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ANNOTATED COMPUTER OUPUT FOR SPLIT PLOT DESIGN: GENSTAT ANOVA

by

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ABSTRACT

In order to provide an understanding of covariance analyses for split plot designs, analyses are first conducted on a data set without the covariate. Then an analysis with the covariate is performed. The third example has a different experiment design for the whole plots and the covariate is constant for all split plots within a whole plot. Computer packages may only give portions of the analysis correctly. Some packages require two procedural calls to obtain a portion of the correct results. GENSTAT requires only one procedural call.

INTRODUCTION

This is part of a continuing project that produces annotated computer output for the analysis of balanced split plot experiments with covariates. The complete project will involve processing three examples on SAS/GLM, BMDP/2V, SPSS-X/MANOVA, GENSTAT/ANOVA, and SYSTAT/MGLH. Only univariate results are considered. We show here the results from GENSTAT ANOVA.

For Example 1, the data are artificial and were constructed for ease of computation; the experiment design for the whole plots is a randomized complete block and the split plot treatments are randomly allocated to the split plot experimental units within <u>each</u> whole plot. Example 2 is the same as Example 1 except that a covariate varies from split plot to split plot. The data for Example 3 come from an experiment wherein the whole plot treatments were laid out in a completely randomized design and the split plot treatments were randomly allotted to the split plot experimental units within each whole plot. The value of the covariate varies from whole plot to whole plot but is constant for all split plots within a whole plot treatment.

We present the elementary computational steps. Simple hypothetical data are used for the first two examples so that it is easy to provide all detailed computations to illustrate how each number is obtained. Some readers may wish to skip the detailed computations. The third example comes from Winer (1971). The detailed computations are given in his book (p. 803).

Data SP-1

Split plot data with whole plots arranged in randomized complete block design (hypothetical data)

	Whole plot treatment									
	W1					W2				
	split plot treatment					split plot treatment				
Block	S_1	S_2	S_3	S_4	Total	S_1	S_2	S_3	S_4	Total
1	3	4	7	6	20	3	2	1	14	20
2	6	10	1	11	28	8	8	2	18	36
3	6	10	4	4	24	10	8	9	13	40
Total	15	24	12	21	72	21	18	12	45	96

Total and Means

	Blocks (8 observations) Total	Mean		W(whole plots) (12 observations) Total	Mean		S(split plot) (6 observations) Total	Mean
1	40	5	W1	72	6	S_1	36	6
2	64	8	W2	96	8	S_2	42	7
3	64	8				S_3	24	4
Grand Total		168				S_4	66	11
Grand Mean		7						

Model: $Y_{ijk} = \mu + \rho_j + \tau_i + \delta_{ij} + \alpha_k + (\alpha\tau)_{ik} + \epsilon_{ijk}$

μ = mean

ρ_j = effect of block j

δ_{ij} = error (a)

ϵ_{ijk} = error (b)

τ_i = effect of whole plot i

α_k = effect of split plot k

$(\alpha\tau)_{ik}$ = effect of interaction of whole plot i and split plot k

Analysis of Variance

Source	(*)	df	SS
B (Blocks)	$= R(\rho \mid \mu,\tau,\alpha,\alpha\tau)$	2	48
W (whole plot treatments)	$= R(\tau \mid \mu,\rho,\alpha,\alpha\tau)$	1	24
B×W (error (a))	$= R(\delta \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	2	16
S (split plot treatments)	$= R(\alpha \mid \mu,\rho,\tau,\alpha\tau)$	3	156
S×W (interaction of S and W)	$= R(\alpha\tau \mid \mu,\alpha,\tau,\rho)$	3	84
(**) S×B:W (error (b))	$= R(\epsilon \mid \mu,\alpha,\tau,\alpha\tau,\rho)$	12	112
Total (Corrected for mean)	$= R(\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon \mid \mu)$	23	440
Mean	$= R(\mu)$	1	1176
Total (Uncorrected for mean)	$= R(\mu,\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon)$	24	1616

(*)Notation follows that of Searle(1971); since the design is balanced, $R(\rho \mid \mu,\tau,\alpha,\alpha\tau) = R(\rho \mid \mu)$, etc. The simpler notation is used later.
(**) S×B:W means S×B within W.

