written as a conditional probability:

$$
\begin{equation*}
F_{Y_{1}}\left(Y_{1}=\alpha_{s} \mid X_{i}=\alpha_{s}\right)=\operatorname{Pr}\left(Y_{1} \leq \alpha_{s} \mid X_{i}=\alpha_{s}\right) \tag{1.14}
\end{equation*}
$$

To estimate $F_{Y_{1}}$ using simulation, I again draw 500 observations of $\vec{x}_{s}$. I then proceed to identify all observations that contain at least an element which falls in bin $s$. Out of these observations, I calculate the share of observations which do not contain any element that falls in a bin higher than $s$. This share provides an approximation of $F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)$. It can be easily shown that $F_{Y_{1}}$ is increasing in $\alpha_{s}$. This pattern is generally observed for the estimates of $F_{Y_{1}}$ for the higher bins. However, for the lower bins, the increasing pattern is often broken. This is because the probability of drawing an observation of $\vec{x}_{s}$ of which the largest element falls in bin $s$ decreases as $s$ decreases. The accuracy of the estimates
therefore decreases as a result of a smaller number of observations for the lower bins. In some cases, the estimate falls to zero. Unfortunately this issue does not readily go away when the number of observations drawn increases. The probability distribution function $f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)$ is computed based on the estimates of the cdf's. Specifically,

$$
\begin{equation*}
f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)=\left(F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)-F_{Y_{1}}\left(\alpha_{s-1} \mid \alpha_{s}\right)\right) / \omega_{s} \tag{1.15}
\end{equation*}
$$

$L\left(\alpha_{s} \mid X_{i}\right)=\exp \left(-\int_{\alpha_{s}}^{X_{i}} \frac{f_{Y_{1}}(s \mid s)}{F_{Y_{1}}(s \mid s)} d s\right)$ is an integral. To approximate it, I again rewrite it into a summation:

$$
\begin{equation*}
L\left(\alpha_{s} \mid X_{i}\right) \approx \exp \left(-\sum_{m=s}^{N_{i}} \frac{f_{Y_{1}}\left(\alpha_{m} \mid \alpha_{m}\right)}{F_{Y_{1}}\left(\alpha_{m} \mid \alpha_{m}\right)} \omega_{m}\right) \tag{1.16}
\end{equation*}
$$

where $N_{i}$ is the number of the bin into which $X_{i}$ falls. The computation of this summation for each bin $s$ makes use of approximations of $\frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}$. However, unlike $\frac{f_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}{F_{Y_{1}}\left(\alpha_{s} \mid \alpha_{s}\right)}$ which is independent of $X_{i}$, the approximation of $L\left(\alpha_{s} \mid X_{i}\right)$ depends on $X_{i}$ through $N_{i}$. For $s>N_{i}$, define $L\left(\alpha_{s} \mid X_{i}\right)$ to equal zero.

To reflect the fact that $X_{i}$ falls into bin $N_{i}$, but does not necessarily cover the entire bin, I adjust $\rho_{N_{i}, i}$ by multiplying it with the ratio $\frac{X_{i}-l b_{N_{i}}}{u b_{N_{i}}-l b_{N_{i}}}$, where $l b_{N_{i}}$ and $u b_{N_{i}}$ are the lower and upper bounds of bin $N_{i}$ respectively.

### 1.5.4 Estimation results

The second panel of Table 1.2 shows the estimation results of the multi-round signaling model. The mean of the common value component $m$, is parameterized into 2 parts, a constant and a coefficient on population. I choose this parsimonious parameterization to reduce the burden of computation. In addition, simple regressions of the winning bids

Table 1.2: Simulated nonlinear least-squares estimates

| Coefficient | Open Exit | Signaling |
| :--- | ---: | ---: |
| Components of mean |  |  |
| Constant $^{b}$ | 11.57 | 13.18 |
|  | $(0.22)$ | $(0.14)$ |
| POP (mils) | 0.91 | 0.95 |
|  | $(0.29)$ | $(0.27)$ |
| Standard deviations |  |  |
| r0 (common value comp.) | 12.75 | 2.66 |
|  | $(5.96)$ | $(0.29)$ |
| t (private value comp.) | 2.84 | 3.58 |
|  | $(0.13)$ | $(0.26)$ |
| s (unobserved error) | 3.95 | 0.65 |
|  | $(0.27)$ | $(0.05)$ |
| \# auctions (T) | 491 | 491 |

Note:
${ }^{a}$ Bootstrapped standard errors in brackets, computed from empirical distribution of parameter estimates from 100 parametric bootstrap resamples
b Not separately identified from $\bar{a}$
against a selection market attributes, including population, population density and area suggest that population is the best predictor. The mean of the private value component, $\bar{a}$ is not separately identified from the constant associated with the common value component.

In order to understand the impact on valuation from a potential model misspecification when bidders play according to the signaling model but the econometrician fits the data using an open exit model, I repeat the SNLS estimation using an open exit model with the same data. In this case, $b_{i, s}^{t}\left(\vec{x}_{s} ; \theta\right)$, the model predicted dropout price for a random draw of private observations $\vec{x}_{s}$ as defined in Section 1.5 .1 is simply equal to the closed-form analytical solution in Section 1.5.2. The estimation results are shown in the first panel of Table 1.2 .

Comparing the estimates of the components of the $m$, the mean of the common value, the values for both parts are higher under the multi-round signaling model than the open
exit model. Consider a market with a population of 200 k (the median population of all 493 markets is 187 k ), the mean of the $\log$ value of the license estimated by the signaling model is $13.4,14 \%$ higher than that estimated by the open exit model at 11.8. This suggests that if bidders are using jump bids as signals, ignoring them leads to underestimation of the mean.

### 1.6 Counterfactual Analysis

Table 1.3: Counterfactual mean log prices and total revenues

|  | "Jump bid" auctions |  | $\begin{array}{r} \text { All } \\ \text { auctions } \end{array}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Mean actual log prices (\$) |  |  |  |  |
| Highest/winning bid | 15.95 |  |  |  |
| Second highest bid (jump bid) | 15.90 |  |  |  |
| Mean predicted log prices ${ }^{( }$(\$) |  |  |  |  |
| Multi-round signaling | 16.58 |  |  |  |
| Single-round signaling | 16.90 |  |  |  |
| Open exit auction (no signaling) | 17.18 |  |  |  |
| Predicted total revenues (\$bn) |  | $\% \Delta$ |  | $\% \Delta$ |
| Multi-round signaling | 17 |  | 170 |  |
| Single-round signaling | 23 | 33\% | 176 | $3 \%$ |
| Open exit auction (no signaling) | 31 | 76\% | 183 | 8\% |
| \# auctions | 81 |  | 491 |  |

Note:
a Average model predicted log prices of 1000 simulated draws

In the counterfactual analysis, I assess the revenue impact of jump bidding using the estimates from the signaling model. In particular, I estimate the changes in revenues from an auction with multi-round signaling to one with single-round signaling, and to an open exit auction where no signaling is allowed.

If jump bids were prohibited, the auctions that would have different prices would be the ones where the winning bids were jump bids. However, the empirical model is not
able to make any prediction on the winning bid because it is not the dropout price of the winner. To get around the limitation, I will instead focus on auctions with second highest bids being jump bids, and make counterfactual predictions on these bids to approximate a change in revenue.

In the data, 81 auctions out of 491 fit this description. I refer to these auctions as the "jump bid" auctions. As shown in Table 1.3, the mean log winning bid for the jump-bid auctions is at 15.95 and the mean $\log$ second highest bid is 15.90 . The small difference between these two numbers suggest that the latter serves as a fairly close estimate to the former.

The multi-round signaling model predicts the mean log second highest bid to be at 16.58. If we only allow the bidders a one-off opportunity to signal using jump bid, the auction is transformed into a first-price sealed-bid auction as shown by the theoretical model. The mean log price is predicted to increase to 16.90 . It continues to go up to 17.18 if all jump bids are forbidden, i.e. in an open exit auction.

The revenues from the 81 jump bid auctions are estimated to increase by $33 \%$ when the auction moves from one that allows multi-round signaling to one that only allows one round of signaling. The revenues increase by another $35 \%$ when signaling through jump bids are prohibited. The total increase in revenues amounts to $76 \%$ of those from the auction with multi-round signaling. Overall, this increase account for an $8 \%$ increase in total revenues across all 491 auctions.

### 1.7 Conclusions

The existing empirical research on jump bidding adopts either a descriptive or a reduced form approach. While both approaches help confirm the pervasiveness of jump bidding
and identify its correlation with a number of economic factors, they are unable to quantify the revenue effect of jump bidding. This paper generalizes a theoretical signaling model on jump bidding and uses it as the basis for a structural estimation that uncovers the underlying bidder value distribution. A counterfactual analysis using these estimates finds an overall improvement of revenue by $8 \%$ for the FCC broadband PCS auction (C-block).

## CHAPTER 2

# ECONOMIES OF DENSITY AND CONGESTION IN THE SHARING ECONOMY 

Coauthored with Julieta Caunedo and Namrata Kala

With the intention to reap the benefits from sharing capital services across small-scale producers, governments in the developing world are increasingly subsidizing equipment rental markets. Service provision is cheaper in dense areas and for higher machine-hours contracted because equipment needs to be moved in space. Using our own census of 40,000 farmers in India, we document that costly delays and price dispersion in rentals are ubiquitous, and that small-scale producers are rationed out by market providers. This rationing could be detrimental to aggregate productivity if small farmers have the highest marginal return to capital. A government subsidized first-come-first-served (fcfs) dispatch system grants small-scale producers timely access to equipment at the expense of travel time. In a calibrated model of frictional rental services, optimal queueing and service dispatch we show that, while the constrained efficient allocation prioritizes large-holder farmers, small-scale farmers in dense areas are valuable because they help maximize capacity utilization. Through counterfactuals, we show that when the induced increase in subsidized equipment supply is high enough, service finding rates for small-farmers increase relative to large-holders farmers even when providers prioritize large-scale.

### 2.1 Introduction

From housing services, to construction machinery to car rides, the sharing economy, boosted by information technology, is expanding to the developing world. Governments are increasing engaged in stimulating markets for equipment sharing by either subsidizing new capital purchases, or by subsidizing set up costs to the creation of marketplaces for rentals through public/private partnerships. These interventions have distributional
effects as well as efficiency effects that are not well understood, and that this paper studies.

The development of rental markets for equipment is an important mechanism that grants small scale producers access to capital and its technology. Indeed, the history of the path of mechanization for currently rich economies suggests that equipment rental markets were a stepping stone to that process. But where contract enforcement is weak and overall wealth is low, rental markets may not emerge, or if they do, they may ration productive users.

In this paper, we study subsidies to the creation of rental markets for agricultural equipment, which have the potential to unravel productivity gains and employment reallocation away from agriculture, typical of the development process. Fairness concerns may lead government supported platforms to servicing farmers in relatively less dense areas, incurring high transportation costs per service, or to servicing relatively low productivity farmers in detriment of high productivity ones. Due to the time-sensitivity of agricultural activities demand is synchronous, inducing congestion and delays. By subsidizing increases in equipment supply the government can lower congestion, but it could also induce marginal farmers to demand equipment services, slowing down service provision for productive farmers. The balance of these effects dictates the efficiency and distributional impact of these rental markets.

Our paper combines the first available detailed transaction level dataset of equipment rental markets in the developing world, a census of 130 villages characterizing the entire demand for services, and a novel model of frictional rental markets to assess market efficiency and the distributional impact of government subsidies. We model and study two prevalent types of provision of mechanization rentals. The first one is the provision of services by single owners of equipment, often times farmers who own agricultural equipment and rent their excess capacity out (i.e. "market" providers). Given service capacity
constraints, these providers prioritize large orders or orders in densely populated locations. The second one is a private-public partnership that sets up custom hiring centers (CHCs) with the objective of providing machinery for small and marginal farmers. These providers allocate equipment on a first-come-first-served basis, so that order size and location is uncorrelated with the timing of service ("fcfs" providers). Marginal farmers are driven into the government subsidized platform, but the additional transportation costs associated to moving equipment in space may well overturn the gains in productivity from timely access to capital for those farmers. Through counterfactual exercises, we quantify the strength of these channels and argue that general equilibrium forces in prices and the equilibrium sorting of farmers to providers are quantitatively important to assess the allocative efficiency of rental markets.

We start by characterizing rental markets in Karnataka, India. We use detailed transaction data from the private-public partnership since 2017, including hours requested, acreage serviced, implement and location. This data is linked to our newly collected survey data on demographics, farmer assets, productivity and engagement in the rental markets.First, we document that requests for capital rental services are spread across space and that there is higher engagement in rental markets by large-scale farmers (more than 4 acres of farmland) than by low-scale farmers (cropping less than 4 acres). However, the distribution of these large and small farmers in space is such that once we control for equipment ownership levels in neighbouring villages, there is no differential access between large and small scale farmers. Second, we document queuing for service fulfillment which varies throughout the season. This congestion effect is of first-order importance for the organic development of rental markets, and a key margin to assess the efficiency and distributional effects of the markets induced by the government subsidies. Third, because agricultural activities are highly time sensitive, the delay in the arrival of inputs associated to these queues is potentially costly. We quantify the costs of delays for productivity
and profitability by estimating the optimal date for land preparation during a season. We choose land preparation because those are the processes for which the rental markets in Karnataka are most active. We compare profits for farmers that prepared the land away from that optimal date. We find that the average productivity cost for delay within a 10-day window of the optimal planting date is $8.5 \%$ per day.

To assess the merits of different dispatch systems for efficiency and productivity, we pose a novel model of frictional rental markets for equipment. There are two types of providers that differ in their order dispatch technology and face capacity constraints for service. There are two types of farmers that differ in their demand of equipment-hours and their locations in space. Providers set rental rates and farmers choose in which provider to queue at, as in a standard directed search model, Shi (2002). 1 The main difference to these models is the interaction of search frictions with service capacity constraints, and the empirically relevant ability of providers to serve multiple orders (of different types) within a period. Consistently with the data, providers can serve up to three orders within a day, posing an interesting combinatorics problem given the provider's capacity. The main predictions of the model are that when small and large farmers are equally distributed in space, small scale farmers should be more likely to approach the fcfs provider than the market provider. Large farmers should be more likely to approach market providers than fcfs ones. This sorting induces disparities in service finding rates across the population of farmers. In equilibrium, rental rates adjust so that farmers face identical expected values of service from either provider. Heterogeneity in the allocation of farmers in space induces additional disparities in the cost of provision, and in the incentives of providers to serve orders, through the opportunity cost of traveled time, per serviced hour.

To bring the model to the data we allow for additional heterogeneity in farmers

[^7]equipment-hours requests and locations. We solve and calibrate our structural model of optimal dispatch, using administrative data from the fcfs platform, the Census data for market providers and our survey data on farmers' productivity for detailed characteristics of demand. The link between equipment-hours demanded, productivity and plot location is inferred directly from the data. Through a simulation exercise, we find that the government subsidized provision, benefited small-holder farmers through declines in waiting times, lower rental costs and increase in service finding rates. However, this dispatch system entails equipment transportation costs that often overturn the gains to those farmers. Large farmers face lower delays when renting from market providers, and their location is such that when market providers minimize transportation costs, larger farmers disproportionally benefit from queuing with them.

Using the model as a laboratory we run a number of counterfactual scenarios. First, we ask how does the current status quo compares to the equilibrium prior to the government intervention, assuming that market providers (less than $10 \%$ of the total supply) accommodate the entry of subsidized providers. We find that the subsidy generated a substantial increase in service capacity, which lead to two-to-three fold increases in service findings rates, and declines in the cost of service provision of more than $20 \%$ per hour serviced. This additional supply was particularly beneficial to smallholder farmers. Second, we investigate the nature of the equilibrium when subsidized providers are allowed to behave as their market counterparts, i.e. a market deregulation. The short run response of the economy (with no endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistently with the profit maximizing strategy of market providers. However, their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers. The long run
response of the economy (with endogenous entry and exit) restores rental costs to their baseline levels, and generates allocations where service findings rates are higher for small holder farmers than large holder farmers, despite market providers prioritizing the latter. The reason is that small-scale request are valuable when there are service capacity constraints, because they allow for better capacity utilization.

A unique dimension of our problem is that location-specific demand congestion generates losses in productivity due to delays in service provision. Delays are often overlooked as a barrier to technology adoption, yet is a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of farming activities. Delays are typically the result of synchronous queuing and equipment movement in space. The notion that there might be scale economies associated to concentrating production in certain locations is explored by Holmes and Lee (2012) in the context of crop choices of adjacent plots. Bassi et al. (2020) documents the workings of rental markets for small carpentry producers in urban Uganda and argue that frictions in their setup are relatively limited. Distinctively, agricultural equipment needs to be moved in space to reap its benefits and the time sensitivity of agricultural production makes demand synchronous. We show that these two margins are key determinants of the efficiency of usage of capital services through the rental market, and its distributional and productivity impact.

This paper also relates to the literature that emphasized barriers to adoption of technology in agriculture (Suri, 2011) and of mechanization in particular. Pingali, Bigot and Binswanger (1988) highlighted the role of contractual enforcement problems while Foster and Rosenzweig (2017) emphasizes economies of scale. Small poor farmers do not find it optimal to adopt capital intensive practices when they entail the purchase of equipment, a relative large expense whose services are used for a limited period in the season. High cost of capital relative to labor have been emphasized as a deterrent to technology adoption, (Pingali, 2007; Yang et al., 2013; Yamauchi, 2016; Manuelli and Seshadri, 2014). In this
paper we focus on the role of geography and density in assuring access to capital services through rental markets.

The role of geography for the efficient allocation of factors of production in agriculture has been studied by Adamopoulos and Restuccia (2018). In their paper they focus on the characteristics of the soil and the potential yields of different plots to assess the degree of misallocation of factors of production. Spatial misallocation has also been studied in the context of the rural-urban gap in labor income by Gollin, Lagakos and Waugh (2013); Lagakos, Mobarak and Waugh (2015).In this paper we focus on the distributional effects across land-sizes and locations of equipment rental markets.

### 2.2 Equipment rental markets.

### 2.2.1 Data description

We study agricultural rental markets in rural India. The study focuses on the state of Karnataka, one of the least mechanized states in India (Satyasai and Balanarayana, 2018), and where the majority of agriculture occurs on small farms (less than 4 acres). Our own census of farming households in 150 villages across the state shows that equipment ownership rates are low and that farmers rely on informal equipment rental markets within each village.

In 2016, the state government partnered with the largest manufacturer of agricultural equipment in India to design and manage a platform through which farmers could rent equipment. They generated a subsidy scheme for equipment purchases to create custom hiring centers (CHCs, also known as "hubs") in 25 districts throughout the state. In exchange for these government subsidies, the service provider committed to rental rates
of about $10 \%$ below their market counterparts. Figure 2.1 plots currently active hubs in space. These CHCs provide rental services in nearby areas and farmers can access these services through a call-center, via an app on a smart-phone, or in person at the CHC.

Figure 2.1: Locations of CHCs and Demand


Triangles indicate CHCs. We aggregate demand following the village where each farmer is registered. Green dots correspond to demand for the smallest plots (1 acre or less), red dots correspond to demand for the largest plots (4 acres or more).

Our first source of data comes from the universe of transaction-level data maintained electronically by these hubs. This administrative data consists of all the transactions completed through that platform since 2016 for about 60 hubs in Karnataka (27 hubs entered in 2016, 29 in 2018 and 10 in 2019). Over the time period covered by the data (October 2016-May 2019), over 17,000 farmers from 840 villages rented equipment from these CHCs. The data contains information on number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Equipment available varies across CHCs (hubs) but the median hub provides equipment that ranges from sprayers to Rotavators or ploughs. Rental rates in the platform are about $10 \%$ below market prices on average, as part of the government's initiative to increase mechanization access. The service provision is first-come-first-served. When a service
is fulfilled, a professional driver brings and operates the equipment at the farmer's plot. Equipment arrives within a lapse of two days in most cases. If delays are longer, farmers are informed at the moment of booking (and may choose to cancel). Table 2.1 reports the 10 most commonly rented implements for the years 2017 and 2018, the number of transactions recorded for each implement, their per-hour rental price, and month where the implement is most commonly rented (has the highest number of transactions).