Calculations of sums of squares:

$N = 2\cdot 3\cdot 4 = 24$, $\bar{Y} = 7$

$$R(\mu,\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon) = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}^2 = (3^2 + 6^2 + 6^2 + \cdots + 18^2 + 13^2) = 1616$$

$R(\mu) = N\bar{Y}^2 = 24\cdot(7)^2 = 1176$

$R(\rho,\tau,\delta,\alpha,\alpha\tau,\epsilon \mid \mu) = 1616 - 1176 = 440$

$$R(\rho|\mu) = R(\mu,\rho) - R(\mu) = \frac{(40^2+64^2+64^2)}{8} - 1176 = 1224 - 1176 = 48$$

$$R(\tau|\mu) = R(\mu,\tau) - R(\mu) = \frac{(72^2+96^2)}{12} - 1176 = 1200 - 1176 = 24$$

$$R(\delta|\mu,\rho,\tau) = R(\delta,\mu,\rho,\tau) - R(\mu,\rho) - R(\tau,\mu) + R(\mu)$$

$$= \frac{(20^2+28^2+24^2+20^2+36^2+40^2)}{4} - 1224 - 1200 + 1176$$

$$= 1264 - 1224 - 1200 + 1176 = 16$$

$$R(\alpha|\mu) = R(\alpha,\mu) - R(\mu) = \frac{(36^2+42^2+24^2+66^2)}{6} - 1176 = 1332 - 1176 = 156$$

$$R(\alpha\tau|\mu,\alpha,\tau) = R(\alpha\tau,\mu,\alpha,\tau) - R(\mu,\alpha) - R(\mu,\tau) + R(\mu)$$

$$= \frac{(15^2+24^2+12^2+21^2+21^2+18^2+12^2+45^2)}{3} - 1332 - 1200 + 1176$$

$$= 1440 - 1332 - 1200 + 1176 = 84$$

$$R(\epsilon|\mu,\rho,\delta,\alpha,\tau,\alpha\tau) = R(\epsilon,\mu,\alpha,\rho,\delta,\tau,\alpha\tau) - R(\mu,\rho,\tau,\delta) - R(\mu,\alpha,\tau,\alpha\tau) + R(\tau,\mu)$$

$$= 1616 - 1264 - 1440 + 1200 = 112$$

Data SP-2

Data SP-2: Data SP-1 with the following covariate Z which varies with split plot

Covariate (Z)

	whole plot									
	W1					W2				
	S_1	S_2	S_3	S_4	Total	S_1	S_2	S_3	S_4	Total
B_1	1	2	1	2	6	2	0	2	4	8
B_2	2	2	0	4	8	4	1	3	4	12
B_3	3	5	2	0	10	3	2	4	7	16
Total	6	9	3	6	24	9	3	9	15	36

Totals and Means

blocks (8 observations)	Total	Mean	W (whole plot) (12 observations)	Total	Mean	S (split plot) (6 observations)	Total	Mean
1	14	14/8	1	24	2.0	1	15	2.5
2	20	20/8	2	36	3.0	2	12	2.0
3	26	26/8				3	12	2.0
Grand						4	21	3.5
Total	60	2.5						

Model: $Y_{ijk} = \mu+\rho_j+\tau_i+\delta_{ij}+\alpha_k+(\alpha\tau)_{ik}+\beta_1(\bar{Z}_{ij.}-\bar{Z}_{...})+\beta_2(Z_{ijk}-\bar{Z}_{ij.})+\epsilon_{ijk}$

ρ_j = effect of jth block $\qquad \tau_i$ = effect of ith whole plot

α_k = effect of kth split plot $\qquad \beta_1$ = whole plot regression slope

β_2 = split plot regression slope $\qquad \delta_{ij}$ = error a $\qquad \epsilon_{ijk}$ = error b

Table of sum of squares and products

Source	df	YY	YZ	ZZ
B	2	48	18	9
W	1	24	12	6
B×W (error a)	2	16	4	1
S	3	156	33	9
S×W	3	84	33	21
S×B:W (error b)	12	112	17	20
Mean	1	1176	420	150
Total	24	1616	537	216

YY column is the same as in SP-1, ZZ column is computed in the same fashion. Thus, only computations for YZ column are illustrated.

$$\text{Total}_{YZ} = \sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}\cdot Z_{ijk}$$

$$= 3(1)+ 6(2)+\cdots+ 14(4)+ 18(4)+ 13(7) = 537$$

$$\text{Mean}_{YZ} = N\bar{Y}_{...}\bar{Z}_{...} = \frac{168\cdot 60}{24} = 420$$

$$B_{YZ} = \frac{\sum_{j=1}^{3}\left(\sum_{i=1}^{2}\sum_{k=1}^{4} Y_{ijk}\right)\left(\sum_{i=1}^{2}\sum_{k=1}^{4} Z_{ijk}\right)}{2\cdot 4} - 420 = \frac{40(14)+64(20)+64(26)}{8} - 420$$

$$= 438 - 420 = 18$$

$$W_{YZ} = \frac{\sum_{i=1}^{2}\left(\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}\right)\left(\sum_{j=1}^{3}\sum_{k=1}^{4} Z_{ijk}\right)}{3(4)} - 420 = 432 - 420 = 12$$

$$B\times W_{YZ} = \frac{\sum_{i=1}^{2}\sum_{j=1}^{3}(\sum_{k=1}^{4} Y_{ijk})(\sum_{k=1}^{4} Z_{ijk})}{4} - 438 - 432 + 420$$

$$= 454 - 438 - 432 + 420 = 4$$

$$S_{YZ}: \sum_{k=1}^{4} \frac{(\sum_{i=1}^{2}\sum_{j=1}^{3} Y_{ijk})(\sum_{i=1}^{2}\sum_{j=1}^{3} Z_{ijk})}{2(3)} - 420 = 453 - 420 = 33$$

$$S\times W_{YZ}: \frac{\sum_{i=1}^{2}\sum_{k=1}^{4}(\sum_{j=1}^{3} Y_{ijk})(\sum_{j=1}^{3} Z_{ijk})}{3} - 453 - 432 + 420$$

$$= 498 - 453 - 432 + 420 = 33$$

$S\times B:W_{YZ}$:

$$\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{4} Y_{ijk}Z_{ijk} - 454 - 498 + 432$$

$$= 537 - 454 - 498 + 432 = 17$$

Analysis of Variance and Covariance

Source		df	SS
B (block)	$= R(\rho \mid \mu,\tau)$	2	48
W (whole plot treatment)	$= R(\tau \mid \mu,\rho,\beta_1)$	1	3.4286
Regression (a)	$= R(\beta_1 \mid \mu,\rho,\tau)$	1	16.0
B×W (error (a))	$= R(\delta \mid \mu,\rho,\tau,\beta_1)$	1	0.0
S (split plot treatment)	$= R(\alpha \mid \mu,\rho,\tau,\alpha\tau,\beta_2)$	3	84.243
S×W (interaction of S and W)	$= R(\alpha\tau \mid \mu,\rho,\tau,\alpha,\beta_2)$	3	37.474
Regression (b)	$= R(\beta_2 \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	1	14.450
S×B: W (error (b))	$= R(\epsilon \mid \mu,\rho,\alpha,\tau,\alpha\tau,\beta_2)$	11	97.550

$$\hat{\beta}_1 = B\times W_{YZ} / B\times W_{ZZ} = 4/1 = 4$$

$$\hat{\beta}_2 = S\times B:W_{YZ} / S\times B:W_{ZZ} = 17/20 = 0.85$$

The SS's adjusted by regression on Z are illustrated below:

$R(\rho|\mu) = 48$, remains same since it is not of interest to adjust for Z on the blocks.

$$R(\tau,\delta|\mu,\rho,\beta_1) = (W_{YY} + B\times W_{YY}) - \frac{(W_{YZ} + B\times W_{YZ})^2}{W_{ZZ} + B\times W_{ZZ}}$$

$$= (24 + 16) - \frac{(12 + 4)^2}{6 + 1} = 40 - \frac{256}{7} = 3.4286$$

$$R(\delta|\mu,\rho,\tau,\beta_1) = B\times W_{YY} - \frac{(B\times W_{YZ})^2}{B\times W_{ZZ}} = 16 - \frac{4^2}{1} = 0$$

$$R(\tau|\mu,\rho,\beta_1) = R(\tau,\delta|\mu,\rho,\beta_1) - R(\delta|\mu,\rho,\tau,\beta_1)$$

$$= 40 - \frac{256}{7} - 0 = 3.4286$$

$$R(\beta_1|\mu,\tau,\rho) = \frac{(B\times W_{YZ})^2}{B\times W_{ZZ}} = \frac{4^2}{1} = 16$$