Table 2.1: Summary Statistics of Commonly Rented Implements from Rental Database

|  | Commonly Rented Implements |  |  |
| :--- | :---: | :---: | :---: |
|  | Number of Transactions | Median Price Per Hour | Peak Month |
| Rotavator 6 Feet | 11,239 | 770 | July |
| Cultivator Duckfoot | 7,287 | 550 | April |
| Cultivator 9 Tyne | 5,245 | 525 | May |
| Plough 2MB Hydraulic Reversible | 3,716 | 450 | February |
| Trolley 2 WD | 2,436 | 250 | January |
| Harvester Tangential Axial Flow (TAF)-Trac | 2,048 | 1800 | May |
| Rotavator 5 Feet | 1,811 | 700 | September |
| Blade Harrow Cross | 1,793 | 360 | March |
| Knapsack Sprayer 20 Litres | 1,688 | 22.5 | October |
| Blade Harrow 5 Blade | 1,600 | 360 | June |

We use two additional sources of data in Karnataka. The first is survey data collected in June-July 2019 around a representative sample of the equipment hubs in the main sample. The survey includes approximately 7,000 farmers, and asks for detailed information on input and output per plot which allow us to generate measures of productivity at the plot level. We also ask about their engagement in rental markets, rental rates, and perceptions on barriers to participation in the market. The second is a census of agricultural farmers including more than 40,000 farmers in the villages covered by the survey data. This census allows us to validate the characteristics of the farmers in the survey relative to the population. It also allows us to measure inventories of equipment potentially available to farmers from nearby villages.

Finally, we use data from a broader set of Indian states as recorded in ICRISAT's household-level panel data. This data which contains detailed agricultural information, including, season-level agricultural operations, their timing, costs and total revenues. The data covers eighteen villages over 2009-2014 in Andhra Pradesh, Gujarat, Karnataka,

Maharashtra, and Madhya Pradesh.

### 2.2.2 Motivating facts

We start by describing the characteristics of the service demand and farmers equipment supply. Then, we focus on a handful of outcomes that are informative to the theory that we describe in Section 2.3. First, because agricultural activities are highly time sensitive, the timing of demand is synchronous leading to endogenous waiting times as a function of service capacity. The service capacity includes farmers' ownership as well as CHCs capacity. Second, because equipment needs to travel for transactions to take place, the joint distribution between travel times and the scale of demand, i.e. equipment-hours per request, is a key input when optimizing service provision. Third, we document substantial price dispersion in rental rates after controlling for observable household characteristics and village/market characteristics, consistent with frictional rental markets. Fourth, delays in service provision are costly to farmers, because they affect field productivity. In what follows we document each of these features.

## Service Capacity and Service Demand.

We start by reporting patterns of ownership (service capacity by farmers) and rentals of equipment across the farmers in our survey (see Figure 2.2..$^{2}$ Most farmers report to own hand tools and animal pulled equipment. Less than $10 \%$ of the farmers report to own larger equipment such as tractors, or rotavators and cultivators. At the same time, tractors and cultivators are among the pieces of equipment with the highest equipmenthours rented. The average hours rented in a season per farmer is 12 hours for tractors and 10 hours for cultivators. These rental transactions mostly entail relational contracts. We

[^8]collect information on the typical customer for a farmer that rents out his/her equipment. We find that $72 \%$ of owners report to rent out to people they know from the village or who they have worked with in the past.

Delays are the most common issue faced by farmers when renting equipment, with $78 \%$ of farmers reporting it as an issue. Importantly, larger farmers (cultivating at the 75th percentile of the land size distribution, i.e. larger landholdings) are nearly 5 percentage points less likely to report delays as an issue. Hence, delays in accessing mechanization are more pervasive among smaller farmers.

Figure 2.2: Ownership and rentals by implement.


The ownership rate is the share of farmers that report to own a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season.

Given the disparities in value of agricultural implements as well as their contribution to production, it is useful to construct a measure of equipment services from rentals and owned equipment. We measure these services as the product of average hours of usage during a season $h_{i}$, market rental rates, $r_{i}$ and the number of implements $i$ owned or rented, $N_{i}$. Hence, equipment services in a farm $k$ are

$$
k=\sum_{i} N_{i} r_{i} h_{i}
$$

The main hypothesis behind this measure is that differences in rental rates across implements shall reflect differences in the services they provide, and that therefore, more expensive equipment provides higher services to production. The main challenge in constructing such a measure is the availability of data on market rental rates. We exploit our transaction level dataset to construct mean rental rates per implement at the village level. Figure 2.3 displays $\log$ owned and rented services. Harvesters (the most expensive implement in our bundle) is reported to be only rented. For those farmers using tractors, more than $60 \%$ of the services available in the farming sector come from rentals whereas the remaining $40 \%$ stem from ownership. Services associated to smaller and cheaper equipment, such as sprayers, are equally accounted for by rentals and ownership.

It is worth noting that given land holdings, ownership of equipment is not cost-effective for most farmers. For instance, the rental price of a rotavator is between ₹ 750 and $₹ 1,000$ per hour (including tractor, a driver and fuel) and the average farmer demands about 6 hours of rotavator services in the season or between ₹ 4500 and ₹ 6000 in services. The purchase price of a new rotavator is over ₹ 110,000 which means that, absent maintenance costs (which are certainly non-negligible), the average farmer needs 19 years to amortize the investment. The rental rate for an inferior technology that serves a similar purpose, i.e. a harrow, is half of the rental rate of the rotavator ( $₹ 360$ ) and the cost of purchase is about ₹50000. Overall, these price differentials are consistent with the observed extensive engagement in rental markets for equipment.

## Heterogeneous Queuing by Production Scale

The demand for equipment rental services vary by agricultural process and therefore throughout the agricultural season. The synchronous nature of many of these processes across farmers induces queuing in the market. Our transaction level data allows us to

Figure 2.3: Capital services from ownership and rentals, by implement.


Shares of log capital services by implement and ownership/rental status. Average rental rates for an hour of service (in ₹) are reported next to each implement.
measure demand fluctuations by computing hours outstanding for service at a daily frequency. We focus on two commonly rented implements for land preparation, rotavators and cultivators. Indeed, our survey data indicates that farmers are most likely to engage in the rental market for land preparation.

Figure 2.4 shows hours of unfulfilled orders for each of these implements over the 2018 kharif season. Queueing peaks by the end of July for rotavators and beginning of August for cultivators. At the peak of the season, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders on average at a point in time.

Demand moves distinctively between large and small requests, measured in service hours (Figure 2.4). A large portion of hours outstanding are accounted for by small orders (less than 4 hours of service), although at pick time the share of hours accounted for by large and small holder farmers equalizes. This is not explained by a higher number of large orders but rather by larger orders altogether.

Figure 2.4: Hours outstanding in the queue.


Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

As demand fluctuations over the season in a somehow predictable manner, it is expected that service supply may adjust. To explore these movements we compute service rates as the fraction of serviced hours within a day divided by the number of service hours outstanding. On average, we see up to 3 orders being fulfilled during a day per equipment piece. We find that service rates move during the season, and that they positively correlate with hours demanded (see Figure 2.5). However, they do not move enough to avoid queueing and congestion in service provision, likely due to capacity constraints or frictions related to finding alternative providers, as we discuss in Section 2.2.2.

Figure 2.5: Service rates.


Averages across hubs (rotavator) Kharif 2018.

## Spatial distribution of rental services

We first document that service delays are negatively associated with cultivated size suggesting that even if the productivity costs of delays are of same magnitude between smalland large-holder farmers, the incidence of those delays is disproportionally bear by those with small plot sizes, columns (1) and (3) in Table 2.2. It is possible that these delays are explained by the geographical location of plots since equipment needs to travel to generate services. Columns (2) and (4) Table 2.2 show that delays have an important spatial dimension-adding village fixed effects substantially attenuates the coefficient on the $\log$ of land size, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays, but if this clustering is not accounted for, smaller farmers face longer delays.

Table 2.2: Delays as a Function of Land Area and Location Fixed Effects

|  | Delays (Sum of Average Delays Over the Season) |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| Log(Area) | $-0.215^{*}$ | -0.144 | $-0.319^{* *}$ | -0.128 |
|  | $(0.115)$ | $(0.0926)$ | $(0.145)$ | $(0.108)$ |
| Observations | 5,615 | 5,615 | 4,345 | 4,345 |
| R-squared | 0.002 | 0.182 | 0.003 | 0.252 |
| Village Fixed Effects | No | Yes | No | Yes |
| Mean Delays | 2.158 | 2.158 | 2.789 | 2.789 |

Estimated coefficients from a regression of reported delays in service provision and the $\log$ (area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

## Frictional Rental Markets

But why are there delays to begin with? Is this a consequence of low ownership rates and service capacity, or rather the consequence of frictions in the rental market that prevent farmers and providers to contract services when desired? There are two features of the market that indicate the presence of frictions in the rental market.

The first one is that the current supply of equipment seem adequate to serve market demand. To compute supply we turn to a Census of 150 villages from the same area, which includes information on over 40,000 farmers. We assume that the equipment has a catchment area of about 10km, since transporting equipment over large distances is time-consuming and expensive (particularly for farmers whose main activity are not rentals). For a median village, the number of available cultivators within a 10 km radius can serve 2016 orders per season, while average demand is 1190 orders. For rotavators, available supply can serve 1008 orders in the season while market demand is 450 for the median village..$^{3}$ Hence, the observation of pervasive delays in service provision paired with adequate supply within each geographical market suggest that congestion does not necessarily arise due to supply shortages.

[^9]The second one is the presence price dispersion in rental prices of equipment within a 10 km catchment area of each village. Using information from our survey of 7,000 farmers, Figure 2.6 shows the distribution of residualized rental rates during land preparation from village effects, i.e. that is the variation in rental rates per hour serviced across farmers within a village. The interquartile range is 1.7 while the coefficient of variation is 0.7 . Burdett and Judd (1983) were the first to show that price dispersion can arise in an environment with identical agents where consumers/farmers find it costly to search for providers. Price dispersion can also be related to informational asymmetries Varian (1980) or to consumer preferences for certain providers over others Rosenthal 1980). Overall, the exchange of identical goods for heterogeneous prices is typically a sign of frictions in the market, which we entertain through the structural model that we study next.

Figure 2.6: Hourly rent distribution (residualized by location)


### 2.3 A frictional model of capital rental services in space

To build a quantitative assessment of the workings of these rental markets, we extend the two-sided heterogeneity framework of Shi (2002) along two dimensions. First, we allow for multiple orders to be served within each period. Second, we build in capacity
constraints for service providers. The first feature is important to assess implications for travel times across orders, i.e. optimal service routes and the costs associated to them. The second feature is important to quantify congestion and the equilibrium service delays as a function of the composition of the service queue in terms of the size and geographical location of the plots being served.

### 2.3.1 Environment

Consider an economy populated by F farmers that use capital services for production. Capital services are available for rent in frictional rental markets.

Farmers differ in their demand for equipment-hours and in their geographic location. A fraction $s$ of them are "large-scale" farmers and demand $k_{s}$ hours, while the remaining $(1-s)$ fraction are "small-scale" farmers, and demand $k_{s^{-}}$hours. We think of the demand scale as being determined by land-holdings. Farmer's productivity gets realizes once inputs have been committed, e.g., weather shocks or delays in equipment rental service provision that affect the output from the farm. These shocks induce revenue costs to farmers. For simplicity, we focus on ex-post shocks related to delays in service provision, which are idiosyncratic to the farm.

There are H rental service providers that can serve up to oorders a day. Providers $j$ differ in their service capacity, up to $\bar{k}_{j}$ machine-hours a day, and in their technology for service provision. A fraction $h$ of providers use a first-come-first-served (fcfs) technology, while the remaining fraction $1-h$ has access to a selection technology that allows them to prioritize certain type of service requests (mkt) $\|^{4}$ We assume no systematic differences in providers' geographical location, i.e. the expected travel time for service provision

[^10]conditional on the machine-hours demanded is the same for both providers.

Denote the ratio of farmers to service providers, $f=\frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \rightarrow \infty$ and neither side is infinitely larger than the other, $f \in(0, \infty)$. Providers post prices $r_{i j}$ and a selection criteria (with commitment) simultaneously at the beginning of each period. Geographical considerations for service provision are included into the opportunity cost of moving equipment from a provider to the plot. Then farmers decide whether and which provider to approach, generating queues for each available provider. Finally, providers decide which orders to serve given the selection criteria and farming production takes place. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize over the set of preferable providers. The key assumption is that agents find it difficult to coordinate their decisions in a large market.

A type i-farmer's strategy is a vector of probabilities $P_{i} \equiv\left(p_{i, \mathrm{fcfs}}, \ldots ; p_{i \mathrm{mkt}}, \ldots.\right)$ where $p_{i j}$ is the probability of applying to each type j-provider. A type-j provider's strategy consists of rental rates per hour serviced, $r_{i j}$ and a selection rule $\chi_{j} \in[0,1]$ for the market provider. The selection rule applies only when a provider receives requests from both type of farmers, in which case the provider prefers the large scale farm if $\chi_{j}=1$, prefers a small scale farm if $\chi_{j}=0$, and he is indifferent between them for $\chi_{j} \in(0,1)$. When the provider receives requests from a single type of farmer type, he randomly selects one farmer for service. Those that request a service from the fcfs provider face the same probability of being first in line irrespective of the machine-hours demanded.

### 2.3.2 Queue lengths as strategies

Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service. The characteristics of their demand (or ex-ante productivity) and geographical locations are as in Assumption 1.

Assumption 1: Equipment-hours demanded by small and large scale farmers satisfy $k_{s}>k_{s^{-}}$. The expected travel time to servicing small-scale farmers is weakly higher than that for large-scale farmers, $E\left(d_{s}\right) \leq E\left(d_{s^{-}}\right)$.

A convenient object for analysis is the queue length, i.e., the expected number of farmers requesting a service from a given provider ${ }^{5}$ Let $q_{i j}$ be the queue length of type $i$ farmers that apply to a type $j$ provider, where $i \in\left\{s, s^{-}\right\}$and $j \in\{\mathrm{fcfs}, \mathrm{mkt}\}$. Then, $q_{s j}=s F p_{s j}$ and $q_{s^{-} j}=(1-s) F p_{s^{-} j}$. The probabilities of approaching different providers for a single farmer should add up to one, which leads to

$$
\begin{gather*}
H\left(h q_{s, \mathrm{fcfs}}+(1-h) q_{s, \mathrm{mkt}}\right)=F s  \tag{2.1}\\
H\left(h q_{s^{-}, \mathrm{fcfs}}+(1-h) q_{s^{-}, \mathrm{mkt}}\right)=F(1-s) \tag{2.2}
\end{gather*}
$$

Assumption 2: The demand for machine-hours satisfies,

$$
\begin{gathered}
o\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)<=\bar{k}_{j} \quad \text { and } \quad(o+1)\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)>\bar{k}_{j} \\
(o-1)\left(k_{s}+E\left(d_{s}\right)\right)<=\bar{k}_{j} \quad \text { and } \quad(o-1)\left(k_{s}+E\left(d_{s}\right)\right)+k_{s^{-}}+E\left(d_{s^{-}}\right)>\bar{k}_{j}
\end{gathered}
$$

[^11]$(o-1)\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)+k_{s}+E\left(d_{s}\right)<=\bar{k}_{j} \quad$ and $\quad(o-1)\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)+2\left(k_{s}+E\left(d_{s}\right)\right)>\bar{k}_{j}$

In other words, for tractability we assume that the service capacity $\bar{k}_{j}$, is enough to serve at most " o " orders; and that if the provider serves only large-scale orders, it can serve "o-1" orders. Finally, we assume enough capacity to serve $o-1$ small scale orders and 1 large scale order.

A farmer of scale i that requests service from provider j gets served with probability $\Delta_{i j}$. This conditional probability depends on the provider's selection criteria and the number of orders it can potentially serve within each period, i.e. its capacity $\bar{k}_{j}$; as well as on the machine-hours demanded $k_{i}$ and the expected travel time for service $E\left(d_{i}\right)$. We assume the empirically relevant service capacity which implies that, on average, $o=3$ orders can be fulfilled within the period $\sqrt{6}$ Hence, $\Delta_{i j}$ is the sum across all possible number of orders of type $i$ being served, $\bar{o}_{i}$, of the probability of servicing $\bar{o}_{i}$ type $i$ farmers, $\phi_{i j}\left(\bar{o}_{i}\right)$, times the probability that a certain farmer of type $i$ is chosen, $\tilde{\Delta}_{i j}\left(\bar{o}_{i}\right)$,

$$
\begin{equation*}
\Delta_{i j}=\sum_{\bar{o}_{i} \in\{1,2,3\}} \phi_{i j}\left(\bar{o}_{i}\right) \tilde{\Delta}_{i j}\left(\bar{o}_{i}\right) \cdot{ }^{7} \tag{2.3}
\end{equation*}
$$

Using the definition of the probability of service, equation 2.3, it is possible to show that the probability of type $i$ being served (weakly) declines in the queue length of type $i^{\prime} \neq i$ farmers. For the first-come-first-served provider the result is straightforward because service probabilities decline with the number of machine-hours in the queue, irrespective of their type. For the market provider with a selection criteria that favours type $i^{\prime}$ farmers, the decline in the probability of service for type $i \neq i^{\prime}$ is strict as the number of type $i^{\prime}$ farmers in the queue increases. The service probability for type $i^{\prime}$ farmers is independent of the queue length of type $i \neq i^{\prime}$ due to the selection criteria.

[^12]
### 2.3.3 Market cost of provision.

We follow Burdett, Shi and Wright (2001) and describe a farmer's decision as a function of the market price it would get for the rental service, $r_{i j}$, which in turn determines its expected "market" profit, $U_{i}$. The agents take the value of the market profit as given when the number of agents in the economy is large, $F, H \rightarrow \infty$. Let $q_{j} \equiv\left\{q_{s j}, q_{s^{-} j}\right\}$ be the queue at provider $j$. Each farmer chooses the service provider to minimize costs given $U_{i}$ and the production technology: $8^{8}$

$$
C_{i}\left(r_{i j}, q_{j}, k_{i}\right)=\min _{j^{\prime}} C_{i}\left(k_{i}, r_{i j^{\prime}}, q_{j^{\prime}}\right) \equiv \min _{j^{\prime}} r_{i j^{\prime}} k_{i}
$$

subject to

$$
\Delta_{i j} \pi_{i j}\left(k_{i}, r_{i j}, q_{j}\right) \equiv \Delta_{i j}\left(E\left(z\left(\Delta_{i j}\right)\right) k_{i}^{\alpha}-r_{i j} k_{i}\right) \geq U_{i}
$$

where $E\left(z\left(\Delta_{i j}\right)\right)$ is the expected productivity in the farm.

Farm's productivity depends on the realization of a random shock related to mistakes in the timing of agricultural activities. To ease the exposition we summarize the optimal timing for agricultural activities as the optimal "land preparation" date, $\theta^{\star}$, and relate deviations from this optimal timing to productivity costs. The realization of the land preparation date is a random draw, $\theta$, from a known distribution $G\left(\bar{\theta}\left(\Delta_{i j}\right)\right)$ with mean $\bar{\theta}\left(\Delta_{i j}\right)$ that depends on provider $j$ 's probability of service. We assume that $\frac{\partial \bar{\theta}\left(\Delta_{i j}\right)}{\partial \Delta}<0$ so that a high probability of service induces shorter average wait time to service. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date as follows, $z=1-\eta\left(\theta-\theta^{\star}\right) I_{\theta^{\star} \leq \theta}$,

[^13]where $\eta$ is the productivity cost per delayed service day. Expected productivity is
$$
E\left(z\left(\Delta_{i j}\right)\right)=1-\eta\left(\bar{\theta}\left(\Delta_{i j}\right)-\theta^{\star}\right) I_{\theta^{\star} \leq \bar{\theta}\left(\Delta_{i j}\right)} .
$$

Given that the productivity function is asymmetric, the expected productivity is independent of the choice of provider whenever the expected wait times are relatively low, i.e. the probability of service is high. We assume this is the case in the reminder of the analysis, i.e. $E\left(z\left(\Delta_{i j}\right)\right)=1$. Appendix B. 2 includes the analysis when there are expected losses from timing at the beginning of the season. Finally, because the draw of the service provision is idiosyncratic, there is no aggregate uncertainty in the economy and factor prices are time independent.

A type i farmer requests a service from a type j firm with positive probability if the expected profits are weakly larger than $U_{i}$. The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1 , yielding $q_{i j} \rightarrow \infty$ as the number of farmers grows large. Then, $\Delta_{i j} \rightarrow 0$ contradicting $\Delta_{i j} \pi_{i j}\left(r_{i j}, k_{i}\right)>\tilde{U}_{i}$. The farmers' strategy is

$$
\begin{array}{rll}
q_{i j} \in(0, \infty) & \text { if } & \Delta_{i j} \pi_{i}\left(r_{i j}, k_{i}\right)=\tilde{U}_{i} \\
q_{i j}=0 & \text { if } & \Delta_{i j} \pi_{i}\left(r_{i j}, k_{i}\right)<\tilde{U}_{i} \tag{2.5}
\end{array}
$$

This expression summarizes the tradeoff between lower provision cost and higher profits; and a lower probability of service. Given the shape of the probability function, there exist a unique queue length $q\left(r_{i j}, U_{i}\right)$ that satisfies the problem of the farmer. The farmer decides his queueing strategy as a function of his capital demand, $k_{i}$ and market prices $r_{i j}$.