$$R(\alpha,\epsilon|\mu,\rho,\tau,\alpha\tau,\beta_2) = (S_{YY} + S\times B{:}W_{YY}) - \frac{(S_{YZ} + S\times B{:}W_{YZ})^2}{S_{ZZ} + S\times B{:}W_{ZZ}}$$

$$= (156 + 112) - \frac{(33+17)^2}{9+20}$$

$$= 268 - 86.207 = 181.793$$

$$R(\alpha\tau,\epsilon|\mu,\rho,\alpha,\tau,\beta_2) = (S\times W_{YY} + S\times B{:}W_{YY}) - \frac{(S\times W_{YZ} + S\times B{:}W_{YZ})^2}{S\times W_{ZZ} + S\times B{:}W_{ZZ}}$$

$$= 84 + 112 - \frac{(33+17)^2}{21+20} = 196 - 60.976 = 135.024$$

Note: $R(\alpha,\epsilon|\mu,\beta_2)$ and $R(\alpha\tau,\epsilon|\mu,\alpha,\tau,\beta_2)$ are intermediate steps for later use.

$$R(\beta_2|\mu,\rho,\alpha,\tau,\alpha\tau) = \frac{(S\times B:W_{YZ})^2}{S\times B:W_{ZZ}} = \frac{17^2}{20} = 14.450$$

$$R(\epsilon|\mu,\rho,\alpha,\tau,\alpha\tau,\beta_2) = S\times B:W_{YY} - \frac{(S\times B:W_{YZ})^2}{S\times B:W_{ZZ}} = 112 - \frac{17^2}{20} = 112 - 14.45 = 97.55$$

$$R(\alpha|\mu,\rho,\tau,\alpha\tau,\beta_2) = R(\alpha,\epsilon|\mu,\rho,\tau,\alpha\tau,\beta_2) - \text{SS error b} = 181.793 - 97.55$$
$$= 84.243$$

$$R(\alpha\tau|\mu,\rho,\alpha,\tau,\beta_2) = R(\alpha\tau,\epsilon|\mu,\rho,\alpha,\tau,\beta_2) - R(\epsilon|\mu,\rho,\alpha,\tau,\alpha\tau,\beta_2)$$

$$= 135.024 - 97.55 = 37.474$$

Data SP-3

Split plot data with plots arranged in a completely randomized design and a covariate Z that is constant within the whole plot. (Winer, 1971, p. 803)

whole plot	Subject	Split plots B_1	B_2	Z	Total
		Y	Y		Y
A_1					
	1	10	8	3	18
	2	15	12	5	27
	3	20	14	8	34
	4	12	6	2	18
A_2					
	5	15	10	1	25
	6	25	20	8	45
	7	20	15	10	35
	8	15	10	2	25
	Total	132	95	39	227
	Mean	16.5	11.9	4.88	

Model: $y_{ijk} = \mu + \tau_i + \delta_{ij} + \alpha_k + (\tau\alpha)_{ik} + \beta_1(Z_{ij}-\bar{Z}_{..}) + \epsilon_{ijk}$

τ_i = A effect (whole plot) δ_{ij} = error (a) ϵ_{ijk} = error (b)

α_k = B effect (split plot) β_1 = whole plot regression slope

Analysis of variance and covariance

Source		df	SS
A (whole plot)	$= R(\tau \mid \mu,\beta_1)$	1	44.492
Regression	$= R(\beta_1 \mid \mu,\tau)$	1	166.577
Error (a)	$= R(\delta \mid \mu,\tau,\beta_1)$	5	61.298
B (split plot)	$= R(\alpha \mid \mu,\tau,\alpha\tau)$	1	85.563
A×B (interaction)	$= R(\tau\alpha \mid \mu,\tau,\alpha)$	1	0.563
Error (b)	$= R(\epsilon \mid \mu,\tau,\alpha,\tau\alpha)$	6	6.375

Table of SS and products

Symbol	Y^2	ZY	Z^2
W	68.06	12.38	2.25
E(a)	227.88	163.00	159.50
S	85.563	0	0
WS	0.563	0	0
E(b)	6.375	0	0

$$\hat{\beta}_1 = \frac{163.00}{159.50} = 1.02$$

Since the computations are illustrated in Winer (1971, p. 803-5) we have omitted them here.

References

Federer. W.T. (1955), Experimental Design, Theory and Application. The Macmillan Co., New York, Chapter 16.

Federer, W.T., and Henderson, H.V. (1979), Covariance Analysis of Designed Experiments X Statistical Packages: An Update, Proc., Comp. Sci. and Stat.: 12th Ann. Sym. on the Interface.

GENSTAT A General Statistical Program Release 4.04 (1983), Rothamsted Experimental Station.

Searle, S.R., (1971), Linear Models, Wiley, N.Y., 532pp.

Searle, S.R., Hudson, G.F.S., and Federer, W.T. (1985), Annotated Computer Output for Covariance-Text, BU-780-M, Biometrics Unit Mimeo Ser., Cornell University, Ithaca, NY.

Winer, B.J., (1971), Statistical Principles in Experimental Design, McGraw-Hill Book Company, New York: 907pp.

SP-1 and SP-2: Control Language

Control Language is typed in upper case and comments are in bold face.

```
'REFE' SPLIT  ⟹ file name
'UNITS' $ 24  ⟹ number of observations
'FACTOR' BLOCKS $3
      : PLOTS $2            } define levels of each factor
      : SUBPLOTS $4
'READ/FORM=P, PRIN=D' BLOCKS, PLOTS, SUBPLOTS, X, Y  ⟹ Input variables
'RUN'
1 1 1 1 3
1 1 2 2 4
1 1 3 1 7
1 1 4 2 6
1 2 1 2 3
1 2 2 0 2
1 2 3 2 1
1 2 4 4 14
2 1 1 2 6
2 1 2 2 10
2 1 3 0 1
2 1 4 4 11
2 2 1 4 8
2 2 2 1 8
2 2 3 3 2
2 2 4 4 18
3 1 1 3 6
3 1 2 5 10
3 1 3 2 4
3 1 4 0 4
3 2 1 3 10
3 2 2 2 8
3 2 3 4 9
3 2 4 7 13
'EOD'  ⟹ tell GENSTAT that data flow ends
'BLOCKS' BLOCKS/PLOTS/SUBPLOTS  ⟹ define error terms (strata) of a
                                   factorial model
'TREATMENTS' PLOTS*SUBPLOTS  ⟹ treatment terms of a factorial model
'COVARIATES' X  ⟹ covariate
'ANOVA' Y  ⟹ invokes analysis of variance on Y variable
'RUN'
'STOP'
'
```

SP-3: Control Language

```
OK, SLIST SPLIT3.OGEN

'REFE' SPLIT3
'UNITS' $ 16
'FACTOR' PLOT $2
       : SUBPLOT $2
      : SUBJECT $8
'READ/FORM=P, PRIN=D' SUBJECT, PLOT, SUBPLOT, Y, Z
'RUN'
1 1 1 10 3
1 1 2 8 3
2 1 1 15 5
2 1 2 12 5
3 1 1 20 8
3 1 2 14 8
4 1 1 12 2
4 1 2 6 2
5 2 1 15 1
5 2 2 10 1
6 2 1 25 8
6 2 2 20 8
7 2 1 20 10
7 2 2 15 10
8 2 1 15 2
8 2 2 10 2
'EOD'
'BLOCKS' PLOT/SUBJECT/SUBPLOT
'TREATMENT' PLOT*SUBPLOT
'COVARIATES' Z
'ANOVA' Y
'RUN'
'STOP'
```

SP-1: Split plots with whole plots arranged in RCB design
SP-2: Split plots with whole plots arranged in RCB with a covariate with split plot

Analysis of variance table on covariate Z

***** ANALYSIS OF VARIANCE *****

VARIATE: Z

SOURCE OF VARIATION	DF	SS	SS%	MS	VR
BLOCKS STRATUM	2	9.000	13.64	4.500	
BLOCKS.PLOTS STRATUM					
PLOTS	1	6.000	9.09	6.000	12.000
RESIDUAL	2	1.000	1.52	0.500	
TOTAL	3	7.000	10.61	2.333	
BLOCKS.PLOTS.SUBPLOTS STRATUM					
SUBPLOTS	3	9.000	13.64	3.000	1.800
PLOTS.SUBPLOTS	3	21.000	31.82	7.000	4.200
RESIDUAL	12	20.000	30.30	1.667	
TOTAL	18	50.000	75.76	2.778	
GRAND TOTAL	23	66.000	100.00		