### 2.3.4 Service Providers

A service provider with capacity $j$ maximizes expected returns. The stock of machinehours available to a provider are exogenously given at $\bar{k}$. For simplicity, we assume the same service capacity across providers, no depreciation or capital accumulation and no maintenance costs $\int^{9}$ The cost of servicing a farmer depends on its location relative to the provider. The location of each plot is indexed by $d_{i}$ and $\mathbf{d}_{\hat{q}_{t}}$ is a vector collecting the locations of the orders completed within a period, $\hat{q}_{t}$. Providers choose the cost of service $r_{i j}$ taking the the machine-hours demanded by each type of farmer as given. For a vector $\left\{U_{i}, k_{i}, E\left(d_{i}\right)\right\}_{i=s, s^{-}}$, he chooses the queue lengths by picking the cost of service and service strategy, i.e. whether to prioritize certain farmer types. The cost of travel time includes the foregone services that could have been provided if the equipment was not travelling as well as the opportunity cost of the driver, which commands a wage $w$ per hour. Finally, the queue length is reset each period and therefore the service provision problem is static ${ }^{10}$

Consider the problem of a first-come-first-served provider. His value is the expected return from servicing at most $o=3$ orders within each period. Let $\bar{o}_{i}$ be the number of orders of type $i$ being served within the period. The per period return $\tilde{V}$ from facing queue $q_{\mathrm{fcfs}}$ depends on the numbers of orders of each type being served, $\left\{\bar{o}_{s}, \bar{o}_{s^{-}}\right\}$and the revenue per type net of labor and transportation costs, $\left\{r_{i, \text { fcfs }} k_{i}-w k_{i}-w E\left(d_{i}\right)\right\}{ }^{111}$ The

[^14]value for a first-come-first-served provider is
\[

$$
\begin{equation*}
V\left(q_{\mathrm{fcfs}}\right)=\max _{\left\{r_{i, f \mathrm{ffs}}\right\}_{i=s, s s^{-}}} \tilde{V}\left(\left\{\bar{o}_{s}, \bar{o}_{s^{-}}\right\}_{q_{\mathrm{fffs}}},\left\{r_{i, \mathrm{fcfs}} k_{i}-w k_{i}-w E\left(d_{i}\right)\right\}_{i=s, s^{-}}\right) \tag{2.6}
\end{equation*}
$$

\]

subject to farmers' strategies, equation 2.4, and feasibility

$$
\begin{equation*}
\sum_{i \in q_{\mathrm{fcfs}}} k_{s}(i)+E\left(d_{s}(i)\right) \leq \bar{k}_{\mathrm{fcfs}} . \tag{2.7}
\end{equation*}
$$

Consider now the problem of a market provider who, in addition to choosing the cost of provision, $r_{\text {mkt }}$, chooses a selection criteria $\chi$. This choice in turn determines the type of orders being served and their quantity, given service capacity. The value of a market provider is

$$
\begin{equation*}
V\left(q_{\mathrm{mkt}, t}\right)=\max _{\chi,\left\{r_{i, \mathrm{mkt}}\right\}_{i=s, s^{-}}} \tilde{V}\left(\left\{\bar{o}_{s}, \bar{o}_{s^{-}}\right\}_{\left(q_{\mathrm{mkt}}, \chi\right)},\left\{r_{i, \mathrm{mkt}} k_{i}-w k_{i}-w E\left(d_{i}\right)\right\}_{i=s, s^{-}}\right) \tag{2.8}
\end{equation*}
$$

subject to farmers' strategies, equation 2.4, and feasibility

$$
\begin{equation*}
\sum_{i \in q_{\mathrm{mkt}}} k_{s}(i)+E\left(d_{s}(i)\right) \leq \bar{k}_{\mathrm{mkt}} . \tag{2.9}
\end{equation*}
$$

### 2.4 Symmetric Equilibrium

Consider the ratio of farmers to hubs, $\frac{F}{H}$, as well as the fraction of providers that serve on a first-come-first-served basis, $h$, as exogenously given. A symmetric equilibrium consists of farmers expected profits $U_{s}, U_{s^{-}}$, provider strategies $r_{i j}, \chi$, and farmer strategies, $q_{i j}$ for $i=\left\{s, s^{-}\right\}$and $j=\{\mathrm{fcfs}, \mathrm{mkt}\}$, that satisfy:

1. given $U_{s}, U_{s^{-}}$and other providers' strategies, each type provider maximizes value, equations 2.6 or 2.8 .
2. observing the providers' decision, farmers of productivity $z_{i}$ choose who to queue with, equation 2.4, and
3. the values $U_{s}, U_{s^{-}}$, through $q_{i j}$, are consistent with feasibility, equations 2.1 and 2.2 .

Proposition 2.1. In all symmetric equilibria where providers serve both types of farmers, the selection process is $\chi=1$ and the per period profit from farmers of type $i$ is $V_{i}^{j}$ :

$$
V_{i}^{j}=\gamma_{1 i}^{j}\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)+\gamma_{2 i}^{j}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right),
$$

where $\gamma_{1, i}^{j}, \gamma_{2, i}^{j}$ are non-linear functions of the queue lengths and the elasticity of the service probabilities with respect to the length of the queue.

The expected per period value of servicing large-scale farmers is higher than for lowscale farmers, $V_{s}^{j}>V_{s^{-}}^{j}$. If the surplus from large-scale orders is sufficiently larger than from small-scale orders, the expected profit from service for large-scale farmers is greater than for small-scale farmers, $U_{s}>U_{s^{-}}{ }^{12}$

A few characteristics are worth highlighting. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. In other words, for two plots located at the same distance to the provider, the marginal cost of service is lower for larger scale farmers. Second, the farmers' expected profit from equipment services depends on the return to his own demand for services and on the equilibrium rental rates. The surplus from service provision is the difference between the revenue accrued to the farmer and the cost of service provision.

[^15]In equilibrium, farmers that are served from both provider types shall be indifferent between them. For the expected profits to be equalized across providers, the product between the probability of service conditional on machine-hours demanded and the cost of service should be the same across providers.

### 2.4.1 Social optimum

Before quantitatively assessing the efficiency implications of different market arrangements it is useful to study the planner's allocation as a benchmark. The social planner chooses $\chi,\left\{q_{i j}\right\}_{i=s, s^{-}, j=\mathrm{fcfs}, \mathrm{mkt}}$ to maximize total surplus in the economy, i.e.

$$
\begin{equation*}
W=\max _{\chi,\left\{q_{i j}\right\}} \sum_{j=\mathrm{fcfs}, \mathrm{mkt}} \sum_{i=s, s^{-}} \Delta_{i j}\left(z k_{i}^{\alpha}-w\left(k_{i}+E\left(d_{i}\right)\right)\right) \tag{2.10}
\end{equation*}
$$

subject to the service capacity constraints, equation 2.7 and 2.9 , and the feasibility constraints, equations 2.1 and 2.2 . The probabilities of service are defined analogously to those described in the decentralized allocation, equation 2.3 .

Proposition 2.2. The decentralized allocation is constrained efficient.

This result follows from the directed nature of the search in our environment, mimicking previous results in Shi (2002). Giving priority to large-scale orders among providers that have a technology to select farmer's types is optimal because those orders have the lowest marginal cost of service per unit of time traveled. The queue lengths are optimal because the expected profits of the farms participating in the market equal the farms' social marginal contribution, which account for their contribution to output and the displacement of other farms from service provision. One way to see this is to notice that by maximizing surplus, the planner effectively is maximizing the sum of the
expected profits to the farm and the expected value of service for each provider, i.e. $W=\sum_{j=\mathrm{fcfs}, \mathrm{mkt}} \sum_{i=s, s^{-}} \Delta_{i j}\left(\pi_{i j}^{\star}+\bar{V}_{i j}^{\star}\right)$ which follows from the definition of the equilibrium expected profits in the decentralized allocation (see Appendix B.2.1). Because farms maximize profits and providers maximize value, these values can be taken as constants. It is trivial from here that the queueing behaviour that solves the decentralized problem by definition maximizes 2.10 which proves the result.

### 2.5 Government regulation in equipment rental markets

In this section, we bring the model to the data to characterize how allocations change with the presence of a first-come-first-served dispatch system relative to the market dispatch system, both in space and across farmers of different production scales. The key outcomes of interest are the selection of farmer types across providers, the equilibrium delays and therefore farming productivity costs, as well as provider profitability. We later study how these outcomes change under two counterfactual market arrangements: first, we allow first-come-first-serve providers to have access to a service selection technology, i.e., behave as market providers to mimick a market deregulation; second, we characterize a long-run equilibrium where in response to market deregulation providers' are also allowed to enter or exit the market.

### 2.5.1 Bringing the model to the data.

The quantitative assessment of the impact of the government intervention in the rental markets for equipment consists of two blocks. The first block solves the model in Section 2.3 for the equilibrium market rental rates and queue lengths, given the empirical supply and demand for equipment services. The second block simulates queues and service
provision strategies for farmers with different scale and geographical location.

Solving the first block requires taking a stand on the heterogeneity in machine-hours demanded. We construct two groups of farmers following their average machine-hours requests in the transaction data: those with requests of more than 3.5 machine-hours per order are denominated large-scale while those with requests of less than 3.5 machine-hours are denominated small-scale. Then, we solve for an equilibrium in which both type of farmers are served by both types of providers, as in the data. We call this equilibrium the "status quo".

The second block involves finding the expected delay and subsequent productivity costs as well as provider profitability under alternative dispatch systems using equilibrium rental rates and queue lengths from the first block. In theory, the queue length itself yields the expected wait time by farm type. However, we recognize that empirically, farmer heterogeneity is richer than the one accommodated by the stylized theoretical model both in terms of machine-hours demanded and in the spatial allocation of demand. We simulate 300 paths of queues of length $q^{\star}$ and composition $\left(q_{s}^{\star}, q_{s^{-}}^{\star}\right)$ as dictated by the equilibrium of the selection model. The actual sample paths for queues $\left(q_{s}^{\star}, q_{s^{-}}^{\star}\right)$ are drawn from the joint empirical distribution of machine-hours and geographical location. Then, given the equilibrium rental rates and the technology for dispatch, we let the provider optimize service delivery. The optimization of service provision in space is effectively the solution to a traveling salesman problem, conditional on the set of orders in the queue.

## Data

Consistently with the evidence in Section 2.2 we use data for the Kharif season (June to October) in year 2018. We exploit four sources of data: (1) detailed transaction data from the government subsidized service provider, (2) our own survey of farmers, (3) our own
census of farming households in the catchment area of the subsidized service providers and (4) high-frequency data from ICRISAT.

The first source of data contains information on machine-hours requested by service, the acreage serviced, the total cost of the service, the implement rented, and the nearest village to the farmer's service request. We focus again on commonly used implements, i.e. rotavators and cultivators; and we narrow the set of provider hubs to those with more than a 100 transactions within the season. We are left with 15 hubs for analysis. We use information on the closest village to each request as well as service hours requested to compute the empirical joint distribution of machine-hours demanded and service travel time in the catchment area of each hub. We geolocate villages and hubs and compute travel times using information from a commonly used Application Programming Interface (API), which factors in road connectivity across locations. Rural areas are not always well connected and while geographical distances may seem short, travel times can increase rapidly with poor road conditions.

The second source of data allows us to characterize the productivity (output per acre) of the farmers being served. We compute the empirical joint distribution of productivity and production scale (i.e., cultivated area) within the catchment area of each provider hub. The survey covers more than 7500 households along 180 villages and includes detailed information on inputs and outputs in farming.

The third source of data allows us to characterize equipment supply, including not only the government subsidized equipment supply but also equipment owners' supply of rental services. We also use the census to measure the total demand of services by production scale within the catchment area of each hub, including demand towards market providers. The census covers more than 40000 households along 150 villages that overlap with the catchment area of the hubs, and entails a brief set of questions about rental market
engagement and farming engagement.

The fourth source of data allows us to measure the productivity costs of delays. The data entails 6200 plots in 18 villages in India during 2009-2014 with daily detailed measures of inputs and output in farming.

## Parameterization

There are 10 parameters per hub that need to be calibrated, as shown in Table 2.3. 8 of these parameters are calibrated directly from the data while the remaining 2 are calibrated internally by solving the model. From those parameters measured in the data, 4 of them are common across hubs: the providers' discount factor $\beta$, and their opportunity cost of moving equipment in space $w$, the productivity cost of delays $\eta$, and the curvature of the farming profit function $\alpha$. The remaining 6 parameters are hub specific and include the share of first-come-first-serve providers relative to the total supply of equipment in the catchment area of a hub $h$, the parameters characterizing the joint distribution of productivity and machine-hours demand within the catchment area of the hub (i.e., mean and standard deviation of productivity and the correlation between productivity and machine-hours), the ratio of farmers demanding service to the providers in the catchment area of each hub $f$, as well as the share of large farmers in the population of farmers demanding equipment in the catchment area of the hub $s$. The latter two model-calibrated parameters are chosen to match the queue length of small-scale farmers at first-come-firstserved providers, and to make sure the equilibrium displays positive queues of small and large-scale farmers with both providers, as we observe in the data. In addition to these 10 parameters, we feed the distribution of plots in space (and their corresponding travel-time) as measured from the platform data.

We set the discount factor to $\beta=0.96$ with an implied daily discount rate of $4 \%$.

Table 2.3: Parameterization

| Parameter | Description | Value | Source/Moment |
| :---: | :---: | :---: | :---: |
| Measured directly in the data common across hubs |  |  |  |
| $\alpha$ | Curvature of the profits function | 0.6 | Survey data |
| $\beta$ | Discount factor | 0.99 | Interest rate |
| $w$ | Travel/op. cost (INR/hr) | 75 | Platform data |
| $\eta$ | Productivity loss/day | 3.4\% | ICRISAT sample |
| $\bar{z}$ | Provider profitability |  | Platform, 10\% |
| hub specific |  | Method |  |
| $h$ | Share of fcfs providers |  | Census data |
| $\mu$ | Log-normal mean of productivity | MLE | Survey data |
| $\sigma$ | Log-normal s.d. of productivity | MLE | Survey data |
|  | Correlation order size and productivity |  | Survey + Platform data |
| $k_{i}, E\left(d_{i}\right)$ | Joint-distribution of order size and travel time | B-splines | Platform data |
| Calibrated using the model (hub-specific) |  |  |  |
| $s$ | Share of large farmers |  | Census data |
| $f$ | No. of farmers/No. of equipment |  | Small-scale queue, fcfs |

Productivity is measured as output per acre.

The opportunity cost of travel time equals the hourly wage of a driver which is directly observed from the platform data, at $w=₹ 75$. The curvature of the profit function is set to 0.6 , as estimated from our own survey data on farm profitability. We exploit the fact that farming profits are proportional to this parameter, i.e. $\pi_{i}=(1-\alpha) y_{i}$ and estimate $\alpha$ from the average ratio of profits to value added as reported by farming households.

To discipline the productivity costs of delays, $\eta$, we use high frequency data from ICRISAT. We study profitability and value-added per acre of about We define an optimal planting time as the date that maximizes the profits per acre in a given village year. Then, we define the cost of the delay as the difference in average value added per acre or profit per acre (depending on the variable of interest) as we move away from the optimal planting date. Formally, we estimate

$$
Y_{i, \text { year }}=\beta_{0}+\beta_{1}^{+}(\text {Planting Date-Optimal })_{>0}+\beta_{1}^{-}(\text {Planting Date-Optimal })_{\leq 0}+\alpha X_{i, \text { year }}+\epsilon_{i, \text { year }}
$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Stan-
dard errors are clustered at the village level. Our estimates for the costs in value added per acre are reported in Table B.1. They indicate that within a 5 -day windows, each additional day away from the profit maximizing date entails a cost of $3.4 \%$ in terms of value added per acre. If we compute the cost at a 10-day window, the cost raises to $8.5 \%$ per day. For our assessment we focus on the former, more conservative estimate of the cost of delay, $\eta=3.4 \%$.

Then, we calibrate hub-specific parameters. We use our census, to compute the share of machinery available from government-subsidized hubs and that available from machineowners (i.e. we count inventory of implements per hub and implements owned by farmers within the catchment area of each hub).To characterize the productivity of farmers requesting different machine-hours we use the subsample of transactions that overlaps with the survey data (approximately, 1300 observations) and compute the underlying correlation between farm productivity, measured as output per acre, and machine-hours requested. Their correlation ranges from -0.28 to 0.35 displaying the wide-heterogeneity in demand characteristics across hubs, column (5) in Table 2.4. When machine-hours requested are proportional plot size a negative correlation between output per acre and machine-hours follows from the negative correlation between productivity and farm size, as has been documented by others in the literature, e.g. Foster and Rosenzweig (2017). A positive correlation is consistent with more productive mechanized farms. Our data is rich enough to display both patterns. We assume that the distribution of productivity is log-normal, $\ln (\bar{z}) \sim \mathcal{N}(\mu, \sigma)$ and fit the empirical distribution of value-added per acre for survey farmers in the catchment area of each hub via maximum likelihood. The estimated mean of productivity suggests differences in log productivity across hubs of $36 \%$ (from 7.4 to 9.3 ) on average, and a log-variance ranging from 1.1 to 2.9 , columns (3-4) in Table 2.4. Finally, we fit the joint distribution of machine-hours demanded and travel time to services from the platform data for each hub using B-splines, akin to a
non-parametric estimation of the distribution. On the travel dimension, the distribution is typically bimodal, with orders bunching at less than 10 -minutes travel time from the hub and 30-minutes travel time ${ }^{13}$

Table 2.4: Hub specific characteristics.

| Measured Directly |  |  |  |  |  | Calibrated |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Supply |  | Demand |  |  |  | Farmers |
| Hub | sh. fcfs <br> $h$ | ${ }_{\bar{k}}$ | $\begin{aligned} & \text { Prodı } \\ & \text { mean } \end{aligned}$ | uctivity variance | Correlation prod - hours | sh. large <br> $s$ | per provider <br> $f$ |
| 1 | 0.86 | 6 | 1.08 | 0.06 | 9.35 | 0.4 | 3.2 |
| 2 | 0.80 | 4 | 1.50 | 0.08 | 8.82 | 0.3 | 3.9 |
| 3 | 0.50 | 8 | 1.12 | 0.21 | 9.83 | 0.3 | 6.2 |
| 4 | 0.86 | 6 | 1.57 | 0.16 | 7.43 | 0.4 | 6.1 |
| 5 | 0.86 | 4 | 1.07 | 0.09 | 8.85 | 0.3 | 4.6 |
| 6 | 0.86 | 4 | 1.33 | 0.07 | 8.89 | 0.3 | 5.3 |
| 7 | 0.86 | 5 | 1.33 | 0.07 | 8.89 | 0.3 | 4.4 |
| 8 | 0.86 | 7 | 1.33 | 0.07 | 8.89 | 0.3 | 5.3 |
| 9 | 0.75 | 11 | 1.49 | 0.17 | 8.67 | 0.4 | 3.4 |
| 10 | 0.75 | 14 | 1.52 | 0.10 | 9.08 | 0.4 | 3.6 |
| 11 | 0.86 | 6 | 2.56 | 0.12 | 8.85 | 0.4 | 3.2 |
| 12 | 0.63 | 8 | 2.56 | 0.12 | 8.85 | 0.5 | 3.8 |
| 13 | 0.86 | 5 | 1.52 | 0.10 | 9.08 | 0.3 | 4.6 |
| 14 | 0.86 | 5 | 1.52 | 0.10 | 9.08 | 0.3 | 5.2 |
| 15 | 0.67 | 11 | 2.89 | 0.14 | 8.15 | 0.3 | 4.7 |
| average | 0.79 | 7 | 1.63 | 0.11 | 8.85 | 0.3 | 4.5 |

Notes: Hub-specific parameters for each hub-implement combination, "Hub" in Column (1). Hubs labeled $5-7,11$ and 13 correspond to Cultivators while the remaining hubs contain information for Rotavators. Information for hubs labeled 6-8 are correspond to different implements in a single government subsidized hub, and therefore demand characteristics are the same. Column (2) reports the share of first-come-firstserve providers in the total equipment supply within each catchment area. Columns (3)-(5) report demand characteristics for each hub, including the characteristics of the distribution of productivity across farmers and its correlation between hours demanded. Columns (6)-(7) report parameters calibrated jointly in the model.

Parameters calibrated jointly include the ratio of farmers to providers in the catchment area of each hub, as well as the the share of large-scale requests in that catchment area. We minimize the distance of the latter to its empirical counterpart, while generating an equilibrium allocation that displays service request from both types of farmers to both type of providers (as in the data). The former is calibrated to minimize the distance between the model predicted queue of small-scale farmers at the fcfs provider and the

[^16]Table 2.5: Moments

|  | Share of large scale $s$ |  | Queue <br> $q_{s^{-}}$fcfs |  | Queue untargeted $q_{s f c f s} / q_{s^{-}} f c f s$ data model |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.12 | 0.35 | 1.3 | 2.0 | 0.7 | 0.8 |
| 2 | 0.29 | 0.30 | 1.5 | 2.0 | 0.5 | 1.1 |
| 3 | 0.45 | 0.25 | 2.3 | 2.3 | 0.8 | 1.6 |
| 4 | 0.36 | 0.40 | 2.7 | 4.0 | 1.3 | 1.6 |
| 5 | 0.25 | 0.30 | 2.3 | 2.3 | 1.2 | 1.5 |
| 6 | 0.19 | 0.25 | 4.3 | 4.3 | 4.3 | 2.2 |
| 7 | 0.19 | 0.30 | 3.3 | 3.3 | 3.3 | 1.9 |
| 8 | 0.19 | 0.25 | 7.3 | 4.3 | 1.5 | 2.2 |
| 9 | 0.39 | 0.40 | 0.7 | 2.0 | 0.2 | 0.9 |
| 10 | 0.28 | 0.35 | 1.0 | 2.0 | 1.0 | 0.4 |
| 11 | 0.35 | 0.35 | 2.0 | 2.0 | 2.0 | 0.8 |
| 12 | 0.35 | 0.45 | 1.0 | 2.0 | 0.3 | 1.4 |
| 13 | 0.28 | 0.30 | 2.3 | 2.3 | 2.3 | 1.7 |
| 14 | 0.28 | 0.30 | 4.0 | 4.0 | 4.0 | 2.4 |
| 15 | 0.31 | 0.30 | 2.0 | 2.0 | 0.5 | 1.1 |

Notes: Calibration moments, data and model counterparts, Columns (2-5). Untargeted queue length for large-scale farmers relative to small-scale farmers, Columns (6-7).
data. To do so, we take the stand that small-scale farmers are those with requests of up to 4 machine-hours per order, and large-scale farmers are those with orders of more than 4 machine-hours per order. These two parameters are calibrated jointly for each hub.

The calibrated ratio of farmers to providers ranges from 2.3 to 4.9 , where a provider should be interpreted as a piece of equipment, column (7) in Table 2.4. The calibrated shares of large-scale farmers are higher than in the data ( 0.31 vs. 0.11 on average across hubs). ${ }^{14}$ When there are few large-scale requests, The model generates queue lengths for small farmers that are broadly in line in the data, Table 2.5. Queues that are "too short" (less than 2 orders) fail to generate equilibria where both type of farmers request service from both providers because a queue length of 1 order for the market provider implies that small farmers are served with probability one there, given capacity. If the queue length is instead "too long" (more than 5 orders) the model benefits an equilibrium

[^17]or small-farmers only request service from fcfs providers, which is inconsistent with the data.

For completeness, we report the (untargeted) ratio of queue lengths of large-scale and small-scale farmers. On average, this ratio is lower in the data than in the model, but this difference is mostly driven for hubs where the differences in queue lengths between small and large-scale farmers is small. Otherwise, the model performs well in explaining the relative length of large-scale queues relative to small-scale queues across hubs.

### 2.5.2 Status quo equilibrium

We solve for the rental rates and queue lengths when both types of farmers have access to both types of providers, i.e. the status quo equilibrium. Each type of farmer is indifferent between being served by a fcfs provider or by a market provider (see equation B. 14 in Appendix B.2.1 for details).

The rental rates for both types of farmers are lower at the first-come-first-served provider than at the market provider to compensate for the longer queues (see Table B. 2 in Appendix (B.4). Rental rates are particularly lower for small farmers queueing with fcfs providers, which makes these providers attractive. Consistently, the service finding rate for small-scale farmers is larger with the fcfs than with the market providers, while the opposite is true for large-scale farmers, as show in Figure 2.7. The level of the rental rates are higher for small-scale than for large-scale farmers due do the higher cost of service per machine-hour rented (i.e. machinery travel costs).

Figure 2.7: Equilibrium


### 2.5.3 Accommodating empirically relevant heterogeneity

The stylized equilibrium queueing model does not capture for full extent of the observed heterogeneity in location and machine-hours demand. We accommodate this heterogeneity through simulation exercises where service queues to each provider are drawn from the empirical joint distribution of location and machine-hours observed in the catchment area of each hub. In other words, the queue lengths of small and large scale orders are the equilibrium ones, but their composition is allowed to vary following the empirical distribution of machine-hours and location observed in the data. We take the stand that a small order corresponds to less than 4 machine-hours. We sample, with replacement, 1000 queues per provider. Each order in the queue is a three dimensional object that includes the machine-hours demanded, the location of the plot, and the productivity of the farmer that requested service. This productivity level (output per acre) is then used to compute the cost of equilibrium delays in service provision ${ }^{15}$

[^18]Figure 2.8: Demand characteristics by provider


On the supply side, we solve for service dispatch system through two possible delivery routes. One follows a "hub and spoke" pattern, under which the equipment must return to the CHC between two consecutive orders. The other allows for a solution to a "Traveling Salesman Problem (TSP)", where the implement travels optimally from order to order within the day. Under fcfs, the provider follows the route that minimizes the travel time within a given day for a given set of requests (and their order) in the queue. Under the market allocation, the type of requests being served is jointly determined with the best service route. The value of an order in the market allocation depends on the density of orders around them, and the size of the order relative to the CHC serving capacity.

We fix the number of orders each provider can serve in a day to 3 , in line with the maximum number of orders that we observe being served by a driver in the administrative data.$^{16}$ We Then we estimate the value function for each provider, i.e. a function that maps any queue of orders to their service value, conditional on the dispatch system and the delivery route.

[^19]
## Farmer allocation across providers

We start by describing the sorting of farmers into different providers, classifying farmers by order size, i.e. the acreage of the plot for service; and by location, i.e. the travel time for service, see Figure 2.8 .

The average order size served by a market provider is 3 acres while the average order size served by a fcfs provider is 2.2 acres. This differential is a consequence of the disparities in the queue composition discussed before. At the same time, there are systematic differences in the travel time to locations. During a service day, market providers travel on average .8 hours ( 48 minutes), while fcfs providers travel twice as much. This differential travel time reflects not only differences in the queue composition along the location dimension, but also disparities in the ability to prioritize orders . While equal-accessconcerns may favor a fcfs service arrangement, it is possible to improve upon the baseline allocation by allowing government subsidized hubs to optimize service delivery within a day. Figure 2.9 displays travel times when providers are allowed to solve a TSP among the orders served within each day. This option is particularly beneficial to fcfs providers that are not allowed to prioritize order sizes. The average travel time (as \% of service time) is $11 \%$ for the market provider and it declines to $9 \%$ once optimizing routes. The average travel time (as \% of service time) is $15 \%$ for the fcfs provider and it declines to $11.5 \%$ once solving the TSP.

## Efficiency in service delivery

Figure 2.10 compares waiting times for different dispatch system and demand characteristics. On average, the mean wait times faced by farmers queueing with market providers is longer that with fcfs. This feature is in part due to differences in the composition of the queue. Market providers have higher service rates for large-scale farmers, but those

Figure 2.9: Travel times

farmers are only $30 \%$ of the population of farmers in the economy on average. For the reminder of the farmers, market providers have lower service rates than fcfs providers, which is reflected in higher waiting times among those queueing with market providers.

Despite the improvements in waiting times for farmers queueing with fcfs providers, these providers face equipment transportation costs (in terms of the opportunity cost of time) that can double those of the market providers. In other words, their inability to prioritize order sizes also affects their ability to service demand in space, traveling "too much" relative to their market counterparts. It is not surprising then that the value of optimizing service routes is higher for fcfs providers (as we have seen from the differential in travel time as percentage of service time in Figure 2.9.).

Indeed, the value of participating in the market is higher for market providers than for fcfs providers. In other words, if given the choice, subsidized providers would choose to operate the technology that allows them to prioritize large-scale orders. One way to measure the willingness to pay for this technology is to compute the difference in net present value of market participation across providers, which we show in Figure 2.11.

Figure 2.10: Time management by hub


Figure 2.11: Providers' value of market participation


The cost of delays and of service provision

Delays in service provision are costly for farmers. Delays are an equilibrium outcome in our economy given the nature of search in the market for rental equipments and the disparities in service provision across providers. Figure 2.12 left panel reports the productivity cost per acre across providers. These are measured as the decline in revenue per acre relative to the average revenue per acre in the catchment area of a hub.

Figure 2.12: Farmer and Providers' costs


Small farmers queueing with fcfs providers face higher service rates, but these providers travel larger distances (relative to service time) than market providers. These travel costs may well overturn the productivity gains to small-holder farmers from timely access to service. Those costs should be valued at the opportunity cost of time, which corresponds to the wages of a driver. Figure 2.12 right panel reports the travel cost per hour serviced, which reflects the "inefficiency" associated to the fcfs provision.

Figure 2.13: Relative Surplus


### 2.6 The value of the subsidies

The cost per acre of service is driven by the delays in service provision and the productivity costs to farmers. While the additional transportation costs from a fcfs arrangement are sizable, the cost of additional time (in terms of the wages of the driver) is relatively low. However, the opportunity cost of travel time is indeed the productivity costs of the delays induced on others. A direct comparison across different dispatch system could be done by computing the surplus generated across providers per day of service, e.g. the terms summing into the surplus of the economy, equation 2.10 .

### 2.6.1 The market prior to the subsidy

We now study how the status quo equilibrium compares to the equilibrium prior to the intervention. While data for the pre-subsidy market is unavailable, we can account for this effect by running a counterfactual analysis where we shut down the supply of equipment
stemming from fcfs, i.e. the government subsidized supply. This counterfactual is valid under the assumption that market participants accommodated the entry of new supply in the market. This is plausible scenario given the relatively low ownership of equipment in the population and the desire of most farmers to have some engagement in the market for equipment.

The substantial increase in equipment supply due to the subsidy implies an increase in the service finding rates, moving from $15 \%$ prior to the subsidy to between $40 \%$ and $55 \%$ after the subsidy depending on the provider, see Table 2.6. Gains in service findings rates are mostly accounted for small farmers, which prior to the subsidy faced a service finding rate of $6 \%$ and after the subsidy face a service finding rate of $56 \%$, with a bit more than half of it being accounted for the fcfs providers. Market providers' service probability also increase in equilibrium because the cost of service declines in response to stronger competition. Rental rates falls by $18 \%$ for small scale farmers and by $5 \%$ for large farmers. The differential effect is a consequence of the implicit priority given to small-scale farmers by the fcfs subsidized provider.

Table 2.6: Effect of the subsidy

|  | pre | post subsidy <br> fcfs |  |
| :--- | ---: | ---: | ---: |
|  |  | mkt |  |
| Service Finding Rate |  |  |  |
| all | 0.15 | 0.41 | 0.56 |
| small | 0.06 | 0.3 | 0.26 |
| large | 0.09 | 0.11 | 0.3 |
| Rental Rate |  |  |  |
| small | 148.5 | 119.3 | 124.5 |
| large | 97.7 | 92.2 | 93.5 |

### 2.6.2 Market Deregulation

One of the findings in Section 2.5 .3 is that fcfs providers could benefit for operating a technology that allows them to optimize equipment in space as well as optimize the type of
orders being served. We study the effect of a market deregulation through counterfactuals. We first explore a scenario in which the technology that allows the market providers to prioritize large farmers is made available to the fcfs providers. Because the prioritization is costless and the fcfs providers are at least as well off as before (i.e. they can now prioritize the high marginal return orders), a profit driven fcfs provider would choose to adopt the technology, i.e. $h=0 .{ }^{17}$ In other words, there is no longer any differentiation between these two types of providers. The nature of the equilibrium may however change. We study the equilibrium effects in two scenarios: first we do not allow providers to exit or enter the market, i.e. the short-run; and second, we allow entry/exit until the expected value of market participation for farmers equals zero, i.e. their outside option.

Table 2.7: Effect of Market Deregulation

|  | Benchmark <br> fcfs |  | mkt | Deregulation <br> short-run |
| :--- | ---: | ---: | ---: | ---: |
| Service Finding Rate |  |  |  |  |
| long-run |  |  |  |  |
| all | 0.41 | 0.56 | 0.49 |  |
| small | 0.3 | 0.26 | 0.51 |  |
| large | 0.11 | 0.3 | 0.31 | 0.33 |
| Rental Rate |  |  |  | 0.17 |
| small | 119.3 | 124.5 | 125.5 | 119.6 |
| large | 92.2 | 93.5 | 94 | 92.3 |

The short run response of the economy (without endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistently with the profit maximizing strategy of market providers. However, their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for smallholder farmers do not change relative to the baseline, the rental costs increase for all farmers. The long run response of the economy (with endogenous entry and exit) restores rental costs to their baseline levels, and generates allocations where service findings rates are higher for small holder farmers than large holder farmers, despite market providers

[^20]Figure 2.14: Impact of Deregulation


Figure 2.15: Impact of Deregulation


prioritizing the latter. The reason is that small-scale request are valuable when there are service capacity constraints, because they allow for better capacity utilization.

Figure 2.15 plots the change in average farm sizes served and the travel time for service, i.e. the change in the distribution of served location, as the market deregulates. Exit of providers induces an increase in the average size of the farm served by each provider, and a reduction in the travel time to service, consistently with providers prioritizing services with low marginal cost of provision. For most hubs, the travel time as a proportion of the service time more than halves.

### 2.7 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spacial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, pose interesting trade offs between efficiency and market access. The returns to these rental markets depend crucially on factors such as density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize traveling equipment time. In this paper, we document and quantify how these factors determine the allocative efficiency and distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment from rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. Indeed, when equipment owners are allowed to behave optimally, by prioritizing larger scale orders (which are the most cost effective) the equilibrium returns to one where smallholder farmers are rationed out of the market.

## CHAPTER 3

## MADE TO BREAK? PLANNED OBSOLESCENCE WITH PRESENT-BIASED CONSUMERS

Planned obsolescence is a common phenomenon in the market of durable goods. It refers to the practice of firms choosing durability levels for their products below the cost-efficient ones. Motivated by the Phoebus cartel, whose reason for engaging in planned obsolescence cannot be explained by existing theories, I offer an alternative explanation that centers on an important concept from behavioral economics: present-biased preferences. I construct a theoretical model which demonstrates that when consumers are present biased, that is, when they exhibit time-inconsistent preference in favor of immediate gratification, a monopolist chooses a profit-maximizing level of durability below that chosen by a perfectly competitive market with the same production technology.

### 3.1 Introduction

In April 2022, the Biden administration announced new energy efficiency regulations that would phase out incandescent light bulbs in the United States. This means after dominating the market for over a century since its commercialization in the late 1800s, the incandescent light bulb will finally give way to its more energy-efficient substitutes.

The history of how the structure of the incandescent light bulb market evolved is a fascinating one. In particular, it witnessed the formation of arguably the first cartel on an international scale - the Phoebus cartel. It was formed in December 1924 by all the major manufacturers of incandescent light bulbs in the world. Evidence suggests that during its 16 years of operation, the Phoebus cartel successfully carried out planned obsolescence. Planned obsolescence refers to the choice of a level of durability by a manufacturer with market power lower than that chosen by a perfectly competitive market with the same
production technology ${ }^{1}$ After coming into operation, the Phoebus cartel steadily brought down the average life span of a light bulb from around 1,800 hours to 1,200 hours for reasons other than cost reduction. The profit motive behind such an action appears to contradict the prediction of existing economic theories on planned obsolescence.

Planned obsolescence by firms with market power - in the extreme case, by a monopolist - is well studied by economists. The existing theoretical literature offers two main reasons why a monopolist may choose a lower level of durability. The first is to mitigate a commitment problem (Coase, 1972, Bulow, 1982). If a monopolist of a durable good cannot commit to future prices, it will set the prices "too low" when the future arrives and push down the overall profit level. By reducing the durability and making the durable good more like a non-durable one, the severity of the commitment problem is reduced. The second reason applies when services delivered by the new product units and the used product units are not perfect substitutes (Waldman, 1998; Hendel and Lizzeri, 1999). Specifically, the quality of services provided by a new unit is superior to that of a used unit. In this case, the monopolist reduces durability to further lower the quality of the used units in order to charge higher prices for the new units and increase the overall profits.

Absent these two reasons, that is, if the monopolist is able to commit not to lower prices in the future by means such as drawing up future contracts or establishing the relevant reputation, and if the services delivered by the new and used product units are perfect substitutes, Swan (1971, 1972) finds that the monopolist chooses the same level of durability as a perfectly competitive market with the same production technology in order to maximize profits. In other words, planned obsolescence is incompatible with profit maximization for a monopolist under these conditions. The intuition behind this

[^21]is that the choice of durability has no impact on the present value of the revenue stream received by the monopolist. Consumers pay for the flow of services delivered by the durable good. The level of durability only determines how often the revenue is collected. With time-consistent discounting, the present value of the total revenues to perpetuity remains unchanged regardless of the level of durability. As a result, the monopolist chooses the cost-minimizing level of durability, which coincides with the one chosen by a perfectly competitive market with the same production technology.

However, if we apply these observations to the Phoebus cartel, it appears to suggest that it should have no economic incentive to carry out planned obsolescence. The Phoebus cartel successfully maintained prices at a stable level despite declining production costs by enforcing strict quantity quotas on its members. During the operation of the cartel, there were no substantive technological advances in the design of the light bulb, therefore services from a new unit could be considered a perfect substitute for services from a used unit. Since neither of the two reasons that justify planned obsolescence applies to the Phoebus cartel, the profit-maximizing level of durability for the Phoebus cartel, according to Swan (1971, 1972), should have been the same as in a perfectly competitive market. The existing theories fail to explain why the Phoebus cartel reduced the life span of a light bulb by a third.

This paper proposes an alternative theory on planned obsolescence to explain the behavior of the Phoebus cartel. It rests on an important concept from the field of behavioral economics - time-inconsistent preferences. Specifically, instead of assuming that consumers apply time-consistent discounting when making intertemporal decisions as in existing theories of planned obsolescence, this paper assumes that consumers are present biased.

In standard economic models, consumers' intertemporal preferences are assumed to
be time consistent. A consumer's relative preference for her well-being at an earlier date over a later date is the same no matter when she is asked. However, empirical evidence shows that this relative preference gets stronger as the earlier date gets closer. Thaler (1981), Loewenstein and Thaler (1989), among many others, provide empirical evidence that discount rates are lower for longer delays. These findings suggest that consumers are present biased. They tend to favor immediate gratification by giving bigger weight to payoffs that are closer to the present time when considering trade-offs between two future periods. Such a feature could be described by a hyperbolic discounting model.

This paper finds that under perfect competition, consumers' present-biased preferences have no effect on durability choice. The reason is that the level of durability is determined by cost minimization while a change in consumer's preferences only affects demand and therefore revenues. However, in the case of monopoly, the present value of the revenue stream is no longer independent of the durability choice when consumers are present biased. Consumers assign a bigger weight to the utilities in the present, which coincides with the period of the purchase action. By reducing durability, the monopolist can induce consumers to repeat the purchase action more often, thus increasing the overall utilities, which is directly translated into higher willingness to pay and thus higher revenues.

The rest of the paper is organized as follows. Section 2 briefly reviews the literature on planned obsolescence and present-biased preferences; Section 3 offers a brief history of the Phoebus cartel; Section 4 introduces the theoretical model; Section 5 concludes.

### 3.2 Literature Review

This paper is closely related to two strands of literature. The first is theoretical works on planned obsolescence. The commitment problem, or time-inconsistency problem faced by
a monopolist of a durable good was first recognized by Coase (1972) and later formalized by Bulow $(\overline{1982)}$. The problem arises as future prices affect the future values of units sold today. This effect acts as an externality. In the absence of the ability to commit, the monopolist does not internalize this externality and sets prices which are too low in the future. In a simple two period model, if the monopolist sells the durable good, which lasts exactly two periods, at the monopoly price in the first period, she will have an incentive to lower the price and serve the residual demand in the second period. Unless she can commit herself not to lower the prices in the second period, consumers' willingness to pay in the first period is reduced, and overall monopoly profits fall. One way to resolve this commitment problem is to reduce the durability of the good. In the two period case, if the durability of the good is reduced from 2 periods to 1 , the durable good effectively becomes a non-durable one and the externality is eliminated. In a multi-period model, or when time is continuous, the shorter the product lasts, the more limited the externality is.