GRAND MEAN 2.50
TOTAL NUMBER OF OBSERVATIONS 24

ANOVA for Y variable without covariate Z

***** ANALYSIS OF VARIANCE *****

VARIATE: Y

SOURCE OF VARIATION	DF		SS MS	SS% VR (F-statistics)
BLOCKS STRATUM	2	$R(\rho \mid \mu,\tau,\alpha,\alpha\tau)$	48.000 24.000	10.91
BLOCKS.PLOTS STRATUM				
PLOTS	1	$R(\tau \mid \mu,\rho,\alpha,\alpha\tau)$	24.000 24.000	5.45 3.000 $= \frac{24.00}{8.00}$
RESIDUAL	2	$R(\delta \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	16.000 8.000	3.64
TOTAL	3		40.000 13.333	9.09
BLOCKS.PLOTS.SUBPLOTS STRATUM				
SUBPLOTS	3	$R(\alpha \mid \mu,\rho,\tau,\alpha\tau,\delta)$	156.000 52.000	35.45 5.571
PLOTS.SUBPLOTS	3	$R(\alpha\tau \mid \mu,\alpha,\tau,\rho,\delta)$	84.000 28.000	19.09 3.000
RESIDUAL	12	$R(\epsilon \mid \mu,\alpha,\tau,\alpha\tau,\rho,\delta)$	112.000 9.333	25.45
TOTAL	18		352.000 19.556	80.00
GRAND TOTAL	23		440.000	100.00

GRAND MEAN 7.00
TOTAL NUMBER OF OBSERVATIONS 24

***** ANALYSIS OF VARIANCE *****
(ADJUSTED FOR COVARIATE)

VARIATE: Y

SOURCE OF VARIATION	DF		SS	SS%	MS	VR (F-statistic)	COV EF = covariance efficiency factor
BLOCKS STRATUM							
COVARIATE	1	(*) pooled	36.000	8.18	36.000	3.000	
RESIDUAL	1		12.000	2.73	12.000		2.000
TOTAL	2	$R(\rho \mid \mu,\tau)$	48.000	10.91	24.000		
BLOCKS.PLOTS STRATUM							
PLOTS	1	$R(\tau \mid \mu,\rho,\beta_1)$	3.429	0.78	3.429		0.143
COVARIATE	1	$R(\beta_1 \mid \mu,\rho,\tau)$	16.000	3.64	16.000		
RESIDUAL	1	$R(\delta \mid \mu,\rho,\tau,\beta_1)$	0.000	0.00	0.000		
TOTAL	3		19.429	4.42	6.476		
BLOCKS.PLOTS.SUBPLOTS STRATUM							
SUBPLOTS	3	$R(\alpha \mid \mu,\rho,\tau,\alpha\tau,\beta_2)$	84.243	19.15	28.081	3.166	0.870
PLOTS.SUBPLOTS	3	$R(\alpha\tau \mid \mu,\rho,\tau,\alpha,\beta_2)$	37.474	8.52	12.491	1.409	0.741
COVARIATE	1	$R(\beta_2 \mid \mu,\rho,\tau,\alpha,\alpha\tau)$	14.450	3.28	14.450	1.629	
RESIDUAL	11	$R(\epsilon \mid \mu,\rho,\alpha,\tau,\alpha\tau,\beta_2)$	97.550	22.17	8.868		1.052
TOTAL	18		233.717	53.12	12.984		
GRAND TOTAL	23	301.146	68.44				

GRAND MEAN 7.00
TOTAL NUMBER OF OBSERVATIONS 24

NOTE: low value of COV EF (usually close to unity) indicates high correlation between treatment and covariate e.g. COV EF for plots is .143; indicating correlation between $\bar{W}_i$ and $\bar{Z}_i$ ($\bar{W}_1 = 6$, $\bar{W}_2 = 8$, $\bar{Z}_1 = 2$, $\bar{Z}_2 = 3$)

(*) Block is just a nuisance factor; hence only $R(\rho \mid \mu,\tau) = 48$ is used in Analysis of variance

***** COVARIANCE REGRESSIONS *****

COVARIATE (Z)	COEFFICIENT	SE
BLOCKS STRATUM		
Z (β_0 is not of interest)	$\hat{\beta}_0 = 2.0 = \frac{B_{YZ}}{B_{ZZ}} = \frac{18}{9}$	1.15
BLOCKS.PLOTS STRATUM		
Z	$\hat{\beta}_1 = 4$	0
BLOCKS.PLOTS.SUBPLOTS STRATUM		
Z	$\hat{\beta}_2 = 0.85$	0.666