A second explanation for planned obsolescence focuses on imperfect substitutability between the services provided by an old and a new unit of the durable good (Waldman, 1998; Hendel and Lizzeri, 1999). Here, durability is defined as how fast the quality of services provided by a product deteriorates. A secondhand market for used units exists. By making the used unit deteriorate faster, the monopolist makes it less appealing to the high-value consumers thereby allowing the monopolist to charge a higher price for new units and improve the overall profit level. This explanation shares similar intuition with the literature on pricing a product line (Mussa and Rosen, 1978; Maskin and Riley, 1984; Moorthy and Png, 1992). The monopolist's choice of a sub-optimal durability level for the used unit is analogical to the choice of sub-optimal quality for products sold to low-valuation consumers.

If a monopolist can perfectly commit to future prices, either through establishing
a reputation in a repeated game setting, or through contractual arrangements such as price guarantees, and if the services from the old and new units are perfect substitutes, then Swan (1971, 1972) shows that a monopolist chooses the same level of durability as a perfectly competitive market. The reason is that the present value of total revenues to perpetuity does not depend on the choice of durability when consumers have timeconsistent preferences. Consumers are paying for the services derived from the product and will adjust their willingness to pay when durability changes. From the revenue perspective, durability simply affects the frequency of revenue collection. It does not affect the present value of total revenues to perpetuity. Therefore the monopolist chooses the level of durability which minimizes the present value of the total costs to perpetuity, which is exactly how a perfectly competitive market chooses the level of durability. A monopolist can still exercise its market power by charging higher prices and selling a smaller quantity, but the level of durability is the same as under perfect competition.

The second strand of literature this paper is related to is on time-inconsistent preferences in behavioral economics. The standard model used to analyze consumers' intertemporal decisions is the exponential discounting model, first proposed by Ramsey (1928) and Samuelson (1937). Exponential discounting implies time consistency - a consumer's relative preference over two periods is unchanged regardless of when the consumer is asked. The main appeal of the exponential discounting model is its simplicity and tractability. However, it has long been criticized for being unrealistic (Loewenstein, 1992). Ample empirical evidence has emerged that indicates that consumers are present biased (Thaler, 1981; Loewenstein and Thaler, 1989). The discount rate between the present and the near future is higher than that between the near future and the far future. In other words, consumers are more impatient over short delays than over long delays. For example, Thaler (1981) finds that participants in his experiment were indifferent between receiving $\$ 15$ now and receiving $\$ 30$ in 3 months, implying a $277 \%$ annual discount rate; the same
participants were also indifferent between receiving $\$ 15$ now and receiving $\$ 100$ in 3 years, implying a $63 \%$ annual discount rate.

To account for consumers' "observed tendencies for immediate gratification" (O'Donoghue and Rabin, 1999), some behavioral economists have proposed hyperbolic discounting as an alternative functional form to exponential discounting (Chung and Herrnstein, 1967; Ainslie and Herrnstein, 1981; Ainslie, 1991, 1992; Ainslie and Haslam, 1992; Loewenstein and Prelec, 1992). O'Donoghue and Rabin (1999) propose a simple twoparameter model, the $(\beta, \delta)$-preferences, that modifies the exponential discounting model to capture the most basic form of present-biased preferences.

### 3.3 A Brief History of the Phoebus Cartel

### 3.3.1 Market structure pre the Phoebus cartel

Scientific research that led to the invention of the incandescent light bulb could be traced back to the work by the English scientist and inventor Ebenezer Kinnersley, who demonstrated in 1761 that a brass wire became red hot when passing electricity through it (Blake-Coleman, 1992). Further research was actively conducted throughout the 19th century. One of the earliest patents for an incandescent lamp was received by Frederick de Moleyns of England in 1841 (Friedel, Israel and Finn, 2010) . By the end of the 19th century, active effort to commercialize the incandescent light bulb was carried out across the Atlantic in both Europe (mainly Britain) and the United States.

The markets for incandescent light bulbs in the late 19th century and the early 20th century were concentrated, largely attributed to patent protections. Cartels, or industrial alliances at national, even continental levels were common. For example, General Electric
fixed prices with Westinghouse Electric Company, its primary competitor, through cross licensing (United States v. General Electric Co., 272 U.S. 476 (1926)). Formed in 1903, the Verkaufsstelle Vereinigter Gühlampenfabriken was a European cartel of carbon-filament lamp manufacturers (Krajewski, 2014). However, such cartel arrangements were often short lived due to constant innovation. For example, in 1906, two European companies introduced a new light bulb with filament made from tungsten paste. This new bulb lasted longer and was brighter than the carbon-filament lamp, thus rendering the Verkaufsstelle Vereinigter Gühlampenfabriken superfluous (Krajewski, 2014).

Technological advancement in incandescent light bulbs reached its peak when General Electric introduced a light bulb with the filament made of tungsten metal and filled with a noble gas ("the basic bulb"). Alliances formed based on the patent for the basic bulb were interrupted by World War I. As soon as the war ended, a new cartel, the Internationale Glühlampen Preisvereinigung, was established in 1921 to control prices for much of continental Europe (Krajewski, 2014). This is the predecessor of the Phoebus cartel.

### 3.3.2 Creation and dissolution of the Phoebus cartel

In December 1924, the Phoebus cartel, or formally the Phoebus S.A. Compagnie Industrielle pour le Développement de l'Éclairage, was created in Geneva. All major light bulb manufacturers in the world, including Germany's Osram, the Netherlands' Philips and France's Compagnie des Lampes, were members. General Electric was represented by its British subsidiary, International General Electric, as well as the Overseas Group, which consisted of its subsidiaries in Brazil, China and Mexico (Krajewski, 2014). ${ }^{2}$

[^22]The officially stated goals of the Phoebus cartel were "securing the cooperation of all parties to the agreement, ensuring the advantageous exploitation of their manufacturing capabilities in the production of lamps, ensuring and maintaining a uniformly high quality, increasing the effectiveness of electric lighting and increasing light use to the advantage of the consumer" (Krajewski, 2014).

The Phoebus cartel was intended to be dissolved in 1955. However, its dominant position was weakened by the expiration of General Electric's patents for the basic bulb in 1929, 1930 and 1933. Conflicts among its members and legal battles in the United States also threatened the cartel's stability. After the outbreak of World War II, the cartel's agreement was nullified in 1940 (Krajewski, 2014).

### 3.3.3 Key operational features

The Phoebus cartel exercised its market power through strict quantity control. The cartel divided the world market into national and regional zones, and assigned a sales quota to each of its member companies. Companies that exceeded their quotas were fined. While the Phoebus cartel did not directly fix prices, the quantity control allowed it to maintain stable prices despite falling manufacturing costs (Krajewski, 2014). The strictly enforced quota system over the years ensured that the Phoebus cartel was not subject to the commitment issue. Consumers therefore would not anticipate the cartel to reduce prices in the future in an attempt to sell more.

Technological innovation in incandescent light bulbs had significantly slowed down before the creation of the Phoebus cartel. During the years the cartel operated, there was no major improvement in the design of the bulbs. Therefore the services received by a consumer from a new bulb was arguably identical to the services from the old one.

The absence of the commitment issue and perfect substitutability between the services of new and used units suggests that the Phoebus cartel would not have any incentive to shorten the durability of the light bulb, according to Swan (1971, 1972). However, the best known legacy of the Phoebus cartel today is arguably the first successful implementation of planned obsolescence in modern manufacturing.


Figure 3.1: Average operational hours of incandescent light bulbs: 1879-1934 Source: Chhatre (2021)

### 3.3.4 Evidence of planned obsolescence

Before the creation of the Phoebus Cartel, the average life of an incandescent light bulb was around 1,800 hours. From 1925 to 1934, the average life decreased steadily to around 1,200 hours (Chhatre, 2021) (see Fig. 3.1). This reduction in durability was achieved by the Phoebus cartel through rigorous research, close monitoring and strict enforcement.

In the corporate archives of the cartel member Osram in Berlin, "meticulous correspondence" between the cartel's factories and the laboratories on research to shorten the life span of the bulbs was found. The measures included modifying the filament and adjusting the current or voltage at which the bulbs operated. Each factory bound by the cartel


Figure 3.2: Archived document showing average operational hours of incandescent light bulbs produced by member factories of the Phoebus cartel from 1926 to 1934 Source: Krajewski (2014)
agreement had to regularly send samples of its bulbs to a central testing laboratory in Switzerland, where they underwent thorough tests against the cartel standard, including the life span. Fig. 3.2 is a piece of historical document discovered in the Osram archive. It documents detailed testing results of the average lives of the light bulb samples sent by member factories from 1926 to 1934. The document confirms that such tests took place on an annual or biannual basis on a large scale. The testing results depict a decreasing trend in the average life span of an incandescent light bulb. Last, the cartel enforced the shortened life span by fining any factory that submitted samples lasting longer or shorter than the regulated standard (Krajewski, 2014) ${ }^{3}$

There is no evidence suggesting that the reduction in durability led to lower production costs. Even the cartel members did not claim that cost reduction was the goal. Instead, they argued that the shortened life span of the light bulb was the unfortunate byproduct of the attempt to offer the consumers a light bulb that was "of a higher quality, more efficient and brighter" (Krajewski, 2014). However, quotes from top management of a cartel member suggest that this may not be the genuine reason behind the reduced life span. After discovering an incidence where some cartel members attempted to secretly introduce a longer lasting bulb, Anton Philips, head of Philips warned that "after the very strenuous efforts we made to emerge from a period of long life lamps, it is of the greatest importance that we do not sink back into the same mire ... supplying lamps that will have a very prolonged life" (Krajewski, 2014).

The history of the Phoebus cartel provides us with a prime example of "made to break" - the manufacturer makes a product that breaks down faster in an attempt to increase profitability. The Phoebus cartel succeeded in engineering such a product, but there is little information to confirm if such a move indeed improved profitability. During

[^23]the years the cartel was in operation, it lost market share to cheaper bulbs imported mainly from Japan (Krajewski, 2014). While this is likely a result of the quantity quota and high prices, the effect of a shorter life span is unclear. A simple way to reconcile the discrepancy between the behavior of the Phoebus cartel and the existing theories on planned obsolescence is that the cartel made a mistake. Its profitability would have been higher if it kept the durability unchanged. In this paper, I will argue that the reduced level of durability was indeed the profit maximizing level and propose an alternative theory to explain the behavior of the Phoebus cartel.

### 3.4 An Alternative Model of Planned Obsolescence

### 3.4.1 Assumptions

Let the structure of the market for a certain durable good be a monopoly. The good has a "sudden death" property. Specifically, each unit of the durable good provides 1 unit of homogeneous service in all discrete time periods $t<N$ and provides no service in any $t \geq N$, where $N$ is an integer and $N \in[1, \bar{N}]$. There are no fixed costs of production. Marginal cost $c(N)$ is a function of the durability of the product where $c^{\prime}(N)>0$. For a fixed durability $N$, marginal cost $c(N)$ remains constant for any quantity $Q>0$ produced. At the beginning of period $t=0$, the monopolist chooses a durability $N$. The monopolist then announces a sale price $P_{N}$ (the rental option is not available). The monopolist is able to commit to the price announced for all subsequent periods. The discount rate faced by the monopolist is constant at $\delta$.

The same durable good market could alternatively operate under perfect competition. The large number of perfectly competitive firms have the same cost structure as the
monopolist: zero fixed cost and marginal cost $c(N) \cdot{ }^{4}$ The perfectly competitive firms also chooses a durability $N$, where $N$ is an integer and $N \in[1, \bar{N}]$ at the beginning of period $t=0$. They do not set prices. Instead, they act as price takers and sell at $p=c(N)$.

There is a continuum of consumers of mass 1 in the market, each consumes a maximum of 1 unit of service per period and values it at $v . v$ is sufficiently large such that at the beginning of each period in which a purchase decision needs to be made (either to buy the product for the first time, or to replace the used unit which broke down at the end of the last period), each consumer purchases 1 unit of the product in equilibrium regardless the market structure ${ }^{5}$ The quantity sold at the market level is therefore 1 in period $t=0, N, 2 N, \ldots$; and 0 in all other periods. Consumers are assumed to exhibit timeinconsistent preferences, specifically ( $\beta, \delta$ )-preferences (O'Donoghue and Rabin, 1999). This type of preference is characterized by a bias for the "present" over the "future". Let $u_{t}$ be a consumer's instantaneous utility in period $t$ and let $U^{t}\left(u_{t}, u_{t+1}, \ldots, u_{T}\right)$ represent a consumer's intertemporal utility from the perspective of period $t$. Formally the $(\beta, \delta)$ preferences can be represented by:

For all $t$,

$$
U^{t}\left(u_{t}, u_{t+1}, \ldots, u_{T}\right) \equiv \delta^{t} u_{t}+\beta \sum_{\tau=t+1}^{T} \delta^{\tau} u_{\tau}
$$

where $0<\beta, \delta \leq 1$.
Compared to the time-consistent preferences represented by exponential discounting, the $(\beta, \delta)$-preferences differ by applying an additional discounting factor $\beta$ to all future periods relative to the current period $t$. The exponential discounting is a special case of $(\beta, \delta)$-preferences with $\beta=1$.

[^24]
### 3.4.2 Durability choice under perfect competition

First, I will examine the durability choice by a perfectly competitive market with the same production technology.

As discussed in Section 3.4.1, in each period $t=0, N, 2 N, \ldots, 1$ unit of the durable good is produced and sold. The present value of the total costs of production to perpetuity is

$$
\begin{equation*}
\kappa(N) \equiv c(N)\left(1+\delta^{N}+\delta^{2 N}+\ldots\right)=\frac{c(N)}{1-\delta^{N}} . \tag{3.1}
\end{equation*}
$$

Under perfect competition, firms choose a level of durability $N^{p c}$ such that it minimizes the present value of the total costs to perpetuity $\kappa(N)$, i.e., $N^{p c}=\operatorname{argmin} \kappa(N)$. For any fixed $\delta, N^{p c}$ always exists since $\kappa(N)$ is continuous in $N$ and $N$ is bounded. The uniqueness of $N^{p c}$ will depend on the functional form of $c(N)$. A sufficient condition that guarantees the uniqueness of $N^{p c}$ is that $\kappa(N)$ is convex. The intuition behind a convex $\kappa(N)$ is that while the per period marginal cost $c(N)$ increases in $N$, the present value of the total costs may be reduced by increasing $N$ and postponing production to a further future. To simplify the discussion, I assume that $\kappa(N)$ is convex. For further discussions on the uniqueness of $N^{p c}$, refer to Appendix C.1. The perfectly competitive market charges a unit of the durable good at its marginal cost, $\mathcal{P}^{p c}=c\left(N^{p c}\right)$. Firms earn zero profit in each period hence the present value of the total profits to perpetuity, $\pi^{p c} \equiv \pi\left(N^{p c}\right)$, is also zero.

As illustrated in Fig. 3.3, the present value of the total costs to perpetuity $\kappa(N)$ is convex $\left(-\kappa(N)\right.$ is concave and is maximized at $\left.N^{p c}\right)$. Since the equilibrium market price is fixed at $\mathcal{P}^{p c}=c\left(N^{p c}\right)$ and quantity is equal to 1 , the revenue each period and the present value of the total revenues to perpetuity are independent of $N$. As a result, $\pi(N)$, the present value of the total profits to perpetuity is an upward parallel shift of $-\kappa(N)$


Figure 3.3: Choice of durability by a perfectly competitive market
and is maximized at $N^{p c}$. At its maximum, $\pi(N)=0$ due to the zero profit condition.

Notice that the durability choice by a perfectly competitive market is unchanged whether the consumers' preferences are time consistent or time inconsistent. This is because cost minimization is independent of consumer preferences.

### 3.4.3 Durability choice under monopoly

Next, I will determine the level of durability chosen by a monopolist when consumers have $(\beta, \delta)$-preferences. I will first examine the special case with $\beta=1$, i.e., when consumers have time-consistent preferences, and then analyze the general case with $0<\beta<1$ when consumers are present biased.

Like in a perfectly competitive market, 1 unit of the durable good is produced and sold in each period $t=0, N, 2 N, \ldots$, as long as the price charged by the monopolist does not exceed $V(N)$, the present value of the total value derived from the flow of services from 1 unit of the good to the consumers with $(\beta, \delta)$-preferences. To maximize profits, the monopolist prices the good at $V(N)$, i.e.,

$$
\begin{aligned}
\mathcal{P}^{m}(N) & =V(N) \\
& =v+\beta \sum_{t=1}^{N-1} \delta^{t} v \\
& =v\left(1+\frac{\beta\left(\delta-\delta^{N}\right)}{1-\delta}\right)
\end{aligned}
$$

The present value of the total revenues to perpetuity $\gamma(N)=v\left(1+\frac{\beta\left(\delta-\delta^{N}\right)}{1-\delta}\right)\left(\frac{1}{1-\delta^{N}}\right)$. Note that this is a present value to the monopolist, whose preferences are time consistent with a single discount factor $\delta$. The present value of the total profits to perpetuity is

$$
\begin{equation*}
\pi(N)=v\left(1+\frac{\beta\left(\delta-\delta^{N}\right)}{1-\delta}\right)\left(\frac{1}{1-\delta^{N}}\right)-c(N)\left(\frac{1}{1-\delta^{N}}\right) \tag{3.2}
\end{equation*}
$$

When $\beta=1$, that is when discounting is time-consistent, $\pi(N)$ can be simplified to $\pi(N)=\frac{v}{1-\delta}-c(N)\left(\frac{1}{1-\delta^{N}}\right)$. In this case, the present value of total revenues to perpetuity $\gamma(N)$ becomes independent of the monopolist's choice of durability $N$. Notice that this result critically depends on the monopolist and the consumers having the same discount factor $\delta$. This is usually guaranteed by a perfect capital market. When the monopolist and the consumers are equally patient, the present value a consumer places on the services derived from one unit of the product in any period is exactly equal to the present value of the revenues received by the monopolist for that period. As a result, the level of durability $N$ only determines how often the revenues are collected, not the total level measured by
the present value. If, for example, the monopolist is more patient than the consumers, i.e. $\delta_{m}>\delta_{c}, \gamma(N)$ is no longer independent of $N$. This gives rise to the opportunity for the monopolist to manipulate the present value of the total revenues to perpetuity by changing the level of durability ${ }^{6}$

When the monopolist and the consumers face the same discount factor, the choice of $N$ when $\beta=1$ is once again a cost minimization problem and the monopolist would choose the same level of durability as the perfectly competitive market, i.e.,

$$
N^{m} \equiv \operatorname{argmax} \pi(N)=\operatorname{argmin} \kappa(N) \equiv N^{p c} .
$$

This is consistent with the result in Swan (1971, 1972).

When $0<\beta<1$, that is when discounting is present-biased, it is straightforward to show that the present value of the total revenues to perpetuity $\gamma(N)$ decreases in $N$ and is convex, i.e.,

$$
\frac{\partial \gamma(N)}{\partial N}=\frac{v(1-\beta) \delta^{N} \log (\delta)}{\left(1-\delta^{N}\right)^{2}}<0, \quad \frac{\partial^{2} \gamma(N)}{\partial N^{2}}>0
$$

$N^{m}$ always exists for the same reasons discussed for $N^{p c}$. It is also unique if $\pi(N)$ is concave, i.e. $\forall N \in[1, \bar{N}]$,

$$
\begin{gather*}
\frac{\partial^{2} \pi(N)}{\partial N^{2}}=\frac{\partial^{2} \gamma(N)}{\partial N^{2}}-\frac{\partial^{2} \kappa(N)}{\partial N^{2}}<0 \\
\frac{\partial^{2} \gamma(N)}{\partial N^{2}}<\frac{\partial^{2} \kappa(N)}{\partial N^{2}} \tag{3.3}
\end{gather*}
$$

Equation 3.3 again relies on the convexity of $\kappa(N)$. In addition, $\kappa(N)$ must be "more convex" than $\gamma(N)$ in order to guarantee the concavity of $\pi(N)$. I assume that $\pi(N)$ is

[^25]concave for simplicity.