***** TABLES OF MEANS *****
(ADJUSTED FOR COVARIATE)

VARIATE: Y

GRAND MEAN 7.00

PLOTS	1	2
	8.00	6.00

$6.00 = \bar{Y}_{2..} - \hat{\beta}_1(\bar{Z}_{2..} - \bar{Z}_{...}) = 8-4.0(3-2.5)=6$

SUBPLOTS	1	2	3	4
	6.00	7.43	4.43	10.15

$$10.15 = \bar{Y}_{..4} - \hat{\beta}_2(\bar{Z}_{..4} - \bar{Z}_{...}) = 11-0.85(\frac{21}{6} - \frac{60}{24}) = 11-.85 = 10.15$$

SUBPLOTS / PLOTS	1	2	3	4
1	7.00	9.15	6.85	9.00
2	5.00	5.70	2.00	11.30

$$11.30 = \bar{Y}_{2.4} - \hat{\beta}_1(\bar{Z}_{2..} - \bar{Z}_{...}) - \hat{\beta}_2(\bar{Z}_{2.4} - \bar{Z}_{2..}) = 15-4(3 - 2.5) - .85(\frac{15}{3} -3) = 11.30$$

***** STANDARD ERRORS OF DIFFERENCES OF MEANS *****

TABLE	PLOTS	SUBPLOTS	PLOTS SUBPLOTS
REP	12	6	3
SED	0.000	1.844	2.718

***** STRATUM STANDARD ERRORS AND COEFFICIENTS OF VARIATION *****

STRATUM	DF	SE	CV%
BLOCKS	1	1.225	17.5
BLOCKS.PLOTS	1	0.000	0.0
BLOCKS.PLOTS.SUBPLOTS	11	**2.978**	42.5

$$2.978 = \frac{\text{unadjusted standard error}}{\sqrt{\text{COV EF of Residual}}} = \frac{\sqrt{9.333}}{\sqrt{1.052}}$$

NOTE:

$$S.E\,(\hat{\bar{Y}}_{i\cdot\cdot} - \hat{\bar{Y}}_{i'\cdot\cdot}) = \left\{ \frac{2E_a}{r(s)} \left(1 + \frac{W_{zz}/(w-1)}{B\times W_{zz}}\right)\right\}^{1/2}$$

$$= 0.0$$

$$S.E\,(\hat{\bar{Y}}_{\cdot\cdot k} - \hat{\bar{Y}}_{\cdot\cdot k'}) = \left\{ \frac{2E_b}{r(w)} \left(1 + \frac{S_{zz}/(s-1)}{S\times B:W_{zz}}\right)\right\}^{1/2}$$

$$= \left\{ \frac{2(97.55)}{3(2)(11)} \left(1 + \frac{9/(4-1)}{20}\right)\right\}^{1/2} = 1.844$$

$$S.E\,(\hat{\bar{Y}}_{i\cdot k} - \hat{\bar{Y}}_{i\cdot k'}) = \left\{ \frac{2E_b}{r} \left(1 + \frac{(S_{zz} + S\times W_{zz})/w(s-1)}{S\times B:W_{zz}}\right)\right\}^{1/2}$$

$$= \left\{ \frac{2(97.55)}{3(11)} \left(1 + \frac{(9+21)/(2)(3)}{20}\right)\right\}^{1/2} = 2.718$$

where W = no. of whole plot = 2, r = no. of blocks = 3, E_b = error b = 97.55/11

S = no. of split plot = 4, E_a = error a = 0

SP-3: Split plots with whole plot arranged in CRD with a covariate constant within whole plot

***** ANALYSIS OF VARIANCE ***** ANOVA on Covariate Z

VARIATE: Z

SOURCE OF VARIATION	DF	SS	SS%	MS	VR
PLOT.SUBJECT STRATUM					
PLOT	1	2.250E 0	1.39	2.250E 0	0.085
RESIDUAL	6	1.595E 2	98.61	2.658E 1	
TOTAL	7	1.618E 2	100.00	2.311E 1	
PLOT.SUBJECT.SUBPLOT STRATUM					
SUBPLOT	1	0.000E 0	0.00	0.000E 0	
PLOT.SUBPLOT	1	0.000E 0	0.00	0.000E 0	
RESIDUAL	6	0.000E 0	0.00	0.000E 0	
TOTAL	8	0.000E 0	0.00	0.000E 0	
GRAND TOTAL	15	1.618E 2	100.00		

GRAND MEAN 4.88
TOTAL NUMBER OF OBSERVATIONS 16

Notice that SS in plot.subject.subplot stratum are all zeroes since Z is constant within whole plot.