Figure 3.4: Choice of durability by a monopolist

As shown in Fig. 3.4, the present value of total profits to perpetuity $\pi(N)$ is concave by assumption. $\pi(N)$ is no longer a parallel upward shift of $-\kappa(N)$, the negative of the present value of the total costs to perpetuity, like in the case of perfect competition. Since the present value of total revenues to perpetuity $\gamma(N)$ decreases in $N, \pi(N)$ is maximized at a smaller $N$ than $-\kappa(N)$. In other words, with present-biased discounting, the monopolist chooses a lower level of durability compared to the one chosen by a perfectly
competitive market, i.e., $N^{m}<N^{p c}$.

The above analysis demonstrates that when consumers have time-consistent preferences, the monopolist is not able to increase the present value of total revenues to perpetuity by manipulating the level of durability. If the monopolist shortens the durability of the product, consumers will adjust their willingness to pay accordingly since what they are paying for is the stream of services delivered by the product. This results in no change in the present value of the total revenues to perpetuity. The monopolist therefore chooses the level of durability to minimize costs, which is exactly how the level of durability under perfect competition is determined. However, when consumers are present biased, they assign a bigger weight to the "present" periods, defined as the periods in which a unit of the product is purchased. By reducing the durability level therefore inducing the consumers to repeat the purchase action more frequently, the monopolist can achieve higher revenues measured by the present value, thus justifying the decision to carry out planned obsolescence by the monopolist.

### 3.4.4 A numerical example

Next, I will illustrate the reduction in the level of durability by a monopolist when consumers are present biased using a simple numerical example. Let marginal cost $c(N)$ take on the form of a power function, i.e. $c(N)=N^{a}$, where $a \in(0,1)$. As discussed in Appendix C.1, this specification of $c(N)$ on the given parameter value ranges guarantees the uniqueness of the solution to the minimization problem of $\kappa(N)$. Assign the following values to the parameters: $a=0.5, \beta=0.8, \delta=0.8, v=1$.

To find the level of durability chosen by a perfectly competitive market, i.e., the cost minimizing level of durability, take the first order condition of Equation 3.1 and obtain
the following function:

$$
a(N)=-\frac{N \delta^{N} \log \delta}{1-\delta^{N}}
$$

This function is invertible for the given parameter values (see Appendix C.1 for details). While there is no closed-form solution for $N, N$ can be solved numerically and the integer solution is $N^{p c}=6.7$

The durability choice of a monopolist when serving consumers with present-biased preferences is given by the solution to the maximization of Equation 3.2. The first order condition is:

$$
v \delta^{N} \log \delta(1-\beta-\delta+\beta \delta)-(1-\delta) N^{a-1}\left(\left(1-\delta^{N}\right) a+N \delta^{N} \log \delta\right)=0
$$

Solve for $N$ numerically and the integer solution is $N^{m}=5.8$ The durability compared to $N^{p c}$ is reduced by $16.7 \%$. The reduction increases with consumer's per period value $v$ and a corner solution of $N^{m}=1$ is reached when $v$ increases to 2.44 or higher.

It can be verified that the second order condition is negative, confirming that the solution is a local and global maximum. Fig. 3.5 illustrates the monopolist's profit level at different levels of durability.

[^26]

Figure 3.5: Profit of a monopolist at different durability levels
Parameter values: $a=0.5, \beta=0.8, \delta=0.8, v=1$.

### 3.5 Conclusions

The Phoebus cartel is remembered today not only for being the first international cartel, but for giving us one of the finest examples of a product that was "made to break". The cartel members carefully engineered a light bulb with a shorter lifespan in an attempt to improve profits. While there was not enough information to confirm whether the Phoebus cartel succeeded in it, should they have done so, the existing theories on planned obsolescence appear to be unable to explain why. The Phoebus cartel was able to adopt and enforce a strict quota system, which removed members' incentives to reduce prices in the future. With the ability to commit to future prices at high levels, the cartel did not need to reduce the product's durability to protect its pricing power. The slowdown in
technological innovation before and during the years that the cartel operated suggests that the services from new and old units of the light bulbs were perfect substitutes. Hence making the old units breakdown sooner and incentivizing consumers to buy new units will not improve overall profitability. When consumers' preferences are time-consistent, they adjust their willingness to pay accordingly with any change in the durability of products. The present value of total revenue to perpetuity is therefore independent of the monopolist's choice of durability. As a result, the monopolist is expected to choose the level of durability the same way as a perfectly competitive market - by cost minimization.

Behavioral economists have presented ample empirical evidence to show that consumers often exhibit present-biasedness when making intertemporal decision. When consumers are present biased, they assign a bigger weight to the "present" periods in which they make a purchase. This paper shows that by assuming consumers are present biased, a monopolist is able to increase revenues and the overall profitability by reducing the product's durability from the level chosen by a perfectly competitive market. This alternative theory on planned obsolescence provides a plausible explanation for the Phoebus cartel's decision to carry out planned obsolescence.

## APPENDIX A

## APPENDIX TO CHAPTER 1

## A. 1 Additional Figures



Figure A.1: Density distribution of jump bids by size I


Figure A.2: Density distribution of jump bids by size II


Figure A.3: Density distribution of jump bids by size III


Figure A.4: Density distribution of jump bids by size IV


Figure A.5: Density distribution of jump bids by size V

## APPENDIX B

## APPENDIX TO CHAPTER 2

## B. 1 Characterization of the probability of service.

## B.1.1 Probability of service conditional on queueing.

Consider a farmer of type i that requests a service from a provider j given that other farmers request services with probability $p_{i, j}$. The probability that a given farmer is being served given that the provider chooses one farmer of type i and he is queuing with this provider is $\tilde{\Delta}_{i j}(1)$, i.e.

$$
\tilde{\Delta}_{i j}(1)=\sum_{n=0}^{f_{i}-1}\binom{f_{i}-1}{n} p_{i j}^{n}\left(1-p_{i j}\right)^{f_{i}-1-n} \frac{1}{n+1}
$$

where $f_{i}$ is the number of farmers of type i searching for a provider, $f_{s}=s F$ and $f_{s^{-}}=$ $(1-s) F$ and $\binom{f_{i}-1}{n}=\frac{f_{i}-1!}{n!\left(f_{i}-1-n\right)!}$. Hence,

$$
\tilde{\Delta}_{i j}(1)=\frac{1-\left(1-p_{i j}\right)^{f_{i}}}{f_{i} p_{i j}}
$$

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$
\tilde{\Delta}_{i j}(1)=\frac{1-e^{-q_{i j}}}{q_{i j}} .
$$

That is, a given farmer of type $i$ is served if at least one farmer of type $i$ has requested a service, which occurs with probability $1-e^{-q_{i j}}$, divided by the number of requests of a
given type, $q_{i j}$.

Next, consider the probability of a given farmer being served when the provider serves $\bar{o}=2$ orders of type i, $\tilde{\Delta}_{i j}(2)$. Similar computations to those above yield a service probability as follows

$$
\tilde{\Delta}_{i j}(2)=2\left(\frac{1-e^{-q_{i j}}}{q_{i j}}\right)-e^{-q_{i j}} .
$$

Finally, consider the probability of a given farmer being served when the provider serves $\bar{o}=3$ orders of type i, $\tilde{\Delta}_{i j}(3)$, which follows

$$
\tilde{\Delta}_{i j}(3)=3\left(\frac{1-e^{-q_{i j}}}{q_{i j}}\right)-2 e^{-q_{i j}}-e^{-q_{i j}} q_{i j}
$$

In what follows we characterize the probability that a provider of type " $j$ " services a farmer of type " $i$ " given that a farmer of type " $i$ " is standing in the queue.

First-come-first-served. The fcfs provider only considers feasibility and the position in the queue. Let the probability of serving $\bar{o}$ farmer of type i be $\phi_{i, f c f s}(\bar{o})$. Given the queue lengths at this provider, there are ${ }^{q_{s}+q_{s}-P_{o}}=\frac{q_{s}+q_{s}-!}{\left(q_{s}+q_{s}--o\right)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a fcfs provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue and at least one has applied. Let this probability be $\hat{\phi}_{s, f c f s}(1) \equiv 3 q_{s} \frac{q_{s}-P_{2}}{q_{s}+q_{s}-P_{3}}$.

$$
\phi_{s, f c f s}(1)=\psi_{s, f c f s}(1) \hat{\phi}_{s, f c f s}(1), 1
$$

where $\psi_{s, f c f s}(1) \equiv\left(1-e^{-q_{s^{-}, f c f s}}-q_{s^{-}, f c f s} e^{-q_{s^{-}, f c f s}}\right)$ is the probability of having at least three orders in the queue of which at least two are of type $s^{-}$, when a sin-

[^27]gle farmer of type $s$ has requested service. To this probability we should add the probability of service when less than $o=3$ farmers apply for service, $\hat{\psi}_{s, f c f s}(1) \equiv$ $\left(e^{-q_{s, f c f s}}\left(e^{-q_{s^{-}}, f c f s}+q_{s^{-}, f c f s} e^{-q_{s^{-}}, f c f s}\right)\right)$ which is the probability of service of large scale order when there are no other service request or there is exactly one additional order requested.

A fcfs provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have applied . Let the first probability be $\hat{\phi}_{s, f c f s}(2) \equiv 3 q_{s}\left(q_{s}-1\right) \frac{q_{s}-2+q_{s}-P_{1}}{q_{s}+q_{s}-P_{3}}$

$$
\phi_{s, f c f s}(2)=\psi_{s, f c f s}(2) \hat{\phi}_{s, f c f s}(2),
$$

where $\psi_{s, f c f s}(2)=\left(1-e^{-q_{s, f c f s}}-e^{-q_{s},, f c f s} q_{s} e^{-q_{s, f c f s}}\right)$ is the probability that there are at least three orders in the queue conditional of a farmer of type $s$ requesting service, of which at least two are of type $s$ (including the one requesting service). ${ }^{2}$ To this probability we should add the probability that there are only two large-scale farmers in the queue $\hat{\psi}_{s, f c f s}(2) \equiv\left(q_{s, f c f s} e^{-q_{s, f c f s}} e^{-q_{s}-, f c f s}\right)$.

Given feasibility, the fcfs provider never serves 3 large-scale orders, $\phi_{s, f c f s}(3)=0$.

A fcfs provider serves a single small-scale farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders, exchanging indexes,

$$
\phi_{s^{-}, f c f s}(1)=\psi_{s^{-}, f c f s}(1) \hat{\phi}_{s^{-}, f c f s}(1),
$$

and adding the probability $\hat{\psi}_{s^{-}, f c f s}(1)$ when there are less than three orders.

[^28]A fcfs provider services 2 small-scale farmers if at least two small-scale orders in the first $o$ positions of the queue, $\hat{\phi}_{s^{-}, f c f s}(2) \equiv 3 \frac{q_{s-}\left(q_{s-}-1\right) q_{s}}{q_{s}+q_{s}-P_{3}}$

$$
\phi_{s^{-}, f c f s}(2)=\psi_{s^{-}, f c f s}(2) \hat{\phi}_{s^{-}, f c f s}(2),
$$

where $\psi_{s^{-}, f c f s}(2)=\left(1-e^{-q_{s, f c f s}}\right)\left(1-e^{-q_{s^{-}, f c f s}}\right)$ is the probability that there are at least three orders in the queue conditional of a farmer of type $s^{-}$requesting service, of which at least one is of type $s$ and at least two are of type $s^{-}$(including the one requesting the service). To this probability we should add the probability that there are only two orders in the queue, $\hat{\psi}_{s^{-}, f c f s}(2)$ defined analogously than for large-scale farmers.

A fcfs provider services 3 small-scale farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^{-}, f c f s}(3)=\frac{q_{s}-P_{3}}{q_{s}+q_{s}-P_{3}}$

$$
\phi_{s^{-}, f c f s}(3)=\psi_{s^{-}, f c f s}(3) \hat{\phi}_{s^{-}, f c f s}(3),
$$

where $\psi_{s^{-}, f c f s}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^{-}, f c f s}(3)=\left(1-e^{-q_{s^{-}, f c f s}}-q_{s^{-}, f c f s} e^{-q_{s^{-}, f c f s}}\right)$.

The general form for the probability of service is,

$$
\begin{equation*}
\Delta_{i, f c f s}=\sum_{\bar{o}=1}^{3} \hat{\psi}_{i, f c f s}(\bar{o})+\phi_{i, f c f s}(\bar{o}) \tilde{\Delta}_{i, f c f s}(\bar{o}), \tag{B.1}
\end{equation*}
$$

where we have defined $\hat{\psi}_{s, f c f s}(3) \equiv 0$ to ease notation.

The main difference in the probability of service for large and small relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^{3} \phi_{s^{-}, f c f s}(\bar{o})=\sum_{\bar{o}=2}^{3} \phi_{s, f c f s}(\bar{o})$.

Market. The market provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly $\bar{o}$ farmers of type i are served conditional on the farmer under consideration having applied and at least 3 farmers of either type requesting service to the provider.

Conditional on a large farmer having applied, a single large-scale farmer is served by a market provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first $o$ available positions, which happens with probability $(1-\chi) \tilde{\phi}_{s, m k t}(1)=(1-\chi) \phi_{s, f c f s}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi \psi_{s, m k t}(1)=\chi e^{-q_{s, m k t}}\left(1-e^{-q_{s^{-}, m k t}}-\right.$ $\left.q_{s^{-}, m k t} e^{-q_{s^{-}, m k t}}\right)$. These service probabilities add up to,

$$
\phi_{s, m k t}(1)=\chi\left(\psi_{s, m k t}(1)\right)+(1-\chi) \tilde{\phi}_{s, m k t}(1)
$$

where we should the events when there are less than three orders $\hat{\psi}_{i, m k t}(1)=\hat{\psi}_{i, f c f s}(1)$ for any $\mathrm{i}=s, s^{-}$by definition.

Two large-scale farmers are served by a market provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability $(1-\chi) \tilde{\phi}_{s, m k t}(2)=(1-\chi) \phi_{s, f c f s}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\left.\chi \psi_{s, m k t}(2)=\chi\left(1-e^{-q_{s, m k t}}-e^{-q_{s-, m k t}} q_{s} e^{-q_{s, m k t}}\right) \cdot\right]^{3}$ These service probabilities add up to

$$
\phi_{s, m k t}(2)=\chi\left(\psi_{s, m k t}(2)\right)+(1-\chi) \tilde{\phi}_{s, m k t}(2)
$$

to this probability we add $\hat{\psi}_{s, m k t}(2)$ defined analogously than for the fcfs provider.

Feasibility prevents three large-scale orders to be served within the period and there-

[^29]fore, $\phi_{s, m k t}(3)=0$.

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-scale farmer is always served by a market provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi \psi_{s^{-}, m k t}(1)=\chi\left(1-e^{-q_{s, m k t}}-\right.$ $q_{s, m k t} e^{-q_{s, m k t}}$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi) \tilde{\phi}_{s^{-}, m k t}(1)$, where $\tilde{\phi}_{s^{-}, m k t}(1)=\phi_{s^{-}, f c f s}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to served at most $o-1$ orders leaving always and idle slot. To these probabilities we add those associated to the event when there are strictly less than two orders in the queue.

$$
\phi_{s^{-}, m k t}(1)=\chi\left(\psi_{s^{-}, m k t}(1)\right)+(1-\chi) \tilde{\phi}_{s^{-}, m k t}(1),
$$

to what we add $\hat{\psi}_{s^{-}, m k t}(1)$.

Two small-scale farmers are served by a market provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi \psi_{s^{-}, m k t}(2)=\chi q_{s, m k t} e^{-q_{s, m k t}}(1-$ $\left.e^{-q_{s^{-}, m k t}}\right)$. Alternatively, two small-scale farmers are served if the provider does not prioritize large-scale orders and there are two small-scale orders among the first three orders in the queue.

$$
\phi_{s^{-}, m k t}(2)=\chi\left(\psi_{s^{-}, m k t}(2)\right)+(1-\chi) \tilde{\phi}_{s^{-}, m k t}(2) .
$$

To these probabilities we add those associated to the event when there are strictly less than two orders in the queue, $\hat{\psi}_{s^{-}, m k t}(2)$.

Three small-scale farmers are served by the market provider if it prioritizes high-
scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi \psi_{s^{-}, m k t}(3)=\chi e^{-q_{s, m k t}}\left(1-e^{-q_{s^{-}, m k t}}-\right.$ $\left.q_{s^{-}, m k t} e^{-q_{s^{-}, m k t}}\right)$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$
\phi_{s^{-}, m k t}(3)=\chi \psi_{s^{-}, m k t}(3)+(1-\chi) \tilde{\phi}_{s^{-}, m k t}(3) .
$$

In sum, the probability of service for a market provider follows

$$
\begin{equation*}
\Delta_{i, m k t}=\sum_{\bar{o}=1}^{3} \hat{\psi}_{i, m k t}(\bar{o})+\phi_{i, m k t}(\bar{o}) \tilde{\Delta}_{i, m k t}(\bar{o}) \tag{B.2}
\end{equation*}
$$

Following equations B.1 and B.2, the probability of being served for a given type $i$ (weakly) declines in the queue length of the other type of farmers. In the first-come-firstserved provider the result is straightforward. For the market provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

## B.1.2 Unconditional probabilities of service.

The unconditional probabilities of service are important in characterizing the value of service for each provider. We consider alternative scenarios, i.e. when the provider serves at capacity ( $o=3$ orders) and when the provider serves less than capacity.

The probability of serving at least an order is

$$
\hat{\Phi}=\left(1-e^{-q_{s j}}\right)+\left(1-e^{-q_{s}-j}\right)
$$

The first-come-first-served provider can serve three orders of small scale (given feasibility) with a service probability of

$$
\Phi_{s^{-}, f c f s}(3)=\left(1-e^{-q_{s^{-}, j}}\left(1+q_{s^{-}, j}+\frac{1}{2} q_{s^{-}, j}^{2}\right)\right) \frac{q_{s}-, j P_{3}}{q_{s, j}+q_{s^{-}, j} P_{3}}
$$

or to serve two orders of one type and one of another, with probability

$$
\bar{\Phi}_{i, f c f s}(1)=\left(1-e^{-q_{i, j}}\right)\left(1-e^{-q_{i^{\prime}, j}}-q_{i^{\prime}, j} e^{-q_{i^{\prime}, j}}\right) \frac{3 q_{i, j} q_{i^{\prime}, j}\left(q_{i^{\prime}, j}-1\right)}{q_{i, j}+q_{i^{\prime}, j} P_{3}},
$$

and $\bar{\Phi}_{i^{\prime}, f c f s}(2)=\bar{\Phi}_{i, f c f s}(1)$ for $i^{\prime} \neq i$.

The provider can also serve two orders of large size, (either because he received only two orders, or because all orders in the queue are of large scale)

$$
\tilde{\Phi}_{s, f c f s}(2)=\left(1-e^{-q_{s, f c f s}}-q_{s, f c f s} e^{-q_{s, f c f s}}\right) e^{-q_{s-, f c f s}},
$$

or it can receive exactly two orders of small size and serve those,

$$
\tilde{\Phi}_{s^{-}, f c f s}(2)=\frac{1}{2} q_{s^{-}, j}^{2} e^{-q_{s^{-}, j}}\left(e^{-q_{s, j}}\right) .
$$

Finally, the provider can serve two orders, one of each type

$$
\tilde{\Phi}_{i, f c f s}\left(1_{2}\right)=\left(q_{i, j} e^{-q_{i, j}} q_{i^{\prime}, j} e^{-q_{i^{\prime}, j}}\right) .
$$

or only one order, with occurs with probability

$$
\tilde{\Phi}_{i, f c f s}\left(1_{1}\right)=\left(q_{i, j} e^{-q_{i, j}} e^{-q_{i^{\prime}, j}}\right)
$$

The probabilities for the market provider are similar to the ones above, except that we need to account for the market provider's ability to select large scale orders.

The market provider can serve three orders of small scale,
$\bar{\Phi}_{s^{-}, m k t}(3)=\chi e^{-q_{s, m k t}}\left(1-e^{-q_{s^{-}, m k t}}-q_{s^{-}, m k t} e^{-q_{s^{-}, m k t}}-\frac{1}{2} q_{s^{-}, m k t}^{2} e^{-q_{s^{-}, m k t}}\right)+(1-\chi) \Phi_{s^{-}, f c f s}(3)$,
or the orders of large scale and one small,

$$
\begin{equation*}
\bar{\Phi}_{s, m k t}(2)=\chi\left(\left(1-e^{-q_{s, m k t}}-q_{s, m k t} e^{-q_{s, m k t}}\right)\left(1-e^{-q_{s}-, m k t}\right)+(1-\chi) \bar{\Phi}_{s, f c f s}(2),\right. \tag{B.4}
\end{equation*}
$$

or two orders of small scale and on large,

$$
\begin{equation*}
\bar{\Phi}_{s^{-}, m k t}(2)=\chi q_{s, m k t} e^{-q_{s, m k t}}\left(1-e^{-q_{s^{-}}, m k t}-q_{s^{-}, m k t} e^{-q_{s^{-}, m k t}}\right)+(1-\chi) \bar{\Phi}_{s^{-}, f c f s}(2) \tag{B.5}
\end{equation*}
$$

When there are less than three orders in the queue there is no need to prioritize orders, and therefore the probabilities of service are identical to those characterized for the FCFS problem, i.e. $\tilde{\Phi}_{i, m k t}=\tilde{\Phi}_{i, f c f s}$.

Expected value of service provision The characterization of the probability allows
us to compute the expected value of service provision:

$$
\begin{array}{r}
\tilde{V}\left(\left\{\bar{o}_{s}, \bar{o}_{s^{-}}\right\}_{\mathbf{q}_{\mathrm{fcfs}}},\left\{\left(r_{i, \text { fcfs }}-w\right) k_{i}-w E\left(d_{i}\right)\right\}_{i=s, s^{-}}\right) \equiv \\
\hat{\Phi}^{-1}\left\{\sum_{i=s, s^{-}} \bar{\Phi}_{i, f c f s}(2)\left[2\left(\left(r_{i, f c f s}-w\right) k_{i}-w E\left(d_{i}\right)\right)+\left(r_{i^{\prime}, f c f s}-w\right) k_{i^{\prime}}-w E\left(d_{i^{\prime}}\right)\right]+\right. \\
\Phi_{s^{-}, f c f s}(3) 3\left(\left(r_{s^{-}, f c f s}-w\right) k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)+ \\
\tilde{\Phi}_{i, f c f s}\left(1_{1}\right)\left(\left(r_{i, f c f s}-w\right) k_{i}-w E\left(d_{i}\right)\right)+ \\
+\tilde{\Phi}_{i, f c f s}\left(1_{2}\right)\left(\left(r_{i, f c f s}-w\right) k_{i}-w E\left(d_{i}\right)+\left(r_{i^{\prime}, f c f s}-w\right) k_{i^{\prime}}-w E\left(d_{i^{\prime}}\right)\right)+ \\
\left.\tilde{\Phi}_{i, f c f s}(2) 2\left(\left(r_{i, f c f s}-w\right) k_{i}-w E\left(d_{i}\right)\right)\right\} \tag{B.6}
\end{array}
$$

$$
\tilde{V}\left(\left\{\bar{o}_{s}, \bar{o}_{s^{-}}\right\}_{\left(\mathbf{q}_{\mathrm{mkt}}, \chi\right)},\left\{\left(r_{i, \mathrm{mkt}}-w\right) k_{i}-w E\left(d_{i}\right)\right\}_{i=s, s^{-}}\right) \equiv
$$

$$
\hat{\Phi}^{-1}\left\{\sum_{i=s, s^{-}} \bar{\Phi}_{i, m k t}(2)\left[2\left(\left(r_{i, m k t}-w\right) k_{i}-w E\left(d_{i}\right)\right)+\left(r_{i^{\prime}, m k t}-w\right) k_{i^{\prime}}-w E\left(d_{i^{\prime}}\right)\right]+\right.
$$

$$
\Phi_{s^{-}, m k t}(3) 3\left(\left(r_{s^{-}, m k t}-w\right) k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)+
$$

$$
\tilde{\Phi}_{i, m k t}\left(1_{1}\right)\left(\left(r_{i, m k t}-w\right) k_{i}-w E\left(d_{i}\right)\right)+
$$

$$
+\tilde{\Phi}_{i, m k t}\left(1_{2}\right)\left(\left(r_{i, m k t}-w\right) k_{i}-w E\left(d_{i}\right)+\left(r_{i^{\prime}, f c f s}-w\right) k_{i^{\prime}}-w E\left(d_{i^{\prime}}\right)\right)+
$$

$$
\begin{equation*}
\left.\tilde{\Phi}_{i, m k t}(2) 2\left(\left(r_{i, m k t}-w\right) k_{i}-w E\left(d_{i}\right)\right)\right\} . \tag{B.7}
\end{equation*}
$$

where the first two terms in either expression correspond to the expected value of serving three orders or different types, while the remaining terms correspond to the expected returns of serving strictly less than three orders. Also note that the scaler $\hat{\Phi}^{-1}$ is inconsequential to the optimality conditions of the problem (and therefore omitted in the derivations that follow).

## B. 2 Proofs

## B.2.1 Proposition 1

First, we solve for the equilibrium value of service when both farmers queue with both providers. Then, we show the value when the service provider serves a single type of farmers. Then we show that the guess that the selection criteria for the market provider should be to prioritize large scale orders. Finally, we show that the expected value of service is higher for large scale farmers.

## Value of service when serving both type of farmers.

Proof. Using the definition of the expected value of service provision (equations B. 6 and B.7) and rearranging terms, as well as the participation constraint for the farmers, equation 2.4 , the problem of the provider is

$$
\begin{aligned}
& \max _{q_{s, j}, q_{s},, j} \sum_{i=s, s^{-}} \Phi_{i, j}(2)\left[z k_{i}^{\alpha}-\frac{U_{i}}{\Delta_{i j}}-w\left(k_{i}+E\left(d_{i}\right)\right)\right]+ \\
& \sum_{i=s, s^{-}} \Phi_{i, j}(1)\left[z k_{i^{\prime}}^{\alpha}-\frac{U_{i^{\prime}}}{\Delta_{i^{\prime} j}}-w\left(k_{i^{\prime}}+E\left(d_{i^{\prime}}\right)\right)\right]+ \\
& \Phi_{s^{-}, j}(3) 3\left[z k_{s^{-}}^{\alpha}-\frac{U_{s^{-}}}{\Delta_{s^{-} j}}-w\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)\right]
\end{aligned}
$$

where $\Phi_{i, j}(2)=\left(\bar{\Phi}_{i, j}(2) 2+\tilde{\Phi}_{i, j}(2) 2+\tilde{\Phi}_{i, j}\left(1_{1}\right)+\tilde{\Phi}_{i, j}\left(1_{2}\right)\right)$ and $\Phi_{i, j}(1)=\left(\bar{\Phi}_{i, j}(2)+\tilde{\Phi}_{i, j}\left(1_{2}\right)\right)$.
Let $\bar{V}_{i j}$ be the profit per order of type $i$ for provider $j$, i.e. $\bar{V}_{i j} \equiv$ $\left(\left(r_{i, j}-w\right) k_{i}-w E\left(d_{i}\right)\right)$. The optimality condition swith respect to the queue length of
large-scale and small-scale farmers are

$$
\begin{gather*}
\sum_{i=s, s^{-}}\left(\frac{\partial \Phi_{i, j}(2)}{\partial q_{s, j}} \bar{V}_{i}+\frac{\partial \Phi_{i, j}(1)}{\partial q_{s, j}} \bar{V}_{i^{\prime}}\right)+3 \frac{\partial \Phi_{s^{-}, j}(3)}{\partial q_{s, j}} \bar{V}_{s^{-}}+ \\
\sum_{i=s, s^{-}}\left(\Phi_{i, j}(2) \frac{\partial \bar{V}_{i}}{\partial q_{s, j}}+\Phi_{i, j}(1) \frac{\partial \bar{V}_{i^{\prime}}}{\partial q_{s, j}}\right)+\Phi_{s^{-}, j}(3) 3 \frac{\partial \bar{V}_{s^{-}}}{\partial q_{s, j}}=0  \tag{B.8}\\
\sum_{i=s, s^{-}}\left(\frac{\partial \Phi_{i, j}(2)}{\partial q_{s^{-}, j}} \bar{V}_{i}+\frac{\partial \Phi_{i, j}(1)}{\partial q_{s^{-}, j}} \bar{V}_{i^{\prime}}\right)+3 \frac{\partial \Phi_{s^{-}, j}(3)}{\partial q_{s^{-}, j}} \bar{V}_{s^{-}}+ \\
\sum_{i=s, s^{-}}\left(\Phi_{i, j}(2) \frac{\partial \bar{V}_{i}}{\partial q_{s^{-}, j}}+\Phi_{i, j}(1) \frac{\partial \bar{V}_{i^{\prime}}}{\partial q_{s^{-}, j}}\right)+\Phi_{s^{-}, j}(3) 3 \frac{\partial \bar{V}_{s^{-}}}{\partial q_{s^{-}, j}}=0, \tag{B.9}
\end{gather*}
$$

where

$$
\frac{\partial \bar{V}_{i}}{\partial q_{i, j}}=-\left[\frac{\bar{V}_{i}+w\left(k_{i}+E\left(d_{i}\right)\right)-z k_{i}^{\alpha}}{\Delta_{i j}}\right]\left(\frac{\partial \Delta_{i, j}}{\partial q_{i j}}\right) \cdot[]
$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q \Delta}(o) \equiv$ $-\frac{\partial \Delta_{s j}}{\partial q} \frac{q_{s j}}{\Delta_{s j}(o)}$, let the elasticity of the value of service to the queue length be $\zeta_{\bar{V} q} \equiv \frac{\partial \bar{V}}{\partial q_{i j}} \frac{q_{i j}}{V}$ and that of the probability of arrival of $o$ orders to the queue length be $\zeta_{q \psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$.

Then, the envelope condition indicates that the elasticity of the value of service to the queue length is an inversely proportional function of the elasticity of the probability of service to the queue length, $\zeta_{\Delta i j}$.

$$
\zeta_{\bar{V}_{i}}=\left(1+\frac{w\left(k_{i}+E\left(d_{i}\right)\right)-z k_{i}^{\alpha}}{\bar{V}_{i}}\right) \zeta_{\Delta i j} .
$$

Equations B. 8 and B. 9 form a system of linear equations that can be solved for the

[^30]two unknowns $\bar{V}_{s^{-}}, \bar{V}_{s}$ as a function of the queue lengths. $5^{5}$
\[

\Gamma\left[$$
\begin{array}{c}
\bar{V}_{s} \\
\bar{V}_{s^{-}}
\end{array}
$$\right]=a\left[$$
\begin{array}{c}
z k_{s}^{\alpha}-w\left(k_{s}+E\left(d_{s}\right)\right) \\
z k_{s^{-}}^{\alpha}-w\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)
\end{array}
$$\right]
\]

where $\Gamma=\left[\begin{array}{l}\Gamma_{1} \Gamma_{2} \\ \Gamma_{3} \Gamma_{4}\end{array}\right]$, for

$$
\begin{gathered}
\Gamma_{1}=\frac{\partial \Phi_{s, j}(2)}{\partial q_{s, j}}+\frac{\partial \Phi_{s^{-}, j}(1)}{\partial q_{s, j}}+\zeta_{\Delta_{s} q_{s}}\left(\frac{\Phi_{s, j}(2)}{q_{s, j}}+\frac{\Phi_{s^{-}, j}(1)}{q_{s, j}}\right) \\
\Gamma_{2}=\frac{\partial \Phi_{s^{-}, j}(2)}{\partial q_{s, j}}+\frac{\partial \Phi_{s, j}(1)}{\partial q_{s, j}}+3 \frac{\partial \Phi_{s^{-}, j}(3)}{\partial q_{s, j}}+ \\
\zeta_{\Delta_{s-} q_{s}}\left(\frac{\Phi_{s, j}(1)}{q_{s, j}}+\frac{\Phi_{s^{-}, j}(2)}{q_{s, j}}+3 \frac{\Phi_{s^{-}, j}(3)}{q_{s, j}}\right) \\
\Gamma_{3}=\frac{\partial \Phi_{s, j}(2)}{\partial q_{s^{-}, j}}+\frac{\partial \Phi_{s^{-}, j}(1)}{\partial q_{s^{-}, j}}+\zeta_{\Delta_{s} q_{s-}}\left(\frac{\Phi_{s^{-}, j}(1)}{q_{s^{-}, j}}+\frac{\Phi_{s, j}(2)}{q_{s^{-}, j}}\right)
\end{gathered}
$$

$$
\Gamma_{4}=\frac{\partial \Phi_{s^{-}, j}(2)}{\partial q_{s^{-}, j}}+\frac{\partial \Phi_{s, j}(1)}{\partial q_{s^{-}, j}}+3 \frac{\partial \Phi_{s^{-}, j}(3)}{\partial q_{s^{-}, j}}+
$$

$$
\zeta_{\Delta_{s-} q_{s-}}\left(\frac{\Phi_{s, j}(1)}{q_{s^{-}, j}}+\frac{\Phi_{s^{-}, j}(2)}{q_{s^{-}, j}}+3 \frac{\Phi_{s^{-}, j}(3)}{q_{s^{-}, j}}\right)
$$

[^31]and in the LHS
\[

a=\left[$$
\begin{array}{cc}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}
$$\right]=\left[$$
\begin{array}{ll}
\zeta_{\Delta_{s} q_{s}}\left(\frac{\Phi_{s, j}(2)}{q_{s, j}}+\frac{\Phi_{s^{-}, j}(1)}{q_{s, j}}\right) & \zeta_{\Delta_{s-} q_{s}}\left(\frac{\Phi_{s, j}(1)}{q_{s, j}}+\frac{\Phi_{s-, j}(2)}{q_{s, j}}+3 \frac{\Phi_{s-, j}(3)}{q_{s, j}}\right) \\
\zeta_{\Delta_{s} q_{s-}}\left(\frac{\Phi_{s^{-}, j}(1)}{q_{s^{-}, j}}+\frac{\Phi_{s, j}(2)}{q_{s^{-}, j}}\right) & \zeta_{\Delta_{s-}-q_{s-}}\left(\frac{\Phi_{s, j}(1)}{q_{s^{-}, j}}+\frac{\Phi_{s^{-}, j}(2)}{q_{s^{-}, j}}+3 \frac{\Phi_{s^{-, j}}(3)}{q_{s^{-}, j}}\right)
\end{array}
$$\right]
\]

The last vector on the LHS corresponds to the surplus from trade for each farmer type.

Standard matrix algebra implies that expected value to the providers satisfies

$$
\begin{equation*}
\bar{V}_{s}^{j}=\gamma_{1 s}^{j}\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)+\gamma_{2 s}^{j}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right) \tag{B.10}
\end{equation*}
$$

$$
\begin{equation*}
\bar{V}_{s^{-}}^{j}=\gamma_{1 s^{-}}^{j}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-E\left(d_{s^{-}}\right)\right)+\gamma_{2 s^{-}}^{j}\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right) \tag{B.11}
\end{equation*}
$$

for $\gamma_{1 s^{-}}^{j}=\frac{\Gamma_{1} a_{22}-a_{12} \Gamma_{3}}{\Gamma_{1} \Gamma_{4}-\Gamma_{2} \Gamma_{3}}$ and $\gamma_{2, s^{-}}^{j}=\frac{a_{21} \Gamma_{1}-a_{11} \Gamma_{3}}{\Gamma_{1} \Gamma_{4}-\Gamma_{2} \Gamma_{3}}$ while $\gamma_{1, s}^{j}=\frac{a_{11} \Gamma_{4}-\Gamma_{2} a_{21}}{\Gamma_{1} \Gamma_{4}-\Gamma_{2} \Gamma_{3}}$ and $\gamma_{2 s}^{j}=\frac{a_{12} \Gamma_{4}-\Gamma_{2} a_{22}}{\Gamma_{1} \Gamma_{4}-\Gamma_{2} \Gamma_{3}}$. Notice that the denominator of each of the $\gamma$ parameters shifts depending on the provider as a function of the probability of service. This heterogeneity changes the value for the derivatives in $\Gamma$.

## Value of service when serving only large scale farmers.

If a provider $\boldsymbol{j}$ attracts only large-scale farmers, i.e. $q_{s^{-} j}=0$, then the expected per period profit of the provider satisfies

$$
\bar{V}_{s}=\gamma\left(q_{s j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right)\left(z k_{s}^{\alpha}-w\left(k_{s}+E\left(d_{s}\right)\right)\right)
$$

where the second term corresponds to the surplus associated to the transaction and $\gamma \in$ $(0,1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, $\zeta$, and the share of capital in farming production.

Proof. The problem of the supplier when it only receives large scale orders is

$$
\max _{q_{s j}, r_{s j}} \psi \bar{V}_{s}
$$

subject to

$$
\begin{gathered}
\tilde{\Delta}_{s j} \pi_{s}\left(r_{s j}, k_{s}\right) \geq U_{s} \\
\sum_{i \in \hat{q}_{j}} k_{s}(i)+E\left(d_{s}(i)\right) \leq \bar{k}_{j}
\end{gathered}
$$

where $\psi=2\left(1-e^{-q_{s, \text { mkt }}}\left(1+q_{s, \mathrm{mkt}}\right)\right)+e^{-q_{s, \mathrm{mkt}}} q_{s, \mathrm{mkt}}$ because there are no small-scale orders and either the supplier serves one or two orders of large scale.

Using the definition of profits to the farmers, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$
\max _{q_{s j}} \psi\left[z k_{s}^{\alpha}-\frac{U_{s}}{\Delta_{s j}}-w\left(k_{s}+E\left(d_{s}\right)\right)\right]
$$

Note that the properties of the probabilities $\psi$ and $\Delta$ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$
\begin{equation*}
\frac{\partial \psi}{\partial q} \bar{V}_{s}-\psi\left[\frac{\bar{V}_{s}+w\left(k_{s}+E\left(d_{s}\right)\right)-z k_{s}^{\alpha}}{\tilde{\Delta}_{s j}(2)}\right] \frac{\partial \tilde{\Delta}_{s j}(2)}{\partial q}=0 . \tag{B.12}
\end{equation*}
$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q \Delta}(o) \equiv-\frac{\partial \Delta_{s j}}{\partial q} \frac{q_{s j}}{\Delta_{s j}(o)}$ and let the elasticity of the probability of arrival of $o$ orders to the
queue length be $\zeta_{q \psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. Finally, let $\gamma\left(q_{s, j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right) \equiv \frac{\zeta_{q \Delta}}{\zeta_{q \psi}+\zeta_{q \Delta}}$,

$$
\begin{equation*}
\bar{V}_{s}=\gamma\left(q_{s j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right)\left(z k_{s}^{\alpha}-w\left(k_{s}+E\left(d_{s}\right)\right)\right) \tag{B.13}
\end{equation*}
$$

which proves the result. For the value to be positive we require $\gamma>0$ which is true by construction. The provider takes a fraction of the surplus from the transaction.

If a provider $\boldsymbol{j}$ attracts only small-scale farmers, i.e. $q_{s, j}=0$, then the expected per period profit of the provider satisfies

$$
\bar{V}_{s^{-}}=\gamma\left(q_{s^{-}}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right)\left(z k_{s^{-}}^{\alpha}-w\left(k_{s^{-}}+E\left(d_{s^{-}}\right)\right)\right)
$$

where $\gamma \in(0,1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, $\zeta$, and the share of capital in farming production.

The derivations when the provider serves only small-scale providers follow the same steps as the ones above, so we omit them for brevity.

The market provider wants to prioritize large scale orders: Compute $\frac{\partial \Pi_{\text {mkt }}}{\partial \chi}$, which are strictly positive, given the definition for the unconditional probabilities of service, B.3 to B.5, and the value of the provider, equations 2.8 and B.7. Then, the optimal selection rule is at the corner, $\chi=1$.

Expected profits to the farmers The expected profits to the farmers depend on the equilibrium being realized, i.e. whether providers serve both type of farmers or providers specialize in a single type. The reason is that the expected profits to the farmer depend
on the cost of service, which can be in turn expressed as a function of the value of service using the definition of the value per period, $z k_{i}^{\alpha}-\frac{U_{i}}{\Delta_{i j}}-w\left(k_{i}+E\left(d_{i}\right)\right)=\bar{V}_{i}^{j}$

$$
\begin{equation*}
U_{i}=\left(z k_{i}^{\alpha}-w\left(k_{i}+E\left(d_{i}\right)\right)-\bar{V}_{i}^{j}\right) \Delta_{i j} \tag{B.14}
\end{equation*}
$$

Replacing the values of expected profits for the providers we obtain

1. If providers serve both type of farms,

$$
\begin{gathered}
U_{s}=\Delta_{s j}\left(\left(1-\gamma_{1 s}^{j}\right)\left(z k_{s}^{\alpha}-w k_{s}+w E\left(d_{s}\right)\right)-\gamma_{2 s}^{j}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}+w E\left(d_{s^{-}}\right)\right)\right) \\
U_{s^{-}}=\Delta_{s^{-}}\left(\left(1-\gamma_{1 s^{-}}^{j}\right)\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}+w E\left(d_{s^{-}}\right)\right)-\gamma_{2 s^{-}}^{j}\left(z k_{s}^{\alpha}-w k_{s}+w E\left(d_{s}\right)\right)\right)
\end{gathered}
$$

2. If a provider serves only large scale farmers,

$$
U_{s}=\tilde{\Delta}_{s j}\left(1-\gamma\left(q_{s j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right)\right)\left(z k_{s}^{\alpha}-w k_{s}+w E\left(d_{s}\right)\right)
$$

3. If a provider serves only small scale farmers,

$$
U_{s^{-}}=\tilde{\Delta}_{s^{-} j}\left(1-\gamma\left(q_{s^{-}}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha\right)\right)\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}+w E\left(d_{s^{-}}\right)\right)
$$

When the providers specialize in service provision, they determine the expected profits to the farmer.