ANOVA on Y without covariate Z

***** ANALYSIS OF VARIANCE *****

VARIATE: Y

SOURCE OF VARIATION	DF	SS	SS%	MS	VR
PLOT.SUBJECT STRATUM					
PLOT	1	68.063	17.52	68.063	1.792
RESIDUAL	6	227.875	58.66	37.979	
TOTAL	7	295.938	76.19	42.277	
PLOT.SUBJECT.SUBPLOT STRATUM					
SUBPLOT	1	85.563	22.03	85.563	80.529
PLOT.SUBPLOT	1	0.563	0.14	0.563	0.529
RESIDUAL	6	6.375	1.64	1.063	
TOTAL	8	92.500	23.81	11.563	
GRAND TOTAL	15	388.438	100.00		

GRAND MEAN 14.19
TOTAL NUMBER OF OBSERVATIONS 16

***** ANALYSIS OF VARIANCE *****
(ADJUSTED FOR COVARIATE)

VARIATE: Y

SOURCE OF VARIATION	DF		SS	SS%	MS	VR	COV EF
PLOT.SUBJECT STRATUM							
PLOT	1	$R(\tau \mid \mu, \beta_1)$	44.492	11.45	44.492	3.629	0.986
COVARIATE	1	$R(\beta_1 \mid \mu, \tau)$	166.577	42.88	166.577	13.587	
RESIDUAL	5	$R(\delta \mid \mu, \tau, \beta_1)$	61.298	15.78	12.260		3.098
TOTAL	7		272.367	70.12	38.910		
PLOT.SUBJECT.SUBPLOT STRATUM							
SUBPLOT	1	$R(\alpha \mid \mu, \tau, \alpha\tau)$	85.563	22.03	85.563	80.529	1.000
PLOT.SUBPLOT	1	$R(\tau\alpha \mid \mu, \tau, \alpha)$	0.563	0.14	0.563	0.529	1.000
RESIDUAL	6	$R(\epsilon \mid \mu, \tau, \alpha, \tau\alpha)$	6.375	1.64	1.063		1.000
TOTAL	8		92.500	23.81	11.563		
GRAND TOTAL	15		364.867	93.93			

GRAND MEAN 14.19
TOTAL NUMBER OF OBSERVATIONS 16

***** COVARIANCE REGRESSIONS *****

COVARIATE	COEFFICIENT	SE
PLOT.SUBJECT STRATUM		
Z	$\hat{\beta}_1 = 1.02$	0.277

***** TABLES OF MEANS *****
(ADJUSTED FOR COVARIATE)

VARIATE: Y

GRAND MEAN 14.19

PLOT	1	2
	12.51	15.87

$15.87 = \bar{Y}_{2..} - \hat{\beta}_1(\bar{Z}_{2.} - \bar{Z}_{..}) = 16.25 - 1.02(5.25-4.88) = 15.87$

SUBPLOT	1	2	
	16.50	11.88	same as unadjusted mean

SUBPLOT	1	2
PLOT		
1	14.63	10.38
2	18.37	13.37

$13.37 = \bar{Y}_{2.2} - \hat{\beta}_1(\bar{Z}_{2.} - \bar{Z}_{..}) = 13.75 - 1.02(5.25-4.88) = 13.37$

***** STANDARD ERRORS OF DIFFERENCES OF MEANS *****

TABLE	PLOT	SUBPLOT	PLOT SUBPLOT
REP	8	8	4
SED	1.763	0.515	1.829
EXCEPT WHEN COMPARING MEANS WITH SAME LEVEL(S) OF:			
PLOT			0.731

***** STRATUM STANDARD ERRORS AND COEFFICIENTS OF VARIATION *****

STRATUM	DF	SE	CV%
PLOT.SUBJECT	5	2.476	17.5
PLOT.SUBJECT.SUBPLOT	6	$1.031 = \dfrac{\sqrt{1.063}}{\sqrt{1.000}}$	7.3

$$= \left[\frac{\text{residual MS of unadjusted}}{\text{COV EF of adjusted residual}}\right]^{1/2}$$