Equilibrium queue lengths. In an equilibrium where farmers reach out to both providers, they should be indifferent across them and the feasibility constraints of the economy should be satisfied $\left[\begin{array}{l}6 \\ \text { We describe the indifference condition for large scale farm- }\end{array}\right.$

[^32]ers, the ones for small scale farmers are analogous.
$$
\frac{\Delta_{s \mathrm{mkt}}}{\Delta_{s \mathrm{fcfs}}}=\frac{\left(1-\gamma_{1 s}^{\mathrm{fcfs}}\right)\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)-\gamma_{2 s}^{\mathrm{fcfs}}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)}{\left(1-\gamma_{1 s}^{\mathrm{mkt}}\right)\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)-\gamma_{2 s}^{\mathrm{mkt}}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)}
$$

When small scale farmers queue only with fcfs providers, the indifference condition for the large farmer is

$$
\frac{\Delta_{s \mathrm{mkt}}}{\Delta_{s \mathrm{fffs}}}=\frac{\left(1-\gamma_{1 s}^{\mathrm{fcfs}}\right)\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)-\gamma_{2 s}^{\mathrm{fcfs}}\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)}{\left(1-\gamma_{s}^{\mathrm{mkt}}\right)\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)}
$$

These indifference conditions jointly with the feasibility constraints of the economy, equations 2.1 and 2.2. yield the optimal queue lengths by provider and type.

Rental rates The rental rates can be computed from the definition of $U$ once the optimal queues have been solved for.

Value of service for farmers $U_{s} \geq U_{s^{-}}$whenever the different in the surplus of service for large-scale providers is large enough. Because in equilibrium the market value of service is the same irrespective of the provider, it is w.l.o.g. to use the values from the fcfs providers.

$$
\begin{array}{r}
U_{s}-U_{s^{-}} \geq 0 \\
\left(\Delta_{s j}\left(1-\gamma_{1 s}^{j}\right)+\Delta_{s^{-} j} \gamma_{2 s^{-}}^{j}\right)\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)- \\
\left.\left(\Delta_{s j} \gamma_{2 s}^{j}+\Delta_{s^{-} j}\left(1-\gamma_{1 s^{-}}^{j}\right)\right)\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)\right) \geq 0
\end{array}
$$

If the surplus is weakly higher for large scale farmers, $z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right) \geq z k_{s^{-}}^{\alpha}-$ feasibility only, which in turn determines the expected value for farmers.
$w k_{s^{-}}-w E\left(d_{s^{-}}\right)$, then it is sufficient that

$$
\begin{equation*}
\frac{\left(\Delta_{s j}\left(1-\gamma_{1 s}^{j}\right)+\Delta_{s^{-}} \gamma_{2 s^{-}}^{j}\right)}{\left(\Delta_{s j} \gamma_{2 s}^{j}+\Delta_{s^{-} j}\left(1-\gamma_{1 s^{-}}^{j}\right)\right)} \geq \frac{\left(z k_{s^{-}}^{\alpha}-w k_{s^{-}}-w E\left(d_{s^{-}}\right)\right)}{\left(z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)\right)} \tag{B.15}
\end{equation*}
$$

The above is a condition on the elasticities of the probability of service to the queue lengths (in $\gamma$ ) relative to the values of the surplus.

Value of service for providers The value of serving large farmers is higher than the value of serving small-farmers for any provider For $\bar{V}_{s}>\bar{V}_{s^{-}}$it is sufficient that $\left(\gamma_{1 s}-\gamma_{2 s^{-}}\right)>0$ and $\left(\gamma_{2 s}-\gamma_{1 s^{-}}\right)>0$, see equations B. 10 and. This is the same as

$$
\begin{aligned}
& a_{11}\left(\Gamma_{4}+\Gamma_{3}\right)-a_{21}\left(\Gamma_{2}+\Gamma_{1}\right) \\
& a_{12}\left(\Gamma_{4}+\Gamma_{3}\right)-a_{22}\left(\Gamma_{2}+\Gamma_{1}\right)
\end{aligned}
$$

If the queue lengths are the same $\Gamma_{4}+\Gamma_{3}=\Gamma_{2}+\Gamma_{1}$ and also $a_{11}>a_{21}$ and $a_{12}>a_{22}$ because the elasticity of the probability of service to the queue large farmers is higher than for the queue of small farmers. By continuity, if the queue lengths are not too different the above result holds. Intuitively, the reason is that if there are no systematic differences in travel time across farmers, then the provider's marginal cost of provision is higher for the smaller farmers and therefore the provider finds them less valuable.

## B. 3 Numerical Solution and Output

## B.3.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order $i$ in the queue comprises two dimensions: $h_{i}$, the number of hours demanded discretized to 6 bins, and $d_{i}$ the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3 , the value function is a mapping from $R^{6}$ to $R^{1}$.

$$
V\left(\left\{\left(k_{1}, \nu_{1}\right),\left(k_{2}, \nu_{2}\right),\left(k_{3}, \nu_{3}\right)\right\}\right): R^{6} \rightarrow[0, \infty]
$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see Judd et al. (2014) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2 . Integration is carried out using Newton-Cotes quadrature.

## B.3.2 Simulations

We simulate the expected wait time and productivity cost under the fcfs arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a $\log$ normal distribution where the
parameters are obtained by fitting the actual productivity information to a $\log$ normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the fcfs arrangement and the market arrangement respectively. In any period if one or more out of these three orders are not served, the queue is filled going through the bootstrap sample. To avoid having a large order "jamming" the queue, we assume every order is feasible. This implies that the maximum wait time under the fcfs arrangement is 3 . We cap the wait time at 5 for the market arrangement. The productivity cost is then calculated by multiplying the simulated productivity by a percentage loss as described in the table 2.3 .

## B. 4 Additional Tables and Figures

Table B.1: Costs of Delays Relative to Optimal Planting Time, Value Added per Acre

|  | Cost per day, value added per acre |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Whole Sample | 5-day around optimal |  | 10-day around optimal |  |
|  |  | Before | After | Before | After |
| $\beta_{1}$ | -41.97 | $391.1^{* *}$ | -215.7 | $1,166^{* * *}$ | $-931.1^{* * *}$ |
|  | $(26.33)$ | $(140.2)$ | $(146.9)$ | $(338.7)$ | $(298.5)$ |
| Observations | 6,034 | 1,461 | 1,882 | 1,010 | 1,221 |
| R-squared | 0.408 | 0.625 | 0.584 | 0.706 | 0.659 |
| Mean of Productivity | 10228 | 11425 | 10694 | 11921 | 10998 |

Figure B.1: Ownership and rentals by implement.


The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed.

Figure B.2: Value Functions
fig 1

fio?
Value by Discretized Demand and Travel Time - Optim TSP Hullahalli Cultivator Duckfoot Jun-Oct


Table B.2: Status Quo Equilibrium Outcomes

| (a) Average Queue Length |  |  |  |  | (b) Rental Rates, ₹ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mall |  | large |  | small |  |  | large |  |
|  | fcfs |  | fcfs | mkt |  | fcfs | mkt | fcfs | mkt |
| 1 | 2.0 | 2.6 | 1.0 | 1.7 | 1 | 125 | 126 | 92 | 93 |
| 2 | 3.0 | 1.8 | 1.0 | 2.1 | 2 | 108 | 117 | 89 | 93 |
| 3 | 3.5 | 1.4 | 2.0 | 5.4 | 3 | 123 | 131 | 101 | 101 |
| 4 | 4.0 | 1.6 | 2.1 | 4.6 | 4 | 115 | 122 | 92 | 92 |
| 5 | 3.5 | 1.5 | 1.1 | 3.0 | 5 | 127 | 133 | 100 | 101 |
| 6 | 4.3 | 1.9 | 1.1 | 2.6 | 6 | 118 | 123 | 91 | 91 |
| 7 | 3.3 | 1.8 | 1.2 | 2.4 | 7 | 113 | 119 | 90 | 90 |
| 8 | 4.3 | 2.0 | 1.1 | 2.5 | 8 | 116 | 120 | 90 | 91 |
| 9 | 2.0 | 2.2 | 1.1 | 2.2 | 9 | 137 | 138 | 101 | 101 |
| 10 | 2.0 | 4.8 | 1.2 | 0.1 | 10 | 142 | 144 | 104 | 104 |
| 11 | 2.0 | 2.6 | 1.0 | 1.6 | 11 | 131 | 132 | 92 | 93 |
| 12 | 2.5 | 1.4 | 1.1 | 2.8 | 12 | 114 | 123 | 90 | 91 |
| 13 | 3.5 | 1.4 | 1.1 | 3.3 | 13 | 102 | 111 | 89 | 90 |
| 14 | 4.0 | 1.6 | 1.2 | 3.9 | 14 | 109 | 114 | 84 | 84 |
| 15 | 4.0 | 1.9 | 1.0 | 2.3 | 15 | 118 | 124 | 91 | 96 |

Panel (a) reports equilibrium average queue lengths by hub for farmers of different scale and different providers. Panel (b) reports the equilibrium rental rates per hour for the relevant implement.

## APPENDIX C

## APPENDIX TO CHAPTER 3

## C. 1 Uniqueness of Solution

The optimal level of durability is determined by the cost minimization problem under perfect competition and under monopoly when consumers have time-consistent preferences. A sufficient condition for the uniqueness of the solution to the cost minimization problem is for the present value of the total costs to perpetuity $\kappa(N)=\frac{c(N)}{1-\delta^{N}}$ to be convex on the relevant parameter value ranges. Whether this condition holds depends on the specific functional form of the marginal cost $c(N)$. By assumption, $c^{\prime}(N)>0$. One class of functions that satisfies this assumption is the power functions $c(N)=N^{a}$, where $a>0$. Let $\kappa(N)=\frac{N^{a}}{1-\delta^{N}}$, where $N \in[1, \bar{N}], a>0$, and $\delta \in(0,1)$. Assume local extrema (maxima or minima) exist, then at these points,

$$
\begin{gather*}
\frac{\partial \kappa(N)}{\partial N}=\frac{N^{a-1}\left(\left(1-\delta^{N}\right) a+N \delta^{N} \log \delta\right)}{\left(1-\delta^{N}\right)^{2}}=0 \\
a=-\frac{N \delta^{N} \log \delta}{1-\delta^{N}} \tag{C.1}
\end{gather*}
$$

There is no closed-form solution for $N$ at the local extrema. For the local extrema to be unique, Equation C.1 needs to be invertible on the specified parameter ranges for $N$ and $\delta$. It can be shown that $\frac{\partial a}{\partial N}$ is negative for $N \in[1, \bar{N}]$ and $\delta \in(0,1)$. The strict monotonicity guarantees the invertibility. This implies that there exists a unique local extremum for $\kappa(N)$ when $\delta \in(0,1)$ and $a$ falls in its range within which Equation C. 1 is invertible. We have $a>0$ by assumption. To find out the upper bound of $a$, it can be similarly shown that for any fixed $N \in[1, \bar{N}]$ and $\delta \in(0,1), a$ increases in $\delta$, and $a \rightarrow 1$
as $\delta \rightarrow 1$. Thus there exists a unique local extremum for $\kappa(N)$ when $\delta \in(0,1), a \in(0,1)$ and $N \in[1, \bar{N}]$.

The last step is to verify the unique local extremum is a minimum for the specified parameter ranges. The second order condition is

$$
\begin{aligned}
\frac{\partial^{2} \kappa(N)}{\partial N^{2}} & =\frac{N^{a-1} \delta^{N} \log \delta(1-a+N \log \delta)}{\left(1-\delta^{N}\right)^{2}} \\
& +\frac{(a-1) N^{a-2}\left(\left(1-\delta^{N}\right) a+N \delta^{N} \log \delta\right)}{\left(1-\delta^{N}\right)^{2}} \\
& +\frac{2 \delta^{N} \log \delta N^{a-1}\left(\left(1-\delta^{N}\right) a+N \delta^{N} \log \delta\right)}{\left(1-\delta^{N}\right)^{3}}
\end{aligned}
$$

Substitute Equation C. 1 into the second order condition, it can be simplified to

$$
\frac{\partial^{2} \kappa(N)}{\partial N^{2}}=\underbrace{-N^{a-1} \delta^{N} \log \delta\left(1-\delta^{N}\right)^{-3}}_{>0} \underbrace{\left(-1+\delta^{N}-N \log \delta\right)}_{?}
$$

Let $f(\delta)=-1+\delta^{N}-N \log \delta$. The sign of $f(\delta)$ is not immediately clear. Take the first derivative,

$$
\frac{\partial f(\delta)}{\partial \delta}=N\left(\delta^{N-1}-\frac{1}{\delta}\right)<0
$$

for $\delta \in(0,1)$ and $N \in[1, \bar{N}]$. In other words, $f(\delta)$ decreases in $\delta$ for the specified parameter range. Further, as $\delta$ approaches $1, f(\delta)$ approaches 0 from above. This shows $f(\delta)>0$ for $\delta \in(0,1)$ and $N \in[1, \bar{N}]$. The second order condition is therefore positive and the extremum is a minimum.

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[^0]:    ${ }^{1}$ For a general definition of affiliation, see the Appendix of Milgrom and Weber (1982).

[^1]:    ${ }^{2}$ See Avery (1998), Sections 3 and 4 for a more general discussion and more details.

[^2]:    ${ }^{3}$ If a small transaction cost of $\epsilon$ is introduced every time a bid is placed, the bidder will strictly prefer $S_{a}$.

[^3]:    ${ }^{4}$ The revenue equivalence theorem specifies a class of auctions that would generate the same expected revenue to the seller, in particular the independent private values model must apply. See Milgrom and Weber (1982), Theorem 0.
    ${ }^{5}$ This corresponds to Theorem 4.7 in Avery (1998).

[^4]:    ${ }^{6}$ Code bidding refers to the practice of attaching market numbers in the trailing digits of bids to signal key interest.

[^5]:    ${ }^{7} \rho^{r}+\bar{\kappa}-\epsilon$ is strictly dominated by $\rho^{r}+\bar{\kappa}$ if $\rho^{r}+\bar{\kappa} \leq \tilde{S}_{1 s t}(\cdot)$, and is strictly dominated by $\rho^{r}$ if $\rho^{r}+\bar{\kappa}>\tilde{S}_{1 s t}(\cdot)$.

[^6]:    ${ }^{8}$ The empirical distribution changes in every iteration of the minimization algorithm due to changes in parameter values. To make sure that the definitions of bins are consistent across iterations, I make draws from the standard normal distribution to obtain the bounds for each bin, and rescale them based on the parameter values of a particular iteration. The number of observations drawn is 100 million.

[^7]:    ${ }^{1}$ As highlighted by Lagos (2000) and Sattinger (2002), queueing models are powerful to micro-found a matching process between, in this case, farmers' orders and service provision.

[^8]:    ${ }^{2}$ Appendix B. 4 reports similar statistics using data from the Census.

[^9]:    ${ }^{3}$ We assume a six-week season, and that each piece of equipment serves three orders a day. The number of orders served in the day is consistent with services per equipment per driver at peak utilization that we observe in the transaction dataset. Even when assuming a shorter 4 week season, supply would account for 1344 per season for cultivators and 672 orders per season for rotavators, well under seasonal demand for each equipment.

[^10]:    ${ }^{4}$ Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

[^11]:    ${ }^{5}$ From a theory standpoint, when the number of providers and firms grow large, the probability of requesting a service to a given provider approaches zero and it is inconvenient to work with. From an empirical standpoint, queues are directly observable from our rental requests data while probabilities are not.

[^12]:    ${ }^{6}$ This is consistent with the median number of orders served within a day in our administrative data.
    ${ }^{7}$ The full derivation can be found in Appendix B.1.

[^13]:    ${ }^{8}$ Notice that the farmer takes the capital demand as given for convenience. We could trivially model the link between land-holdings and capital demand through a Leontief production function between capital and land.

[^14]:    ${ }^{9}$ In mapping the model to the data, each provider correspond to a piece of equipment and therefore it is reasonable to assume that service capacity per machine is the same irrespective of the dispatch system used.
    ${ }^{10}$ This feature allow us to handle the high dimensionality of the combinatorics problem when providers are allowed to prioritize certain farmer types.
    ${ }^{11}$ We assume the revenue is separable in the number of orders and relax this assumption in the quantitative exercise when the provider optimizes service provision in space, i.e. minimizes transportation cost across orders.

[^15]:    ${ }^{12}$ The ratio of the surpluses $\frac{z k_{s}^{\alpha}-w k_{s}-w E\left(d_{s}\right)}{z k_{s}^{\alpha}-w k_{s}--w E\left(d_{s^{-}}\right)}$must be larger than a constant that depends on the elasticity of the probability of service, see equation B.15 in Appendix B.2.

[^16]:    ${ }^{13}$ We could have alternatively calibrated a joint distribution of productivity, machine-hours requested and travel time. However, the overlap of the survey data and platform accounts for $20 \%$ of the survey data and we therefore benefited from including the fullness of the distribution of machine-hours and travel time. The latter is a key input into the costs of service and therefore the incentives to service large and small scale farmers.

[^17]:    ${ }^{14}$ Alternatively, we could have targeted the queue of large-scale farmers in the first-come-first-serve providers which we currently use as an untargeted moment. Results are qualitatively similar to those reported here and available upon request.

[^18]:    ${ }^{15}$ As robustness, we simulate outcomes when we assume no correlation between farm productivity and order sizes, and when we flip the sign of the empirical correlation between machine-hours and productivity within the catchment area of a hub. These results are available upon request.

[^19]:    ${ }^{16}$ As we explain in Appendix B.3 this is a high dimensional problem, and the number of possible combinations of orders to be served within a period grows exponentially with the number of orders in the queue and its characteristics (including hours serviced and location, i.e., latitude and longitude).

[^20]:    ${ }^{17}$ As we pointed out before, the value of service for market providers is always above the one for first-come-first-served providers, and therefore each provider has incentives to adopt a dispatch system that prioritizes large orders.

[^21]:    ${ }^{1}$ This paper does not focus on planned obsolescence in the context of new product introduction (see Waldman (1993) for details).

[^22]:    ${ }^{2}$ Other members included Hungary's Tungsram, the UK's Associated Electrical Industries, and Japan's Tokyo Electric.

[^23]:    ${ }_{3}$ Krajewski (2014) does not provide direct evidence that shows at which level this regulated standard was. However, from other evidences such as Fig. 3.2, it can be inferred that this regulated standard was below the average life span of light bulbs at the beginning of the formation of the cartel.

[^24]:    ${ }^{4}$ To make the comparison between two market structures - monopoly and perfect competition - easy, the monopoly could be seen as when the firms under perfect competition collude as a cartel with no new entry.
    ${ }^{5}$ Demand is perfectly inelastic up to a certain price.

[^25]:    ${ }^{6}$ For a detailed discussion on relaxing the assumption that the monopolist and the consumers having the same discount rate, see Schemalensee (1979).

[^26]:    ${ }^{7}$ The real number solution to the minimization problem is $N=5.63$.
    ${ }^{8}$ The real number solution to the maximization problem is $N=4.92$.

[^27]:    ${ }^{1}$ Note that $\hat{\phi}_{s, f c f s}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \rightarrow \infty$ these two are arbitrarily close.

[^28]:    ${ }^{2}$ This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

[^29]:    ${ }^{3}$ If there are more large-scale orders the provider still serves two because of its capacity constraints.

[^30]:    ${ }^{4}$ If we account for the cost of expected delays, then the envelope condition is $-\frac{\partial \Delta_{i, j}}{\partial q_{i j}} \frac{1}{\Delta_{i j}}\left[\frac{U_{i}}{\Delta_{i j}}-\frac{\partial z}{\partial \Delta_{i j}} \frac{\Delta_{i j}}{z} z k_{i}^{\alpha}\right]$

[^31]:    ${ }^{5}$ Note that equation B. 8 reduces to B. 12 when there are no small-scale orders.

[^32]:    ${ }^{6}$ If they choose to reach out to a single provider, then the equilibrium queue length is determined by

